8 Problems on Ages

SOLVED EXAMPLES

Ex. 1. ***The ratio of the ages of Tina and Rakesh is 9 : 10 respectively. Ten years ago, the ratio of their ages was 4 : 5 respectively. What is the present age of Rakesh?*** **(Bank P.O., 2010)**

Sol. Let Tina's age be $9x$ years. Then, Rakesh's age = $10x$ years.

$\therefore \frac{9x-10}{10x-10}=\frac{4}{5} \Rightarrow 5(9x-10)=4(10x-10) \Rightarrow 45x-40x=50-40 \Rightarrow 5x=10 \Rightarrow x=2.$

$\therefore$ Present age of Rakesh = (10×2) years = 20 years.

Ex.2. ***Samir's age is one-fourth of his father's age and two-third of his sister Reema's age. What is the ratio of the ages of Samir, Reema and their father respectively?*** **(Bank P.O., 2009)**

Sol. Samir's age = $\frac{1}{4}\times$ Father's age = $\frac{2}{3}\times$ Reema's age = x years (say).

Then, Samir's age = x years, Reema's age = $\frac{3}{2}x$ years and Father's age = $4x$ years.

Ratio of the ages of Samir, Reema and father = $x:\frac{3}{2}x:4x=2:3:8.$

Ex.3. ***The age of father 10 years ago was thrice the age of his son. 10 years hence father's age will be twice that of his son. Find the ratio of their present ages.*** **(L.I.C., 2003)**

Sol. Let son's age 10 years ago be x years. Then, father's age 10 years ago = $3x$ years.

Son's age 10 years hence = $(x + 20)$ years.

Father's age 10 years hence = $(3x + 20)$ years.

$\therefore \quad 3x + 20 = 2(x + 20) \Rightarrow x = (40 - 20) = 20.$

Ratio of father's age and son's age at present = $(3x + 10) : (x + 10)$

$= (3 \times 20 + 10) : (20 + 10) = 70 : 30 = 7 : 3.$

Ex. 4. ***A man's present age is two-fifths of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present?*** **(M.A.T., 2009)**

Sol. Let mother's age be x years. Then, man's age = $\frac{2x}{5}$ years.

$\frac{2x}{5}+8 = \frac{1}{2}(x+8) \Rightarrow \frac{2x}{5}+8=\frac{1}{2}x+4 \Rightarrow \frac{1}{2}x-\frac{2x}{5} = 4 \Rightarrow 5x - 4x = 40 \Rightarrow x = 40.$

$\therefore$ Mother's age = 40 years.

Ex. 5. ***The ages of two persons differ by 16 years. If 6 years ago, the elder one be three times as old as the younger one, find their present ages.*** **(L.I.C., 2009)**

Sol. Let their ages be x years and $(x - 16)$ years. Then,

$(x - 6) = 3\{(x - 16) - 6\} \Rightarrow x - 6 = 3(x - 22) \Rightarrow 3x - x = 66 - 6 \Rightarrow 2x = 60 \Rightarrow x = 30.$

So, their ages are 30 years and $(30 - 16) = 14$ years.

Ex. 6. ***The product of the ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, then find Nikita's age.***

Sol. Let Ankit's age be x years. Then, Nikita's age = $\frac{240}{x}$ years. $2\times\frac{240}{x}-x = 4 \Rightarrow \frac{480}{x}-x=4 \Rightarrow 480 - x^2 = 4x$

$\Rightarrow x^2 + 4x - 480 = 0$

$\Rightarrow x^2 + 24x - 20x - 480 = 0 \Rightarrow x(x + 24) - 20(x + 24) = 0$

$\Rightarrow (x + 24)(x - 20) = 0 \Rightarrow x = 20$ [$\because x \neq -24$]

$\therefore$ Nikita's age $= \frac{240}{20}$ years $= 12$ years.

Ex. 7. ***Reenu's age after 6 years will be three–sevenths of her father's age. 10 years ago, the ratio of their ages was 1 : 5. What is Reenu's father's age at present?***

Sol. Let Reenu's age 10 years ago be x years.

Then, her father's age 10 years ago = $5x$ years.

$(x + 10) + 6 = \frac{3}{7} \times [(5x + 10) + 6] \Rightarrow x + 16 = \frac{3}{7}(5x + 16) \Rightarrow 7x + 112 = 15x + 48 \Rightarrow 8x = 64 \Rightarrow x = 8.$

$\therefore$ Reenu's father's present age = $(5x + 10)$ = $(5 \times 8 + 10)$ years = 50 years.

EXERCISE–A
(OBJECTIVE TYPE QUESTIONS)

Directions: ***Mark (√) against the correct answer.***

1. The ratio of the present ages of a mother and her daughter is 7 : 1. Four years ago, the ratio of their ages was 19 : 1. What will be the mother's age four years from now? **(S.B.I. P.O., 2010)**

(*a*) 42 years (*b*) 38 years
(*c*) 46 years (*d*) 36 years
(*e*) None of these

2. The ages of Nishi and Vinnee are in the ratio 6 : 5 respectively. After 9 years, the ratio of their ages will be 9 : 8. What is the difference in their ages now ? **(Bank P.O., 2008)**

(*a*) 3 years (*b*) 5 years
(*c*) 7 years (*d*) 9 years
(*e*) None of these

3. The present ages of Amit and his father are in the ratio 2 : 5 respectively. Four years hence, the ratio of their ages becomes 5 : 11 respectively. What was the father's age five years ago? **(Bank P.O., 2009)**

(*a*) 30 years (*b*) 35 years
(*c*) 40 years (*d*) 45 years
(*e*) None of these

4. The ratio of the ages of a father and his son is 17 : 7 respectively. Six years ago, the ratio of their ages was 3 : 1 respectively. What is the father's present age ? **(Bank P.O., 2009)**

(*a*) 64 years (*b*) 51 years
(*c*) 48 years (*d*) Cannot be determined
(*e*) None of these

5. The ages of Shakti and Kanti are in the ratio of 8 : 7 respectively. After 10 years, the ratio of their ages will be 13 : 12. What is the difference between their ages ? **(Bank P.O., 2008)**

(*a*) 2 years (*b*) 4 years
(*c*) 8 years (*d*) 6 years
(*e*) None of these

6. The ages of A and B are in the ratio 6 : 5 and the sum of their ages is 44 years. What will be the ratio of their ages after 8 years? **(P.C.S., 2008)**

(*a*) 7 : 6 (*b*) 8 : 7
(*c*) 9 : 8 (*d*) 3 : 4

7. Farah got married 8 years ago. Today her age is $1\frac{2}{7}$ times her age at the time of her marriage. At present her daughter's age is one-sixth of her age. What was her daughter's age 3 years ago? **(Bank P.O., 2009)**

(*a*) 4 years (*b*) 3 years
(*c*) 6 years (*d*) Cannot be determined
(*e*) None of these

8. The age of a mother today is thrice that of her daughter. After 12 years, the age of the mother will be twice that of her daughter. The present age of the daughter is: **(S.S.C., 2006)**

(*a*) 12 years (*b*) 14 years
(*c*) 16 years (*d*) 18 years

9. The present age of Mr. Sanyal is three times the age of his son. Six years hence, the ratio of their ages will be 5 : 2. What is the present age of Mr. Sanyal? **(L.I.C., 2007)**

(*a*) 48 years (*b*) 50 years
(*c*) 54 years (*d*) 60 years
(*e*) None of these

10. The average of the ages of a man and his daughter is 34 years. If the respective ratio of their ages four years from now is 14 : 5, what is daughter's present age? **(Bank P.O., 2008)**

(*a*) 10 years (*b*) 12 years
(*c*) 18 years (*d*) Cannot be determined
(*e*) None of these

11. Ratio of Rani's and Komal's ages is 3 : 5 respectively. Ratio of Komal's and Pooja's ages is 2 : 3 respectively.

If Rani is two-fifth of Pooja's age, what is Rani's age? **(Bank P.O., 2009)**

(*a*) 10 years (*b*) 15 years
(*c*) 14 years (*d*) Cannot be determined
(*e*) None of these

12. The age of a father 10 years ago was thrice the age of his son. 10 years hence, the father's age will be twice that of his son. The ratio of their present ages is **(P.C.S., 2009)**

(*a*) 8 : 5 (*b*) 7 : 3
(*c*) 9 : 5 (*d*) 5 : 2

13. The ratio between the ages of Ram and Mohan is 4 : 5 and that between the ages of Ram and Anil is 5 : 6. If the sum of the ages of the three is 69 years, what is Mohan's age? **(Railways, 2008)**

(*a*) 20 years (*b*) 24 years
(*c*) 25 years (*d*) 30 years

14. At present, Suresh's age is twice the age of his daughter. After 6 years from now, the ratio of the ages of Suresh and his daughter will be 23 : 13. What is the present age of Suresh? **(Bank P.O., 2008)**

(*a*) 36 years (*b*) 40 years
(*c*) 46 years (*d*) Cannot be determined
(*e*) None of these

15. The difference between the present ages of Arun and Deepak is 14 years. Seven years ago, the ratio of their ages was 5 : 7 respectively. What is Deepak's present age? **(Bank P.O., 2008)**

(*a*) 35 years (*b*) 42 years
(*c*) 49 years (*d*) 56 years
(*e*) None of these

16. Ten years ago, a man was seven times as old as his son. Two years hence, twice his age will be equal to five times the age of his son. What is the present age of the son? **(Railways, 2006)**

(*a*) 12 years (*b*) 13 years
(*c*) 14 years (*d*) 15 years

17. The ages of Samina and Suhana are in the ratio of 7 : 3 respectively. After 6 years, the ratio of their ages will be 5 : 3. What is the difference in their ages? **(Bank P.O., 2008)**

(*a*) 6 years (*b*) 8 years
(*c*) 10 years (*d*) 12 years
(*e*) None of these

18. The ages of Sulekha and Arunima are in the ratio of 9 : 8 respectively. After 5 years, the ratio of their ages will be 10 : 9. What is the difference in their ages? **(Bank P.O., 2008)**

(*a*) 4 years (*b*) 5 years
(*c*) 6 years (*d*) 7 years
(*e*) None of these

19. The ages of A and B are presently in the ratio of 5 : 6 respectively. Six years hence, this ratio will become 6 : 7 respectively. What was B's age 5 years ago? **(Bank P.O., 2009)**

(*a*) 25 years (*b*) 30 years
(*c*) 31 years (*d*) 36 years
(*e*) None of these

20. The age of the mother today is thrice that of her daughter. After 12 years, the age of the mother will be twice that of her daughter. The age of the daughter today is **(S.S.C., 2006)**

(*a*) 12 years (*b*) 14 years
(*c*) 16 years (*d*) 18 years

21. The sum of the ages of a daughter and her mother is 56 years. After four years, the age of the mother will be three times that of the daughter. At present their ages are **(S.S.C., 2006)**

(*a*) 10 years, 46 years (*b*) 12 years, 44 years
(*c*) 11 years, 45 years (*d*) 13 years, 43 years

22. The present age of son is half of the present age of his mother. Ten years ago, his mother's age was thrice the age of her son. What is the present age of the son? **(Railways, 2006)**

(*a*) 20 years (*b*) 25 years
(*c*) 30 years (*d*) 40 years

23. Ram's son's age is $\frac{1}{3}$ of Ram's wife's age. Ram's wife's age is $\frac{4}{5}$ of Ram's age and Ram's age is $\frac{3}{5}$ of Ram's father's age. Find the age of Ram's son, if Ram's father is 50 years old. **(Railways, 2005)**

(*a*) 6 years (*b*) 8 years
(*c*) 10 years (*d*) 12 years

24. Ratio between the ages of Subhash, Prasad and Amar is 3 : 6 : 7. If the difference between the ages of Prasad and Amar is 10 years, then what is the difference between the ages of Subhash and Prasad? **(Railways, 2006)**

(*a*) 5 years (*b*) 10 years
(*c*) 20 years (*d*) 30 years

25. Rajan got married 8 years ago. His present age is $\frac{6}{5}$ times his age at the time of his marriage. Rajan's sister was 10 years younger to him at the time of his marriage. The age of Rajan's sister is **(M.A.T., 2005)**

(*a*) 32 years (*b*) 36 years
(*c*) 38 years (*d*) 40 years

26. The ages of two persons differ by 20 years. If 5 years ago, the older one be 5 times as old as the younger one, then their present ages are **(M.A.T., 2004)**

(*a*) 25 years, 5 years (*b*) 30 years, 10 years
(*c*) 35 years, 15 years (*d*) 50 years, 30 years

27. A couple has a son and a daughter. The age of the father is four times that of the son and the age of the daughter is one-third of that of her mother. The wife is 6 years younger to her husband and the sister is 3 years older than her brother. The mother's age is **(P.C.S., 2008)**

(*a*) 42 years (*b*) 48 years
(*c*) 54 years (*d*) 63 years

28. The present ages of three persons are in the proportion 4 : 7 : 9. Eight years ago, the sum of their ages was 56 years. The present age of the eldest person is

(*a*) 28 years (*b*) 36 years
(*c*) 45 years (*d*) None of these

29. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is

(*a*) 19 years (*b*) 29 years
(*c*) 39 years (*d*) 49 years
(*e*) None of these

30. Reenu's father was 38 years of age when she was born while her mother was 36 years old when her brother 4 years younger to her was born. What is the difference between the ages of her parents?

(*a*) 2 years (*b*) 4 years
(*c*) 6 years (*d*) 8 years
(*e*) None of these

31. The sum of the ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child?

(*a*) 4 years (*b*) 6 years
(*c*) 8 years (*d*) 10 years
(*e*) None of these

32. A man was asked to state his age in years. His reply was, "Take my age 3 years hence, multiply it by 3 and then subtract 3 times my age 3 years ago and you will know how old I am." What is the age of the man? **(S.S.C., 2004)**

(*a*) 18 years (*b*) 20 years
(*c*) 24 years (*d*) 32 years

33. The sum of the ages of Jayant, Prem and Paras is 93 years. Ten years ago, the ratio of their ages was 2 : 3 : 4. What is the present age of Paras?

(*a*) 24 years (*b*) 28 years
(*c*) 32 years (*d*) 34 years
(*e*) 38 years

34. The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was 34. The man's age is

(*a*) 40 years (*b*) 45 years
(*c*) 50 years (*d*) 55 years
(*e*) None of these

35. The ratio of a man's age and his son's age is 7 : 3 and the product of their ages is 756. The ratio of their ages after 6 years will be

(*a*) 5 : 2 (*b*) 2 : 1
(*c*) 11 : 7 (*d*) 13 : 9
(*e*) None of these

36. Sonal is 40 years old and Nitya is 60 years old. How many years ago was the ratio of their ages 3 : 5?

(*a*) 5 years (*b*) 10 years
(*c*) 20 years (*d*) 37 years
(*e*) None of these

37. The ratio between the present ages of A and B is 5 : 3 respectively. The ratio between A's age 4 years ago and B's age 4 years hence is 1 : 1. What is the ratio between A's age 4 years hence and B's age 4 years ago?

(*a*) 1 : 3 (*b*) 3 : 1
(*c*) 2 : 1 (*d*) 4 : 1
(*e*) None of these

38. The ratio of the ages of a man and his wife is 4 : 3. After 4 years, this ratio will be 9 : 7. If at the time of their marriage, the ratio of their ages was 5 : 3, then how many years ago were they married?

(*a*) 8 years (*b*) 10 years
(*c*) 12 years (*d*) 15 years
(*e*) None of these

39. The ratio between the ages of Neelam and Shiny is 5 : 6 respectively. If the ratio between the one-third age of Neelam and half of Shiny's age is 5 : 9, then what is Shiny's age? **(Bank P.O., 2002)**

(*a*) 25 years (*b*) 30 years
(*c*) 36 years (*d*) Cannot be determined
(*e*) None of these

40. 18 years ago, a man was three times as old as his son. Now, the man is twice as old as his son. The sum of the present ages of the man and his son is **(S.S.C., 2003)**

(*a*) 54 years (*b*) 72 years
(*c*) 105 years (*d*) 108 years

41. A man is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years, how many times would he be of Ronit's age?

(*a*) 2 times (*b*) $2\frac{1}{2}$ times

(*c*) $2\frac{3}{4}$ times (*d*) 3 times

42. One year ago, Promila was four times as old as her daughter Sakshi. Six years hence, Promila's age will exceed her daughter's age by 9 years. The ratio of the present ages of Promila and her daughter is

(*a*) 9 : 2 (*b*) 11 : 3

(*c*) 12 : 5 (*d*) 13 : 4

(*e*) None of these

43. The age of a man 10 years ago was thrice the age of his son. 10 years hence, the man's age will be twice the age of his son. The ratio of their present ages is **(L.I.C., 2003)**

(*a*) 5 : 2 (*b*) 7 : 3

(*c*) 9 : 2 (*d*) 13 : 4

44. Tanya's grandfather was 8 times older to her 16 years ago. He would be 3 times of her age 8 years from now. 8 years ago, what was the ratio of Tanya's age to that of her grandfather? **(S.S.C., 2003)**

(*a*) 1 : 2 (*b*) 1 : 5

(*c*) 3 : 8 (*d*) None of these

45. The difference between the ages of two men is 10 years. 15 years ago, the elder one was twice as old as the younger one. The present age of the elder man is

(*a*) 25 years (*b*) 35 years

(*c*) 45 years (*d*) 52 years

(*e*) 55 years

46. 6 years ago, the ratio of the ages of Kunal and Sagar was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sagar's age at present?

(Bank P.O., 2009)

(*a*) 16 years (*b*) 18 years

(*c*) 20 years (*d*) Cannot be determined

(*e*) None of these

47. Sneh's age is $\frac{1}{6}$th of her father's age. Sneh's father's age will be twice of Vimal's age after 10 years. If Vimal's 8th birthday was celebrated 2 years ago, then what is Sneh's present age?

(*a*) $6\frac{2}{3}$ years (*b*) 10 years

(*c*) 12 years (*d*) 15 years

(*e*) None of these

48. The ages of Samina and Suhana are in the ratio of 7 : 3 respectively. After 6 years, the ratio of their ages will be 5 : 3. What is the difference in their ages? **(Bank P.O., 2008)**

(*a*) 6 years (*b*) 8 years

(*c*) 10 years (*d*) 12 years

(*e*) None of these

49. The ages of Sulekha and Arunima are in the ratio 9 : 8 respectively. After 5 years, the ratio of their ages will be 10 : 9. What is the difference in their ages? **(Bank P.O., 2008)**

(*a*) 4 years (*b*) 5 years (*c*) 6 years

(*d*) 7 years (*e*) None of these

50. Three years ago, the ratio of the ages of Amisha and Nimisha was 8 : 9 respectively. 3 years hence, the ratio of their ages will be 11 : 12 respectively. What is the present age of Amisha? **(Bank P.O., 2009)**

(*a*) 16 years (*b*) 19 years

(*c*) 21 years (*d*) Cannot be determined

(*e*) None of these

51. If 10 years are subtracted from the present age of Mr. Roy and the remainder divided by 14, then you would get the present age of his grandson Sachin. If Sachin is 9 years younger to Saloni whose age is 14 years, then what is the present age of Mr. Roy?

(*a*) 60 years (*b*) 70 years

(*c*) 74 years (*d*) 80 years

(*e*) None of these

52. X's age 3 years ago was three times the present age of Y. At present, Z's age is twice the age of Y. Also Z is 12 years younger than X. What is the present age of Z? **[IBPS—RRB Officer's Gr. 'B' Exam, 2015]**

(*a*) 15 year (*b*) 24 year

(*c*) 12 year (*d*) 18 year

(*e*) 6 year

53. Eight year ago, Poorvi's age was equal to the sum of the present ages of her one son and one daughter. Five years hence, the respective ratio between the ages of her daughter and her son that time will be 7 : 6. If Poorvi's husband is 7 years elder to her and his present age is three times the present age of their son, what is the present age of the daughter? (in years) **[RBI Gr. 'B' (Phase I) Exam, 2015]**

(*a*) 15 years (*b*) 23 years

(*c*) 19 years (*d*) 27 years

(*e*) 13 years

54. The sum of present ages of a father and his son is 8 years more than the present age of the mother. The mother is 22 years older than the son. What will be the age of the father after 4 years? **[United India Insurance Co. Ltd., (UIICL) Assistant (Online) Exam, 2015]**

(*a*) 34 years (*b*) 36 years
(*c*) 40 years (*d*) 38 years
(*e*) 28 years

55. Rahul is as much younger than Sagar as he is older than Purav. If the sum of the ages of Purav and Sagar is 66 years, and Sagar's age is 48 years, then what is Purav's age? (in years)
[NICL—AAO Exam, 2015]
(*a*) 18 (*b*) cannot be determined
(*c*) 16 (*d*) 20
(*e*) 12

56. 4 years ago, the ratio of 1/2 of A's age at that time and four times of B's age at the time was 5 : 12. Eight years hence, 1/2 of A's age at that time will be less than B's age at that time by 2 years. What is B's present age?
[IBPS—RRB Officers Exam, 2015]
(*a*) 10 years (*b*) 14 years
(*c*) 12 years (*d*) 5 years
(*e*) 8 years

57. Ten years hence, the respective ratio between Simmi's age and Niti's age will be 7 : 9. Two years ago, the respective ratio between Simmi's age and Niti's age was 1 : 3. If Abhay is 4 years older to his sister Niti, what is Abhay's present age? (in years)
[CET—Maharashtra (MBA) Exam, 2016]
(*a*) 8 (*b*) 4
(*c*) 16 (*d*) 12
(*e*) 20

ANSWERS

1. (*c*)	**2.** (*a*)	**3.** (*b*)	**4.** (*b*)	**5.** (*a*)	**6.** (*b*)	**7.** (*b*)	**8.** (*a*)	**9.** (*c*)	**10.** (*e*)
11. (*d*)	**12.** (*b*)	**13.** (*c*)	**14.** (*b*)	**15.** (*d*)	**16.** (*c*)	**17.** (*b*)	**18.** (*b*)	**19.** (*c*)	**20.** (*a*)
21. (*b*)	**22.** (*a*)	**23.** (*b*)	**24.** (*d*)	**25.** (*c*)	**26.** (*b*)	**27.** (*c*)	**28.** (*b*)	**29.** (*c*)	**30.** (*c*)
31. (*a*)	**32.** (*a*)	**33.** (*e*)	**34.** (*e*)	**35.** (*b*)	**36.** (*b*)	**37.** (*b*)	**38.** (*c*)	**39.** (*d*)	**40.** (*d*)
41. (*a*)	**42.** (*d*)	**43.** (*b*)	**44.** (*d*)	**45.** (*b*)	**46.** (*a*)	**47.** (*a*)	**48.** (*b*)	**49.** (*b*)	**50.** (*b*)
51. (*d*)	**52.** (*d*)	**53.** (*b*)	**54.** (*a*)	**55.** (*a*)	**56.** (*a*)	**57.** (*d*)			

SOLUTIONS

1. Let mother's age be $7x$ years. Then, daughter's age = x years. $\frac{7x-4}{x-4}=\frac{19}{1}$

$\Rightarrow 7x - 4 = 19\ (x - 4)$
$\Rightarrow 19x - 7x = 76 - 4$
$\Rightarrow 12x = 72$
$\Rightarrow x = 6.$
Mother's age after 4 years = $(7x + 4)$
$= (7 \times 6 + 4)$ years = 46 years.

2. Let Nishi's age be $6x$ years. Then, Vinnee's age = $5x$ years.
$\therefore \frac{6x+9}{5x+9}=\frac{9}{8} \Rightarrow 8\ (6x + 9) = 9\ (5x + 9)$
$\Rightarrow 48x - 45x = 81 - 72$
$\Rightarrow 3x = 9$
$\Rightarrow x = 3.$
Difference in their ages = $(6x - 5x) = x$ years = 3 years.

3. Let Amit's age be $2x$ years. Then, his father's age = $5x$ years.
$\therefore \frac{2x+4}{5x+4}=\frac{5}{11} \Rightarrow 11\ (2x + 4) = 5\ (5x + 4)$
$\Rightarrow 22x + 44 = 25x + 20$
$\Rightarrow 3x = 24$
$\Rightarrow x = 8.$
Father's age 5 years ago = $(5x - 5)$ years
$= (5 \times 8 - 5)$ years = 35 years.

4. Let father's age be $17x$ years. Then, son's age = $7x$ years.
$\frac{17x-6}{7x-6}=\frac{3}{1} \Rightarrow 3\ (7x - 6) = 17x - 6$
$\Rightarrow 21x - 18 = 17x - 6$
$\Rightarrow 4x = 12$
$\Rightarrow x = 3.$
$\therefore$ Father's present age = $17x$ years
$= (17 \times 3)$ years = 51 years.

5. Let Shakti's age be $8x$ years. Then, Kanti's age = $7x$ years.
$\therefore \frac{8x+10}{7x+10}=\frac{13}{12} \Rightarrow 12(8x + 10) = 13(7x + 10)$
$\Rightarrow 96x + 120 = 91x + 130$
$\Rightarrow 5x = 10$
$\Rightarrow x = 2.$
Difference between their ages = $(8x - 7x)$ years
$= x$ years = 2 years.

6. A's age = $\left(44 \times \frac{6}{11}\right)$ years = 24 years and B's age
$= (44 - 24)$ years = 20 years.
Ratio of their ages after 8 years = $\frac{(24+8)}{(20+8)}=\frac{32}{28}=\frac{8}{7}=8:7.$

7. Let Farah's age 8 years ago be x years. Then, her present age = $(x + 8)$ years.

$\therefore\ x+8=\frac{9}{7}x \Rightarrow 7x + 56 = 9x$

$\Rightarrow 2x = 56$

$\Rightarrow x = 28.$

$\therefore$ Farah's age now = $(x + 8)$ years = $(28 + 8)$ years = 36 years.

Her daughter's age now = $\left(\frac{1}{6}\times 36\right)$ years = 6 years.

Her daughter's age 3 years ago = $(6 - 3)$ years = 3 years.

8. Let the daughter's age be x years. Then, mother's age = $3x$ years.

$3x + 12 = 2(x + 12)$

$\Rightarrow 3x + 12 = 2x + 24$

$\Rightarrow x = 12.$

Present age of daughter = 12 years.

9. Let the son's age be x years. Then, Mr. Sanyal's age = $3x$ years.

$\therefore\ \frac{3x+6}{x+6}=\frac{5}{2} \Rightarrow 2(3x + 6) = 5(x + 6)$

$\Rightarrow 6x + 12 = 5x + 30$

$\Rightarrow x = 18.$

$\therefore$ Present age of Mr. Sanyal = $3x$ years = (3×18) years = 54 years.

10. Average age of man and his daughter = 34 years.

Their total age = (34×2) years = 68 years.

Let man's age be x years. Then, daughter's age = $(68 - x)$ years.

$\therefore\ \frac{x+4}{68-x+4}=\frac{14}{5} \Rightarrow 5(x + 4) = 14(72 - x)$

$\Rightarrow 5x + 20 = 1008 - 14x$

$\Rightarrow 19x = 988$

$\Rightarrow x = 52.$

$\therefore$ Daughter's present age = $(68 - 52)$ years = 16 years.

11. Rani's age : Komal's age = 3 : 5 = $\frac{3}{5}:1.$

Komal's age : Pooja's age = 2 : 3 = $1:\frac{3}{2}.$

Rani's age : Komal's age : Pooja's age = $\frac{3}{5}:1:\frac{3}{2}=6:10:15.$

Let Rani's age be $6x$ years. Then, Komal's age = $10x$ years and Pooja's age = $15x$ years.

Rani's age = $\frac{2}{5}$ of Pooja's age $\Rightarrow 6x=\frac{2}{5}\times 15x.$

Thus, we can not find the value of x and therefore of $6x$.

So, the answer cannot be determined.

12. Let son's age 10 years ago be x years.

Then, father's age 10 years ago = $3x$ years.

Son's age now = $(x + 10)$ years, Father's age now = $(3x + 10)$ years.

$(3x + 10) + 10 = 2[(x + 10) + 10]$

$\Rightarrow 3x + 20 = 2(x + 20)$

$\Rightarrow 3x + 20 = 2x + 40$

$\Rightarrow x = 20.$

Ratio of present ages of father and son

$= \frac{3x+10}{x+10}=\frac{3\times 20+10}{20+10}=\frac{70}{30}=\frac{7}{3}=7:3.$

13. Ram's age : Mohan's age = 4 : 5 = $1:\frac{5}{4}.$

Ram's age : Anil's age = 5 : 6 = $1:\frac{6}{5}.$

Let Ram's age be x years. Then, Mohan's age = $\frac{5x}{4}$ years.

And, Anil's age = $\frac{6x}{5}$ years.

$\therefore\ x+\frac{5x}{4}+\frac{6x}{5}=69$

$\Rightarrow 20x + 25x + 24x = 1380$

$\Rightarrow 69x = 1380$

$\Rightarrow x = 20.$

Mohan's age = $\frac{5x}{4}\text{ years}=\frac{5\times 20}{4}$ years = 25 years.

14. Let daughter's age be x years. Suresh's age = $2x$ years.

$\therefore\ \frac{2x+6}{x+6}=\frac{23}{13} \Rightarrow 13(2x + 6) = 23(x + 6)$

$\Rightarrow 26x + 78 = 23x + 138$

$\Rightarrow 3x = 60$

$\Rightarrow x = 20.$

Present age of Suresh = $2x$ years

$= (2 \times 20)$ years = 40 years.

15. Let the ages of Arun and Deepak 7 years ago be $5x$ years and $7x$ years respectively. Then,

Arun's present age = $(5x + 7)$ years, Deepak's present age = $(7x + 7)$ years.

$\therefore (7x + 7) - (5x + 7) = 14$

$\Rightarrow 2x = 14$

$\Rightarrow x = 7.$

Deepak's present age = $(7 \times 7 + 7)$ years = 56 years.

16. Let son's age 10 years ago be x years. Then, man's age 10 years ago = $7x$ years.

Son's present age = $(x + 10)$ years, Man's present age = $(7x + 10)$ years.

$\therefore 2[(7x + 10) + 2] = 5[(x + 10) + 2]$

$\Rightarrow 2(7x + 12) = 5(x + 12)$

$\Rightarrow 14x + 24 = 5x + 60$

$\Rightarrow 9x = 36$

$\Rightarrow x = 4.$

$\therefore$ Son's present age = $(x + 10)$ years = $(4 + 10)$ years = 14 years.

17. Let Samina's age be $7x$ years. Then, Suhana's age = $3x$ years.

$\therefore\ \frac{7x+6}{3x+6}=\frac{5}{3} \Rightarrow 3(7x + 6) = 5(3x + 6)$

$\Rightarrow 21x + 18 = 15x + 30$

$\Rightarrow 6x = 12$

$\Rightarrow x = 2.$

Difference in their ages = $(7x - 3x)$ years

$= 4x$ years= (4×2) years = 8 years.

18. Let Sulekha's age be $9x$ years. Then, Arunima's age = $8x$ years.

$\therefore \frac{9x+5}{8x+5} = \frac{10}{9} \Rightarrow 9(9x + 5) = 10(8x + 5)$

$\Rightarrow 81x + 45 = 80x + 50$

$\Rightarrow x = 5.$

Difference in their ages = $(9x - 8x)$ years

$= x$ years = 5 years.

19. Let A's age be $5x$ years. Then, B's age = $6x$ years.

$\therefore \frac{5x+6}{6x+6} = \frac{6}{7} \Rightarrow 7(5x + 6) = 6(6x + 6)$

$\Rightarrow 35x + 42 = 36x + 36$

$\Rightarrow x = 6.$

B's age 5 years ago = $(6x - 5)$ years

$= (6 \times 6 - 5)$ years = 31 years.

20. Let daughter's age be x years. Then, mother's age = $3x$ years.

$(3x + 12) = 2(x + 12)$

$\Rightarrow 3x + 12 = 2x + 24$

$\Rightarrow x = 12.$

$\therefore$ Daughter's age today = 12 years.

21. Let daughter's age be x years. Then, mother's age = $(56 - x)$ years.

$(56 - x) + 4 = 3(x + 4)$

$\Rightarrow 60 - x = 3x + 12$

$\Rightarrow 4x = 48$

$\Rightarrow x = 12.$

$\therefore$ Daughter's age = 12 years, Mother's age = 44 years.

22. Let mother's age be $2x$ years. Then, son's age = x years.

$(2x - 10) = 3(x - 10)$

$\Rightarrow 2x - 10 = 3x - 30$

$\Rightarrow x = 20.$

Son's age = 20 years.

23. Ram's father's age = 50 years, Ram's age

$= \left(\frac{3}{5} \times 50\right)$ years = 30 years.

Ram's wife's age = $\left(\frac{4}{5} \times 30\right)$ years = 24 years.

Ram's son's age = $\left(\frac{1}{3} \times 24\right)$ years = 8 years.

24. Let Subhash's age be $3x$ years. Then, Prasad's age = $6x$ years and Amar's age = $7x$ years.

$\therefore \quad 7x - 6x = 10$

$\Rightarrow x = 10.$

Required difference = $(6x - 3x)$ years

$= 3x$ years = (3×10) years = 30 years.

25. Let Rajan's age 8 years ago be x years. His present age = $(x + 8)$ years.

$\therefore \quad x + 8 = \frac{6}{5}x \Rightarrow 5x + 40 = 6x \Rightarrow x = 40.$

Rajan's sister's age 8 years ago = $(40 - 10)$ years = 30 years.

His sister's age now = $(30 + 8)$ years = 38 years.

26. Let their present ages be x years and $(x - 20)$ years.

$(x - 5) = 5\,[(x - 20) - 5]$

$\Rightarrow (x - 5) = 5(x - 25)$

$\Rightarrow (x - 5) = 5x - 125$

$\Rightarrow 4x = 120$

$\Rightarrow x = 30.$

$\therefore$ Their present ages are 30 years and 10 years.

27. M–Mother, F–Father, S–Son and D–Daughter.

$F = 4S,\ D = \frac{1}{3}M,\ M = F - 6$ and $S = D - 3$

$\therefore \quad M = 3\ D = 3(S + 3)$

$= 3S + 9 = \frac{3}{4}F + 9 = \frac{3}{4}(M + 6) + 9$

$= \frac{3}{4}M + \frac{3}{4} \times 6 + 9$

$\Rightarrow \quad \left(M - \frac{3}{4}M\right) = \left(\frac{9}{2} + 9\right) \Rightarrow \frac{1}{4}M = \frac{27}{2}$

$\Rightarrow M = \left(\frac{27}{2} \times 4\right) = 54$ years.

$\therefore \quad$ The mother is 54 years old.

28. A : B : C = 4 : 7 : 9 and (A + B + C)

$= 56 + (8 + 8 + 8) = 80.$

$\therefore \quad$ C's age = $\left(80 \times \frac{9}{20}\right)$ years = 36 years.

29. Let B's age be x years. Then, A's age = $(x + 9)$ years.

$(x + 9) + 10 = 2(x - 10)$

$\Rightarrow x + 19 = 2x - 20$

$\Rightarrow x = 39.$

B's age = 39 years.

30. Mother's age when Reenu's brother was born = 36 years.

Father's age when Reenu's brother was born = $(38 + 4)$ years = 42 years.

Required difference = $(42 - 36)$ years = 6 years.

31. Let the ages of children be x, $(x + 3)$, $(x + 6)$, $(x + 9)$ and $(x + 12)$ years.

Then, $x + x + 3 + x + 6 + x + 9 + x + 12 = 50$

$\Rightarrow 5x = 20 \Rightarrow x = 4.$

$\therefore \quad$ Age of youngest child = 4 years.

32. Let the present age of the man be x years. Then

$3(x + 3) - 3(x - 3) = x$

$\Rightarrow (3x + 9) - (3x - 9) = x$

$\Rightarrow x = 18.$

$\therefore \quad$ The present age of the man is 18 years.

33. Let their ages 10 years ago be $2x$ years, $3x$ years and $4x$ years respectively.

Then, $(2x + 10) + (3x + 10) + (4x + 10) = 93$

$\Rightarrow 9x + 30 = 93$

$\Rightarrow 9x = 63 \Rightarrow x = 7.$

Present age of Paras = (4 × 7 + 10) years = 38 years.

34. Let the man's age be x years.

Then, son's age = $(45 - x)$ years.

$(x - 5)(45 - x - 5) = 34$

$\Rightarrow (x - 5)(40 - x) = 34$

$\therefore \quad 40x - x^2 - 200 + 5x = 34$

$\Rightarrow x^2 - 45x + 234 = 0$

$\Rightarrow \quad x^2 - 39x - 6x + 234 = 0$

$\Rightarrow x(x - 39) - 6(x - 39) = 0$

$\Rightarrow \quad (x - 39)(x - 6) = 0$

$\Rightarrow x = 39$ or $x = 6.$

$\therefore$ Man's age = 39 years.

35. Let the man's age be $7x$ years. Then, son's age = $3x$ years.

$\therefore \quad 7x \times 3x = 756$

$\Rightarrow 21x^2 = 756$

$\Rightarrow x^2 = 36 = 6^2$

$\Rightarrow x = 6.$

The ratio of their ages after 6 years = $(7x + 6) : (3x + 6)$

$= (7 \times 6 + 6) : (3 \times 6 + 6)$

$= 48 : 24 = 2 : 1.$

36. Let x years ago the ratio of their ages be 3 : 5.

Then, $\dfrac{40 - x}{60 - x} = \dfrac{3}{5} \Rightarrow 3(60 - x) = 5(40 - x)$

$\Rightarrow 180 - 3x = 200 - 5x$

$\Rightarrow 2x = 20$

$\Rightarrow x = 10.$

$\therefore$ 10 years ago, their ages were in the ratio 3 : 5.

37. Let A' s age be $5x$ years. Then, B's age = $3x$ years.

$$\frac{5x-4}{3x+4} = \frac{1}{1} \Rightarrow 5x - 4 = 3x + 4 \Rightarrow 2x = 8 \Rightarrow x = 4.$$

$$\therefore \quad \frac{\text{A's age 4 years hence}}{\text{B's age 4 years ago}} = \frac{5x+4}{3x-4}$$

$$= \frac{5\times 4+4}{3\times 4-4} = \frac{24}{8} = \frac{3}{1} = 3:1.$$

38. Let the man's age be $4x$ years. Then, his wife's age = $3x$ years.

Then, $\dfrac{4x + 4}{3x + 4} = \dfrac{9}{7} \Rightarrow 7(4x + 4) = 9(3x + 4)$

$\Rightarrow 28x + 28 = 27x + 36 \Rightarrow x = 8.$

Man's age = (4 × 8) years

= 32 years, Wife's age

= (3 × 8) years = 24 years.

Let they be married y years ago. Then,

$\dfrac{32 - y}{24 - y} = \dfrac{5}{3} \Rightarrow 3(32 - y) = 5(24 - y$

$\Rightarrow 96 - 3y = 120 - 5y$

$\Rightarrow 2y = (120 - 96) = 24$

$\Rightarrow y = 12.$

So, they were married 12 years ago.

39. Let Neelam's age be $5x$ years and Shiny's age be $6x$ years.

$$\left(\frac{1}{3}\times 5x\right):\left(\frac{1}{2}\times 6x\right) = 5:9 \Rightarrow \frac{5x}{3\times 3x} = \frac{5}{9}.$$

Thus, Shiny's age cannot be determined.

40. Let the son's age 18 years ago be x years. Then, man's age 18 years ago = $3x$ years.

$(3x + 18) = 2(x + 18)$

$\Rightarrow 3x + 18 = 2x + 36$

$\Rightarrow x = 18.$

Sum of their present ages = $(3x + 18 + x + 18)$ years

$= (4x + 36)$ years

$= (4 \times 18 + 36)$ years = 108 years.

41. Let Ronit's present age be x years.

Then, the man's age = $(x + 3x)$ years = $4x$ years.

$$4x + 8 = \frac{5}{2}(x + 8) \Rightarrow 8x + 16 = 5x + 40 \Rightarrow 3x = 24 \Rightarrow x = 8.$$

$$\therefore \quad \text{Required ratio} = \frac{(4x+16)}{(x+16)} = \frac{(4\times 8+16)}{(8+16)} = \frac{48}{24} = 2 \text{ times.}$$

42. Let Sakshi's age 1 year ago be x years. Then, Promila's age 1 year ago = $4x$ years.

$\therefore$ Sakshi's age now = $(x + 1)$ years, Promila's age now = $(4x + 1)$ years.

$(4x + 1) + 6 = (x + 1 + 6) + 9$

$\Rightarrow 4x + 7 = x + 16$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3.$

Ratio of Promila's age and Sakshi's age now

$$= \frac{(4x+1)}{(x+1)} = \frac{13}{4} = 13:4.$$

43. Let son's age 10 years ago be x years. Then, man's age 10 years ago = $3x$ years.

Son's present age = $(x + 10)$ years, Man's present age = $(3x + 10)$ years.

$(3x + 10) + 10 = 2(x + 10 + 10)$

$\Rightarrow 3x + 20 = 2(x + 20)$

$\Rightarrow 3x + 20 = 2x + 40$

$\Rightarrow x = 20.$

Ratio of present ages of man and the son

$$= \frac{3x+10}{x+10} = \frac{3\times 20+10}{20+10} = \frac{70}{30} = 7:3.$$

44. 16 years ago, let $T = x$ years and $G = 8x$ years.

After 8 years from now, $T = (x + 16 + 8)$ years and G

$= (8x + 16 + 8)$ years.

$\therefore \quad 8x + 24 = 3(x + 24)$

$\Rightarrow 8x - 3x = 72 - 24$

$\Rightarrow 5x = 48.$

$$8 \text{ years ago, } \frac{T}{G} = \frac{x+8}{8x+8} = \frac{\frac{48}{5}+8}{8\times\frac{48}{5}+8} = \frac{48+40}{384+40} = \frac{88}{424} = \frac{11}{53}.$$

45. Let their ages be x years and $(x + 10)$ years.

Then $(x + 10 - 15) = 2(x - 15)$

$\Rightarrow x - 5 = 2x - 30$

$\Rightarrow x = 25.$

Present age of the elder man = $(x + 10)$ years = $(25 + 10)$ years = 35 years.

46. Let the ages of Kunal and Sagar 6 years ago be $6x$ and $5x$ years.

Then, $\frac{(6x+6)+4}{(5x+6)+4} = \frac{11}{10} \Rightarrow \frac{6x+10}{5x+10} = \frac{11}{10}$

$\Rightarrow 10(6x + 10) = 11(5x + 10)$

$\Rightarrow 60x + 100 = 55x + 110$

$\Rightarrow 5x = 10$

$\Rightarrow x = 2.$

Sagar's present age = $(5x + 6)$ years = $(5 \times 2 + 6)$ years = 16 years.

47. Vimal's present age = $(8 + 2)$ years = 10 years.

Sneh's father's age = $2(10 + 10)$ years = 40 years.

Sneh's age = $\left(\frac{1}{6} \times 40\right)$ years $= \frac{20}{3}$ years $= 6\frac{2}{3}$ years.

48. Let Samina's age be $7x$ years. Then, Suhana's age = $3x$ years.

$\therefore \frac{7x+6}{3x+6} = \frac{5}{3} \Rightarrow 3(7x + 6) = 5(3x + 6)$

$\Rightarrow 21x + 18 = 15x + 30$

$\Rightarrow 6x = 12$

$\Rightarrow x = 2.$

Difference in their ages = $(7x - 3x)$ years = $4x$ years = (4×2) years = 8 years.

49. Let Sulekha's age be $9x$ years. Then, Arunima's age = $8x$ years.

$\frac{9x+5}{8x+5} = \frac{10}{9} \Rightarrow 9(9x + 5) = 10(8x + 5)$

$\Rightarrow 81x + 45 = 80x + 50$

$\Rightarrow x = 5.$

Difference in their ages = $(9x - 8x)$ years = x years = 5 years.

50. Let Amisha's age 3 years ago be $8x$ years. Then, Nimisha's age 3 years ago = $9x$ years.

Present age of Amisha = $(8x + 3)$ years.

Present age of Nimisha = $(9x + 3)$ years.

$\frac{(8x+3)+3}{(9x+3)+3} = \frac{11}{12} \Rightarrow \frac{8x+6}{9x+6} = \frac{11}{12}$

$\Rightarrow 12(8x + 6) = 11(9x + 6)$

$\Rightarrow 96x + 72 = 99x + 66$

$\Rightarrow 3x = 6$

$\Rightarrow x = 2.$

Amisha's present age = $(8 \times 2 + 3)$ years = 19 years.

51. Saloni's age = 14 years $\Rightarrow$ Sachin's age = $(14 - 9)$ years = 5 years.

Let the present age of Mr. Roy be x years.

$\frac{x-10}{14} = 5 \Rightarrow x - 10 = 70 \Rightarrow x = 80$ years.

52. Let the present age of Y be a years.

Three years ago X's age = $3a$ years

Then, present age of X is $(3a + 3)$

Z's present age = $2a$

According to the given information

Now, $(3a + 3) - 2a = 12 \Rightarrow a = 9$ year

$\therefore$ Present age of $Z = 2a = 2 \times 9 = 18$ years

53. Let the age of the son and the daughter of Poorvi be $6a$ years and $7a$ years respectively. 5 years hence, present age of son = $6a - 5$ and present age of daughter = $7a - 5$

According to the question,

Eight years ago, the age of Poorvi $= 6a - 5 + 7a - 5$

$= 13a - 10$

So, present age of Poorvi = $13a - 10 + 8 = 13a - 2$.

Since, present age of Poorvi husband = $3(6a - 5)$

The difference of present age of Poorvi husband and Poorvi = 7, (given)

$3(6a-5)-(13a-2) = 7, \Rightarrow 18a - 15 - 13a + 2 = 7$

$\Rightarrow 5a = 20 \Rightarrow a = 4$

The present age of daughter = $(7a - 5) = 7 \times 4 - 5 = 23$ years

54. Let present age of father, mother and son be x, y and z respectively

Sum of present ages of father and son = (Mother's present age + 8 years)

$\Rightarrow x + z = y + 8$ years ...(i)

Mother's present age = (Son's present age + 22)

$\Rightarrow y = z + 22$ ...(ii)

Put the value of y in equation (i) we get

$x + z = z + 22 + 8$

$\Rightarrow x + z = z + 30$

$\Rightarrow x = 30$ years

$\therefore$ Father's present age = 30 years

Age of father after four years = 30 + 4 = 34 years

$\therefore$ Required age of father

= 34 years

55. Let the age of Rahul, Sagar and Purav be x, y and z respectively

According to the given information

Age of Sagar − Age of Rahul = Age of Rahul − Age of Purav

$\Rightarrow y - x = x - z$

$\Rightarrow 2x = y + z$ ...(i)

Also $y + z = 66$ years

From (i) $x = 33$ years

Also as per Eq (i) we have Purav's age + Sagar's age = 66 years.

By going through option (a) given Purav = 18, and Rahul = 33 years, Sagar = 48 years

Difference between Rahul's and Purav's age = 18 years

56. Let the present age of A be a years and that of B be b years.

Then, 4 years ago,

A's age = $(a - 4)$ years

B's age = $(b - 4)$ years

Now, according to the given information in question,

$\frac{\frac{a-4}{2}}{4(b-4)}=\frac{5}{12}$ or $\frac{a-4}{2(4b-16)}=\frac{5}{12}$ or $\frac{a-4}{4b-16}=\frac{5}{6}$

By cross multiplying we get

or, $6a - 24 = 20b - 80$

or, $6a - 20b = -56$

or $10b - 3a = 28$

After 8 years,

$\frac{a+8}{2}+2=b+8$

or, $\frac{a}{2}+4+2=b+8$

or, $b-\frac{a}{2}=-2$

or, $2b-a=-4$ …(i)

or, $a=2b+4$ ….(ii)

Putting the value of a in equation (i), we get

$10b-3(2b+4)=28$

Or, $10b - 6b - 12 = 28$

Or, $4b = 40$

$\therefore b = 10$

Hence, the present age of B is 10 years.

57. Let present ages of Simmi and Niti be a and b years, respectively.

Ten years hence, the ratio between Simmi's age and Niti's age = 7 : 9

According to the question, $\frac{a+10}{b+10}=\frac{7}{9}$

By cross multiplying we get

$\Rightarrow 9a+90=7b+70$

$\Rightarrow 7b-9a=20$ …..(i)

Also, $\frac{a-2}{b-2}=\frac{1}{3}$

By cross multiplying we get

$3a-6=b-2$

$\Rightarrow 3a-b=4$ …(ii)

Multiplying equation (ii) by 3

$9a - 3b = 12$ ...(iii)

Adding equation (ii) and (iii) we get

$-9a+7b=20$

$4b = 32 \Rightarrow b = 8$year

From equation (ii) we get

$a=\frac{4+b}{3}=\frac{4+8}{3}=\frac{12}{3}=4$ years

Since, Abhay is 4 years older to Niti.

So, Abhay present age = 8 + 4 = 12 years

EXERCISE–B

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions *(Questions 1 to 8): Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Read both the statements and*

Give answer (a) if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;

Give answer (b) if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;

Give answer (c) if the data either in Statement I or in Statement II alone are sufficient to answer the question;

Give answer (d) if the data even in both Statements I and II together are not sufficient to answer the question;

Give answer (e) if the data in both Statements I and II together are necessary to answer the question.

1. The sum of the ages of P, Q and R is 96 years. What is the age of Q ?

I. P is 6 years older than R.

II. The total of the ages of Q and R is 56 years.

2. What is Sonia's present age ?

I. Sonia's present age is five times Deepak's present age.

II. Five years ago her age was twenty-five times Deepak's age at that time.

3. How old is C now ?

I. Three years ago, the average of A and B was 18 years.

II. With C joining them now, the average becomes 22 years.

4. What is Reena's present age ?

I. Reena's present age is five times her son's present age.

II. Reena's age two years hence will be three times her daughter's age at that time.

5. What is the average age of A and B ?

(Bank P.O., 2007)

I. The ratio between one-fifth of A's age and one-fourth of B's age is 1 : 2.

II. The product of their ages is 20 times B's age.

6. Average age of employees working in a department is 30 years. In the next year, ten workers will retire. What will be the average age in the next year ?

I. Retirement age is 60 years.

II. There are 50 employees in the department.

7. What is the ratio between the ages of the father and the son ?

I. The sum of their ages is 50 years.

II. 3 times the sum of their ages is equal to 5 times the father's age.

8. Divya is twice as old as Shruti. What is the difference in their ages ?

I. Five years hence, the ratio of their ages would be 9 : 5.

II. Ten years back, the ratio of their ages was 3 : 1.

Directions (*Questions 9 to 13*): *Each of the questions given below consists of a question followed by three statements. You have to study the question and the statements and decide which of the statements is/are necessary to answer the question.*

9. What is the present age of A ?

I. The sum of the ages of A and B is 21 years.

II. The difference of the ages of A and B is 5 years.

III. The product of the ages of A and B is 104 years.

(*a*) I and II only (*b*) II and III only

(*c*) I and III only

(*d*) Any two of the three (*e*) None of these

10. What is the present age of Tanya ? **(Bank P.O., 2008)**

I. The ratio between the present ages of Tanya and her brother Rahul is 3 : 4 respectively.

II. After 5 years the ratio between the ages of Tanya and Rahul will be 4 : 5.

III. Rahul is 5 years older than Tanya.

(*a*) I and II only (*b*) II and III only

(*c*) I and III only (*d*) All I, II and III

(*e*) Any two of the three

11. What is the difference between the ages of Y and X?

I. The ratio between the ages of X and Y is 2 : 3.

II. Y's age is 50% more than X's age.

III. One-fourth of X's age is equal to one-sixth of Y's age.

(*a*) All I, II and III (*b*)Any two of the three

(*c*) III, and either I or II (*d*) Only I and II

(*e*) Question cannot be answered even with information in all three statements

12. What is Arun's present age ?

I. Five years ago, Arun's age was double that of his son's age at that time.

II. Present ages of Arun and his son are in the ratio of 11 : 6 respectively.

III. Five years hence, the respective ratio of Arun's age and his son's age will become 12 : 7.

(*a*) Only I and II (*b*) Only II and III

(*c*) Only I and III (*d*)Any two of the three

(*e*) None of these

13. What is Ravi's present age ?

I. The present age of Ravi is half of that of his father.

II. After 5 years, the ratio of Ravi's age to that of his father's age will be 6 : 11.

III. Ravi is 5 years younger than his brother.

(*a*) I and II only (*b*) II and III only

(*c*) I and III only (*d*) All I, II and III

(*e*) Even with all the three statements answer cannot be given.

Directions (*Questions 14 to 16*): *Each of these questions is followed by three statements. You have to study the question and all the three statements given to decide whether any information provided in the statement(s) is redundant and can be dispensed with while answering the given question.*

14. What is the ratio of the present ages of Anna and her mother ?

I. The sum of the ages of Anna, her mother and her father is 62.

II. Five years ago, Anna's age was one-fifth of her father's age.

III. Two years ago, the sum of the ages of Anna and her father was 36.

(*a*) I or II only (*b*) II or III only

(*c*) III only (*d*) I or III only

(*e*) All I, II and III are required.

15. What will be the ratio between the ages of Sam and Albert after 5 years ?

I. Sam's present age is more than Albert's present age by 4 years.

II. Albert's present age is 20 years.

III. The ratio of Albert's present age to Sam's present age is 5 : 6.

(*a*) I or II or III only (*b*) II only

(*c*) III only (*d*) I or III only

(*e*) II or III only.

16. What is the difference between the present ages of Ayush and Deepak ?

I. The ratio between Ayush's present age and his age after 8 years is 4 : 5.

II. The ratio between the present ages of Ayush and Deepak is 4 : 3.

III. The ratio between Deepak's present age and his age four years ago is 6 : 5.

(*a*) Any two of I, II and III

(*b*) I or III only

(*c*) Any one of the three

(*d*) All I, II and III are required

(*e*) Even with all I, II and III, the answer cannot be obtained.

ANSWERS

1. (*e*)	**2.** (*e*)	**3.** (*e*)	**4.** (*d*)	**5.** (*e*)	**6.** (*e*)	**7.** (*b*)	**8.** (*c*)	**9.** (*d*)	**10.** (*e*)
11. (*e*)	**12.** (*d*)	**13.** (*a*)	**14.** (*e*)	**15.** (*a*)	**16.** (*c*)				

SOLUTIONS

1. Given : P + Q + R = 96 ...(*i*)

I. P = R + 6 ...(*ii*)

II. Q + R = 56 ...(*iii*)

On subtracting (*iii*) from (*i*), we get P = 40.

Putting P = 40 in (*ii*), we get R = 34. Putting R = 34 in (*iii*), we get Q = 22.

Thus, I and II both together give the answer. So, correct answer is (*e*).

2. **I.** $S = 5D \Rightarrow D = \frac{S}{5}$ ...(*i*)

II. $S - 5 = 25\,(D - 5) \Leftrightarrow S = 25D - 120$ ...(*ii*)

Using (*i*) in (*ii*), we get

$$S = \left(25 \times \frac{S}{5}\right) - 120 \Leftrightarrow 4S = 120 \Leftrightarrow S = 30.$$

Thus, I and II both together give the answer. So, correct answer is (*e*).

3. **I.** 3 years ago, $\frac{1}{2}(A + B) = 18$

$\Rightarrow$ 3 years ago, (A + B) = 36

Now, (A + B) = (36 + 3 + 3) = 42

$\Rightarrow$ A + B = 42 ...(*i*)

II. Now, $\frac{1}{3}(A + B + C) = 22$

$\Rightarrow$ A + B + C = 66 ...(*ii*)

From (*i*) and (*ii*), we get C = (66 – 42) = 24.

Thus, I and II both together give the answer. So, correct answer is (*e*).

4. **I.** Reena's present age = 5 × (Her son's present age).

II. Reena's age 2 years hence = 3 times her daughter's age at that time.

Clearly, data even in I and II is not sufficient to get Reena's present age.

$\therefore$ Correct answer is (*d*).

5. **I.** $\frac{A}{5} : \frac{B}{4} = 1 : 2 \Leftrightarrow \frac{A}{5} \times \frac{4}{B} = \frac{1}{2}$

$$\Leftrightarrow \frac{A}{B} = \left(\frac{1}{2} \times \frac{5}{4}\right) = \frac{5}{8} \Leftrightarrow A : B = 5 : 8.$$

II. 20B = AB.

Let A's age be 5*x* years. Then, B's age is 8*x* years.

$\therefore$ $20 \times 8x = 5x \times 8x \Leftrightarrow 40x = 160 \Leftrightarrow x = 4$.

$\therefore$ A = 20 and B = 32.

Thus, I and II together give the answer. So, correct answer is (*e*).

6. **I.** Retirement age is 60 years.

II. There are 50 employees in the department.

Average age of 50 employees = 30 years.

Total age of 50 employees = (50 × 30) years = 1500 years.

Number of employees next year = 40.

Total age of 40 employees next year

= (1500 + 50 – 60 × 10) = 940.

Average age next year = $\frac{940}{40}$ years = $23\frac{1}{2}$ years.

Thus, I and II together give the answer. So, correct answer is (*e*).

7. **I.** F + S = 50 ...(*i*)

II. 3 (F + S) = 5F ...(*ii*)

From II, we get $2F = 3S \Leftrightarrow \frac{F}{S} = \frac{3}{2}$.

Thus, II alone gives the answer, but I alone does not give the answer.

$\therefore$ Correct answer is (*b*).

8. Let Divya's present age be D years and Shruti's present age be S years.

Then, $D = 2 \times S \Leftrightarrow D - 2S = 0$ ...(*i*)

I. $\frac{D + 5}{S + 5} = \frac{9}{5}$ ...(*ii*)

II. $\frac{D - 10}{S - 10} = \frac{3}{1}$ ...(*iii*)

From (*ii*), we get 5D + 25 = 9S + 45

$\Leftrightarrow$ 5D – 9S = 20 ...(*iv*)

From (*iii*), we get D – 10 = 3S – 30

$\Leftrightarrow$ D – 3S = – 20 ...(*v*)

Thus from (*i*) and (*iv*), we get the answer.

Also, from (*i*) and (*v*), we get the answer.

$\therefore$ I alone as well as II alone gives the answer. Hence, the correct answer is (*c*).

9. **I.** A + B = 21.

II. A – B = 5.

III. AB = 104.

Clearly, any two of three will give the answer. So, correct answer is (*d*).

10. **I.** Let the present ages of Tanya and Rahul be 3*x* years and 4*x* years respectively.

II. After 5 years, (Tanya's age) : (Rahul's age) = 4 : 5.

III. (Rahul's age) = (Tanya's age) + 5.

From I and II, we get $\frac{3x + 5}{4x + 5} = \frac{4}{5}$. This gives *x*.

$\therefore$ Tanya's age = 3*x* can be found. Thus, I and II give the answer.

From I and III, we get 4*x* = 3*x* + 5. This gives *x*.

$\therefore$ Tanya's age = 3*x* can be found. Thus, I and III give the answer.

From III : Let Tanya's present age be *t* years.

Then, Rahul's present age = (*t* + 5) years.

Thus, **from II and III,** we get : $\frac{t + 5}{t + 10} = \frac{4}{5}$. This gives *t*.

Thus, II and III give the answer.

$\therefore$ Correct answer is (*e*).

11. I. $X : Y = 2 : 3 \Rightarrow \frac{X}{Y} = \frac{2}{3} \Rightarrow 3X = 2Y.$

II. $Y = \frac{150}{100}X \Rightarrow Y = \frac{3X}{2} \Rightarrow 3X = 2Y.$

III. $\frac{1}{4}X = \frac{1}{6}Y \Rightarrow 6X = 4Y \Rightarrow 3X = 2Y.$

Thus, even I, II and III together do not give the answer.

$\therefore$ Correct answer is (*e*).

12. II. Let the present ages of Arun and his son be $11x$ and $6x$ years respectively.

I. 5 years ago, Arun's age = 2 × His son's age $\Rightarrow 11x - 5 = 2(6x - 5)$.

III. 5 years hence, $\frac{\text{Arun's age}}{\text{Son's age}} = \frac{12}{7}$.

Clearly, any two of the above will give Arun's present age.

$\therefore$ Correct answer is (*d*).

13. I. Let Ravi's present age be x years. Then, his father's present age = $2x$ years.

II. From I and II, we get $\frac{x+5}{2x+5}$

$= \frac{6}{11}$. This gives x, the answer.

III. Ravi is younger than his brother.

From I and II, we get $\frac{x+5}{2x+5}$

$= \frac{6}{11}$. This gives x, the answer.

Thus, I and II together give the answer. Clearly, III is redundant.

$\therefore$ Correct answer is (*a*).

14. I. A + M + F = 62.

II. $(A - 5) = \frac{1}{5}(F - 5)$.

III. (A – 2) + (F – 2) = 36.

From II and III, we may get A and F.

Putting these values in I, we get M.

Thus, all I, II and III are required to get the answer.

$\therefore$ Correct answer is (*e*).

15. Clearly, any two of the given statements will give the answer and in each case, the third is redundant.

$\therefore$ Correct answer is (*a*).

16. Clearly, any two of the given statements will give the answer and in each case, the third is redundant.

$\therefore$ Correct answer is (*c*).

9 Surds and Indices

IMPORTANT FACTS AND FORMULAE

I. Laws of Indices:

(i) $a^m \times a^n = a^{m+n}$ **(ii)** $\frac{a^m}{a^n} = a^{m-n}$ **(iii)** $(a^m)^n = a^{mn}$

(iv) $(ab)^n = a^n b^n$ **(v)** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ **(vi)** $a^0 = 1$

II. Surds: *Let a be a rational number and n be a positive integer such that* $a^{\frac{1}{n}} = \sqrt[n]{a}$ *is irrational. Then,* $\sqrt[n]{a}$ *is called a surd of order n.*

III. Laws of Surds:

(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$ **(ii)** $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ **(iii)** $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iv) $(\sqrt[n]{a})^n = a$ **(v)** $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ **(vi)** $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

SOLVED EXAMPLES

Ex. 1. *Simplify* : **(i)** $(27)^{\frac{2}{3}}$ **(ii)** $(1024)^{-\frac{4}{5}}$ **(iii)** $\left(\frac{8}{125}\right)^{-\frac{4}{3}}$.

Sol. (*i*) $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\left(3\times\frac{2}{3}\right)} = 3^2 = 9.$

(*ii*) $(1024)^{-\frac{4}{5}} = (4^5)^{-\frac{4}{5}} = 4^{\left\{5\times\frac{(-4)}{5}\right\}} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}.$

(*iii*) $\left(\frac{8}{125}\right)^{-\frac{4}{3}} = \left\{\left(\frac{2}{5}\right)^3\right\}^{-\frac{4}{3}} = \left(\frac{2}{5}\right)^{\left\{3\times\frac{(-4)}{3}\right\}} = \left(\frac{2}{5}\right)^{-4} = \left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4} = \frac{625}{16}.$

Ex. 2. *What will come in place of both the question marks in the following question?* **(Bank Recruitment, 2010)**

$$(?)^{\frac{1}{4}} = \frac{48}{(?)^{\frac{3}{4}}}$$

Sol. Let $x^{\frac{1}{4}} = \frac{48}{x^{\frac{3}{4}}}$. Then, $x^{\frac{1}{4}} . x^{\frac{3}{4}} = 48 \Leftrightarrow x^{\left(\frac{1}{4}+\frac{3}{4}\right)} = 48 \Leftrightarrow x = 48.$

Ex. 3. *Evaluate* : **(i)** $(.00032)^{\frac{3}{5}}$ **(ii)** $(256)^{0.16} \times (16)^{0.18}$.

Sol. (*i*) $(0.00032)^{\frac{3}{5}} = \left(\frac{32}{100000}\right)^{\frac{3}{5}} = \left(\frac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{3}{5}} = \left(\frac{1}{5}\right)^{\left(5\times\frac{3}{5}\right)} = \left(\frac{1}{5}\right)^3 = \frac{1}{125}.$]

(*ii*) $(256)^{0.16} \times (16)^{0.18} = \{(16)^2\}^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18}$

$= (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32+0.18)} = (16)^{0.5} = (16)^{\frac{1}{2}} = 4.$

Ex. 4. ***Solve : $9^{8.6} \times 8^{3.9} \times 72^{4.4} \times 9^{3.9} \times 8^{8.6} = 72^x$.*** **(L.I.C., 2005)**

Sol. Let $9^{8.6} \times 8^{3.9} \times 72^{4.4} \times 9^{3.9} \times 8^{8.6} = 72^x$.

Then, $9^{(8.6 + 3.9)} \times 8^{(3.9 + 8.6)} \times 72^{4.4} = 72^x$

$\Leftrightarrow \quad 9^{12.5} \times 8^{12.5} \times 72^{4.4} = 72^x \Leftrightarrow (9 \times 8)^{12.5} \times 72^{4.4} = 72^x$

$\Leftrightarrow \quad 72^{12.5} \times 72^{4.4} = 72^x \Leftrightarrow 72^{(12.5 + 4.4)} = 72^x \Leftrightarrow 72^{16.9} = 72^x \Leftrightarrow x = 16.9.$

Ex. 5. ***Solve : $(0.064) \times (0.4)^7 = (0.4)^2 \times (0.0256)^2$.*** **(Bank P.O., 2010)**

Sol. Let $(0.064) \times (0.4)^7 = (0.4)^x \times (0.0256)^2$.

Then, $(0.4)^3 \times (0.4)^7 = (0.4)^x \times [(0.4)^4]^2$

$\Leftrightarrow \quad (0.4)^{(3 + 7)} = (0.4)^x \times (0.4)^8 \Leftrightarrow (0.4)^{10} = (0.4)^{x + 8} \Leftrightarrow x + 8 = 10 \Leftrightarrow x = 2.$

Ex. 6. ***What is the quotient when $(x^{-1} - 1)$ is divided by $(x - 1)$?***

Sol. $\dfrac{x^{-1} - 1}{x - 1} = \dfrac{\dfrac{1}{x} - 1}{x - 1} = \dfrac{(1 - x)}{x} \times \dfrac{1}{(x - 1)} = -\dfrac{1}{x}.$

Hence, the required quotient is $-\dfrac{1}{x}$.

Ex. 7. ***If $2^{x-1} + 2^{x+1} = 1280$, then find the value of x.***

Sol. $2^{x-1} + 2^{x+1} = 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280$

$\Leftrightarrow \quad 2^{x-1} = \dfrac{1280}{5} = 256 = 2^8 \quad \Leftrightarrow \quad x - 1 = 8 \quad \Leftrightarrow \quad x = 9.$

Hence, $x = 9$.

Ex. 8. ***Find the value of*** $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$.

Sol. $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left\{(2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{4}} = \left[5\left\{2^{\left(3 \times \frac{1}{3}\right)} + 3^{\left(3 \times \frac{1}{3}\right)}\right\}^3\right]^{\frac{1}{4}}$

$= \{5(2 + 3)^3\}^{\frac{1}{4}} = (5 \times 5^3)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{\left(4 \times \frac{1}{4}\right)} = 5^1 = 5.$

Ex. 9. ***Find the value of*** $\left\{(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right\}$.

Sol. $\left[(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right] = \left[(4^2)^{\frac{3}{2}} + (4^2)^{-\frac{3}{2}}\right] = 4^{\left(2 \times \frac{3}{2}\right)} + 4^{\left\{2 \times \frac{(-3)}{2}\right\}}$

$= 4^3 + 4^{-3} = 4^3 + \dfrac{1}{4^3} = \left(64 + \dfrac{1}{64}\right) = \dfrac{4097}{64}.$

Ex. 10. ***If*** $\left(\dfrac{1}{5}\right)^{3y} = 0.008$, ***then find the value of $(0.25)^y$.***

Sol. $\left(\dfrac{1}{5}\right)^{3y} = 0.008 = \dfrac{8}{1000} = \dfrac{1}{125} = \left(\dfrac{1}{5}\right)^3 \quad \Leftrightarrow \quad 3y = 3 \quad \Leftrightarrow \quad y = 1.$

$\therefore \ (0.25)^y = (0.25)^1 = 0.25.$

Ex. 11. *Simplify :* $\dfrac{(6.25)^{\frac{1}{2}} \times (0.0144)^{\frac{1}{2}} + 1}{(0.027)^{\frac{1}{3}} \times (81)^{\frac{1}{4}}}$. (S.S.C., 2005)

Sol. Given expression $= \dfrac{\left\{(2.5)^2\right\}^{\frac{1}{2}} \times \left\{(0.12)^2\right\}^{\frac{1}{2}} + 1}{\left\{(0.3)^3\right\}^{\frac{1}{3}} \times (3^4)^{\frac{1}{4}}} = \dfrac{(2.5)^{\left(2\times\frac{1}{2}\right)} \times (0.12)^{\left(2\times\frac{1}{2}\right)} + 1}{(0.3)^{\left(3\times\frac{1}{3}\right)} \times 3^{\left(4\times\frac{1}{4}\right)}}$

$= \dfrac{2.5 \times 0.12 + 1}{0.3 \times 3} = \dfrac{0.3 + 1}{0.9} = \dfrac{1.3}{0.9} = \dfrac{13}{9} = 1.444\ldots = 1.\overline{4}.$

Ex. 12. *Find the value of* $\dfrac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\dfrac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}} = \dfrac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \dfrac{3^{\left(5\times\frac{n}{5}\right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \dfrac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$

$= \dfrac{3^{n+(2n+1)}}{3^{2n+n-1}} = \dfrac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9.$

Ex. 13. *Find the value of* $\left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right)$. (N.I.F.T., 2003)

Sol. Putting $2^{\frac{1}{4}} = x$, we get :

$\left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right) = (x-1)(x^3 + x^2 + x + 1)$

$= (x - 1)\,[x^2\,(x + 1) + (x + 1)]$

$= (x - 1)\,(x + 1)\,(x^2 + 1) = (x^2 - 1)\,(x^2 + 1)$

$= (x^4 - 1) = \left[\left(2^{\frac{1}{4}}\right)^4 - 1\right] = \left[2^{\left(\frac{1}{4}\times 4\right)} - 1\right] = (2 - 1) = 1.$

Ex. 14. *Find the value of* $\dfrac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

Sol. $\dfrac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \dfrac{6^{\frac{2}{3}} \times (6^7)^{\frac{1}{3}}}{(6^6)^{\frac{1}{3}}} = \dfrac{6^{\frac{2}{3}} \times 6^{\left(7\times\frac{1}{3}\right)}}{6^{\left(6\times\frac{1}{3}\right)}} = \dfrac{6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}\right)}}{6^2}$

$= 6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}-2\right)} = 6^{\frac{2}{3}} \times 6^{\frac{1}{3}} = 6^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 6^1 = 6.$

Ex. 15. *If* $\left(\dfrac{p}{q}\right)^{rx-s} = \left(\dfrac{q}{p}\right)^{px-q}$, *then find the value of x.*

Sol. $\left(\dfrac{p}{q}\right)^{rx-s} = \left(\dfrac{q}{p}\right)^{px-q} \Leftrightarrow \left(\dfrac{p}{q}\right)^{rx-s} = \left(\dfrac{p}{q}\right)^{-(px-q)}$

$\Leftrightarrow rx - s = -(px - q) \Leftrightarrow rx - s = -px + q$

$\Leftrightarrow rx + px = q + s \Leftrightarrow x\,(p + r) = q + s$

$\Leftrightarrow x = \dfrac{q+s}{p+r}.$

Ex. 16. ***If $x = y^a$, $y = z^b$ and $z = x^c$, then find the value of abc.***

Sol. $z^1 = x^c = (y^a)^c \quad [\because x = y^a]$

$= y^{(ac)} = (z^b)^{ac} \quad [\because y = z^b]$

$= z^{b(ac)} = z^{abc}$

$\therefore abc = 1.$

Ex. 17. ***Simplify :*** $\left(\frac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \times \left(\frac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \times \left(\frac{x^c}{x^a}\right)^{(c^2+a^2+ca)}$.

Sol. Given Expression $= \{x^{(a-b)}\}^{(a^2+b^2+ab)} \cdot \{x^{(b-c)}\}^{(b^2+c^2+bc)} \cdot \{x^{(c-a)}\}^{(c^2+a^2+ca)}$

$= x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ca)}$

$= x^{(a^3-b^3)} \cdot x^{(b^3-c^3)} \cdot x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.$

Ex. 18. ***If $8^x \cdot 2^y = 512$ and $3^{3x+2y} = 9^6$, then what is the value of x and y?*** **(M.A.T., 2004)**

Sol. $8^x\, 2^y = 512 \Leftrightarrow (2^3)^x \cdot 2^y = 2^9 \Leftrightarrow 2^{3x+y} = 2^9 \Leftrightarrow 3x + y = 9$ ...(*i*)

And, $3^{3x+2y} = 9^6 \Leftrightarrow 3^{3x+2y} = (3^2)^6 = 3^{12} \Leftrightarrow 3x + 2y = 12$ ...(*ii*)

Subtracting (*i*) from (*ii*), we get: $y = 3$.

Putting $y = 3$ in (*i*), we get: $3x = 6$ or $x = 2$.

Hence, $x = 2$ and $y = 3$.

Ex. 19. ***Find the largest from among $\sqrt[4]{6}$, $\sqrt{2}$ and $\sqrt[3]{4}$.***

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C.M. is 12.

Changing each to a surd of order 12, we get :

$\sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = \left(6^{\frac{3}{12}}\right) = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}.$

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = \left(2^{\frac{6}{12}}\right) = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}.$

$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = \left(4^{\frac{4}{12}}\right) = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}.$

Clearly, $(256)^{\frac{1}{12}} > (216)^{\frac{1}{12}} > (64)^{\frac{1}{12}}.$

$\therefore$ Largest one is $(256)^{\frac{1}{12}}$ *i.e.*, $\sqrt[3]{4}$.

Ex. 20. ***Find the square root of $(3+\sqrt{5})$.*** **(L.I.C.A.A.O., 2007)**

Sol. $\sqrt{3+\sqrt{5}} = \sqrt{3+2\sqrt{\frac{5}{4}}} = \sqrt{\frac{5}{2}+\frac{1}{2}+2\sqrt{\frac{5}{2}\times\frac{1}{2}}} = \sqrt{\left(\sqrt{\frac{5}{2}}\right)^2+\left(\sqrt{\frac{1}{2}}\right)^2+2\sqrt{\frac{5}{2}}\sqrt{\frac{1}{2}}}$

$= \sqrt{\left(\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}\right)^2} = \left(\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}\right).$

Ex. 21. ***If $x = 3+2\sqrt{2}$, find the value of $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$.***

Sol. $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = x+\frac{1}{x}-2 = (3+2\sqrt{2})+\frac{1}{(3+2\sqrt{2})}-2$

$= (3+2\sqrt{2})+\frac{1}{(3+2\sqrt{2})}\times\frac{(3-2\sqrt{2})}{(3-2\sqrt{2})}-2 = (3+2\sqrt{2})+(3-2\sqrt{2})-2 = 4.$

$\therefore \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right) = 2.$

Ex. 22. *If* $2^x = 3^y = 6^{-z}$, *find the value of* $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$.

Sol. Let $2^x = 3^y = 6^{-z} = k$. Then, $2 = k^{\frac{1}{x}}$, $3 = k^{\frac{1}{y}}$ and $6 = k^{-\frac{1}{z}}$.

Now, $2 \times 3 = 6 \Leftrightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}} \Leftrightarrow k^{\left(\frac{1}{x}+\frac{1}{y}\right)} = k^{-\frac{1}{z}}$

$$\Leftrightarrow \frac{1}{x}+\frac{1}{y} = -\frac{1}{z} \Leftrightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0.$$

Ex. 23. *Given* $t = 2+\sqrt[3]{4}+\sqrt[3]{2}$, *determine the value of* $t^3 - 6t^2 + 6t - 2$. **(M.A.T., 2005)**

Sol. $t = 2+\sqrt[3]{4}+\sqrt[3]{2} \Rightarrow t-2 = \sqrt[3]{4}+\sqrt[3]{2} \Rightarrow (t-2)^3 = (\sqrt[3]{4}+\sqrt[3]{2})^3$

$\Rightarrow t^3 - 8 - 6t\,(t-2) = 4 + 2 + 3\,\sqrt[3]{8}\,(\sqrt[3]{4}+\sqrt[3]{2})$

$\Rightarrow t^3 - 8 - 6t^2 + 12t = 6 + 3 \times 2\,(t-2)$

$\Rightarrow t^3 - 6t^2 + 12t - 8 = 6 + 6t - 12 \Rightarrow t^3 - 6t^2 + 6t - 2 = 0.$

EXERCISE
(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (✓) against the correct answer:*

1. $\sqrt[3]{5}$ is a surd of the order **(R.R.B., 2008)**

(*a*) $\frac{1}{3}$ (*b*) 1

(*c*) 2 (*d*) 3

2. $5^0 \times 8$ =? **(R.R.B., 2006)**

(*a*) 0 (*b*) 8

(*c*) 40 (*d*) 200

3. Which of the following are equal in value?

I. 4^1 II. 1^4

III. 4^0 IV. 0^4

(*a*) I and II (*b*) II and III

(*c*) III and IV (*d*) I and IV

4. If $289 = 17^{\frac{1}{5}x}$, then x =? **(Bank P.O., 2009)**

(*a*) $\frac{2}{5}$ (*b*) 8

(*c*) 16 (*d*) 32

(*e*) None of these

5. $(81)^4 \div (9)^5$ =? **(Agriculture Officers', 2009)**

(*a*) 9 (*b*) 81

(*c*) 729 (*d*) 6561

(*e*) None of these

6. The value of $\left(\frac{9^2 \times 18^4}{3^{16}}\right)$ is **(R.R.B., 2006)**

(*a*) $\frac{3}{2}$ (*b*) $\frac{4}{9}$

(*c*) $\frac{16}{81}$ (*d*) $\frac{32}{243}$

7. $[4^3 \times 5^4] \div 4^5$ =? **(Bank Recruitment, 2008)**

(*a*) 29.0825 (*b*) 30.0925

(*c*) 35.6015 (*d*) 39.0625

(*e*) None of these

8. $9^3 \times 6^2 \div 3^3$ =? **(L.I.C.A.D.O., 2007)**

(*a*) 948 (*b*) 972

(*c*) 984 (*d*) 1012

(*e*) None of these

9. $(19)^{12} \times (19)^8 \div (19)^4 = (19)^?$ **(Bank Recruitment, 2008)**

(*a*) 6 (*b*) 8

(*c*) 12 (*d*) 24

(*e*) None of these

10. $(64)^4 \div (8)^5$ =? **(Agriculture Officer's, 2008)**

(*a*) $(8)^8$ (*b*) $(8)^2$

(*c*) $(8)^{12}$ (*d*) $(8)^4$

(*e*) None of these

11. $(1000)^{12} \div (10)^{30}$ =? **(Bank P.O., 2008)**

(*a*) $(1000)^2$ (*b*) 10

(*c*) 100 (*d*) $(100)^2$

(*e*) None of these

12. $(3)^8 \times (3)^4$ =? **(Bank P.O., 2009)**

(*a*) $(27)^3$ (*b*) $(27)^5$

(*c*) $(729)^2$ (*d*) $(729)^3$

(*e*) None of these

13. $\frac{343 \times 49}{216 \times 16 \times 81}$ =? **(Bank P.O., 2010)**

(*a*) $\frac{7^5}{6^7}$ (*b*) $\frac{7^5}{6^8}$

(*c*) $\frac{7^6}{6^7}$ (*d*) $\frac{7^4}{6^8}$

(*e*) None of these

14. $\frac{16\times 32}{9\times 27\times 81}$ =? (Bank P.O., 2009)

(a) $\left(\frac{2}{3}\right)^{9}$ (b) $\left(\frac{2}{3}\right)^{11}$

(c) $\left(\frac{2}{3}\right)^{12}$ (d) $\left(\frac{2}{3}\right)^{13}$

(e) None of these

15. $9^3 \times (81)^2 \div (27)^3 = (3)^?$ (Bank P.O., 2010)

(a) 3 (b) 4

(c) 5 (d) 6

(e) None of these

16. $(6)^4 \div (36)^3 \times 216 = 6^{(?-5)}$ (Bank Recruitment, 2010)

(a) 1 (b) 4

(c) 6 (d) 7

(e) None of these

17. $(0.2)^2, \frac{1}{100}, (0.01)^{\frac{1}{2}}, (0.008)^{\frac{1}{3}}$. Of these, which one is the greatest? (P.C.S., 2004)

(a) $(0.008)^{\frac{1}{3}}$ (b) $(0.01)^{\frac{1}{2}}$

(c) $(0.2)^2$ (d) $\frac{1}{100}$

18. Which of the following expressions has the greatest value?

(a) $[(2^{-1})^0]^2$ (b) $\left[(4^0)^{-\frac{1}{2}}\right]^2$

(c) $[(2^{-2})^{-1}]^2$ (d) $[(2^{-1})^2]^2$

19. $(10)^{24} \times (10)^{-21} = ?$ (Bank Recruitment, 2008)

(a) 3 (b) 10

(c) 100 (d) 1000

(e) None of these

20. The value of $(256)^{\frac{5}{4}}$ is

(a) 512 (b) 984

(c) 1024 (d) 1032

21. The value of $(\sqrt{8})^{\frac{1}{3}}$ is

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) 8

22. The value of $\left(\frac{32}{243}\right)^{-\frac{4}{5}}$ is

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$

(c) $\frac{16}{81}$ (d) $\frac{81}{16}$

23. The value of $\left(-\frac{1}{216}\right)^{-\frac{2}{3}}$ is

(a) 36 (b) − 36

(c) $\frac{1}{36}$ (d) $-\frac{1}{36}$

24. The value of $27^{-\frac{2}{3}}$ lies between (C.D.S., 2002)

(a) 0 and 1 (b) 1 and 2

(c) 2 and 3 (d) 3 and 4

25. The value of $\sqrt[3]{2^4\sqrt{2^{-5}\sqrt{2^6}}}$ is (S.S.C., 2005)

(a) 1 (b) 2

(c) $2^{\frac{5}{3}}$ (d) 2^5

26. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ = ? (R.R.B., 2007)

(a) $2^{\frac{29}{31}}$ (b) $2^{\frac{31}{32}}$

(c) $2^{\frac{9}{2}}$ (d) $2^{\frac{11}{2}}$

27. The value of $(0.03125)^{-\frac{2}{5}}$ is (R.R.B., 2006)

(a) 4 (b) 9

(c) 12 (d) 31.25

28. $\left(\frac{1}{2}\right)^{-\frac{1}{2}}$ is equal to (Section Officer's, 2005)

(a) $\frac{1}{\sqrt{2}}$ (b) $2\sqrt{2}$

(c) $-\sqrt{2}$ (d) $\sqrt{2}$

29. Simplified form of $\left[\left(\sqrt[5]{x^{-\frac{3}{5}}}\right)^{-\frac{5}{3}}\right]^5$ is (S.S.C., 2010)

(a) $\frac{1}{x}$ (b) x

(c) x^{-5} (d) x^5

30. What will come in place of both the question marks in the following question? (Bank Recruitment, 2010)

$$\frac{(?)^{\frac{2}{3}}}{42} = \frac{5}{(?)^{\frac{1}{3}}}$$

(a) 10 (b) $10\sqrt{2}$

(c) $\sqrt{20}$ (d) 20

(e) 210

31. The value of $5^{\frac{1}{4}} \times (125)^{0.25}$ is :

(a) $\sqrt{5}$ (b) 5

(c) $5\sqrt{5}$ (d) 25

32. The value of $\frac{1}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{1}{(32)^{-\frac{1}{5}}}$ is **(M.B.A., 2003)**

(a) 102 (b) 105
(c) 107 (d) 109

33. $(2.4 \times 10^3) \div (8 \times 10^{-2}) = ?$

(a) 3×10^{-5} (b) 3×10^4
(c) 3×10^5 (d) 30

34. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} \div \left(\frac{1}{27}\right)^{-\frac{4}{3}} = ?$

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$
(c) $\frac{4}{9}$ (d) $\frac{1}{8}$

35. $(48)^{-\frac{2}{7}} \times (16)^{-\frac{5}{7}} \times (3)^{-\frac{5}{7}} = ?$ **(P.C.S., 2008)**

(a) $\frac{1}{3}$ (b) $\frac{1}{48}$
(c) 1 (d) 48

36. If $10^x = \frac{1}{2}$, then $10^{-8x} = ?$ **(P.C.S., 2008)**

(a) $\frac{1}{256}$ (b) 16
(c) 80 (d) 256

37. If $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$, then x is equal to **(S.S.C., 2010)**

(a) -2 (b) -1
(c) 1 (d) 2

38. $49 \times 49 \times 49 \times 49 = 7^?$

(a) 4 (b) 7
(c) 8 (d) 16

39. The value of $(8^{-25} - 8^{-26})$ is

(a) 7×8^{-25} (b) 7×8^{-26}
(c) 8×8^{-26} (d) None of these

40. $(64)^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}} = ?$

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$
(c) $\frac{1}{16}$ (d) $\frac{3}{16}$
(e) None of these

41. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is **(P.C.S., 2009)**

(a) $\frac{1}{2}$ (b) 1
(c) 2 (d) $\frac{7}{2}$

42. If $2^{2n-1} = \frac{1}{8^{n-3}}$, then the value of n is

(a) -2 (b) 0
(c) 2 (d) 3

43. If $5^a = 3125$, then the value of $5^{(a-3)}$ is

(a) 25 (b) 125
(c) 625 (d) 1625

44. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is **(M.B.A., 2006)**

(a) 4 (b) 5
(c) 6 (d) 8

45. If $\sqrt{2^n} = 64$, then the value of n is

(a) 2 (b) 4
(c) 6 (d) 12

46. If $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3}$, then the value of n is

(a) 2 (b) 3
(c) 4 (d) 5

47. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then the value of n is

(a) 0 (b) 2
(c) 3 (d) 4

48. If $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, then the value of x is

(a) 1 (b) 2
(c) 3 (d) 4

49. If $2^x = \sqrt[3]{32}$, then x is equal to

(a) 5 (b) 3
(c) $\frac{3}{5}$ (d) $\frac{5}{3}$

50. If $2^x \times 8^{\frac{1}{5}} = 2^{\frac{1}{5}}$, then x is equal to

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$
(c) $\frac{2}{5}$ (d) $-\frac{2}{5}$

51. If $5^{(x+3)} = 25^{(3x-4)}$, then the value of x is

(a) $\frac{5}{11}$ (b) $\frac{11}{5}$
(c) $\frac{11}{3}$ (d) $\frac{13}{5}$

52. $\frac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$ when simplified is **(M.B.A., 2011)**

(a) $2^{n+1} - \frac{1}{8}$ (b) -2^{n+1}
(c) $1 - 2^n$ (d) $\frac{7}{8}$

53. Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$; the result is **(M.B.A., 2011)**

(*a*) a^4 (*b*) a^8
(*c*) a^{12} (*d*) a^{16}

54. $(256)^{0.16} \times (256)^{0.09} = ?$ **(S.S.C., 2004)**

(*a*) 4 (*b*) 16
(*c*) 64 (*d*) 256.25

55. $(0.04)^{-1.5} = ?$ **(Bank P.O., 2003)**

(*a*) 25 (*b*) 125
(*c*) 250 (*d*) 625

56. $(17)^{3.5} \times (17)^{?} = 17^8$ **(Bank P.O., 2003)**

(*a*) 2.29 (*b*) 2.75
(*c*) 4.25 (*d*) 4.5

57. $6^{1.2} \times 36^{?} \times 30^{2.4} \times 25^{1.3} = 30^5$ **(Specialist Officers', 2006)**

(*a*) 0.1 (*b*) 0.7
(*c*) 1.4 (*d*) 2.6
(*e*) None of these

58. $2^{3.6} \times 4^{3.6} \times 4^{3.6} \times (32)^{2.3} = (32)^{?}$ **(Specialist Officers', 2007)**

(*a*) 5.9 (*b*) 7.7
(*c*) 9.5 (*d*) 13.1
(*e*) None of these

59. $3^{3.5} \times 21^2 \times 42^{2.5} \div 2^{2.5} \times 7^{3.5} = 21^{?}$ **(Bank P.O., 2006)**

(*a*) 6.5 (*b*) 8
(*c*) 10 (*d*) 12.5
(*e*) None of these

60. $8^{0.4} \times 4^{1.6} \times 2^{1.6} = ?$ **(Agriculture Officers', 2009)**

(*a*) 48 (*b*) 52
(*c*) 64 (*d*) 76
(*e*) None of these

61. $8^7 \times 2^6 \div 8^{2.4} = 8^{?}$ **(Bank P.O., 2009)**

(*a*) 6.6 (*b*) 8.6
(*c*) 9.6 (*d*) 10.6
(*e*) None of these

62. $25^{2.7} \times 5^{4.2} \div 5^{5.4} = 25^{?}$ **(Bank Recruitment, 2010)**

(*a*) 1.6 (*b*) 1.7
(*c*) 3.2 (*d*) 3.6
(*e*) None of these

63. $8^{2.4} \times 2^{3.7} - (16)^{1.3} = 2^{?}$ **(Bank Recruitment, 2010)**

(*a*) 4.8 (*b*) 5.7
(*c*) 5.8 (*d*) 7.1
(*e*) None of these

64. $(0.04)^2 \div (0.008) \times (0.2)^6 = (0.2)^{?}$ **(Bank Recruitment, 2010)**

(*a*) 5 (*b*) 6
(*c*) 8 (*d*) 9
(*e*) None of these

65. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^{?}$ **(Bank P.O., 2003)**

(*a*) 3.5 (*b*) 4.5
(*c*) 6 (*d*) 7
(*e*) None of these

66. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^{?}$ **(Bank P.O., 2003)**

(*a*) 8.5 (*b*) 13
(*c*) 16 (*d*) 17.5
(*e*) None of these

67. The value of $\dfrac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$ is **(C.B.I., 2003)**

(*a*) $\dfrac{3}{7}$ (*b*) $\dfrac{7}{3}$
(*c*) $1\dfrac{3}{7}$ (*d*) $2\dfrac{2}{7}$

68. $(64x^3 \div 27\, a^{-3})^{-\frac{2}{3}} = ?$ **(R.R.B., 2006)**

(*a*) $\dfrac{9ax}{16}$ (*b*) $\dfrac{9}{16ax}$
(*c*) $\dfrac{9}{16x^2\, a^2}$ (*d*) $\dfrac{3}{4}x^{-2}\, a^{-2}$

69. If $2^{n+4} - 2^{n+2} = 3$, then n is equal to

(*a*) 0 (*b*) 2
(*c*) – 1 (*d*) – 2

70. If $2^{n-1} + 2^{n+1} = 320$, then n is equal to

(*a*) 6 (*b*) 8
(*c*) 5 (*d*) 7

71. If $3^x - 3^{x-1} = 18$, then the value of x^x is

(*a*) 3 (*b*) 8
(*c*) 27 (*d*) 216

72. $\dfrac{2^{n+4} - 2 \times 2^n}{2 \times 2^{(n+3)}} + 2^{-3}$ is equal to

(*a*) 2^{n+1} (*b*) $\left(\dfrac{9}{8} - 2^n\right)$
(*c*) $\left(-2^{n+1} + \dfrac{1}{8}\right)$ (*d*) 1

73. The value of $\dfrac{2^{3x+4} + 8^{x+1}}{8^{x+1} - 2^{3x+2}}$ is

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

74. The value of $\dfrac{2^{n-1} - 2^n}{2^{n+4} + 2^{n+1}}$ is

(*a*) $-\dfrac{1}{36}$ (*b*) $\dfrac{2}{3}$
(*c*) $\dfrac{1}{13}$ (*d*) $\dfrac{5}{13}$

75. If $x = 5 + 2\sqrt{6}$, then $\sqrt{x} - \frac{1}{\sqrt{x}}$ is (A.A.O. Exam, 2009)

(a) $2\sqrt{2}$ (b) $2\sqrt{3}$

(c) $\sqrt{3} + \sqrt{2}$ (d) $\sqrt{3} - \sqrt{2}$

76. $(4 + \sqrt{7})$, expressed as a perfect square, is equal to (Section Officers', 2005)

(a) $(2 + \sqrt{7})^2$ (b) $\left(\frac{\sqrt{7}}{2} + \frac{1}{2}\right)^2$

(c) $\left\{\frac{1}{2}(\sqrt{7} + 1)^2\right\}$ (d) $(\sqrt{3} + \sqrt{4})^2$

77. $\sqrt{8 - 2\sqrt{15}}$ is equal to (C.P.O., 2007)

(a) $3 - \sqrt{5}$ (b) $\sqrt{5} - \sqrt{3}$

(c) $5 - \sqrt{3}$ (d) $\sqrt{5} + \sqrt{3}$

78. $\sqrt{6 - 4\sqrt{3} + \sqrt{16 - 8\sqrt{3}}}$ is equal to (A.A.O. Exam, 2010)

(a) $1 - \sqrt{3}$ (b) $\sqrt{3} - 1$

(c) $2(2 - \sqrt{3})$ (d) $2(2 + \sqrt{3})$

79. The value of $\frac{1}{\sqrt{12 - \sqrt{140}}} - \frac{1}{\sqrt{8 - \sqrt{60}}} - \frac{2}{\sqrt{10 + \sqrt{84}}}$ is (S.S.C., 2005)

(a) 0 (b) 1

(c) 2 (d) 3

80. The value of the expression $\sqrt{4 + \sqrt{15}} + \sqrt{4 - \sqrt{15}} - \sqrt{12 - 4\sqrt{5}}$ is

(a) an irrational number

(b) a negative integer

(c) a natural number

(d) a non-integer rational number

81. If $N = \frac{\sqrt{\sqrt{5} + 2} + \sqrt{\sqrt{5} - 2}}{\sqrt{\sqrt{5} + 1}} - \sqrt{3 - 2\sqrt{2}}$, then the value of N is (A.A.O. Exam., 2009)

(a) $2\sqrt{2} - 1$ (b) 3

(c) 1 (d) 2

82. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to

(a) 1.45 (b) 1.88

(c) 2.9 (d) 3.7

83. If m and n are whole numbers suchthat $m^n = 121$, then the value of $(m - 1)^{n+1}$ is (S.S.C., 2001)

(a) 1 (b) 10

(c) 121 (d) 1000

84. Number of prime factors in $(216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}}$ is

(a) 6 (b) 7

(c) 8 (d) None of these

85. Number of prime factors in $\frac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}}$ is

(a) 56 (b) 66

(c) 112 (d) None of these

86. $1 + (3 + 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)$ is equal to (Section Officers', 2005)

(a) $\frac{3^{64} - 1}{2}$ (b) $\frac{3^{64} + 1}{2}$

(c) $3^{64} - 1$ (d) $3^{64} + 1$

87. $\frac{1}{1 + a^{(n-m)}} + \frac{1}{1 + a^{(m-n)}} = ?$ (M.B.A., 2003; NMAT, 2006)

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) a^{m+n}

88. If $a + b + c = 0$, then the value of $(x^a)^{a^2 - bc} \cdot (x^b)^{b^2 - ca} \cdot (x^c)^{c^2 - ab}$ is equal to

(a) -2 (b) -1

(c) 0 (d) 1

89. $\frac{1}{1 + x^{(b-a)} + x^{(c-a)}} + \frac{1}{1 + x^{(a-b)} + x^{(c-b)}} + \frac{1}{1 + x^{(b-c)} + x^{(a-c)}} = ?$ (M.B.A., 2003)

(a) 0 (b) 1

(c) x^{a-b-c} (d) None of these

90. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\frac{x^a}{x^b}\right)^{(a+b-c)} = ?$

(NMAT, 2005; L.I.C., 2003)

(a) x^{abc} (b) 1

(c) $x^{ab + bc + ca}$ (d) x^{a+b+c}

91. $\left(\frac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a)} = ?$ (M.B.A., 2006)

(a) 0 (b) x^{abc}

(c) x^{a+b+c} (d) 1

92. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = ?$

(a) 1 (b) $x^{\frac{1}{abc}}$

(c) $x^{\frac{1}{(ab + bc + ca)}}$ (d) None of these

93. The expression $\dfrac{\left(x+\frac{1}{y}\right)^a \cdot \left(x-\frac{1}{y}\right)^b}{\left(y+\frac{1}{x}\right)^a \cdot \left(y-\frac{1}{x}\right)^b}$ reduces to

(a) $\left(\frac{x}{y}\right)^{a-b}$ (b) $\left(\frac{y}{x}\right)^{a-b}$

(c) $\left(\frac{x}{y}\right)^{a+b}$ (d) $\left(\frac{y}{x}\right)^{a+b}$

94. The value of $\left(x^{\frac{b+c}{c-a}}\right)^{\frac{1}{a-b}} \cdot \left(x^{\frac{c+a}{a-b}}\right)^{\frac{1}{b-c}} \cdot \left(x^{\frac{a+b}{b-c}}\right)^{\frac{1}{c-a}}$ is

(a) 1 (b) a

(c) b (d) c

95. If $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}}$ and $xyz = 1$, then the value of $p + q + r$ would be **(M.B.A. 2008)**

(a) 0 (b) 1

(c) 2 (d) a rational number

96. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals

(a) $\dfrac{xz}{x+z}$ (b) $\dfrac{xz}{2(x-z)}$

(c) $\dfrac{xz}{2(z-x)}$ (d) $\dfrac{2xz}{(x+z)}$

97. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is **(M.B.A., 2005; R.R.B., 2008)**

(a) 0 (b) 1

(c) $\dfrac{1}{abc}$ (d) abc

98. If $2^x = 4^y = 8^z$ and $\left(\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}\right) = \frac{24}{7}$, then the value of z is

(a) $\dfrac{7}{16}$ (b) $\dfrac{7}{32}$

(c) $\dfrac{7}{48}$ (d) $\dfrac{7}{64}$

99. Suppose $4^a = 5$, $5^b = 6$, $6^c = 7$, $7^d = 8$, then the value of $abcd$ is **(A.A.O. Exam, 2009)**

(a) 1 (b) $\dfrac{3}{2}$

(c) 2 (d) $\dfrac{5}{2}$

100. If $abc = 1$, then $\left(\dfrac{1}{1+a+b^{-1}} + \dfrac{1}{1+b+c^{-1}} + \dfrac{1}{1+c+a^{-1}}\right) = ?$ **(C.D.S. 2004)**

(a) 0 (b) 1

(c) $\dfrac{1}{ab}$ (d) ab

101. If a, b, c are real numbers, then the value of $\sqrt{a^{-1}b}\cdot\sqrt{b^{-1}c}\cdot\sqrt{c^{-1}a}$ is

(a) abc (b) $\sqrt{abc}$

(c) $\dfrac{1}{abc}$ (d) 1

102. If $3^{(x-y)} = 27$ and $3^{(x+y)} = 243$, then x is equal to **(R.R.B., 2003)**

(a) 0 (b) 2

(c) 4 (d) 6

103. If $x^y = y^x$, then $\left(\frac{x}{y}\right)^{\frac{x}{y}}$ is equal to **(M.C.A., 2005)**

(a) $x^{\frac{y}{x}}$ (b) $x^{\frac{x}{y}-1}$

(c) 1 (d) $x^{\frac{x}{y}}$

104. If $4^{x+y} = 1$ and $4^{x-y} = 4$, then the values of x and y respectively are

(a) $-\frac{1}{2}$ and $\frac{1}{2}$ (b) $-\frac{1}{2}$ and $-\frac{1}{2}$

(c) $\frac{1}{2}$ and $-\frac{1}{2}$ (d) $\frac{1}{2}$ and $\frac{1}{2}$

105. If $2^{2x-1} + 4^x = 2^{x-\frac{1}{2}} + 2^{x+\frac{1}{2}}$, then x equals

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{3}$

(c) $\dfrac{3}{2}$ (d) 1

106. If $3^{2x-y} = 3^{x+y} = \sqrt{27}$, the value of y is **(R.R.B., 2005)**

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{3}{2}$ (d) $\dfrac{3}{4}$

107. If $3^x = 5^y = 45^z$, then **(C.D.S., 2002)**

(a) $\dfrac{2}{z} = \dfrac{1}{y} - \dfrac{1}{x}$ (b) $\dfrac{2}{y} = \dfrac{1}{x} - \dfrac{1}{z}$

(c) $\dfrac{2}{x} = \dfrac{1}{z} - \dfrac{1}{y}$ (d) $x + y + z = 0$

108. Given $2^x = 8^{y+1}$ and $9^y = 3^{x-9}$, the value of $x + y$ is **(M.B.A., 2010)**

(a) 18 (b) 21

(c) 24 (d) 27

109. What are the values of x and y that satisfy the equation $2^{0.7x} \cdot 3^{-1.25y} = \dfrac{8\sqrt{6}}{27}$? **(C.A.T., 2006)**

(a) $x = 2.5, y = 6$ (b) $x = 3, y = 5$
(c) $x = 3, y = 4$ (d) $x = 5, y = 2$

110. Let r be the result of doubling both thebase and the exponent of a^b, $b \neq 0$. If r equals the product of a^b by x^b, then x equals **(M.B.A., 2010)**
(a) 2 (b) 4
(c) $2a$ (d) $4a$

111. Which of the following is the greatest? **(Section Officers', 2005)**
(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$
(c) $\sqrt[4]{4}$ (d) $\sqrt[6]{6}$

112. The greatest of $\sqrt{2}, \sqrt[6]{3}, \sqrt[3]{4}, \sqrt[4]{5}$ is **(S.S.C., 2005)**
(a) $\sqrt{2}$ (b) $\sqrt[3]{4}$
(c) $\sqrt[4]{5}$ (d) $\sqrt[6]{3}$

113. The largest number in the sequence $1, 2^{\frac{1}{2}}, 3^{\frac{1}{3}}, 4^{\frac{1}{4}}, \ldots, n^{\frac{1}{n}}$ is **(I.I.F.T., 2005)**
(a) $2^{\frac{1}{2}}$ (b) $3^{\frac{1}{3}}$
(c) $5^{\frac{1}{5}}$ (d) $6^{\frac{1}{6}}$

114. If $x = 5 + 2\sqrt{6}$, then $\dfrac{(x-1)}{\sqrt{x}}$ is equal to
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\sqrt{3}$ (d) $2\sqrt{3}$

115. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$, then $\{(x + y - z)^3 + 27xyz\}$ equals
(a) -1 (b) 0
(c) 1 (d) 27

116. If $x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$, then the value of $x^3 - 6x^2 + 6x$ is **(R.R.B., 2008)**
(a) 1 (b) 2
(c) 3 (d) None of these

117. Find the value of $(-2)^5 \times (2)^{-5} \times (3)^3$ **[ESIC—UDC Exam, 2016]**
(a) –108 (b) 27
(c) $(2)^{25} \times (3)^3$ (d) –27

118. The quotient when 10^{100} is divided by 5^{75} is **[SSC—CHSL (10+2) Exam, 2015]**
(a) $2^{25} \times 10^{75}$ (b) 10^{25}
(c) 2^{75} (d) $2^{75} \times 10^{25}$

119. The exponential form of $\sqrt{\sqrt{2} \times \sqrt{3}}$ is **[SSC—CHSL (10+2) Exam, 2015]**
(a) 6 (b) $6^{\frac{1}{2}}$
(c) $6^{\frac{1}{3}}$ (d) $6^{\frac{1}{4}}$

120. $21^{?} \times 21^{6.5} = 21^{12.4}$ **[United India Insurance Co. Ltd., UIICL—Assistant (Online) Exam, 2015]**
(a) 18.9 (b) 4.4
(c) 5.9 (d) 13.4

121. $\dfrac{5.4 \div 3 \times 16 + 2}{18 \div 5 \times 6 \div 3}$ **[United India Insurance Co. Ltd., UIICL—Assistant (Online) Exam, 2015]**
(a) 2 (b) 4
(c) 6 (d) 8

122. $\left(32 \times 10^{-5}\right)^{-2} \times 64 \div \left(2^{16} \times 10^{-4}\right) = 10^{?}$ **[IBPS—RRB Office Assistant (Online) Exam, 2015]**
(a) 6 (b) 10
(c) –8 (d) –6

ANSWERS

1. (d)	2. (b)	3. (b)	4. (e)	5. (c)	6. (c)	7. (d)	8. (b)	9. (e)	10. (e)
11. (a)	12. (c)	13. (a)	14. (a)	15. (c)	16. (c)	17. (a)	18. (c)	19. (d)	20. (c)
21. (c)	22. (d)	23. (a)	24. (a)	25. (b)	26. (b)	27. (a)	28. (d)	29. (b)	30. (e)
31. (b)	32. (a)	33. (b)	34. (c)	35. (b)	36. (d)	37. (b)	38. (c)	39. (b)	40. (c)
41. (c)	42. (c)	43. (a)	44. (a)	45. (d)	46. (d)	47. (c)	48. (d)	49. (d)	50. (d)
51. (b)	52. (d)	53. (a)	54. (a)	55. (b)	56. (d)	57. (b)	58. (a)	59. (b)	60. (c)
61. (a)	62. (e)	63. (b)	64. (e)	65. (d)	66. (b)	67. (a)	68. (c)	69. (d)	70. (d)
71. (c)	72. (d)	73. (d)	74. (a)	75. (a)	76. (c)	77. (b)	78. (b)	79. (a)	80. (a)
81. (c)	82. (c)	83. (d)	84. (b)	85. (b)	86. (b)	87. (c)	88. (d)	89. (b)	90. (b)
91. (d)	92. (a)	93. (c)	94. (a)	95. (a)	96. (d)	97. (b)	98. (c)	99. (b)	100. (b)
101. (d)	102. (c)	103. (b)	104. (c)	105. (a)	106. (a)	107. (c)	108. (d)	109. (d)	110. (d)
111. (b)	112. (b)	113. (b)	114. (b)	115. (b)	116. (b)	117. (d)	118. (d)	119. (d)	120. (c)
121. (a)	122. (d)								

SOLUTIONS

1. $\sqrt[n]{a}$ is called a surd of order n.

2. $5^0 \times 8 = 1 \times 8 = 8$. $[\because 5^0 = 1]$

3. I. $4^1 = 4$ II. $1^4 = 1$
III. $4^0 = 1$ IV. $0^4 = 0$

4. $289 = 17^{\frac{1}{5}x} \Rightarrow 17^2 = 17^{\frac{1}{5}x} \Rightarrow \frac{1}{5}x = 2 \Rightarrow x = 2 \times 5 = 10.$

5. $(81)^4 \div (9)^5 = \frac{(9^2)^4}{(9^5)} = \frac{9^{(2\times4)}}{9^5} = \frac{9^8}{9^5} = 9^{(8-5)} = 9^3 = 729.$

6. $\left(\frac{9^2 \times 18^4}{3^{16}}\right) = \frac{9^2 \times (9\times2)^4}{3^{16}} = \frac{(3^2)^2 \times (3^2)^4 \times 2^4}{3^{16}}$

$= \frac{3^4 \times 3^8 \times 2^4}{3^{16}} = \frac{3^{(4+8)} \times 2^4}{3^{16}} = \frac{3^{12} \times 2^4}{3^{16}}$

$= \frac{2^4}{3^{(16-12)}} = \frac{2^4}{3^4} = \frac{16}{81}.$

7. $\frac{4^3 \times 5^4}{4^5} = \frac{5^4}{4^{(5-3)}} = \frac{5^4}{4^2} = \frac{625}{16} = 39.0625.$

8. $\frac{9^3 \times 6^2}{3^3} = \frac{(3^2)^3 \times (3\times2)^2}{3^3} = \frac{3^{(2\times3)} \times 3^2 \times 2^2}{3^3} = \frac{3^{(6+2)} \times 2^2}{3^3}$

$= 3^{(8-3)} \times 2^2 = 3^5 \times 2^2 = 243 \times 4 = 972.$

9. $\frac{(19)^{12} \times (19)^8}{(19)^4} = \frac{19^{(12+8)}}{(19)^4} = \frac{(19)^{20}}{(19)^4} = (19)^{(20-4)} = (19)^{16}.$

Hence, missing number = 16.

10. $(64)^4 \div (8)^5 = (8^2)^4 \div (8)^5 = (8)^{(2\times4)} \div 8^5 = \frac{8^8}{8^5} = 8^{(8-5)} = 8^3.$

11. $(1000)^{12} \div (10)^{30} = \frac{(10^3)^{12}}{(10)^{30}} = \frac{(10)^{(3\times12)}}{(10)^{30}} = \frac{(10)^{36}}{(10)^{30}} = (10)^{(36-30)}$

$= 10^6 = (10^3)^2 = (1000)^2.$

12. $3^8 \times 3^4 = 3^{(8+4)} = 3^{12} = (3^6)^2 = (729)^2.$

13. $\frac{343 \times 49}{216 \times 16 \times 81} = \frac{7^3 \times 7^2}{6^3 \times 2^4 \times 3^4} = \frac{7^{(3+2)}}{6^3 \times (2\times3)^4}$

$= \frac{7^5}{6^3 \times 6^4} = \frac{7^5}{6^{(3+4)}} = \frac{7^5}{6^7}.$

14. $\frac{16 \times 32}{9 \times 27 \times 81} = \frac{2^4 \times 2^5}{3^2 \times 3^3 \times 3^4} = \frac{2^{(4+5)}}{3^{(2+3+4)}} = \frac{2^9}{3^9} = \left(\frac{2}{3}\right)^9.$

15. Let $9^3 \times (81)^2 \div (27)^3 = 3^x$. Then,

$3^x = \frac{(3^2)^3 \times (3^4)^2}{(3^3)^3} = \frac{3^{(2\times3)} \times 3^{(4\times2)}}{3^{(3\times3)}} = \frac{3^6 \times 3^8}{3^9} = \frac{3^{(6+8)}}{3^9}$

$\Rightarrow \quad 3^x = \frac{3^{14}}{3^9} = 3^{(14-9)} = 3^5 \Rightarrow x = 5.$

16. Let $6^4 \div (36)^3 \times 216 = 6^{(x-5)}$.
Then, $6^{(x-5)} = 6^4 \div (6^2)^3 \times 6^3$
$= 6^4 \div 6^{(2\times3)} \times 6^3 = 6^4 \div 6^6 \times 6^3 = 6^{(4-6+3)} = 6$
$\Rightarrow \quad x - 5 = 1 \Rightarrow x = 6.$

17. $(0.2)^2 = 0.2 \times 0.2 = 0.04;$

$\frac{1}{100} = 0.01; \qquad (0.01)^{\frac{1}{2}} = \left[(0.1)^2\right]^{\frac{1}{2}} = (0.1)^{\left(2\times\frac{1}{2}\right)} = 0.1;$

$(0.008)^{\frac{1}{3}} = [(0.2)^3]^{\frac{1}{3}} = (0.2)^{\left(3\times\frac{1}{3}\right)} = 0.2.$

Clearly, $0.2 > 0.1 > 0.04 > 0.01.$

So, $(0.008)^{\frac{1}{3}}$ is the greatest.

18. $[(2^{-1})^0]^2 = \left[\left(\frac{1}{2}\right)^0\right]^2 = (1)^2 = 1.$

$\left[(4^0)^{-\frac{1}{2}}\right]^2 = \left[(1)^{-\frac{1}{2}}\right]^2 = (1)^2 = 1.$

$\left[(2^{-2})^{-1}\right]^2 = \left[\left(\frac{1}{2^2}\right)^{-1}\right]^2 = \left[\left(\frac{1}{4}\right)^{-1}\right]^2 = (4)^2 = 16.$

$\left[(2^{-1})^2\right]^2 = \left[\left(\frac{1}{2}\right)^2\right]^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}.$

19. $(10)^{24} \times (10)^{-21} = 10^{(24-21)} = 10^3 = 1000.$

20. $(256)^{\frac{5}{4}} = (4^4)^{\frac{5}{4}} = 4^{\left(4\times\frac{5}{4}\right)} = 4^5 = 1024.$

21. $(\sqrt{8})^{\frac{1}{3}} = \left(8^{\frac{1}{2}}\right)^{\frac{1}{3}} = 8^{\left(\frac{1}{2}\times\frac{1}{3}\right)}$

$= 8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\left(3\times\frac{1}{6}\right)} = 2^{\frac{1}{2}} = \sqrt{2}.$

22. $\left(\frac{32}{243}\right)^{-\frac{4}{5}} = \left\{\left(\frac{2}{3}\right)^5\right\}^{-\frac{4}{5}} = \left(\frac{2}{3}\right)^{5\times\frac{(-4)}{5}}$

$= \left(\frac{2}{3}\right)^{(-4)} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}.$

23. $\left(-\frac{1}{216}\right)^{-\frac{2}{3}} = \left[\left(-\frac{1}{6}\right)^3\right]^{-\frac{2}{3}} = \left(-\frac{1}{6}\right)^{3\times\frac{(-2)}{3}}$

$= \left(-\frac{1}{6}\right)^{-2} = \frac{1}{\left(-\frac{1}{6}\right)^2} = \frac{1}{\left(\frac{1}{36}\right)} = 36.$

24. $(27)^{-\frac{2}{3}} = (3^3)^{-\frac{2}{3}} = 3^{\left[3\times\left(-\frac{2}{3}\right)\right]} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

Clearly, $0 < \frac{1}{9} < 1.$

25. $\sqrt[3]{2^4\sqrt{2^{-5}\sqrt{2^6}}} = \sqrt[3]{2^4\sqrt{2^{-5}(2^6)^{\frac{1}{2}}}} = \sqrt[3]{2^4\sqrt{2^{-5}\,(2)^{\left(6\times\frac{1}{2}\right)}}}$

$= \sqrt[3]{2^4\sqrt{2^{-5}\,.\,2^3}} = \sqrt[3]{2^4\sqrt{2^{(-5+3)}}}$

$= \sqrt[3]{2^4\sqrt{2^{(-2)}}} = \sqrt[3]{2^4\,.\,(2^{-2})^{\frac{1}{2}}} = \sqrt[3]{2^4\,.\,2^{\left(-2\times\frac{1}{2}\right)}}$

$= \sqrt[3]{2^4 . 2^{(-1)}} = \sqrt[3]{2^{(4-1)}} = \sqrt[3]{2^3} = (2^3)^{\frac{1}{3}}$

$= 2^{\left(3\times\frac{1}{3}\right)} = 2.$

26. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}} = \sqrt{2\sqrt{2\sqrt{2\sqrt{2.2^{\frac{1}{2}}}}}} = \sqrt{2\sqrt{2\sqrt{2\sqrt{2^{\left(1+\frac{1}{2}\right)}}}}}$

$= \sqrt{2\sqrt{2\sqrt{2\sqrt{2^{\frac{3}{2}}}}}} = \sqrt{2\sqrt{2\sqrt{2.\left(2^{\frac{3}{2}}\right)^{\frac{1}{2}}}}}$

$= \sqrt{2\sqrt{2\sqrt{2.2^{\left(\frac{3}{2}\times\frac{1}{2}\right)}}}} = \sqrt{2\sqrt{2\sqrt{2.2^{\frac{3}{4}}}}}$

$= \sqrt{2\sqrt{2\sqrt{2^{\left(1+\frac{3}{4}\right)}}}} = \sqrt{2\sqrt{2\sqrt{2^{\frac{7}{4}}}}}$

$= \sqrt{2\sqrt{2.\left(2^{\frac{7}{4}}\right)^{\frac{1}{2}}}} = \sqrt{2\sqrt{2.2^{\left(\frac{7}{4}\times\frac{1}{2}\right)}}}$

$= \sqrt{2\sqrt{2.2^{\frac{7}{8}}}} = \sqrt{2\sqrt{2^{\left(1+\frac{7}{8}\right)}}} = \sqrt{2\sqrt{2^{\frac{15}{8}}}}$

$= \sqrt{2\left(2^{\frac{15}{8}}\right)^{\frac{1}{2}}} = \sqrt{2.2^{\frac{15}{16}}} = \sqrt{2^{\left(1+\frac{15}{16}\right)}}$

$= \sqrt{2^{\frac{31}{16}}} = \left(2^{\frac{31}{16}}\right)^{\frac{1}{2}} = 2^{\left(\frac{31}{16}\times\frac{1}{2}\right)} = 2^{\frac{31}{32}}.$

27. $(0.03125)^{-\frac{2}{5}} = \left[(0.5)^5\right]^{-\frac{2}{5}} = 0.5^{\left[5\times\left(-\frac{2}{5}\right)\right]} = (0.5)^{-2}$

$= \frac{1}{(0.5)^2} = \frac{1}{0.25} = 4.$

28. $\left(\frac{1}{2}\right)^{-\frac{1}{2}} = (2)^{\frac{1}{2}} = \sqrt{2}.$

29. $\left[\left(\sqrt[5]{x^{-\frac{3}{5}}}\right)^{-\frac{5}{3}}\right]^5 = \left[\left\{\left(x^{-\frac{3}{5}}\right)^{\frac{1}{5}}\right\}^{-\frac{5}{3}}\right]^5 = \left[\left(x^{\left\{\left(-\frac{3}{5}\right)\times\frac{1}{5}\right\}}\right)^{-\frac{5}{3}}\right]^5$

$= \left[\left(x^{-\frac{3}{25}}\right)^{-\frac{5}{3}}\right]^5 = \left[x^{\left\{\left(-\frac{3}{25}\right)\times\left(-\frac{5}{3}\right)\right\}}\right]^5$

$= \left(x^{\frac{1}{5}}\right)^5 = x^{\left(\frac{1}{5}\times 5\right)} = x.$

30. Let $\frac{x^{\frac{2}{3}}}{42} = \frac{5}{x^{\frac{1}{3}}}.$

Then, $x^{\frac{2}{3}} . x^{\frac{1}{3}} = 42\times 5 = 210 \Rightarrow x^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 210 \Rightarrow x = 210.$

31. $5^{\frac{1}{4}} \times (125)^{0.25} = 5^{0.25} \times (5^3)^{0.25} = 5^{0.25} \times 5^{(3\times 0.25)}$

$= 5^{0.25} \times 5^{0.75} = 5^{(0.25+0.75)} = 5^1 = 5.$

32. $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(32)^{-\frac{1}{5}}}$

$= \frac{1}{(6^3)^{-\frac{2}{3}}} + \frac{1}{(4^4)^{\left(-\frac{3}{4}\right)}} + \frac{1}{(2^5)^{-\frac{1}{5}}}$

$= \frac{1}{6^{3\times\frac{(-2)}{3}}} + \frac{1}{4^{4\times\frac{(-3)}{4}}} + \frac{1}{2^{5\times\frac{(-1)}{5}}} = \frac{1}{6^{-2}} + \frac{1}{4^{-3}} + \frac{1}{2^{-1}}$

$= (6^2 + 4^3 + 2^1) = (36 + 64 + 2) = 102.$

33. $(2.4\times 10^3) \div (8\times 10^{-2}) = \frac{2.4\times 10^3}{8\times 10^{-2}} = \frac{24\times 10^2}{8\times 10^{-2}} = (3\times 10^4).$

34. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} \div \left(\frac{1}{27}\right)^{-\frac{4}{3}} = (216)^{\frac{2}{3}} \div (27)^{\frac{4}{3}} = \frac{(216)^{\frac{2}{3}}}{(27)^{\frac{4}{3}}} = \frac{(6^3)^{\frac{2}{3}}}{(3^3)^{\frac{4}{3}}}$

$= \frac{6^{\left(3\times\frac{2}{3}\right)}}{3^{\left(3\times\frac{4}{3}\right)}} = \frac{6^2}{3^4} = \frac{36}{81} = \frac{4}{9}.$

35. $(48)^{-\frac{2}{7}} \times (16)^{-\frac{5}{7}} \times (3)^{-\frac{5}{7}} = (16\times 3)^{-\frac{2}{7}} \times (16)^{-\frac{5}{7}} \times (3)^{-\frac{5}{7}}$

$= (16)^{-\frac{2}{7}} \times (3)^{-\frac{2}{7}} \times (16)^{-\frac{5}{7}} \times (3)^{-\frac{5}{7}}$

$= (16)^{\left(-\frac{2}{7}-\frac{5}{7}\right)} \times (3)^{\left(-\frac{2}{7}-\frac{5}{7}\right)}$

$= (16)^{\left(-\frac{7}{7}\right)} \times (3)^{\left(-\frac{7}{7}\right)} = (16)^{-1} \times (3)^{-1}$

$= \frac{1}{16}\times\frac{1}{3} = \frac{1}{48}.$

36. $10^{-8x} = (10^x)^{-8} = \left(\frac{1}{2}\right)^{-8} = 2^8 = 256.$

37. $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1} \Rightarrow \left(\frac{3}{5}\right)^{(3-6)} = \left(\frac{3}{5}\right)^{2x-1}$

$\Rightarrow \left(\frac{3}{5}\right)^{-3} = \left(\frac{3}{5}\right)^{2x-1}$

$\Rightarrow 2x - 1 = -3 \Rightarrow 2x = -2 \Rightarrow x = -1.$

38. $49 \times 49 \times 49 \times 49 = 7^2 \times 7^2 \times 7^2 \times 7^2 = 7^{(2+2+2+2)} = 7^8.$

$\therefore$ Required number = 8.

39. $8^{-25} - 8^{-26} = \left(\frac{1}{8^{25}} - \frac{1}{8^{26}}\right) = \frac{(8-1)}{8^{26}} = 7\times 8^{-26}.$

40. $(64)^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}} = (8^2)^{-\frac{1}{2}} - [(-2)^5]^{-\frac{4}{5}}$

$= 8^{\left[2\times\left(\frac{-1}{2}\right)\right]} - (-2)^{\left[5\times\left(\frac{-4}{5}\right)\right]}$

$= 8^{-1} - (-2)^{-4} = \frac{1}{8} - \frac{1}{(-2)^4}$

$= \left(\frac{1}{8} - \frac{1}{16}\right) = \frac{1}{16}.$

41. $\left(\frac{a}{b}\right)^{x-1}=\left(\frac{b}{a}\right)^{x-3}\Leftrightarrow\left(\frac{a}{b}\right)^{x-1}=\left(\frac{a}{b}\right)^{-(x-3)}=\left(\frac{a}{b}\right)^{(3-x)}$

$\Leftrightarrow \quad x-1=3-x \quad\Leftrightarrow\quad 2x=4 \quad\Leftrightarrow\quad x=2.$

42. $2^{2n-1}=\frac{1}{8^{n-3}}\Leftrightarrow 2^{2n-1}=\frac{1}{(2^3)^{n-3}}$

$=\frac{1}{2^{3(n-3)}}=\frac{1}{2^{(3n-9)}}=2^{(9-3n)}$

$\Leftrightarrow \quad 2n-1=9-3n \quad\Leftrightarrow\quad 5n=10$

$\Leftrightarrow \quad n=2.$

43. $5^a=3125=5^5\Rightarrow a=5.\Rightarrow 5^{(a-3)}=5^{(5-3)}=5^2=25.$

44. $5\sqrt{5}\times5^3\div5^{-\frac{3}{2}}=5^{a+2}\quad\Leftrightarrow\quad\frac{5\times5^{\frac{1}{2}}\times5^3}{5^{-\frac{3}{2}}}=5^{a+2}$

$\Leftrightarrow\quad 5^{\left(1+\frac{1}{2}+3+\frac{3}{2}\right)}=5^{a+2}\Leftrightarrow 5^6=5^{a+2}$

$\Leftrightarrow \quad a+2=6\Leftrightarrow a=4.$

45. $\sqrt{2^n}=64\quad\Leftrightarrow\quad(2^n)^{\frac{1}{2}}=2^6\quad\Leftrightarrow\quad 2^{\frac{n}{2}}=2^6\quad\Leftrightarrow\quad\frac{n}{2}=6$

$\Leftrightarrow\quad n=12.$

46. $(\sqrt{3})^5\times9^2=3^n\times3\sqrt{3}\quad\Leftrightarrow\quad\left(3^{\frac{1}{2}}\right)^5\times(3^2)^2=3^n\times3\times3^{\frac{1}{2}}$

$\Leftrightarrow\quad 3^{\left(\frac{1}{2}\times5\right)}\times3^{(2\times2)}=3^{\left(n+1+\frac{1}{2}\right)}$

$\Leftrightarrow\quad 3^{\left(\frac{5}{2}+4\right)}=3^{\left(n+\frac{3}{2}\right)}\Leftrightarrow n+\frac{3}{2}=\frac{13}{2}$

$\Leftrightarrow\quad n=\left(\frac{13}{2}-\frac{3}{2}\right)=\frac{10}{2}=5.$

47. $\frac{9^n\times3^5\times(27)^3}{3\times(81)^4}=27\quad\Leftrightarrow\quad\frac{(3^2)^n\times3^5\times(3^3)^3}{3\times(3^4)^4}=3^3$

$\Leftrightarrow\quad\frac{3^{2n}\times3^5\times3^{(3\times3)}}{3\times3^{(4\times4)}}=3^3$

$\Leftrightarrow\quad\frac{3^{2n+5+9}}{3\times3^{16}}=3^3\Leftrightarrow\frac{3^{2n+14}}{3^{17}}=3^3$

$\Leftrightarrow\quad 3^{(2n+14-17)}=3^3$

$\Leftrightarrow\quad 3^{2n-3}=3^3\quad\Leftrightarrow\quad 2n-3=3\quad\Leftrightarrow\quad 2n=6\quad\Leftrightarrow\quad n=3.$

48. $\left(\frac{9}{4}\right)^x\cdot\left(\frac{8}{27}\right)^{x-1}=\frac{2}{3}\quad\Leftrightarrow\quad\frac{9^x}{4^x}\times\frac{8^{(x-1)}}{27^{(x-1)}}=\frac{2}{3}$

$\Leftrightarrow\quad\frac{(3^2)^x}{(2^2)^x}\times\frac{(2^3)^{(x-1)}}{(3^3)^{(x-1)}}=\frac{2}{3}\quad\Leftrightarrow\quad\frac{3^{2x}\times2^{3(x-1)}}{2^{2x}\times3^{3(x-1)}}=\frac{2}{3}$

$\Leftrightarrow\quad\frac{2^{(3x-3-2x)}}{3^{(3x-3-2x)}}=\frac{2}{3}\Leftrightarrow\frac{2^{(x-3)}}{3^{(x-3)}}=\frac{2}{3}\Leftrightarrow\left(\frac{2}{3}\right)^{(x-3)}=\left(\frac{2}{3}\right)^1$

$\Leftrightarrow\quad x-3=1\quad\Leftrightarrow x=4.$

49. $2^x=\sqrt[3]{32}\quad\Leftrightarrow\quad 2^x=(32)^{\frac{1}{3}}=(2^5)^{\frac{1}{3}}=2^{\frac{5}{3}}\quad\Leftrightarrow\quad x=\frac{5}{3}.$

50. $2^x\times8^{\frac{1}{5}}=2^{\frac{1}{5}}\quad\Leftrightarrow\quad 2^x\times(2^3)^{\frac{1}{5}}=2^{\frac{1}{5}}\quad\Leftrightarrow\quad 2^x\times2^{\frac{3}{5}}=2^{\frac{1}{5}}$

$\Leftrightarrow\quad 2^{\left(x+\frac{3}{5}\right)}=2^{\frac{1}{5}}$

$\Leftrightarrow\quad x+\frac{3}{5}=\frac{1}{5}\quad\Leftrightarrow\quad x=\left(\frac{1}{5}-\frac{3}{5}\right)=\frac{-2}{5}.$

51. $5^{(x+3)}=25^{(3x-4)}\Leftrightarrow 5^{(x+3)}=(5^2)^{(3x-4)}\Leftrightarrow 5^{(x+3)}$

$=5^{2(3x-4)}\quad\Leftrightarrow\quad 5^{(x+3)}=5^{(6x-8)}$

$\Leftrightarrow\quad x+3=6x-8\quad\Leftrightarrow\quad 5x=11$

$\Leftrightarrow\quad x=\frac{11}{5}.$

52. $\frac{2^{n+4}-2(2^n)}{2(2^{n+3})}=\frac{2^{n+4}-2^{n+1}}{2^{n+4}}=\frac{2^{n+4}}{2^{n+4}}-\frac{2^{n+1}}{2^{n+4}}$

$=1-2^{n+1-(n+4)}=1-2^{-3}=1-\frac{1}{8}=\frac{7}{8}.$

53. $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4\left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4=\left[\left\{(a^9)^{\frac{1}{6}}\right\}^{\frac{1}{3}}\right]^4\cdot\left[\left\{(a^9)^{\frac{1}{3}}\right\}^{\frac{1}{6}}\right]^4$

$=a^{\left(9\times\frac{1}{6}\times\frac{1}{3}\times4\right)}.a^{\left(9\times\frac{1}{3}\times\frac{1}{6}\times4\right)}$

$=a^2\,.\,a^2=a^4.$

54. $(256)^{0.16}\times(256)^{0.09}=(256)^{(0.16+0.09)}=(256)^{0.25}$

$=(256)^{\left(\frac{25}{100}\right)}$

$=(256)^{\frac{1}{4}}=(4^4)^{\frac{1}{4}}=4^{\left(4\times\frac{1}{4}\right)}=4^1=4.$

55. $(0.04)^{-1.5}=\left(\frac{4}{100}\right)^{-1.5}=\left(\frac{1}{25}\right)^{-\frac{3}{2}}=(25)^{\frac{3}{2}}=(5^2)^{\frac{3}{2}}$

$=5^{\left(2\times\frac{3}{2}\right)}=5^3=125.$

56. Let $(17)^{3.5}\times(17)^x=17^8$. Then, $(17)^{(3.5+x)}=(17)^8$.

$\therefore\quad 3.5+x=8\quad\Leftrightarrow\quad x=(8-3.5)$

$\Leftrightarrow\quad x=4.5.$

57. Let $6^{1.2}\times36^x\times30^{2.4}\times25^{1.3}=30^5$.

Then, $6^{1.2}\times(6^2)^x\times(6\times5)^{2.4}\times(5^2)^{1.3}=30^5$

$\Leftrightarrow\quad 6^{1.2}\times6^{2x}\times6^{2.4}\times5^{2.4}\times5^{2.6}=(6\times5)^5$

$\Leftrightarrow\quad 6^{(1.2+2x+2.4)}\times5^{(2.4+2.6)}=6^5\times5^5$

$\Leftrightarrow\quad 6^{(3.6+2x)}\times5^5=6^5\times5^5\Leftrightarrow 3.6+2x=5\Leftrightarrow 2x=1.4$

$\Leftrightarrow\quad x=0.7.$

58. Let $2^{3.6}\times4^{3.6}\times4^{3.6}\times32^{2.3}=32^x$.

Then, $2^{3.6}\times(2^2)^{3.6}\times(2^2)^{3.6}\times(2^5)^{2.3}=(2^5)^x$

$\Leftrightarrow\quad 2^{3.6}\times2^{(2\times3.6)}\times2^{(2\times3.6)}\times(2^5)^{2.3}=(2^5)^x$

$\Leftrightarrow\quad 2^{(3.6+7.2+7.2)}\times(2^5)^{2.3}=(2^5)^x\Leftrightarrow 2^{18}\times(2^5)^{2.3}=(2^5)^x$

$\Leftrightarrow\quad (2^5)^{3.6}\times(2^5)^{2.3}=(2^5)^x\quad\Leftrightarrow\quad(2^5)^{(3.6+2.3)}=(2^5)^x$

$\Leftrightarrow\quad (2^5)^{5.9}=(2^5)^x\Leftrightarrow x=5.9.$

59. Let $3^{3.5}\times(21)^2\times(42)^{2.5}\div2^{2.5}\times7^{3.5}=21^x$.

Then, $3^{3.5}\times7^{3.5}\times(21)^2\times(21\times2)^{2.5}\div2^{2.5}=21^x$

$\Leftrightarrow\quad (21)^x=(3\times7)^{3.5}\times(21)^2\times(21)^{2.5}\times2^{2.5}\div2^{2.5}$

$\Leftrightarrow (21)^x = (21)^{3.5} \times (21)^{(2+2.5)} \Leftrightarrow (21)^x = (21)^{3.5} \times (21)^{(4.5)}$

$\Leftrightarrow (21)^x = (21)^{(3.5+4.5)} = (21)^8$

$\Leftrightarrow x = 8.$

60. $8^{0.4} \times 4^{1.6} \times 2^{1.6} = (2^3)^{0.4} \times (2^2)^{1.6} \times 2^{1.6}$

$= 2^{(3 \times 0.4)} \times 2^{(2 \times 1.6)} \times 2^{1.6}$

$= 2^{1.2} \times 2^{3.2} \times 2^{1.6} = 2^{(1.2+3.2+1.6)} = 2^6 = 64.$

61. Let $8^7 \times 2^6 \div 8^{2.4} = 8^x$.

Then, $8^7 \times (2^3)^2 \div 8^{2.4} = 8^x$

$\Leftrightarrow 8^7 \times 8^2 \div 8^{2.4} = 8^x$

$\Leftrightarrow 8^x = 8^{(7+2-2.4)} \Leftrightarrow 8^x = 8^{6.6} \Leftrightarrow x = 6.6.$

62. Let $25^{2.7} \times 5^{4.2} \div 5^{5.4} = 25^x$.

Then, $(25)^{2.7} \times 5^{(4.2-5.4)} = 25^x$

$\Leftrightarrow (25)^{2.7} \times 5^{(-1.2)} = 25^x \Leftrightarrow (25)^{2.7} \times \frac{1}{5^{1.2}} = 25^x$

$\Leftrightarrow \frac{(25)^{2.7}}{(5^2)^{0.6}} = 25^x \Leftrightarrow \frac{(25)^{2.7}}{(25)^{0.6}} = 25^x$

$\Leftrightarrow 25^x = 25^{(2.7-0.6)} = 25^{2.1} \Leftrightarrow x = 2.1$

63. Let $8^{2.4} \times 2^{3.7} \div (16)^{1.3} = 2^x$.

Then, $(2^3)^{2.4} \times 2^{3.7} \div (2^4)^{1.3} = 2^x$

$\Leftrightarrow 2^{(3 \times 2.4)} \times 2^{3.7} \div 2^{(4 \times 1.3)} = 2^x$

$\Leftrightarrow 2^{7.2} \times 2^{3.7} \div 2^{5.2} = 2^x$

$\Leftrightarrow 2^x = 2^{(7.2+3.7-5.2)} = 2^{5.7} \Leftrightarrow x = 5.7.$

64. Let $(0.04)^2 \div (0.008) \times (0.2)^6 = (0.2)^x$.

Then, $(0.2)^x = [(0.2)^2]^2 \div (0.2)^3 \times (0.2)^6$

$\Leftrightarrow (0.2)^x = (0.2)^{(2 \times 2)} \div (0.2)^3 \times (0.2)^6$

$\Leftrightarrow (0.2)^x = (0.2)^4 \div (0.2)^3 \times (0.2)^6 = (0.2)^{(4-3+6)} = (0.2)^7$

$\Leftrightarrow x = 7.$

65. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^x$

$\Leftrightarrow (18)^{3.5} \times \frac{1}{(27)^{3.5}} \times 6^{3.5} = 2^x$

$\Leftrightarrow (3^2 \times 2)^{3.5} \times \frac{1}{(3^3)^{3.5}} \times (2 \times 3)^{3.5} = 2^x 11$

$\Leftrightarrow 3^{(2 \times 3.5)} \times 2^{3.5} \times \frac{1}{3^{(3 \times 3.5)}} \times 2^{3.5} \times 3^{3.5} = 2^x$

$\Leftrightarrow 3^7 \times 2^{3.5} \times \frac{1}{3^{10.5}} \times 2^{3.5} \times 3^{3.5} = 2^x$

$\Leftrightarrow \frac{3^{(7+3.5)}}{3^{10.5}} \times 2^{(3.5+3.5)} \Leftrightarrow 2^7 = 2^x \Leftrightarrow x = 7.$

66. Let $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^x$.

Then, $\frac{(5^2)^{7.5} \times (5)^{2.5}}{(5^3)^{1.5}} = 5^x \Leftrightarrow \frac{5^{(2 \times 7.5)} \times 5^{2.5}}{5^{(3 \times 1.5)}} = 5^x$

$\Leftrightarrow 1\frac{5^{15} \times 5^{2.5}}{5^{4.5}} = 5^x$

$\Leftrightarrow 5^x = 5^{(15+2.5-4.5)} = 5^{13} \Leftrightarrow x = 13.$

67. $\frac{(243)^{0.13} \times (243)^{0.07}}{7^{0.25} \times (49)^{0.075} \times (343)^{0.2}} = \frac{(243)^{(0.13+0.07)}}{7^{0.25} \times (7^2)^{0.075} \times (7^3)^{0.2}}$

$= \frac{(243)^{0.2}}{7^{0.25} \times 7^{(2 \times 0.075)} \times 7^{(3 \times 0.2)}}$

$= \frac{(3^5)^{0.2}}{7^{0.25} \times 7^{0.15} \times 7^{0.6}} = \frac{3^{(5 \times 0.2)}}{7^{(0.25+0.15+0.6)}} = \frac{3^1}{7^1} = \frac{3}{7}.$

68. $(64x^3 \div 27a^{-3})^{-\frac{2}{3}} = \left(\frac{64x^3}{27a^{-3}}\right)^{-\frac{2}{3}} = \left(\frac{4^3 . x^3}{3^3 . a^{-3}}\right)^{-\frac{2}{3}}$

$= \left(\frac{4^3 .x^3 .a^3}{3^3}\right)^{\frac{2}{3}}$

$= \frac{\{(4ax)^3\}^{-\frac{2}{3}}}{3^{3 \times \left(-\frac{2}{3}\right)}} = \frac{(4ax)^{3 \times \left(-\frac{2}{3}\right)}}{3^{-2}} = \frac{(4ax)^{-2}}{3^{-2}}$

$= \frac{3^2}{(4ax)^2} = \frac{9}{16a^2x^2}.$

69. $2^{n+4} - 2^{n+2} = 3 \Leftrightarrow 2^{n+2}(2^2 - 1) = 3$

$\Leftrightarrow 2^{n+2} = 1 = 2^0$

$\Leftrightarrow n + 2 = 0 \Leftrightarrow n = -2.$

70. $2^{n-1} + 2^{n+1} = 320 \Leftrightarrow 2^{n-1}(1 + 2^2) = 320$

$\Leftrightarrow 5 \times 2^{n-1} = 320$

$\Leftrightarrow 2^{n-1} = \frac{320}{5} = 64 = 2^6$

$\Leftrightarrow n - 1 = 6 \Leftrightarrow n = 7.$

71. $3^x - 3^{x-1} = 18 \Leftrightarrow 3^{x-1}(3 - 1) = 18$

$\Leftrightarrow 3^{x-1} = 9 = 3^2 \Leftrightarrow x - 1 = 2 \Leftrightarrow x = 3.$

$\therefore \quad x^x = 3^3 = 27.$

72. $\frac{2^{n+4} - 2 \times 2^n}{2 \times 2^{n+3}} + 2^{-3} = \frac{2^{n+4} - 2^{n+1}}{2^{n+4}} + \frac{1}{2^3}$

$= \frac{2^{n+1}(2^3 - 1)}{2^{n+4}} + \frac{1}{2^3}$

$= \frac{2^{n+1} \times 7}{2^{n+1} \times 2^3} + \frac{1}{2^3} = \left(\frac{7}{8} + \frac{1}{8}\right) = \frac{8}{8} = 1.$

73. $\frac{2^{3x+4} + 8^{x+1}}{8^{x+1} - 2^{3x+2}} = \frac{2^{3x+4} + (2^3)^{x+1}}{(2^3)^{x+1} - 2^{3x+2}} = \frac{2^{3x+4} + 2^{3(x+1)}}{2^{3(x+1)} - 2^{3x+2}}$

$= \frac{2^{3x+4} + 2^{3x+3}}{2^{3x+3} - 2^{3x+2}} = \frac{2^{3x+3}(2+1)}{2^{3x+2}(2-1)}$

$= 3.2^{(3x+3)-(3x+2)} = 3.2^1$

$= 3 \times 2 = 6.$

74. $\frac{2^{n-1} - 2^n}{2^{n+4} + 2^{n+1}} = \frac{2^{n-1}(1-2)}{2^{n+1}(2^3+1)} = \left(-\frac{1}{9}\right).2^{(n-1)-(n+1)}$

$= \left(-\frac{1}{9}\right).2^{-2} = \left(-\frac{1}{9}\right) \times \frac{1}{2^2} = \left(-\frac{1}{9}\right) \times \frac{1}{4}$

$= -\frac{1}{36}.$

75. $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} - 2 = (5 + 2\sqrt{6}) + \frac{1}{(5+2\sqrt{6})} - 2$

$= (5 + 2\sqrt{6}) + \frac{1}{(5+2\sqrt{6})} \times \frac{(5-2\sqrt{6})}{(5-2\sqrt{6})} - 2$

$= (5 + 2\sqrt{6}) + (5 - 2\sqrt{6}) - 2 = 8.$

$\therefore \quad \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=\sqrt{8}=2\sqrt{2}.$

76. $4+\sqrt{7} = \frac{7}{2}+\frac{1}{2}+2\times\frac{\sqrt{7}}{\sqrt{2}}\times\frac{1}{\sqrt{2}}$

$= \left(\frac{\sqrt{7}}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+2\times\frac{\sqrt{7}}{\sqrt{2}}\times\frac{1}{\sqrt{2}}$

$= \left(\frac{\sqrt{7}}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)^2=\frac{1}{2}(\sqrt{7}+1)^2.$

77. $\sqrt{8-2\sqrt{15}} = \sqrt{5+3-2\times\sqrt{5}\times\sqrt{3}}$

$= \sqrt{(\sqrt{5})^2+(\sqrt{3})^2-2\times\sqrt{5}\times\sqrt{3}}$

$= \sqrt{(\sqrt{5}-\sqrt{3})^2}=(\sqrt{5}-\sqrt{3}).$

78. $\sqrt{6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}} = \sqrt{6-4\sqrt{3}+\sqrt{12+4-8\sqrt{3}}}$

$= \sqrt{6-4\sqrt{3}+\sqrt{(2\sqrt{3})^2+(2)^2-2\times2\sqrt{3}\times2}}$

$= \sqrt{6-4\sqrt{3}+\sqrt{(2\sqrt{3}-2)^2}}=\sqrt{6-4\sqrt{3}+2\sqrt{3}-2}$

$= \sqrt{(\sqrt{3})^2+(1)^2-2\times\sqrt{3}\times1}=\sqrt{(\sqrt{3}-1)^2}=\sqrt{3}-1.$

79. $\frac{1}{\sqrt{12-\sqrt{140}}}-\frac{1}{\sqrt{8-\sqrt{60}}}-\frac{2}{\sqrt{10+\sqrt{84}}}$

$= \frac{1}{\sqrt{12-\sqrt{4\times35}}}-\frac{1}{\sqrt{8-\sqrt{4\times15}}}-\frac{2}{\sqrt{10+\sqrt{4\times21}}}$

$= \frac{1}{\sqrt{12-2\sqrt{35}}}-\frac{1}{\sqrt{8-2\sqrt{15}}}-\frac{2}{\sqrt{10+2\sqrt{21}}}$

$= \frac{1}{\sqrt{7+5-2\sqrt{35}}}-\frac{1}{\sqrt{5+3-2\sqrt{15}}}-\frac{2}{\sqrt{7+3+2\sqrt{21}}}$

$= \frac{1}{\sqrt{(\sqrt{7})^2+(\sqrt{5})^2-2\times\sqrt{7}\times\sqrt{5}}}-\frac{1}{\sqrt{(\sqrt{5})^2+(\sqrt{3})^2-2\times\sqrt{5}\times\sqrt{3}}}-\frac{2}{\sqrt{(\sqrt{7})^2+(\sqrt{3})^2+2\times\sqrt{7}\times\sqrt{3}}}$

$= \frac{1}{\sqrt{(\sqrt{7}-\sqrt{5})^2}}-\frac{1}{\sqrt{(\sqrt{5}-\sqrt{3})^2}}-\frac{2}{\sqrt{(\sqrt{7}+\sqrt{3})^2}}$

$= \frac{1}{(\sqrt{7}-\sqrt{5})}-\frac{1}{(\sqrt{5}-\sqrt{3})}-\frac{2}{(\sqrt{7}+\sqrt{3})}$

$= \frac{1}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}-\frac{1}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}-\frac{2}{\sqrt{7}+\sqrt{3}}\times\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$= \frac{\sqrt{7}+\sqrt{5}}{7-5}-\frac{\sqrt{5}+\sqrt{3}}{5-3}-\frac{2(\sqrt{7}-\sqrt{3})}{7-3}$

$= \frac{(\sqrt{7}+\sqrt{5})}{2}-\frac{(\sqrt{5}+\sqrt{3})}{2}-\frac{(\sqrt{7}-\sqrt{3})}{2}$

$= \frac{\sqrt{7}+\sqrt{5}-\sqrt{5}-\sqrt{3}-\sqrt{7}+\sqrt{3}}{2}=0.$

80. $\sqrt{4+\sqrt{15}} = \sqrt{\frac{5}{2}+\frac{3}{2}+2\times\frac{\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{3}}{\sqrt{2}}}$

$= \sqrt{\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2+2\times\frac{\sqrt{5}}{\sqrt{2}}\times\frac{\sqrt{3}}{\sqrt{2}}}$

$= \sqrt{\left(\frac{\sqrt{5}}{\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}}\right)^2}=\frac{\sqrt{5}}{\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}}.$

Similarly, $\sqrt{4-\sqrt{15}}=\frac{\sqrt{5}}{\sqrt{2}}-\frac{\sqrt{3}}{\sqrt{2}}.$

$\sqrt{12-4\sqrt{5}}=\sqrt{10+2-2\times\sqrt{10}\times\sqrt{2}}$

$= \sqrt{(\sqrt{10})^2+(\sqrt{2})^2-2\times\sqrt{10}\times\sqrt{2}}$

$= \sqrt{(\sqrt{10}-\sqrt{2})^2}=(\sqrt{10}-\sqrt{2}).$

$\therefore$ Given expression

$= \left(\frac{\sqrt{5}}{\sqrt{2}}+\frac{\sqrt{3}}{\sqrt{2}}\right)+\left(\frac{\sqrt{5}}{\sqrt{2}}-\frac{\sqrt{3}}{2}\right)-(\sqrt{10}-\sqrt{2})$

$= \frac{2\sqrt{5}}{\sqrt{2}}-\sqrt{10}+\sqrt{2}=\sqrt{10}-\sqrt{10}+\sqrt{2}=\sqrt{2},$

which is an irrational number.

81. Let $X = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}.$

Then $X^2 = \frac{(\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2})^2}{(\sqrt{\sqrt{5}+1})^2}$

$= \frac{(\sqrt{5}+2)+(\sqrt{5}-2)+2\sqrt{(\sqrt{5}+2)(\sqrt{5}-2)}}{(\sqrt{5}+1)}$

$= \frac{2\sqrt{5}+2\sqrt{(\sqrt{5})^2-(2)^2}}{\sqrt{5}+1}=\frac{2\sqrt{5}+2}{\sqrt{5}+1}$

$= \frac{2(\sqrt{5}+1)}{(\sqrt{5}+1)}=2$

$\Rightarrow \quad X=\sqrt{2}.$

$\therefore \quad N=\sqrt{2}-\sqrt{3-2\sqrt{2}}=\sqrt{2}-\sqrt{(\sqrt{2})^2+1^2-2\times\sqrt{2}\times1}$

$= \sqrt{2}-\sqrt{(\sqrt{2}-1)^2}=\sqrt{2}-(\sqrt{2}-1)=1.$

82. $x^z = y^2 \Leftrightarrow (10^{0.48})^z = (10^{0.70})^2$

$\Leftrightarrow 10^{(0.48z)} = 10^{(2\times0.70)} = 10^{1.40}$

$\Leftrightarrow 0.48z = 1.40$

$\Leftrightarrow z=\frac{140}{48}=\frac{35}{12}=2.9$ (approx.).

83. We know that $11^2 = 121$.

Putting $m = 11$ and $n = 2$, we get :

$(m-1)^{n+1} = (11-1)^{(2+1)} = 10^3 = 1000.$

84. $(216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}} = (3^3 \times 2^3)^{\frac{3}{5}} \times (5^4 \times 2^2)^{\frac{2}{5}} \times (5^2 \times 2^2 \times 3)^{\frac{1}{5}}$

$= 3^{\left(3\times\frac{3}{5}\right)} \times 2^{\left(3\times\frac{3}{5}\right)} \times 5^{\left(4\times\frac{2}{5}\right)} \times 2^{\left(2\times\frac{2}{5}\right)} \times 5^{\left(2\times\frac{1}{5}\right)} \times 2^{\left(2\times\frac{1}{5}\right)} \times 3^{\frac{1}{5}}$

$= 3^{\frac{9}{5}} \times 2^{\frac{9}{5}} \times 5^{\frac{8}{5}} \times 2^{\frac{4}{5}} \times 5^{\frac{2}{5}} \times 2^{\frac{2}{5}} \times 3^{\frac{1}{5}}$

$= 3^{\left(\frac{9}{5}+\frac{1}{5}\right)} \times 2^{\left(\frac{9}{5}+\frac{4}{5}+\frac{2}{5}\right)} \times 5^{\left(\frac{8}{5}+\frac{2}{5}\right)} = 3^2 \times 2^3 \times 5^2.$

Hence, the number of prime factors = (2 + 3 + 2) = 7.

85. $\frac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}} = \frac{(2\times3)^{12} \times (5\times7)^{28} \times (3\times5)^{16}}{(2\times7)^{12} \times (3\times7)^{11}}$

$= \frac{2^{12} \times 3^{12} \times 5^{28} \times 7^{28} \times 3^{16} \times 5^{16}}{2^{12} \times 7^{12} \times 3^{11} \times 7^{11}}$

$= 2^{(12-12)} \times 3^{(12+16-11)} \times 5^{(28+16)} \times 7^{(28-12-11)}$

$= 2^0 \times 3^{17} \times 5^{44} \times 7^5 = 3^{17} \times 5^{44} \times 7^5.$

Number of prime factors = 17 + 44 + 5 = 66.

86. $1 + (3 + 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)$

$= 1 + \frac{1}{2}[(3 - 1)(3 + 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$

$= 1 + \frac{1}{2}[(3^2 - 1)(3^2 + 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$

$= 1 + \frac{1}{2}[(3^4 - 1)(3^4 + 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$

$= 1 + \frac{1}{2}[(3^8 - 1)(3^8 + 1)(3^{16} + 1)(3^{32} + 1)]$

$= 1 + \frac{1}{2}[(3^{16} - 1)(3^{16} + 1)(3^{32} + 1)]$

$= 1 + \frac{1}{2}[(3^{32} - 1)(3^{32} + 1)]$

$= 1 + \frac{1}{2}(3^{64} - 1) = \frac{2 + 3^{64} - 1}{2} = \frac{3^{64} + 1}{2}.$

87. $\frac{1}{1 + a^{(n-m)}} + \frac{1}{1 + a^{(m-n)}} = \frac{1}{\left(1 + \frac{a^n}{a^m}\right)} + \frac{1}{\left(1 + \frac{a^m}{a^n}\right)}$

$= \frac{a^m}{(a^m + a^n)} + \frac{a^n}{(a^m + a^n)} = \frac{(a^m + a^n)}{(a^m + a^n)} = 1.$

88. $(x^a)^{a^2 - bc} \cdot (x^b)^{b^2 - ca} \cdot (x^c)^{c^2 - ab}$

$= x^{[a(a^2 - bc)]} \cdot x^{[b(b^2 - ca)]} \cdot x^{[c(c^2 - ab)]}$

$= x^{(a^3 - abc)} \cdot x^{(b^3 - abc)} \cdot x^{(c^3 - abc)}$

$= x^{(a^3 - abc + b^3 - abc + c^3 - abc)} = x^{(a^3 + b^3 + c^3 - 3abc)}$

$= x^{(3abc - 3abc)} = x^0 = 1.$

[$\because$ If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$]

89. Given Exp. $= \frac{1}{\left(1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}\right)} + \frac{1}{\left(1 + \frac{x^a}{x^b} + \frac{x^c}{x^b}\right)} + \frac{1}{\left(1 + \frac{x^b}{x^c} + \frac{x^a}{x^c}\right)}$

$= \frac{x^a}{(x^a + x^b + x^c)} + \frac{x^b}{(x^a + x^b + x^c)} + \frac{x^c}{(x^a + x^b + x^c)}$

$= \frac{(x^a + x^b + x^c)}{(x^a + x^b + x^c)} = 1.$

90. Given Exp.

$= x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \cdot x^{(a-b)(a+b-c)}$

$= x^{(b-c)(b+c)-a(b-c)} \cdot x^{(c-a)(c+a)-b(c-a)} \cdot x^{(a-b)(a+b)-c(a-b)}$

$= x^{(b^2 - c^2 + c^2 - a^2 + a^2 - b^2)} \cdot x^{-a(b-c)-b(c-a)-c(a-b)}$

$= (x^0 \times x^0) = (1 \times 1) = 1.$

91. Given Exp. $= x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c-a)(c+a)}$

$= x^{(a^2 - b^2)} \cdot x^{(b^2 - c^2)} \cdot x^{(c^2 - a^2)}$

$= x^{(a^2 - b^2 + b^2 - c^2 + c^2 - a^2)} = x^0 = 1.$

92. Given Exp. $= \{x^{(a-b)}\}^{\frac{1}{ab}} \cdot \{x^{(b-c)}\}^{\frac{1}{bc}} \cdot \{x^{(c-a)}\}^{\frac{1}{ca}}$

$= x^{\frac{(a-b)}{ab}} \cdot x^{\frac{(b-c)}{bc}} \cdot x^{\frac{(c-a)}{ca}}$

$= x^{\left\{\frac{(a-b)}{ab} + \frac{(b-c)}{bc} + \frac{(c-a)}{ca}\right\}}$

$= x^{\left(\frac{1}{b} - \frac{1}{a}\right) + \left(\frac{1}{c} - \frac{1}{b}\right) + \left(\frac{1}{a} - \frac{1}{c}\right)} = x^0 = 1.$

93. Given Exp. $= \frac{\left(\frac{xy+1}{y}\right)^a \cdot \left(\frac{xy-1}{y}\right)^b}{\left(\frac{xy+1}{x}\right)^a \cdot \left(\frac{xy-1}{x}\right)^b}$

$= \frac{(xy+1)^a \cdot (xy-1)^b \cdot x^a \cdot x^b}{(xy+1)^a \cdot (xy-1)^b \cdot y^a \cdot y^b}$

$= \frac{x^{a+b}}{y^{a+b}} = \left(\frac{x}{y}\right)^{a+b}.$

94. Given Exp. $= x^{\frac{b+c}{(a-b)(c-a)}} \cdot x^{\frac{c+a}{(a-b)(b-c)}} \cdot x^{\frac{a+b}{(b-c)(c-a)}}$

$= x^{\frac{(b+c)(b-c)+(c+a)(c-a)+(a+b)(a-b)}{(a-b)(b-c)(c-a)}}$

$= x^{\frac{(b^2 - c^2)+(c^2 - a^2)+(a^2 - b^2)}{(a-b)(b-c)(c-a)}} = x^0 = 1.$

95. Let $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}} = k$. Then, $x = k^p$, $y = k^q$, $z = k^r$.

$\therefore \quad xyz = 1 \Rightarrow k^p \cdot k^q \cdot k^r = 1 = k^0$

$\Rightarrow \quad k^{(p+q+r)} = k^0 \Rightarrow p + q + r = 0.$

96. Let $a^x = b^y = c^z = k$.

Then, $a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}$ and $c = k^{\frac{1}{z}}$.

$\therefore \quad b^2 = ac \Leftrightarrow \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}} \Leftrightarrow k^{\left(\frac{2}{y}\right)} = k^{\left(\frac{1}{x}+\frac{1}{z}\right)}$

$\therefore \quad \frac{2}{y} = \frac{(x+z)}{xz} \Leftrightarrow \frac{y}{2} = \frac{xz}{(x+z)} \Leftrightarrow y = \frac{2xz}{(x+z)}.$

97. $a^1 = c^z = (b^y)^z = b^{yz} = (a^x)^{yz} = a^{xyz} \Rightarrow xyz = 1.$

98. $2^x = 4^y = 8^z \Leftrightarrow 2^x = 2^{2y} = 2^{3z} \Leftrightarrow x = 2y = 3z.$

$\therefore \quad \frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7} \Leftrightarrow \frac{1}{6z} + \frac{1}{6z} + \frac{1}{6z} = \frac{24}{7}$

$\Leftrightarrow \quad \frac{3}{6z} = \frac{24}{7} \Leftrightarrow z = \left(\frac{3}{6} \times \frac{7}{24}\right) = \frac{7}{48}.$

99. $8 = 7^d = (6^c)^d = 6^{cd} = (5^b)^{cd} = 5^{bcd} = (4^a)^{bcd} = 4^{abcd}$

$\Rightarrow \quad 4^{abcd} = 8 \Rightarrow (2^2)^{abcd} = 2^3 \Rightarrow 2abcd = 3$

$\Rightarrow \quad abcd = \frac{3}{2}.$

100. Given Exp. $= \frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$

$= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{b^{-1}+1+b^{-1}c^{-1}} + \frac{a}{a+ac+1}$

$= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{1+b^{-1}+a} + \frac{a}{a+b^{-1}+1}$

$= \frac{1+a+b^{-1}}{1+a+b^{-1}} = 1.$

$[\because abc = 1 \Rightarrow (bc)^{-1} = a \Rightarrow b^{-1}c^{-1} = a \text{ and } ac = b^{-1}]$

101. $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a} = (a^{-1})^{\frac{1}{2}} \cdot b^{\frac{1}{2}} \cdot (b^{-1})^{\frac{1}{2}} \cdot c^{\frac{1}{2}} \cdot (c^{-1})^{\frac{1}{2}} \cdot a^{\frac{1}{2}}$

$= (a^{-1}a)^{\frac{1}{2}} \cdot (b \cdot b^{-1})^{\frac{1}{2}} \cdot (c \cdot c^{-1})^{\frac{1}{2}}$

$= (1)^{\frac{1}{2}} \cdot (1)^{\frac{1}{2}} \cdot (1)^{\frac{1}{2}} = (1 \times 1 \times 1) = 1.$

102. $3^{x-y} = 27 = 3^3$

$\Leftrightarrow \quad x - y = 3 \qquad \ldots (i)$

$3^{x+y} = 243 = 3^5$

$\Leftrightarrow \quad x + y = 5 \qquad \ldots (ii)$

On solving (i) and (ii), we get $x = 4$.

103. Let $x^y = y^x = k$.

Then, $x = k^{\frac{1}{y}}$ and $y = k^{\frac{1}{x}}$.

$\therefore \quad \left(\frac{x}{y}\right)^{\frac{x}{y}} = \left(\frac{k^{\frac{1}{y}}}{k^{\frac{1}{x}}}\right)^{\frac{x}{y}} = k^{\left[\left(\frac{1}{y}-\frac{1}{x}\right)\frac{x}{y}\right]} = k^{\left(\frac{x-y}{xy}\right)\frac{x}{y}} = k^{\left(\frac{x-y}{y^2}\right)}$

$= (x^y)^{\left(\frac{x-y}{y^2}\right)} = x^{\left(\frac{x-y}{y}\right)} = x^{\left(\frac{x}{y}-1\right)}.$

104. $4^{x+y} = 1 = 4^0 \Leftrightarrow x + y = 0 \qquad \ldots (i)$

$4^{x-y} = 4 = 4^1 \Rightarrow x - y = 1 \qquad \ldots (ii)$

Adding (i) and (ii), we get : $2x = 1$ or $x = \frac{1}{2}$.

Putting $x = \frac{1}{2}$ in (i), we get : $y = -\frac{1}{2}$.

105. $2^{2x-1} + 4^x = 2^{x-\frac{1}{2}} + 2^{x+\frac{1}{2}} \Leftrightarrow 2^{2x-1} + 2^{2x} = 2^{x-\frac{1}{2}} + 2^{x+\frac{1}{2}}$

$\Leftrightarrow \quad 2^{(2x-1)}(1+2) = 2^{\left(x-\frac{1}{2}\right)}(1+2)$

$\Leftrightarrow \quad 2^{(2x-1)} = 2^{\left(x-\frac{1}{2}\right)} \Leftrightarrow 2x - 1 = x - \frac{1}{2} \Leftrightarrow x = \frac{1}{2}.$

106. $3^{2x-y} = 3^{x+y} = \sqrt{3^3} = 3^{\frac{3}{2}} \Leftrightarrow 2x - y = \frac{3}{2}$ and $x + y = \frac{3}{2}$

$\Leftrightarrow \quad 3x = \frac{3}{2} + \frac{3}{2} = 3 \Leftrightarrow x = 1.$

$\therefore \quad y = \left(\frac{3}{2} - 1\right) = \frac{1}{2}.$

107. Let $3^x = 5^y = 45^z = k$. Then, $3 = k^{\frac{1}{x}}, 5 = k^{\frac{1}{y}}, 45 = k^{\frac{1}{z}}$.

$45 = 3^2 \times 5$

$\Leftrightarrow \quad k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^2 \cdot \left(k^{\frac{1}{y}}\right) = k^{\frac{2}{x}} \cdot k^{\frac{1}{y}} = k^{\left(\frac{2}{x}+\frac{1}{y}\right)}$

$\Leftrightarrow \quad \frac{1}{z} = \frac{2}{x} + \frac{1}{y} \Leftrightarrow \frac{2}{x} = \frac{1}{z} - \frac{1}{y}.$

108. $2^x = 8^{y+1} \Leftrightarrow 2^x = (2^3)^{y+1} = 2^{(3y+3)}$

$\Leftrightarrow x = 3y + 3 \Leftrightarrow x - 3y = 3 \qquad \ldots (i)$

$9^y = 3^{x-9} \Leftrightarrow (3^2)^y = 3^{x-9}$

$\Leftrightarrow 2y = x - 9 \Leftrightarrow x - 2y = 9 \qquad \ldots (ii)$

Subtracting (i) from (ii), we get: $y = 6$. Putting $y = 6$ (i), we get $x = 21$.

$\therefore \quad x + y = 21 + 6 = 27.$

109. $2^{0.7x} \cdot 3^{-1.25y} = \frac{8\sqrt{6}}{27}$

$\Leftrightarrow \quad \frac{2^{0.7x}}{3^{1.25y}} = \frac{2^3 \cdot 2^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}}{3^3} = \frac{2^{\left(3+\frac{1}{2}\right)}}{3^{\left(3-\frac{1}{2}\right)}} = \frac{2^{\frac{7}{2}}}{3^{\frac{5}{2}}} = \frac{2^{3.5}}{3^{2.5}}.$

$\therefore \quad 0.7x = 3.5 \Rightarrow x = \frac{3.5}{0.7} = 5$ and $1.25y = 2.5$

$\Rightarrow \quad y = \frac{2.5}{1.25} = 2.$

110. $r = (2a)^{2b} = 2^{2b} \times a^{2b} = (2^2)^b \times (a^b)^2 = 4^b \times (a^b)^2.$

Also, $r = a^b \times x^b.$

$\therefore \quad a^b \times x^b = 4^b \times (a^b)^2 \Leftrightarrow x^b = 4^b \times a^b = (4a)^b \Leftrightarrow x = 4a.$

111. L.C.M. of 2, 3, 4, 6 is 12.

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = 2^{\frac{6}{12}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}.$

$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = 3^{\frac{4}{12}} = (3^4)^{\frac{1}{12}} = (81)^{\frac{1}{12}} = \sqrt[12]{81}.$

$\sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = 4^{\frac{3}{12}} = (4^3)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}.$

$\sqrt[6]{6} = 6^{\frac{1}{6}} = 6^{\left(\frac{1}{6}\times\frac{2}{2}\right)} = 6^{\frac{2}{12}} = (6^2)^{\frac{1}{12}} = (36)^{\frac{1}{12}} = \sqrt[12]{36}.$

Clearly, $\sqrt[12]{81}$ *i.e.*, $\sqrt[3]{3}$ is the greatest.

112. L.C.M of 2, 3, 4, 6 is 12.

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{6}{6}\right)} = 2^{\frac{6}{12}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}.$

$\sqrt[6]{3} = 3^{\frac{1}{6}} = 3^{\left(\frac{1}{6}\times\frac{2}{2}\right)} = 3^{\frac{2}{12}} = (3^2)^{\frac{1}{12}} = (9)^{\frac{1}{12}} = \sqrt[12]{9}.$

$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3}\times\frac{4}{4}\right)} = 4^{\frac{4}{12}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}} = \sqrt[12]{256}.$

$\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\left(\frac{1}{4}\times\frac{3}{3}\right)} = 5^{\frac{3}{12}} = (5^3)^{\frac{1}{12}} = (125)^{\frac{1}{12}} = \sqrt[12]{125}.$

Clearly, $\sqrt[12]{256}$ *i.e.*, $\sqrt[3]{4}$ is the greatest.

113. ....

114. $x = 5 + 2\sqrt{6} = 3 + 2 + 2\sqrt{6} = (\sqrt{3})^2 + (\sqrt{2})^2 + 2\times\sqrt{3}\times\sqrt{2}$

$= (\sqrt{3} + \sqrt{2})^2.$

Also, $(x - 1) = 4 + 2\sqrt{6} = 2(2 + \sqrt{6}) = 2\sqrt{2}(\sqrt{2} + \sqrt{3}).$

$\therefore \quad \dfrac{(x-1)}{\sqrt{x}} = \dfrac{2\sqrt{2}(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = 2\sqrt{2}.$

115. $x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}} \Rightarrow \left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)^3 = \left(z^{\frac{1}{3}}\right)^3$

$\Rightarrow \quad x + y + 3x^{\frac{1}{3}}y^{\frac{1}{3}}\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right) = z$

$\Rightarrow \quad x + y + 3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}} = z$

$\Rightarrow \quad x + y - z = -3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$

$\Rightarrow \quad (x + y - z)^3 = \left(-3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}\right)^3$

$\Rightarrow \quad (x + y - z)^3 = -27xyz \Rightarrow (x + y - z)^3 + 27\,xyz = 0.$

116. $x = 2 + 2^{\frac{2}{3}} + 2^{\frac{1}{3}} \Rightarrow (x - 2) = 2^{\frac{2}{3}} + 2^{\frac{1}{3}}$

$\Rightarrow \quad (x - 2)^3 = \left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}\right)^3$

$= 2^2 + 2 + 3.2^{\frac{2}{3}}\cdot 2^{\frac{1}{3}}\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}\right)$

$\Rightarrow \quad (x - 2)^3 = 6 + 6(x - 2) = 6 + 6x - 12$

$\Rightarrow \quad (x - 2)^3 = 6x - 6 \Rightarrow x^3 - 8 - 6x(x - 2) = 6x - 6$

$\Rightarrow \quad x^3 - 8 - 6x^2 + 12x = 6x - 6 \Rightarrow x^3 - 6x^2 + 6x = 2.$

117. Given expression $(-2)^5 \times (2)^{-5} \times (3)^3$

$\dfrac{(-2)^5}{2^5} \times (3)^3 \qquad \left\{\because a^{-m} = \dfrac{1}{a^m}\right\}$

$\dfrac{(-1)^5(2)^5\times(3)^3}{(2^5)} = (-3)^3 = -27$

118. Expression $= \dfrac{(10)^{100}}{(5)^{75}}$

$= \dfrac{(2\times5)^{100}}{(5)^{75}} = \dfrac{(2)^{100}\times(5)^{100}}{(5)^{75}} = 2^{100}\times\dfrac{5^{100}}{5^{75}} = 2^{100}\times5^{(100-75)}$

$\left\{\because \dfrac{a^m}{a^n} = a^{m-n}\right\}$

$= 2^{100} \times 5^{25}$

$= 2^{25}\times5^{25}\times2^{75} \qquad \{\because a^m \times a^n = a^{m+n}\}$

$= (10)^{25}\times2^{75} \qquad \{\because a^m \times b^m = ab^m\}$

119. Expression $= \sqrt{\sqrt{2}\times\sqrt{3}}$

$= \left(\sqrt{2}\times\sqrt{3}\right)^{\frac{1}{2}} = \left(2^{\frac{1}{2}}\times3^{\frac{1}{2}}\right)^{\frac{1}{2}} \qquad \{\because a^m \times b^m = ab^m\}$

$= (6)^{\frac{1}{2}\times\frac{1}{2}} = (6)^{\frac{1}{4}} \qquad \left\{\because \left(a^m\right)^n = a^{mn}\right\}$

120. $21^{?}\times21^{6.5} = 21^{12.4} \qquad \{\because a^m \times a^n = a^{m+n}\}$

$\Rightarrow 21^{?+6.5} = 21^{12.4}$

$\Rightarrow ? + 6.5 = 12.4$

$\Rightarrow ? = 12.4 - 6.5 = 5.9$

121. $\dfrac{5.4\div3\times16\div2}{18\div5\times6\div3}$

$= \dfrac{\frac{5.4}{3}\times\frac{16}{2}}{\frac{18}{5}\times\frac{6}{3}} = \dfrac{1.8\times8}{3.6\times2} = 2$

122. $\left(32\times10^{-5}\right)^2\times64\div\left(2^{16}\times10^{-4}\right) = 10^{?}$

$\Rightarrow \left(2^5\times10^{-5}\right)^2\times2^6\div\left(2^{16}\times10^{-4}\right) = 10^{?} \qquad \left\{\because \left(a^m\right)^n = a^{mn}\right\}$

$\Rightarrow \dfrac{2^{10}\times10^{-10}\times2^6}{2^{16}\times10^{-4}} = 10^{?} \qquad \{\because a^m \times a^n = a^{m+n}\}$

$\Rightarrow \dfrac{2^{16}\times10^4}{2^{16}\times10^{10}} = 10^{?} \qquad \left\{\because a^{-m} = \dfrac{1}{a^m}\right\}$

$\Rightarrow 10^{4-10} = 10^{?}$

$\Rightarrow 10^{-6} = 10^{?}$

$\Rightarrow ? = -6$