$$\begin{cases} f'' + \lambda f = 0 \\ f'(0) - f(0) = 0 \quad , \quad f(1) = 0 \end{cases}$$

Chapter 3.11 #3

Find e-vals & e-functions

Determine e-vals graphically

soln $\lambda = 0 \Rightarrow f = ax + b \qquad f' = a$

$f'(0) - f(0) = 0 \Rightarrow a - b = 0 \Rightarrow f = ax + a \qquad f' = a$

$f(1) = 0 \Rightarrow 2a = 0 \Rightarrow a = 0 \Rightarrow b = 0 \Rightarrow$ $\lambda = 0$ not an e-val

$\lambda > 0 \qquad f = c_1 \cos\sqrt{\lambda}\, x + c_2 \sin\sqrt{\lambda}\, x \qquad f' = -c_1\sqrt{\lambda}\sin\sqrt{\lambda}\, x + c_2\sqrt{\lambda}\cos\sqrt{\lambda}\, x$

$f'(0) - f(0) = 0 \Rightarrow c_2\sqrt{\lambda}\cos\sqrt{\lambda} - c_1 = 0$

$\Rightarrow c_1 = c_2\sqrt{\lambda}\cos\sqrt{\lambda}$

$\Rightarrow$ $f = c_2\sqrt{\lambda}\cos\sqrt{\lambda}\, x + c_2 \sin\sqrt{\lambda}\, x = c_2\left(\sqrt{\lambda}\cos\sqrt{\lambda}\, x + \sin\sqrt{\lambda}\, x\right)$

$f(1) = 0 \Rightarrow c_2\left(\sqrt{\lambda}\cos\sqrt{\lambda} + \sin\sqrt{\lambda}\right) = 0$

$\Rightarrow$ $\tan\sqrt{\lambda} = -\sqrt{\lambda}$

$\lambda < 0$; let $\lambda = -a^2$ with $a > 0 \qquad f = c_1\cosh ax + c_2 \sinh ax$

$f' = c_1 a \sinh ax + c_2 a \cosh ax$

$f'(0) - f(0) = 0 \Rightarrow c_2 a \cosh a - c_1 = 0$

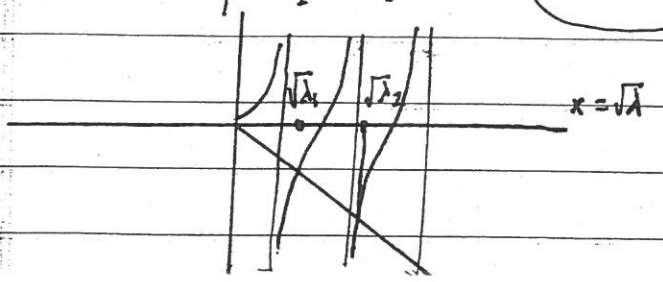
$c_1 = c_2 a \cosh a$

$\Rightarrow f = c_2 a\cosh a + c_2 \sinh a = c_2(a\cosh a + \sinh a)$

$f(1) = 0 \Rightarrow$ $\tanh a = -a$ no solutions

so the e-vals of the problem are the positive solns $\lambda_1, \lambda_2, \ldots$ to $\tan\sqrt{\lambda} = -\sqrt{\lambda}$

and the e-functions are $f_n \sim \sqrt{\lambda_n}\cos\sqrt{\lambda_n}\, x + \sin\sqrt{\lambda_n}\, x$



$\sqrt{\lambda_1} \approx 2.03 \qquad \sqrt{\lambda_2} \approx 4.91$

$\sqrt{\lambda_n} \approx \frac{2n-1}{2}\pi$ n large

§3.1.1 #4 Find e-values and e-functions

$f'' + \lambda f = 0 \qquad f'(0)=0 \qquad f'(1)+f(1)=0$

$\lambda = 0 \Rightarrow f = a+bx \Rightarrow f' = b$

$f'(0)=0 \Rightarrow b=0 \Rightarrow f=a \Rightarrow f'=0$

$f'(1)+f(1)=0 \Rightarrow a=0, \quad a=0, b=0 \Rightarrow$ $\lambda=0$ not an e-val

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$\lambda > 0 \Rightarrow f = c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x \qquad f' = -c_1\sqrt{\lambda}\sin\sqrt{\lambda}x + c_2\sqrt{\lambda}\cos\sqrt{\lambda}x$

$f'(0)=0 \Rightarrow c_2=0 \Rightarrow$ $f = c_1\cos\sqrt{\lambda}x$ $\qquad f' = -c_1\sqrt{\lambda}\sin\sqrt{\lambda}x$

$f'(1)+f(1)=0 \Rightarrow -c_1\sqrt{\lambda}\sin\sqrt{\lambda} + c_1\cos\sqrt{\lambda} = 0 \Rightarrow$ $\tan\sqrt{\lambda} = \frac{1}{\sqrt{\lambda}}$

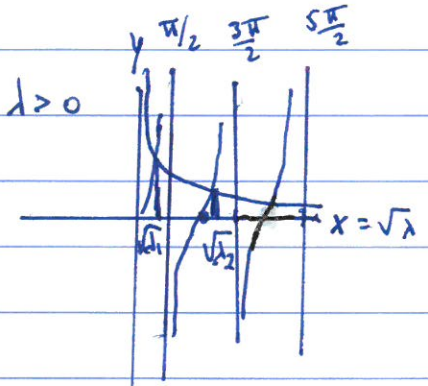
$\lambda < 0, \quad \lambda = -a^2, \ a>0 \Rightarrow f = c_1\cosh ax + c_2 \sinh ax$

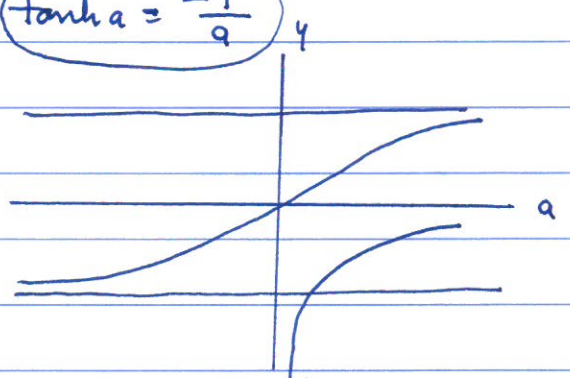
$f' = c_1 a \sinh ax + c_2 a \cosh ax$

$f'(0)=0 \Rightarrow c_2=0 \Rightarrow f = c_1\cosh ax \qquad f' = c_1 a \sinh ax$

$f'(1)+f(1)=0 \Rightarrow c_1 a\sinh a + c_1\cosh a = 0$

$\Rightarrow$ $\tanh a = \frac{-1}{a}$





no intersections

estimates $\sqrt{\lambda_1} \approx .86 \qquad \sqrt{\lambda_2} \approx 3.4$

$\sqrt{\lambda_n} \approx (n-1)\pi \quad n$ large

e-values The e-values of the problem are the positive solns λ_n to $\tan\sqrt{\lambda} = \frac{1}{\sqrt{\lambda}}$

e-functions The e-functions of the problem are $f_n \sim \cos\sqrt{\lambda_n}x$