34

Heights and Distances

IMPORTANT FACTS AND FORMULAE

I. We already know that:

In a right angled $\triangle OAB$, where $\angle BOA = \theta$,

(i)
$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{OB}};$$

(ii)
$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{OA}}{\text{OB}};$$

(iii)
$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{AB}}{\text{OA}};$$

(iv)
$$\csc \theta = \frac{1}{\sin \theta} = \frac{OB}{AB};$$

(v)
$$\sec \theta = \frac{1}{\cos \theta} = \frac{OB}{OA};$$

$$(vi) \cot \theta = \frac{1}{\tan \theta} = \frac{OA}{AB}.$$



$$(i) \sin^2 \theta + \cos^2 \theta = 1.$$

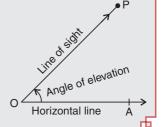
(ii)
$$1 + \tan^2 \theta = \sec^2 \theta$$
.

(iii)
$$1 + \cot^2 \theta = \csc^2 \theta$$
.

III. Values of T-ratios:

θ	0°	(π / 6) 30°	(π / 4) 45°	(π/3) 60°	(π / 2) 90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

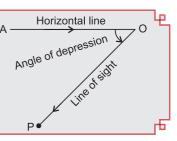
- **IV. Angle of Elevation:** Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.
 - \therefore Angle of elevation of P from O = \angle AOP.



Perpendicular

2√3 m

➡ V. Angle of Depression: Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



SOLVED EXAMPLES

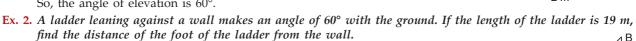
- Ex. 1. If the height of a pole is $2\sqrt{3}$ metres and the length of its shadow is 2 metres, find the angle of elevation
 - Sol. Let AB be the pole and AC be its shadow.

Let angle of elevation, $\angle ACB = \theta$.

Then,
$$AB = 2\sqrt{3}$$
 m, $AC = 2$ m.

$$\tan \theta = \frac{AB}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3} \implies \theta = 60^{\circ}.$$

So, the angle of elevation is 60°.

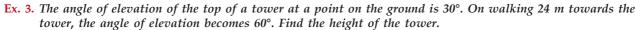


Then,
$$\angle$$
 ACB = 60° and BC = 19 m.

Let
$$AC = x$$
 metres

$$\frac{AC}{BC} = \cos 60^{\circ} \implies \frac{x}{19} = \frac{1}{2} \implies x = \frac{19}{2} = 9.5.$$

:. Distance of the foot of the ladder from the wall = 9.5 m.



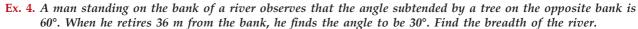
$$\frac{AB}{AD} = \tan 60^{\circ} = \sqrt{3} \implies AD = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}.$$

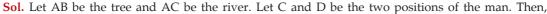
$$\frac{AB}{AC} = \tan 30^{\circ} = \frac{1}{\sqrt{3}} \implies AC = AB \times \sqrt{3} = h\sqrt{3}.$$

$$CD = (AC - AD) = \left(h\sqrt{3} - \frac{h}{\sqrt{3}}\right).$$

$$h\sqrt{3} - \frac{h}{\sqrt{3}} = 24 \implies h = 12\sqrt{3} = (12 \times 1.73) = 20.76.$$

Hence, the height of the tower is 20.76 m.



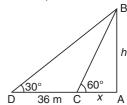


$$\angle$$
ACB = 60°, \angle ADB = 30° and CD = 36 m.

Let AB = h metres and AC = x metres.

Then,
$$AD = (36 + x)$$
 metres.

$$\frac{AB}{AD} = \tan 30^{\circ} = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{36 + x} = \frac{1}{\sqrt{3}}$$



$$\Rightarrow h = \frac{36 + x}{\sqrt{3}}$$
...(i)

$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3} \implies \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow h = \sqrt{3} x$$
 ...(ii)

From (i) and (ii), we get: $\frac{36+x}{\sqrt{3}} = \sqrt{3} x \implies x = 18 \text{ m}.$

So, the breadth of the river = 18 m.

- Ex. 5. A man on the top of a tower, standing on the seashore, finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.
 - Sol. Let AB be the tower and C and D be the two positions of the boat.

Let AB = h, CD = x and AD = y.
$$\frac{h}{y}$$
 = tan 60° = $\sqrt{3}$ \Rightarrow $y = \frac{h}{\sqrt{3}}$.

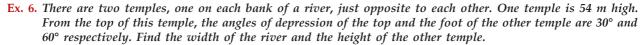
$$\frac{h}{x+y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \implies x+y = \sqrt{3} \ h.$$

$$\therefore \quad x = (x+y) - y = \left(\sqrt{3} \ h - \frac{h}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}.$$

Now, $\frac{2h}{\sqrt{3}}$ is covered in 10 min.

$$\therefore \quad \frac{h}{\sqrt{3}} \text{ will be covered in } \left(10 \times \frac{\sqrt{3}}{2h} \times \frac{h}{\sqrt{3}}\right) = 5 \text{ min.}$$

Hence, required time = 5 minutes.





Then,
$$AB = 54 \text{ m}$$
.

Let AC =
$$x$$
 metres and CD = h metres. \angle ACB = 60°, \angle EDB = 30°.

$$\frac{AB}{AC}$$
 = tan 60° = $\sqrt{3}$

$$\Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{54}{\sqrt{3}} = \left(\frac{54}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right) = 18\sqrt{3} \text{ m}.$$

$$DE = AC = 18\sqrt{3} \text{ m}$$

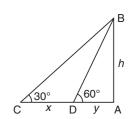
$$\frac{BE}{DE} = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$$

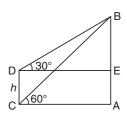
$$\Rightarrow$$
 BE = $\left(18\sqrt{3} \times \frac{1}{\sqrt{3}}\right)$ = 18 m.

$$\therefore$$
 CD = AE = AB - BE = (54 - 18) m = 36 m.

So, Width of the river =
$$AC = 18\sqrt{3} \text{ m} = (18 \times 1.73) \text{ m} = 31.14 \text{ m}.$$

Height of the other temple = CD = 18 m.





HEIGHTS AND DISTANCES 873

EXERCISE

(OBJECTIVE TYPE QUESTIONS)

Directions:	Mark	(\(\sigma\)	against	the	correct	answer
--------------------	------	---------------------	---------	-----	---------	--------

1. The angle of elevation of the sun, when the length of the shadow of a tree is $\sqrt{3}$ times the height of the tree, is (R.R.B., 2008)

(a) 30°

(b) 45°

(c) 60°

(d) 90°

2. From a point P on a level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, the distance of point P from the foot of the tower is

(R.R.B., 2006)

(a) 149 m

(b) 156 m

(c) 173 m

(d) 200 m

3. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is :

(a) 2.3 m

(b) 4.6 m

(c) 7.8 m

(d) 9.2 m

4. An observer 1.6 m tall is $20\sqrt{3}$ m away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The height of the tower is :

(a) 21.6 m

(b) 23.2 m

(c) 24.72 m

(d) None of these

5. Two ships are sailing in the sea on the two sides of a lighthouse. The angles of elevation of the top of the lighthouse as observed from the two ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is

(a) 173 m

(b) 200 m

(c) 273 m

(d) 300 m

6. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° with the man's eye. The man walks some distance towards the tower to watch its top and the angle of elevation becomes 60°. What is the distance between the base of the tower and the point P?

(Bank P.O., 2007)

(a) $4\sqrt{3}$ units

(b) 8 units

(c) 12 units

(d) Data inadequate

(e) None of these

7. The angle of elevation of the top of a tower from a certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top of the tower increases by 15°. The height of the tower is (S.S.C., 2005)

(a) 17.3 m

(b) 21.9 m

(c) 27.3 m

(d) 30 m

8. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?

(a) 32 kmph

(b) 36 kmph

(c) 38 kmph

(d) 40 kmph

(e) 42 kmph

9. On the same side of a tower, two objects are located. Observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, the distance between the objects is

(a) 63.5 m

(b) 76.9 m

(c) 86.7 m

(d) 90 m

10. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower? (R.R.B., 2008)

(a) 14 min. 35 sec.

(b) 15 min. 49 sec.

(c) 16 min. 23 sec.

(d) 18 min. 5 sec.

11. The top of a 15 metre high tower makes an angle of elevation of 60° with the bottom of an electric pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

(R.R.B., 2009)

(a) 5 metres

(b) 8 metres

(c) 10 metres

(*d*) 12 metres

(e) None of these

12. The angle of depression of a point situated at a distance of 70m from the base of a tower is 60°. The height of the tower is [SSC—CHSL (10+2) Exam, 2015]

(a) $35\sqrt{3}$ m

(b) $70\sqrt{3} \text{ m}$

(c) $\frac{70\sqrt{3}}{3}$ m

(d) 70 m

13. TF is a tower with F on the ground. The angle of elevation of T from A is x° such that $\tan x^{\circ} = \frac{2}{5}$ and AF = 200m. The angle of elevation of T from a nearer point B is y° with BF = 80m. The value of y° is

[SSC—CHSL (10+2) Exam, 2015]

(a) 75°

(b) 45°

(c) 60°

 $(d) 30^{\circ}$

- 14. A boy is standing at the top of the tower and another boy is at the ground at some distance from the foot of the tower, then the angle of elevation and depression between the boys when both look at each other will be [CLAT, 2016]
 - (a) Equal
 - (b) Angle of elevation will be greater
 - (c) Cannot be predicted for relation
 - (d) Angle of depression will be greater
- 15. The angles of elevation of the top of a tower from two points P and Q at distances m^2 and n^2 respectively, from the base and in the same straight line with it are complementary. The height of the tower is

[CDS, 2016]

- (a) $(mn)^{1/2}$
- (b) $mn^{1/2}$
- (c) $m^{1/2}n$
- (d) mn
- 16. The angle of elevation of a cloud from a point 200 m above a lake is 30° and the angle of depression

- of its reflection in the lake is 60°. The height of the cloud is [CDS, 2016]
- (a) 200 m
- (b) 300 m
- (c) 400 m
- (d) 600 m
- 17. From the top of a tower, the angles of depression of two objects P and Q (situated on the ground on the same side of the tower) separated at a distance of $(3-\sqrt{3})$ m are 45° and 60° respectively. The height of the tower is [CDS, 2016]
 - (a) 200 m
- (b) 250 m
- (c) 300 m
- (d) None of these
- 18. If a 30 m ladder is placed against a 15 m wall such that it just reaches the top of the wall, then the elevation of the wall is equal to

[DMRC-Customer Relationship Assistant (CRA) Exam, 2016]

- (a) 45°
- (b) 30°
- $(c) 60^{\circ}$
- (d) 50°

ANSWERS

1. (a) **2.** (c) **3.** (*d*) **4.** (a) **5.** (c) **6.** (*d*) **7.** (c) **8.** (a) **9.** (a) **10.** (c) **11.** (c) **12.** (b) **13.** (*b*) **17.** (c) **14.** (a) **15.** (*d*) **16.** (a) **18.** (a)

SOLUTIONS

В

В

1. Let AB be the tree and AC be its shadow. Let $\angle ACB = \theta$

Then,
$$\frac{AC}{AB} = \sqrt{3}$$

$$\Rightarrow$$
 cot $\theta = \sqrt{3}$ \Rightarrow $\theta = 30^{\circ}$.

2. Let AB be the tower.

Then, $\angle APB = 30^{\circ}$ and AB = 100 m. $C \stackrel{\checkmark}{\triangle} \theta$



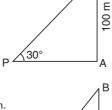
$$\Rightarrow AP = (AB \times \sqrt{3})$$
$$= 100\sqrt{3} \text{ m}.$$

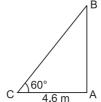
 $(100 \times 1.73) \text{ m} = 173 \text{ m}.$

3. Let AB be the wall and BC be the ladder. Then, $\angle ACB = 60^{\circ}$ and AC = 4.6 m.

$$\frac{AC}{BC} = \cos 60^{\circ} = \frac{1}{2}$$

 $BC = 2 \times AC = (2 \times 4.6) \text{ m}$ = 9.2 m.





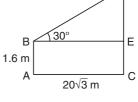
4. Let AB be the observer and CD be the tower. Draw BE \perp CD.

Then,
$$CE = AB = 1.6 \text{ m}$$
,

BE = AC =
$$20\sqrt{3}$$
 m.

$$\frac{DE}{BE} = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow DE = \frac{20\sqrt{3}}{\sqrt{3}} m = 20 m.$$



8

D

- \therefore CD = CE + DE = (1.6 + 20) m = 21.6 m.
- 5. Let AB be the lighthouse and C and D be the positions of the ships.

Then, AB = 100 m,
$$\angle$$
ACB = 30° and \angle ADB = 45°.

$$\frac{AB}{AC} = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AC = AB \times \sqrt{3} = 100\sqrt{3} \text{ m.} \sqrt{30^{\circ}}$$



$$\Rightarrow$$
 AD = AB = 100 m.

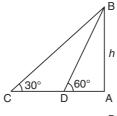
$$\therefore$$
 CD = (AC + AD) = $(100\sqrt{3} + 100)$ m

=
$$100 (\sqrt{3} + 1) \text{ m} = (100 \times 2.73) \text{ m} = 273 \text{ m}.$$

6. One of AB, AD and CD must have been given.

So, the data is inadequate.

7. Let AB be the tower and C and D be the points of observation. Then, $\angle ACB = 30^{\circ}$, $\angle ADB = 45^{\circ}$ and CD = 20 m.

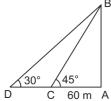


Let AB = h.
Then,
$$\frac{AB}{AC}$$
 = tan 30° = $\frac{1}{\sqrt{3}}$

$$\Rightarrow$$
 AC = AB $\times \sqrt{3}$ = $h\sqrt{3}$.

And,
$$\frac{AB}{AD} = \tan 45^\circ = 1$$

$$\Rightarrow$$
 AD = AB = h .



$$CD = 20 \Rightarrow (AC - AD) = 20 \Rightarrow h\sqrt{3} - h = 20.$$

$$h = \frac{20}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$

=
$$10(\sqrt{3} + 1)$$
 m = (10×2.73) m = 27.3 m.

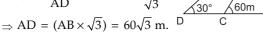
8. Let AB be the tower and C and D be the two positions of the boats.

Then, $\angle ACB = 45^{\circ}$, $\angle ADB = 30^{\circ}$ and AC = 60 m. Let AB = h.

Then,
$$\frac{AB}{AC} = \tan 45^\circ = 1$$

$$\Rightarrow$$
 AB = AC \Rightarrow h = 60 m.

And,
$$\frac{AB}{AD} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$



:.
$$CD = (AD - AC) = 60 (\sqrt{3} - 1) \text{ m}.$$

Hence, required speed

=
$$\left[\frac{60(\sqrt{3}-1)}{5}\right]$$
 m/s = (12×0.73) m/s
= $\left(12 \times 0.73 \times \frac{18}{5}\right)$ km/hr
= 31.5 km/hr ≈ 32 km/hr.

9. Let AB be the tower and C and D be the objects. Then, AB = 150 m, $\angle ACB = 45^{\circ}$ and $\angle ADB = 60^{\circ}$.

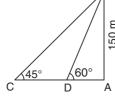
$$\frac{AB}{AD} = \tan 60^{\circ} = \sqrt{3}$$

$$AD = \frac{AB}{AD} = \frac{150}{D} \text{ m}$$

$$\Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{150}{\sqrt{3}} \text{ m}.$$

$$\frac{AB}{AC} = \tan 45^{\circ} = 1$$

$$\Rightarrow$$
 AC = AB = 150 m.
 \therefore CD = (AC - AD)



$$= \left(150 - \frac{150}{\sqrt{3}}\right) \text{ m} = \left[\frac{150(\sqrt{3} - 1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right] \text{ m}$$

$$= 50 (3 - \sqrt{3}) \text{ m} = (50 \times 1.27) \text{ m} = 63.5 \text{ m}.$$

10. Let AB be the tower and C and D be the two positions of the car.

Then, $\angle ACB = 45^{\circ}$, $\angle ADB = 30^{\circ}$.

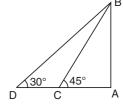
Let AB = h, CD = x and AC = y.

$$\frac{AB}{AC} = \tan 45^\circ = 1 \Rightarrow \frac{h}{y} = 1 \Rightarrow y = h.$$

$$\frac{AB}{AD} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{x+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x + y = \sqrt{3} h.$$



$$x = (x + y) - y = \sqrt{3} h - h = h(\sqrt{3} - 1).$$

Now, $h(\sqrt{3}-1)$ is covered in 12 min.

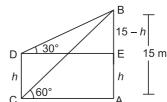
So, h will be covered in
$$\left[\frac{12}{h(\sqrt{3}-1)} \times h\right] = \frac{12}{(\sqrt{3}-1)} \min$$
.

$$= \left(\frac{1200}{73}\right) \text{ min.} \approx 16 \text{ min. } 23 \text{ sec.}$$

11. Let AB be the tower and CD be the electric pole.

Then, $\angle ACB = 60^{\circ}$, $\angle EDB = 30^{\circ}$ and AB = 15 m. Let CD = h. Then, BE = (AB - AE) = (AB - CD) = (15 - h).

$$\frac{AB}{AC} = \tan 60^{\circ} = \sqrt{3} \implies AC = \frac{AB}{\sqrt{3}} = \frac{15}{\sqrt{3}}.$$



And,
$$\frac{BE}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow DE = (BE \times \sqrt{3})$$
$$= \sqrt{3} (15 - h).$$

$$AC = DE \Rightarrow \frac{15}{\sqrt{3}} = \sqrt{3} (15 - h)$$

$$\Rightarrow$$
 3h = (45 - 15) \Rightarrow h = 10 m.

12. Length of the tower AB = h meter

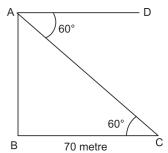
$$\angle DAC = \angle ACB = 60^{\circ}$$

BC = 70 meterIn ΔABC,

$$\tan 60^{\circ} = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{70}$$

$$\Rightarrow h = 70\sqrt{3}$$
 meter.



13. Given $\tan x^{\circ} = \frac{2}{5}$; AF = 200 meter

$$\therefore \frac{2}{5} = \frac{TF}{AF}$$

$$\Rightarrow TF = \frac{2 \times 200}{5}$$

$$\Rightarrow TF = 80 \text{ m}$$
We have, BF = 80 m
$$\therefore \tan y^{\circ} = \frac{TF}{BF}$$

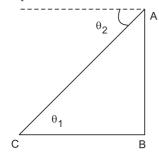
200 m

$$\Rightarrow \tan y^{\circ} = \frac{80}{80}$$

$$\Rightarrow \tan y^{\circ} = 1 = \tan 45^{\circ}$$

$$\therefore y^{\circ} = 45^{\circ}$$

14. Here, AD is parallel to BC and AC is a transversal.



$$\Rightarrow \theta_1 = \theta_2$$

$$15. \tan \theta = \frac{H}{m^2}$$

$$\tan(90^{\circ} - \theta) = \frac{H}{n^2}$$

$$\Rightarrow \cot \theta = \frac{H}{n^2}$$

$$\Rightarrow \tan \theta . \cot \theta = \frac{H}{m^2} \times \frac{H}{n^2}$$

$$\Rightarrow \frac{H}{m^2} \times \frac{H}{n^2} = 1$$

$$\Rightarrow H^2 = m^n n^2$$

$$\Rightarrow$$
 H = mn

16. Let AC = H m

$$OA = a \text{ m}$$
$$\tan 30^{\circ} = \frac{1}{\sqrt{3}} = \frac{H}{a}$$

$$\tan 60^{\circ} = \sqrt{3} = \frac{H + 200 + 200}{a}$$

$$\Rightarrow \frac{H + 400}{\sqrt{3}H} = \sqrt{3}$$

$$\Rightarrow$$
 H + 400 = 3H

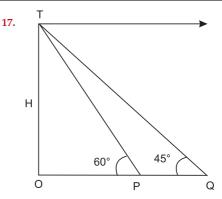
$$\Rightarrow$$
 2H = 400 m

$$\Rightarrow$$
 H = 200 m

Height of cloud above lake = 200 m

√60°





Let OP =
$$a$$

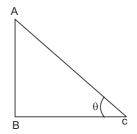
 $\tan 60^{\circ} = \frac{H}{a}$
 $\Rightarrow H = \sqrt{3}a$
 $\Rightarrow \frac{H}{\sqrt{3}} = a$...(i)
 $\tan 45^{\circ} = 1 = \frac{H}{a + 100(3 - \sqrt{3})}$
 $\Rightarrow a + 100(3 - \sqrt{3}) = H$
From (i) $\frac{H}{\sqrt{3}} + 100(3 - \sqrt{3}) = H$
 $H + 300\sqrt{3} - 300 = \sqrt{3}H$
 $\Rightarrow 300\sqrt{3} - 300 = \sqrt{3}H - H$
 $(\sqrt{3} - 1)H = 300(\sqrt{3} - 1)$
 $H = 300 \text{ m}$

18.

Cloud

- Lake

Reflection



AC = 30 meter
AB = 15 meter

$$\angle$$
 ACB = θ
 $\therefore \sin \theta = \frac{AB}{AC} = \frac{15}{30} = \frac{1}{2}$
 $\Rightarrow \sin \theta = \sin 30^{\circ}$
 $\Rightarrow \theta = 30^{\circ}$