

IMPORTANT CONCEPTS

Suppose a man has to pay ₹ 156 after 4 years and the rate of interest is 14% per annum. Clearly, ₹ 100 at 14% will amount to ₹ 156 in 4 years. So, the payment of ₹ 100 now will clear off the debt of ₹ 156 due 4 years hence. We say that :

Sum due = ₹ 156 due 4 years hence;

Present Worth (P.W.) = ₹ 100;

True Discount (T.D.) = ₹ (156 - 100) = ₹ 56 = (Sum due) - (P.W.).

We define : T.D. = Interest on P.W.

$$\text{Amount} = (\text{P.W.}) + (\text{T.D.}).$$

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

$$\text{I. P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \frac{100 \times \text{T.D.}}{R \times T} \quad \text{II. T.D.} = \frac{(\text{P.W.}) \times R \times T}{100} = \frac{\text{Amount} \times R \times T}{100 + (R \times T)}$$

$$\text{III. Sum} = \frac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})} \quad \text{IV. (S.I.)} - (\text{T.D.}) = \text{S.I. on T.D.}$$

$$\text{V. When the sum is put at compound interest, then } \text{P.W.} = \frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}.$$

SOLVED EXAMPLES

Ex. 1. Find the present worth of ₹ 930 due 3 years hence at 8% per annum. Also find the discount.

$$\text{Sol. P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = ₹ \left[\frac{100 \times 930}{100 + (8 \times 3)} \right] = ₹ \left(\frac{100 \times 930}{124} \right) = ₹ 750.$$

$$\text{T.D.} = (\text{Amount}) - (\text{P.W.}) = ₹ (930 - 750) = ₹ 180.$$

Ex. 2. The true discount on a bill due 9 months hence at 12% per annum is ₹ 540. Find the amount of the bill and its present worth.

Sol. Let amount be ₹ x. Then,

$$\frac{x \times R \times T}{100 + (R \times T)} = \text{T.D.} \Rightarrow \frac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)} = 540 \Rightarrow x = \left(\frac{540 \times 109}{9} \right) = ₹ 6540.$$

$$\therefore \text{Amount} = ₹ 6540.$$

$$\text{P.W.} = ₹ (6540 - 540) = ₹ 6000.$$

Ex. 3. The true discount on a certain sum of money due 3 years hence is ₹ 250 and the simple interest on the same sum for the same time and at the same rate is ₹ 375. Find the sum and the rate percent.

Sol. T.D. = ₹ 250 and S.I. = ₹ 375.

$$\therefore \text{Sum due} = \frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = ₹ \left(\frac{375 \times 250}{375 - 250} \right) = ₹ 750.$$

$$\text{Rate} = \left(\frac{100 \times 375}{750 \times 3} \right) \% = 16\frac{2}{3}\%.$$

Ex. 4. The difference between the simple interest and true discount on a certain sum of money for 6 months at $12\frac{1}{2}\%$ per annum is ₹ 25. Find the sum.

Sol. Let the sum be ₹ x . Then,

$$\text{T.D.} = \frac{x \times \frac{25}{2} \times \frac{1}{2}}{100 + \left(\frac{25}{2} \times \frac{1}{2} \right)} = \left(x \times \frac{25}{4} \times \frac{4}{425} \right) = \frac{x}{17}.$$

$$\text{S.I.} = \left(x \times \frac{25}{2} \times \frac{1}{2} \times \frac{1}{100} \right) = \frac{x}{16}.$$

$$\therefore \frac{x}{16} - \frac{x}{17} = 25 \Rightarrow 17x - 16x = 25 \times 16 \times 17 \Rightarrow x = 6800.$$

Hence, sum due = ₹ 6800.

Ex. 5. A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement he gains ₹ 40. What is the amount of the bill, if the money be worth $12\frac{1}{2}\%$?

Sol. Let the sum be ₹ x . Then,

$$\left[\frac{x}{2} + \frac{\frac{x}{2} \times 100}{100 + \left(\frac{25}{2} \times 2 \right)} \right] - \frac{x \times 100}{100 + \left(\frac{25}{2} \times 1 \right)} = 40 \Rightarrow \frac{x}{2} + \frac{2x}{5} - \frac{8x}{9} = 40 \Rightarrow x = 3600.$$

\therefore Amount of the bill = ₹ 3600.

EXERCISE

(OBJECTIVE TYPE QUESTIONS)

Directions: Mark (✓) against the correct answer:

- The present worth of ₹ 2310 due $2\frac{1}{2}$ years hence, the rate of interest being 15% per annum, is
 (a) ₹ 1750 (b) ₹ 1680
 (c) ₹ 1840 (d) ₹ 1443.75
- If the true discount on a sum due 2 years hence at 14% per annum be ₹ 168, the sum due is
 (a) ₹ 768 (b) ₹ 968
 (c) ₹ 1960 (d) ₹ 2400
- The true discount on ₹ 2562 due 4 months hence is ₹ 122. The rate percent is
 (a) 12% (b) $13\frac{1}{3}\%$
 (c) 15% (d) 14%

- The true discount on ₹ 1760 due after a certain time at 12% per annum is ₹ 160. The time after which it is due is
 (a) 6 months (b) 8 months
 (c) 9 months (d) 10 months
- The true discount on a bill due 9 months hence at 16% per annum is ₹ 189. The amount of the bill is
 (a) ₹ 1386 (b) ₹ 1764
 (c) ₹ 1575 (d) ₹ 2268
- The interest on ₹ 750 for 2 years is the same as the true discount on ₹ 960 due 2 years hence. If the rate of interest is the same in both cases, it is
 (a) 12% (b) 14%
 (c) 15% (d) $16\frac{2}{3}\%$

7. The simple interest and the true discount on a certain sum for a given time and at a given rate are ₹ 85 and ₹ 80 respectively. The sum is
 (a) ₹ 1800 (b) ₹ 1450
 (c) ₹ 1360 (d) ₹ 6800
8. If ₹ 10 be allowed as true discount on a bill of ₹ 110 due at the end of a certain time, then the discount allowed on the same sum due at the end of double the time is
 (a) ₹ 20 (b) ₹ 21.81
 (c) ₹ 22 (d) ₹ 18.33
9. A man wants to sell his scooter. There are two offers, one at ₹ 12,000 cash and the other at a credit of ₹ 12,880 to be paid after 8 months, money being at 18% per annum. Which is the better offer?
 (a) ₹ 12,000 in cash (b) ₹ 12,880 at credit
 (c) Both are equally good
10. Goods were bought for ₹ 600 and sold the same day for ₹ 688.50 at a credit of 9 months and thus gaining 2%. The rate of interest per annum is
 (a) $16\frac{2}{3}\%$ (b) $14\frac{1}{2}\%$
 (c) $13\frac{1}{3}\%$ (d) 15%
11. The present worth of ₹ 1404 due in two equal half-yearly instalments at 8% per annum simple interest is
 (a) ₹ 1325 (b) ₹ 1300
 (c) ₹ 1350 (d) ₹ 1500
12. A trader owes a merchant ₹ 10,028 due 1 year hence. The trader wants to settle the account after 3 months. If the rate of interest is 12% per annum, how much cash should he pay?
 (a) ₹ 9025.20 (b) ₹ 9200
 (c) ₹ 9600 (d) ₹ 9560
13. A man buys a watch for ₹ 1950 in cash and sells it for ₹ 2200 at a credit of 1 year. If the rate of interest is 10% per annum, the man
 (a) gains ₹ 55 (b) gains ₹ 50
 (c) loses ₹ 30 (d) gains ₹ 30
14. A man purchased a cow for ₹ 3000 and sold it the same day for ₹ 3600, allowing the buyer a credit of 2 years. If the rate of interest be 10% per annum, then the man has a gain of
 (a) 0% (b) 5%
 (c) 7.5% (d) 10%
15. A owes B, ₹ 1573 payable $1\frac{1}{2}$ years hence. Also B owes A, ₹ 1444.50 payable 6 months hence. If they want to settle the account forthwith, keeping 14% as the rate of interest, then who should pay and how much?
 (M.A.T. 2006; G.B.O., 2007)
 (a) A, ₹ 28.50 (b) B, ₹ 37.50
 (c) A, ₹ 50 (d) B, ₹ 50
16. A has to pay ₹ 220 to B after 1 year. B asks A to pay ₹ 110 in cash and defer the payment of ₹ 110 for 2 years. A agrees to it. If the rate of interest be 10% per annum, in this mode of payment
 (a) There is no gain or loss to any one
 (b) A gains ₹ 7.34 (c) A loses ₹ 7.34
 (d) A gains ₹ 11
17. ₹ 20 is the true discount on ₹ 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same?
 (a) ₹ 10 (b) ₹ 10.40
 (c) ₹ 15.20 (d) ₹ 13
18. The true discount on a certain sum of money due $2\frac{2}{3}$ years hence, is ₹ 150 and the simple interest on the same sum for the same time and at the same rate is ₹ 200. Interest rate per annum is [MAT, 2011]
 (a) 10% (b) 12%
 (c) $12\frac{1}{2}\%$ (d) $8\frac{1}{2}\%$
19. ₹ 20 is the true discount on ₹ 260 due after a certain time. What will be the true discount on the same sum due after half of the former time, the rate of interest being the same? [MAT, 2011]
 (a) ₹ 15.20 (b) ₹ 10.40
 (c) ₹ 10.83 (d) ₹ 13
20. The total discount on ₹ 1860 due after a certain time at 5% is ₹ 60. Find the time after which it is due [SSC—CHSL (10+2) Exam, 2015]
 (a) 9 months (b) 10 months
 (c) 7 months (d) 8 months
21. The difference between simple interest and the true discount on ₹ 2400 due 4 years hence at 5% per annum simple interest is [SSC—CHSL (10+2) Exam, 2015]
 (a) ₹ 70 (b) ₹ 30
 (c) ₹ 50 (d) ₹ 80

ANSWERS

1. (b) 2. (a) 3. (c) 4. (d) 5. (b) 6. (b) 7. (c) 8. (d) 9. (a) 10. (a)
 11. (a) 12. (b) 13. (b) 14. (a) 15. (d) 16. (b) 17. (b) 18. (c) 19. (b) 20. (d)
 21. (d)

SOLUTIONS

1. P.W. = ₹ $\left[\frac{100 \times 2310}{100 + \left(15 \times \frac{5}{2} \right)} \right]$ = ₹ 1680.
2. P.W. = $\frac{100 \times \text{T.D.}}{R \times T} = \frac{100 \times 168}{14 \times 2} = 600$.
 \therefore Sum = (P.W. + T.D.) = ₹ (600 + 168) = ₹ 768.
3. P.W. = ₹ (2562 - 122) = ₹ 2440.
 \therefore S.I. on ₹ 2440 for 4 months = ₹ 122.
 \therefore Rate = $\left(\frac{100 \times 122}{2440 \times \frac{1}{3}} \right) \% = 15\%$.
4. P.W. = ₹ (1760 - 160) = ₹ 1600.
 \therefore S.I. on ₹ 1600 at 12% = ₹ 160.
 \therefore Time = $\left(\frac{100 \times 160}{1600 \times 12} \right) = \frac{5}{6}$ years = $\left(\frac{5}{6} \times 12 \right)$ months
 = 10 minutes.
5. Let P.W. be ₹ x .
 Then, S.I. on ₹ x at 16% for 9 months = ₹ 189.
 $\therefore x \times 16 \times \frac{9}{12} \times \frac{1}{100} = 189$ or $x = 1575$.
 \therefore P.W. = ₹ 1575.
 \therefore Sum due = P.W. + T.D. = ₹ (1575 + 189) = ₹ 1764.
6. S.I. on ₹ 750 = T.D. on ₹ 960.
 This means P.W. of ₹ 960 due 2 years hence is ₹ 750.
 \therefore T.D. = ₹ (960 - 750) = ₹ 210.
 Thus, S.I. on ₹ 750 for 2 years is ₹ 210.
 \therefore Rate = $\left(\frac{100 \times 210}{750 \times 2} \right) \% = 14\%$.
7. Sum = $\frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \frac{85 \times 80}{(85 - 80)} = ₹ 1360$.
8. S.I. on ₹ (110 - 10) for a certain time = ₹ 10.
 S.I. on ₹ 100 for double the time = ₹ 20.
 T.D. on ₹ 120 = ₹ (120 - 100) = ₹ 20.
 T.D. on ₹ 110 = ₹ $\left(\frac{20}{120} \times 110 \right) = ₹ 18.33$.
9. P.W. of ₹ 12,880 due 8 months hence
 = ₹ $\left[\frac{12880 \times 100}{100 + \left(18 \times \frac{8}{12} \right)} \right] = ₹ \left(\frac{12880 \times 100}{112} \right) = ₹ 11500$.
 Clearly, ₹ 12,000 in cash is a better offer.
10. S.P. = 102% of ₹ 600 = ₹ $\left(\frac{102}{100} \times 600 \right) = ₹ 612$.
 Now, P.W. = ₹ 612 and sum = ₹ 688.50.
 \therefore T.D. = ₹ (688.50 - 612) = ₹ 76.50.
 Thus, S.I. on ₹ 612 for 9 months is ₹ 76.50.

$$\therefore \text{Rate} = \left(\frac{100 \times 76.50}{612 \times \frac{3}{4}} \right) \% = 16\frac{2}{3}\%.$$

11. Required sum = P.W. of ₹ 702 due 6 months hence + P.W. of ₹ 702 due 1 year hence
 = ₹ $\left[\left(\frac{100 \times 702}{100 + 8 \times \frac{1}{2}} \right) + \left(\frac{100 \times 702}{100 + (8 \times 1)} \right) \right] = ₹ (675 + 650)$
 = ₹ 1325.
12. Required money = P.W. of ₹ 10028 due 9 months hence
 = ₹ $\left[\frac{10028 \times 100}{100 + \left(12 \times \frac{9}{12} \right)} \right] = ₹ 9200$.
13. S.P. = P.W. of ₹ 2200 due 1 year hence
 = ₹ $\left[\frac{2200 \times 100}{100 + (10 \times 1)} \right] = ₹ 2000$.
 \therefore Gain = ₹ (2000 - 1950) = ₹ 50.
14. C.P. = ₹ 3000.
 S.P. = ₹ $\left[\frac{3600 \times 100}{100 + (10 \times 2)} \right] = ₹ 3000$.
 Gain = 0%.
15. A owes = P.W. of ₹ 1573 due $\frac{3}{2}$ years hence
 = ₹ $\left[\frac{1573 \times 100}{100 + \left(14 \times \frac{3}{2} \right)} \right] = ₹ \left(\frac{1573 \times 100}{121} \right) = ₹ 1300$.
 B owes = P.W. of ₹ 1444.50 due 6 months hence
 = ₹ $\left[\frac{1444.50 \times 100}{100 + \left(14 \times \frac{1}{2} \right)} \right] = ₹ \left(\frac{1444.50 \times 100}{107} \right) = ₹ 1350$.
 \therefore B must pay ₹ 50 to A.
16. A has to pay = P.W. of ₹ 220 due 1 year hence
 = ₹ $\left[\frac{220 \times 100}{100 + (10 \times 1)} \right] = ₹ 200$.
 A actually pays = ₹ 110 + P.W. of ₹ 110 due 2 years hence
 = $\left[110 + \frac{110 \times 100}{100 + (10 \times 2)} \right] = ₹ 192.66$.
 \therefore A gains = ₹ (200 - 192.66) = ₹ 7.34.
17. S.I. on ₹ (260 - 20) for a given time = ₹ 20.
 S.I. on ₹ 240 for half the time = ₹ 10.
 T.D. on ₹ 250 = ₹ 10.
 \therefore T.D. on ₹ 260 = ₹ $\left(\frac{10}{250} \times 260 \right) = ₹ 10.40$.

TRUE DISCOUNT

18. TD = ₹ 150 and SI = ₹ 200

$$\begin{aligned}\text{Sum due} &= \frac{\text{SI} \times \text{TD}}{(\text{SI}) - (\text{TD})} \\ &= ₹ \left(\frac{200 \times 150}{200 - 150} \right) = ₹ 600\end{aligned}$$

$$\begin{aligned}\text{Rate} &= \left(\frac{100 \times 200}{600 \times \frac{8}{3}} \right) \% \\ &= \left(\frac{20000}{1600} \right) \% \\ &= \left(\frac{200}{16} \right) \% \\ &= 12\frac{1}{2} \%\end{aligned}$$

19. SI on ₹ (260 - 20) for a given time = ₹ 20

SI on ₹ 240 for half the time = ₹ 20

TD on ₹ 250 = ₹ 10

$$\text{TD on ₹ 260} = \frac{10}{250} \times 260 = ₹ 10.40$$

20. Here, A = 1860, R = 5%

TD = 60

$$\therefore \text{TD} = \frac{A \times R \times T}{100 + R \times T}$$

$$\Rightarrow 60 = \frac{1860 \times 5 \times T}{100 + 5T}$$

$$\Rightarrow 6000 + 300T = 9300T$$

$$\Rightarrow 6000 = 9300T - 300T$$

$$\Rightarrow 6000 = 9000T$$

$$\Rightarrow T = \frac{6000}{9000}$$

$$\Rightarrow T = \frac{2}{3} \text{ year}$$

$$\Rightarrow T = \frac{2}{3} \times 12 = 8 \text{ months}$$

21. Given A = 2400, R = 5%, T = 4 year

$$\therefore \text{TD} = \frac{A \times R \times T}{100 + R \times T}$$

$$= \frac{2400 \times 5 \times 4}{100 + 20} = ₹ 400$$

$$\therefore \text{SI} - \text{TD} = \frac{\text{TD} \times R \times T}{100}$$

$$= \frac{400 \times 5 \times 4}{100} = ₹ 80$$