



1. Define Experiment, Sample space, Outcome and Event.
2. What is probability and explain different types of probability?
3. In loan defaulters older people make up only 1.4%. Now the probability that someone defaults on a loan is 0.184, Find the probability of default on loan knowing that he is an old person. Older people make up only 0.8%
4. Define Bayes theorem and write the formulae.
5. Solve the below problem using Bayes theorem:

Spam Assassin works by having users train the system. It looks for patterns in the words in emails marked as spam by the user.

For example, it may have learned that the word "free" appears in 30% of the mails marked as spam, i.e., $P(\text{Free} | \text{Spam}) = 0.30$. Assuming 1% of non-spam mail includes the word "free" and 50% of all mails received by the user are spam, find the probability that a mail is spam if the word "free" appears in it.

Probability Assignment

1) Experiment 1-

Experimental probability is probability that is determined on the basis of the results of an experiment repeated many times

Ex 1- probability of occurrence of tails =

$$\frac{\text{number of times tails occurs}}{\text{number of times coin is tossed}}$$

Sample space 1- Sample space is a set of possible outcomes of a random experiment

Ex 1- Rolling Two dice

$$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

Outcome :-

The outcomes of a process are the possible results.

Ex :- When a die is rolled the possible outcomes are 1, 2, 3, 4, 5, 6

Event :- The set of outcomes from an experiment is known as event.

2) What is probability and explain different types of probability?

A) probability means possibility of occurring any event.

Formula :-

$$\text{Probability} = \frac{\text{no of ways an event can occur}}{\text{Total possible events}}$$

Types of probability :-

- 1) Theoretical probability
- 2) Experimental probability
- 3) Axiomatic probability

3)

Given that

$$P(\text{older people} \mid \text{loan defaulters}) = 1.4\%$$

$$= 0.014$$

$$P(\text{loan default}) = 0.184$$

$$P(\text{older people}) = 0.8\%$$

$$= 0.008$$

$$P(\text{loan default} \mid \text{older people}) = ?$$

from Bayes Theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(\text{loan default} \mid \text{older people}) = \frac{0.014 \times 0.184}{0.008}$$

$$= 0.322$$

4) Bayes, Theorem

1) from conditional probability we know that

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{--- (1)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \quad \text{--- (2)}$$

from eq (1)

$$P(A \cap B) = P(A|B) \cdot P(B)$$

from eq (2)

$$P(B \cap A) = P(B|A) \cdot P(A)$$

$$P(A \cap B) = P(B \cap A)$$

$$\therefore P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

5) Given that

$$P(\text{free} | \text{spam}) = 0.30$$

$$P(\text{free} | \text{not spam}) = 1\% = 0.01$$

$$P(\text{spam}) = 50\% = 0.50$$

$$\begin{aligned} P(\text{not spam}) &= 1 - P(\text{spam}) \\ &= 1 - 0.50 \\ &= 0.50 \end{aligned}$$

$$P(\text{free}) = P(\text{free} | \text{spam}) \times P(\text{spam}) + P(\text{free} | \text{not spam}) \times P(\text{not spam})$$

$$\begin{aligned} P(\text{free}) &= 0.30 \times 0.50 + 0.01 \times 0.50 \\ &= 0.165 \end{aligned}$$

from Bayes Theorem

$$P(\text{spam} | \text{free}) = \frac{P(\text{free} | \text{spam}) \times P(\text{spam})}{P(\text{free})}$$

$$= \frac{0.30 \times 0.50}{0.165}$$

$$= 0.909$$