MATH 4545 QUIZ #1 NAME _____

Find and Sketch the Fourier Cosine Series and Fourier Sine Series of $f(x) = \begin{cases} 1-x & 0 \le x \le 1 \\ 0 & 1 < x \le 2 \end{cases}$.

SOLN $L = 2$

F.C.S. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$

$u = 1-x \quad dv = \cos \frac{n\pi x}{2}\,dx$

$du = -dx \quad v = \frac{2}{n\pi} \sin \frac{n\pi x}{2}$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{1}{2}$$

$$a_n = \frac{2}{L}\int_0^L f(x) \cos \frac{n\pi x}{L}\,dx = \int_0^1 (1-x) \cos \frac{n\pi x}{2}\,dx$$

$$= (1-x)\cdot \frac{2}{n\pi} \sin \frac{n\pi x}{2}\Big|_0^1 + \frac{2}{n\pi}\int_0^1 \sin \frac{n\pi x}{2}\,dx$$

$$= -\frac{4}{n^2\pi^2} \cos \frac{n\pi x}{2}\Big|_0^1 = -\frac{4}{n^2\pi^2}\cos \frac{n\pi}{2} + \frac{4}{n^2\pi^2}$$

$$f(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \left(\frac{4}{n^2\pi^2} - \frac{4}{n^2\pi^2} \cos \frac{n\pi}{2} \right) \cos \frac{n\pi x}{2}$$

F.S.S. $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$

$u = 1-x \quad dv = \sin \frac{n\pi x}{2}$

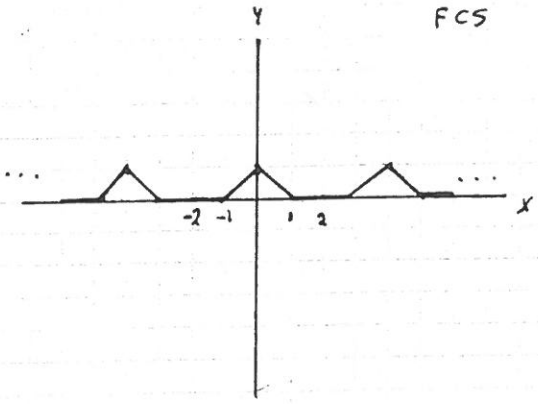
$du = -dx \quad v = -\frac{2}{n\pi} \cos \frac{n\pi x}{2}$

$$b_n = \frac{2}{L}\int_0^L f(x) \sin \frac{n\pi x}{L}\,dx = \int_0^1 (1-x) \sin \frac{n\pi x}{2}\,dx$$

$$= (1-x)\left(-\frac{2}{n\pi} \cos \frac{n\pi x}{2} \right)\Big|_0^1 - \frac{2}{n\pi}\int_0^1 \cos \frac{n\pi x}{2}\,dx$$

$$= \frac{2}{n\pi} - \frac{4}{n^2\pi^2} \sin \frac{n\pi x}{2}\Big|_0^1 = \frac{2}{n\pi} - \frac{4}{n^2\pi^2} \sin \frac{n\pi}{2}$$

$$f(x) = \sum_{n=1}^{\infty} \left(\frac{2}{n\pi} - \frac{4}{n^2\pi^2} \sin \frac{n\pi}{2} \right) \sin \frac{n\pi x}{2}$$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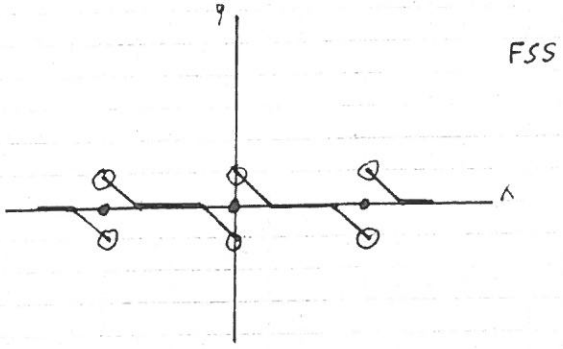
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A SL BVP

Given the S-L BVP $\begin{cases} f''+\lambda f=0, & f=f(x) \quad 0<x<1 \\ f(0)=0 & 3f(1)-2f'(1)=0 \end{cases}$

Find e-vals and e-functions

SOLN: $\lambda=0$ $f(x)=\alpha x+\beta$ $f'(x)=\alpha$

$f(0)=0 \Rightarrow \beta=0 \Rightarrow f(x)=\alpha x$ $f'(x)=\alpha$

$3f(1)-2f'(1)=0 \Rightarrow 3\alpha-2\alpha=0 \Rightarrow \alpha=0$ $\beta=\alpha=0 \Rightarrow \lambda=0$ not an e-val.

$\lambda>0$ $f(x)=c_1\cos\sqrt{\lambda}x+c_2\sin\sqrt{\lambda}x \Rightarrow f'(x)=-c_1\sqrt{\lambda}\sin\sqrt{\lambda}x+c_2\sqrt{\lambda}\cos\sqrt{\lambda}x$

$f(0)=0 \Rightarrow c_1=0 \Rightarrow f(x)=c_2\sin\sqrt{\lambda}x$ $f'(x)=c_2\sqrt{\lambda}\cos\sqrt{\lambda}x$

$3f(1)-2f'(1)=0 \Rightarrow 3c_2\sin\sqrt{\lambda}-2c_2\sqrt{\lambda}\cos\sqrt{\lambda}=0$

$\tan\sqrt{\lambda}=\frac{2}{3}\sqrt{\lambda}$ eqn for e-vals

$\lambda<0$ let $\lambda=-a^2$, $a>0$ $f(x)=c_1\cosh ax+c_2\sinh ax$ $f'(x)=ac_1\sinh ax+ac_2\cosh ax$

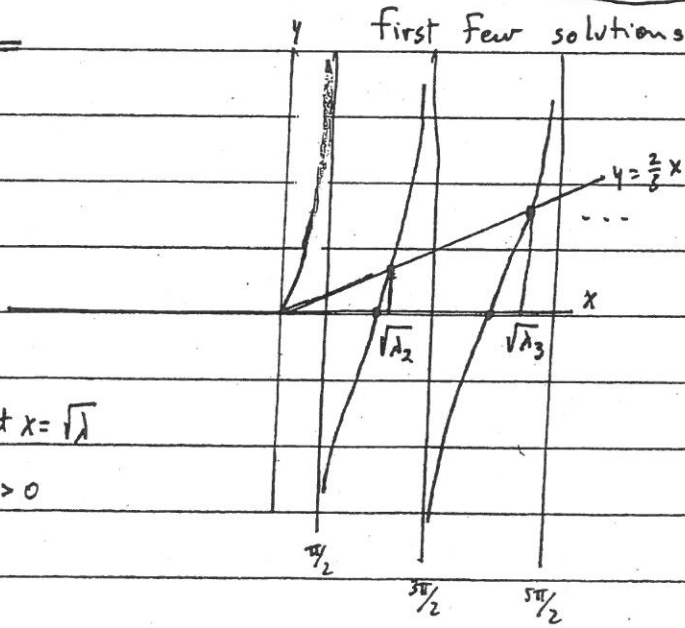
$f(0)=0 \Rightarrow c_1=0 \Rightarrow f(x)=c_2\sinh ax$

$f'(x)=ac_2\cosh ax$

$3f(1)-2f'(1)=0 \Rightarrow 3c_2\sinh a-2ac_2\cosh a=0$

eqn for e-vals: $\tanh a=\frac{2}{3}a$ $a=\sqrt{-\lambda}$

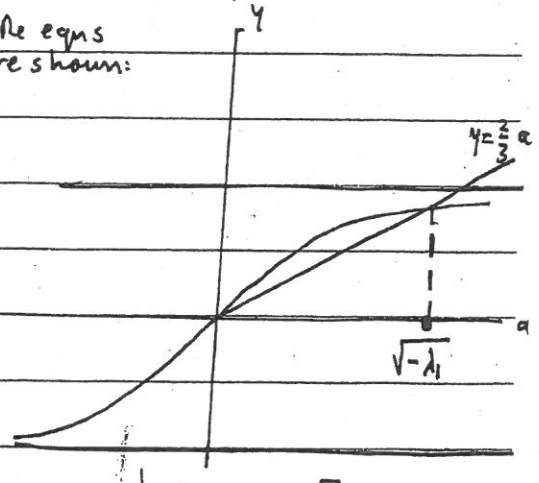
first few solutions to the eqns are shown:



let $x=\sqrt{\lambda}$

$\lambda>0$

$\sqrt{\lambda_2}\approx 4.4$ $\sqrt{\lambda_3}\approx 7.7\ldots$



$\lambda<0$ $a=\sqrt{-\lambda}$

$\sqrt{-\lambda_1}\approx 1.3$

$\lambda_1\approx -1.69$

A SL BVP

Given the S-L BVP $\begin{cases} f'' + \lambda f = 0, & f = f(x) \quad 0 < x < 1 \\ f(0) = 0 & 3f(1) - 2f'(1) = 0 \end{cases}$

Find e-vals and e-functions

SOLN:

$\lambda = 0$ $\quad f(x) = \alpha x + \beta \quad f'(x) = \alpha$

$f(0) = 0 \Rightarrow \beta = 0 \Rightarrow f(x) = \alpha x \quad f'(x) = \alpha$

$3f(1) - 2f'(1) = 0 \Rightarrow 3\alpha - 2\alpha = 0 \Rightarrow \alpha = 0 \quad \beta = \alpha = 0 \Rightarrow \lambda = 0$ not an e-val.

$\lambda > 0$ $\quad f(x) = c_1 \cos\sqrt{\lambda}\, x + c_2 \sin\sqrt{\lambda}\, x \Rightarrow f'(x) = -c_1\sqrt{\lambda}\sin\sqrt{\lambda}\, x + c_2\sqrt{\lambda}\cos\sqrt{\lambda}\, x$

$f(0) = 0 \Rightarrow c_1 = 0 \Rightarrow f(x) = c_2 \sin\sqrt{\lambda}\, x \quad f'(x) = c_2\sqrt{\lambda}\cos\sqrt{\lambda}\, x$

$3f(1) - 2f'(1) = 0 \Rightarrow 3c_2 \sin\sqrt{\lambda} - 2c_2\sqrt{\lambda}\cos\sqrt{\lambda} = 0$

$\tan\sqrt{\lambda} = \frac{2}{3}\sqrt{\lambda}$ eqn for + e-vals

$\lambda < 0$ let $\lambda = -a^2$, $a > 0 \quad f(x) = c_1 \cosh ax + c_2 \sinh ax \quad f'(x) = ac_1 \sinh ax + ac_2 \cosh ax$

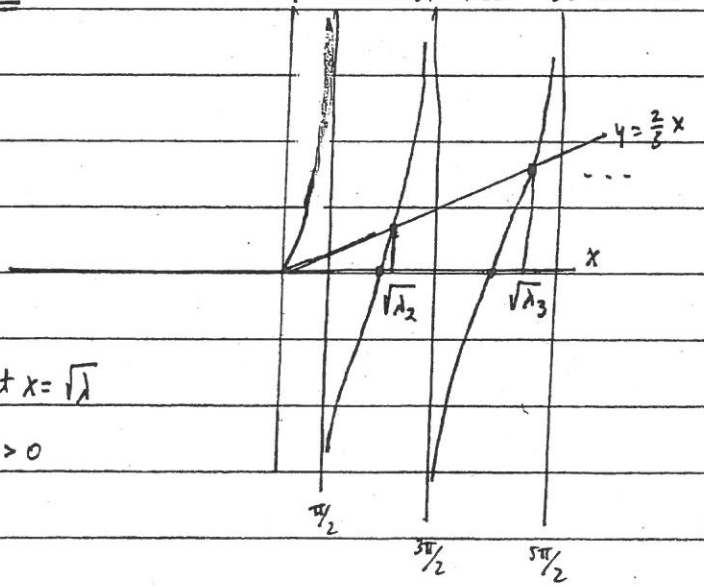
$f(0) = 0 \Rightarrow c_1 = 0 \Rightarrow f(x) = c_2 \sinh ax$

$f'(x) = ac_2 \cosh ax$

$3f(1) - 2f'(1) = 0 \Rightarrow 3c_2 \sinh a - 2ac_2 \cosh a = 0$

eqn for - e-vals: $\tanh a = \frac{2}{3}a$ $\quad a = \sqrt{-\lambda}$

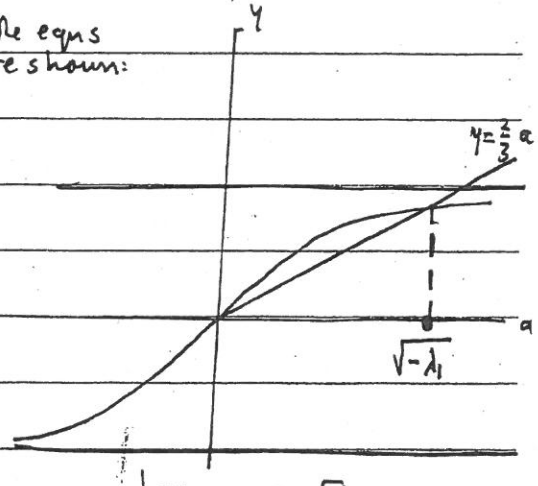
first few solutions to the eqns are shown:



Let $x = \sqrt{\lambda}$

$\lambda > 0$

$\sqrt{\lambda_2} \approx 4.4 \quad \sqrt{\lambda_3} \approx 7.7 \cdots$



$\lambda < 0 \quad a = \sqrt{-\lambda}$

$\sqrt{-\lambda_1} \approx 1.3$

$\lambda_1 \approx -1.69$