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Mechatronics 2

ACTIVE DAMPING OF SOLAR PANELS USING PIEZOELECTRIC TRANSDUCERS

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Electromechanical Engineering

Contents

1 Introduction

This report shows the work done to actively damp the five first resonant modes of a solar panel. Everything has been done using MATLAB on a linear model provided in the state-space representation.
This model is based on a finite element simulation, allowing to extract a truncated model including the 10 first vibrations mode.
This is necessary as real collocated systems will always have an infinite number of modes, making computation impossible. The modes of interest for this project are only the first five ones, but as the truncation will introduce some distortion near the cutoff frequency, it has been placed further away from the fifth mode. This allows to keep a good fidelity on all the modes of interest. The 5 last will be ignored for the rest of this report. One can see, on the figure 1, the top view of this analysis showing the patches position and, on the figure 2, the geometry of the panel.

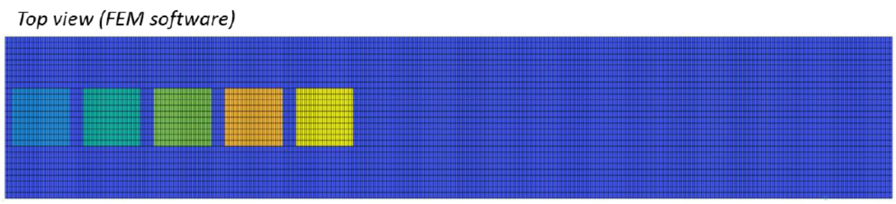

Figure 1: Finite element analysis model

The patches are collocated, meaning that the sensors and actuators are located at the same position. This is a very good feature to have, as it will create an alternating pole/zero configuration in the system, providing very good robustness of the system, when controlled.

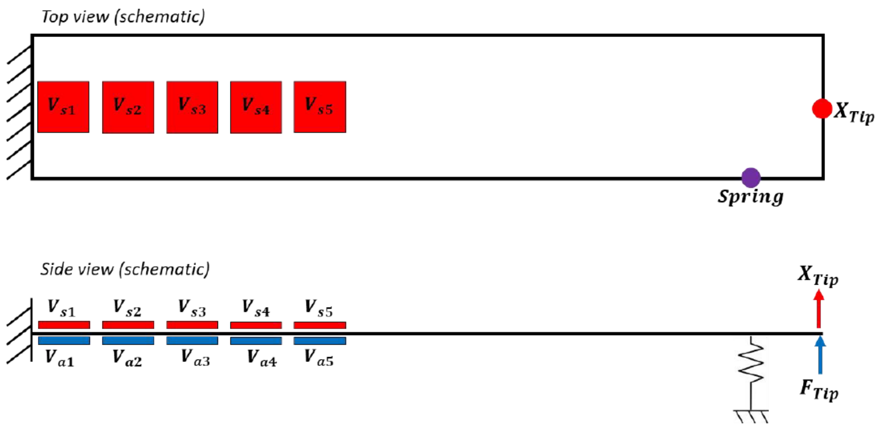

Figure 2: Schematic view of the system, the locations of each sensor and actuators.

To assess the performance of the control laws that will be presented, a performance index will be used. This represents an excitation at $Ftip$, measured at $Xtip$. By inspecting the bode plot, one will be able to assess the damping achieved on each mode, and compare the performances of several control strategies. This pair of sensor-actuator will be not be used in the control laws but only to inspect their performances.
One can also notice that the position of the patches is set and cannot be changed. This will have implication further away on the performances of the controllers. Indeed, poorly placed actuators will never be able to perform well even with very advanced control laws. To place them, one should normally look at the strain map, found using the finite element analysis, to see where each mode is creating the most strain.
As piezoelectric patches will react to deformations, it is important that they are placed at certain locations where these strains are high. Unfortunately, the modes are giving very different strain map, and it is therefore hard to place patches that can act on all the modes at once. This is why several patches can be used, allowing to

place each of them at a relevant position, and maximise their performances. The strain map of the first mode is presented in the figure 3.

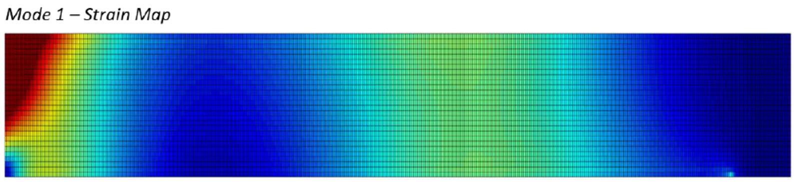

Figure 3: Strain map of the first mode. The dark red area indicates where the highest strains are acting. To be able to actively detect and act on this mode, one must place its patches around this location.

On the figure 4, one can see the bode plot of each of the collocated pair. This will be useful to assess which pair of patches can detect and influence which mode.

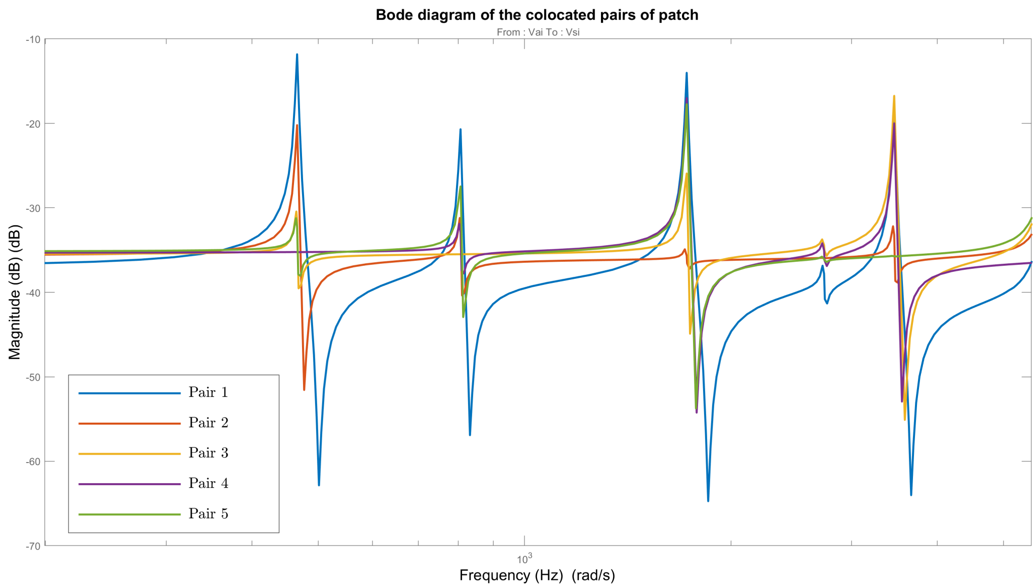

Figure 4: Bode plot of the different pair of patches. The influence that each patch can have on each mode can be compared. When a peak is higher for one pair than for the others, it means that the pair has a greater influence on the mode.

The fourth mode appears to be very badly detected by any of the available patches. As explained earlier, this could be because no patches are placed in an area where the strains due to this mode are high enough. A better choice could be to put a patch where a high deformation is occurring due to this mode. However, as one can see in the figure 5, the patch three is located in an orange region.
The hypothesis made by the group is that the absolute value of the strain at this location is still too low. The consequence would be that this mode will be very hard to damp, regardless the control strategy adopted. This has to be verified, as the scale and the numerical values of the strains were not given.
That is why these strain map will not be further used to assess the capability of a pair of patches to damp a certain mode. It is better to base any further choice on the bode plots of the figure 4 than on these incomplete strain information.

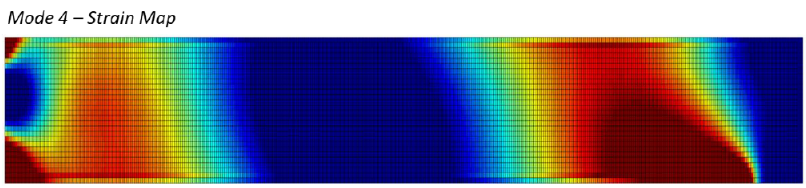

Figure 5: Strain map of the fourth mode. Highest strain zones are not covered by any patches.

Being sure that the mode are well detected and impacted by the patches is normally part of the job of the mechatronic engineer in charge, but for this project it was not possible to tweak the model to modify the patch positions.

2 SISO controllers

In this section, several ”single input single output”, or ”SISO”, controllers will be used. These controllers are used only on one of the patches, that must be selected first.
One can see in the figure 4 that the first pair of patches can affect all the mode of interest (except the 4th one that no patch can properly affect, as explained in the introduction), and is therefore a very good candidate for a SISO controller. One can see that it is indeed the most capable for the 1^{st}, 2^{nd} and 3^{rd} modes, while being reasonably good for the 5^{th} one.
On the figure 6, one can see the control scheme used for the following controller. There is no reference to track, as the goal here is to stabilise the plant around 0. One has to be careful while stating the sign of the controller's used, as there is no sum symbol as in the mode classic control scheme, used with PID for example. If the controller is used in a negative feedback architecture (the more classic type), the minus sign must be visible in the transfer function itself to avoid confusion with the positive feedback based controllers, presented later on.

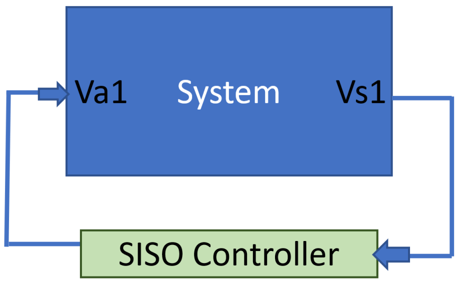

Figure 6: General SISO controller scheme, applied on the first pair of patches.

2.1 Lead and Direct velocity feedback controller

The Lead controller and the Direct Velocity Feedback controller (DVF) are very similar controller as one can see by comparing their transfer function :

DVF :

$$C(s) = -gs \tag{1}$$

Lead :

$$C(s) = -\frac{gs}{s+\omega_p} \tag{2}$$

The difference between these two controllers is that one of them has a proper transfer function (Lead) while the other one does not (DVF).
If the sensor used is measuring speeds, a DVF controller can be used as the controller would just be proportional to the measurement ($u = K * y$). On the other hand if the sensor is not a speed sensor, a Lead controller should be used to ensure causality. Also, as the phase of the system will always tend to decrease when going to higher

frequencies, due to all the delays encountered in the sensors and actuators, it is better to counteract the phase drop induced by the zero of the controller by adding a pole to it.
In the case of this study, piezo electric sensor are used so a Lead controller should be used.

2.1.1 Design

As seen in the previous section, a DVF or a Lead controller consist of a zero at the origin of the root-locus. For the Lead controller a real pole is also present. It must be far enough to not impact the damping capability of the controller but not too far depending on the processing speed of the controller.
To design an active damping controller, one can take a look at the root locus of the closed loop system where each lobe correspond to each mode pole path. By playing with the Gain of the controller, the damping of each mode can be accentuated by moving the pole positions in order to have the highest damping possible.

As seen on Figures 7 and 8, a compromise must be made depending on what is aimed to be damped. On Fig.7, the first mode is aimed and so its damping (its complex angle as seen on the grid) is maximised while the other modes are also damped but not maximised.
On Fig.8 on the other hand, all 5 first modes are aimed but none are maximally damped.

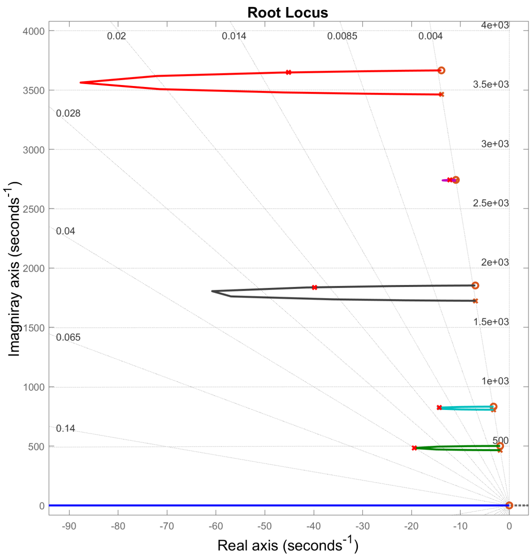

Figure 7: Root Locus of a Lead controller aimed to damp optimally the first mode

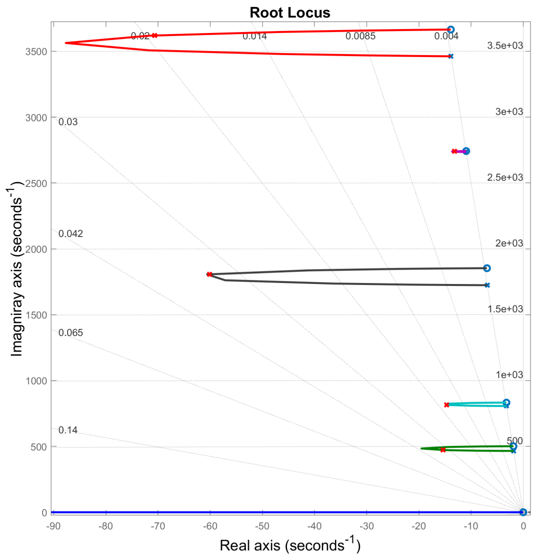

Figure 8: Root Locus of a Lead controller aimed to damp as well as possible the first five modes

The controller designed on Fig.7 was chosen as the first mode has the highest magnitude and thus needs special attention.
As seen on the performance index of the system (Fig.9), this SISO controller actively damps the five first modes with an emphasis on the first mode. As only one patch as been used, this can be a cheap and easy solution depending on the damping necessities.

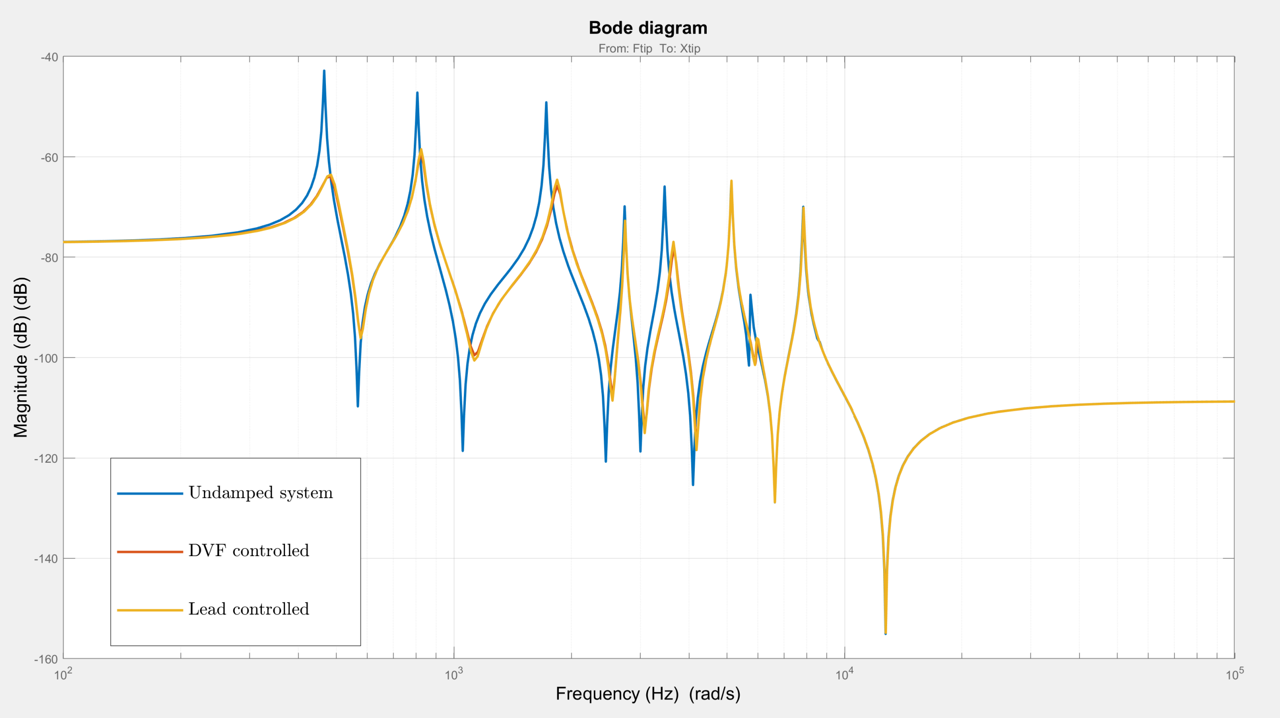

Figure 9: Performance index of the Lead and DVF SISO controllers

One can also note that the performance difference between DVF and Lead controllers is negligible, as expected, because they are numerically nearly identical in terms of damping and theoretically a Lead controller tends to a DVF controller when its pole tends to infinity.

2.2 Positive Position Feedback, second order

The next SISO controller that will be presented is called Positive Position Feedback (PPF) of the second order. As its name indicates, its sign is reverse compared to classical feedback controller, as one can see in the following formula:

$$\frac{+g}{s^2 + 2\xi_f \omega_f s + \omega_f^2} \tag{3}$$

This controller is able to damp a very narrow frequency range, so it has to be tuned precisely in order to be efficient. Moreover, as it is a positive feedback loop, as indicated by the plus sign, so the gain that can be applied is always limited. Indeed, when increasing the gain, one will see further on root locus, see figure 11 that a pole can cross the imaginary axis and become unstable. The stability condition can be expressed mathematically with the following equation.

$$gH(0)G(0) < 1 \tag{4}$$

A reasonable safety margin is also always good to maintain a certain robustness in the design, by not getting too close to violate this equation.
Despite these characteristics, the 2^{order} PPF has a major advantage in case the plant has no high frequency roll-off, i.e. a decrease in the magnitude bode plot at high frequencies.
Indeed, when there is no such natural attenuation, this could limit the performances of any classical controller. As the phase of the system will always tend to decrease when going to higher frequencies, due to all the delays encountered in the sensors actuators, it will at some point cross the -180 degrees line. This is inevitable, and if the magnitude at this location is too close to 0dB, the gain margin of the controlled system will be extremely limited. This is avoided if the magnitude bode plot always decrease when going to higher frequencies. However, this is not the case for collocated piezo/piezo systems, as the one of this project. In this situation, a 2^{order} PPF can be used to raise (due to its positive feedback) the phase of the system. Fortunately, the magnitude bode plot is already far away from 0dB even at low frequencies. This issue is therefore not crucial, but the 2^{nd} order

PPF controller can still be a good candidate to focus the damping on a precise frequency range.

2.2.1 Design parameters

When tuning such controller, one must position a pair of conjugate poles, and then adjust the gain of the controller accordingly. The idea is to try to attract a given pole to the left by adding the pole of the controller in front of it (i.e. at the same frequency). This can be done by changing the resonance frequency of the filter's equation or by inspecting the root locus and moving manually the poles. This later technique is less accurate but allows to directly monitor the shape of the locus. One can see on the figure 10 the desired shape of the root locus. To maximise the damping, the two poles (i.e. the one of the controller and the one of the mode that is targeted) must be at the same location.

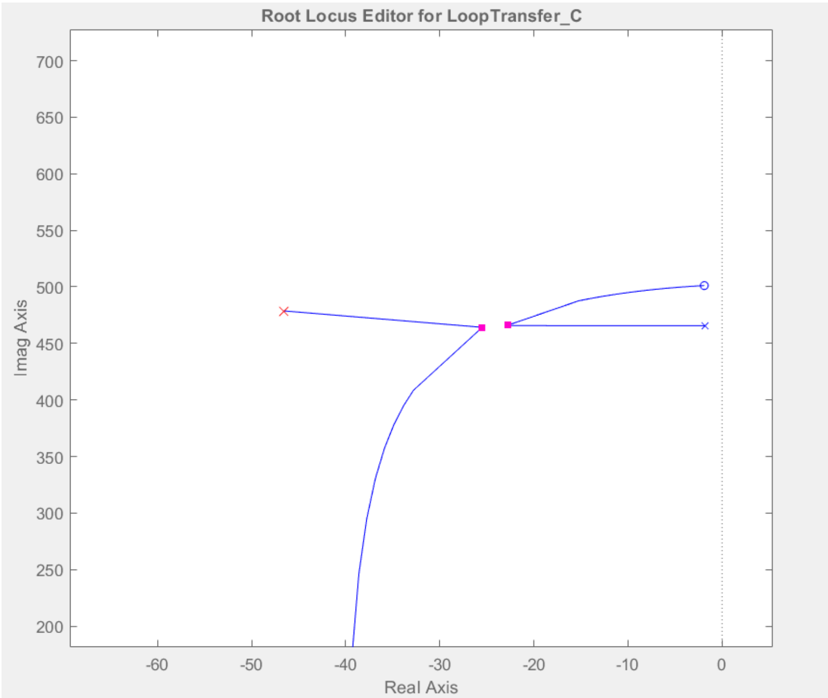

Figure 10: Close view on a root locus with the desired shape, that almost maximise the damping of one mode. In order to have the ideal maximum damping, the two lobes should coincide, and the two poles must be at the exact same location.

Then, by increasing the gain, one can position the two poles at the edge of the lobes, to achieve the best damping.
In practice this can be hard to achieve manually (i.e. with sisotool, moving the pole position before adjusting the gain), but a good result can still be obtained in a reasonable time, as shown on the figure 10 and 11
.

2.2.2 Tuning

For this first SISO controller, the first mode has been targeted, using the first pair of patches as usual.
One can see on the figure 11 that the other modes are not affected by the damping of the first one.

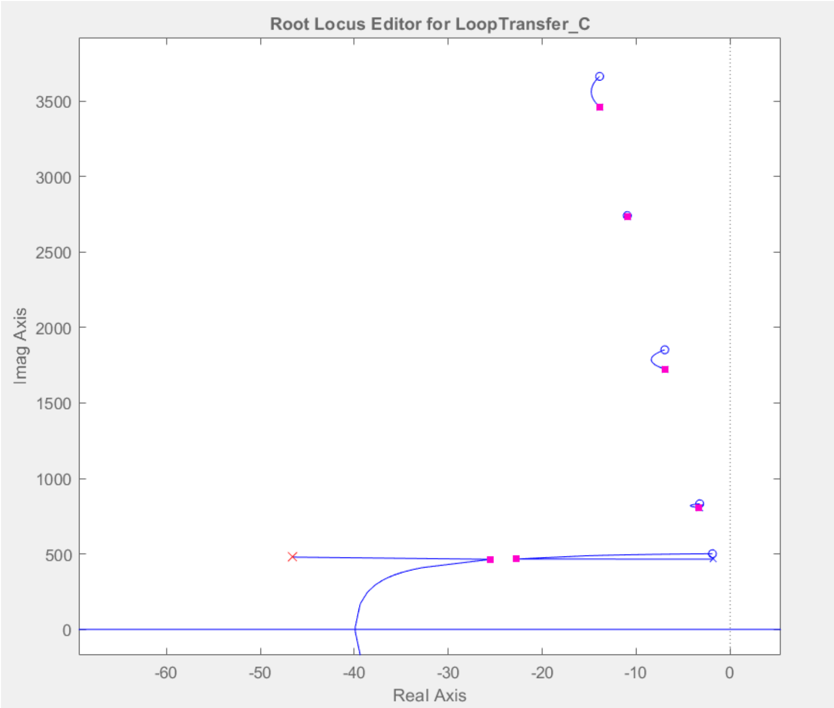

Figure 11: Root locus obtained with a 2^{nd} order PPF tuned on the first mode.

The performance index of this closed loop can be seen on the figure 12.

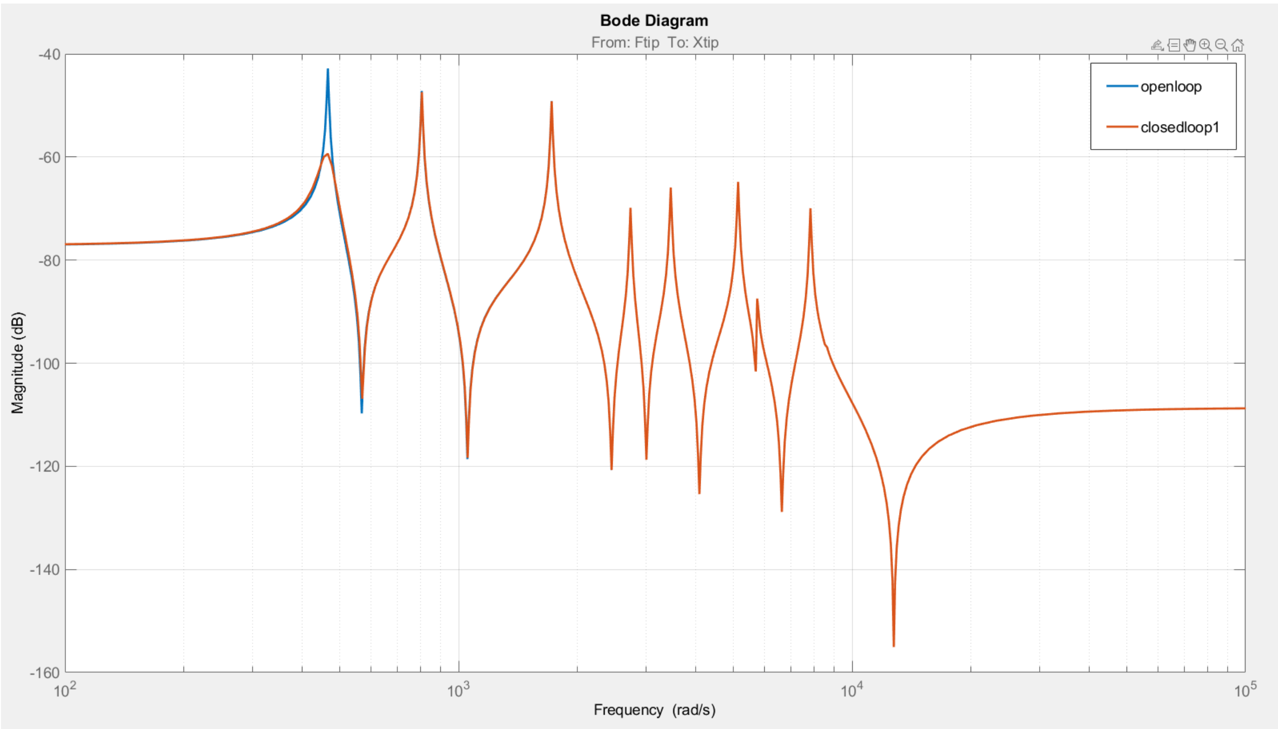

Figure 12: Bode plot of the closed loop obtained with the 2^{nd} order PPF designed to damp the first mode.

Even if the damping of the first mode is rather good, this can obviously not be used as a SISO controller. And since the absence of roll-off do not seem too problematic in this case, one might question the use of this section. The reason it is presented here is rather that it could be useful in a decentralised MIMO controller, to be aimed at damping a specific mode, while others loop are focusing on the other modes, see section 3.
For a more precise tuning, optimisation methods exist to precisely place the conjugate poles at the maximum damping location, such as H_2 or H_∞. However, they can be way more time-consuming to apply, for a very small improvement of the performances, and therefore will not be used for this controller.

2.3 Positive Position Feedback, first order

The last SISO controller that will be presented here is the Positive Position Feedback of first order. As one can deduce from its name, it's the same category of controller as the previous one, but this time with only a real pole instead of a conjugate pair of poles for the second order version. Its transfer function is of the following form :

$$\frac{+g}{1+\tau s} \tag{5}$$

and as for its 2^{nd} order version, the plus sign before the gain is here to indicate the positive sign of the feedback loop.
The advantage of this controller compared to the previous one is that it doesn't have to be tuned on one specific mode. Indeed, it can damp (about) equally every mode, as long as the selected pair of patches can reach it.
For the tuning, there are still two parameters to take into account : the position of the pole and the static gain. As for the 2^{nd} order PPF, a stability condition can be found, as if the gain is set too high, one can lose stability. This can be determined by either applying the following formula

$$gG(0) < 1 \tag{6}$$

or by simply inspecting the poles of the root locus. As the tuning of the controller will be done on the root locus, a special care was put into making sure that no pole is crossing the Imaginary axis. A small gain margin will also always be taken to be sure that if the real plant is a bit different from the model (e.g. evolving in time, dependent of the temperature or simply due to an inaccurate modelling).
As there is only one real pole, the high frequency roll-off will be less abrupt (only 20dB/decade) than for the 2^{nd} order version, but as previously explained, it is not a problem with this plant (as we don't need to add a high frequency roll-off).

2.3.1 Design parameters

This part will cover the design process of a positive feedback controller of the first order. The different trade-off's encountered when tuning will also be interpreted, so that a choice can be easily made depending on the performances one wants to achieve. The two parameters one can change in the controller is τ , i.e. the position of the pole, and the gain g.
When changing these values, the root locus has been inspected closely to monitor the displacement of the poles and the stability margin.

The first observation that was made is that when putting the pole close to the origin, the damping achieved by the controller was very small. Indeed, putting it further away increased the maximal gain one can set before going instable, which allowed to move the poles much more to the left, and by doing that, increase their damping. One can see the root locus of such a case in the figure 13. As usual, these control laws are tested on the first pair of patches, as it is the most indicated one for a SISO controller.

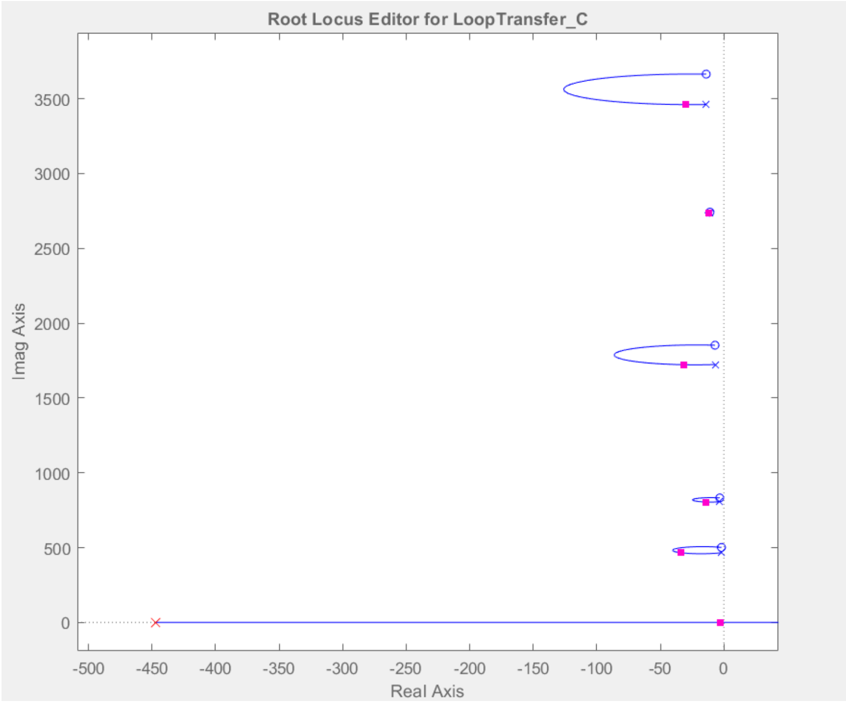

Figure 13: Root locus illustrating the tuning of a PPF controller of the first order, applied on the first pair of patches.

The pole close to zero is the one that must be closely monitored during the tuning as it is very close to crossing the Imaginary axis line, and lead the plant to instability.
Unfortunately, when doing so, an amplification of the low frequencies is occurring, as one can see on the figure 14, representing the bode plot of associated with the root locus shown in the previous figure. This low frequency amplification is due to the value of the gain, that is set very high and close to instability.

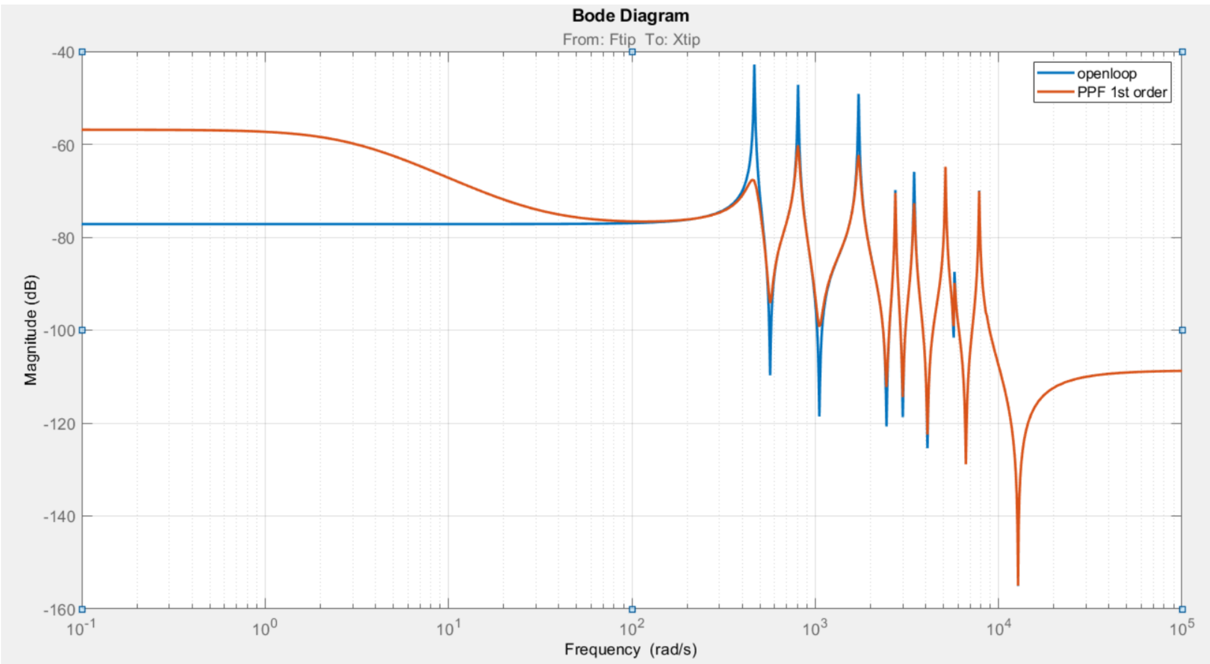

Figure 14: PPF first order bode TO CHANGE FOR MORE READABLE DATA

For this controller, the pole was set around the same frequency as the first mode.
One can see that good damping is achieved in almost all the mode of interest (except the 4$^{\text{th}}$ one, as usual).

2.3.2 Tuning

Now that the design parameters and their effect are known, one can try to find a compromise to design the most adapted controller. Several choices will be compared here.
First, the performances depending on the position of the controller's pole (i.e. the frequency at which the pole is set) will be compared on the figure 15. These plots have all been done with the gain pushed near to the maximal allowed value but still ensuring a small margin, and three values for the pole location.

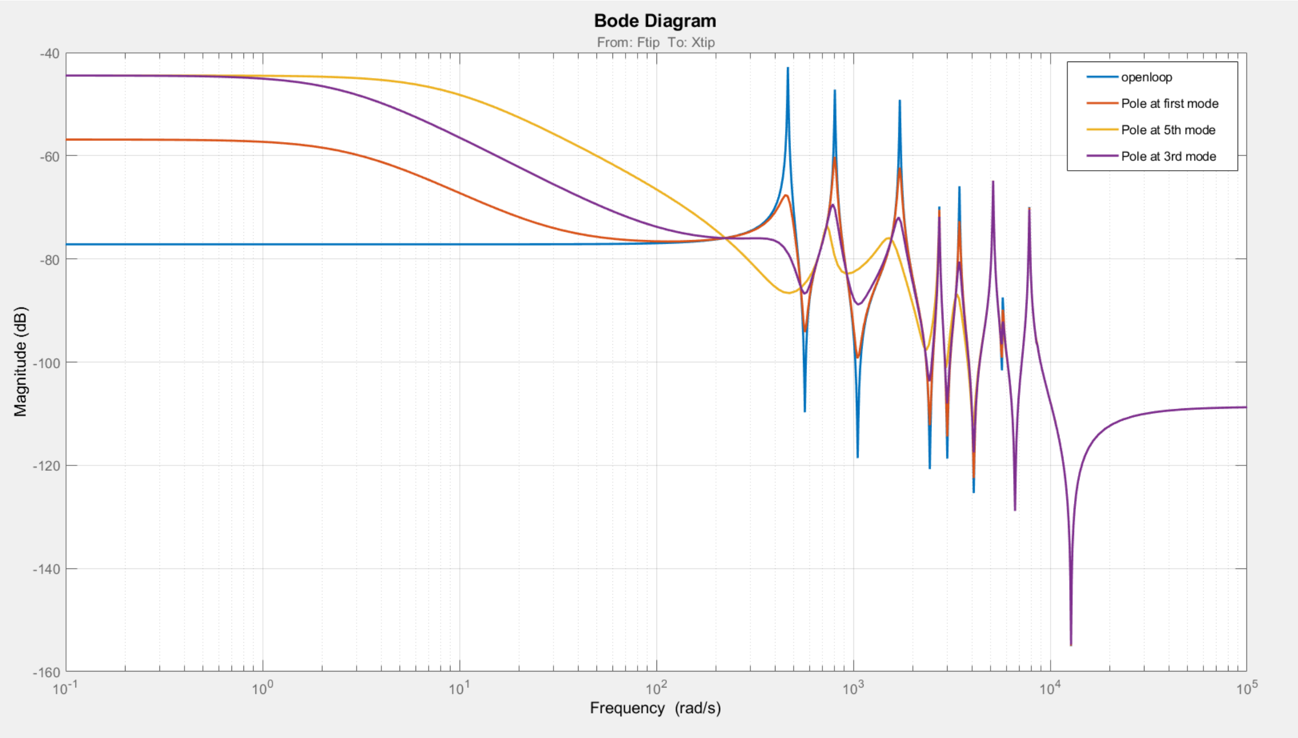

Figure 15: Bode plot comparing the performances of 3 PPF first order controllers, illustrating the influence of the position of the controller's pole. The gain is pushed to the stability limit for all of these controllers.

One can clearly see that the further the pole is set, the higher the damping on each mode. However, the amplification at low frequencies, i.e. the DC gain, is also higher. To deal with that, one can play on the value of the gain. When the pole is set at high frequencies, one can push the gain much higher than for a pole closer to zero. This can help to damp better, but it can also allow to not push the pole too close to the origin when increasing the gain. Indeed, one can see in the figure 16 that good performances can be achieved without being too close to instability.

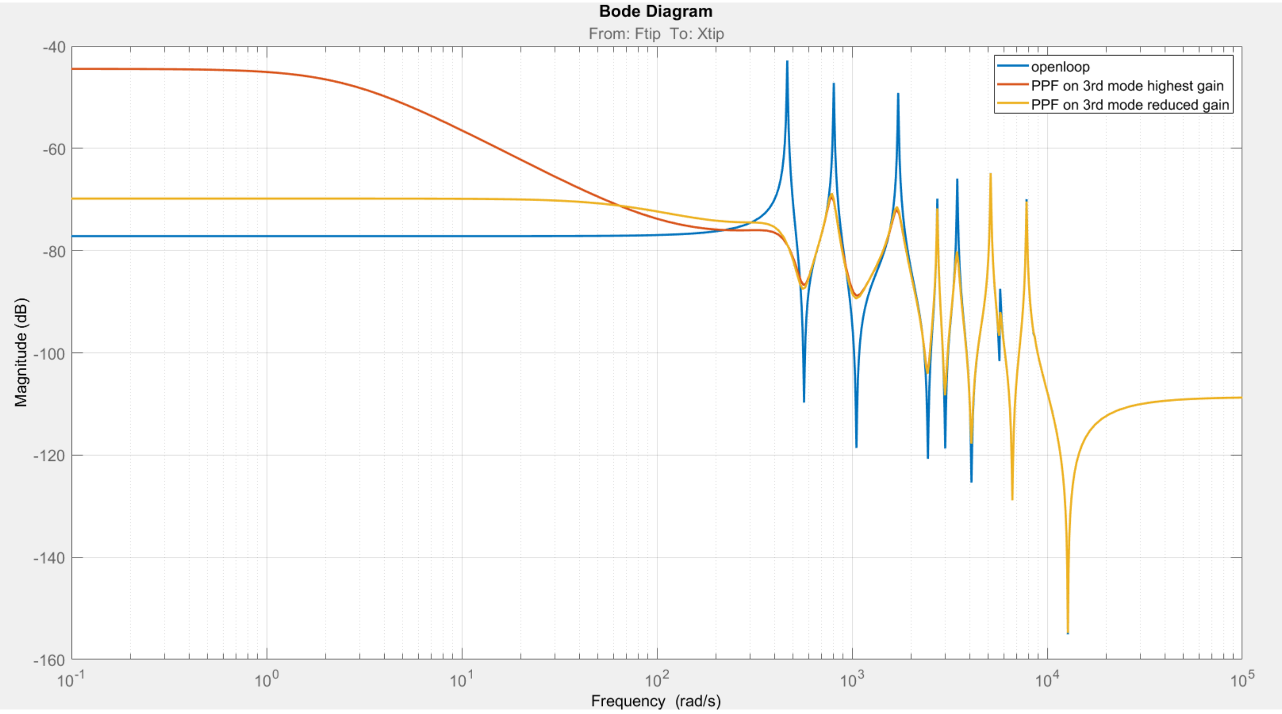

Figure 16: Comparing different values of the gain for the PPF controller, with its pole located around the 3rd mode frequency.

This was not the case when the pole of the controller was set very close to the origin, as the gain needed to be pushed as much as possible to get a good damping of the pole. However, with this last controller, the DC gain stabilised around -55dB compared to the two others, where it was stabilising around a much higher value. This can be a problem as low frequency excitation are very common. It can also be problematic if an object is put on the panel, leading to a static excitation. If this is amplified by the controller, it can lead to some damages on the actuators. It is, therefore, best to keep it relatively low, as a safety measure.

By setting a lower gain, as shown in the root locus of the figure 17, one can see the high damping on several modes, and the location of the real pole, relatively far away from the unstable right plane. In the following paragraphs, this decrease in gain will be referred by a small or bigger gain margin. However, even if this pole is moving to the left quickly, by decreasing by a very little the value of the gain, the gain margin value will not be much higher. It will only help with the low frequency amplification problem described earlier. Indeed, the value of the gain margin will still close to 0dB to achieve a good damping.

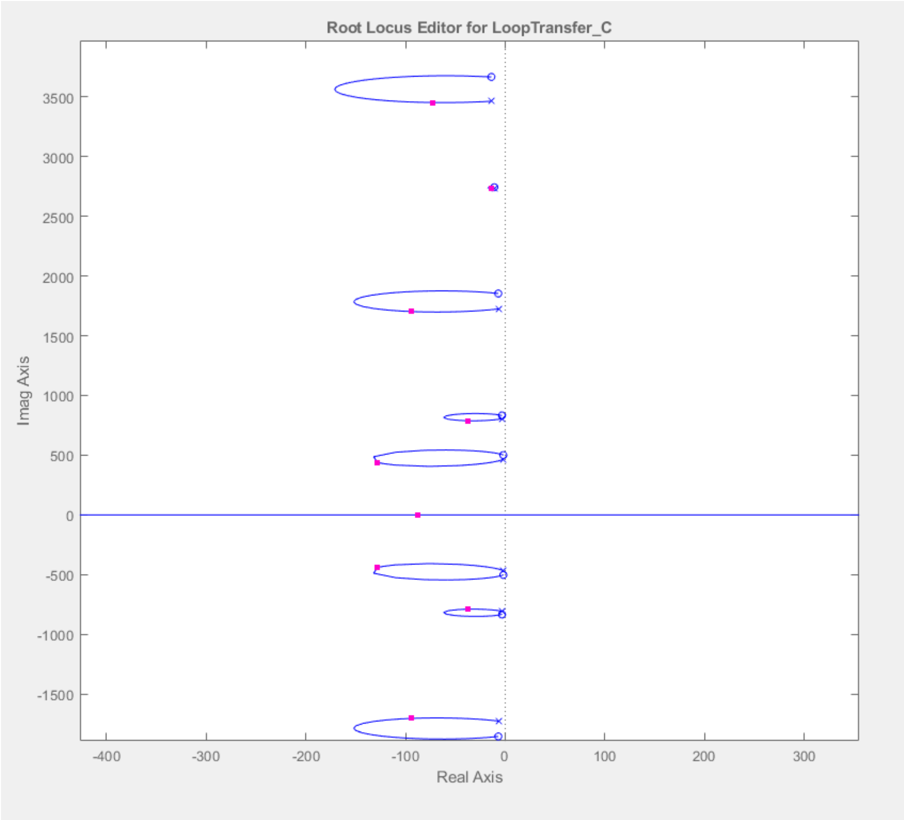

Figure 17: Root locus of the plant and the controller, tuned with its pole around the 3rd mode, and a slightly reduced gain, to ensure low frequency gain is not too high.

As a good trade-off between the performances and this low-frequency amplification can be found, for example the yellow curve on the figure 16. This controller has a pole located at the frequency of the 3^{rd} mode, and moderate gain, allowing to have sufficiently good damping and a rather small rise in the low frequencies. According to different requirements, this choice can be further refined, but for this project, this was considered acceptable.

2.4 Comparison and conclusions on SISO controller

The performance of a well tuned first order PPF seems to be higher than the lead controller and the second order PPF. Indeed, despite being a SISO controller applied on the first pair of patches only, a good damping of four of the five modes of interest is achieved with it, which is not the case for the PPF of second order, that must be designed for one mode only. The lead controller present similar results than the PPF first order, allowing to damp several modes at once. It can also be tuned to focus only one mode, as for the PPF of second order. These two can nevertheless be used in MIMO controllers, where each sub loop would focus on a different mode.
However, one must be careful when implementing a SISO PPF of first order on a real plant, as the gain margin in the simulated model can be rather small, when the performances are pushed to the limit (which was often the case in the presented plots). If the model is not exactly known, or some parameters vary over time and temperature, this could lead to instability and damage to the plant. Moreover, another issue could be that the gain of the controller is too big, and only works in simulation with a linear model. Indeed, if a large gain ask more voltage to the actuators, that the limit they can provide, they will saturate. Not only the control law will not work as good, but having actuators saturated for a long time could also damage them. One must be aware of these issues when designing it, and maybe choose to be a bit more robust, which will decrease the performances.
The fourth mode seems really hard to damp with a SISO controller (any of them). Indeed, none of the patches can really reach this peak. A MIMO controller could combine the effort of several patches to try to effectively damp it. This will be presented in the next section.

3 MIMO controllers

In this section, multiple input multiple output, or MIMO, controllers will be presented. Which means that not only the first pair of patches will be used, but as many as needed. This will further depend on the budget and the requirements for the performances, to see if some improvement is worth the price of additional sensor actuator pairs.
All the following controllers are designed as decentralised controller. This means that the loops are closed between one sensor and its collocated actuator only. Therefore, there will not be any non-collocated base control laws, e.g. between the first actuator and the last sensors. This could be very dangerous, as non-collocated controllers are not robust at all, and could diverge quickly due to the absence of alternating pole and zero. Another advantages of using decentralised MIMO, is that one can tune independently all the SISO controllers, using for example "sisotool" in Matlab.

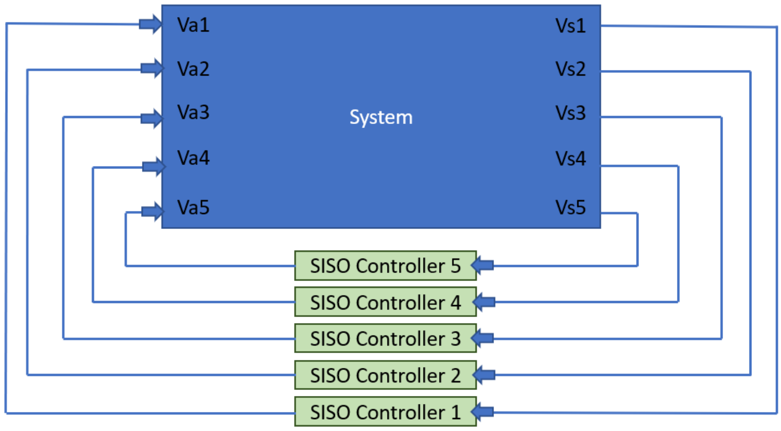

Figure 18: Scheme of a decentralised MIMO controller.

3.1 Lead based decentralised MIMO

3.1.1 Lazy Lead based decentralised MIMO

The first decentralised MIMO controller implemented uses Lead controllers.
The name Lazy Lead based decentralised MIMO was chosen because, as it is sometimes done in real life, after finding that one good controller for the first patch, it was then applied to all other patches.
Although not optimal, it achieves good results and takes a lot less time to design.

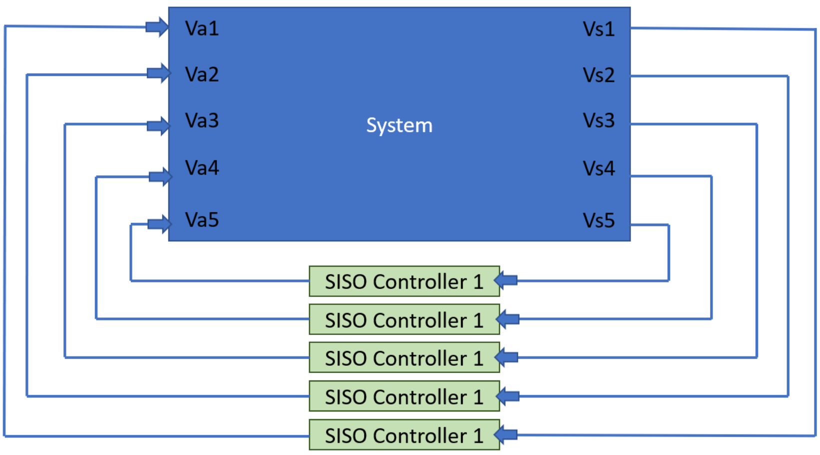

Figure 19: Scheme of a lazy decentralised MIMO controller.

3.1.2 Lead based decentralised MIMO

After designing the lazy MIMO controller, a Lead controller was tuned for each patch focusing specific modes as follows :

Patch	Mode aimed
1	1
2	2
3	5
4	3
5	2

Table 1: SISO controllers designs

As explained in Section 1, none of the patches have a good effect on the 4th mode so none were design to specifically damp it, but it is still damped a bit compared to the undamped system.

It is also worth noticing that some of the patches contribute very little to the damping in this MIMO controller and could be removed for a more cost effective choice.
In the performance index of the system (Fig.20), it can be observed that the Lead MIMO controller achieves a better overall damping on the 5 five modes but that the Lazy Lead MIMO controller achieves a better damping on the first mode. This can be explained by the fact that the controller applied to all the patches in the Lazy MIMO focus more or less the first mode.

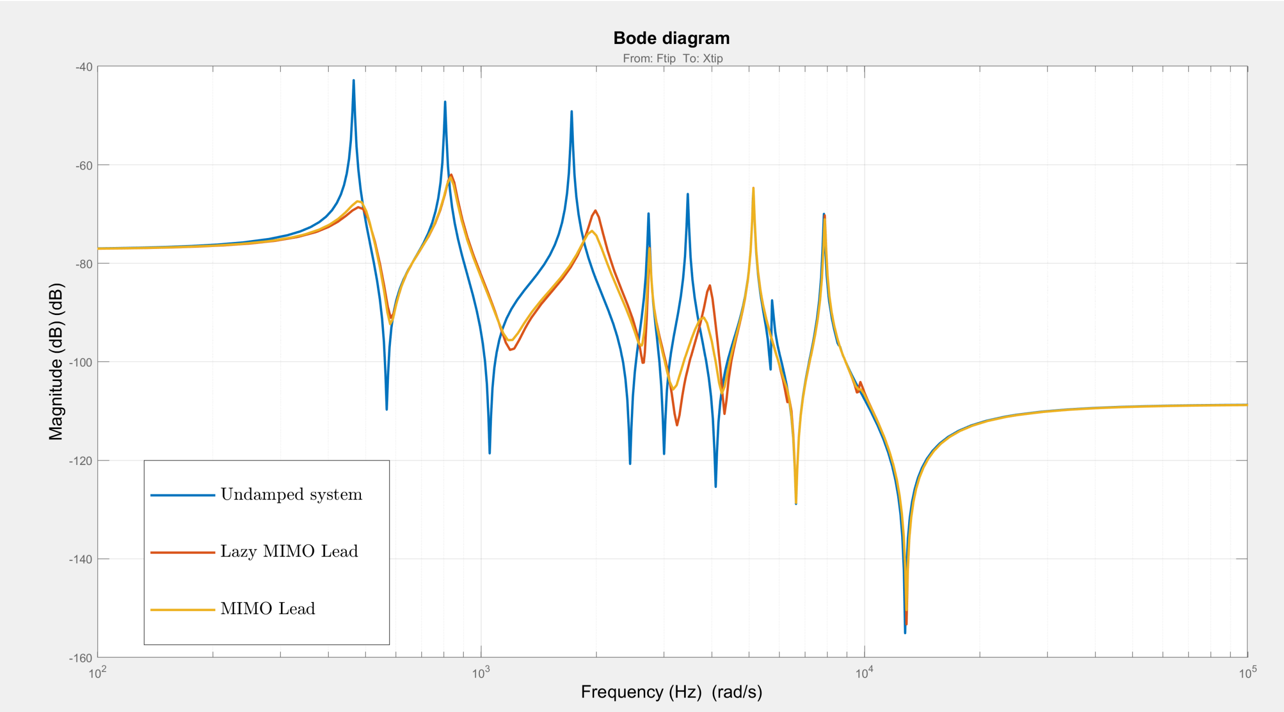

Figure 20: Performance index of the Lead based decentralised MIMO controllers

3.2 PPF based decentralised MIMO

The idea of this section is to propose MIMO controllers, completing the performances of the first order PPF controller presented earlier. Indeed, this SISO controller, applied on the first pair of patches, was not able to damp correctly the 4^{th} mode. Therefore, adding other SISO controller with other pair of patches, tuned on this remaining peak could help the overall performances. To do so, one must use SISO controller that can be tuned to a specific frequency range, such as the Lead and 2^{nd} order PPF.
In order to select which patch is the best candidate to damp this 4^{th} mode, a bode plot of the performance index, focused on the fourth mode, is shown on the figure 21.

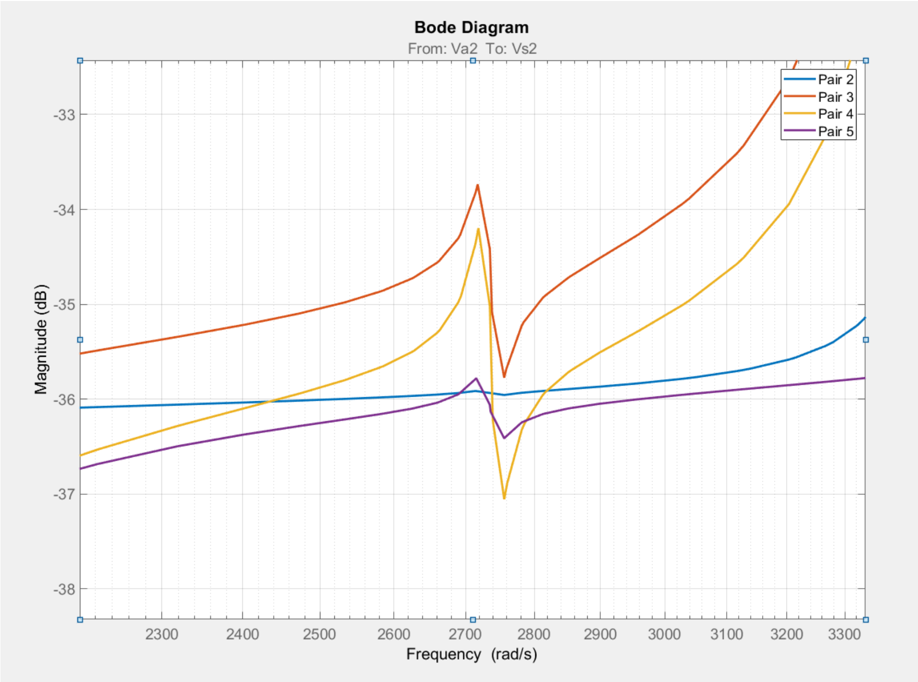

Figure 21: Comparison of the incidence of all the remaining patches on the 4th mode

One can see that, even thought they are all very limited, the patch 3 seems to be the most capable of detecting and influencing the targeted mode. If adding a control law on this pair is not sufficient, another one can be added on the 4th pair as well. If this is still not damped enough, one could try to also use the last pair, but the improvement will probably be very limited. One has to take into account that adding pairs will increase the cost of the system, and if the performances are almost identical, it might be better without it. Finally, the second one seems incapable of influencing this mode and will therefore not be used.

3.2.1 MIMO with 2nd order PPF

The figure 22 shows the root locus of the first controller, designed for the third pair of actuators and sensor. One can see that the 4th mode has been pulled to the left, increasing its damping.

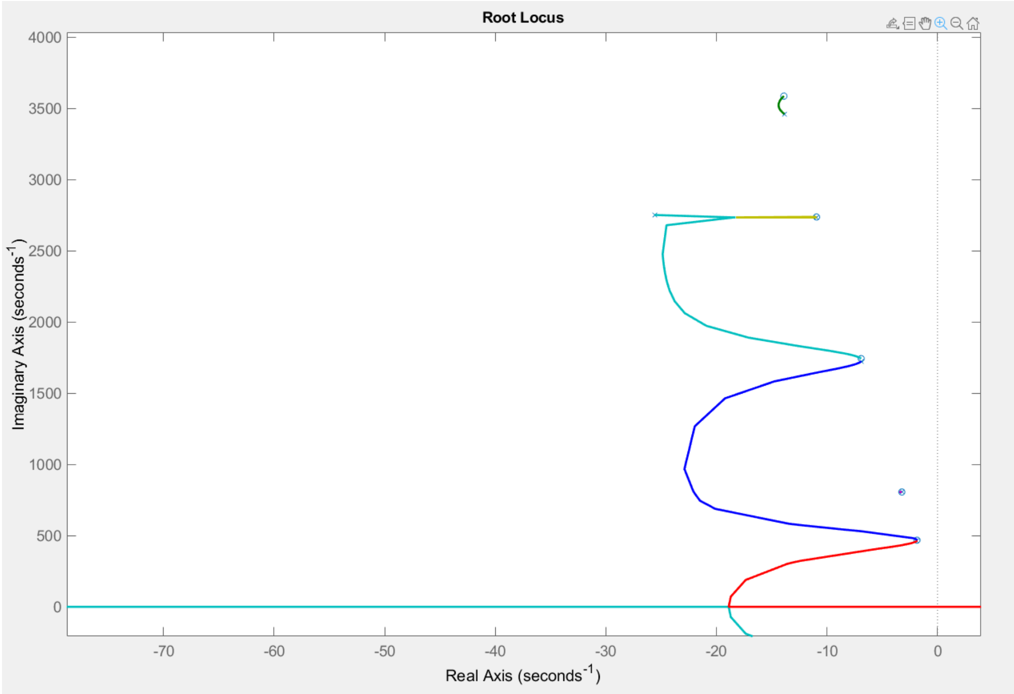

Figure 22: Root locus of the first 2nd order controllers implemented alongside the PPF of first order.

A similar procedure has been presented earlier with more details in section 2.2. The same design has been done for the two remaining pair of patches, the fourth and the fifth one. The following figure illustrates the relatively small performance gain achieved with this strategy.

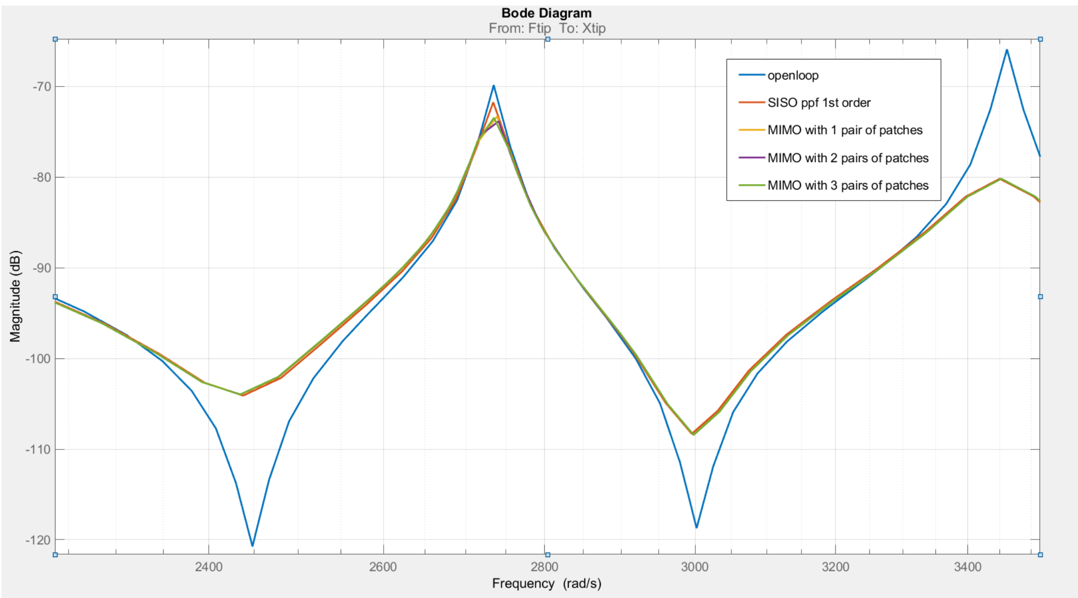

Figure 23: Comparison of the performance of the MIMO controller when additional patches are used. Focus on the 4th mode as it is the targeted.

One can notice that with only the control loop around the 3rd pair of actuators/sensors, the results are already almost as good as the one with 2 additional pairs of patches. If one thinks that damping a bit more this 4th mode is absolutely necessary, using only the 3rd pair should be enough, and saving money on the two others is probably a better option. If this increase of damping is judged too small, one should only use the SISO first order PPF, and save even more money on actuators. This will depend on the project requirements.

4 Conclusion and concerns about practical implementation

In this report, several control strategies have been presented and compared as well as the main tuning parameters to consider when implementing them.
As no precise requirements are stated (e.g. price, performances to achieve) no controller is chosen among these, as no objective criterion can lead to a justifiable choice. Depending on the situation, one might need to spend more to have better performances, using multiple pair of patches and a MIMO controller, or chose a SISO controller to save money and only use 1 pair of patches. Besides this, in a real life project, the engineer in charge must also locate its patches in the best way possible, to avoid the issues stated with the 4$^{\text{th}}$ mode.
When implementing these controllers on a real plant, one must still be cautious about some parts. First, the model has probably a mismatch with the real plant, as it comes from a simulation and not a real measurement. Moreover, it is only a linear model, so if it is approximating the reality (i.e. linearisation), it will create a mismatch when away from the origin. The modes might be slightly different (e.g. not exactly the same frequency or amplitude), and the controller will not be tuned perfectly for them. Even with that, the actual sensors (e.g. noise) and actuators (e.g. saturation) will also differ. This could be a problem (e.g.saturation leads to a slower stabilisation) when the gains are pushed too much and can even lead to damages (overheating of the actuators).
The controller will be implemented on either a digital or analog controller, leading to different problems and advantages (e.g. bandwidth, delay or price). But in all these case, it will never be as designed in simulation.
The safety margin might also be enhanced, to ensure stability at all cost, but reducing the overall performances.
To finish with, the figure 24 illustrate the impulse response of the system controlled by the PPF based MIMO controller.

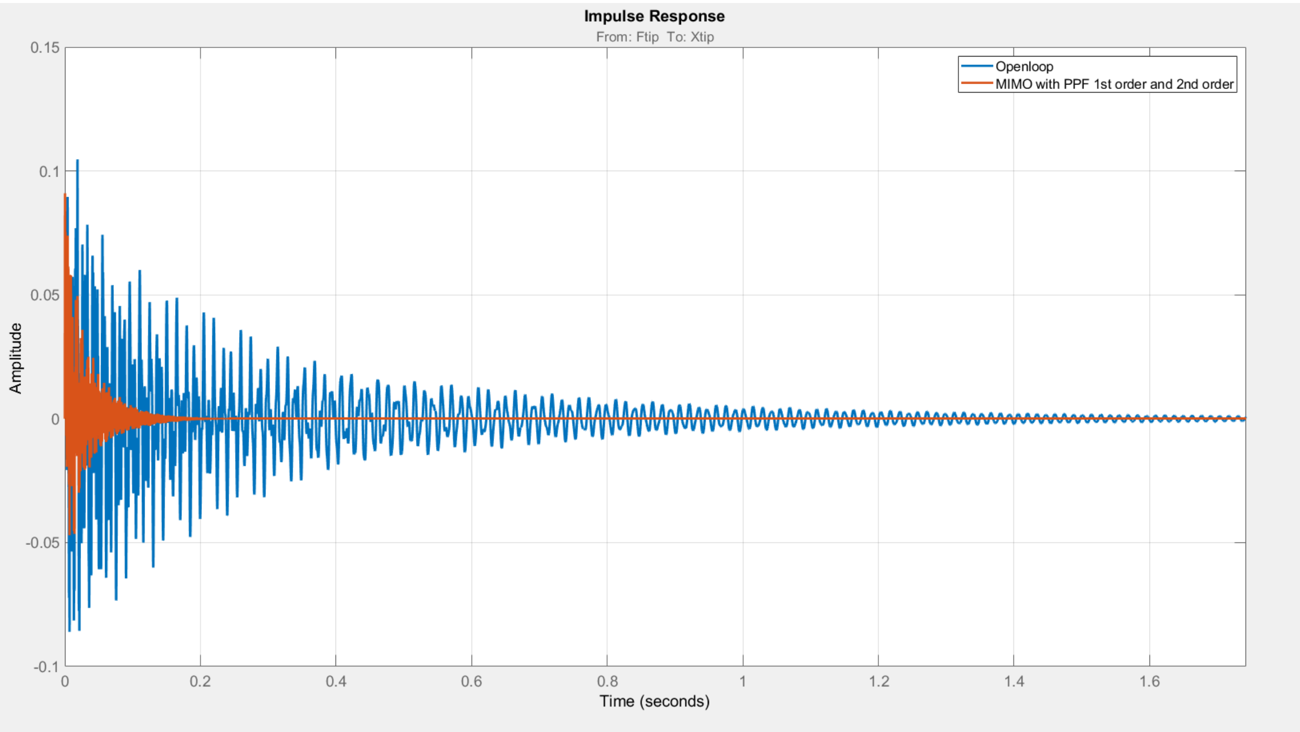

Figure 24: Impulse response on the performance index, to illustrate the damping of one of the controllers (i.e. The PPF based MIMO)

The vibrations are very limited and the reaction time is improved, thanks to the added damping.