

Question

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Theoretical Solution

Let the speed of the stream be x km/h.

Therefore, the speed of the boat upstream = $(18 - x)$ km/h and the speed of the boat downstream = $(18 + x)$ km/h.

The time taken to go upstream is:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{24}{18 - x} \text{ hours.} \quad (1)$$

Similarly, the time taken to go downstream is:

$$\frac{24}{18 + x} \text{ hours.} \quad (2)$$

According to the question,

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1 \quad (3)$$

Multiplying throughout by $24(18 + x)(18 - x)$, we get:

$$24(18 + x) - 24(18 - x) = (18 - x)(18 + x) \quad (4)$$

$$x^2 + 48x - 324 = 0 \quad (5)$$

Using the quadratic formula:

$$x = \frac{-48 \pm \sqrt{48^2 + 1296}}{2} = \frac{-48 \pm \sqrt{3600}}{2} \quad (6)$$

$$= \frac{-48 \pm 60}{2} \quad (7)$$

$$= 6 \text{ or } -54 \quad (8)$$

Since x is the speed of the stream, it cannot be negative. So, we ignore the root $x = -54$.

Therefore, $x = 6$ gives the speed of the stream as **6 km/h**.

Theorem: (9)

Let $x = s$ be a solution of $x = g(x)$ and suppose that g has a continuous derivative in some interval J containing s . Then if $|g'| \leq K < 1$ in J , the iteration process defined above converges for any x_0 in J . The limit of the sequence $[x_n]$ is s

Since there is no solution (evident by quadratic formula) there exists no interval J for which the process converges to a point.

The same behaviour is shown by the Newton-Raphson Method,
Start with an initial guess x_0 , and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (10)$$

where ,

$$f(x) = x^2 + 48x - 324 \quad (11)$$

$$f'(x) = 2x + 48 \quad (12)$$

SOLVING USING A MATRIX NUMERICAL METHOD (FIXED-POINT ITERATION)

To solve using a matrix numerical method, we rewrite the equation as:

$$x = \frac{324}{x} - 48$$

This allows us to express the iteration process in matrix form.

Step 1: Fixed-Point Iteration Formula

$$x_{n+1} = \frac{324}{x_n} - 48$$

Step 2: Matrix Representation

We can express the iterative process in a vector form:

$$X_{n+1} = [f(X_n)] = \left[\frac{324}{X_n} - 48 \right]$$

Step 3: Iterations

Using an initial guess $x_0 = 5$:

$$x_1 = \frac{324}{5} - 48$$

$$x_1 = 64.8 - 48 = 16.8$$

$$x_2 = \frac{324}{16.8} - 48$$

$$x_2 = 19.2857 - 48 = 6.2857$$

x

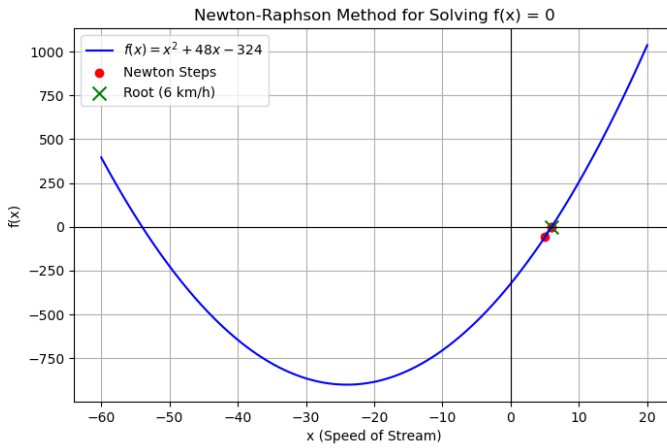


Fig. 0.1: Plot showing the relationship between $f(x)$ and speed of stream