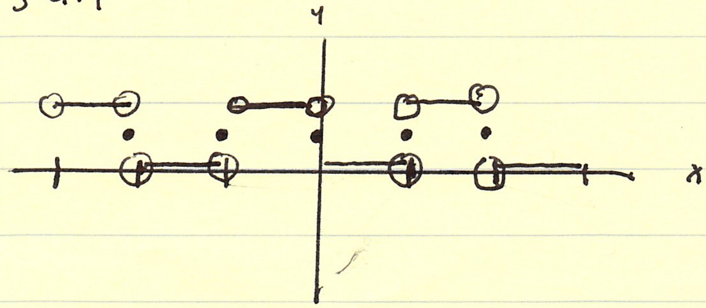
HW Solns §2.1

1) $f(x) = \begin{cases} 1 & -1 \le x \le 0 \\ 0 & 0 < x \le 1 \end{cases}$



Find and graph the F.S.

Soln: $L = 1$

$a_0 = \frac{1}{L}\int_{-L}^{L} f(x) = 1\cdot$ "area" $= 1$

$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx = \int_{-1}^{0}\cos n\pi x\,dx = \frac{1}{n\pi}\sin n\pi x\Big|_{-1}^{0} = 0$

$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx = \int_{-1}^{0}\sin n\pi x\,dx = -\frac{1}{n\pi}\cos n\pi x\Big|_{-1}^{0} = -\frac{1}{n\pi} + \frac{1}{n\pi}\cos n\pi$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{1}{n\pi}(\cos n\pi - 1)\sin n\pi x$$

3) $f(x) = \begin{cases} -2 & -1 \le x \le 0 \\ 3 & 0 < x \le \end{cases}$ Find the F.S. of $f(x)$

Soln: $L = 1$

$a_0 = \int_{-1}^{0} -2\,dx + \int_{0}^{1} 3\,dx = -2x\Big|_{-1}^{0} + 3x\Big|_{0}^{1} = -2 + 3 = 1$

$a_n = \int_{-1}^{1} f(x)\cos n\pi x\,dx = \int_{-1}^{0} -2\cos n\pi x\,dx + \int_{0}^{1} 3\cos n\pi x\,dx =$

$-\frac{2}{n\pi}\sin n\pi x\Big|_{-1}^{0} + \frac{3}{n\pi}\sin n\pi x\Big|_{0}^{1} = 0$

$b_n = \int_{-1}^{0} -2\sin n\pi x\,dx + \int_{0}^{1} 3\sin n\pi x\,dx = \frac{2}{n\pi}\cos n\pi x\Big|_{-1}^{0} - \frac{3}{n\pi}\cos n\pi x\Big|_{0}^{1}$

$= \frac{2}{n\pi} - \frac{2}{n\pi}\cos n\pi - \frac{3}{n\pi}\cos n\pi + \frac{3}{n\pi} = \frac{5}{n\pi}(1 - \cos n\pi)$

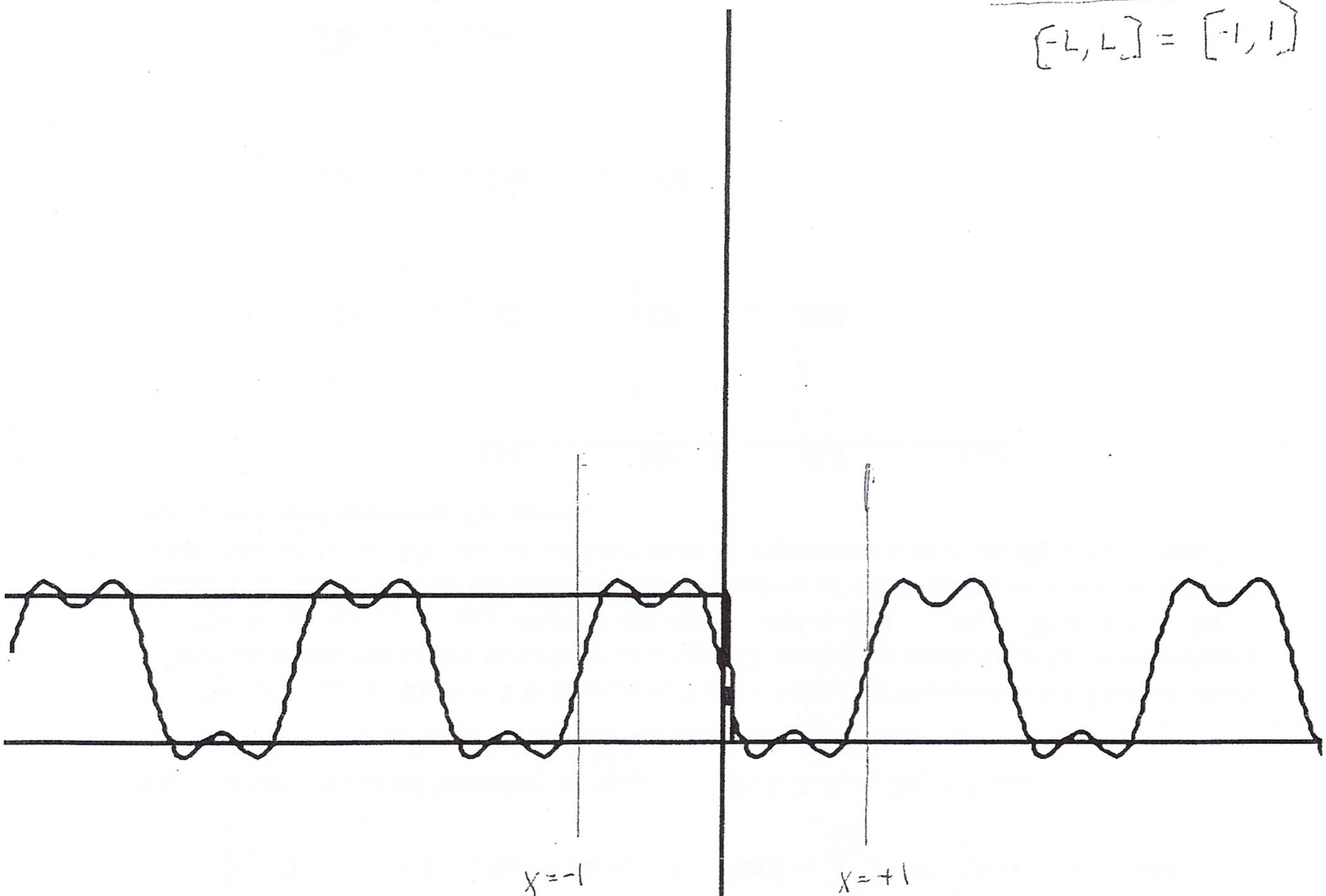
$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{5}{n\pi}(1 - \cos n\pi)\sin n\pi x$$

y = (-x+abs(x))/(2*abs(x)) $= \begin{cases} +1 & x < 0 \\ 0 & x > 0 \end{cases}$

-5.000 < x < 5.000 Order: 4

-5.000 < y < 5.000 Period: 2.00

$[-L, L] = [-1, 1]$



Fourier Series for

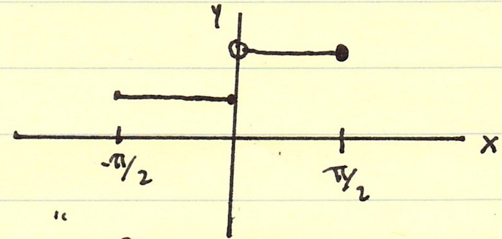
$$\frac{-x+|x|}{2|x|} = \begin{cases} 0 & x>0 \\ +1 & x<0 \end{cases} \quad \text{on } [-1,1]$$

4 terms

2.1 #4 Done in class

$$f(x) = \begin{cases} 1 & -\frac{\pi}{2} \le x \le 0 \\ 2 & 0 < x \le \frac{\pi}{2} \end{cases}$$

Write and sketch the F.S.



Sol $L = \frac{\pi}{2}$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{\pi/2} \cdot \text{"area"} = 3$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx = \frac{2}{\pi}\left[\int_{-\pi/2}^{0} \cos 2nx\,dx + \int_{0}^{\pi/2} 2\cos 2nx\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{2n}\sin 2nx\Big|_{-\pi/2}^{0} + \frac{1}{n}\sin 2nx\Big|_{0}^{\pi/2}\right] = 0$$

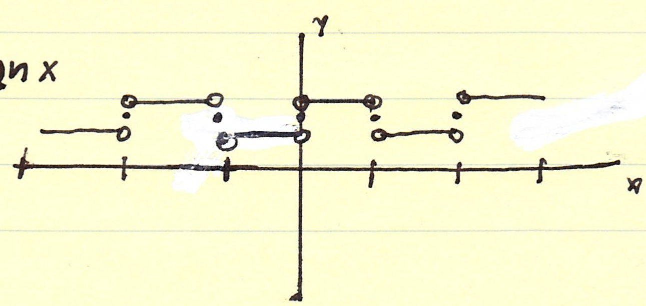
$$b_n = \frac{2}{\pi}\left[\int_{-\pi/2}^{0} \sin 2nx\,dx + \int_{0}^{\pi/2} 2\sin 2nx\,dx\right]$$

$$= \frac{2}{\pi}\left[-\frac{1}{2n}\cos 2nx\Big|_{-\pi/2}^{0} - \frac{1}{n}\cos 2nx\Big|_{0}^{\pi/2}\right]$$

$$= \frac{2}{\pi}\left[-\frac{1}{2n} + \frac{1}{2n}\cos n\pi - \frac{1}{n}\cos n\pi + \frac{1}{n}\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{2n} - \frac{1}{2n}\cos n\pi\right] = \frac{1}{n\pi}\left[1 - \cos n\pi\right]$$

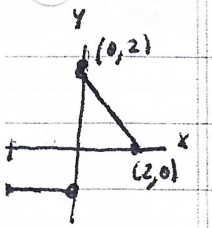
$$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi}\left[1 - \cos n\pi\right]\sin 2nx$$



Exs

A Full Fourier Series

#7 $f(x) = \begin{cases} -1 & -2 \le x \le 0 \\ 2-x & 0 < x \le 2 \end{cases}$ Construct and graph the F.S.



Soln: $L = 2$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2}\left(\int_{-2}^{0} -dx + \int_0^2 (2-x)\,dx\right) = 0$$

$n \ge 1$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{2}\,dx = \frac{1}{2}\left[\int_{-2}^{0} -\cos\frac{n\pi x}{2}\,dx + \int_0^2 (2-x)\cos\frac{n\pi x}{2}\,dx\right]$$

1st integral $\quad -\frac{2}{n\pi}\sin\frac{n\pi x}{2}\Big|_{-2}^{0} = 0;$ $\quad$ parts $\quad u = 2-x \quad dv = \cos\frac{n\pi x}{2}$ $\quad du = -dx \quad v = \frac{2}{n\pi}\sin\frac{n\pi x}{2}$

2nd integral $\quad (2-x)\cdot\frac{2}{n\pi}\sin\frac{n\pi x}{2}\Big|_0^2 + \frac{2}{n\pi}\int_0^2 \sin\frac{n\pi x}{2}\,dx = -\frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2}\Big|_0^2$

$$= -\frac{4}{n^2\pi^2}\cos n\pi + \frac{4}{n^2\pi^2} \Longrightarrow \boxed{a_n = \frac{2}{n^2\pi^2} - \frac{2}{n^2\pi^2}\cos n\pi}$$

$$b_n = \frac{1}{2}\left[\int_{-2}^{0} -\sin\frac{n\pi x}{2}\,dx + \int_0^2 (2-x)\sin\frac{n\pi x}{2}\,dx\right]$$

$u = 2-x \quad dv = \sin\frac{n\pi x}{2}$ $\quad du = -dx \quad v = -\frac{2}{n\pi}\cos\frac{n\pi x}{2}$

$$= \frac{1}{2}\left[\frac{2}{n\pi}\cos\frac{n\pi x}{2}\Big|_{-2}^{0} - \frac{2(2-x)}{n\pi}\cos\frac{n\pi x}{2}\Big|_0^2 - \frac{2}{n\pi}\underbrace{\int_0^2 \cos\frac{n\pi x}{2}\,dx}_{0}\right]$$

$$= \frac{1}{2}\left[\frac{2}{n\pi} - \frac{2}{n\pi}\cos n\pi + \frac{4}{n\pi}\right] = \boxed{\frac{3}{n\pi} - \frac{1}{n\pi}\cos n\pi}$$

$$f(x) = \sum_{n=1}^{\infty}\left[\left(\frac{2}{n^2\pi^2} - \frac{2}{n^2\pi^2}\cos n\pi\right)\cos\frac{n\pi x}{2} + \left(\frac{3}{n\pi} - \frac{1}{n\pi}\cos n\pi\right)\sin\frac{n\pi x}{2}\right]$$