26 Races and Games of Skill

IMPORTANT FACTS

Race : *A contest of speed in running, riding, driving, sailing or rowing is called a race.*

Race Course : *The ground or path on which contests are made is called a race course.*

Starting Point : *The point from which a race begins is known as a starting point.*

Winning Point or Goal : *The point set to bound a race is called a winning point or a goal.*

Winner : *The person who first reaches the winning point is called a winner.*

Dead Heat Race : *If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.*

Start : Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that 'A gives B, a start of 12 metres'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only (100 – 12) = 88 metres.

In a 100 m race, 'A can give B 12 m' or 'A can give B a start of 12 m' or 'A beats B by 12 m' means that while A runs 100 m, B runs (100 – 12) = 88 m.

Games : 'A game of 100, means that the person among the contestants who scores 100 points first is the winner'.

If A scores 100 points while B scores only 80 points, then we say that 'A can give B 20 points'.

SOLVED EXAMPLES

Ex. 1. ***In a km race, A beats B by 28 metres or 7 seconds. Then, find A's time over the course.***

Sol. Clearly, B covers 28 m in 7 seconds.

$\therefore$ B's time over the course = $\left(\frac{7}{28}\times 1000\right)\text{sec} = 250\text{ sec}.$

$\therefore$ *A*'s time over the course

= (250 – 7) sec = 243 sec = 4 min 3 sec.

Ex. 2. ***A runs $1\frac{3}{4}$ times as fast as B. If A gives B a start of 84 m, how far the winning post be so that A and B might reach it at the same time?***

Sol. $A : B = \frac{7}{4} : 1 = 7 : 4.$

Thus, in a race of 7 m, *A* gains 3 m over *B*.

3 m are gained by *A* in a race of 7 m.

84 m are gained by *A* in a race of $\left(\frac{7}{3}\times 84\right)\text{m} = 196\,\text{m}.$

$\therefore$ Winning post must be 196 m away from the starting point.

Ex. 3. ***A can run 1 km in 3 min 10 sec and B can cover the same distance in 3 min 20 sec. By what distance can A beat B?*** **(Railways, 2005)**

Sol. *A* beats *B* by (200 – 190) sec = 10 sec.

Distance covered by *B* in 10 sec

$= \left(\frac{1000}{200}\times 10\right)\text{m} = 50\,\text{m}.$

$\therefore$ *A* beats *B* by 50 m.

Ex. 4. ***In a 100 m race, A beats B by 25 m and B beats C by 4 m. In the same race, find the distance by which A beats C.***

Sol. $A : B = 100 : 75$ and $B : C = 100 : 96$.

$$\therefore A : C = \left(\frac{A}{B} \times \frac{B}{C}\right) = \left(\frac{100}{75} \times \frac{100}{96}\right) = \frac{100}{72} = 100 : 72.$$

A beats C by $(100 - 72)$ m $= 28$ m.

Ex. 5. ***In a 100 m race, A runs at 8 km per hour. If A gives B a start of 5 m and still beats him by 15 seconds, what is B's speed?***

Sol. Time taken by A to cover 100 m

$$= \left(\frac{60 \times 60}{8000} \times 100\right) \text{sec} = 45 \text{ sec}.$$

$\therefore$ B covers $(100 - 5)$ m

$= 95$ m in $(45 + 15)$ sec $= 60$ sec.

$$\therefore \quad B\text{'s speed} = \left(\frac{95}{60}\right) \text{m/sec} = \left(\frac{95}{60} \times \frac{18}{5}\right) \text{km/hr} = \frac{57}{10} \text{km/hr} = 5.7 \text{ km/hr}.$$

Ex. 6. ***A, B and C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many metres start can B give C?***

Sol. $A : B = 1000 : 960$ and $A : C = 1000 : 936$.

$$\therefore \frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{960}{1000} \times \frac{1000}{936}\right) = \frac{1000}{975} \Rightarrow B : C = 1000 : 975.$$

$\therefore$ B can give C a start of 25 m.

Ex. 7. ***In a 100 m race for children, A runs at 1.66 m/s. If A gives B a start of 4 m and still beats him by 12 seconds, what is B's speed?*** **(C.P.O., 2007)**

Sol. Time taken by A to run 100 m

$$= \frac{100}{1.66} \text{sec} = \frac{100 \times 100}{166} \text{sec} = \frac{5000}{83} \text{sec}.$$

Time taken by B to run 96 m

$$= \left(\frac{5000}{83} + 12\right) \text{sec} = \frac{(5000 + 996)}{83} \text{sec} = \frac{5996}{83} \text{sec}.$$

B's speed

$$= \left(\frac{96 \times 83}{5996}\right) \text{m/sec} = \left(\frac{24 \times 83}{1499}\right) \text{m/s} = \frac{1992}{1499} \text{m/s}$$

$= 1.328$ m/s $= 1.33$ m/s.

Ex. 8. ***In a game of 80 points, A can give B 5 points and C 15 points. How many points B can give C in a game of 60?***

(a) 8 points (b) 10 points

(c) 20 points (d) 12 points

Sol. $A : B = 80 : 75$ and $A : C = 80 : 65$

$$\frac{B}{C} = \left(\frac{B}{A} \times \frac{A}{C}\right) = \left(\frac{75}{80} \times \frac{80}{65}\right) = \frac{15}{13} = \left(\frac{15 \times 4}{13 \times 4}\right) = \frac{60}{52}$$

$\Rightarrow B : C = 60 : 52$.

Hence, in a game of 60, B can give C 8 points.

EXERCISE
(OBJECTIVE TYPE QUESTIONS)

Directions: *Mark (√) against the correct answer in each of the following :*

1. In a 100 m race, *A* covers the whole distance in 36 seconds and *B* in 45 seconds. In this race, *A* beats *B* by
(*a*) 20 m (*b*) 25 m
(*c*) 22.5 m (*d*) 9 m

2. In a kilometre race, A beats B by 100 m and B beats C by 150 m. In the same race, by how many metres does A beat C? **(M.B.A., 2004)**
(*a*) 225 m (*b*) 235 m
(*c*) 240 m (*d*) 250 m

3. A can run 22.5 m while *B* runs 25 m. In a kilometre race, *B* beats A by **(M.B.A., 2006)**
(*a*) 100 m (*b*) $111\frac{1}{9}$ m
(*c*) 25 m (*d*) 50 m

4. In a kilometre race, *A*, *B* and *C* are three participants. *A* can give *B* a start of 50 m and *C* a start of 69 m. The start which *B* can allow *C*, is **(S.S.C., 2006)**
(*a*) 17 m (*b*) 18 m
(*c*) 19 m (*d*) 20 m

5. In a 1000 m race, *A* can beat *B* by 100 m. In a race of 400 m, *B* can beat *C* by 40 m. By how many metres will *A* beat *C* in a race of 500 m? **(Railways, 2006)**
(*a*) 85 m (*b*) 95 m
(*c*) 105 m (*d*) 115 m

6. In a 100 m race, *A* beats *B* by 10 m and *C* by 13 m. In a race of 180 m, *B* will beat *C* by
(*a*) 5.4 m (*b*) 4.5 m
(*c*) 5 m (*d*) 6 m

7. In a 200 m race, *A* can beat *B* by 31 m and *C* by 18 m. In a race of 350 m, *C* will beat *B* by:
(*a*) 22.75 m (*b*) 25 m
(*c*) 19.5 m (*d*) 13 m

8. In a race of 200 m, *B* can give a start of 10 m to *A* and *C* can give a start of 20 m to *B*. The start that *C* can give to *A* in the same race is **(S.S.C., 2007)**
(*a*) 27 m (*b*) 29 m
(*c*) 30 m (*d*) 25 m

9. In a 200 m race, A beats B by 35 m or 7 seconds. *A*'s time over the course is
(*a*) 40 sec (*b*) 47 sec
(*c*) 33 sec (*d*) None of these

10. In a 500 m race, the ratio of the speeds of two contestants *A* and *B* is 3 : 4. If A has a start of 140 m, then A wins by
(*a*) 60 m (*b*) 40 m
(*c*) 20 m (*d*) 10 m

11. A runs $1\frac{2}{3}$ times as fast as *B*. If *A* gives *B* a start of 80 m, how far must the winning post be so that *A* and *B* might reach it at the same time?
(*a*) 200 m (*b*) 300 m
(*c*) 270 m (*d*) 160 m

12. In a 100 m race, *A* can beat *B* by 25 m and *B* can beat *C* by 4 m. In the same race *A* can beat *C* by
(*a*) 21 m (*b*) 26 m
(*c*) 28 m (*d*) 29 m

13. *A* and *B* take part in a 100 m race. *A* runs at 5 km an hour. *A* gives *B* a start of 8 m and still beats him by 8 seconds. The speed of *B* is **(Railways, 2007)**
(*a*) 4.45 km/hr (*b*) 4.14 km/hr
(*c*) 4.15 km/hr (*d*) 4.25 km/hr

14. In a 400 m race, *A* gives *B* a start of 5 seconds and beats him by 15 m. In another race of 400 m, *A* beats *B* by $7\frac{1}{7}$ seconds. Their respective speeds are **(M.A.T. 2009)**
(*a*) 6 m/sec, 7 m/sec (*b*) 5 m/sec, 7 m/sec
(*c*) 8 m/sec, 7 m/sec (*d*) 9 m/sec, 7 m/sec

15. In a kilometre race, *A* beats *B* by 30 seconds and *B* beats *C* by 15 seconds. If *A* beats *C* by 180 m, the time taken by *A* to run 1 kilometre, is **(S.S.C., 2006)**
(*a*) 200 sec (*b*) 205 sec
(*c*) 210 sec (*d*) 250 sec

16. In a 800 metre race, *A* defeated *B* by 15 seconds. If *A*'s speed was 8 km/hr, the speed of B was **(S.S.C., 2004)**
(*a*) $\frac{16}{27}$ km/hr (*b*) $\frac{27}{16}$ km/hr
(*c*) $7\frac{17}{25}$ km/hr (*d*) $8\frac{17}{25}$ km/hr

17. *A* and *B* can cover a 200 m race in 22 seconds and 25 seconds respectively. When *A* finished the race, then *B* is at what distance from the finishing line?
(*a*) 24 m (*b*) 30 m
(*c*) 48 m (*d*) 54 m **(Railways, 2004)**

18. In a game of 100 points, *A* can give *B* 20 points and *C* 28 points. Then, *B* can give *C*:
(*a*) 8 points (*b*) 10 points
(*c*) 14 points (*d*) 40 points

19. At a game of billiards, *A* can give *B* 15 points in 60 and *A* can give *C* 20 points in 60. How many points can *B* give *C* in a game of 90?
(*a*) 30 points (*b*) 20 points
(*c*) 10 points (*d*) 12 points

20. Four sisters Suvarna, Tara, Uma and Vibha are playing a game such that the loser doubles the money of each of the other players from her share. They played four games and each sister lost one game in alphabetical order. At the end of fourth game, each sister had ₹ 32. How much money did Suvarna start with?

(*a*) ₹ 60 (*b*) ₹ 34
(*c*) ₹ 66 (*d*) ₹ 28

[SSC—CGL (Tier I) Exam, 2012]

21. A team played 40 games in a season and won in 24 of them. What percent of games played did the team win?

(*a*) 70% (*b*) 40%
(*c*) 60% (*d*) 35%

(SSC—CGL (Tier I) Exam, 2012]

Directions (22 and 25): At the start of a game of cards, J and B together had four times as much money as T, while T and B together had three times as much as J. At the end of the evening, J and B together had three times as much money as T, while T and B together had twice as much as J, B lost ₹ 200.

22. What fraction of the total money did T have at the beginning of the game? **[SNAP, 2012]**

(*a*) 1/3 (*b*) 1/8
(*c*) 2/9 (*d*) 1/5

23. What fraction of the total money did J win/lose? **[SNAP, 2012]**

(*a*) Won 1/12 (*b*) Lost 1/6
(*c*) Lost 1/3 (*d*) Won 1/5

24. In racing over a distance *d* at uniform speed, A can beat B by 20 metres, B can beat C by 10 metres, and A can beat C by 28 metres. Then d, in metres, is **[GBO Exam, 2012]**

(*a*) 50 (*b*) 75
(*c*) 100 (*d*) 120

25. A runs $1\frac{2}{3}$ times as fast as B. If A gives B a start of 80m, how far must the winning post from the starting point be so that A and B might reach it at the same time? **[CDS Exam, 2016]**

(*a*) 200m (*b*) 300m
(*c*) 270m (*d*) 160m

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*a*)	**4.** (*d*)	**5.** (*b*)	**6.** (*d*)	**7.** (*b*)	**8.** (*b*)	**9.** (*c*)	**10.** (*c*)
11. (*a*)	**12.** (*c*)	**13.** (*b*)	**14.** (*c*)	**15.** (*b*)	**16.** (*c*)	**17.** (*a*)	**18.** (*b*)	**19.** (*c*)	**20.** (*c*)
21. (*c*)	**22.** (*d*)	**23.** (*a*)	**24.** (*c*)	**25.** (*a*)					

SOLUTIONS

1. Clearly, *A* beats *B* by 9 seconds.

Distance covered by *B* in 9 sec. = $\left(\frac{100}{45}\times 9\right)\text{m} = 20\,\text{m}$.

$\therefore$ *A* beats *B* by 20 m.

2. *A* : *B* = 1000 : 900 and *B* : *C* = 1000 : 850.

$\frac{A}{C}=\frac{A}{B}\times\frac{B}{C}=\frac{1000}{900}\times\frac{1000}{850}=\frac{1000}{765} \Rightarrow A : C = 1000 : 765.$

$\therefore$ *A* beats *C* by (1000 – 765) m = 235 m.

3. $B : A = 25 : \frac{45}{2} = 50 : 45 = (50 \times 20) : (45 \times 20) = 1000 : 900.$

$\therefore$ In a km race, *B* beats *A* by (1000 – 900) m = 100 m.

4. *A* : *B* : *C* = 1000 : (1000 – 50) : (1000 – 69)
= 1000 : 950 : 931.

In a 950 m race, *B* can give *C* a start of (950 – 931) m = 19 m.

In a 1000 m race, *B* can give C a start of $\left(\frac{19}{950}\times 1000\right)\text{m} = 20\,\text{m}$.

5. *A* : *B* = 1000 : 900, *B* : *C* = 400 : 360 = 100 : 90 = 900 : 810
$\Rightarrow$ *A* : *B* : *C* = 1000 : 900 : 810 $\Rightarrow$ *A* : *C* = 1000 : 810 = 500 : 405
$\Rightarrow$ In a 500 m race, A beats C by (500 – 405) m = 95 m.

6. $A:B=100:90 \text{ and } A:C=100:87 \Rightarrow \frac{A}{B}=\frac{100}{90}=\frac{10}{9} \text{ and } \frac{A}{C}=\frac{100}{87}$

$\Rightarrow \frac{B}{C}=\left(\frac{B}{A}\times\frac{A}{C}\right)=\left(\frac{9}{10}\times\frac{100}{87}\right)=\frac{90}{87}=\frac{90\times 2}{87\times 2}=\frac{180}{174}$

$\Rightarrow$ B : C = 180 : 174
$\Rightarrow$ In a 180 m race, B beats C by (180 – 174) m = 6 m.

7. *A* : *B* = 200 : 169 and *A* : *C* = 200 : 182
$\Rightarrow \frac{C}{B}=\left(\frac{C}{A}\times\frac{A}{B}\right)=\left(\frac{182}{200}\times\frac{200}{190}\right)=\frac{182}{169}.$

When *C* covers 182 m, *B* covers = 169 m.

When *C* covers 350 m, *B* covers = $\left(\frac{169}{182}\times 350\right)\text{m} = 325\,\text{m}$.

$\therefore$ *C* beats *B* by 25 m.

8. *B* : *A* = 200 : 190, *C* : *B* = 200 : 180

$\frac{C}{A}=\left(\frac{C}{B}\times\frac{B}{A}\right)=\left(\frac{200}{180}\times\frac{200}{190}\right)=\frac{200}{171}.$

$\therefore$ *C* can give to *A*, a start of (200 – 171) m = 29 m.

9. *B* covers 35 m in 7 seconds.

B covers 200 m in $\left(\frac{7}{35}\times 200\right)\text{sec} = 40\,\text{sec}$.

Time taken by *A* = (40 – 7) sec = 33 sec.

10. To reach the winning post A has to cover
$= (500 - 140)$ m $= 360$ m.
While A covers 3 m, B covers 4 m.
While A covers 360 m, B covers $\left(\frac{4}{3}\times 360\right)\text{m} = 480\,\text{m}$.
Thus, when A reaches the winning post, B covers 480 m.
$\therefore$ A wins by $(500 - 480)$ m $= 20$ m.

11. Ratio of speeds of A and $B = \frac{5}{3}:1 = 5:3$.
Thus, in a race of 5 m, A gains 2 m over B.
2 m are gained by B in a race of 5 m.
80 m will be gained by B in a race of $\left(\frac{5}{2}\times 80\right)\text{m} = 200\,\text{m}$.
$\therefore$ Winning post is 200 m away from the starting point

12. $A : B = 100 : 75$ and $B : C = 100 : 96$.
$A : C = \left(\frac{A}{B}\times\frac{B}{C}\right) = \left(\frac{100}{75}\times\frac{100}{96}\right) = \frac{100}{72}$.
$\therefore$ A beats C by $(100 - 72)$ m $= 28$ m.

13. A's speed $= 5$ km/hr $= \left(5\times\frac{5}{18}\right)\text{m/sec} = \frac{25}{18}\text{m/sec}$.
Time taken by A to cover 100 m $= \left(100\times\frac{18}{25}\right)\text{sec} = 72\,\text{sec}$.
Time taken by B to cover 92 m $= (72 + 8)$ sec $= 80$ sec.
B's speed $=$
$\left(\frac{92}{80}\right)\text{m/sec} = \left(\frac{92}{80}\times\frac{18}{5}\right)\text{km/hr} = \frac{414}{100}\text{km/hr} = 4.14\,\text{km/hr}$.

14. Suppose A covers 400 m in t sec. Then, B covers 385 m in $(t + 5)$ sec.
$\therefore$ B covers 400 m in $\left\{\frac{(t+5)}{385}\times 400\right\}\text{sec} = \frac{80(t+5)}{77}\text{sec}$.
Also, B covers 400 m in $\left(t + 7\frac{1}{7}\right)\text{sec} = \frac{(7t+50)}{7}\text{sec}$.
$\therefore$ $\frac{80(t+5)}{77} = \frac{7t+50}{7} \Rightarrow 80(t + 5) = 11(7t + 50)$
$\Rightarrow (80t - 77t) = (550 - 400) \Rightarrow 3t = 150 \Rightarrow t = 50$.
$\therefore$ A's speed $=$
$\frac{400}{50}\text{m/sec} = 8\,\text{m/sec}$, B's speed $= \frac{385}{55}\text{m/sec} = 7\,\text{m/sec}$.

15. In a km race, suppose A takes t sec. Then, B takes $(t + 30)$ sec. and C takes $(t + 45)$ sec.
180 m is covered by C in 45 sec.
$\therefore$ 1000 m is covered by C in $\left(\frac{45}{180}\times 1000\right)\text{sec} = 250\,\text{sec}$.
$\therefore$ A covers 1000 m in $(250 - 45)$ sec $= 205$ sec.

16. A's speed $= 8$ km/hr $= \left(8\times\frac{5}{18}\right)\text{m/sec} = \frac{20}{9}\text{m/sec}$.
Time taken by A to cover 800 m $= \left(800\times\frac{9}{20}\right)\text{sec} = 360\,\text{sec}$.
Time taken by B to cover 800 m $= (360 + 15)$ sec
$= 375$ sec.
Speed of
$B = \frac{800}{375}\text{m/sec} = \left(\frac{800}{375}\times\frac{18}{5}\right)\text{km/hr} = \frac{192}{25}\text{km/hr} = 7\frac{17}{25}\text{km/hr}$.

17. Distance covered by B in 25 sec $= 200$ m.
Distance covered by B in 22 sec. $= \left(\frac{200}{25}\times 22\right)\text{m} = 176\,\text{m}$.
$\therefore$ B was at a distance of $(200 - 176)$ m $= 24$ m from the finishing line.

18. $A : B = 100 : 80$ and $A : C = 100 : 72$.
$\therefore$ $\frac{B}{C} = \left(\frac{B}{A}\times\frac{A}{C}\right) = \left(\frac{80}{100}\times\frac{100}{72}\right) = \frac{10}{9}\times\frac{10}{10} = \frac{100}{90} = 100:90$.
$\therefore$ B can give C 10 points.

19. $A : B = 60 : 45$ and $A : C = 60 : 40$
$\therefore$ $\frac{B}{C} = \left(\frac{B}{A}\times\frac{A}{C}\right) = \left(\frac{45}{60}\times\frac{60}{40}\right) = \frac{45}{40}\times\frac{2}{2} = \frac{90}{80} = 90:80$.
$\therefore$ B can give C 10 points in a game of 90.

20.

End of	Suvarna	Tara	Uma	Vibha
4th Game	32	32	32	32
3rd Game	16	16	16	80
2nd Game	8	8	72	40
1st Game	4	68	36	20
Original Money	62	34	18	10

Suvarna starts with ₹ 66

21. Number of games played = 40
Number of won games = 24
Percentage of games played $= \frac{24}{40}\times 100 = 60\%$

Direction for Solution: (3 and 4)
J and B have 4 times as much as T. $\frac{1}{5}$th.
So, T so total.
Similarly, J is $1/4^{th}$ and so on.
Taking total as $60x$:
T has $12x$, J has $15x$ so B has $33x$
Finally T has $15x$, J has $20x$ so B has $25x$.

22. So T has $1/5^{th}$ initially. **23.** J won $5/60 = 1/12$

24. Let distance be $= x$ metre A runs $= x$ metre
B runs $= (x - 20)$ metres. B can beat C by 10 metres
C runs $= (x - 10)$ metres A can beat by C 28 metres
C runs $= (x - 28)$ metres B runs $= x - 20$
C runs $= \left(\frac{x-10}{x}\right)\times(x-20)$
'A' can beat C by 28 metre $x - \left[\frac{(x-10)(x-20)}{x}\right] = 28$
$x^2 - x^2 + 30x - 200 = 28x$ $30x - 200 = 28x$
$\Rightarrow 30x - 28x = 200$ $2x = 200$
$x = 100$ metres.

25. Let speed of B $= a$ m/s
Speed of A $= \frac{5}{3}a$ m/s
Let x be the distance $= \frac{x\times 3}{5a} = \frac{x-80}{a}$
$\Rightarrow 3x = 5x - 400$ $\Rightarrow 2x = 400$
$\Rightarrow x = 200$m