Proof: Let $A = \{x : p(x) > 0\}$ be the support set of $p(x)$. Then

$$-D(p||q) = -\sum_{x \in A} p(x) \log \frac{p(x)}{q(x)} \tag{2.83}$$

$$= \sum_{x \in A} p(x) \log \frac{q(x)}{p(x)} \tag{2.84}$$

$$\leq \log \sum_{x \in A} p(x) \frac{q(x)}{p(x)} \tag{2.85}$$

$$= \log \sum_{x \in A} q(x) \tag{2.86}$$

$$\leq \log \sum_{x \in \mathcal{X}} q(x) \tag{2.87}$$

$$= \log 1 \tag{2.88}$$

$$= 0, \tag{2.89}$$

where (2.85) follows from Jensen's inequality. Since $\log t$ is a strictly concave function of t, we have equality in (2.85) if and only if $q(x)/p(x)$ is constant everywhere [i.e., $q(x) = cp(x)$ for all x]. Thus, $\sum_{x \in A} q(x) = c \sum_{x \in A} p(x) = c$. We have equality in (2.87) only if $\sum_{x \in A} q(x) = \sum_{x \in \mathcal{X}} q(x) = 1$, which implies that $c = 1$. Hence, we have $D(p||q) = 0$ if and only if $p(x) = q(x)$ for all x. $\square$