CUNY Data 609 HW 11

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529 #1

Verify that the given function pair is a solution to the first order system:

$x = -e^t, y = e^t$

$\frac{dx}{dt} = -y, \frac{dy}{dt} = -x$

We need to verify: $\frac{dx}{dt} + y = 0$

We know that: $\frac{dx}{dt} = -e^t$

we subsitute: $-e^t + e^t ? = ?0$ Yes

Check for second side of pair:

we need to verify : $\frac{dy}{dt} + y = -0$

We know that: $\frac{dy}{dt} = e^t$

we subsitute: $e^t + e^t ? = ?0$ NO

The pair is **NOT** a solution

529 #6

Find and classify the points of the given autonomous system:

$\frac{dx}{dt} = -(y-1), \frac{dy}{dt} = x - 2$

Set both to zero and solve:

-(y-1) = 0

-y+1=0

1=y

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x-2 = 0

x=2

The ony critical point is at (2,1).

Since the first derivative is linear, the second derivative will be a constant. Since the second derivative is a constant the

p 546 #1

Apply the first and second derivative tests to the function $f(y) = \frac{y^a}{e^{by}}$ by to show that y=a/b is a unique critical point that yields the relative maximum f(a/b). Show also that f(y) approaches zero as y tends to infinity

First Derivative:

$f'(y) = y^{(a-1)}e^{(-by)}(a - by)$

First Derivative test:

At what point does the first derivative = 0?
$y^{(a-1)}e^{(-by)}(a - by) = 0$

$y = \frac{a}{b}$

Since the first derivative is 0 at $y = \frac{a}{b}$ then $\frac{a}{b}$ is a critical point

Second Derivative:

$f''(y) = y^{(a-2)}e^{-by}(a^2 - a(2by + 1) + b^2y^2)$

Second Derivative Test:

$f''(\frac{a}{b}) = y^{(a-2)}e^{-by}(a^2 - a(2by + 1) + b^2y^2)$

$f''(\frac{a}{b}) = a^2y^{(a-2)}e^{-by} + b^2y^ae^{-by} - ay^{(a-2)}e^{-by} - 2aby^{(a-1)}e^{-by}$

Since leading term is aˆ2 and this is >0 and the leading term aˆ2 approaches infinity, therefore as f(y) approaches zero y goes to infinity.

NOTE: In this (546 #1) I uses Wolfram Alpha to preform the rote manipulations, However the interperatations are my own.