Problem #2 This problem works for any L; I took $L = \pi$

$f(x) = x^2 \qquad 0 \le x \le \pi$

The F.C.S. is

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \left(\frac{4 \cos n\pi}{n^2} \right) \cos nx$$

Letting $x = \pi$, $f(\pi) = \pi^2 \Rightarrow$

$$\pi^2 = \frac{\pi^2}{3} + 4 \left(\sum_1^{\infty} \frac{1}{n^2} \right) \quad \text{since } (\cos n\pi)^2 = 1$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Extra:

The F.S.S. is

$$f(x) = 2\pi \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} - \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3}$$

The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$ converges to $\frac{\pi}{4}$ by the example we did in class.

Substituting $x = \frac{\pi}{2}$ gives

$$\frac{\pi^2}{4} = 2\pi \cdot \frac{\pi}{4} - \frac{8}{\pi} \left(1 - \frac{1}{3^3} + \frac{1}{5^3} - \cdots \right)$$

$$\Rightarrow \frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \cdots$$