1 Number System

FUNDAMENTAL CONCEPTS

I. Numbers

In Hindu-Arabic system, we have ten digits, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

A number is denoted by a group of digits, called ***numeral***.

For denoting a numeral, we use the place-value chart, given below.

	Ten-Crores	Crores	Ten-Lakhs	Lakhs	Ten-Thousands	Thousands	Hundreds	Tens	Ones
(*i*)				5	2	8	6	7	9
(*ii*)			4	3	8	0	9	6	7
(*iii*)		3	5	2	1	8	0	0	9
(*iv*)	5	6	1	3	0	7	0	9	0

The four numerals shown above may be written in words as:

(*i*) Five lakh twenty-eight thousand six hundred seventy-nine
(*ii*) Forty-three lakh eighty thousand nine hundred sixty-seven
(*iii*) Three crore fifty-two lakh eighteen thousand nine
(*iv*) Fifty-six crore thirteen lakh seven thousand ninety

Now, suppose we are given the following four numerals in words:

(*i*) Nine crore four lakh six thousand two
(*ii*) Twelve crore seven lakh nine thousand two hundred seven
(*iii*) Four lakh four thousand forty
(*iv*) Twenty-one crore sixty lakh five thousand fourteen

Then, using the place-value chart, these may be written in figures as under:

	Ten-Crores	Crores	Ten-Lakhs	Lakhs	Ten-Thousands	Thousands	Hundreds	Tens	Ones
(*i*)		9	0	4	0	6	0	0	2
(*ii*)	1	2	0	7	0	9	2	0	7
(*iii*)				4	0	4	0	4	0
(*iv*)	2	1	6	0	0	5	0	1	4

II. Face value and Place value (or Local Value) of a Digit in a Numeral

(*i*) The face value of a digit in a numeral is its own value, at whatever place it may be.

Ex. ***In the numeral 6872, the face value of 2 is 2, the face value of 7 is 7, the face value of 8 is 8 and the face value of 6 is 6.***

(*ii*) In a given numeral:

Place value of ones digit = (ones digit) × 1,
Place value of tens digit = (tens digit) × 10,
Place value of hundreds digit = (hundreds digit) × 100 and so on.

Ex. ***In the numeral 70984, we have***

Place value of 4 = (4 × 1) = 4,
Place value of 8 = (8 × 10) = 80,
Place value of 9 = (9 × 100) = 900,
Place value of 7 = (7 × 10000) = 70000.

Note: Place value of 0 in a given numeral is 0, at whatever place it may be.

III. Various Types of Numbers

1. Natural Numbers: *Counting numbers are called natural numbers.*

Thus, 1, 2, 3, 4, ... are all natural numbers.

2. Whole Numbers: *All counting numbers, together with 0, form the set of whole numbers.*

Thus, 0, 1, 2, 3, 4, ... are all whole numbers.

3. Integers: *All counting numbers, zero and negatives of counting numbers, form the set of integers.*
Thus, ..., – 3, – 2, – 1, 0, 1, 2, 3, ... are all integers.
Set of positive integers = {1, 2, 3, 4, 5, 6, ...}
Set of negative integers = { – 1, – 2, – 3, – 4, – 5, – 6, ...}
Set of all non-negative integers = {0, 1, 2, 3, 4, 5, ...}

4. Even Numbers: *A counting number divisible by 2 is called an even number.*
Thus, 0, 2, 4, 6, 8, 10, 12, ... etc. are all even numbers.

5. Odd Numbers: *A counting number not divisible by 2 is called an odd number.*
Thus, 1, 3, 5, 7, 9, 11, 13, ... etc. are all odd numbers.

6. Prime Numbers: *A counting number is called a prime number if it has exactly two factors, namely itself and 1.*
Ex. All prime numbers less than 100 are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

7. Composite Numbers: *All counting numbers, which are not prime, are called composite numbers.*
A composite number has more than 2 factors.

8. Perfect Numbers: *A number, the sum of whose factors (except the number itself), is equal to the number, is called a perfect number,* e.g. 6, 28, 496
The factors of 6 are 1, 2, 3 and 6. And, 1 + 2 + 3 = 6.
The factors of 28 are 1, 2, 4, 7, 14 and 28. And, 1 + 2 + 4 + 7 + 14 = 28.

9. Co-primes (or Relative Primes): *Two numbers whose H.C.F. is 1 are called co-prime numbers,*
Ex. (2, 3), (8, 9) are pairs of co-primes.

10. Twin Primes: *Two prime numbers whose difference is 2 are called twin-primes,*
Ex. (3, 5), (5, 7), (11, 13) are pairs of twin-primes.

11. Rational Numbers: *Numbers which can be expressed in the form* $\frac{p}{q}$*, where p and q are integers and* $q \neq 0$*, are called rational numbers.*
Ex. $\frac{1}{8}, \frac{-8}{11}, 0, 6, 5\frac{2}{3}$ etc.

12. Irrational Numbers: *Numbers which when expressed in decimal would be in non-terminating and non-repeating form, are called irrational numbers.*
Ex. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi, e, 0.231764735......$

IV. Important Facts:

1. All natural numbers are whole numbers.
2. All whole numbers are not natural numbers.
   0 is a whole number which is not a natural number.
3. Even number + Even number = Even number
   Odd number + Odd number = Even number
   Even number + Odd number = Odd number
   Even number – Even number = Even number
   Odd number – Odd number = Even number
   Even number – Odd number = Odd number
   Odd number – Even number = Odd number
   Even number × Even number = Even number
   Odd number × Odd number = Odd number
   Even number × Odd number = Even number
4. The smallest prime number is 2.
5. The only even prime number is 2.
6. The first odd prime number is 3.
7. 1 is a unique number – neither prime nor composite.
8. The least composite number is 4.
9. The least odd composite number is 9.
10. **Test for a Number to be Prime:**
    Let p be a given number and let n be the smallest counting number such that $n^2 \geq p$.
    Now, test whether p is divisible by any of the prime numbers less than or equal to n. If yes, then p is not prime otherwise, p is prime.

Ex. *Test, which of the following are prime numbers?*

(i) 137 *(ii) 173* *(iii) 319* *(iv) 437* *(v) 811*

Sol. (*i*) We know that $(12)^2 > 137$.
Prime numbers less than 12 are 2, 3, 5, 7, 11.
Clearly, none of them divides 137.
$\therefore$ 137 is a prime number.

(*ii*) We know that $(14)^2 > 173$.
Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.
Clearly, none of them divides 173.
$\therefore$ 173 is a prime number.

(*iii*) We know that $(18)^2 > 319$.
Prime numbers less than 18 are 2, 3, 5, 7, 11, 13, 17.
Out of these prime numbers, 11 divides 319 completely.
$\therefore$ 319 is not a prime number.

(*iv*) We know that $(21)^2 > 437$.
Prime numbers less than 21 are 2, 3, 5, 7, 11, 13, 17, 19.
Clearly, 437 is divisible by 19.
$\therefore$ 437 is not a prime number.

(*v*) We know that $(30)^2 > 811$.
Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
Clearly, none of these numbers divides 811.
$\therefore$ 811 is a prime number.

V. Important Formulae:

(*i*) $(a + b)^2 = a^2 + b^2 + 2ab$

(*ii*) $(a - b)^2 = a^2 + b^2 - 2ab$

(*iii*) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(*iv*) $(a + b)^2 - (a - b)^2 = 4ab$

(*v*) $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

(*vi*) $(a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

(*vii*) $a^2 - b^2 = (a + b)\,(a - b)$

(*viii*) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(*ix*) $a^3 + b^3 = (a + b)\,(a^2 + b^2 - ab)$

(*x*) $a^3 - b^3 = (a - b)\,(a^2 + b^2 + ab)$

(*xi*) $a^3 + b^3 + c^3 - 3abc = (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - ca)$

(*xii*) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

TESTS OF DIVISIBILITY

1. Divisibility By 2:

A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

Ex. 58694 is divisible by 2, while 86945 is not divisible by 2.

2. Divisibility By 3:

A number is divisible by 3 only when the sum of its digits is divisible by 3.

Ex. (*i*) In the number 695421, the sum of digits = 27, which is divisible by 3.
$\therefore$ 695421 is divisible by 3.

(*ii*) In the number 948653, the sum of digits = 35, which is not divisible by 3.
$\therefore$ 948653 is not divisible by 3.

3. Divisibility By 9:

A number is divisible by 9 only when the sum of its digits is divisible by 9.

Ex. (*i*) In the number 246591, the sum of digits = 27, which is divisible by 9.
$\therefore$ 246591 is divisible by 9.

(*ii*) In the number 734519, the sum of digits = 29, which is not divisible by 9.
$\therefore$ 734519 is not divisible by 9.

4. Divisibility By 4:

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Ex. (*i*) 6879376 is divisible by 4, since 76 is divisible by 4.

(*ii*) 496138 is not divisible by 4, since 38 is not divisible by 4.

5. Divisibility By 8:

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

Ex. (*i*) In the number 16789352, the number formed by last 3 digits, namely 352 is divisible by 8.
$\therefore$ 16789352 is divisible by 8.
(*ii*) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.
$\therefore$ 576484 is not divisible by 8.

6. Divisibility By 10:
A number is divisible by 10 only when its unit digit is 0.
Ex. (*i*) 7849320 is divisible by 10, since its unit digit is 0.
(*ii*) 678405 is not divisible by 10, since its unit digit is not 0.

7. Divisibility By 5:
A number is divisible by 5 only when its unit digit is 0 or 5.
Ex. (*i*) Each of the numbers 76895 and 68790 is divisible by 5.

8. Divisibility By 11:
A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a number divisible by 11.
Ex. (*i*) Consider the number 29435417.
(Sum of its digits at odd places) – (Sum of its digits at even places)
= (7 + 4 + 3 + 9) – (1 + 5 + 4 + 2) = (23 – 12) = 11, which is divisible by 11.
$\therefore$ 29435417 is divisible by 11.
(*ii*) Consider the number 57463822.
(Sum of its digits at odd places) – (Sum of its digits at even places)
= (2 + 8 + 6 + 7) – (2 + 3 + 4 + 5) = (23 – 14) = 9, which is not divisible by 11.
$\therefore$ 57463822 is not divisible by 11.

9. Divisibility By 25:
A number is divisible by 25 if the number formed by its last two digits is either 00 or divisible by 25.
Ex. (*i*) In the number 63875, the number formed by last 2 digits, namely 75 is divisible by 25.
$\therefore$ 63875 is divisible by 25.
(*ii*) In the number 96445, the number formed by last 2 digits, namely 45 is not divisible by 25.
$\therefore$ 96445 is not divisible by 25.

10. Divisibility By 7 or 13:
Divide the number into groups of 3 digits (starting from right) and find the difference between the sum of the numbers in odd and even places. If the difference is 0 or divisible by 7 or 13 (as the case may be), it is divisible by 7 or 13.
Ex. (*i*) 4537792 $\rightarrow$ 4 / 537 / 792
(792 + 4) – 537 = 259, which is divisible by 7 but not by 13.
$\therefore$ 4537792 is divisible by 7 and not by 13.
(*ii*) 579488 $\rightarrow$ 579 / 488
579 – 488 = 91, which is divisible by both 7 and 13.
$\therefore$ 579488 is divisible by both 7 and 13.

11. Divisibility By 16:
A number is divisible by 16, if the number formed by its last 4 digits is divisible by 16.
Ex. (*i*) In the number 463776, the number formed by last 4 digits, namely 3776, is divisible by 16.
$\therefore$ 463776 is divisible by 16.
(*ii*) In the number 895684, the number formed by last 4 digits, namely 5684, is not divisible by 16.
$\therefore$ 895684 is not divisible by 16.

12. Divisibility By 6: *A number is divisible by 6, if it is divisible by both 2 and 3.*

13. Divisibility By 12: *A number is divisible by 12, if it is divisible by both 3 and 4.*

14. Divisibility By 15: *A number is divisible by 15, if it is divisible by both 3 and 5.*

15. Divisibility By 18: *A number is divisible by 18, if it is divisible by both 2 and 9.*

16. Divisibility By 14: *A number is divisible by 14, if it is divisible by both 2 and 7.*

17. Divisibility By 24: *A given number is divisible by 24, if it is divisible by both 3 and 8.*

18. Divisibility By 40: *A given number is divisible by 40, if it is divisible by both 5 and 8.*

19. Divisibility By 80: *A given number is divisible by 80, if it is divisible by both 5 and 16.*

Note: *If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.*

If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

Ex. 36 is divisible by both 4 and 6, but it is not divisible by $(4 \times 6) = 24$, since 4 and 6 are not co-primes.

VI. Factorial of a Number

Let n be a positive integer.

Then, the continued product of first n natural numbers is called factorial n, denoted by n ! or $\underline{|n}$.

Thus, $n\,! = n\,(n-1)\,(n-2)\,..........\,3.2.1$

Ex. $5\,! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$

Note: $0\,! = 1$

VII. Modulus of a Number

$$|x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

Ex. $|-5| = 5$, $|4| = 4$, $|-1| = 1$, etc.

VIII. Greatest Integral Value

The greatest integral value of an integer x, denoted by [x], is defined as the greatest integer not exceeding x.

Ex. $[1.35] = 1$, $\left[\frac{11}{4}\right] = \left[2\frac{3}{4}\right] = 2$, etc.

IX. Multiplication BY Short cut Methods

1. Multiplication By Distributive Law:

(*i*) $a \times (b + c) = a \times b + a \times c$ (*ii*) $a \times (b - c) = a \times b - a \times c$

Ex. (*i*) $567958 \times 99999 = 567958 \times (100000 - 1) = 567958 \times 100000 - 567958 \times 1$
$= (56795800000) - 567958) = 56795232042.$

(*ii*) $978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000.$

2. Multiplication of a Number By 5^n:

Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

Ex. $975436 \times 625 = 975436 \times 5^4 = \frac{9754360000}{16} = 609647500.$

X. Division Algorithm or Euclidean Algorithm

If we divide a given number by another number, then:

Dividend = (Divisor × Quotient) + Remainder

Important Facts:

1. (*i*) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

(*ii*) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

(*iii*) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

2. To find the highest power of a prime number p in n !

Highest power of p in $n! = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ... + \left[\frac{n}{p^r}\right]$, where $p^r \leq n < p^{r+1}$

SOLVED EXAMPLES

Ex. 1. *Simplify:* (*i*) 8888 + 888 + 88 + 8

(*ii*) 715632 − 631104 − 9874 − 999 **(LIC, ADO, 2007)**

Sol. (*i*)

$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 9872 \\ \hline \end{array}$$

(*ii*) Given exp $= 715632 - (631104 + 9874 + 99)$
$= 715632 - 641077 = 74555.$

$$\begin{array}{r} 631104 \\ 9874 \\ +\quad 99 \\ \hline 641077 \\ \hline \end{array} \qquad \begin{array}{r} 715632 \\ -\ 641077 \\ \hline 74555 \\ \hline \end{array}$$

Ex. 2. ***What value will replace the question mark in each of the following questions?***
(i)? – 1936248 = 1635773 (ii) 9587 –? = 7429 – 4358

Sol. (*i*) Let $x - 1936248 = 1635773$. Then, $x = 1635773 + 1936248 = 3572021$.
(*ii*) Let $9587 - x = 7429 - 4358$.
Then, $9587 - x = 3071 \Rightarrow x = 9587 - 3071 = 6516$.

Ex. 3. ***What could be the maximum value of Q in the following equation?***
5P9 + 3R7 + 2Q8 = 1114

Sol. We may analyse the given equation as shown:

$$\begin{array}{r} ①② \\ 5\ P\ 9 \\ 3\ R\ 7 \\ 2\ Q\ 8 \\ \hline 1\ 1\ 1\ 4 \end{array}$$

Clearly, $2 + P + R + Q = 11$.
So, the maximum value of Q can be $(11 - 2)$, i.e. 9 (when $P = 0$, $R = 0$).

Ex. 4. ***Simplify: (i) 5793405 × 9999 (ii) 839478 × 625***

Sol. (*i*) $5793405 \times 9999 = 5793405 \times (10000 - 1) = 57934050000 - 5793405 = 57928256595$.

(*ii*) $839478 \times 625 = 839478 \times 5^4 = 839478 \times \left(\frac{10}{2}\right)^4 = \frac{839478 \times 10^4}{2^4} = \frac{8394780000}{16} = 524673750.$

Ex. 5. ***Evaluate: (i) 986 × 137 + 986 × 863 (ii) 983 × 207 – 983 × 107***

Sol. (*i*) $986 \times 137 + 986 \times 863 = 986 \times (137 + 863) = 986 \times 1000 = 986000$.
(*ii*) $983 \times 207 - 983 \times 107 = 983 \times (207 - 107) = 983 \times 100 = 98300$.

Ex. 6. ***Simplify: (i) 1605 × 1605 (ii) 1398 × 1398***

Sol. (*i*) $1605 \times 1605 = (1605)^2 = (1600 + 5)^2 = (1600)^2 + 5^2 + 2 \times 1600 \times 5$
$= 2560000 + 25 + 16000 = 2576025$.
(*ii*) $1398 \times 1398 = (1398)^2 = (1400 - 2)^2 = (1400)^2 + 2^2 - 2 \times 1400 \times 2$
$= 1960000 + 4 - 5600 = 1954404$.

Ex. 7. ***Evaluate: (i) 475 × 475 + 125 × 125 (ii) 796 × 796 – 204 × 204***

Sol. (*i*) We have $(a^2 + b^2) = \frac{1}{2}[(a+b)^2 + (a-b)^2]$

$\therefore\ (475)^2 + (125)^2 = \frac{1}{2}.[(475+125)^2 + (475-125)^2] = \frac{1}{2}.[(600)^2 + (350)^2]$

$= \frac{1}{2}[360000 + 122500] = \frac{1}{2} \times 482500 = 241250.$

(*ii*) $796 \times 796 - 204 \times 204 = (796)^2 - (204)^2 = (796 + 204)(796 - 204)$ $\quad \boxed{[\because (a-b)^2 = (a+b)(a-b)]}$
$= (1000 \times 592) = 592000$.

Ex. 8. ***Simplify: (i) (387 × 387 + 113 × 113 + 2 × 387 × 113)***
(ii) (87 × 87 + 61 × 61 – 2 × 87 × 61)

Sol. (*i*) Given Exp. $= (387)^2 + (113)^2 + 2 \times 387 \times 113 = (a^2 + b^2 + 2ab)$, where $a = 387$ and $b = 113$
$= (a + b)^2 = (387 + 113)^2 = (500)^2 = 250000$.
(*ii*) Given Exp. $= (87)^2 + (61)^2 - 2 \times 87 \times 61 = (a^2 + b^2 - 2ab)$, where $a = 87$ and $b = 61$
$= (a - b)^2 = (87 - 61)^2 = (26)^2 = (20 + 6)^2 = (20)^2 + 6^2 + 2 \times 20 \times 6 = (400 + 36 + 240)$
$= (436 + 240) = 676$.

Ex. 9. ***Find the square root of $4a^2 + b^2 + c^2 + 4ab - 2bc - 4ac$.*** **(Campus Recruitment, 2010)**

Sol. $\sqrt{4a^2 + b^2 + c^2 + 4ab - 2bc - 4ac} = \sqrt{(2a)^2 + b^2 + (-c)^2 + 2 \times 2a \times b + 2 \times b \times (-c) + 2 \times (2a) \times (-c)}$
$= \sqrt{(2a + b - c)^2} = (2a + b - c).$

Ex. 10. ***A is counting the numbers from 1 to 31 and B from 31 to 1. A is counting the odd numbers only. The speed of both is the same. What will be the number which will be pronounced by A and B together?*** **(Campus Recruitment, 2010)**

Sol. The numbers pronounced by A and B in order are:

A	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
B	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16

Clearly both A and B pronounce the number 21 together.

Ex. 11. ***Simplify:*** *(i)* $\dfrac{\mathbf{789 \times 789 \times 789 + 211 \times 211 \times 211}}{\mathbf{789 \times 789 - 789 \times 211 + 211 \times 211}}$ *(ii)* $\dfrac{\mathbf{658 \times 658 \times 658 - 328 \times 328 \times 328}}{\mathbf{658 \times 658 + 658 \times 328 + 328 \times 328}}$

Sol. (*i*) Given exp. $= \dfrac{(789)^3 + (211)^3}{(789)^2 - (789 \times 211) + (211)^2} = \dfrac{a^3 + b^3}{a^2 - ab + b^2}$, (where $a = 789$ and $b = 211$)

$= (a + b) = (789 + 211) = 1000.$

(*ii*) Given exp. $= \dfrac{(658)^3 - (328)^3}{(658)^2 + (658 \times 328) + (328)^2} = \dfrac{a^3 - b^3}{a^2 + ab + b^2}$, (where $a = 658$ and $b = 328$)

$= (a - b) = (658 - 328) = 330.$

Ex. 12. ***Simplify:*** $\dfrac{\mathbf{(893 + 786)^2 - (893 - 786)^2}}{\mathbf{(893 \times 786)}}$.

Sol. Given exp. $= \dfrac{(a+b)^2 - (a-b)^2}{ab}$ (where $a = 893, b = 786$) $= \dfrac{4ab}{ab} = 4.$

Ex. 13. ***Which of the following are prime numbers?***

(i) 241 ***(ii) 337*** ***(iii) 391*** ***(iv) 571***

Sol. (*i*) Clearly, $16 > \sqrt{241}$.

Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.

241 is not divisible by any of them.

$\therefore$ 241 is a prime number.

(*ii*) Clearly, $19 > \sqrt{337}$. Prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17.

337 is not divisible by any one of them.

$\therefore$ 337 is a prime number.

(*iii*) Clearly, $20 > \sqrt{391}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.

We find that 391 is divisible by 17.

$\therefore$ 391 is not a prime number.

(*iv*) Clearly, $24 > \sqrt{571}$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19,23.

571 is not divisible by any one of them.

$\therefore$ 571 is a prime number.

Ex. 14. ***If Δ stands for the operation 'adding first number to twice the second number', then find the value of (1 Δ 2) Δ 3.***

Sol. $(1 \,\Delta\, 2) \,\Delta\, 3 = (1 + 2 \times 2) \,\Delta\, 3 = 5 \,\Delta\, 3 = 5 + 2 \times 3 = 5 + 6 = 11.$

Ex. 15. ***Given that $1^2 + 2^2 + 3^2 + ... + 10^2 = 385$, then find the value of $2^2 + 4^2 + 6^2 + ... + 20^2$.***

Sol. $2^2 + 4^2 + 6^2 + ... + 20^2 = 2^2 (1^2 + 2^2 + 3^2 + ... + 10^2) = 2^2 \times 385 = 4 \times 385 = 1540.$

Ex.16. ***Which of the following numbers is divisible by 3?***

(i) 541326 ***(ii) 5967013***

Sol. (*i*) Sum of digits in 541326 = (5 + 4 + 1 + 3 + 2 + 6) = 21, which is divisible by 3.

Hence, 541326 is divisible by 3.

(*ii*) Sum of digits in 5967013 = (5 + 9 + 6 + 7 + 0 + 1 + 3) = 31, which is not divisible by 3.

Hence, 5967013 is not divisible by 3.

Ex. 17. ***What least value must be assigned to * so that the number 197*5462 is divisible by 9?***

Sol. Let the missing digit be x.

Sum of digits $= (1 + 9 + 7 + x + 5 + 4 + 6 + 2) = (34 + x)$.

For $(34 + x)$ to be divisible by 9, x must be replaced by 2.

Hence, the digit in place of * must be 2.

Ex. 18. ***Which of the following numbers is divisible by 4?***

(i) 67920594 ***(ii) 618703572***

Sol. (*i*) The number formed by the last two digits in the given number is 94, which is not divisible by 4.

Hence, 67920594 is not divisible by 4.

(*ii*) The number formed by the last two digits in the given number is 72, which is divisible by 4.

Hence, 618703572 is divisible by 4.

Ex. 19. ***Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5?***

Sol. Since the given number is divisible by 5, so 0 or 5 must come in place of $. But, a number ending with 5 is never divisible by 8. So, 0 will replace $.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and $ are 4 and 0 respectively.

Ex. 20. ***Show that 4832718 is divisible by 11.***

Sol. (Sum of digits at odd places) – (Sum of digits at even places) = (8 + 7 + 3 + 4) – (1 + 2 + 8) = 11, which is divisible by 11.

Hence, 4832718 is divisible by 11.

Ex. 21. ***Is 52563744 divisible by 24?***

Sol. 24 = 3 × 8, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8. Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes.

So, it is divisible by 3 × 8, i.e. 24.

Ex. 22. ***What are the values of M and N respectively if M39048458N is divisible by both 8 and 11, where M and N are single-digit integers?***

Sol. Since the given number is divisible by 8, it is obvious that the number formed by the last three digits, i.e. 58N is divisible by 8, which is possible only when $N = 4$.

Now, (sum of digits at even places) – (sum of digits at odd places)

$= (8 + 4 + 4 + 9 + M) - (4 + 5 + 8 + 0 + 3)$

$= (25 + M) - 20 = M + 5$, which must be divisible by 11.

So, $M = 6$.

Hence, $M = 6$, $N = 4$.

Ex. 23. ***Find the number of digits in the smallest number which is made up of digits 1 and 0 only and is divisible by 225.***

Sol. 225 = 9 × 25, where 9 and 25 are co-primes.

Clearly, a number is divisible by 225 if it is divisible by both 9 and 25.

Now, a number is divisible by 9 if the sum of its digits is divisible by 9 and a number is divisible by 25 if the number formed by the last two digits is divisible by 25.

∴ The smallest number which is made up of digits 1 and 0 and divisible by 225 = 11111111100.

Hence, number of digits = 11.

Ex. 24. ***If the number 3422213pq is divisible by 99, find the missing digits p and q.***

Sol. 99 = 9 × 11, where 9 and 11 are co-primes.

Clearly, a number is divisible by 99 if it is divisible by both 9 and 11.

Since the number is divisible by 9, we have:

$(3 + 4 + 2 + 2 + 2 + 1 + 3 + p + q)$ = a multiple of 9

$\Rightarrow 17 + (p + q) = 18$ or 27

$\Rightarrow p + q = 1$ ...(*i*) or $p + q = 10$ ...(*ii*)

Since the number is divisible by 11, we have:

$(q + 3 + 2 + 2 + 3) - (p + 1 + 2 + 4) = 0$ or a multiple of 11

$\Rightarrow (10 + q) - (7 + p) = 0$ or 11

$\Rightarrow 3 + (q - p) = 0$ or 11

$\Rightarrow q - p = -3$ or $q - p = 8$

$\Rightarrow p - q = 3$ ...(*iii*) or $q - p = 8$ ...(*iv*)

Clearly, if (*i*) holds, then neither (*iii*) nor (*iv*) holds. So, (*i*) does not hold.

Also, solving (*ii*) and (*iii*) together, we get: $p = 6.5$, which is not possible.

Solving (*ii*) and (*iv*) together, we get: $p = 1$, $q = 9$.

Ex. 25. ***x is a positive integer such that $x^2 + 12$ is exactly divisible by x. Find all the possible values of x.***

Sol. $\dfrac{x^2 + 12}{x} = \dfrac{x^2}{x} + \dfrac{12}{x} = x + \dfrac{12}{x}.$

Clearly, 12 must be completely divisible by x.

So, the possible values of x are 1, 2, 3, 4, 6 and 12.

Ex. 26. ***Find the smallest number to be added to 1000 so that 45 divides the sum exactly.***

Sol. On dividing 1000 by 45, we get 10 as remainder.

$\therefore$ Number to be added = (45 – 10) = 35.

Ex. 27. ***What least number must be subtracted from 2000 to get a number exactly divisible by 17?***

Sol. On dividing 2000 by 17, we get 11 as remainder.

$\therefore$ Required number to be subtracted = 11.

Ex. 28. ***Find the number which is nearest to 3105 and is exactly divisible by 21.***

Sol. On dividing 3105 by 21, we get 18 as remainder.

$\therefore$ Number to be added to 3105 = (21 – 18) = 3.

Hence, required number = 3105 + 3 = 3108.

Ex. 29. ***Find the smallest number of five digits which is exactly divisible by 476.***

Sol. Smallest number of 5 digits = 10000.

On dividing 10000 by 476, we get 4 as remainder.

$\therefore$ Number to be added = (476 – 4) = 472.

Hence, required number = 10472.

Ex. 30. ***Find the greatest number of five digits which is exactly divisible by 47.***

Sol. Greatest number of 5 digits is 99999.

On dividing 99999 by 47, we get 30 as remainder.

$\therefore$ Required number = (99999 – 30) = 99969.

Ex. 31. ***When a certain number is multiplied by 13, the product consists entirely of fives. Find the smallest such number.***

Sol. Clearly, we keep on dividing 55555....... by 13 till we get 0 as remainder.

$\therefore$ Required number = 42735.

```
    42735
13)555555(
   52
    35
    26
     95
     91
      45
      39
       65
       65
        ×
```

Ex. 32. ***When a certain number is multiplied by 18, the product consists entirely of 2's. What is the minimum number of 2's in the product?***

Sol. We keep on dividing 22222......... by 18 till we get 0 as remainder.

Clearly, number of 2's in the product = 9.

```
    12345679
18)222222222 (
   18
    42
    36
     62
     54
      82
      72
      102
       90
       122
       108
        142
        126
         162
         162
           ×
```

Ex. 33. ***Find the smallest number which when multiplied by 9 gives the product as 1 followed by a certain number of 7s only.***

Sol. The least number having 1 followed by 7s, which is divisible by 9, is 177777, as 1 + 7 + 7 + 7 + 7 + 7 = 36 (which is divisible by 9).

$\therefore$ Required number = 177777 ÷ 9 = 19753.

Ex. 34. ***What is the unit's digit in the product?***

81 × 82 × 83 × .... 89?

Sol. Required unit's digit = Unit's digit in the product 1 × 2 × 3 ×.......× 9 = 0

[$\because$ 2 × 5 = 10]

Ex. 35. ***Find the unit's digit in the product $(2467)^{153} \times (341)^{72}$.***

Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.

Now, 7^4 gives unit digit 1.

$\therefore$ 7^{152} gives unit digit 1.

$\therefore$ 7^{153} gives unit digit (1 × 7) = 7. Also, 1^{72} gives unit digit 1.

Hence, unit digit in the product = (7 × 1) = 7.

Ex. 36. ***Find the unit's digit in $(264)^{102} + (264)^{103}$.***

Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103}$.

Now, 4^2 gives unit digit 6.

$\therefore$ $(4)^{102}$ gives unit digit 6.

$(4)^{103}$ gives unit digit of the product (6 × 4) i.e., 4.

Hence, unit's digit in $(264)^{102} + (264)^{103}$ = unit's digit in (6 + 4) = 0.

Ex. 37. ***Find the total number of prime factors in the expression $(4)^{11} \times (7)^5 \times (11)^2$.***

Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{11} \times 2^{11} \times 7^5 \times 11^2 = 2^{22} \times 7^5 \times 11^2$.

$\therefore$ Total number of prime factors = (22 + 5 + 2) = 29.

Ex. 38. ***What is the number of zeros at the end of the product of the numbers from 1 to 100?***

Sol. Let $N = 1 \times 2 \times 3 \times \ldots\ldots\ldots \times 100$.

Clearly, only the multiples of 2 and 5 yield zeros on multiplication.

In the given product, the highest power of 5 is much less than that compared to 2. So, we shall find the highest power of 5 in N.

Highest power of 5 in $N = \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20 + 4 = 24.$

Hence, required number of zeros = 24.

Ex. 39. ***What is the number of zeros at the end of the product $5^5 \times 10^{10} \times 15^{15} \times \ldots\ldots\ldots \times 125^{125}$?***

Sol. Clearly, the highest power of 2 is less than that of 5 in N.

So, the highest power of 2 in N shall give us the number of zeros at the end of N.

Highest power of 2 = Number of multiples of 2 + Number of multiples of 4 (i.e. 2^2) + Number of multiples of 8 (i.e. 2^3) + Number of multiples of 16 (i.e. 2^4)

$= [(10 + 20 + 30 + \ldots\ldots + 120) + (20 + 40 + 60 + \ldots\ldots + 120) + (40 + 80 + 120) + 80]$

$= (780 + 420 + 240 + 80) = 1520.$

Hence, required number of zeros = 1520.

Ex. 40. ***On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.***

Sol. Divisor $= \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}} = \frac{15968 - 37}{89} = 179.$

Ex. 41. ***A number when divided by 114, leaves remainder 21. If the same number is divided by 19, find the remainder.*** **(S.S.C., 2010)**

Sol. On dividing the given number by 114, let k be the quotient and 21 the remainder.

Then, number $= 114\,k + 21 = 19 \times 6k + 19 + 2 = 19\,(6k + 1) + 2.$

$\therefore$ The given number when divided by 19 gives remainder = 2.

Ex. 42. ***A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 and 7 respectively. Find the respective remainders if the order of divisors be reversed.***

Sol.

$$\begin{array}{r|l} 3 & x \\ \hline 5 & y - 1 \\ \hline 8 & z - 4 \\ \hline & 1 - 7 \end{array}$$

$\therefore z = (8 \times 1 + 7) = 15$; $y = (5\,z + 4) = (5 \times 15 + 4) = 79$;

$x = (3y + 1) = (3 \times 79 + 1) = 238.$

Now,

$$\begin{array}{r|l} 8 & 238 \\ \hline 5 & 29 - 6 \\ \hline 3 & 5 - 4 \\ \hline & 1 - 2 \end{array}$$

$\therefore$ Respective remainders are 6, 4, 2.

Ex. 43. ***Three boys A, B, C were asked to divide a certain number by 1001 by the method of factors. They took the factors in the orders 13, 11, 7; 7, 11, 13 and 11, 7, 13 respectively. If the first boy obtained 3, 2, 1 as successive remainders, then find the successive remainders obtained by the other two boys B and C.***

Sol.

$$\begin{array}{r|l} 13 & x \\ \hline 11 & y - 3 \\ \hline 7 & z - 2 \\ \hline & 1 - 1 \end{array}$$

$\therefore z = 7 \times 1 + 1 = 8,$

$y = 11\,z + 2 = 11 \times 8 + 2 = 90;$

$x = 13\,y + 3 = 13 \times 90 + 3 = 1173.$

Now,

$$\begin{array}{r|l} 7 & 1173 \\ \hline 11 & 167 - 4 \\ \hline 13 & 15 - 2 \\ \hline & 1 - 2 \end{array}$$

So, B obtained 4, 2 and 2 as successive remainders.

And,

$$\begin{array}{r|l} 11 & 1173 \\ \hline 7 & 106 - 7 \\ \hline 13 & 15 - 1 \\ \hline & 1 - 2 \end{array}$$

C obtained 7, 1 and 2 as successive remainders.

Ex. 44. ***In a division sum, the divisor is ten times the quotient and five times the remainder. If the remainder is 46, determine the dividend.***

Sol. Remainder = 46 ; Divisor = 5 × 46 = 230 ; Quotient = $\frac{230}{10} = 23$.

∴ Dividend = Divisor × Quotient + Remainder = 230 × 23 + 46 = 5336.

Ex. 45. ***If three times the larger of the two numbers is divided by the smaller one, we get 4 as quotient and 3 as remainder. Also, if seven times the smaller number is divided by the larger one, we get 5 as quotient and 1 as remainder. Find the numbers.***

Sol. Let the larger number be x and the smaller number be y.

Then, $3x = 4y + 3 \Rightarrow 3x - 4y = 3$ ...(*i*)

And, $7y = 5x + 1 \Rightarrow -5x + 7y = 1$ ...(*ii*)

Multiplying (*i*) by 5 and (*ii*) by 3, we get:

$15x - 20y = 15$ ...(*iii*) and $-15x + 21y = 3$ ...(*iv*)

Adding (*iii*) and (*iv*), we get: $y = 18$.

Putting $y = 18$ in (*i*), we get: $x = 25$.

Hence, the numbers are 25 and 18.

Ex. 46. ***A number when divided by 6 leaves remainder 3. When the square of the same number is divided by 6, find the remainder.***

Sol. On dividing the given number by 6, let k be the quotient and 3 the remainder.

Then, number = $6k + 3$.

Square of the number $= (6k + 3)^2 = 36k^2 + 9 + 36k = 36k^2 + 36k + 6 + 3$

$= 6(6k^2 + 6k + 1) + 3$, which gives a remainder 3 when divided by 6.

Ex. 47. ***Find the remainder when $9^6 + 7$ is divided by 8.***

Sol. $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

So, $(9^6 - 1)$ is divisible by $(9 - 1)$, i.e. $8 \Rightarrow (9^6 - 1) + 8$ is divisible by $8 \Rightarrow (9^6 + 7)$ is divisible by 8.

Hence, required remainder = 0.

Ex. 48. ***Find the remainder when $(397)^{3589} + 5$ is divided by 398.***

Sol. $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

So, $[(397)^{3589} + 1]$ is divisible by $(397 + 1)$, i.e. 398

$\Rightarrow [\{(397)^{3589} + 1\} + 4]$ gives remainder 4 when divided by 398

$\Rightarrow [(397)^{3589} + 5]$ gives remainder 4 when divided by 398.

Ex. 49. ***If 7^{126} is divided by 48, find the remainder.***

Sol. $7^{126} = (7^2)^{63} = (49)^{63}$.

Now, since $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n,

so $[(49)^{63} - 1]$ or $(7^{126} - 1)$ is divisible by $(49 - 1)$ i.e. 48.

∴ Remainder obtained when $(7)^{126}$ is divided by 48 = 1.

Ex. 50. ***Find the remainder when $(257^{166} - 243^{166})$ is divided by 500.***

Sol. $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.

∴ $(257^{166} - 243^{166})$ is divisible by $(257 + 243)$, i.e. 500.

Hence, required remainder = 0.

Ex. 51. ***Find a common factor of $(127^{127} + 97^{127})$ and $(127^{97} + 97^{97})$.***

Sol. $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

∴ $(127^{127} + 97^{127})$ as well as $(127^{97} + 97^{97})$ is divisible by $(127 + 97)$, i.e. 224.

Hence, required common factor = 224.

Ex. 52. ***A 99–digit number is formed by writing the first 59 natural numbers one after the other as:***
1234567891011121314...........5859
Find the remainder obtained when the above number is divided by 16.

Sol. The required remainder is the same as that obtained on dividing the number formed by the last four digits i.e. 5859 by 16, which is 3.

EXERCISE
(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (✓) against the correct answer in each of the following:*

1. What is the place value of 5 in 3254710? **(CLAT, 2010)**
   (*a*) 5 (*b*) 10000
   (*c*) 50000 (*d*) 54710
2. The face value of 8 in the number 458926 is **(R.R.B., 2006)**
   (*a*) 8 (*b*) 1000
   (*c*) 8000 (*d*) 8926
3. The sum of the place values of 3 in the number 503535 is **(M.B.A., 2005)**
   (*a*) 6 (*b*) 60
   (*c*) 3030 (*d*) 3300
4. The difference between the place values of 7 and 3 in the number 527435 is
   (*a*) 4 (*b*) 5
   (*c*) 45 (*d*) 6970
5. The difference between the local value and the face value of 7 in the numeral 32675149 is
   (*a*) 5149 (*b*) 64851
   (*c*) 69993 (*d*) 75142
   (*e*) None of these
6. The sum of the greatest and smallest number of five digits is **(M.C.A., 2005)**
   (*a*) 11,110 (*b*) 10,999
   (*c*) 109,999 (*d*) 111,110
7. If the largest three-digit number is subtracted from the smallest five-digit number, then the remainder is
   (*a*) 1 (*b*) 9000
   (*c*) 9001 (*d*) 90001
8. The smallest number of 5 digits beginning with 3 and ending with 5 will be **(R.R.B., 2006)**
   (*a*) 31005 (*b*) 30015
   (*c*) 30005 (*d*) 30025
9. What is the minimum number of four digits formed by using the digits 2, 4, 0, 7? **(P.C.S., 2007)**
   (*a*) 2047 (*b*) 2247
   (*c*) 2407 (*d*) 2470
10. All natural numbers and 0 are called the .... numbers. **(R.R.B., 2006)**
    (*a*) rational (*b*) integer
    (*c*) whole (*d*) prime
11. Consider the following statements about natural numbers:
    (1) There exists a smallest natural number.
    (2) There exists a largest natural number.
    (3) Between two natural numbers, there is always a natural number.
    Which of the above statements is/are correct?
    (*a*) None (*b*) Only 1
    (*c*) 1 and 2 (*d*) 2 and 3
12. Every rational number is also
    (*a*) an integer (*b*) a real number
    (*c*) a natural number (*d*) a whole number
13. The number π is **(R.R.B., 2005)**
    (*a*) a fraction (*b*) a recurring decimal
    (*c*) a rational number (*d*) an irrational number
14. $\sqrt{2}$ is a/an
    (*a*) rational number (*b*) natural number
    (*c*) irrational number (*d*) integer
15. The number $\sqrt{3}$ is
    (*a*) a finite decimal
    (*b*) an infinite recurring decimal
    (*c*) equal to 1.732
    (*d*) an infinite non-recurring decimal
16. There are just two ways in which 5 may be expressed as the sum of two different positive (non-zero) integers, namely 5 = 4 + 1 = 3 + 2. In how many ways, 9 can be expressed as the sum of two different positive (non-zero) integers?
    (*a*) 3 (*b*) 4
    (*c*) 5 (*d*) 6
17. P and Q are two positive integers such that $PQ = 64$. Which of the following cannot be the value of $P + Q$?
    (*a*) 16 (*b*) 20
    (*c*) 35 (*d*) 65
18. If $x + y + z = 9$ and both y and z are positive integers greater than zero, then the maximum value x can take is **(Campus Recruitment, 2006)**
    (*a*) 3 (*b*) 7
    (*c*) 8 (*d*) Data insufficient
19. What is the sum of the squares of the digits from 1 to 9?
    (*a*) 105 (*b*) 260
    (*c*) 285 (*d*) 385
20. If n is an integer between 20 and 80, then any of the following could be $n + 7$ except
    (*a*) 47 (*b*) 58
    (*c*) 84 (*d*) 88
21. Which one of the following is the correct sequence in respect of the Roman numerals: C, D, L and M? **(Civil Services, 2008)**
    (*a*) $C > D > L > M$ (*b*) $M > L > D > C$
    (*c*) $M > D > C > L$ (*d*) $L > C > D > M$
22. If the numbers from 1 to 24, which are divisible by 2 are arranged in descending order, which number will be at the 8th place from the bottom? **(CLAT, 2010)**

(*a*) 10 (*b*) 12
(*c*) 16 (*d*) 18

23. $2 - 2 + 2 - 2 +$ .... 101 terms =? **(P.C.S., 2008)**
(*a*) – 2 (*b*) 0
(*c*) 2 (*d*) None of these

24. 98th term of the infinite series 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, ... is **(M.C.A., 2005)**
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

25. If x, y, z be the digits of a number beginning from the left, the number is
(*a*) xyz (*b*) $x + 10y + 100z$
(*c*) $10x + y + 100z$ (*d*) $100x + 10y + z$

26. If x, y, z and w be the digits of a number beginning from the left, the number is
(*a*) $xyzw$
(*b*) $wzyx$
(*c*) $x + 10y + 100z + 1000w$
(*d*) $10^3x + 10^2y + 10z + w$

27. If n and p are both odd numbers, which of the following is an even number?
(*a*) $n + p$ (*b*) $n + p + 1$
(*c*) $np + 2$ (*d*) np

28. For the integer n, if n^3 is odd, then which of the following statements are true?
I. n is odd. II. n^2 is odd.
III. n^2 is even.
(*a*) I only (*b*) II only
(*c*) I and II only (*d*) I and III only

29. If $(n - 1)$ is an odd number, what are the two other odd numbers nearest to it?
(*a*) $n, n - 1$ (*b*) $n, n - 2$
(*c*) $n - 3, n + 1$ (*d*) $n - 3, n + 5$

30. Which of the following is always odd?
(*a*) Sum of two odd numbers
(*b*) Difference of two odd numbers
(*c*) Product of two odd numbers
(*d*) None of these

31. If x is an odd integer, then which of the following is true?
(*a*) $5x - 2$ is even (*b*) $5x^2 + 2$ is odd
(*c*) $5x^2 + 3$ is odd (*d*) None of these

32. If a and b are two numbers such that $ab = 0$, then **(R.R.B., 2006)**
(*a*) $a = 0$ and $b = 0$ (*b*) $a = 0$ or $b = 0$ or both
(*c*) $a = 0$ and $b \neq 0$ (*d*) $b = 0$ and $a \neq 0$

33. If A, B, C, D are numbers in increasing order and D, B, E are numbers in decreasing order, then which one of the following sequences need neither be in a decreasing nor in an increasing order?
(*a*) E, C, D (*b*) E, B, C
(*c*) D, B, A (*d*) A, E, C

34. If m, n, o, p and q are integers, then $m (n + o) (p - q)$ must be even when which of the following is even?
(*a*) m (*b*) p
(*c*) $m + n$ (*d*) $n + p$

35. If n is a negative number, then which of the following is the least?
(*a*) 0 (*b*) $-n$
(*c*) $2n$ (*d*) n^2

36. If $x - y = 8$, then which of the following must be true?
I. Both x and y are positive.
II. If x is positive, y must be positive.
III. If x is negative, y must be negative.
(*a*) I only (*b*) II only
(*c*) I and II (*d*) III only

37. If x and y are negative, then which of the following statements is/are always true?
I. $x + y$ is positive.
II. xy is positive.
III. $x - y$ is positive.
(*a*) I only (*b*) II only
(*c*) III only (*d*) I and III only

38. If $n = 1 + x$, where x is the product of four consecutive positive integers, then which of the following is/are true?
I. n is odd. II. n is prime.
III. n is a perfect square.
(*a*) I only (*b*) I and II only
(*c*) I and III only (*d*) None of these

39. If $x = \frac{2}{5}y + 3$, how does y change when x increases from 1 to 2?
(*a*) y increases from -5 to $-\frac{5}{2}$
(*b*) y increases from $\frac{2}{5}$ to 5
(*c*) y increases from $\frac{5}{2}$ to 5
(*d*) y decreases from -5 to $-\frac{5}{2}$

40. If x is a rational number and y is an irrational number, then
(*a*) both $x + y$ and xy are necessarily rational
(*b*) both $x + y$ and xy are necessarily irrational
(*c*) xy is necessarily irrational, but $x + y$ can be either rational or irrational
(*d*) $x + y$ is necessarily irrational, but xy can be either rational or irrational

41. The difference between the square of any two consecutive integers is equal to
(*a*) sum of two numbers
(*b*) difference of two numbers
(*c*) an even number
(*d*) product of two numbers

42. Between two distinct rational numbers a and b, there exists another rational number which is **(P.C.S., 2006)**

(*a*) $\frac{a}{2}$ (*b*) $\frac{b}{2}$

(*c*) $\frac{ab}{2}$ (*d*) $\frac{a+b}{2}$

43. If $B > A$, then which expression will have the highest value (given that A and B are positive integers)? **(Campus Recruitment, 2007)**

(*a*) $A - B$ (*b*) AB

(*c*) $A + B$ (*d*) Can't say

44. If $0 < x < 1$, which of the following is greatest? **(Campus Recruitment, 2007)**

(*a*) x (*b*) x^2

(*c*) $\frac{1}{x}$ (*d*) $\frac{1}{x^2}$

45. If p is a positive fraction less than 1, then

(*a*) $\frac{1}{p}$ is less than 1 (*b*) $\frac{1}{p}$ is a positive integer

(*c*) p^2 is less than p

(*d*) $\frac{2}{p} - p$ is a positive number

46. If x is a real number, then $x^2 + x + 1$ is

(*a*) less than $\frac{3}{4}$

(*b*) zero for at least one value of x

(*c*) always negative

(*d*) greater than or equal to $\frac{3}{4}$

47. Let n be a natural number such that $\frac{1}{2}+\frac{1}{3}+\frac{1}{7}+\frac{1}{n}$ is also a natural number. Which of the following statements is not true? **(A.A.O. Exam, 2009)**

(*a*) 2 divides n (*b*) 3 divides n

(*c*) 7 divides n (*d*) $n > 84$

48. If n is an integer, how many values of n will give an integral value of $\left(\frac{16n^2+7n+6}{n}\right)$?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) None of these

49. If $p > q$ and $r < 0$, then which is true?

(*a*) $pr < qr$ (*b*) $p - r < q - r$

(*c*) $p + r < q + r$ (*d*) None of these

50. If $X < Z$ and $X < Y$, which of the following is necessarily true?

I. $Y < Z$ II. $X^2 < YZ$

III. $ZX < Y + Z$

(*a*) Only I (*b*) Only II

(*c*) Only III (*d*) None of these

51. In the relation $x > y + z$, $x + y > p$ and $z < p$, which of the following is necessarily true? **(Campus Recruitment, 2008)**

(*a*) $y > p$ (*b*) $x + y > z$

(*c*) $y + p > x$ (*d*) Insufficient data

52. If a and b are positive integers and $\frac{(a-b)}{3.5}=\frac{4}{7}$, then **(Campus Recruitment, 2010)**

(*a*) $b > a$ (*b*) $b < a$

(*c*) $b = a$ (*d*) $b \geq a$

53. If $13 = \frac{13\,w}{(1-w)}$, then $(2w)^2 = ?$ **(Campus Recruitment, 2009)**

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$

(*c*) 1 (*d*) 2

Directions (Questions 54–57): *For a 5–digit number, without repetition of digits, the following information is available.* **(B.B.A., 2006)**

(*i*) The first digit is more than 5 times the last digit.

(*ii*) The two-digit number formed by the last two digits is the product of two prime numbers.

(*iii*) The first three digits are all odd.

(*iv*) The number does not contain the digits 3 or 0 and the first digit is also the largest.

54. The second digit of the number is

(*a*) 5 (*b*) 7

(*c*) 9

(*d*) Cannot be determined

55. The last digit of the number is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

56. The largest digit in the number is

(*a*) 5 (*b*) 7

(*c*) 8 (*d*) 9

57. Which of the following is a factor of the given number?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 9

58. The least prime number is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

59. Consider the following statements:

1. If x and y are composite numbers, then $x + y$ is always composite.
2. There does not exist a natural number which is neither prime nor composite.

Which of the above statements is/are correct?

(*a*) 1 only (*b*) 2 only

(*c*) Both 1 and 2 (*d*) Neither 1 nor 2

60. The number of prime numbers between 0 and 50 is

(*a*) 14 (*b*) 15

(*c*) 16 (*d*) 17

61. The prime numbers dividing 143 and leaving a remainder of 3 in each case are
(*a*) 2 and 11 (*b*) 11 and 13
(*c*) 3 and 7 (*d*) 5 and 7

62. The sum of the first four primes is
(*a*) 10 (*b*) 11
(*c*) 16 (*d*) 17

63. The sum of all the prime numbers from 1 to 20 is
(*a*) 75 (*b*) 76
(*c*) 77 (*d*) 78

64. A prime number N, in the range 10 to 50, remains unchanged when its digits are reversed. The square of such a number is
(*a*) 121 (*b*) 484
(*c*) 1089 (*d*) 1936

65. The remainder obtained when any prime number greater than 6 is divided by 6 must be **(Campus Recruitment, 2007)**
(*a*) either 1 or 2 (*b*) either 1 or 3
(*c*) either 1 or 5 (*d*) either 3 or 5

66. Which of the following is not a prime number? **(CLAT, 2010)**
(*a*) 21 (*b*) 23
(*c*) 29 (*d*) 43

67. Which of the following is a prime number? **(CLAT, 2010)**
(*a*) 19 (*b*) 20
(*c*) 21 (*d*) 22

68. Which of the following is a prime number? **(Campus Recruitment, 2008)**
(*a*) 115 (*b*) 119
(*c*) 127 (*d*) None of these

69. Which of the following is a prime number? **(R.R.B., 2006)**
(*a*) 143 (*b*) 289
(*c*) 117 (*d*) 359

70. The smallest value of natural number n, for which $2n + 1$ is not a prime number, is
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) None of these

71. The smallest three-digit prime number is
(*a*) 101 (*b*) 103
(*c*) 107 (*d*) None of these

72. How many of the integers between 110 and 120 are prime numbers? **(M.B.A., 2006)**
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3
(*e*) 4

73. Four prime numbers are arranged in ascending order. The product of first three is 385 and that of last three is 1001. The largest prime number is **(R.R.B., 2006)**
(*a*) 9 (*b*) 11
(*c*) 13 (*d*) 17

74. The sum of three prime numbers is 100. If one of them exceeds another by 36, then one of the numbers is
(*a*) 7 (*b*) 29
(*c*) 41 (*d*) 67

75. Which one of the following is a prime number?
(*a*) 161 (*b*) 221
(*c*) 373 (*d*) 437

76. The smallest prime number, that is the fifth term of an increasing arithmetic sequence in which all the four preceding terms are also prime, is
(*a*) 17 (*b*) 29
(*c*) 37 (*d*) 53

77. The number of prime numbers between 301 and 320 are
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

78. Consider the following statements:
1. If $p > 2$ is a prime, then it can be written as $4n + 1$ or $4n + 3$ for a suitable natural number n.
2. If $p > 2$ is a prime, then $(p - 1)(p + 1)$ is always divisible by 4.

Of these statements,
(*a*) (1) is true but (2) is false
(*b*) (1) is false but (2) is true
(*c*) (1) and (2) are false
(*d*) (1) and (2) are true

79. What is the first value of n for which $n^2 + n + 41$ is not a prime?
(*a*) 1 (*b*) 10
(*c*) 20 (*d*) 40

80. Let $X_k = (p_1 p_2 \ldots\ldots p_k) + 1$, where $p_1, p_2, \ldots\ldots, p_k$ are the first k primes.
Consider the following:
1. X_k is a prime number.
2. X_k is a composite number.
3. $X_k + 1$ is always an even number.

Which of the above is/are correct?
(*a*) 1 only (*b*) 2 only
(*c*) 3 only (*d*) 1 and 3

81. $6 \times 3\ (3 - 1)$ is equal to **(CLAT, 2010)**
(*a*) 19 (*b*) 20
(*c*) 36 (*d*) 53

82. 1234 + 2345 − 3456 + 4567 =? **(Bank Recruitment, 2010)**
(*a*) 4590 (*b*) 4670
(*c*) 4680 (*d*) 4690
(*e*) None of these

83. 5566 − 7788 + 9988 =? + 4444 **(Bank Recruitment, 2010)**
(*a*) 3223 (*b*) 3232
(*c*) 3322 (*d*) 3333
(*e*) None of these

84. 38649 – 1624 – 4483 =? **(Bank Recruitment, 2009)**
(*a*) 32425 (*b*) 32452
(*c*) 34522 (*d*) 35422
(*e*) None of these

85. 884697 – 773697 – 102479 =? **(Bank Recruitment, 2009)**
(*a*) 8251 (*b*) 8512
(*c*) 8521 (*d*) 8531
(*e*) None of these

86. 10531 + 4813 – 728 =? × 87 **(Bank Recruitment, 2008)**
(*a*) 168 (*b*) 172
(*c*) 186 (*d*) 212
(*e*) None of these

87. What is 394 times 113?
(*a*) 44402 (*b*) 44522
(*c*) 44632 (*d*) 44802
(*e*) None of these

88. 1260 ÷ 14 ÷ 9 =? **(Bank P.O., 2009)**
(*a*) 9 (*b*) 10
(*c*) 81 (*d*) 810
(*e*) None of these

89. 136 × 12 × 8 =? **(Bank P.O., 2009)**
(*a*) 12066 (*b*) 13046
(*c*) 13064 (*d*) 13066
(*e*) None of these

90. 8888 + 848 + 88 –? = 7337 + 737 **(Bank P.O., 2009)**
(*a*) 1450 (*b*) 1550
(*c*) 1650 (*d*) 1750
(*e*) None of these

91. 414 ×? × 7 = 127512 **(Bank P.O., 2009)**
(*a*) 36 (*b*) 40
(*c*) 44 (*d*) 48
(*e*) None of these

92. Product of 82540027 and 43253 is
(*a*) 3570103787831 (*b*) 3570103787832
(*c*) 3570103787833 (*d*) 3570103787834

93. (46351 – 36418 – 4505) ÷? = 1357 **(Bank P.O., 2009)**
(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 6
(*e*) None of these

94. 6 × 66 × 666 =? **(Bank Recruitment, 2007)**
(*a*) 263376 (*b*) 263763
(*c*) 263736 (*d*) 267336
(*e*) None of these

95. If you subtract – 1 from + 1, what will be the result? **(R.R.B., 2006)**
(*a*) – 2 (*b*) 0
(*c*) 1 (*d*) 2

96. 8 + 88 + 888 + 8888 + 88888 + 888888 =?
(*a*) 897648 (*b*) 896748
(*c*) 986748 (*d*) 987648
(*e*) None of these

97. From the sum of 17 and – 12, subtract 48. **(E.S.I.C., 2006)**
(*a*) – 43 (*b*) – 48
(*c*) – 17 (*d*) – 20

98. 60840 ÷ 234 =?
(*a*) 225 (*b*) 255
(*c*) 260 (*d*) 310
(*e*) None of these

99. 3578 + 5729 –? × 581 = 5821
(*a*) 3 (*b*) 4
(*c*) 6 (*d*) None of these

100. – 95 ÷ 19 =?
(*a*) – 5 (*b*) – 4
(*c*) 0 (*d*) 5

101. 12345679 × 72 is equal to
(*a*) 88888888 (*b*) 888888888
(*c*) 898989898 (*d*) 999999998

102. 8899 – 6644 – 3322 =? – 1122
(*a*) 55 (*b*) 65
(*c*) 75 (*d*) 85
(*e*) None of these

103. 74844 ÷? = 54 × 63 **(Bank P.O., 2009)**
(*a*) 22 (*b*) 34
(*c*) 42 (*d*) 54
(*e*) None of these

104. 1256 × 3892 =?
(*a*) 4883852 (*b*) 4888532
(*c*) 4888352 (*d*) 4883582
(*e*) None of these

105. What is 786 times 964? **(Bank P.O., 2008)**
(*a*) 757704 (*b*) 754164
(*c*) 759276 (*d*) 749844
(*e*) None of these

106. What is 348 times 265? **(S.B.I.P.O., 2008)**
(*a*) 88740 (*b*) 89750
(*c*) 92220 (*d*) 95700
(*e*) None of these

107. (71 × 29 + 27 × 15 + 8 × 4) equals **(S.S.C., 2007)**
(*a*) 2496 (*b*) 3450
(*c*) 3458 (*d*) None of these

108. $? \times (|a| \times |b|) = -ab$
(*a*) 0 (*b*) –1
(*c*) 1 (*d*) None of these

109. $(46)^2 - (?)^2 = 4398 - 3066$
(*a*) 16 (*b*) 28
(*c*) 36 (*d*) 42
(*e*) None of these

110. (800 ÷ 64) × (1296 ÷ 36) =?
(*a*) 420 (*b*) 460
(*c*) 500 (*d*) 540
(*e*) None of these

111. 5358 × 51 =?
(*a*) 273258 (*b*) 273268
(*c*) 273348 (*d*) 273358

112. 587 × 999 =?
(*a*) 586413 (*b*) 587523
(*c*) 614823 (*d*) 615173

113. 3897 × 999 =?
(*a*) 3883203 (*b*) 3893103
(*c*) 3639403 (*d*) 3791203
(*e*) None of these

114. 72519 × 9999 =?
(*a*) 725117481 (*b*) 674217481
(*c*) 685126481 (*d*) 696217481
(*e*) None of these

115. 2056 × 987 =?
(*a*) 1936372 (*b*) 2029272
(*c*) 1896172 (*d*) 1923472
(*e*) None of these

116. 1904 × 1904 =?
(*a*) 3654316 (*b*) 3632646
(*c*) 3625216 (*d*) 3623436
(*e*) None of these

117. 1397 × 1397 =?
(*a*) 1951609 (*b*) 1981709
(*c*) 18362619 (*d*) 2031719
(*e*) None of these

118. 107 × 107 + 93 × 93 =?
(*a*) 19578 (*b*) 19418
(*c*) 20098 (*d*) 21908
(*e*) None of these

119. 217 × 217 + 183 × 183 =? **(R.R.B., 2007)**
(*a*) 79698 (*b*) 80578
(*c*) 80698 (*d*) 81268
(*e*) None of these

120. 106 × 106 – 94 × 94 =?
(*a*) 2400 (*b*) 2000
(*c*) 1904 (*d*) 1906
(*e*) None of these

121. 8796 × 223 + 8796 × 77 =?
(*a*) 2736900 (*b*) 2738800
(*c*) 2658560 (*d*) 2716740
(*e*) None of these

122. 287 × 287 + 269 × 269 – 2 × 287 × 269 =?
(*a*) 534 (*b*) 446
(*c*) 354 (*d*) 324
(*e*) None of these

123. $\{(476 + 424)^2 - 4 \times 476 \times 424\}$ =?
(*a*) 2906 (*b*) 3116
(*c*) 2704 (*d*) 2904
(*e*) None of these

124. The value of 112×5^4 is
(*a*) 6700 (*b*) 70000
(*c*) 76500 (*d*) 77200

125. Multiply 5746320819 by 125.
(*a*) 718,290,102,375 (*b*) 728,490,301,375
(*c*) 748,290,103,375 (*d*) 798,290,102,975

126. 935421 × 625 =?
(*a*) 575648125 (*b*) 584638125
(*c*) 584649125 (*d*) 585628125

127. $(999)^2 - (998)^2$ =? **(R.R.B., 2008)**
(*a*) 1992 (*b*) 1995
(*c*) 1997 (*d*) 1998

128. $(80)^2 - (65)^2 + 81$ =?
(*a*) 306 (*b*) 2094
(*c*) 2175 (*d*) 2256
(*e*) None of these

129. $(24 + 25 + 26)^2 - (10 + 20 + 25)^2$ =?
(*a*) 352 (*b*) 400
(*c*) 752 (*d*) 2600
(*e*) None of these

130. $(65)^2 - (55)^2$ =?
(*a*) 10 (*b*) 100
(*c*) 120 (*d*) 1200

131. If a and b be positive integers such that $a^2 - b^2 = 19$, then the value of a is **(S.S.C., 2010)**
(*a*) 9 (*b*) 10
(*c*) 19 (*d*) 20

132. If a and b are positive integers, $a > b$ and $(a + b)^2 - (a - b)^2 > 29$, then the smallest value of a is
(*a*) 3 (*b*) 4
(*c*) 6 (*d*) 7

133. 397 × 397 + 104 × 104 + 2 × 397 × 104 =?
(*a*) 250001 (*b*) 251001
(*c*) 260101 (*d*) 261001

134. If $(64)^2 - (36)^2 = 20 \times x$, then x =?
(*a*) 70 (*b*) 120
(*c*) 180 (*d*) 140
(*e*) None of these

135. $\dfrac{(489+375)^2-(489-375)^2}{(489\times 375)} = ?$
(*a*) 144 (*b*) 864
(*c*) 2 (*d*) 4
(*e*) None of these

136. $\dfrac{(963+476)^2+(963-476)^2}{(963\times 963+476\times 476)} = ?$
(*a*) 2 (*b*) 4

(*c*) 497 (*d*) 1449
(*e*) None of these

137. $\dfrac{768\times768\times768+232\times232\times232}{786\times768-768\times232+232\times232}=?$
(*a*) 1000 (*b*) 536
(*c*) 500 (*d*) 268
(*e*) None of these

138. $\dfrac{854\times854\times854-276\times276\times276}{854\times854+854\times276+276\times276}=?$
(*a*) 1130 (*b*) 578
(*c*) 565 (*d*) 1156
(*e*) None of these

139. $\dfrac{753\times753+247\times247-753\times247}{753\times753\times753+247\times247\times247}=?$
(*a*) $\dfrac{1}{1000}$ (*b*) $\dfrac{1}{506}$
(*c*) $\dfrac{253}{500}$ (*d*) None of these

140. $\dfrac{256\times256-144\times144}{112}$ is equal to **(S.S.C., 2010)**
(*a*) 420 (*b*) 400
(*c*) 360 (*d*) 320

141. If $a = 11$ and $b = 9$, then the value of $\left(\dfrac{a^2+b^2+ab}{a^3-b^3}\right)$ is **(S.S.C., 2010)**
(*a*) $\dfrac{1}{2}$ (*b*) $\dfrac{1}{20}$
(*c*) 2 (*d*) 20

142. If $a + b + c = 0$, $(a + b)(b + c)(c + a)$ equals **(M.C.A., 2005)**
(*a*) $ab(a + b)$ (*b*) $(a + b + c)^2$
(*c*) $-abc$ (*d*) $a^2 + b^2 + c^2$

143. If $a = 7$, $b = 5$, $c = 3$, then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ is
(*a*) – 12 (*b*) 0
(*c*) 8 (*d*) 12

144. Both addition and multiplication of numbers are operations which are
(*a*) neither commutative nor associative
(*b*) associative but not commutative
(*c*) commutative but not associative
(*d*) commutative and associative

145. Which of the following digits will replace the *H* marks in the following equation?
$9H + H8 + H6 = 230$
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 9
(*e*) None of these

146. Find the missing number in the following addition problem:

$$\begin{array}{cccc} & 8 & 3 & 5 \\ & 4 & * & 8 \\ + & 9 & * & 4 \\ \hline 2 & 2 & * & 7 \\ \hline \end{array}$$

(*a*) 0 (*b*) 4
(*c*) 6 (*d*) 9

147. What number should replace *M* in this multiplication problem?

$$\begin{array}{cccc} & 3 & M & 4 \\ & & \times & 4 \\ \hline 1 & 2 & 1 & 6 \\ \hline \end{array}$$

(*a*) 0 (*b*) 2
(*c*) 4 (*d*) 8

148. If *p* and *q* represent digits, what is the maximum possible value of *q* in the statement **(S.S.C., 2010)**
$5p9 + 327 + 2q8 = 1114$?
(*a*) 6 (*b*) 7
(*c*) 8 (*d*) 9

149. What would be the maximum value of *Q* in the following equation?
$5P7 + 8Q9 + R32 = 1928$
(*a*) 6 (*b*) 8
(*c*) 9 (*d*) Data inadequate
(*e*) None of these

150. What should come in place of * mark in the following equation?
1*5$4 ÷ 148 = 78
(*a*) 1 (*b*) 4
(*c*) 6 (*d*) 8
(*e*) None of these

151. If 6*43 – 46@9 = 1904, which of the following should come in place of *?
(*a*) 4 (*b*) 6
(*c*) 9 (*d*) Cannot be determined
(*e*) None of these

152. What should be the maximum value of *Q* in the following equation?
$5P9 - 7Q2 + 9R6 = 823$
(*a*) 5 (*b*) 6
(*c*) 7 (*d*) 9
(*e*) None of these

153. In the following sum, '?' stands for which digit?
? + 1? + 2? +? 3 +? 1 = 21?
(*a*) 4 (*b*) 6
(*c*) 8 (*d*) 9
(*e*) None of these

Directions (Questions 154–155): *These questions are based on the following information:*
CBA + *CCA* = *ACD*, where *A*, *B*, *C* and *D* stand for distinct digits and *D* = 0.

154. *B* takes the value
(*a*) 0 (*b*) 5
(*c*) 9 (*d*) 0 or 9

155. *C* takes the value
(*a*) 0 (*b*) 2
(*c*) 2 or 3 (*d*) 5

156. A 3–digit number $4a3$ is added to another 3–digit number 984 to give the four-digit number $13b7$, which is divisible by 11. Then, $(a + b)$ is **(M.B.A., 2006)**
(*a*) 10 (*b*) 11
(*c*) 12 (*d*) 15

157. If $ab\overline{)252}(ba$, the values of *a* and *b* are **(I.A.M., 2007)**

$$\begin{array}{r} 24 \\ \hline 12 \\ 12 \\ \hline \times \end{array}$$

(*a*) 1, 2 (*b*) 2, 3
(*c*) 1, 3 (*d*) None of these

158.

$$\begin{array}{r} * \; * \; * \\ \times \quad * \\ \hline 8 \; * \; * \; 1 \end{array}$$

In the above multiplication problem, * is equal to
(*a*) 1 (*b*) 3
(*c*) 7 (*d*) 9

159. If * means adding 6 times the second number to the first number, then (1 * 2) * 3 equals
(*a*) 21 (*b*) 31
(*c*) 91 (*d*) 93

160. If $1 \times 2 \times 3 \times .... \times n$ is denoted by $\underline{|n}$, then $\underline{|8} - \underline{|7} - \underline{|6}$ is equal to
(*a*) $6 \times 7 \times \underline{|8}$ (*b*) $7 \times 8 \times \underline{|7}$
(*c*) $6 \times 8 \times \underline{|6}$ (*d*) $7 \times 8 \times \underline{|6}$

161. The highest power of 9 dividing 99! completely is
(*a*) 11 (*b*) 20
(*c*) 22 (*d*) 24

162. For an integer *n*, $n! = n(n - 1)(n - 2)$........... 3.2.1. **(P.C.S., 2008)**
Then, 1! + 2! + 3! + ... + 100! when divided by 5 leaves remainder
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

163. The number of prime factors in the expression $6^{10} \cdot 7^{17} \cdot 11^{27}$ is equal to
(*a*) 54 (*b*) 64
(*c*) 71 (*d*) 81

164. What is the number of prime factors contained in the product $30^7 \times 22^5 \times 34^{11}$?
(*a*) 49 (*b*) 51
(*c*) 52 (*d*) 53

165. What number multiplied by 48 will give the same product as 173 multiplied by 240?
(*a*) 495 (*b*) 545
(*c*) 685 (*d*) 865

166. A positive number, which when added to 1000, gives a sum which is greater than when it is multiplied by 1000. This positive integer is
(*a*) 1 (*b*) 3
(*c*) 5 (*d*) 7

167. 7 is added to a certain number ; the sum is multiplied by 5 ; the product is divided by 9 and 3 is subtracted from the quotient. Thus, if the remainder left is 12, what was the original number? **(S.S.C., 2005)**
(*a*) 20 (*b*) 30
(*c*) 40 (*d*) 60

168. Symbiosis runs a Corporate Training Programme. At the end of running the first programme, its total takings were ₹ 38950. There were more than 45 but less than 100 participants. What was the participant fee for the programme? **(SNAP, 2005)**
(*a*) ₹ 410 (*b*) ₹ 450
(*c*) ₹ 500 (*d*) ₹ 510

169. The sum of four consecutive even numbers *A, B, C* and *D* is 180. What is the sum of the set of next four consecutive even numbers? **(Bank Recruitment, 2008)**
(*a*) 196 (*b*) 204
(*c*) 212 (*d*) 214
(*e*) None of these

170. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9 and then back to the index figure for 10, middle finger for 11, and so on. She counted upto 1994. She ended on her
(*a*) thumb (*b*) index finger
(*c*) middle finger (*d*) ring finger

171. Given $n = 1 + x$ and *x* is the product of four consecutive integers. Then which of the following is true?
I. *n* is an odd integer. II. *n* is prime.
III. *n* is a perfect square.
(*a*) Only I is correct
(*b*) Only III is correct
(*c*) Both I and II are correct
(*d*) Both I and III are correct

172. If $x + y = 15$ and $xy = 56$, then what is the value of $x^2 + y^2$? **(L.I.C.A.D.O., 2007)**
(*a*) 110 (*b*) 113
(*c*) 121 (*d*) Cannot be determined
(*e*) None of these

173. Given that $(1^2 + 2^2 + 3^2 + ... + 20^2) = 2870$, the value of $(2^2 + 4^2 + 6^2 + ... + 40^2)$ is
(*a*) 2870 (*b*) 5740
(*c*) 11480 (*d*) 28700

174. The value of $5^2 + 6^2 + ... + 10^2 + 20^2$ is
(*a*) 755 (*b*) 760
(*c*) 765 (*d*) 770

175. Given that $1 + 2 + 3 + 4 + ... + 10 = 55$, then the sum $6 + 12 + 18 + 24 + ... + 60$ is equal to
(*a*) 300 (*b*) 330
(*c*) 455 (*d*) 655

176. If m and n are natural numbers such that $2^m - 2^n = 960$, what is the value of m? **(M.A.T., 2007)**
(*a*) 10 (*b*) 12
(*c*) 15
(*d*) Cannot be determined

177. On multiplying a number by 7, all the digits in the product appear as 3's. The smallest such number is **(C.P.O., 2006)**
(*a*) 47619 (*b*) 46719
(*c*) 48619 (*d*) 47649

178. The number of digits in the smallest number, which when multiplied by 7 yields all nines, is
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

179. A boy multiplies 987 by a certain number and obtains 559981 as his answer. If in the answer both 9's are wrong but the other digits are correct, then the correct answer will be
(*a*) 553681 (*b*) 555181
(*c*) 555681 (*d*) 556581

180. The numbers 1, 3, 5, ..., 25 are multiplied together. The number of zeros at the right end of the product is **(R.R.B., 2006)**
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

181. The numbers 1, 2, 3, 4, ..., 1000 are multiplied together. The number of zeros at the end (on the right) of the product must be
(*a*) 30 (*b*) 200
(*c*) 211 (*d*) 249

182. First 100 multiples of 10 i.e. 10, 20, 30, ..., 1000 are multiplied together. The number of zeros at the end of the product will be
(*a*) 100 (*b*) 111
(*c*) 124 (*d*) 125

183. The number of zeros at the end of the product
$5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50$ is
(*a*) 5 (*b*) 7
(*c*) 8 (*d*) 10

184. The number of zeros at the end of 60! is
(*a*) 12 (*b*) 14
(*c*) 16 (*d*) 18

185. The numbers 1, 3, 5, 7, ..., 99 and 128 are multiplied together. The number of zeros at the end of the product must be
(*a*) Nil (*b*) 7
(*c*) 19 (*d*) 22

186. The numbers 2, 4, 6, 8, ..., 98, 100 are multiplied together. The number of zeros at the end of the product must be
(*a*) 10 (*b*) 11
(*c*) 12 (*d*) 13

187. Let S be the set of prime numbers greater than or equal to 2 and less than 100. Multiply all the elements of S. With how many consecutive zeros will the product end?
(*a*) 1 (*b*) 4
(*c*) 5 (*d*) 10

188. Find the number of zeros at the end of the result
$3 \times 6 \times 9 \times 12 \times 15 \times .... \times 99 \times 102$.
(*a*) 4 (*b*) 6
(*c*) 7 (*d*) 10

189. The unit's digit of 13^{2003} is **(A.A.O. Exam, 2010)**
(*a*) 1 (*b*) 3
(*c*) 7 (*d*) 9

190. The digit in the unit's place of the number 123^{99} is **(I.A.M., 2007)**
(*a*) 1 (*b*) 4
(*c*) 7 (*d*) 8

191. Match List I with List II and select the correct answer:

List I (Product)	**List II (Digit in the unit's place)**
A. $(1827)^{16}$	(1) 1
B. $(2153)^{19}$	(2) 3
C. $(5129)^{21}$	(3) 5
	(4) 7
	(5) 9

(*a*) A B C: 1 4 3 (*b*) A B C: 4 2 3
(*c*) A B C: 1 4 5 (*d*) A B C: 4 2 5

192. The digit in the unit's place of the number $(67)^{25} - 1$ must be
(*a*) 0 (*b*) 6
(*c*) 8 (*d*) None of these

193. The unit's digit in the product $274 \times 318 \times 577 \times 313$ is
(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5

194. In the product $459 \times 46 \times 28^* \times 484$, the digit in the unit place is 8. The digit to come in place of * is
(*a*) 3 (*b*) 5
(*c*) 7 (*d*) None of these

195. The digit in the unit place of the number represented by $(7^{95} - 3^{58})$ is
(*a*) 0 (*b*) 4
(*c*) 6 (*d*) 7

196. Unit's digit in $(784)^{126} + (784)^{127}$ is
(*a*) 0 (*b*) 4
(*c*) 6 (*d*) 8

197. The digit in the unit's place of $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - 16^4 + 259]$ is
(*a*) 1 (*b*) 4
(*c*) 5 (*d*) 6

198. The digit in the unit's place of the product $(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(*a*) 0 (*b*) 2
(*c*) 3 (*d*) 5

199. If x is an even number, then x^{4n}, where n is a positive integer, will always have
(*a*) zero in the unit's place
(*b*) 6 in the unit's place
(*c*) either 0 or 6 in the unit's place
(*d*) None of these

200. If m and n are positive integers, then the digit in the unit's place of $5^n + 6^m$ is always
(*a*) 1 (*b*) 5
(*c*) 6 (*d*) $n + m$

201. The number formed from the last two digits (ones and tens) of the expression $2^{12n} - 6^{4n}$, where n is any positive integer is **(S.S.C., 2005)**
(*a*) 10 (*b*) 00
(*c*) 30 (*d*) 02

202. The last digit in the decimal representation of $\left(\frac{1}{5}\right)^{2000}$ is **(Hotel Management, 2009)**
(*a*) 2 (*b*) 4
(*c*) 5 (*d*) 6

203. Let x be the product of two numbers 3,659,893,456,789,325,678 and 342,973,489,379,256. The number of digits in x is **(A.A.O., 2010)**
(*a*) 32 (*b*) 34
(*c*) 35 (*d*) 36

204. Let a number of three digits have for its middle digit the sum of the other two digits. Then it is a multiple of **(C.P.F., 2008)**
(*a*) 10 (*b*) 11
(*c*) 18 (*d*) 50

205. What least value must be given to n so that the number $6135n2$ becomes divisible by 9? **(L.I.C.A.D.O., 2008)**
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

206. Find the multiple of 11 in the following numbers. **(R.R.B., 2006)**
(*a*) 112144 (*b*) 447355
(*c*) 869756 (*d*) 978626

207. 111,111,111,111 is divisible by
(*a*) 3 and 37 only
(*b*) 3, 11 and 37 only
(*c*) 3, 11, 37 and 111 only
(*d*) 3, 11, 37, 111 and 1001

208. Which of the following numbers is not divisible by 18?
(*a*) 34056 (*b*) 50436
(*c*) 54036 (*d*) 65043

209. The number 89715938* is divisible by 4. The unknown non-zero digit marked as * will be
(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 6

210. Which one of the following numbers is divisible by 3?
(*a*) 4006020 (*b*) 2345678
(*c*) 2876423 (*d*) 9566003

211. A number is divisible by 11 if the difference between the sums of the digits in odd and even places respectively is
(*a*) a multiple of 3
(*b*) a multiple of 5
(*c*) zero or a multiple of 7
(*d*) zero or a multiple of 11

212. Which one of the following numbers is divisible by 11?
(*a*) 4823718 (*b*) 4832718
(*c*) 8423718 (*d*) 8432718

213. Which one of the following numbers is divisible by 15?
(*a*) 17325 (*b*) 23755
(*c*) 29515 (*d*) 30560

214. 7386038 is divisible by
(*a*) 3 (*b*) 4
(*c*) 9 (*d*) 11

215. Consider the following statements:
The numbers 24984, 26784 and 28584 are
(1) divisible by 3 (2) divisible by 4
(3) divisible by 9
Which of these are correct?
(*a*) 1 and 2 (*b*) 2 and 3
(*c*) 1 and 3 (*d*) 1, 2 and 3

216. Which of the following numbers is a multiple of 8?
(*a*) 923872 (*b*) 923972
(*c*) 923862 (*d*) 923962

217. If 78*3945 is divisible by 11, where * is a digit, then * is equal to
(*a*) 0 (*b*) 1
(*c*) 3 (*d*) 5

218. If m and n are integers divisible by 5, which of the following is not necessarily true?
(*a*) $m + n$ is divisible by 10
(*b*) $m - n$ is divisible by 5
(*c*) $m^2 - n^2$ is divisible by 25
(*d*) None of these

219. An integer is divisible by 16 if and only if its last X digits are divisible by 16. The value of X would be

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

220. Which of the following numbers is divisible by 3, 7, 9 and 11?
(*a*) 639 (*b*) 2079
(*c*) 3791 (*d*) 37911

221. A number 476**0 is divisible by both 3 and 11. The non-zero digits in the hundred's and ten's place respectively are
(*a*) 7, 4 (*b*) 5, 3
(*c*) 5, 2 (*d*) None of these

222. How many of the following numbers are divisible by 3 but not by 9?
2133, 2343, 3474, 4131, 5286, 5340, 6336, 7347, 8115, 9276
(*a*) 5 (*b*) 6
(*c*) 7 (*d*) None of these

223. If the number 357*25* is divisible by both 3 and 5, then the missing digits in the unit's place and the thousandth's place respectively are
(*a*) 0, 6 (*b*) 5, 1
(*c*) 5, 4 (*d*) None of these

224. 6897 is divisible by
(*a*) 11 only (*b*) 19 only
(*c*) both 11 and 19 (*d*) neither 11 nor 19

225. Which of the following numbers is exactly divisible by 24?
(*a*) 35718 (*b*) 63810
(*c*) 537804 (*d*) 3125736

226. The number 311311311311311311311 is
(*a*) neither divisible by 3 nor by 11
(*b*) divisible by 11 but not by 3
(*c*) divisible by 3 but not by 11
(*d*) divisible by both 3 and 11

227. 325325 is a six-digit number. It is divisible by
(*a*) 7 only (*b*) 11 only
(*c*) 13 only (*d*) all 7, 11 and 13

228. If the seven-figure number 30X0103 is a multiple of 13, then X is
(*a*) 1 (*b*) 6
(*c*) 7 (*d*) 8

229. If a number is divisible by both 11 and 13, then it must be necessarily
(*a*) 429 (*b*) divisible by (11×13)
(*c*) divisible by $(11 + 13)$ (*d*) divisible by $(13 - 11)$

230. Which of the following numbers are completely divisible by 7?
I. 195195 II. 181181
III. 120120 IV. 891891
(*a*) Only I and II (*b*) Only II and III
(*c*) Only I and IV (*d*) Only II and IV
(*e*) All are divisible

231. If x and y are two digits of the number $653xy$ such that the number is divisible by 80, then $x + y$ is equal to
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

232. The six-digit number $5ABB7A$ is a multiple of 33 for non-zero digits A and B. Which of the following could be possible value of $A + B$? **(A.A.O. Exam, 2010)**
(*a*) 8 (*b*) 9
(*c*) 10 (*d*) 14

233. Which of the following numbers is divisible by 99?
(*a*) 114345 (*b*) 913464
(*c*) 135792 (*d*) 3572404

234. The digits indicated by * in 3422213** so that this number is divisible by 99 are **(R.R.B., 2010)**
(*a*) 1, 9 (*b*) 3, 7
(*c*) 4, 6 (*d*) 5, 5

235. If 37X3 is a four-digit natural number divisible by 7, then the place marked as X must have the value
(*a*) 0 (*b*) 3
(*c*) 5 (*d*) 9

236. If the seven-digit number $876p37q$ is divisible by 225, then the values of p and q respectively are
(*a*) 0 and 0 (*b*) 9 and 0
(*c*) 0 and 5 (*d*) 9 and 5

237. If a number $774958A96B$ is divisible by 8 and 9, the respective values of A and B will be
(*a*) 5 and 8 (*b*) 7 and 8
(*c*) 8 and 0 (*d*) None of these

238. How many of the following numbers are divisible by 132?
264, 396, 462, 792, 968, 2178, 5184, 6336
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

239. If x and y are positive integers such that $(3x + 7y)$ is a multiple of 11, then which of the following is also a multiple of 11?
(*a*) $5x - 3y$ (*b*) $9x + 4y$
(*c*) $4x + 6y$ (*d*) $x + y + 6$

240. If n be any natural number then by which largest number $(n^3 - n)$ is always divisible? **(S.S.C., 2010)**
(*a*) 3 (*b*) 6
(*c*) 12 (*d*) 18

241. If a and b are two odd positive integers, by which of the following integers is $(a^4 - b^4)$ always divisible? **(S.S.C., 2010)**
(*a*) 3 (*b*) 6
(*c*) 8 (*d*) 12

242. The difference between the squares of any two consecutive integers is equal to
(*a*) an even number
(*b*) difference of given numbers
(*c*) sum of given numbers
(*d*) product of given numbers

243. The number $6n^2 + 6n$ for natural number n is always divisible by **(M.A.T., 2007)**
(*a*) 6 only (*b*) 6 and 12
(*c*) 12 only (*d*) 18 only

244. The difference of a number consisting of two digits and the number formed by interchanging the digits is always divisible by
(*a*) 5 (*b*) 7
(*c*) 9 (*d*) 11

245. The sum of a number consisting of two digits and the number formed by interchanging the digits is always divisible by
(*a*) 7 (*b*) 9
(*c*) 10 (*d*) 11

246. The largest natural number, which exactly divides the product of any four consecutive natural numbers, is **(S.S.C., 2007)**
(*a*) 6 (*b*) 12
(*c*) 24 (*d*) 120

247. If n is a whole number greater than 1, then $n^2 (n^2 - 1)$ is always divisible by
(*a*) 8 (*b*) 10
(*c*) 12 (*d*) 16

248. If n is any odd number greater than 1, then $n (n^2 - 1)$ is
(*a*) divisible by 24 always (*b*) divisible by 48 always
(*c*) divisible by 96 always (*d*) None of these

249. The sum of the digits of a 3-digit number is subtracted from the number. The resulting number is always
(*a*) not divisible by 9 (*b*) divisible by 9
(*c*) not divisible by 6 (*d*) divisible by 6

250. A number is multiplied by 11 and 11 is added to the product. If the resulting number is divisible by 13, the smallest original number is
(*a*) 12 (*b*) 22
(*c*) 26 (*d*) 53

251. The product of any three consecutive natural numbers is always divisible by
(*a*) 3 (*b*) 6
(*c*) 9 (*d*) 15

252. The sum of three consecutive odd numbers is always divisible by
I. 2 II. 3
III. 5 IV. 6
(*a*) Only I (*b*) Only II
(*c*) Only I and II (*d*) Only I and III

253. The greatest number by which the product of three consecutive multiples of 3 is always divisible is
(*a*) 54 (*b*) 81
(*c*) 162 (*d*) 243

254. If p is a prime number greater than 3, then $(p^2 - 1)$ is always divisible by
(*a*) 6 but not 12 (*b*) 12 but not 24
(*c*) 24 (*d*) None of these

255. The difference between the squares of two consecutive odd integers is always divisible by
(*a*) 3 (*b*) 6
(*c*) 7 (*d*) 8

256. A 4–digit number is formed by repeating a 2–digit number such as 2525, 3232 etc. Any number of this form is exactly divisible by **(S.S.C., 2005, 2010)**
(*a*) 7 (*b*) 11
(*c*) 13
(*d*) Smallest 3–digit prime number

257. A 6-digit number is formed by repeating a 3-digit number; for example, 256256 or 678678 etc. Any number of this form is always exactly divisible by
(*a*) 7 only (*b*) 11 only
(*c*) 13 only (*d*) 1001

258. The sum of the digits of a natural number $(10^n - 1)$ is 4707, where n is a natural number. The value of n is **(Hotel Management, 2010)**
(*a*) 477 (*b*) 523
(*c*) 532 (*d*) 704

259. $(x^n - a^n)$ is divisible by $(x - a)$
(*a*) for all values of n
(*b*) only for even values of n
(*c*) only for odd values of n
(*d*) only for prime values of n

260. Which one of the following is the number by which the product of 8 consecutive integers is divisible?
(*a*) 4 ! (*b*) 6 !
(*c*) 7 ! (*d*) 8 !
(*e*) All of these

261. Consider the following statements:
For any positive integer n, the number $10^n - 1$ is divisible by
(1) 9 for n = odd only (2) 9 for n = even only
(3) 11 for n = odd only (4) 11 for n = even only
Which of the above statements are correct?
(*a*) 1 and 3 (*b*) 2 and 3
(*c*) 1 and 4 (*d*) 2 and 4

262. If n is any positive integer, $3^{4n} - 4^{3n}$ is always divisible by
(*a*) 7 (*b*) 12
(*c*) 17 (*d*) 145

263. If the square of an odd natural number is divided by 8, then the remainder will be
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

264. The largest number that exactly divides each number of the sequence $1^5 - 1, 2^5 - 2, 3^5 - 3, ..., n^5 - n,....$ is
(*a*) 1 (*b*) 15
(*c*) 30 (*d*) 120

265. The difference of the squares of two consecutive even integers is divisible by
(*a*) 3 (*b*) 4
(*c*) 6 (*d*) 7

266. The difference of the squares of two consecutive odd integers is divisible by

(*a*) 3 (*b*) 6
(*c*) 7 (*d*) 8

267. The smallest 4-digit number exactly divisible by 7 is **(P.C.S., 2009)**

(*a*) 1001 (*b*) 1007
(*c*) 1101 (*d*) 1108

268. What least number must be added to 1056 to get a number exactly divisible by 23? **(P.C.S., 2009)**

(*a*) 2 (*b*) 3
(*c*) 21 (*d*) 25

269. Which of the following numbers should be added to 8567 to make it exactly divisible by 4? **(Bank Recruitment, 2008)**

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6
(*e*) None of these

270. Find the least 6-digit number which is exactly divisible by 349. **(R.R.B., 2006)**

(*a*) 100163 (*b*) 101063
(*c*) 160063 (*d*) None of these

271. Which is the greatest 5-digit number exactly divisible by 279? **(R.R.B., 2006)**

(*a*) 99603 (*b*) 99550
(*c*) 99882 (*d*) None of these

272. The least number, which must be added to the greatest 6-digit number so that the sum may be exactly divisible by 327 is

(*a*) 194 (*b*) 264
(*c*) 292 (*d*) 294

273. The least number more than 5000 which is divisible by 73 is

(*a*) 5009 (*b*) 5037
(*c*) 5073 (*d*) 5099

274. The nearest integer to 58701 which is exactly divisible by 567 is

(*a*) 55968 (*b*) 58068
(*c*) 58968 (*d*) None of these

275. The smallest number which must be subtracted from 8112 to make it exactly divisible by 99 is

(*a*) 91 (*b*) 92
(*c*) 93 (*d*) 95

276. The smallest number that must be added to 803642 in order to obtain a multiple of 11 is

(*a*) 1 (*b*) 4
(*c*) 7 (*d*) 9

277. The number of times 99 is subtracted from 1111 so that the remainder is less than 99 is

(*a*) 10 (*b*) 11
(*c*) 12 (*d*) 13

278. The smallest number by which 66 must be multiplied to make the result divisible by 18 is

(*a*) 3 (*b*) 6
(*c*) 9 (*d*) 18

279. The smallest 6-digit number exactly divisible by 111 is

(*a*) 111111 (*b*) 110011
(*c*) 100011 (*d*) 110101
(*e*) None of these

280. The sum of all 2-digit numbers divisible by 5 is

(*a*) 945 (*b*) 1035
(*c*) 1230 (*d*) 1245
(*e*) None of these

281. How many 3-digit numbers are completely divisible by 6?

(*a*) 149 (*b*) 150
(*c*) 151 (*d*) 166

282. The number of terms between 11 and 200 which are divisible by 7 but not by 3 are **(C.P.F., 2008)**

(*a*) 18 (*b*) 19
(*c*) 27 (*d*) 28

283. Out of the numbers divisible by 3 between 14 and 95 if the numbers with 3 at unit's place are removed, then how many numbers will remain? **(R.R.B., 2006)**

(*a*) 22 (*b*) 23
(*c*) 24 (*d*) 25

284. How many numbers less than 1000 are multiples of both 10 and 13?

(*a*) 6 (*b*) 7
(*c*) 8 (*d*) 9

285. How many integers between 100 and 150, both inclusive, can be evenly divided by neither 3 nor 5?

(*a*) 26 (*b*) 27
(*c*) 28 (*d*) 33

286. If all the numbers from 501 to 700 are written, what is the total number of times the digit 6 appears? **(Civil Services, 2007)**

(*a*) 138 (*b*) 139
(*c*) 140 (*d*) 141

287. How many 3-digit numbers are there in between 100 and 300, having first and the last digit as 2?

(*a*) 9 (*b*) 10
(*c*) 11 (*d*) 12

288. The total number of integers between 200 and 400, each of which either begins with 3 or ends with 3 or both is **(S.S.C., 2007)**

(*a*) 10 (*b*) 100
(*c*) 110 (*d*) 120

289. While writing all the numbers from 700 to 1000, how many numbers occur in which the first digit is greater than the second digit, and the second digit is greater than the third digit?

(*a*) 61 (*b*) 64
(*c*) 78 (*d*) 85

290. A 9-digit number in which zero does not appear and no digits are repeated has the following properties: The number comprising the left most two digits is divisible by 2, that comprising the left most three digits is divisible by 3, and so on.
The number is
(*a*) 183654729 (*b*) 381654729
(*c*) 983654721 (*d*) 981654723

291. If 11,109,999 is divided by 1111, then what is the remainder? **(M.A.T., 2007)**
(*a*) 1098 (*b*) 1010
(*c*) 1110 (*d*) 1188

292. A number divided by 68 gives the quotient 260 and remainder zero. If the same number is divided by 65, the remainder is
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

293. Which of the following prime numbers while dividing 2176 leaves 9 as remainder?
(*a*) 17 (*b*) 29
(*c*) 167 (*d*) 197

294. Match List I with List II and select the correct answer:

List I (*a*, *b* as given in Euclidean algorithm $a = bq + r$)	**List II** (Values of q and r)
A. $a = -112, b = -7$	1. $q = -13, r = 1$
B. $a = 118, b = -9$	2. $q = 14, r = 3$
C. $a = -109, b = 6$	3. $q = -19, r = 5$
D. $a = 115, b = 8$	4. $q = 16, r = 0$

A B C D A B C D
(*a*) 3 1 4 2 (*b*) 3 2 4 1
(*c*) 4 1 3 2 (*d*) 4 2 3 1

295. The number 534677 is divided by 777. The difference of divisor and remainder is
(*a*) 577 (*b*) 676
(*c*) 687 (*d*) 789

296. In a division sum, the quotient, dividend and remainder are 15, 940 and 25 respectively. The divisor is
(*a*) 31 (*b*) 50
(*c*) 60 (*d*) 61

297. In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 48, then the dividend is
(*a*) 2404 (*b*) 3648
(*c*) 4808 (*d*) 4848

298. The divisor is 25 times the quotient and 5 times the remainder. If the quotient is 16, then the dividend is
(*a*) 400 (*b*) 480
(*c*) 6400 (*d*) 6480

299. A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is
(*a*) 1220 (*b*) 1250
(*c*) 22030 (*d*) 220030

300. In doing a question of division with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. The correct quotient is
(*a*) 0 (*b*) 12
(*c*) 13 (*d*) 20

301. In a division problem, the divisor is 7 times of quotient and 5 times of remainder. If the dividend is 6 times of remainder, then the quotient is equal to
(*a*) 0 (*b*) 1
(*c*) 7 (*d*) None of these
(Hotel Management, 2007)

302. On dividing a number by 19, the difference between quotient and remainder is 9. The number is
(*a*) 352 (*b*) 361
(*c*) 370 (*d*) 371

303. A number when divided by 136 leaves remainder 36. If the same number is divided by 17, the remainder will be **(S.S.C., 2010)**
(*a*) 2 (*b*) 3
(*c*) 7 (*d*) 9

304. A number when divided by 195 leaves a remainder 47. If the same number is divided by 15, the remainder will be **(Hotel Management, 2010)**
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

305. A certain number when divided by 899 gives a remainder 63. What is the remainder when the same number is divided by 29? **(R.R.B., 2008)**
(*a*) 5 (*b*) 25
(*c*) 27 (*d*) None of these

306. A number when divided by 5 leaves the remainder 3. What is the remainder when the square of the same number is divided by 5?
(*a*) 0 (*b*) 3
(*c*) 4 (*d*) 9

307. The difference between two numbers is 1365. When the larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. What is the smaller number?
(*a*) 240 (*b*) 270
(*c*) 295 (*d*) 360

308. When n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4?
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 6

309. When a number is divided by 13, the remainder is 11. When the same number is divided by 17, the remainder is 9. What is the number?

(*a*) 339 (*b*) 349
(*c*) 369 (*d*) Data inadequate
(*e*) None of these

310. In a division sum, the remainder was 71. With the same divisor but twice the dividend, the remainder is 43. Which one of the following is the divisor?
(*a*) 86 (*b*) 93
(*c*) 99 (*d*) 104

311. When a certain positive integer P is divided by another positive integer, the remainder is r_1. When a second positive integer Q is divided by the same integer, the remainder is r_2 and when $(P + Q)$ is divided by the same divisor, the remainder is r_3. Then the divisor may be
(*a*) $r_1 r_2 r_3$ (*b*) $r_1 + r_2 + r_3$
(*c*) $r_1 - r_2 + r_3$ (*d*) $r_1 + r_2 - r_3$
(*e*) Cannot be determined

312. Two numbers when divided by a certain divisor leave the remainders 4375 and 2986 respectively but when the sum of two numbers is divided by the same divisor, the remainder is 2361. The divisor in question is
(*a*) 4675 (*b*) 4900
(*c*) 5000 (*d*) None of these

313. A number divided by 13 leaves a remainder 1 and if the quotient, thus obtained, is divided by 5, we get a remainder of 3. What will be the remainder if the number is divided by 65?
(*a*) 16 (*b*) 18
(*c*) 28 (*d*) 40

314. The numbers 2272 and 875 are divided by a three-digit number N, giving the same remainder. The sum of the digits of N is
(*a*) 10 (*b*) 11
(*c*) 12 (*d*) 13

315. A number when divided by three consecutive numbers 9, 11, 13 leaves the remainders 8, 9 and 8 respectively. If the order of divisors is reversed, the remainders will be **(R.R.B., 2008)**
(*a*) 10, 8, 9 (*b*) 10, 1, 6
(*c*) 8, 9, 8 (*d*) 9, 8, 8

316. After the division of a number successively by 3, 4 and 7, the remainders obtained are 2, 1 and 4 respectively. What will be the remainder if 84 divides the same number?
(*a*) 41 (*b*) 53
(*c*) 75 (*d*) 80

317. A number is successively divided by 8, 7 and 3 giving residues 3, 4 and 2 respectively and quotient 31. The number is
(*a*) 3555 (*b*) 5355
(*c*) 5535 (*d*) 5553

318. A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6?
(*a*) 2 (*b*) 3
(*c*) 4 (*d*) 5

319. When the square of any odd number, greater than 1, is divided by 8, it always leaves remainder **(I.A.M., 2007)**
(*a*) 1 (*b*) 6
(*c*) 8
(*d*) Cannot be determined

320. The numbers from 1 to 29 are written side by side as follows:
1234567891011121314..........2829
If this number is divided by 9, then what is the remainder? **(M.A.T., 2006)**
(*a*) 0 (*b*) 1
(*c*) 3 (*d*) None of these

321. If 17^{200} is divided by 18, the remainder is
(*a*) 1 (*b*) 2
(*c*) 16 (*d*) 17

322. What is the remainder when 2^{31} is divided by 5?
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

323. Consider the following statements:
(1) $a^n + b^n$ is divisible by $a + b$ if $n = 2k + 1$, where k is a positive integer.
(2) $a^n - b^n$ is divisible by $a - b$ if $n = 2k$, where k is a positive integer.
Which of the statements given above is/are correct?
(*a*) 1 only (*b*) 2 only
(*c*) Both 1 and 2 (*d*) Neither 1 nor 2

324. $(7^{19} + 2)$ is divided by 6. The remainder is
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 5

325. If $(10^{12} + 25)^2 - (10^{12} - 25)^2 = 10^n$, then the value of n is
(*a*) 5 (*b*) 10
(*c*) 14 (*d*) 20

326. $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ is divisible by
(*a*) 11 (*b*) 16
(*c*) 25 (*d*) 30

327. $(4^{61} + 4^{62} + 4^{63} + 4^{64})$ is divisible by
(*a*) 3 (*b*) 11
(*c*) 13 (*d*) 17

328. $(9^6 + 1)$ when divided by 8, would leave a remainder of
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

329. If n is even, $(6^n - 1)$ is divisible by
(*a*) 6 (*b*) 30
(*c*) 35 (*d*) 37

330. If $(12^n + 1)$ is divisible by 13, then n is
(*a*) 1 only (*b*) 12 only
(*c*) any odd integer (*d*) any even integer

331. 25^{25} is divided by 26, the remainder is
(*a*) 1 (*b*) 2
(*c*) 24 (*d*) 25

332. If $(67^{67} + 67)$ is divided by 68, the remainder is
(*a*) 1 (*b*) 63
(*c*) 66 (*d*) 67

333. One less than $(49)^{15}$ is exactly divisible by
(*a*) 8 (*b*) 14
(*c*) 50 (*d*) 51

334. The remainder when 7^{84} is divided by 342 is
(*a*) 0 (*b*) 1
(*c*) 49 (*d*) 341

335. The remainder when 2^{60} is divided by 5 equals
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

336. By how many of the following numbers is $2^{12} - 1$ divisible?
2, 3, 5, 7, 10, 11, 13, 14
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

337. The remainder when $(15^{23} + 23^{23})$ is divided by 19, is
(*a*) 0 (*b*) 4
(*c*) 15 (*d*) 18

338. When 2^{256} is divided by 17, the remainder would be
(*a*) 1 (*b*) 14
(*c*) 16 (*d*) None of these

339. $7^{6n} - 6^{6n}$, where n is an integer > 0, is divisible by
(*a*) 13 (*b*) 127
(*c*) 559 (*d*) All of these

340. It is given that $(2^{32} + 1)$ is exactly divisible by a certain number. Which of the following is also definitely divisible by the same number?
(S.S.C., 2007)
(*a*) $2^{16} + 1$ (*b*) $2^{16} - 1$
(*c*) 7×2^{33} (*d*) $2^{96} + 1$

341. The number $(2^{48} - 1)$ is exactly divisible by two numbers between 60 and 70. The numbers are
(A.A.O. Exam, 2010)
(*a*) 63 and 65 (*b*) 63 and 67
(*c*) 61 and 65 (*d*) 65 and 67

342. n being any odd number greater than 1, $n^{65} - n$ is always divisible by
(*a*) 5 (*b*) 13
(*c*) 24 (*d*) None of these

343. Let $N = 55^3 + 17^3 - 72^3$. Then, N is divisible by
(*a*) both 7 and 13 (*b*) both 3 and 13
(*c*) both 17 and 7 (*d*) both 3 and 17

Directions (Questions 344–345): *These questions are based on the following information:*
Given $N = \underline{|1} + \underline{|2} + \underline{|3} + \ldots\ldots + \underline{|99} + \underline{|100}$.

344. Find the last two digits of N.
(*a*) 00 (*b*) 13
(*c*) 19 (*d*) 23

345. Find the remainder when N is divided by 168.
(*a*) 33 (*b*) 67
(*c*) 129 (*d*) 153

346. What is the remainder when 4^{61} is divided by 51?
(*a*) 20 (*b*) 41
(*c*) 50 (*d*) None of these

347. What is the remainder when 17^{36} is divided by 36?
(*a*) 1 (*b*) 7
(*c*) 19 (*d*) 29

348. Which one of the following is the common factor of $(47^{43} + 43^{43})$ and $(47^{47} + 43^{47})$?
(*a*) $(47 - 43)$ (*b*) $(47 + 43)$
(*c*) $(47^{43} + 43^{43})$ (*d*) None of these

349. Find the product of all odd natural numbers less than 5000.
(*a*) $\dfrac{5000!}{2500 \times 2501}$ (*b*) $\dfrac{5000!}{2^{2500} \times 2500!}$
(*c*) $\dfrac{5000!}{2^{5000}}$ (*d*) None of these

350. How many zeros will be required to number the pages of a book containing 1000 pages?
(*a*) 168 (*b*) 184
(*c*) 192 (*d*) 216

351. If $a^2 + b^2 + c^2 = 1$, what is the maximum value of abc?
(*a*) $\dfrac{1}{3}$ (*b*) $\dfrac{1}{3\sqrt{3}}$
(*c*) $\dfrac{2}{\sqrt{3}}$ (*d*) 1

352. Find the unit's digit in the sum of the fifth powers of the first 100 natural numbers.
(*a*) 0 (*b*) 2
(*c*) 5 (*d*) 8

353. If the symbol $[x]$ denotes the greatest integer less than or equal to x, then the value of
$$\left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{50}\right] + \left[\frac{1}{4} + \frac{2}{50}\right] + \ldots\ldots\ldots + \left[\frac{1}{4} + \frac{49}{50}\right] \text{ is}$$
(*a*) 0 (*b*) 9
(*c*) 12 (*d*) 49

354. When $100^{25} - 25$ is written in decimal notation, the sum of its digits is
(*a*) 444 (*b*) 445
(*c*) 446 (*d*) 448

355. What is the number of digits in the number $(1024)^4 \times (125)^{11}$?
(*a*) 35 (*b*) 36
(*c*) 37 (*d*) 38

356. How many numbers will be there between 300 and 500, where 4 comes only one time?
(*a*) 89 (*b*) 99
(*c*) 110 (*d*) 120
[UPSSSC Lower Subordinate (Pre.) Exam, 2016]

357. Which is not a prime number?
[Indian Railways Gr. 'D' Exam, 2014]
(*a*) 13 (*b*) 19
(*c*) 21 (*d*) 17

358. If $x = a(b - c)$, $y = b(c - a)$, $z = c(a - b)$, then the value of $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$ is
[SSC—CHSL (10 + 2) Exam, 2015]
(*a*) $\frac{2xyz}{abc}$ (*b*) $\frac{xyz}{abc}$
(*c*) 0 (*d*) $\frac{3xyz}{abc}$

359. Among the following statements, the statement which is not correct is: **[SSC—CHSL (10 + 2) Exam, 2015]**
(*a*) Every natural number is an integer.
(*b*) Every natural number is a real number.
(*c*) Every real number is a rational number.
(*d*) Every integer is a rational number.

360. If $a + b + c = 6$ and $ab + bc + ca = 10$ then the value of $a^3 + b^3 + c^3 - 3abc$ is
[SSC—CHSL (10 + 2) Exam, 2015]
(*a*) 36 (*b*) 48
(*c*) 42 (*d*) 40

361. If (1001 × 111) = 110000 + (11 × ____), then the number in the blank space is
(*a*) 121 (*b*) 211
(*c*) 101 (*d*) 1111
[CTET, 2016]

Direction (Question 362): The following question consists of a question and two statements I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the question. Given answer

(A) The data in statement I alone is sufficient to answer the question while II alone is not sufficient to answer the questions.
(B) Data in statement II alone is sufficient to answer the question while data in statement I alone is not sufficient to answer the question.
(C) The data in statement I alone or statement II alone is sufficient to answer the question.
(D) The data in both Statement I and II is insufficient to answer the question.
(E) The data in both Statement I and II is sufficient to answer the question.

362. If (the place value of 5 in 15201) + (the place value of 6 in 2659) = 7 × ____, then the number of the blank space is:
(*a*) 800 (*b*) 80
(*c*) 90 (*d*) 900
[CTET, 2016]

363. The sum of digits of a two – digit number is 12 and the difference between the two – digits of the two – digit number is 6. What is the two – digit number?
[IBPS—RRB Office Assistant (Online) Exam, 2015]
(*a*) 39 (*b*) 84
(*c*) 93
(*d*) Other than the given options
(*e*) 75

364. The difference between the greatest and the least four digit numbers that beings with 3 and ends with 5 is
[SSC—CHSL (10 + 2) Exam, 2015]
(*a*) 900 (*b*) 909
(*c*) 999 (*d*) 990

365. The sum of the perfect squares between 120 and 300 is
[SSC—CHSL (10 + 2) Exam, 2015]
(*a*) 1204 (*b*) 1024
(*c*) 1296 (*d*) 1400

366. If $p^3 - q^3 = (p - q)(p - q)^2 - xpq$, then find the value of x **[SSC—CHSL (10 + 2) Exam, 2015]**
(*a*) 1 (*b*) –3
(*c*) 3 (*d*) –1

367. What minimum value should be assigned to *, so that 2361*48 is exactly divisible by 9?
[ESIC—UDC Exam, 2016]
(*a*) 2 (*b*) 3
(*c*) 9 (*d*) 4

368. If p, q, r are all real numbers then $(p-q)^3 + (q-r)^3 + (r-p)^3$ is equal to **[SSC—CAPF/CPOExam, 2016]**
(*a*) $(p-q)(q-r)(r-p)$ (*b*) $3(p-q)(q-r)(r-p)$
(*c*) 1 (*d*) 0

369. If $(a^2 - b^2) \div (a + b) = 25$, then $(a + b) = ?$
(*a*) 30 (*b*) 25
(c) 125 (*d*) 150
[RRB—NTPC Exam, 2016]

370. How many prime numbers are there between 100 to 200? **[CMAT, 2017]**
(*a*) 21 (*b*) 20
(*c*) 16 (*d*) 11

371. The least number of five digit is exactly divisible by 88 is
(*a*) 10032 (*b*) 10132
(*c*) 10088 (*d*) 10023
[SSC Multi-Tasking Staff Exam -2017]

372. The number of three digit numbers which are multiples of 9 are

[CLAT, 2016]

(*a*) 100 (*b*) 99
(*c*) 98 (*d*) 101

373. Two consecutive even positive integers, sum of the squares of which is 1060, are

[CLAT, 2016]

(*a*) 12 and 14 (*b*) 20 and 22
(*c*) 22 and 24 (*d*) 15 and 18

Directions (Question 374): The question below consists of a question and two statements numbered I and II given below it. You have to decide whether the data given in the statements are sufficient to answer the questions. Read both the statements and given answer.

(A) If the data in statement I alone is sufficient to answer the question, while the data in statement II alone is not sufficient to answer the question.

(B) If the data in statement II alone is sufficient to answer the question, while the data in statement I alone is not sufficient to answer the question.

(C) If the data either in statement I alone or statement II alone is sufficient to answer the questions.

(D) If the data given in both Statement I and II together are not sufficient to answer the question.

(E) If the data in both Statement I and II together are necessary to answer the question.

374. What is the number of trees planted in the field in row and column?

[IBPS—Bank Spl. Officer (Marketing) Exam, 2016]

I. Number of columns is more than the number of rows by 4.

II. Number of columns is 20.

375. If n is a natural number and $n = p_1^{x_1} p_2^{x_2} p_3^{x_3}$, where p_1, p_2, p_3 are distinct prime factors, then the number of prime factors for n is

[CDS, 2016]

(*a*) $x_1 + x_2 + x_3$ (*b*) x_1, x_2, x_3
(*c*) $(x_1 + 1)(x_2 + 1)(x_3 + 1)$ (*d*) None of the above

376. Consider the following statements for the sequence of numbers given below:
11, 111, 1111, 11111, ….

1. Each number can be expressed in the form $(4m + 3)$, where m is a natural number.
2. Some numbers are squares.

Which of the above statements is/are correct?

[CDS, 2016]

(*a*) 1 only (*b*) 2 only
(*c*) Both 1 and 2 (*d*) neither 1 nor 2)

377. If the sum of two numbers is 14 and their difference is 10. Find the product of these two numbers.

[UPSSSC Lower Subordinate (Pre.) Exam, 2016]

(*a*) 24 (*b*) 22
(*c*) 20 (*d*) 18

378. If $m = -4$, $n = -2$, then the value of $m^3 - 3m^2 + 3m + 3n + 3n^2 + n^3$ is

(*a*) −120 (*b*) −124
(*c*) −126 (*d*) −128

[SSC—CGL (Tier-I) Exam, 2015]

379. If the sum of two numbers is 14 and their difference is 10, find the product of these two numbers.

[UPSSSC—Lower Subordinate (Pre.) Exam 2016]

(*a*) 18 (*b*) 20
(*c*) 24 (*d*) 22

380. What is the sum of all natural numbers from 1 to 100? **[CLAT-2016]**

(*a*) 5050 (*b*) 6000
(c) 5000 (*d*) 5052

ANSWERS

1. (*c*)	**2.** (*a*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*a*)	**10.** (*c*)
11. (*b*)	**12.** (*b*)	**13.** (*d*)	**14.** (*c*)	**15.** (*d*)	**16.** (*b*)	**17.** (*c*)	**18.** (*b*)	**19.** (*c*)	**20.** (*d*)
21. (*c*)	**22.** (*c*)	**23.** (*c*)	**24.** (*b*)	**25.** (*d*)	**26.** (*d*)	**27.** (*a*)	**28.** (*c*)	**29.** (*c*)	**30.** (*c*)
31. (*b*)	**32.** (*b*)	**33.** (*d*)	**34.** (*a*)	**35.** (*c*)	**36.** (*d*)	**37.** (*b*)	**38.** (*c*)	**39.** (*a*)	**40.** (*d*)
41. (*a*)	**42.** (*d*)	**43.** (*d*)	**44.** (*d*)	**45.** (*d*)	**46.** (*d*)	**47.** (*d*)	**48.** (*c*)	**49.** (*a*)	**50.** (*b*)
51. (*b*)	**52.** (*b*)	**53.** (*c*)	**54.** (*d*)	**55.** (*b*)	**56.** (*d*)	**57.** (*b*)	**58.** (*c*)	**59.** (*d*)	**60.** (*b*)

61. (*d*)	**62.** (*d*)	**63.** (*c*)	**64.** (*a*)	**65.** (*c*)	**66.** (*a*)	**67.** (*a*)	**68.** (*c*)	**69.** (*d*)	**70.** (*b*)
71. (*a*)	**72.** (*b*)	**73.** (*c*)	**74.** (*d*)	**75.** (*c*)	**76.** (*b*)	**77.** (*c*)	**78.** (*b*)	**79.** (*d*)	**80.** (*d*)
81. (*c*)	**82.** (*d*)	**83.** (*c*)	**84.** (*e*)	**85.** (*c*)	**86.** (*a*)	**87.** (*b*)	**88.** (*b*)	**89.** (*e*)	**90.** (*d*)
91. (*c*)	**92.** (*a*)	**93.** (*c*)	**94.** (*c*)	**95.** (*d*)	**96.** (*d*)	**97.** (*a*)	**98.** (*c*)	**99.** (*c*)	**100.** (*a*)
101. (*b*)	**102.** (*a*)	**103.** (*a*)	**104.** (*c*)	**105.** (*a*)	**106.** (*c*)	**107.** (*a*)	**108.** (*b*)	**109.** (*b*)	**110.** (*e*)
111. (*a*)	**112.** (*a*)	**113.** (*b*)	**114.** (*a*)	**115.** (*b*)	**116.** (*c*)	**117.** (*a*)	**118.** (*c*)	**119.** (*b*)	**120.** (*a*)
121. (*e*)	**122.** (*d*)	**123.** (*c*)	**124.** (*b*)	**125.** (*a*)	**126.** (*b*)	**127.** (*c*)	**128.** (*d*)	**129.** (*d*)	**130.** (*d*)
131. (*b*)	**132.** (*b*)	**133.** (*b*)	**134.** (*d*)	**135.** (*d*)	**136.** (*a*)	**137.** (*a*)	**138.** (*b*)	**139.** (*a*)	**140.** (*b*)
141. (*a*)	**142.** (*c*)	**143.** (*d*)	**144.** (*d*)	**145.** (*e*)	**146.** (*c*)	**147.** (*a*)	**148.** (*b*)	**149.** (*c*)	**150.** (*a*)
151. (*e*)	**152.** (*c*)	**153.** (*c*)	**154.** (*c*)	**155.** (*b*)	**156.** (*a*)	**157.** (*a*)	**158.** (*d*)	**159.** (*b*)	**160.** (*c*)
161. (*d*)	**162.** (*d*)	**163.** (*b*)	**164.** (*d*)	**165.** (*d*)	**166.** (*a*)	**167.** (*a*)	**168.** (*a*)	**169.** (*c*)	**170.** (*b*)
171. (*d*)	**172.** (*b*)	**173.** (*c*)	**174.** (*a*)	**175.** (*b*)	**176.** (*a*)	**177.** (*a*)	**178.** (*d*)	**179.** (*c*)	**180.** (*a*)
181. (*d*)	**182.** (*c*)	**183.** (*c*)	**184.** (*b*)	**185.** (*b*)	**186.** (*c*)	**187.** (*a*)	**188.** (*c*)	**189.** (*c*)	**190.** (*c*)
191. (*c*)	**192.** (*b*)	**193.** (*a*)	**194.** (*a*)	**195.** (*b*)	**196.** (*a*)	**197.** (*b*)	**198.** (*a*)	**199.** (*c*)	**200.** (*a*)
201. (*b*)	**202.** (*d*)	**203.** (*b*)	**204.** (*b*)	**205.** (*a*)	**206.** (*d*)	**207.** (*d*)	**208.** (*d*)	**209.** (*c*)	**210.** (*a*)
211. (*d*)	**212.** (*b*)	**213.** (*a*)	**214.** (*d*)	**215.** (*d*)	**216.** (*a*)	**217.** (*d*)	**218.** (*a*)	**219.** (*b*)	**220.** (*b*)
221. (*c*)	**222.** (*b*)	**223.** (*d*)	**224.** (*c*)	**225.** (*d*)	**226.** (*a*)	**227.** (*d*)	**228.** (*d*)	**229.** (*b*)	**230.** (*e*)
231. (*d*)	**232.** (*b*)	**233.** (*a*)	**234.** (*a*)	**235.** (*a*)	**236.** (*c*)	**237.** (*c*)	**238.** (*a*)	**239.** (*a*)	**240.** (*b*)
241. (*c*)	**242.** (*c*)	**243.** (*b*)	**244.** (*c*)	**245.** (*d*)	**246.** (*c*)	**247.** (*c*)	**248.** (*a*)	**249.** (*b*)	**250.** (*a*)
251. (*b*)	**252.** (*b*)	**253.** (*c*)	**254.** (*c*)	**255.** (*d*)	**256.** (*d*)	**257.** (*d*)	**258.** (*b*)	**259.** (*a*)	**260.** (*e*)
261. (*d*)	**262.** (*c*)	**263.** (*a*)	**264.** (*c*)	**265.** (*b*)	**266.** (*d*)	**267.** (*a*)	**268.** (*a*)	**269.** (*e*)	**270.** (*a*)
271. (*c*)	**272.** (*d*)	**273.** (*b*)	**274.** (*c*)	**275.** (*c*)	**276.** (*c*)	**277.** (*b*)	**278.** (*a*)	**279.** (*c*)	**280.** (*a*)
281. (*b*)	**282.** (*a*)	**283.** (*c*)	**284.** (*b*)	**285.** (*b*)	**286.** (*c*)	**287.** (*b*)	**288.** (*c*)	**289.** (*d*)	**290.** (*b*)
291. (*c*)	**292.** (*a*)	**293.** (*d*)	**294.** (*c*)	**295.** (*b*)	**296.** (*d*)	**297.** (*d*)	**298.** (*d*)	**299.** (*d*)	**300.** (*d*)
301. (*b*)	**302.** (*d*)	**303.** (*a*)	**304.** (*b*)	**305.** (*a*)	**306.** (*c*)	**307.** (*b*)	**308.** (*b*)	**309.** (*b*)	**310.** (*c*)
311. (*d*)	**312.** (*c*)	**313.** (*d*)	**314.** (*a*)	**315.** (*b*)	**316.** (*b*)	**317.** (*b*)	**318.** (*c*)	**319.** (*a*)	**320.** (*c*)
321. (*a*)	**322.** (*c*)	**323.** (*c*)	**324.** (*c*)	**325.** (*c*)	**326.** (*d*)	**327.** (*d*)	**328.** (*c*)	**329.** (*c*)	**330.** (*c*)
331. (*d*)	**332.** (*c*)	**333.** (*a*)	**334.** (*b*)	**335.** (*b*)	**336.** (*a*)	**337.** (*a*)	**338.** (*a*)	**339.** (*d*)	**340.** (*d*)
341. (*a*)	**342.** (*c*)	**343.** (*d*)	**344.** (*b*)	**345.** (*a*)	**346.** (*d*)	**347.** (*a*)	**348.** (*b*)	**349.** (*b*)	**350.** (*c*)
351. (*b*)	**352.** (*a*)	**353.** (*c*)	**354.** (*a*)	**355.** (*b*)	**356.** (*b*)	**357.** (*c*)	**358.** (*d*)	**359.** (*c*)	**360.** (*a*)
361. (*c*)	**362.** (*a*)	**363.** (*d*)	**364.** (*d*)	**365.** (*d*)	**366.** (*b*)	**367.** (*b*)	**368.** (*b*)	**369.** (*b*)	**370.** (*a*)
371. (*a*)	**372.** (*a*)	**373.** (*c*)	**374.** (*d*)	**375.** (*b*)	**376.** (*a*)	**377.** (*a*)	**378.** (*c*)	**379.** (*c*)	**380.** (*a*)

SOLUTIONS

1.

TL	L	T-Th	Th	H	T	O
3	2	5	4	7	1	0

Place value of 5 = (5 × 10000) = 50000.

2. The face value of 8 in the given numeral is 8.

3. Sum of the place values of 3 = (3000 + 30) = 3030.

4. Difference between the place values of 7 and 3 in given numeral = (7000 – 30) = 6970.

5. Difference between the local value and the face value of 7 in the given numeral = (70000 – 7) = 69993.

6. Required sum = (99999 + 10000) = 109999.

7. Required difference = (10000 – 999) = 9001.

8. Required number = 30005.

9. Required number = 2047.

10. All natural numbers and 0 are called the whole numbers.

11. Clearly, there exists a smallest natural number, namely 1. So statement (1) is true.
Natural numbers are counting numbers and the counting process never ends. So, the largest nature number is not known. Thus, statement (2) is false.
Thre is no natural number between two consecutive natural numbers. So, statement (3) is false.

12. Clearly, every rational number is also a real number.

13. Clearly, π is an irrational number.

14. Since $\sqrt{2}$ is a non-terminating and non-repeating decimal, so it is an irrational number.

15. $\sqrt{3}$ is an infinite non-recurring decimal.

16. We can write 9 = (1 + 8) ; 9 = (2 + 7) ;
9 = (3 + 6), 9 = (4 + 5).
Thus, it can be done in 4 ways.

17. We may have (64 and 1), (32 and 2), (16 and 4) and (8 and 8).
In any case, the sum is not 35.

18. We may take the least values of y and z as $y = 1$ and $z = 1$.
So, the maximum value of x is 7.

19. We know that: $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6} n(n+1)(2n+1)$.

$$\therefore (1^2 + 2^2 + 3^2 + ...+ 9^2) = \left(\frac{1}{6} \times 9 \times 10 \times 19\right) = 285.$$

20. $(n + 7) = 88 \Rightarrow n = (88 - 7) = 81$, which is false as $20 < n < 80$.
So, the required number is 88.

21. We have, $M = 1000$; $D = 500$; $C = 100$ and $L = 50$.
$\therefore M > D > C > L$ is the correct sequence.

22. These numbers are 24, 22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2. 8th number from the bottom is 16.

23. The given series is such that the sum of first hundred terms is zero, and 101st term is 2. So, the sum of 101 terms is 2.

24. Given series is 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, ...
$\therefore$ 96th term is 4. So, $T_{97} = 1$ and $T_{98} = 2$. Hence, the 98th term is 2.

25. Let the hundred's, ten's and one's digits be x, y and z respectively.
Then, the given number is $100x + 10y + z$.

26. Let the thousand's, hundred's, ten's and one's digits be x, y, z, w respectively.
Then, the number is $1000x + 100y + 10z + w = 10^3 x + 10^2 y + 10z + w$.

27. We know that the sum of two odd numbers is even.
$\therefore$ (n is odd, p is odd) $\Rightarrow (n + p)$ is even.

28. n^3 is odd $\Rightarrow n$ is odd and n^2 is odd.
$\therefore$ I and II are true.

29. $(n - 1)$ is odd $\Rightarrow (n - 1) - 2$ and $(n - 1) + 2$ are odd $\Rightarrow (n - 3)$ and $(n + 1)$ are odd.

30. The product of two odd numbers is always odd.

31. x is odd $\Rightarrow x^2$ is odd $\Rightarrow 5x^2$ is odd $\Rightarrow (5x^2 + 2)$ is odd.

32. $ab = 0 \Rightarrow a = 0$ or $b = 0$ or both are zero.

33. $A < B < C < D \Rightarrow D > B > A$
Also, $A < B < C < D$ and $D > B > E \Rightarrow E < B < C < D$
$\Rightarrow E < C < D$ and $E < B < C$.
Clearly, we cannot arrange A, E, C is an increasing or decreasing order.

34. When m is even, then m (n + 'o') $(p - q)$ is even.

35. $n < 0 \Rightarrow 2n < 0, -n > 0$ and $n^2 = (-n)^2 > 0$.
Thus, out of the numbers $0, -n, 2n$ and n^2 we find that $2n$ is the least.

36. It is given that $x - y = 8$.
I. We may have $x = 5$ and $y = -3$.
So, it is not necessary that both x and y are positive.
II. If x is positive, then it is not necessary that y is positive, as $x = 5 \Rightarrow y = -3$.
III. If $x < 0$, then $y = x - 8$ which is clearly less than 0.
So, if x is negative, then y must be negative.

37. If $x < 0$ and $y < 0$, then clearly $xy > 0$.
So, whenever x and y are negative, then xy is positive.
Note that $(x < 0, y < 0)$ does not imply that $(x + y)$ is positive.
e.g. If $x = -2$ and $y = -3$, then $(x + y) = -5 < 0$.
Note that $(x < 0, y < 0)$ does not imply that $(x - y) > 0$.
e.g. If $x = -5$ and $y = -2$, then $x - y = -5 - (-2)$
$= -5 + 2 = -3 < 0$.

38. Let $n = 1 + x = 1 + m(m + 1)(m + 2)(m + 3)$, where m is a positive integer.
Then, clearly two of m, $(m + 1)$, $(m + 2)$, $(m + 3)$ are even and so their product is even. Thus, x is even and hence $n = 1 + x$ is odd.
Also, $n = 1 + m(m + 3)(m + 1)(m + 2) = 1 + (m^2 + 3m)(m^2 + 3m + 2)$
$\Rightarrow n = 1 + y(y + 2)$, where $m^2 + 3m = y$
$\Rightarrow n = 1 + y^2 + 2y = (1 + y)^2$, which is a perfect square.
Hence, I and III are true.

39. $1 \le x \le 2 \Rightarrow 1 \le \frac{2}{5} y + 3 \le 2$

$$\Rightarrow (1 - 3) \le \frac{2}{5} y \le (2 - 3) \Rightarrow \frac{5}{2}(1 - 3) \le y \le \frac{5}{2}(2 - 3)$$

$$\Rightarrow -5 \le y \le \frac{-5}{2}.$$

Hence, y increases from -5 to $\frac{-5}{2}$.

40. (a) Let $x = 0$ and $y = \sqrt{2}$. Then, x is rational and y is irrational.
$\therefore \quad x + y = 0 + \sqrt{2} = \sqrt{2}$ which is irrational.
Thus, $x + y$ is not rational.

(*b*) Let $x = 0$ and $y = \sqrt{2}$. Then, x is rational and y is irrational.

$\therefore$ $xy = 0 \times \sqrt{2} = 0$, which is rational.

Hence, xy is not irrational.

(*c*) As shown in (*b*), xy is not necessarily irrational.

(*d*) $x + y$ is necessary irrational. But xy can be either rational or irrational.

Hence, (*d*) is true.

41. Let x and $(x + 1)$ be two consecutive integers. Then $(x + 1)^2 - x^2 = (x + 1 + x)(x + 1 - x) = (x + 1 + x) \times 1 = (x + 1 + x) =$ sum of given numbers.

42. If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number lying between a and b.

43. $B > A \Rightarrow A < B \Rightarrow (A - B) < 0$.

Since A and B are both positive integers, we have $(A + B) > 0$ and $AB > 0$.

If ($A = 1$ and $B = 2$), we have $AB < (A + B)$.

If ($A = 2$ and $B = 3$), we have $(A + B) < AB$.

Thus, we cannot say which one of $A + B$ and AB has the highest value.

44. $0 < x < 1 \Rightarrow \quad x^2 < x < 1$ ...(*i*)

$$\Rightarrow \frac{1}{x^2} > \frac{1}{x} > 1 > x > x^2 \quad \text{[using } (i)]$$

Hence, $\frac{1}{x^2}$ is the greatest.

45. $p < 1 \Rightarrow \frac{1}{p} > 1 \Rightarrow \frac{2}{p} > 2 \Rightarrow \frac{2}{p} - p > 2 - p > 0 \quad [\because p < 1]$

Hence, $\left(\frac{2}{p} - p\right)$ is a positive number.

46. $(x^2 + x + 1) = \left(x^2 + x + \frac{1}{4}\right) + \frac{3}{4}$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4} \qquad \left[\because \left(x + \frac{1}{2}\right)^2 \geq 0\right]$$

Hence, $(x^2 + x + 1)$ is greater than or equal to $\frac{3}{4}$.

47. $\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{7}\right) + \frac{1}{n} = \frac{(21 + 14 + 6)}{42} + \frac{1}{n} = \left(\frac{41}{42} + \frac{1}{n}\right)$.

This sum is a natural number when $n = 42$.

So, each one of the statements that 2 divides n ; 3 divides n and 7 divides n is true.

Hence, $n > 84$ is false.

48. $\frac{(16n^2 + 7n + 6)}{n} = \left(\frac{16n^2}{n} + \frac{7n}{n} + \frac{6}{n}\right) = \left(16n + 7 + \frac{6}{n}\right)$.

For $\left(16n + 7 + \frac{6}{n}\right)$ to be an integer, we may have $n = 1$ or $n = 2$ or $n = 3$ or $n = 6$.

Hence, 4 values of n will give the desired result.

49. $(p > q$ and $r < 0) \Rightarrow pr < qr$ is true.

50. $(X < Z$ and $X < Y) \Rightarrow X^2 < YZ$.

51. $x + y > p$ and $p > z \Rightarrow x + y > z$.

52. $\frac{(a-b)}{3.5} = \frac{4}{7} \Rightarrow (a - b) = \frac{4}{7} \times \frac{7}{2} = 2 \Rightarrow b < a$.

53. $\frac{13}{1} = \frac{13\,w}{(1-w)} \Rightarrow \frac{w}{(1-w)} = 1 \Rightarrow w = 1 - w \Rightarrow 2w = 1 \Rightarrow w = \frac{1}{2}$.

$\therefore$ $(2w)^2 = 4w^2 = 4 \times \frac{1}{4} = 1$.

Questions 54 to 57

Let the digits of the number in order be A, B, C, D, E.

Then, $A > 5\ E \Rightarrow E = 1$.

A, B, C are all odd and none of the digits is $3 \Rightarrow A, B, C$ are the digits from 5, 7, 9.

Since A is the largest digit, so $A = 9$.

$\therefore$ $B = 5$ or 7 and $C = 5$ or 7.

Now, the number DE is the product of two prime numbers.

$E = 1$ and $D = 2, 4, 6$ or 8.

41 and 61 are prime numbers and 81 cannot be expressed as product of two primes.

Only 21 can be expressed as the product of two prime numbers $(21 = 3 \times 7)$.

So, $D = 2$.

Hence, the number is 95721 or 97521.

54. The second digit of the number is either 5 or 7.

55. The last digit of the number is 1.

56. The largest digit in the number is 9.

57. The number is odd. So, it is not divisible by 2 or 4.

Sum of digits $= 9 + 7 + 5 + 2 + 1 = 24$, which is divisible by 3 but not by 9.

So, the given number is divisible by 3.

58. The least prime number is 2.

59. **Statement 1.** Let $x = 4$ and $y = 15$. Then, each one of x and y is a composite number.

But, $x + y = 19$, which is not composite.

$\therefore$ Statement 1 is not true.

Statement 2. We know that 1 is neither prime nor composite.

$\therefore$ Statement 2 is not true.

Thus, neither 1 nor 2 is correct.

60. Prime numbers between 0 and 50 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

Their number is 15.

61. Each of the required numbers must divide $(143 - 3) = 140$ exactly.

Now, $140 = 5 \times 7 \times 4$.

Hence, the required prime numbers are 5 and 7.

62. Sum of first four prime numbers $= (2 + 3 + 5 + 7) = 17$.

63. Sum of all prime numbers from 1 to 20 $= (2 + 3 + 5 + 7 + 11 + 13 + 17 + 19) = 77$.

64. Clearly, 11 is a prime number which remains unchanged when its digits are interchanged. And, $(11)^2 = 121$.

Hence, the square of such a number is 121.

65. Let the required prime number be p. Let p when divided by 6 give n as quotient and r as remainder. Then

$p = 6n + r$, where $0 \leq r < 6$

Now, $r = 0$, $r = 2$, $r = 3$ and $r = 4$ do not give p as prime.

$\therefore$ $r \neq 0$, $r \neq 2$, $r \neq 3$ and $r \neq 4$.

Hence, $r = 1$ or $r = 5$.

66. Clearly, $21 = 3 \times 7$, so 21 is not prime.

67. Clearly, 19 is a prime number.

68. We know that 115 is divisible by 5. So, it is not prime.

119 is divisible by 7. So, it is not prime.

$127 < (12)^2$ and prime numbers less than 12 are 2, 3, 5, 7, 11.

Clearly, 127 is not exactly divisible by any of them. Hence, 127 is a prime number.

69. Clearly, 143 is divisible by 11. So, 143 is not prime.
289 is divisible by 17. So, 289 is not prime.
117 is divisible by 3. So, 117 is not prime.
$359 < (20)^2$ and prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
And, 359 is not exactly divisible by any of them. Hence, 359 is a prime number.

70. Putting $n = 1, 2, 3, 4$ respectively in $(2n + 1)$ we get:
$(2 \times 1 + 1) = 3$, $(2 \times 2 + 1) = 5$, $(2 \times 3 + 1) = 7$ and $(2 \times 4 + 1) = 9$,
where 3, 5, 7 are prime numbers.
$\therefore$ The smallest value of n for which $(2n + 1)$ is not prime, is $n = 4$.

71. Clearly, 100 is divisible by 2. So, 100 is not prime.
$(101) < (11)^2$ and prime numbers less than 11 are 2, 3, 5, 7.
Clearly, 101 is not divisible by any of 2, 3, 5 and 7.
Hence, 101 is the smallest 3-digit prime number.

72. Each one of 112, 114, 116, 118 is divisible by 2. So, none is prime.
Each one of 111, 114, 117 is divisible by 3. So, none is prime.
Clearly, 115 is divisible by 5. So, it is not prime.
Each one of 112 and 119 is divisible by 7. So, none is prime.
Hence, there is only 1 prime number between 110 and 120, which is 113.

73. Let the given prime numbers be p, q, r and s. Then
$$p \times q \times r = 385 \text{ and } q \times r \times s = 1001$$
$$\Rightarrow \frac{p\times q\times r}{q\times r\times s} = \frac{\cancel{385}^{\cancel{55}\,5}}{\cancel{1001}_{\cancel{143}\,13}} = \frac{5}{13} \Rightarrow \frac{p}{s} = \frac{5}{13} \Rightarrow p = 5 \text{ and } s = 13.$$
Hence the largest of these prime numbers is 13.

74. Let the required prime numbers be $x, y, y + 36$. Then,
$$x + y + y + 36 = 100 \Rightarrow x + 2y = 64$$
Let $x = 2$. Then, $2y = 62 \Rightarrow y = 31$.
So, these prime numbers are 2, 31 and 67.
In given choices 67 is the answer.

75. $\sqrt{437} > 20$
All prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. 161 is divisible by 7, 221 is divisible by 13 and 437 is divisible by 19. 373 is not divisible by any of the above prime numbers.
$\therefore$ 373 is prime.

76. The required arithmetic sequence of five prime numbers is 5, 11, 17, 23, 29 and therefore, the required 5th term is 29.

77. Each of the numbers 302, 303, 304, 305, 306, 308, 309, 310, 312, 314, 315, 316 and 318 is clearly a composite number. Out of 307, 311, 313, 317 and 319 clearly every one is prime. Hence, there are 5 prime numbers between 301 and 320.

78. Clearly, $3 \neq 4n + 1$ and $3 \neq 4n + 3$ for any natural number n.
$\therefore$ Statement (1) is false.
Putting $p = 3, 5, 7, 11, 13, 17$ etc. we get:
$(p - 1)(p + 1) = (2 \times 4), (4 \times 6), (6 \times 8), (10 \times 12), (12 \times 14), (16 \times 18)$ etc., each one of which is divisible by 4.
$\therefore$ Statement (2) is true.
Hence, (1) is false and (2) is true.

79. $n = 1 \Rightarrow (n^2 + n + 41) = (1 + 1 + 41) = 43$, which is prime.
$n = 10 \Rightarrow (n^2 + n + 41) = (100 + 10 + 41) = 151$, which is prime.
$n = 20 \Rightarrow (n^2 + n + 41) = (400 + 20 + 41) = 461$, which is prime.
$n = 40 \Rightarrow (n^2 + n + 41) = (1600 + 40 + 41) = 1681$, which is divisible by 41.
Thus, 1681 is not a prime number.
Hence $n = 40$ for which $(n^2 + n + 41)$ is not prime.

80. $X_2 = (2 \times 3) + 1 = 7$, which is prime and $X_2 + 1 = 8$, which is even.
$X_3 = (2 \times 3 \times 5) + 1 = 31$, which is prime and $X_3 + 1 = 32$, which is even.
$X_4 = (2 \times 3 \times 5 \times 7) + 1 = 211$, which is prime and $X_4 + 1 = 212$, which is even and so on.
Thus X_k is prime and $(X_k + 1)$ is even.
Hence, 1 and 3 are true statements.

81. $6 \times 3\,(3 - 1) = 6 \times 3(2) = 6 \times 6 = 36$.

82. Given Expression $= (1234 + 2345 + 4567) - 3456$
$= (8146 - 3456) = 4690$.
$$\begin{array}{r} 1\,2\,3\,4 \\ 2\,3\,4\,5 \\ +\,4\,5\,6\,7 \\ \hline 8\,1\,4\,6 \\ -\,3\,4\,5\,6 \\ \hline 4\,6\,9\,0 \end{array}$$

83. Let $5566 - 7788 + 9988 = x + 4444$. Then
$(5566 + 9988) - 7788 = x + 4444$
$\Rightarrow 15554 - 7788 = x + 4444 \Rightarrow x + 4444 = 7766$
$\Rightarrow x = (7766 - 4444) = 3322$.
$$\begin{array}{r} 7\,7\,6\,6 \\ -\,4\,4\,4\,4 \\ \hline 3\,3\,2\,2 \end{array}$$

84. Given Expression $= 38649 - (1624 + 4483)$
$= (38649 - 6107) = 32542$.

85. Given Expression $= 884697 - (773697 + 102479)$
$= 884697 - 876176$
$= 8521$.
$$\begin{array}{r} 8\,8\,4\,6\,9\,7 \\ -\,8\,7\,6\,1\,7\,6 \\ \hline 8\,5\,2\,1 \end{array}$$

86. Let $10531 + 4813 - 728 = x \times 87$. Then
$(15344 - 728) = 87 \times x \Rightarrow x = \dfrac{14616}{87} = 168$.
$$\begin{array}{r} 1\,0\,5\,3\,1 \\ +\,4\,8\,1\,3 \\ \hline 1\,5\,3\,4\,4 \\ -\,7\,2\,8 \\ \hline 1\,4\,6\,1\,6 \end{array}$$
$$\begin{array}{r} 168 \\ 87\overline{)14616(} \\ 87 \\ \hline 591 \\ 522 \\ \hline 696 \\ 696 \\ \hline \times \end{array}$$

87. $394 \times 113 = 394 \times (100 + 10 + 3)$
$= (394 \times 100) + (394 \times 10) + (394 \times 3)$
$= 39400 + 3940 + 1182$
$= 44522$.
$$\begin{array}{r} 3\,9\,4\,0\,0 \\ 3\,9\,4\,0 \\ +\,1\,1\,8\,2 \\ \hline 4\,4\,5\,2\,2 \end{array}$$

88. $1260 \div 14 \div 9 = \left(1260 \times \dfrac{1}{14} \times \dfrac{1}{9}\right) = 10$.

89. $136 \times 12 \times 8 = 136 \times 96 = 136 \times (100 - 4)$
$= (136 \times 100) - (136 \times 4)$
$= 13600 - 544 = 13056$.
$$\begin{array}{r} 1\,3\,6\,0\,0 \\ -\;\;\,5\,4\,4 \\ \hline 1\,3\,0\,5\,6 \end{array}$$

90. Let $8888 + 848 + 88 - x = 7337 + 737$.
Then $9824 - x = 8074$

$\Rightarrow x = 9824 - 8074 \quad \Rightarrow x = 1750.$

$$\begin{array}{r} 9824 \\ -8074 \\ \hline 1750 \end{array} \quad \begin{array}{r} 8888 \\ 848 \\ +88 \\ \hline 9824 \end{array} \quad \begin{array}{r} 7337 \\ +737 \\ \hline 8074 \end{array}$$

91. Let $414 \times x \times 7 = 127512$. Then

$$x = \frac{127512}{414 \times 7} = \frac{18216}{414} = \frac{2024}{46} = \frac{1012}{23} = 44.$$

$$\begin{array}{r} 44 \\ 23)\overline{1012}(\\ 92 \\ \hline 92 \\ 92 \\ \hline \times \end{array}$$

92. $82540027 \times 43253 = 82540027 \times (40000 + 3000 + 200 + 50 + 3)$
$= (82540027 \times 40000) + (82540027 \times 3000)+(82540027 \times 200)$
$+ (82540027 \times 50) + (82540027 \times 3)$

$$\begin{array}{r} 3301601080000 \\ 247620081000 \\ 16508005400 \\ 4127001350 \\ +\quad 247620081 \\ \hline 3570103787831 \end{array}$$

Shortcut Method:

Product of unit's digits of the given numbers = 7 × 3 = 21

Clearly, the required product will have 1 as the unit's digit, which is 3570103787831.

93. Let $\frac{46351 - 36418 - 4505}{x} = 1357$. Then

$$x = \frac{46351 - (36418 + 4505)}{1357}$$

$$\Rightarrow x = \frac{(46351 - 40923)}{1357} = \frac{5428}{1357} = 4.$$

$$\begin{array}{r} 46351 \\ -40923 \\ \hline 5428 \end{array} \quad \begin{array}{r} 36418 \\ +4505 \\ \hline 40923 \end{array}$$

94. $6 \times 66 \times 666 = 6 \times (6 \times 11) \times (6 \times 111)$
$= (6 \times 6 \times 6) \times (11 \times 111) = (216 \times 1221)$
$= (1221 \times 216) = 1221 \times (200 + 10 + 6)$
$= (1221 \times 200) + (1221 \times 10) + (1221 \times 6)$
$= 244200 + 12210 + 7326 = 263736.$

$$\begin{array}{r} 244200 \\ 12210 \\ +\quad 7326 \\ \hline 263736 \end{array}$$

95. $(+ 1) - (- 1) = (+ 1 + 1) = 2.$

96.
$$\begin{array}{r} 888888 \\ 88888 \\ 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 987648 \end{array}$$

97. Given Expression $= 17 + (- 12) - 48 = 17 - 60 = - 43.$

98. $\frac{60840}{234} = \frac{30420}{117} = \frac{3380}{13} = 260.$

99. Let $3578 + 5729 - x \times 581 = 5821$. Then

$x \times 581 = (3578 + 5729) - 5821 \Rightarrow x \times 581 = (9307 - 5821)$

$= 3486 \Rightarrow x = \frac{3486}{581} = 6.$

100. $- 95 \div 19 = \frac{-95}{19} = -5.$

101. $12345679 \times 72 = 12345679 \times (70 + 2)$
$= (12345679 \times 70) + (12345679 \times 2)$
$= (864197530 + 24691358)$
$= 888888888.$

$$\begin{array}{r} 864197530 \\ +\quad 24691358 \\ \hline 888888888 \end{array}$$

102. $8899 - 6644 - 3322 = x - 1122 \Rightarrow 2255 - 3322 + 1122 = x$
$\Rightarrow x = 3377 - 3322 = 55.$

103. Let $74844 \div x = 54 \times 63$. Then, $\frac{74844}{x} = 54 \times 63$

$$\Rightarrow x = \frac{74844}{54 \times 63} = \frac{8316}{6 \times 63} = \frac{1386}{63} = \frac{198}{9} = 22.$$

104. $1256 \times 3892 = (1000 + 200 + 50 + 6) \times 3892$
$= (1000 \times 3892) + (200 \times 3892) +$
$(50 \times 3892) + (6 \times 3892)$
$= 3892000 + 778400 + 194600 + 23352$
$= 4888352.$

$$\begin{array}{r} 3892000 \\ 778400 \\ 194600 \\ +23352 \\ \hline 4888352 \end{array}$$

105. $786 \times 964 = (800 - 14) \times 964$
$= (800 \times 964) - (14 \times 964)$
$= (771200 - 13496) = 757704.$

$$\begin{array}{r} 771200 \\ -13496 \\ \hline 757704 \end{array}$$

106. $348 \times 265 = (350 - 2) \times 265 = (350 \times 265) - (2 \times 265)$
$= \{(300 + 50) \times 265\} - 530 = (300 \times 265)$
$+ (50 \times 265) - 530$
$= 79500 + 13250 - 530 = 92750 - 530$
$= 92220.$

107. $(71 \times 29 + 27 \times 15 + 8 \times 4) = (80 - 9) \times 29 + 405 + 32 = (80 \times 29) - (9 \times 29) + 437 = 2320 - 261 + 437 = 2757 - 261 = 2496.$

108. Let $x \times (|a| \times |b|) = - ab$. Then, $x = \frac{-(ab)}{|ab|} = -1.$

109. Let $(46)^2 - x^2 = 4398 - 3066$
Then, $(46)^2 - x^2 = 1332 \Rightarrow x^2 = (46)^2 - 1332$
$\therefore x^2 = (50 - 4)^2 - 1332 = (50)^2 + 4^2 - 2 \times 50 \times 4 - 1332$
$\Rightarrow x^2 = 2500 + 16 - 400 - 1332 = 2516 - 1732 = 784$
$\Rightarrow x = \sqrt{784} = 28.$

$$\begin{array}{l|l} 2 & \overline{784}\,(28 \\ & 4 \\ \hline 48 & 384 \\ & 384 \\ \hline & \times \end{array} \qquad \begin{array}{r} 2516 \\ -1732 \\ \hline 784 \end{array}$$

110. $(800 \div 64) \times (1296 \div 36) = \frac{\overset{50}{\cancel{800}}}{\underset{4}{\cancel{64}}} \times \frac{\overset{9}{\cancel{27}}\;\cancel{108}\;\cancel{1296}}{\underset{3}{\cancel{36}}} = 450.$

111. $5358 \times 51 = 5358 \times (50 + 1) = 5358 \times 50 + 5358 \times 1$

$= \dfrac{5358 \times 100}{2} + 5358$

$= \dfrac{535800}{2} + 5358 = 267900 + 5358 = 273258.$

112. $587 \times 999 = 587 \times (1000 - 1) = (587 \times 1000) - (587 \times 1)$
$= 587000 - 587 = 586413.$

113. $3897 \times 999 = 3897 \times (1000 - 1) = (3897 \times 1000) - (3897 \times 1)$
$= 3897000 - 3897 = 3893103.$

114. $72519 \times 9999 = 72519 \times (10000 - 1)$
$= (72519 \times 10000) - (72519 \times 1)$
$= 725190000 - 72519 = 725117481.$

$$\begin{array}{r} 725190000 \\ -\ 72519 \\ \hline 725117481 \end{array}$$

115. $2056 \times 987 = 2056 \times (1000 - 13)$
$= (2056 \times 1000) - (2056 \times 13)$
$= 2056000 - 26728 = 2029272.$

116. $1904 \times 1904 = (1904)^2 = (1900 + 4)^2$
$= (1900)^2 + 4^2 + 2 \times 1900 \times 4$
$= 3610000 + 16 + 15200 = 3625216.$

117. $1397 \times 1397 = (1397)^2 = (1400 - 3)^2$
$= (1400)^2 + 3^2 - 2 \times 1400 \times 3 = 1960000 + 9 - 8400$
$= 1951609.$

118. $(107 \times 107) + (93 \times 93) = (107)^2 + (93)^2$
$= (100 + 7)^2 + (100 - 7)^2$
$= (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
$= 2[(100)^2 + 7^2] = 2[10000 + 49]$
$= 2 \times 10049 = 20098.$

119. $(217 \times 217) + (183 \times 183) = (217)^2 + (183)^2$
$= (200 + 17)^2 + (200 - 17)^2 = (a + b)^2 + (a - b)^2$
$= 2(a^2 + b^2)$, where $a = 200$, $b = 17$
$= 2\ [(200)^2 + (17)^2] = 2\ [40000 + 289]$
$= (2 \times 40289) = 80578.$

120. $(106 \times 106 - 94 \times 94) = (106)^2 - (94)^2$
$= (106 + 94)\ (106 - 94) = (200 \times 12) = 2400.$

121. $(8796 \times 223 + 8796 \times 77) = 8796 \times (223 + 77)$
[by distributive law]
$= (8796 \times 300) = 2638800.$

122. $(287 \times 287 + 269 \times 269 - 2 \times 287 \times 269)$
$= (287)^2 + (269)^2 - (2 \times 287 \times 269)$
$= (287 - 269)^2 = (18)^2 = 324.$
$[\because a^2 + b^2 - 2ab = (a - b)^2]$

123. $(476 + 424)^2 - 4 \times 476 \times 424 = (a + b)^2 - 4ab = (a - b)^2$, where $a = 476$ & $b = 424 = (476 - 424)^2 = (52)^2 = (50 + 2)^2$
$= (50)^2 + 2^2 + 2 \times 50 \times 2$
$= (2500 + 4 + 200) = 2704.$

124. $(112 \times 5^4) = 112 \times \left(\dfrac{10}{2}\right)^4 = \dfrac{112 \times 10000}{16} = 70000.$

125. $(5746320819 \times 125) = \dfrac{5746320819 \times (125 \times 8)}{8}$

$= \dfrac{5746320819 \times 1000}{8} = \dfrac{5746320819000}{8} = 718290102375.$

126. $935421 \times 625 = \dfrac{935421 \times 625 \times 16}{16} = \dfrac{935421 \times 10000}{16}$

$= \dfrac{9354210000}{16} = 584638125.$

127. $(999)^2 - (998)^2 = (999 + 998)\ (999 - 998) = (1997 \times 1) = 1997.$

128. $(80)^2 - (65)^2 + 81 = (80 + 65)\ (80 - 65) + 81 = (145 \times 15) + 81 = (2175 + 81) = 2256.$

129. $(24 + 25 + 26)^2 - (10 + 20 + 25)^2 = (75)^2 - (55)^2$
$= (75 + 55)\ (75 - 55) = (130 \times 20) = 2600.$

130. $(65)^2 - (55)^2 = (65 + 55)\ (65 - 55) = (120 \times 10) = 1200.$

131. $(a^2 - b^2) = 19 \Rightarrow (a + b)\ (a - b) = 19.$
Clearly, $a = 10$ and $b = 9$.

132. Since $(a + b)^2 - (a - b)^2 = 4ab$, so the given expression should be a multiple of 4.
So, the least value of $4ab$ is 32 and so the least value of ab is 8.
Hence, the smallest value of a is 4 and that of b is 2.
Hence, $a = 4$.

133. Given Expression $= (397)^2 + (104)^2 + 2 \times 397 \times 104$
$= (397 + 104)^2 = (501)^2 = (500 + 1)^2$
$= (500)^2 + 1^2 + 2 \times 500 \times 1 = 250000 + 1 + 1000 = 251001.$

134. $(64)^2 - (36)^2 = 20 \times x \Rightarrow (64 + 36)\ (64 - 36) = 20 \times x$

$\Rightarrow x = \dfrac{100 \times 28}{20} = 140.$

135. Given Expression

$= \dfrac{(a+b)^2 - (a-b)^2}{ab}$ (where $a = 489$, $b = 375$) $= \dfrac{4ab}{ab} = 4$

136. Given Expression $= \dfrac{(a+b)^2 + (a-b)^2}{(a^2+b^2)}$,

(where $a = 963$ and $b = 476$) $= \dfrac{2\,(a^2+b^2)}{(a^2+b^2)} = 2.$

137. Given Expression

$= \dfrac{(a^3+b^3)}{(a^2-ab+b^2)}$, (where $a = 768$ and $b = 232$) $= (a + b)$
$= (768 + 232) = 1000.$

138. Given Expression $= \dfrac{(854)^3 - (276)^3}{(854)^2 + (854 \times 276) + (276)^2}$

$= \dfrac{(a^3-b^3)}{(a^2+ab+b^2)}$, where $a = 854$ and $b = 276$
$= (a - b) = (854 - 276) = 578.$

139. Given Expression $= \dfrac{(753)^2 + (247)^2 - (753 \times 247)}{(753)^3 + (247)^3}$

$= \dfrac{(a^2+b^2-ab)}{(a^3+b^3)}$, where $a = 753$ and $b = 247$

$= \dfrac{1}{(a+b)} = \dfrac{1}{(753+247)} = \dfrac{1}{1000}.$

140. Given Expression $= \dfrac{(256)^2 - (144)^2}{112} = \dfrac{(256+144)\,(256-144)}{112}$

$= \dfrac{(400 \times 112)}{112} = 400.$

141. $\dfrac{(a^2+b^2+ab)}{(a^3-b^3)} = \dfrac{a^2+b^2+ab}{(a-b)(a^2+b^2+ab)} = \dfrac{1}{(a-b)} = \dfrac{1}{(11-9)} = \dfrac{1}{2}.$

142. $a + b + c = 0 \Rightarrow a + b = -c$, $(b + c) = -a$ and $(c + a) = -b$
$\Rightarrow (a + b)\ (b + c)\ (c + a) = (-c) \times (-a) \times (-b) = -(abc).$

143. $(a^2 + b^2 + c^2 - ab - bc - ca) = (a + b + c)^2 - 3\ (ab + bc + ca)$
$= (7 + 5 + 3)^2 - 3\ (35 + 15 + 21)$
$= (15)^2 - 3 \times 71$
$= (225 - 213) = 12.$

144. Both addition and multiplication of numbers are commutative and associative.

145. $9H + H8 + H6 = 230 \Rightarrow \{(9 \times 10) + H\} + (10H + 8) + (10H + 6) = 230$
$\Rightarrow 21\,H + 104 = 230 \Rightarrow 21H = 126$
$\Rightarrow H = 6.$

146. Let the missing digit be x. Then, 1 (carried over) + 3 + x + x = 10 + $x \Rightarrow x = 6$.

147. Clearly, $M = 0$ since $304 \times 4 = 1216$.

148. $5p9 + 327 + 2q8 = 1114 \Rightarrow (500 + 10p + 9) + (327) + (200 + 10q + 8) = 1114$
$\Rightarrow 10(p + q) + 1044 = 1114$
$\Rightarrow 10\,(p + q) = 70 \Rightarrow (p + q) = 7$
$\Rightarrow$ Maximum value of q is 7
[As minimum value of $p = 0$]

149. $5P7 + 8Q9 + R32 = 1928 \Rightarrow (500 + 10P + 7) + (800 + 10Q + 9) + (100R + 30 + 2) = 1928$
$\Rightarrow 10P + 10Q + 100R + 1348 = 1928$
$\Rightarrow 10(P + Q + 10R) = 580$
$\Rightarrow P + Q + 10R = 58 \Rightarrow R = 5$ and $P + Q = 8$ or $R = 4$ and $P + Q = 18$
$\Rightarrow$ Maximum value of Q is 9
[for $P = 9$ in second case]

150. Let $\dfrac{1x\,5y4}{148} = 78$.

Then, $10000 + 1000\,x + 500 + 10y + 4 = 148 \times 78$
$\Rightarrow 10000 + 1000x + 500 + 10y + 4 = 11544$
$= 10000 + 1000 + 500 + 40 + 4$
$\Rightarrow 1000x = 1000 \Rightarrow x = 1.$

151. Let $6x43 - 46y9 = 1904$.
Clearly, $y = 3$ and $x = 5$.
Hence * must be replaced by 5.

$$\begin{array}{r} ③ \\ 6\;x\;\cancel{4}\;3 \\ -\;4\;6\;y\;9 \\ \hline 1\;9\;0\;4 \\ \hline \end{array}$$

152. $5\,P\,9 - 7\,Q\,2\ + 9\,R\,6 = 823$
$\Rightarrow \quad (500 + 10P + 9) - (700 + 10Q + 2) + (900 + 10R + 6) = 823$
$\Rightarrow \ (500 + 900 - 700) + 10(P + R - Q) + (9 + 6 - 2) = 823$
$\Rightarrow \ 700 + 10(P + R - Q) = 810 = 700 + 110$
$\Rightarrow \ 10\,(P + R - Q) = 110$
$\Rightarrow \ P + R - Q = 11$
$\Rightarrow \ Q = (P + R - 11).$
To get maximum value of Q we take $P = 9$ and $R = 9$.
This gives $Q = (9 + 9 - 11) = 7$.
Hence, the maximum value of Q is 7.

153. Let the required digit be x. Then
$x + 1x + 2x + x3 + x1 = 21x$
$\Rightarrow \quad x + 10 + x + 20 + x + 10x + 3 + 10x + 1 = 200 + 10 + x$
$\Rightarrow \quad 22x = 210 - 34 = 176 \Rightarrow x = 8.$
Hence, the required digit is 8.

154. It is given that $D = 0$. So, we have $A = 5$.
So, 1 is carried over.
$1 + B + C = 10 + C \Rightarrow 1 + B = 10 \Rightarrow B = 9.$
Now, $1 + C + C = A \Rightarrow 1 + 2C = 5 \Rightarrow 2\,C = 4 \Rightarrow C = 2$.
Hence $B = 9$.

$$\begin{array}{r} ①① \quad\;\; \\ C\;B\;A = 5 \\ +\;C\;C\;A = 5 \\ \hline A\;C\;0 \quad\;\; \end{array}$$

155. We have, $C = 2$.

156. Since $13b7$ is divisible by 11, we have
$(7 + 3) - (b + 1) = 0 \Rightarrow 9 - b = 0 \Rightarrow b = 9.$
Putting $b = 9$, $a + 8 = 9$ we get $a = 1$. Hence, $(a + b) = (1 + 9) = 10$.

$$\begin{array}{r} 4\;a\;3 \\ 9\;8\;4 \\ \hline 1\;3\;b\;7 \\ \hline \end{array}$$

157. Clearly, we have $ab \times b = 24$ and $ab \times a = 12$.
$\therefore \ \dfrac{ab \times b}{ab \times a} = \dfrac{24}{12} \Rightarrow \dfrac{b}{a} = \dfrac{2}{1} \Rightarrow a = 1, b = 2.$

158. Clearly, $111 \times 1 = 111 \neq 8111$.
But, $999 \times 9 = 8991$. Hence, we have 9 in place of *.

159. $(1 * 2) = 1 + 6 \times 2 = 1 + 12 = 13.$
$(1 * 2) * 3 = 13 * 3 = 13 + 6 \times 3 = 13 + 18 = 31.$

160. $\underline{|8} - \underline{|7} - \underline{|6} = 8 \times 7 \times \underline{|6} - 7 \times \underline{|6} - \underline{|6}$
$= (56 - 7 - 1) \times \underline{|6} = 48 \times \underline{|6} = 6 \times 8 \times \underline{|6}$.

161. Highest power of 3 in 99! $= \left[\dfrac{99}{3}\right] + \left[\dfrac{99}{3^2}\right] + \left[\dfrac{99}{3^3}\right] + \left[\dfrac{99}{3^4}\right]$
$= \left[\dfrac{99}{3}\right] + \left[\dfrac{99}{9}\right] + \left[\dfrac{99}{27}\right] + \left[\dfrac{99}{81}\right]$
$= 33 + 11 + 3 + 1 = 48.$
Since $9 = 3^2$, so highest power of 9 dividing 99! $= \dfrac{48}{2} = 24.$

162. Every number from $\underline{|5}$ onwards is completely divisible by 5.
$\therefore \ (\underline{|5} + \underline{|6} + \underline{|7} + \ldots\ldots.. + \underline{|100})$ is completely divisible by 5.
And,
$(\underline{|1} + \underline{|2} + \underline{|3} + \underline{|4}) = (1 + 2 + 3 \times 2 \times 1 + 4 \times 3 \times 2 \times 1) = (1 + 2 + 6 + 24) = 33.$
Clearly, 33 when divided by 5 leaves a remainder 3.
Hence, $(\underline{|1} + \underline{|2} + \underline{|3} + \underline{|4} + \underline{|5} + \ldots\ldots.. + \underline{|100})$ when divided by 5 leaves a remainder 3.

163. $6^{10} \times 7^{17} \times 11^{27} = (2 \times 3)^{10} \times 7^{17} \times 11^{27} = 2^{10} \times 3^{10} \times 7^{17} \times 11^{27}$.
Number of prime factors in the given expression = (10 + 10 + 17 + 27) = 64.

164. $(30)^7 \times (22)^5 \times (34)^{11} = (2 \times 3 \times 5)^7 \times (2 \times 11)^5 \times (2 \times 17)^{11}$
$= 2^{(7 + 5 + 11)} \times 3^7 \times 5^7 \times 11^5 \times 17^{11}$
$= (2^{23} \times 3^7 \times 5^7 \times 11^5 \times 17^{11}).$
Number of prime factors = (23 + 7 + 7 + 5 + 11) = 53.

165. Let $x \times 48 = 173 \times 240$. Then, $x = \dfrac{173 \times 240}{48} = 173 \times 5 = 865.$

166. $(1000 + x) > (1000 \times x)$. Clearly, $x = 1$.

167. Let the original number be x. Then,
$$\frac{(x + 7) \times 5}{9} - 3 = 12 \Rightarrow \frac{5x + 35}{9} = 15 \Rightarrow 5x + 35 = 135$$
$\Rightarrow 5x = 100 \Rightarrow x = 20.$

168. 38950 is completely divisible by 410 only.
Hence ₹ 410 is the correct answer.

169. Let the four consecutive even numbers be a, $a + 2$, $a + 4$ and $a + 6$.
Then, $a + a + 2 + a + 4 + a + 6 = 180 \Rightarrow 4a = 168$
$\Rightarrow a = 42.$
So, these numbers are 42, 44, 46 and 48.
Sum of next four consecutive even numbers = (50 + 52 + 54 + 56) = 212.

170. Number on thumbs = 1, 9, 17, 25, ...
This is an AP in which $a = 1$ and $d = (9 - 1) = 8$.

$\therefore$ $T_n = a + (n - 1)d = 1 + 8(n - 1) = (8n - 7)$.
$8n - 7 = 1994 \Rightarrow 8n = 2001 \Rightarrow n = 250$.
$T_{250} = 1 + (250 - 1) \times 8 = 1 + 249 \times 8 = 1993$.
So, 1993 lies on thumb and 1994 on index finger.

171. Out of four consecutive integers two are even and therefore, their product is even and on adding 1 to it, we get an odd integer. So, n is odd. Some possible values of n are as under:
$n = 1 + (1 \times 2 \times 3 \times 4) = (1 + 24) = 25 = 5^2$,
$n = 1 + (2 \times 3 \times 4 \times 5) = (1 + 120) = 121 = (11)^2$,
$n = 1 + (3 \times 4 \times 5 \times 6) = (1 + 360) = 361 = (19)^2$,
$n = 1 + (4 \times 5 \times 6 \times 7) = 841 = (29)^2$ and so on.
Hence, n is odd and a perfect square.

172. $(x^2 + y^2) = (x + y)^2 - 2xy = (15)^2 - 2 \times 56$
$= (225 - 112) = 113$.

173. $(2^2 + 4^2 + 6^2 + \ldots + 40^2) = (1 \times 2)^2 + (2 \times 2)^2 + (2 \times 3)^2 + \ldots + (2 \times 20)^2 = 2^2 \times (1^2 + 2^2 + 3^2 + \ldots + 20^2)$
$= (4 \times 2870) = 11480$.

174. $5^2 + 6^2 + \ldots + 10^2 + 20^2 = (1^2 + 2^2 + 3^2 + \ldots + 10^2) - (1^2 + 2^2 + 3^2 + 4^2) + 400$

$= \frac{1}{6}n(n+1)(2n+1) - (1+4+9+16) + 400$, where $n = 10$

$= \left(\frac{1}{6} \times 10 \times 11 \times 21\right) - 30 + 400 = (385 - 30 + 400) = 755$.

175. $(6 + 12 + 18 + 24 + \ldots + 60) = 6 \times (1 + 2 + 3 + 4 + \ldots + 10) = 6 \times 55 = 330$.

176. $2^m > 960$ when the least value of m is 10.
Then, $2^{10} = 1024$ and $1024 - 960 = 64 = 2^6$.
$\therefore$ $m = 10$ and $n = 6$.

177. We keep on dividing 33333... by 7 till we get 0 as remainder.

```
      47619
   7)333333(
     28
      53
      49
       43
       42
        13
         7
         63
         63
          ×
```

$\therefore$ Required number = 47619

178. We keep on dividing 99999.... by 7 till we get 0 as remainder.

```
     142857
  7)999999(
     7
     29
     28
      19
      14
       59
       56
        39
        35
         49
         49
          ×
```

$\therefore$ Required number = 142857.
Number of digits = 6.

179.
```
        56
  987) 559981(
       4935
        6648
        5922
         7261
         2961
           ×
```

Clearly, 7261 must be replaced by 2961, which is possibvle if 6648 is replaced by 6218, which in turn is possible if 5599 is replaced by 5556.
Thus, the correct number is 555681.

180. Clearly, multiples of 2 and 5 together yield 0.
Since the product of odd numbers contains no power of 2, so the given product does not give 0 at the unit place.

181. Let $N = 1 \times 2 \times 3 \times 4 \times \ldots \times 1000 = 1000!$
Clearly, the highest power of 2 in N is very high as compared to that of 5.
So, the number of zeros in N will be equal to the highest power of 5 in N.
$\therefore$ Required number of zeros

$$= \left[\frac{1000}{5}\right] + \left[\frac{1000}{5^2}\right] + \left[\frac{1000}{5^3}\right] + \left[\frac{1000}{5^4}\right]$$

$= 200 + 40 + 8 + 1 = 249$.

182. Let $N = 10 \times 20 \times 30 \times \ldots \times 1000 = 10^{100} \times (1 \times 2 \times 3 \times 4 \times \ldots 100) = 10^{100} \times 100!$
Number of zeros in 100 ! = Highest power of 5 in 100 !

$= \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right] = 20 + 4 = 24$.

$\therefore$ Number of zeros in $N = 100 + 24 = 124$.

183. Let $N = 5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50$
$= 5^{10} \times (1 \times 2 \times 3 \times 4 \times \ldots \times 10) = 5^{10} \times 10!$

Highest power of 2 in 10! $= \left[\frac{10}{2}\right] + \left[\frac{10}{2^2}\right] + \left[\frac{10}{2^3}\right] = 5 + 2 + 1 = 8$.

Highest power of 5 in 10! $= \left[\frac{10}{5}\right] = 2$.

$\therefore$ $N = 2^8 \times 5^{12} \times k$.
Since highest power of 2 is less than that of 5, so required number of zeros = 8.

184. Clearly, highest power of 2 is much higher as compared to that of 5 in 60!, so
Required number of zeros = Highest power of

$5 = \left[\frac{60}{5}\right] + \left[\frac{60}{5^2}\right] = 12 + 2 = 14$.

185. Let $N = (1 \times 3 \times 5 \times 7 \times \ldots \times 99) \times 128$.
Clearly, N contains 10 multiples of 5 (5, 15, 25, 35, ..., 95) and only one multiple of 2 i.e. 128 or 2^7.
Clearly, highest power of 5 in N is greater than that of 2.
$\therefore$ Number of zeros in N = Highest power of 2 in $N = 7$.

186. $N = 2 \times 4 \times 6 \times 8 \times \ldots \times 98 \times 100$
$= 2^{50} \times (1 \times 2 \times 3 \times \ldots \times 49 \times 50) = 2^{50} \times 50!$
Clearly, the highest power of 2 in N is much higher than that of 5.
$\therefore$ Number of zeros in N = Highest power of 5 in N =

$\left[\frac{50}{5}\right] + \left[\frac{50}{5^2}\right] = 10 + 2 = 12$.

187. Clearly, the list of prime numbers from 2 to 99 has only 1 multiple of 2 and only 1 multiple of 5.
So, number of zeros in the product = 1.

188. Let $N = 3 \times 6 \times 9 \times 12 \times .... \times 102 = 3^{34} \times (1 \times 2 \times 3 \times 4 \times .... \times 34) = 3^{34} \times 34!$
Clearly, highest power of 2 in 34 ! is much greater than that of 5.
So, number of zeros in N = Highest power of 5 in 34! = $\left[\frac{34}{5}\right]+\left[\frac{34}{5^2}\right]=6+1=7.$

189. 3^4 gives unit digit 1. So, $(3^4)^{500}$ gives unit digit 1.
And, 3^3 gives unit digit 7.
$\therefore$ $(13)^{2003}$ gives unit digit = $(1 \times 7) = 7$.

190. 3^4 gives unit digit 1. So, $(3^4)^{24}$ gives unit digit 1.
And, 3^3 gives unit digit 7.
$\therefore$ $3^{99} = (3^4)^{24} \times 3^3$ gives unit digit (1×7) i.e. 7.

191. (A) 7^4 gives unit digit 1. So, $7^{16} = (7^4)^4$ gives unit digit 1.
$\therefore$ $(1827)^{16}$ gives unit digit 1. So, $A \to (1)$.
(B) 3^4 gives unit digit 1. So, $(3^4)^4$ gives unit digit 1.
$\therefore$ $3^{19} = 3^{16} \times 3^3$ gives unit digit = $(1 \times 7) = 7$.
$\therefore$ $(2153)^{19}$ gives unit digit 7. So, $B \to (4)$.
(C) 9^2 gives unit digit 1. So, $(9^2)^{10}$ gives unit digit 1.
$\therefore$ $9^{21} = (9^{20} \times 9)$ gives unit digit = $(1 \times 9) = 9$.
$\therefore$ $(5129)^{21}$ gives unit digit = 9. So, $C \to (5)$.
Hence, $\begin{matrix} A & B & C \\ 1 & 4 & 5 \end{matrix}$ is the correct result.

192. Unit digit of $(67)^{25}$ = Unit digit of 7^{25}.
Unit digit of 7^4 is 1 and so the unit digit of $(7^4)^6$ is 1.
$\therefore$ Unit digit of $7^{25} = (1 \times 7) = 7$.
Hence, the unit digit of $(7^{25} - 1)$ is $(7 - 1) = 6$.

193. Unit digit in the given product = Unit digit of $(4 \times 8 \times 7 \times 3)$, which is 2.

194. Unit digit in the given product = 8.
Unit digit of $(9 \times 6 \times x \times 4)$ is 8. So, $x = 3$.

195. Unit digit of 7^4 is 1. So, the unit digit of $(7^4)^{23}$ is 1.
$\therefore$ Unit digit of 7^{95} = Unit digit of $(7^{92} \times 7^3) = (1 \times 3) = 3$.
Unit digit of 3^4 is 1. So, the unit digit of $(3^4)^{14}$ is 1.
$\therefore$ Unit digit of 3^{58} = Unit digit of $(3^{56} \times 3^2) = (1 \times 9) = 9$.
Hence, the unit digit of $(7^{95} - 3^{58}) = (13 - 9) = 4$.

196. Unit digit of 4^2 is 6. So, unit digit in $(4^2)^{63}$ is 6.
$\therefore$ Unit digit of $(784)^{126}$ = Unit digit in 4^{126}, which is 6.
Unit digit in 4^{127} = Unit digit in $(4^{126} \times 4)$ = Unit digit in (6×4), which is 4.
$\therefore$ Unit digit in $(784)^{127}$ is 4.
Hence, unit digit of $\{(784)^{126} + (784)^{127}\}$ = Unit digit of $(6 + 4)$ = Unit digit of 10, which is 0.

197. Unit digit in $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + 259]$ = Unit digit in $(1 + 1 - 6 + 5 - 6 + 9) = 4$.

198. Unit digit in the given product
= Unit digit in $(4^{1793} \times 5^{317} \times 1^{491})$
Unit digit in 4^2 is 6 and so the unit digit in $(4^2)^{896}$ is 6.
$\therefore$ Unit digit of 4^{1793} = Unit digit in (6×4) = Unit digit in 24, which is 4.
Unit digit in 5^{317} is 5 and the unit digit in 1^{491} is 1.
$\therefore$ Unit digit in the given product = Unit digit in $(4 \times 5 \times 1)$, which is 0.

199. Let $x = 2y$. Then, $x^{4n} = (2y)^{4n} = \{(2y)^4\}^n = (16\ y^4)^n$.
y = 1, 2, 3, 4, 5, 6, 7, 8, 9 gives unit digit as 6 in $(16y^4)^n$.
But, $y = 5$, gives unit digit 0 in $(16\ y^4)^n$.
Hence, the unit digit is 0 or 6.

200. In 5^n we have 5 as unit digit and in 6^m we have 6 as unit digit.
$\therefore$ Unit digit in $(5^n + 6^m)$ = Unit digit in $(5 + 6)$ = Unit digit in 11 = 1.

201. We have: $(2^{12n} - 6^{4n}) = (2^{12n} - 2^{4n} \times 3^{4n}) = 2^{4n}\ (2^{8n} - 3^{4n})$.
Putting $n = 1$, we get the number $2^4\ (2^8 - 3^4)$
$= 16\ (256 - 81) = (16 \times 175) = 2800$
Hence, the number formed by last two digits is 00.

202. $\left(\frac{1}{5}\right)^{2000} = (0.2)^{2000}$.
Last digit of $(0.2)^{2000}$ = Last digit of $(0.2)^4 = 6$.

203. Sum of digits in the two numbers = 19 + 15 = 34.
So, the product will have 33 or 34 digits.
Since $36 \times 34 = 1224$ (i.e. product has 2 + 2 = 4 digits), so the number of digits in x is 34.

204. We know that a number having x,y,z as its digits, is a multiple of 11, if $z + x - y = 0$
Hence, $y = z + x$.

205. $6135n2$ is divisible by 9 if $(6 + 1 + 3 + 5 + n + 2) = (17 + n)$ is divisible by 9.
This happens when the least value of n is 1.

206. In 978626, we have $(6 + 6 + 7) - (2 + 8 + 9) = 0$.
Hence, 978626 is completely divisible by 11.

207. Sum of all digits = 12, which is divisible by 3. So, the given number is divisible by 3.
(Sum of digits at odd places) – (Sum of digits at even places) = 6 – 6 = 0.
So, the given number is divisible by 11.
The given number when divided by 37 gives 3003003003.
So, the given number is divisible by 37.
The given number when divided by 111 gives 1001001001.
Clearly, it is divisible by 111 as well as by 1001.
Hence, the given number is divisible by each one of 3, 11, 37, 111 and 1001.

208. We have $18 = 2 \times 9$, where 2 and 9 are co-primes.
But, 65043 is not divisible by 2. So, it is not divisible by 18.

209. Let the missing digit be x. Then, $(80 + x)$ must be divisible by 4. Hence, $x = 4$.

210. Sum of the digits in 4006020 is 12, which is divisible by 3. Hence, 4006020 is divisible by 3.

211. Clearly, (d) is true.

212. For 4823718 we have
$(8 + 7 + 2 + 4) - (1 + 3 + 8) = (21 - 12) = 9$, which is not a multiple of 11.
$\therefore$ 4823718 is not divisible by 11.
Consider the number 4832718.
We have $(8 + 7 + 3 + 4) - (1 + 2 + 8) = (22 - 11) = 11$, which is a multiple of 11.
Hence, 4832718 is divisible by 11.

213. Consider the number 17325.
Its unit digit is 5. So, it is divisible by 5.
Sum of its digits = $(5 + 2 + 3 + 7 + 1) = 18$, which is divisible by 3.
So, the given number is divisible by 3.
And, since 5 and 3 are co-primes, so the given number is divisible by (5×3), i.e. 15.

214. Given number is 7386038.
Sum of its digits = 35, which is not divisible by any of 3 and 9.
So, the given number is not divisible by any of 3 and 9.
Also, 38 is not divisible by 4. So, the given number is not divisible by 4.

Also, $(8 + 0 + 8 + 7) - (3 + 6 + 3) = (23 - 12) = 11$.
So, the given number is divisible by 11.

215. $(2 + 4 + 9 + 8 + 4) = 27$, $(2 + 6 + 7 + 8 + 4) = 27$ and $(2 + 8 + 5 + 8 + 4) = 27$.
So, each one is divisible by 3 and 9 both.
Also, 84 is divisible by 4. So, each one is divisible by 4.
Hence, each given number is divisible by 3, 9 and 4.
So, the statements 1, 2 and 3 are all correct.

216. We know that 872 is divisible by 8. Hence 923872 is divisible by 8.

217. Here $(5 + 9 + x + 7) - (4 + 3 + 8) = 6 + x$. So, we must have $x = 5$.

218. Take $m = 15$ and $n = 20$. Then, each one of m and n is divisible by 5. But, $(m + n)$ is not divisible by 10.
Hence, $(m + n)$ is divisible by 10 is not true.

219. An integer is divisible by 16, if the number formed by last 4 digits is divisible by 16.

220. Clearly, 639 is not divisible by 7.
Consider 2079. Sum of its digits $= (2 + 0 + 7 + 9) = 18$.
So, it is divisible by both 3 and 9.
Also, $(79 - 2) = 77$, which is divisible by 7.
So, 2079 is divisible by 7.
Also, $(9 + 0) - (7 + 2) = 0$. So, 2079 is divisible by 11.
Hence, 2079 is divisible by each one of 3, 7, 9 and 11.

221. Let the given number be $476xy0$. Then
$(0 + x + 7) - (y + 6 + 4) = 0 \Rightarrow x - y - 3 = 0 \Rightarrow x - y = 3$.
Also, $(4 + 7 + 6 + x + y + 0) = (17 + x + y)$ must be divisible by 3.
Since $x \neq 0$, $y \neq 0$, so $x + y \neq 1$.
$\therefore\ x + y = 4$ or 7 or 10 etc.
$(x + y = 4$ and $x - y = 3) \Rightarrow x = 7/2$, which is not admissible.
$(x + y = 7$ and $x - y = 3) \Rightarrow x = 5$ and $y = 2$.

222. Sum of the digits in respective numbers is:
9, 12, 18, 9, 21, 12, 18, 21, 15 and 24.
Out of these 12, 21, 12, 21, 15, 24 are divisible by 3 but not by 9.
So, the number of required numbers is 6.

223. Let the unit's place be x and the thousand's place be y.
Then, $357y25x$ is divisible by 5 only when $x = 0$ or $x = 5$.
Also, this number is divisible by 3 only when sum of its digits is divisible by 3.
So, $(22 + x + y)$ must be divisible by 3.
$\therefore\ x + y = 2$
Taking $x = 0$, we get $y = 2$.
So, the unit place = 0 and thousand's place = 2.

224. Clearly, $(7 + 8) - (9 + 6) = 0$. So, 6897 is divisible by 11.
Also, $\frac{6897}{19} = 363$. So, 6897 is divisible by 19.
Hence, 6897 is divisible by both 11 and 19.

225. We have $24 = 3 \times 8$, where 3 and 8 are co-primes.
Clearly, 718 is not divisible by 8. So, 35718 is not divisible by 8.
810 is not divisible by 8. So, 63810 is not divisible by 8.
804 is not divisible by 8. So, 537804 is not divisible by 8.
736 is divisible by 8. So, 3125736 is divisible by 8.
Also, sum of its digits $= (3 + 1 + 2 + 5 + 7 + 3 + 6) = 27$, which is divisible by 3.
So, 3125736 is divisible by 3 also.
Hence, it is divisible by 24.

226. Sum of the digits of the given number $= (7 \times 3) + (14 \times 1) = (21 + 14) = 35$, which is not divisible by 3.
So the given number is not divisible by 3.
Also, $(1 + 3 + 1 + 1 + 3 + 1 + 1 + 3 + 1 + 1 + 3) - (1 + 1 + 3 + 1 + 1 + 3 + 1 + 1 + 3 + 1) = (19 - 16) = 3$, which is neither 0 nor divisible by 11.
So, the given number is not divisible by 11 also.
Hence, it is divisible by neither 3 nor 11.

227. $(325 - 325) = 0$, which is divisible by 7.
So, the given number is divisible by 7.
$(5 + 3 + 2) - (2 + 5 + 3) = 0$. So, the given number is divisible by 11.
And, $\frac{325325}{13} = 25025$. So, 325325 is divisible by 13.
Hence, it is divisible by all 7, 11 and 13.

228. We first divide the number into groups of 3 digits from the right $\to$ 3 0X0 103
Difference of sum of numbers at odd and even places = $(103 + 3) - 0X0 = 106 - 0X0$, which must be divisible by 13.
$106 - 0X0$ is divisible by 13 only for $X = 8$.

229. We know that 11 and 13 are co-prime.
So, a number divisible by both 11 and 13 will be divisible by (11×13).

230. I. We have$(195 - 195)$ 0
$\therefore$ 195195 is divisible by 7.
II. We have $(181 - 181) = 0$
$\therefore$ 181181 is divisible by 7.
III. We have $(120 - 120) = 0$
$\therefore$ 120120 is divisible by 7.
IV. We have $(891 - 891) = 0$
$\therefore$ 891891 is divisible by 7.
Hence, all are divisible by 7.

231. Since $653xy$ is divisible by 80, we must have $y = 0$.
Now, $653x0$ must be divisible by both 5 and 16.
Clearly, it is divisible by 5 for all values of x.
Now, the number $53x0$ must be divisible by 16.
The least value of x is clearly 6. So, $x + y = 6 + 0 = 6$.

232. We know that $33 = 11 \times 3$, where 11 and 3 are co-primes.
So, the given number must be divisible by both 11 and 3.
Since $5ABB7A$ is divisible by 11, we have
$(A + B + A) - (7 + B + 5) = (2A - 12)$ is either 0 or 11.
$\Rightarrow\quad 2A - 12 = 0$ or $2A - 12 = 11 \Rightarrow A = 6 \quad \left[\because A \neq \frac{23}{2}\right]$
So, the number becomes $56BB76$, which is divisible by 3.
$\therefore\ (5 + 6 + B + B + 7 + 6) = (24 + 2B)$ must be divisible by 3.
$\therefore\ 2B = 6 \Rightarrow B = 3 \qquad \left[\because B \neq 0 \text{ and } B \neq \frac{3}{2}\right]$
Hence, $(A + B) = (6 + 3) = 9$.

233. We have, $99 = (11 \times 9)$, where 11 and 9 are co-primes.
Consider the number 114345.
Clearly, $(5 + 3 + 1) - (4 + 4 + 1) = 0$. So, 114345 is divisible by 11.
Also, sum of its digits $= (1 + 1 + 4 + 3 + 4 + 5) = 18$, which is divisible by 9.
$\therefore$ 114345 is divisible by 9.
Hence, it is divisible by (11×9), i.e. 99.

234. Let the unit's digit be x and ten's digit be y. Then, the number is $3422213yx$.
Also, $99 = (11 \times 9)$, where 11 and 9 are co-primes.
Since the given number is divisible by 9, it follows that $(3 + 4 + 2 + 2 + 2 + 1 + 3 + y + x) = (17 + y + x)$ must be divisible by 9.
So, $y + x = 1$ or $y + x = 10$.

Again, the given number is divisible by 11.
So, $(x + 3 + 2 + 2 + 3) - (y + 1 + 2 + 4) = x - y + 3$ is either 0 or 11.
$\therefore$ $(x - y + 3 = 0$ or $x - y + 3 = 11) \Rightarrow (y - x = 3$ or $x - y = 8)$
Now, $(y + x = 1$ and $y - x = 3) \Rightarrow y = 2$ and $x = -1$.

$(y + x = 1$ and $x - y = 8) \Rightarrow x = \frac{9}{2}$.

$(y + x = 10$ and $y - x = 3) \Rightarrow y = \frac{13}{2}$.

$(y + x = 10$ and $x - y = 8) \Rightarrow x = 9$ and $y = 1$.

Thus, $x = 9$, $y = 1$. So, the required number is 342221319.

235. 37×3 is divisible by 7
$\Rightarrow$ $(7 \times 3 - 3)$ is either 0 or divisible by 7)
$\Rightarrow$ 7×0 is divisible by 7.
$\Rightarrow$ X = 0 or X = 7.

236. $225 = 9 \times 25$, where 9 and 25 are co-primes.
So, a number is divisible by 225 if it is divisible by both 9 and 25.
Given number is divisible by 25, only if $7q$ is divisible by 25, i.e. if $q = 5$.
Sum of digits of given number $= (8 + 7 + 6 + p + 3 + 7 + 5) = 36 + p$, which must be divisible by 9.
This is possible if $p = 0$.
Hence, $p = 0$, $q = 5$.

237. Since $774958A96B$ is divisible by 8, so the number $96B$ must be divisible by 8. So, $B = 0$, as 960 is divisible by 8. Now, $774958A960$ is divisible by 9. So, $(55 + A)$ must be divisible by 9.
This happens when $A = 8$. Hence, $A = 8$ and $B = 0$.

238. $132 = 11 \times 3 \times 4$. So, the required number must be divisible by 3, 4 and 11.
264 $\rightarrow$ divisible by 3, 4 and 11.
396 $\rightarrow$ divisible by 3, 4 and 11.
462 $\rightarrow$ not divisible by 4.
792 $\rightarrow$ divisible by 3, 4 and 11.
968 $\rightarrow$ not divisible by 3
2178 $\rightarrow$ not divisible by 4
5184 $\rightarrow$ not divisible by 11.
6336 $\rightarrow$ divisible by 3, 4 and 11.
Hence, out of the given numbers 4 are divisible by 3, 4 and 11.

239. Let $3x + 7y = 11k$. Then, $y = \frac{(11k - 3x)}{7}$.

Then, $5x - 3y = 5x - \frac{3(11k - 3x)}{7} = \frac{35x - 33k + 9x}{7}$

$= \frac{44x - 33k}{7} = \frac{11(4x - 3k)}{7}$, which is divisible by 11.

240. $(1^3 - 1) = 0$, $(2^3 - 2) = 6$, $(3^3 - 3) = 24$, $(4^3 - 4) = 60$ and so on, each one of which is divisible by 6.

241. Let $a = 2k + 1$ & $b = 2m + 1$. Then,
$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a + b)(a - b)(a^2 + b^2)$
$= (2k + 2m + 2)(2k - 2m)(4k^2 + 4m^2 + 2 + 4k + 4m)$
$= 8(k + m + 1)(k - m)(2k^2 + 2m^2 + 1 + 2k + 2m)$,
which is divisible by 8.

242. Let the two consecutive integers be a and $(a + 1)$. Then, $(a + 1)^2 - a^2 = a^2 + 1 + 2a - a^2 = (2a + 1) = (a + a + 1) =$ sum of given integers.

243. $6n^2 + 6n = 6n(n + 1)$
$(n = 1 \Rightarrow 6n^2 + 6n = 12)$, $(n = 2 \Rightarrow 6n^2 + 6n = 36)$, $(n = 3 \Rightarrow 6n^2 + 6n = 72)$,
each one of which is divisible by 6 and 12 both.

244. Let the ten's digit be x and the unit's digit be y. Then, $(10x + y) - (10y + x) = 9x - 9y = 9(x - y)$, which is divisible by 9.

245. Let the ten's digit be x and the unit's digit be y.
Then, $(10x + y) + (10y + x) = 11(x + y)$, which is divisible by 11.

246. Let $P = n(n + 1)(n + 2)(n + 3)$. Then, $n = 1$ gives $P = (1 \times 2 \times 3 \times 4) = 24$.
Hence, the required number is 24.

247. Let $N = n^2(n^2 - 1) = n^2(n - 1)(n + 1)$.
Then, $n = 2 \Rightarrow N = 2^2 \times (2 - 1) \times (2 + 1) = (4 \times 1 \times 3) = 12$.
Hence, the required number is 12.

248. Let $n = (2m + 1)$. Then,
$N = n(n^2 - 1) = n(n - 1)(n + 1) = (2m + 1)(2m)(2m + 2) = 4m(m + 1)(2m + 1)$.
Now, $m = 1 \Rightarrow N = (4 \times 1 \times 2 \times 3) = 24$.
So, N is always divisible by 24.

249. Let the hundreds, tens and unit digits be x, y and z respectively.
Then, $(100x + 10y + z) - (x + y + z) = 99x + 9y = 9(11x + y)$.
So, the resulting number is divisible by 9.

250. Let the required number be x. Then $\frac{11x + 11}{13} =$ a whole number.
So, $(11x + 11)$ must be divisible by 13. By hit and trial, we get $x = 12$.
Hence, the smallest original number is 12.

251. Let the required product be $n(n + 1)(n + 2)$. Then,
$n = 1 \Rightarrow n(n + 1)(n + 2) = (1 \times 2 \times 3) = 6$.
$n = 2 \Rightarrow n(n + 1)(n + 2) = (2 \times 3 \times 4) = 24$.
$n = 3 \Rightarrow n(n + 1)(n + 2) = (3 \times 4 \times 5) = 60$.
So, each such product is divisible by 6.

252. Let the required odd numbers be n, $(n + 2)$ and $(n + 4)$.
Then $n + (n + 2) + (n + 4) = 3n + 6 = 3(n + 2)$, which is always divisible by 3.
But $n = 1 \Rightarrow 3(n + 2) = 3 \times 3 = 9$ which is not divisible by any of 2, 5 and 6.
Hence only II is true.

253. Three consecutive multiples of 3 are $3m$, $3(m + 1)$ and $3(m + 2)$.
Their product $= 3m \times 3(m + 1) \times 3(m + 2) = 27 \times m \times (m + 1) \times (m + 2)$.
Putting $m = 1$, this product is $(27 \times 1 \times 2 \times 3) = 162$.
So, this product is always divisible by 162.

254. $p = 5 \Rightarrow (p^2 - 1) = (25 - 1) = 24$, which is divisible by 24.
$p = 7 \Rightarrow (p^2 - 1) = (49 - 1) = 48$, which is divisible by 24.
$p = 11 \Rightarrow (p^2 - 1) = (121 - 1) = 120$, which is divisible by 24.
Hence, $(p^2 - 1)$ is always divisible by 24.

255. Let the two consecutive odd integers be $(2m + 1)$ and $(2m + 3)$.
Then, $(2m + 3)^2 - (2m + 1)^2 = [(2m + 3) + (2m + 1)][(2m + 3) - (2m + 1)] = (4m + 4) \times 2 = 8m + 8 = 8(m + 1)$, which is always divisible by 8.

256. Clearly, 2525 is not divisible by any of the numbers 7, 11 and 13.
The smallest 3-digit prime number is 101.

```
      25             32
101)2525(       101)3232(
    202             303
     505             202
     505             202
      ×               ×
```

Hence (d) is true.

257. Numbers like 2525, 3636 etc. are divisible by 101.
Numbers like 256256, 678678 etc. are divisible by 1001.
Numbers like 32163216, 43754375 etc. are divisible by 10001 and so on.

258. 10^n has $(n + 1)$ digits. Then, 9 will appear n times in $(10^n - 1)$.
So, sum of digits in $(10^n - 1) = 9n$.
$\therefore\ 9n = 4707 \Rightarrow n = \frac{4707}{9} = 523.$

259. We know that $(x^n - a^n)$ is always divisible by $(x - a)$ for all values of n.

260. The product of 8 consecutive numbers is divisible by each one of 8!, 7!, 6!, 5!, 4!, 3! and 2!

261. We know that $(x^n - a^n)$ is divisible by $(x - a)$, when n is even and $(x^n - a^n)$ is divisible by $(x + a)$, when n is even.
$\therefore$ $(10^n - 1)$ is divisible by $(10 - 1) = 9$, when n is even.
And, $(10^n - 1)$ is divisible by $(10 + 1) = 11$, when n is even.
$\therefore$ Statements 2 and 4 are correct.

262. Putting $n = 1$, we get $(3^{4n} - 4^{3n}) = (3^4 - 4^3) = (81 - 64) = 17$, which is divisible by 17.

263. Let the odd natural number be $(2n + 1)$.
$n = 1$ gives $(2n + 1)^2 = (2 + 1)^2 = 9$.
This when divided by 8 gives 1 as remainder.
$n = 2$ gives $(2n + 1)^2 = 25$.
This when divided by 8 gives 1 as remainder and so on.

264. Required number $= (2^5 - 2) = (32 - 2) = 30$.

265. Two consecutive even integers are $2n$ and $(2n + 2)$.
$\therefore\ (2n + 2)^2 - (2n)^2 = (4n^2 + 4 + 8n - 4n^2) = 4 + 8n = 4(1 + 2n)$, which is always divisible by 4.

266. Two consecutive odd integers are $(2m + 1)$ and $(2m + 3)$.
$\therefore\ (2m + 3)^2 - (2m + 1)^2 = (2m + 3 + 2m + 1)\ (2m + 3 - 2m - 1) = (4m + 4) \times 2 = 8(m + 1)$, which is always divisible by 8.

267. The smallest 4-digit number is 1000.
This when divided by 7 leaves 6 as remainder.
$\therefore$ 1001 is the smallest 4-digit number exactly divisible by 7.

268. On dividing 1056 by 23, we get 21 as remainder.
$\therefore$ Required number to be added $= (23 - 21) = 2$.

```
23)1056(45
    92
    136
    115
     21
```

269. On dividing 8567 by 4, the remainder is 3.
To make it divisible by 4, we must add 1 to it.

270. The least 6-digit number = 100000.
Required number = 100000 + (349 – 186) = 100000 + 163 = 100163.

```
349)100000(286
     698
     3020
     2792
      2280
      2094
       186
```

271. The greatest 5-digit number = 99999.
On dividing 99999 by 279, we get 117 as remainder.
$\therefore$ Required number = (99999 – 117) = 99882.

```
279)99999(358
    837
    1629
    1395
     2349
     2232
      117
```

272. The greatest 6-digit number is 999999.
Required number to be added = (327 – 33) = 294.

```
327)999999(3058
    981
     1899
     1635
      2649
      2616
        33
```

273. On dividing 5000 by 73, we get
Required number = 5000 + (73 – 36) = 5037.

```
73)5000(68
   438
    620
    584
     36
```

274. On dividing 58701 by 567, we get 300 as remainder.
$\therefore$ Required number = 58701 + (567 – 300) = 58701 + 267 = 58968.

```
567)58701(103
    567
     2001
     1701
      300
```

275. On dividing 8112 by 99, we get 93 as remainder.
So, the required number to be subtracted is 93.

```
99)8112(81
   792
    192
     99
     93
```

276. On dividing 803642 by 11, we get 4 as remainder.
Required number to be added = (11 – 4) = 7.

277. On dividing 1111 by 99, the quotient is 11 and the remainder is 22.
Hence, the required number is 11.

```
99)1111( 11
   99
    121
     99
     22
```

278. $66 = 11 \times 6$
In order to get a number divisible by 18, the above product must be multiplied by 3.
Hence 66 must be multiplied by 3.

279. The smallest 6-digit number is 100000. On dividing 100000 by 111, we get 100 as remainder.
So, the number to be added = (111 – 100) = 11.
Hence, the required number = 100011.

```
111)100000(900
     999
      100
```

280. All 2-digit numbers divisible by 5 are 10, 15, 20, 25, ..., 95.
This is an *A.P.* in which $a = 10$, $d = 5$ and $T_n = 95$.
$T_n = 95 \Rightarrow a + (n - 1)\,d = 95 \Rightarrow 10 + (n - 1) \times 5 = 95$
$\Rightarrow (n - 1) = \frac{85}{5} = 17 \;\Rightarrow n = 18.$
$\therefore$ Sum $= \frac{n}{2}(a + l) = \frac{18}{2}(10 + 95) = (9 \times 105) = 945.$

281. 3-digit numbers divisible by 6 are 102, 108, 114, ..., 996.
This is an A.P. in which $a = 102$, $d = 6$ and $T_n = 996$.
$\therefore T_n = a + (n - 1)\,d \Rightarrow 102 + (n - 1) \times 6 = 996 \Rightarrow (n - 1) \times 6 = 894$
$\Rightarrow (n - 1) = 149 \Rightarrow n = 150.$
Hence, there are 150 such numbers.

282. Multiples of 7 between 11 and 200 are 14, 21, 28, 35, 42, ..., 189, 196.
$T_m = 196 \Rightarrow 14 + (m - 1) \times 7 = 196 \Rightarrow (m - 1) \times 7 = 182$
$\Rightarrow (m - 1) = 26 \Rightarrow m = 27.$
Multiples of 7 and 3 both, i.e. that of 21 are 21, 42, 63, ..., 189
$T_n = 189 \Rightarrow 21 + (n - 1) \times 21 = 189 \Rightarrow (n - 1) \times 21 = 168$
$\Rightarrow (n - 1) = 8 \Rightarrow n = 9.$
$\therefore$ Required number of terms $= (27 - 9) = 18.$

283. Numbers between 14 and 95 and divisible by 3 are 15, 18, 21 24, ..., 93.
$T_m = 93 \Rightarrow 15 + (n - 1) \times 3 = 93 \Rightarrow (n - 1) \times 3 = 78$
$\Rightarrow (n - 1) = 26 \Rightarrow n = 27.$
Numbers to be deleted are 33, 63, 93.
They are 3 in number.
Required number of numbers $= (27 - 3) = 24.$

284. Required numbers are multiples of (10×13), i.e. 130.
These numbers are 130, 260, 390, 520, 650, 780 and 910. They are 7 in number.

285. Number of integers between 100 and 150 (including both) = 51.
Numbers divisible by 3 are 102, 105, 108, ..., 150.
Let $T_m = 150$. Then, $a + (m - 1)\,d = T_m$.
$\therefore 102 + (m - 1) \times 3 = 150 \Rightarrow (m - 1) \times 3 = 48 \Rightarrow m - 1 = 16 \Rightarrow m = 17.$
Numbers divisible by 5 are 100, 105, 110, 115, ..., 150.
Let $T_n = 150$. Then, $a + (n - 1)\,d = T_n$.
$\therefore 100 + (n - 1) \times 5 = 150 \Rightarrow (n - 1) \times 5 = 50 \Rightarrow (n - 1) = 10 \Rightarrow n = 11.$
Numbers divisible by both 3 and 5 are 105, 120, 135, 150. They are four in number.
Number of numbers divisible by 3 or 5 = (17 + 11 – 4) = 24.
Number of numbers neither divisible by 3 nor by 5 are (51 – 24) = 27.

286. Numbers from 501 to 599 which have 6 as digit are 506, 516, 526, 536, 546, 556, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 576, 586 and 596, i.e. 6 occurs 20 times.
Number of times 6 occurs from 600 to 699 = 100 + 20 = 120.
$\therefore$ Total number of times 6 occurs = 20 + 120 = 140.

287. Such numbers are 202, 212, 222, 232, 242, 252, 262, 272, 282, 292. There are 10 such numbers.

288. Such numbers are 203, 213, 223, 233, 243, 253, 263, 273, 283, 293 and all numbers from 300 to 399. Clearly, number of such numbers = 10 + 100 = 110.

289. When the second digit is 1, third digit can be 0, i.e. there is one such number.
When the second digit is 2, third digit can be 0 or 1, i.e. there are 2 such numbers.
When the second digit is 3, third digit can be 0, 1 or 2 i.e. there are 3 such numbers, and so on.
When the first digit is 7, second digit can be 1, 2, 3, 4, 5 or 6. So, there are
(1 + 2 + 3 + 4 + 5 + 6) = 21 such numbers between 700 and 799.
When the first digit is 8, second digit can be 1, 2, 3, 4, 5, 6 or 7. So, there are
(1 + 2 + 3 + 4 + 5 + 6 + 7) = 28 such numbers between 800 and 899.
When the first digit is 9, second digit can be 1, 2, 3, 4, 5, 6, 7 or 8. So, there are
(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 36 such numbers between 900 and 999.
Hence, the required number = (21 + 28 + 36) = 85.

290. (*a*) In 183654729, 1836547 is not divisible by 7.
(*b*) In 381654729, 38 is divisible by 2, 381 is divisible by 3, 3816 is divisible by 4, 38165 is divisible by 5, 381654 is divisible by 6, 3816547 is divisible by 7, 38165472 is divisible by 8 and 381654729 is divisible by 9.
(*c*) In 983654721, 983 is not divisible by 3.
(*d*) In 981654723, 9816547 is not divisible by 7.

291. On dividing 11109999 by 1111, we get:
So, the required remainder is 1110.

```
1111)11109999(9999
      9999
      11109
       9999
       11109
        9999
        11109
         9999
         1110
```

292. Number = (68 × 260) = 17680. On dividing this number by 65, we get zero as remainder.

```
65)17680(272
   130
    468
    455
     130
     130
      ×
```

293. (2176 – 9) = 2167 = (11 × 197).
So, the required number is 197.

294. (*a*)
```
–7)–112(16
   –112
      0
```
(*b*)
```
–9)118(–13
    9
    28
    27
     1
```
(*c*)
```
6)–109(–19
   –6
   – 49
   – 54
      5
```
(*d*)
```
8)115(14
   8
   35
   32
    3
```

$q = 16, r = 0$ $\quad$ $q = -13, r = 1.$
$\therefore A \to 4$ $\quad$ $\therefore B \to 1$

$q = -19, r = 5$ $\quad$ $q = 14, r = 3$
$\therefore C \to 3$ $\quad$ $\therefore D \to 2$

295. On dividing 534677 by 777 we get 101 as remainder.
$\therefore$ (Divisor) – (Remainder) = (777 – 101) = 676.

```
777) 534677 (688
     4662
      6847
      6216
       6317
       6216
        101
```

296. $\dfrac{\text{(Dividend)} - \text{(Remainder)}}{\text{Quotient}} = \text{Divisor.}$

$\therefore$ Divisor $= \dfrac{(940-25)}{15} = \dfrac{915}{15} = 61.$

297. Remainder = 48, Divisor = (5 × 48) = 240.

12 × Quotient = Divisor $\Rightarrow$ Quotient $= \dfrac{240}{12} = 20.$

Dividend = (240 × 20) + 48 = 4848.

298. Quotient = 16, Divisor = (25 × 16) = 400.

5 × Remainder = Divisor $\Rightarrow$ Remainder $= \dfrac{400}{5} = 80$.

Dividend = (400 × 16) + 80 = 6480.

299. Divisor = (555 + 445) = 1000, Quotient = 2(555 – 445) = 2 × 110 = 220 and Remainder = 30.
$\therefore$ Required number = (1000 × 220) + 30 = 220000 + 30 = 220030.

300. Divisor taken = 12, Quotient obtained = 35, Remainder = 0.
$\therefore$ Dividend = (12 × 35) = 420.
Now, dividend = 420, divisor = 21, remainder = 0.
$\therefore$ Quotient $= \dfrac{420}{21} = 20$.

301. Divisor = 7 × quotient = 5 × remainder and Dividend = 6 × remainder.

```
5x) 6x (1
    5x
     x
```

Let remainder be x. Then, divisor = $5x$ and dividend = $6x$.
On dividing $6x$ by $5x$, we get 1 as quotient and x as remainder.
$\therefore$ Quotient = 1.

302. Since the required number is a 3-digit number, so on dividing by 19, it would yield a two-digit quotient which means that the quotient is greater than the remainder.
Let the remainder be x. Then, quotient = $x + 9$.
So, number $N = 19(x + 9) + x = 20x + 171$.
$\therefore$ $(N - 171)$ must be divisible by 20.
Clearly, (371 – 171) = 200, which is divisible by 20.
Hence, the required number = 371.

303. Let the number be x. Let x when divided by 136 give q as quotient and 36 as remainder. Then, $x = 136\,q + 36 = (17 \times 8q) + (17 \times 2) + 2 = 17 \times (8q + 2) + 2.$
So, the given number when divided by 17 gives 2 as remainder.

304. Let the number be x and the quotient be q.
Then, $x = 195\,q + 47 = (15 \times 13\,q) + (15 \times 3) + 2 = 15(13q + 3) + 2.$
So, the given number when divided by 15 gives 2 as remainder.

305. Let the number be x and the quotient be q.
Then, $x = 899\,q + 63 = (29 \times 31q) + (29 \times 2) + 5 = 29(31q + 2) + 5.$
So, the given number when divided by 29 gives 5 as remainder.

306. Let the number be x and on dividing by 5, we get q as quotient and 3 as remainder.
Then, $x = 5q + 3 \Rightarrow x^2 = (5q + 3)^2 = (25q^2 + 30q + 9)$
$= 5(5q^2 + 6q + 1) + 4.$
Thus, on dividing x^2 by 5, we get 4 as remainder.

307. Let the smaller number be x. Then, larger number $= (x + 1365).$
$\therefore$ $x + 1365 = 6x + 15 \Rightarrow 5x = 1350 \Rightarrow x = 270.$
Hence, the smaller number = 270.

308. Let $n = 4k + 3$.
Then, $2n = 2(4k + 3) = 8k + 6 = 4 \times 2k + 4 \times 1 + 2 = 4(2k + 1) + 2.$
Thus, on dividing $2n$ by 4, we get 2 as remainder.

309. Let $x = 13p + 11$ and $x = 17q + 9$.
Then, $13p + 11 = 17q + 9 \Rightarrow 17q - 13p = 2 \Rightarrow q = \dfrac{2 + 13p}{17}$
The least value of p for which $q = \dfrac{2 + 13p}{17}$ is a whole number, is $p = 26$.
$\therefore$ $x = (13 \times 26 + 11) = 338 + 11 = 349.$

310. Let the dividend be $(x + 71)$ and the divisor be y.
Then, $[2(x + 71) - 43]$ is divisible by $y \Rightarrow (2x + 142 - 43)$ is divisible by y
$\Rightarrow (2x + 99)$ is divisible by y.
$\therefore$ Divisor = 99

Shortcut Method:
Divisor = (2 × 71 – 43) = (142 – 43) = 99.

311. Let $P = x + r_1$ and $Q = y + r_2$, where each of x and y are divisible by the common divisor.
Then, $P + Q = (x + r_1) + (y + r_2) = (x + y) + (r_1 + r_2).$
$(P + Q)$ leaves remainder r_3 when divided by the common divisor.
$\Rightarrow [(x + y) + (r_1 + r_2) - r_3]$ is divisible by the common divisor.
Since $(x + y)$ is divisible by the common divisor, so divisor $= r_1 + r_2 - r_3$.

312. As proved in the above question, divisor = 4375 + 2986 – 2361 = 5000.

313. The number is of the form $(13k + 1)$, where k is of the form $(5m + 3)$.
$\therefore$ Number $= 13k + 1 = 13(5m + 3) + 1 = 65m + 40.$
Clearly when the number is divided by 65, we get 40 as remainder.

314. Clearly, (2272 – 875) = 1397, is exactly divisible by N.
Now, 1397 = 11 × 127.
$\therefore$ The required 3-digit number is 127, the sum of whose digits is 10.

315.

```
 9 | x
11 | y – 8
13 | z – 9
   | 1 – 8
```

$z = 13 \times 1 + 8 = 21.$
$y = 11 \times z + 9 = 11 \times 21 + 9 = 240.$
$x = 9 \times y + 8 = 9 \times 240 + 8 = 2168.$

Now, when order of divisors is reversed, we have:

$$\begin{array}{r|l} 13 & 2168 \\ \hline 11 & 166 - 10 \\ \hline 9 & 15 - 1 \\ \hline & 1 - 6 \end{array}$$

$\therefore$ Respective remainders are 10, 1 and 6.

316. $$\begin{array}{r|l} 3 & x \\ \hline 4 & y - 2 \\ \hline 7 & z - 1 \\ \hline & 1 - 4 \end{array}$$

$z = 7 \times 1 + 4 = 11.$

$y = 4 \times z + 1 = 4 \times 11 + 1 = 45.$

$x = 3 \times y + 2 = 3 \times 45 + 2 = 137.$

When 137 is divided by 84, the remainder obtained is 53.

317. $$\begin{array}{r|l} 8 & x \\ \hline 7 & y - 3 \\ \hline 3 & z - 4 \\ \hline & 31 - 2 \end{array}$$

$z = 3 \times 31 + 2 = 95.$

$y = 7 \times z + 4 = 7 \times 95 + 4 = 669.$

$x = 8 \times y + 3 = 8 \times 669 + 3 = 5355.$

$\therefore$ Required number = 5355.

318. $$\begin{array}{r|l} 3 & x \\ \hline 2 & y - 1 \\ \hline & 1 - 1 \end{array}$$

$y = 2 \times 1 + 1 = 3$

$x = 3 \times y + 1 = 3 \times 3 + 1 = 10.$

Clearly, 10 when divided by 6, leaves a remainder 4.

319. Let the number be $N = 2x + 1$.

$N^2 = (2x + 1)^2 = 4x^2 + 1 + 4x = 4x\,(x + 1) + 1.$

Clearly, $4x\,(x + 1)$ is always divisible by 8 since one of x and $(x + 1)$ is even which when multiplied by 4 is always divisible by 8.

Hence, required remainder = 1.

320. Sum of digits of numbers from 1 to 10 = 46.

Sum of digits of numbers from 11 to 20 = 56.

Sum of digits of numbers from 21 to 29 = 63.

Sum of digits of the given number = 46 + 56 + 63 = 165.

So, the required remainder is the remainder obtained on dividing 165 by 9, which is 3.

321. When n is even, $(x^n - a^n)$ is divisible by $(x + a)$.

$\therefore$ $(17^{200} - 1^{200})$ is divisible by $(17 + 1)$

$\Rightarrow$ $(17^{200} - 1)$ is divisible by 18

$\Rightarrow$ On dividing 17^{200} by 18, we get 1 as remainder.

322. $2^{31} = 2 \times 2^{30} = 2 \times (2^2)^{15} = 2 \times 4^{15}.$

When n is odd, $(x^n + a^n)$ is divisible by $(x + a)$.

$\therefore$ $(4^{15} + 1^{15})$ is divisible by $(4 + 1)$

$\Rightarrow$ $(4^{15} + 1)$ is divisible by 5 $\Rightarrow$ $(2^{30} + 1)$ is divisible by 5

$\Rightarrow$ On dividing 2^{30} by 5, we get $(5 - 1)$ i.e. 4 as remainder.

$\therefore$ Remainder obtained on dividing 2^{31} by 5

= Remainder obtained on dividing (2×4) i.e. 8 by 5 = 3.

323. Clearly,

(1) when n is odd, $(a^n + b^n)$ is divisible by $(a + b)$. So, (1) is true.

(2) $(a^n - b^n)$ is divisible by $(a - b)$ for all values of n. So, (2) is true.

324. $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

$\therefore$ $(7^{19} - 1^{19})$ is divisible by $(7 - 1)$

$\Rightarrow$ $(7^{19} - 1)$ is divisible by 6

$\Rightarrow$ On dividing $(7^{19} + 2)$ by 6, remainder obtained = 3.

325. $(10^{12} + 25)^2 - (10^{12} - 25)^2 = 4 \times 10^{12} \times 25$

$[\because (a + b)^2 - (a - b)^2 = 4ab]$

$= 10^{12} \times 100 = 10^{12} \times 10^2 = 10^{14}.$

Hence, $n = 14$.

326. $(3^{25} + 3^{26} + 3^{27} + 3^{28}) = 3^{25}\,(1 + 3 + 3^2 + 3^3)$

$= 3^{25}\,(1 + 3 + 9 + 27) = 3^{25} \times 40$

$= (3 \times 10) \times (3^{24} \times 4) = 30 \times (3^{24} \times 4),$

which is divisible by 30.

327. $(4^{61} + 4^{62} + 4^{63} + 4^{64}) = 4^{61}\,(1 + 4 + 4^2 + 4^3) = 4^{61} \times 85,$ which is divisible by 17.

328. $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n

$\therefore$ $(9^6 - 1^6)$ is divisible by $(9 - 1)$

$\Rightarrow$ $(9^6 - 1)$ is divisible by 8

$\Rightarrow$ On dividing $(9^6 + 1)$ by 8, we get 2 as remainder.

329. When n is even, $(x^n - a^n)$ is divisible by both $(x - a)$ and $(x + a)$.

So, $(6^n - 1)$ is divisible by both $(6 - 1)$ and $(6 + 1)$

$\Rightarrow$ $(6^n - 1)$ is divisible by both 5 and 7

$\Rightarrow$ $(6^n - 1)$ is divisible by (5×7), i.e. 35.

[$\because$ 5 and 7 are co-primes]

330. $(x^n + a^n)$ is divisible by $(x + a)$ when n is odd.

$\therefore$ $(12^n + 1)$ is divisible by $(12 + 1)$ i.e. 13 when n is odd.

331. $(x^n + a^n)$ is divisible by $(x + a)$ when n is odd.

$\therefore$ $(25^{25} + 1^{25})$ is divisible by $(25 + 1)$

$\Rightarrow$ $(25^{25} + 1)$ is divisible by 26

$\Rightarrow$ On dividing 25^{25} by 26, we get $(26 - 1) = 25$ as remainder.

332. $(x^n + a^n)$ is divisible by $(x + a)$ when n is odd.

$\therefore$ $(67^{67} + 1^{67})$ is divisible by $(67 + 1)$

$\Rightarrow$ $(67^{67} + 1)$ is divisible by 68

$\Rightarrow$ On dividing $(67^{67} + 67)$ by 68, we get $(67 - 1) = 66$ as remainder.

333. $(49^{15} - 1) = (7^2)^{15} - 1 = 7^{30} - 1.$

Now, when n is even, $(x^n - a^n)$ is divisible by both $(x - a)$ and $(x + a)$.

$\therefore$ $(7^{30} - 1)$ is divisible by both $(7 - 1)$ and $(7 + 1)$ i.e. by both 6 and 8. Thus, $[(49)^{15} - 1]$ is divisible by both 6 and 8.

334. $7^{84} = (7^3)^{28} = (343)^{28}.$

Now, $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

$\therefore$ $[(343)^{28} - 1]$ is divisible by $(343 - 1)$

$\Rightarrow$ $[(343)^{28} - 1]$ is divisible by 342

$\Rightarrow$ $(7^{84} - 1)$ is divisible by 342

$\Rightarrow$ On dividing 7^{84} by 342, we get 1 as remainder.

335. $2^{60} = (2^2)^{30} = 4^{30}.$

When n is even, $(x^n - a^n)$ is divisible by $(x + a)$.

$\therefore$ $(4^{30} - 1^{30})$ is divisible by $(4 + 1)$

$\Rightarrow$ $(4^{30} - 1)$ is divisible by 5 $\Rightarrow$ $(2^{60} - 1)$ is divisible by 5.

$\Rightarrow$ On dividing 2^{60} by 5, we get 1 as remainder.

336. $(2^{12} - 1) = (4096 - 1) = 4095$, which is clearly divisible by 3, 5, 7 and 13 i.e. four numbers in all.

337. $(x^n + a^n)$ is divisible by $(x + a)$ when n is odd.

$\therefore$ $(15^{23} + 23^{23})$ is divisible by $(15 + 23)$

$\Rightarrow (15^{23} + 23^{23})$ is divisible by 38 and hence by 19

$\Rightarrow$ On dividing $(15^{23} + 23^{23})$ by 19, we get 0 as remainder.

338. When n is even, $(x^n - a^n)$ is divisible by $(x + a)$.

Now, $2^{256} = (2^4)^{64} = (16)^{64}$.

$\therefore (16^{64} - 1^{64})$ is divisible by $(16 + 1)$

$\Rightarrow (16^{64} - 1)$ is divisible by 17

$\Rightarrow (2^{256} - 1)$ is divisible by 17

$\Rightarrow$ On dividing 2^{256} by 17, we get 1 as remainder.

339. When n is even, $(x^n - a^n)$ is divisible by both $(x - a)$ as well as $(x + a)$.

Now, $(7^{6n} - 6^{6n}) = [(7^3)^{2n} - (6^3)^{2n}] = [(343)^{2n} - (216)^{2n}]$.

$\therefore (7^{6n} - 6^{6n})$ is divisible by both $(7 - 6)$ and $(7 + 6)$

$\Rightarrow (7^{6n} - 6^{6n})$ is divisible by 13.

And, $[(343)^{2n} - (216)^{2n}]$ is divisible by both $(343 - 216)$ and $(343 + 216)$

$\Rightarrow (7^{6n} - 6^{6n})$ is divisible by both 127 and 559.

340. Let $2^{32} = x$. Then, $(2^{32} + 1) = (x + 1)$.

Let $(x + 1)$ be completely divisible by the natural number N.

Then, $(2^{96} + 1) = [(2^{32})^3 + 1] = (x^3 + 1) = (x + 1)(x^2 - x + 1)$, which is completely divisible by N since $(x + 1)$ is divisible by N.

341. $(2^{48} - 1) = [(2^6)^8 - 1] = (64)^8 - 1$.

When n is even, $(x^n - a^n)$ is completely divisible by both $(x - a)$ and $(x + a)$.

$\therefore (64^8 - 1^8)$ is divisible by both $(64 - 1)$ and $(64 + 1)$

$\Rightarrow (2^{48} - 1)$ is divisible by both 63 and 65.

342. $n^{65} - n = n(n^{64} - 1) = n(n^{32} - 1)(n^{32} + 1)$

$= n(n^{16} - 1)(n^{16} + 1)(n^{32} + 1)$

$= n(n^8 - 1)(n^8 + 1)(n^{16} + 1)(n^{32} + 1)$

$= n(n^4 - 1)(n^4 + 1)(n^8 + 1)(n^{16} + 1)(n^{32} + 1)$

$= n(n^2 - 1)(n^2 + 1)(n^4 + 1)(n^8 + 1)(n^{16} + 1)(n^{32} + 1)$

$= (n - 1)\,n\,(n + 1)(n^2 + 1)(n^4 + 1)(n^8 + 1)(n^{16} + 1)(n^{32} + 1)$.

Clearly, $(n - 1)$, n and $(n + 1)$ are three consecutive numbers and they have to be multiples of 2, 3 and 4 as n is odd.

Thus, the given number is definitely a multiple of 24.

343. Let $a = 55$, $b = 17$.

Then, $N = a^3 + b^3 - (a + b)^3 = a^3 + b^3 - [a^3 + b^3 + 3ab(a + b)]$

$= -3ab(a + b) = -3 \times 55 \times 17 \times 72$.

Clearly, N is divisible by both 3 and 17.

344. Taking out $\underline{|10}$ common from $[\underline{|10} + \underline{|11} + ... + \underline{|100}]$, we get this expression in the form of a multiple of $\underline{|10}$ which has zeros as its last two digits. So, the last two digits of the expression $[\underline{|10} + \underline{|11} + ... + \underline{|100}]$ are zeros.

Thus, the last two digits of N must be the last two digits of the sum $(\underline{|1} + \underline{|2} + ... + \underline{|9})$.

Now, $\underline{|1} + \underline{|2} + ... + \underline{|9} = 1 + 2 + 6 + 24 + 120 + 720 + 5040 + 40320 + 362880$

It has clearly 13 as the last two digits.

So, the last two digits of N are 13.

345. $168 = 2^3 \times 3 \times 7$ and $\underline{|7} = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7 \times 5 \times 3^2 \times 2^4 = 168 \times 30$

Hence, $\underline{|7}$ and all the factorials greater than $\underline{|7}$ are divisible by 168.

Now, $N = \underline{|1} + \underline{|2} + \underline{|3} + ... + \underline{|99} + \underline{|100}$

$= \underline{|1} + \underline{|2} + \underline{|3} + \underline{|4} + \underline{|5} + \underline{|6} +$ a multiple of 168.

So, the remainder obtained on dividing N by 168 is the same as that obtained on dividing $(\underline{|1} + \underline{|2} + \underline{|3} + \underline{|4} + \underline{|5} + \underline{|6})$ by 168.

Now, $\underline{|1} + \underline{|2} + \underline{|3} + \underline{|4} + \underline{|5} + \underline{|6} = 1 + 2 + 6 + 24 + 120 + 720$

$= 873 = (168 \times 5) + 33$.

Hence, the required remainder is 33.

346. $4^{61} = 4 \times 4^{60} = 4 \times (4^4)^{15} = 4 \times (256)^{15}$.

Now, $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.

$\therefore (256^{15} - 1)$ is divisible by $(256 - 1)$ i.e. 255 and hence by 51.

$\Rightarrow$ On dividing $(256)^{15}$ by 51, we get 1 as remainder

$\Rightarrow$ On dividing 4^{60} by 51, we get 1 as remainder

$\Rightarrow$ On dividing 4^{61} by 51, remainder obtained $= (4 \times 1) = 4$.

347. $17^{36} = (17^2)^{18} = (289)^{18}$.

Now, $[(289)^{18} - 1]$ is divisible by $(289 - 1)$, i.e. 288

$\Rightarrow (17^{36} - 1)$ is divisible by 288 and hence by 36

$\Rightarrow$ On dividing 17^{36} by 36, we get 1 as remainder.

348. When n is odd, $(x^n + a^n)$ is always divisible by $(x + a)$.

$\therefore$ Each one of $(47^{43} + 43^{43})$ and $(47^{47} + 43^{47})$ is divisible by $(47 + 43)$.

349. Product of all odd natural numbers less than 5000

$= 1 \times 3 \times 5 \times 7 \times .... \times 4999$

$$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times .... \times 5000}{2 \times 4 \times 6 \times 8 \times .... \times 5000}$$

$$= \frac{5000\,!}{2^{2500}\,(1 \times 2 \times 3 \times 4 \times .... \times 2500)} = \frac{5000\,!}{2^{2500}.\,2500\,!}.$$

350. The pages of the book may be divided into 10 groups: (1 – 100), (101 – 200), (201 – 300),..........., (901 – 1000).

Clearly, for the first group, one needs 11 zeros.

For second to ninth groups, one needs 20 zeros each.

For the tenth group, one needs 21 zeros.

So, total number of zeros required $= 11 + 8 \times 20 + 21 = 192$.

351. $a^2 + b^2 + c^2 = 1$.

So, the maximum value of $a^2\,b^2\,c^2 = \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{1}{27}$.

($\because$ when sum of three positive quantities is fixed, the product will be maximum when the quantities are equal)

Hence, maximum value of $abc = \frac{1}{\sqrt{27}} = \frac{1}{3\sqrt{3}}$.

352. Let $N = 1^5 + 2^5 + 3^5 + ... + (100)^5$.

Then, $N = (1^5 + 2^5 + 3^5 + ... + 10^5) + (11^5 + 12^5 + ... + 20^5) + (21^5 + 22^5 + ... + 30^5) + ... + (91^5 + 92^5 + ... + 100^5) = N_1 + N_2 + N_3 + ... + N_{10}$.

Since each one of N_1, N_2, N_3, ..., N_{10} has the same unit's digit of its terms, so unit's digit of each one of N_1, N_2,......, N_{10} is also the same.

$\therefore$ Unit's digit in $N = 10 \times$ Unit's digit of $N_1 = 0$.

353. Clearly, each of the 38 terms $\left(\frac{1}{4}\right),\left(\frac{1}{4}+\frac{1}{50}\right),\left(\frac{1}{4}+\frac{2}{50}\right),\ldots\ldots,\left(\frac{1}{4}+\frac{37}{50}\right)$ has a value lying between 0 and 1, while each one of the 12 terms $\left(\frac{1}{4}+\frac{38}{50}\right),\left(\frac{1}{4}+\frac{39}{50}\right),\ldots\ldots,\left(\frac{1}{4}+\frac{49}{50}\right)$ has a value lying between 1 and 2.

Hence, the given expression = (0 × 38) + (1 × 12) = 12.

354. $100^{25} - 25 = (10^2)^{25} - 25 = 10^{50} - 25$

$$= \underbrace{1000\ldots..00}_{50\text{ zeros}} - 25 = \underbrace{9999\ldots\ldots.9975}_{48\text{ times}}$$

$\therefore$ Sum of digits = (48 × 9) + 7 + 5 = 432 + 7 + 5 = 444.

355. $(1024)^4 \times (125)^{11} = (2^{10})^4 \times (5^3)^{11} = 2^{40} \times 5^{33} = 2^7 \times (2^{33} \times 5^{33}) = 2^7 \times 10^{33} = 128 \times 10^{33}$.

Clearly, the number has 1, 2, 8 and thirty-three zeros, i.e. (3 + 33) = 36 digits in all.

356. From 300 to 399, we note that when '4' comes only one time = 19 such instances.

From 400 to 499, we note that when '4' comes only one time = 80 such instances.

So, total = (19 + 80) = 99 such instances

357. 21 = 3 × 7 is not a prime number because 21 is a composite number.

358. Given $x = a(b - c)$, $y = b(c - a)$; $z = (a - b)$

$x = a(b - c)$

$\Rightarrow \frac{x}{a} = b - c$ ... (*i*)

Similarly, $y = b(c - a)$

$\Rightarrow \frac{y}{b} = c - a$ (ii) and similarly $z = c(a-b)\frac{z}{c} = c - a$ (iii)

Adding (i), (ii) and (iii) we get

$\because \frac{x}{a}+\frac{y}{b}+\frac{z}{c} = b-c+c-a+a-b = 0$

$\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 0$

$\because \left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3+\left(\frac{z}{c}\right)^3$

$= 3\times\frac{x}{a}\times\frac{y}{b}\times\frac{z}{c} = \frac{3xyz}{abc}$

[If $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$]

359. Every real number is a rational number is not a correct statement.

360. Given

$a + b + c = 6$

$ab + bc + ca = 10$

$\therefore (a + b + c)^2 = 36$

$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 36$

$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 36$

$\Rightarrow a^2 + b^2 + c^2 + 2 \times 10 = 36$

$\Rightarrow a^2 + b^2 + c^2 = 16$

As we know $\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca} = (a+b+c)$

$\frac{a^3+b^3+c^3-3abc}{16-(ab+bc+ca)} = 6$

$\Rightarrow \frac{a^3+b^3+c^3-3abc}{16-10} = 6$

$\Rightarrow a^3+b^3+c^3-3abc = 6\times6$

$\Rightarrow a^3+b^3+c^3-3abc = 36$

361. Let the blank space is x

$\therefore$ 1001 × 111 = 110000 + 11x

Now find the value of x

(1000 + 1) × 111 = 110000 + 11x

111000 + 111 = 110000 + 11x

111111 = 110000 + 11x

11x = 111111 − 110000

11x = 1111

$x = \frac{1111}{11} = 101$

362. The place value of 5 in 15201 = 5000

Place value of 6 in 2659 = 600

$\therefore$ we have 5000 + 600 = 7x

7x = 5600

$x = \frac{5600}{7} = 800$

363. Let the two − digit number be $10a + b$ where $a > b$

According to the question,

$a + b = 12$ ....(i)

$a - b = 6$.....(ii)

On adding equation (i) and (ii).

$2a = 19$

$\Rightarrow a = 9$

From equation (i),

$9 + b = 12$

$\Rightarrow b = 12 - 9 = 3$

$\therefore$ Number is $10a + b$

= 9 × 10 + 3 = 93

$\therefore$ When $a < b$. Then required number is = 39

364. Greatest four digit number that begins with 3 and ends with 5 = 3995

Least four digit number that begins with 3 and ends with 5 = 3005

$\Rightarrow (p-q)(p^2+q^2-pq)=(p-q)\{(p-q)^2-xpq\}$

$\{\because a^3-b^3=(a-b)(a^2-ab+b^2)\}$

by cancelling same terms of both sides

$\Rightarrow p^2+q^2-pq=p^2+q^2-2pq-xpq$

$\{(a-b^2)=a^2+b^2-2ab\}$

$\Rightarrow pq = -xpq$

$\Rightarrow x = -1$

367. 2361*48 will be divisible by 9 if the sum of the digits of the given number is divisible by 9.
2 + 3 + 6 + 1 + * + 4 + 8 i.e. (24 + *) is divisible by 9.
Clearly, * = 3 because 27 is divisible by 9.

368. Time taken by A to complete colouring a picture $=\frac{3}{4}$ hours
= 0.75 hours

Time taken by B to complete colouring a picture $\frac{7}{12}=$ hours
= 0.58 hours

Time taken by C to complete colouring a picture $\frac{5}{8}=$ hours
= 0.625 hours

Time taken by D to complete colouring a picture $\frac{6}{7}$ = hours = 0.85 hours

Hence, B took least time in colouring the picture.

369. Let number be a
According question, $\frac{4}{5}$ of a – 45% of a = 56

$\frac{4}{5}a-\frac{45}{100}\times a=56$

$\frac{35a}{100}=56 \Rightarrow a=\frac{100\times 56}{35}$

$\Rightarrow a = 160$

$= 65\%$ of a = $\frac{65}{100}\times 160=104$

370. Given $x+y:y+z:z+x=6:7:8$

$\frac{x+y}{6}=\frac{y+z}{7}=\frac{z+x}{8}=a$

(let)

$\Rightarrow x + y = 6a$ .... (i)

$y + z = 7a$ …(ii)

$z + x = 8a$ …(iii)

On adding all three equations

$x+y+y+z+z+x=6a+7a+8a$

$\Rightarrow 2(x+y+z)=21a$

$\Rightarrow 2\times 14=21a\ [\because x+y+z=14]$

$\therefore a=\frac{2\times 14}{21}=\frac{4}{3}$

$\therefore x+y=6a=6\times\frac{4}{3}=8$

$\therefore z=(x+y+z)-(x+y)$

= 14 – 8 = 6

371. A megabyte is 1.048, 576 bytes or 1,024 kilobytes. It conveniently expression the binary multiples inherent in digital computer memory architectures. However, megabyte architectures. However, megabyte is also taken to mean 1000 × 1024 (1024000) bytes.

372. The first 3-digit number which is divisible by 9 is 108 and last three digit number which is divisible by 9 is 999.
So, we have an AP with a = 108, d = 9 and a_n = 999

$\therefore a_n = a + (n-1)d$

$\Rightarrow 999 = 108 + (n-1)\,9$

$\Rightarrow 999 - 108 = (n-1)\,9$

$\Rightarrow 891 = (n-1)\,9$

$\Rightarrow (n-1) = \frac{891}{9} = 99$

$\Rightarrow n + 99 + 1 = 100$

373. Let, the two consecutive even numbers are a and $(a + 2)$, respectively
According to question,

$a^2+(a+2)^2=1060\ \{\because (a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow a^2 + a^2 + 4 + 4a = 1060$

$\Rightarrow 2a^2+4a-1056=0$

$\Rightarrow a^2+2a-528=0$

$\Rightarrow a^2+24a-22a-528=0$

$\Rightarrow a(a+24)-22(a+24)=0$

$\Rightarrow (a-22)(a+24)=0$

$\Rightarrow a = -24, 22$

374. The data in both the statements I and II are not sufficient to answer the question.

375. Given $n = p_1^{x_1}\, p_2^{x_2}\, p_3^{x_3}$ where p_1, p_2, p_3 are distinct prime factors
Number of prime factors form = $(x_1 \times x_2 \times x_3) = x_1\, x_2\, x_3$
Hence, option (b) is correct

376. 11, 111, 1111, 11111, …..
Let $m = 2 \Rightarrow 4 \times 2 + 3 = 11$
$m = 27 \Rightarrow 4 \times 27 + 3 = 111$
Each number can be expressed in the form $(4m + 3)$ where m is a natural number
Hence, statement 1 is only correct.

377. Maximum daily wages of an officers
= H.C.F. of ₹ 4,956; and ₹ 3,894
= ₹ 354
Illustration:

$$
\begin{array}{l}
3894\overline{)4956}(1 \\
\quad 3894 \\
\quad 1062\overline{)3894}(3 \\
\qquad 3186 \\
\qquad 708\overline{)1062}(1 \\
\qquad\quad 708 \\
\qquad\quad 354\overline{)708}(2 \\
\qquad\qquad \underline{708} \\
\qquad\qquad \times
\end{array}
$$

Maximum daily wages of an officer = ₹ 354

Number of Maximum days to attend the duty = $\frac{4956}{354} = 14$

Number of days present on duty = $\frac{3894}{354} = 11$

Number of absent days = 14 − 11 = 3 days

378. Let, the two natural numbers be a and b
According to given information
$\therefore a^2 - b^2 = 19$
$\Rightarrow (a + b)(a - b) = 19 \times 1$
$\Rightarrow a + b = 19$ ….(i)
$a - b = 1$ …. (ii)
On adding,
$2a = 20$
$\Rightarrow a = 10$
From equation (i),
$\therefore 10 + b = 19$
$\Rightarrow b = 19 - 10 = 9$
$\therefore a^2 + a^2 = (10)^2 + (9)^2$
$= 100 + 81 = 181$

379. Let the two numbers be are a and b
$\therefore a + b = 14$ ….(i)
$a - b = 10$…(ii)
By adding equation (i) and (ii) we get
$2a = 24$
$\therefore a = 12$ and $b = 2$
$\therefore$ product of these two numbers = 12 × 2 = 24

380. Let A is to be added then $2x^2 + 3x - 5 + A = x^2 + x + 1$
$\Rightarrow A = x^2 - x + 1 - (2x^2 + 3x - 5) \Rightarrow A = x^2 - x + 1 - 2x^2 - 3x + 5$
$= -x^2 - 4x + 4$