Lab5 Transient Response of First-Order RC Circuits

Objectives

1. To understand the basic characteristics of an RC circuit.
2. To understand the time constant in a RC circuit and how it can be changed.
3. To measure the time constant of a RC circuit.

Equipment

1. Breadboard and connector wires
2. Capacitors
3. Resistors
4. Signal generator (to supply Square wave)
5. Oscilloscope (observe time behavior of an RC circuit)

Background

The three common passive circuit elements are resistor, capacitor and inductor. Unlike resistors, which dissipate energy, capacitors and the inductors are two important passive linear circuit elements which do not dissipate but store energy. For this reason, capacitors and inductors are called storage elements. A circuit comprising a resistor and capacitor is called RC circuits. A first-order circuit is characterized by a first-order differential equation. The differential equation resulting from analyzing RC circuit is of the first order. Hence, the RC circuit is a first order circuit with a resistor and a capacitor in series connected to a voltage source such as a battery.

Natural response of an RC circuit

As shown in Fig.1, when t<0, the switch is set to position①, the capacitor has been charged:

$$u_C(0^-) = U_s$$

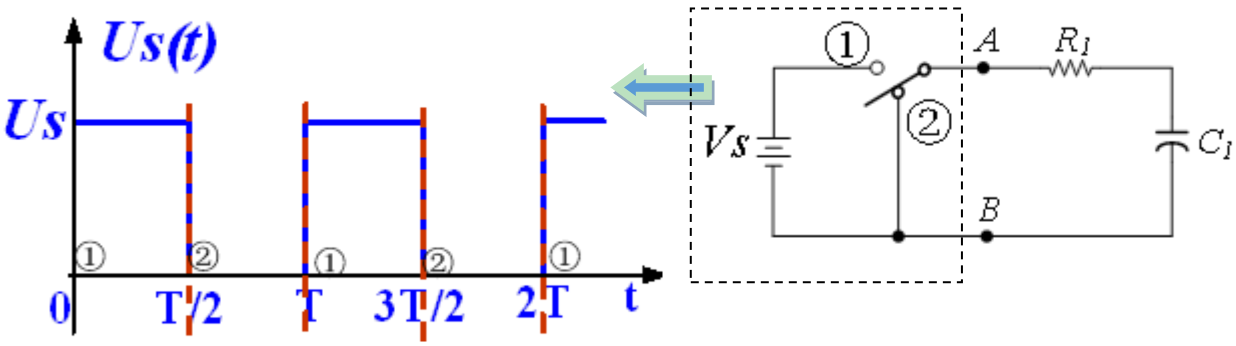

Fig.1 first order RC circuit

At $t=0$, the switch is flipped to position②, so that voltage source with a constant voltage of U_s is no longer included in the circuit, the capacitor is discharged, and the charge stored on the capacitor is free to leave the plates and will cause a current to flow. The current will be the largest at the beginning, t=0, and will decay away as charge leaves the capacitor's plates. Since the current is decreasing the voltage difference across the resistor is also decreasing.

When $t \geq 0$时，from KVL: $RC\frac{du_C}{dt} + u_C = 0 \quad t \geq 0$

We have $u_C(0_+) = u_C(0_-) = U_s$

$$u_C(t) = u_C(0_+)e^{-\frac{t}{\tau}} \quad t \geq 0^+$$

$$i_C(t) = -\frac{u_C(0_+)}{R}e^{-\frac{t}{\tau}} \quad t \geq 0$$

Fig2graphs the behavior of the voltage across the capacitor and resistor as a function of the time constant, of the circuit for a discharging capacitor.

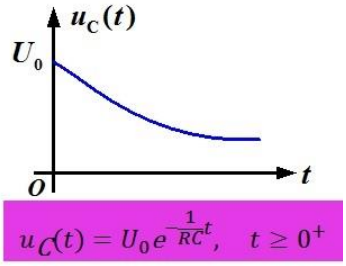

Fig.2(a)voltage across the capacitor u_C

@ Natural response

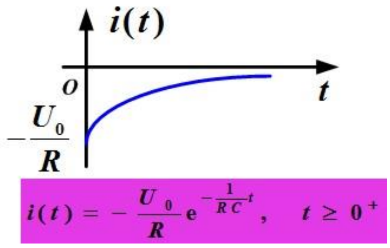

Fig.2(b)current across the capacitor i_C

@ Natural response

The time constant τ is an important parameter for first-order circuits The time constant for the RC circuit equals the product of R (in ohms) and C(farads), in which R is the total resistance and C is the total capacitance of the circuit. $u_C(t) = u_C(0_+)e^{-\frac{t}{\tau}} \quad t \geq 0^+$ indicates that the natural response of an RC circuit is an exponential decay of the initial voltage. The time constant RC governs the rate of decay. The time constant of a circuit is the time required for the response to decay to a factor of 1/e or 36.8% of its initial value. It is evident that the voltage is less than percent of after (five time constants). Thus, while in theory it takes an infinite time for "final state" conditions to be reached, in practical terms, after five time constants it is nearly impossible to observe further changes. As a result, it is customary to assume that the capacitor is fully discharged (or charged) after five time constants. Generally speaking, a circuit with a small time constant gives a fast response in that it reaches the final state quickly due to quick dissipation of energy stored, whereas a circuit with a large time constant gives a slow response because it takes longer to reach final state. But, no matter the time constant is small or large, the circuit reaches final state in five time constants.

Forced response of an RC circuit

As shown in Fig.1, when t<0, the switch is set to position②, the capacitor is not charged: $u_C(0^-)=0$. At t=0, the switch is transferred from position② to position①, and the voltage source with a constant voltage of U_s starts charging capacitor C through resistance R. The process of capacitor charging is the establishment of energy storage in the capacitor. At this time, the response of the circuit is called the forced response of an RC circuit.

When t>0, from KVL:

$$Ri_C(t)+u_C(t)=u_s(t)$$

$$RC\frac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=u_s(t)\quad t\geq 0^+$$

Initial value $u_C(0^-)=0$, It can be obtained that the voltage and current of the **capacitor** change with time:

$$u_C(t)=U_s(1-e^{-\frac{t}{\tau}})\quad t\geq 0^+$$

$$i_C(t)=\frac{U_s}{R}\mathrm{e}^{-\frac{t}{\tau}}\quad t\geq 0^+$$

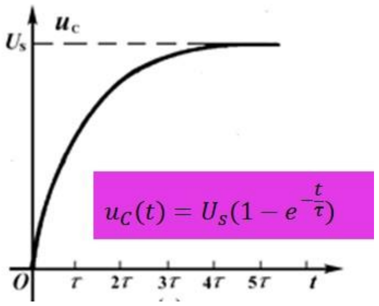

Fig.3(a)voltage cross the capacitor u_C

@ Forced response

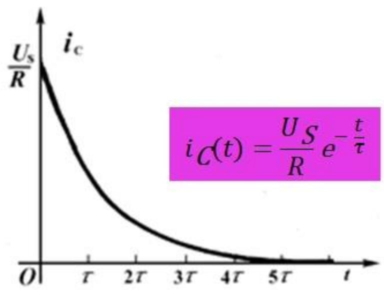

Fig.3(b)current across the capacitor i_C

@ Forced response

From Fig.3 we can find the transient response of voltage and current across the capacitor. At time t=τ=RC (one time constant), the voltage across the capacitor has grown to a value0.63Us. It will take an infinite amount of time for the capacitor to fully charge to its maximum value. For practical purposes we will assume that after five time constants the capacitor is fully charged.

Transient Response of First-Order RC Circuits@ square-wave voltage input

Suppose we connect a voltage source U_s, across a resistor and capacitor in series as shown in Figure1, which is commonly known as an RC circuit. When the switch is moved to position① , the voltage source U_s is connected to the circuit and a time-varying current begins flowing through the circuit as the capacitor charges. When the switch is then moved to position② , the voltage source U_s is taken out of the circuit and the capacitor discharges through the resistor. If the switch is moved alternately between positions ①and②, the voltage across points A and B can be plotted and would be like a square wave left. As a result, the components in the dotted box are analogous to a square-wave generator, which can be produced by a function generator with outputs at points A and B.

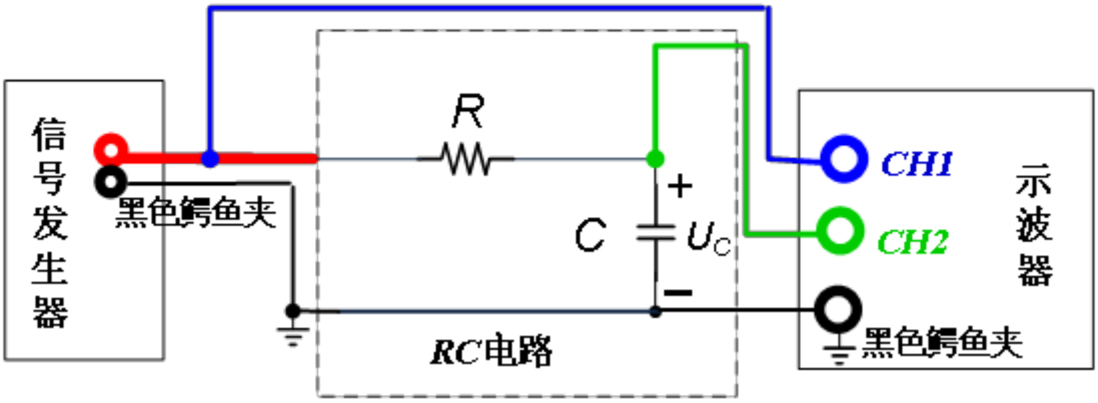

Fig.4 RC Circuit measurement setup

As shown in figure4, the input voltage is generated from the square-wave generator which is monitored by CH1, and the voltage across the capacitor is monitored by channel CH2, the wave is shown in Fig.5.

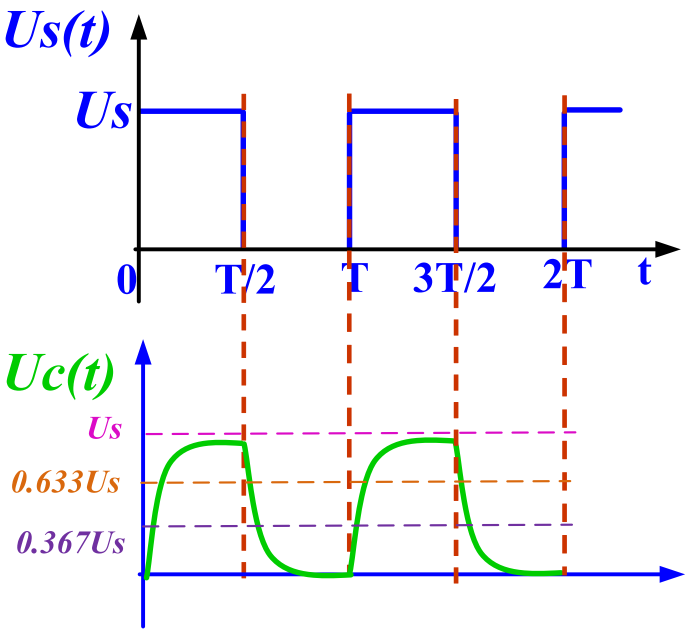

Fig.5 Transient Response of First-Order RC Circuits@ square-wave voltage input

The charging and the discharging curves of an RC circuit is shown in figure4. Time constant of an RC circuit can be measured using a digital oscilloscope. The time constant theoretically given by $\tau = RC$, is the time taken by the circuit to charge the capacitor from 0 to 0.632 times of the maximum voltage. In case of discharging, the time constant is the amount of time required to reduce the voltage across the capacitor from the maximum value to 0.368 of the maximum value.