The role of clustering in the efficient solution of small Traveling Salesperson Problems (TSPs)

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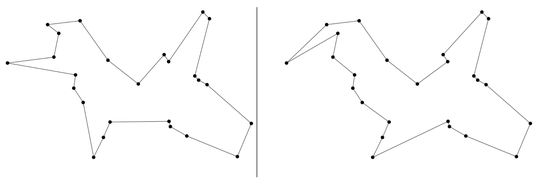
Introduction

- The TSP involves connecting a set of points to each other while visiting each point only once.
- It is an NP-hard problem, i.e., that computers find difficult to solve optimally.
- However humans are surprisingly efficient, with solutions typically within 10% of optimal (Dry et al., 2006; Graham et al., 2000; MacGregor & Ormerod, 1996; Vickers et al., 2003).
- We propose that people use the available cluster structure of the stimulus to inform their TSP solution (Best & Simon, 2000; Dry et al., 2012; Kong & Schunn, 2007).
- We have previously shown that clusters humans draw for TSP-like stimuli are reliable, i.e., similar to each other. This might provide a stable basis for TSP solutions.
- **Research question**: Does higher cluster structure in a stimulus lead humans to more reliably solve the TSP?

Methods

- **N = 46 undergraduate students** completed **32 TSP problems**, **two times** each.
  Example from one participant:



- Measures calculated:
  - Reliability of TSP solutions (similarity of two solutions)
  - Percentage above optimality
  - Reaction time

References

Best, B. J., & Simon, H. A. (2000). Simulating human performance on the traveling salesman problem. *Proceedings of the Third International Conference on Cognitive Modeling*, 42–49.

Dry, M., Lee, M., Vickers, D., & Hughes, P. (2006). Human Performance on Visually Presented Traveling Salesperson Problems with Varying Numbers of Nodes. *The Journal of Problem Solving*, *1*(1). https://doi.org/10.7771/1932-6246.1004

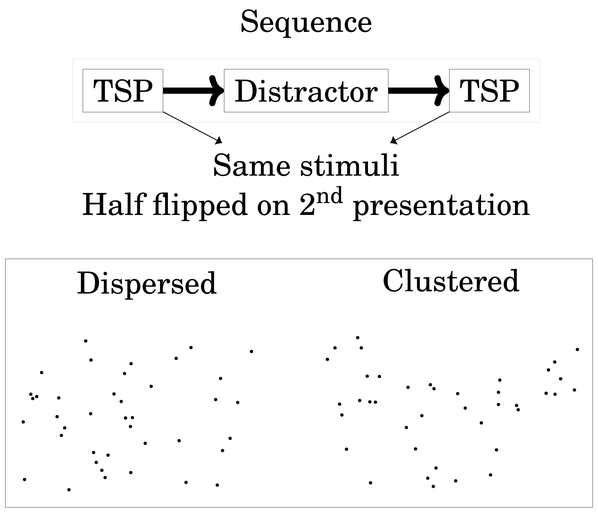
Dry, M., Preiss, K., & Wagemans, J. (2012). Clustering, randomness, and regularity: Spatial distributions and human performance on the traveling salesperson problem and minimum spanning tree problem. *The Journal of Problem Solving*, *4*(1), 2.

Graham, S. M., Joshi, A., & Pizlo, Z. (2000). The traveling salesman problem: A hierarchical model. *Memory & cognition*, *28*(7), 1191–1204.

Kong, X., & Schunn, C. D. (2007). Global vs. local information processing in visual/spatial problem solving: The case of traveling salesman problem. *Cognitive Systems Research*, *8*(3), 192–207.

MacGregor, J. N., & Ormerod, T. (1996). Human performance on the traveling salesman problem. *Perception & psychophysics*, *58*(4), 527–539.

Procedure



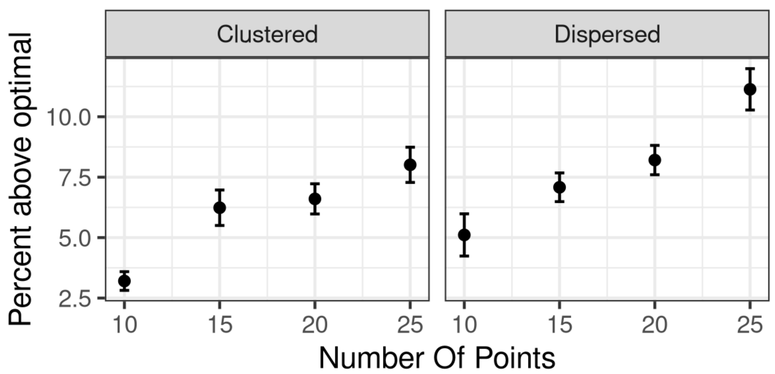
Structure: Half clustered, half dispersed
Numbers of points: 10, 15, 20, 25

Measure of TSP reliability

$$\text{Reliability} = \frac{\text{Number of shared edges}}{\text{Number of total edges}}$$

Ranges from 0 (completely different TSP tour) to 1 (exact same TSP tour).

Result: % above optimal

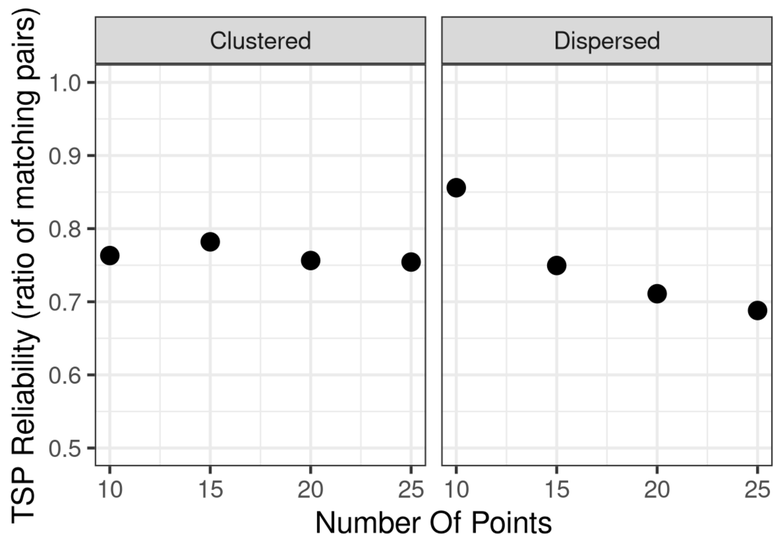


Dispersed stimuli were 1.87% less optimal than clustered stimuli ($p < 0.001$).

Takeaway

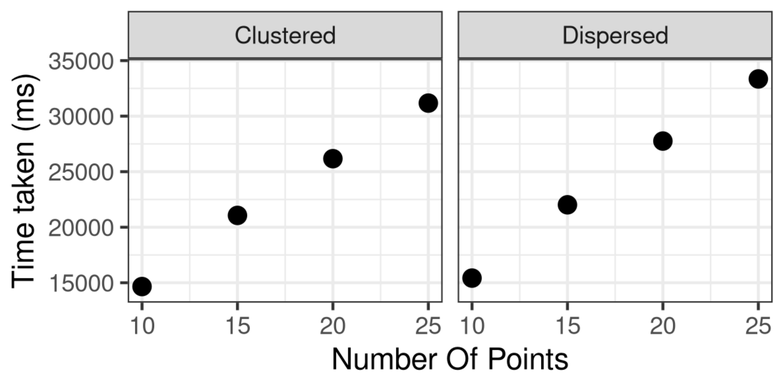
Reliability between two TSPs on the same stimulus is higher when it is clustered. Thus, humans might be using cluster structure for TSP solutions.

Result: TSP Reliability



- Lower reliability for dispersed stimuli (M = 0.71) than clustered stimuli (M = 0.75).
- We found evidence for an interaction between Cluster Structure and Number Of Points ($p < 0.001$). For each additional point, TSP reliability for dispersed stimuli drops by 0.011, but only 0.001 for clustered stimuli.

Result: Time Taken



- No difference in time taken to complete TSP between clustered and disperse stimuli ($p = 0.08$).
- Time taken was linear with Number Of Points ($p < 0.001$).

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