Last Wave Problem 5.2.2 # 3

$c=1 \quad L=1$

Formal Soln: $u(x,t) = \sum_{n=1}^{\infty} \sin n\pi x \left(b_{1n} \cos n\pi t + b_{2n} \sin n\pi t \right)$

$u(x,0) = 2 \sin 3\pi x \implies$

$$2 \sin 3\pi x = \sum_{n=1}^{\infty} b_{1n} \sin n\pi x$$

match up $n=3$ $\quad b_{13} = 2$ $\quad$ all other b_{1n}'s are $\underline{0}$

$$u_t(x,t) = \sum_{n=1}^{\infty} \sin n\pi x \left(-n\pi b_{1n} \sin n\pi t + n\pi b_{2n} \cos n\pi t \right)$$

$u_t(x,0) = 2 \implies 2 = \sum_{n=1}^{\infty} n\pi b_{2n} \sin n\pi x$

$$\implies n\pi b_{2n} = \frac{\int_0^1 2 \sin n\pi x \, dx}{\int_0^1 \sin^2 n\pi x \, dx} = 2 \int_0^1 2 \sin n\pi x \, dx$$

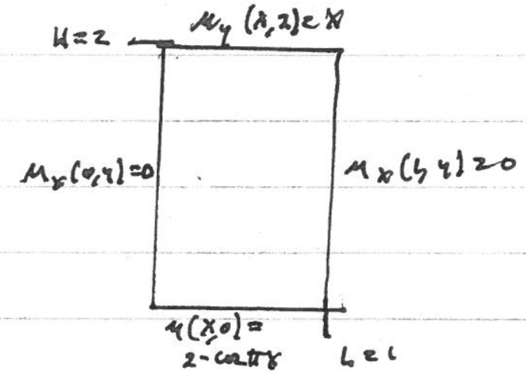
$$= -\frac{4}{n\pi} \cos n\pi x \Big|_0^1 = \frac{4}{n\pi} (1 - \cos n\pi)$$

or $b_{2n} = \frac{4}{n^2\pi^2} (1 - \cos n\pi)$

Assemble

$$u(x,t) = 2 \sin 3\pi x \cos 3\pi t + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2} (1 - \cos n\pi) \sin n\pi x \sin n\pi t$$

Laplace in a Rectangle



§5.3.1 #4 $\quad u_x(0,y)=0 \quad u_x(1,y)=0$

$u(x,0)=2-\cos\pi x \quad u_y(x,2)=x$

Soln: translate boundary conds.

$$X''+\lambda X=0 \qquad Y''-\lambda Y=0$$

$$X'(0)=0 \qquad X'(1)=0$$

$$\Rightarrow \lambda_n = n^2\pi^2 \quad X_n \sim \cos n\pi x \quad \Rightarrow \quad Y_0 = a_0 y + b_0$$

$n=0,1,2,\ldots \qquad n>0,\ Y_n = \sinh n\pi y + \delta_n \cosh n\pi(y-2)$

formal soln: $$u(x,y) = aY_0 + b_0 + \sum_{n=1}^{\infty} \cos n\pi x \left(a_n \sinh n\pi y + b_n \cosh n\pi(y-2)\right)$$

$u(x,0)=2-\cos\pi x \quad \Rightarrow$ by matching up $\quad b_0=2 \quad b_1 = \dfrac{-1}{\cosh 2\pi}$

$$u_y(x,y) = a_0 + \sum_{n=1}^{\infty} \cos n\pi x \left(a_n \cdot n\pi \cosh n\pi y + b_n n\pi \sinh n\pi(y-2)\right)$$

$$u_y(x,2) = x \quad \Longrightarrow \quad \boxed{x = a_0 + \sum_{n=1}^{\infty} (a_n \cdot n\pi \cosh 2n\pi) \cos n\pi x}$$

$$a_0 = \frac{\int_0^1 x\,dx}{\int_0^1 dx} = \frac{1}{2} \qquad n\pi\cosh(2n\pi)\, a_n = \frac{\int_0^1 x \cos n\pi x\,dx}{\int_0^1 \cos^2 n\pi x\,dx}$$

after integration and solving for a_n: $$a_n = \frac{2}{n^3\pi^3 \cosh 2n\pi}(\cos n\pi - 1)$$

Assembling the soln:

$$u(x,y) = \frac{1}{2}y + 2 + \sum_{n=1}^{\infty} \frac{2(\cos n\pi - 1)}{n^3\pi^3 \cosh 2n\pi} \cos n\pi x \sinh n\pi y - \frac{1}{\cosh 2\pi} \cos \pi x \cosh \pi(y-2)$$