2 heat equation problems with basic

SL BVP 2: $u_x(0,t)=0 \quad u_x(1,t)=0 \qquad 0<x<1$

S.1.2 #1 $\quad u(x,0)=f(x)=3-2\cos 4\pi x$

Soln: $k=1 \quad L=1$

The formal solution to this problem is

$$u(x,t)=\sum_{n=0}^{\infty} a_n e^{-n^2\pi^2 t}\cos n\pi x$$

$$u(x,0)=f(x) \implies 3-2\cos 4\pi x = \sum_{n=0}^{\infty} a_n \cos n\pi x$$

match up: $a_0=3 \quad a_4=-2$ all other a_n's are 0.

$$\implies u(x,t)=3-2e^{-16\pi^2 t}\cos 4\pi x$$

#3 $\quad u(x,0)=f(x)=2-3x$

The formal solution is $u(x,t)=\sum_{n=0}^{\infty} a_n e^{-n^2\pi^2 t}\cos n\pi x$

$$2-3x=\sum_{n=0}^{\infty} a_n \cos n\pi x \implies$$

$$a_0=\frac{\int_0^1(2-3x)\,dx}{\int_0^1 dx}=2x-\frac{3}{2}x^2\Big|_0^1=\frac{1}{2}$$

$u=2-3x \quad dv=\cos n\pi x\,dx$

$du=-3dx \quad v=\frac{1}{n\pi}\sin n\pi x$

$n\geq 1$ $\quad a_n=\frac{\int_0^1(2-3x)\cos n\pi x\,dx}{\int_0^1\cos^2 n\pi x\,dx}=2\left[(2-3x)\frac{1}{n\pi}\sin n\pi x\Big|_0^1+\frac{3}{n\pi}\int_0^1\sin n\pi x\,dx\right]$

denom is $\frac{1}{2}$

$$=\frac{6}{n\pi}\left(-\frac{1}{n\pi}\cos n\pi x\right)\Big|_0^1=-\frac{6}{n^2\pi^2}(\cos n\pi-1)$$

$$u(x,t)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{6}{n^2\pi^2}(1-\cos n\pi)e^{-n^2\pi^2 t}\cos n\pi x$$