Les Formules

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Stage de CSTB, 2019

1 Related formulas to the surface component

1.1 Related formulas to collective dwellings

1.1.1 Related formulas to the surface component M

$$D_{surface\_component\_m} = \frac{B_{surface\_m}}{B_{dwelling\_total}}$$

- *Where:*

$$B_{surface\_m} = \left(1 - D_{share\_of\_window}\right) \times \sqrt{D_{surface} \times B_{dwelling\_per\_floor} \times 4 \times D_{ceiling\_height}} \\ \times \left(B_{floors} + 1\right) \tag{1}$$

$$B_{dwelling\_total} = B_{dwelling\_per\_floor} \times \left(B_{floors} + 1\right) \tag{2}$$

1.1.2 Related formulas to the surface component W

$$D_{surface\_component\_w} = \frac{B_{surface\_w}}{B_{dwelling\_total}}$$

- *Where:*

$$B_{surface\_w} = D_{share\_of\_window} \times \sqrt{D_{surface} \times B_{dwelling\_per\_floor} \times 4 \times D_{ceiling\_height}} \\ \times \left(B_{floors} + 1\right) \tag{3}$$

1.1.3 Related formulas to the surface components R and F

$$D_{surface\_component\_r} = \frac{B_{surface\_r}}{B_{dwelling\_total}}$$

$$= \frac{D_{surface} \times B_{dwelling\_per\_floor}}{B_{dwelling\_per\_floor} \times (B_{floors} + 1)}$$

$$= \frac{D_{surface}}{B_{floors} + 1}$$

$$D_{surface\_component\_f} = \frac{B_{surface\_f}}{B_{dwelling\_total}}$$

$$= \frac{D_{surface} \times B_{dwelling\_per\_floor}}{B_{dwelling\_per\_floor} \times (B_{floors} + 1)}$$

$$= \frac{D_{surface}}{B_{floors} + 1}$$

$$= D_{surface\_component\_r}$$

- *Where:*

$$B_{surface\_r} = D_{surface} \times B_{dwelling\_per\_floor} \tag{4}$$

$$B_{surface\_f} = D_{surface} \times B_{dwelling\_per\_floor}$$

$$= B_{surface\_r}$$

1.2 Related formulas to individual dwellings

1.2.1 Related formulas to the surface component M

$$D_{surface\_component\_m} = \frac{B_{surface\_m}}{B_{dwelling\_total}}$$

$$= B_{surface\_m}$$

- *Where:*

$$B_{surface\_m} = \left(1 - D_{share\_of\_window}\right) \times \sqrt{\frac{D_{surface}}{B_{floors} + 1}} \times 4 \times D_{ceiling\_height} \times \left(B_{floors} + 1\right) \quad (5)$$

$$B_{dwelling\_total} = 1 \quad (6)$$

1.2.2 Related formulas to the surface component W

$$D_{surface\_component\_w} = \frac{B_{surface\_w}}{B_{dwelling\_total}}$$

$$= B_{surface\_w}$$

- *Where:*

$$B_{surface\_w} = D_{share\_of\_window} \times \sqrt{\frac{D_{surface}}{\left(B_{floors} + 1\right)}} \times 4 \times D_{ceiling\_height} \times \left(B_{floors} + 1\right) \quad (7)$$

1.2.3 Related formulas to the surface components R and F

$$D_{surface\_component\_r} = B_{surface\_r}$$

$$D_{surface\_component\_f} = B_{surface\_f}$$

$$= D_{surface\_component\_r}$$

- *Where:*

$$B_{surface\_r} = \frac{D_{surface}}{B_{floors} + 1} \quad (8)$$

$$B_{surface\_f} = \frac{D_{surface}}{B_{floors} + 1}$$

$$= B_{surface\_f}$$

2   Related formulas to the overall heat transfer coefficient (Ug)

$$D_{ug} = \frac{B_{surface\_m} \times D_{um} + B_{surface\_w} \times D_{uw} + B_{surface\_r} \times D_{ur} + B_{surface\_f} \times D_{uf}}{B_{surface\_envelope}}$$

$$= \frac{B_{surface\_m} \times D_{um} + B_{surface\_w} \times D_{uw} + B_{surface\_r} \times \left(D_{ur} + D_{uf}\right)}{B_{surface\_envelope}}$$

$$= \frac{B_{weighted\_surface\_total}}{B_{surface\_envelope}}$$

- *Where:*

$$B_{weighted\_surface\_total} = B_{weighted\_surface\_m\_w} + B_{weighted\_surface\_r\_f} \tag{9}$$

$$B_{weighted\_surface\_m\_w} = B_{surface\_m} \times D_{um} + B_{surface\_w} \times D_{uw} \tag{10}$$

$$B_{weighted\_surface\_r\_f} = B_{surface\_r} \times D_{ur} + B_{surface\_r} \times D_{uf} \tag{11}$$

$$B_{surface\_envelope} = B_{surface\_m} + B_{surface\_w} + B_{surface\_r} + B_{surface\_f}$$

3   Related formulas to the useful energy needs (UEN)

3.1   Related formulas to collective dwellings

3.1.1   Related formulas to the UEN per m^2

Original related formulas to the UEN per m^2

$$D_{uen\_per\_metre2} = \frac{D_{intermittence\_adjusted} \times D_{ug} \times B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

$$= D_{intermittence\_adjusted} \times D_{ug} \times \frac{B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

- *Where:*

$$D_{intermittence\_adjusted} = \frac{D_{intermittence}}{1 + 0.1 \times \left(D_{ug} \times \frac{B_{surface\_envelope}}{B_{surface\_habitable}} \times \frac{2.5}{D_{ceiling\_height}} - 1\right)}$$

$$B_{surface\_envelope} = B_{surface\_m} + B_{surface\_w} + B_{surface\_r} + B_{surface\_f}$$

$$B_{surface\_habitable} = B_{surface\_r} \times \left(B_{floors} + 1\right)$$

Simplified related formulas to the UEN per m^2

$$D_{uen\_per\_metre2} = D_{intermittence\_adjusted} \times D_{ug} \times \frac{B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

$$= \frac{D_{intermittence}}{1 + 0.1 \times \left(\frac{B_{weighted\_surface\_total}}{B_{surface\_envelope}} \times \frac{B_{surface\_envelope}}{B_{surface\_habitable}} \times \frac{2.5}{D_{ceiling\_height}} - 1 \right)}$$
$$\times \frac{B_{weighted\_surface\_total}}{B_{surface\_envelope}} \times \frac{B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

$$= \frac{D_{intermittence}}{1 + \frac{1}{10} \times \left(\frac{B_{weighted\_surface\_total}}{B_{surface\_habitable}} \times \frac{2.5}{D_{ceiling\_height}} - 1 \right)}$$
$$\times \frac{B_{weighted\_surface\_total} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

$$= \frac{10 \times B_{surface\_habitable} \times D_{ceiling\_height} \times D_{intermittence}}{9 \times B_{surface\_habitable} \times D_{ceiling\_height} + 2.5 \times B_{weighted\_surface\_total}}$$
$$\times \frac{B_{weighted\_surface\_total} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

$$= \frac{B_{weighted\_surface\_total}}{9 \times B_{surface\_habitable} \times D_{ceiling\_height} + 2.5 \times B_{weighted\_surface\_total}}$$
$$\times \frac{D_{ceiling\_height} \times D_{intermittence} \times D_{degrees\_of\_reference}}{100}$$

- *Where:*

$$B_{surface\_habitable} = B_{surface\_r} \times \left(B_{floors} + 1 \right)$$

3.1.2 Related formulas to the UEN per dwelling

$$D_{uen\_per\_dwelling} = D_{uen\_per\_metre2} \times D_{surface}$$

$$= \frac{B_{weighted\_surface\_total}}{9 \times B_{surface\_habitable} \times D_{ceiling\_height} + 2.5 \times B_{weighted\_surface\_total}}$$
$$\times \frac{D_{ceiling\_height} \times D_{intermittence} \times D_{degrees\_of\_reference}}{100} \times D_{surface}$$

$$= D_{uen\_per\_dwelling\_1} \times D_{uen\_per\_dwelling\_2} \tag{12}$$

- *Where:*

$$D_{uen\_per\_dwelling\_1} \quad = \quad \frac{B_{weighted\_surface\_total}}{B_{enlarged\_weighted\_surface\_total}} \tag{13}$$

$$B_{enlarged\_weighted\_surface\_total} \quad = \quad 9 \times B_{heightened\_surface\_habitable} + 2.5 \times B_{weighted\_surface\_total} \tag{14}$$

$$B_{heightened\_surface\_habitable} \quad = \quad B_{surface\_r} \times \left(B_{floors} + 1\right) \times D_{ceiling\_height} \tag{15}$$

$$D_{uen\_per\_dwelling\_2} \quad = \quad D_{ceiling\_height} \times D_{intermittence} \times D_{degrees\_of\_reference} \times D_{surface}$$
$$\times\, 0.01 \tag{16}$$

3.2 Related formulas to individual dwellings

3.2.1 Related formulas to the UEN per m^2

Original related formulas to the UEN per m^2

$$D_{uen\_per\_metre2} \quad = \quad \frac{D_{intermittence\_adjusted} \times D_{ug} \times B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

$$= \quad D_{intermittence\_adjusted} \times D_{ug} \times \frac{B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}$$

- *Where:*

$$D_{intermittence\_adjusted} \quad = \quad \frac{D_{intermittence}}{1 + 0.1 \times \left(D_{ug} \times \frac{B_{surface\_envelope}}{B_{surface\_habitable}} \times \frac{2.5}{D_{ceiling\_height}} - 1\right)}$$

$$B_{surface\_envelope} \quad = \quad B_{surface\_m} + B_{surface\_w} + B_{surface\_r} + B_{surface\_f}$$

$$B_{surface\_habitable} \quad = \quad D_{surface}$$

Simplified related formulas to the UEN per m^2

$$
D_{uen\_per\_metre2} = D_{intermittence\_adjusted} \times D_{ug} \times \frac{B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}
$$

$$
= \frac{D_{intermittence}}{1 + 0.1 \times \left(\frac{B_{weighted\_surface\_total}}{B_{surface\_envelope}} \times \frac{B_{surface\_envelope}}{B_{surface\_habitable}} \times \frac{2.5}{D_{ceiling\_height}} - 1 \right)}
$$
$$
\times \frac{B_{weighted\_surface\_total}}{B_{surface\_envelope}} \times \frac{B_{surface\_envelope} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}
$$

$$
= \frac{D_{intermittence}}{1 + \frac{1}{10} \times \left(\frac{B_{weighted\_surface\_total}}{B_{surface\_habitable}} \times \frac{2.5}{D_{ceiling\_height}} - 1 \right)}
$$
$$
\times \frac{B_{weighted\_surface\_total} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}
$$

$$
= \frac{10 \times B_{surface\_habitable} \times D_{ceiling\_height} \times D_{intermittence}}{9 \times B_{surface\_habitable} \times D_{ceiling\_height} + 2.5 \times B_{weighted\_surface\_total}}
$$
$$
\times \frac{B_{weighted\_surface\_total} \times D_{degrees\_of\_reference}}{B_{surface\_habitable} \times 1000}
$$

$$
= \frac{B_{weighted\_surface\_total}}{9 \times B_{surface\_habitable} \times D_{ceiling\_height} + 2.5 \times B_{weighted\_surface\_total}}
$$
$$
\times \frac{D_{ceiling\_height} \times D_{intermittence} \times D_{degrees\_of\_reference}}{100}
$$

- *Where:*

$$
B_{surface\_habitable} = D_{surface}
$$

3.2.2   Related formulas to the UEN per dwelling

$$
D_{uen\_per\_dwelling} = D_{uen\_per\_metre2} \times D_{surface}
$$

$$
= \frac{B_{weighted\_surface\_total}}{9 \times B_{surface\_habitable} \times D_{ceiling\_height} + 2.5 \times B_{weighted\_surface\_total}}
$$
$$
\times \frac{D_{ceiling\_height} \times D_{intermittence} \times D_{degrees\_of\_reference}}{100} \times D_{surface}
$$

$$
= D_{uen\_per\_dwelling\_1} \times D_{uen\_per\_dwelling\_2} \tag{17}
$$

- *Where:*

$$D_{uen\_per\_dwelling\_1} = \frac{B_{weighted\_surface\_total}}{B_{enlarged\_weighted\_surface\_total}} \tag{18}$$

$$B_{enlarged\_weighted\_surface\_total} = 9 \times B_{heightened\_surface\_habitable} + 2.5 \times B_{weighted\_surface\_total} \tag{19}$$

$$B_{heightened\_surface\_habitable} = D_{surface} \times D_{ceiling\_height} \tag{20}$$

$$D_{uen\_per\_dwelling\_2} = D_{ceiling\_height} \times D_{intermittence} \times D_{degrees\_of\_reference} \times D_{surface}$$
$$\times 0.01 \tag{21}$$

4 Related formulas to the global heating costs (GHC)

4.1 Related formulas to the dwellings with individual heating system

4.1.1 Related formulas for the case I

- *When:*

  - $HS_{initial} = HS_{to\_switch}$

To Switch / Initial	ER	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
ER	$\oplus$								
GB		$\oplus$							
CGB			$\oplus$						
OB				$\oplus$					
COB					$\oplus$				
EHP2						$\oplus$			
WB							$\oplus$		
WPB								$\oplus$	
EHP									$\oplus$

$$
\begin{aligned}
D_{GHC\_I} \quad = \quad & \frac{D_{investment\_cost} \times HS_{inflation\_rate\_1st}}{HS_{discount\_1st}} + \frac{D_{investment\_cost} \times HS_{inflation\_rate\_2nd}}{HS_{discount\_2nd}} \\
& + \frac{D_{uen\_per\_dwelling}}{HS_{efficiency}} \times HS_{energy\_bill} + HS_{maintenance} + D_{insulation\_cost} \\[1.5em]
= \quad & \left(HS_{a\_investment} + HS_{b\_investment} \times D_{uen\_per\_dwelling} \right) \\
& \times \frac{HS_{inflation\_rate\_1st} \times HS_{discount\_2nd} + HS_{inflation\_rate\_2nd} \times HS_{discount\_1st}}{HS_{discount\_1st} \times HS_{discount\_2nd}} \\
& + D_{uen\_per\_dwelling} \times \frac{HS_{energy\_bill}}{HS_{efficiency}} + HS_{maintenance} + D_{insulation\_cost} \\[1.5em]
= \quad & HS_{ghc\_uen\_condition\_1} + D_{unvariable\_cost\_1}
\end{aligned}
\tag{22}
$$

- *Where:*

$$
D_{investment\_cost} \quad = \quad HS_{a\_investment} + HS_{b\_investment} \times D_{uen\_per\_dwelling}
\tag{23}
$$

$$HS_{ghc\_uen\_condition\_1} = HS_{uen\_cost\_1} \times D_{uen\_per\_dwelling} \tag{24}$$

$$
\begin{aligned}
&if \quad HS_{to\_switch} \in \{ER,\ GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\} \\
&\Rightarrow \quad HS_{initial} \in \{ER,\ GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\} \\
&and \quad HS_{to\_switch} = HS_{initial}
\end{aligned}
$$

$$HS_{uen\_cost\_1} = HS_{b\_investment} \times HS_{time\_factor\_1} + \frac{HS_{energy\_bill}}{HS_{efficiency}} \tag{25}$$

$$HS_{time\_factor\_1} = \frac{HS_{inflation\_rate\_1st} \times HS_{discount\_2nd} + HS_{inflation\_rate\_2nd} \times HS_{discount\_1st}}{HS_{discount\_1st} \times HS_{discount\_2nd}} \tag{26}$$

$$HS_{inflation\_rate\_1st} = \left(1 + HS_{inflation\_rate}\right)^{18 - HS_{initial\_age} + year},\ year \in [1, 20] \tag{27}$$

$$HS_{inflation\_rate\_2nd} = \left(1 + HS_{inflation\_rate}\right)^{38 - HS_{initial\_age} + year},\ year \in [1, 20] \tag{28}$$

$$HS_{discount\_1st} = \left(1 + HS_{discount\_rate}\right)^{19 - HS_{initial\_age}} \tag{29}$$

$$HS_{discount\_2nd} = \left(1 + HS_{discount\_rate}\right)^{39 - HS_{initial\_age}} \tag{30}$$

$$HS_{energy\_bill} = \sum_{i=1}^{40} \frac{HS_{energy\_price} \times \left(1 + HS_{annual\_variation}\right)^{i + year - 2} + HS_{carbon\_tax}}{\left(1 + D_{discount\_rate}\right)^{i}}$$
$$,\ year \in [1, 20] \tag{31}$$

$$HS_{carbon\_tax} = HS_{emission\_factor} \times HS_{carbon\_price} \tag{32}$$

$$D_{unvariable\_cost\_1} = D_{insulation\_cost} + HS_{fixed\_cost\_1} \tag{33}$$

$$
\begin{aligned}
if \quad & HS_{to\_switch} \in \{ER,\ GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\} \\
\Rightarrow \quad & HS_{initial} \in \{ER,\ GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\} \\
and \quad & HS_{to\_switch} = HS_{initial}
\end{aligned}
$$

$$D_{insulation\_cost} = D_{insulation\_cost\_m\_w} + D_{insulation\_cost\_r\_f} \tag{34}$$

$$
\begin{aligned}
D_{insulation\_cost\_m\_w} = \ & \frac{B_{surface\_m}}{B_{dwelling\_total}} \times D_{price\_including\_tax\_m} \\
& + \frac{B_{surface\_w}}{B_{dwelling\_total}} \times D_{price\_including\_tax\_w}
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
D_{insulation\_cost\_r\_f} = \ & \frac{D_{surface}}{B_{floors} + 1} \times D_{price\_including\_tax\_r} \\
& + \frac{D_{surface}}{B_{floors} + 1} \times D_{price\_including\_tax\_f}
\end{aligned}
\tag{36}
$$

$$HS_{fixed\_cost\_1} = HS_{a\_investment} \times HS_{time\_factor\_1} + HS_{maintenance} \tag{37}$$

$$HS_{maintenance} = \sum_{i=0}^{39} \frac{HS_{maintenance\_cost} \times \left(1 + HS_{inflation\_rate}\right)^{i+year-1}}{\left(1 + D_{discount\_rate}\right)^i},\ year \in [1, 20] \tag{38}$$

4.1.2   Related formulas for the case II

- *When:*

  - $HS_{initial} \neq HS_{to\_switch}$
  - $HS_{initial} \in \{ER,\ WB,\ EHP\}$ *and* $HS_{to\_switch} \in \{ER,\ WB,\ EHP\}$
  - $HS_{initial} \in \{GB,\ CGB\}$ *and* $HS_{to\_switch} \in \{GB,\ CGB\}$
  - $HS_{initial} \in \{OB,\ COB\}$ *and* $HS_{to\_switch} \in \{OB,\ COB\}$

To Switch / Initial	ER	GB	CGB	OB	COB	EHP2WB	WPB	EHP
ER	$\times$						$\oplus$	$\oplus$
GB		$\times$	$\oplus$					
CGB		$\oplus$	$\times$					
OB				$\times$	$\oplus$			
COB				$\oplus$	$\times$			
EHP2						$\times$		
WB	$\oplus$						$\times$	$\oplus$
WPB							$\times$	
EHP	$\oplus$					$\oplus$		$\times$

$$
\begin{aligned}
D_{GHC\_II} \;=\;& D_{investment\_cost} \times HS_{inflation\_rate\_0} + \frac{D_{investment\_cost} \times HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\[2mm]
& + \frac{D_{uen\_per\_dwelling}}{HS_{efficiency}} \times HS_{energy\_bill} + HS_{maintenance} + D_{insulation\_cost} \\[2mm]
& + HS_{same\_switch\_cost} \\[4mm]
\;=\;& \left(HS_{a\_investment} + HS_{b\_investment} \times D_{uen\_per\_dwelling} \right) \\[1mm]
& \times \frac{HS_{discount\_20} \times HS_{inflation\_rate\_0} + HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\[2mm]
& + D_{uen\_per\_dwelling} \times \frac{HS_{energy\_bill}}{HS_{efficiency}} + HS_{maintenance} + D_{insulation\_cost} \\[2mm]
& + HS_{same\_switch\_cost} \\[4mm]
\;=\;& HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_2}
\end{aligned}
\tag{39}
$$

• *Where:*

$$HS_{ghc\_uen\_condition\_2} \quad = \quad HS_{uen\_cost\_2} \times D_{uen\_per\_dwelling} \tag{40}$$

$$if \quad HS_{to\_switch} \in \{ER, OB, COB, EHP2, WB, WPB, EHP\}$$
$$\Rightarrow \quad HS_{initial} \in \{ER, GB, CGB, OB, COB, EHP2, WB, WPB, EHP\}$$
$$and \quad HS_{to\_switch} \neq HS_{initial}$$

$$if \quad HS_{to\_switch} \in \{GB, CGB\}$$
$$\Rightarrow \quad HS_{initial} \in \{ER, GB, CGB, EHP2, WB, WPB, EHP\}$$
$$and \quad HS_{to\_switch} \neq HS_{initial}$$

$$HS_{uen\_cost\_2} \quad = \quad HS_{b\_investment} \times HS_{time\_factor\_2} + \frac{HS_{energy\_bill}}{HS_{efficiency}} \tag{41}$$

$$HS_{time\_factor\_2} \quad = \quad \frac{HS_{discount\_20} \times HS_{inflation\_rate\_0} + HS_{inflation\_rate\_20}}{HS_{discount\_20}} \tag{42}$$

$$HS_{discount\_20} \quad = \quad \left(1 + HS_{discount\_rate}\right)^{20} \tag{43}$$

$$HS_{inflation\_rate\_0} \quad = \quad \left(1 + HS_{inflation\_rate}\right)^{year-1}, \; year \in [0, 20] \tag{44}$$

$$HS_{inflation\_rate\_20} \quad = \quad \left(1 + HS_{inflation\_rate}\right)^{19+year}, \; year \in [0, 20] \tag{45}$$

$$D_{unvariable\_cost\_2} \quad = \quad D_{insulation\_cost} + HS_{fixed\_cost\_2} \tag{46}$$

$$if \quad HS_{to\_switch} \in \{ER, WB, EHP\}$$
$$\Rightarrow \quad HS_{initial} \in \{ER, WB, EHP\}$$
$$and \quad HS_{to\_switch} \neq HS_{initial}$$

$$if \quad HS_{to\_switch} \in \{GB, CGB\}$$
$$\Rightarrow \quad HS_{initial} \in \{GB, CGB\}$$
$$and \quad HS_{to\_switch} \neq HS_{initial}$$

$$if \quad HS_{to\_switch} \in \{OB, COB\}$$
$$\Rightarrow \quad HS_{initial} \in \{OB, COB\}$$
$$and \quad HS_{to\_switch} \neq HS_{initial}$$

$$HS_{fixed\_cost\_2} \quad = \quad HS_{a\_investment} \times HS_{time\_factor\_2} + HS_{maintenance}$$
$$+ HS_{same\_switch\_cost} \tag{47}$$

4.1.3 Related formulas for the case III

- *When:*

  - $HS_{initial} \neq HS_{to\_switch}$
  - $HS_{initial} = WB$ and $HS_{to\_switch} = WPB$

To Switch / Initial	ER	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
ER	×						×		×
GB		×	×						
CGB		×	×						
OB				×	×				
COB				×	×				
EHP2						×			
WB	×						×	⊕	×
WPB								×	
EHP	×						×		×

$$
\begin{aligned}
D_{GHC\_III} \quad = \quad & D_{investment\_cost} \times HS_{inflation\_rate\_0} + \frac{D_{investment\_cost} \times HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
& + \frac{D_{uen\_per\_dwelling}}{HS_{efficiency}} \times HS_{energy\_bill} + HS_{maintenance} + D_{insulation\_cost} \\
& + HS_{same\_switch\_cost} + HS_{control\_cost} \\[2mm]
= \quad & \left(HS_{a\_investment} + HS_{b\_investment} \times D_{uen\_per\_dwelling} \right) \\
& \times \frac{HS_{discount\_20} \times HS_{inflation\_rate\_0} + HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
& + D_{uen\_per\_dwelling} \times \frac{HS_{energy\_bill}}{HS_{efficiency}} + HS_{maintenance} + D_{insulation\_cost} \\
& + HS_{same\_switch\_cost} + HS_{control\_cost} \\[2mm]
= \quad & HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_3} \tag{48}
\end{aligned}
$$

$$
\tag{49}
$$

- *Where:*

$$
D_{unvariable\_cost\_3} \quad = \quad D_{insulation\_cost} + HS_{fixed\_cost\_3} \tag{50}
$$

$$
\begin{aligned}
if & \quad HS_{to\_switch} = WPB \\
\Rightarrow & \quad HS_{initial} = WB
\end{aligned}
$$

$$
HS_{fixed\_cost\_3} \quad = \quad HS_{fixed\_cost\_2} + HS_{control\_cost} \tag{51}
$$

$$
HS_{control\_cost} \quad = \quad HS_{delevery\_type\_cost} \times D_{surface} \tag{52}
$$

4.1.4 Related formulas for the case IV

- *When:*

  - $HS_{initial} \neq HS_{to\_switch}$
  - $HS_{initial} \in \{GB, CGB\}$ and $HS_{to\_switch} \in \{OB, COB, EHP2, WPB\}$
  - $HS_{initial} \in \{OB, COB\}$ and $HS_{to\_switch} \in \{EHP2, WPB\}$
  - $HS_{initial} \in \{EHP2, WPB\}$ and $HS_{to\_switch} \in \{GB, CGB, OB, COB, EHP2, WPB\}$

To Switch / Initial	ER	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
ER	$\times$						$\times$		$\times$
GB		$\times$	$\times$	$\oplus$	$\oplus$	$\oplus$		$\oplus$	
CGB		$\times$	$\times$	$\oplus$	$\oplus$	$\oplus$		$\oplus$	
OB				$\times$	$\times$	$\oplus$		$\oplus$	
COB				$\times$	$\times$	$\oplus$		$\oplus$	
EHP2		$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\times$		$\oplus$	
WB	$\times$						$\times$	$\times$	$\times$
WPB		$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$		$\times$	
EHP	$\times$						$\times$		$\times$

$$
\begin{aligned}
D_{GHC\_IV} &= D_{investment\_cost} \times HS_{inflation\_rate\_0} + \frac{D_{investment\_cost} \times HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
&\quad + \frac{D_{uen\_per\_dwelling}}{HS_{efficiency}} \times HS_{energy\_bill} + HS_{maintenance} + D_{insulation\_cost} \\
&\quad + HS_{different\_switch\_cost} \\[2ex]
&= \left(HS_{a\_investment} + HS_{b\_investment} \times D_{uen\_per\_dwelling} \right) \\
&\quad \times \frac{HS_{discount\_20} \times HS_{inflation\_rate\_0} + HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
&\quad + D_{uen\_per\_dwelling} \times \frac{HS_{energy\_bill}}{HS_{efficiency}} + HS_{maintenance} + D_{insulation\_cost} \\
&\quad + HS_{different\_switch\_cost} \\[2ex]
&= HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_4}
\end{aligned}
$$

$$(53)$$

$$(54)$$

- *Where:*

$$D_{unvariable\_cost\_4} = D_{insulation\_cost} + HS_{fixed\_cost\_4} \tag{55}$$

$$\begin{aligned}
&if \quad HS_{to\_switch} \in \{GB,\ CGB\} \\
&\Rightarrow \quad HS_{initial} \in \{OB,\ COB,\ EHP2,\ WPB\}
\end{aligned}$$

$$\begin{aligned}
&if \quad HS_{to\_switch} \in \{OB,\ COB\} \\
&\Rightarrow \quad HS_{initial} \in \{GB,\ CGB,\ EHP2,\ WPB\}
\end{aligned}$$

$$\begin{aligned}
&if \quad HS_{to\_switch} \in \{EHP2,\ WPB\} \\
&\Rightarrow \quad HS_{initial} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WPB\} \\
&and \quad HS_{to\_switch} \neq HS_{initial}
\end{aligned}$$

$$HS_{fixed\_cost\_4} = HS_{a\_investment} \times HS_{time\_factor\_2} + HS_{maintenance} + HS_{different\_switch\_cost} \tag{56}$$

4.1.5 Related formulas for the case V

- *When:*

  - $HS_{initial} \neq HS_{to\_switch}$
  - $HS_{initial} \in \{ER,\ EHP\}$ and $HS_{to\_switch} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WPB\}$
  - $HS_{initial} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WPB\}$ and $HS_{to\_switch} \in \{ER,\ WB,\ EHP\}$
  - $HS_{initial} = WB$ and $HS_{to\_switch} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2\}$

To Switch / Initial	ER	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
ER	×	⊕	⊕	⊕	⊕	⊕	×	⊕	×
GB	⊕	×	×	×	×	×	⊕	×	⊕
CGB	⊕	×	×	×	×	×	⊕	×	⊕
OB	⊕			×	×	×	⊕	×	⊕
COB	⊕			×	×	×	⊕	×	⊕
EHP2	⊕	×	×	×	×	×	⊕	×	⊕
WB	×	⊕	⊕	⊕	⊕	⊕	×	×	×
WPB	⊕	×	×	×	×	×	⊕	×	⊕
EHP	×	⊕	⊕	⊕	⊕	⊕	×	⊕	×

$$
\begin{aligned}
D_{GHC\_V} \quad = \quad & D_{investment\_cost} \times HS_{inflation\_rate\_0} + \frac{D_{investment\_cost} \times HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
& + \frac{D_{uen\_per\_dwelling}}{HS_{efficiency}} \times HS_{energy\_bill} + HS_{maintenance} + D_{insulation\_cost} \\
& + HS_{different\_switch\_cost} + HS_{control\_cost} \\[2ex]
= \quad & \left(HS_{a\_investment} + HS_{b\_investment} \times D_{uen\_per\_dwelling} \right) \\
& \times \frac{HS_{discount\_20} \times HS_{inflation\_rate\_0} + HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
& + D_{uen\_per\_dwelling} \times \frac{HS_{energy\_bill}}{HS_{efficiency}} + HS_{maintenance} + D_{insulation\_cost} \\
& + HS_{different\_switch\_cost} + HS_{control\_cost} \\[2ex]
= \quad & HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_5} \quad\quad\quad\quad\quad (57)
\end{aligned}
$$

$$(58)$$

- *Where:*

$$
D_{unvariable\_cost\_5} \quad = \quad D_{insulation\_cost} + HS_{fixed\_cost\_5} \quad\quad\quad (59)
$$

$$
\begin{aligned}
&if \quad\quad HS_{to\_switch} \in \{ER,\ WB,\ EHP\} \\
&\Rightarrow \quad\quad HS_{initial} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WPB\} \\[1.5ex]
&if \quad\quad HS_{to\_switch} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2\} \\
&\Rightarrow \quad\quad HS_{initial} \in \{ER,\ WB,\ EHP\} \\[1.5ex]
&if \quad\quad HS_{to\_switch} = WPB \\
&\Rightarrow \quad\quad HS_{initial} \in \{ER,\ EHP\}
\end{aligned}
$$

$$
HS_{fixed\_cost\_5} \quad = \quad HS_{fixed\_cost\_4} + HS_{control\_cost} \quad\quad\quad (60)
$$

4.1.6 Related formulas for the case VI

- *When:*

  – $HS_{initial} \neq HS_{to\_switch}$
  – $HS_{initial} \in \{OB,\ COB\}$ *and* $HS_{to\_switch} \in \{GB,\ CGB\}$

To Switch / Initial	ER	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
ER	×	×	×	×	×	×	×	×	×
GB	×	×	×	×	×	×	×	×	×
CGB	×	×	×	×	×	×	×	×	×
OB	×	⊕	⊕	×	×	×	×	×	×
COB	×	⊕	⊕	×	×	×	×	×	×
EHP2	×	×	×	×	×	×	×	×	×
WB	×	×	×	×	×	×	×	×	×
WPB	×	×	×	×	×	×	×	×	×
EHP	×	×	×	×	×	×	×	×	×

$$
\begin{aligned}
D_{GHC\_VI} &= D_{investment\_cost} \times HS_{inflation\_rate\_0} + \frac{D_{investment\_cost} \times HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
&\quad + \frac{D_{uen\_per\_dwelling}}{HS_{efficiency}} \times HS_{no\_gas\_40} + HS_{maintenance} + D_{insulation\_cost} \\
&\quad + HS_{different\_switch\_cost} \\[2mm]
&= \left(HS_{a\_investment} + HS_{b\_investment} \times D_{uen\_per\_dwelling} \right) \\
&\quad \times \frac{HS_{discount\_20} \times HS_{inflation\_rate\_0} + HS_{inflation\_rate\_20}}{HS_{discount\_20}} \\
&\quad + D_{uen\_per\_dwelling} \times \frac{HS_{no\_gas\_40}}{HS_{efficiency}} + HS_{maintenance} + D_{insulation\_cost} \\
&\quad + HS_{different\_switch\_cost} \\[2mm]
&= HS_{ghc\_uen\_condition\_3} + D_{unvariable\_cost\_4}
\end{aligned}
$$

$$(61)$$

$$(62)$$

- *Where:*

$$HS_{ghc\_uen\_condition\_3} = HS_{uen\_cost\_3} \times D_{uen\_per\_dwelling} \tag{63}$$

$$if \quad HS_{to\_switch} \in \{GB, CGB\}$$
$$\Rightarrow \quad HS_{initial} \in \{OB, COB\}$$

$$HS_{uen\_cost\_3} = HS_{b\_investment} \times HS_{time\_factor\_2} + \frac{HS_{no\_gas\_prices}}{HS_{efficiency}} \tag{64}$$

$$HS_{no\_gas\_prices} = \sum_{i=1}^{40} \frac{HS_{specific\_gas\_price} \times (1 + HS_{annual\_variation})^{i+year-2} + HS_{carbon\_tax}}{(1 + D_{discount\_rate})^i}, \; year \in [1, 20] \tag{65}$$

4.2 Related formulas to the dwellings with collective heating system

4.2.1 Related formulas for the case I

- *When:*

  – $HS_{initial} = HS_{to\_switch}$

To Switch / Initial	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
GB	$\oplus$							
CGB		$\oplus$						
OB			$\oplus$					
COB				$\oplus$				
EHP2					$\oplus$			
WB						$\oplus$		
WPB							$\oplus$	
EHP								$\oplus$

$$D_{GHC\_I} = HS_{ghc\_uen\_condition\_1} + D_{unvariable\_cost\_1} \tag{66}$$

- *Where:*

$$HS_{ghc\_uen\_condition\_1} \quad = \quad HS_{uen\_cost\_1} \times D_{uen\_per\_dwelling} \tag{67}$$

$$if \quad HS_{to\_switch} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\}$$
$$\Rightarrow \quad HS_{initial} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\}$$
$$and \quad HS_{to\_switch} = HS_{initial}$$

$$D_{unvariable\_cost\_1} \quad = \quad D_{insulation\_cost} + HS_{fixed\_cost\_1} \tag{68}$$

$$if \quad HS_{to\_switch} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\}$$
$$\Rightarrow \quad HS_{initial} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WB,\ WPB,\ EHP\}$$
$$and \quad HS_{to\_switch} = HS_{initial}$$

4.2.2 Related formulas for the case II

- *When:*

  - $HS_{initial} \neq HS_{to\_switch}$
  - $HS_{initial} \in \{GB, CGB\}$ and $HS_{to\_switch} \in \{GB, CGB\}$
  - $HS_{initial} \in \{OB, COB\}$ and $HS_{to\_switch} \in \{OB, COB\}$

To Switch Initial	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
GB	$\times$	$\oplus$						
CGB	$\oplus$	$\times$						
OB			$\times$	$\oplus$				
COB			$\oplus$	$\times$				
EHP2					$\times$			
WB						$\times$		
WPB							$\times$	
EHP								$\times$

$$D_{GHC\_II} \quad = \quad HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_2} \tag{69}$$

- *Where:*

$$HS_{ghc\_uen\_condition\_2} \quad = \quad HS_{uen\_cost\_2} \times D_{uen\_per\_dwelling} \qquad (70)$$

$$
\begin{aligned}
&if \quad && HS_{to\_switch} \in \{OB, COB, EHP2, WB, WPB, EHP\} \\
&\Rightarrow && HS_{initial} \in \{GB, CGB, OB, COB, EHP2, WB, WPB, EHP\} \\
&and \quad && HS_{to\_switch} \neq HS_{initial}
\end{aligned}
$$

$$
\begin{aligned}
&if \quad && HS_{to\_switch} \in \{GB, CGB\} \\
&\Rightarrow && HS_{initial} \in \{GB, CGB, EHP2, WB, WPB, EHP\} \\
&and \quad && HS_{to\_switch} \neq HS_{initial}
\end{aligned}
$$

$$D_{unvariable\_cost\_2} \quad = \quad D_{insulation\_cost} + HS_{fixed\_cost\_2} \qquad (71)$$

$$
\begin{aligned}
&if \quad && HS_{to\_switch} \in \{GB, CGB\} \\
&\Rightarrow && HS_{initial} \in \{GB, CGB\} \\
&and \quad && HS_{to\_switch} \neq HS_{initial}
\end{aligned}
$$

$$
\begin{aligned}
&if \quad && HS_{to\_switch} \in \{OB, COB\} \\
&\Rightarrow && HS_{initial} \in \{OB, COB\} \\
&and \quad && HS_{to\_switch} \neq HS_{initial}
\end{aligned}
$$

4.2.3   Related formulas for the case III

- *When:*

  - $HS_{initial} \neq HS_{to\_switch}$
  - $HS_{initial} \in \{WB, WPB\}$ and $HS_{to\_switch} \in \{WB, WPB\}$

To Switch / Initial	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
GB	$\times$	$\times$						
CGB	$\times$	$\times$						
OB			$\times$	$\times$				
COB			$\times$	$\times$				
EHP2					$\times$			
WB						$\times$	$\oplus$	
WPB						$\oplus$	$\times$	
EHP								$\times$

$$D_{GHC\_III} = HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_3} \tag{72}$$

$$\tag{73}$$

- *Where:*

$$D_{unvariable\_cost\_3} = D_{insulation\_cost} + HS_{fixed\_cost\_3} \tag{74}$$

$$\begin{aligned} if & \quad HS_{to\_switch} \in \{WB,\ WPB\} \\ \Rightarrow & \quad HS_{initial} \in \{WB,\ WPB\} \\ and & \quad HS_{to\_switch} \neq HS_{initial} \end{aligned}$$

4.2.4   Related formulas for the case IV

- *When:*

  - $HS_{initial} \neq HS_{to\_switch}$
  - $HS_{initial} \in \{GB, CGB\}$ and $HS_{to\_switch} \in \{OB, COB, EHP2, WPB\}$
  - $HS_{initial} \in \{OB, COB\}$ and $HS_{to\_switch} \in \{EHP2, WPB\}$
  - $HS_{initial} \in \{EHP2, WPB\}$ and $HS_{to\_switch} \in \{GB, CGB, OB, COB, EHP2, WPB\}$
  - $HS_{initial} \in \{WB, EHP\}$ and $HS_{to\_switch} \in \{WB, EHP\}$

To Switch / Initial	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
GB	×	×	⊕	⊕	⊕		⊕	
CGB	×	×	⊕	⊕	⊕		⊕	
OB			×	×	⊕		⊕	
COB			×	×	⊕		⊕	
EHP2	⊕	⊕	⊕	⊕	×		⊕	
WB						×	×	⊕
WPB	⊕	⊕	⊕	⊕	⊕	×	×	
EHP						⊕		×

$$D_{GHC\_IV} = HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_4} \tag{75}$$

- *Where:*

$$D_{unvariable\_cost\_4} \quad = \quad D_{insulation\_cost} + HS_{fixed\_cost\_4} \tag{76}$$

$$if \quad HS_{to\_switch} \in \{GB,\ CGB\}$$
$$\Rightarrow \quad HS_{initial} \in \{OB,\ COB,\ EHP2,\ WPB\}$$

$$if \quad HS_{to\_switch} \in \{OB,\ COB\}$$
$$\Rightarrow \quad HS_{initial} \in \{GB,\ CGB,\ EHP2,\ WPB\}$$

$$if \quad HS_{to\_switch} \in \{EHP2,\ WPB\}$$
$$\Rightarrow \quad HS_{initial} \in \{GB,\ CGB,\ OB,\ COB,\ EHP2,\ WPB\}$$
$$and \quad HS_{to\_switch} \neq HS_{initial}$$

$$if \quad HS_{to\_switch} \in \{WB,\ EHP\}$$
$$\Rightarrow \quad HS_{initial} \in \{WB,\ EHP\}$$
$$and \quad HS_{to\_switch} \neq HS_{initial}$$

4.2.5 Related formulas for the case V

- *When:*

    - $HS_{initial} \neq HS_{to\_switch}$
    - $HS_{initial} \in \{GB, CGB, OB, COB, EHP2\}$ and $HS_{to\_switch} \in \{WB, EHP\}$
    - $HS_{initial} = WB$ and $HS_{to\_switch} \in \{GB, CGB, OB, COB, EHP2\}$
    - $HS_{initial} = WPB$ and $HS_{to\_switch} = EHP$
    - $HS_{initial} = EHP$ and $HS_{to\_switch} \in \{GB, CGB, OB, COB, EHP2, WPB\}$

To Switch / Initial	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
GB	×	×	×	×	×	⊕	×	⊕
CGB	×	×	×	×	×	⊕	×	⊕
OB			×	×	×	⊕	×	⊕
COB			×	×	×	⊕	×	⊕
EHP2	×	×	×	×	×	⊕	×	⊕
WB	⊕	⊕	⊕	⊕	⊕	×	×	×
WPB	×	×	×	×	×	×	×	⊕
EHP	⊕	⊕	⊕	⊕	⊕	×	⊕	×

$$D_{GHC\_V} \quad = \quad HS_{ghc\_uen\_condition\_2} + D_{unvariable\_cost\_5} \tag{77}$$

- *Where:*

$$D_{unvariable\_cost\_5} \quad = \quad D_{insulation\_cost} + HS_{fixed\_cost\_5} \tag{78}$$

$$if \quad HS_{to\_switch} \in \{GB, CGB, OB, COB, EHP2\}$$
$$\Rightarrow \quad HS_{initial} \in \{WB, EHP\}$$

$$if \quad HS_{to\_switch} = WB$$
$$\Rightarrow \quad HS_{initial} \in \{GB, CGB, OB, COB, EHP2\}$$

$$if \quad HS_{to\_switch} = WPB$$
$$\Rightarrow \quad HS_{initial} = EHP$$

$$if \quad HS_{to\_switch} = EHP$$
$$\Rightarrow \quad HS_{initial} \in \{GB, CGB, OB, COB, EHP2, WPB\}$$

4.2.6 Related formulas for the case VI

- *When:*

    - $HS_{initial} \neq HS_{to\_switch}$
    - $HS_{initial} \in \{OB, COB\}$ and $HS_{to\_switch} \in \{GB, CGB\}$

To Switch / *Initial*	GB	CGB	OB	COB	EHP2	WB	WPB	EHP
GB	×	×	×	×	×	×	×	×
CGB	×	×	×	×	×	×	×	×
OB	⊕	⊕	×	×	×	×	×	×
COB	⊕	⊕	×	×	×	×	×	×
EHP2	×	×	×	×	×	×	×	×
WB	×	×	×	×	×	×	×	×
WPB	×	×	×	×	×	×	×	×
EHP	×	×	×	×	×	×	×	×

$$D_{GHC\_VI} \quad = \quad HS_{ghc\_uen\_condition\_3} + D_{unvariable\_cost\_4} \tag{79}$$

- *Where:*

$$HS_{ghc\_uen\_condition\_3} = HS_{uen\_cost\_3} \times D_{uen\_per\_dwelling} \quad (80)$$

$$if \quad HS_{to\_switch} \in \{GB,\ CGB\}$$
$$\Rightarrow \quad HS_{initial} \in \{OB,\ COB\}$$