NCERT-10.4.ex.15

EE24BTECH11039 - MANDALA RANJITH

Question

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Theoritical Solution

Let the speed of the stream be x km/h.

Therefore, the speed of the boat upstream = $(18 - x)$ km/h and the speed of the boat downstream = $(18 + x)$ km/h.

The time taken to go upstream is:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{24}{18 - x} \text{ hours.} \tag{1}$$

Similarly, the time taken to go downstream is:

$$\frac{24}{18 + x} \text{ hours.} \tag{2}$$

According to the question,

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1 \tag{3}$$

Multiplying throughout by $24(18 + x)(18 - x)$, we get:

$$24(18 + x) - 24(18 - x) = (18 - x)(18 + x) \tag{4}$$

$$x^2 + 48x - 324 = 0 \tag{5}$$

Using the quadratic formula:

$$x = \frac{-48 \pm \sqrt{48^2 + 1296}}{2} = \frac{-48 \pm \sqrt{3600}}{2} \tag{6}$$

$$= \frac{-48 \pm 60}{2} \tag{7}$$

$$= 6 \text{ or } -54 \tag{8}$$

Since x is the speed of the stream, it cannot be negative. So, we ignore the root $x = -54$.

Therefore, $x = 6$ gives the speed of the stream as **6 km/h**.

Theorem: (9)

Let $x = s$ be a solution of $x = g(x)$ and suppose that g has a continuous derivative in some interval J containing s. Then if $|g'| \le K < 1$ in J, the iteration process defined above converges for any x_0 in J. The limit of the sequence $[x_n]$ is s

Since there is no solution (evident by quadratic formula) there exists no interval J for which the process converges to a point.

The same behaviour is shown by the Newton-Raphson Method,
Start with an initial guess x_0, and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{10}$$

where ,

$$f(x) = x^2 + 48x - 324 \tag{11}$$

$$f'(x) = 2x + 48 \tag{12}$$

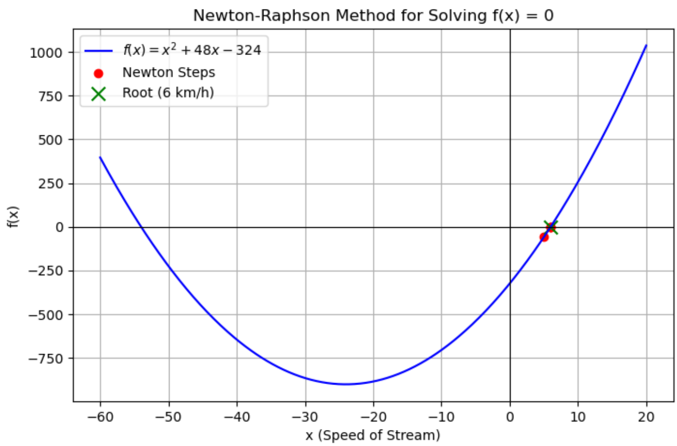

Fig. 0.1: Plot showing the relationship between $f(x)$ and speed of stream