Math 112B/212B, Introduction to Mathematical Biology, Homework 4

1. Consider the transport equation,

$$\frac{\partial u}{\partial t} + c(x,t)\frac{\partial u}{\partial x} = 0,$$

with $c(x,t) = 3$. (a) Find the characteristics and draw several of them in the $x-t$ plane. (b) Find the solution of the initial-value problem with $u(x,0) = \exp(-x^2)$. (c) Draw this solution as a function of x for $t = 0, t = 1, t = 2$.

2. Consider the same transport equation with $c(x,t) = e^{-t}$. (a) Find the characteristics, and draw them in the $x-t$ plane. (b) Find the solution to the initial-value problem where $u(x,0) = exp(-x^2)$. *Hint: to find the equations for the characteristics lines, first divide the PDE by* e^{-t}.

3. Consider the inviscid Burgers' equation with the initial condition $u(x,0) = f_0(x)$ given by the following function: $f_0(x) = 1$ for $x < 0$, $f_0(x) = 1 + x$ for $0 \leq x \leq 1$, and $f_0(x) = 2$ for $x > 1$. Find the equations for the characteristics, and draw the characteristics in the (x,t) plane. Will a shock form in this case?

4. Same as before, but the function $f_0(x)$ is given by $f_0(x) = 2$ for $x < 0$, $f_0(x) = 2 - x$ for $0 \leq x \leq 1$, and $f_0(x) = 1$ for $x > 1$.