**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans** –

Here work begin on the transmission of a customer’s car 10 minutes after the car is dropped off, hence from 60 minutes 10 minutes are already gone and remaining 50 min is left to complete a job.

Let X be the time required for servicing transmissions.

Now we find out the probability that the service time exceeds 50 minutes P(X>50).

The Z-score for a given value X in a normal distribution is calculated as:

Z= (X−μ)/​ σ --------1)

Here, X= 50

μ= 45

σ= 8

Put all values in equation 1)

Z= (50-45)/8

= **0.625**

Now, we find out the probability of Z=0.625score by using the standard normal distribution table(z-table).

P(Z>0.625)

Probability that the service time exceeds 50 minutes P(X>50).

P(X > 50)=1−P(Z ≤ 0.625)

Using Z-score table

P(Z ≤ 0.625)≈0.7323

Therefore,

P(X>50)≈1−0.7323≈0.2676

Hence B) is the right answer from above given options.

**B. 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans** - ﻿

Here,

μ= 38 & σ=6.

a) Probability of employees >44= Pr(x>44)=1-Pr(x<=44)

Z=(X-μ)/σ = (x-38)/6

Pr(x<=44)=Pr(z<=(44-38)/6)=Pr(Z<=1)=0.84134-84.134%

Probability that employees will be greater than 44 = 100-84.134 = 15.866

Probability of employess between 38 & 44 = Pr(x<=44) - Pr(x>=38)

Here, Pr(x<=44) = 0.84134

Pr(x>=38)=Pr(z>=(38-38)/6)=Pr(z>=0)=0.5

Therefore, Pr(x<=44) - Pr(x>=38) = 0.84134 −0.5 = 0.34134 = 34.134%

So, the statement "More employees at the processing center are older than 44 than between 38 and 44" is TRUE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans** -For calculation of the number of employees under the age of 30, we need to find

the probability that an employee's age is less than 30(P<30).

Firstly, we will calculate Z-value

Z30​=(30−38​)/6=**−1.33**

P(< 30)=P(Z<−1.33) = **0.09176**

The expected number of employees under 30 = Probability\*Total number of employees.

= 0.09176\*400

= **36**

Hence**, “**A training program for employees under the age of 30 at the center would be expected to attract about 36 employees” is **True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans** - If *X*1​∼*N*(*μ*,*σ*2) and *X*2​∼*N*(*μ*,*σ*2) are identically distributed (iid) normal random variables, let's will check the distributions and parameters of 2X1​ and 2X1​+X2​:

1. **Distribution of 2X1​:**
   * The distribution of 2*X*1​ is also normal. Specifically, if *X* has a normal distribution with mean *μ* and variance *σ*2, then 2*X* will have a normal distribution with mean 2*μ* and variance 4*σ*2.
   * So, for 2*X*1​, the mean is 2*μ* and the variance is 4*σ*2.
   * 2*X*1​∼*N*(2*μ*,4*σ*2)
2. **Distribution of 2*X*1​+*X*2​:**
   * The sum of two independent normal random variables is also normal. If *X*1​ and *X*2​ are independent normal random variables with means *μ* and variances *σ*2, then *X*1​+*X*2​ will be a normal random variable with mean *μ*+*μ*=2*μ* and variance *σ*2+*σ*2=2*σ*2.
   * So, for *X*1​+*X*2​, the mean is2*μ* and the variance is 2*σ*2.
   * *X*1​+*X*2​∼*N*(2*μ*,2*σ*2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans** - The probability of getting value between a & b is 0.99

So, the probability of getting value outside a & b is 1-0.99 = 0.01

The probability towards left of a = -0.01/2=-0.05

The probability towards right of b = 0.01/2=0.05

Since we have probabilities of a & b, we need calulcate the probability of X - the random

variable at a & b which has these probabilities

By finding Standard Normal Variable (z),need to calculate X:

Z=(X- *μ*)/ *σ ---------------1)*

For a probability of 0.005, z values is -2.57

Z\* σ+ μ=x

-(-2.57)\*20+100=151.4

(-2.57)\*20+100=48.6

Hence**, Option D** (48.5, 151.5) is the correct answer

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans** - Mean profit is RS 540 Million

Std deviation is RS 225 Million

Range is RS

State norms interval (0.95, 540, 225)

Range is RS 99.0081034, 980.991896

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans** –

formula X= X=μ + Zo;

where in from z table, 5 percentile = -1.645

X = 540(-1.645)\*225

X = 169.875

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans**-

Stats. norm. cdf (0, 5, 3)

0.0477903

Probability od division 2 making a loss p(X<0) Stats. norm. cdf(0, 7, 4)

0.0400591.