Interplay between resource dynamics, network structure and spatial propagation of transient explosive synchronization in an adaptively coupled mouse brain network model

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9 Abstract

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Generalized epileptic attacks, which exhibit widespread disruption of brain activity, are characterized by recurrent, spontaneous and synchronized bursts of neural activity that self-initiate and self-terminate through critical transitions. Here we utilize the general framework of explosive synchronization (ES) from complex systems science to study the role of network structure and resource dynamics in the generation and propagation of seizures. We show that a combination of resource constraint and adaptive coupling in a Kuramoto network oscillator model can reliably generate seizure-like synchronization activity across different network topologies, including a biologically derived mesoscale mouse brain network. The model, coupled with a novel algorithm for tracking seizure propagation, provides mechanistic insight into the dynamics of transition to the synchronized state and its dependence on resources; and identifies key brain areas that may be involved in the initiation and spatial propagation of the seizure. The model, though minimal, efficiently recapitulates several experimental and theoretical predictions from more complex models, and makes novel experimentally testable predictions.

22 Significance statement / Author Summary

23 Understanding seizure dynamics at the whole-brain level is crucial for controlling abnormal hypersynchronous 24 activity. Currently, complete brain coverage recordings are lacking in both patients and animal models. We 25 employ network science tools to investigate epileptic seizure-like synchronization in a mouse whole brain 26 network, leveraging network structure and supported dynamics as the basis for seizure evolution. Our results 27 align with experimental findings, suggesting that seizure activity initiates in the cortico-thalamic circuit. 28 Importantly, our novel analysis identifies key nodes, primarily in the cortex, driving this hypersynchronous 29 activity. Our findings highlight network structure's role in shaping seizure dynamics and the techniques 30 developed here could enhance our control of generalized seizures when combined with patient-specific data.

31 Keywords

- 32 epilepsy, mouse brain connectome, explosive synchronization, adaptive coupling, seizure propagation,
- 33 resource dynamics

34 Introduction

- Epileptic seizures, characterized by bursts of excessive neuronal synchronization which usually self-initiate and self-terminate, are considered as a dynamical disease of brain networks ¹. Seizures can be classified into distinct subtypes, broadly including those that are confined to a circumscribed area (focal) and those which involve larger sections of the brain (generalized) ². A wide range of microscopic mechanisms contribute to this limited repertoire of seizure types ³. Although seizures can originate from different brain regions in patients, they may still manifest similar macroscopic features, as seen in EEG recordings^{3–5}. Furthermore, studies have suggested that seizures with similar microscopic mechanisms can present as either focal or generalized depending on the macroscopic network structure ⁶. This decoupling between microscopic and macroscopic dynamics underscores the importance of directly modeling emergent properties of seizures and highlights the significance of adopting a network level approach to studying epilepsy, which has also been recognized by the International League Against Epilepsy ⁷.
- 46 Experimental evidence shows that network structure alone is not sufficient but the dynamics supported by it 47 also plays an important role in seizure generation and propagation in a brain network ^{8,9}. Seizures have been 48 hypothesized to exist in the bistable regime of dynamical networks that exhibits multiple stable states – normal 49 (unsynchronized) and abnormal (hyper-synchronized). In such a system, random fluctuations (noise) or 50 resource availability can transition the network between the different states, giving rise to transient 51 hyper-synchronized activity seen during epileptic seizures ^{1,10}.
- The phenomenon of explosive synchronization (**ES**) that is widely studied in complex systems and network science can provide a general framework to understand the role of network structure in facilitating seizure dynamics ^{11–14}. ES is characterized by first order, discontinuous and irreversible transitions between globally coherent and incoherent states. These features are highly relevant to seizure dynamics, which also show signatures of critical transitions at both onset and termination across multiple spatial scales ^{15,16}. Consequently, ES models have been employed to study abrupt transitions in brain networks ^{17,18}. Moreover, complete brain coverage recordings, which can elucidate the dynamics of generalized seizures, are lacking in both patient and animal models. Thus, integrating ES with biological networks enables the study of seizure-like synchronization dynamics at the whole-brain level.
- 61 Although ES in complex networks has been successfully modeled using two common microscopic mechanisms the presence of microscopic correlation features, such as frequency-degree coupling (**FDC**); and adaptive coupling ^{12,14} they do not explain the transient and recurrent nature of seizures. Experimental and computational studies have linked this transient nature of seizures with the dynamics of energy metabolism ^{4,10}. Consistent with this observation, a recent study has shown the occurrence of transient ES (**tES**) in a resource-constrained scale-free network with FDC ¹⁹, where the time-varying nature of resource consumption is shown to cause the transient behavior.
- 68 While resource constrained networks with FDC exhibit tES for certain scale-free networks, there are several other frequently occurring families of network structures (Fig. 1a-c) across which this mechanism does not appear to generalize. Especially in the context of neural dynamics, the network structure of brains (Fig. 1d) is often found to show characteristics of small-world networks (SWNs) as well as scale-free networks (SFNs) (Fig. 1e,f). Moreover, adaptive coupling schemes have been shown to exhibit ES more generally across several network topologies, and in fact ES has been suggested to be a generic property of networks with adaptive coupling ¹⁴. Adaptive coupling is biologically plausible and has often been used to model interaction in biological systems ²⁰. Therefore in this study, we combine resource-constraint with adaptive coupling in a model (Fig. 1g) that can manifest tES across several types of network structures, including classic small-world

77 and scale-free networks as well as a real biological network (a mesoscale mouse brain network, MBN,

78 obtained from the Allen Institute public dataset ²¹).

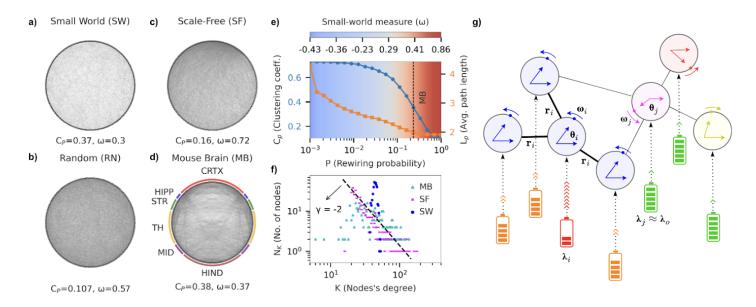


Figure 1: A model with adaptive coupling and resource constraint manifests transient explosive synchronization (tES) in the mesoscale mouse brain network (MBN), that shows both scale-free (SFN) and small-world (SWN) properties

a) Visualization of a small-world (SW) network generated using the Watts-Strogatz algorithm (number of nodes, N=400, average degree, $\langle k \rangle$ =40, rewiring probability, p=0.232). The generated network has average path length (L_n=1.97) and clustering coefficient (C_a=0.37) similar to mouse brain (MB) network; and small-world measure, ω=0.3. b) Random network with path length and average degree matched to MB (L_n =1.89, $\langle k \rangle$ = 45). c) Whole mouse brain mesoscale network having 426 nodes (213 in each hemisphere), each representing a region in mesoscale connectome from Allen mouse brain atlas. The color coded ring around the MB network groups the 426 nodes into 6 major regions (Cortex (CRTX), Hippocampus (HIPP), Striatum (STR), Thalamus and Hypothalamus (TH), Midbrain (MID), Hindbrain (HIND)). The graph (L_x=2.1, ω=0.37) is generated using a binarized version of the weighted network to allow for comparison with SW and scale-free (SF) networks. d) Scale-free (L_n=1.96) network generated using Barabasi-Albert algorithm with preferential attachment parameter, m = 20. e) Average path length and clustering coefficient for SW network as a function of rewiring probability p. The small-world measure ranges between -1 (fully ordered network, blue) to 1 (fully random network, red), with values close to zero (white) corresponding to a perfect small-world network. The clustering coefficient and average path length for MB correspond to a small-world measure of 0.37 (vertical black line), close to 0. f) Degree distribution of SW, SF, and MB networks. Both MB and SF network degree distributions fit the power law distribution with exponent y=-2. SW shows a Gaussian degree distribution. g) (Top) Network of interconnected Kuramoto oscillators, where each oscillator is coupled with every other oscillator through adaptive coupling (a,). (Bottom) Resource constraint implies that each oscillator is connected to individual resource reserves (battery), which define the excitability resources of the system (λ). The local synchrony determines the rate of energy consumption as well as the strength of local interactions.

80 Depending on the resource availability, our model exhibits desynchronized activity, bistability of

- 81 desynchronized and hyper-synchronized activity (i.e. tES) as well as steady-state hyper-synchronized activity
- 82 in SFNs, SWNs and the MBN. Furthermore, during the sudden transition to the synchronized state, we observe
- 83 a wave-like propagation of synchronization across subnetworks within the MBN, beginning with
- 84 cortico-thalamic subnetworks, followed by subcortical and deeper subnetworks. We also develop a novel
- 85 algorithm to analyze how the synchronization propagates across individual nodes (brain areas) in the MBN and
- 86 identify key brain areas that may be responsible for initiation, sustenance and propagation of the
- 87 hyper-synchronized state. Our results agree with several observations from experimental studies, suggesting
- 88 that a few key parameters can successfully capture the network level phenomenology of seizure dynamics.
- 89 Finally the model allows us to study the relationship between the hyper-synchronized state and the resource
- 90 consumption to recovery rate ratio. Specifically, the model predicts an optimal intermediate ratio for which the
- 91 likelihood of tES, i.e. epileptic attacks, is minimal. This and related predictions of our model should be directly
- 92 testable in experiments.

93 Results

94 An oscillator network model for transient explosive synchronization (tES) based on adaptive coupling

95 and resource constraint

- 96 Our model consists of N sinusoidal oscillators that form the nodes of a network. Following the Kuramoto model,
- 97 the interactions between connected oscillators depend on their phase difference. The interaction strength is
- 98 determined by the structural weight of the connection, and is further modulated by both the synchronization
- 99 levels of neighboring nodes and the availability of excitability resources (Fig. 1g).
- 100 The dynamics of the network are governed by the following equations:

$$\dot{\theta_i} = \omega_i + \lambda_i r_i \sum_{j=1}^N A_{ij} \sin(\theta_j - \theta_i) \quad \text{(Eqn. 1)}$$

- 102 Here, $i \in [1, N]$, θ_i and $\dot{\theta_i}$ are instantaneous phase and angular velocity of the i^{th} oscillator, and ω_i is its natural
- 103 frequency, uniformly distributed in [-1, 1]. The adjacency matrix A_{ii} encodes the network structure.
- 104 The interaction strengths are modulated by the local synchrony parameter,

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$$r_i = (1/k_i) \left| \sum_{i=1}^N A_{ij} e^{i\theta_j} \right|, \text{ where } k_i = \sum_{i=1}^N A_{ij} \text{ is the degree,} \quad \text{(Eqn. 2)}$$

- 106 giving rise to adaptive coupling, whereby nodes with higher local synchrony get coupled more strongly.
- 107 The interactions are also modulated by the availability of resources to individual nodes, λ_i . Following the model
- 108 by Frolov & Hramov¹⁹, we model the time-varying nature of excitability through diffusive coupling as follows:

$$\dot{\lambda}_{i} = \alpha(\lambda_{o} - \lambda_{i}) - \beta r_{i} \quad (Eqn. 3)$$

- 110 where the first term represents the recovery of excitability resources at a rate α , and the second term
- 111 represents the local synchrony-dependent resource consumption at a rate βr_i . β is the maximal consumption
- rate (when $r_i = 1$). The capacity of the resource reserve for each node is denoted by λ_o (size of resource bath).
- 113 The macroscopic behavior of the network is characterized by the global synchrony parameter,

114
$$R = (1/N) | \sum_{i=1}^{N} e^{i\theta_i} |, \quad (Eqn. 4)$$

115 which ranges from 0 (complete desynchronization) to 1 (complete synchronization).

116 A Small World Network (SWN) shows tES with adaptive coupling and resource constraint

- 117 We begin by investigating the properties of our model in an SWN. For this, we generate an SWN comprising
- 118 400 nodes while maintaining parameters such as average degree ($\langle k \rangle$ =40), average path length (L₀=1.97), and
- 119 clustering coefficient (C_n=0.37) similar to the MBN for later comparison (Fig. 1a).
- 120 We first characterize the resource-dependence of the network dynamics in the absence of resource dynamics
- 121 (Fig. 2a). Thus, in equation 1, λ_i is replaced by Λ , a fixed resource available to each node at all times. We

simulate this model with varying Λ and observe the steady-state behavior. For this, we begin with $\Lambda=0$, adiabatically increase (decrease) Λ with increment (decrement) of $\Delta\Lambda=0.003$, simulate the model for 1000 time steps and compute the stationary value of global synchrony (R) (see methods), going up to $\Lambda=0.12$. For very small Λ the network exhibits normal activity, and for very high values of Λ , it goes into the hypersynchronized state.

Interestingly for intermediate values, as Λ is slowly varied, we observe an abrupt first-order irreversible transition, with the presence of a hysteresis region – depending on the direction of change of Λ , or in other words, depending on the current state of the dynamics, the network goes into either the normal or the hyper-synchronized state (Fig. 2a).

Such existence of hysteresis has been shown to give rise to tES when resource constraint is imposed (Eqn. 3) when sufficient resources become available, the network transitions to the hyper-synchronized state. This transition is characterized by a sharp decrease in available resources due to increased consumption (Fig. 2d,e). Depending on the resource availability, the network spends a finite amount of time in this state before returning to the incoherent state as resources become depleted. The system replenishes its resources while in the incoherent state and transitions back to the hyper-synchronized state once sufficient resources become available.

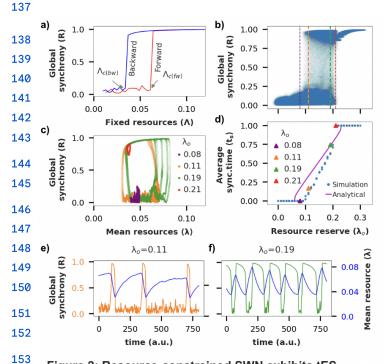


Figure 2: Resource-constrained SWN exhibits tES

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a) Forward (red) and backward (blue) transition curves for an adiabatically increasing (decreasing) resource level, Λ. Arrows indicate critical points corresponding to forward (or backward) transition from unordered (ordered) to ordered (unordered) state of the network. **b)** Bifurcation diagram of global synchrony (R) vs λ for SWN with resource constraint. c) ($\langle \lambda \rangle$,R) state space trajectory of activity for different values of λ_o . d) Fraction of time spent in the synchronized state (average synchronized time) vs λ_0 . **e, f)** Global synchrony and instantaneous mean resource level as a function of time for two different values of λ_{α} chosen from the bistable region in (b). Note: λ_{α} (fixed) is the fixed resources and $\langle \lambda \rangle$ (varying) is the instantaneous mean resources (averaged over all nodes) of the system.

Towards testing this hypothesized mechanism for tES, we first identify the parameter range for which the full model shows bistability, i.e. the network spends time in two (meta)stable states (Fig. 2b). We impose resource constraint at a consumption rate β (= 0.002) and again simulate the full model for λ_a , the size of the resource bath, varying between 0.01 and 0.3 with an increment of 0.01. For each value of λ_{a} , we simulate the model for 1000 time steps, and observe the range of values taken by the global synchrony parameter over the simulated period. The resulting bifurcation diagram reveals that the SWN exhibits a globally incoherent state for λ_{a} < 0.095, where resources are too limited to allow hyper-synchronization; and a hyper-synchronized state for $\lambda_{a} > 0.21$. For intermediate values of λ_{a} the network shows a coexistence of both states, reflecting the presence of tES, as seen in the timeseries of the global synchrony parameter (Fig. 2d,e). Consistent with the proposed mechanism, the transitions to and from the hyper-synchronized state occur when the mean resource availability across all nodes is close to the corresponding critical values of Λ , as revealed in the state-space trajectory of the system (Fig. 2c).

Since the transition back to the incoherent state occurs due to resource depletion, we expect the network to spend longer time in the

165 hyper-synchronized state as the size of the resource bath, λ_o , increases. This prediction is supported in our 166 simulation results (Fig. 2d) as well as with a simple analytical calculation (see supplement).

167 Finally we note that the phenomenon of ES or tES is not observed with SWN when correlation feature-based 168 connectivity, such as FDC, is used 19.

169 The mesoscale mouse brain network (MBN) shows partial tES with adaptive coupling

170 Next we use our model to study tES in a real biological neural network, using the mouse brain mesoscale 171 connectivity data from the Allen Brain Atlas. The mesoscale atlas is constructed by injecting viral vectors to 172 trace axonal projections across pairs of brain regions in mice ²¹. The dataset consists of detailed and accurate 173 connectivity information across 426 brain areas spanning both hemispheres (see Methods) in healthy mice. 174 The resulting network (Fig. 1d) comprises 11,000 directed edges, with weights rescaled between 0 and 1.

175 We repeat the analyses from SWN on MBN with an added nuance: while we assumed the SWNs to be binary 176 undirected networks, the dataset we use allows us to define the MBN as a weighted directed network. To 177 account for this, we use a modified version of equations 1 and 2 (see supplement). We analyze all three 178 variants of the MBN – binary-undirected, binary-directed, and weighted-directed. The results presented below 179 refer to the most complete weighted-directed variant unless mentioned otherwise, while the results for the 180 other two variants are qualitatively similar (Fig. S4).

181 We again begin by characterizing the system with fixed resource availability (see Methods for details). 182 Surprisingly, unlike the SWN which showed hysteresis, the MBN shows bistability for intermediate values of 183 fixed resources ($\Lambda \in [2.58, 2.65]$) (Fig. 3a). Even binary-undirected and binary-directed versions of MBN, 184 show hysteresis but not bistability for fixed resources (Fig. S4a, S4b). It is important to note that when 185 examining a network with fixed resources, the hysteresis region indicates the potential for tES, but does not 186 guarantee it. On the other hand, the presence of the bistability region directly confirms the existence of tES for 187 the weighted-directed MBN even without the need for resource constraint. We further confirm the existence of

188 tES with simulations of the full model that includes 189 resource constraint (Fig. 3b-d, S3). Note that with the 190 FDC model with or without resource constraints we 191 are unable to induce tES in the MBN (Fig. S4f, S4g).

192 Our analysis of the MBN recapitulates the
193 observations from SWN, including the increasing time
194 spent in the synchronized state with increasing size of
195 resource bath (Fig. 3b inset). An interesting deviation
196 is that the synchronized state is only partially
197 synchronized, as reflected in the global synchrony
198 parameter only reaching up to 0.6 instead of 1 (Fig.
199 3a-d). This happens because certain subnetworks
200 within the MBN never participate in the
201 synchronization. We thus define actively participating
202 nodes as those with a local synchrony greater than
203 the threshold value of 0.7 during the partially
204 synchronized state. We then compare the state space
205 trajectory obtained by averaging the available

206 resources over all nodes versus averaging only over

208 space trajectory shifts to the right in the former case.

207 the actively participating nodes (Fig. S2). The state

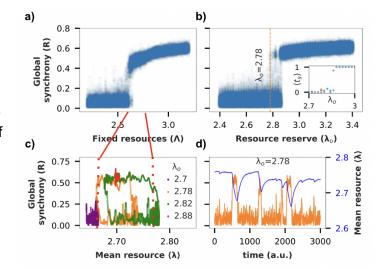
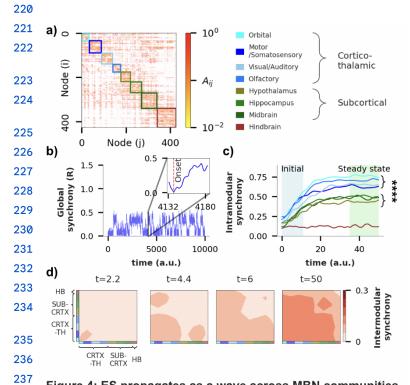


Figure 3: Mesoscale MBN with excitability resource constraint exhibits tES

a) Bifurcation diagram of global synchrony (R) vs Λ for the weighted MB network. b) Bifurcation diagram of global order (R) vs λ_o for the resource constrained model. (Inset) Fraction of time spent in synchronized state vs λ_o . c) ($\langle \lambda \rangle$,R) state space trajectory of activity for different values of λ_o . d) Time series of global synchrony vs instantaneous resources.

suggesting that inactive nodes are pushing the average available resources ($\langle \lambda \rangle$) to higher value. We thus conclude that only the nodes that actively participate in the synchronization cluster increase their energy consumption during tES.

Another deviation from SWNs is that the average resource level (even after restricting to participating nodes) at the time of transitions is observed to be higher than the corresponding resource level in the fixed resource model (Fig. 3a,c). One hypothesis is that even within the synchronized cluster, there may be a core subcluster that drives tES, for which the average energies at transition may be lower, but the peripheral nodes in the synchronized cluster, with higher energy availability, push the apparent average transition energy levels higher. This, however, does not seem to be the case. We conclude that the higher complexity of biological networks, perhaps due to their modular, hierarchical and heterogeneous nature compared to the SWN, makes the relationship between the fixed resource and resource constrained dynamics less predictable.



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Figure 4: ES propagates as a wave across MBN communities a) Adjacency matrix of MBN where coloured boxes group the 426 nodes into 8 distinct communities obtained using the Louvian algorithm with a resolution parameter of 1. Communities are named based on underlying circuitry (see Supplementary figure 5) and can be further grouped into broader categories: cortico-thalamic (blue) and subcortical (green). Modularity score (Q) of the obtained community partition is 0.53. b) The global synchrony parameter reflects several tES events in a weighted MBN for λ_0 = 2.87. (Inset) activity for one of the transients (out of 52 total) starting from the 'onset index' (red vertical line), characterized by the point where complete desynchronization occurs just before the abrupt transition. c) Intramodular synchrony (synchronization level within each community averaged over all 52 transients) during the transition shown in (b). Colors represent communities from (a). d) Contour map of intermodular synchrony for 4 different time points, spanning from a time close to 'onset index' (t=2.2) to the time post abrupt transition (t=50) reveals a wave propagating across the network.

In the following sections we investigate the dynamics of propagation of synchronization in the MBN.

ES propagates as a wave from cortico-thalamic to subcortical subnetworks within the MBN

We group the 426 nodes in MBN into distinct communities purely based on the network structure, using the Louvain algorithm (see Methods) ²². We hypothesize that such structure-based partitioning of the nodes will group together nodes that are highly likely to form a synchronization cluster. The process naturally partitions the MBN into 8 communities which can be clubbed into three broad classes: cortico-thalamic (4 communities), subcortical (3 communities) and hindbrain (Fig. 4a).

We then generate a set of 52 transitions from the incoherent to the hyper-synchronized state by running 4 long simulations, with fixed $\lambda_o=2.87$, but with different initial conditions (see Methods). Each transition is characterized by the presence of an 'onset index', a point where nearly complete desynchronization occurs just before the abrupt transition is about to begin (Fig. 4b, inset), subsequent to which synchronization rapidly increases. To study the spatial propagation of synchronization within the short temporal window in which it occurs, we analyze the transient dynamics over a 50 time unit window following the onset index.

We first study the dynamics of synchronization during the transient window within each community, by computing the intra-modular synchrony (a measure of phase alignment across nodes within the community, see Methods) averaged over all 52 transients. Based on the temporal evolution of the intramodular synchrony, we can group the 8 communities into three distinct cohorts that overlap perfectly with the structural classes defined earlier: cortico-thalamic, subcortical, and hindbrain (Fig. 4c). The steady-state intra-modular synchrony is significantly higher for the cortico-thalamic communities, followed by subcortical, and finally hindbrain, which shows no internal synchronization ($p < 10^{-4}$ for all the three pairs).

At the start of the transient process, the orbital, olfactory and sensorimotor areas exhibit significantly higher intra-modular synchrony than the rest of the communities (p < 0.05 for each pair, Fig. S8), and also show a steeper rate of increase (Fig. S8), indicating their potential role in initiating and driving the abrupt transition. As the transient progresses, orbital and olfactory areas consistently maintain a significantly higher level of synchronization compared to the rest (p < 0.05, Fig. S8). These results suggest that, while the onset sites of tES can be the orbital, olfactory, or sensorimotor areas, it is likely that the orbital and olfactory areas play a dominant role in driving the transition throughout the entire duration of the transient process. We note that, while intramodular synchrony in the visual area does not seem to significantly differ from the orbital area (Fig. S8), it shows much higher trial to trial variation in terms of its participation. This reinforces previous observations in the literature that the visual area is not critical for the propagation of synchronization²³.

The dynamics of synchronization propagation can be understood by analyzing the inter-modular synchrony between community pairs (see Methods). Consistent with what we found earlier, ES initiates in the orbital and sensorimotor areas (Fig. 4d, t = 2.2), and then spreads across all cortical regions (Fig. 4d, t = 4.4). Notably, the hippocampal / midbrain areas synchronize with cortical areas before synchronizing among themselves (Fig. 4d, t = 4.4, 6, 50), indicating that the cortical areas are driving their synchronization. At steady state (t = 50), all cortical, thalamic, and subcortical regions achieve synchrony, while hindbrain exhibits minimal participation throughout. Overall, these findings suggest a hierarchical synchronization process, with cortical areas potentially driving synchronization among subcortical regions.

276 Propagation of ES across individual nodes in the MBN reveals critical nodes that drive ES

Going beyond the community level, below we assess synchronization propagation at a single node level.
Assessing the synchronization between individual nodes typically involves computing the correlation of activity
of the pair over short time windows ^{24,25}. However, given the extremely short-lived transition window, this
technique cannot provide sufficient temporal resolution for studying abrupt transitions. We therefore came up
with a novel algorithm that we call the Synchronization Cluster Tracking Algorithm (SCTA) (see methods) for
this analysis. The SCTA employs a two-step procedure for quantifying the participation of individual nodes in
the synchronization process: (1) generating synchronization clusters by starting from a seed node and
expanding the cluster till the local synchrony drops below a fixed threshold; and (2) tracking the
synchronization clusters progressively through time to get a 'cluster lineage' for each node (see Methods). We
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The transition typically begins with multiple small synchronization clusters (cluster size < 10) during the early phase (Fig. 5a, t \approx 17). As the transition progresses, we observe the emergence of a 'main synchronization cluster' that quickly comes to dominate in size (Fig. 5a) throughout the rest of the transition. The size of the main cluster varies as the small peripheral clusters or individual nodes continue to join or leave it. This indicates that while the specific onset sites may vary across different trials initially, once the abrupt transition begins, it rapidly spreads from that initial onset site to encompass a broad set of core nodes that hold it together.

To test the 'core' nodes hypothesis, we quantify the time spent by individual nodes in the main synchronization cluster across the 52 transients. We find that although a majority of the nodes exhibit participation in the main

297 synchronization cluster, certain nodes consistently
298 spend a significant amount of time in the main cluster
299 (median time spent > 35, std. dev. < 10, Fig. 5b). This
300 core spans across multiple brain areas including the
301 cortex (19 nodes), hippocampus (6), striatum (9),
302 thalamus (10) and the midbrain (3). In contrast, nodes
303 with high variability likely represent peripheral nodes
304 that frequently attach and detach from the main cluster,
305 contributing to the observed variation in cluster size.

306 This analysis allows us to hypothesize the existence of 307 driver nodes for hyper-synchronization, as nodes with 308 consistent, high participation, and a high out-degree. 309 By spending longer times in the main synchronization 310 cluster, and influencing several downstream nodes they 311 are likely to play a key role in the propagation of 312 synchronization (Fig. 5c). Numerous other nodes with 313 lower out degrees also consistently spend more time in 314 the main cluster, indicating their higher susceptibility to 315 the influence of the drivers, rather than themselves 316 influencing other nodes. These driver areas include the 317 Perirhinal (PERI), Entorhinal (ENTI), Orbital (ORBI), 318 Reticular Nucleus (RE), Basolateral Amygdala (BLA), 319 Piriform (PIR) and Agranular insular area (Ai). Notably, 320 a majority of these "driver nodes" are located in cortical 321 areas. The hindbrain spends the least time in the main 322 cluster, consistent with our earlier findings.

323 An intermediate resource recovery-to-consumption 324 ratio is optimal

325 Since we model resource dynamics explicitly, our 326 model allows us to investigate the impact of resource 327 dynamics on the propensity of tES for the network. In

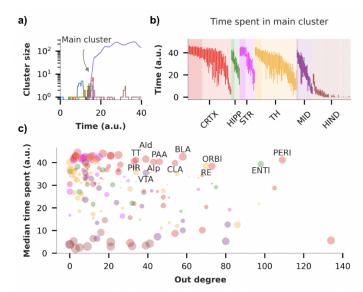


Figure 5: Participation of individual nodes in the main synchronization cluster during the transition

a) Synchronization clusters for the transient shown in Fig. 4b obtained using Synchronization Cluster Tracking Algorithm (SCTA) synchronization threshold (Supplementary material). The figure illustrates one main synchronization cluster (purple, ~200 nodes) along with several smaller clusters (~2-20 nodes). b) Box plot of the time spent by each node in the main synchronization cluster for 52 different transients (only left hemisphere nodes are shown for clarity). Nodes are color coded as per the major regions defined in Fig. 1c. Dark shaded area in each region shows node with median time spent > 35, and variance in time spent < 49 (σ < 7). c) Median time spent in the main synchronization cluster vs out degree of nodes in the left hemisphere (limited to 213 nodes). The size of the scatter points is inversely proportional to the variance in time spent across 52 transients, with larger circles indicating low variance. Labeled points indicate nodes with out-degree > 30, median time spent > 35, and variance in time spent < 49.

328 particular, we study the impact of the resource 329 recovery-to-consumption rate ratio (α/β) by fixing the recovery rate ($\alpha=0.01$) and varying the consumption 330 coefficient ($\beta\in\{0.01,\,0.005,\,0.002,\,0.001,\,0.0005,\,0.00025,\,0.0002\}$). For this set of parameters, we identify 331 the range of resource bath size (λ_o) that support tES. We find that as the recovery-to-consumption rate ratio 332 increases, both SWN and MBN exhibit a shift in the range boundaries to lower values, indicating a higher 333 propensity for tES at low resource levels (Fig. 6a,b). The exponential decrease in range boundaries with an 334 increase in the ratio suggests that even a slight change in the recovery-to-consumption ratio can significantly 335 alter the network's propensity to generate tES.

336 In contrast, as the recovery-to-consumption rate ratio increases, the width of the bistability region decreases.

337 Moreover, within the bistable region also, the duration of time spent in the synchronized state decreases,

338 resulting in a decreased probability of tES occurrence (Fig. 6c).

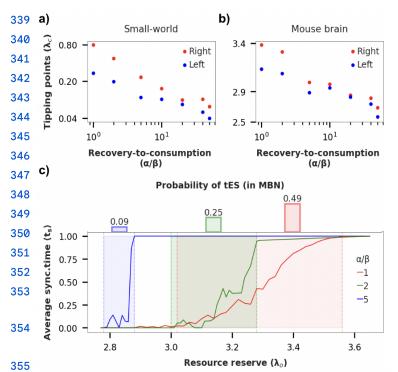


Figure 6: Effect of resource consumption coefficient (β) on network dynamics

a, b) Critical value of λ_o corresponding to left (blue) and right (red) tipping points of the bistability region as function of recovery-to-consumption ratio (or metabolism-to-uptake ratio). Note: Both x and y-axis are on log scale. **c)** (bottom) Fraction of time spent in synchronized state, (average synchronized time) $\langle t_s \rangle$ vs λ_o (bistable region shaded). (top) Probability of tES occurrence as function of recovery-to-consumption rate ratio (α/β) (estimated by measuring area under curve in shaded region normalized by area of shaded region).

367 seizure generation.

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Together, these results suggest an optimal intermediate ratio for a healthy brain as follows (Fig. 6d): Assume that the brain always operates at a resource bath size such that it is near the bistable region (criticality hypothesis ^{17,26}). Moreover, the resource bath size also undergoes fluctuations. In this scenario, a very low ratio would mean a high probability of tES, i.e. a high likelihood of seizures. If the ratio is very high, the width of the bistable region is so small that fluctuations in the resource bath size push the brain into the monostable hypersynchronized state, again increasing the likelihood of seizures. But an intermediate value of the ratio ensures a balance between these two extremes, and should be observable in a healthy brain.

Discussion

We first show in an idealized SWN (and other topologies such as SFN) how adaptive coupling can give rise to resource level-dependent hysteresis. Upon the addition of resource dynamics, this gives rise to tES for intermediate sizes of the resource bath. Recent research comparing diffusive and adaptive coupling, common modeling choices in networks of neural masses, has demonstrated a higher likelihood of networks with adaptive coupling to generate seizures ²⁰. Our findings reinforce this preference for adaptive coupling in exhibiting a higher tendency for

368 Our results hold well qualitatively when we apply the same model to a real biological neural network – the 369 mesoscale mouse brain network from the Allen Brain Atlas (Fig. 3). Although the structural network comes 370 from healthy, rather than epileptic mice, our results demonstrate the ability of the model to generate 371 seizure-like dynamics in a biologically realistic network. The framework can be used to further study how 372 perturbations to this network can increase their susceptibility to seizures, thus understanding the specific 373 potential structural elements in diseased mice (or humans) that lead to epilepsy. The choice of a mouse brain 374 as the model system is made because of the greater precision and completeness with which the anatomical 375 connectivity can be measured, compared to non-invasive methods employed in humans.

At the same time, for the MBN we observe some very interesting deviations compared to SWNs. The MBN 377 reaches only a partially synchronized state, with specifically the hindbrain subnetwork never participating in the 378 synchronization (Fig. 3). Even within the synchronized cluster, we hypothesize the presence of a core that 379 becomes fully synchronized, and drives the tES event, and a periphery that does not necessarily reach full 380 synchronization. The energy levels of the nodes at the time of transition may provide a means to identify the 381 core and the periphery. Moreover, the constitution of this core likely depends on the size of the resource bath, 382 so that for different bath sizes, we observe different average transition energies (Fig. 3c). We speculate that

383 this added complexity is a result of the weighted, hierarchical and modular network structure in real brains 384 compared to our idealized SWN. These hypotheses and speculations are areas for further study to understand 385 how the network structure affects its susceptibility to tES.

386 We then study the dynamics of synchronization propagation across the network at the level of communities 387 (aka subnetworks) and individual nodes. A salient feature we observe is that preceding each abrupt transition, 388 there is a point of near-complete desynchronization across the network (Fig. 4b-inset). This is accompanied by 389 the formation of several small clusters which later merge into the main synchronization cluster. These 390 phenomena are consistent with results obtained through mean field analyses ^{14,25}, as well as experimental 391 observations at micro- and macroscopic levels ^{27–30}.

392 At the intra-community level, we find that the cortico-thalamic networks (particularly the orbital, olfactory and 393 sensorimotor areas) exhibit a higher starting synchrony, faster increase of synchrony and a higher steady-state 394 synchrony during the transitions, compared to subcortical communities (Fig. 4c). This suggests their role in 395 initiating and propagating the hyper-synchronized state, consistent with extensive observations and predictions 396 in literature 31–36. Additionally, we find certain cortical networks (orbital, sensorimotor) to be more critical for 397 synchronization propagation than others (visual) 23.

398 Quantification of inter-modular synchronization reveals that the synchronization expands in a hierarchical 399 manner, as a propagating wave from the cortical to subcortical regions (Fig. 4d), so that the subcortical areas 400 synchronize with cortical areas before they synchronize among themselves. Although whole-brain recordings 401 during generalized epilepsy are lacking, this would be an interesting hypothesis to test in model organisms with 402 invasive electrophysiology.

We develop a novel algorithm to track the synchronization cluster lineages for individual nodes, which reveals the existence of a single large synchronization cluster during the transition, with several small clusters that dynamically join or leave it. Based on the consistency and time spent by the nodes in the main cluster, and their out degrees, we find a set of 'core' nodes that hold the cluster together, irrespective of the initiating site. This driver set includes Perirhinal, Entorhinal, Orbital, Reticular Nucleus, Basolateral Amygdala, Piriform and Agranular insular areas. These predictions are supported by several experimental findings ^{37,38}: for instance, the entorhinal, perirhinal, and piriform cortex form a highly interconnected network with other limbic structures and have been shown to possess characteristics that make them susceptible to the initiation and spread of epileptic seizures ³⁹.

According to theoretical analysis¹⁴ and our cluster tracking results, abrupt transitions during tES are preceded by the formation of numerous small synchronization clusters. This is consistently preceded by almost complete desynchronization. The more of these clusters, the more abrupt the transition¹⁴. A similar phenomenon of synchronization cluster formation and interictal/preictal desynchronization is observed before critical transitions during seizures ^{28,29,41}. Experimental evidence shows that these individual clusters exhibit high-frequency oscillations (HFOs) of 80-500 Hz⁴¹. These observations suggest that the preictal/interictal dynamics of HFO may vary depending on the seizure class that exhibits preictal desynchronization. Testing this hypothesis is intriguing, as it could emphasize the importance of considering seizure type when using HFO as a biomarker formation and interictal dynamics of the considering seizure type when using HFO as a biomarker formation and interictal dynamics of the considering seizure type when using HFO as a biomarker formation and interictal desynchronization.

Lastly, our mechanistic model highlights the importance of an intermediate resource recovery-to-consumption ratio, effectively balancing the heightened tES likelihood and the occurrence of monostable hypersynchronous activity. This implies an optimal recovery-to-consumption range where seizures are infrequent 42, and a constant hypersynchronous state is improbable. Deviations from this range may trigger abnormal brain states, suggesting a testable hypothesis for the susceptibility to epileptic attacks in relation to ATP demand and

426 oxygen consumption rates observed during ictal and interictal epileptiform activity 43,44.

To summarize, our mesoscale network model for generalized epilepsy applied to a real biological brain network makes several predictions that are consistent with experimental data and more biologically realistic and complex models. The simplicity, coupled with the generality, of the model holds significant value for two key reasons: first, its simplicity allows for the simulation of large-scale brain networks without significant concerns about computational load; and second, it potentially enables the study of seizure dynamics in a wide range of whole-brain networks, and could have applicability from a translational perspective. By identifying the propagation pattern during seizures, we can potentially identify strategies to halt the propagation. Therefore, the model and techniques developed here can be applied to connectome data from actual epileptic brains, with the hope of identifying the seizure onset site and its progression.

436 Methods

437 Simulations

438 To evaluate the dynamics of the model with different network structures (small-world, scale-free, mouse brain),

439 we perform two types of simulations: adiabatic progression and bifurcation diagram construction. In the

440 adiabatic progression, we systematically increase or decrease the fixed resource Λ to observe the global order

441 of the conventional adaptive coupling model with fixed resources. This allows us to determine the hysteresis

442 region of the system. For the bifurcation diagram construction, we increase the resource bath size parameter

443 λ_{0} and measure the global order at each time point. For all simulations, we use $\alpha=0.01,\,\beta=0.002$ (unless

444 otherwise specified) and the initial phases θ_i are distributed uniformly in the range [0, 2π). Equations are

445 simulated using the Euler method with a step size of 0.05.

446 To construct hysteresis (bifurcation) diagram in MBN, unlike SWN, we run simulations for a duration of 2000

447 time units for each value of Λ / λ_a through adiabatic progression with $\Delta\Lambda=0.02$ / $\Delta\lambda_a=0.02$.

448 To study the progression of ES in a weighted MBN, we conduct four separate runs with a fixed value of 449 $\lambda_0 = 2.87$, each using distinct initial conditions for (phase) θ and (frequency) ω . The simulations spanned a

450 duration of 20000 time units. From these simulations, we extract a total of 52 transients by selecting segments

451 of length 50-time units, starting from the "onset index" just before the abrupt transition.

452 Data Analysis

453 Intramodular synchrony:

454 At any particular time unit during the simulation, to asses synchrony level among nodes belonging to one 455 community (obtained from community detection algorithm), intramodular (within community) synchrony, for

456 each community, is computed as the average coherence of phase alignments of all the nodes:

$$IMS_c = 1/N_c \left| \sum_{i=1}^{N_c} e^{i\theta_i} \right|$$

458 where, IMS_c is intra-modular synchrony is for the c^{th} community and N_c is the number of nodes. The average 459 intra-modular synchrony ($\overline{IMS_c}$) is computed by averaging the synchronization level (IMS_c) within each 460 community across 52 transients:

461
$$\overline{IMS}_{c} = \sum_{tr=1}^{52} (1/N_{c} | \sum_{i=1}^{N_{c}} e^{i\theta_{i}^{tr}} |)$$

462 This calculation is performed for each time point within a 50-time unit window to capture the temporal evolution 463 of intra-modular synchrony during the period of abrupt transition.

464 Intermodular synchrony:

465 To compute coherence between two distinct communities, intermodular synchrony is computed as the average 466 absolute value of pairwise sum of phase alignment between nodes belonging to different communities:

$$IMS_{c_1c_2} = 1/(N_{c_1}N_{c_2}) \left| \sum_{i=1}^{N_{c_1}} \sum_{j=1}^{N_{c_2}} 0.5 * (e^{i\theta_i} + e^{i\theta_j}) \right|$$

- 468 where, N_{c_1} and N_{c_2} is number of nodes in community c_1 and c_2 . To get temporal evolution of inter-modular
- 469 synchrony, the average value across 52 transients is computed using a similar methodology as employed for 470 intramodular synchrony analysis.
- 471 Synchronization cluster tracking algorithm:
- 472 The Synchronization cluster tracking algorithm (SCTA) performs two major tasks: i) finds the synchronization
- 473 clusters at each time unit (Fig. SM1), ii) tracks the temporal evolution of identified clusters across different time
- 474 units (Fig. SM2).
- 475 The SCTA aims to expand synchronization clusters within the network based on a given synchronization
- 476 threshold. It follows three key steps:
- 1. Expansion around Central Nodes: The algorithm begins by expanding the synchronization clusters around the central nodes, which are selected from the previous time step. These central nodes act as trackers for the clusters across different time steps. Nodes with local synchronization exceeding a predetermined threshold become part of the cluster, which terminates with nodes that fall below the threshold (Fig. SM1).
- 2. Expansion for Unassigned Nodes: Next, the algorithm expands the synchronization clusters for any nodes that have not yet been assigned to a cluster. This step ensures that all nodes are considered and included in appropriate clusters based on the synchronization threshold (Fig. SM2).
- Update Central Node List: Finally, once all nodes have been traversed or become part of some cluster, the algorithm updates the list of central nodes for each cluster. Central nodes are the top 5 nodes in each cluster with highest local synchrony. These central nodes are important reference points for the clusters, preserving their cluster membership over time, and acting as seed nodes for the expansion of clusters in the next time step (Fig. SM2).
- 490 By following these steps iteratively, the algorithm progressively identifies, expands, and tracks synchronization 491 clusters within the network.
- 492 Time spent in main synchronization cluster:
- 493 By executing the SCTA over a 50-time unit window for a specific transient, we obtain the cluster sizes as a
- 494 function of time. The largest cluster is defined as the main synchronization cluster, and we measure the time
- 495 spent by each node as part of the main cluster within the 50 time step window.

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588 Author contributions

- 589 AKR and SRG conceived the study, designed the model and computational framework, analyzed the data, and
- 590 wrote the manuscript. AKR carried out the implementation. SRG conceived the idea for the novel algorithm,
- 591 and in consultation with SRG, AKR further developed and refined the algorithm.

592 Code availability

- 593 The code and parameters that have provided the results presented here are available at GitHub
- 594 https://github.com/csndl-iitd/tES_mesoscale_connectivity_model.git