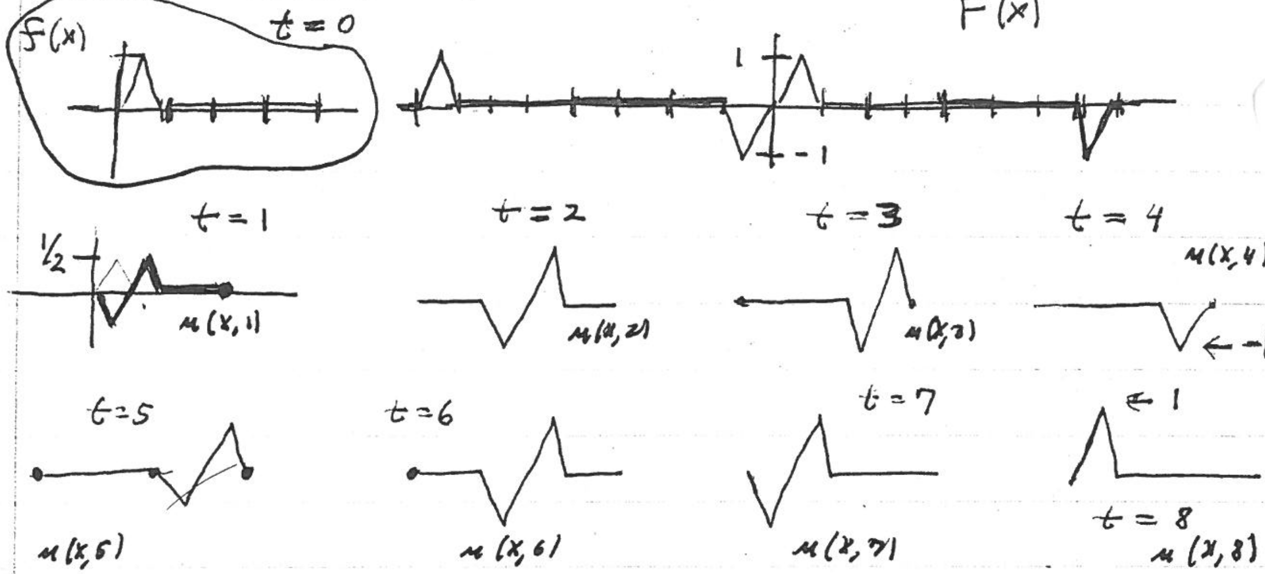
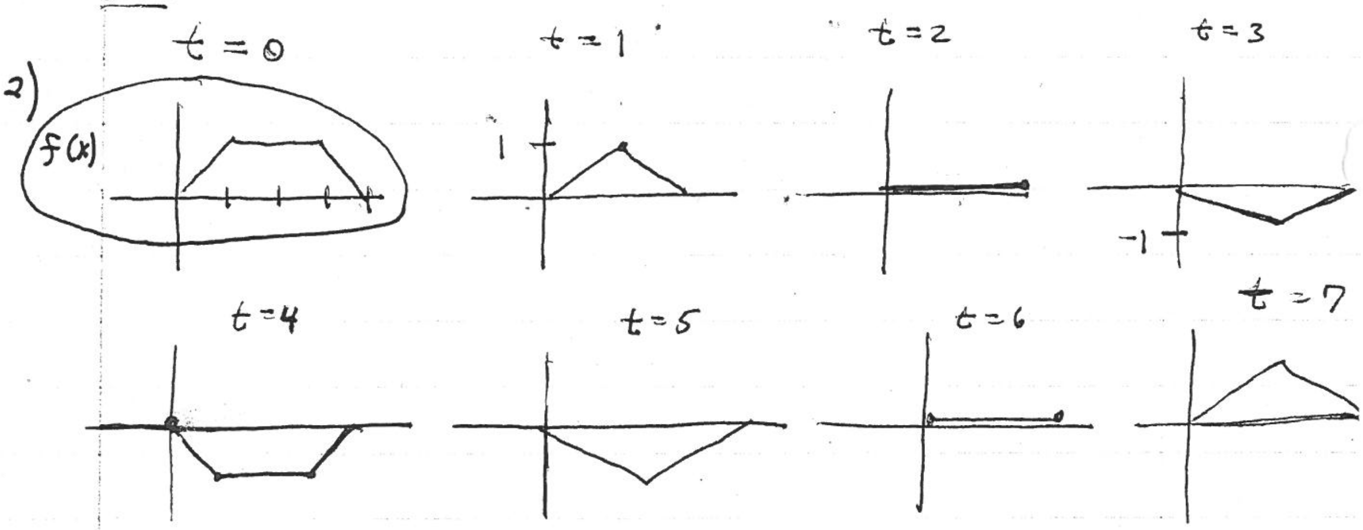
Two travelling wave examples $c = 1$

1)



all amplitudes between $t=1$ and $t=8$ are $1/2$
except at $t=4$ and $t=8$

2)



We're using d'Alembert's solution

$$u(x,t) = \frac{1}{2}\Big(F(x-ct) + F(x+ct)\Big)$$

F is the odd periodic extension of $f(x) = u(x,0)$
Note the period of $u(x,t)$ is $\dfrac{2\pi}{\frac{n\pi c}{L}}$ which is 8 in these examples.

Practice

d'Alembert: The solution $u(x,t)$ to the wave eqn $u_{tt} = c^2 u_{xx}$, $0<x<L$ with ends held fixed and starting from rest with initial position $u(x,0) = f(x)$ can be written as

$$u(x,t) = \frac{1}{2}\left(F(x-ct) + F(x+ct)\right)$$

where $F(x)$ is the odd periodic extension of $f(x)$.

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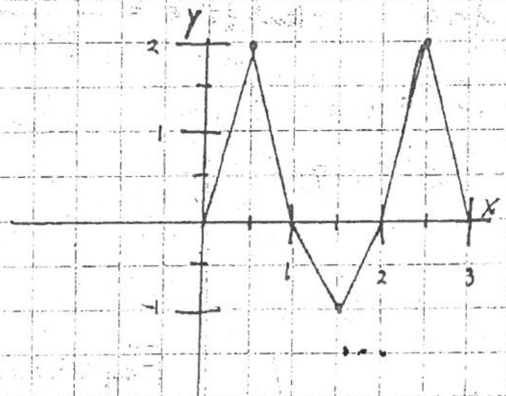
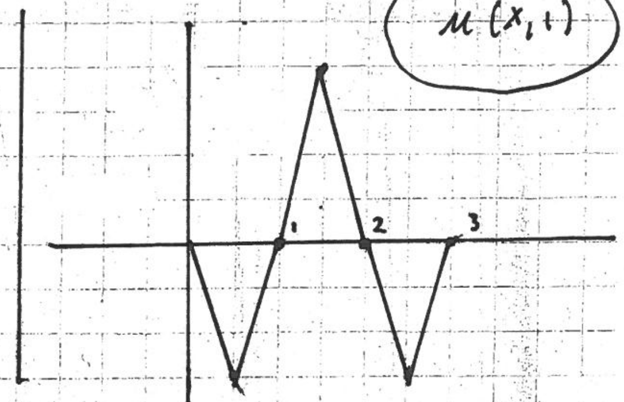
1) Ex. Given $f(x)$ $0<x<3$ as shown, below.

assume $c=1$ and sketch $u(x,1)$.



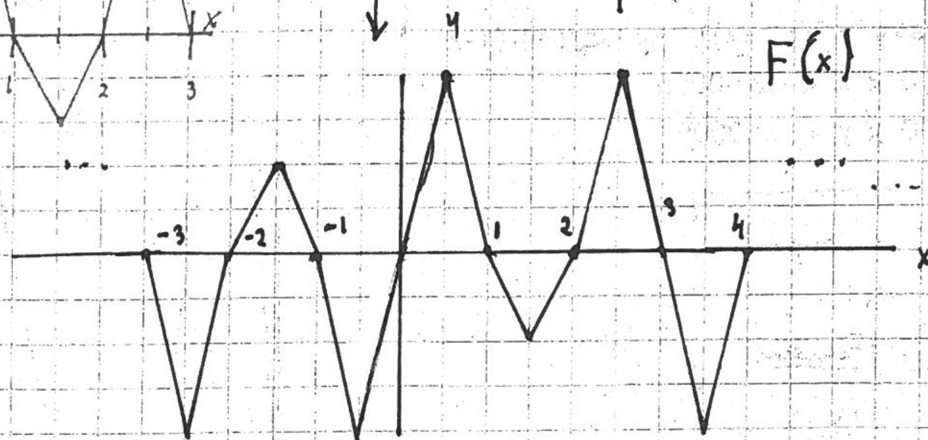
$u(x,1)$



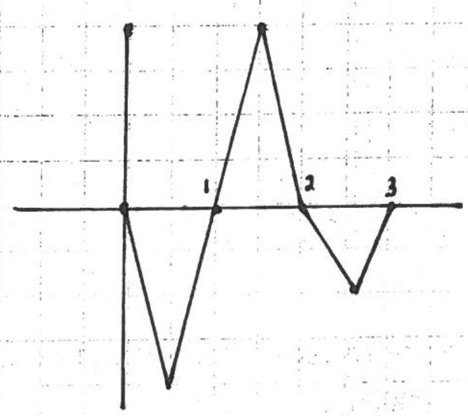
f(x)

F(x)

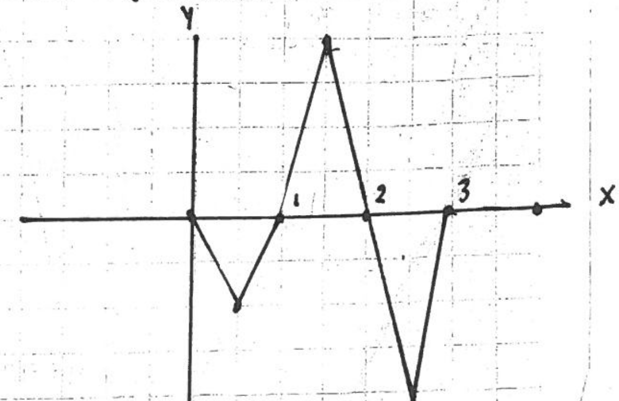
Soln



F(x-1)
(translate to right)



F(x+1)
(translate to left)



$u(x,1)$ is above

$$u(x,1) = \frac{1}{2}F(x-1) + \frac{1}{2}F(x+1)$$