Preliminary results – Modelling choices

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1 Obtain monthly payments in annuity mortgage

The annuity mortgage is characterize by its equal payments. Suppose payments $B_1, \ldots, B_n$, for these payments the relation

$$B_1 = \ldots = B_n$$

holds. Note that at t_0 there is no payments (as the mortgage is granted). Let us denote the interest at time t as r_t. Since we look at fixed mortgages the relation

$$r_1 = \ldots = r_n$$

holds for the interest rates. Let us assume that we are in a risk neutral market. When we valuate the loan in the risk neutral market, the present value of the outstanding debt, should be equal to the loan and hence

$$L = B(1+r)^{-t_1} + \ldots B(1+r)^{-t_n}$$

where L is the granted loan. Using the geometric series, the value of each payment B can be determined:

$$B\left[\sum_{j=1}^{n}\left((1+r)^{-1}\right)^{t_j}\right] = B\left[\sum_{j=0}^{n}\left((1+r)^{-1}\right)^{t_j} - 1\right] = B\left[\frac{1-\left(\frac{1}{1+r}\right)^{n+1}}{1-\frac{1}{(1+r)}} - 1\right].$$

From this the monthly payments B can be determined:

$$B = \frac{L}{\left[\frac{1-\left(\frac{1}{1+r}\right)^{n+1}}{1-\frac{1}{(1+r)}} - 1\right]}$$

The term

$$\frac{1-\left(\frac{1}{1+r}\right)^{n+1}}{1-\frac{1}{(1+r)}} - 1$$

will we call the monthly factor. Note that the interest rate given is a yearly rate. The corresponding monthly rate can be calculated by:

$$(1+r)^{\frac{1}{12}}.$$

Now to get the outstanding debt at time t_j,

$$U_{t_j} = B\left((1+r)^1 + (1+r)^2 + \ldots + (1+r)^{t_n - t_j}\right)$$

where t_n is the last payment date. This yields a payment of the upb at time t_j of

$$U_{t_{j-1}} - U_{t_j} = B(1+r)^{t_n - t_j + 1}$$