H.W #1

$$A = \begin{bmatrix} x & y \\ 2.2 & 2.1 \\ 0.4 & 0.6 \\ 1.6 & 2.3 \\ 2.0 & 2.1 \\ 2.9 & 2.8 \\ 0.8 & 1.1 \\ 1.3 & 1.4 \\ 1.1 & 1.0 \\ 2.3 & 2.6 \end{bmatrix}$$

$$\bar{A} = [1.6 \quad 1.7]$$

$$Cov(X,Y) = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})$$

$Var(x) = 0.36 + 1.44 + 0 + 0.16 + 1.69 + 0.64 + 0.09$

$+ 0.25 + 0.49 = 5.12$

$$Var(Y) = 4.85$$

$$Cov \begin{bmatrix} 5.12 & 4.7 \\ 4.7 & 4.85 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{pmatrix} 5.12 & 4.7 \\ 4.7 & 4.85 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$\det\begin{pmatrix} 5.12 - \lambda & 4.7 \\ 4.7 & 4.85 - \lambda \end{pmatrix}$$

$$(5.12 - \lambda)(4.85 - \lambda) - 4.7^2 = 0$$

$$24.8 - 5.12\lambda - 4.85\lambda + \lambda^2 - 22.09$$

$$\lambda^2 - 9.97\lambda + 2.7 = 0$$

$$\lambda = \frac{-(-9.97) \pm \sqrt{99.4 - 10.8}}{2}$$

$$\lambda = \frac{9.97 \pm \sqrt{88.6}}{2} = \frac{9.97 \pm 9.4}{2}$$

$$\lambda \approx 0.25, 9.65$$

Find eigen vector

for eigen value 0.25

$$A \cdot v_1 = \lambda_1 v_1$$

$$(A - \lambda_1) \cdot V_1 = 0$$

$$\begin{bmatrix} 4.87 & 4.7 \\ 4.7 & 4.6 \end{bmatrix} \begin{bmatrix} V_{1,1} \\ V_{1,2} \end{bmatrix} = 0$$

Similarly, for eigen value 9.65

$$\begin{bmatrix} -4.53 & 4.7 \\ 4.7 & -4.8 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = 0$$

$$= 0$$

$$-4.53 V_{21} + 4.7 V_{22} + 4.7 V_{21} - 4.8 V_{22} = 0$$

$$4.7 V_{21} - 4.8 V_{22} = 0$$

$$V_{21} = V_{22} (1.03)$$

$$V_2 = K \begin{bmatrix} 1.03 \\ 0.96 \end{bmatrix}$$