TD01

1) Rappels expo. et log.

$a^{-n} = 1/a^n \quad a^n \times a^m = a^{n+m} \quad a^n / a^m = a^{n-m}$

$\log_a (xy) = \log_a x + \log_a y$

$\log_a (x/y) = \log_a x - \log_a y$

$\log_a x^n = n \log_a x$

$\log_a (e_a^x) = x \quad | \log_a a^x = x$

$e_a (\log_a x) = x \quad a^{\log_a x} = x$

1) $a^3 \times a^4 = a^7 \quad (a^3)^4 = a^{12} \quad a^2 \times \frac{1}{a^3} = a^{-1}$

$6^6 \times \left(\frac{1}{3}\right)^6 = 2^6 = 64$

2) $\log_a a^{10} = 10 \log_a a = 10$

$\log_2 64 = \log_2 2^6 = 6 \log_2 2 = 6$

$\log_2 a + \log_4 a$:

On note : $X = \log_2 a$ et $Y = \log_4 a$

On a : $2^X = a$ et $4^Y = a$

$\rightarrow 2^X = 4^Y = (2^2)^Y$

$\rightarrow X = 2Y \quad \rightarrow \log_2 (a^{3/2}) = 3/2 \log_2 a$

$a^{\log_a 11} = 11$

2) Conversions de base

1. $(101010)_2 = 0 \times 2^0 + 1 \times 2^1 + 0 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + 1 \times 2^5$

$= 2 + 8 + 32 = (42)_{10}$

$(1021)_3 = 1\times3^0 + 2\times3^1 + 0\times3^2 + 1\times3^3$

$= 1+6+27 = (34)_{10}$

$(483)_{10} = 2\times241+1 \longrightarrow (111100011)_2$

$\longrightarrow 241 = 2\times120+1$

$\longrightarrow 120 = 2\times60+0$

$\longrightarrow 60 = 2\times30+0$

$\longrightarrow 30 = 2\times15+0$

$\longrightarrow 15 = 2\times7+1$

$\longrightarrow 7 = 2\times3+1$

$\longrightarrow 3 = 2\times1+1$

$\longrightarrow 1 = 2\times0+1$

483 |3 $\longrightarrow (122220)_3$

0   161 |3

2   53 |3

2   17 |3

2   5 |3

2   1 |3

1   0

2^8 2^4 2^0

$(1|1110|0011)_2 = 0001\ 0000\ 0000$ car $16=2^4$

$0000\ 1110\ 0000$

$0000\ 0000\ 0011$

$= (0001)\times2^8 = (0001)\times16^2 = 1\times16^2$

$(1110)\times2^4 = (1110)\times16^1 = E\times16^1$

$(0011) = (0011)\times16^0 = 3\times16^0$

Donc, $1\times16^2+E\times16^1+3\times16^0 = (1E3)_{16}$

$(A341B)_{16}$ A   3   1   B

$(1010)_2\ (0011)_2\ (0001)_2\ (1011)_2$

$= (1010\ 0011\ 0001\ 1011)_2$

$(10212)_3 \Rightarrow 01\ 02\ 12$ $(12)_3 = 5$

1   2   5 $(02)_3 = 2$

$\boxed{\text{car } 9 = 3^2} \rightarrow (125)_9$ $(01)_3 = 1$

2. $\rightarrow (102013)_4 + (1)_4 \rightarrow$

$$\begin{array}{r} 102013 \\ + \quad 1 \\ \hline (102020)_4 \end{array}$$

$(4)_{10} = (10)_4$

$$\rightarrow \begin{array}{r} 102013 \\ \times \quad 10 \\ \hline 0 \\ +1020130 \\ \hline (1020130)_4 \end{array}$$

$\overset{a}{(102013)} \div (4)_{12} = \overset{b}{(10)_4}$

car $a = qb + r$ car $q + r = a \div b$

$\rightarrow q = 10201$ et $r = 3$

$\rightarrow (10201)_4 \times (10)_4 + (3)_4$

$= (102010)_4 + (3)_4$

$= (102013)_4$

$\rightarrow (3)_4$

$(101011)_2 + (2A)_{16}$ et $(101011)_2 = (2B)_{16}$

$\rightarrow (55)_{16}$

2   A

$(0010)_2 (1010)_2 \rightarrow$

$$\begin{array}{r} 101011 \\ +101010 \\ \hline 1010101 \end{array}$$

$(10110)_2 + (111)_2 = (11101)_2$

$(10110)_2 \times (101)_2$

$$
\begin{array}{r}
\rightarrow \quad 10110 \\
\times \quad 101 \\
\hline
10110 \\
+ \quad 0 \\
+1011000 \\
\hline
(1101110)_2
\end{array}
$$

$((100)_2)^4 = (10\ 000)_2^2 = (100000000)_2$

3) Propriétés

3, soit $(11)_2$ et n chiffres $\rightarrow 2^n - 1$

On suppose que m s'écrit avec n chiffres binaires

$m = (a_{n-1}, a_{n-2}, ---, a_1, a_0)_2$

On cherche n : $m = a_{n-1} \times 2^{n-1} + a_{n-2} \times 2^{n-2} + \ldots + a_1 \times 2 + a_0$

suppose que $a_{n-1} = 1$ sinon écrit avec - de bits

On a : $m = 2^{n-1} + a_{n-2} \times 2^{n-2} + \ldots + a_0$

$\rightarrow m \geqslant 2^{n-1} \rightarrow \log_2(m) \geqslant \log_2(2^{n-1}) = n-1$

$\rightarrow n \leqslant \log_2(m) + 1$

A.N :
$n = 6$
et $n = 2$

On sait que $m < 2^n \rightarrow \log_2(m) < \log_2(2^n)$

$\rightarrow \log_2(m) < n$

Au final, $\log_2(m) < n \leqslant \log_2(m) + 1$ donc

$n = \log_2(m) + 1$