10 Logarithms

IMPORTANT FACTS AND FORMULAE

I. Logarithm: If a is a positive real number, other than 1 and $a^m = x$, then we write: $m = \log_a x$ and we say that the value of $\log x$ to the base a is m.

Example:

(*i*) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$ (*ii*) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(*ii*) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$ (*iv*) $(.1)^2 = .01 \Rightarrow \log_{(.1)} \cdot 01 = 2$

II. Properties of Logarithms:

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_x x = 1$
4. $\log_a 1 = 0$
5. $\log_a (x^p) = p (\log_a x)$
6. $\log_a x = \frac{1}{\log_x a}$
7. $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}$
8. $a^{\log_a x} = x$
9. $x^{\log_a y} = y^{\log_a x}$
10. $\log_{a^q} x^p = \frac{p}{q} \log_a x$

Remember: When base is not mentioned, it is taken as 10.

III. Common Logarithms: Logarithms to the base 10 are known as common logarithms.

IV. The logarithm of a number contains two parts, namely *characteristic* and *mantissa.*

Characteristic: The integral part of the logarithm of a number is called its ***characteristic.***

Case I : When the number is greater than 1.

In this case, the characteristic is one less than the number of digits to the left of the decimal point in the given number.

Case II : When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of – 1, – 2, etc. we write, $\bar{1}$ (one bar), $\bar{2}$ (two bar), etc.

Example:

Number	*Characteristic*	*Number*	*Characteristic*
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa: The decimal part of the logarithm of a number is known as its ***mantissa.***

For mantissa, we look through log table.

SOLVED EXAMPLES

Ex. 1. ***Evaluate : (i)*** $\log_3 27$ ***(ii)*** $\log_7 \left(\frac{1}{343}\right)$ ***(iii)*** $\log_{100} (0.01)$ ***(iv)*** $\log_8 128$

Sol. (*i*) $\log_3 27 = \log_3 3^3 = 3 \log_3 3 = 3.$ [$\because \log_3 3 = 1$]

(*ii*) $\log_7 \left(\frac{1}{343}\right) = \log_7 \left(\frac{1}{7^3}\right) = \log_7 7^{-3} = -3 \log_7 7 = -3.$

(*iii*) Let $\log_{100} (0.01) = \log_{100} \left(\frac{1}{100}\right) = \log_{100}(100)^{-1} = -1 \log_{100} 100 = -1.$

(*iv*) $\log_8 128 = \log_{2^3} (2^7) = \frac{7}{3} \log_2 2 = \frac{7}{3}.$

Ex. 2. ***Evaluate: (i)*** $\log_7 1 = 0$ ***(ii)*** $\log_{34} 34$ ***(iii)*** $36^{\log 64}$

Sol. (*i*) We know that $\log_a 1 = 0$, so $\log_7 1 = 0$.

(*ii*) We know that $\log_a a = 1$, so $\log_{34} 34 = 1$.

(*iii*) We know that $a^{\log a\ x} = x$.

Now, $36^{\log 64} = 6^{2\ (\log 6\ 4)} = 6^{\log 6\ (42)} = 6^{\log 616} = 16.$

Ex. 3. ***If*** $\log_{\sqrt{8}} x = 3\frac{1}{3}$, ***find the value of x.***

Sol. $\log_{\sqrt{8}} x = \frac{10}{3} \Leftrightarrow x = (\sqrt{8})^{10/3} = (2^{3/2})^{10/3} = 2^{\left(\frac{3}{2} \times \frac{10}{3}\right)} = 2^5 = 32.$

Ex. 4. ***Evaluate:*** **(i)** $\log_5 3 \times \log_{27} 25$ **(ii)** $\log_9 27 - \log_{27} 9$

Sol. (*i*) $\log_5 3 \times \log_{27} 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 27} = \frac{\log 3}{\log 5} \times \frac{\log (5^2)}{\log (3^3)} = \frac{\log 3}{\log 5} \times \frac{2 \log 5}{3 \log 3} = \frac{2}{3}.$

(*ii*) Let $\log_9 27 = n$. Then, $9^n = 27 \Leftrightarrow 3^{2n} = 3^3 \Leftrightarrow 2n = 3 \Leftrightarrow n = \frac{3}{2}.$

Again, let $\log_{27} 9 = m$.

Then, $27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m = 2 \Leftrightarrow m = \frac{2}{3}.$

$\therefore \quad \log_9 27 - \log_{27} 9 = (n - m) = \left(\frac{3}{2} - \frac{2}{3}\right) = \frac{5}{6}.$

Ex. 5. ***Simplify:*** $\left(\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243}\right)$

Sol. $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} = \log \frac{75}{16} - \log \left(\frac{5}{9}\right)^2 + \log \frac{32}{243}$

$= \log \frac{75}{16} - \log \frac{25}{81} + \log \frac{32}{243} = \log \left(\frac{75}{16} \times \frac{32}{243} \times \frac{81}{25}\right) = \log 2.$

Ex. 6. ***If*** $\log_2 [\log_3 (\log_2 x)] = 1$, ***find the value of x.*** **(M.B.A., 2005)**

Sol. $\log_2 [\log_3 (\log_2 x)] = 1 \Rightarrow \log_3 (\log_2 x) = 2^1 = 2 \Rightarrow \log_2 x = 3^2 = 9.$

Ex. 7. ***If*** $\log_{10}(x^2 - 6x + 45) = 2$, ***find the value of x.*** **(R.R.B., 2006)**

Sol. $\log 10\ (x^2 - 6x + 45) = 2 \Rightarrow x^2 - 6x + 45 = 10^2 = 100$

$\Rightarrow x^2 - 6x - 55 = 0 \Rightarrow x^2 - 11x + 5x - 55 = 0$

$\Rightarrow x\ (x - 11) + 5\ (x - 11) = 0 \Rightarrow (x - 11)\ (x + 5) = 0$

$\Rightarrow x = 11$ or $x = -5$.

Ex. 8. ***Find the value of x which satisfies the relation*** $\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$ **(M.B.A., 2002)**

Sol. $\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$

$\Leftrightarrow \log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + \log_{10} 10$

$\Leftrightarrow \log_{10} [3\ (4x + 1)] = \log_{10} [10\ (x + 1)]$

$\Leftrightarrow 3\ (4x + 1) = 10\ (x + 1) \Leftrightarrow 12x + 3 = 10x + 10 \Leftrightarrow 2x = 7 \Leftrightarrow x = \frac{7}{2}.$

Ex. 9. ***Simplify:*** $\left[\frac{1}{\log_{xy} (xyz)} + \frac{1}{\log_{yz} (xyz)} + \frac{1}{\log_{zx} (xyz)}\right]$ **(M.A.T., 2005)**

Sol. Given expression $= \log_{xyz}(xy) + \log_{xyz}(yz) + \log_{xyz}(zx)$

$= \log_{xyz}(xy \times yz \times zx) = \log_{xyz}(xyz)^2$

$= 2\log_{xyz}(xyz) = 2 \times 1 = 2$ $\left[\because \log_a x = \dfrac{1}{\log_x a}\right]$

Ex. 10. *If* $\log_a b = \dfrac{1}{2}, \log_b c = \dfrac{1}{3}$ *and* $\log_c a = \dfrac{k}{5}$*, find the value of k.* **(M.A.T., 2006)**

Sol. $\log_a b = \dfrac{1}{2}, \log_b c = \dfrac{1}{3}, \log_c a = \dfrac{k}{5} \Rightarrow \dfrac{\log b}{\log a} = \dfrac{1}{2}, \dfrac{\log c}{\log b} = \dfrac{1}{3}, \dfrac{\log a}{\log c} = \dfrac{k}{5}$

$\Rightarrow \dfrac{\log b}{\log a} \times \dfrac{\log c}{\log b} \times \dfrac{\log a}{\log c} = \dfrac{1}{2} \times \dfrac{1}{3} \times \dfrac{k}{5} \Rightarrow \dfrac{k}{30} = 1 \Rightarrow k = 30.$

Ex. 11. *If* $\log_{10} 2 = 0.30103$*, find the value of* $\log_{10} 50$.

Sol. $\log_{10} 50 = \log_{10}\left(\dfrac{100}{2}\right) = \log_{10} 100 - \log_{10} 2 = 2 - 0.30103 = 1.69897.$

Ex. 12. *If* $\log 2 = 0.3010$ *and* $\log 3 = 0.4771$*, find the values of :*

(i) $\log 25$ **(ii)** $\log 4.5$

Sol. (i) $\log 25 = \log\left(\dfrac{100}{4}\right) = \log 100 - \log 4 = 2 - 2\log 2 = (2 - 2 \times 0.3010) = 1.398.$

(ii) $\log 4.5 = \log\left(\dfrac{9}{2}\right) = \log 9 - \log 2 = 2\log 3 - \log 2$

$= (2 \times 0.4771 - 0.3010) = 0.6532$

Ex. 13. *If* $\log 2 = 0.30103$*, find the number of digits in* 2^{56}. **(M.A.T. 2005)**

Sol. $\log(2^{56}) = 56\log 2 = (56 \times 0.30103) = 16.85768.$

Its characteristic is 16. Hence, the number of digits in 2^{56} is 17.

EXERCISE

(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (✓) against the correct answer:*

1. The value of $\log_2 16$ is
   (a) $\dfrac{1}{8}$ (b) 4
   (c) 8 (d) 16
2. The value of $\log_{343} 7$ is
   (a) $\dfrac{1}{3}$ (b) -3
   (c) $-\dfrac{1}{3}$ (d) 3
3. The value of $\log_5 \dfrac{(125)(625)}{25}$ is equal to **(M.B.A., 2011)**
   (a) 725 (b) 5
   (c) 3125 (d) 6
4. The value of $\log_{\sqrt{2}} 32$ is
   (a) $\dfrac{5}{2}$ (b) 5
   (c) 10 (d) $\dfrac{1}{10}$
5. Determine the value of $\log_{3\sqrt{2}}\left(\dfrac{1}{18}\right)$. **(M.A.T., 2005)**
   (a) 2 (b) -2
   (c) $\sqrt{2}$ (d) $\sqrt{3}$
6. The value of $\log_{10}(.0001)$ is
   (a) $\dfrac{1}{4}$ (b) $-\dfrac{1}{4}$
   (c) -4 (d) 4
7. The value of $\log_{.01}(1000)$ is
   (a) $\dfrac{1}{3}$ (b) $-\dfrac{1}{3}$
   (c) $\dfrac{3}{2}$ (d) $-\dfrac{3}{2}$
8. What is the value of $[\log_{10}(5\log_{10} 100)]^2$? **(M.B.A., 2010)**
   (a) 1 (b) 2
   (c) 10 (d) 25
9. The logarithm of 0.0625 to the base 2 is
   (a) -4 (b) -2
   (c) 0.25 (d) 0.5
10. The logarithm of 0.00001 to the base 0.01 is equal to
    (a) $-\dfrac{5}{2}$ (b) $\dfrac{5}{2}$
    (c) 3 (d) 5

11. If $\log_3 x = -2$, then x is equal to
(*a*) -9 (*b*) -6
(*c*) -8 (*d*) $\frac{1}{9}$

12. If $\log_8 x = \frac{2}{3}$, then the value of x is
(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$
(*c*) 3 (*d*) 4

13. If $\log_8 p = 25$ and $\log_2 q = 5$, then
(*a*) $p = q^{15}$ (*b*) $p^2 = q^3$
(*c*) $p = q^5$ (*d*) $p^3 = q$

14. If $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2}$, then x is equal to
(*a*) $-\frac{3}{4}$ (*b*) $\frac{3}{4}$
(*c*) $\frac{81}{256}$ (*d*) $\frac{256}{81}$

15. If $\log_x 4 = 0.4$, then the value of x is
(*a*) 1 (*b*) 4
(*c*) 16 (*d*) 32

16. If $\log_{10000} x = -\frac{1}{4}$, then x is equal to **(N.M.A.T., 2006)**
(*a*) $\frac{1}{10}$ (*b*) $\frac{1}{100}$
(*c*) $\frac{1}{1000}$ (*d*) $\frac{1}{10000}$

17. If $\log_x 4 = \frac{1}{4}$, then x is equal to
(*a*) 16 (*b*) 64
(*c*) 128 (*d*) 256

18. If $\log_x (0.1) = -\frac{1}{3}$, then the value of x is
(*a*) 10 (*b*) 100
(*c*) 1000 (*d*) $\frac{1}{1000}$

19. If $\log_{32} x = 0.8$, then x is equal to
(*a*) 25.6 (*b*) 16
(*c*) 10 (*d*) 12.8

20. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is
(*a*) 2^{10} (*b*) 2^{100}
(*c*) 2^{1000} (*d*) 2^{10000}

21. The value of $\log_{(-1/3)} 81$ is equal to
(*a*) -27 (*b*) -4
(*c*) 4 (*d*) 27

22. The value of $\log_{2\sqrt{3}} (1728)$ is
(*a*) 3 (*b*) 5
(*c*) 6 (*d*) 9

23. $\frac{\log \sqrt{8}}{\log 8}$ is equal to **(M.B.A. 2004, I.A.F., 2002)**
(*a*) $\frac{1}{\sqrt{8}}$ (*b*) $\frac{1}{4}$
(*c*) $\frac{1}{2}$ (*d*) $\frac{1}{8}$

24. Which of the following statements is not correct ? **(M.B.A., 2003)**
(a) $\log_{10} 10 = 1$
(b) $\log (2 + 3) = \log (2 \times 3)$
(c) $\log_{10} 1 = 0$
(d) $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

25. The value of $\frac{6 \log_{10} 1000}{3 \log_{10} 100}$ is equal to **(C.D.S., 2002)**
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

26. $\log_{10} (10 \times 10^2 \times 10^3 \times \ldots\ldots \times 10^9)$ is
(*a*) 10 (*b*) 20
(*c*) 45 (*d*) 55

27. The value of $\log_2 (\log_5 625)$ is :
(*a*) 2 (*b*) 5
(*c*) 10 (*d*) 15

28. If $\log_2 [\log_3 (\log_2 x)] = 1$, then x is equal to : **(M.B.A., 2007)**
(*a*) 0 (*b*) 12
(*c*) 128 (*d*) 512

29. $\log_{10} \log_{10} \log_{10} (10^{10^{10}})$ is equal to
(*a*) 0 (*b*) 1
(*c*) 10 (*d*) 100

30. The value of $\log_2 \log_2 \log_3 \log_3 27^3$ is
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

31. The value of $\log_2 [\log_2 \{\log_4 (\log_4 256^4)\}]$ is
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 4

32. If $a^x = b^y$, then
(*a*) $\log \frac{a}{b} = \frac{x}{y}$ (*b*) $\frac{\log a}{\log b} = \frac{x}{y}$
(*c*) $\frac{\log a}{\log b} = \frac{y}{x}$ (*d*) None of these

33. log 360 is equal to
(*a*) 2 log 2 + 3 log 3
(*b*) 3 log 2 + 2 log 3
(*c*) 3 log 2 + 2 log 3 – log 5
(*d*) 3 log 2 + 2 log 3 + log 5

34. $\log_{10} \frac{26}{51} + \log_{10} \frac{119}{91} - \log_{10} \frac{13}{32} - \log_{10} \frac{64}{39}$ is equal to
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

35. The value of $\left(\frac{1}{3}\log_{10}125 - 2\log_{10}4 + \log_{10}32\right)$ is

(*a*) 0 (*b*) $\frac{4}{5}$

(*c*) 1 (*d*) 2

36. The value of $\log_{10}1\frac{1}{2} + \log_{10}1\frac{1}{3} + \ldots$ up to 198 terms is equal to

(*a*) 0 (*b*) 2

(*c*) 10 (*d*) 100

37. What is the value of the following expression?

$\log\left(\frac{9}{14}\right) - \log\left(\frac{15}{16}\right) + \log\left(\frac{35}{24}\right)$ **(I.I.F.T., 2005)**

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

38. $2\log_{10}5 + \log_{10}8 - \frac{1}{2}\log_{10}4 = ?$ **(M.B.A., 2002)**

(*a*) 2 (*b*) 4

(*c*) $2 + 2\log_{10}2$ (*d*) $4 - 4\log_{10}2$

39. The value of $\log_{10}2 + 16\log_{10}\frac{16}{15} + 12\log_{10}\frac{25}{24} + 7\log_{10}\frac{81}{80}$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

40. If $\log_a(ab) = x$, then $\log_b(ab)$ is **(M.A.T., 2002)**

(*a*) $\frac{1}{x}$ (*b*) $\frac{x}{x+1}$

(*c*) $\frac{x}{1-x}$ (*d*) $\frac{x}{x-1}$

41. If $\log_a m = x$, then $\log_{\frac{1}{a}}\left(\frac{1}{m}\right)$ equals

(*a*) $-\frac{1}{x}$ (*b*) $\frac{1}{x}$

(*c*) $-x$ (*d*) x

42. If $\log_{10}2 = a$ and $\log_{10}3 = b$, then $\log_5 12$ equals **(M.B.A., 2010)**

(*a*) $\frac{a+b}{1+a}$ (*b*) $\frac{2a+b}{1+a}$

(*c*) $\frac{a+2b}{1+a}$ (*d*) $\frac{2a+b}{1-a}$

43. If $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, then the value of $\log\left(4.\sqrt[3]{63}\right)$ is

(*a*) $2x + \frac{2}{3}y - \frac{1}{3}z$ (*b*) $2x + \frac{2}{3}y + \frac{1}{3}z$

(*c*) $2x - \frac{2}{3}y + \frac{1}{3}z$ (*d*) $-2x + \frac{2}{3}y + \frac{1}{3}z$

44. If $\log_4 x + \log_2 x = 6$, then x is equal to

(*a*) 2 (*b*) 4

(*c*) 8 (*d*) 16

45. If $\log_{10}(x^2 - 6x + 10) = 0$, then the value of x is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

46. If $\log_{10}x + \log_{10}y = 3$ and $\log_{10}x - \log_{10}y = 1$, then x and y are respectively

(*a*) 10 and 100 (*b*) 100 and 10

(*c*) 1000 and 100 (*d*) 100 and 1000

47. If $\log_{10}x + \log_{10}5 = 2$, then x equals

(*a*) 15 (*b*) 20

(*c*) 25 (*d*) 100

48. If $\log_8 x + \log_8\frac{1}{6} = \frac{1}{3}$, then the value of x is

(*a*) 12 (*b*) 16

(*c*) 18 (*d*) 24

49. If $\log_{10}125 + \log_{10}8 = x$, then x is equal to **(M.B.A., 2005)**

(*a*) $\frac{1}{3}$ (*b*) .064

(*c*) -3 (*d*) 3

50. The value of $(\log_9 27 + \log_8 32)$ is

(*a*) $\frac{7}{2}$ (*b*) $\frac{19}{6}$

(*c*) 4 (*d*) 7

51. $(\log_5 3) \times (\log_3 625)$ equals

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

52. $(\log_5 5)(\log_4 9)(\log_3 2)$ is equal to

(*a*) 1 (*b*) $\frac{3}{2}$

(*c*) 2 (*d*) 5

53. If $\log_{12}27 = a$, then $\log_6 16$ is

(*a*) $\frac{3-a}{4(3+a)}$ (*b*) $\frac{3+a}{4(3-a)}$

(*c*) $\frac{4(3+a)}{(3-a)}$ (*d*) $\frac{4(3-a)}{(3+a)}$

54. If $\log_{10}5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$, then x is equal to

(*a*) 1 (*b*) 3

(*c*) 5 (*d*) 10

55. If $\log_5(x^2 + x) - \log_5(x + 1) = 2$, then the value of x is **(M.B.A., 2007)**

(*a*) 5 (*b*) 10

(*c*) 25 (*d*) 32

56. $\frac{1}{2}(\log x + \log y)$ will equal $\log\left(\frac{x+y}{2}\right)$ if **(R.R.B., 2005)**

(a) $y = 0$ (b) $x = \sqrt{y}$

(c) $x = y$ (d) $x = \frac{y}{2}$

57. The value of $\left(\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}\right)$ is

(a) 0 (b) 1

(c) 5 (d) 60

58. The value of $(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)(\log_8 9)$ is

(a) 2 (b) 7

(c) 8 (d) 33

59. The value of $16^{\log 45}$ is

(a) $\frac{5}{64}$ (b) 5

(c) 16 (d) 25

60. If $\log x + \log y = \log(x + y)$, then

(a) $x = y$ (b) $xy = 1$

(c) $y = \frac{x-1}{x}$ (d) $y = \frac{x}{x-1}$

61. If $\log\frac{a}{b} + \log\frac{b}{a} = \log(a + b)$, then **(M.B.A., 2007)**

(a) $a + b = 1$ (b) $a - b = 1$

(c) $a = b$ (d) $a^2 - b^2 = 1$

62. $\left[\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ac}\right) + \log\left(\frac{c^2}{ab}\right)\right]$ is equal to **(M.B.A., 2006)**

(a) 0 (b) 1

(c) 2 (d) abc

63. $\frac{1}{\log_a b} \times \frac{1}{\log_b c} \times \frac{1}{\log_c a}$ is equal to **(Hotel Management, 2010)**

(a) $a + b + c$ (b) abc

(c) 0 (d) 1

64. $\left[\frac{1}{(\log_a bc) + 1} + \frac{1}{(\log_b ca) + 1} + \frac{1}{(\log_c ab) + 1}\right]$ is equal to **(I.I.F.T., 2005)**

(a) 1 (b) $\frac{3}{2}$

(c) 2 (d) 3

65. The value of $\left[\frac{1}{\log_{(p/q)} x} + \frac{1}{\log_{(q/r)} x} + \frac{1}{\log_{(r/p)} x}\right]$ is

(a) 0 (b) 1

(c) 2 (d) 3

66. If $\log_{10} 7 = a$, then $\log_{10}\left(\frac{1}{70}\right)$ is equal to **(C.D.S., 2003)**

(a) $-(1 + a)$ (b) $(1 + a)^{-1}$

(c) $\frac{a}{10}$ (d) $\frac{1}{10a}$

67. If $a = b^x$, $b = c^y$ and $c = a^z$, then the value of xyz is equal to

(a) -1 (b) 0

(c) 1 (d) abc

68. If $\log x - 5 \log 3 = -2$, then x equals **(M.B.A., 2011)**

(a) 0.8 (b) 0.81

(c) 1.25 (d) 2.43

69. If $a = b^2 = c^3 = d^4$, then the value of $\log_a(abcd)$ would be **(M.B.A., 2008)**

(a) $\log_a 1 + \log_a 2 + \log_a 3 + \log_a 4$ (b) $\log_a 24$

(c) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ (d) $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$

70. If $\log_3 x + \log_9 x^2 + \log_{27} x^3 = 9$, then x equals **(M.B.A., 2010)**

(a) 3 (b) 9

(c) 27 (d) None of these

71. If $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$, what is the value of x? **(M.B.A., 2009)**

(a) 2 (b) 3

(c) 4 (d) 5

72. If $a = \log_8 225$ and $b = \log_2 15$, then a in terms of b, is **(M.B.A., 2010)**

(a) $\frac{b}{2}$ (b) $\frac{2b}{3}$

(c) b (d) $\frac{3b}{2}$

73. If $\log 27 = 1.431$, then the value of $\log 9$ is

(a) 0.934 (b) 0.945

(c) 0.954 (d) 0.958

74. If $\log_{10} 2 = 0.3010$, then $\log_2 10$ is equal to

(a) $\frac{699}{301}$ (b) $\frac{1000}{301}$

(c) 0.3010 (d) 0.6990

75. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 5$ is

(a) 0.3241 (b) 0.6911

(c) 0.6990 (d) 0.7525

76. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 80$ is

(a) 1.6020 (b) 1.9030

(c) 3.9030 (d) None of these

77. If log 3 = 0.477 and $(1000)^x = 3$, then x equals
(*a*) 0.0159 (*b*) 0.0477
(*c*) 0.159 (*d*) 10

78. If $\log_{10} 2 = 0.3010$, the value of $\log_{10} 25$ is
(*a*) 0.6020 (*b*) 1.2040
(*c*) 1.3980 (*d*) 1.5050

79. If $\log_{10}20 = 1.3010$ and $\log_{10}30 = 1.4771$, then $\log_{10}$ (60000) is equal to
(*a*) 0.7781 (*b*) 1.7781
(*c*) 2.7781 (*d*) 4.7781

80. If log 2 = 0.3010 and log 3 = 0.4771, the value of $\log_5 512$ is **(M.A.T., 2002)**
(*a*) 2.870 (*b*) 2.967
(*c*) 3.876 (*d*) 3.912

81. If $\log_{10}3 = 0.4771$ and $\log_{10}7 = 0.8451$, then the value of $\log_{10}\left(23\frac{1}{3}\right)$ is equal to
(*a*) 0.368 (*b*) 1.356
(*c*) 1.368 (*d*) 1.477

82. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, then the value of $\log_{10} 1.5$ is
(*a*) 0.1761 (*b*) 0.7116
(*c*) 0.7161 (*d*) 0.7611

83. If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is
(*a*) 0.4471 (*b*) 1.4471
(*c*) 2.4471 (*d*) None of these

84. If log (0.57) = $\bar{1}.756$, then the value of log 57 + log $(0.57)^3$ + log $\sqrt{0.57}$ is
(*a*) 0.902 (*b*) $\bar{2}.146$
(*c*) 1.902 (*d*) $\bar{1}.146$

85. If the logarithm of a number is – 3.153, what are characteristic and mantisa? **(M.B.A., 2009)**
(*a*) Characteristic = – 4, mantissa = 0.847
(*b*) Characteristic = – 3, mantissa = – 0.153
(*c*) Characteristic = 4, mantissa = – 0.847
(*d*) Characteristic = 3, mantissa = – 0.153

86. If log 2 = 0.30103, the number of digits in 2^{64} is
(*a*) 18 (*b*) 19
(*c*) 20 (*d*) 21

87. If log 2 = 0.30103, the number of digits in 4^{50} is **(Hotel Management, 2009)**
(*a*) 30 (*b*) 31
(*c*) 100 (*d*) 200

88. If log 2 = 0.30103, then the number of digits in 5^{20} is
(*a*) 14 (*b*) 16
(*c*) 18 (*d*) 25

89. If log 2 = 0.30103, log 3 = 0.47712, then the number of digits in 6^{20} is
(*a*) 15 (*b*) 16
(*c*) 17 (*d*) 18

90. The number of digits in $4^9 \times 5^{17}$, when expressed in usual form, is **(A.A.O. Exam, 2009)**
(*a*) 16 (*b*) 17
(*c*) 18 (*d*) 19

91. If log 3 log $(3^x - 2)$ and log $(3^x + 4)$ are in arithmetic progression, then x is equal to **(IIFT, 2012)**
(*a*) 8/3 (*b*) log 3^8
(*c*) log 2^3 (*d*) 8

92. If $\log_{10}a = p$ and $\log_{10}b = q$, then what is $\log_{10} (a^p b^q)$ equal to ? **(CDS, 2012)**
(*a*) $p^2 + q^2$ (*b*) $p^2 - q^2$
(*c*) p^2q^2 (*d*) $\frac{p^2}{q^2}$

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*b*)	**4.** (*c*)	**5.** (*b*)	**6.** (*c*)	**7.** (*d*)	**8.** (*a*)	**9.** (*a*)	**10.** (*b*)
11. (*d*)	**12.** (*d*)	**13.** (*a*)	**14.** (*d*)	**15.** (*d*)	**16.** (*a*)	**17.** (*d*)	**18.** (*c*)	**19.** (*b*)	**20.** (*c*)
21. (*b*)	**22.** (*c*)	**23.** (*c*)	**24.** (*b*)	**25.** (*d*)	**26.** (*c*)	**27.** (*a*)	**28.** (*d*)	**29.** (*b*)	**30.** (*a*)
31. (*a*)	**32.** (*c*)	**33.** (*d*)	**34.** (*a*)	**35.** (*c*)	**36.** (*b*)	**37.** (*a*)	**38.** (*a*)	**39.** (*b*)	**40.** (*d*)
41. (*d*)	**42.** (*d*)	**43.** (*b*)	**44.** (*d*)	**45.** (*c*)	**46.** (*b*)	**47.** (*b*)	**48.** (*a*)	**49.** (*d*)	**50.** (*b*)
51. (*d*)	**52.** (*a*)	**53.** (*d*)	**54.** (*b*)	**55.** (*c*)	**56.** (*c*)	**57.** (*b*)	**58.** (*a*)	**59.** (*d*)	**60.** (*d*)
61. (*a*)	**62.** (*a*)	**63.** (*d*)	**64.** (*a*)	**65.** (*a*)	**66.** (*a*)	**67.** (*c*)	**68.** (*d*)	**69.** (*c*)	**70.** (*c*)
71. (*c*)	**72.** (*b*)	**73.** (*c*)	**74.** (*b*)	**75.** (*c*)	**76.** (*b*)	**77.** (*c*)	**78.** (*c*)	**79.** (*d*)	**80.** (*c*)
81. (*c*)	**82.** (*a*)	**83.** (*a*)	**84.** (*a*)	**85.** (*a*)	**86.** (*c*)	**87.** (*b*)	**88.** (*a*)	**89.** (*b*)	**90.** (*c*)
91. (*c*)	**92.** (*a*)								

SOLUTIONS

1. $\log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4.$

2. $\log_{343} 7 = \log_{7^3} 7 = \frac{1}{3}\log_7 7 = \frac{1}{3}.$

3. $\log_5 \left(\frac{125 \times 625}{25}\right) = \log_5 \left(\frac{5^3 \times 5^4}{5^2}\right) = \log_5 5^5 = 5 \log_5 5 = 5.$

4. $\log_{\sqrt{2}} 32 = \log_{2^{1/2}} 2^5 = \frac{5}{\left(\frac{1}{2}\right)}\log_2 2 = 5 \times 2 = 10.$

5. $\log_{3\sqrt{2}} \left(\frac{1}{18}\right) = \log_{3\sqrt{2}} \left[\frac{1}{(3\sqrt{2})^2}\right] = \log_{3\sqrt{2}} (3\sqrt{2})^{-2}$

$= (-2) \log_{3\sqrt{2}} 3\sqrt{2} = -2.$

6. $\log_{10} (.0001) = \log_{10} \left(\frac{1}{10000}\right) = \log_{10} \left(\frac{1}{10^4}\right) = \log_{10} 10^{-4}$

$= -4 \log_{10} 10 = -4.$

7. $\log_{.01} (1000) = \log_{10^{-2}} (10^3) = -\frac{3}{2}\log_{10} 10 = -\frac{3}{2}.$

8. $[\log_{10} (5 \log_{10} 100)]^2 = [\log_{10}\{5 \log_{10} (10)^2\}]^2 = [\log_{10} (5 \times 2)]^2 = (\log_{10} 10)^2 = 1.$

9. $\log_2 0.0625 = \log_2 \left(\frac{625}{10000}\right) = \log_2 \left(\frac{1}{16}\right) = \log_2 2^{-4}$

$= (-4) \log_2 2 = -4.$

10. $\log_{0.01} 0.0001 = \log_{10^{-2}} 10^{-5} = \left(\frac{-5}{-2}\right)\log_{10} 10 = \frac{5}{2}.$

11. $\log_3 x = -2 \Rightarrow x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$

12. $\log_8 x = \frac{2}{3} \Rightarrow x = 8^{2/3} = (2^3)^{\frac{2}{3}} = 2^{\left(3 \times \frac{2}{3}\right)} = 2^2 = 4.$

13. $\log_8 p = 25$ and $\log_2 q = 5 \Rightarrow p = 8^{25}$ and $q = 2^5$

$\Rightarrow p = (2^3)^{25}$ and $q = 2^5.$

$\Rightarrow p = 2^{75}$ and $q = 2^5$

$\Rightarrow p = (2^5)^{15}$ and $q = 2^5$

$\Rightarrow p = q^{15}.$

14. $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2} \Leftrightarrow x^{-1/2} = \frac{9}{16} \Leftrightarrow \frac{1}{\sqrt{x}} = \frac{9}{16} \Leftrightarrow \sqrt{x} = \frac{16}{9}$

$\Leftrightarrow x = \left(\frac{16}{9}\right)^2 = \frac{256}{81}.$

15. $\log_x 4 = 0.4 \Leftrightarrow \log_x 4 = \frac{4}{10} = \frac{2}{5} \Leftrightarrow x^{2/5} = 4$

$\Leftrightarrow x = 4^{5/2} = (2^2)^{5/2}$

$\Leftrightarrow x = 2^{\left(2 \times \frac{5}{2}\right)} = 2^5 \Leftrightarrow x = 32.$

16. Let $\log_{10000} x = -\frac{1}{4} \Leftrightarrow x = (10000)^{-\frac{1}{4}} = (10^4)^{-1/4}$

$= 10^{-1} = \frac{1}{10}.$

17. $\log_x 4 = \frac{1}{4} \Leftrightarrow x^{1/4} = 4 \Leftrightarrow x = 4^4 = 256.$

18. $\log_x (0.1) = -\frac{1}{3} \Leftrightarrow x^{-1/3} = 0.1 \Leftrightarrow \frac{1}{x^{1/3}} = 0.1$

$\Leftrightarrow x^{1/3} = \frac{1}{0.1} = 10 \Leftrightarrow x = (10)^3 = 1000.$

19. $\log_{32} x = 0.8 \Leftrightarrow x = (32)^{0.8} = (2^5)^{4/5} = 2^4 = 16.$

20. $\log_2 x = 10 \Rightarrow x = 2^{10}.$

$\therefore \log_x y = 100 \Rightarrow y = x^{100} = (2^{10})^{100} \Rightarrow y = 2^{1000}.$

21. Let $\log_{(-1/3)} 81 = x$. Then,

$\left(-\frac{1}{3}\right)^x = 81 = 3^4 = (-3^4) = \left(-\frac{1}{3}\right)^4$

$\therefore x = -4$ *i.e.*, $\log_{(-1/3)} 81 = -4.$

22. Let $\log_{2\sqrt{3}} (1728) = x$. Then,

$(2\sqrt{3})^x = 1728 = (12)^3 = [(2\sqrt{3})^2]^3 = (2\sqrt{3})^6.$

$\therefore x = 6$, *i.e.*, $\log_{2\sqrt{3}} (1728) = 6.$

23. $\frac{\log \sqrt{8}}{\log 8} = \frac{\log (8)^{1/2}}{\log 8} = \frac{\frac{1}{2}\log 8}{\log 8} = \frac{1}{2}.$

24. (*a*) Since $\log_a a = 1$, so $\log_{10} 10 = 1.$

(*b*) $\log (2 + 3) = 5$ and $\log (2 \times 3) = \log 6 = \log 2 + \log 3.$

$\therefore \quad \log (2 + 3) \neq \log (2 \times 3).$

(*c*) Since $\log_a 1 = 0$, so $\log_{10} 1 = 0.$

(*d*) $\log (1 + 2 + 3) = \log 6 = \log (1 \times 2 \times 3)$

$= \log 1 + \log 2 + \log 3.$

So, (*b*) is incorrect.

25. $\frac{6 \log_{10} 1000}{3 \log_{10} 100} = \frac{6 \log_{10} 10^3}{3 \log_{10} 10^2} = \frac{6 \times 3 \log_{10} 10}{3 \times 2 \log_{10} 10} = \frac{18}{6} = 3.$

26. $\log_{10} (10 \times 10^2 \times 10^3 \times \ldots \times 10^9)$

$= \log_{10} 10^{(1 + 2 + 3 + \ldots + 9)} = \log_{10} 10^{45} = 45 \log_{10} 10 = 45.$

27. $\log_2 (\log_5 625) = \log_2 (\log_5 5^4) = \log_2 (4 \log_5 5) = \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2.$

28. $\log_2 [\log_3 (\log_2 x)] = 1 \Rightarrow \log_3 (\log_2 x) = 2^1 = 2 \Rightarrow \log_2 x = 3^2 = 9 \Rightarrow x = 2^9 = 512.$

29. $\log_{10} \log_{10} \log_{10} (10^{10^{10}})$

$= \log_{10} \log_{10} (10^{10} \log_{10} 10) = \log_{10} \log_{10}(10^{10})$

$= \log_{10} (10 \log_{10} 10) = \log_{10} 10 = 1.$

30. $\log_2 \log_2 \log_3 (\log_3 27^3)$

$= \log_2 \log_2 \log_3 [\log_3 (3^3)^3] = \log_2 \log_2 \log_3 [\log_3 (3)^9]$

$= \log_2 \log_2 \log_3 (9 \log_3 3) = \log_2 \log_2 \log_3 9$

$[\because \log_3 3 = 1]$

$= \log_2 \log_2 [\log_3 (3)^2] = \log_2 \log_2 (2 \log_3 3)$

$= \log_2 \log_2 2 = \log_2 1 = 0.$

31. $\log_2 [\log_2 \{\log_4 (\log_4 256^4)\}] = \log_2 \log_2 [\log_4 \{\log_4 (4^4)^4\}] = \log_2 \log_2 [\log_4 \{\log_4 4^{16}\}]$

$= \log_2 \log_2 [\log_4 \{16 \log_4 4\}] = \log_2 \log_2 \log_4 16 = \log_2 \log_2 \log_4 (4^2) = \log_2 \log_2 (2 \log_4 4)$

$= \log_2 \log_2 2 = \log_2 1 = 0.$

32. $a^x = b^y \Rightarrow \log a^x = \log b^y$
$\Rightarrow x \log a = y \log b \Rightarrow \dfrac{\log a}{\log b} = \dfrac{y}{x}.$

33. $360 = (2 \times 2 \times 2) \times (3 \times 3) \times 5.$
So, $\log 360 = \log (2^3 \times 3^2 \times 5) = \log 2^3 + \log 3^2 + \log 5 = 3 \log 2 + 2 \log 3 + \log 5.$

34. $\log_{10}\dfrac{26}{51}+\log_{10}\dfrac{119}{91}-\log_{10}\dfrac{13}{32}-\log_{10}\dfrac{64}{39}$
$=\left(\log_{10}\dfrac{26}{51}+\log_{10}\dfrac{119}{91}\right)-\left(\log_{10}\dfrac{13}{32}+\log_{10}\dfrac{64}{39}\right)$
$=\log_{10}\left(\dfrac{26}{51}\times\dfrac{119}{91}\right)-\log_{10}\left(\dfrac{13}{32}\times\dfrac{64}{39}\right)=\log_{10}\dfrac{2}{3}-\log_{10}\dfrac{2}{3}=0.$

35. $\dfrac{1}{3}\log_{10} 125 - 2 \log_{10} 4 + \log_{10} 32 = \log_{10} (125)^{1/3} - \log_{10} (4)^2 + \log_{10} 32$
$= \log_{10} 5 - \log_{10} 16 + \log_{10} 32 = \log_{10}\left(\dfrac{5\times 32}{16}\right)$
$= \log_{10} 10 = 1.$

36. $\log_{10} 1\dfrac{1}{2}+\log_{10} 1\dfrac{1}{3}+\cdots$ upto 198 terms $= \log_{10}\left(1\dfrac{1}{2}\times 1\dfrac{1}{3}\times\cdots\times 1\dfrac{1}{199}\right)$
$= \log_{10}\left(\dfrac{3}{2}\times\dfrac{4}{3}\times\cdots\times\dfrac{200}{199}\right) = \log_{10}\left(\dfrac{200}{2}\right)$
$= \log_{10} 100 = \log_{10} 10^2 = 2 \log_{10} 10 = 2.$

37. $\log\left(\dfrac{9}{14}\right)-\log\left(\dfrac{15}{16}\right)+\log\left(\dfrac{35}{24}\right)$
$= \log\left(\dfrac{9}{14}\div\dfrac{15}{16}\times\dfrac{35}{24}\right)=\log\left(\dfrac{9}{14}\times\dfrac{16}{15}\times\dfrac{35}{24}\right)=\log 1 = 0.$

38. $2\log_{10} 5+\log_{10} 8-\dfrac{1}{2}\log_{10} 4 = \log_{10} (5^2) + \log_{10} 8 - \log_{10} (4^{1/2})$
$= \log_{10} 25 + \log_{10} 8 - \log_{10} 2 = \log_{10}\left(\dfrac{25\times 8}{2}\right)$
$= \log_{10} 100 = 2.$

39. $\log_{10} 2 + 16\log_{10}\dfrac{16}{15}+12\log_{10}\dfrac{25}{24}+7\log_{10}\dfrac{81}{80}$
$= \log_{10} 2 + \log_{10}\left(\dfrac{16}{15}\right)^{16}+\log_{10}\left(\dfrac{25}{24}\right)^{12}+\log_{10}\left(\dfrac{81}{80}\right)^{7}$
$= \log_{10}\left[2\times\left(\dfrac{16}{15}\right)^{16}\times\left(\dfrac{25}{24}\right)^{12}\times\left(\dfrac{81}{80}\right)^{7}\right]$
$= \log_{10}\left[2\times\dfrac{(2^4)^{16}}{5^{16}\times 3^{16}}\times\dfrac{(5^2)^{12}}{(2^3)^{12}\times 3^{12}}\times\dfrac{(3^4)^7}{(2^4)^7\times 5^7}\right]$
$= \log_{10}\left[\dfrac{2\times 2^{64}\times 5^{24}\times 3^{28}}{5^{16}\times 3^{16}\times 2^{36}\times 3^{12}\times 2^{28}\times 5^7}\right]$
$= \log_{10} [2^{(1 + 64 - 36 - 28)} \times 5^{(24 - 16 - 7)} \times 3^{(28 - 16 - 12)}]$
$= \log_{10} (2^1 \times 5^1 \times 3^0) = \log_{10} 10 = 1.$

40. $\log_a (ab) = x \Leftrightarrow \dfrac{\log ab}{\log a} = x \Leftrightarrow \dfrac{\log a + \log b}{\log a} = x$
$\Leftrightarrow 1+\dfrac{\log b}{\log a} = x \Leftrightarrow \dfrac{\log b}{\log a} = x-1 \Leftrightarrow \dfrac{\log a}{\log b}=\dfrac{1}{x-1}$
$\Leftrightarrow 1+\dfrac{\log a}{\log b}=1+\dfrac{1}{x-1} \Leftrightarrow \dfrac{\log b}{\log b}+\dfrac{\log a}{\log b}=\dfrac{x}{x-1}$
$\Leftrightarrow \dfrac{\log b+\log a}{\log b}=\dfrac{x}{x-1} \Leftrightarrow \dfrac{\log (ab)}{\log b}=\dfrac{x}{x-1}$
$\Leftrightarrow \log_b (ab) = \dfrac{x}{x-1}.$

41. $\log_{\frac{1}{a}}\left(\dfrac{1}{m}\right)=\log_{a^{-1}} m^{-1}=\dfrac{-1}{-1}\log_a m = \log_a m = x.$

42. $\log_5 12 = \log_5 (3 \times 4) = \log_5 3 + \log_5 4 = \log_5 3 + 2 \log_5 2$
$=\dfrac{\log_{10} 3}{\log_{10} 5}+\dfrac{2\log_{10} 2}{\log_{10} 5}=\dfrac{\log_{10} 3}{\log_{10} 10-\log_{10} 2}+\dfrac{2\log_{10} 2}{\log_{10} 10-\log_{10} 2}$
$= \dfrac{b}{1-a}+\dfrac{2a}{1-a}=\dfrac{2a+b}{1-a}.$

43. $\log (4\cdot\sqrt[3]{63})=\log 4+\log (\sqrt[3]{63}) = \log 4 + \log(63)^{1/3} = \log (2^2) + \log (7 \times 3^2)^{1/3}$
$= 2\log 2+\dfrac{1}{3}\log 7+\dfrac{2}{3}\log 3 = 2x+\dfrac{1}{3}z+\dfrac{2}{3}y.$

44. $\log_4 x+\log_2 x = 6 \Leftrightarrow \dfrac{\log x}{\log 4}+\dfrac{\log x}{\log 2}=6 \Leftrightarrow \dfrac{\log x}{2\log 2}+\dfrac{\log x}{\log 2}=6$
$\Leftrightarrow 3 \log x = 12 \log 2 \Leftrightarrow \log x = 4 \log 2$
$\Leftrightarrow \log x = \log (2^4) = \log 16 \Leftrightarrow x = 16.$

45. $\log_{10} (x^2 - 6x + 10) = 0 \Rightarrow (x^2 - 6x + 10) = 10^0 = 1$
$\Rightarrow x^2 - 6x + 9 = 0$
$\Rightarrow (x - 3)^2 = 0 \Rightarrow x = 3.$

46. $\log_{10} x + \log_{10} y = 3$ ...(*i*)
$\log_{10} x - \log_{10} y = 1$ ...(*ii*)
Adding (*i*) and (*ii*), we get:
$2 \log_{10} x = 4$ or $\log_{10} x = 2 \Rightarrow \therefore x = 10^2 = 100.$
Also, $\log_{10} y = 3 - \log_{10} x = 3 - 2 = 1 \Rightarrow y = 10^1 = 10.$
Hence, $x = 100$, $y = 10$.

Another method:

$\log_{10} x + \log_{10} y = 3 \Rightarrow \log_{10} (xy) = 3 \Rightarrow xy = 10^3 = 1000.$
$\log_{10} x - \log_{10} y = 1$
$\Rightarrow \log_{10}\left(\dfrac{x}{y}\right)=1 \Rightarrow \dfrac{x}{y}=10^1=10 \Rightarrow x = 10y \Rightarrow 10y.y = 1000$
$\Rightarrow 10y^2 = 1000 \Rightarrow y^2 = 100 \Rightarrow y = 10.$
$\therefore x = 10y = 10 \times 10 = 100.$

47. $\log_{10} x + \log_{10} 5 = 2 \Rightarrow \log_{10} 5x = 2$
$\Rightarrow 5x = 10^2 = 100 \Rightarrow x = 20.$

48. $\log_8 x+\log_8\left(\dfrac{1}{6}\right)=\dfrac{1}{3}$
$\Leftrightarrow \dfrac{\log x}{\log 8}+\dfrac{\log \frac{1}{6}}{\log 8}=\dfrac{1}{3} \Leftrightarrow \log x+\log\dfrac{1}{6}=\dfrac{1}{3}\log 8$

$\Leftrightarrow \quad \log x + \log \frac{1}{6} = \log (8^{1/3}) = \log 2 \quad \Leftrightarrow \quad \log x = \log 2$

$- \log \frac{1}{6} = \log \left(2 \times \frac{6}{1}\right) = \log 12. \qquad \therefore\ x = 12.$

49. $\log_{10} 125 + \log_{10} 8 = x \Rightarrow \log_{10} (125 \times 8) = x$

$\Rightarrow x = \log_{10} (1000) = \log_{10} (10)^3 = 3 \log_{10} 10 = 3.$

50. $\log_9 27 + \log_8 32 = \log_{3^2}(3^3) + \log_{2^3}(2^5)$

$= \frac{3}{2} \log_3 3 + \frac{5}{3} \log_2 2 = \frac{3}{2} + \frac{5}{3} = \frac{9+10}{6} = \frac{19}{6}.$

51. Given expression

$= \left(\frac{\log 3}{\log 5} \times \frac{\log 625}{\log 3}\right) = \frac{\log 625}{\log 5} = \frac{\log (5^4)}{\log 5} = \frac{4 \log 5}{\log 5} = 4.$

52. Given expression

$= \frac{\log 9}{\log 4} \times \frac{\log 2}{\log 3} = \frac{\log 3^2}{\log 2^2} \times \frac{\log 2}{\log 3} = \frac{2 \log 3}{2 \log 2} \times \frac{\log 2}{\log 3} = 1.$

$[\because \log_5 5 = 1]$

53. $\log_{12} 27 = a \quad \Rightarrow \frac{\log 27}{\log 12} = a \Rightarrow \frac{\log 3^3}{\log (3 \times 2^2)} = a$

$\Rightarrow \frac{3 \log 3}{\log 3 + 2 \log 2} = a \Rightarrow \frac{\log 3 + 2 \log 2}{3 \log 3} = \frac{1}{a}$

$\Rightarrow \frac{\log 3}{3 \log 3} + \frac{2 \log 2}{3 \log 3} = \frac{1}{a} \Rightarrow \frac{2 \log 2}{3 \log 3} = \frac{1}{a} - \frac{1}{3} = \left(\frac{3-a}{3a}\right)$

$\Rightarrow \frac{\log 2}{\log 3} = \left(\frac{3-a}{2a}\right) \Rightarrow \log 3 = \left(\frac{2a}{3-a}\right) \log 2.$

$\log_6 16 = \frac{\log 16}{\log 6} = \frac{\log 2^4}{\log (2 \times 3)} = \frac{4 \log 2}{\log 2 + \log 3} = \frac{4 \log 2}{\log 2 \left[1 + \left(\frac{2a}{3-a}\right)\right]}$

$= \frac{4}{\left(\frac{3+a}{3-a}\right)} = \frac{4(3-a)}{(3+a)}.$

54. $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

$\Rightarrow \log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + \log_{10} 10$

$\Rightarrow \log_{10} [5 (5x + 1)] = \log_{10} [10 (x + 5)]$

$\Rightarrow 5 (5x + 1) = 10 (x + 5)$

$\Rightarrow 5x + 1 = 2x + 10 \Rightarrow 3x = 9 \Rightarrow x = 3.$

55. $\log_5 (x^2 + x) - \log_5 (x + 1) = 2$

$\Rightarrow \log_5 \left(\frac{x^2 + x}{x + 1}\right) = 2 \Rightarrow \log_5 \left[\frac{x (x+1)}{x+1}\right] = 2$

$\Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25.$

56. $\frac{1}{2} (\log x + \log y) = \log \left(\frac{x+y}{2}\right) \Rightarrow \frac{1}{2} \log (xy) = \log \left(\frac{x+y}{2}\right)$

$\Rightarrow \log (xy)^{\frac{1}{2}} = \log \left(\frac{x+y}{2}\right) \Rightarrow (xy)^{\frac{1}{2}} = \left(\frac{x+y}{2}\right) \Rightarrow xy = \left(\frac{x+y}{2}\right)^2$

$\Rightarrow 4xy = x^2 + y^2 + 2xy$

$\Rightarrow x^2 + y^2 - 2xy = 0$

$\Rightarrow (x - y)^2 = 0 \Rightarrow x - y = 0 \Rightarrow x = y.$

57. Given expression

$= \log_{60} 3 + \log_{60} 4 + \log_{60} 5 = \log_{60} (3 \times 4 \times 5)$

$= \log_{60} 60 = 1.$

58. Given Expression

$= \left(\frac{\log 4}{\log 3} \times \frac{\log 5}{\log 4} \times \frac{\log 6}{\log 5} \times \frac{\log 7}{\log 6} \times \frac{\log 8}{\log 7} \times \frac{\log 9}{\log 8}\right)$

$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2 \log 3}{\log 3} = 2.$

59. We know that : $a^{\log_a x} = x.$

$\therefore 16^{\log_4 5} = (4^2)^{\log_4 5} = 4^{2 \log_4 5} = 4^{\log_4 (5^2)} = 4^{\log_4 25} = 25.$

60. $\log x + \log y = \log (x + y)$

$\Rightarrow \log (x + y) = \log (xy)$

$\Rightarrow x + y = xy \Rightarrow y (x - 1) = x \Rightarrow y = \frac{x}{x-1}.$

61. $\log \frac{a}{b} + \log \frac{b}{a} = \log (a+b) \Rightarrow \log (a+b) = \log \left(\frac{a}{b} \times \frac{b}{a}\right) = \log 1.$

So, $a + b = 1.$

62. Given expression $= \log \left(\frac{a^2}{bc} \times \frac{b^2}{ac} \times \frac{c^2}{ab}\right) = \log 1 = 0.$

63. Given expression $= \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

64. Given expression

$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a (abc)} + \frac{1}{\log_b (abc)} + \frac{1}{\log_c (abc)}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} (abc) = 1.$

65. Given expression

$= \log_x \left(\frac{p}{q}\right) + \log_x \left(\frac{q}{r}\right) + \log_x \left(\frac{r}{p}\right)$

$= \log_x \left(\frac{p}{q} \times \frac{q}{r} \times \frac{r}{p}\right) = \log_x 1 = 0.$

66. $\log_{10} \left(\frac{1}{70}\right) = \log_{10} 1 - \log_{10} 70 = - \log_{10}$

$(7 \times 10) = - (\log_{10} 7 + \log_{10} 10) = - (a + 1).$

67. $a = b^x, b = c^y, c = a^z \Rightarrow x = \log_b a, y = \log_c b, z = \log_a c$

$\Rightarrow xyz = (\log_b a) \times (\log_c b) \times (\log_a c)$

$\Rightarrow xyz = \left(\frac{\log a}{\log b} \times \frac{\log b}{\log c} \times \frac{\log c}{\log a}\right) = 1.$

68. $\log x - 5 \log 3 = - 2 \Rightarrow \log x - \log 3^5 = - 2$

$\Rightarrow \log \left(\frac{x}{3^5}\right) = -2 \Rightarrow \frac{x}{243} = 10^{-2} = \frac{1}{100} \Rightarrow x = \frac{243}{100} = 2.43$

69. $a = b^2 = c^3 = d^4 \Rightarrow b = a^{\frac{1}{2}}, c = a^{\frac{1}{3}}, d = a^{\frac{1}{4}}.$

$\therefore \log_a (abcd) = \log_a \left(a \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{4}}\right) = \log_a a^{\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)}$

$= \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \log_a a = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}.$

70. $\log_3 x + \log_a x^2 + \log_{27} x^3 = 9 \Rightarrow \log_3 x + \log_{3^2} x^2 + \log_{3^3} x^3 = 9$

$\Rightarrow \log_3 x + \frac{2}{2}\log_3 x + \frac{3}{3}\log_3 x = 9 \Rightarrow 3\log_3 x = 9 \Rightarrow \log_3 x = 3 \Rightarrow x = 3^3 = 27.$

71. $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0 \Rightarrow \log_5(\sqrt{x+5}+\sqrt{x}) = 7^0 = 1$

$\Rightarrow \sqrt{x+5}+\sqrt{x} = 5^1 = 5 \Rightarrow (\sqrt{x+5}+\sqrt{x})^2 = 25$

$\Rightarrow (x+5)+x+2\sqrt{x+5}\sqrt{x} = 25 \Rightarrow 2x + 2\sqrt{x}\sqrt{x+5} = 20$

$\Rightarrow \sqrt{x}\sqrt{x+5} = 10 - x \Rightarrow x(x+5) = (10-x)^2$

$\Rightarrow x^2 + 5x = 100 + x^2 - 20x \Rightarrow 25x = 100 \Rightarrow x = 4.$

72. $a = \log_8 225 = \log_{2^3}(15^2) = \frac{2}{3}\log_2 15 = \frac{2b}{3}.$

73. $\log 27 = 1.431$
$\Rightarrow \log(3^3) = 1.431 \Rightarrow 3\log 3 = 1.431 \Rightarrow \log 3 = 0.477.$
$\therefore \log 9 = \log(3^2) = 2\log 3 = (2 \times 0.477) = 0.954.$

74. $\log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{0.3010} = \frac{10000}{3010} = \frac{1000}{301}.$

75. $\log_{10} 5 = \log_{10}\left(\frac{10}{2}\right) = \log_{10} 10 - \log_{10} 2 = 1 - \log_{10} 2$
$= (1 - 0.3010) = 0.6990.$

76. $\log_{10} 80 = \log_{10}(8 \times 10) = \log_{10} 8 + \log_{10} 10 = \log_{10}(2^3) + 1 = 3\log_{10} 2 + 1 = (3 \times 0.3010) + 1 = 1.9030.$

77. $(1000)^x = 3 \Rightarrow \log[(1000)^x] = \log 3 \Rightarrow x \log 1000 = \log 3$

$\Rightarrow x\log(10^3) = \log 3 \Rightarrow 3x \log 10 = \log 3$

$\Rightarrow 3x = \log 3$

$\Rightarrow x = \frac{0.477}{3} = 0.159.$

78. $\log_{10} 25 = \log_{10}\left(\frac{100}{4}\right) = \log_{10} 100 - \log_{10} 4$

$= 2 - 2\log_{10} 2 = (2 - 2 \times 0.3010)$
$= (2 - 0.6020) = 1.3980.$

79. $\log_{10}(60000) = \log_{10}(20 \times 30 \times 100) = \log_{10} 20 + \log_{10} 30 + \log_{10} 100$
$= 1.3010 + 1.4771 + 2 = 4.7781.$

80. $\log_5 512 = \frac{\log 512}{\log 5} = \frac{\log 2^9}{\log\left(\frac{10}{2}\right)} = \frac{9\log 2}{\log 10 - \log 2}$

$= \frac{(9 \times 0.3010)}{1 - 0.3010} = \frac{2.709}{0.699} = \frac{2709}{699} = 3.876.$

81. $\log_{10}\left(23\frac{1}{3}\right) = \log_{10}\left(\frac{70}{3}\right) = \log_{10} 70 - \log_{10} 3 = \log_{10}(7 \times 10) - \log_{10} 3$
$= \log_{10} 7 + \log_{10} 10 - \log_{10} 3 = 0.8451 + 1 - 0.4771 = 1.368.$

82. $\log_{10}(1.5) = \log_{10}\left(\frac{3}{2}\right) = \log_{10} 3 - \log_{10} 9$

$2 = (0.4771 - 0.3010) = 0.1761.$

83. $\log_{10}(2.8) = \log_{10}\left(\frac{28}{10}\right) = \log_{10} 28 - \log_{10} 10 = \log_{10}(7 \times 2^2) - 1 = \log_{10} 7 + 2\log_{10} 2 - 1$
$= 0.8451 + 2 \times 0.3010 - 1 = 0.8451 + 0.602 - 1 = 0.4471.$

84. $\log(0.57) = \bar{1}.756 \Rightarrow \log 57 = 1.756$

[$\because$ mantissa will remain the same]

$\therefore \log 57 + \log(0.57)^3 + \log\sqrt{0.57}$

$= \log 57 + 3\log\left(\frac{57}{100}\right) + \log\left(\frac{57}{100}\right)^{1/2}$

$= \log 57 + 3\log 57 - 3\log 100 + \frac{1}{2}\log 57 - \frac{1}{2}\log 100$

$= \frac{9}{2}\log 57 - \frac{7}{2}\log 100 = \frac{9}{2} \times 1.756 - \frac{7}{2} \times 2 = 7.902 - 7$
$= 0.902.$

85. Let $\log x = -3.153$.
Then, $\log x = -3.153 = -3 + (-0.153)$
$= (-3-1) + (1 - 0.153) = -4 + 0.847 = \bar{4}.847.$
Hence, characteristic $= -4$, mantissa $= 0.847$.

86. $\log(2^{64}) = 64 \times \log 2 = (64 \times 0.30103) = 19.26592.$
Its characteristic is 19. Hence, the number of digits in 2^{64} is 20.

87. $\log 4^{50} = 50\log 4 = 50\log 2^2 = (50 \times 2)\log 2 = 100 \times \log 2 = (100 \times 0.30103) = 30.103.$
$\therefore$ Characteristic $= 30$. Hence, the number of digits in $4^{50} = 31$.

88. $\log 5^{20} = 20\log 5 = 20 \times \left[\log\left(\frac{10}{2}\right)\right] = 20(\log 10 - \log 2)$
$= 20(1 - 0.3010) = 20 \times 0.6990 = 13.9800.$
$\therefore$ Characteristic $= 13$. Hence, the number of digits in 5^{20} is 14.

89. $\log 6^{20} = 20\log(2 \times 3) = 20(\log 2 + \log 3)$
$= 20(0.30103 + 0.47712)$
$= 20 \times 0.77815 = 15.563.$
$\therefore$ Characteristic $= 15$. Hence, the number of digits in $6^{20} = 16$.

90. $\log(4^9 \times 5^{17}) = \log(4^9) + \log(5^{17}) = \log(2^2)^9 + \log(5^{17})$
$= \log(2^{18}) + \log(5^{17})$
$= 18\log 2 + 17\log 5 = 18\log 2 + 17(\log 10 - \log 2)$
$= 18\log 2 + 17\log 10 - 17\log 2 = \log 2 + 17\log 10$
$= 0.3010 + 17 \times 1 = 17.3010.$
$\therefore$ Characteristic $= 17$. Hence, the number of digits in $(4^9 \times 5^{17}) = 18$.

91. In arithmetic progression common ratios are equal to
$\log(3^x - 2) - \log 3 = \log(3^x + 4) - \log(3^x - 2)$

$\frac{\log(3^x - 2)}{\log 3} = \frac{\log(3^x + 4)}{\log(3^x - 2)} \quad \left(\therefore \log a - \log b = \log\frac{a}{b}\right)$

$\frac{\log 3^x}{\log 2 \log 3} = \frac{x\log 3 \log 4 \log 2}{x \log 3}$

$\frac{x\log 3}{\log 2 \log 3} = \frac{x\log 3\log 4\log 2}{x\log 2}$

$\frac{x}{\log 2} = \log 4 \log 2$

$x = \log 4 \log 2 \log 2$
$x = \log 8$
$x = \log 2^3$

92. Given $\log_{10} a = p$, $\log_{10} b = q$
$\log_{10}(a^p b^q) = \log_{10} a^p + \log_{10} b^q$
$= p\log_{10} a + q\log_{10} b = p^2 + q^2$