NUMBER SYSTEM

BASIC (LEVEL 1)

1. A teacher when distributed certain number of chocolates to 4 children, 5 children and 7 children, he always left with 1 chocolate. Find the least number of chocolates the teacher brought to the class?
   To get the least number of chocolates, we get the LCM of 4, 5, and 7
   = 140
   Thus, minimum number of chocolates she brought to the class = 140 + 1 = 141

2. Find the remainder when 1201 x 1203 x 1205 x 1207 is divided by 6.
   Dividing each number separately by 6, we get 1, 3, 5, and 1 as the remainders.
   Multiplying, (1, 3, 5, and 1) we get 15 and when 15 is divided by 6, we get 3 as the remainder.

3. Find the remainder when 111222333444 is divided by 7, 11 and 13?
   Make triplets from the right hand side and apply the rule,
   (444+222) – (333+111) = 222
   Divide 222 by 7, 11 and 13, we get the remainders 5, 2, and 1 respectively.

4. Find the maximum power of 5 in 60!
   Applying the formula, 60/5 + 60/25 + 60/125 …. = 12+2 = 14

MODERATE (LEVEL 1)

5. How many 0’s are there at the end of 100!
   100/5 + 100/25 + 100/125…. = 20 + 4 = 24 0’s at the end of 100!

6. What is the unit digit of the expression 317^{171}?
   Cyclicity for 7 is 4, thus by dividing the power, i.e. 171 by 4 we get 3 as remainder. Thus, 7^3 gives 3 as the unit digit answer.

7. Find the remainder when 1! + 2! + 3! + 4! ………..100! is divided by 24?
   By applying the rule, we divide the terms of the above expression individually, and add them to get the final remainder. But from 4! Onwards all the terms will be divisible by 24, leaving 0 remainder.
   So remainder = 1 + 2 + 6 + 0 + 0 + …… = 9

8. Find the remainder when 8^{80} is divided by 17?
   $8^{80}/17 = (2^3)^{80} / 17 = 2^{240}/17 = (2^4)^{60}/17 = (16)^{60}/17 = (17\text{-}1)^{60}/17 = (\text{-}1)^{60}/17 = 1$
9. Find the highest power of 12 that can divide 49!
   $12 = 3 \times 4 = 3 \times 2^2$

We find the maximum power of 2 in 49! = 49/2 + 49/4 + 49/8 + 49/16 + 49/32 = 46
So, maximum power of 2^2 in 49! = 23
Now, we find the maximum power of 3 in 49! = 49/3 + 49/9 + 49/27 = 22
Now, we take the minimum of 23 and 22, which is 22

10. Find the total number of factors, even factors and odd factors of 1200 respectively.
$1200 = 2^4 \times 3^1 \times 5^2$
This is in the form of $a^p \times b^q \times c^r$
Now, applying the formula,
Total Number of Factors = (p+1)(q+1)(r+1) = (4+1)(1+1)(2+1) = 30
Even Factors = p(q+1)(r+1) = 4(1+1)(2+1) = 24
Odd Factors = (q+1)(r+1) = (1+1)(2+1) = 6

11. Find the unit place of $456^{456} \times 234^{234} \times 567^{567} \times 912^{912}$
The question can be reduced to $6^{456} \times 4^{234} \times 7^{567} \times 2^{912}$
Dividing the powers with the cyclicity of 6, 4, 7 and 2 respectively, we get
$6^{456/1} \times 4^{234/4} \times 7^{567/4} \times 2^{912/4}$
$= 6^0 \times 4^2 \times 7^3 \times 2^0$ [wherever the remainder is 0, we will pick the last digit of the cyclicity]
= 6 x 6 x 3 x 6 = 8 (as the unit digit)

12. Find the unit place of $47^{23} - 23^{47}$
$47^{23/4} - 23^{47/4}$
$7^3 – 3^3 = 3 – 7 = 6$ (since $47^{23} > 23^{47}$)

13. Find the remainders when 50^{51}, 51^{51} & 52^{51} are divided by 7 respectively.
$50^{51}/7 = (49 + 1)^{51}/7 = 1$
$51^{51}/7 = (49 + 2)^{51}/7 = (2^3)^{17}/7 = (7+1)^{17}/7 = 1$
$52^{51}/7 = (49 + 3)^{51}/7 = (3^3)^{17}/7 = (28-1)^{17}/7 = -1/7 = 6$

14. How many factors of $2^5 \times 5^3 \times 7^4$ have odd number of factors?
Number of odd factors = (3+1)(4+1) = 20

15. Find the value of |A-B| if 32A4873B is divisible by 72.
Since the number is divisible by 72, means it should be divisible by 8 & 9 also.
Checking for the divisibility of 8, (last 3 digits should be divisible), thus 73B is divisible by 8.
Thus, B = 6
Applying the divisibility test for 9, we get (33 + A) should be divisible by 9
A = 3
Thus, |A-B| = 3