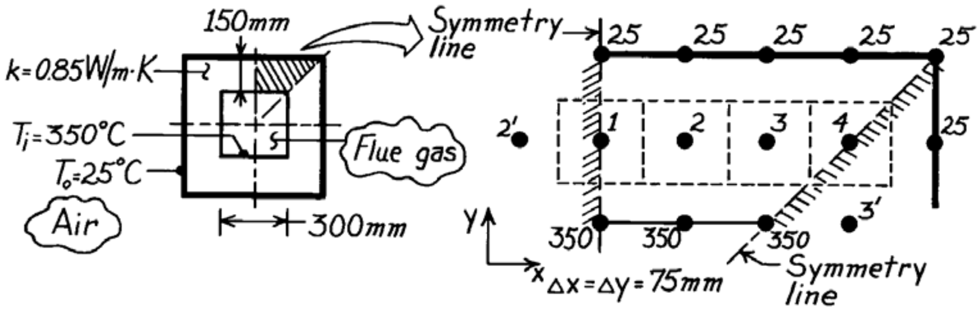
PROBLEM 4.54

KNOWN: Flue of square cross section with prescribed geometry, thermal conductivity and inner and outer surface temperatures.

FIND: Heat loss per unit length from the flue, q'.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties, (3) No internal generation.

ANALYSIS: Taking advantage of symmetry, the nodal network using the suggested 75mm grid spacing is shown above. To obtain the heat rate, we first need to determine the unknown temperatures T_1, T_2, T_3 and T_4. Recognizing that these nodes may be treated as interior nodes, the nodal equations from Eq. 4.29 are

$$(T_2 + 25 + T_2 + 350) - 4T_1 = 0$$
$$(T_1 + 25 + T_3 + 350) - 4T_2 = 0$$
$$(T_2 + 25 + T_4 + 350) - 4T_3 = 0$$
$$(T_3 + 25 + 25 + T_3) - 4T_4 = 0.$$

The Gauss-Seidel iteration method is convenient for this system of equations and following the procedures of Section 4.5.2, they are rewritten as,

$$T_1^k = 0.50\ T_2^{k-1} + 93.75$$
$$T_2^k = 0.25\ T_1^k + 0.25\ T_3^{k-1} + 93.75$$
$$T_3^k = 0.25\ T_2^k + 0.25\ T_4^{k-1} + 93.75$$
$$T_4^k = 0.50\ T_3^k + 12.5.$$

The iteration procedure is implemented in the table on the following page, one row for each iteration k. The initial estimates, for k = 0, are all chosen as (350 + 25)/2 ≈ 185°C. Iteration is continued until the maximum temperature difference is less than 0.2°C, i.e., $\varepsilon < 0.2°C$.

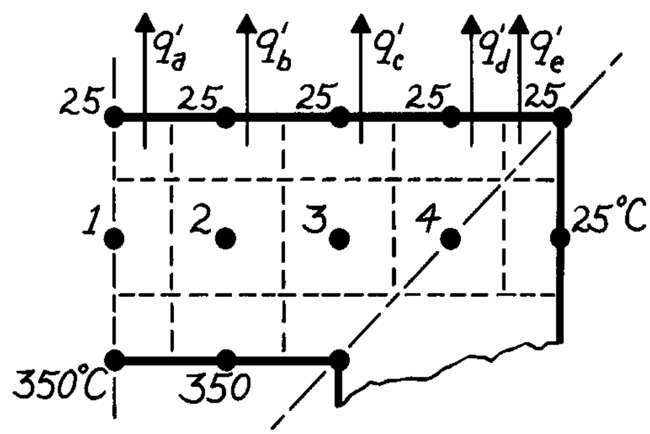
Note that if the system of equations were organized in matrix form, Eq. 4.48, diagonal dominance would exist. Hence there is no need to reorder the equations since the magnitude of the diagonal element is greater than that of other elements in the same row.

Continued …..

PROBLEM 4.54 (Cont.)

k	T_1(°C)	T_2(°C)	T_3(°C)	T_4(°C)	
0	185	185	185	185	← initial estimate
1	186.3	186.6	186.6	105.8	
2	187.1	187.2	167.0	96.0	
3	187.4	182.3	163.3	94.2	
4	184.9	180.8	162.5	93.8	
5	184.2	180.4	162.3	93.7	
6	184.0	180.3	162.3	93.6	
7	183.9	180.3	162.2	93.6	← ε <0.2°C

From knowledge of the temperature distribution, the heat rate may be obtained by summing the heat rates across the nodal control volume surfaces, as shown in the sketch.



The heat rate leaving the outer surface of this flue section is,

$$q' = q'_a + q'_b + q'_c + q'_d + q'_e$$
$$q' = k\frac{\Delta x}{\Delta y}\left[\frac{1}{2}(T_1 - 25) + (T_2 - 25) + (T_3 - 25) + (T_4 - 25) + 0\right]$$
$$q' = 0.85\frac{W}{m\cdot K}\left[\frac{1}{2}(183.9 - 25) + (180.3 - 25) + (162.2 - 26) + (93.6 - 25)\right]$$
$$q' = 374.5 \text{ W/m}.$$

Since this flue section is 1/8 the total cross section, the total heat loss from the flue is

$$q' = 8 \times 374.5 \text{ W/m} = 3.00 \text{ kW/m}. \qquad <$$

COMMENTS: The heat rate could have been calculated at the inner surface, and from the above sketch has the form

$$q' = k\frac{\Delta x}{\Delta y}\left[\frac{1}{2}(350 - T_1) + (350 - T_2) + (350 - T_3)\right] = 374.5 \text{ W/m}.$$

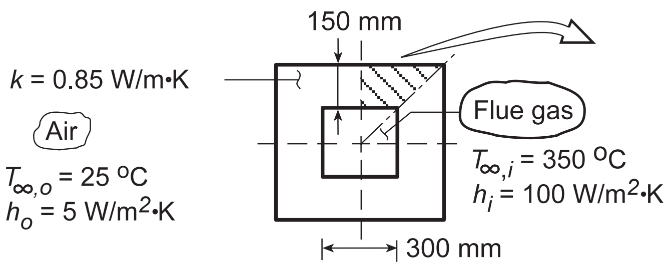
This result should compare very closely with that found for the outer surface since the conservation of energy requirement must be satisfied in obtaining the nodal temperatures.

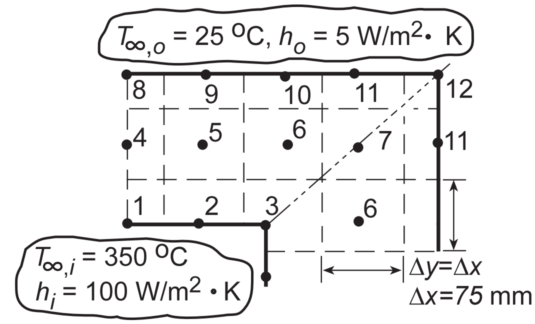
PROBLEM 4.55

KNOWN: Flue of square cross section with prescribed geometry, thermal conductivity and inner and outer surface convective conditions.

FIND: (a) Heat loss per unit length, q', by convection to the air, (b) Effect of grid spacing and convection coefficients on temperature field; show isotherms.

SCHEMATIC:





Schematic (a)

ASSUMPTIONS: (1) Steady-state, two-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) Taking advantage of symmetry, the nodal network for a 75 mm grid spacing is shown in schematic (a). To obtain the heat rate, we need first to determine the temperatures T_i. Recognize that there are four types of nodes: interior (4-7), plane surface with convection (1, 2, 8-11), internal corner with convection (3), and external corner with convection (12). Using the appropriate relations from Table 4.2, the finite-difference equations are

Node		Equation
1	$\left(2T_4 + T_2 + T_2\right) + \frac{2h_i\Delta x}{k}T_{\infty,i} - 2\left(\frac{h_i\Delta x}{k} + 2\right)T_1 = 0$	4.42
2	$\left(2T_5 + T_3 + T_1\right) + \frac{2h_i\Delta x}{k}T_{\infty,i} - 2\left(\frac{h_i\Delta x}{k} + 2\right)T_2 = 0$	4.42
3	$2\left(T_6 + T_6\right) + \left(T_2 + T_2\right) + \frac{2h_i\Delta x}{k}T_{\infty,i} - 2\left(3 + \frac{h_i\Delta x}{k}\right)T_3 = 0$	4.41
4	$\left(T_8 + T_5 + T_1 + T_5\right) - 4T_4 = 0$	4.29
5	$\left(T_9 + T_6 + T_2 + T_4\right) - 4T_5 = 0$	4.29
6	$\left(T_{10} + T_7 + T_3 + T_5\right) - 4T_6 = 0$	4.29
7	$\left(T_{11} + T_{11} + T_6 + T_6\right) - 4T_7 = 0$	4.29
8	$\left(2T_4 + T_9 + T_9\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_8 = 0$	4.42
9	$\left(2T_5 + T_{10} + T_8\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_9 = 0$	4.42
10	$\left(2T_6 + T_{11} + T_9\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_{10} = 0$	4.42
11	$\left(2T_7 + T_{12} + T_{10}\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 2\right)T_{11} = 0$	4.42
12	$\left(T_{11} + T_{11}\right) + \frac{2h_o\Delta x}{k}T_{\infty,o} - 2\left(\frac{h_o\Delta x}{k} + 1\right)T_{12} = 0$	4.43

Continued...

The Gauss-Seidel iteration is convenient for this system of equations. Following procedures of Section 4.5.2, the system of equations is rewritten in the proper form. Note that diagonal dominance is present; hence, no re-ordering is necessary.

$$T_1^k = 0.09239T_2^{k-1} + 0.09239T_4^{k-1} + 285.3$$
$$T_2^k = 0.04620T_1^k + 0.04620T_3^{k-1} + 0.09239T_5^{k-1} + 285.3$$
$$T_3^k = 0.08457T_2^k + 0.1692T_6^{k-1} + 261.2$$
$$T_4^k = 0.25T_1^k + 0.50T_5^{k-1} + 0.25T_8^{k-1}$$
$$T_5^k = 0.25T_2^k + 0.25T_4^k + 0.25T_6^{k-1} + 0.25T_9^{k-1}$$
$$T_6^k = 0.25T_3^k + 0.25T_5^k + 0.25T_7^{k-1} + 0.25T_9^{k-1}$$
$$T_7^k = 0.50T_6^k + 0.50T_{11}^{k-1}$$
$$T_8^k = 0.4096T_4^k + 0.4096T_9^{k-1} + 4.52$$
$$T_9^k = 0.4096T_5^k + 0.2048T_8^k + 0.2048T_{10}^{k-1} + 4.52$$
$$T_{10}^k = 0.4096T_6^k + 0.2048T_9^k + 0.2048T_{11}^{k-1} + 4.52$$
$$T_{11}^k = 0.4096T_7^k + 0.2048T_{10}^k + 0.2048T_{12}^{k-1} + 4.52$$
$$T_{12}^k = 0.6939T_{11}^k + 7.65$$

The initial estimates (k = 0) are carefully chosen to minimize calculation labor; let $\varepsilon < 1.0$.

k	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}
0	340	330	315	250	225	205	195	160	150	140	125	110
1	338.9	336.3	324.3	237.2	232.1	225.4	175.2	163.1	161.7	155.6	130.7	98.3
2	338.3	337.4	328.0	241.4	241.5	226.6	178.6	169.6	170.0	158.9	130.4	98.1
3	338.8	338.4	328.2	247.7	245.7	230.6	180.5	175.6	173.7	161.2	131.6	98.9
4	339.4	338.8	328.9	251.6	248.7	232.9	182.3	178.7	176.0	162.9	132.8	99.8
5	339.8	339.2	329.3	254.0	250.5	234.5	183.7	180.6	177.5	164.1	133.8	100.5
6	340.1	339.4	329.7	255.4	251.7	235.7	184.7	181.8	178.5	164.7	134.5	101.0
7	340.3	339.5	329.9	256.4	252.5	236.4	185.5	182.7	179.1	165.6	135.1	101.4

The heat loss to the outside air for the upper surface (Nodes 8 through 12) is of the form

$$q' = h_o \Delta x \left[\frac{1}{2}\left(T_8 - T_{\infty,o}\right) + \left(T_9 - T_{\infty,o}\right) + \left(T_{10} - T_{\infty,o}\right) + \left(T_{11} - T_{\infty,o}\right) + \frac{1}{2}\left(T_{12} - T_{\infty,o}\right) \right]$$

$$q' = 5\,\mathrm{W/m^2 \cdot K} \times 0.075\,\mathrm{m} \left[\frac{1}{2}(182.7-25) + (179.1-25) + (165.6-25) + (135.1-25) + \frac{1}{2}(101.4-25) \right] {}^\circ\mathrm{C} = 195\,\mathrm{W/m}$$

Hence, for the entire flue cross-section, considering symmetry,

$$q'_{tot} = 8 \times q' = 8 \times 195\,\mathrm{W/m} = 1.57\,\mathrm{kW/m}$$ <

The convection heat rate at the inner surface is

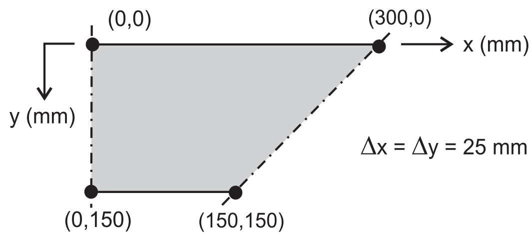
$$q'_{tot} = 8 \times h_i \Delta x \left[\frac{1}{2}\left(T_{\infty,i} - T_1\right) + \left(T_{\infty,i} - T_2\right) + \frac{1}{2}\left(T_{\infty,i} - T_3\right) \right] = 8 \times 190.5\,\mathrm{W/m} = 1.52\,\mathrm{kW/m}$$

which is within 2.5% of the foregoing result. The calculation would be identical if $\varepsilon = 0$.

Continued...

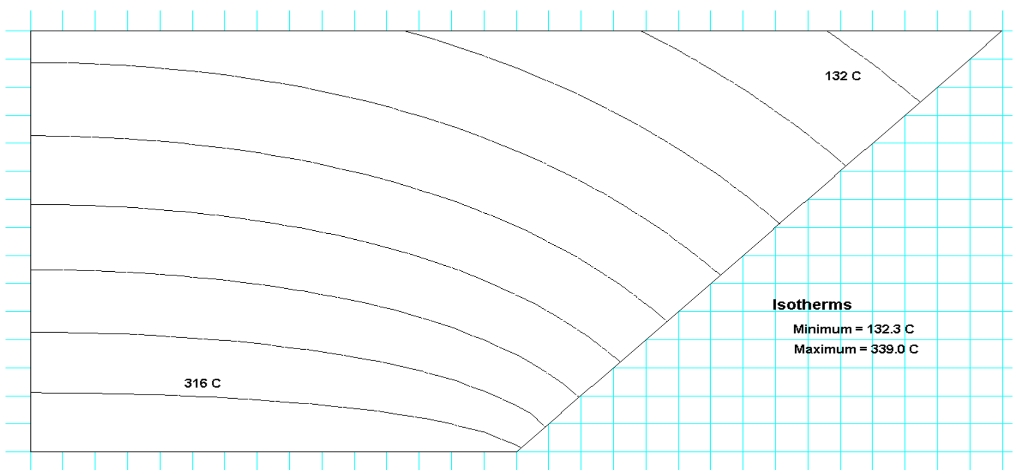
PROBLEM 4.55 (Cont.)

(b) Using the *Finite-Difference Equations* option from the *Tools* portion of the IHT menu, the following two-dimensional temperature field was computed for the grid shown in the schematic below, where x and y are in mm and the temperatures are in °C.



y\x	0	25	50	75	100	125	150	175	200	225	250	275	300
0	180.7	180.2	178.4	175.4	171.1	165.3	158.1	149.6	140.1	129.9	119.4	108.7	98.0
25	204.2	203.6	201.6	198.2	193.3	186.7	178.3	168.4	157.4	145.6	133.4	121.0	
50	228.9	228.3	226.2	222.6	217.2	209.7	200.1	188.4	175.4	161.6	147.5		
75	255.0	254.4	252.4	248.7	243.1	235.0	223.9	209.8	194.1	177.8			
100	282.4	281.8	280.1	276.9	271.6	263.3	250.5	232.8	213.5				
125	310.9	310.5	309.3	307.1	303.2	296.0	282.2	257.5					
150	340.0	340.0	339.6	339.1	337.9	335.3	324.7						

Agreement between the temperature fields for the (a) and (b) grids is good, with the largest differences occurring at the interior and exterior corners. Ten isotherms generated using *FEHT* are shown on the symmetric section below. Note how the heat flow is nearly normal to the flue wall around the mid-section. In the corner regions, the isotherms are curved and we'd expect that grid size might influence the accuracy of the results. Convection heat transfer to the inner surface is



$$q' = 8h_i\Delta x\Big[\big(T_{\infty,i} - T_1\big)/2 + \big(T_{\infty,i} - T_2\big) + \big(T_{\infty,i} - T_3\big) + \big(T_{\infty,i} - T_4\big)$$
$$+\big(T_{\infty,i} - T_5\big) + \big(T_{\infty,i} - T_6\big) + \big(T_{\infty,i} - T_7\big)/2\Big] = 1.52\,\text{kW/m}$$

and the agreement with results of the coarse grid is excellent.

The heat rate increases with increasing h_i and h_o, while temperatures in the wall increase and decrease, respectively, with increasing h_i and h_o.