Regular S-L problems

$$
\boxed{
\begin{array}{lll}
\text{Eqn} & (p f')' + q f + \lambda \sigma f = 0 & a < x < b \\
\text{Bdy conds} & \begin{cases} K_1 f(a) + K_2 f'(a) = 0 \\ K_3 f(b) + K_2 f'(b) = 0 \end{cases} &
\end{array}
}
$$

p, q, σ are functions of x
p, q, σ are required to be cont on $[a, b]$
p' is required to be cont on $[a, b]$
p and σ are required to be positive
K_1, K_2, K_3, K_4 are constants

S-L Theory says

1) There are always an infinite number of real e-vals for the problem $\lambda_1 < \lambda_2 < \cdots < \lambda_n \cdots$ so that $\lim_{n \to \infty} \lambda_n = \infty$.

2) Eigenfunctions corresponding to distinct e-values are orthogonal with respect to σ on $[a, b]$. This means

$$\int_a^b f_i(x) f_j(x) \sigma(x)\, dx = 0 \quad \text{for all } i \neq j.$$

Every p.w. differentiable function $g(x)$ on $[a, b]$ can be "written" as a sum of a complete set of e-functions:

$$g(x) \sim \sum_{n=1}^{\infty} a_n f_n(x) \quad \text{— \underline{a generalized F.S. of } g(x)}$$

The coefficients are computed as follows

$$a_n = \frac{\int_a^b g(x) \cdot f_n(x)\, \sigma(x)\, dx}{\int_a^b \sigma(x) f_n^2(x)\, dx}$$