MATH 4545 Quiz #3 NAME _____

1) Find $u(x,t)$ $\quad 0<x<2$ $\quad t>0$

given $\begin{cases} u_t = u_{xx} & L=2 \quad c=1 \\ u(0,t) = u_x(2,t) = 0 \\ u(x,0) = 100 & 0 \le x \le 2 \end{cases}$

SOLN: $u = X \cdot T$

X satisfies $X'' + \lambda X = 0$ $\quad X(0)=0$ $\quad X'(2)=0$

e-values $\lambda_n = \left(\frac{(2n-1)\pi}{4}\right)^2$ $\quad$ e-functions $X_n = \sin\frac{(2n-1)\pi x}{4}$ $\quad n=1,2,\dots$

formal soln $u(x,t) = \sum_{n=1}^{\infty} a_n \sin\frac{(2n-1)\pi x}{4}\, e^{-\left(\frac{(2n-1)\pi}{4}\right)^2 t}$

$$a_n = \frac{\int_0^2 100 \sin\frac{(2n-1)\pi x}{4}\,dx}{\int_0^2 \sin^2\frac{(2n-1)\pi x}{4}\,dx}$$

num: $\frac{-400}{(2n-1)\pi}\cos\frac{(2n-1)\pi x}{4}\Big|_0^2 = \frac{400}{(2n-1)\pi}$ $\quad \cos\frac{(2n-1)}{2}\pi = 0$

den. $\int_0^2 \sin^2\frac{2n-1}{4}\pi x\,dx = \int_0^2 \left(\frac{1}{2} - \frac{1}{2}\cos\frac{(2n-1)\pi}{2}x\right)dx = \frac{1}{2}x - \frac{1}{(2n-1)\pi}\sin(2n-1)\pi x\Big|_0^2 = 1$

$\Rightarrow a_n = \frac{400}{(2n-1)\pi}$

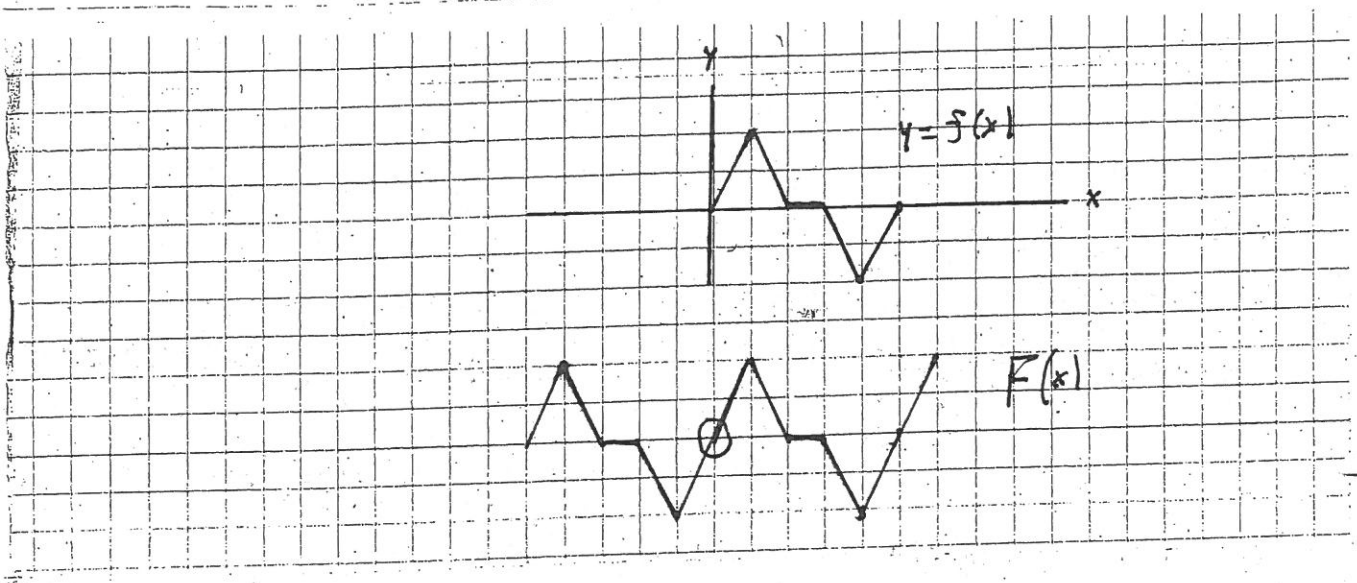
$$u(x,t) = \sum \frac{400}{(2n-1)\pi}\sin\frac{(2n-1)\pi x}{4}\, e^{-\left(\frac{(2n-1)\pi}{4}\right)^2 t}$$

2) Suppose $u(x,t)$ satisfies the wave equation $u_{tt} = u_{xx}$ $0 \le x \le 5$

$u(0,t) = u(5,t) = 0$

$u(x,0) = f(x)$ below

$u_t(x,0) = 0$



Use d'Alembert's solution to graph $u(x,1)$.