Department of Statistics

STAT 4261/5261

Final Project of Stats for Finance

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1 Summary

In this project, we have implemented a statistical analysis of a set of 15 stocks.
We scrapped from Yahoo finance the monthly closing prices of the 15 following stocks from September 2016 to November 2021 (using the **tidyquant** library on R) : Apple, Netflix, Amazon, JP Morgan, Sony, Visa, Mastercard, Microsoft, Adobe, Tesla, Rakuten, Walmart, Dow Jones, Nasdaq, Alibaba.
Our analysis has revealed interesting information that we will deepen in the next section :

- The stocks we chose have well performed during last five years, especially Tesla, Netflix and Amazon
- Most of our stocks have monthly returns normally distributed
- Covid-19 epidemic has significantly affected the market
- Diversification of investments allows to considerably reduce volatility and the probability of an extreme loss
- When we target a low expected return, investing a high part of our capital in a risk free asset is a good and safe strategy.
- To reduce dimension, we can cluster our company per sector
- Joint distribution of the monthly returns of our stocks is close to multivariate normal distribution.

2 Descriptive Statistics

2.1 Sample statistics

	Apple	Netflix	Amazon	JP Morgan	Sony
Mean	3.231	3.523	2.640	1.658	2.341
Std	8.4	10.0	8.0	7.0	6.8
Skewness	-0.246	0.871	0.363	-0.315	-0.083
Kurtosis	2.6	5.3	4.2	4.5	2.5
Sharpes's Ratio	0.33	0.31	0.27	0.17	0.28
	Visa	Mastercard	Microsoft	Adobe	Tesla
Mean	1.558	2.070	2.986	3.209	5.761
Std	6.0	6.8	5.2	6.9	22.2
Skewness	-0.245	-0.502	0.158	-0.157	-0.036
Kurtosis	2.6	3.4	3.2	2.5	6.2
Sharpes's Ratio	0.19	0.24	0.49	0.40	0.34
	Rakuten	Walmart	Dow Jones	Nasdaq	Alibaba
Mean	0.078	1.216	1.128	1.864	0.785
Std	10.0	5.2	4.5	4.9	9.9
Skewness	0.488	-0.309	-0.625	-0.228	-0.093
Kurtosis	2.4	3.3	4.4	3.6	2.5
Sharpes's Ratio	-0.04	0.15	0.15	0.29	0.03

FIGURE 1 – Monthly returns statistics

The chart above summarizes important statistics of the monthly returns of our stocks. We can notice that all of our assets have a positive average monthly return : for some assets, this value is high (almost 6 % for Tesla !) whereas for others it is very low (less than 0.1 % for Rakuten). The risk differs a lot from one asset to another : it can reach high amd low values (22.2 for Tesla and less than 5 for Dow Jones and Nasdaq).
Regarding skewness and kurtosis, most of our stocks have skewness close to 0 and kurtosis close to 3 : this is a first hint that they follow a normal distribution (excepted Tesla and Netflix which have a kurtosis of 6.2 and 5.3).

	Apple	Netflix	Amazon	JP Morgan	Sony
Annual Mean	38.77	42.27	31.68	19.89	28.09
Annual Std	29	35	28	24	23
	Visa	Mastercard	Microsoft	Adobe	Tesla
Annual Mean	18.70	24.85	35.83	38.51	85.03
Annual Std	21	24	18	24	67
	Rakuten	Walmart	Dow Jones	Nasdaq	Alibaba
Annual Mean	0.93	14.59	13.54	22.36	9.42
Annual Std	35	18	16	17	34

FIGURE 2 – Annual returns statistics

The differences of returns among the stocks are even more obvious : we notice for example a mean annual growth of 85.03 % for Tesla whereas it is less than 1% for Rakuten!

The above charts summarizes the mean annual returns and annual standard deviation of our stocks.

2.2 Evolution of the stocks

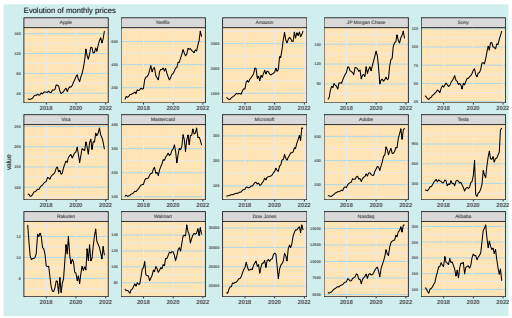


FIGURE 3 – Evolution of prices from 09/30/2016 to 11/30/201

We observe on this figure that most of our assets have considerably increased from September 2016. However, we notice a significant fall in approximately March 2020. We deduced that most of our assets have been affected by the Covid-19 epidemic and the quarantine between March and May 2020 all around the world.
Nevertheless, we also observe that this period corresponds to the beginning of an amazing rise for two companies : Netflix and Amazon. This is easily understandable : indeed, when people were locked at home, the consumption of film has grown drastically which explains the evolution of the Netflix stock. Concerning Amazon, people were required to make their purchases online, causing the rise of Amazon stock.

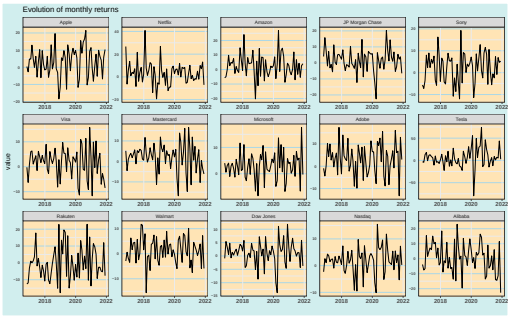


FIGURE 4 – Monthly returns from 10/30/2016 to 11/30/201

This graph shows the monthly returns of our stocks. Again, we observe the effects of the Covid-19 during Spring 2020 : excepted Netflix and Amazon, all the assets have (very) negative returns during this period. We can also observe the impact of other events on these curve : Apple stock has significantly fallen in November 2018, which corresponds to the publication of the bad result concerning the Iphone sales : it shows how sensitive is the Apple stock to the Iphone sales.

Finally, to deepen our study on the particular period at the beginning of 2020 due to the Covid-19, let focus on the equity curves of our stocks from January to June 2020.

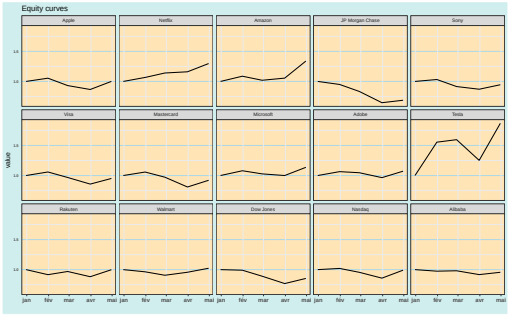
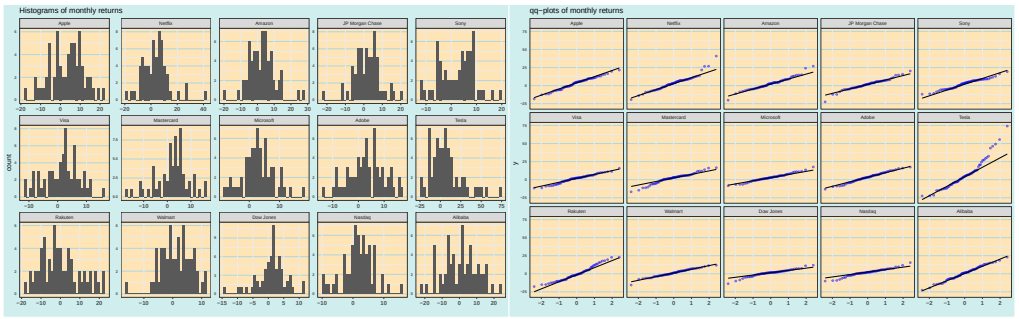


FIGURE 5 – The evolution of 1 $ from January to June 2020

These plots allow to confirm what we observed previously : for most of our stocks, investing during that period would not have been profitable. However, we observe again that for Netflix and Amazon, at the opposite of the others stocks, investing at this period would have been beneficial.

2.3 Distribution of returns

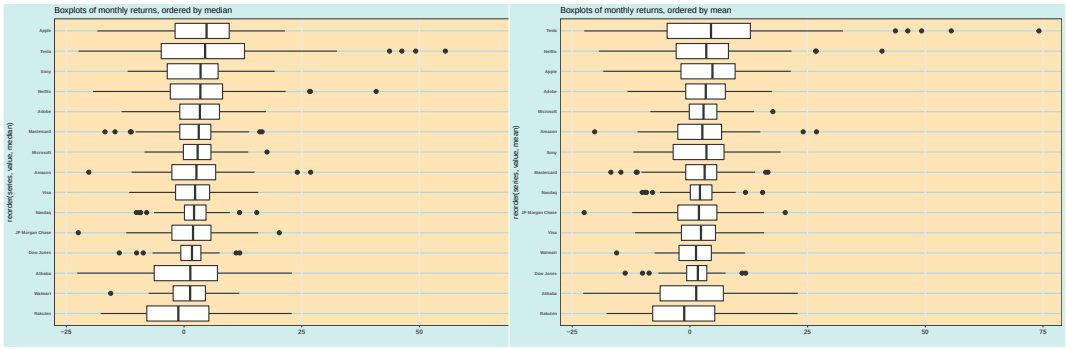
FIGURE 6 – Histograms & qq-plots



(a) Histograms of stocks **(b)** qq-plots

These figures allow to deepen our study about the normality of our data. Previously by analyzing skewness and kurtosis, we did the assumptions that our stocks have a "gaussian" behaviour excepted Netflix and Tesla.
Moreover, most of our returns histograms have a "gaussian" shape, excepted Netflix and Tesla. However, it allows to detect that Sony's stock does not follow a normal distribution.
The qq-plots are more precise : they confirm that Tesla and Netflix have not a normal distribution but also catch a doubt over the normality of other stocks like Mastercard, Sony or Rakuten.

FIGURE 7 – Boxplots



(a) boxplots of stocks ordered by median **(b)** boxplots of stocks ordered by median

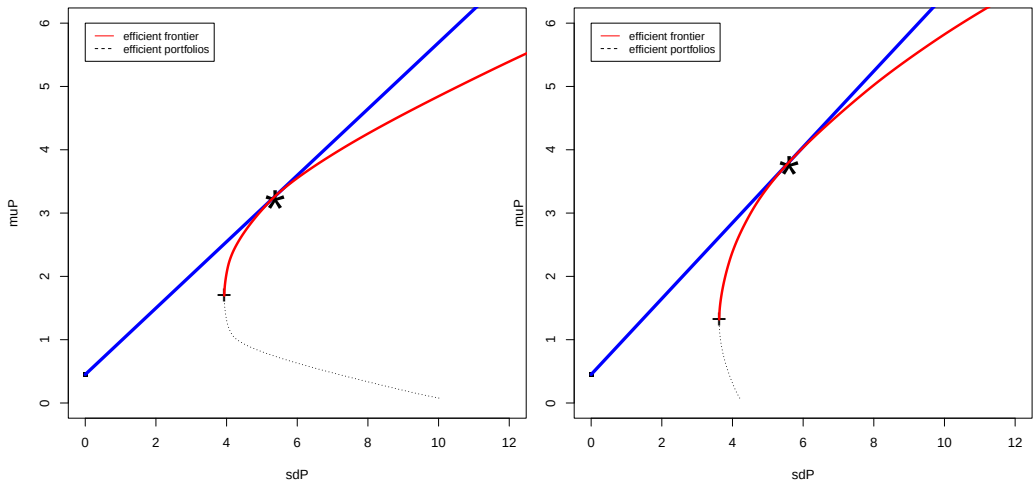
We can observe on the boxplots above that our stocks contains outliers : some have several outliers returns, like Tesla and Mastercard whereas some do not have any outliers like Apple or Rakuten.
The figure 7A) represents the boxplots ordered by median whereas the figure 7B) represents the boxplots ordered by mean. It is interesting to notice that when ordered by median, some stocks are at the top of the rank, which reveals a certain regularity of the values of the monthly returns. For example it is frequent that Apple has the highest monthly returns compared to other stocks. However, when ordered by mean, the ranking

is different : Apple is third and Tesla is first, followed by Netflix. Indeed, it is a consequence of some abnormal high returns (outliers) which allows to Tesla and Netflix to be the assets with the highest average monthly returns.

Also, we notice that companies in the tech field are on the top of the figures above. Indeed, the companies with the highest mean and median returns are Apple, Tesla, Netflix whereas companies of other sectors like JP Morgan or Walmart have low average monthly returns. Thus, we conclude the tech industry is in an amazing growth period compared to other sectors.

3 Portfolio Theory

FIGURE 8 – Expected return versus risk



(a) Short not allowed

(b) Short allowed

The figure 7A) represents the expected return of portfolios versus their risk when shorting is not allowed.
The minimum variance portfolio is represented by a + and the tangency portfolio by a *.

The figure 7B) represents the expected return of portfolios versus their risk when shorting is allowed.
To obtain coherent portfolios, without extreme values of short, we decided to limit the short on an asset to 10%.

Without short	Min Var	Tangency
Mean	1.67	3.27
Std	3.93	5.37
Sharpe's ratio	0.31	0.52
Value at Risk	4329	5748
With short	Min Var	Tangency
Mean	1.30	3.92
Std	3.62	5.67
Sharpe's ratio	0.23	0.61
Value at Risk	4902	5748

FIGURE 9 – Statistics of portfolios (5 % VaR with $S_0 = 100,000$ over a one month investment horizon)

The differences between the statistics when shorting is allowed or not are interesting. Indeed, when we allow shorting, the investors have less constraints to respect, which explains that the minimum variance portfolio is less risky when shorting is possible. Similarly, the tangency portfolio has a lower Sharpe's ratio when we cannot short.

We also notice that the risks of these two portfolios are significantly lower than the risk of each stocks (figure 1), although theirs expected returns stay comparable to the expected returns of each stock : diversification when investing is very useful to make risk decrease while maintaining a reasonable return .
When considering the Sharpe's ratios of each individual stocks and each portfolios, the advantages of diversification seem obvious : the Sharpe's ratios of our portfolios is higher than for each assets, especially the one of the tangency portfolio.

	Apple	Netflix	Amazon	JP Morgan	Sony
VaR	11599	9045	7821	8585	8526
	Visa	Mastercard	Microsoft	Adobe	Tesla
VaR	8642	11157	6545	8807	14866
	Rakuten	Walmart	Dow Jones	Nasdaq	Alibaba
VaR	13603	5945	6605	7856	11422

FIGURE 10 – 5% non parametric Value at Risk with $S_0 = 100,000$ over a one month investment horizon

The comparison between 5% Value at Risk with initial investment of $100,000 over a one month investment horizon also illustrates perfectly how diversifying asset in a portfolio is profitable for investors : for our four portfolios (minimum variance and tangency portfolios with and without shorting), the VaR is significantly smaller than for each individual assets.

4 Asset Allocation

In this section, we will study the assets allocation when we want efficient portfolios which achieve a target expected return of 6% per year.

	Without risk free asset	With risk free asset
Std	6.83	0.095
Value at Risk	8503	5.37
Expected Shortfall	10528	0.52

FIGURE 11 – Statistics of portfolios with targeted 6% expected return

First of all, it is interesting to see in the chart above the huge difference between the indicators of the two portfolios. We can see a significant decrease of the risk measures (standard deviation, Value at Risk and Expected Shortfall) when we combine our risky assets with T-Bills.
Indeed, it is due to the proximity of the monthly risk free rate (0.45%) and the monthly target expected return (0.5 %) : to achieve such a target, we just need to allocate a low proportion of our capital to risky assets, and a high proportion to the T-Bills. That is why the risk measures stay so low.

	T-Bills	Apple	Netflix	Amazon
Weights	98.22	0	0.18	0
	JP Morgan	Sony	Visa	Mastercard
Weights	0	0.26	0	0
	Microsoft	Adobe	Tesla	Rakuten
Weights	1.20	0	0.14	0
	Walmart	Dow Jones	Nasdaq	Alibaba
Weights	0	0	0	0

FIGURE 12 – Weights (in percents) of portfolios of risky assets and T-Bills with targeted 6% expected return

The chart above confirms our previous prediction after observing the figure 11 : most of our capital have been invested in the risk free asset (98 %).

5 Principal Component Analysis

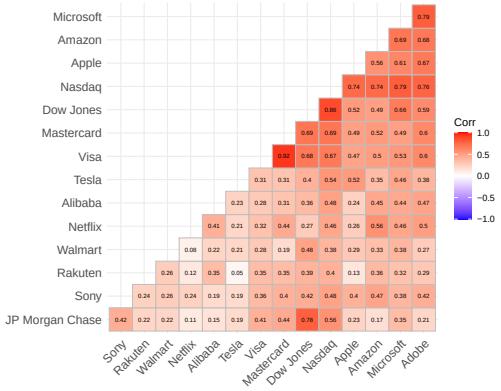


FIGURE 13 – Correlation matrix our monthly returns of the 15 stocks

This heatmap correlation matrix is interesting because we can read on it the companies that are interdependent. Indeed, ad expected, we can see that the returns of Apple are strongly correlated to companies of the same sector (Amazon, Microsoft ...), Visa and Mastercard have a high correlation, like Dow Jones and Nasdaq.

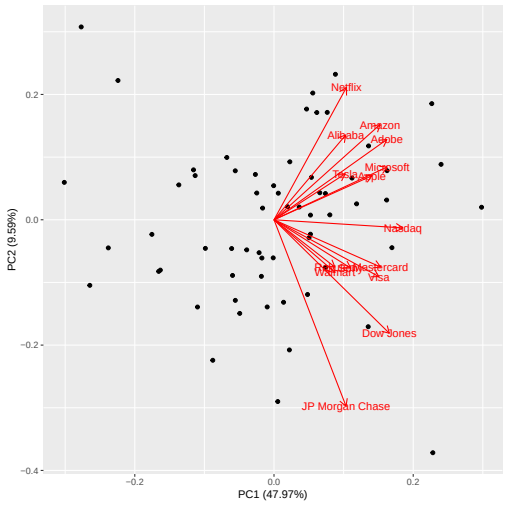


FIGURE 14 – Two directions projection using principal component analysis

The PCA plot (**14**) allows to visualize our previous analysis. The projection of our stocks on the two first components allows to detect two clusters : the tech companies and the others.

6 Risk Management

	Apple	Netflix	Amazon	JP Morgan	Sony
non parametric VaR	11599	9045	7821	8585	8526
parametric VaR	13629	13563	11993	13909	10674
	Visa	Mastercard	Microsoft	Adobe	Tesla
non parametric VaR	8642	11157	6545	8807	14866
parametric VaR	10196	13503	7092	10219	20320
	Rakuten	Walmart	Dow Jones	Nasdaq	Alibaba
non parametric VaR	13603	5945	6605	7856	11422
parametric VaR	15332	8932	9790	9186	17871

FIGURE 15 – 5% Value at Risk with $S_0 = 100,000$ over a one month investment horizon

First, the Value at Risk computed in this chart shows us which stocks are exposed to extreme decreases.
Tesla has the highest VaR, no matter if we consider the parametric or non-parametric estimation. Microsoft is much more steady and has the lowest estimation of VaR, which proves that someone who invests in Microsoft stock is protected from a sudden huge loss.
It is also interesting to analyze differences between parametric and non parametric estimation of the Value at Risk. Indeed, for some stocks like Microsoft or Nasdaq, both estimations are close whereas for some other stocks, like Netflix and Tesla, there is a significant gap between both estimations. We notice that the two latter companies are the ones that we conjectured as non normal in the descriptive statistics section, using qq-plots and histograms (6) and skewness & kurtosis analysis : thus, this difference of estimation is probably due to the fact that the parametric estimation supposes each asset as Normal, whereas Netflix and Tesla are not.

7 Copulas

In this section, we try, using copulas, to model the joint distributions of our 15 stocks.
Using pseudo-likelihood method estimation, we fitted our data to several famous copulas and then determined which copula was the most suited to our stocks.

	Normal	Frank	Clayton	Gumbel	Joe
Maximum likelihood	175.5	133.3	148.5	135.5	100.2

FIGURE 16 – Maximum Likelihood Estimation of several copulas, using pseudo maximum likelihood estimation

The maximum likelihood estimation of our stocks for each copula gives relevant information. Although we previously detected that most of our stocks were normally distributed, we were not able to provide an estimation of the joint distribution of our data.
Copulas are interesting because they allow to provide trustworthy estimations. We tried to fit our data to five famous copulas : Normal, Frank, Clayton, Gumbel and Joe.
Finally, normal copula is the one which fits the best to the 15 stocks because it is the copula with the highest maximum likelihood estimation.

8 Conclusion

To conclude, the analysis of our 15 stocks bring a lot of relevant information and is a primordial step before investing.
Indeed, it is important to know how risky is our investment and several indicators like standard deviation, value at risk and expected shortfall are useful to measure the risk of an investment.
Stocks have similar behaviours in the market when they are in the same industry and that is why it is important investing in companies from different sectors, in order to have uncorrelated assets. Finally, it is very important to have a global understanding of the market and to be informed of the political and economical current situation. Indeed, stock prices are influenced by external events, like the sales of the company (like for Apple in 2018) or other unexpected event like the Covid-19 epidemic.