30 Permutations and Combinations

IMPORTANT FACTS AND FORMULAE

I. Factorial *n*: *Let n be a positive integer. Then, factorial n is denoted by* $\lfloor n$ *or* $n!$, *defined as* $\lfloor n = n(n-1)(n-2)(n-3)\ldots\ldots 4\cdot3\cdot2\cdot1$.

Ex. (*i*) $\lfloor 4 = (4\times3\times2\times1) = 24$. (*ii*) $\lfloor 5 = (5\times4\times3\times2\times1) = 120$.

Note: We define, $\lfloor 0 = 1$.

II. (*i*) **Permutations:** *The different arrangements of a given number of things by taking some or all at a time are called permutations.*

Ex.1. All permutations or arrangements made with the letters *a, b, c* by taking two at a time are (*ab, ba, ac, ca, bc, cb*).

Ex.2. All permutations made with the letters *a, b, c* by taking 3 at a time are (*abc, acb, bac, bca, cab, cba*).

(*ii*) ***Number of Permutations of n things, taking r at a time is given by:***

$$^{n}P_{r} = n(n-1)(n-2)\ldots\ldots(n-r+1) = \frac{\lfloor n}{\lfloor n-r}.$$

Ex. (*i*) $^{8}P_{2} = (8\times7) = 56$. (*ii*) $^{7}P_{3} = (7\times6\times5) = 210$.

(*iii*) ***Number of all permutations of n things, taking all at a time is*** $\lfloor n$.

(*iv*) If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind such that $(p_1 + p_2 + \ldots + p_r) = n$, then **number of permutations** $= \dfrac{\lfloor n}{\lfloor p_1 \cdot \lfloor p_2 \cdot \lfloor p_3 \cdot \ldots\ldots \lfloor p_r}$.

III. (*i*) **Combinations:** *Each of the different groups or selections which can be formed by taking some or all at a time, is called a combination.*

Ex. 1. *Out of three boys A, B, C we want to select two.*

The possible selections are (AB, BC, CA).

Note that AB and BA represent the same combination.

Ex. 2. *The only combination of three letters A, B, C taken all at a time is* ***ABC.***

Ex. 3. *Various groups of two out of 4 persons A, B, C, D are AB, AC, AD, BC, BD, CD.*

Important Note: AB and BA are two different permutations.

But, they represent the same combination.

(*ii*) ***Number of all combinations of n things, taken r at a time, is***

$$^{n}C_{r} = \frac{\lfloor n}{(\lfloor r)(\lfloor n-r)} = \frac{n(n-1)(n-2)\ldots\ldots \text{to } r \text{ factors}}{\lfloor r}$$

(*iii*) $^{n}C_{n} = 1$ and $^{n}C_{0} = 1$ (*iv*) $^{n}C_{r} = {}^{n}C_{(n-r)}$

Ex. (*i*) $^{8}C_{3} = \dfrac{8\times7\times6}{3\times2\times1} = 56$. (*ii*) $^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_{3} = \dfrac{16\times15\times14}{3\times2\times1} = 560$.

SOLVED EXAMPLES

Ex. 1. ***Evaluate :*** $\dfrac{\underline{|50}}{\underline{|48}}$.

Sol. $\dfrac{\underline{|50}}{\underline{|48}} = \dfrac{50 \times 49 \times \underline{|48}}{\underline{|48}} = (50 \times 49) = 2450.$

Ex. 2. ***Evaluate : (i)*** $^5P_5$ ***(ii)*** $^{50}P_3$

Sol. (*i*) $^5P_5 = \underline{|5} = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

(*ii*) $^{50}P_3 = \dfrac{\underline{|50}}{\underline{|(50-3)}} = \dfrac{\underline{|50}}{\underline{|47}} = \dfrac{50 \times 49 \times 48 \times (\underline{|47})}{\underline{|47}} = (50 \times 49 \times 48) = 117600.$

Ex. 3. ***Evaluate : (i)*** $^{10}C_3$ ***(ii)*** $^{100}C_{98}$ ***(iii)*** $^{60}C_{60}$

Sol. (*i*) $^{10}C_3 = \dfrac{10 \times 9 \times 8}{\underline{|3}} = \dfrac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120.$

(*ii*) $^{100}C_{98} = {}^{100}C_{(100-98)} = {}^{100}C_2 = \dfrac{(100 \times 99)}{(2 \times 1)} = 4950.$

(*iii*) $^{60}C_{60} = 1$ $[\because {}^nC_n = 1]$

Ex. 4. ***In how many different ways can the letters of the word 'FIGHT' be arranged?*** **(Bank P.O., 2008)**

Sol. The given word contains 5 different letters.

Required number of ways = $^5P_5 = \underline{|5} = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

Ex. 5. ***In how many different ways can the letters of the word 'PRESENT' be arranged?*** **(Bank P.O., 2006)**

Sol. The word 'PRESENT' contains 7 letters, namely 2E and all other 5 are different.

$\therefore$ Required number of ways = $\dfrac{\underline{|7}}{\underline{|2}} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times (\underline{|2})}{\underline{|2}} = 2520.$

Ex. 6. ***How many arrangements can be made out of the letters of the word 'ENGINEERING'?***

Sol. The word 'ENGINEERING' contains 11 letters, namely 3E, 3N, 2G, 2I and 1R.

$\therefore$ Required number of ways = $\dfrac{\underline{|11}}{(\underline{|3})(\underline{|3})(\underline{|2})(\underline{|2})(\underline{|1})}$

$= \dfrac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 6 \times 2 \times 2 \times 1}$

$= (11 \times 10 \times 7 \times 6 \times 5 \times 4 \times 3) = 277200.$

Ex. 7. ***In how many different ways can the letters of the word 'DESIGN' be arranged so that the vowels are at the two ends?*** **(Bank P.O., 2009)**

Sol. The given word 'DESIGN' contains 4 consonants and 2 vowels.

At the two ends the two vowels can be arranged in 2 ways.

Remaining 4 letters can be arranged in $\underline{|4} = (4 \times 3 \times 2 \times 1) = 24$ ways.

Total number of ways = $(24 \times 2) = 48.$

$\therefore$ Required number of ways = 48.

Ex. 8. ***In how many different ways can the letters of the word 'DAUGHTER' be arranged so that the vowels always come together?***

Sol. The given word contains 8 different letters.

When the vowels AUE are taken together, we may treat them as 1 letter.

Then, the letters to be arranged are DGHTR (AUE)

The vowels can be arranged in $^6P_6 = \underline{|6} = 720$ ways.

The vowels AUE may be arranged in $\underline{|3} = 6$ ways.

Required number of ways = $(720 \times 6) = 4320$ ways.

Ex. 9. ***In how many different ways can the letters of the word 'DIRECTOR' be arranged so that the vowels are always together?***

Sol. In the given word, we treat the vowels IEO as 1 letter.

Thus, we have DRCTR (IEO).

This group has 6 letters in which R occurs 2 times and other are all different.

Number of ways of arranging these letters = $\dfrac{\lfloor 6}{\lfloor 2} = \dfrac{(6 \times 5 \times 4 \times 3 \times 2 \times 1)}{2} = 360.$

Now, 3 vowels can be arranged among themselves in $\lfloor 3 = 6$ ways.

$\therefore$ Required number of ways = (360 × 6) = 2160.

Ex. 10. ***In how many different ways can the letters of the word 'DIGEST' be arranged so that the vowels are never together?*** **(Bank P.O., 2004)**

Sol. In the given word DIGEST, we take the vowels IE as one letter.

Then, we can write it as DGST (IE).

This word has 5 letters which can be arranged among themselves in

$\lfloor 5$ = (5 × 4 × 3 × 2 × 1) = 120 ways.

The letters of IE can be arranged in 2 ways.

$\therefore$ Number of ways of arranging the letters of given word with vowels together

= (120 × 2) = 240 ways.

Number of ways of arranging all the letters of the given word

= $\lfloor 6$ = (6 × 5 × 4 × 3 × 2 × 1) = 720 ways.

$\therefore$ Number of ways of arrangements so that the vowels are never together

= (720 – 240) = 480.

Ex. 11. ***In how many different ways can the letters of the word 'DETAIL' be arranged so that the vowels occupy only the odd positions?*** **(Bank P.O., 2002)**

Sol. There are 6 letters in the given word, out of which there are 3 consonants and 3 vowels.

Let us mark these positions as (1) (2) (3) (4) (5) (6).

Now, 3 vowels can be placed at any of 3 places, marked 1, 3, 5.

Number of these arrangements = ${}^3P_3 = \lfloor 3 = 6.$

Also, 3 consonants can be placed at the remaining 3 places.

Number of these arrangements = ${}^3P_3 = \lfloor 3 = 6.$

Total number of ways = (6 × 6) = 36.

Ex. 12. ***In how many ways can a cricket eleven be chosen out of 14 players?***

Sol. Required number of ways = ${}^{14}C_{11} = {}^{14}C_{(14-11)} = {}^{14}C_3 = \dfrac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364.$

Ex. 13. ***In how many ways, a committee of 6 members be selected from 7 men and 5 ladies, consisting of 4 men and 2 ladies?***

Sol. We have to select (4 men out of 7) and (2 ladies out of 5).

$\therefore$ Required number of ways = ${}^7C_4 \times {}^5C_2 = {}^7C_3 \times {}^5C_2 = \left(\dfrac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \dfrac{5 \times 4}{2 \times 1}\right) = 350.$

EXERCISE

(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (√) against the correct answer in each of the following:*

1. $({}^{75}P_2 - {}^{75}C_2)$ =?

(*a*) 0 (*b*) 75

(*c*) 150 (*d*) 2775

(*e*) 5550

2. In how many different ways can the letters of the word DISPLAY be arranged? **(Bank P.O., 2009)**

(*a*) 720 (*b*) 1440

(*c*) 2520 (*d*) 5040

(*e*) None of these

3. In how many different ways can the letters of the word SMART be arranged? (Bank P.O., 2009)
(a) 25 (b) 60
(c) 180 (d) 200
(e) None of these

4. In how many different ways can the letters of the word FORMULATE be arranged? (Bank P.O., 2008)
(a) 8100 (b) 40320
(c) 153420 (d) 362880
(e) None of these

5. In how many different ways can the letters of the word GAMBLE be arranged? (Bank P.O., 2010)
(a) 15 (b) 25
(c) 60 (d) 125
(e) None of these

6. In how many different ways can the letters of the word RIDDLED be arranged? (Bank P.O., 2006)
(a) 840 (b) 1680
(c) 2520 (d) 5040
(e) None of these

7. In how many different ways can the letters of the word CREATE be arranged? (Bank P.O., 2011)
(a) 25 (b) 36
(c) 360 (d) 720
(e) None of these

8. In how many different ways can the letters of the word TOTAL be arranged? (Bank P.O., 2009)
(a) 45 (b) 60
(c) 72 (d) 120
(e) None of these

9. In how many different ways can the letters of the word OFFICES be arranged? (Bank P.O., 2010)
(a) 2520 (b) 5040
(c) 1850 (d) 1680
(e) None of these

10. In how many different ways can the letters of the word BANANA be arranged?
(a) 60 (b) 120
(c) 360 (d) 720
(e) None of these

11. In how many different ways can the letters of the word WEDDING be arranged? (Bank P.O., 2008)
(a) 2500 (b) 2520
(c) 5000 (d) 5040
(e) None of these

12. In how many different ways can the letters of the word INCREASE be arranged? (Bank P.O., 2009)
(a) 40320 (b) 10080
(c) 20160 (d) 64
(e) None of these

13. In how many different ways can the letters of the word ABSENTEE be arranged? (Bank P.O., 2006)
(a) 512 (b) 6720
(c) 9740 (d) 40320
(e) None of these

14. In how many different ways can the letters of the word AWARE be arranged? (Bank P.O., 2010)
(a) 40 (b) 60
(c) 120 (d) 150
(e) None of these

15. In how many different ways can the letters of the word DAILY be arranged? (Bank P.O., 2008)
(a) 48 (b) 60
(c) 120 (d) 160
(e) None of these

16. In how many different ways can the letters of the word RUMOUR be arranged?
(a) 30 (b) 90
(c) 180 (d) 720
(e) None of these

17. In how many different ways can the letters of the word OPERATE be arranged? (Bank P.O., 2009)
(a) 360 (b) 720
(c) 5040 (d) 2520
(e) None of these

18. In how many different ways can the letters of the word PUNCTUAL be arranged? (Bank P.O., 2009)
(a) 64 (b) 960
(c) 20160 (d) 40320
(e) None of these

19. In how many different ways can the letters of the word CREAM be arranged? (Bank P.O., 2008)
(a) 25 (b) 120
(c) 260 (d) 480
(e) None of these

20. Out of 5 men and 3 women, a committee of three members is to be formed so that it has 1 woman and 2 men. In how many different ways can it be done? (Bank P.O., 2009)
(a) 10 (b) 20
(c) 23 (d) 30
(e) None of these

21. Out of 5 women and 4 men, a committee of three members is to be formed in such a way that at least one member is a woman. In how many different ways can it be done? (Bank P.O., 2009)
(a) 76 (b) 80
(c) 84 (d) 96
(e) None of these

22. A committee of 5 members is to be formed out of 3 trainees, 4 professors and 6 research associates. In how many different ways can this be done, if the committee should have 4 professors and 1 research associate or all 3 trainees and 2 professors? **(S.B.I. P.O., 2010)**

(*a*) 12 (*b*) 13
(*c*) 24 (*d*) 52
(*e*) None of these

23. A committee of 5 members is to be formed out of 3 trainees, 4 professors and 6 research associates. In how many different ways can this be done if the committee should have 2 trainees and 3 research associates? **(S.B.I. P.O., 2010)**

(*a*) 15 (*b*) 45
(*c*) 60 (*d*) 9
(*e*) None of these

24. In how many ways can a committee of 4 people be chosen out of 8 people? **(Bank P.O., 2007)**

(*a*) 32 (*b*) 70
(*c*) 110 (*d*) 126
(*e*) None of these

25. A committee of 5 members is to be formed by selecting out of 4 men and 5 women. In how many different ways the committee can be formed if it should have 2 men and 3 women? **(Bank P.O., 2005)**

(*a*) 16 (*b*) 36
(*c*) 45 (*d*) 60
(*e*) None of these

26. A committee of 5 members is to be formed by selecting out of 4 men and 5 women. In how many different ways the committee can be formed if it should have at least 1 man? **(Bank P.O., 2011)**

(*a*) 115 (*b*) 120
(*c*) 125 (*d*) 140
(*e*) None of these

27. In how many ways a committee consisting of 5 men and 6 women can be formed from 8 men and 10 women? **(Bank P.O., 2009)**

(*a*) 266 (*b*) 5040
(*c*) 11760 (*d*) 86400
(*e*) None of these

28. A select group of 4 is to be formed from 8 men and 6 women in such a way that the group must have at least 1 woman. In how many different ways can it be done? **(Bank P.O., 2005)**

(*a*) 364 (*b*) 728
(*c*) 931 (*d*) 1001
(*e*) None of these

29. From a group of 7 men and 6 women, 5 persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

(*a*) 564 (*b*) 645
(*c*) 735 (*d*) 756
(*e*) None of these

30. A box contains 2 white, 3 black and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least 1 black ball is to be included in the draw?

(*a*) 32 (*b*) 48
(*c*) 64 (*d*) 96
(*e*) None of these

31. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?

(*a*) 45 (*b*) 63
(*c*) 90 (*d*) 126
(*e*) None of these

32. In how many different ways can the letters of the word ENGINEERING be arranged?

(*a*) 277200 (*b*) 92400
(*c*) 69300 (*d*) 23100
(*e*) None of these

33. In how many different ways can the letters of the word ALLAHABAD be arranged?

(*a*) 3780 (*b*) 1890
(*c*) 7560 (*d*) 2520
(*e*) None of these

34. In how many different ways can the letters of the word JUDGE be arranged in such a way that the vowels always come together?

(*a*) 48 (*b*) 120
(*c*) 124 (*d*) 160
(*e*) None of these

35. In how many different ways can the letters of the word AUCTION be arranged in such a way that the vowels always come together? **(Bank P.O., 2010)**

(*a*) 30 (*b*) 48
(*c*) 144 (*d*) 576
(*e*) None of these

36. In how many different ways can the letters of the word SOFTWARE be arranged in such a way that the vowels always come together? **(Bank P.O., 2009)**

(*a*) 120 (*b*) 360
(*c*) 1440 (*d*) 13440
(*e*) 4320

37. In how many different ways can the letters of the word OPTICAL be arranged in such a way that the vowels always come together?

(*a*) 120 (*b*) 720
(*c*) 2160 (*d*) 4320
(*e*) None of these

38. In how many different ways can the letters of the word BANKING be arranged in such a way that the vowels always come together? **(Bank P.O., 2009)**

(*a*) 120 (*b*) 240
(*c*) 360 (*d*) 540
(*e*) 720

39. In how many different ways can the letters of the word CAPITAL be arranged so that the vowels always come together? **(Bank P.O. 2012)**
(*a*) 120 (*b*) 360
(*c*) 720 (*d*) 840
(*e*) None of these

40. In how many ways can the letters of the word MATHEMATICS be arranged so that all the vowels always come together?
(*a*) 10080 (*b*) 120960
(*c*) 4989600 (*d*) 20160
(*e*) None of these

41. In how many different ways can the letters of the word CORPORATION be arranged so that the vowels always come together? **(Bank P.O. 2011)**
(*a*) 810 (*b*) 1440
(*c*) 2880 (*d*) 50400
(*e*) 5760

42. In how many different ways can the letters of the word MACHINE be arranged so that the vowels may occupy only the odd positions?
(*a*) 210 (*b*) 576
(*c*) 144 (*d*) 1728
(*e*) 3456

43. In how many different ways can the letters of the word EXTRA be arranged so that the vowels are never together?
(*a*) 120 (*b*) 48
(*c*) 72 (*d*) 168
(*e*) None of these

44. In an examination there are three multiple choice questions and each question has 4 choices. The number of ways in which a student can fail to get all answers correct is **[MAT, 2012]**
(*a*) 11 (*b*) 27
(*c*) 12 (*d*) 63

45. There are six teachers. Out of them two are primary teachers and two are secondary teachers. They are to stand in a row, so as the primary teachers, middle teachers and secondary teachers are always in a set. The number of ways in which they can do So, is **[MAT, 2011]**
(*a*) 52 (*b*) 48
(*c*) 34 (*d*) None of these

46. In how many different ways can the letters of the word 'BAKERY' be arranged? **[SBI— Bank Clerical Exam, 2012]**
(*a*) 2,400 (*b*) 2,005
(*c*) 720 (*d*) 5,040
(*e*) None of these

47. In how many different ways can the letters of the word 'TRANSPIRATION' be arranged so that the vowels always come together? **[DMRC—Train Operator/Station Controller Exam, 2016]**
(*a*) 2429500 (*b*) 1360800
(*c*) 1627800 (*d*) None of these

48. In how many ways can the letters of the word 'MOMENT' be arranged? **[UPSSSC—Lower Subordinate (Pre.) Exam, 2016]**
(*a*) 360 (*b*) 60
(*c*) 720 (*d*) 120

ANSWERS

1. (*d*)	**2.** (*d*)	**3.** (*e*)	**4.** (*d*)	**5.** (*e*)	**6.** (*a*)	**7.** (*c*)	**8.** (*b*)	**9.** (*a*)	**10.** (*a*)
11. (*b*)	**12.** (*c*)	**13.** (*b*)	**14.** (*b*)	**15.** (*c*)	**16.** (*c*)	**17.** (*d*)	**18.** (*c*)	**19.** (*b*)	**20.** (*d*)
21. (*b*)	**22.** (*e*)	**23.** (*c*)	**24.** (*b*)	**25.** (*d*)	**26.** (*c*)	**27.** (*c*)	**28.** (*c*)	**29.** (*a*)	**30.** (*c*)
31. (*b*)	**32.** (*a*)	**33.** (*c*)	**34.** (*a*)	**35.** (*d*)	**36.** (*e*)	**37.** (*b*)	**38.** (*e*)	**39.** (*b*)	**40.** (*b*)
41. (*d*)	**42.** (*b*)	**43.** (*c*)	**44.** (*d*)	**45.** (*b*)	**46.** (*c*)	**47.** (*b*)	**48.** (*a*)		

SOLUTIONS

1. $({}^{75}P_2 - {}^{75}C_2) = \left\{\frac{\underline{|75}}{\underline{|75-2}} - \frac{75 \times 74}{2}\right\} = \frac{\underline{|75}}{\underline{|73}} - (75 \times 37)$

$= \frac{75 \times 74 \times \underline{|73}}{\underline{|73}} - (75 \times 37)$

$= (75 \times 74 - 75 \times 37) = 75 \times 37 \times (2 - 1)$

$= (75 \times 37) = 2775.$

2. The given word contains 7 letters, all different.

$\therefore$ Required number of ways $= {}^7P_7 = \underline{|7}$

$= (7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) = 5040.$

3. The given word contains 5 letters, all different.

$\therefore$ Required number of ways $= {}^5P_5 = \underline{|5}$

$= (5 \times 4 \times 3 \times 2 \times 1) = 120.$

4. The given word contains 9 letters, all different.

$\therefore$ Required number of ways $= {}^9P_9 = \underline{|9}$

$= (9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) = 362880.$

5. The given word contains 6 letters, all different.

$\therefore$ Required number of ways $= {}^6P_6 = \underline{|6}$

$= (6 \times 5 \times 4 \times 3 \times 2 \times 1) = 720.$

6. The given word contains 7 letters of which D is taken 3 times.

$\therefore$ Required number of ways $= \dfrac{\underline{|7}}{\underline{|3}} = \dfrac{7 \times 6 \times 5 \times 4 \times \underline{|3}}{\underline{|3}}$

$= (7 \times 6 \times 5 \times 4) = 840.$

7. The given word contains 6 letters of which E is taken 2 times.

$\therefore$ Required number of ways $= \dfrac{\underline{|6}}{\underline{|2}} = \dfrac{6 \times 5 \times 4 \times 3 \times \underline{|2}}{\underline{|2}} = 360.$

8. The given word contains 5 letters of which T is taken 2 times.

$\therefore$ Required number of ways $= \dfrac{\underline{|5}}{\underline{|2}} = \dfrac{5 \times 4 \times 3 \times \underline{|2}}{\underline{|2}} = 60.$

9. The given word contains 7 letters of which F is taken 2 times.

$\therefore$ Required number of ways

$= \dfrac{\underline{|7}}{\underline{|2}} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 2520.$

10. The given word contains 6 letters of which A is taken 3 times, N is taken 2 times and the rest are all different.

$\therefore$ Required number of ways

$= \dfrac{\underline{|6}}{\underline{|3}\,.\,\underline{|2}} = \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 2} = 60.$

11. The given word contains 7 letters of which D is taken 2 times.

$\therefore$ Required number of ways

$= \dfrac{\underline{|7}}{\underline{|2}} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 2520.$

12. The given word contains 8 letters of which E is taken 2 times.

$\therefore$ Required number of ways

$= \dfrac{\underline{|8}}{\underline{|2}} = \dfrac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2} = 20160.$

13. The given word contains 8 letters of which E is taken 3 times.

$\therefore$ Required number of ways

$= \dfrac{\underline{|8}}{\underline{|3}} = \dfrac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6} = 6720.$

14. The given word contains 5 letters of which A is taken 2 times.

$\therefore$ Required number of ways $= \dfrac{\underline{|5}}{\underline{|2}} = \dfrac{5 \times 4 \times 3 \times 2 \times 1}{2} = 60.$

15. The given word contains 5 letters, all different.

$\therefore$ Required number of ways $= \underline{|5} = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

16. The given word contains 6 letters out of which R is taken 2 times, U is taken 2 times and other letters are all different.

$\therefore$ Required number of ways

$= \dfrac{\underline{|6}}{\underline{|2} \cdot \underline{|2}} = \dfrac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 2} = 180.$

17. The given word contains 7 letters out of which E is taken 2 times and all other letters are different.

$\therefore$ Required number of ways

$= \dfrac{\underline{|7}}{\underline{|2}} = \dfrac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2} = 2520.$

18. The given word contains 8 letters out of which U is taken 2 times and all other letters are different.

$\therefore$ Required number of ways

$= \dfrac{\underline{|8}}{\underline{|2}} = \dfrac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2} = 20160.$

19. The given word contains 5 letters, all different.

$\therefore$ Required number of ways $= \underline{|5} = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

20. Required number of ways $= \left({}^3C_1 \times {}^5C_2\right) = 3 \times \dfrac{5 \times 4}{2 \times 1} = 30.$

21. Required number of ways $= ({}^5C_1 \times {}^4C_2) + ({}^5C_2 \times {}^4C_1) + ({}^5C_3)$

$= \left(5 \times \dfrac{4 \times 3}{2 \times 1}\right) + \left(\dfrac{5 \times 4}{2 \times 1} \times 4\right) + \left(\dfrac{5 \times 4 \times 3}{3 \times 2 \times 1}\right)$

$= (30 + 40 + 10) = 80.$

22. Required number of ways $= ({}^4C_4 \times {}^6C_1) + ({}^3C_3 \times {}^4C_2)$

$= (1 + 6) + \left(1 + \dfrac{4 \times 3}{2}\right) = (7 + 7) = 14.$

23. Required number of ways $= ({}^3C_2 \times {}^6C_3) = ({}^3C_1 \times {}^6C_3)$

$= \left(3 \times \dfrac{6 \times 5 \times 4}{3 \times 2 \times 1}\right) = 60.$

24. Required number of ways $= {}^8C_4 = \dfrac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 70.$

25. Required number of ways $= ({}^4C_2 \times {}^5C_3) = ({}^4C_2 \times {}^5C_2)$

$= \left(\dfrac{4 \times 3}{2 \times 1} \times \dfrac{5 \times 4}{2 \times 1}\right) = 60.$

26. The committee should have

(1 man, 4 women) or (2 men, 3 women) or (3 men, 2 women) or (4 men, 1 woman).

Required number of ways $= ({}^4C_1 \times {}^5C_4) + ({}^4C_2 \times {}^5C_3) + ({}^4C_3 \times {}^5C_2) + ({}^4C_6 \times {}^5C_1)$

$= ({}^4C_1 \times {}^5C_1) + ({}^4C_2 \times {}^5C_2) + ({}^4C_1 \times {}^5C_2) + ({}^4C_4 \times {}^5C_1)$

$= (4 \times 5) + \left(\dfrac{4 \times 3}{2 \times 1} \times \dfrac{5 \times 4}{2 \times 1}\right) + \left(4 \times \dfrac{5 \times 4}{2 \times 1}\right) + (1 \times 5)$

$= (20 + 60 + 40 + 5) = 125.$

27. Required number of ways $= ({}^8C_5 \times {}^{10}C_6) + ({}^8C_3 \times {}^{10}C_4)$

$= \dfrac{8 \times 7 \times 6}{\underline{|3}} \times \dfrac{10 \times 9 \times 8 \times 7}{\underline{|4}}$

$= \dfrac{8 \times 7 \times 6}{6} \times \dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 11760.$

28. Required number of ways

$= ({}^6C_1 \times {}^8C_3) + ({}^6C_2 \times {}^8C_2) + ({}^6C_3 \times {}^8C_1) + ({}^6C_4 \times {}^8C_0)$

$$= \left\{6 \times \frac{8 \times 7 \times 6}{\lfloor 3}\right\} + \left(\frac{6 \times 5}{2 \times 1} \times \frac{8 \times 7}{2 \times 1}\right) + \left(\frac{6 \times 5 \times 4}{\lfloor 3} \times 8\right) + ({}^6C_2 \times 1)$$

$$= \left\{6 \times \frac{8 \times 7 \times 6}{3 \times 2 \times 1}\right\} + 420 + \left(\frac{6 \times 5 \times 4}{6} \times 8\right) + \left(\frac{6 \times 5}{2 \times 1} \times 1\right)$$

$= (336 + 420 + 160 + 15) = 931.$

29. Required number of ways

$= ({}^7C_3 \times {}^6C_2) + ({}^7C_4 \times {}^6C_1) + ({}^7C_5 \times {}^6C_0)$

$$= \left\{\frac{7 \times 6 \times 5}{\lfloor 3} \times \frac{6 \times 5}{\lfloor 2}\right\} + ({}^7C_3 \times {}^6C_1) + ({}^7C_2 \times 1)$$

$$= \left(\frac{7 \times 6 \times 5}{6} \times \frac{6 \times 5}{2 \times 1}\right) + \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 6\right) + \left(\frac{7 \times 6}{2 \times 1} \times 1\right)$$

$= (525 + 210 + 21) = 756.$

30. We may have (1 black and 2 non-black) or (2 black and 1 non-black) or (3 black).

Required no. of ways $= ({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3)$

$$= \left(3 \times \frac{6 \times 5}{2 \times 1}\right) + \left(\frac{3 \times 2}{2 \times 1} \times 6\right) + 1 = (45 + 18 + 1) = 64.$$

31. Required no. of ways

$$= ({}^7C_5 \times {}^3C_2) = ({}^7C_2 \times {}^3C_1) = \frac{7 \times 6}{2 \times 1} \times 3 = 63.$$

32. The given word contains 11 letters, namely 3E, 3N, 2G, 2I and 1R.

$\therefore$ Required number of ways $= \dfrac{\lfloor 11}{(\lfloor 3)(\lfloor 3)(\lfloor 2)(\lfloor 2)(\lfloor 1)}$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 6 \times 2 \times 2 \times 1}$$

$= (11 \times 10 \times 9 \times 8 \times 7 \times 5) = 277200.$

33. The given word contains 9 letters, namely 4A, 2L, 1H, 1B and 1D.

$\therefore$ Required number of ways $= \dfrac{\lfloor 9}{(\lfloor 4)(\lfloor 2)(\lfloor 1)(\lfloor 1)(\lfloor 1)}$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2} = 7560.$$

34. The given word contains 5 different letters.

Keeping the vowels UE together, we suppose them as 1 letter.

Then, we have to arrange the letters JDG(UE).

Now, 4 letters can be arranged in $\lfloor 4 = 24$ ways.

The vowels (UE) can be arranged among themselves in 2 ways.

$\therefore$ Required no. of ways $= (24 \times 2) = 48.$

35. The given word contains 7 different letters.

Keeping the vowels (AUIO) together, we take them as 1 letter.

Then, we have to arrange the letters CTN(AUIO).

Now, 4 letters can be arranged in $\lfloor 4 = 24$ ways.

The vowels (AUIO) can be arranged among themselves in $\lfloor 4 = 24$ ways.

$\therefore$ Required number of ways $= (24 \times 24) = 576.$

36. The given word contains 8 different letters.

We keep the vowels (OAE) together and treat them as 1 letter.

Thus, we have to arrange the 6 letters SFTWR(OAE).

These can be arranged in $\lfloor 6 = 720$ ways.

The vowels (OAE) can be arranged among themselves in $\lfloor 3 = 6$ ways.

$\therefore$ Required number of ways $= (720 \times 6) = 4320.$

37. The given word contains 7 different letters.

We keep the vowels (OIA) together and treat them as 1 letter.

Thus, we have to arrange the letters PTCL(OIA).

These can be arranged in $\lfloor 5 = 120$ ways.

The vowels (OIA) can be arranged among themselves in $\lfloor 3 = 6$ ways.

$\therefore$ Required number of ways $= (120 \times 6) = 720.$

38. The given word contains 7 letters of which N is taken 2 times.

We keep the vowels (AI) together and treat them as 1 letter.

Thus, we have to arrange 6 letters BNKNG(AI) of which N occurs 2 times and the rest are different.

These can be arranged in $\dfrac{\lfloor 6}{\lfloor 2} = (6 \times 5 \times 4 \times 3) = 360$ ways.

Now 2 vowels (AI) can be arranged among themselves in 2 ways.

$\therefore$ Required number of ways $= (360 \times 2) = 720.$

39. Keeping the vowels (AIA) together, we have CPTL (AIA).

We treat (AIA) as 1 letter.

Thus, we have to arrange 5 letters.

These can be arranged in $\lfloor 5 = (5 \times 4 \times 3 \times 2 \times 1)$ ways $= 120$ ways.

Now, (AIA) are 3 letters with 2A and 1I.

These can be arranged among themselves in $\dfrac{\lfloor 3}{\lfloor 2} = \dfrac{3 \times 2 \times 1}{2 \times 1} = 3$ ways.

$\therefore$ Required number of ways $= (120 \times 3) = 360.$

40. Keeping the vowels (AEIA) together, we have MTHMTCS(AEAI).

Now, we have to arrange 8 letters, out of which we have 2M, 2T and the rest are all different.

Number of ways of arranging these letters

$$= \frac{\lfloor 8}{\lfloor 2 . \lfloor 2} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 10080.$$

Now, (AEAI) has 4 letters, out of which we have 2A, 1E and 1I.

Number of ways of arranging these letters

$$= \frac{\lfloor 4}{\lfloor 2} = \frac{4 \times 3 \times 2 \times 1}{2} = 12.$$

$\therefore$ Required number of ways $= (10080 \times 12) = 120960.$

41. Keeping the vowels (OOAIO) together as one letter we have CRPRTN(OOAIO).

This has 7 letters, out of which we have 2R, 1C, 1P, 1T and 1N.

Number of ways of arranging these letters

$= \frac{\lfloor 7}{\lfloor 2} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 2520.$

Now, (OOAIO) has 5 letters, out of which we have 3O, 1A and 1I.

Number of ways of arranging these letters

$= \frac{\lfloor 5}{\lfloor 3} = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 20.$

$\therefore$ Required number of ways = (2520 × 20) = 50400.

42. There are 7 letters in the given word, out of which there are 3 vowels and 4 consonants.

Let us mark the positions to be filled up as follows:

$(^1)(^2)(^3)(^4)(^5)(^6)(^7)$

Now, 3 vowels can be placed at any of the three places out of four marked 1, 3, 5, 7.

Number of ways of arranging the vowels = $^4P_3$

= (4 × 3 × 2) = 24.

4 consonants at the remaining 4 positions may be arranged in $^4P_4 = \lfloor 4 = 24$ ways.

Required number of ways = (24 × 24) = 576.

43. Taking the vowels (EA) as one letter, the given word has the letters XTR (EA), i.e., 4 letters.

These letters can be arranged in $\lfloor 4$ = 24 ways.

The letters *EA* may be arranged amongst themselves in 2 ways.

Number of arrangements having vowels together

= (24 × 2) = 48 ways.

Total arrangements of all letters = $\lfloor 5 = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

Number of arrangements not having vowels together

= (120 – 48) = 72.

44. Number of ways of attempting 1st, 2nd, 3rd questions are each.

Total number of ways = $4^3 = 4 \times 4 \times 4 = 64$

Number of ways, getting all correct answers = $1^3 = 1$

$\therefore$ Number of ways of not getting all answer correct

= 64 – 1 = 63

45. There are 2 primary teachers.

They can stand in a row in P(2, 2) = 2!

= 2 × 1 ways = 2 ways

$\therefore$ Two middle teachers.

They can stand in a row in P(2, 2) = 2!

= 2 × 1 = 2 ways

There are two secondary teachers.

They can stand in a row in

P(2, 2) = 2! = 2 × 1 = 2 ways

These three sets can be arranged themselves in 3!

Ways = 3 × 2 × 1 = 6 ways

Hence, the required number of ways

= 2 × 2 × 2 × 6 = 48 ways

46. The letters of the word 'BAKERY'' be arranged in 6! ways st.

6! = 6 × 5 × 4 × 3 × 2 × 1 = 720

47. The word TRANSPIRATION has 13 letters in which each of T, R, A, N and I has come two time

We have to arrange TT RR NN PS (AA II O).

There are five vowels in the given words.

$\therefore$ we consider these give vowels as one letter.

$\therefore$ Required number of arrangements.

$= \frac{9! \times 5!}{2!2!2!2!2!}$

$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 5 \times 4 \times 3 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$

= 1360800

48. There are six letters in the given word MOMENT and letter 'M' has come twice.

$\therefore$ Required number of ways

$= \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 360$