HW1 Group 1.rmd

My Tran, Kevin Sheard, Pooja Patel

2023-01-31

Supposed the following data: 0, 2, 0, 2, 0 are observed from the distribution with pmf

$$\mathrm{p}(X=0;\theta)=\frac{1-\theta}{3}$$

$$\mathrm{p}(X=1;\theta)=\frac{1}{3}$$

$$\mathrm{p}(X=2;\theta)=\frac{1+\theta}{3};$$

and 0, otherwise.

Find the MLE of θ. Calculate the MLE of $\mathrm{P}(X=2)$.

Solution:

$$\mathrm{L}(\theta)=\mathrm{p}(X=0;\theta)\mathrm{p}(X=2;\theta)\mathrm{p}(X=0;\theta)\mathrm{p}(X=2;\theta)\mathrm{p}(X=0;\theta)$$

$$\Rightarrow \mathrm{L}(\theta)=(\mathrm{p}(X=0;\theta))^3(\mathrm{p}(X=2;\theta))^2$$

$$\Rightarrow \mathrm{L}(\theta)=(\frac{1-\theta}{3})^3*(\frac{1+\theta}{3})^2$$

$$\Rightarrow \ln(\mathrm{L}(\theta))=3*(\ln(\frac{1-\theta}{3}))+2*(\ln(\frac{1+\theta}{3}))$$

$$\Rightarrow \frac{\partial \ln(\mathrm{L}(\theta))}{\partial\theta}=\frac{-3}{1-\theta}+\frac{2}{1+\theta}$$

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uniroot(function(x) -3/(1-x) + 2/(1+x), c(-0.999,0.999)) $root
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## [1] -0.2
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MLE for $\hat{\theta}$ is -0.2

$\hat{\theta}$ for $\mathrm{P}(X=2)$ is $\frac{1+\hat{\theta}}{3}=\frac{1+(-0.2)}{3}=\frac{0.8}{3}\approx 0.2\overline{6}$