Wave Problems 5.2.1 ends held fixed, $c=1$, $L=1$

All these problems have the formal solution

$$u(x,t) = \sum_{n=1}^{\infty}\left(b_{1n}\cos\frac{n\pi ct}{L} + b_{2n}\sin\frac{n\pi ct}{L}\right)\sin\frac{n\pi x}{L}$$

1) $u(x,0) = f(x) = -3\sin 2\pi x + 4\sin 7\pi x$ $\qquad u_t(x,0) = \sin 3\pi x$

$u(x,0)=f(x) \implies -3\sin 2\pi x + 4\sin 7\pi x = \sum_{n=1}^{\infty} b_{1n}\sin n\pi x$

$\implies b_{12} = -3 \qquad b_{17} = 4$ $\qquad$ all other $b_{1n}$'s $= 0$

* $u_t(x,t) = \sum_{n=1}^{\infty}\left(-n\pi\, b_{1n}\sin n\pi t + n\pi b_{2n}\cos n\pi t\right)\sin n\pi x$

$u_t(x,0) = \sin 3\pi x \implies \sin 3\pi x = \sum_{n=1}^{\infty} n\pi b_{2n}\sin n\pi x \implies n=3,$

$3\pi b_{23} = 1$

$\implies u(x,t) = -3\sin 2\pi x\cos 2\pi t + 4\sin 7\pi x\cos 7\pi t + \frac{1}{3\pi}\sin 3\pi x\sin 3\pi t$

2) $u(x,0) = f(x) = -1$ $\qquad u_t(x,0) = 3\sin\pi x = g(x)$

$u(x,0) = f(x) \implies -1 = \sum_{n=1}^{\infty} b_{1n}\sin n\pi x$

$$b_{1n} = \frac{\int_0^1 -\sin n\pi x\,dx}{\int_0^1 \sin^2 n\pi x\,dx} = -2\int_0^1 \sin n\pi x\,dx = \frac{+2}{n\pi}\cos n\pi x\Big|_0^1 = \frac{2}{n\pi}(\cos n\pi - 1)$$

Using $u_t(x,t)$ as in * above

$u_t(x,0) = g(x) \implies 3\sin\pi x = \sum_{n=1}^{\infty} n\pi\, b_{2n}\sin n\pi x \implies$ $n=1$, $b_{21} = \frac{3}{\pi}$

$$\implies u(x,t) = \sum_{n=1}^{\infty}\frac{2}{n\pi}(\cos n\pi - 1)\cos n\pi t + \frac{3}{\pi}\sin\pi x\sin\pi t$$