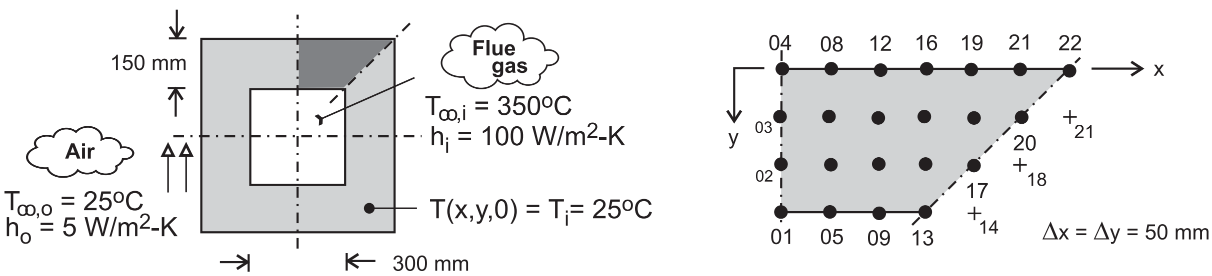
PROBLEM 5.125

KNOWN: Flue of square cross-section, initially at a uniform temperature is suddenly exposed to hot flue gases. See Problem 4.55.

FIND: Temperature distribution in the wall 5, 10, 50 and 100 hours after introduction of gases using the *implicit* finite-difference method.

SCHEMATIC:



ASSUMPTIONS: (1) Two-dimensional transient conduction, (2) Constant properties.

PROPERTIES: Flue (given): $k = 0.85$ W/m·K, $\alpha = 5.5 \times 10^{-7}$ m²/s.

ANALYSIS: The network representing the flue cross-sectional area is shown with Δx = Δy = 50mm. Initially all nodes are at T_i = 25°C when suddenly the interior and exterior surfaces are exposed to convection processes, ($T_{\infty,i}$, h_i) and ($T_{\infty,o}$, h_o), respectively Referring to the network above, note that there are four types of nodes: interior (02, 03, 06, 07, 10, 11, 14, 15, 17, 18, 20); plane surfaces with convection (interior – 01, 05, 09); interior corner with convection (13), plane surfaces with convection (exterior – 04, 08, 12, 16, 19, 21); and, exterior corner with convection. The system of finite-difference equations representing the network is obtained using *IHT*|*Tools*|*Finite-difference equations*|*Two-dimensional*|*Transient*. The *IHT* code is shown in Comment 2 and the results for t = 5, 10, 50 and 100 hour are tabulated below.

Node 17 $$(1+4\mathrm{Fo})T_{17}^{p+1} - \mathrm{Fo}\left(T_{18}^{p+1} + T_{14}^{p+1} + T_{18}^{p+1} + T_{14}^{p+1}\right) = T_{17}^{p}$$

Node 13 $$\left[1+4\mathrm{Fo}\left[1+\frac{1}{3}\mathrm{Bi_i}\right]\right]T_{13}^{p+1} - \frac{2}{3}\mathrm{Fo}\left(2T_{14}^{p+1} + T_9 + 2T_{14}^{p+1} + T_9^{p+1}\right) = T_{13}^{p} + \frac{4}{3}\mathrm{Bi_i}\cdot\mathrm{Fo}\cdot T_{\infty,i}$$

Node 12 $$\left(1+2\mathrm{Fo}\left(2+\mathrm{Bi_o}\right)\right)T_{12}^{p+1} - \mathrm{Fo}\left(2T_{11}^{p+1} + T_{16}^{p+1} + T_{8}^{p+1}\right) = T_{12}^{p} + 2\mathrm{Bi_o}\cdot\mathrm{Fo}\cdot T_{\infty,o}$$

Node 22 $$\left(1+4\mathrm{Fo}\left(1+\mathrm{Bi_o}\right)\right)T_{22}^{p+1} - 2\mathrm{Fo}\left(T_{21}^{p+1} + T_{21}^{p+1}\right) = T_{22}^{p} + 4\mathrm{Bi_o}\cdot\mathrm{Fo}\cdot T_{\infty,o}$$

Numerical values for the relevant parameters are:

$$\mathrm{Fo} = \frac{\alpha\Delta t}{\Delta x^2} = \frac{5.5\times10^{-6}\mathrm{m}^2/\mathrm{s}\times3600\mathrm{s}}{(0.050\mathrm{m})^2} = 7.92000$$

$$\mathrm{Bi_o} = \frac{h_o\Delta x}{k} = \frac{5\ \mathrm{W/m^2\cdot K}\times0.050\mathrm{m}}{0.85\ \mathrm{W/m\cdot K}} = 0.29412$$

$$\mathrm{Bi_i} = \frac{h_i\Delta x}{k} = \frac{100\ \mathrm{W/m^2\cdot K}\times0.050\mathrm{m}}{0.85\ \mathrm{W/m\cdot K}} = 5.88235$$

The system of FDEs can be represented in matrix notation, [A][T] = [C]. The coefficient matrix [A] and terms for the right-hand side matrix [C] are given on the following page.

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PROBLEM 5.125 (Cont.)

The coefficient matrix [A] and *RHS matrix [C]*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	RHS matrix [C]
1	E	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_1^p - 7331.1765$
2	1	F	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_2^p$
3	0	1	F	1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_3^p$
4	0	0	2	G	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_4^p - 175.38235$
5	1	0	0	0	E	2	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_5^p - 7331.1765$
6	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_6^p$
7	0	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	$-0.12626T_7^p$
8	0	0	0	1	0	0	2	G	0	0	0	1	0	0	0	0	0	0	0	0	0	0	$-0.12626T_8^p - 175.37235$
9	0	0	0	0	1	0	0	0	E	2	0	0	1	0	0	0	0	0	0	0	0	0	$-0.12626T_9^p - 7331.1765$
10	0	0	0	0	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	0	$-0.12626T_{10}^p$
11	0	0	0	0	0	0	1	0	0	1	F	1	0	0	1	0	0	0	0	0	0	0	$-0.12626T_{11}^p$
12	0	0	0	0	0	0	0	1	0	0	2	G	0	0	0	1	0	0	0	0	0	0	$-0.12626T_{12}^p - 175.38235$
13	0	0	0	0	0	0	0	0	4	0	0	0	H	8	0	0	0	0	0	0	0	0	$-0.37879T_{13}^p - 14,658.824$
14	0	0	0	0	0	0	0	0	0	1	0	0	1	F	1	0	1	0	0	0	0	0	$-0.12626T_{14}^p$
15	0	0	0	0	0	0	0	0	0	0	1	0	0	1	F	1	0	1	0	0	0	0	$-0.12626T_{15}^p$
16	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2	G	0	0	1	0	0	0	$-0.12626T_{16}^p - 175.38235$
17	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	F	2	0	0	0	0	$-0.12626T_{17}^p$
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	F	1	1	0	0	$-0.12626T_{18}^p$
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	G	0	1	0	$-0.12626T_{19}^p - 175.38235$
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	F	2	0	$-0.12626T_{20}^p$
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	G	1	$-0.12626T_{21}^p - 175.38235$
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	K	$-0.12626T_{22}^p - 350.76471$

$E = -15.89096$ $F = -4.12626$ $G = -4.71450$ $H = -35.90819$ $K = -5.30274$

For this problem a stock computer program was used to obtain the solution matrix [T]. The initial temperature distribution was $T_m^0 = 298\text{K}$. The results are tabulated below.

Node/time (h)	T(m,n) (C) 0	5	10	50	100
T01	25	335.00	338.90	340.20	340.20
T02	25	248.00	274.30	282.90	282.90
T03	25	179.50	217.40	229.80	229.80
T04	25	135.80	170.30	181.60	181.60
T05	25	334.50	338.50	339.90	339.90
T06	25	245.30	271.90	280.80	280.80
T07	25	176.50	214.60	227.30	227.30
T08	25	133.40	168.00	179.50	179.50
T09	25	332.20	336.60	338.20	338.20
T10	25	235.40	263.40	273.20	273.20
T11	25	166.40	205.40	219.00	219.00
T12	25	125.40	160.40	172.70	172.70
T13	25	316.40	324.30	327.30	327.30
T14	25	211.00	243.00	254.90	254.90
T15	25	146.90	187.60	202.90	202.90
T16	25	110.90	146.70	160.20	160.20
T17	25	159.80	200.50	216.20	216.20
T18	25	117.40	160.50	177.50	177.50
T19	25	90.97	127.40	141.80	141.80
T20	25	90.62	132.20	149.00	149.00
T21	25	72.43	106.70	120.60	120.60
T22	25	59.47	87.37	98.89	98.89

COMMENTS: (1) Note that the steady-state condition is reached by t = 5 hours; this can be seen by comparing the distributions for t = 50 and 100 hours. Within 10 hours, the flue is within a few degrees of the steady-state condition.

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PROBLEM 5.125 (Cont.)

(2) The *IHT* code for performing the numerical solution is shown in its entirety below. Use has been made of symmetry in writing the FDEs. The tabulated results above were obtained by copying from the *IHT Browser* and pasting the desired columns into EXCEL.

```
// From Tools|Finite-difference equations|Two-dimensional|Transient
// Interior surface nodes, 01, 05, 09, 13
/* Node 01: plane surface node, s-orientation; e, w, n labeled 05, 05, 02 . */
rho * cp * der(T01,t) = fd_2d_psur_s(T01,T05,T05,T02,k,qdot,deltax,deltay,Tinfi,hi,q''a)
q''a = 0          // Applied heat flux, W/m^2; zero flux shown
qdot = 0
rho * cp * der(T05,t) = fd_2d_psur_s(T05,T09,T01,T06,k,qdot,deltax,deltay,Tinfi,hi,q''a)
rho * cp * der(T09,t) = fd_2d_psur_s(T09,T13,T05,T10,k,qdot,deltax,deltay,Tinfi,hi,q''a)
/* Node 13: internal corner node, w-s orientation; e, w, n, s labeled 14, 09, 14, 09. */
rho * cp * der(T13,t) = fd_2d_ic_ws(T13,T14,T09,T14,T09,k,qdot,deltax,deltay,Tinfi,hi,q''a)

// Interior nodes, 02, 03, 06, 07, 10, 11, 14, 15, 18, 20
/* Node 02: interior node; e, w, n, s labeled 06, 06, 03, 01. */
rho * cp * der(T02,t) = fd_2d_int(T02,T06,T06,T03,T01,k,qdot,deltax,deltay)
rho * cp * der(T03,t) = fd_2d_int(T03,T07,T07,T04,T02,k,qdot,deltax,deltay)
rho * cp * der(T06,t) = fd_2d_int(T06,T10,T02,T07,T05,k,qdot,deltax,deltay)
rho * cp * der(T07,t) = fd_2d_int(T07,T11,T03,T08,T06,k,qdot,deltax,deltay)
rho * cp * der(T10,t) = fd_2d_int(T10,T14,T06,T11,T09,k,qdot,deltax,deltay)
rho * cp * der(T11,t) = fd_2d_int(T11,T15,T07,T12,T10,k,qdot,deltax,deltay)
rho * cp * der(T14,t) = fd_2d_int(T14,T17,T10,T15,T13,k,qdot,deltax,deltay)
rho * cp * der(T15,t) = fd_2d_int(T15,T18,T11,T16,T14,k,qdot,deltax,deltay)
rho * cp * der(T17,t) = fd_2d_int(T17,T18,T14,T18,T14,k,qdot,deltax,deltay)
rho * cp * der(T18,t) = fd_2d_int(T18,T20,T15,T19,T17,k,qdot,deltax,deltay)
rho * cp * der(T20,t) = fd_2d_int(T20,T21,T18,T21,T18,k,qdot,deltax,deltay)

// Exterior surface nodes, 04, 08, 12, 16, 19, 21, 22
/* Node 04: plane surface node, n-orientation; e, w, s labeled 08, 08, 03. */
rho * cp * der(T04,t) = fd_2d_psur_n(T04,T08,T08,T03,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T08,t) = fd_2d_psur_n(T08,T12,T04,T07,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T12,t) = fd_2d_psur_n(T12,T16,T08,T11,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T16,t) = fd_2d_psur_n(T16,T19,T12,T15,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T19,t) = fd_2d_psur_n(T19,T21,T16,T18,k,qdot,deltax,deltay,Tinfo,ho,q''a)
rho * cp * der(T21,t) = fd_2d_psur_n(T21,T22,T19,T20,k,qdot,deltax,deltay,Tinfo,ho,q''a)
/* Node 22: external corner node, e-n orientation; w, s labeled 21, 21. */
rho * cp * der(T22,t) = fd_2d_ec_en(T22,T21,T21,k,qdot,deltax,deltay,Tinfo,ho,q''a)

// Input variables
deltax = 0.050
deltay = 0.050
Tinfi = 350
hi = 100
Tinfo = 25
ho = 5
k = 0.85
alpha = 5.55e-7
alpha = k / (rho * cp)
rho = 1000                    // arbitrary value
```

(3) The results for t = 50 hour, representing the steady-state condition, are shown below, arranged according to the coordinate system.

	Tmn (C)						
x/y (mm)	0	50	100	150	200	250	300
0	181.60	179.50	172.70	160.20	141.80	120.60	98.89
50	229.80	227.30	219.00	202.90	177.50	149.00	
100	282.90	280.80	273.20	172.70	216.20		
150	340.20	339.90	338.20	327.30			

In Problem 4.55, the temperature distribution was determined using the FDEs written for steady-state conditions, but with a finer network, $\Delta x = \Delta y = 25$ mm. By comparison, the results for the coarser network are slightly higher, within a fraction of 1°C, along the mid-section of the flue, but notably higher in the vicinity of inner corner. (For example, node 13 is 2.6°C higher with the coarser mesh.)