Rocket science notes

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Project 9: Rocket design with python

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Contents

1 Introduction **1**
1.1 Assumptions 1
1.2 How to use this document 1

2 Fundamentals **2**
2.1 Newton's second law 2
2.2 Thrust and effective exhaust velocity 4
2.3 Specific impulse 4
2.4 The rocket equation 5
2.5 Final mass and burning time 5

3 Rockets characteristics **6**
3.1 Mass distribution in rockets 6
3.2 Multi-staged rockets 7
3.3 Geometry 7

1 Introduction

This booklet is an introduction to rocket science. It contains the necessary material that you will use for developing your project. The content can be updated as the course advanced to make explanations more clear or to add more material that could be useful for certain implementations in your code.

1.1 Assumptions

Througout this document, some assumptions are done in order to simplify the derivations of the equations. These assumptions are:

- Net lift acting on the rocket is null.
- Atmospheric effects (i.e. drag, temperature changes) are not considered.

1.2 How to use this document

At the beginning of each chapter there is a short description on its contents and what you will. There are some demonstrations and derivations that I consider to be interesting for knowing where some concepts come from. However, these derivations are not at all necessary for carrying on the project, and you can skip them if you wish. The only necessary equations that you will need are contained within boxes; see, for instance, Equation (18).

If you don't understand something that you think it is essential for advancing, write me an email as soon as possible. The physics contained in this document must not suppose a barrier for you to proceed, as they are not the objective but a tool for developing your code.

2 Fundamentals

This chapter presents the background for any rocket scientist. It is organised in four different sections with the necessary information for analyzing single-stage rockets.

Section 2.1 contains the derivation of Newton's second law applied to rockets, which is useful for the definitions stated in the following sections. However, a total understanding of this derivation is not essential, and you can skip this section if you wish. You can jump to section 2.2 and further for getting the formulas you will use, and go back to 2.1 if you are curious or want more extra information.

2.1 Newton's second law

Rockets are **variable-mass systems** that produce propulsion by expelling mass (the mass of burnt propellants). Their motion is dominated by Newton's second law. In its most fundamental form, this law takes the following form

$$\sum \boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} \tag{1}$$

where $\boldsymbol{F}$ are the external forces applied to the rocket and $\boldsymbol{p}$ is the momentum.

Momentum change

Figure 1 shows a rocket at two time consequtive time instants: t and $t + dt$. At the first one, the rocket with mass m is moving at a velocity $\boldsymbol{v}$. After a time some time dt, some mass dm has been expelled at velocity $\boldsymbol{u}$, producing a velocity increase in the rocket to $\boldsymbol{v} + d\boldsymbol{v}$ and a mass decrease to $m - dm$.

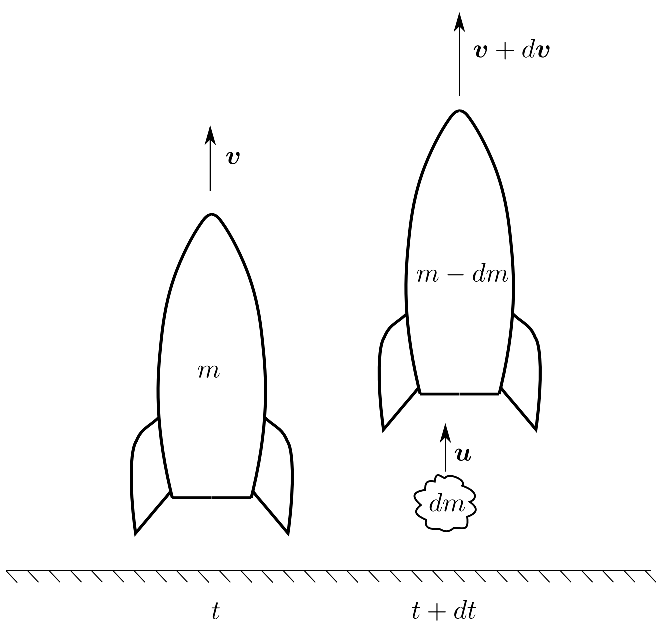

Figure 1: Momentum change in a variable-mass system.

The momentum of the system at the first time instant is:

$$\boldsymbol{p}_t = m\boldsymbol{v} \tag{2}$$

At the second time instant, the momentum is:

$$\boldsymbol{p}_{t+dt} = (m - dm)\,(\boldsymbol{v} + d\boldsymbol{v}) + dm\boldsymbol{u} \tag{3}$$

The difference in momentum between both time instants is:

$$d\boldsymbol{p} = \boldsymbol{p}_{t+dt} - \boldsymbol{p}_t = m \cdot d\boldsymbol{v} + (\boldsymbol{u} - \boldsymbol{v})\, dm \tag{4}$$

To simplify this expression, another velocity can be defined: the **relative velocity of expelled propellants with respect to the rocket**, denoted by $\mathbf{c}$ and defined as $\mathbf{c} = \mathbf{v} - \mathbf{u}$. With this definition, the last expression is:

$$d\boldsymbol{p} = m d\boldsymbol{v} - \boldsymbol{c} dm \tag{5}$$

And finally the derivative with respect to time can be taken, yielding the right-hand side of Equation (1):

$$\frac{d\boldsymbol{p}}{dt} = m\frac{d\boldsymbol{v}}{dt} - \boldsymbol{c}\frac{dm}{dt} \tag{6}$$

In order to produce motion, rockets burn propellant and expell exhaust gases. This means that the mass reduction of the rocket is the mass expulsion of propellants. In this way, the **propellant mass flow rate** (i.e. the rate of mass expulsion of the rocket) $\dot{m}$ can be defined as follows:

$$\dot{m} = \frac{dm}{dt} \tag{7}$$

So the derivative of momentum with respect to time can finally be rewritten as follows:

$$\frac{d\boldsymbol{p}}{dt} = m\frac{d\boldsymbol{v}}{dt} - \dot{m}\boldsymbol{c} \tag{8}$$

External forces

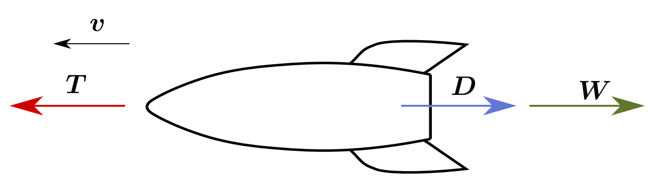

Figure 2: External forces acting on rocket: thrust $\mathbf{T}$, drag $\mathbf{D}$ and weight $\mathbf{W}$.

For determining the left-hand side of Equation (1), the external forces are divided into two contributions (see Figure 2):

1. Aerodynamic forces $\boldsymbol{F_a}$. Assuming that net lift acting on the rocket is zero, external forces are reduced to two contributions: **drag** ($\boldsymbol{D}$) and a net thrust force acting on the nozzle exit, called **pressure thrust** ($\boldsymbol{T_p}$):

$$\boldsymbol{F_a} = \boldsymbol{T_p} + \boldsymbol{D} = A_e\,(p_e - p_0)\,\boldsymbol{n} + \boldsymbol{D} \tag{9}$$

   with $\boldsymbol{T_p} = A_e\,(p_e - p_0)\,\boldsymbol{n}$ where: A_e is the exit area of the nozzle, p_e is the exit pressre of the nozzle, p_0 is the atmospheric pressure and $\boldsymbol{n}$ is the normal vector to the nozzle exit surface. Details on drag and on the derivation of pressure thrust are not given in this document.

2. Weight $\boldsymbol{W}$. It is given by the following expression:

$$\boldsymbol{W} = m\boldsymbol{g} \tag{10}$$

So the forces' term is given by:

$$\sum \boldsymbol{F} = \boldsymbol{F_a} + \boldsymbol{W} = A_e\,(p_e - p_0)\,\boldsymbol{n} + \boldsymbol{D} + m\boldsymbol{g} \tag{11}$$

And eventually, both expressions (8) and (11) can be plugged into (1) to yield Newton's second law:

$$m\frac{d\boldsymbol{v}}{dt} - \dot{m}\boldsymbol{c} = A_e\left(p_e - p_0\right)\boldsymbol{n} + \boldsymbol{D} + m\boldsymbol{g} \tag{12}$$

This equation can be rearranged by putting the term $\dot{m}\boldsymbol{c}$ into the right hand side:

$$m\frac{d\boldsymbol{v}}{dt} = \underbrace{\dot{m}\boldsymbol{c} + A_e\left(p_e - p_0\right)\boldsymbol{n}}_{\text{Thrust}} + \boldsymbol{D} + m\boldsymbol{g} \tag{13}$$

In the last equation, there are two terms that generate propulsive force known as **thrust**: the **impulse thrust** produced by mass expulsion and given by the term $\dot{m}\boldsymbol{c}$, and the **pressure thrust** already defined. If thrust is denoted by $\boldsymbol{T}$, then Newton's second law is:

$$m\frac{d\boldsymbol{v}}{dt} = \boldsymbol{T} + \boldsymbol{D} + m\boldsymbol{g} \tag{14}$$

Simplifications

The equations derived previously are vectorial, so they can be applied to any direction in space. For the purpose of this project, the main direction of interest is the direction of motion, represented by the vertical coordinate z as in Figures 1 and 2. Therefore, for the sake of simplicity the bold symbols denoting vectors will be removed, and all quantities will refer to direction z unless otherwise stated. With these considerations, Newton's second law is expressed as:

$$m\frac{dv}{dt} = T + D + mg \tag{15}$$

When calculating flight trajectories, two or three directions must be considered (for 2D or 3D trajectories respectively). In this project, the trajectories we will calculate are 2D. Therefore, distinction in terms of horizontal and vertical directions (x and z coordinates) has to be done. We will deal with this at the due moment in a later chapter.

2.2 Thrust and effective exhaust velocity

Thrust is the propulsive force of the rocket. According to Equation (13), it can be defined as (eliminating vectors):

$$T = \dot{m}c + A_e\left(p_e - p_0\right) \tag{16}$$

In this project, we will not deal neither with pressures (p_e, p_0) nor with geometric characteristics of the nozzles (A_e). For getting rid of these magnitudes, a magnitude name **effective exhaust velocity** (c_{eff}) can be defined:

$$c_{\text{eff}} = c + \frac{A_e\left(p_e - p_0\right)}{\dot{m}} \tag{17}$$

This magnitude can be substituted in Equation (16) to produce a more simple definition of thrust:

$$\boxed{T = \dot{m}c_{\text{eff}}} \tag{18}$$

Thrust will usually be expressed in kN, $\dot{m}$ in kg/s and c_{eff} in m/s.

2.3 Specific impulse

The **specific impulse** is a very important definition in rockets. It can be defined from thrust:

$$\boxed{I_{\text{sp}} = \frac{T}{\dot{m}g_0} = \frac{c_{\text{eff}}}{g_0}} \tag{19}$$

where g_0 is the gravity acceleration at sea level: $g_0 = 9.81\ m/s^2$. I_{sp}, which is expressed in s, is a figure of merit that indicates how efficiently a rocket burns its propellant.

2.4 The rocket equation

One of the most important relations in rocket propulsion is the **rocket equation**, also called Tsiolkovsky's equation on behalf of its author. This expression relates the change of velocity of a variable-mass system with the mass difference:

$$\boxed{\Delta v = v_f - v_0 = I_{\mathrm{sp}} g_0 \ln\left(\frac{m_0}{m_f}\right)} \tag{20}$$

where:

- v_0 if the initial velocity of the rocket.
- v_f is the final velocity of the rocket after all the propellant has been burnt.
- m_0 if the initial mass of the rocket.
- m_f is the final mass of the rocket after all the propellant has been burnt

The rocket equation (20) shows that the higher the I_{sp} of a rocket, the larger its final velocity.

2.5 Final mass and burning time

Let the propellant mass be m_p, which is a known parameter. Thus, the final mass of the rocket is simply the subtraction of the initial and propellant masses:

$$\boxed{m_f = m_0 - m_p} \tag{21}$$

The propellant mass will be consumed at a rate $\dot{m}$. Then, the **burning time** can be calculated as:

$$\boxed{t_b = \frac{m_p}{\dot{m}}} \tag{22}$$

3 Rockets characteristics

This chapter contains the geometric and mass definitions needed for defining and sketching rockets. All the concepts in this chapter appear in the rocket database which is at your disposal in github.

For simplifying the visualization of the geometric characteristics and mass distribution inside rockets, Figure 3 is used. In the left a rocket with a single stage is shown, while the rocket in the right shows a two-stages rocket.

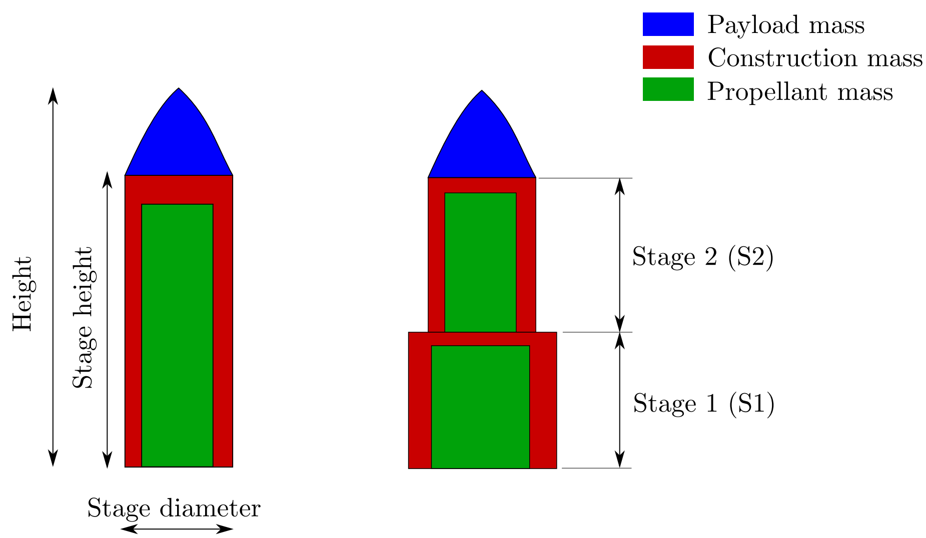

Figure 3: Simplified geometry of rockets with a schematic of mass distribution. *Left*: single-staged rocket. *Right*: multi-staged rocket with two stages.

3.1 Mass distribution in rockets

Before launch, a rocket has an initial mass m_0, also called **lift-off mass**. This mass is the addition of three contributions:

$$\boxed{m_0 = m_c + m_p + m_u} \tag{23}$$

where:

- m_c is the **construction mass**: the mass of the structure and equipment that belongs to the rocket. This mass is not expelled during flight.
- m_p is the **propellant mass**: the mass that is expelled during flight.
- m_u is the **payload mass** (also called useful mass): the mass carried by the rocket for achieving a given mission. Examples of payload masses are satellites which are put into orbit, astronauts, probes which are sent to the outer space, etc.

These three masses are depicted in Figure 3 in different colours. In the mission, the propellant mass is expelled at a rate $\dot{m}$ during a time t_b. For calculating these values, you can do the following:

1. As you know the specific impulse I_{sp} and the thrust T (they are defined in the database), then you can calculate $\dot{m}$ from Equation (19).
2. Then, Equation (22) is applied for obtaining t_b.

3.2 Multi-staged rockets

In reality there are not many single-staged rockets being used, as they can not reach velocities which are high enough to overcome the effect of gravity and leave the Earth. Most rockets are **multi-staged**.

The idea of multi-staging is that staged are stacked one onto each other. Each staged has its own construction and propellant masses: once a stage has consumed all its propellant, its corresponding construction mass is expelled and the next stage can start to burn its fuel. On top of the last stage there is the payload mass of the rocket. Therefore, in a rocket there will be as many propellant and construction masses as stages, but there will be only one payload mass.

In this project, we will work with rockets of up to two stages. Therefore, the **lift-off mass for a two-stages rocket** is given by:

$$\boxed{m_0 = m_{c_1} + m_{p_1} + m_{c_2} + m_{p_2} + m_u} \tag{24}$$

where each subindex refers to each stage. The bottom stage will be stage 1 (**S1**), while the upper stage will be stage 2 (**S2**), see Figure 3.

All rockets contained in the database have two stages except one, the rocket Miura 1.

3.3 Geometry

Rockets will be sketched as shown in Figure 3: they will be considered as a rectangle plus a triangle. The rectangle will be the stage containing the construction and propellant masses, and the triangle will represent the contained of the useful mass (this part is known as nosecone).

The rockets will have a **diameter** and a **height**. A **stage height** is defined too, so that the difference between this value and the height is the length of the nosecone.

For two-stages rockets, each stage is defined by its own length and diameter: **S1 length**, **S1 diameter**, **S2 length** and **S2 diameter**. One way of testing the coherence of the rocket's geometry is by checking that the sum of both lenghts is not larger than the height of the rocket: in other words, that the length of the nosecone is negative. You can implement this functionality as a test that is performed when a new rocket is defined and added into the database.

For single-staged rockets (Miura 1 and the ones that you add), all the characteristics from stage 2 are not defined. In the database, these attributes are empty values.