

**b.**

$\hat{Z} = \begin{bmatrix} \hat{u}_t & \hat{v}_t \\ \Delta \hat{u} & \Delta \hat{v} \end{bmatrix} - \begin{bmatrix} \hat{\mu}_u & 0 \\ 0 & \hat{\mu}_v \end{bmatrix}$

$\hat{\Phi} = \begin{bmatrix} 1 & \hat{u} & \hat{v} & \hat{u}^2 & \hat{v}^3 & \xi_u & \xi_v \end{bmatrix}$

$\hat{\Xi} = \begin{bmatrix} \xi_u & \xi_v \end{bmatrix}$

$\hat{\Xi} = \arg \min_{\Xi} \left[ \|\hat{\Phi} \Xi - \hat{Z}\|_2^2 + \lambda \|\Xi\|_0 \right]$

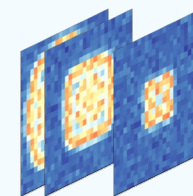
**Sparse Regression:**  $\hat{\Xi} = \arg \min_{\Xi} \left[ \|\hat{\Phi} \Xi - \hat{Z}\|_2^2 + \lambda \|\Xi\|_0 \right]$

**c.** Identified PDE Formulation

$$u_t = 2.001 \times 10^{-5} \Delta u - 1.003 uv^2 - 0.04008u + 0.04008$$

$$v_t = 5.042 \times 10^{-6} \Delta v + 1.009 uv^2 - 0.1007v$$

Low-res Noisy Measurement



Physics-based model

Coefficients fine-tuning

$$u_t = c_1 \Delta u + c_2 uv^2 + c_3 u + c_4$$

$$v_t = c_5 \Delta v + c_6 uv^2 + c_7 v$$

Identified PDE Structure

**Note:**  $c_1 \sim c_7$  are PDE coefficients for further tuning

	$\xi_u$	$\xi_v$
1	0.0356	0
$u$	0.0271	0
$v$	0	-0.0735
$u^2$	0	0
$\vdots$	$\vdots$	$\vdots$
$uv^2$	-0.8644	0.9203
$\vdots$	$\vdots$	$\vdots$
$v^3$	0	0

After Thresholding