Choosing a Statistical Model Amidst Ceiling and Floor Effects

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NASP 2021

Research Questions

This file contains R code and output to accompany *Choosing a statistical model amidst ceiling and floor effects* by Rao, Running, and Codding (2021). Specifically, it will diagnose a ceiling or floor effect (CFE), determine whether it is inappropriate to use an ANOVA model and if so whether it is appropriate to use a Tobit model, and subsequently fit and interpret a Tobit model.

This file assumes that readers are already familiar with the basics of using R, as well as the basics of the ANOVA model and general linear models.

Required Packages

This code utilizes the following packages in R:

- ggplot2
- cbPallete
- dplyr
- VGAM
- sm
- MASS

DATA

First, download the dataset `Fraction Knowledge Dataset.csv`. You can find the dataset and other supplementary material by visiting `https://github.com/RaoVNV/NASP2021/`.

Importing into R

Next, load the dataset into R. The following code creates a pop-up window. Use the pop-up window to navigate to the `Fraction Knowledge Dataset.csv` file on your computer.

```
fraction_knowledge <- read.csv(file=file.choose())
```

To ensure the file has been imported correctly, take a peak at the characteristics of the dataset.

```
#take a look at the first 5 observations
head(fraction_knowledge)
```

```
##   ï..student_ID site_ID teacher_ID      condition concepts_pre concepts_post
## 1           319   Site1   Teacher1        Control           17            17
## 2           312   Site1   Teacher1        Control           27            40
## 3           306   Site1   Teacher1 Concepts-First            6            40
## 4           315   Site1   Teacher1      Iterative           16            32
## 5           321   Site1   Teacher1      Iterative           26            37
## 6           311   Site1   Teacher1 Concepts-First           25            23
##   procedures_pre procedures_post
## 1             21              24
## 2             13              33
## 3             35              39
## 4             32              34
## 5             23              36
## 6             NA              38
```

```
#take a look at the last 5 observations
tail(fraction_knowledge)
```

```
##     ï..student_ID site_ID teacher_ID      condition concepts_pre concepts_post
## 109           215   Site2   Teacher5      Iterative           36            37
## 110           208   Site2   Teacher5 Concepts-First           27            40
## 111           218   Site2   Teacher5 Concepts-First           33            40
## 112           216   Site2   Teacher5        Control           30            40
## 113           222   Site2   Teacher5        Control           39            40
## 114           217   Site2   Teacher5      Iterative           40            40
##     procedures_pre procedures_post
## 109             23              29
## 110             40              39
## 111             32              39
## 112             35              40
## 113             35              39
## 114             40              40
```

```r
#take a look at all the variable names
names(fraction_knowledge)
```

```
## [1] "ï..student_ID"   "site_ID"         "teacher_ID"      "condition"
## [5] "concepts_pre"    "concepts_post"   "procedures_pre"  "procedures_post"
```

```r
#create quick summaries of all variables
summary(fraction_knowledge)
```

```
##  ï..student_ID     site_ID           teacher_ID         condition
##  Min.   :101.0   Length:114         Length:114         Length:114
##  1st Qu.:209.2   Class :character   Class :character   Class :character
##  Median :320.5   Mode  :character   Mode  :character   Mode  :character
##  Mean   :327.1
##  3rd Qu.:423.8
##  Max.   :528.0
##
##   concepts_pre   concepts_post   procedures_pre  procedures_post
##  Min.   : 2.00   Min.   :10.00   Min.   : 0.00   Min.   : 0.00
##  1st Qu.:22.00   1st Qu.:33.00   1st Qu.:21.00   1st Qu.:33.00
##  Median :28.00   Median :39.00   Median :29.00   Median :39.00
##  Mean   :28.28   Mean   :35.36   Mean   :26.55   Mean   :34.98
##  3rd Qu.:38.00   3rd Qu.:40.00   3rd Qu.:34.00   3rd Qu.:40.00
##  Max.   :40.00   Max.   :40.00   Max.   :40.00   Max.   :40.00
##  NA's   :1       NA's   :4       NA's   :3       NA's   :4
```

Initial Processing

When we will analyze differences in scores between each of the experimental groups based on *condition* variable, we want to use the `control` group as a reference group. We can set the order that R processes each group by specifying levels of a factor.

```r
#Make sure the group variable is in the order we want
fraction_knowledge$condition <-
  factor(fraction_knowledge$condition,
    levels=c("Control","Concepts-First","Iterative"))
```

Visit `https://github.com/RaoVNV/NASP2021/` to review the `Fraction Knowledge Data Dictionary` for more information about each of the variables contained in this dataset, and the `Fraction Knowledge Data Introduction` for more information about the experimental study for which this data was collected.

DESCRIPTIVE STATISTICS

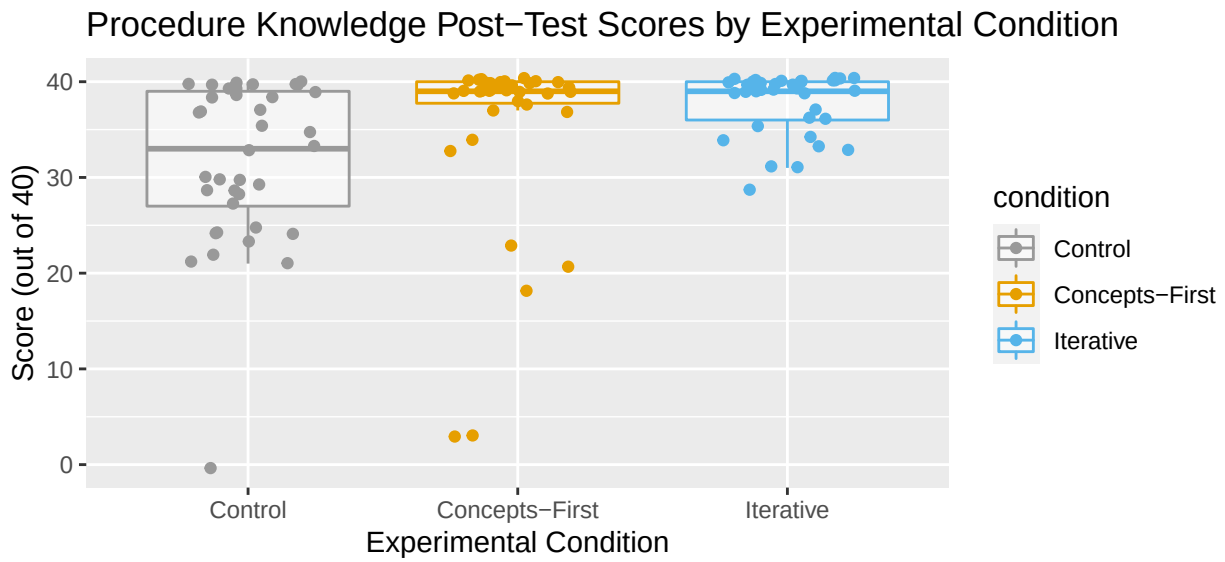
In this example, we will examine the students' scores on the procedures assessment at post-test. Our main analysis goal is to identify whether there were group differences between each of three experimental conditions. One important covariates that may affect students' post-test scores are their pre-test scores. The study used block randomization to assign students within each classroom to each experimental condition based on their pre-test score. Therefore, the design matches that of the Analysis of Covariance (ANCOVA) model, with post scores modeled as a function of experimental condition and pre-test scores: `procedures_post ~ condition + procedures_pre`.

However, before we fit the ANCOVA model, we must explore the data with descriptive summaries and visualizations, which will help us examine ANCOVA's suitability.

Diagnosing the CFE

First, let's look at the distribution for procedure knowledge post-test scores by group:

```
ggplot(data=fraction_knowledge,
       aes(y=procedures_post, x=condition, color=condition)
)  +
  scale_fill_manual(values=cbPalette) + scale_colour_manual(values=cbPalette) +
  geom_boxplot(outlier.shape=NA, alpha=0.5) +
  geom_jitter(width=0.25)+
  ggtitle("Procedure Knowledge Post-Test Scores by Experimental Condition") +
  ylab("Score (out of 40)") +
  xlab("Experimental Condition")
```

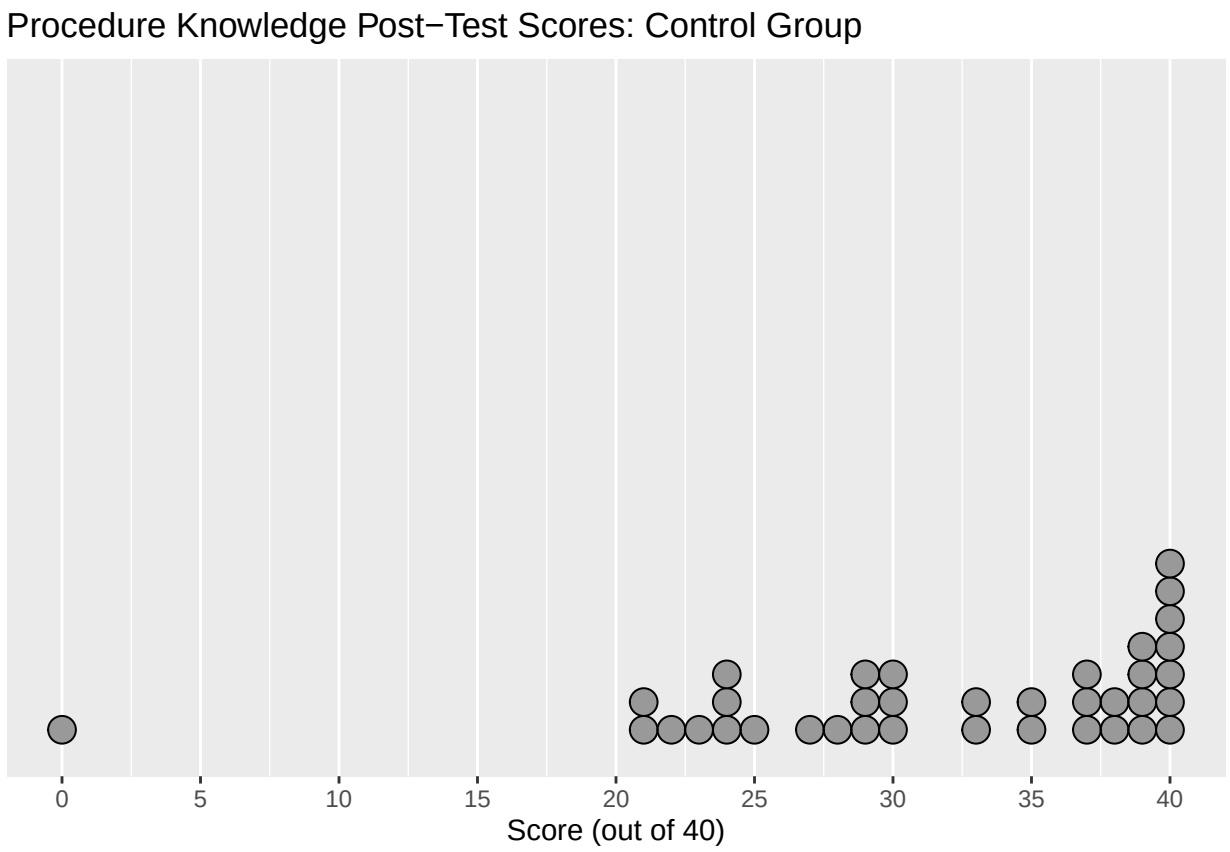


We can see that there are quite a few *dots* bunched up right around a value of 40. That signifies that several students have achieved the maximum score. This is evidence of a ceiling effect. ANOVA and ANCOVA models are based on the assumption that scores are normally distributed within groups. While *true* procedural knowledge scores may still be normally distributed, our observed scores are not, due to the ceiling effect.

We can further examine the extent of the ceiling effect by creating dotplots for each group.

Control Group

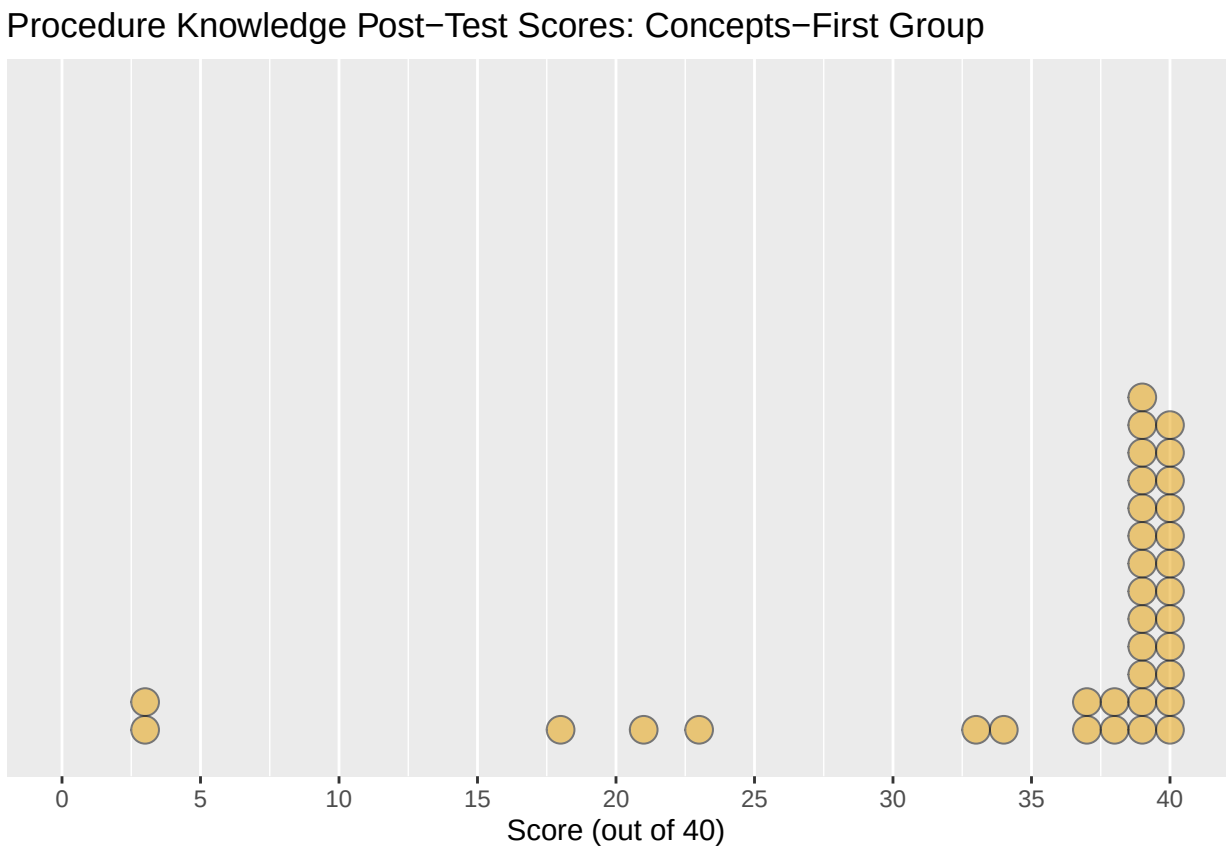
```
ggplot(
  data=(fraction_knowledge %>% filter(condition=="Control")),
  aes(x=procedures_post, fill=condition)
) +
  scale_fill_manual(values=cbPalette[1]) + #Matches Color used in the previous graph
  theme(legend.position = "none") + #Hides the Legend
  scale_y_continuous(NULL, breaks = NULL) + #Hides the Y-Axis
  scale_x_continuous(limits=c(0,40), breaks = seq(0,40,5)) + #Specifies the X-axis
  geom_dotplot(binwidth=1) + #Creates a dotplot
  ggtitle("Procedure Knowledge Post-Test Scores: Control Group") + #Adds a main title
  ylab("Relative Frequency") + #Adds a y-axis label
  xlab("Score (out of 40)") #Adds an x-axis label
```



Seven of the 38 students in the Control group scored the maximum score.

Concepts First Group

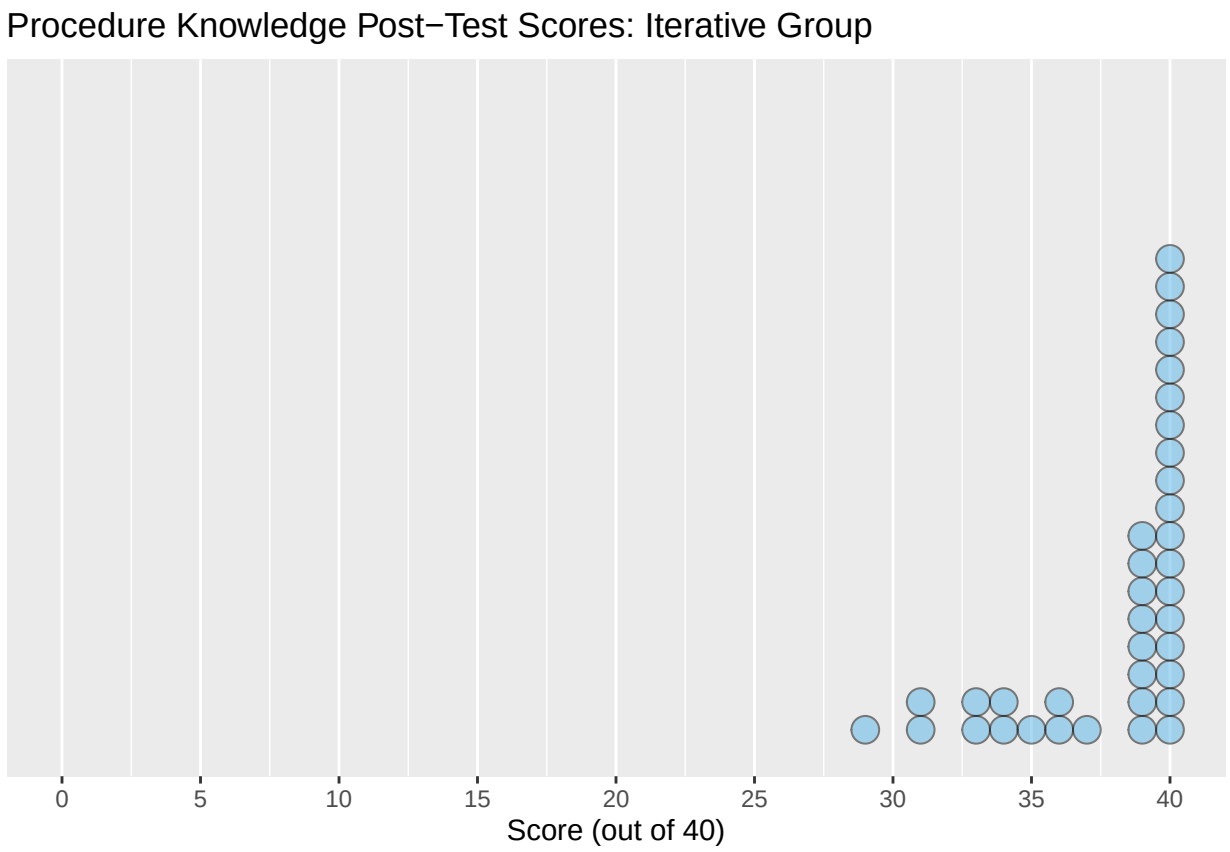
```
ggplot(
  data=(fraction_knowledge %>% filter(condition=="Concepts-First")),
  aes(x=procedures_post, fill=condition)
) +
  scale_fill_manual(values=cbPalette[2]) +
  theme(legend.position = "none") +
  scale_y_continuous(NULL, breaks = NULL) +
  scale_x_continuous(limits=c(0,40), breaks = seq(0,40,5)) +
  geom_dotplot(alpha=0.5, binwidth=1) +
  ggtitle("Procedure Knowledge Post-Test Scores: Concepts-First Group") +
  ylab("Relative Frequency") +
  xlab("Score (out of 40)")
```



Twelve of the 38 students in the Concepts-First group scored the maximum score, and another 13 students were only one point shy of the maximum.

Iterative Group

```
ggplot(
  data=(fraction_knowledge %>% filter(condition=="Iterative")),
  aes(x=procedures_post, fill=condition)
) +
  scale_fill_manual(values=cbPalette[3]) +
  theme(legend.position = "none") +
  scale_y_continuous(NULL, breaks = NULL) +
  scale_x_continuous(limits=c(0,40), breaks = seq(0,40,5)) +
  geom_dotplot(alpha=0.5, binwidth=1) +
  ggtitle("Procedure Knowledge Post-Test Scores: Iterative Group") +
  ylab("Relative Frequency") +
  xlab("Score (out of 40)")
```



Eighteen of the 38 students in the Iterative group scored the maximum score, and another 8 students were only one point shy of the maximum.

Quantifying the CFE

Having diagnosed ceiling effects in all three groups, we need to quantify the magnitude of the ceiling effect, in order to help determine which statistical model to utilize.

```
#Percentage of observations at ceiling by group
fraction_knowledge %>%
  group_by(condition) %>%
  filter(!is.na(procedures_post)) %>%
  summarise(
      p.atCFE = count(procedures_post==40)/n(),
      p.nearCFE= count(procedures_post>=39)/n()
  )
```

```
## # A tibble: 3 x 3
##   condition      p.atCFE p.nearCFE
##   <chr>            <dbl>     <dbl>
## 1 Control          0.189     0.297
## 2 Concepts-First   0.333     0.694
## 3 Iterative        0.486     0.703
```

We see in this table that approximately 19% of scores in the `Control` group are at the ceiling, and 30% are within 1 point of the ceiling. Similarly, approximately 33% of scores in the `Concepts-First` group are at the ceiling and 49% of scores in the `Iterative` group are at the ceiling.

The 30-20 rule says that to use ANOVA, no group should have more than 30% of it's observations at the ceiling. This is violated by both the `Concepts-First` and `Iterative` groups. Furthermore, the difference in the percentage of observations at the ceiling between two groups should not be more than 20%. This is violated by the difference between the `Iterative` group and the `Control` group, which is nearly 30 percentage points. Therefore, we should not use ANOVA or ANCOVA to analyze this data.

To decide whether to use Tobit, we can follow the 70% rule. If no more than 70% of observations are at the ceiling in each group, then we can use Tobit regression. Since the proportions are all well below 70%, we can use Tobit regression to analyze this data.

TOBIT REGRESSION

Tobit regression carries the same model assumptions as generalized linear models: normality of residuals, and homoscedasticity. We will first fit the model before examining the model diagnostics.

Fitting the Model

Fitting a tobit model is very similar to fitting a generalized linear model with the `glm()` function, and only has a few small differences compared to fitting an ANOVA model.

We will be using the `vglm()` function from the `{VGAM}` package. We first specify the name of the file the data is contained within, with the `data=` option, just as we have in previous functions.

The second argument to the function is the model specification. The general form of a model is `response variable ~ explanatory variables + covariates`. This is the same format used by the `glm()` and `anova()` functions in R. In this case, we want to model procedure post-test scores as a function of experimental condition while controlling for procedure pre-test scores.

The last argument is where we specify that we are fitting a Tobit model. We do this by using the `family=tobit()` option. We must also specific the minimum possible score and the maximum possible score with the `Lower=` and `Upper=` options.

```
tb.mdl <- vglm(
  data = fraction_knowledge,
  procedures_post ~ condition + procedures_pre,
  family=tobit(Lower=0, Upper=40)
)
```

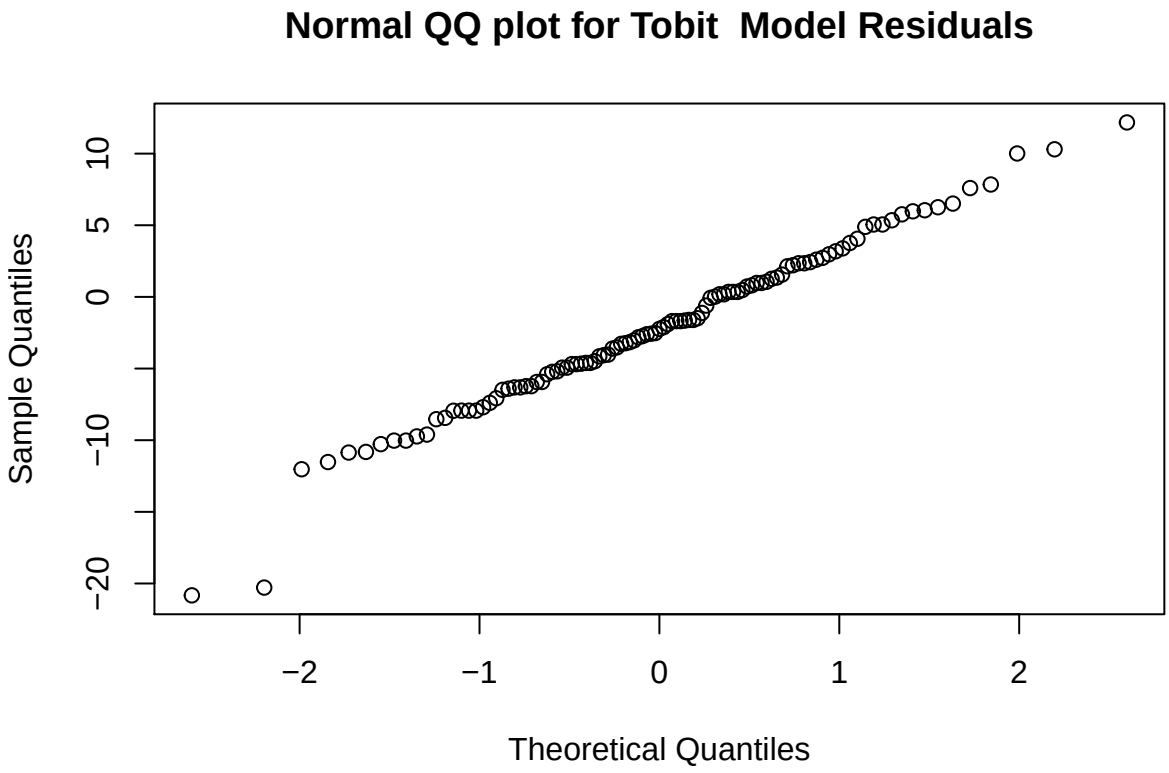
Residual Analysis

Before we inspect the model summary, we must ensure that the model assumptions are met. There are two assumptions to Tobit regression, the same as to all general linear regression including ANOVA: residuals must be normally distributed, and the variables of the residuals should not change as a function of the explanatory variables.

Normality

In order to check the normality of residuals, we use the Normal QQ Plot. When the points form a straight line, it is an indication that the residuals can indeed be appropriately modeled by a normal distribution.

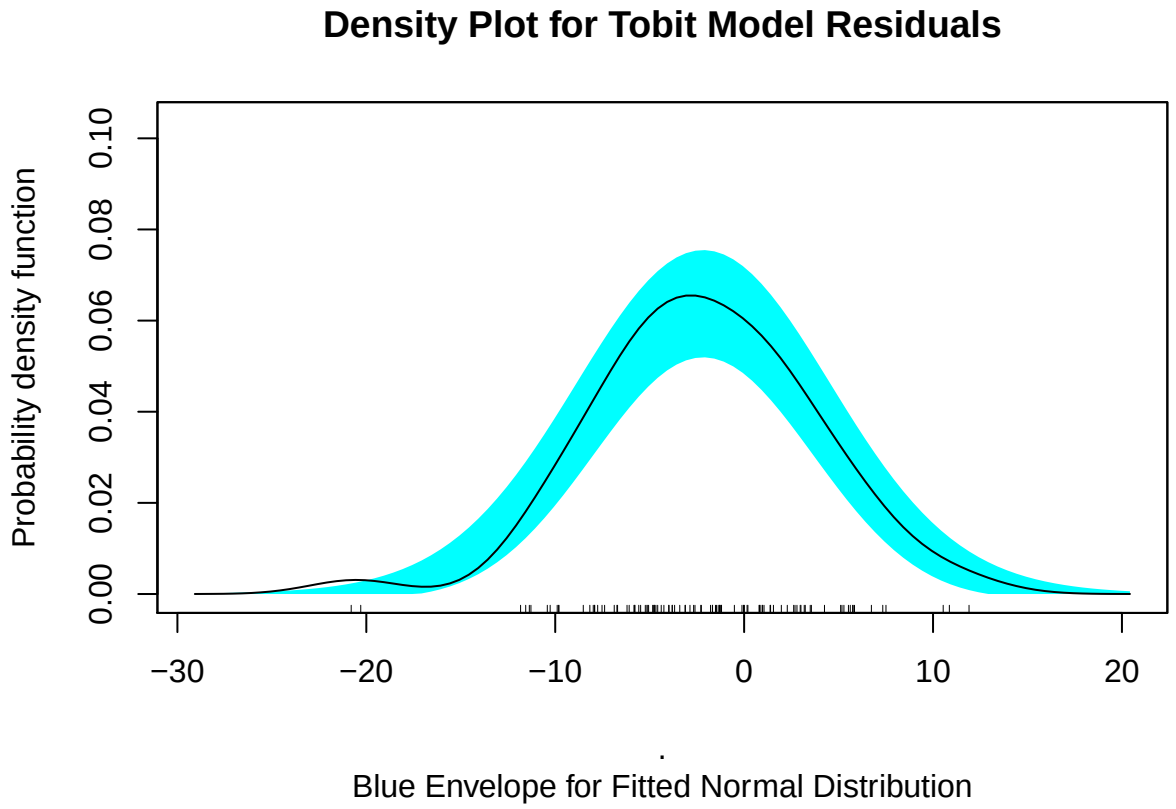
```
resid(tb.mdl, type = "response") %>% qqnorm(main="Normal QQ plot for Tobit  Model Residuals")
```



In this case, it does not appear that there are any major violations of the assumption that the residuals should be normally distribution.

We can also examine the distribution via a density plot. When the line based on our data falls entirely within the blue region (representing what we'd expect under a normal distribution), then we have no reason to worry that the normality assumption is violated.

```
resid(tb.mdl, type = "response") %>% sm.density(model = "normal")
title(main="Density Plot for Tobit Model Residuals",
      sub="Blue Envelope for Fitted Normal Distribution")
```



Indeed, it does not appear that the normality assumption is violated, as the black line representing the distribution of residuals for our data falls entirely within the blue region.

Homoscedasticity

Interpreting homoscedasticity from residual vs fitted plots in censored models is a little bit tricky because of the ceiling effect. For example, it is not possible to have a positive residual for a predicted score at the ceiling. Similarly, there is an upper limit to the positive residuals for any given predicted score, as scores are limited by the ceiling.

Therefore, the main goal of the residual vs fitted plot is to ensure there are no obvious patterns in the plot.

```
data.frame(
 r=resid(tb.mdl, type = "response"),
 f=fitted(tb.mdl, type = "censored")
) %>%
ggplot(aes(x=f, y=r)) +
  geom_jitter(alpha=0.5, width=0.5, height=0.5) +
  ggtitle("Residual v Fitted Plot for Tobit Regression Model") +
  ylab("Residuals") +
  xlab("Fitted Scores (out of 40)") +
  geom_abline(slope=-1, intercept=40, alpha=0.5)
```

We see that residuals are mainly limited by the presence of the ceiling (denoted by the sloping line). However, residuals generally fall between +10 and -10, and we can only assume that the they fail to reach as high as +10 due to the ceiling for fitted scores above 30.

No other obvious patterns in the residuals exist, such as an obvious curvilinear pattern, and therefore, it appears that the homoscedasticity assumption is not violated.

There are two potential extreme values with residuals near -20, but in-person follow-up determined that these are not data errors, and thus the observations were retained in the dataset.

Interpreting the model

Now that we have verified that no model assumptions are violated, we can therefore conclude that the Tobit model is an appropriate and useful model to help us analyze the dataset, and can begin interpreting the fitted model.

Experimental Condition

The first step in interpreting the model is to determine whether there are differences in post-test scores by condition. To do this, we utilize the ANOVA Type 3 Sums of Squares. This test examines the ratio of the variance between groups to the variance within groups. Note, this test, though it bears the name `ANOVA`, is *not* the same thing as fitting an ANOVA model to compare a difference in group means, although it is similar.

```
anova.vglm(tb.mdl, type="III")
```

```
## Analysis of Deviance Table (Type III tests: each term added last)
##
## Model: 'tobit', 'VGAMcategorical'
##
## Links: 'identitylink', 'loglink'
##
## Response: procedures_post
##
##                Df 2 * LogLik Diff. Resid. Df  LogLik  Pr(>Chi)
## condition       2           12.541       211 -268.44  0.001892 **
## procedures_pre  1           39.862       210 -282.11 2.725e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We see from the Type III tests that there appears to be strong evidence indicating that there are differences in post-test scores by experimental condition (p=0.002). Also note that pre-test scores appear to be related to post-test scores.

Estimated group differences

What are the estimated differences in the mean scores for each group? We can examine this by simply viewing the model summary.

```
summary(tb.mdl)
```

```
##
## Call:
## vglm(formula = procedures_post ~ condition + procedures_pre,
##     family = tobit(Lower = 0, Upper = 40), data = fraction_knowledge)
##
## Coefficients:
##                           Estimate Std. Error z value Pr(>|z|)
## (Intercept):1            19.70660    2.38589   8.260  < 2e-16 ***
## (Intercept):2             1.97167    0.09183  21.470  < 2e-16 ***
## conditionConcepts-First   3.57895    1.83072   1.955 0.050591 .
## conditionIterative        6.57917    1.86897   3.520 0.000431 ***
## procedures_pre            0.54128    0.07927   6.828 8.59e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Names of linear predictors: mu, loglink(sd)
##
## Log-likelihood: -262.1748 on 209 degrees of freedom
##
## Number of Fisher scoring iterations: 7
##
## No Hauck-Donner effect found in any of the estimates
```

The first two values reported as Intercept1 and Intercept2 are the predicted mean when covariates=0 and the log of the residual standard error. In this case, for students in the control group with a pre-test score of 0 and whole number knowledge score of 0, the model predicts that they will have an average post-test score of 19.71. Similarly, by exponentiating 1.972, we can estimate that the residual standard error for students scores is approximately 7.18.

The next two values reported are indicator variables for the `Concepts-First` and the `Iterative` groups. In this case, the model estimates that the average score for students in the `Concepts-First` group is 3.58 points higher than the average score for students in the `Control` group ($p = 0.051$; approximately 9% points), while the average score in the `Iterative` is 6.58 points higher than the `Control` group's average ($p = 0.0004$; approximately 16% points).

The final value reported is the estimate for the regression coefficients for the covariate in the model, pre-test scores.

Recall that the ANOVA-based model would have likely underestimated the true differences in this case. Indeed, the ANCOVA model estimates the differences between the `Concepts-First` group and the `Control` group as 2.33 and between the `Iterative` group and the `Control` group as 4.17 after adjusting for pre-test scores, which are both near a 35% underestimation compared to the Tobit model estimated differences.

We can also create confidence intervals for the regression coefficients using the `confintvglm()` function, used here to create 95% confidence intervals for the difference in mean scores for the two experimental conditions compared to the `Control` Group.

```
confintvglm(tb.mdl)[3:4,]
```

```
##                                 2.5 %    97.5 %
## conditionConcepts-First -0.009203548  7.167104
## conditionIterative       2.916058345 10.242279
```

Effect sizes

We can convert these estimates into effect sizes akin to Cohen's d by utilizing the model estimated standard deviation of post-test scores or the standard deviation of pre-test scores. We cannot use the standard deviation of observed post-test scores as the ceiling effect will lead to an underestimate of the true standard deviation. We can only use the standard deviation of pre-test scores so long as there is no ceiling effect in the pre-test scores. There are also other more advanced measures of effect sizes that may be more appropriate with advanced regression models such as the Tobit model.

```
coef(tb.mdl)[3:4] / exp(coef(tb.mdl)[2])
```

```
## conditionConcepts-First      conditionIterative
##               0.4982769               0.9159802
```

There appears to be a medium effect on post-test scores based on the Concepts-First condition, and a large effect on post-test scores in the Iterative condition.

It should be noted that Tobit regression *can* result in slight over-estimations of the effect size. However, with the `<70%` rule clearly met, the overestimation should not be any more than 0.025 in terms of the effect size, and therefore would not alter our interpretations.

Multiple Comparisons

While the standard model output allowed us to compare both experimental conditions to the `Control` Group, we may be interested in comparing the experimental conditions to each other. We can achieve this by specifying contrasts. The `contr.sdif` function from the `{MASS}` package will allow us to interpret regression coefficients in terms of successive differences. That is, the first estimate will provide the difference between the `Concepts-First` and the `Control` group, while the second estimate will provide the difference between the `Iterative` and the `Concepts-First` group, based on the order of the levels in our variable.

```
tb.mdl.2 <- vglm(
  data = fraction_knowledge,
  procedures_post ~ condition + procedures_pre,
  family=tobit(Lower=0, Upper=40),
  contrasts = list(condition="contr.sdif")
)
summary(tb.mdl.2)
```

```
##
## Call:
## vglm(formula = procedures_post ~ condition + procedures_pre,
##     family = tobit(Lower = 0, Upper = 40), data = fraction_knowledge,
##     contrasts = list(condition = "contr.sdif"))
##
```

```
## Coefficients:
##                                  Estimate Std. Error z value Pr(>|z|)
## (Intercept):1                    23.09264    2.15296  10.726  < 2e-16 ***
## (Intercept):2                     1.97167    0.09183  21.470  < 2e-16 ***
## conditionConcepts-First-Control   3.57895    1.83072   1.955   0.0506 .
## conditionIterative-Concepts-First 3.00022    1.91714   1.565   0.1176
## procedures_pre                    0.54128    0.07927   6.828 8.59e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Names of linear predictors: mu, loglink(sd)
##
## Log-likelihood: -262.1748 on 209 degrees of freedom
##
## Number of Fisher scoring iterations: 7
##
## No Hauck-Donner effect found in any of the estimates
```

```
confintvglm(tb.mdl.2)[4,]
```

```
##      2.5 %     97.5 %
## -0.7573098  6.7577458
```

```
coef(tb.mdl.2)[4] / exp(coef(tb.mdl.2)[2])
```

```
## conditionIterative-Concepts-First
##                         0.4177033
```

We now see that the estimated difference in the mean scores between the `Iterative` group and the `Concepts-First` group is approximately 3.00 (p=0.118; approximately 7.5% points; moderate effect size of 0.418).

SUMMARY

When faced with a CFE, four easy steps can be taken to confidently and more precisely estimate differences between group means:

- Diagnose the CFE with a dotplot
- Quantify the magnitude of the CFE
- Determine the appropriateness of an ANOVA and/or Tobit model
- Fit and Interpret a Tobit Regression Model if appropriate

All four of these steps can easily be performed in R, as shown in this document. While Tobit models have more advanced extensions (i.e. generalized Tobit regression), the basic linear Tobit regression model can help school psychology researchers mitigate measurement problems and while providing more precise estimates of intervention effects.