Chapter 3 Digital Logic Structures

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How do we represent data in a computer?

At the lowest level, a computer has electronic "plumbing"

· Operates by controlling the flow of electrons

Easy to recognize two conditions:

- 1. Presence of a voltage we'll call this state "1"
- 2. Absence of a voltage we'll call this state "0"



Computer use transistors as switches to manipulate bits

- Before transistors: tubes, electro-mechanical relays (pre 1950s)
- · Mechanical adders (punch cards, gears) as far back as mid-1600s

Before describing transistors, we present an analogy...

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Transistor: Building Block of Computers

Microprocessors contain millions of transistors

- Intel Pentium 4 (2000): 48 million
- IBM PowerPC 750FX (2002): 38 million
- IBM/Apple PowerPC G5 (2003): 58 million

Logically, each transistor acts as a switch Combined to implement logic functions

· AND, OR, NOT

Combined to build higher-level structures

· Adder, multiplexer, decoder, register, ...

Combined to build processor

• LC-3

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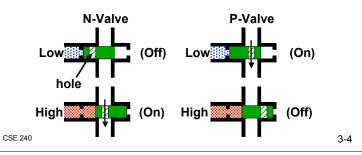
A Transistor Analogy: Computing with Air

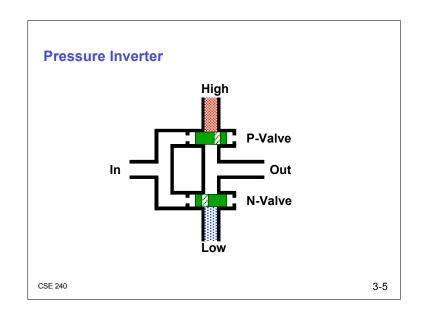
Use air pressure to encode values

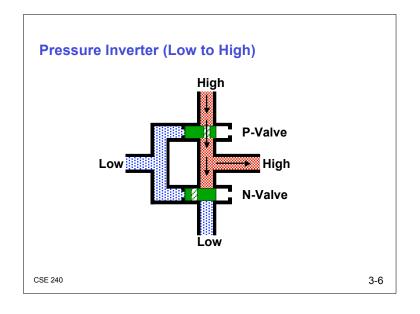
- High pressure represents a "1" (blow)
- · Low pressure represents a "0" (suck)

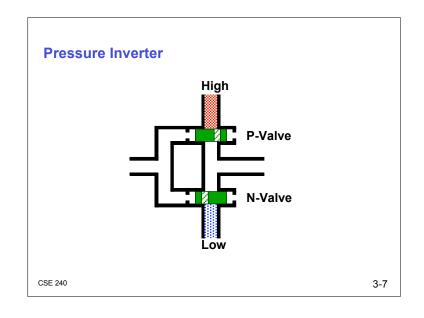
Valve can allow or disallow the flow of air

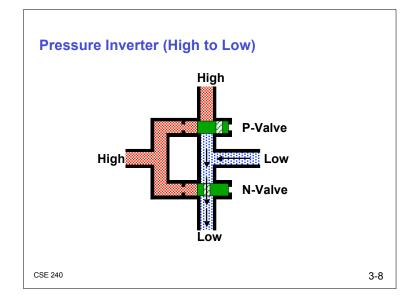
· Two types of valves











Analogy Explained

Pressure differential → electrical potential (voltage)

- Air molecules → electrons
- High pressure → high voltage
- Low pressure → low voltage

Air flow → electrical current

- Pipes → wires
- · Air only flows from high to low pressure
- · Electrons only flow from high to low voltage
- · Flow only occurs when changing from 1 to 0 or 0 to 1

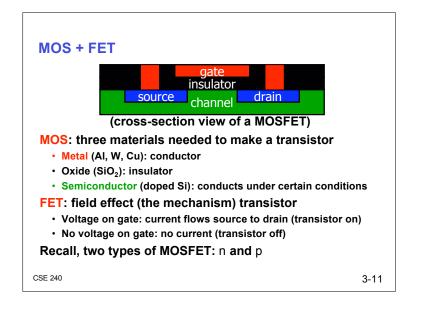
Valve → transistor

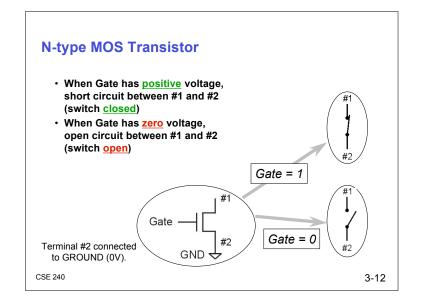
• The transistor: one of the century's most important inventions

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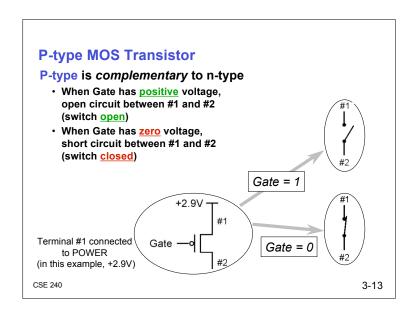
Transistors as Switches Two types N-type Properties Solid state (no moving parts) Reliable (low failure rate) Small (90nm channel length) Fast (<0.1ns switch latency)

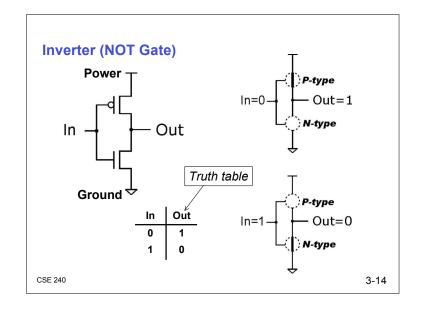
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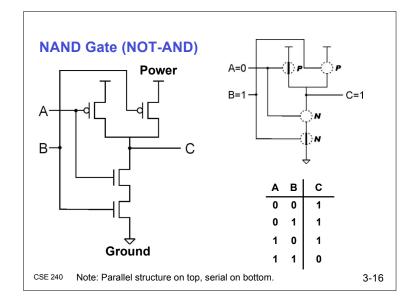
CMOS Circuit

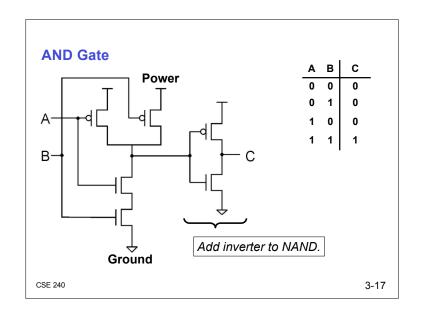
Inverter is an example of Complementary MOS (CMOS)

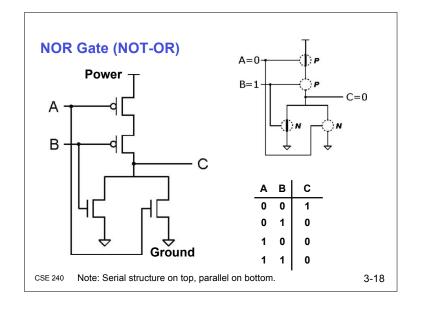
Uses both n-type and p-type MOS transistors

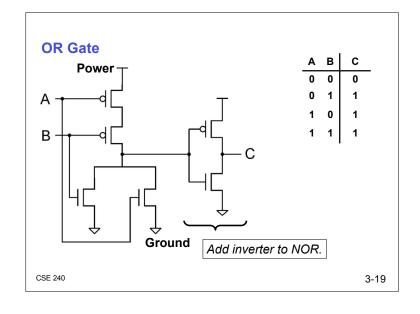
- p-type
 - > Attached to POWER (high voltage)
 - > Pulls output voltage UP when input is zero
- n-type
 - > Attached to GROUND (low voltage)
 - > Pulls output voltage DOWN when input is one

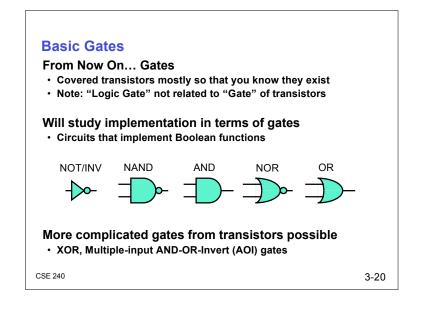
For all inputs, make sure that output is either connected to GROUND or to POWER, but not both! (why?)











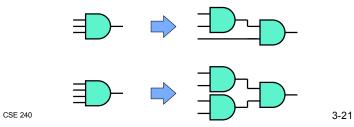
More than 2 Inputs?

AND/OR can take any number of inputs

- AND = 1 if all inputs are 1
- OR = 1 if any input is 1
- · Similar for NAND/NOR

Implementation

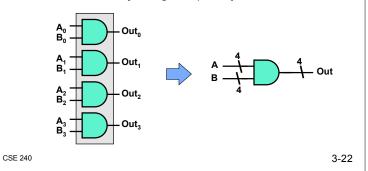
Multiple two-input gates or single CMOS circuit

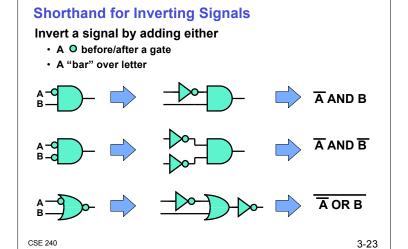


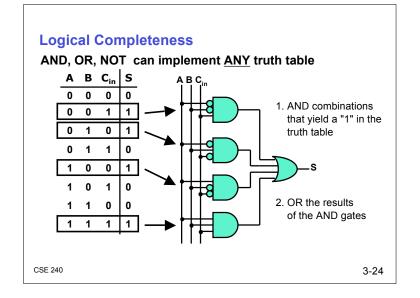
Visual Shorthand for Multi-bit Gates

Use a cross-hatch mark to group wires

- Example: calculate the AND of a pair of 4-bit numbers
- A₃ is "high-order" or "most-significant" bit
- If "A" is 1000, then A₃ = 1, A₂ = 0, A₄ = 0, A₀ = 0







Logical Completeness via PLAs

Any truth table as a Programmable Logic Array (PLA)

- · Traditionally a grid of AND and OR gates
- · Configurable by removing wires

Single-output custom PLA (as on previous slide):

- One AND gate per row with "1" in output in truth table
- Maximum number of AND gates: 2ⁿ for n inputs
- One OR gate

Multiple-output custom PLA:

- Build multiple single-output PLAs
- · Share AND gates "in common"
- · One OR gate per output column in truth table

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Summary

MOS transistors are used as switches to implement logic functions

- n-type: connect to GROUND, turn on (with 1) to pull down to 0
- p-type: connect to POWER, turn on (with 0) to pull up to 1

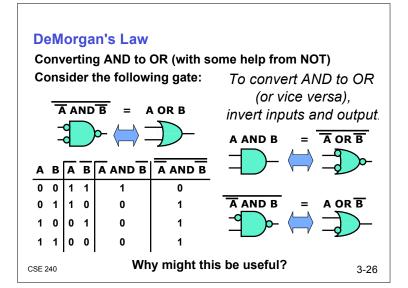
Basic gates: NOT, NOR, NAND

- Logic functions are usually expressed with AND, OR, and NOT
- Universal: any truth table to simple gates (via a PLA)

DeMorgan's Law

· Convert AND to OR (and vice versa) by inverting inputs/output

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Next Time

Lecture

· Combinational Logic Circuits

Reading

Chapter 3.3

Quiz

· Online (as always)

Upcoming

· HW2 due next Friday

Chapter 3 Digital Logic Structures

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AND, OR, NOT Gates: What Good Are They?

Last time:

- · Transistors and gates
- Can implement any logical function using gates (using PLAs)

Today:

- We'll use gates to create some building blocks of a processor
- One goal: automate binary arithmetic from Chapter 2
- · Continuing on our bottom-up journey

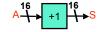
Next time:

- Storing bits (memory)
- · Circuits with "state"

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Incrementer

Let's create a incrementer



- Input: A (as a 16-bit 2's complement integer)
- Output: A+1 (also as a 16-bit 2's complement integer)

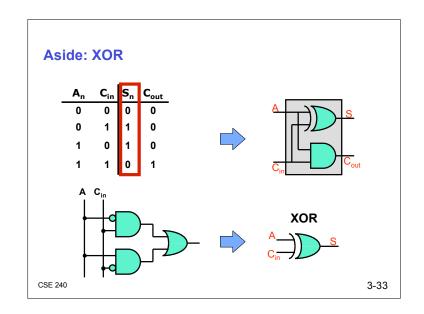
Approach #1 (impractical):

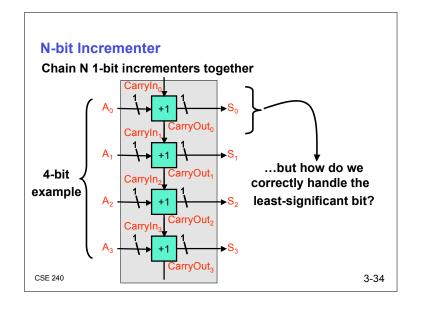
- · Use PLA-like techniques to implement circuit
- Problem: 216 or 65536 rows, 16 output columns
- · In theory, possible; in practice, intractable

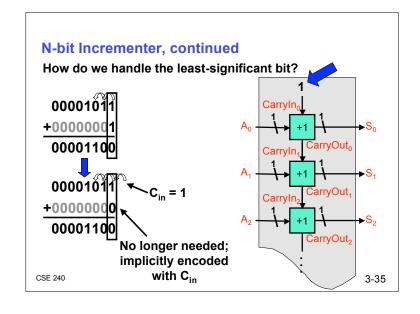
Approach#2 (pragmatic):

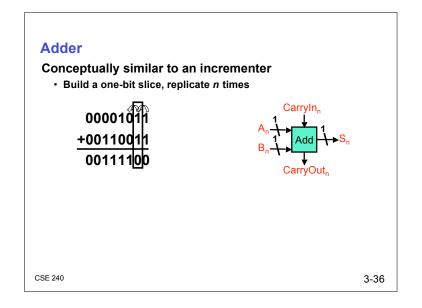
- · Create a 1-bit incrementer circuit
- · Replicate it 16 times

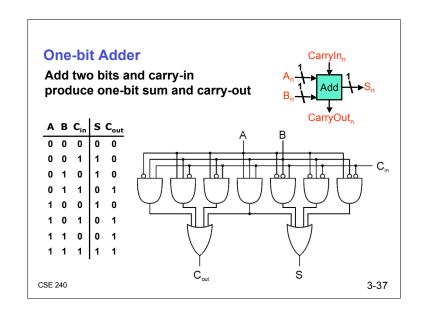
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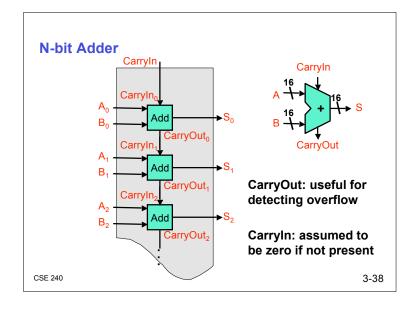












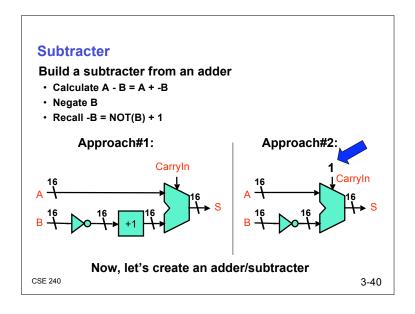
Aside: Efficient Adders

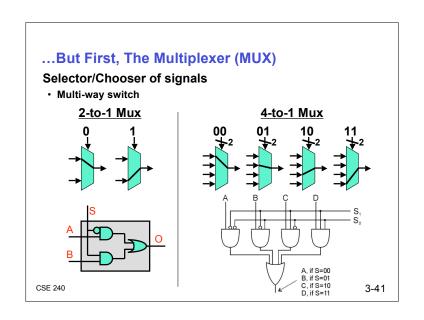
Full disclosure:

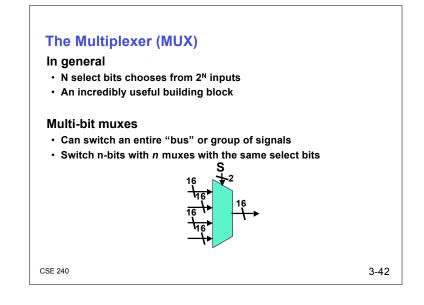
- · Our adder: Ripple-carry adder
- · No one (sane) actually uses ripple-carry adders
- · Why? way too slow
- Latency proportional to n

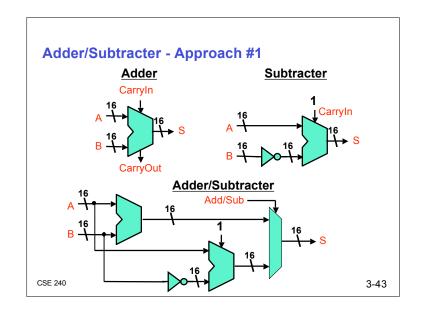
We can do better

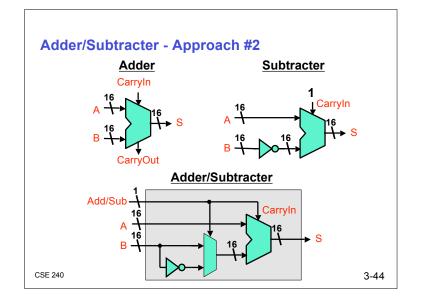
- Many ways to create adders with latency proportional to $log_2(n)$
- In theory: constant latency (build a big PLA)
- In practice: too much hardware, too many high-degree gates
- · "Constant factor" matters, too
- · More on this topic in CSE371











Ok, So We Can Add and Subtract

Other arithmetic operations similar

· Even floating point operations

We can calculate; but we can't remember

- · Next time: storage and memory
- · After that: simple "state machines"
- · After that: a simple processor

Remember: Readings, Quizzes, and Homework

- · We'll return Homework 1 on Wednesday
- Homework 2 due Friday

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Chapter 3 Digital Logic Structures

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Combinational vs. Sequential Logic

Combinational Circuit

- · Always gives the same output for a given set of inputs
 - > For example, adder always generates sum and carry, regardless of previous inputs

Sequential Circuit

- Stores information
- · Output depends on stored information (state) plus input
 - > Given input might produce different outputs, depending on stored information
- · Example: ticket counter
 - > Advances when you push the button
 - > Output depends on previous state
- · Useful for building "memory" elements and "state machines"

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Storage - Cross-Coupled Inverters

Cross-coupled inverters (INV) gates

• Holds value Q and Q' (Q' is the same as Q)

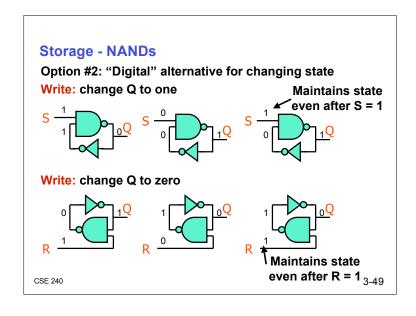


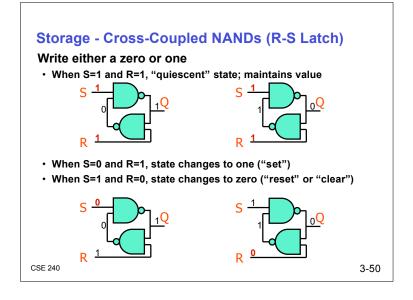


Read: get value from either Q or Q'

Maintains its "state", but how do we change the state?

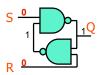
Write: Option #1: put opposite values on Q and Q' simultaneously
 Requires "analog" overdriving of Q and Q'





Storage - Cross-Coupled NANDs (R-S Latch) What happens with S=0 and R=0?

- Short answer: bad things
- · Long answer: value stored will depend on timing on circuit



- · Does S or R go to one first?
 - > If they change at the same time?
 - ➤ Oscillation or meta-stability can result
- · Let's make sure this can never happen...

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Gated D-Latch

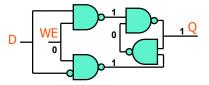
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Add logic to an R-S latch

· Create a better interface

Two inputs: D (data) and WE (write enable)

- When WE = 1, latch is set to value of D
 S = NOT(D), R = D
- When WE = 0, latch continues to hold previous value
 S = R = 1
- · Does not allow S=0, R=0 case to occur

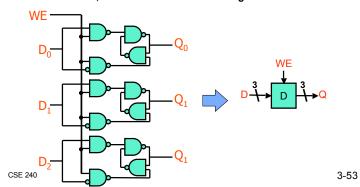


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Register

A register stores a multi-bit value

- A collection of D-latches, controlled by a common WE
- · When WE=1, n-bit value D is written to register

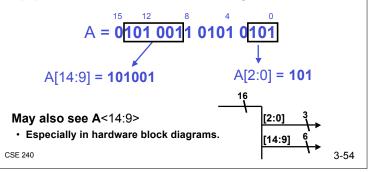


Aside: More on Representing Multi-bit Values

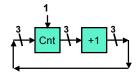
Number bits from right (0) to left (n-1)

• Just a convention -- could be left to right, but must be *consistent*Use brackets to denote range:

D[l:r] denotes bit I to bit r, from left to right



Let's Try to Build a Counter



How quickly will this count?

Timing dependent

Will it even work?

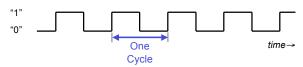
- · Probably not
- · D-latches are "transparent"
 - > Allows next input to immediately flow to output
 - > Outputs will never be "stable"

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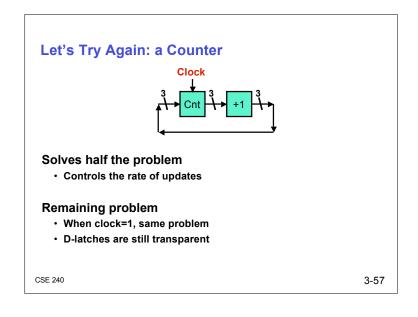
What's Missing? The Clock

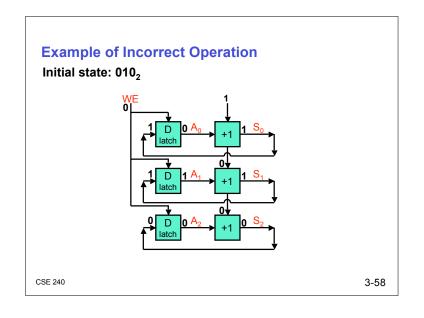
A clock controls when registers are "updated"

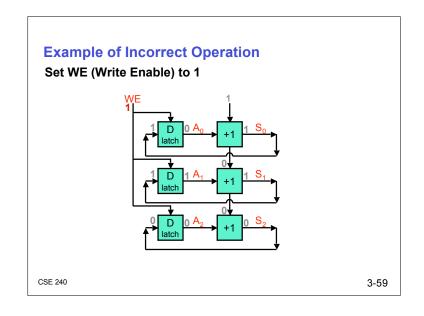
- · Oscillating global signal with fixed period
- · Typical clock frequencies today: a couple of gigahertz

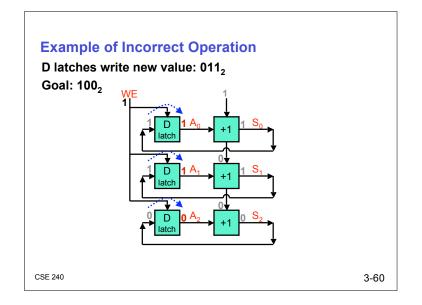


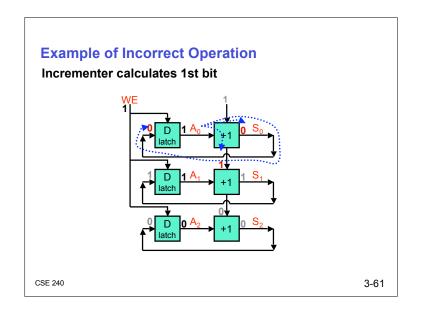
- Corresponds to <1 nanosecond between one rise and the next
- Generated on-chip by special circuitry (for example, oscillating ring of inverters)

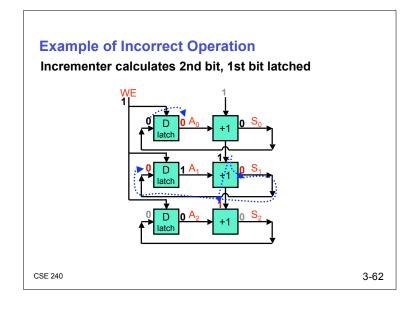


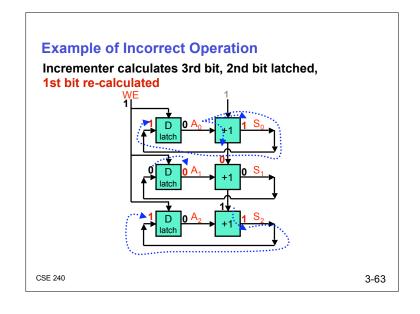


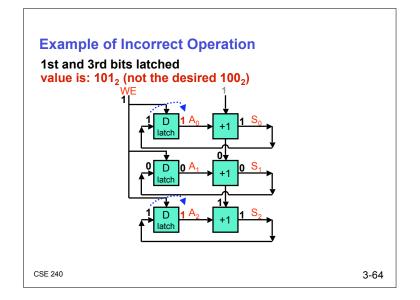


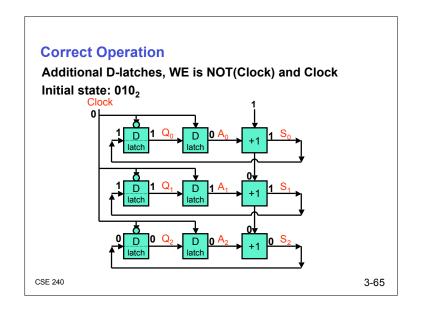


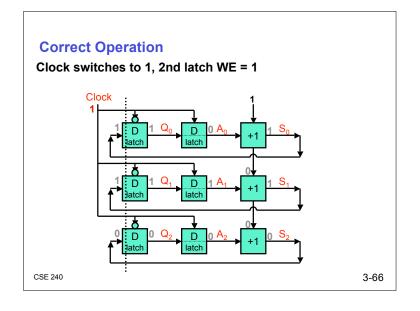


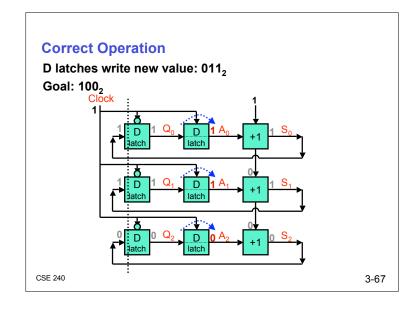


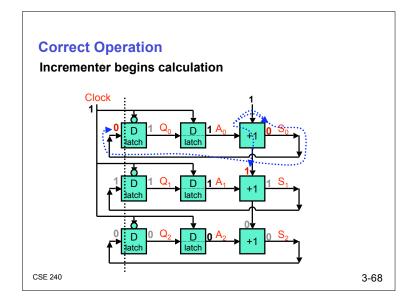


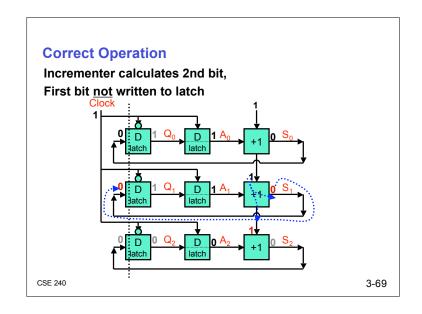


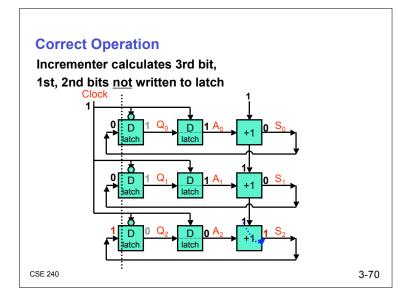


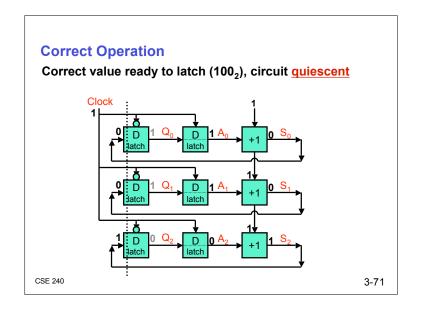


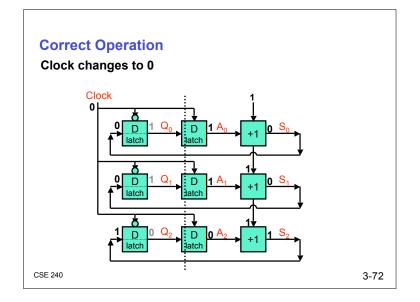


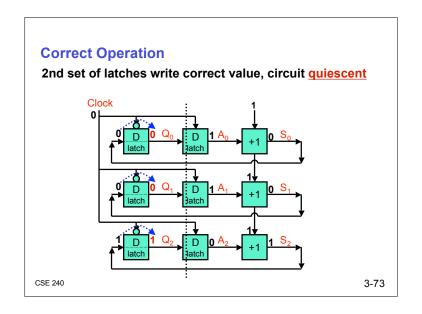


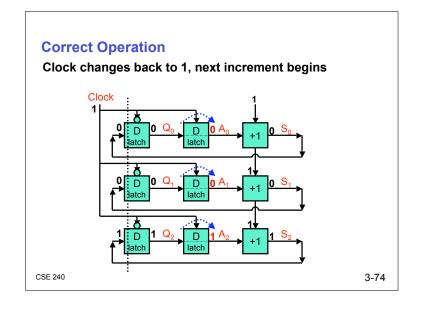


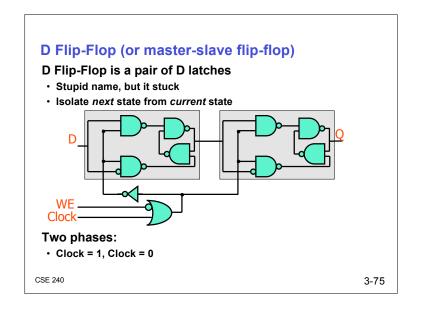


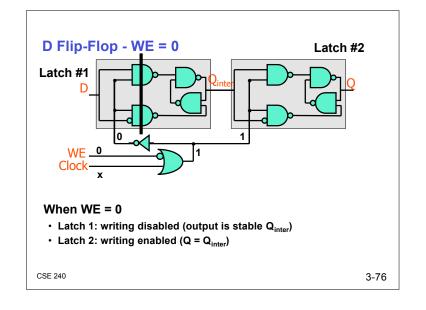


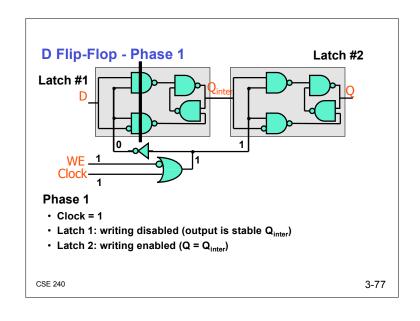


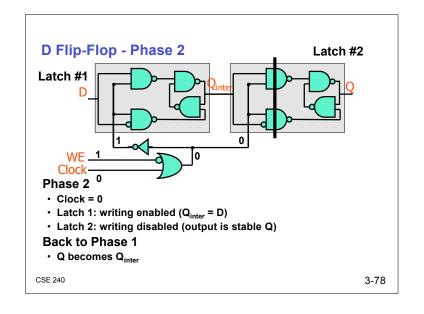


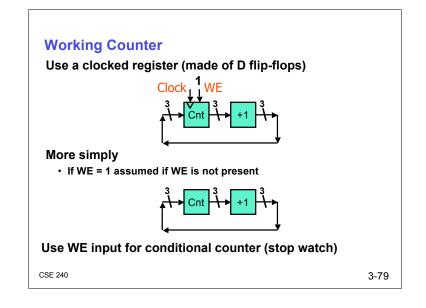


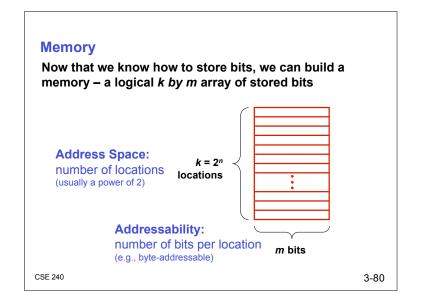


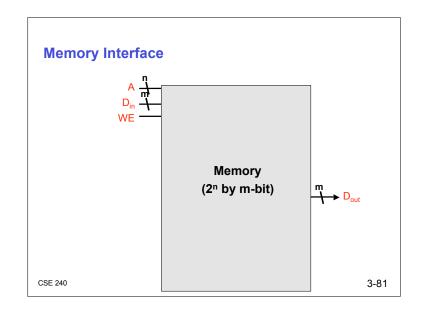


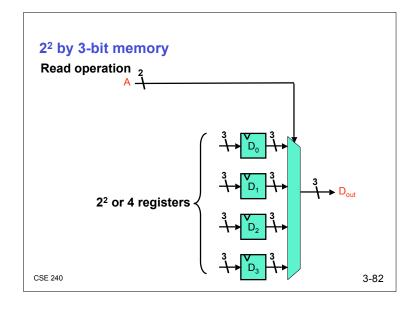


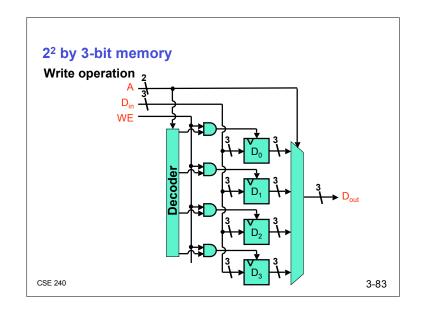


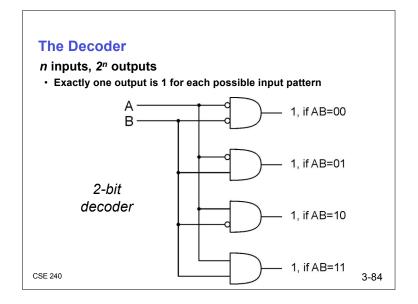


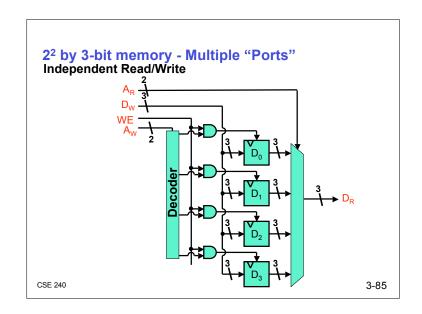


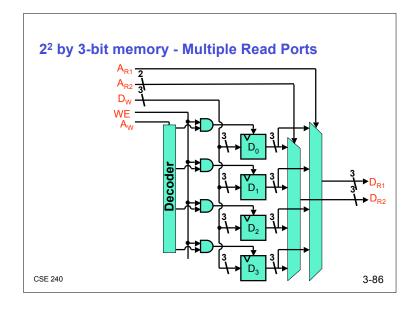


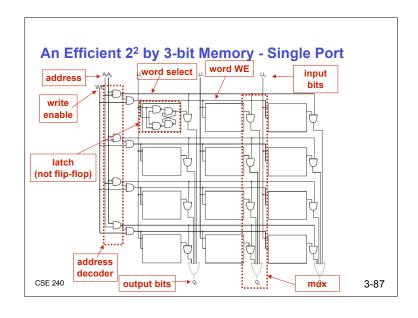












More Memory Details

This is still not the way actual memory is implemented

Real mem: Fewer transistors, much more dense, relies on analog properties

But the logical structure is similar

- · Address decoder
- · Word select line, word write enable
- Bit line

Two basic kinds of RAM (Random Access Memory)

Static RAM (SRAM)

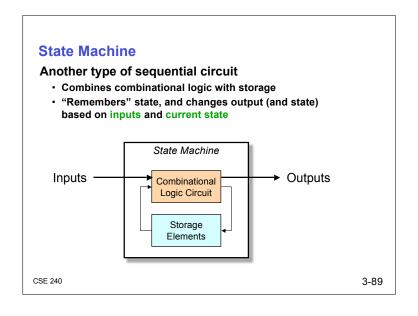
· Fast, maintains data as long as power applied

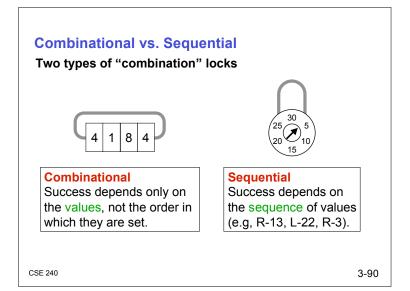
Dynamic RAM (DRAM)

 Slower but denser, bit storage decays – must be periodically refreshed

CSE 240 Also, non-volatile memories: ROM, PROM, flash, ...

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State

The state of system is snapshot of all relevant elements of system at moment snapshot is taken

Examples

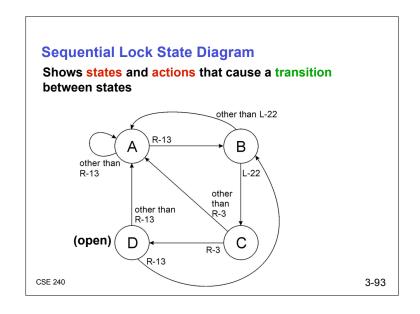
- The state of a basketball game can be represented by the scoreboard
 - > Number of points, time remaining, possession, etc.
- The state of a tic-tac-toe game can be represented by the placement of X's and O's on the board (and turn)

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State of Sequential Lock

Our lock example has four different states, labeled A-D:

- A: The lock is not open, and no relevant operations have been performed
- B:The lock is not open, and the user has completed the R-13 operation
- C:The lock is not open, and the user has completed R-13, followed by L-22
- D:The lock is open



Implementing a Finite State Machine Combinational logic • Determine outputs and next state. Storage elements • Maintain state representation. State Machine Combinational Logic Circuit Clock Clock Storage Elements 3-95

Finite State Machine

A description of a system with the following components:

- 1. A finite number of states
- 2. A finite number of external inputs
- 3. A finite number of external outputs
- 4. An explicit specification of all state transitions
- An explicit specification of what determines each external output value

Often described by a state diagram

- · Inputs trigger state transitions
- · Outputs are associated with each state (or with each transition)

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Storage

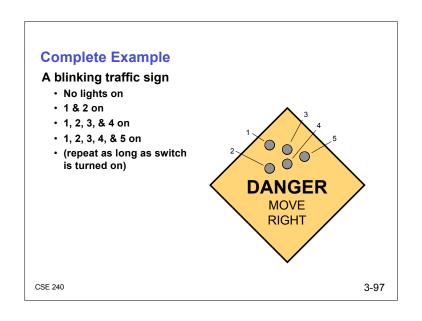
Master-slave flip-flop stores one state bit

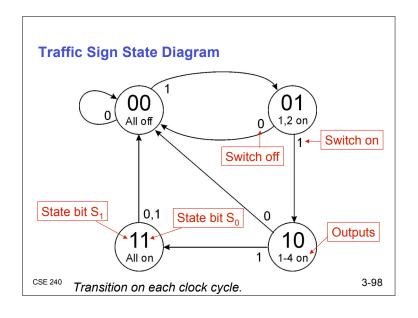
Number of storage elements (flip-flops)

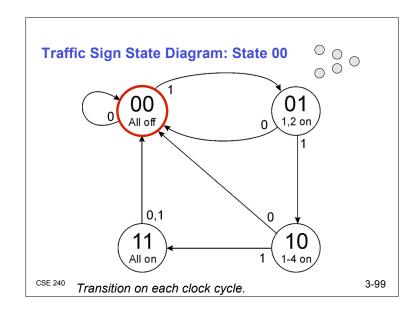
· Determined by number of states (and representation of states)

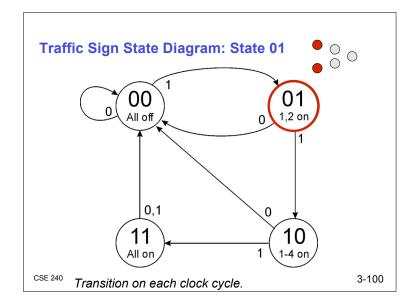
Examples

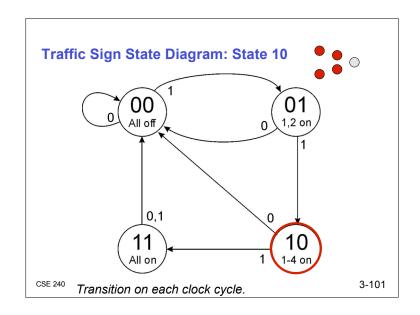
- · Sequential lock
 - > Four states two bits
- · Basketball scoreboard
 - >8 bits for each score, 5 bits for minutes, 6 bits for seconds, 1 bit for possession arrow, 1 bit for half, ...

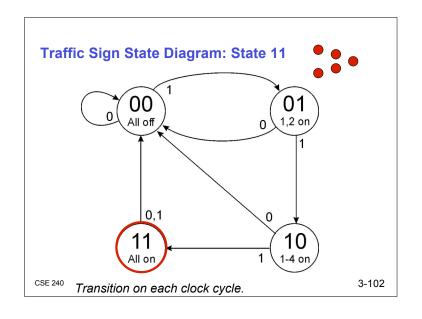


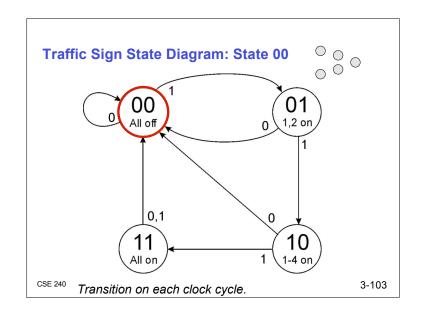


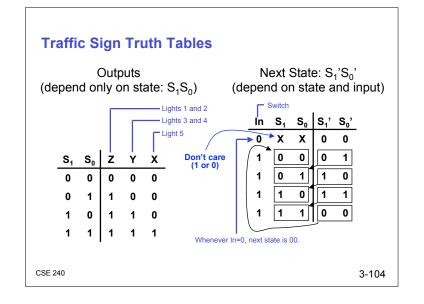


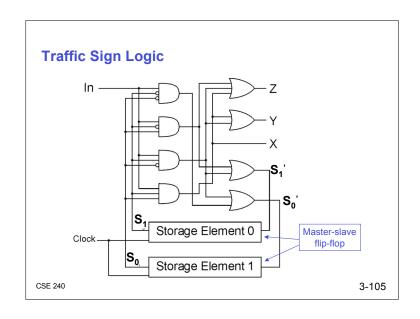


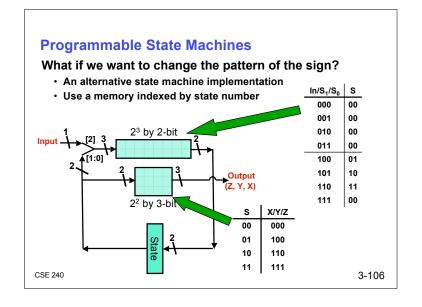


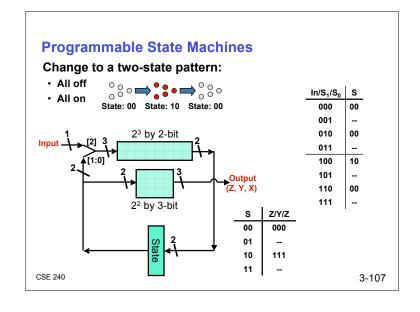












From Logic to Data Path

The data path of a computer is all the logic used to process information.

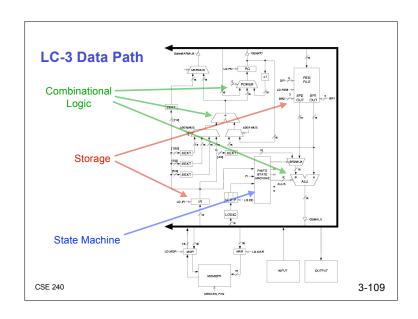
· See the data path of the LC-3 on next slide

Combinational Logic

- · Decoders -- convert instructions into control signals
- · Multiplexers -- select inputs and outputs
- · ALU (Arithmetic and Logic Unit) -- operations on data

Sequential Logic

- State machine -- coordinate control signals and data movement
- · Registers and latches -- storage elements



Next Time

Topic

· The von Neumann Model

Readings

• Chapter 4.0 - 4.2

Online quiz

· You know the drill!

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Looking Forward...

We've touched upon basic digital logic

- Transistors
- Gates
- · Storage (latches, flip-flops, memory)
- State machines

Build some simple circuits

- Incrementer, adder, subtracter, adder/subtracter
- Counter (consisting of register and incrementer)
- · Hard-coded traffic sign state machine
- Programmable traffic sign state machine

Up next: a computer as a (simple?) state machine