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MAT 135B

Investigating the Fairness of Coin Tosses: An Experimental Approach

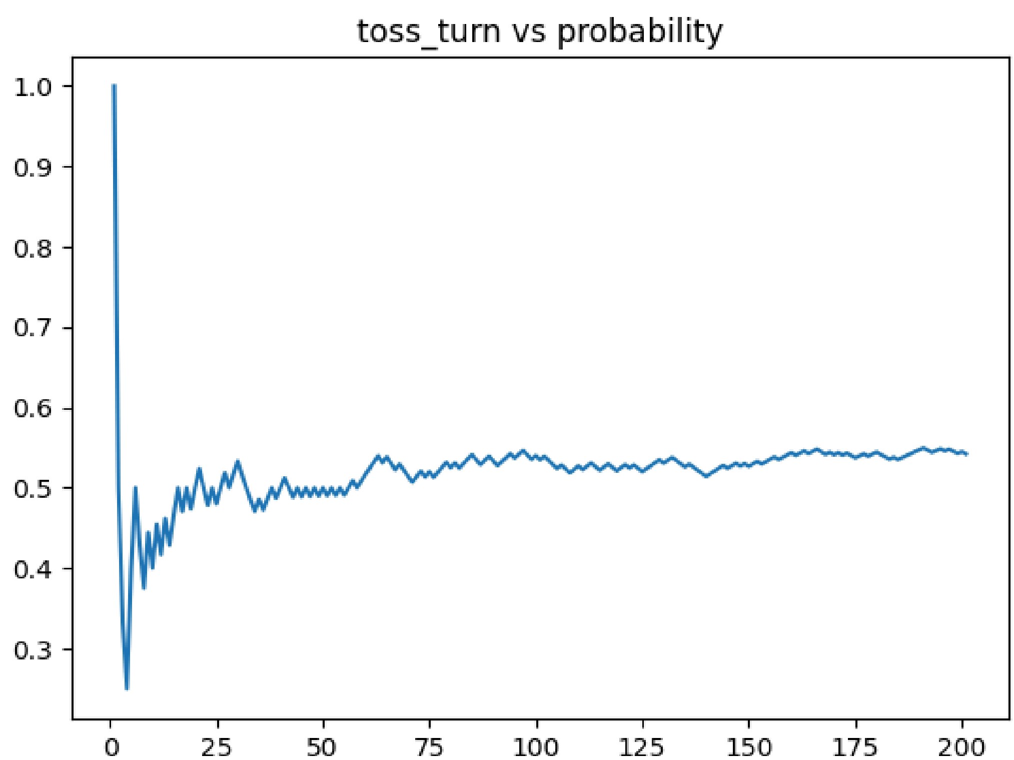
Abstract: This paper aims to use rigorous experimental methods to investigate the inherent fairness of coin tossing. Prior research, most notably that of Persi Diaconis and others, has demonstrated that what has historically been thought of as a random process can be biased by physical elements like the coin's starting position, angular momentum, air resistance, and the surface it lands on. This study tried to quantify any bias and determine its effect on the perceived fairness of coin tossing by controlling for variables including toss method, coin type, and landing surface.

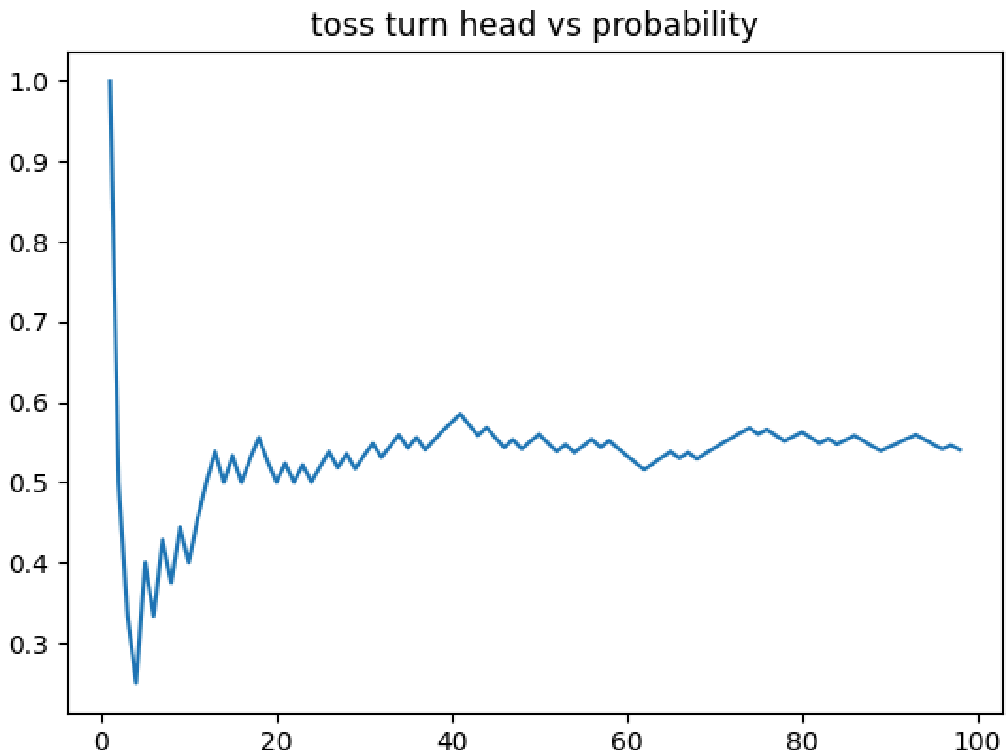
Our methodology includes repetitive coin flipping under varied, yet controlled conditions to emulate real-world scenarios while isolating specific variables. We record the outcomes to construct a large dataset, which is then statistically analyzed to identify any inherent bias. The resulting data, represented in the attached graphs, exhibit the empirical probabilities of obtaining heads or tails across numerous trials.

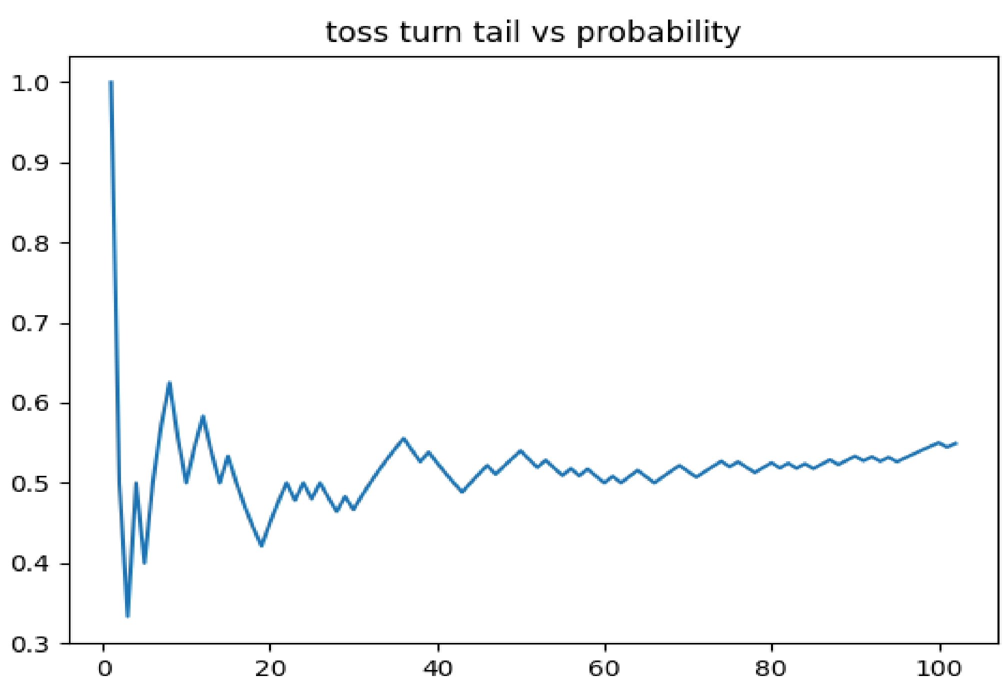
Throughout ancient times, people have made decisions by doing the seemingly straightforward process of tossing a coin. It is a representation of justice and chance, a binary decision left up to fate. But mathematicians, physicists, and even casual observers have been captivated by the topic of whether a coin flip is truly random, and hence fair. Famously proving that a coin's physical characteristics and the circumstances surrounding its toss can affect its result, Persi Diaconis and his co-authors(2007) raise the possibility that the coin toss is not as fair as previously thought. By performing a number of controlled experiments to look at the effects of several factors on the result of a coin toss, this research seeks to further explore this idea.

Experimental Procedure:

The experiment involved writing Python code to randomly determine the initial side (heads or tails) of each coin toss and to record both the number of flips and the outcomes. Moreover, the code conducted a probability analysis and generated visual representations to analyze the connection between the initial face and the final face. Four participants engaged in the experiment using the same coin with each participant executing 50 tosses. After the initial side of the coin was randomly assigned by the code, the participants proceeded to toss the coin. The results of each toss were entered into the Python. Total tosses are up to 200 times. The entire tossing process was also recorded through video.




toss turn head vs probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
20
40
60
80
100


toss turn tail vs probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
20
40
60
80
100

Python code

python data and analysis

toss coin .mov

Program code

Statistical Analysis

Null Hypothesis H_0: There is no same-side bias, $\beta = 0.5$.

Alternative Hypothesis H_1: There is a same-side bias, where β is characterized by a Beta(100, 95) distribution.

【we choose Beta(100, 95), reflecting our relatively neutral prior belief with a slight inclination towards a bias.】

Given that we observed 109same-side outcomes in 200 tosses（102 initial face T with 56result face T, 98 initial face H with 53 result face H）

1. **Prior Distribution:**
    - The alternative hypothesis (H_1) assumes a prior of Beta(100, 95).
2. **Observed Data:**
    - 109 same-side outcomes were observed in 200 coin tosses.
3. **Posterior Distribution:**
    - The updated posterior distribution after observing the data is Beta(100+109, 95+200-109)=Beta(209,186)

Approximating BF_{10}:

- **Calculating the likelihood under the Null Hypothesis (H_0):**
    - The probability of observing 104 successes (same-side outcomes) when $\beta = 0.5$ is calculated using the binomial distribution formula

      $$L(k|\beta, N) = \binom{N}{k} \beta^k (1-\beta)^{N-k}$$

      For k=109, N=200, β=0.5
      likelihood under H_0 is approximately 0.048
- **Approximating the likelihood under Alternative Hypothesis (H_1):**
    - Using mean of the posterior distribution Beta(209, 186) as an approximation of likelihood under H_1. likelihood under $H_1$is 209/(209+186)=0.529
- **For BF_{10}:**
    - value of BF_{10} is obtained by dividing the likelihood under H_1 by the likelihood under H_0.
      0.529/0.048=11.02

As a conclusion, based on our recorded data and prior belief, the hypothesis of bias (H_1) is approximately 11.02 times more than the hypothesis of no bias (H_0),which provides strong evidence in favor of our alternative hypothesis: There is a same-side bias.