12.2 #1   Redone

(There's an algebra error in my solution — corrected below.)

$$u_t - (2u+1)u_x = 3 \qquad\qquad u(x,0) = f(x) = 1 - x$$

Soln $\quad \dfrac{dx}{dt} = -(2u+1) \qquad \dfrac{du}{dt} = 3 \qquad ;\ \text{let } x_0 = x(0)$

$$u = 3t + K$$
$$\begin{cases} u(x_0,0) = K \\ u(x_0,0) = 1 - x_0 \end{cases} \Rightarrow \quad K = 1 - x_0$$

$$\dfrac{dx}{dt} = -\left(2(3t + 1 - x_0) + 1\right)$$

$$\dfrac{dx}{dt} = -6t + 2x_0 - 3 \qquad \leftarrow \text{my mistake was at this step}$$

$$x = -3t^2 + 2x_0 t - 3t + C$$

$x(0) = x_0 \Rightarrow \quad x = -3t^2 + 2x_0 t - 3t + x_0$

$\Rightarrow \quad x_0 = \dfrac{x + 3t^2 + 3t}{2t + 1}$

$\Rightarrow \quad \boxed{u = 3t + 1 - \dfrac{x + 3t^2 + 3t}{2t + 1}}$

$\cap \quad u = \dfrac{6t^2 + 5t + 1 - x - 3t^2 - 3t}{2t + 1}$

$\boxed{u = \dfrac{3t^2 + 2t - x + 1}{2t + 1}}$