MATH 4545 Quiz #2 NAME _____

Given the SL-BVP $\begin{cases} f''+\lambda f=0, & f=f(x) \quad 0\le x\le 1 \\ f'(0)=0 & f(1)-f'(1)=0. \end{cases}$

a) Determine all e-values of the problem graphically.

b) Determine all e-Functions of the problem.

c) No estimates required but show the 1st three e-values on your graphs.

Soln

$\lambda=0 \Rightarrow f=ax+b \quad f'=a$

$f'(0)=0 \Rightarrow (a=0) \Rightarrow f=b \quad f'=0$

$f(1)-f'(1)=0 \Rightarrow (b=0)$ So $\lambda=0$ is <u>not</u> an e-value

$\lambda>0 \Rightarrow f=c_1\cos\sqrt{\lambda}x+c_2\sin\sqrt{\lambda}x \quad f'=-c_1\sqrt{\lambda}\sin\sqrt{\lambda}x+c_2\sqrt{\lambda}\cos\sqrt{\lambda}x$

$f'(0)=0 \Rightarrow c_2=0 \Rightarrow (f=c_1\cos\sqrt{\lambda}x) \quad f'=-c_1\sqrt{\lambda}\sin\sqrt{\lambda}x$

$f(1)-f'(1)=0 \Rightarrow \cos\sqrt{\lambda}+\sqrt{\lambda}\sin\sqrt{\lambda}=0 \Rightarrow$

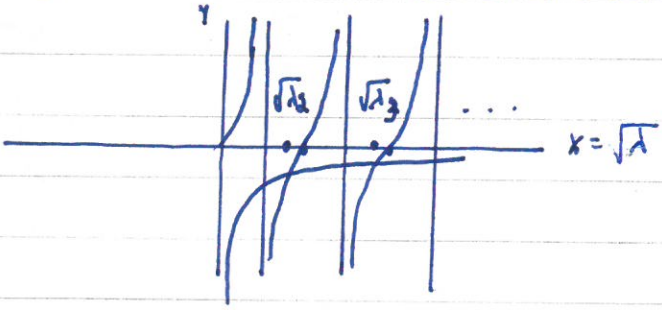
$(\tan\sqrt{\lambda}=-1/\sqrt{\lambda})$

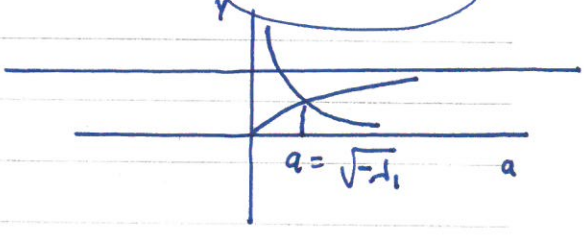
$\lambda<0 \quad \lambda=-a^2 \quad a>0$

$f=c_1\cosh ax+c_2\sinh ax \quad f'=c_1 a\sinh ax+c_2 a\cosh ax$

$f'(0)=0 \Rightarrow c_2=0 \Rightarrow (f=c_1\cosh ax) \quad f'=c_1 a\sinh ax$

$f(1)-f'(1)=0 \Rightarrow \cosh a-a\sinh a=0 \Rightarrow (\tanh a=\frac{1}{a})$





The positive e-values are the solutions $\lambda_2, \lambda_3 \ldots$ to $\tan\sqrt{\lambda_n}=-\frac{1}{\sqrt{\lambda_n}}$

The e-functions $(f_n\sim\cos\sqrt{\lambda_n}x)$ $n=2,3,\ldots$

There is one negative e-value $(\lambda_1=-a^2)$ where a solves $(\tanh a=\frac{1}{a})$

The corresponding e-function is $(f_1\sim\cosh ax)$

(estimates $\lambda_1\approx -1.44 \quad \lambda_2\approx 7.8 \quad \lambda_3\approx 39.2$)