Trabalho1_V9

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2 1 - Regressao Linear com uma Variavel

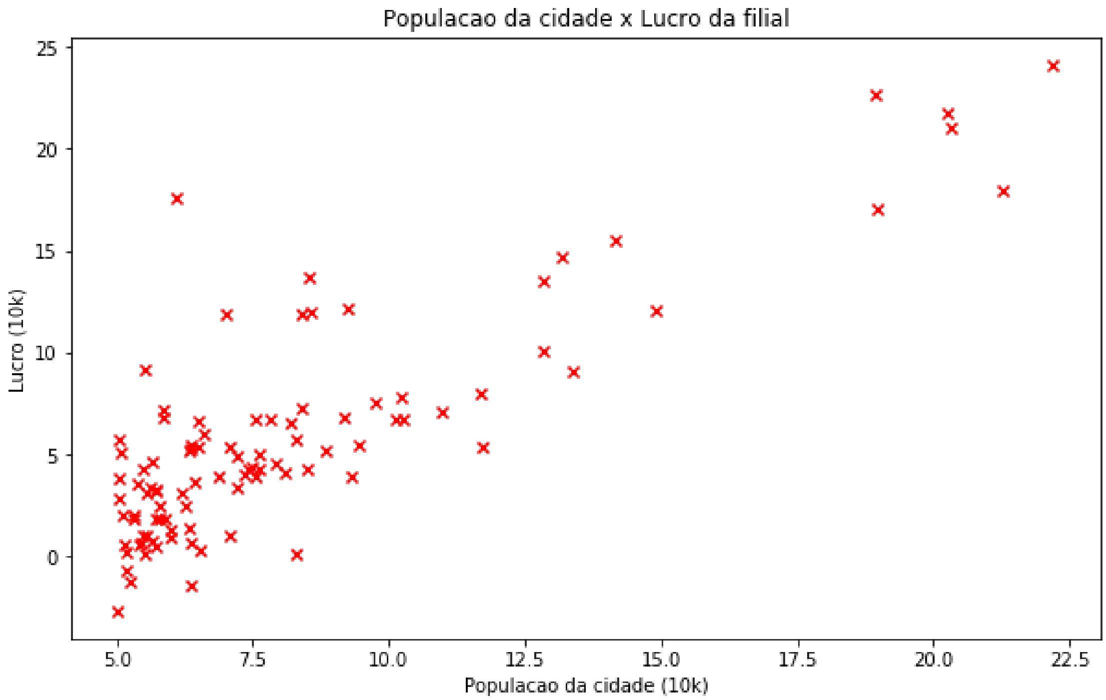
Importando os dados do arquivo ex1data1

```
In [1]: import os

        import numpy as np

        import pandas as pd

        import matplotlib.pyplot as plt

        %matplotlib inline


        def importarDados(filepath, names):

            path = os.getcwd() + filepath

            data = pd.read_csv(path, delimiter=",", header=None, names=names)


            X = data.iloc[:,0:-1].values

            y = data.iloc[:, -1:].values


            # Incluir o valor de 1 em x, pois theta0 = 1

            X = np.c_[np.ones((X.shape[0], 1)), X]


            data.X = X

            data.y = y
```

```
    return data

data = importarDados(filepath="\ex1data1.txt", names=["Population","Profit"])
```

1.1 Visualizacao dos Dados
Plotando o grafico População da cidade x Lucro da Filial usando o script plot_ex1data1.py

```
In [2]: %run -i "plot_ex1data1.py"

        plot()
```



1.2 Gradiente Descendente
Iniciado theta com o valor 0 e passando o theta como parâmetro para a função custo_regrlin

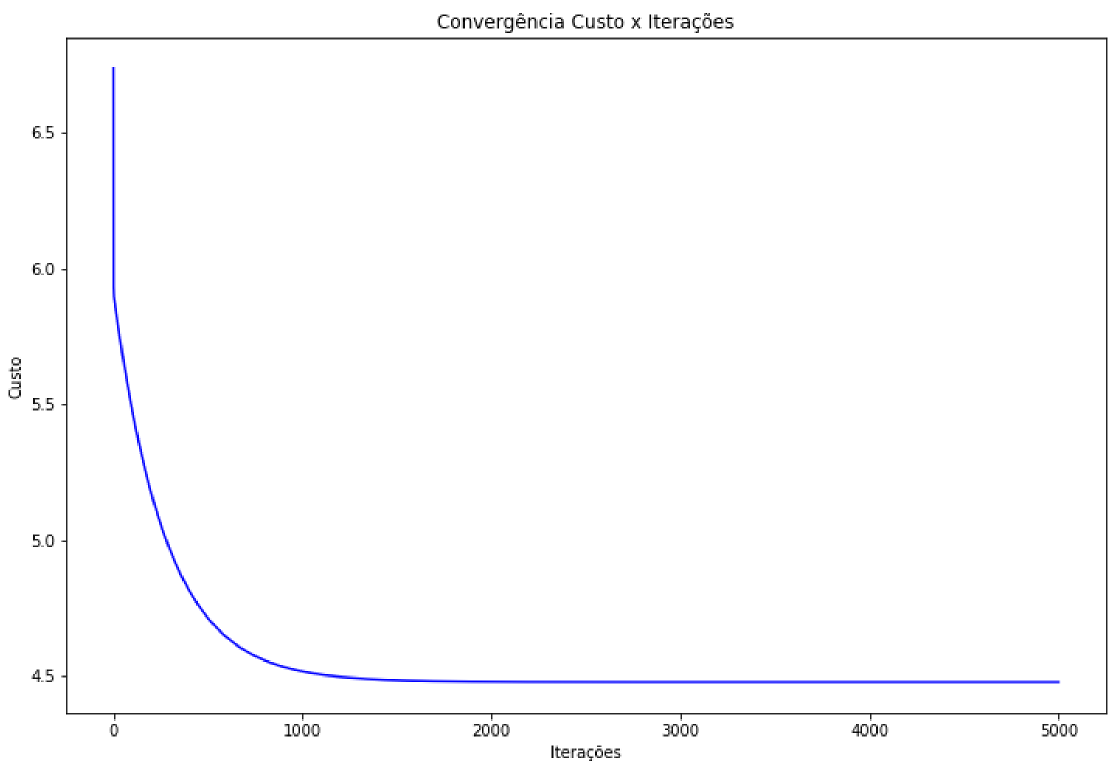
```
In [3]: %run -i "custo_reglin_uni.py"

        # Iniciando Theta com o Valor 0
        theta = np.array([0,0]).reshape(-1, 1)
        #executando a função de custo passando os parâmetros
        result = (custo_regrlin(data.X,data.y, theta))

        print ("Com todos os parâmetros iguais a zero, a função gera um valor igual a %.2f." % r
```

```
Com todos os parâmetros iguais a zero, a função gera um valor igual a 32.07.
```

```
<matplotlib.figure.Figure at 0x26226309b70>
```

O GD e o calculo da funcao de custo foram implementados na função gd_reglin_uni. Para comprovoção, foi criado um parâmetro booleano grafico para gerar o grafico de convergência com os valores sucessivos do gradiente descendente que nunca devem crescer, com o valor de custo produzindo o valor aproximado de 4,47. São passados como parâmetro:taxa de aprendizado=0,01,nº de iterações 5000, parâmetro grafico para visualização do grafico de convergência custo x iterações.

```
In [4]: %run -i "gd_reglin_uni.py"


        custo, theta = gd_reglin_uni(data.X,data.y, 0.01,5000,grafico=True)

        print(custo)
```

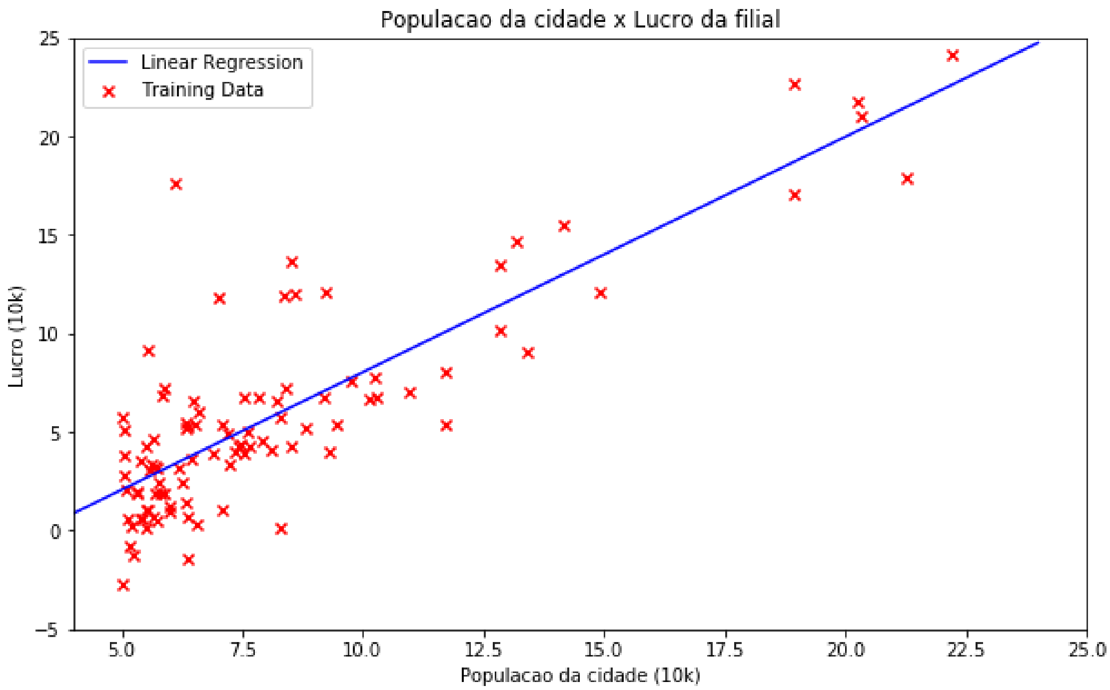



```
4.47697139698
```

Utilizando o script visualizar_reta.py para visualizar a reta correspondente aos parametros determinados. O resultado da execução desse script é o grafico da reta após a regressão linear.

```
In [5]: %run -i "visualizar_reta.py"

        plot("\ex1data1.txt",theta)
```

```
<matplotlib.figure.Figure at 0x262205fcac8>
```



utilizando o modelo de regressao linear produzido para predizer o lucro em regioes com populações de 35.000 e 70.000 habitantes. O codigo foi implementado na função theta_gd.py

```
In [6]: %run -i "theta_gd"

        theta_gd(theta)
```

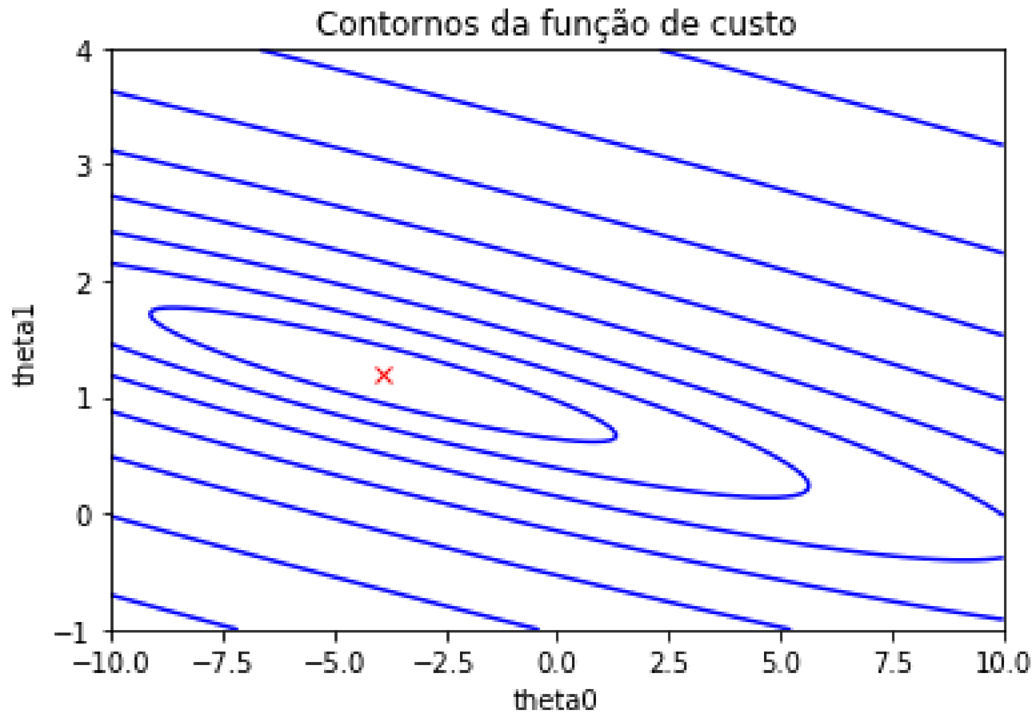
```
Lucro em regiões com populações de 35.000 habitantes = $ 2801.48
Lucro em regiões com populações de 70.000 habitantes = $ 44555.97


<matplotlib.figure.Figure at 0x262265e2160>
```

1.3 Visualizacão de J(theta)

Utiliza a função Plot() do Script visualizar_J_contour.py(descrição da implementação no código) passando os dados importados de ex1data1.txt e o valor de theta como parâmetros.

```
In [7]: %run -i "visualizar_J_contour.py"

        plot(data, theta)
```

```
<matplotlib.figure.Figure at 0x2622660c4e0>
```

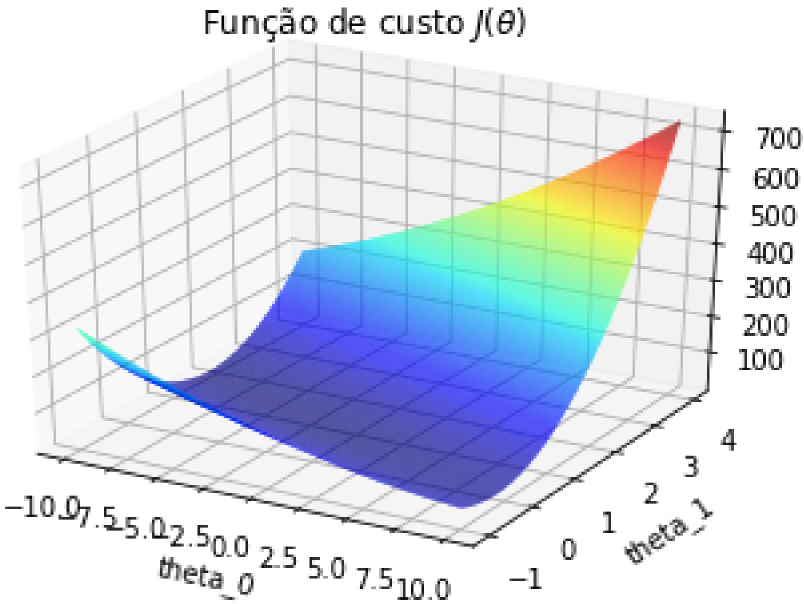



```
Out[7]: array([[ 328.09290555,  327.8529662 ,  327.61312685, ...,   48.23462257,
           48.19438322,   48.15424387],
         [ 325.81366782,  325.57454445,  325.33552108, ...,   47.5848969 ,
           47.54547353,   47.50615016],
         [ 323.54257049,  323.3042631 ,  323.06605571, ...,   46.94331163,
           46.90470424,   46.86619685],
         ...,
         [ 198.66419212,  198.82979482,  198.99549753, ...,  728.67340296,
          729.03870567,  729.40410837],
         [ 200.43073034,  200.59714903,  200.76366772, ...,  732.06945325,
          732.43557193,  732.80179062],
         [ 202.20540897,  202.37264364,  202.53997831, ...,  735.47364393,
          735.8405786 ,  736.20761327]])
```

Utiliza a função Plot() do Script visualizar_J_surface.py(descrição da implementação no código) passando os dados importados de ex1data1.txt como parâmetros.

```
In [8]: %run -i "visualizar_J_surface.py"

        plot(data)
```

```
<matplotlib.figure.Figure at 0x262265f9198>
```




```
Out[8]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x26220434470>
```

3 2 - Regressao Linear com Multiplas Variaveis

Importando os dados do arquivo ex1data2

```
In [9]: def importarDados(filepath):

            path = os.getcwd() + filepath

            data = pd.read_csv(path, delimiter=",", header=None)


            X = data.iloc[:,0:-1].values

            y = data.iloc[:, -1:].values


            return X,y


        X,y = importarDados(filepath="\ex1data2.txt")
```

2.1 Normalizacao das caracteristicas

É passado como parâmetro X e y. Onde cada coluna da matriz de dados X passada para a função normalizar caracteristica corresponde a uma característica não existindo limites para o numero de características e/ou exemplos. Como retorno é armazenado valor medio e desvio padrão para utilização.

```
In [10]: %run -i "normalizacao.py"

         x_norm,y_norm,valor_mediaX,desvio_padraoX= normalizar_caracteristica(X,y)

         print(x_norm)
```

```
[[  1.00000000e+00   1.31415422e-01  -2.26093368e-01]
 [  1.00000000e+00  -5.09640698e-01  -2.26093368e-01]
 [  1.00000000e+00   5.07908699e-01  -2.26093368e-01]
 [  1.00000000e+00  -7.43677059e-01  -1.55439190e+00]
 [  1.00000000e+00   1.27107075e+00   1.10220517e+00]
 [  1.00000000e+00  -1.99450507e-02   1.10220517e+00]
 [  1.00000000e+00  -5.93588523e-01  -2.26093368e-01]
 [  1.00000000e+00  -7.29685755e-01  -2.26093368e-01]
 [  1.00000000e+00  -7.89466782e-01  -2.26093368e-01]
 [  1.00000000e+00  -6.44465993e-01  -2.26093368e-01]
 [  1.00000000e+00  -7.71822042e-02   1.10220517e+00]
 [  1.00000000e+00  -8.65999486e-04  -2.26093368e-01]
 [  1.00000000e+00  -1.40779041e-01  -2.26093368e-01]
 [  1.00000000e+00   3.15099326e+00   2.43050370e+00]
 [  1.00000000e+00  -9.31923697e-01  -2.26093368e-01]
 [  1.00000000e+00   3.80715024e-01   1.10220517e+00]
 [  1.00000000e+00  -8.65782986e-01  -1.55439190e+00]
 [  1.00000000e+00  -9.72625673e-01  -2.26093368e-01]
 [  1.00000000e+00   7.73743478e-01   1.10220517e+00]
 [  1.00000000e+00   1.31050078e+00   1.10220517e+00]
 [  1.00000000e+00  -2.97227261e-01  -2.26093368e-01]
 [  1.00000000e+00  -1.43322915e-01  -1.55439190e+00]
 [  1.00000000e+00  -5.04552951e-01  -2.26093368e-01]
 [  1.00000000e+00  -4.91995958e-02   1.10220517e+00]
 [  1.00000000e+00   2.40309445e+00  -2.26093368e-01]
 [  1.00000000e+00  -1.14560907e+00  -2.26093368e-01]
 [  1.00000000e+00  -6.90255715e-01  -2.26093368e-01]
 [  1.00000000e+00   6.68172729e-01  -2.26093368e-01]
 [  1.00000000e+00   2.53521350e-01  -2.26093368e-01]
 [  1.00000000e+00   8.09357707e-01  -2.26093368e-01]
 [  1.00000000e+00  -2.05647815e-01  -1.55439190e+00]
 [  1.00000000e+00  -1.27280274e+00  -2.88269044e+00]
 [  1.00000000e+00   5.00114703e-02   1.10220517e+00]
 [  1.00000000e+00   1.44532608e+00  -2.26093368e-01]
 [  1.00000000e+00  -2.41262044e-01   1.10220517e+00]
 [  1.00000000e+00  -7.16966387e-01  -2.26093368e-01]
```

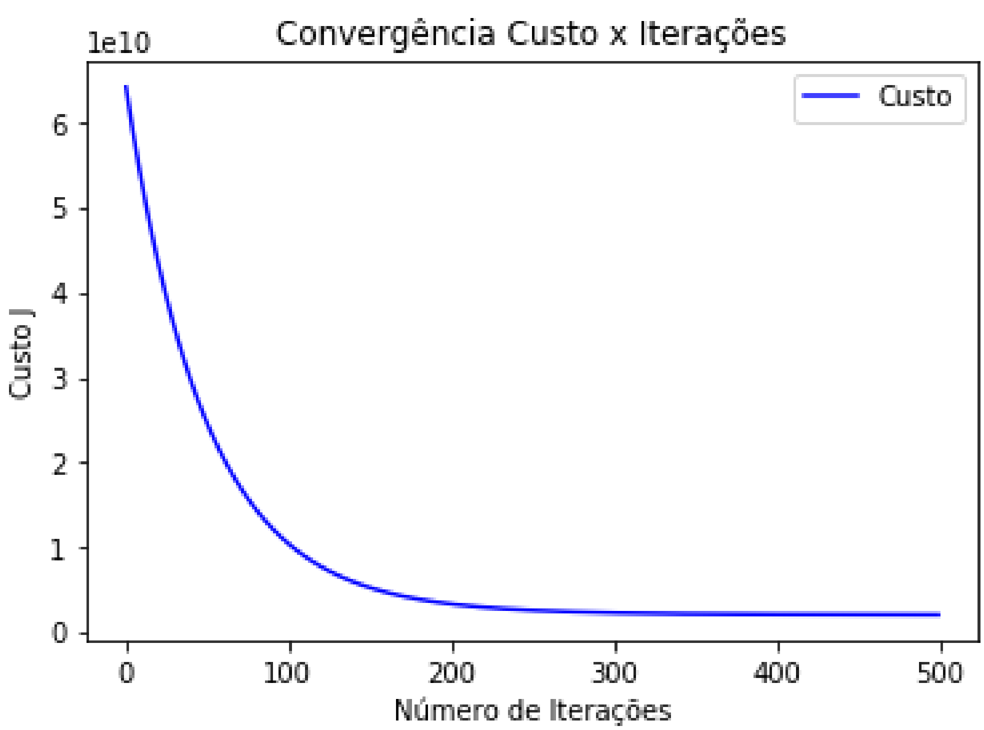
```
[  1.00000000e+00  -9.68809863e-01  -2.26093368e-01]
[  1.00000000e+00   1.67029651e-01   1.10220517e+00]
[  1.00000000e+00   2.81647389e+00   1.10220517e+00]
[  1.00000000e+00   2.05187753e-01   1.10220517e+00]
[  1.00000000e+00  -4.28236746e-01  -1.55439190e+00]
[  1.00000000e+00   3.01854946e-01  -2.26093368e-01]
[  1.00000000e+00   7.20322135e-01   1.10220517e+00]
[  1.00000000e+00  -1.01841540e+00  -2.26093368e-01]
[  1.00000000e+00  -1.46104938e+00  -1.55439190e+00]
[  1.00000000e+00  -1.89112638e-01   1.10220517e+00]
[  1.00000000e+00  -1.01459959e+00  -2.26093368e-01]]
```

```
<matplotlib.figure.Figure at 0x26226305518>
```

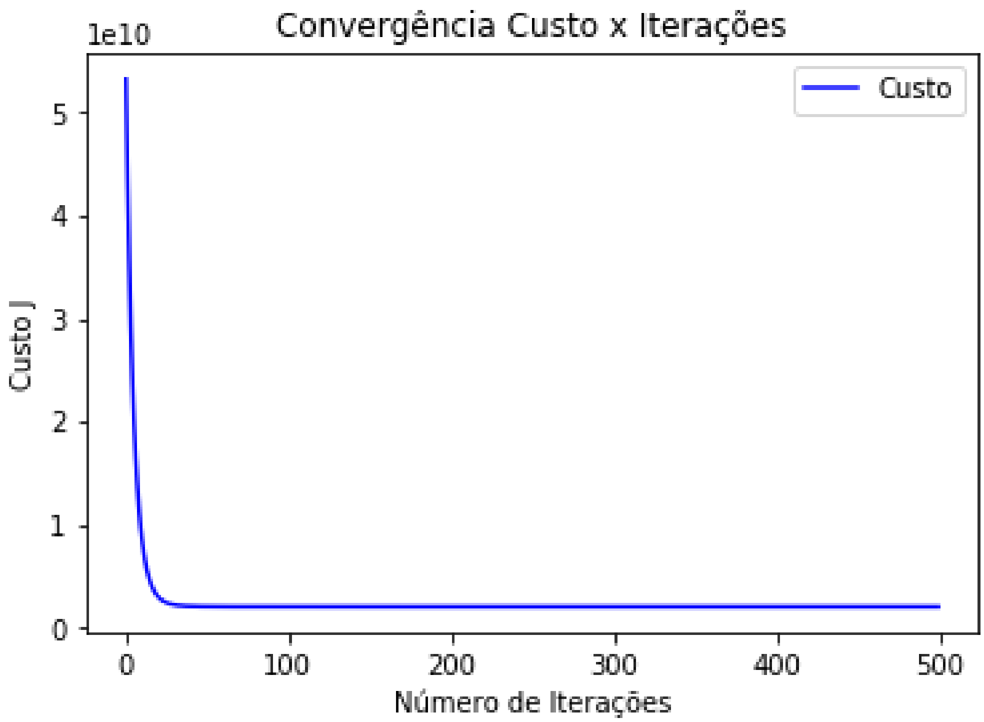
2.2 Gradiente descendente

É passado como parâmetro x_norm e y, nº de iterações, épocas. Como retorno custo e gradiente.

```
In [11]: %run -i "gd_reglin_multi.py"

         custo, theta = gd(x_norm,y, 0.01, 500)
```



```
In [12]: %run -i "gd_reglin_multi.py"
         custo, theta = gd(x_norm,y, 0.1, 500)
```



Nesse caso, com uma taxa de aprendizado maior, houve uma convergência mais rápida.

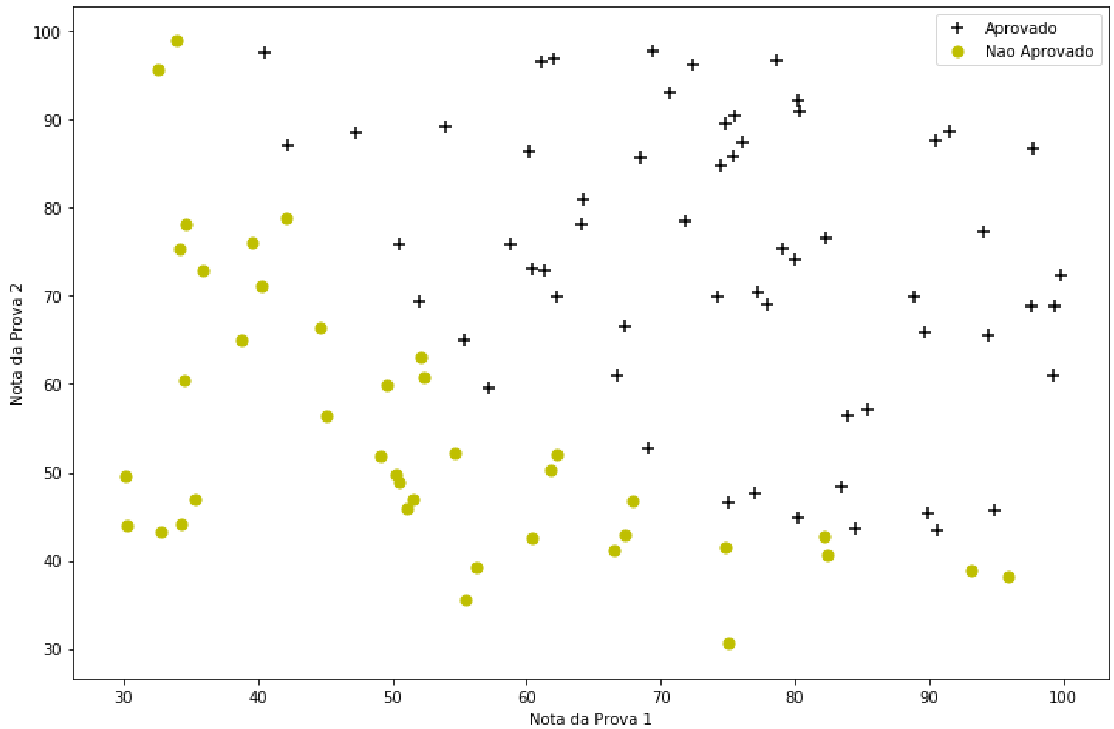
4 3 Regressao Logistica

Importando os dados do arquivo ex2data1.txt

```
In [13]: %run -i "plot_ex2data1.py "

         data,examData,labels = importarDados(filepath="\ex2data1.txt", names=['Prova 1', 'Prova
```

3.1 Visualizacao dos dados
Utilizando a função plot_ex2data1 para gerar o grafico a partir dos dados coletados

```
In [14]: %run -i "plot_ex2data1.py "

         plot(data)
```



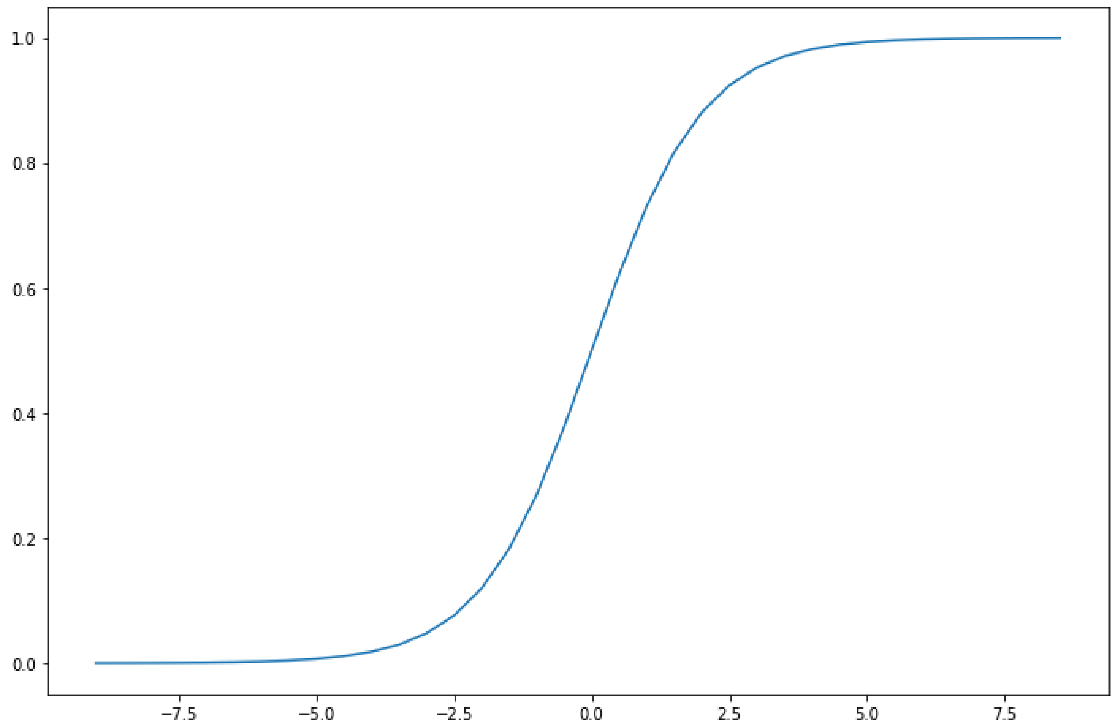
3.2.1 Funcão sigmoide

Como primeiro passo nessa parte, é implementada a função em Python que calcula o valor da função sigmoide.

```
In [15]: %run -i "sigmoide.py "

         print (sigmoide(0))
         print (sigmoide(9))
         print (sigmoide(-9))

         testesigmoide = np.arange(-9, 9, step=.5)

         fig, ax = plt.subplots(figsize=(12,8))
         ax.plot(testesigmoide, sigmoide(testesigmoide))
         filename = 'target/plot3.2.1.png'
         plt.savefig(filename)
```

```
0.5
0.999876605424
0.000123394575986
```

```
<matplotlib.figure.Figure at 0x262265ec630>
```



3.2.2 Funcão de custo e gradiente

Uma vez que implementada essa função, ao realizar uma chamada usando o valor inicial de θ, se deve confirmar que o valor produzido é aproximadamente 0,693.

```
In [130]: %run -i "custo_reglog.py "
          %run -i "NormalizacaoCaracteristicas.py "

          examData_norm,labels_norm,mean_examData,std_examData,mean_labels,std_labels = normaliz

          theta=np.array([0,0,0],ndmin = 2)
          J = funcaoCustoRegressaoLogistica(theta, examData_norm.T, labels_norm)
          print(J)
```

```
0.69314718056


NormalizacaoCaracteristicas.py :19: RuntimeWarning: invalid value encountered in true_divide
  X_norm = (X - mean_X) / std_X
```

3.2.3 Aprendizado dos parâmetros

Theta foi minimizado com relacão ao vetor de parâmetros. Implementação gera o vetor que minimiza a função de custo utilizando implementação do gradiente descendente. Usando a API de otimização (optimize) da biblioteca SciPy para aprendizado dos parâmetros das funções para calcular o custo e os gradientes.

```
In [16]: import scipy.optimize as opt
         %run -i "plot_ex2data1.py "
         %run -i "custo_reglog.py "
         %run -i "gd_reglog.py "

         data,X,y = importarDados(filepath="\ex2data1.txt", names=['Prova 1', 'Prova 2', 'Aprova
         theta=np.array([0,0,0],ndmin = 2)
         #print(data)

         result= opt.fmin_tnc(func=funcaoCustoRegressaoLogistica, x0=theta,fprime=gd_reglog ,arg
         #print(result[0])
         print("O valor do custo otimizado é: %.4f" %funcaoCustoRegressaoLogistica(result[0], X,
```

```
O valor do custo otimizado é: 0.2035
```

```
<matplotlib.figure.Figure at 0x262205fc978>
```

```
In [17]: theta_min = np.matrix(result[0])
         print ('Theta otimizado: %s' % theta_min)
```

```
Theta otimizado: [[-25.16131872   0.20623159   0.20147149]]
```

3.2.4 Avaliacão do modelo

Após o aprendizado dos parâmetros, você pode usar o modelo correspondente para predizer se um candidato qualquer serrá aprovado. Para um candidato com notas 45 e 85 na primeira e segunda avaliações, respectivamente, você deve esperar que ele seja aprovado com probabilidade aproximada de 0.80.

Para isso foi feito a operação do vetor de valor correspondente as notas com o theta min.

```
In [18]: valor = [1, 45, 85]
         probabilidade = sigmoide(valor * theta_min.T)
         print("Probabilidade: ", probabilidade[0,0])
```

```
Probabilidade:  0.776290625527
```

É passado como parêmetro os valores de X,y e o resultado do custo otimizado para definição da porcentagem de acertos.

```
In [19]: %run -i "predizer_aprovacao.py "

         precisao = acuracia(X, y, result)
         print ('Porcentagem de acertos do classificador sobre o conjunto de treinamento = %.2f
```

```
Porcentagem de acertos do classificador sobre o conjunto de treinamento = 89.00 %
```

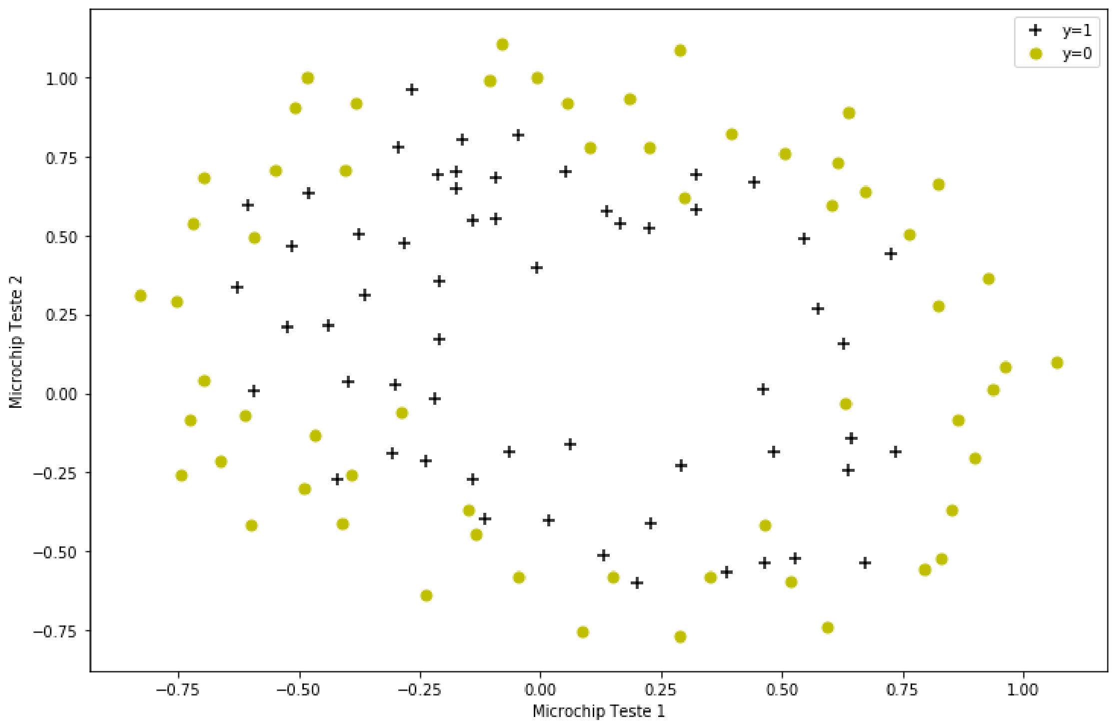
5 4 Regressão Logistica com Regularização

Importando os dados do arquivo ex2data2.txt

```
In [20]: %run -i "plot_ex2data2.py "

         data,X,y = importarDados(filepath="\ex2data2.txt", names=['Teste 1', 'Teste 2', 'Aceito
```

4.1 Visualização dos Dados
Utilizando a função plot_ex2data2 para gerar o grafico a partir dos dados coletados

```
In [21]: %run -i "plot_ex2data2.py "

         plot(data)
```



4.2 Mapeamento de características (feature mapping)
Essa função é implementada em um arquivo de nome mapFeature.py, que irá mapear as características para todos os termos polinomiais de x1 e x2, até a sexta potência. Ao final são gerados 28 elementos após a criação de features de 1,x1,x2,x1x2,x1^{2,x1x2,x2}2... até x2ˆ6

```
In [22]: %run -i "mapFeature.py "

         x1 = X[:, 0]
         x2 = X[:, 1]
         #Definindo a potência dos termos
```

```python
p=6
X = mapFeature(x1,x2,p)
print(X)
```

```
[array([ 1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.]), array([ 0.051267
       -0.52477  , -0.39804  , -0.30588  ,  0.016705 ,  0.13191  ,
        0.38537  ,  0.52938  ,  0.63882  ,  0.73675  ,  0.54666  ,
        0.322    ,  0.16647  , -0.046659 , -0.17339  , -0.47869  ,
       -0.60541  , -0.62846  , -0.59389  , -0.42108  , -0.11578  ,
        0.20104  ,  0.46601  ,  0.67339  , -0.13882  , -0.29435  ,
       -0.26555  , -0.16187  , -0.17339  , -0.28283  , -0.36348  ,
       -0.30012  , -0.23675  , -0.06394  ,  0.062788 ,  0.22984  ,
        0.2932   ,  0.48329  ,  0.64459  ,  0.46025  ,  0.6273   ,
        0.57546  ,  0.72523  ,  0.22408  ,  0.44297  ,  0.322    ,
        0.13767  , -0.0063364, -0.092742 , -0.20795  , -0.20795  ,
       -0.43836  , -0.21947  , -0.13882  ,  0.18376  ,  0.22408  ,
        0.29896  ,  0.50634  ,  0.61578  ,  0.60426  ,  0.76555  ,
        0.92684  ,  0.82316  ,  0.96141  ,  0.93836  ,  0.86348  ,
        0.89804  ,  0.85196  ,  0.82892  ,  0.79435  ,  0.59274  ,
        0.51786  ,  0.46601  ,  0.35081  ,  0.28744  ,  0.085829 ,
        0.14919  , -0.13306  , -0.40956  , -0.39228  , -0.74366  ,
       -0.69758  , -0.75518  , -0.69758  , -0.4038   , -0.38076  ,
       -0.50749  , -0.54781  ,  0.10311  ,  0.057028 , -0.10426  ,
       -0.081221 ,  0.28744  ,  0.39689  ,  0.63882  ,  0.82316  ,
        0.67339  ,  1.0709   , -0.046659 , -0.23675  , -0.15035  ,
       -0.49021  , -0.46717  , -0.28859  , -0.61118  , -0.66302  ,
       -0.59965  , -0.72638  , -0.83007  , -0.72062  , -0.59389  ,
       -0.48445  , -0.0063364,  0.63265  ]), array([ 0.69956 ,  0.68494 ,  0.69225 ,  0.50219 ,
        0.034357, -0.19225 , -0.40424 , -0.51389 , -0.56506 , -0.5212  ,
       -0.24342 , -0.18494 ,  0.48757 ,  0.5826  ,  0.53874 ,  0.81652 ,
        0.69956 ,  0.63377 ,  0.59722 ,  0.33406 ,  0.005117, -0.27266 ,
       -0.39693 , -0.60161 , -0.53582 , -0.53582 ,  0.54605 ,  0.77997 ,
        0.96272 ,  0.8019  ,  0.64839 ,  0.47295 ,  0.31213 ,  0.027047,
       -0.21418 , -0.18494 , -0.16301 , -0.41155 , -0.2288  , -0.18494 ,
       -0.14108 ,  0.012427,  0.15863 ,  0.26827 ,  0.44371 ,  0.52412 ,
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 1.03255971e-01,   1.24749066e-01,   6.01860826e-01,
 5.46641107e-01,   1.24749066e-01,   2.25147637e-01,
 6.01860826e-01,   9.52719292e-01,   1.85932068e+00,
 1.64959475e+00,   3.12626377e-01,   4.95709484e-01,
 8.49416069e-02,   6.94182034e-02,   1.00903382e-06,
```

```
         3.79428457e-02,   6.75425533e-02,   2.47110336e-03,
         7.57144336e-04,   5.72997019e-06,   4.98853262e-08,
         9.87105617e-08,   9.65324848e-05,   5.40024334e-03,
         3.17645990e-07,   9.24726012e-04,   2.44498012e-02,
         1.46892430e-02,   9.95627986e-01,   9.95627986e-01,
         8.22905998e-10]))]
```

```
<matplotlib.figure.Figure at 0x2624732d9e8>
```

4.3 Função de custo e gradiente

```
In [23]: %run -i "costFunctionReg.py "
         %run -i "gradientReg.py "
         %run -i "plot_ex2data2.py "
         from scipy.optimize import fmin
         from scipy.optimize import fmin_bfgs


         data,X,y = importarDados(filepath="\ex2data2.txt", names=['Teste 1', 'Teste 2', 'Aceito
         theta_ini = np.zeros(X.shape[1])
         lmd = 1
         custo = costFunctionReg(theta_ini,X,y,lmd)
         grad = gradientReg(theta_ini,X,y,lmd)
         print(custo,grad)
         print()
         print('Realizando otimização de theta: ')
         print()
         theta = fmin(costFunctionReg, x0=theta_ini, args=(X,y,lmd))
         #theta = fmin_bfgs(costFunctionReg, x0=theta, args=(X,y,lmd), full_output=False)
```

```
0.69314718056 [  1.87880932e-02   7.77711864e-05]

Realizando otimização de theta:

Optimization terminated successfully.
         Current function value: 0.690290
         Iterations: 57
         Function evaluations: 108
```

Testes para Lambda = 0

```
In [24]: lmd = 0
         custo = costFunctionReg(theta_ini,X,y,lmd)
         grad = gradientReg(theta_ini,X,y,lmd)
         print(custo)
```

```
0.69314718056
```

Testes para Lambda = 100

```
In [25]: lmd = 100
         custo = costFunctionReg(theta_ini,X,y,lmd)
         grad = gradientReg(theta_ini,X,y,lmd)
         print(custo)
```

```
0.69314718056
```

Neste caso mesmo alterando o valor de lambda entre 0, 1 e 100 não existe variação no custo, pois como theta é zero a parte da regularização dará sempre zero. Então o custo será so a função da primeira parte do somatório independente de lambda.

6 5 Regressão Linear com Regularização

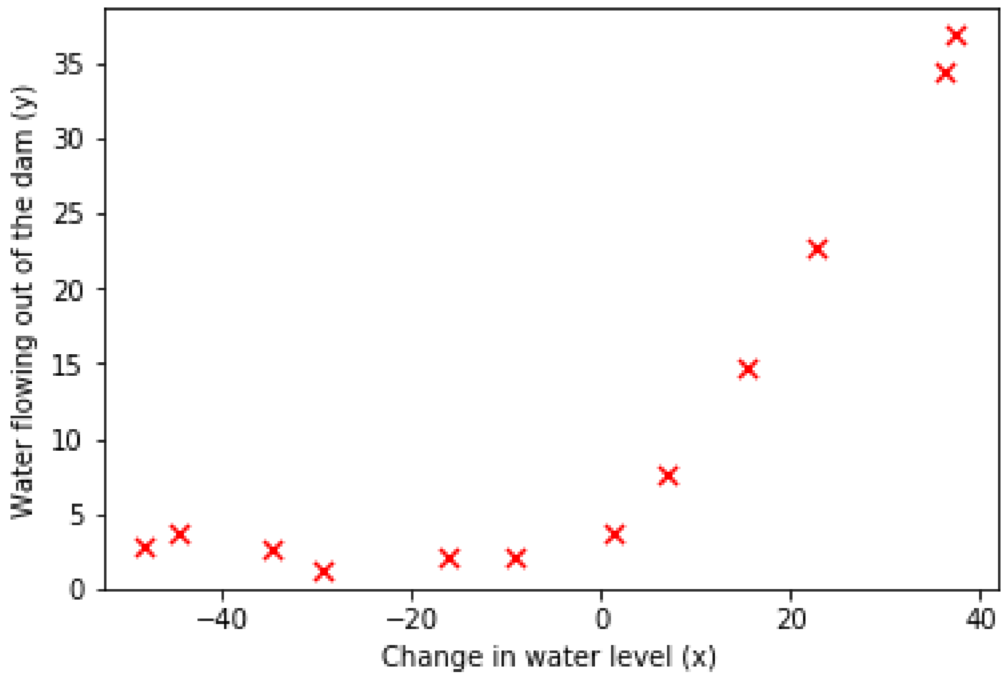
Importando os dados do arquivo ex5data1.txt

```
In [26]: from scipy.io import loadmat

         matriz = loadmat("ex5data1.mat")
         #data.keys()
         y_train = matriz['y']
         X_train = np.c_[np.ones_like(matriz['X']), matriz['X']]

         yval = matriz['yval']
         Xval = np.c_[np.ones_like(matriz['Xval']), matriz['Xval']]


         print('X_train:', X_train.shape)
         print('y_train:', y_train.shape)
         print('Xval:', Xval.shape)
         print('yval:', yval.shape)
```

```
X_train: (12, 2)
y_train: (12, 1)
Xval: (21, 2)
yval: (21, 1)
```

5.1 Visualização dos Dados
Gráfico dos registros históricos na mudançaa no nível da água, x, e da quantidade de água que sai da barragem, y.

```
In [27]: plt.scatter(X_train[:,1], y_train, s=50, c='r', marker='x', linewidths=1)
         plt.xlabel('Change in water level (x)')
         plt.ylabel('Water flowing out of the dam (y)')
         plt.ylim(ymin=0);
```



5.2 Função de custo da regressão linear regularizada

Utilizada a função linearRegCostFunction, passando theta inicial começando com 1,1 , dados de treinamento e lambda =0 como parâmetro

```
In [28]: %run -i "linearRegCostFunction.py"


         initial_theta = np.ones((X_train.shape[1],1))
         lmd=0
         custo = regCostFunction(initial_theta,X_train, y_train, lmd)

         print("Com theta inicializado com (1; 1), a função de custo retorna ",custo)
```

```
Com theta inicializado com (1; 1), a função de custo retorna  303.951525554


<matplotlib.figure.Figure at 0x26247326940>
```

5.3 Gradiente na regressão linear regularizada

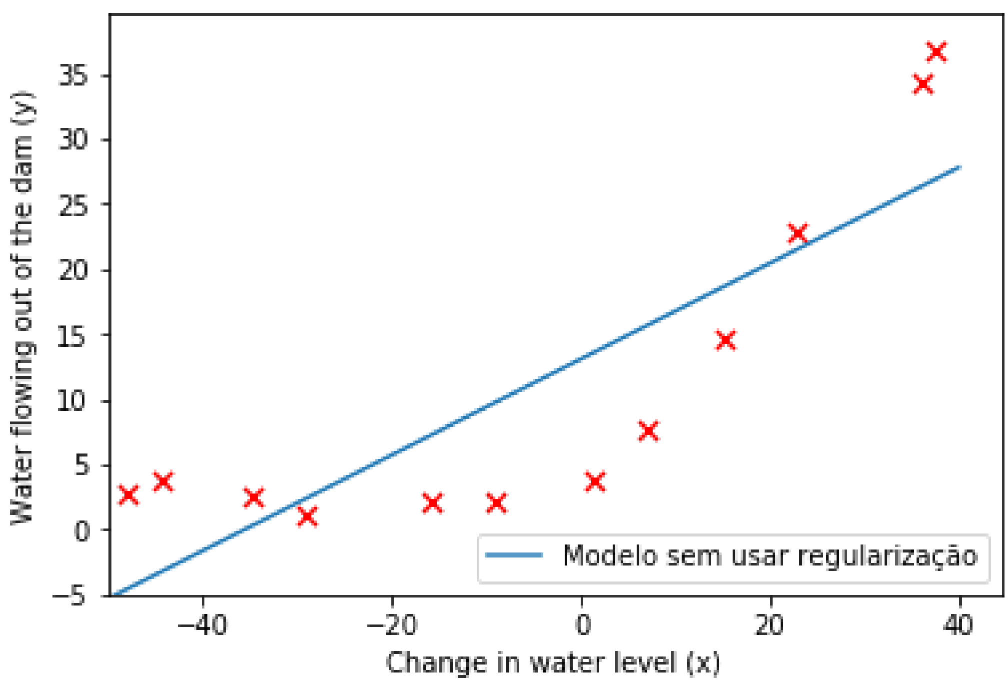
Adicionado código para calcular o gradiente e testando a corretude usando theta inicializado em (1; 1)

```
In [29]: %run -i "linearRegCostFunction.py"
         lmd =0
         gradient = gradientReg(initial_theta, X_train, y_train, lmd)
         print("Com theta inicializado com (1; 1), o gradiente é ", gradient)
```

```
Com theta inicializado com (1; 1), o gradiente é  [ -15.30301567  598.16741084]
```

5.4 Ajustando os parâmetros da regressão linear
Criado a função linearRegCostFunction para computar os valores otimos para theta, mas sem usar regularizacão com lambda = 0.

```
In [30]: %run -i "linearRegCostFunction.py"

         lmd=0

         ajuste = linearRegCostFunction(X_train, y_train, lmd)

         plt.plot(np.linspace(-50,40), (ajuste.x[0]+ (ajuste.x[1]*np.linspace(-50,40))), label='
         plt.scatter(X_train[:,1], y_train, s=50, c='r', marker='x', linewidths=1)
         plt.xlabel('Change in water level (x)')
         plt.ylabel('Water flowing out of the dam (y)')
         plt.ylim(ymin=-5)
         plt.xlim(xmin=-50)
         plt.legend(loc=4);
```



7 6 Vies-Variância

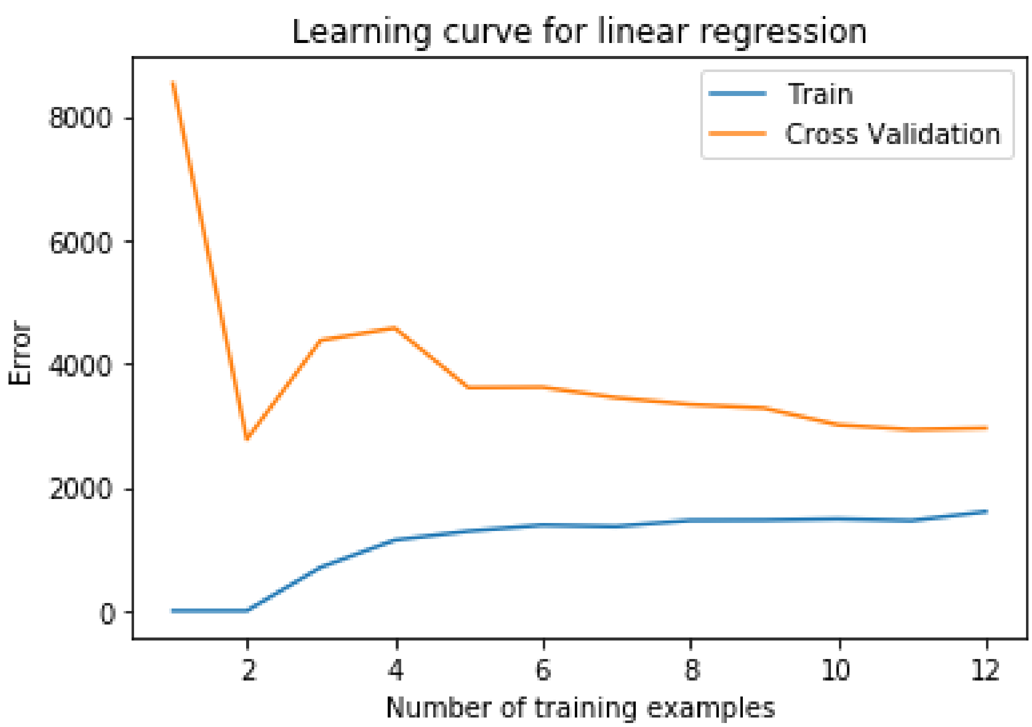
6.1 Curvas de Aprendizado

Utilizando a função Para learningCurve para traçaar a curva de aprendizado,e definindo um conjunto de treinamento e validação cruzada erro para diferentes tamanhos de conjuntos de treinamento. É passado lambda =0 como parâmetro. Retorna dois vetores com os respectivos erros

```
In [31]: %run -i "learningCurve.py"
         lmd = 0
         t_error, v_error = learningCurve(X_train, y_train, Xval, yval,lmd)
```

```
<matplotlib.figure.Figure at 0x262265d3c50>
```

```
In [32]: plt.plot(np.arange(1,13), t_error, label='Train')
         plt.plot(np.arange(1,13), v_error, label='Cross Validation')
         plt.title('Learning curve for linear regression')
         plt.xlabel('Number of training examples')
         plt.ylabel('Error')
         plt.legend();
```



8 7 Regressão Polinomial

implementação de código em um arquivo de nome poly_features.py

```
In [33]: %run -i "poly_features.py"
```

<matplotlib.figure.Figure at 0x2623f4f81d0>

7.1 Regressao Polinomial - aprendizado

```
In [46]: %run -i "poly_features.py"
         %run -i "normalizarCaracteristica.py"


         p = 8
         m = X.shape[0]
         X_test = matriz['Xtest']
         X_val = matriz['Xval']
         y = matriz['y'].ravel()
         y_test = matriz['ytest'].ravel()
         y_val = matriz['yval'].ravel()
         m_test = X_test.shape[0]
         m_val = X_val.shape[0]


         X_poly = polyFeatures(X, p)
         X_poly, mu, sigma = normalizarCaracteristica(X_poly)
         X_poly = np.hstack((np.ones((m, 1)), X_poly))

         X_poly_test = polyFeatures(X_test, p)
         X_poly_test, dummy_mu, dummy_sigma = normalizarCaracteristica(X_poly_test, mu, sigma)
         X_poly_test = np.hstack((np.ones((m_test, 1)), X_poly_test))

         X_poly_val = polyFeatures(X_val, p)
         X_poly_val, dummy_mu, dummy_sigma = normalizarCaracteristica(X_poly_val, mu, sigma)
         X_poly_val = np.hstack((np.ones((m_val, 1)), X_poly_val))

         print ('Exemplo de Treino normalizado 1:')
         print (X_poly[0, :])
```

```
Exemplo de Treino normalizado 1:
[ 1.         -0.00710129  0.99791805 -0.98977681  0.66359293 -0.21829255
  0.61587084 -0.58801313  0.2439574  -0.24294194  0.17403955 -0.40598636
 -0.02886215 -0.23180187 -0.06697    -0.30185289 -0.15515712]
```