Correction to handout
with solution to 3.1.1 #3

On the 2 lines below $\lambda > 0$
cross out $\cos\sqrt{\lambda}$
The lines should read

$$c_2\sqrt{\lambda} - c_1 = 0$$
$$c_1 = c_2\sqrt{\lambda}$$

Everything else in the problem is correct

In class problem

$$f'' + \lambda f = 0 \qquad 0 \le x \le 1$$
$$f(0) = 0 \qquad 3f(1) - 2f'(1)$$

Write $u(x) = 1$ as a sum of the 1st two e-functions.

Soln: We saw $f_1 \sim \sinh\sqrt{-\lambda_1}\,x \qquad f_2 \sim \sin\sqrt{\lambda_2}\,x$

$$\sqrt{-\lambda_1} \approx 1.3 \qquad \sqrt{\lambda_2} \approx 4.4$$

$$u(x) \approx a_1 \sinh 1.3x + a_2 \sin 4.4x$$

We'll use $\sinh^2 x = \frac{1}{2}\cosh 2x - \frac{1}{2}$

$$a_1 = \frac{\int_0^1 u(x)\sinh 1.3x\,dx}{\int_0^1 \sinh^2 1.3x\,dx} = \frac{\int_0^1 \sinh 1.3x\,dx}{\int_0^1 \sinh^2 1.3x\,dx}$$

num: $\frac{1}{1.3}\cosh 1.3x\Big|_0^1 = \frac{1}{1.3}(\cosh 1.3 - 1) \approx .75$

den. $\int_0^1 \left(\frac{1}{2}\cosh 2.6x - \frac{1}{2}\right)dx = \frac{1}{5.2}\sinh 2.6x\Big|_0^1 - \frac{x}{2}\Big|_0^1$

$$= \frac{\sinh 2.6}{5.2} - \frac{1}{2} \approx .79 \qquad \Rightarrow a_1 \approx .95$$

$$a_2 = \frac{\int_0^1 \sin\sqrt{\lambda_2}\,x\,dx}{\int_0^1 \sin^2\sqrt{\lambda_2}\,x\,dx}$$

num: $-\frac{1}{\sqrt{\lambda_2}}\cos\sqrt{\lambda_2}\,x\Big|_0^1 = -\frac{1}{\sqrt{\lambda_2}}\left(\cos\sqrt{\lambda_2} - 1\right)$

den. $\int_0^1 \left(\frac{1}{2} - \frac{1}{2}\cos 2\sqrt{\lambda_2}\,x\right)dx =$

$$\frac{1}{2}x - \frac{1}{4\sqrt{\lambda_2}}\sin 2\sqrt{\lambda_2}\,x\Big|_0^1 = \frac{1}{2} - \frac{1}{4\sqrt{\lambda_2}}\sin 2\sqrt{\lambda_2} = \frac{2\sqrt{\lambda_2} - \sin 2\sqrt{\lambda_2}}{4\sqrt{\lambda_2}}$$

$$\Rightarrow a_2 = \frac{-4\left(\cos\sqrt{\lambda_2} - 1\right)}{2\sqrt{\lambda_2} - \sin 2\sqrt{\lambda_2}} \approx \frac{-4(\cos 4.4 - 1)}{2(4.4) - \sin 8.8} \approx .64$$

$$\Rightarrow u(x) \approx .95\sinh(1.3x) + .64\sin(4.4x)$$

Completion of problem #7 in 3.1.1

$$f'' + 4f' + 3\lambda f = 0 \qquad 0 \le x \le 1$$

$$f(0) = 0 \qquad f(1) = 0$$

Find e-values and e-functions

Soln We considered the cases

1) $\lambda = \frac{4}{3}$ $\qquad f = c_1 e^{-2x} + c_2 x e^{-2x}$

we saw $\lambda = \frac{4}{3}$ is not an e-val

2) $\lambda < \frac{4}{3}$ $\qquad f = c_1 e^{(-2+\sqrt{4-3\lambda})x} + c_2 e^{(-2-\sqrt{4-3\lambda})x}$

we saw that $\lambda < \frac{4}{3}$ produced no e-vals

3) $\lambda > 4/3$ $\qquad f \sim e^{-2x}\left(c_1 \cos\sqrt{3\lambda-4}\,x + c_2 \sin\sqrt{3\lambda-4}\,x\right)$

$f(0) = 0 \Rightarrow c_1 = 0 \Rightarrow f(x) = c_2 e^{-2x} \sin\sqrt{3\lambda-4}\,x$

$f(1) = 0 \Rightarrow c_2 e^{-2} \sin\sqrt{3\lambda-4} = 0$

$\Rightarrow \sqrt{3\lambda-4} = n\pi \qquad n = 1, 2, \ldots$

$3\lambda - 4 = n^2\pi^2$

e-vals $\quad \lambda_n = \dfrac{4 + n^2\pi^2}{3} \qquad n = 1, 2, \ldots$

e-functions $\quad f_n \sim e^{-2x} \sin n\pi x$

3.1.2 #10

Write $u(x) = x+1$ in a generalized F.S

in the e-functions $\cos\frac{(2n-1)\pi x}{2}$ $\quad n=1,2,\dots \quad 0 \le x \le 1$

Solution :

$$x+1 \approx \sum_{n=1}^{\infty} a_n \cos\frac{(2n-1)\pi x}{2}$$

$$a_n = \frac{\int_0^1 (x+1)\cos\frac{(2n-1)\pi x}{2}\,dx}{\int_0^1 \cos^2\frac{(2n-1)\pi x}{2}\,dx}$$

$u = x+1 \qquad dv = \cos\frac{(2n-1)\pi x}{2}dx$

$du = dx \qquad v = \frac{2}{(2n-1)\pi}\sin\frac{(2n-1)\pi x}{2}$

num. $$(x+1)\frac{2}{(2n-1)\pi}\sin\frac{(2n-1)\pi x}{2}\Big|_0^1 - \frac{2}{(2n-1)\pi}\int_0^1 \sin\frac{(2n-1)\pi x}{2}dx$$

$$= \frac{4}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2} + \frac{4}{(2n-1)^2\pi^2}\cos\frac{(2n-1)\pi x}{2}\Big|_0^1$$

$$= \frac{4}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2} - \frac{4}{(2n-1)^2\pi^2} \qquad \left(\cos\frac{(2n-1)\pi}{2} = 0\right)$$

denom: $\frac{1}{2}$

$$x+1 \approx \sum_{n=1}^{\infty}\left(\frac{8}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2} - \frac{8}{(2n-1)^2\pi^2}\right)\cos\frac{(2n-1)\pi x}{2}$$

MATH 4545 TAKE HOME #1 DUE 2/17

1) Consider the SL BVP
$$\begin{cases} f'' + \lambda f = 0 & f = f(x) \\ f(0) + f'(0) = 0 & 0 \le x \le p \\ f(p) - f'(p) = 0 & p > 0. \end{cases}$$

This problem investigates the negative e-values and how they depend on p.

a) Let $p = 1$ and estimate the <u>negative</u> e-value(s).

b) Do the same for $p = 2$.

c) " " " " $p = 3$.

Extra: Make and justify a statement about how the number of negative e-values depends on a general real $p > 0$. The justification does not have to be rigorous, but it should be mathematical.

2) In class we used the FSS of $f(x) = x$ to show that
$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$
Use the F.S.S. of $f(x) = x^2$ to find $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

Extra: Find $1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$

You may work in groups up to 3, and each group should turn in a common paper.