5 Square Roots and Cube Roots

IMPORTANT FACTS AND FORMULAE

I. Square Root: If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.

Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.

II. Cube Root: The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.

Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note:

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ **2.** $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}} = \dfrac{\sqrt{x}}{\sqrt{y}} \times \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{xy}}{y}$.

SOLVED EXAMPLES

Ex. 1. *Evaluate $\sqrt{6084}$ by factorization method.*

2	6084
2	3042
3	1521
3	507
13	169
	13

Sol. Method: Express the given number as the product of prime factors. Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

Thus, resolving 6084 into prime factors, we get:

$6084 = 2^2 \times 3^2 \times 13^2$

$\therefore \quad \sqrt{6084} = (2 \times 3 \times 13) = 78.$

Ex. 2. *Find the square root of 1471369.*

$$
\begin{array}{r|l}
1 & \overline{1}\,\overline{47}\,\overline{13}\,\overline{69}\ (1213 \\
 & 1 \\ \hline
22 & 47 \\
 & 44 \\ \hline
241 & 313 \\
 & 241 \\ \hline
2423 & 7269 \\
 & 7269 \\ \hline
 & \times
\end{array}
$$

Sol. Method: In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.

Now, $1^2 = 1$. On subtracting, we get 0 as remainder.

Now, bring down the next period i.e., 47.

Now, trial divisor is $1 \times 2 = 2$ and trial dividend is 47.

So, we take 22 as divisor and put 2 as quotient.

The remainder is 3.

Next, we bring down the next period which is 13.

Now, trial divisor is $12 \times 2 = 24$ and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.

The remainder is 72.

Bring down the next period i.e., 69.

Now, the trial divisor is $121 \times 2 = 242$ and the trial dividend is 7269. So, we take 3 as quotient and 2423 as divisor. The remainder is then zero.

Hence, $\sqrt{1471369} = 1213$.

Ex. 3. *Evaluate:* $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$. **(P.C.S., 2009)**

Sol. Given expression $= \sqrt{248 + \sqrt{52 + 12}} = \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16.$

Ex. 4. *Simplify :* $\dfrac{112}{\sqrt{196}} \times \dfrac{\sqrt{576}}{12} \times \dfrac{\sqrt{256}}{8}$.

Sol. Given expression $= \dfrac{112}{14} \times \dfrac{24}{12} \times \dfrac{16}{8} = 8 \times 2 \times 2 = 32.$

Ex. 5. *If* $a * b * c = \dfrac{\sqrt{(a+2)(b+3)}}{c+1}$, *then find the value of* $6 * 15 * 3$.

Sol. $6 * 15 * 3 = \dfrac{\sqrt{(6+2)(15+3)}}{3+1} = \dfrac{\sqrt{8 \times 18}}{4} = \dfrac{\sqrt{144}}{4} = \dfrac{12}{4} = 3.$

Ex. 6. *Find the value of* $\sqrt{1\dfrac{9}{16}}$.

Sol. $\sqrt{1\dfrac{9}{16}} = \sqrt{\dfrac{25}{16}} = \dfrac{\sqrt{25}}{\sqrt{16}} = \dfrac{5}{4} = 1\dfrac{1}{4}.$

Ex. 7. *What is the square root of 0.0009?*

Sol. $\sqrt{0.0009} = \sqrt{\dfrac{9}{10000}} = \dfrac{\sqrt{9}}{\sqrt{10000}} = \dfrac{3}{100} = 0.03.$

Ex. 8. *Evaluate* $\sqrt{175.2976}$.

Sol. **Method:** We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$\therefore \quad \sqrt{175.2976} = 13.24.$

1	$\overline{1}\,\overline{75}.\overline{29}\,\overline{76}$ (13.24
	1
23	75
	69
262	629
	524
2644	10576
	10576
	×

Ex. 9. *What will come in place of question mark in each of the following questions?*

(i) $\sqrt{\dfrac{32.4}{?}} = 2$ *(ii)* $\sqrt{86.49} + \sqrt{5 + (?)^2} = 12.3$

Sol. (*i*) Let $\sqrt{\dfrac{32.4}{x}} = 2$. Then, $\dfrac{32.4}{x} = 4 \Leftrightarrow 4x = 32.4 \Leftrightarrow x = 8.1.$

(*ii*) Let $\sqrt{86.49} + \sqrt{5 + x^2} = 12.3.$

Then, $9.3 + \sqrt{5+x^2} = 12.3 \Leftrightarrow \sqrt{5+x^2} = 12.3 - 9.3 = 3$

$\Leftrightarrow 5 + x^2 = 9 \Leftrightarrow x^2 = 9 - 5 = 4 \Leftrightarrow x = \sqrt{4} = 2.$

Ex. 10. *Find the value of* $\sqrt{\dfrac{0.289}{0.00121}}$. **(IGNOU, 2003)**

Sol. $\sqrt{\dfrac{0.289}{0.00121}} = \sqrt{\dfrac{0.28900}{0.00121}} = \sqrt{\dfrac{28900}{121}} = \dfrac{170}{11}.$

Ex. 11. *If* $\sqrt{841} = 29$, *then find the value of* $\sqrt{841} + \sqrt{8.41} + \sqrt{0.0841} + \sqrt{0.000841}$. **(M.B.A., 2006)**

Sol. Given expression $= \sqrt{841} + \sqrt{\dfrac{841}{10^2}} + \sqrt{\dfrac{841}{10^4}} + \sqrt{\dfrac{841}{10^6}} = \sqrt{841} + \dfrac{\sqrt{841}}{10} + \dfrac{\sqrt{841}}{10^2} + \dfrac{\sqrt{841}}{10^3}$

$= 29 + \dfrac{29}{10} + \dfrac{29}{100} + \dfrac{29}{1000} = 29 + 2.9 + 0.29 + 0.029 = 32.219.$

Ex. 12. *If* $\sqrt{1 + \dfrac{x}{144}} = \dfrac{13}{12}$, *then find the value of* x.

Sol. $\sqrt{1+\dfrac{x}{144}} = \dfrac{13}{12} \Rightarrow \left(1+\dfrac{x}{144}\right) = \left(\dfrac{13}{12}\right)^2 = \dfrac{169}{144} \Rightarrow \dfrac{x}{144} = \dfrac{169}{144} - 1$

$\Rightarrow \dfrac{x}{144} = \dfrac{25}{144} \Rightarrow x = 25.$

Ex. 13. *Simplify :* $\dfrac{1}{\sqrt{100}-\sqrt{99}} - \dfrac{1}{\sqrt{99}-\sqrt{98}} + \dfrac{1}{\sqrt{98}-\sqrt{97}} - \dfrac{1}{\sqrt{97}-\sqrt{96}} + \ldots + \dfrac{1}{\sqrt{2}-\sqrt{1}}.$ **(Section Officers', 2005)**

Sol. Given expression

$$= \frac{1}{\sqrt{100}-\sqrt{99}} \times \frac{\sqrt{100}+\sqrt{99}}{\sqrt{100}+\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}} \times \frac{\sqrt{99}+\sqrt{98}}{\sqrt{99}+\sqrt{98}} + \frac{1}{\sqrt{98}-\sqrt{97}} \times \frac{\sqrt{98}+\sqrt{97}}{\sqrt{98}+\sqrt{97}}$$

$$- \frac{1}{\sqrt{97}-\sqrt{96}} \times \frac{\sqrt{97}+\sqrt{96}}{\sqrt{97}+\sqrt{96}} + \ldots + \frac{1}{\sqrt{2}-\sqrt{1}} \times \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}+\sqrt{1}}$$

$$= \frac{\sqrt{100}+\sqrt{99}}{(100-99)} - \frac{\sqrt{99}+\sqrt{98}}{(99-98)} + \frac{\sqrt{98}+\sqrt{97}}{(98-97)} - \frac{\sqrt{97}+\sqrt{96}}{(97-96)} + \ldots + \frac{\sqrt{2}+\sqrt{1}}{(2-1)}$$

$$= (\sqrt{100}+\sqrt{99}) - (\sqrt{99}+\sqrt{98}) + (\sqrt{98}+\sqrt{97}) - (\sqrt{97}+\sqrt{96}) + \ldots + (\sqrt{2}+\sqrt{1})$$

$$= \sqrt{100}+\sqrt{1} = 10 + 1 = 11.$$

Ex. 14. *Find the sum :* $3 + \dfrac{1}{\sqrt{3}} + \dfrac{1}{3+\sqrt{3}} - \dfrac{1}{3-\sqrt{3}}.$ **(S.S.C., 2007)**

Sol. $3 + \dfrac{1}{3} + \dfrac{1}{3+\sqrt{3}} - \dfrac{1}{3-\sqrt{3}} = 3 + \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} + \dfrac{1}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}} - \dfrac{1}{3-\sqrt{3}} \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}}$

$$= 3 + \frac{\sqrt{3}}{3} + \frac{3-\sqrt{3}}{(9-3)} - \frac{3+\sqrt{3}}{(9-3)} = 3 + \frac{\sqrt{3}}{3} + \frac{3-\sqrt{3}}{6} - \frac{3+\sqrt{3}}{6}$$

$$= \frac{18 + 2\sqrt{3} + 3 - \sqrt{3} - 3 - \sqrt{3}}{6} = \frac{18}{6} = 3.$$

Ex. 15. *Find the value of* $\sqrt{3}$ *upto three places of decimal.*

Sol.

$$\begin{array}{r|l}
1 & \overline{3}.\overline{00}\,\overline{00}\,\overline{00}\ (\ 1.732 \\
 & 1 \\ \hline
27 & 200 \\
 & 189 \\ \hline
343 & \ \ 1100 \\
 & \ \ 1029 \\ \hline
3462 & \ \ \ \ 7100 \\
 & \ \ \ \ 6924
\end{array}$$

$\therefore \sqrt{3} = 1.732.$

Ex. 16. *If* $\sqrt{3} = 1.732$*, find the value of* $\sqrt{192} - \dfrac{1}{2}\sqrt{48} - \sqrt{75}$ *correct to 3 places of decimal.* **(S.S.C., 2004)**

Sol. $\sqrt{192} - \dfrac{1}{2}\sqrt{48} - \sqrt{75} = \sqrt{64\times 3} - \dfrac{1}{2}\sqrt{16\times 3} - \sqrt{25\times 3} = 8\sqrt{3} - \dfrac{1}{2}\times 4\sqrt{3} - 5\sqrt{3}$

$$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732.$$

Ex. 17. *If* $\sqrt{0.05 \times 0.5 \times a} = 0.5 \times 0.05 \times \sqrt{b}$*, then find the value of* $\dfrac{a}{b}$. **(M.B.A., 2006)**

Sol. Clearly, we have: $\dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{0.5\times 0.05}{\sqrt{0.05\times 0.5}} \Rightarrow \dfrac{a}{b} = \dfrac{0.5\times 0.05\times 0.5\times 0.05}{0.05\times 0.5} = 0.5 \times 0.05 = 0.025.$

Ex. 18. *Evaluate :* $\sqrt{\dfrac{9.5 \times .0085 \times 18.9}{.0017 \times 1.9 \times 0.021}}$.

Sol. Given exp. = $\sqrt{\dfrac{9.5 \times .0085 \times 18.900}{.0017 \times 1.9 \times 0.021}}$.

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

$\therefore$ Given exp. = $\sqrt{\dfrac{95 \times 85 \times 18900}{17 \times 19 \times 21}} = \sqrt{5 \times 5 \times 900} = 5 \times 30 = 150.$

Ex. 19. *Simplify :* $\sqrt{[(12.1)^2 - (8.1)^2] \div [(0.25)^2 + (0.25)\,(19.95)]}$. **(C.B.I., 2003)**

Sol. Given exp. = $\sqrt{\dfrac{(12.1+8.1)\,(12.1-8.1)}{(0.25)\,(0.25+19.95)}} = \sqrt{\dfrac{20.2 \times 4}{0.25 \times 20.2}} = \sqrt{\dfrac{4}{0.25}} = \sqrt{\dfrac{400}{25}} = \sqrt{16} = 4.$

Ex. 20. *If* $x = 1 + \sqrt{2}$ *and* $y = 1 - \sqrt{2}$, *find the value of* $(x^2 + y^2)$.

Sol. $x^2 + y^2 = (1+\sqrt{2})^2 + (1-\sqrt{2})^2 = 2\,[(1)^2 + (\sqrt{2})^2] = 2 \times 3 = 6.$

Ex. 21. *Evaluate* $\sqrt{0.9}$ *upto 3 places of decimal.*

Sol.

$$\begin{array}{r|l}
9 & 0.\overline{90}\,\overline{00}\,\overline{00}\ (0.948 \\
 & \underline{81} \\
184 & 900 \\
 & \underline{736} \\
1888 & 16400 \\
 & 15104
\end{array}$$

$\therefore \sqrt{0.9} = 0.948.$

Ex. 22. *Find the square root of* $0.\dot{1}$.

Sol. $\sqrt{0.\dot{1}} = \sqrt{\dfrac{1}{9}} = \dfrac{1}{3} = 0.333\cdots = 0.\dot{3}.$

Ex. 23. *If* $\sqrt{15} = 3.88$, *find the value of* $\sqrt{\dfrac{5}{3}}$.

Sol. $\sqrt{\dfrac{5}{3}} = \sqrt{\dfrac{5 \times 3}{3 \times 3}} = \dfrac{\sqrt{15}}{3} = \dfrac{3.88}{3} = 1.2933..... = 1.29\overline{3}.$

Ex. 24. *Find the least square number which is exactly divisible by 10, 12, 15 and 18.*

Sol. L.C.M. of 10, 12, 15, 18 = 180. Now, 180 = 2 × 2 × 3 × 3 × 5 = $2^2 \times 3^2 \times 5$.

To make it a perfect square, it must be multiplied by 5.

$\therefore$ Required number = $(2^2 \times 3^2 \times 5^2) = 900$.

Ex. 25. *Find the greatest number of five digits which is a perfect square.*

Sol. Greatest number of 5 digits is 99999.

$$\begin{array}{r|l}
3 & \overline{9}\,\overline{99}\,\overline{99}\ (316 \\
 & \underline{9} \\
61 & 99 \\
 & \underline{61} \\
626 & 3899 \\
 & \underline{3756} \\
 & 143
\end{array}$$

$\therefore$ Required number = (99999 − 143) = 99856.

Ex. 26. ***Find the smallest number that must be added to 1780 to make it a perfect square.***

Sol.

$$\begin{array}{r|l} 4 & \overline{17}\,\overline{80}\ (42 \\ & 16 \\ \hline 82 & \ \ 180 \\ & \ \ 164 \\ \hline & \ \ \ \ 16 \end{array}$$

$\therefore$ Number to be added $= (43)^2 - 1780 = 1849 - 1780 = 69.$

Ex. 27. ***If*** $\sqrt{2} = 1.4142$***, find the value of*** $\dfrac{\sqrt{2}}{(2+\sqrt{2})}$.

Sol. $\dfrac{\sqrt{2}}{(2+\sqrt{2})} = \dfrac{\sqrt{2}}{(2+\sqrt{2})} \times \dfrac{(2-\sqrt{2})}{(2-\sqrt{2})} = \dfrac{2\sqrt{2}-2}{(4-2)} = \dfrac{2(\sqrt{2}-1)}{2} = (\sqrt{2}-1) = (1.4142 - 1)$

$= 0.4142.$

Ex. 28. ***If*** $x = \left(\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)$ ***and*** $y = \left(\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$***, find the value of*** $(x^2 + y^2)$.

Sol. $x = \dfrac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})} \times \dfrac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \dfrac{(\sqrt{5}+\sqrt{3})^2}{(5-3)} = \dfrac{5+3+2\sqrt{15}}{2} = 4+\sqrt{15}.$

$y = \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})} \times \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} = \dfrac{(\sqrt{5}-\sqrt{3})^2}{(5-3)} = \dfrac{5+3-2\sqrt{15}}{2} = 4-\sqrt{15}.$

$\therefore\ x^2 + y^2 = (4+\sqrt{15})^2 + (4-\sqrt{15})^2 = 2[(4)^2 + (\sqrt{15})^2] = 2 \times 31 = 62.$

Ex. 29. ***Find the value of:*** $\sqrt{6+\sqrt{6+\sqrt{6+....}}}$ **(S.S.C., 2006)**

Sol. Let $\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}} = x.$

Then, $\sqrt{6+x} = x \Leftrightarrow 6 + x = x^2 \Rightarrow x^2 - x - 6 = 0 \Leftrightarrow x^2 - 3x + 2x - 6 = 0$

$\Leftrightarrow x(x-3) + 2(x-3) = 0 \Leftrightarrow (x-3)(x+2) = 0 \Leftrightarrow x = 3.$ $[\because x \neq -2]$

Hence, $\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}} = 3.$

Ex. 30. ***Find the cube root of 2744.***

Sol. **Method :** Resolve the given number as the product of prime factors and take the product of prime factors, choosing one out of three of the same prime factors.

Resolving 2744 as the product of prime factors, we get :

2	2744
2	1372
2	686
7	343
7	49
	7

$2744 = 2^3 \times 7^3.$

$\therefore\ \sqrt[3]{2744} = 2 \times 7 = 14.$

Ex. 31. ***By what least number 4320 be multiplied to obtain a number which is a perfect cube?***

Sol. Clearly, $4320 = 2^3 \times 3^3 \times 2^2 \times 5.$

To make it a perfect cube, it must be multiplied by 2×5^2 *i.e.*, 50.

EXERCISE
(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (✓) against the correct answer:*

1. $\sqrt{53824} = ?$

(*a*) 202 (*b*) 232
(*c*) 242 (*d*) 332

2. The square root of 41209 is equal to **(L.I.C.A.D.O., 2008)**

(*a*) 103 (*b*) 203
(*c*) 303 (*d*) 403

3. The square root of 123454321 is **(P.C.S., 2008)**

(*a*) 111111 (*b*) 12341
(*c*) 11111 (*d*) 11211

4. The number of digits in the square root of 625685746009 is **(S.S.C., 2007)**

(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

5. $\sqrt{\sqrt{17956}+\sqrt{24025}} = ?$ **(Bank P.O., 2008)**

(*a*) 19 (*b*) 155
(*c*) 256 (*d*)289
(*e*) None of these

6. $\sqrt{\sqrt{44944}+\sqrt{52441}} = ?$ **(Bank P.O., 2008)**

(*a*) 17 (*b*) 312
(*c*) 441 (*d*) 485
(*e*) None of these

7. One-fourth of the sum of prime numbers, greater than 4 but less than 16, is the square of **(P.C.S., 2009)**

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 7

8. The value of $\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}$ is

(*a*) 4 (*b*) 6
(*c*) 8 (*d*) 10

9. Evaluate $\sqrt{41-\sqrt{21+\sqrt{19-\sqrt{9}}}}$.

(*a*) 3 (*b*) 5
(*c*) 6 (*d*) 6.4

10. $\sqrt{176+\sqrt{2401}}$ is equal to

(*a*) 14 (*b*) 15
(*c*) 18 (*d*) 24

11. $\dfrac{\sqrt{196}}{7}\times\dfrac{\sqrt{441}}{7}\times\dfrac{120}{\sqrt{225}} = ?$ **(R.R.B., 2008)**

(*a*) 48 (*b*) 54
(*c*) 58 (*d*) 84

12. $\left(\sqrt{\dfrac{225}{729}}-\sqrt{\dfrac{25}{144}}\right)\div\sqrt{\dfrac{16}{81}} = ?$

(*a*) $\dfrac{1}{48}$ (*b*) $\dfrac{5}{48}$
(*c*) $\dfrac{5}{16}$ (*d*) None of these

13. $(15)^2 + (18)^2 - 20 = \sqrt{?}$ **(Bank P.O., 2006)**

(*a*) 22 (*b*) 23
(*c*) 529 (*d*) 279841
(*e*) None of these

14. $\sqrt{?}\times\sqrt{484} = 1034$ **(L.I.C.A.D.O., 2007)**

(*a*) 2025 (*b*) 2209
(*c*) 2304 (*d*) 2401
(*e*) None of these

15. $\sqrt{11881}\times\sqrt{?} = 10137$ **(Bank Recruitment, 2008)**

(*a*) 8281 (*b*) 8649
(*c*) 9216 (*d*) 9409
(*e*) None of these

16. In the equation $\dfrac{4050}{\sqrt{x}} = 450,$ the value of x is **(L.I.C.A.D.O., 2008)**

(*a*) 9 (*b*) 49
(*c*) 81 (*d*) 100

17. $\sqrt{\dfrac{16}{25}}\times\sqrt{\dfrac{?}{25}}\times\dfrac{16}{25} = \dfrac{256}{625}$ **(R.R.B. 2008)**

(*a*) 5 (*b*) 8
(*c*) 16 (*d*) None of these

18. The square root of $(272^2 - 128^2)$ is

(*a*) 144 (*b*) 200
(*c*) 240 (*d*) 256

19. If $x * y = x + y + \sqrt{xy}$, the value of $6 * 24$ is

(*a*) 41 (*b*) 42
(*c*) 43 (*d*) 44

20. If $y = 5$, then what is the value of $10y\sqrt{y^3 - y^2}$?

(*a*) $50\sqrt{2}$ (*b*) 100
(*c*) $200\sqrt{5}$ (*d*) 500

21. $\sqrt{110\dfrac{1}{4}} = ?$

(*a*) 10.25 (*b*) 10.5
(*c*) 11.5 (*d*) 19.5

22. $\sqrt{\frac{25}{81}-\frac{1}{9}}=?$ **(Hotel Management, 2002)**

(*a*) $\frac{2}{3}$ (*b*) $\frac{4}{9}$

(*c*) $\frac{16}{81}$ (*d*) $\frac{25}{81}$

23. $[(\sqrt{81})^2]^2=(?)^2$ **(Specialist Officers, 2006)**

(*a*) 8 (*b*) 9

(*c*) 4096 (*d*) 6561

(*e*) None of these

24. The digit in the unit's place in the square root of 15876 is

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8

25. Which of the following is closest to $\sqrt{3}$? **(S.S.C., 2005)**

(*a*) 1.69 (*b*) $\frac{173}{100}$

(*c*) 1.75 (*d*) $\frac{9}{5}$

26. How many two-digit numbers satisfy this property: The last digit (unit's digit) of the square of the two-digit number is 8?

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) None of these

27. What percentage of the numbers from 1 to 50 have squares that end in the digit 1? **(M.B.A., 2006)**

(*a*) 1 (*b*) 5

(*c*) 10 (*d*) 11

(*e*) 20

28. While solving a mathematical problem, Samidha squared a number and then subtracted 25 from it rather than the required i.e., first subtracting 25 from the number and then squaring it. But she got the right answer. What was the given number? **(Bank P.O., 2006)**

(*a*) 13 (*b*) 38

(*c*) 48

(*d*) Cannot be determined (*e*) None of these

29. How many perfect squares lie between 120 and 300? **(S.S.C., 2010)**

(*a*) 5 (*b*) 6

(*c*) 7 (*d*) 8

30. The number of perfect square numbers between 50 and 1000 is

(*a*) 21 (*b*) 22

(*c*) 23 (*d*) 24

(Section Officers', 2003)

31. A man born in the first half of the nineteenth century was x years old in the year x^2. He was born in **(M.B.A., 2011)**

(*a*) 1806 (*b*) 1812

(*c*) 1825 (*d*) 1836

32. R is a positive number. It is multiplied by 8 and then squared. The square is now divided by 4 and the square root is taken. The result of the square root is Q. What is the value of Q? **(SNAP, 2010)**

(*a*) $3R$ (*b*) $4R$

(*c*) $7R$ (*d*) $9R$

33. The smallest natural number which is a perfect square and which ends in 3 identical digits lies between

(*a*) 1000 and 2000 (*b*) 2000 and 3000

(*c*) 3000 and 4000 (*d*) 4000 and 5000

34. $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2$ is equal to **(S.S.C., 2005)**

(*a*) $2\frac{1}{2}$ (*b*) $3\frac{1}{2}$

(*c*) $4\frac{1}{2}$ (*d*) $5\frac{1}{2}$

35. If the product of four consecutive natural numbers increased by a natural number p, is a perfect square, then the value of p is **(C.P.O., 2006)**

(*a*) 1 (*b*) 2

(*c*) 4 (*d*) 8

36. What is the square root of 0.16?

(*a*) 0.004 (*b*) 0.04

(*c*) 0.4 (*d*) 4

37. The value of $\sqrt{0.000441}$ is **(S.S.C., 2002)**

(*a*) 0.00021 (*b*) 0.0021

(*c*) 0.021 (*d*) 0.21

38. $\sqrt{0.00004761}$ equals **(C.B.I., 2003)**

(*a*) 0.00069 (*b*) 0.0069

(*c*) 0.0609 (*d*) 0.069

39. $1.5^2\times\sqrt{0.0225}=?$ **(Bank P.O., 2002)**

(*a*) 0.0375 (*b*) 0.3375

(*c*) 3.275 (*d*) 32.75

40. $\sqrt{0.01+\sqrt{0.0064}}=?$

(*a*) 0.03 (*b*) 0.3

(*c*) 0.42 (*d*) None of these

41. The value of $\sqrt{0.01}+\sqrt{0.81}+\sqrt{1.21}+\sqrt{0.0009}$ is **(S.S.C., 2002)**

(*a*) 2.03 (*b*) 2.1

(*c*) 2.11 (*d*) 2.13

42. $\sqrt{.0025} \times \sqrt{2.25} \times \sqrt{.0001} = ?$

(*a*) .000075 (*b*) .0075
(*c*) .075 (*d*) None of these

43. $\sqrt{1.5625} = ?$ **(S.B.I.P.O., 2003)**

(*a*) 1.05 (*b*) 1.25
(*c*) 1.45 (*d*) 1.55

44. If $\sqrt{.00000676} = .0026$, the square root of 67,60,000 is:

(*a*) $\frac{1}{26}$ (*b*) 26
(*c*) 260 (*d*) 2600

45. If $\sqrt{18225} = 135$, then the value of $(\sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225} + \sqrt{0.00018225})$ is

(*a*) 1.49985 (*b*) 14.9985
(*c*) 149.985 (*d*) 1499.85

46. If $\sqrt{4096} = 64$, then the value of $\sqrt{40.96} + \sqrt{0.4096} + \sqrt{0.004096} + \sqrt{0.00004096}$ up to two places of decimals is **(S.S.C., 2005)**

(*a*) 7.09 (*b*) 7.10
(*c*) 7.11 (*d*) 7.12

47. Given that $\sqrt{13} = 3.605$ and $\sqrt{130} = 11.40$, find the value of $\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013}$.

(*a*) 36.164 (*b*) 36.304
(*c*) 37.164 (*d*) 37.304

48. If $\frac{52}{x} = \sqrt{\frac{169}{289}}$, the value of x is

(*a*) 52 (*b*) 58
(*c*) 62 (*d*) 68

49. For what value of * the statement $\left(\frac{*}{15}\right)\left(\frac{*}{135}\right) = 1$ is true? **(S.S.C., 2002)**

(*a*) 15 (*b*) 25
(*c*) 35 (*d*) 45

50. Which number should replace both the question marks in the following equation? **(Bank P.O., 2008)**

$$\frac{?}{1776} = \frac{111}{?}$$

(*a*) 343 (*b*) 414
(*c*) 644 (*d*) 543
(*e*) None of these

51. Which number can replace both the question marks in the equation $\frac{4\frac{1}{2}}{?} = \frac{?}{32}$.

(*a*) 1 (*b*) 7
(*c*) $7\frac{1}{2}$ (*d*) None of these

52. What should come in place of both the question marks in the equation $\frac{?}{\sqrt{128}} = \frac{\sqrt{162}}{?}$.

(*a*) 12 (*b*) 14
(*c*) 144 (*d*) 196

53. If $\sqrt{x + \frac{x}{y}} = x\sqrt{\frac{x}{y}}$, where x and y are positive real numbers, then y is equal to **(Hotel Mgmt, 2010)**

(*a*) $x + 1$ (*b*) $x - 1$
(*c*) $x^2 + 1$ (*d*) $x^2 - 1$

54. The number $25^{64} \times 64^{25}$ is the square of a natural number n. The sum of the digits of n is **(A.A.O. Exam., 2010)**

(*a*) 7 (*b*) 14
(*c*) 21 (*d*) 28

55. If $0.13 \div p^2 = 13$, then p equals

(*a*) 0.01 (*b*) 0.1
(*c*) 10 (*d*) 100

56. What number should be divided by $\sqrt{0.25}$ to give the result as 25? **(C.B.I., 2003)**

(*a*) 12.5 (*b*) 25
(*c*) 50 (*d*) 125

57. If $\sqrt{3^n} = 729$, then the value of n is **(Section Officers, 2003)**

(*a*) 6 (*b*) 8
(*c*) 10 (*d*) 12

58. If $\sqrt{18 \times 14 \times x} = 84$, then x equals

(*a*) 22 (*b*) 24
(*c*) 28 (*d*) 32

59. $28\sqrt{?} + 1426 = \frac{3}{4}$ of 2872

(*a*) 576 (*b*) 676
(*c*) 1296 (*d*) 1444

60. $\sqrt{\frac{?}{169}} = \frac{54}{39}$

(*a*) 108 (*b*) 324
(*c*) 2916 (*d*) 4800

61. If $\sqrt{x} \div \sqrt{441} = 0.02$, then the value of x is

(*a*) 0.1764 (*b*) 1.764
(*c*) 1.64 (*d*) 2.64

62. $\sqrt{\frac{.0196}{?}} = 0.2$

(*a*) 0.49 (*b*) 0.7
(*c*) 4.9 (*d*) None of these

63. $\sqrt{0.0169 \times ?} = 1.3$ **(Hotel Management, 2001)**

(*a*) 10 (*b*) 100
(*c*) 1000 (*d*) None of these

64. If $\sqrt{1369}+\sqrt{.0615+x}$ = 37.25, then x is equal to

(*a*) 10^{-1} (*b*) 10^{-2}
(*c*) 10^{-3} (*d*) None of these

65. If $\sqrt{(x-1)(y+2)}=7$, x and y being positive whole numbers, then the values of x and y respectively are

(*a*) 8, 5 (*b*) 15, 12
(*c*) 22, 19 (*d*) None of these

66. If $\sqrt{.04\times.4\times a}=.004\times.4\times\sqrt{b}$, then $\frac{a}{b}$ is

(*a*) 16×10^{-3} (*b*) 16×10^{-4}
(*c*) 16×10^{-5} (*d*) None of these

67. Three-fifth of the square of a certain number is 126.15. What is the number?

(*a*) 14.5 (*b*) 75.69
(*c*) 145 (*d*) 210.25

68. $\sqrt{\frac{0.361}{0.00169}}=?$

(*a*) $\frac{1.9}{13}$ (*b*) $\frac{19}{13}$
(*c*) $\frac{1.9}{130}$ (*d*) $\frac{190}{13}$

69. $\sqrt{\frac{48.4}{0.289}}$ is equal to **(S.S.C., 2004)**

(*a*) $1\frac{5}{17}$ (*b*) $12\frac{1}{17}$
(*c*) $12\frac{16}{17}$ (*d*) $129\frac{7}{17}$

70. If $\sqrt{1+\frac{x}{169}}=\frac{14}{13}$, then x is equal to

(*a*) 1 (*b*) 13
(*c*) 27 (*d*) None of these

71. $\sqrt{1+\frac{55}{729}}=1+\frac{x}{27}$, then the value of x is **(C.D.S., 2003)**

(*a*) 1 (*b*) 3
(*c*) 5 (*d*) 7

72. $\sqrt{\frac{4}{3}}-\sqrt{\frac{3}{4}}=?$ **(R.R.B., 2005)**

(*a*) $\frac{4\sqrt{3}}{6}$ (*b*) $\frac{1}{2\sqrt{3}}$
(*c*) 1 (*d*) $-\frac{1}{2\sqrt{3}}$

73. The value of $\sqrt{2}$ upto three places of decimal is

(*a*) 1.410 (*b*) 1.412
(*c*) 1.413 (*d*) 1.414

74. $(2\sqrt{27}-\sqrt{75}+\sqrt{12})$ is equal to

(*a*) $\sqrt{3}$ (*b*) $2\sqrt{3}$
(*c*) $3\sqrt{3}$ (*d*) $4\sqrt{3}$

75. By how much does $\sqrt{12}+\sqrt{18}$ exceed $\sqrt{3}+\sqrt{2}$?

(*a*) $\sqrt{2}-4\sqrt{3}$ (*b*) $\sqrt{3}+2\sqrt{2}$
(*c*) $2(\sqrt{3}-\sqrt{2})$ (*d*) $3(\sqrt{3}-\sqrt{2})$

76. $\frac{\sqrt{24}+\sqrt{216}}{\sqrt{96}}=?$

(*a*) $2\sqrt{6}$ (*b*) 2
(*c*) $6\sqrt{2}$ (*d*) $\frac{2}{\sqrt{6}}$

77. The value of $\frac{\sqrt{80}-\sqrt{112}}{\sqrt{45}-\sqrt{63}}$ is

(*a*) $\frac{3}{4}$ (*b*) $1\frac{1}{3}$
(*c*) $1\frac{7}{9}$ (*d*) $1\frac{3}{4}$

78. If $3\sqrt{5}+\sqrt{125}=17.88$, then what will be the value of $\sqrt{80}+6\sqrt{5}$?

(*a*) 13.41 (*b*) 20.46
(*c*) 21.66 (*d*) 22.35

79. $\sqrt{50}\times\sqrt{98}$ is equal to

(*a*) 63.75 (*b*) 65.95
(*c*) 70 (*d*) 70.25

80. Given $\sqrt{2}=1.414$. The value of $\sqrt{8}+2\sqrt{32}-3\sqrt{128}+4\sqrt{50}$ is: **(S.S.C., 2003)**

(*a*) 8.426 (*b*) 8.484
(*c*) 8.526 (*d*) 8.876

81. The approximate value of $\frac{3\sqrt{12}}{2\sqrt{28}}\div\frac{2\sqrt{21}}{\sqrt{98}}$ is **(Section Officers, 2003)**

(*a*) 1.0605 (*b*) 1.0727
(*c*) 1.6007 (*d*) 1.6026

82. $\sqrt{110.25}\times\sqrt{0.01}\div\sqrt{0.0025}-\sqrt{420.25}$ equals **(SNAP, 2010)**

(*a*) 0.50 (*b*) 0.64
(*c*) 0.73 (*d*) 0.75

83. $\sqrt{\frac{.081\times.484}{.0064\times6.25}}$ is equal to

(*a*) 0.9 (*b*) 0.99
(*c*) 9 (*d*) 99

84. $\sqrt{\dfrac{0.204 \times 42}{0.07 \times 3.4}}$ is equal to

(a) $\dfrac{1}{6}$ (b) 0.06
(c) 0.6 (d) 6

85. $\sqrt{\dfrac{0.081 \times 0.324 \times 4.624}{1.5625 \times 0.0289 \times 72.9 \times 64}}$ is equal to

(a) 0.024 (b) 0.24
(c) 2.4 (d) 24

86. $\sqrt{\dfrac{9.5 \times .085}{.0017 \times .19}}$ equals

(a) .05 (b) 5
(c) 50 (d) 500

87. The value of $\sqrt{\dfrac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}}$ is **(S.S.C., 2002)**

(a) 0.1 (b) 10
(c) 10^2 (d) 10^3

88. The square root of $(7 + 3\sqrt{5})(7 - 3\sqrt{5})$ is **(S.S.C., 2004)**

(a) $\sqrt{5}$ (b) 2
(c) 4 (d) $3\sqrt{5}$

89. $\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right)^2$ simplifies to

(a) $\dfrac{3}{4}$ (b) $\dfrac{4}{\sqrt{3}}$
(c) $\dfrac{4}{3}$ (d) None of these

90. If $a = 0.1039$, then the value of $\sqrt{4a^2 - 4a + 1} + 3a$ is **(C.B.I., 2003)**

(a) 0.1039 (b) 0.2078
(c) 1.1039 (d) 2.1039

91. The square root of $\dfrac{(0.75)^3}{1 - 0.75} + [0.75 + (0.75)^2 + 1]$ is

(a) 1 (b) 2
(c) 3 (d) 4

92. If $3a = 4b = 6c$ and $a + b + c = 27\sqrt{29}$, then $\sqrt{a^2 + b^2 + c^2}$ is

(a) $3\sqrt{29}$ (b) 81
(c) 87 (d) None of these

93. The square root of $0.\overline{4}$ is **(S.S.C., 2004)**

(a) $0.\overline{6}$ (b) $0.\overline{7}$
(c) $0.\overline{8}$ (d) $0.\overline{9}$

94. Which one of the following numbers has rational square root?

(a) 0.4 (b) 0.09
(c) 0.9 (d) 0.025

95. The value of $\sqrt{0.4}$ is

(a) 0.02 (b) 0.2
(c) 0.51 (d) 0.63

96. $\sqrt{0.2} = ?$ **(R.R.B., 2007)**

(a) 0.02 (b) 0.2
(c) 0.447 (d) 0.632

97. The value of $\sqrt{0.121}$ is

(a) 0.011 (b) 0.11
(c) 0.347 (d) 1.1

98. The value of $\sqrt{0.064}$ is

(a) 0.008 (b) 0.08
(c) 0.252 (d) 0.8

99. The value of $\sqrt{\dfrac{0.16}{0.4}}$ is **(IGNOU, 2003)**

(a) 0.02 (b) 0.2
(c) 0.63 (d) None of these

100. The value of $\dfrac{1 + \sqrt{0.01}}{1 - \sqrt{0.1}}$ is close to

(a) 0.6 (b) 1.1
(c) 1.6 (d) 1.7

101. The square root of 535.9225 is **(R.R.B., 2006)**

(a) 23.15 (b) 23.45
(c) 24.15 (d) 28.25

102. If $\sqrt{5} = 2.236$, then the value of $\dfrac{1}{\sqrt{5}}$ is

(a) .367 (b) .447
(c) .745 (d) None of these

103. If $\sqrt{24} = 4.899$, the value of $\sqrt{\dfrac{8}{3}}$ is

(a) 0.544 (b) 1.333
(c) 1.633 (d) 2.666

104. If $\sqrt{6} = 2.449$, then the value of $\dfrac{3\sqrt{2}}{2\sqrt{3}}$ is

(a) 0.6122 (b) 0.8163
(c) 1.223 (d) 1.2245

105. If $\sqrt{5} = 2.236$, then the value of $\dfrac{\sqrt{5}}{2} - \dfrac{10}{\sqrt{5}} + \sqrt{125}$ is equal to

(a) 5.59 (b) 7.826
(c) 8.944 (d) 10.062

106. If 2 * 3 = $\sqrt{13}$ and 3 * 4 = 5, then the value of 5 * 12 is

(*a*) $\sqrt{17}$ (*b*) $\sqrt{29}$
(*c*) 12 (*d*) 13

107. If 1537* is a perfect square, then the digit which replaces * is **(Hotel Management, 2007)**

(*a*) 2 (*b*) 4
(*c*) 5 (*d*) 6

108. The smallest perfect square that is divisible by 7! is **(I.I.F.T., 2010)**

(*a*) 19600 (*b*) 44100
(*c*) 176400 (*d*) 705600

109. The least perfect square number divisible by 3, 4, 5, 6 and 8 is

(*a*) 900 (*b*) 1200
(*c*) 2500 (*d*) 3600

110. The least perfect square, which is divisible by each of 21, 36 and 66, is

(*a*) 213444 (*b*) 214344
(*c*) 214434 (*d*) 231444

111. The least number by which 294 must be multiplied to make it a perfect square, is

(*a*) 2 (*b*) 3
(*c*) 6 (*d*) 24

112. Find the smallest number by which 5808 should be multiplied so that the product becomes a perfect square.

(*a*) 2 (*b*) 3
(*c*) 7 (*d*) 11

113. The least number by which 1470 must be divided to get a number which is a perfect square, is

(*a*) 5 (*b*) 6
(*c*) 15 (*d*) 30

114. What is the smallest number to be subtracted from 549162 in order to make it a perfect square?

(*a*) 28 (*b*) 36
(*c*) 62 (*d*) 81

115. What is the least number which should be subtracted from 0.000326 to make it a perfect square?

(*a*) 0.000002 (*b*) 0.000004
(*c*) 0.02 (*d*) 0.04

116. What is the least number to be added to 7700 to make it a perfect square?

(*a*) 77 (*b*) 98
(*c*) 131 (*d*) 221
(*e*) None of these **(Bank Recruitment, 2008)**

117. The smallest number to be added to 680621 to make the sum a perfect square is **(S.S.C., 2005)**

(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 8

118. The greatest four-digit perfect square number is

(*a*) 9000 (*b*) 9801
(*c*) 9900 (*d*) 9981

119. The least number of 4 digits which is a perfect square is

(*a*) 1000 (*b*) 1016
(*c*) 1024 (*d*) 1036

120. The sum of 18 consecutive natural numbers is a perfect square. What is the smallest possible value of this sum? **(A.A.O. Exam, 2009)**

(*a*) 169 (*b*) 225
(*c*) 289 (*d*) 441

121. $\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$ is equal to

(*a*) 1 (*b*) 2
(*c*) 4 (*d*) $\sqrt{6}$

122. Given $\sqrt{5}=2.2361, \sqrt{3}=1.7321$, then $\dfrac{1}{\sqrt{5}-\sqrt{3}}$ is equal to

(*a*) 1.98 (*b*) 1.984
(*c*) 1.9841 (*d*) 2

123. $\dfrac{1}{(\sqrt{9}-\sqrt{8})}-\dfrac{1}{(\sqrt{8}-\sqrt{7})}+\dfrac{1}{(\sqrt{7}-\sqrt{6})}-\dfrac{1}{(\sqrt{6}-\sqrt{5})}+\dfrac{1}{(\sqrt{5}-\sqrt{4})}$ is equal to **(M.B.A., 2007)**

(*a*) 0 (*b*) $\dfrac{1}{3}$
(*c*) 1 (*d*) 5

124. Determine the value of

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{120}+\sqrt{121}}.$$

(M.A.T., 2005)

(*a*) 8 (*b*) 10
(*c*) $\sqrt{120}$ (*d*) $12\sqrt{2}$

125. The expression $1-\dfrac{1}{1+\sqrt{3}}+\dfrac{1}{1-\sqrt{3}}$ equals **(M.B.A., 2011)**

(*a*) $1-\sqrt{3}$ (*b*) 1
(*c*) $-\sqrt{3}$ (*d*) $\sqrt{3}$

126. $\left(2+\sqrt{2}+\dfrac{1}{2+\sqrt{2}}+\dfrac{1}{\sqrt{2}-2}\right)$ simplifies to **(M.B.A., 2007)**

(*a*) $2-\sqrt{2}$ (*b*) 2
(*c*) $2+\sqrt{2}$ (*d*) $2\sqrt{2}$

127. What is the value of $\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{2}{3+\sqrt{5}}-\frac{3}{3+\sqrt{3}}$? **(I.I.F.T., 2005)**

(*a*) $-\frac{1}{2}$ (*b*) 0
(*c*) $\frac{1}{2}$ (*d*) 1

128. If $\sqrt{2}$ = 1.4142, the value of $\frac{7}{(3+\sqrt{2})}$ is **(R.R.B., 2005)**

(*a*) 1.5858 (*b*) 3.4852
(*c*) 3.5858 (*d*) 4.4142

129. $\left[\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}-\frac{6}{\sqrt{8}-\sqrt{12}}\right]=?$ **(Teachers' Exam, 2010)**

(*a*) $\sqrt{3}-\sqrt{2}$ (*b*) $\sqrt{3}+\sqrt{2}$
(*c*) $5\sqrt{3}$ (*d*) 1

130. $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$ is equal to

(*a*) 1 (*b*) 2
(*c*) $6-\sqrt{35}$ (*d*) $6+\sqrt{35}$

131. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$, then **(R.R.B., 2010)**

(*a*) $a = -11, b = -6$ (*b*) $a = -11, b = 6$
(*c*) $a = 11, b = -6$ (*d*) $a = 6, b = 11$

132. If $\sqrt{2}$ = 1.414, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to

(*a*) 0.172 (*b*) 0.414
(*c*) 0.586 (*d*) 1.414

133. Given that $\sqrt{3}$ = 1.732, the value of $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is **(S.S.C., 2007)**

(*a*) 1.414 (*b*) 1.732
(*c*) 2.551 (*d*) 4.899

134. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ simplifies to **(S.S.C., 2008)**

(*a*) $16-\sqrt{3}$ (*b*) $4-\sqrt{3}$
(*c*) $2-\sqrt{3}$ (*d*) $2+\sqrt{3}$

135. If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then $(x + y)$ equals **(S.S.C., 2005)**

(*a*) $2(\sqrt{5}+\sqrt{3})$ (*b*) $2\sqrt{15}$
(*c*) 8 (*d*) 16

136. $\frac{\sqrt{2}\,(2+\sqrt{3})}{\sqrt{3}\,(\sqrt{3}+1)}\times\frac{\sqrt{2}\,(2-\sqrt{3})}{\sqrt{3}\,(\sqrt{3}-1)}$ is equal to **(C.P.O., 2006)**

(*a*) $3\sqrt{2}$ (*b*) $\frac{\sqrt{2}}{3}$
(*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

137. $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to **(S.S.C., 2007)**

(*a*) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$
(*b*) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$
(*c*) $1+\sqrt{5}-\sqrt{2}+\sqrt{10}$
(*d*) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

138. $\left[\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}\right]$ in simplified form equals **(S.S.C., 2005)**

(*a*) 0 (*b*) $\frac{1}{\sqrt{2}}$
(*c*) 1 (*d*) $\sqrt{2}$

139. If $x = (7-4\sqrt{3})$, then the value of $\left(x+\frac{1}{x}\right)$ is

(*a*) $3\sqrt{3}$ (*b*) $8\sqrt{3}$
(*c*) 14 (*d*) $14+8\sqrt{3}$

140. If $x = 3+\sqrt{8}$, then $x^2+\frac{1}{x^2}$ is equal to **(S.S.C., 2007)**

(*a*) 30 (*b*) 34
(*c*) 36 (*d*) 38

141. If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}, b=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then the value of $a^2 + b^2$ would be **(M.B.A., 2008)**

(*a*) 10 (*b*) 98
(*c*) 99 (*d*) 100

142. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\left(\frac{a^2+ab+b^2}{a^2-ab+b^2}\right)$ is

(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$
(*c*) $\frac{3}{5}$ (*d*) $\frac{5}{3}$

143. If $x = \sqrt{1+\sqrt{1+\sqrt{1+\cdots\infty}}}$ then the positive value of x is **(M.B.A., 2006)**

(*a*) $\frac{\sqrt{7}+1}{2}$ (*b*) $\frac{\sqrt{6}+1}{2}$

(*c*) $\frac{\sqrt{5}+1}{2}$ (*d*) $\frac{\sqrt{3}+1}{2}$

144. $\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}$ is equal to **(S.S.C., 2005)**

(*a*) 1 (*b*) 1.5

(*c*) 2 (*d*) 2.5

145. If $a = \sqrt{3+\sqrt{3+\sqrt{3+\cdots}}}$, then which of the following is true? **(M.B.A., 2007)**

(*a*) $2 < a < 3$ (*b*) $a > 3$

(*c*) $3 < a < 4$ (*d*) $a = 3$

146. $\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}-\sqrt{3-x}} = 2.$ Then x is equal to **(S.S.C., 2010)**

(*a*) $\frac{5}{7}$ (*b*) $\frac{7}{5}$

(*c*) $\frac{5}{12}$ (*d*) $\frac{12}{5}$

147. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels. **(M.A.T., 2005)**

(*a*) 32 (*b*) 34

(*c*) 35 (*d*) 36

148. A gardener plants 17956 trees in such a way that there are as many rows as there are trees in a row. The number of trees in a row are **(M.B.A., 2006)**

(*a*) 134 (*b*) 136

(*c*) 144 (*d*) 154

149. The number of trees in each row of a garden is equal to the total number of rows in the garden. After 111 trees have been uprooted in a storm, there remain 10914 trees in the garden. The number of rows of trees in the garden is **(C.P.O., 2007)**

(*a*) 100 (*b*) 105

(*c*) 115 (*d*) 125

150. 1250 oranges were distributed among a group of girls of a class. Each girl got twice as many oranges as the number of girls in that group. The number of girls in the group was **(P.C.S., 2006)**

(*a*) 25 (*b*) 45

(*c*) 50 (*d*) 100

151. A General wishes to draw up his 36581 soldiers in the form of a solid square. After arranging them, he found that some of them are left over. How many are left?

(*a*) 65 (*b*) 81

(*c*) 100 (*d*) None of these

152. A group of students decided to collect as many paise from each member of the group as is the number of members. If the total collection amounts to ₹ 59.29, the number of members in the group is

(*a*) 57 (*b*) 67

(*c*) 77 (*d*) 87

153. A mobile company offered to pay the Indian Cricket Team as much money per run scored by the side as the total number it gets in a one-dayer against Australia. Which one of the following cannot be the total amount to be spent by the company in this deal? **(P.C.S., 2008)**

(*a*) 21,904 (*b*) 56,169

(*c*) 1,01,761 (*d*) 1,21,108

154. $\sqrt[3]{148877} = ?$ **(L.I.C.A.D.O., 2007)**

(*a*) 43 (*b*) 49

(*c*) 53 (*d*) 59

155. $\sqrt[3]{681472} = ?$ **(Bank P.O., 2009)**

(*a*) 76 (*b*) 88

(*c*) 96 (*d*) 98

156. $1728 \div \sqrt[3]{262144} \times ? - 288 = 4491$ **(Bank P.O., 2008)**

(*a*) 148 (*b*) 156

(*c*) 173 (*d*) 177

157. $99 \times 21 - \sqrt[3]{?} = 1968$ **(NABARD, 2008)**

(*a*) 1367631 (*b*) 111

(*c*) 1366731 (*d*) 1367

158. The cube root of .000216 is

(*a*) .6 (*b*) .06

(*c*) .006 (*d*) None of these

159. $\sqrt[3]{4\frac{12}{125}} = ?$

(*a*) $1\frac{2}{5}$ (*b*) $1\frac{3}{5}$

(*c*) $1\frac{4}{5}$ (*d*) $2\frac{2}{5}$

160. $\sqrt[3]{\sqrt{.000064}} = ?$

(*a*) .02 (*b*) .2

(*c*) 2 (*d*) None of these

161. The smallest positive integer n, for which $864n$ is a perfect cube, is **(C.P.O., 2007)**

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

162. Value of $\sqrt{.01} \times \sqrt[3]{.008} - .02$ is **(P.C.S., 2006)**

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

163. The value of $\sqrt[3]{\dfrac{0.2\times0.2\times0.2+0.04\times0.04\times0.04}{0.4\times0.4\times0.4+0.08\times0.08\times0.08}}$ is **(S.S.C., 2005)**

(*a*) 0.125 (*b*) 0.25
(*c*) 0.5 (*d*) 0.75

164. A rationalising factor of $(\sqrt[3]{9}-\sqrt[3]{3}+1)$ is **(S.S.C., 2007)**

(*a*) $\sqrt[3]{3}-1$ (*b*) $\sqrt[3]{3}+1$
(*c*) $\sqrt[3]{9}-1$ (*d*) $\sqrt[3]{9}+1$

165. The largest four-digit number which is a perfect cube, is

(*a*) 8000 (*b*) 9261
(*c*) 9999 (*d*) None of these

166. By what least number must 21600 be multiplied so as to make it a perfect cube? **(M.A.T., 2002)**

(*a*) 6 (*b*) 10
(*c*) 20 (*d*) 30

167. What is the smallest number by which 3600 be divided to make it a perfect cube?

(*a*) 9 (*b*) 50
(*c*) 300 (*d*) 450

168. Which smallest number must be added to 710 so that the sum is a perfect cube? **(S.S.C., 2005)**

(*a*) 11 (*b*) 19
(*c*) 21 (*d*) 29

169. Solve $\sqrt{7921}=?$ **[Indian Railway Gr. 'D' Exam, 2014]**

(*a*) 89 (*b*) 87
(*c*) 37 (*d*) 47

170. Solve $\sqrt[4]{(625)^3}=?$ **[Indian Railway Gr. 'D' Exam, 2014]**

(*a*) $\sqrt[3]{1875}$ (*b*) 25
(*c*) 125 (*d*) None of these

171. If $\sqrt{y}=4x$, then $\dfrac{x^2}{y}$ is **[SSC—CHSL (10+2) Exam, 2015]**

(*a*) 2 (*b*) $\dfrac{1}{16}$
(*c*) $\dfrac{1}{4}$ (*d*) 4

Direction (*Q. No. 172*): What approximate value will come in place of question mark(?) in the given question? (You are not expected to calculate the exact value)

172. $\sqrt{2025.11}\times\sqrt{256.04}+\sqrt{399.95}\times\sqrt{?}=33.98\times40.11$

[IBPS—Bank Spl. Officer (IT) Exam, 2015]

(*a*) 1682 (*b*) 1024
(*c*) 1582 (*d*) 678

173. If $\sqrt{7}$ = 2.645, then the value of $\dfrac{1}{\sqrt{28}}$ up to three places of decimal is **[SSC—CHSL (10+2) Exam, 2015]**

(*a*) 0.183 (*b*) 0.185
(*c*) 0.187 (*d*) 0.189

174. Solve: $\left(\sqrt{\dfrac{25}{9}}-\sqrt{\dfrac{64}{81}}\right)\div\sqrt{\dfrac{16}{324}}=?$

(*a*) 4.5 (*b*) 2.5
(*c*) 1.5 (*d*) 3.5

[United India Insurance Co. Ltd. (UIICL) Assistant (Online) Exam, 2015]

175. Solve $1728\div\sqrt[3]{262144}\times?-288=4491$

(*a*) 148 (*b*) 156
(*c*) 173 (*d*) 177
(e) 185

[United India Insurance Co. Ltd. (UIICL) Assistant (Online) Exam, 2015]

176. Solve: $\left(\sqrt{7}+11\right)^2=(?)^{\frac{1}{3}}+2\sqrt{847}+122$

(*a*) $36+44\sqrt{7}$ (*b*) 6
(*c*) 216 (*d*) 36

[IDBI Bank Executive Officers Exam, 2015]

Directions (*Q. No. 177 & 178*): What will come in place of question mark in these questions?

177. $?-\sqrt{(784)}=6\times\sqrt{(324)}$ **(NICL—AAO Exam, 2015]**

(*a*) 128 (*b*) 160
(*c*) 236 (*d*) 136

178. $\sqrt{(2116)}-\sqrt{1600}=\sqrt{(?)}$ **[NICL—AAO Exam, 2015]**

(*a*) 20 (*b*) 64
(*c*) 81 (*d*) 36

179. Solve $\sqrt{(27\div5\times?)+15}=5.4\div6+0.3$

[IBPS—RRB Office Assistant (Online) Exam, 2015]

(*a*) 2 (*b*) 6
(*c*) 10 (*d*) 4

180. $\sqrt{24\div0.5+1}+\sqrt{18\div0.6+6}$ = ?

(*a*) 19 (*b*) 13
(*c*) 12 (*d*) 15

[IBPS—RRB Office Assistant (Online) Exam, 2015]

181. $\left(\sqrt{63}+\sqrt{252}\right)\times\left(\sqrt{175}+\sqrt{28}\right)=?$

[IBPS—RRB Office Assistant (Online) Exam, 2015]

(*a*) $16\sqrt{7}$ (*b*) 441
(*c*) 16 (*d*) $7\sqrt{7}$

182. $9x^2+25-30x$ can be expressed as the square of **[SSC—CHSL (10+2) Exam, 2015]**

(*a*) $3x^2-25$ (*b*) $3x-5$
(*c*) $-3x-5$ (*d*) $3x-5$

183. If $\sqrt{33}=5.745$, then the value of the following is approximately $\sqrt{\dfrac{3}{11}}$ **[SSC—CHSL (10+2) Exam, 2015]**

(*a*) 1 (*b*) 6.32
(*c*) 0.5223 (*d*) 2.035

184. $\sqrt{?}+14=\sqrt{2601}$ **[SBI Jr. Associates (Pre.) Exam, 2016]**

(*a*) 1521 (*b*) 1369

(*c*) 1225 (*d*) 961

185. If $a=\frac{\sqrt{3}}{2}$, then $\sqrt{1+a}+\sqrt{1-a}=?$

[DMRC—Train Operator (Station Controller) Exam, 2016]

(*a*) $\left(2-\sqrt{3}\right)$ (*b*) $\left(2+\sqrt{3}\right)$

(*c*) $\left(\frac{\sqrt{3}}{2}\right)$ (*d*) $\sqrt{3}$

186. What is $\dfrac{5+\sqrt{10}}{5\sqrt{5}-2\sqrt{20}-\sqrt{32}+\sqrt{50}}$ equal to? **[CDS 2016]**

(*a*) 5 (*b*) $5\sqrt{2}$

(*c*) $5\sqrt{5}$ (*d*) $\sqrt{5}$

187. The square root of $\dfrac{(0.75)^3}{1-0.75}+\left[0.75+(0.75)^2+1\right]$ is **[CDS 2016]**

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

188. $\sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}}$ is equal to

(*a*) $\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)$ (*b*) $\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)$

(*c*) $\left(\sqrt{2}+\sqrt{5}-\sqrt{3}\right)$ (*d*) None of these

[DMRC—Customer Relationship Assistant (CRA) Exam, 2016]

ANSWERS

1. (*b*)	**2.** (*b*)	**3.** (*c*)	**4.** (*c*)	**5.** (*e*)	**6.** (*e*)	**7.** (*a*)	**8.** (*a*)	**9.** (*c*)	**10.** (*b*)
11. (*a*)	**12.** (*c*)	**13.** (*d*)	**14.** (*b*)	**15.** (*b*)	**16.** (*c*)	**17.** (*c*)	**18.** (*c*)	**19.** (*b*)	**20.** (*d*)
21. (*b*)	**22.** (*b*)	**23.** (*e*)	**24.** (*c*)	**25.** (*b*)	**26.** (*d*)	**27.** (*e*)	**28.** (*a*)	**29.** (*c*)	**30.** (*d*)
31. (*a*)	**32.** (*b*)	**33.** (*a*)	**34.** (*c*)	**35.** (*a*)	**36.** (*c*)	**37.** (*c*)	**38.** (*b*)	**39.** (*b*)	**40.** (*b*)
41. (*d*)	**42.** (*d*)	**43.** (*b*)	**44.** (*d*)	**45.** (*b*)	**46.** (*c*)	**47.** (*d*)	**48.** (*d*)	**49.** (*d*)	**50.** (*e*)
51. (*d*)	**52.** (*a*)	**53.** (*d*)	**54.** (*b*)	**55.** (*b*)	**56.** (*a*)	**57.** (*d*)	**58.** (*c*)	**59.** (*b*)	**60.** (*b*)
61. (*a*)	**62.** (*a*)	**63.** (*b*)	**64.** (*c*)	**65.** (*a*)	**66.** (*c*)	**67.** (*a*)	**68.** (*d*)	**69.** (*c*)	**70.** (*c*)
71. (*a*)	**72.** (*b*)	**73.** (*d*)	**74.** (*c*)	**75.** (*b*)	**76.** (*b*)	**77.** (*b*)	**78.** (*d*)	**79.** (*c*)	**80.** (*b*)
81. (*a*)	**82.** (*a*)	**83.** (*b*)	**84.** (*d*)	**85.** (*a*)	**86.** (*c*)	**87.** (*b*)	**88.** (*b*)	**89.** (*c*)	**90.** (*c*)
91. (*b*)	**92.** (*c*)	**93.** (*a*)	**94.** (*b*)	**95.** (*d*)	**96.** (*c*)	**97.** (*c*)	**98.** (*c*)	**99.** (*c*)	**100.** (*c*)
101. (*a*)	**102.** (*b*)	**103.** (*c*)	**104.** (*d*)	**105.** (*b*)	**106.** (*d*)	**107.** (*d*)	**108.** (*c*)	**109.** (*d*)	**110.** (*a*)
111. (*c*)	**112.** (*b*)	**113.** (*d*)	**114.** (*d*)	**115.** (*a*)	**116.** (*e*)	**117.** (*a*)	**118.** (*b*)	**119.** (*c*)	**120.** (*b*)
121. (*a*)	**122.** (*c*)	**123.** (*d*)	**124.** (*b*)	**125.** (*a*)	**126.** (*b*)	**127.** (*b*)	**128.** (*a*)	**129.** (*c*)	**130.** (*d*)
131. (*c*)	**132.** (*b*)	**133.** (*b*)	**134.** (*a*)	**135.** (*c*)	**136.** (*d*)	**137.** (*b*)	**138.** (*b*)	**139.** (*c*)	**140.** (*b*)
141. (*b*)	**142.** (*b*)	**143.** (*c*)	**144.** (*c*)	**145.** (*a*)	**146.** (*d*)	**147.** (*d*)	**148.** (*a*)	**149.** (*b*)	**150.** (*a*)
151. (*c*)	**152.** (*c*)	**153.** (*d*)	**154.** (*c*)	**155.** (*b*)	**156.** (*d*)	**157.** (*a*)	**158.** (*b*)	**159.** (*b*)	**160.** (*b*)
161. (*b*)	**162.** (*a*)	**163.** (*c*)	**164.** (*b*)	**165.** (*b*)	**166.** (*b*)	**167.** (*d*)	**168.** (*b*)	**169.** (*a*)	**170.** (*c*)
171. (*b*)	**172.** (*b*)	**173.** (*d*)	**174.** (*d*)	**175.** (*d*)	**176.** (*c*)	**177.** (*d*)	**178.** (*d*)	**179.** (*d*)	**180.** (*b*)
181. (*b*)	**182.** (*d*)	**183.** (*c*)	**184.** (*b*)	**185.** (*d*)	**186.** (*d*)	**187.** (*b*)	**188.** (*a*)		

SOLUTIONS

1.

$$\begin{array}{r|l} 2 & 5\ \overline{38}\ \overline{24}\ (232 \\ & 4 \\ \hline 43 & 1\ 38 \\ & 1\ 29 \\ \hline 462 & \quad 9\ 24 \\ & \quad 9\ 24 \\ \hline & \quad \times \end{array}$$

$\therefore \sqrt{53824}=232.$

2.

$$\begin{array}{r|l} 2 & 4\ \overline{12}\ \overline{09}\ (203 \\ & 4 \\ \hline 40 & 12 \\ & 0 \\ \hline 403 & 12\ 09 \\ & 12\ 09 \\ \hline & \quad \times \end{array}$$

$\therefore \sqrt{41209}=203.$

3.

$$\begin{array}{r|l} 1 & \overline{1}\ \overline{23}\ \overline{45}\ \overline{43}\ \overline{21}\ (11111 \\ & 1 \\ \hline 21 & 23 \\ & 21 \\ \hline 221 & \ 2\ 45 \\ & \ 2\ 21 \\ \hline 2221 & \ \ 24\ 43 \\ & \ \ 22\ 21 \\ \hline 22221 & \ \ \ 222\ 21 \\ & \ \ \ 222\ 21 \\ \hline & \qquad \times \end{array}$$

$\therefore \sqrt{123454321}=11111.$

4. The number of digits in the square root of a perfect square number of n digits is

(*i*) $\frac{n}{2}$, if n is even

(*ii*) $\frac{n+1}{2}$, if n is odd

Here, n = 12. So, required number of digits = $\frac{n}{2} = \frac{12}{2} = 6.$

5. $\sqrt{\sqrt{17956} + \sqrt{24025}} = \sqrt{134 + 155} = \sqrt{289} = 17.$

6. $\sqrt{\sqrt{44944} + \sqrt{52441}} = \sqrt{212 + 229} = \sqrt{441} = 21.$

7. Sum of prime numbers greater than 4 but less than 16 = (5 + 7 + 11 + 13) = 36.

$\therefore \quad \frac{1}{4} \times 36 = 9 = 3^2.$

8. Given exp. $= \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + 15}}}}$

$= \sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{169}}}}$

$= \sqrt{10 + \sqrt{25 + \sqrt{108 + 13}}}$

$= \sqrt{10 + \sqrt{25 + \sqrt{121}}}$

$= \sqrt{10 + \sqrt{25 + 11}} = \sqrt{10 + \sqrt{36}}$

$= = \sqrt{10 + 6} = \sqrt{16} = 4.$

9. Given exp. $= \sqrt{41 - \sqrt{21 + \sqrt{19 - 3}}}$

$= \sqrt{41 - \sqrt{21 + \sqrt{16}}} = \sqrt{41 - \sqrt{21 + 4}}$

$= \sqrt{41 - \sqrt{25}} = \sqrt{41 - 5} = \sqrt{36} = 6.$

10. Given exp. $= \sqrt{176 + 49} = \sqrt{225} = 15.$

11. Given exp. $= \frac{14}{7} \times \frac{21}{7} \times \frac{120}{15} = 2 \times 3 \times 8 = 48.$

12. Given exp. $= \left(\frac{\sqrt{225}}{\sqrt{729}} - \frac{\sqrt{25}}{\sqrt{144}}\right) \div \frac{\sqrt{16}}{\sqrt{81}}$

$= \left(\frac{15}{27} - \frac{5}{12}\right) \div \frac{4}{9} = \left(\frac{15}{108} \times \frac{9}{4}\right) = \frac{5}{16}.$

13. Let $(15)^2 + (18)^2 - 20 = \sqrt{x}$.

Then, $\sqrt{x} = 225 + 324 - 20 = 529$

$\Leftrightarrow x = (529)^2 = 279841.$

14. Let $\sqrt{x} \times \sqrt{484} = 1034$. Then, $\sqrt{x} \times 22 = 1034$

$\Leftrightarrow \sqrt{x} = \frac{1034}{22} = 47$

$\Leftrightarrow x = (47)^2 = 2209.$

15. Let $\sqrt{11881} \times \sqrt{x} = 10137$. Then, $109 \times \sqrt{x} = 10137$

$\Leftrightarrow \quad \sqrt{x} = \frac{10137}{109} = 93$

$\Leftrightarrow \quad x = (93)^2 = 8649.$

16. $\frac{4050}{\sqrt{x}} = 450 \Leftrightarrow \sqrt{x} = \frac{4050}{450} = 9$

$\Leftrightarrow x = 9^2 = 81.$

17. Let $\sqrt{\frac{16}{25}} \times \sqrt{\frac{x}{25}} \times \frac{16}{25} = \frac{256}{625}.$

Then, $\frac{4}{5} \times \frac{\sqrt{x}}{2} \times \frac{16}{25} = \frac{256}{625}$

$\Leftrightarrow \frac{64\sqrt{x}}{625} = \frac{256}{625}$

$\Leftrightarrow \sqrt{x} = \frac{256}{625} \times \frac{625}{64}$

$\Leftrightarrow x = 4^2 = 16.$

18. $\sqrt{(272)^2 - (128)^2} = \sqrt{(272 + 128)(272 - 128)}$

$= \sqrt{400 \times 144} = \sqrt{57600} = 240.$

19. $6 * 24 = 6 + 24 + \sqrt{6 \times 24} = 30 + \sqrt{144}$

$= 30 + 12 = 42.$

20. $10y\sqrt{y^3 - y^2} = 10 \times 5\sqrt{5^3 - 5^2}$

$= 50 \times \sqrt{125 - 25} = 50 \times \sqrt{100}$

$= 50 \times 10 = 500.$

21. $\sqrt{110\frac{1}{4}} = \sqrt{\frac{441}{4}} = \frac{\sqrt{441}}{\sqrt{4}} = \frac{21}{2} = 10.5.$

22. $\sqrt{\frac{25}{81} - \frac{1}{9}} = \sqrt{\frac{25 - 9}{81}} = \sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}} = \frac{4}{9}.$

23. Let $[(\sqrt{81})^2]^2 = x^2$. Then, $x^2 = (81)^2$ or $x = 81$.

24.

1	$\overline{1}\ \overline{58}\ \overline{76}$ (126
	1
22	58
	44
246	14 76
	14 76
	×

$\therefore \quad \sqrt{15876} = 126.$

25.

1	$\overline{3}.\overline{00}\ \overline{00}\ \overline{00}$ (1.732
	1
27	2 00
	1 89
343	11 00
	10 29
3492	71 00
	69 84
	1 16

$\therefore \quad \sqrt{3} = 1.73 = \frac{173}{100}.$

26. A number ending in 8 can never be a perfect square.

27. The squares of numbers having 1 and 9 as the unit's digit end in the digit 1.

Such numbers are: 1, 9, 11, 19, 21, 29, 31, 39, 41, 49 i.e., there are 10 such numbers.

$\therefore$ Required percentage $= \left(\dfrac{10}{50} \times 100\right)\% = 20\%.$

28. Let the given number be x.

Then, $x^2 - 25 = (x - 25)^2 \Leftrightarrow x^2 - 25 = x^2 + 625 - 50x \Leftrightarrow 50x = 650 \Leftrightarrow x = 13.$

29. $(11)^2 = 121$ and $(17)^2 = 289$.

So, the perfect squares between 120 and 300 are the squares of numbers from 11 to 17. Clearly, these are 7 in number.

30. The first perfect square number after 50 is 64 (= 8^2) and the last perfect square number before 1000 is 961 [= $(31)^2$].

So, the perfect squares between 50 and 1000 are the squares of numbers from 8 to 31. Clearly, these are 24 in number.

31. Clearly, the man was born between 1800 and 1850.

The only perfect square number between 1800 and 1850 is 1849. And, 1849 = $(43)^2$.

So, the man was 43 years old in 1849. Thus, he was born in (1849 – 43) = 1806.

32. $Q = \sqrt{\dfrac{(8R)^2}{4}} = \dfrac{\sqrt{(8R)^2}}{\sqrt{4}} = \dfrac{8R}{2} = 4R.$

33. The smallest such number is 1444[= $(38)^2$]. It lies between 1000 and 2000.

34. $\left(\sqrt{2} + \dfrac{1}{\sqrt{2}}\right)^2 = (\sqrt{2})^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 + 2 \times \sqrt{2} \times \dfrac{1}{\sqrt{2}}$

$= 2 + \dfrac{1}{2} + 2 = 4\dfrac{1}{2}.$

35. We have:

$1 \times 2 \times 3 \times 4 = 24$ and $24 + 1 = 25$ [= 5^2].

$2 \times 3 \times 4 \times 5 = 120$ and $120 + 1 = 121$ [= 11^2]

$3 \times 4 \times 5 \times 6 = 360$ and $360 + 1 = 361$ [= 19^2]

$4 \times 5 \times 6 \times 7 = 840$ and $840 + 1 = 841$ [= 29^2].

$\therefore p = 1.$

36. $\sqrt{0.16} = \sqrt{\dfrac{16}{100}} = \dfrac{\sqrt{16}}{\sqrt{100}} = \dfrac{4}{10} = 0.4.$

37. $\sqrt{0.000441} = \sqrt{\dfrac{441}{10^6}} = \dfrac{\sqrt{441}}{\sqrt{10^6}} = \dfrac{21}{10^3} = \dfrac{21}{1000} = 0.021.$

38. $\sqrt{0.00004761} = \sqrt{\dfrac{4761}{10^8}} = \dfrac{\sqrt{4761}}{\sqrt{10^8}} = \dfrac{69}{10^4} = \dfrac{69}{10000} = 0.0069.$

39. $1.5^2 \times \sqrt{0.0225} = 1.5^2 \times \sqrt{\dfrac{225}{10000}} = 2.25 \times \dfrac{15}{100}$

$= 2.25 \times 0.15 = 0.3375.$

40. $\sqrt{0.01 + \sqrt{0.0064}} = \sqrt{0.01 + \sqrt{\dfrac{64}{10000}}}$

$= \sqrt{0.01 + \dfrac{8}{100}} = \sqrt{0.01 + 0.08} = \sqrt{0.09} = 0.3.$

41. Given exp. $= \sqrt{\dfrac{1}{100}} + \sqrt{\dfrac{81}{100}} + \sqrt{\dfrac{121}{100}} + \sqrt{\dfrac{9}{10000}}$

$= \dfrac{1}{10} + \dfrac{9}{10} + \dfrac{11}{10} + \dfrac{3}{100}$

$= 0.1 + 0.9 + 1.1 + 0.03 = 2.13.$

42. Given exp. $= \sqrt{\dfrac{25}{10000}} \times \sqrt{\dfrac{225}{100}} \times \sqrt{\dfrac{1}{10000}}$

$= \dfrac{5}{100} \times \dfrac{15}{10} \times \dfrac{1}{100} = \dfrac{75}{100000} = 0.00075.$

43.

$$\begin{array}{r|l}
1 & 1.\overline{56}\ \overline{25}\ (1.25 \\
 & 1 \\ \hline
22 & \ 56 \\
 & \ 44 \\ \hline
245 & \ 12\ 25 \\
 & \ 12\ 25 \\ \hline
 & \quad \times
\end{array}$$

$\therefore \sqrt{1.5625} = 1.25.$

44. $\sqrt{6760000} = \sqrt{0.00000676 \times 10^{12}} = \sqrt{0.00000676} \times \sqrt{10^{12}}$

$= .0026 \times 10^6 = 2600.$

45. Given exp. $= \sqrt{\dfrac{18225}{10^2}} + \sqrt{\dfrac{18225}{10^4}} + \sqrt{\dfrac{18225}{10^6}} + \sqrt{\dfrac{18225}{10^8}}$

$= \dfrac{\sqrt{18225}}{10} + \dfrac{\sqrt{18225}}{10^2} + \dfrac{\sqrt{18225}}{10^3} + \dfrac{\sqrt{18225}}{10^4}$

$= \dfrac{135}{10} + \dfrac{135}{100} + \dfrac{135}{1000} + \dfrac{135}{10000}$

$= 13.5 + 1.35 + 0.135 + 0.0135 = 14.9985.$

46. Given exp. $= \sqrt{\dfrac{4096}{10^2}} + \sqrt{\dfrac{4096}{10^4}} + \sqrt{\dfrac{4096}{10^6}} + \sqrt{\dfrac{4096}{10^8}}$

$= \dfrac{\sqrt{4096}}{10} + \dfrac{\sqrt{4096}}{10^2} + \dfrac{\sqrt{4096}}{10^3} + \dfrac{\sqrt{4096}}{10^4}$

$= \dfrac{64}{10} + \dfrac{64}{100} + \dfrac{64}{1000} + \dfrac{64}{10000}$

$= 6.4 + 0.64 + 0.064 + 0.0064$

$= 7.1104 \approx 7.11.$

47. Given exp. $= \sqrt{1.30} + \sqrt{1300} + \sqrt{0.0130}$

$= \sqrt{\dfrac{130}{100}} + \sqrt{13 \times 100} + \sqrt{\dfrac{130}{10000}}$

$= \dfrac{\sqrt{130}}{10} + \sqrt{13} \times 10 + \dfrac{\sqrt{130}}{100}$

$= \dfrac{11.40}{10} + 3.605 \times 10 + \dfrac{11.40}{100}$

$= 1.14 + 36.05 + 0.114 = 37.304.$

48. $\dfrac{52}{x} = \sqrt{\dfrac{169}{289}}$

$\Leftrightarrow \dfrac{52}{x} = \dfrac{13}{17}$

$\Leftrightarrow x = \left(\dfrac{52 \times 17}{13}\right) = 68.$

49. Let the missing number be x.

Then, $x^2 = 15 \times 135 \Leftrightarrow x = \sqrt{15 \times 135}$

$= \sqrt{15^2 \times 3^2} = 15 \times 3 = 45.$

50. Let $\frac{x}{1776}=\frac{111}{x}$. Then, x^2

$= 111 \times 1776 = 111 \times 111 \times 16$

$\Rightarrow x = \sqrt{(111)^2 \times (4)^2} = 111 \times 4 = 444.$

51. Let $\frac{4\frac{1}{2}}{x}=\frac{x}{32}$. Then, $x^2 = 32\times\frac{9}{2}=144 \Leftrightarrow x=\sqrt{144}=12.$

52. Let $\frac{x}{\sqrt{128}}=\frac{\sqrt{162}}{x}$. Then, $x^2=\sqrt{128\times162}=\sqrt{64\times2\times18\times9}$

$=\sqrt{8^2\times6^2\times3^2}=8\times6\times3=144.$

$\therefore\ x=\sqrt{144}=12.$

53. $\sqrt{x+\frac{x}{y}}=x\sqrt{\frac{x}{y}} \Rightarrow x+\frac{x}{y}=x^2\cdot\frac{x}{y} \Rightarrow \frac{xy+x}{y}=\frac{x^3}{y}$

$\Rightarrow\ xy + x = x^3$

$\Rightarrow\ y + 1 = x^2$

$\Rightarrow\ y = x^2 - 1.$

54. $n^2 = (25)^{64} \times (64)^{25} = (5^2)^{64} \times (2^6)^{25}$

$= 5^{128} \times 2^{150} = 5^{128} \times 2^{128} \times 2^{22}$

$\Rightarrow n = 5^{64} \times 2^{64} \times 2^{11} = (5 \times 2)^{64} \times 2^{11} = 10^{64} \times 2048.$

$\therefore$ Sum of digits of $n = 2 + 0 + 4 + 8 = 14.$

55. $\frac{0.13}{p^2}=13$

$\Leftrightarrow\ p^2=\frac{0.13}{13}=\frac{1}{100}$

$\Leftrightarrow\ p=\sqrt{\frac{1}{100}}=\frac{1}{10}=0.1.$

56. Let the required number be x. Then, $\frac{x}{\sqrt{0.25}}=25$

$\Leftrightarrow\ \frac{x}{0.5}=25$

$\Leftrightarrow\ x=25\times0.5=12.5.$

57. $\sqrt{3^n}=729=3^6 \Leftrightarrow (\sqrt{3^n})^2=(3^6)^2 \Leftrightarrow 3^n=3^{12} \Leftrightarrow n=12.$

58. $\sqrt{18\times14\times x} = 84 \Leftrightarrow 18 \times 14 \times x = 84 \times 84$

$\Leftrightarrow x = \frac{84\times84}{18\times14}=28.$

59. Let $28\sqrt{x}+1426 = 3 \times 718.$

Then, $28\sqrt{x}=2154-1426 \Leftrightarrow 28\sqrt{x}=728 \Leftrightarrow \sqrt{x} = 26$

$\Leftrightarrow x = (26)^2 = 676.$

60. Let $\sqrt{\frac{x}{169}}=\frac{54}{39}$. Then, $\frac{\sqrt{x}}{13}=\frac{54}{39} \Leftrightarrow \sqrt{x}=\left(\frac{54}{39}\times13\right)=18$

$\Leftrightarrow\ x=(18)^2=324.$

61. $\frac{\sqrt{x}}{\sqrt{441}}=0.02 \Leftrightarrow \frac{\sqrt{x}}{21}=0.02$

$\Leftrightarrow\ \sqrt{x}=0.02\times21=0.42 \Leftrightarrow x$

$= (0.42)^2 = 0.1764.$

62. Let $\sqrt{\frac{.0196}{x}}=0.2$. Then, $\frac{.0196}{x}=0.04$

$\Leftrightarrow\ x=\frac{.0196}{.04}=\frac{1.96}{4}=.49.$

63. Let $\sqrt{0.0169\times x}=1.3$. Then, $0.0169x = (1.3)^2 = 1.69$

$\Rightarrow x = \frac{1.69}{0.0169}=100.$

64. $37+\sqrt{.0615+x}=37.25 \Leftrightarrow \sqrt{.0615+x}=0.25$

$\Leftrightarrow .0615 + x = (0.25)^2 = 0.0625$

$\Leftrightarrow x=.001=\frac{1}{10^3}=10^{-3}.$

65. $\sqrt{(x-1)(y+2)} = 7 \Rightarrow (x - 1)(y + 2) = (7)^2 \Rightarrow (x - 1) = 7$ and $(y + 2) = 7 \Rightarrow x = 8$ and $y = 5.$

66. $\frac{\sqrt{a}}{\sqrt{b}}=\frac{.004\times.4}{\sqrt{.04\times.4}} \Rightarrow \frac{a}{b}=\frac{.004\times.4\times.004\times.4}{.04\times.4}=\frac{.0000064}{.04}$

$\Rightarrow\ \frac{a}{b}=\frac{.00064}{4}=.00016=\frac{16}{10^5}=16\times10^{-5}.$

67. Let the number be x. Then,

$\frac{3}{5}x^2=126.15 \Leftrightarrow x^2=\left(126.15\times\frac{5}{3}\right)=210.25$

$\Leftrightarrow\ x=\sqrt{210.25}=14.5.$

68. $\sqrt{\frac{0.361}{0.00169}}=\sqrt{\frac{0.36100}{0.00169}}=\sqrt{\frac{36100}{169}}=\frac{190}{13}.$

69. $\sqrt{\frac{48.4}{0.289}}=\sqrt{\frac{48.400}{0.289}}=\sqrt{\frac{48400}{289}}=\frac{220}{17}=12\frac{16}{17}.$

70. $\sqrt{1+\frac{x}{169}}=\frac{14}{13} \Rightarrow 1+\frac{x}{169}=\frac{196}{169}$

$\Rightarrow\ \frac{x}{169}=\left(\frac{196}{169}-1\right)=\frac{27}{169} \Rightarrow x=27.$

71. $\sqrt{1+\frac{55}{729}}=1+\frac{x}{27} \Rightarrow \sqrt{\frac{784}{729}}=\frac{27+x}{27}$

$\Rightarrow\ \frac{28}{27}=\frac{27+x}{27} \Rightarrow 27+x=28 \Rightarrow x=1.$

72. $\sqrt{\frac{4}{3}}-\sqrt{\frac{3}{4}}=\frac{\sqrt{4}}{\sqrt{3}}-\frac{\sqrt{3}}{\sqrt{4}}=\frac{\sqrt{4}\times\sqrt{4}-\sqrt{3}\times\sqrt{3}}{\sqrt{12}}=\frac{4-3}{2\sqrt{3}}=\frac{1}{2\sqrt{3}}.$

73.

$$\begin{array}{r|l}
1 & \overline{2}.\overline{00}\ \overline{00}\ \overline{00}\ (1.414 \\
 & 1 \\
\hline
24 & 100 \\
 & 96 \\
\hline
281 & 400 \\
 & 281 \\
\hline
2824 & 11900 \\
 & 11296 \\
\hline
\end{array}$$

$\therefore\ \sqrt{2}=1.414$

74. $2\sqrt{27}-\sqrt{75}+\sqrt{12}=2\sqrt{9\times3}-\sqrt{25\times3}+\sqrt{4\times3}$

$=6\sqrt{3}-5\sqrt{3}+2\sqrt{3}=3\sqrt{3}.$

75. $(\sqrt{12}+\sqrt{18})-(\sqrt{3}+\sqrt{2})=(\sqrt{4\times3}+\sqrt{9\times2})-(\sqrt{3}+\sqrt{2})$

$=(2\sqrt{3}+3\sqrt{2})-(\sqrt{3}+\sqrt{2})$

$=(2\sqrt{3}-\sqrt{3})+(3\sqrt{2}-\sqrt{2})=\sqrt{3}+2\sqrt{2}.$

76. $\dfrac{\sqrt{24}+\sqrt{216}}{\sqrt{96}}=\dfrac{\sqrt{4\times6}+\sqrt{36\times6}}{\sqrt{16\times6}}=\dfrac{2\sqrt{6}+6\sqrt{6}}{4\sqrt{6}}=\dfrac{8\sqrt{6}}{4\sqrt{6}}=2.$

77. $\dfrac{\sqrt{80}-\sqrt{112}}{\sqrt{45}-\sqrt{63}}=\dfrac{\sqrt{16\times5}-\sqrt{16\times7}}{\sqrt{9\times5}-\sqrt{9\times7}}$

$=\dfrac{4\sqrt{5}-4\sqrt{7}}{3\sqrt{5}-3\sqrt{7}}=\dfrac{4(\sqrt{5}-\sqrt{7})}{3(\sqrt{5}-\sqrt{7})}=\dfrac{4}{3}=1\dfrac{1}{3}.$

78. $3\sqrt{5}+\sqrt{125}=17.88 \Rightarrow\ 3\sqrt{5}+\sqrt{25\times5}=17.88$

$\Rightarrow\ 3\sqrt{5}+5\sqrt{5}=17.88\ \Rightarrow\ 8\sqrt{5}=17.88 \Rightarrow\ \sqrt{5}=2.235.$

$\therefore\ \sqrt{80}+6\sqrt{5}=\sqrt{16\times5}+6\sqrt{5}=4\sqrt{5}+6\sqrt{5}$

$=10\sqrt{5}=(10\times2.235)=22.35.$

79. $\sqrt{50}\times\sqrt{98}=\sqrt{50\times98}=\sqrt{4900}=70.$

80. Given exp. $=\sqrt{4\times2}+2\sqrt{16\times2}-3\sqrt{64\times2}+4\sqrt{25\times2}$

$=2\sqrt{2}+8\sqrt{2}-24\sqrt{2}+20\sqrt{2}=6\sqrt{2}=6\times1.414=8.484.$

81. Given exp. $=\dfrac{3\sqrt{12}}{2\sqrt{28}}\times\dfrac{\sqrt{98}}{2\sqrt{21}}=\dfrac{3\sqrt{4\times3}}{2\sqrt{4\times7}}\times\dfrac{\sqrt{49\times2}}{2\sqrt{21}}$

$=\dfrac{6\sqrt{3}}{4\sqrt{7}}\times\dfrac{7\sqrt{2}}{2\sqrt{21}}=\dfrac{21\sqrt{6}}{4\sqrt{7\times21}}=\dfrac{21\sqrt{6}}{28\sqrt{3}}$

$=\dfrac{3}{4}\sqrt{2}=\dfrac{3}{4}\times1.414=3\times0.3535=1.0605.$

82. Given exp. $=\sqrt{\dfrac{11025}{100}}\times\sqrt{\dfrac{1}{100}}\div\sqrt{\dfrac{25}{10000}}-\sqrt{\dfrac{42025}{100}}$

$=\dfrac{105}{10}\times\dfrac{1}{10}\div\dfrac{5}{100}-\dfrac{205}{10}$

$=\dfrac{105}{100}\times\dfrac{100}{5}-\dfrac{205}{10}$

$=21-\dfrac{205}{10}=\dfrac{5}{10}=\dfrac{1}{2}=0.50.$

83. Sum of decimal places in the numerator and denominator under the radical sign being the same, we remove the decimal.

$\therefore$ Given exp. $=\sqrt{\dfrac{81\times484}{64\times625}}=\dfrac{9\times22}{8\times25}=0.99.$

84. Given exp. $=\sqrt{\dfrac{204\times42}{7\times34}}=\sqrt{36}=6.$

85. Given exp. $=\sqrt{\dfrac{81\times324\times4624}{15625\times289\times729\times64}}$

$=\dfrac{9\times18\times68}{125\times17\times27\times8}=\dfrac{3}{125}=0.024.$

86. Given exp. $=\sqrt{\dfrac{9.5\times.08500}{.19\times.0017}}=\sqrt{\dfrac{95\times8500}{19\times17}}$

$=\sqrt{5\times500}=\sqrt{2500}=50.$

87. Given exp. $=\sqrt{\dfrac{(0.03)^2+(0.21)^2+(0.065)^2}{\left(\dfrac{0.03}{10}\right)^2+\left(\dfrac{0.21}{10}\right)^2+\left(\dfrac{0.065}{10}\right)^2}}$

$=\sqrt{\dfrac{100\,[(0.03)^2+(0.21)^2+(0.065)^2]}{(0.03)^2+(0.21)^2+(0.065)^2}}=\sqrt{100}=10.$

88. $\sqrt{(7+3\sqrt{5})\,(7-3\sqrt{5})}=\sqrt{(7)^2-(3\sqrt{5})^2}$

$=\sqrt{49-45}=\sqrt{4}=2.$

89. $\left(\sqrt{3}-\dfrac{1}{\sqrt{3}}\right)^2=(\sqrt{3})^2+\left(\dfrac{1}{\sqrt{3}}\right)^2-2\times\sqrt{3}\times\dfrac{1}{\sqrt{3}}$

$=3+\dfrac{1}{3}-2=1+\dfrac{1}{3}=\dfrac{4}{3}.$

90. $\sqrt{4a^2-4a+1}+3a$

$=\sqrt{(1)^2+(2a)^2-2\times1\times2a}+3a=\sqrt{(1-2a)^2}+3a$

$=(1-2a)+3a=(1+a)=(1+0.1039)=1.1039.$

91. $\sqrt{\dfrac{(0.75)^3}{(1-0.75)}+[0.75+(0.75)^2+1]}$

$=\sqrt{\dfrac{(0.75)^3+(1-0.75)\,[(1)^2+(0.75)^2+1\times0.75]}{1-0.75}}$

$=\sqrt{\dfrac{(0.75)^3+[(1)^3-(0.75)^3]}{1-0.75}}$

$=\sqrt{\dfrac{1}{0.25}}=\sqrt{\dfrac{100}{25}}=\sqrt{4}=2.$

92. $a+b+c=27\sqrt{29}\ \Rightarrow\ 2c+\dfrac{3}{2}c+c=27\sqrt{29}$

$\Rightarrow\dfrac{9}{2}c=27\sqrt{29}\ \Rightarrow\ c=6\sqrt{29}.$

$\sqrt{a^2+b^2+c^2}=\sqrt{(a+b+c)^2-2\,(ab+bc+ca)}$

$=\sqrt{(27\sqrt{29})^2-2\left(2c\times\dfrac{3}{2}c+\dfrac{3}{2}c\times c+c\times2c\right)}$

$=\sqrt{(729\times29)-2\left(3c^2+\dfrac{3}{2}c^2+2c^2\right)}$

$=\sqrt{(729\times29)-2\times\dfrac{13}{2}c^2}$

$=\sqrt{(729\times29)-13\times(6\sqrt{29})^2}=\sqrt{29\,(729-468)}$

$=\sqrt{29\times261}=\sqrt{29\times29\times9}=29\times3=87.$

93. $\sqrt{0.\bar{4}}=\sqrt{\dfrac{4}{9}}=\dfrac{2}{3}=0.666.....=0.\bar{6}.$

94. $\sqrt{0.09}=\sqrt{\dfrac{9}{100}}=\dfrac{3}{10}=0.3$, which is rational.

95.
$$\begin{array}{r|l} 6 & 0.\overline{40}\ \overline{00}\ \overline{00}\ (.63 \\ & 36 \\ \hline 123 & 4\ 00 \\ & 3\ 69 \\ \hline \end{array}$$

$\therefore\ \sqrt{0.4} = 0.63$

96.
$$\begin{array}{r|l} 4 & 0.\overline{20}\ \overline{00}\ \overline{00}\ (.447 \\ & 16 \\ \hline 84 & 4\ 00 \\ & 3\ 36 \\ \hline 887 & 64\ 00 \\ & 62\ 09 \\ \hline \end{array}$$

$\therefore\ \sqrt{0.2} = 0.447$

97.
$$\begin{array}{r|l} 3 & 0.\overline{12}\ \overline{10}\ \overline{00}\ (.347 \\ & 9 \\ \hline 64 & 3\ 10 \\ & 2\ 56 \\ \hline 687 & 54\ 00 \\ & 48\ 00 \\ \hline \end{array}$$

$\therefore\ \sqrt{0.121} = 0.347$

98.
$$\begin{array}{r|l} 2 & 0.\overline{06}\ \overline{40}\ \overline{00}\ (.252 \\ & 4 \\ \hline 45 & 2\ 40 \\ & 2\ 25 \\ \hline 502 & 1500 \\ & 1004 \\ \hline \end{array}$$

$\therefore\ \sqrt{0.064} = 0.252$

99. $\sqrt{\dfrac{0.16}{0.4}} = \sqrt{\dfrac{0.16}{0.40}} = \sqrt{\dfrac{16}{40}} = \sqrt{\dfrac{4}{10}} = \sqrt{0.4} = 0.63.$

100. $\dfrac{1+\sqrt{0.01}}{1-\sqrt{0.1}} = \dfrac{1+0.1}{1-0.316} = \dfrac{1.1}{0.684} = \dfrac{1100}{684} = 1.6.$

$$\boxed{\begin{array}{r|l} 3 & 0.\overline{10}\ \overline{00}\ \overline{00}\ (0.316 \\ & 9 \\ \hline 61 & 1\ 00 \\ & 61 \\ \hline 626 & 39\ 00 \\ & 37\ 56 \\ \hline \end{array}}$$

101. $\sqrt{535.9225} = \sqrt{\dfrac{5359225}{10000}} = \dfrac{2315}{100} = 23.15.$

$$\begin{array}{r|l} 2 & \overline{5}\ \overline{35}\ \overline{92}\ \overline{25}\ (2315 \\ & 4 \\ \hline 43 & 1\ 35 \\ & 1\ 29 \\ \hline 461 & 6\ 92 \\ & 4\ 61 \\ \hline 4625 & 231\ 25 \\ & 231\ 25 \\ \hline & \times \end{array}$$

102. $\dfrac{1}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5}}{5} = \dfrac{2.236}{5} = 0.447.$

103. $\sqrt{\dfrac{8}{3}} = \sqrt{\dfrac{8\times 3}{3\times 3}} = \dfrac{\sqrt{24}}{3} = \dfrac{4.899}{3} = 1.633.$

104. $\dfrac{3\sqrt{2}}{2\sqrt{3}} = \dfrac{3\sqrt{2}}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{6}}{2\times 3} = \dfrac{\sqrt{6}}{2} = \dfrac{2.449}{2} = 1.2245.$

105. $\dfrac{\sqrt{5}}{2} - \dfrac{10}{\sqrt{5}} + \sqrt{125} = \dfrac{(\sqrt{5})^2 - 20 + 2\sqrt{5}\times 5\sqrt{5}}{2\sqrt{5}}$

$= \dfrac{5-20+50}{2\sqrt{5}} = \dfrac{35}{2\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{35\sqrt{5}}{10} = \dfrac{7}{2} \times 2.236$

$= 7 \times 1.118 = 7.826.$

106. Clearly, $a * b = \sqrt{a^2+b^2}$. $\therefore$ $5 * 12$

$= \sqrt{5^2+12^2} = \sqrt{25+144} = \sqrt{169} = 13.$

107. Let the missing digit be x.

$$\begin{array}{r|l} 1 & \overline{1}\ \overline{53}\ \overline{7x}\ (124 \\ & 1 \\ \hline 22 & 53 \\ & 44 \\ \hline 244 & 9\ 7x \\ & 9\ 76 \\ \hline & \times \\ \hline \end{array}$$

Then, $x = 6$.

108. $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 7 \times 2 \times 3 \times 5 \times 2^2 \times 3 \times 2$

$= 2^4 \times 3^2 \times 5 \times 7.$

Thus, the smallest perfect square number which is divisible by 7! is

$(2^4 \times 3^2 \times 5 \times 7) \times (5 \times 7)$

$= 5040 \times 35 = 176400.$

109. L.C.M. of 3, 4, 5, 6, 8 is 120. Now, 120

$= 2 \times 2 \times 2 \times 3 \times 5.$

To make it a perfect square, it must be multiplied by $2 \times 3 \times 5$.

So, required number $= 2^2 \times 2^2 \times 3^2 \times 5^2 = 3600.$

110. L.C.M. of 21, 36, 66

$= 2772$. Now, 2772

$= 2 \times 2 \times 3 \times 3 \times 7 \times 11.$

To make it a perfect square, it must be multiplied by 7×11.

So, required number $= 2^2 \times 3^2 \times 7^2 \times 11^2 = 213444.$

111. $294 = 7 \times 7 \times 2 \times 3.$

To make it a perfect square, it must be multiplied by 2×3 *i.e.*, 6.

$\therefore$ Required number = 6.

112. $5808 = 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11 = 2^2 \times 2^2 \times 3 \times 11^2.$

To make it a perfect square, it must be multiplied by 3.

113. $1470 = 7 \times 7 \times 5 \times 6$. To make it a perfect square, it must be divided by 5×6, *i.e.*, 30.

114.

```
     7 | 54 91 62 (741
       | 49
       |------------
   144 |  5 91
       |  5 76
       |------------
  1481 |    15 62
       |    14 81
       |------------
       |       81
```

$\therefore$ Required number to be subtracted = 81.

115. $0.000326 = \dfrac{326}{10^6}$

```
     1 | 3 26 (18
       | 1
       |--------
    28 | 2 26
       | 2 24
       |--------
       |    2
```

$\therefore$ Required number to be subtracted $= \dfrac{2}{10^6} = 0.000002.$

116.

```
     8 | 77 00 (87
       | 64
       |---------
   167 | 13 00
       | 11 69
       |---------
       |  1 31
```

$\therefore$ Number to be added

$= (88)^2 - 7700 = 7744 - 7700 = 44.$

117.

```
     8 | 68 06 21 (824
       | 64
       |------------
   162 |  4 06
       |  3 24
       |------------
  1644 |    82 21
       |    65 76
       |------------
       |    16 45
```

$\therefore$ Number to be added $= (825)^2 - 680621$

$= 680625 - 680621 = 4.$

118. Greatest number of four digits is 9999.

```
     9 | 99 99 (99
       | 81
       |---------
   189 | 18 99
       | 17 01
       |---------
       |  1 98
```

$\therefore$ Reqd. number = (9999 – 198) = 9801.

119. Least number of 4 digits is 1000.

```
     3 | 10 00 (31
       |  9
       |---------
    61 |  1 00
       |    61
       |---------
       |    39
```

$\therefore (31)^2 < 1000 < (32)^2.$

Hence, required number $= (32)^2 = 1024.$

120. Let the 18 consecutive natural numbers be x, $(x + 1)$ $(x + 2)$, $(x + 3)$, ..., $(x + 17)$. Then, $x + (x + 1) + (x + 2) + \ldots\ldots + (x + 17) = 18x + (1 + 2 + 3 + \ldots + 17) = 18x + 153.$

Putting $x = 1, 2, 3, 4,$ ...... we find that the smallest value of x for which $(18x + 153)$ becomes a perfect square is $x = 4$.

$\therefore$ Required value $= 18 \times 4 + 153 = 72 + 153 = 225.$

121. Given expression

$$= \sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2^2 - \left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)^2}$$

$$= \sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{4-(2+\sqrt{2+\sqrt{3}})}$$

$$= \sqrt{2+\sqrt{3}} \cdot \sqrt{2+\sqrt{2+\sqrt{3}}} \cdot \sqrt{2-\sqrt{2+\sqrt{3}}}$$

$$= \sqrt{2+\sqrt{3}} \cdot \sqrt{2^2-\left(\sqrt{2+\sqrt{3}}\right)^2} = \sqrt{2+\sqrt{3}} \cdot \sqrt{2-\sqrt{3}}$$

$$= \sqrt{2^2-(\sqrt{3})^2} = \sqrt{4-3} = \sqrt{1} = 1.$$

122. $$\frac{1}{(\sqrt{5}-\sqrt{3})} = \frac{1}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} = \frac{(\sqrt{5}+\sqrt{3})}{(5-3)}$$

$$= \frac{(2.2361+1.7321)}{2} = \frac{3.9682}{2} = 1.9841.$$

123. Given expression $$= \frac{1}{(\sqrt{9}-\sqrt{8})} \times \frac{(\sqrt{9}+\sqrt{8})}{(\sqrt{9}+\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} \times \frac{(\sqrt{8}+\sqrt{7})}{(\sqrt{8}+\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} \times \frac{(\sqrt{7}+\sqrt{6})}{(\sqrt{7}+\sqrt{6})}$$

$$- \frac{1}{(\sqrt{6}-\sqrt{5})} \times \frac{(\sqrt{6}+\sqrt{5})}{(\sqrt{6}+\sqrt{5})} + \frac{1}{(\sqrt{5}-\sqrt{4})} \times \frac{(\sqrt{5}+\sqrt{4})}{(\sqrt{5}+\sqrt{4})}$$

$$= \frac{(\sqrt{9}+\sqrt{8})}{(9-8)} - \frac{(\sqrt{8}+\sqrt{7})}{(8-7)} + \frac{(\sqrt{7}+\sqrt{6})}{(7-6)} - \frac{(\sqrt{6}+\sqrt{5})}{(6-5)} + \frac{(\sqrt{5}+\sqrt{4})}{(5-4)}$$

$$= (\sqrt{9}+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+\sqrt{4}) = (\sqrt{9}+\sqrt{4}) = 3+2 = 5.$$

124. Given exp. $$= \frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{121}+\sqrt{120}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{1}} \times \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}} + \cdots + \frac{1}{\sqrt{121}+\sqrt{120}} \times \frac{\sqrt{121}-\sqrt{120}}{\sqrt{121}-\sqrt{120}}$$

$$= \frac{\sqrt{2}-\sqrt{1}}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \cdots + \frac{\sqrt{121}-\sqrt{120}}{121-120}$$

$$= \sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+\cdots+\sqrt{121}-\sqrt{120}$$

$$= -1+\sqrt{121} = -1+11 = 10.$$

125. Given exp. $= \dfrac{(1+\sqrt{3})(1-\sqrt{3})-(1-\sqrt{3})+(1+\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$

$= \dfrac{1-(\sqrt{3})^2-1+\sqrt{3}+1+\sqrt{3}}{1^2-(\sqrt{3})^2}$

$= \dfrac{2\sqrt{3}-2}{1-3} = \dfrac{2(\sqrt{3}-1)}{(-2)} = 1-\sqrt{3}.$

126. Given exp. $= (2+\sqrt{2}) + \dfrac{1}{(2+\sqrt{2})} \times \dfrac{(2-\sqrt{2})}{(2-\sqrt{2})} - \dfrac{1}{(2-\sqrt{2})} \times \dfrac{(2+\sqrt{2})}{(2+\sqrt{2})}$

$= (2+\sqrt{2}) + \dfrac{(2-\sqrt{2})}{(4-2)} - \dfrac{(2+\sqrt{2})}{(4-2)}$

$= (2+\sqrt{2}) + \dfrac{1}{2}(2-\sqrt{2}) - \dfrac{1}{2}(2+\sqrt{2}) = 2.$

127. Given exp. $= \dfrac{1}{(\sqrt{5}+\sqrt{3})} \times \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})} + \dfrac{2}{(3+\sqrt{5})} \times \dfrac{(3-\sqrt{5})}{(3-\sqrt{5})} - \dfrac{3}{(3+\sqrt{3})} \times \dfrac{(3-\sqrt{3})}{(3-\sqrt{3})}$

$= \dfrac{(\sqrt{5}-\sqrt{3})}{(5-3)} + \dfrac{2(3-\sqrt{5})}{(9-5)} - \dfrac{3(3-\sqrt{3})}{(9-3)}$

$= \dfrac{\sqrt{5}-\sqrt{3}}{2} + \dfrac{2(3-\sqrt{5})}{4} - \dfrac{3(3-\sqrt{3})}{6}$

$= \dfrac{6(\sqrt{5}-\sqrt{3})+6(3-\sqrt{5})-6(3-\sqrt{3})}{12} = 0.$

128. $\dfrac{7}{(3+\sqrt{2})} = \dfrac{7}{(3+\sqrt{2})} \times \dfrac{(3-\sqrt{2})}{(3-\sqrt{2})} = \dfrac{7(3-\sqrt{2})}{(9-2)}$

$= (3-\sqrt{2}) = (3-1.4142) = 1.5858.$

129. Given exp. $= \dfrac{3\sqrt{2}}{(\sqrt{6}-\sqrt{3})} \times \dfrac{(\sqrt{6}+\sqrt{3})}{(\sqrt{6}+\sqrt{3})} - \dfrac{4\sqrt{3}}{(\sqrt{6}-\sqrt{2})} \times \dfrac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})} - \dfrac{6}{2(\sqrt{2}-\sqrt{3})}$

$= \dfrac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{(6-3)} - \dfrac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(6-2)} + \dfrac{3}{(\sqrt{3}-\sqrt{2})} \times \dfrac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$

$= \sqrt{2}(\sqrt{6}+\sqrt{3}) - \sqrt{3}(\sqrt{6}+\sqrt{2}) + 3(\sqrt{3}+\sqrt{2})$

$= \sqrt{12}+\sqrt{6}-\sqrt{18}-\sqrt{6}+3\sqrt{3}+3\sqrt{2}$

$= 2\sqrt{3}-3\sqrt{2}+3\sqrt{3}+3\sqrt{2} = 5\sqrt{3}$

130. $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \dfrac{(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})} \times \dfrac{(\sqrt{7}+\sqrt{5})}{(\sqrt{7}+\sqrt{5})} = \dfrac{(\sqrt{7}+\sqrt{5})^2}{(7-5)}$

$= \dfrac{7+5+2\sqrt{35}}{2} = \dfrac{12+2\sqrt{35}}{2} = 6+\sqrt{35}.$

131. $a+b\sqrt{3} = \dfrac{(5+2\sqrt{3})}{(7+4\sqrt{3})} \times \dfrac{(7-4\sqrt{3})}{(7-4\sqrt{3})} = \dfrac{35-20\sqrt{3}+14\sqrt{3}-24}{(7)^2-(4\sqrt{3})^2}$

$= \dfrac{11-6\sqrt{3}}{49-48} = 11-6\sqrt{3}.$ $\therefore\ a = 11,\ b = -6.$

132. $\dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = (\sqrt{2}-1)^2.$

$\therefore\ \sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}} = (\sqrt{2}-1) = (1.414-1) = 0.414.$

133. Given exp. $= \dfrac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}} = \dfrac{(3+\sqrt{6})}{(\sqrt{3}+\sqrt{2})}$

$= \dfrac{(3+\sqrt{6})}{(\sqrt{3}+\sqrt{2})} \times \dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$= \dfrac{3\sqrt{3}-3\sqrt{2}+3\sqrt{2}-2\sqrt{3}}{(3-2)} = \sqrt{3} = 1.732.$

134. Given exp. $= \dfrac{(2+\sqrt{3})}{(2-\sqrt{3})} \times \dfrac{(2+\sqrt{3})}{(2+\sqrt{3})} + \dfrac{(2-\sqrt{3})}{(2+\sqrt{3})} \times \dfrac{(2-\sqrt{3})}{(2-\sqrt{3})} + \dfrac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \dfrac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= \dfrac{(2+\sqrt{3})^2}{(4-3)} + \dfrac{(2-\sqrt{3})^2}{(4-3)} + \dfrac{(\sqrt{3}-1)^2}{(3-1)}$

$= [(2+\sqrt{3})^2 + (2-\sqrt{3})^2] + \dfrac{4-2\sqrt{3}}{2}$

$= 2(4+3)+2-\sqrt{3} = 16-\sqrt{3}.$

135. $x+y = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \dfrac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})} \times \dfrac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})} + \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})} \times \dfrac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$

$= \dfrac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} + \dfrac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \dfrac{(\sqrt{5}+\sqrt{3})^2+(\sqrt{5}-\sqrt{3})^2}{5-3}$

$= \dfrac{2\left[(\sqrt{5})^2+(\sqrt{3})^2\right]}{2} = 5+3 = 8.$

136. Given exp. $= \dfrac{(\sqrt{2})^2}{(\sqrt{3})^2} \cdot \dfrac{(2+\sqrt{3})(2-\sqrt{3})}{(\sqrt{3}+1)(\sqrt{3}-1)} = \dfrac{2}{3}\left[\dfrac{2^2-(\sqrt{3})^2}{(\sqrt{3})^2-1^2}\right]$

$= \dfrac{2}{3}\left(\dfrac{4-3}{3-1}\right) = \dfrac{2}{3} \times \dfrac{1}{2} = \dfrac{1}{3}.$

137. Given exp. $= \dfrac{12}{3+(\sqrt{5}+2\sqrt{2})} \times \dfrac{3-(\sqrt{5}+2\sqrt{2})}{3-(\sqrt{5}+2\sqrt{2})}$

$= \dfrac{12(3-\sqrt{5}-2\sqrt{2})}{3^2-(\sqrt{5}+2\sqrt{2})^2} = \dfrac{12(3-\sqrt{5}-2\sqrt{2})}{9-(5+8+4\sqrt{10})}$

$= \dfrac{12(3-\sqrt{5}-2\sqrt{2})}{(-4-4\sqrt{10})} = \dfrac{3(\sqrt{5}+2\sqrt{2}-3)}{\sqrt{10}+1}$

$= \dfrac{3(\sqrt{5}+2\sqrt{2}-3)}{\sqrt{10}+1} \times \dfrac{\sqrt{10}-1}{\sqrt{10}-1}$

$$= \frac{3\sqrt{50} - 3\sqrt{5} + 6\sqrt{20} - 6\sqrt{2} - 9\sqrt{10} + 9}{10 - 1}$$

$$= \frac{15\sqrt{2} - 3\sqrt{5} + 12\sqrt{5} - 6\sqrt{2} - 9\sqrt{10} + 9}{9}$$

$$= \frac{9\sqrt{2} + 9\sqrt{5} - 9\sqrt{10} + 9}{9} = 1 + \sqrt{2} + \sqrt{5} - \sqrt{10}.$$

138. Given exp. $= \dfrac{1}{(\sqrt{2} - \sqrt{5}) + \sqrt{3}} + \dfrac{1}{(\sqrt{2} - \sqrt{5}) - \sqrt{3}}$

$$= \frac{\left[(\sqrt{2} - \sqrt{5}) - \sqrt{3}\right] + \left[(\sqrt{2} - \sqrt{5}) + \sqrt{3}\right]}{\left[(\sqrt{2} - \sqrt{5}) + \sqrt{3}\right]\left[(\sqrt{2} - \sqrt{5}) - \sqrt{3}\right]}$$

$$= \frac{2(\sqrt{2} - \sqrt{5})}{(\sqrt{2} - \sqrt{5})^2 - (\sqrt{3})^2} = \frac{2(\sqrt{2} - \sqrt{5})}{(2 + 5 - 2\sqrt{10}) - 3}$$

$$= \frac{2(\sqrt{2} - \sqrt{5})}{4 - 2\sqrt{10}} = \frac{\sqrt{2} - \sqrt{5}}{2 - \sqrt{10}}$$

$$= \frac{\sqrt{2} - \sqrt{5}}{\sqrt{2}(\sqrt{2} - \sqrt{5})} = \frac{1}{\sqrt{2}}.$$

139. $x + \dfrac{1}{x} = (7 - 4\sqrt{3}) + \dfrac{1}{(7 - 4\sqrt{3})} \times \dfrac{(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})}$

$$= (7 - 4\sqrt{3}) + \frac{(7 + 4\sqrt{3})}{(49 - 48)}$$

$$= (7 - 4\sqrt{3}) + (7 + 4\sqrt{3}) = 14.$$

140. $x = 3 + \sqrt{8} \Rightarrow x^2 = (3 + \sqrt{8})^2 = 3^2 + \left(\sqrt{8}\right)^2 + 2 \times 3 \times \sqrt{8}$

$$= 9 + 8 + 6\sqrt{8} = 17 + 12\sqrt{2}.$$

$$x^2 + \frac{1}{x^2} = (17 + 12\sqrt{2}) + \frac{1}{(17 + 12\sqrt{2})} \times \frac{(17 - 12\sqrt{2})}{(17 - 12\sqrt{2})}$$

$$= (17 + 12\sqrt{2}) + \frac{(17 - 12\sqrt{2})}{289 - 288}.$$

$$= (17 + 12\sqrt{2}) + (17 - 12\sqrt{2}) = 34.$$

141. $a = \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \times \dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

$$= \frac{(\sqrt{3} + \sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{3 + 2 + 2\sqrt{6}}{3 - 2} = 5 + 2\sqrt{6}.$$

$$b = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{(\sqrt{3} - \sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{3 + 2 - 2\sqrt{6}}{3 - 2} = 5 - 2\sqrt{6}.$$

$$a^2 + b^2 = (5 + 2\sqrt{6})^2 + (5 - 2\sqrt{6})^2 = 2\left[(5)^2 + (2\sqrt{6})^2\right]$$

$$= 2(25 + 24) = 2 \times 49 = 98.$$

142. $a = \dfrac{(\sqrt{5} + 1)}{(\sqrt{5} - 1)} \times \dfrac{(\sqrt{5} + 1)}{(\sqrt{5} + 1)}$

$$= \frac{(\sqrt{5} + 1)^2}{(5 - 1)} = \frac{5 + 1 + 2\sqrt{5}}{4} = \left(\frac{3 + \sqrt{5}}{2}\right).$$

$$b = \frac{(\sqrt{5} - 1)}{(\sqrt{5} + 1)} \times \frac{(\sqrt{5} - 1)}{(\sqrt{5} - 1)} = \frac{(\sqrt{5} - 1)^2}{(5 - 1)}$$

$$= \frac{5 + 1 - 2\sqrt{5}}{4} = \left(\frac{3 - \sqrt{5}}{2}\right).$$

$$\therefore\ a^2 + b^2 = \frac{(3 + \sqrt{5})^2}{4} + \frac{(3 - \sqrt{5})^2}{4}$$

$$= \frac{(3 + \sqrt{5})^2 + (3 - \sqrt{5})^2}{4} = \frac{2(9 + 5)}{4} = 7.$$

Also, $ab = \dfrac{(3 + \sqrt{5})}{2} \cdot \dfrac{(3 - \sqrt{5})}{2} = \dfrac{(9 - 5)}{4} = 1.$

$$\therefore\ \frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{(a^2 + b^2) + ab}{(a^2 + b^2) - ab} = \frac{7 + 1}{7 - 1} = \frac{8}{6} = \frac{4}{3}.$$

143. $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots \infty}}} \Leftrightarrow x = \sqrt{1 + x}$

$\Leftrightarrow x^2 = 1 + x \Leftrightarrow x^2 - x - 1 = 0$

$$\Leftrightarrow x = \frac{1 \pm \sqrt{(-1^2) - 4 \times 1 \times (-1)}}{2}$$

$$\Leftrightarrow x = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}.$$

Hence, positive value of x is $\dfrac{1 + \sqrt{5}}{2}$.

144. Let $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$.

Then, $x = \sqrt{2 + x} \Leftrightarrow x^2 = 2 + x \Leftrightarrow x^2 - x - 2 = 0$

$\Leftrightarrow x^2 - 2x + x - 2 = 0$

$\Leftrightarrow x(x - 2) + (x - 2) = 0 \Leftrightarrow (x - 2)(x + 1) = 0$

$\Leftrightarrow x = 2.\ [\because x \neq -1]$

145. $a = \sqrt{3 + \sqrt{3 + \sqrt{3 + \cdots}}} \Leftrightarrow a = \sqrt{3 + a} \Leftrightarrow a^2 = 3 + a$

$\Leftrightarrow a^2 - a - 3 = 0$

$$\Leftrightarrow a = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-3)}}{2}$$

$$\Leftrightarrow a = \frac{1 \pm \sqrt{1 + 12}}{2} = \frac{1 \pm \sqrt{13}}{2} \Leftrightarrow a = \frac{1 + \sqrt{13}}{2}\ [\because a > 0]$$

$$\Leftrightarrow a = \frac{1 + 3.6}{2} = \frac{4.6}{2} = 2.3.$$

$\therefore\ 2 < a < 3.$

146. $\dfrac{\sqrt{3 + x} + \sqrt{3 - x}}{\sqrt{3 + x} - \sqrt{3 - x}} = 2$

$$\Leftrightarrow \frac{\sqrt{3 + x} + \sqrt{3 - x}}{\sqrt{3 + x} - \sqrt{3 - x}} \times \frac{\sqrt{3 + x} + \sqrt{3 - x}}{\sqrt{3 + x} + \sqrt{3 - x}} = 2$$

$$\Leftrightarrow \frac{(\sqrt{3 + x} + \sqrt{3 - x})^2}{(\sqrt{3 + x})^2 - (\sqrt{3 - x})^2} = 2$$

$$\Leftrightarrow \frac{(3 + x) + (3 - x) + 2\sqrt{(3 + x)(3 - x)}}{(3 + x) - (3 - x)} = 2$$

$$\Leftrightarrow 6 + 2\sqrt{9 - x^2} = 4x \Leftrightarrow 2\sqrt{9 - x^2} = 4x - 6$$

$\Leftrightarrow \sqrt{9-x^2} = 2x-3$

$\Leftrightarrow 9 - x^2 = (2x-3)^2 = 4x^2 + 9 - 12x$

$\Leftrightarrow 5x^2 - 12x = 0 \Leftrightarrow 5x^2 = 12x$

$\Leftrightarrow x = \frac{12}{5}.$

147. Let the total number of camels be x.

Then, $x - \left(\frac{x}{4} + 2\sqrt{x}\right) = 15 \Leftrightarrow \frac{3x}{4} - 2\sqrt{x} = 15$

$\Leftrightarrow 3x - 8\sqrt{x} = 60 \Leftrightarrow 8\sqrt{x} = 3x - 60$

$\Leftrightarrow 64x = (3x-60)^2 \Leftrightarrow 64x = 9x^2 + 3600 - 360x$

$\Leftrightarrow 9x^2 - 424x + 3600 = 0 \Leftrightarrow 9x^2 - 324x - 100x + 3600 = 0$

$\Leftrightarrow 9x(x-36) - 100(x-36) = 0 \Leftrightarrow (x-36)(9x-100) = 0$

$\Leftrightarrow x = 36.$ $\left[\because x \neq \frac{100}{9}\right]$

148.

$$\begin{array}{r|l}
1 & \overline{1}\ \overline{79}\ \overline{56}\ (134 \\
 & 1 \\ \hline
23 & \ \ 79 \\
 & \ \ 69 \\ \hline
264 & \ \ 10\ 56 \\
 & \ \ 10\ 56 \\ \hline
 & \quad \times
\end{array}$$

$\therefore$ Number of rows = 134.

149.

$$\begin{array}{r|l}
1 & \overline{1}\ \overline{10}\ \overline{25}\ (105 \\
 & 1 \\ \hline
20 & \ \ 10 \\
 & \ \ \ 0 \\ \hline
205 & \ \ 10\ 25 \\
 & \ \ 10\ 25 \\ \hline
 & \quad \times
\end{array}$$

Number of rows = $\sqrt{10914 + 111} = \sqrt{11025} = 105.$

150. Let the number of girls in the group be x.
Then, number of oranges given to each girl = $2x$.

$\therefore x \times 2x = 1250 \Leftrightarrow 2x^2 = 1250 \Leftrightarrow x^2 = 625 \Leftrightarrow x = \sqrt{625} = 25.$

151.

$$\begin{array}{r|l}
1 & \overline{3}\ \overline{65}\ \overline{81}\ (191 \\
 & 1 \\ \hline
29 & 2\ 65 \\
 & 2\ 61 \\ \hline
381 & \ \ 4\ 81 \\
 & \ \ 3\ 81 \\ \hline
 & \ \ 1\ 00
\end{array}$$

$\therefore$ Number of men left = 100.

152. Money collected = (59.29 × 100) paise = 5929 paise.

$\therefore$ Number of members = $\sqrt{5929} = 77.$

153. Clearly, the required number must be a perfect square. Since a number having 8 as the unit's digit cannot be a perfect square, so 1,21,108 is not a perfect square.

154. $148877 = 53 \times 53 \times 53$

$\therefore \sqrt[3]{148877} = 53.$

53	148877
53	2809
	53

155. $681472 = 8 \times 8 \times 8 \times 11 \times 11 \times 11 = 8^3 \times (11)^3.$

8	681472
8	85184
8	10648
11	1331
11	121
	11

$\sqrt[3]{681472} = 8 \times 11 = 88.$

156. $262144 = 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^6.$

$\therefore \sqrt[3]{262144} = 8^2 = 64.$

Let $1728 \div \sqrt[3]{262144} \times x - 288 = 4491.$

Then, $1728 \div 64 \times x - 288 = 4491$

$\Leftrightarrow 27x = 4779$

$\Leftrightarrow x = \frac{4779}{27} = 177.$

8	262144
8	32768
8	4096
8	512
8	64
	8

157. Let $99 \times 21 - \sqrt[3]{x} = 1968.$

Then, $2079 - \sqrt[3]{x} = 1968 \Leftrightarrow \sqrt[3]{x} = 2079 - 1968 = 111$

$\Leftrightarrow x = (111)^3 = 1367631.$

158. $(.000216)^{1/3} = \left(\frac{216}{10^6}\right)^{1/3}$

$$= \left(\frac{6 \times 6 \times 6}{10^2 \times 10^2 \times 10^2}\right)^{1/3} = \frac{6}{10^2} = \frac{6}{100} = .06.$$

159. $\sqrt[3]{4\frac{12}{125}} = \sqrt[3]{\frac{512}{125}} = \left(\frac{8 \times 8 \times 8}{5 \times 5 \times 5}\right)^{1/3} = \frac{8}{5} = 1\frac{3}{5}.$

160. $\sqrt{.000064} = \sqrt{\frac{64}{10^6}} = \frac{8}{10^3} = \frac{8}{1000} = .008.$

$\therefore \sqrt[3]{\sqrt{.000064}} = \sqrt[3]{.008} = \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0.2.$

161. $864 = 3 \times 3 \times 3 \times 4 \times 4 \times 2.$

Clearly, 864 when multipled by 2 will become a perfect cube.

Hence, $n = 2$.

4	864
4	216
3	54
3	18
3	18
3	6
	2

162. $\sqrt{.01} \times \sqrt[3]{.008} - .02 = \sqrt{(.1)^2} \times \sqrt[3]{(.2)^3} - .02$

$= .1 \times .2 - .02 = .02 - .02 = 0.$

163. Given exp. $= \sqrt[3]{\dfrac{0.008 + 0.000064}{0.064 + 0.000512}} = \sqrt[3]{\dfrac{0.008064}{0.064512}}$

$= \sqrt[3]{\dfrac{8064}{64512}} = \sqrt[3]{\dfrac{1}{8}} = \dfrac{1}{2} = 0.5.$

164. Let $\sqrt[3]{3} = x.$

Then, $\left(\sqrt[3]{9} - \sqrt[3]{3} + 1\right) = (x^2 - x + 1) = \dfrac{x^3 + 1}{x + 1} = \dfrac{(\sqrt[3]{3})^3 + 1}{\left(\sqrt[3]{3} + 1\right)}.$

$\Rightarrow \left(\sqrt[3]{9} - \sqrt[3]{3} + 1\right)\left(\sqrt[3]{3} + 1\right) = \left(\sqrt[3]{3}\right)^3 + 1$

$= 3 + 1 = 4$, which is rational.

165. Clearly, $(21)^3 = 9261$ and $(22)^3 = 10648$.

So, 9261 is the largest four-digit number which is a perfect cube.

166. $21600 = 2^5 \times 3^3 \times 5^2$.

To make it a perfect cube, it must be multiplied by (2×5), i.e., 10.

167. $3600 = 2^3 \times 5^2 \times 3^2 \times 2$.

To make it a perfect cube, it must be divided by $5^2 \times 3^2 \times 2$, i.e. 450.

168. Required number to be added

$= 9^3 - 710 = 729 - 710 = 19.$

169.

8	$\overline{79}\ \overline{21}$	89
8	64	
169	1521	
9	1521	
	×	

$\Rightarrow \sqrt{7921} = 89$

170. Given $? = \sqrt[4]{(625)^3}$

$= (625)^{\frac{3}{4}} = (5 \times 5 \times 5 \times 5)^{\frac{3}{4}}$

$= \left(5^4\right)^{\frac{3}{4}} = 5^3 = 125$

171. Given $\sqrt{y} = 4x$

$\Rightarrow y = (4x)^2 = 16x^2$

$\Rightarrow y = 16x^2$

$\therefore \dfrac{x^2}{y} = \dfrac{1}{16}$

172. Let the number be x.

Given: $\sqrt{2025.11} \times \sqrt{256.04} + \sqrt{399.95} \times \sqrt{?} = 33.98 \times 40$

$\sqrt{2025} \times \sqrt{256} + \sqrt{400} \times \sqrt{?} = 34 \times 40$

$45 \times 16 + 20 \times \sqrt{?} = 1360$

$20 \times \sqrt{?} = 1360 - 720 = 640$

$\sqrt{?} = \dfrac{640}{20} = 32$

$\Rightarrow ? = (32)^2$

$\therefore ? = 32 \times 32 = 1024$

Hence, the number is 1024.

173. $\dfrac{1}{\sqrt{28}} = \dfrac{1}{\sqrt{2 \times 2 \times 7}} = \dfrac{1}{2\sqrt{7}}$

$= \dfrac{\sqrt{7}}{2\sqrt{7} \times \sqrt{7}} = \dfrac{\sqrt{7}}{14}$

$= \dfrac{2.645}{14} = 0.189$

174. $= \left(\sqrt{\dfrac{25}{9}} - \sqrt{\dfrac{64}{81}}\right) \div \sqrt{\dfrac{16}{324}}$

$= \left(\dfrac{5}{3} - \dfrac{8}{9}\right) \div \dfrac{4}{18}$

LCM of 3 and 9 is 9.

$= \left(\dfrac{15 - 8}{9}\right) \div \dfrac{2}{9} = \dfrac{7}{9} \div \dfrac{2}{9}$

$= \dfrac{7}{9} \times \dfrac{9}{2} = \dfrac{7}{2} = 3.5$

175. Given: $1728 \div \sqrt[3]{262144} \times ? - 288 = 4491$

$\Rightarrow 1728 \div \sqrt[3]{64 \times 64 \times 64} \times ?$

$= 4491 + 288$

$\Rightarrow \dfrac{1728}{64} \times ? = 4779$

$\Rightarrow 27 \times ? = 4779$

$\Rightarrow ? = \dfrac{4779}{27} = 177$

176. Let the number be a.

$\left(\sqrt{7} + 11\right)^2$

$= a^{\frac{1}{3}} + 2\sqrt{847} + 122$

$\Rightarrow 7 + 121 + 22\sqrt{7}$

$= a^{\frac{1}{3}} + 22\sqrt{7} + 122$

$\Rightarrow 128 - 122 = a^{\frac{1}{3}}$

$\Rightarrow a^{\frac{1}{3}} = 6$

$\Rightarrow a = (6)^3 = 216$

Hence, the number is 216.

177. Let the number be x.

$x - \sqrt{784} = 6 \times \sqrt{324}$

$x - 28 = 6 \times 18$

$x - 28 = 108$

$x = 108 + 28 = 136$

178. Let the number be x.

Given $\sqrt{2116} - \sqrt{1600} = \sqrt{(x)}$

$46 - 40 = \sqrt{(x)}$

$\Rightarrow 6 = \sqrt{x}$

$\Rightarrow x = (6)^2 = 36$

Hence, the number is 36.

179. Let the number be x.

$$\sqrt{(27 \div 5 \times x) \div 15} = 5.4 \div 6 + 0.3$$

$$\Rightarrow \sqrt{\frac{(27 \div 5 \times x)}{15}} = \frac{5.4}{6} + 0.3$$

$$\Rightarrow \sqrt{\frac{27 \times x}{5 \times 15}} = \frac{5.4}{6} + 0.3$$

$$\Rightarrow \sqrt{\frac{9 \times x}{5 \times 5}} = 0.9 + 0.3$$

$$\Rightarrow \frac{3}{5}\sqrt{x} = 1.2$$

$$\Rightarrow \sqrt{x} = \frac{1.2 \times 5}{3} = 2 \Rightarrow x = (2)^2$$

$$\Rightarrow x = 2 \times 2 = 4^2$$

180. Given $\sqrt{24 \div 0.5 + 1} + \sqrt{18 \div 0.6 + 6}$

$$= \sqrt{24 \times \frac{1}{0.5} + 1} + \sqrt{18 \times \frac{1}{0.6} + 6}$$

$$= \sqrt{24 \times \frac{10}{5} + 1} + \sqrt{\frac{18 \times 10}{6} + 6}$$

$$= \sqrt{48 + 1} + \sqrt{30 + 6}$$

$$\sqrt{49} + \sqrt{36} = 7 + 6 = 13$$

181. Given $\left(\sqrt{63} + \sqrt{252}\right) \times \left(\sqrt{175} + \sqrt{28}\right)$

$$= \left(\sqrt{63} + \sqrt{4 \times 63}\right) \times \left(\sqrt{25 \times 7} + \sqrt{4 \times 7}\right)$$

$$= \left(\sqrt{7 \times 9} + \sqrt{4 \times 9 \times 7}\right) \times \left(\sqrt{5 \times 5 \times 7} + \sqrt{2 \times 2 \times 7}\right)$$

$$= \left(3\sqrt{7} + 6\sqrt{7}\right) \times \left(5\sqrt{7} + 2\sqrt{7}\right)$$

$$= 9\sqrt{7} \times 7\sqrt{7} = 441$$

182. Given $9x^2 + 25 - 30x$

We have to find $\sqrt{9x^2 + 25 - 30x}$

$$= \sqrt{(3x)^2 - 2.3x \cdot 5 + (-5)^2} \quad \left\{\because a^2 - 2ab + b^2 = (a-b)^2\right\}$$

$$= \sqrt{(3x-5)^2} = 3x - 5$$

183. $\sqrt{\frac{3}{11}} = \sqrt{\frac{3 \times 11}{11 \times 11}} = \frac{\sqrt{33}}{11}$

$$= \frac{5.745}{11} = 0.5223$$

184. Let the number be x.

Given $\sqrt{x} + 14 = \sqrt{2601}$

or, $\sqrt{x} = 51 - 14 = 37$

or $x = 37^2 = 1369$

185. $a = \frac{\sqrt{3}}{2}$ (given)

$$\therefore \sqrt{1+a} + \sqrt{1-a}$$

$$= \sqrt{1 + \frac{\sqrt{3}}{2}} + \sqrt{1 - \frac{\sqrt{3}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{2}} + \sqrt{\frac{2-\sqrt{3}}{2}}$$

$$= \sqrt{\frac{2\left(2+\sqrt{3}\right)}{4}} + \sqrt{\frac{2\left(2-\sqrt{3}\right)}{4}}$$

$$= \sqrt{\frac{4+2\sqrt{3}}{4}} + \sqrt{\frac{4-2\sqrt{3}}{4}}$$

$$= \sqrt{\frac{3+1+2\times\sqrt{3}\times 1}{2}} + \sqrt{\frac{3+1-2\times\sqrt{3}\times 1}{2}}$$

$$\because \left\{\begin{array}{l} \sqrt{3}^2 + (1)^2 - 2\times\sqrt{3}\times 1 = \left(\sqrt{3}-1\right)^2 \\ \left(\sqrt{3}\right)^2 + (1)^2 + 2\times\sqrt{3}\times 1 = \left(\sqrt{3}+1\right)^2 \\ a^2 + b^2 + 2ab = (a+b)^2 \\ a^2 + b^2 - 2ab = (a-b)^2 \end{array}\right\}$$

$$= \frac{\sqrt{\left(\sqrt{3}+1\right)^2}}{2} + \frac{\sqrt{\left(\sqrt{3}-1\right)^2}}{2}$$

$$= \frac{\sqrt{3}+1+\sqrt{3}-1}{2}$$

$$= \frac{2\sqrt{3}}{2} = \sqrt{3}$$

186. Given $\frac{5+\sqrt{10}}{5\sqrt{5} - 2\sqrt{20} - \sqrt{32} + \sqrt{50}}$

$$= \frac{5+\sqrt{10}}{5\sqrt{5} - 2\times 2\sqrt{5} - 2\times 2\sqrt{2} + 5\sqrt{2}}$$

$$= \frac{5+\sqrt{10}}{5\sqrt{5} - 4\sqrt{5} - 4\sqrt{2} + 5\sqrt{2}}$$

$$= \frac{5+\sqrt{10}}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}\left(\sqrt{5}+\sqrt{2}\right)}{\sqrt{5}+\sqrt{2}} = \sqrt{5}$$

187. Given: $\frac{(0.75)^3}{1-0.75} + \left[0.75 + (0.75)^2 + 1\right]$

$$= \frac{(0.75)^2 \times 0.75}{0.25} + [0.75 + 0.5625 + 1]$$

$$= 0.5625 \times 3 + [0.75 + 0.5625 + 1]$$

$$= 1.6875 + 2.3125 = 4$$

Square root of 4 = 2

188. Given $\sqrt{10 + 2\sqrt{6} + 2\sqrt{10} + 2\sqrt{15}}$

$$= \sqrt{10 + 2\times\sqrt{3}\times\sqrt{2} + 2\times\sqrt{2}\times\sqrt{5} + 2\times\sqrt{3}\times\sqrt{5}}$$

$$= \sqrt{2+3+5+2\times\sqrt{2}\times\sqrt{3} + 2\times\sqrt{5}\times\sqrt{2} + 2\times\sqrt{5}\times\sqrt{3}}$$

$$= \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2 + 2\times\sqrt{2}\times\sqrt{3} + 2\times\sqrt{5}\times\sqrt{2} + 2\times\sqrt{5}\times\sqrt{3}}$$

$$\left\{\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2\right\}$$

$$= \sqrt{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)^2}$$

$$= \left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)$$