Lab7 Guide RLC Resonant Circuit

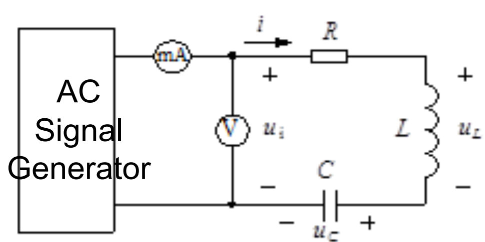
Objectives

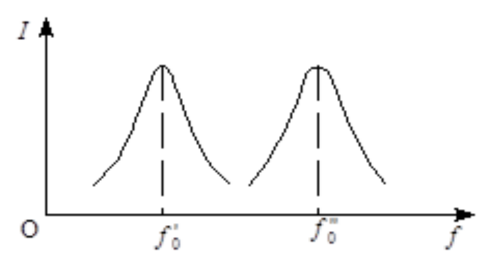
1. To study the behavior of series RLC circuits at resonance.
2. To understand the resonance frequency, cut-off frequency, bandwidth and quality factor of a resonance circuit.
3. To experimentally determine the resonance frequency in a series RLC circuit and compare this to the expected resonance value.
4. Learn to use an oscilloscope and function generator.

Background

1. Series RLC circuits Introduction

The three basic passive components of: *Resistance*, *Inductance*, and *Capacitance* have different phase relationships to each other when connected to a sinusoidal alternating supply.Series RLC circuits consist of a resistor, a capacitor and an inductor connected in series. Series RLC circuits are classed as second-order circuits because they contain two energy storage elements, an inductor L and a capacitor C. Consider the RLC circuit shown in Fig.1.





(a) (b)

Figure 1 (a) the RLC circuit

Figure 1 (b) The magnitude of the Current I reaches a maximum when the circuit is driven at its resonant frequenc

For the series RLC circuit shown in Fig.1, the Impedance of a Series RLC Circuit Z and the current I is:

$$Z = |Z|\angle\varphi = R + \mathrm{j}(X_L + X_C) = R + \mathrm{j}(\omega L - \frac{1}{\omega C})$$

$$I = \frac{U_\mathrm{i}}{|Z|} = \frac{U_\mathrm{i}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

At a certain frequency, the reactance will be equal and cancel, $X_L = -X_C$, that is $\omega L - \frac{1}{\omega C} = 0$, giving minimum value for impedance, and a maximum value for $I_0 = \frac{U_\mathrm{i}}{R}$, This phenomenon is called resonance. Here ω_0 and f_0 is the resonant angular frequency and frequency of an RLC series circuit when $\phi = 0$,that is: $\omega_0 = \frac{1}{\sqrt{LC}}$ $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

In Fig1(b), a graph of current versus frequency for two RLC series circuits differing in the resonant frequency, for the larger value of LC, the lower the resonant frequency.

2. Resonance Curve and the Quality Factor

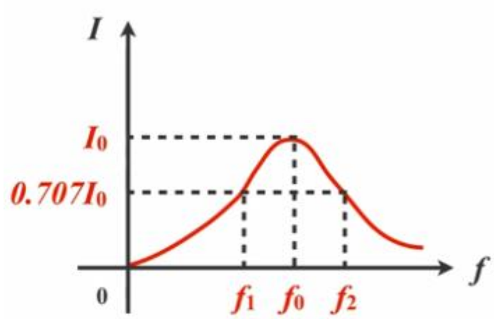
The quality factor of the resonant circuit can be calculated as:

$$Q = \frac{U_L}{U_\mathrm{i}} = \frac{U_C}{U_\mathrm{i}} = \frac{\omega_0 L}{R} = \frac{\sqrt{L/C}}{R}$$

$$U_L = U_C = QU$$

The maximum power dissipated at the resonance frequency is $P = \frac{1}{2}\frac{V_s^2}{R}$. At some certain frequency the power dissipated by the resistor is half of the maximum power. This half power occurs at the cutoff frequencies for which the amplitude of the voltage across the resistor becomes equal to $1/\sqrt{2}$ of the maximum, that is f_1, f_2, the bandwidth is the difference between the half power frequencies:

$$f_{BW} = f_2 - f_1$$



The quality factor, Q can also be defined as the ratio of the center frequency to the bandwidth:

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{f_{BW}}$$

Note that for a fixed L and C, a decrease in R corresponds to a narrower resonance and thus a higher selectivity regarding the frequency range that can be passed by the circuit. Therefore, the quality factor of a circuit is a measure of how sharp the resonance curve is.

3. Methods for determining the resonant frequency of a circuit f0

1. The maximum voltage method at the resistor. Keep the output amplitude of the signal source constant, and make the frequency of the signal source gradually increase from small to large. Observe the voltage U_R, when U_R is the maximum, accordingly the frequency value set at the signal source is the resonant frequency f_0 of the circuit.
2. Lissajous curve method. The default mode of oscilloscope is YT mode, indicating the process of signal changing with time. That is, the vertical Y-axis represents voltage and the horizontal X-axis represents time. At the same time, oscilloscope can be switched to the **XY** mode. In **XY** mode, the oscilloscope display to a volts-versus-volts display using two input channels. Channel 1 is the X-axis input, channel 2 is the Y-axis input. XY display mode is a common use by measuring the phase difference between two signals of the same frequency with the Lissajous method.

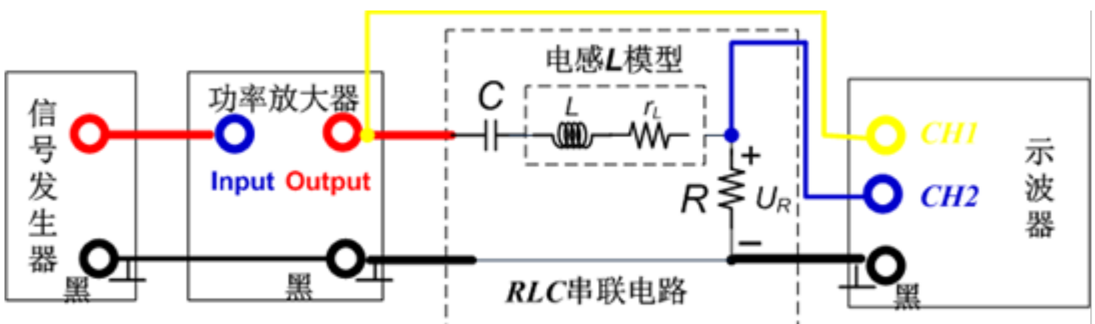


Fig.2 schematic diagram of RLC series circuit test

As shown in Fig.2, sine wave signal generated by signal generator u_i is fed to channel 1 of the oscilloscope, and u_R is to channel 2, we have

$$CH1: \quad x = u_i = U_m \sin(\omega t + \varphi_1)$$
$$CH2: \quad y = u_R = U_{Rm} \sin(\omega t + \varphi_2)$$

According to the characteristic that when the circuit resonates, the input signal and the resistance voltage are in phase, that is $\varphi_1 = \varphi_2$, so that the oscilloscope displays as a diagonal line:

$$y = \frac{U_{Rm}}{U_m} x$$

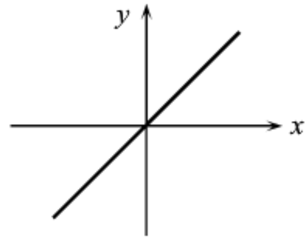


Figure 3 Signals in phase

Any phase difference between the two channels will cause a half-oval trace. According to the characteristic that the input signal and the resistance voltage phase are consistent when the circuit resonates, the two signals are connected to the two channels of the oscilloscope respectively, and the oscilloscope is set in x-y mode, by adjusting the signal frequency of the input signal generator, we can see an ellipse with varying polar distance on the oscilloscope. When the ellipse becomes a straight line, the circuit at this time has resonance, the frequency of the input signal is the resonance frequency.