Lab6 Guide

Study the transition process of Circuits

Objectives

1. To Study the transition process of RC differential circuit and integral circuit;
2. To Study the transition process of RLC second-order circuit.
3. To compare experimental results with theory and Multisim simulations, and to account for possible differences.
4. To be more familiari with oscilloscope and function generator.

Background

The three basic passive components of: *Resistance*, *Inductance*, and *Capacitance*. Capacitors and inductors are two of the three passive elements used in circuit design. These two passive elements are not able to dissipate or generate energy, but can return stored energy into a circuit. If one of these passive elements were in a circuit it would form a first order circuit because a first order differential equation would be required to solve for a voltage or current. A resistor, a capacitor and an inductor connected in series or parallel with either a voltage or current source we will form a second-order circuit. Finding the solution to this second order equation involves finding the roots of its characteristic.

1. RC Integrator

Consider the basic RC circuit in Fig. 1. Applying Ohm's Law across R gives Vin -Vout = IR. The same current I passes through the capacitor according to I = C(dV/dt).

$$\frac{dV_{out}}{dt} + \frac{1}{RC}V_{out} = \frac{1}{RC}V_{in} \tag{1}$$

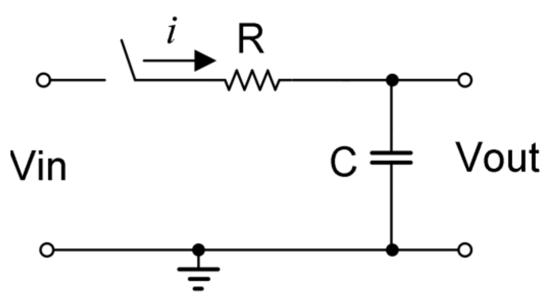


Fig.1. RC Integrator

From Equation (1), we see that if Vout ≪ Vin then the solution to our RC circuit becomes

(2).

Also the limit Vout ≪ Vin corresponds roughly to t ≪ RC. Within this approximation, we see clearly from Equation（2） why the circuit above is sometimes called an “integrator”.

2. RC Differentiator

Consider the basic RC circuit in Fig.2, we have

$$V_{out} = Ri = RC(dVc/dt) = RC\frac{d(V_{in} - V_{out})}{dt}$$

In the limit Vin ≫ Vout, we have a differentiator:

$$V_{out} = \mathrm{RC}\frac{dV_{in}}{dt}$$

The limit of validity is the opposite of the integrator, t ≫ RC.

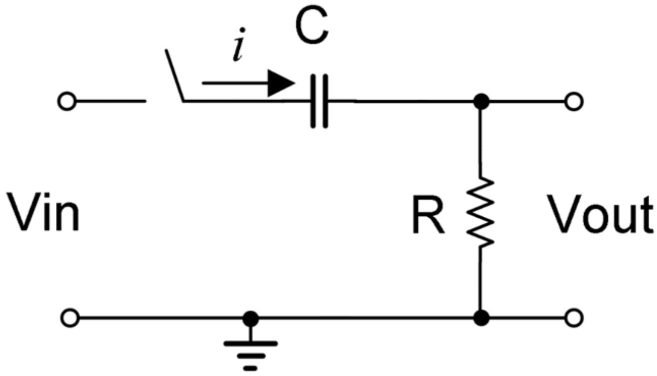


Fig.2. RC Differentiator

3. Transient Capacitor Voltage for a Step Input to a Series RLC Circuit

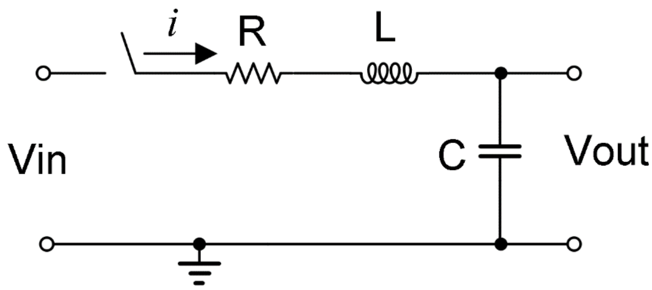


Figure 3 the RLC circuit

The characteristic equation modeling a series RLC is $S^2 + s\left(\frac{R}{L}\right) + \frac{1}{LC} = 0$. By defining Characteristic Equation becomes $S^2 + 2\alpha s + {\omega_0}^2 = 0$.

Roots $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - {\omega_0}^2}$

α is the damping factor or decay constant [s^{-1}]: $\alpha = \frac{R}{2L}$

ω_0 is the resonant frequency or undamped natural frequency [radian/s]: $\omega_0 = \frac{1}{\sqrt{LC}}$

The value of the term $\sqrt{\alpha^2 - {\omega_0}^2}$ determines the behavior of the response. Three types of responses are possible:

- Critically Damped System (no oscillatory behavior):

For $\alpha = \omega_0$ that is $\alpha = \frac{R}{2L} = \frac{1}{\sqrt{LC}}$, $R = 2\sqrt{\frac{L}{C}}$

Note: This will almost hard to observe in the lab.

- Overdamped Damped System (no oscillatory behavior):

For $\alpha > \omega_0$, that is $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$, $R > 2\sqrt{\frac{L}{C}}$, s1 and s2 will be both be real and negative.

- Undamped System(oscillatorybehavior):

For $\alpha < \omega_0$,that is $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$, R< $2\sqrt{\frac{L}{C}}$

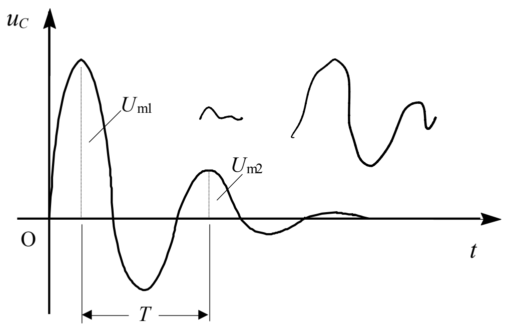
$$S_1 = -\frac{R}{2L} + j\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$$

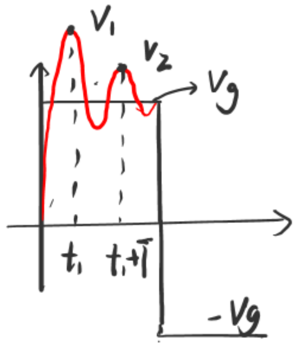
$$S_2 = -\frac{R}{2L} - j\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$$

The two roots are complex conjugates, where $\alpha = \frac{R}{2L}$ is the decay constant or damping factor. It will determine the rate at which the transient response attenuates away and $\omega_0 = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$ is the damped oscillation frequency. It gives the angular of the oscillations that occur.

For the under–damped response, the damping factor or decay constant [s^{-1}] α and the resonant frequency or undamped natural frequency [radian/s] ω_0 can be measured by the graph of the attenuation oscillation curve (see Figure below)

$\omega_0 = \frac{1}{\sqrt{LC}}$





attenuation oscillation curve

T can be read directly from the oscilloscope, then:

$$\omega_0 = \frac{2\pi}{T}$$

Moreover, the ratio of two adjacent maximum values of the attenuation oscillation curve has the following relationship:

$$\frac{U_{m1}}{U_{m2}} = e^{\alpha t}$$

From this we can see that

$$\alpha = \frac{1}{T} ln \frac{U_{m1}}{U_{m2}} = \frac{1}{T} ln \frac{U_1}{U_2}$$

4. How to observe the transition process of circuit with oscilloscope

The transition process in the circuit generally reaches the steady state after a period of time. In order to study the transition process when a circuit is connected to a DC voltage by using oscilloscope, the following methods can be used:

Add a periodic "square wave" voltage to the circuit, as shown in Figure 4.

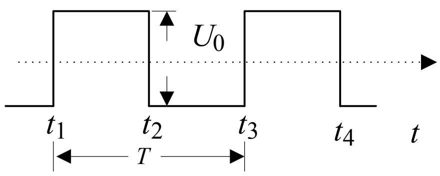


Figure 4 the square wave

Its effect on the circuit can be understood as follows: at t1, t3,..., the input voltage jumps to $\frac{1}{2}U_0$, which is equivalent to suddenly connecting the circuit with a DC voltage $\frac{1}{2}U_0$; At t2, t4,..., the input voltage jumps from $\frac{1}{2}U_0$ to $-\frac{1}{2}U_0$, which is equivalent to suddenly connecting the input end of the circuit with a DC voltage $-\frac{1}{2}U_0$. Because the circuit is continuously connected with $\frac{1}{2}U_0$ and $-\frac{1}{2}U_0$, there will be a repetitive transition process in the circuit, which can be observed with an oscilloscope.