31 Probability

IMPORTANT FACTS AND FORMULAE

I. Experiment: An operation which can produce some well-defined outcomes is called an experiment.

II. Random Experiment: An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples of Performing a Random Experiment:

(*i*) Rolling an unbiased dice

(*ii*) Tossing a fair coin

(*iii*) Drawing a card from a pack of well-shuffled cards

(*iv*) Picking up a ball of certain colour from a bag containing balls of different colours

Details :

(*i*) When we throw a coin, then either a Head (H) or a Tail (T) appears.

(*ii*) A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively.
When we throw a die, the outcome is the number that appears on its upper face.

(*iii*) A pack of cards has 52 cards.
It has 13 cards of each suit, namely **Spades, Clubs, Hearts** and **Diamonds.**
Cards of spades and clubs are **black cards.**
Cards of hearts and diamonds are **red cards.**
There are 4 honours of each suit.
These are **Aces, Kings, Queens** and **Jacks.**
These are called **face cards.**

III. Sample Space: When we perform an experiment, then the set S of all possible outcomes is called the **Sample Space.**

Examples of Sample Spaces:

(*i*) In tossing a coin, S = {H, T}.

(*ii*) If two coins are tossed, then S = {HH, HT, TH, TT}.

(*iii*) In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.

IV. Event : Any subset of a sample space is called an event.

V. Probability of Occurrence of an Event:

Let S be the sample space and let E be an event. Then, $E \subseteq S$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)}.$$

VI. Results on Probability:

(*i*) $P(S) = 1$ (*ii*) $0 \le P(E) \le 1$ (*iii*) $P(\phi) = 0$

(*iv*) For any events A and B, we have :
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(*v*) If $\bar{A}$ denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

SOLVED EXAMPLES

Ex. 1. *In a throw of a coin, find the probability of getting a head.*

Sol. Here $S = \{H, T\}$ and $E = \{H\}$.

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}.$$

Ex. 2. ***Two unbiased coins are tossed. What is the probability of getting at most one head ?***

Sol. Here S = {HH, HT, TH, TT}.

Let E = event of getting at most one head.

$\therefore$ E = {TT, HT, TH}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}.$

Ex. 3. ***An unbiased die is tossed. Find the probability of getting a multiple of 3.***

Sol. Here S = {1, 2, 3, 4, 5, 6}.

Let E be the event of getting a multiple of 3.

Then, E = {3, 6}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}.$

Ex. 4. ***In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.***

Sol. Here, n (S) = (6 × 6) = 36.

Let E = Event of getting a total more than 7

= {(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}.$

Ex. 5. ***A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.***

Sol. Let S be the sample space. Then,

$n(s)$ = Number of ways of drawing 2 balls out of (6 + 4) = ${}^{10}C_2 = \frac{(10 \times 9)}{(2 \times 1)} = 45.$

Let E = Event of getting both balls of the same colour. Then,

$n(E)$ = Number of ways of drawing (2 balls out of 6) or (2 balls out of 4)

$= ({}^6C_2 + {}^4C_2) = \frac{(6 \times 5)}{(2 \times 1)} + \frac{(4 \times 3)}{(2 \times 1)} = (15 + 6) = 21.$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}.$

Ex. 6. ***Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6 ?***

Sol. Clearly, n (S) = 6 × 6 = 36.

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then

E = {(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)}

$\therefore$ $n(E) = 14.$

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}.$

Ex. 7. ***Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queens ?***

Sol. We have $n(s) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326.$

Let A = event of getting both black cards;

B = event of getting both queens.

$\therefore$ $A \cap B$ = event of getting queens of black cards.

$\therefore$ $n(A) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325,\ n(B) = {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6$ and $n(A \cap B) = {}^2C_2 = 1.$

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{325}{1326};\ P(B) = \frac{n(B)}{n(S)} = \frac{6}{1326}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{1326}.$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}\right) = \frac{330}{1326} = \frac{55}{221}.$

EXERCISE

(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (✓) against the correct answer*:

1. In a simultaneous throw of two coins, the probability of getting at least one head is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

2. Three unbiased coins are tossed. What is the probability of getting at least 2 heads?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{8}$

3. Three unbiased coins are tossed. What is the probability of getting at most two heads?

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{7}{8}$

4. In a single throw of a die, what is the probability of getting a number greater than 4 ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{4}$

5. In a simultaneous throw of two dice, what is the probability of getting a total of 7 ?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

6. What is the probability of getting a sum 9 from two throws of a dice ?

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$

(c) $\frac{1}{9}$ (d) $\frac{1}{12}$

7. In a simultaneous throw of two dice, what is the probability of getting a doublet?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{3}{7}$

8. In a simultaneous throw of two dice, what is the probability of getting a total of 10 or 11 ?

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$

(c) $\frac{7}{12}$ (d) $\frac{5}{36}$

9. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even ?

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{3}{8}$ (d) $\frac{5}{16}$

10. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn bears a number which is a multiple of 3 ?

(a) $\frac{3}{10}$ (b) $\frac{3}{20}$

(c) $\frac{2}{5}$ (d) $\frac{1}{2}$

11. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$

(c) $\frac{8}{15}$ (d) $\frac{9}{20}$

12. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize ?

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$

(c) $\frac{2}{7}$ (d) $\frac{5}{7}$

13. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card ?

(a) $\frac{1}{13}$ (b) $\frac{4}{13}$

(c) $\frac{1}{4}$ (d) $\frac{9}{52}$

14. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$

(c) $\frac{1}{26}$ (d) $\frac{1}{52}$

15. One card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card or a king ?

(a) $\frac{1}{2}$ (b) $\frac{6}{13}$

(c) $\frac{7}{13}$ (d) $\frac{27}{52}$

16. From a pack of 52 cards, one card is drawn at random. What is the probability that the card drawn is a ten or a spade ?

(a) $\frac{4}{13}$ (b) $\frac{1}{4}$

(c) $\frac{1}{13}$ (d) $\frac{1}{26}$

17. The probability that a card drawn from a pack of 52 cards will be a diamond or a king, is

(a) $\frac{2}{13}$ (b) $\frac{4}{13}$

(c) $\frac{1}{13}$ (d) $\frac{1}{52}$

18. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings ?

(a) $\frac{1}{15}$ (b) $\frac{25}{57}$

(c) $\frac{35}{256}$ (d) $\frac{1}{221}$

19. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is

(a) $\frac{3}{20}$ (b) $\frac{29}{34}$

(c) $\frac{47}{100}$ (d) $\frac{13}{102}$

20. Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings, is

(a) $\frac{7}{13}$ (b) $\frac{3}{26}$

(c) $\frac{63}{221}$ (d) $\frac{55}{221}$

21. A bag contains 6 black and 8 white balls. One ball is drawn at random. What is the probability that the ball drawn is white ?

(a) $\frac{3}{4}$ (b) $\frac{4}{7}$

(c) $\frac{1}{8}$ (d) $\frac{3}{7}$

(e) None of these

22. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green ?

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{7}{19}$ (d) $\frac{8}{21}$

(e) $\frac{9}{21}$

23. A box contains 4 red, 5 green and 6 white balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green ?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{5}$ (d) $\frac{7}{15}$

(e) None of these

24. A basket contains 4 red, 5 blue and 3 green marbles. If 2 marbles are drawn at random from the basket, what is the probability that both are red?

(S.B.I. P.O., 2010)

(a) $\frac{3}{7}$ (b) $\frac{1}{2}$

(c) $\frac{1}{11}$ (d) $\frac{1}{6}$

(e) None of these

25. An urn contains 6 red, 4 blue, 2 green and 3 yellow marbles. If two marbles are drawn at random from the urn, what is the probability that both are red?

(S.B.I. P.O., 2010)

(a) $\frac{1}{6}$ (b) $\frac{1}{7}$

(c) $\frac{2}{15}$ (d) $\frac{2}{5}$

(e) None of these

26. A basket contains 6 blue, 2 red, 4 green and 3 yellow balls. If three balls are picked up at random, what is the probability that none is yellow? **(Bank P.O., 2009)**

(a) $\frac{3}{455}$ (b) $\frac{1}{5}$

(c) $\frac{4}{5}$ (d) $\frac{44}{91}$

(e) None of these

27. An urn contains 6 red, 4 blue, 2 green and 3 yellow marbles. If three marbles are picked up at random, what is the probability that 2 are blue and 1 is yellow? **(S.B.I. P.O., 2010)**

(a) $\frac{3}{91}$ (b) $\frac{1}{5}$

(c) $\frac{18}{455}$ (d) $\frac{7}{15}$

(e) None of these

28. An urn contains 6 red, 4 blue, 2 green and 3 yellow marbles. If four marbles are picked up at random, what is the probability that 1 is green, 2 are blue and 1 is red ? **(S.B.I. P.O., 2011)**

(a) $\frac{13}{35}$ (b) $\frac{24}{455}$

(c) $\frac{11}{15}$ (d) $\frac{1}{13}$

(e) None of these

29. An urn contains 6 red, 4 blue, 2 green and 3 yellow marbles. If two marbles are picked up at random, what is the probability that either both are green or both are yellow? **(Bank P.O., 2010)**

(a) $\frac{5}{91}$ (b) $\frac{1}{35}$

(c) $\frac{1}{3}$ (d) $\frac{4}{105}$

(e) None of these

30. A basket contains 6 blue, 2 red, 4 green and 3 yellow balls. If four balls are picked up at random, what is the probability that 2 are red and 2 are green ? **(Bank P.O., 2009)**

(a) $\frac{4}{15}$ (b) $\frac{5}{27}$

(c) $\frac{1}{3}$ (d) $\frac{2}{455}$

(e) None of these

31. A basket contains 4 red, 5 blue and 3 green marbles. If three marbles are picked up at random what is the probability that at least one is blue? **(S.B.I. P.O., 2010)**

(a) $\frac{7}{12}$ (b) $\frac{37}{44}$

(c) $\frac{5}{12}$ (d) $\frac{7}{44}$

(e) None of these

32. An urn contains 6 red, 4 blue, 2 green and 3 yellow marbles. If 4 marbles are picked up at random, what is the probability that at least one of them is blue? **(S.B.I. P.O., 2010)**

(a) $\frac{4}{15}$ (b) $\frac{69}{91}$

(c) $\frac{11}{15}$ (d) $\frac{22}{91}$

(e) None of these

33. A basket contains 6 blue, 2 red, 4 green and 3 yellow balls. If 5 balls are picked up at random, what is the probability that at least one is blue? **(Bank P.O., 2009)**

(a) $\frac{137}{143}$ (b) $\frac{18}{455}$

(c) $\frac{9}{91}$ (d) $\frac{2}{5}$

(e) None of these

34. An urn contains 2 red, 3 green and 2 blue balls. If 2 balls are drawn at random, find the probability that no ball is blue. **(Railways, 2006)**

(a) $\frac{5}{7}$ (b) $\frac{10}{21}$

(c) $\frac{2}{7}$ (d) $\frac{11}{21}$

(e) None of these

35. A box contains 10 black and 10 white balls. What is the probability of drawing 2 balls of the same colour?

(a) $\frac{9}{19}$ (b) $\frac{9}{38}$

(c) $\frac{10}{19}$ (d) $\frac{5}{19}$

(e) None of these

36. A box contains 20 electric bulbs, out of which 4 are defective. Two balls are chosen at random from this box. The probability that at least one of them is defective, is

(a) $\frac{4}{19}$ (b) $\frac{7}{19}$

(c) $\frac{12}{19}$ (d) $\frac{21}{95}$

(e) None of these

37. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that the selected students are 2 boys and 1 girl, is:

(a) $\frac{21}{46}$ (b) $\frac{25}{117}$

(c) $\frac{1}{50}$ (d) $\frac{3}{25}$

(e) None of these

38. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The chance that exactly 2 of them are children, is

(a) $\frac{1}{9}$ (b) $\frac{1}{5}$

(c) $\frac{1}{12}$ (d) $\frac{10}{21}$

(e) None of these

39. Two dice are tossed. The probability that the total score is a prime number is

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$

(c) $\frac{5}{12}$ (d) $\frac{7}{9}$

(e) None of these

40. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi ?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{3}{10}$

(e) None of these

41. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and the probability of wife's selection is $\frac{1}{5}$. What is the probability that only one of them is selected ?

(a) $\frac{4}{5}$ (b) $\frac{2}{7}$

(c) $\frac{4}{7}$ (d) $\frac{8}{15}$

(e) None of these

42. A speaks truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other, in narrating the same incident?

(a) 5% (b) 15%

(c) 35% (d) 45%

(e) None of these

43. A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing a single event, is **(Railways, 2006)**

(a) 0.54 (b) 0.56

(c) 0.68 (d) 0.94

(e) None of these

44. A committee of 3 members is to be selected out of 3 men and 2 women. What is the probability that the committee has at least 1 woman? **(Bank P.O. ,2008)**

(a) $\frac{1}{10}$ (b) $\frac{9}{20}$

(c) $\frac{1}{20}$ (d) $\frac{9}{10}$

(e) None of these

45. A bag contains 3 blue, 2 green and 5 red balls. If four balls are picked at random, what is the probability that two are green and two are blue?
[DMRC—Customer Relationship Assistant (CRA) Exam, 2016]

(a) $\frac{1}{18}$ (b) $\frac{1}{70}$

(c) $\frac{3}{5}$ (d) $\frac{1}{2}$

46. Dev can hit a target 3 times in 6 shorts Pawan can hit the target 2 times in 6 shorts and Lakhan can hit the target 4 times in 4 shorts. What is the probability that at least 2 shorts hit the target
[DMRC—Customer Relationship Assistant (CRA) Exam, 2016]

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) None of these

47. A bag contains 10 mangoes out of which 4 are taken out together. If one of them is found to be good, the probability that other is also good is
[DMRC—Train Operator (Station Controller) Exam, 2016]

(a) $\frac{1}{3}$ (b) $\frac{8}{15}$

(c) $\frac{5}{18}$ (d) $\frac{2}{3}$

48. A bag contains 4 red, 5 yellow and 6 pink balls. Two balls are drawn at random. What is the probability that none of the balls drawn are yellow in colour?
[IBPS—Bank PO/MT (Pre.) Exam, 2015]

(a) $\frac{1}{7}$ (b) $\frac{3}{7}$

(c) $\frac{2}{7}$ (d) $\frac{5}{14}$ (e) $\frac{9}{14}$

49. A bag contains 6 red balls 11 yellow balls and 5 pink balls. If two balls are drawn at random from the bag. One after another what is the probability that the first ball is red and second ball is yellow.
[IBPS—Bank PO (Pre.) Exam, 2015]

(a) $\frac{1}{14}$ (b) $\frac{2}{7}$

(c) $\frac{5}{7}$ (d) $\frac{3}{14}$

(e) None of these

50. A bag contains 4 red balls, 6 blue balls and 8 pink balls. One ball is drawn at random and replace with 3 pink balls. A probability that the first ball drawn was either red or blue in colour and the second ball drawn was pink in colour?
[CET—(Maharashtra (MBA) Exam, 2016]

(a) 12/21 (b) 13/17

(c) 11/30 (d) 13/18

(e) None of these

ANSWERS

1. (*d*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*a*)	**6.** (*c*)	**7.** (*a*)	**8.** (*d*)	**9.** (*b*)	**10.** (*a*)
11. (*d*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*c*)	**16.** (*a*)	**17.** (*b*)	**18.** (*d*)	**19.** (*d*)	**20.** (*d*)
21. (*b*)	**22.** (*d*)	**23.** (*b*)	**24.** (*c*)	**25.** (*b*)	**26.** (*d*)	**27.** (*c*)	**28.** (*b*)	**29.** (*d*)	**30.** (*d*)
31. (*b*)	**32.** (*b*)	**33.** (*a*)	**34.** (*b*)	**35.** (*a*)	**36.** (*b*)	**37.** (*a*)	**38.** (*d*)	**39.** (*c*)	**40.** (*a*)
41. (*b*)	**42.** (*c*)	**43.** (*a*)	**44.** (*d*)	**45.** (*b*)	**46.** (*a*)	**47.** (*a*)	**48.** (*b*)	**49.** (*b*)	**50.** (*e*)

SOLUTIONS

1. Here S = {HH, HT, TH, TT}.

Let E = event of getting at least one head = {HT, TH, HH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$.

2. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at least two heads = {THH, HTH, HHT, HHH}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$.

3. Here S = {TTT, TTH, THT, HTT, THH, HTH, HHT, HHH}.

Let E = event of getting at most two heads.

Then, E = {TTT, TTH, THT, HTT, THH, HTH, HHT}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$.

4. When a die is thrown, we have S = {1, 2, 3, 4, 5, 6}.

Let E = event of getting a number greater than 4 = {5, 6}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.

5. We know that in a simultaneous throw of two dice, n (S) = 6 × 6 = 36.

Let E = event of getting a total of 7

= {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.

6. In two throws of a die, n (S) = (6 × 6) = 36.

Let E = event of getting a sum 9

= {(3, 6), (4, 5), (5, 4), (6, 3)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$.

7. In a simultaneous throw of two dice, n (S) = (6 × 6) = 36.

Let E = event of getting a doublet

= {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.

8. In a simultaneous throw of two dice, we have n (S) = (6 × 6) = 36.

Let E = event of getting a total of 10 or 11

= {(4, 6), (5, 5), (6, 4), (5, 6), (6, 5)}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$.

9. In a simultaneous throw of two dice, we have n (S) = (6 × 6) = 36.

Let E = event of getting two numbers whose product is even.

Then, E = {(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

$\therefore \quad n(E) = 27$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}$.

10. Here, S = {1, 2, 3, 4, ..., 19, 20}.

Let E = event of getting a multiple of 3 = {3, 6, 9, 12, 15, 18}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{20} = \frac{3}{10}$.

11. Here, S = {1, 2, 3, 4, ..., 19, 20}.

Let E = event of getting a multiple of 3 or 5 = {3, 6, 9, 12, 15, 18, 5, 10, 20}.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$.

12. P (getting a prize) = $\frac{10}{(10+25)} = \frac{10}{35} = \frac{2}{7}$.

13. Clearly, there are 52 cards, out of which there are 16 face cards.

$\therefore \quad P$ (getting a face card) = $\frac{16}{52} = \frac{4}{13}$.

14. Here, n (S) = 52.

Let E = event of getting a queen of club or a king of heart.

Then, $\quad n(E) = 2$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$.

15. Here, n (S) = 52.

There are 26 red cards (including 2 kings) and there are 2 more kings.

Let E = event of getting a red card or a king.

Then, $\quad n(E) = 28$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}$.

16. Here, $n\,(S) = 52$.

There are 13 spades (including one ten) and there are 3 more tens.

Let E = event of getting a ten or a spade.

Then, $n\,(E) = (13 + 3) = 16$.

$\therefore\quad P\,(E) = \dfrac{n\,(E)}{n\,(S)} = \dfrac{16}{52} = \dfrac{4}{13}.$

17. Here, $n\,(S) = 52$.

There are 13 cards of diamond (including one king) and there are 3 more kings.

Let E = event of getting a diamond or a king.

Then, $n\,(E) = (13 + 3) = 16$.

$\therefore\quad P\,(E) = \dfrac{n\,(E)}{n\,(S)} = \dfrac{16}{52} = \dfrac{4}{13}.$

18. Let S be the sample space.

Then, $n\,(S) = {}^{52}C_2 = \dfrac{(52 \times 51)}{(2 \times 1)} = 1326.$

Let E = event of getting 2 kings out of 4.

$\therefore\quad n\,(E) = {}^{4}C_2 = \dfrac{(4 \times 3)}{(2 \times 1)} = 6.$

$\therefore\quad P\,(E) = \dfrac{n\,(E)}{n\,(S)} = \dfrac{6}{1326} = \dfrac{1}{221}.$

19. Let S be the sample space.

Then, $n\,(S) = {}^{52}C_2 = \dfrac{(52 \times 51)}{(2 \times 1)} = 1326.$

Let E = event of getting 1 spade and 1 heart.

$\therefore\quad n\,(E)$ = number of ways of choosing 1 spade out of 13 and 1 heart out of 13

$= ({}^{13}C_1 \times {}^{13}C_1) = (13 \times 13) = 169.$

$\therefore\quad P\,(E) = \dfrac{n\,(E)}{n\,(S)} = \dfrac{169}{1326} = \dfrac{13}{102}.$

20. Clearly, $n\,(S) = {}^{52}C_2 = \dfrac{(52 \times 51)}{2} = 1326.$

Let E_1 = event of getting both red cards,

E_2 = event of getting both kings.

Then, $E_1 \cap E_2$ = event of getting 2 kings of red cards.

$\therefore\ n\,(E_1) = {}^{26}C_2 = \dfrac{(26 \times 25)}{(2 \times 1)} = 325;\ n\,(E_2) = {}^{4}C_2 = \dfrac{(4 \times 3)}{(2 \times 1)} = 6;$

$n\,(E_1 \cap E_2) = 2C_2 = 1$

$\therefore\quad P\,(E_1) = \dfrac{n\,(E_1)}{n\,(S)} = \dfrac{325}{1326};\ P\,(E_2) = \dfrac{n\,(E_2)}{n\,(S)} = \dfrac{6}{1326};$

$P\,(E_1 \cup E_2) = \dfrac{1}{1326}$

$\therefore\quad P$ (both red or both kings) $= P\,(E_1 \cup E_2) = P\,(E_1) + P\,(E_2) - P\,(E_1 \cap E_2)$

$= \left(\dfrac{325}{1326} + \dfrac{6}{1326} - \dfrac{1}{1326}\right) = \dfrac{330}{1326} = \dfrac{55}{221}.$

21. Total number of balls = (6 + 8) = 14.

Number of white balls = 8.

P(drawing a white ball) $= \dfrac{8}{14} = \dfrac{4}{7}.$

22. Total number of balls = (8 + 7 + 6) = 21.

Let E = Event that the ball drawn is neither red nor green

= Event that the ball drawn is red.

$\therefore\quad n(E) = 8.$

$\therefore\quad P\,(E) = \dfrac{8}{21}.$

23. Total number of balls = (4 + 5 + 6) = 15.

P(drawing a red ball or a green ball) = P(red) + P(green)

$= \left(\dfrac{4}{15} + \dfrac{5}{15}\right) = \dfrac{9}{15} = \dfrac{3}{5}.$

24. Total number of balls = (4 + 5 + 3) = 12.

Let E be the event of drawing 2 red balls.

Then, $n(E) = {}^{4}C_2 = \dfrac{4 \times 3}{2 \times 1} = 6.$

Also, $n(S) = {}^{12}C_2 = \dfrac{12 \times 11}{2 \times 1} = 66.$

$\therefore\ P\,(E) = \dfrac{n(E)}{n(S)} = \dfrac{6}{66} = \dfrac{1}{11}.$

25. Total number of balls = (6 + 4 + 2 + 3) = 15.

Let E be the event of drawing 2 red balls.

Then, $n(E) = {}^{6}C_2 = \dfrac{6 \times 5}{2 \times 1} = 15.$

Also, $n(S) = {}^{15}C_2 = \dfrac{15 \times 14}{2 \times 1} = 105.$

$\therefore\quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{15}{105} = \dfrac{1}{7}.$

26. Total number of balls = (6 + 2 + 4 + 3) = 15.

Let E be the event of drawing 3 non-yellow balls.

Then, $n(E) = {}^{12}C_3 = \dfrac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220.$

Also, $n(S) = {}^{15}C_3 = \dfrac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455.$

$\therefore\quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{220}{455} = \dfrac{44}{91}.$

27. Total number of marbles = (6 + 4 + 2 + 3) = 15.

Let E be the event of drawing 2 blue and 1 yellow marble.

Then, $n(E) = ({}^{4}C_2 \times {}^{3}C_1) = \dfrac{4 \times 3}{2 \times 1} \times 3 = 18.$

Also, $n(S) = {}^{15}C_3 = \dfrac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455.$

$\therefore\quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{18}{455}.$

28. Total number of marbles = (6 + 4 + 2 + 3) = 15.

Let E be the event of drawing 1 green, 2 blue and 1 red marble.

Then, $n(E) = ({}^{2}C_1 \times {}^{4}C_2 \times {}^{6}C_1) = 2 \times \dfrac{4 \times 3}{2 \times 1} \times 6 = 72.$

And, $n(S) = {}^{15}C_4 = \dfrac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365.$

$\therefore\quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{72}{1365} = \dfrac{24}{455}.$

29. Total number of marbles = (6 + 4 + 2 + 3) = 15.

Let E be the event of drawing 2 marbles such that either both are green or both are yellow.

Then, $n(E) = ({}^2C_1 + {}^3C_2) = (1 + {}^3C_1) = (1 + 3) = 4$. And, $n(S) = {}^{15}C_2 = \frac{15 \times 14}{2 \times 1} = 105$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{105}$.

30. Total number of balls = (6 + 2 + 4 + 3) = 15.

Let E be the event of drawing 4 balls such that 2 are red and 2 are green.

Then, $n(E) = ({}^2C_2 \times {}^4C_2) = \left(1 \times \frac{4 \times 3}{2 \times 1}\right) = 6$.

And, $n(S) = {}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{1365} = \frac{2}{455}$.

31. Total number of marbles = (4 + 5 + 3) = 12.

Let E be the event of drawing 3 marbles such that none is blue.

Then, $n(E)$ = number of ways of drawing 3 marbles out of 7 = ${}^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$.

And, $n(S) = {}^{12}C_3 = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{35}{220} = \frac{7}{44}$.

Required probability = $1 - P(E) = \left(1 - \frac{7}{44}\right) = \frac{37}{44}$.

32. Total number of marbles = (6 + 4 + 2 + 3) = 15.

Let E be the event of drawing 4 marbles such that none is blue.

Then, $n(E)$ = number of ways of drawing 4 marbles out of 11 non-blue

$$= {}^{11}C_4 = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = 330.$$

And, $n(S) \quad = {}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365$.

$\therefore \quad P(E) \quad = P(E) = \frac{n(E)}{n(S)} = \frac{330}{1365} = \frac{22}{91}$.

$\therefore \quad$ Required probability = $\left(1 - \frac{22}{91}\right) = \frac{69}{91}$.

33. Total number of balls = (6 + 2 + 4 + 3) = 15.

Let E be the event of drawing 5 balls out of 9 non-blue balls

$$= {}^9C_5 = {}^9C_{(9-5)} = {}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126.$$

And, $n(S) = {}^{15}C_5 = \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} = 3003$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{126}{3003} = \frac{6}{143}$.

$\therefore \quad$ Required probability = $\left(1 - \frac{6}{143}\right) = \frac{137}{143}$.

34. Total number of balls = (2 + 3 + 2) = 7.

Let E be the event of drawing 2 non-blue balls.

Then, $n(E) = {}^5C_2 = \frac{5 \times 4}{2 \times 1} = 10$.

And, $n(S) = {}^7C_2 = \frac{7 \times 6}{2 \times 1} = 21$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{10}{21}$.

35. Total number of balls = (10 + 10) = 20.

Let E be the event of drawing 2 balls of the same colour.

$n(E)$ = number of ways of drawing 2 black balls or 2 white balls

$$n(E) = ({}^{10}C_2 + {}^{10}C_2) = 2 \times {}^{10}C_2 = 2 \times \frac{10 \times 9}{2 \times 1} = 90.$$

$n(S)$ = number of ways of drawing 2 balls out of 20

$$= {}^{20}C_2 = \frac{20 \times 19}{2 \times 1} = 190.$$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{90}{190} = \frac{9}{19}$.

36. $P(\text{none is defective}) = \frac{{}^{16}C_2}{{}^{20}C_2} = \left(\frac{16 \times 15}{2 \times 1} \times \frac{2 \times 1}{20 \times 19}\right) = \frac{12}{19}$.

P(at least 1 is defective) = $\left(1 - \frac{12}{19}\right) = \frac{7}{19}$.

37. Let S be the sample space and let E be the event of selecting 2 boys and 1 girl.

Then, $n(S)$ = number of ways of selecting 3 students out of 25 = ${}^{25}C_3 = \frac{25 \times 24 \times 23}{3 \times 2 \times 1} = 2300$.

And, $n(E) = ({}^{15}C_2 \times {}^{10}C_1) = \left(\frac{15 \times 14}{2 \times 1} \times 10\right) = 1050$.

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1050}{2300} = \frac{21}{46}$.

38. $n(S)$ = number of ways of choosing 4 persons out of 9

$$= {}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126.$$

$n(E)$ = Number of ways of choosing 2 children out of 4 and 2 persons out of (3 + 2) persons

$$n(E) = ({}^4C_2 \times {}^5C_2) = \left(\frac{4 \times 3}{2 \times 1} \times \frac{5 \times 4}{2 \times 1}\right) = 60.$$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{60}{126} = \frac{10}{21}$.

39. Clearly, $n(S) = (6 \times 6) = 36$.

Let E be the event that the sum is a prime number. Then,

$n(E)$ = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)}

$\therefore \quad n(E) = 15$.

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}.$

40. $P(E) = \dfrac{30}{100} = \dfrac{3}{10}, P(H) = \dfrac{20}{100} = \dfrac{1}{5}$ and $P(E \cap H) = \dfrac{10}{100} = \dfrac{1}{10}.$

$P(E \text{ or } H) = P(E \cup H)$

$= P(E) + P(H) - P(E \cap H) = \left(\dfrac{3}{10} + \dfrac{1}{5} - \dfrac{1}{10}\right) = \dfrac{4}{10} = \dfrac{2}{5}.$

41. Let E_1 = Event that the husband is selected
and E_2 = Event that the wife is selected. Then,

$P(E_1) = \dfrac{1}{7}$ and $P(E_2) = \dfrac{1}{5}.$

$\therefore \quad P(\bar{E}_1) = \left(1 - \dfrac{1}{7}\right) = \dfrac{6}{7}$ and $P(\bar{E}_2) = \left(1 - \dfrac{1}{5}\right) = \dfrac{4}{5}.$

$\therefore$ Required probability = P[(A and not B) or (B and not A)]

$= P\left[(E_1 \cap \bar{E}_2) \text{ or } (E_2 \cap \bar{E}_1)\right]$

$= P\left[(E_1 \cap \bar{E}_2) + P(E_2 \cap \bar{E}_1)\right]$

$= P(E_1) \,.\, P(\bar{E}_2) + P(E_2) \,.\, P(\bar{E}_1) = \left(\dfrac{1}{7} \times \dfrac{4}{5}\right) + \left(\dfrac{1}{5} \times \dfrac{6}{7}\right) = \dfrac{10}{35} = \dfrac{2}{7}.$

42. Let E_1 = Event that A speaks the truth
and E_2 = Event that B speaks the truth. Then,

$P(E_1) = \dfrac{75}{100} = \dfrac{3}{4}, P(E_2) = \dfrac{80}{100} = \dfrac{4}{5}, P(\bar{E}_1) = \left(1 - \dfrac{3}{4}\right)$

$= \dfrac{1}{4}, P(\bar{E}_2) = \left(1 - \dfrac{4}{5}\right) = \dfrac{1}{5}.$

P(A and B contradict each other)
= P[(A speaks the truth and B tells a lie) or (A tells a lie and B speaks the truth)]

$= P\left[(E_1 \cap \bar{E}_2) \text{ or } (\bar{E}_1 \cap E_2)\right] = P(E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2)$

$= P(E_1) \,.\, P(\bar{E}_2) + P(\bar{E}_1) \,.\, P(E_2)$

$= \left(\dfrac{3}{4} \times \dfrac{1}{5}\right) + \left(\dfrac{1}{4} \times \dfrac{4}{5}\right) = \left(\dfrac{3}{20} + \dfrac{1}{5}\right) = \dfrac{7}{20}$

$= \left(\dfrac{7}{20} \times 100\right)\% = 35\%.$

43. Let E_1 = Event that A speaks the truth
and E_2 = Event that B speaks the truth.

Then, $P(E_1) = \dfrac{60}{100} = \dfrac{3}{5}, P(E_2) = \dfrac{70}{100} = \dfrac{7}{10}, P(\bar{E}_1)$

$= \left(1 - \dfrac{3}{5}\right) = \dfrac{2}{5}, P(\bar{E}_2) = \left(1 - \dfrac{7}{10}\right) = \dfrac{3}{10}.$

P(A and B say the same thing)
= P[(A speaks the truth and B speaks the truth) or (A tells a lie and B tells a lie)]

$= P\left[(E_1 \cap E_2) \text{ or } (\bar{E}_1 \cap \bar{E}_2)\right] = P(E_1 \cap E_2) + P(\bar{E}_1 \cap \bar{E}_2)$

$= P(E_1) \,.\, P(E_2) + P(\bar{E}_1) \,.\, P(\bar{E}_2)$

$= \left(\dfrac{3}{5} \times \dfrac{7}{10}\right) + \left(\dfrac{2}{5} \times \dfrac{3}{10}\right) = \dfrac{27}{50} = 0.54.$

44. Total number of persons = (3 + 2) = 5.

$\therefore \quad n(S) = {}^5C_3 = {}^5C_2 = \dfrac{5 \times 4}{2 \times 1} = 10.$

Let E be the event of selecting 3 members having at least 1 woman
Then, $n(E)$ = n[(1 woman and 2 men) or (2 women and 1 man)]
= n(1 woman and 2 men) + n(2 women and 1 man)
= $({}^2C_1 \times {}^3C_1) + ({}^2C_2 \times {}^3C_1) = ({}^2C_1 \times {}^3C_1) + (1 \times {}^3C_1)$
= (2 × 3) + (1 × 3) = (6 + 3) = 9.

$\therefore \quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{9}{10}.$

45. Number of blue balls = 3 balls
Number of green balls = 2 balls
Number of red balls = 5 balls
Total balls in the bag = 3 + 2 + 5 = 10
Total possible outcomes = Selection of 4 balls out of 10 balls = ${}^{10}C_4 = \dfrac{10!}{4! \times (10-4)!} = \dfrac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} = 210$

Favorable outcomes = (selection of 2 green balls out of 2 balls) × (selection of 2 balls out of 3 blue balls)

$= {}^2C_2 \times {}^3C_2$

= 1 × 3 = 3

$\therefore$ Required probability = $\dfrac{\text{Favorable out comes}}{\text{Total possible outcomes}}$

$= \dfrac{3}{210} = \dfrac{1}{70}$

46. Probability of hitting the target:

Dev can hit target $\Rightarrow \dfrac{3}{6} = \dfrac{1}{2}$, Lakhan can hit target $= \dfrac{4}{4} = 1$

Pawan can hit target = $\dfrac{2}{6} = \dfrac{1}{3}$

Required probability that at least 2 shorts hit target

$= \dfrac{1}{2} \times \dfrac{2}{3} + \dfrac{1}{2} \times \dfrac{1}{3} + \dfrac{1}{2} \times \dfrac{1}{3}$

$= \dfrac{1}{3} + \dfrac{1}{6} + \dfrac{1}{6} = \dfrac{4}{6} = \dfrac{2}{3}$

47. Out of 10 mangoes, 4 mangoes are rotten

$\therefore$ Required probability = $\dfrac{{}^6C_2}{{}^{10}C_2} = \dfrac{\dfrac{6!}{2!(6-2)!}}{\dfrac{10!}{2!(10-2)!}} = \dfrac{\dfrac{6!}{2!4!}}{\dfrac{10!}{2! \times 8!}}$

$= \dfrac{\dfrac{6 \times 5}{1 \times 2}}{\dfrac{10 \times 9}{1 \times 2}} = \dfrac{6 \times 5}{10 \times 9} = \dfrac{1}{3}$

48. Number of red balls = 4
Number of yellow ball = 5
Number of pink ball = 6
Total number of balls = 4 + 5 + 6 = 15
Total possible outcomes = selection of 2 balls out of 15 balls = ${}^{15}C_2 = \dfrac{15!}{2!(15-2)!} = \dfrac{15!}{2! \times 13!} = \dfrac{15 \times 14}{1 \times 2} = 105$

Total favourable outcomes = selection of 2 balls out of 4 orange and 6 pink balls.

$$^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10\times 9}{1\times 2} = 45$$

$\therefore$ Required probability = $\frac{45}{105} = \frac{3}{7}$

49. Number of red balls = 6

Number of yellow ball = 11

Number of pink balls = 5

Total number of balls = 6 + 11 + 5 = 22

Total possible outcomes $= n(E) = {}^{22}C_2 = \frac{22!}{2!(22-2)!} = \frac{22!}{2!\times 20!}$

$= \frac{22\times 21}{2\times 1} = 231$

Number of favourable outcomes

$= n(s) = {}^{6}C_1 \times {}^{11}C_1 = 6\times 11 = 66$

Required probability = $\frac{n(E)}{n(S)} = \frac{66}{231} = \frac{2}{7}$

50. Number of Red balls = 4

Number of Blue balls = 6

Number of Pink balls = 8

Total number of balls = 4 + 6 + 8 = 18

Required probability

$$= \frac{4}{18}\times\frac{11}{20} + \frac{6}{18}\times\frac{11}{20}$$

$$= \frac{11}{20}\left[\left(\frac{4}{18}+\frac{6}{18}\right)\right]$$

$$= \frac{11}{20}\times\frac{10}{18} = \frac{11}{36}$$