UNIVERSITÉ LIBRE DE BRUXELLES

Active Vibration Control strategies

By: Rasa Jamshidi
Email: r.jamshidi@uliege.be

Topics of Seminar:

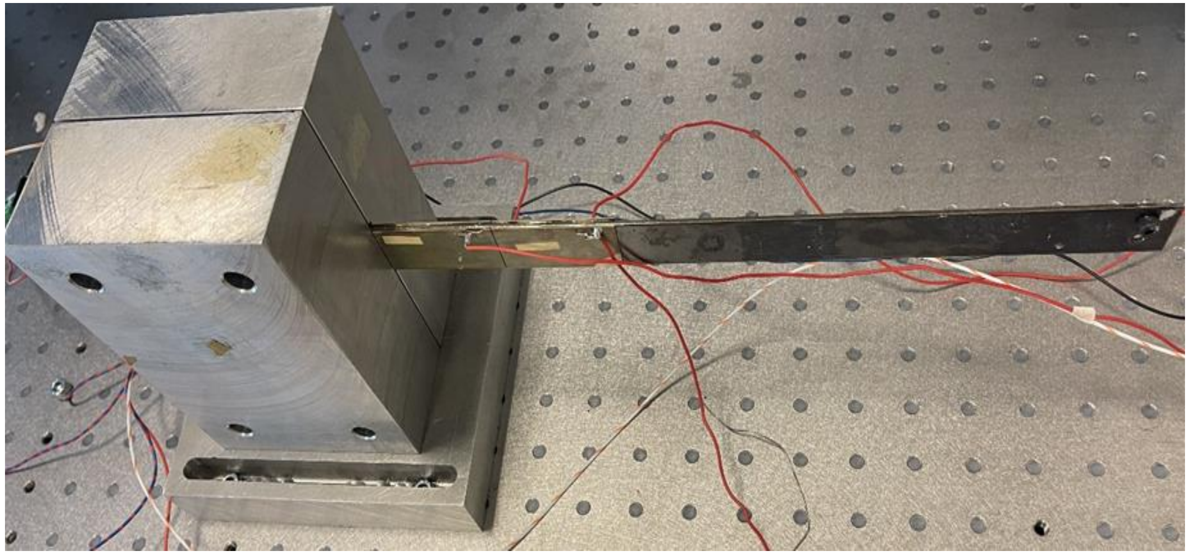
1- Collocated systems

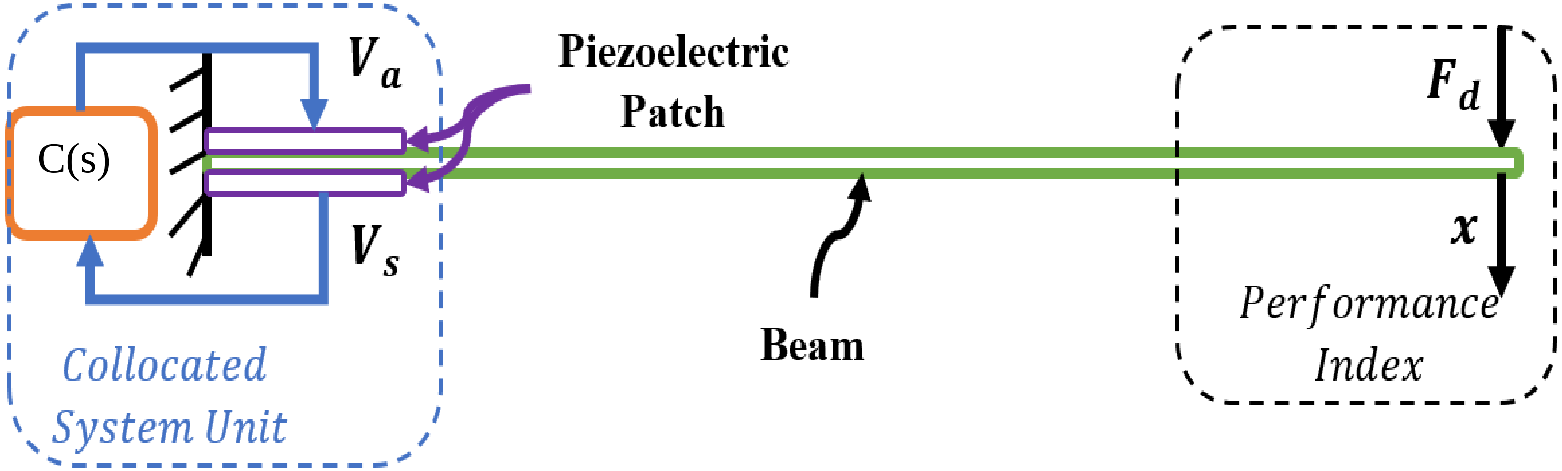
2- Positive position feedback (PPF) Controller

3- Negative Derivative Feedback Controller

4- Application on the beam

5- Application on the Bladed Rail

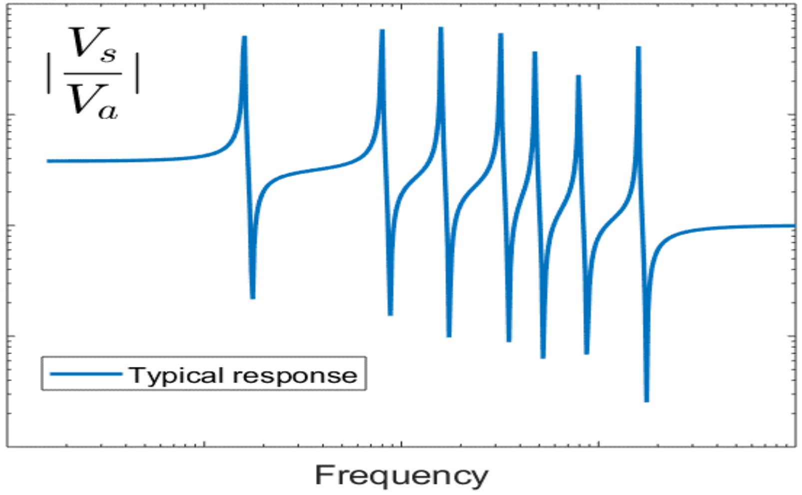




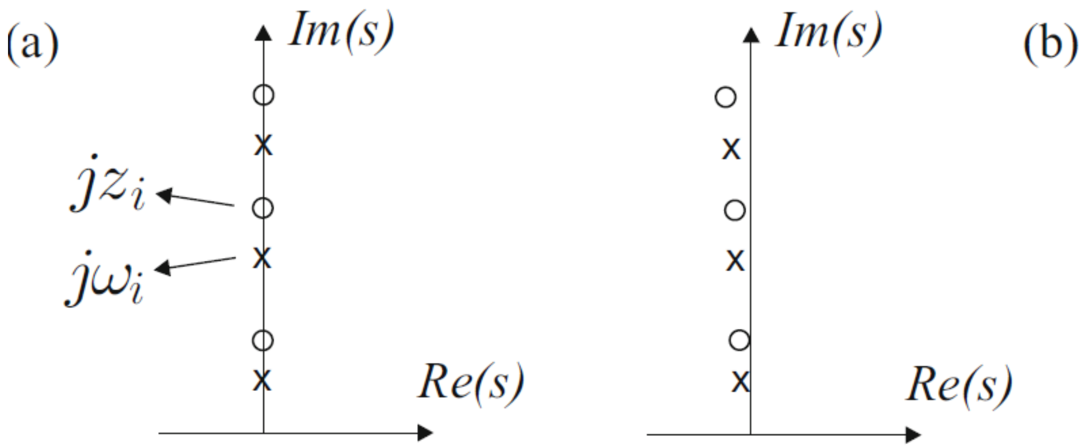
Collocated systems

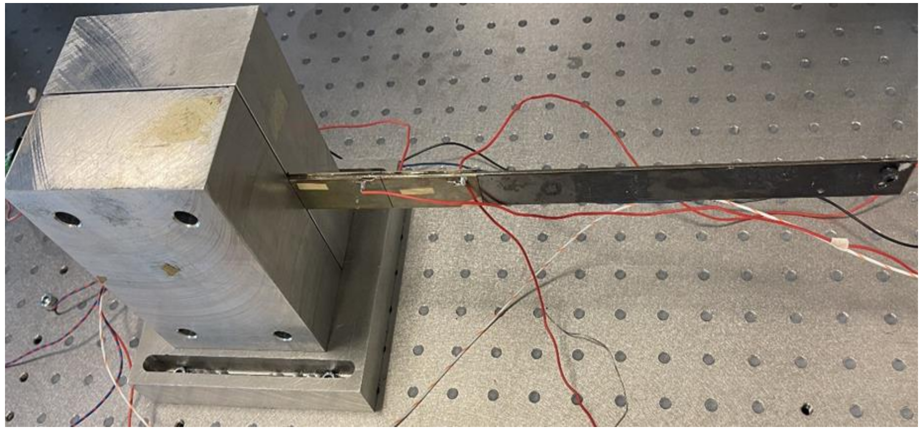
Location of Sensor and actuator are considered very close to each other or the same

This creates an alternating poles and zeros in the open loop
(Frequency Response from actuator to sensor)



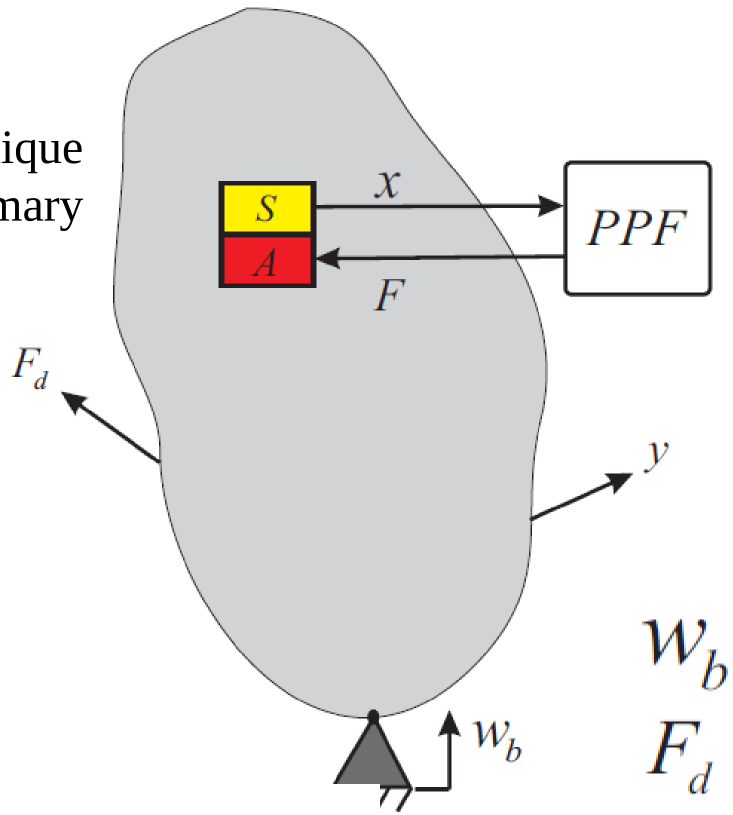
The asymptotic stability of a wide class of single-input single-output (SISO) control systems, even if the system parameters are subject to large perturbations. This is because the root locus plot keeps the same general shape, and remains entirely within the left half plane when the system parameters are changed from their nominal values.
Such a control system is said to be *robust* with respect to stability.

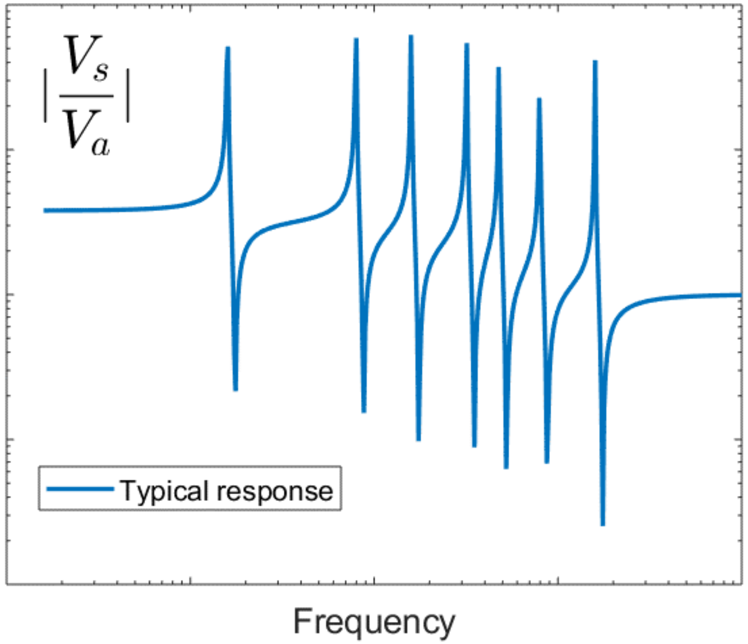




PPF Controller

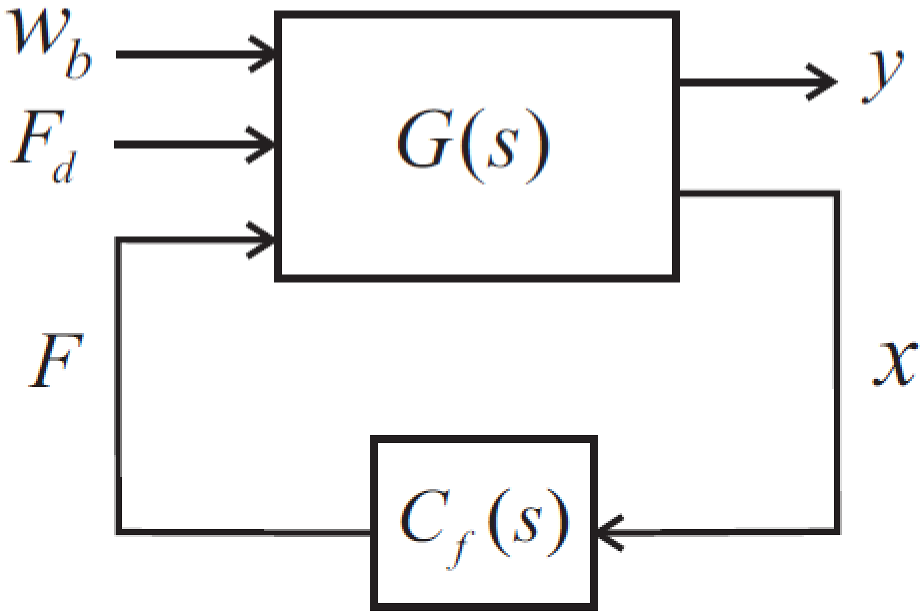
PPF is one of the most effective control technique to damp a certain resonance of collocated primary system with multiple modes.

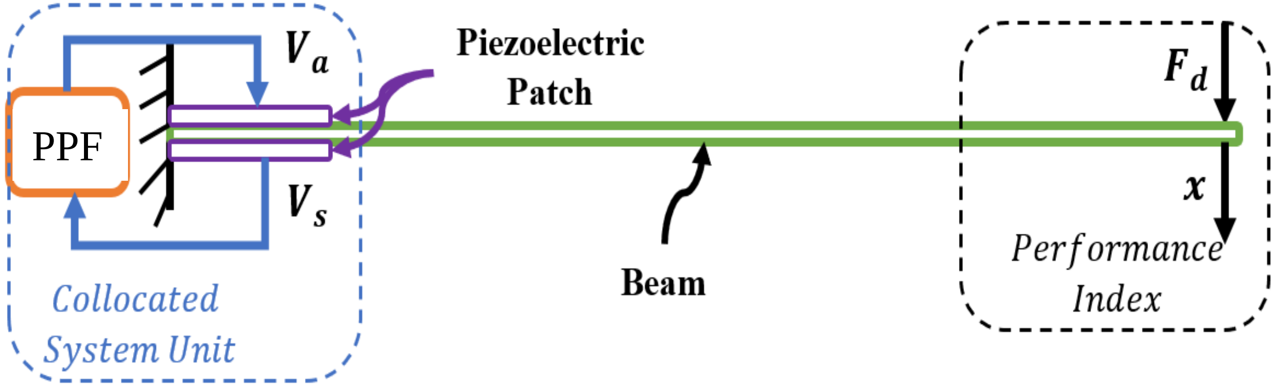




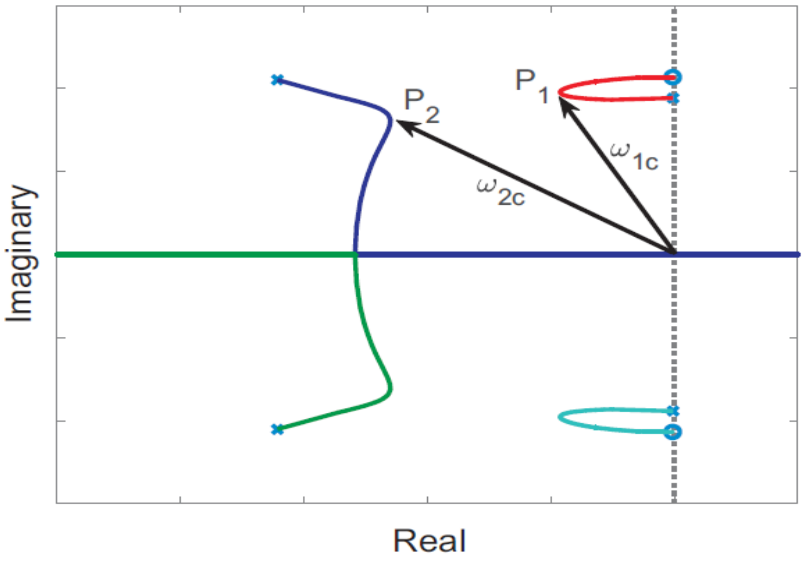
$$G(s) = \frac{V_s}{V_a} = g_0 \frac{s^2 + 2\xi_z \omega_z s + \omega_z^2}{s^2 + 2\xi_p \omega_p s + \omega_p^2}$$

$$C_f(s) = +\frac{g_f}{s^2 + 2\xi_f \omega_f s + \omega_f^2}$$





- $Characteristic\ Eq.\ from\ general\ case$
- $1 + GC = (s^2 + 2\xi_f\omega_f s + \omega_f^2)(s^2 + \omega_p^2) - g_0 g_f(s^2 + \omega_z^2) = s^4 + 2\xi_f\omega_f s^3 + \omega_p^2 + \omega_f^2 - g_0 g_f s^2 + 2\xi_f\omega_f\omega_p^2 s + \omega_f^2\omega_p^2 - g_0 g_f \omega_z$



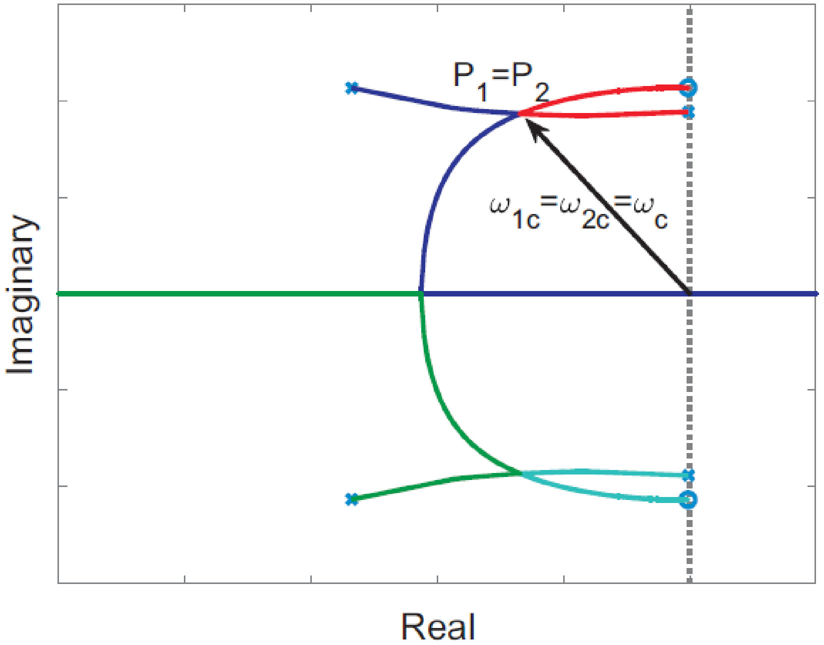
Maximum Damping Method

$$\alpha = \frac{\omega_f}{\omega_p} \qquad \beta = \frac{g_f}{g_0} \qquad \gamma = \frac{\omega_z}{\omega_p}$$

$$(s^2 + 2\xi_c\omega_c s + \omega_c^2)^2 = s^4 + 4\xi_c\omega_c s^3 + (4\xi_c^2\omega_c^2 + 2\omega_c^2)s^2 + 4\xi_c\omega_c^3 s + \omega_c^4$$

$$\boldsymbol{\omega_c = \omega_p}$$

$$\begin{cases} 4\,\xi_c\omega_c = 2\xi_f\alpha\omega_p \\ (4\xi_c^2 + 2)\omega_c^2 = (\alpha^2 + 1)\omega_p^2 - \dfrac{\beta\alpha^2}{\gamma^2} g_0^2\omega_p^2 \\ 4\xi_c\omega_c^3 = 2\xi_f\alpha\omega_p^3 \\ \omega_c^4 = \alpha^2\omega_p^4 - \beta\alpha^2 g_0^2\omega_p^4 \end{cases}$$



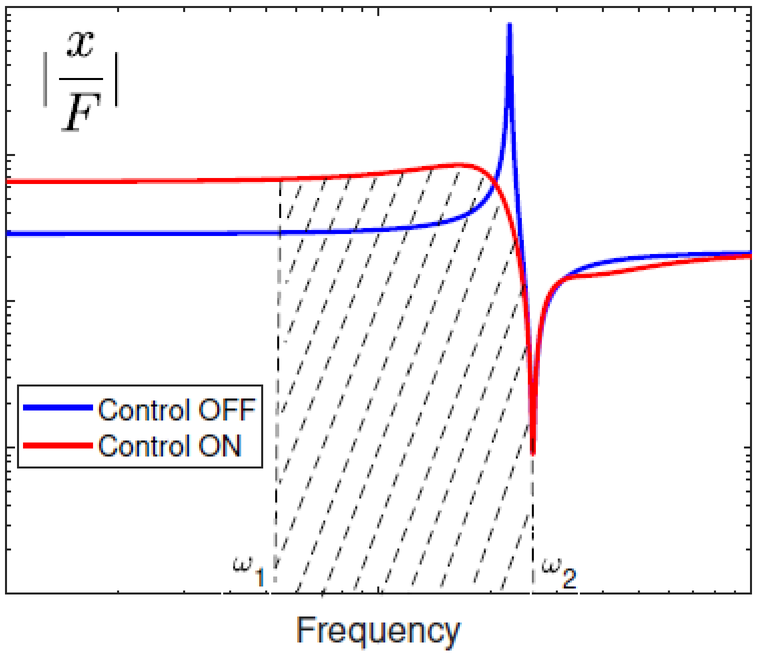
Maximum Damping Method

- $\alpha = \sqrt{\frac{4\gamma^2\xi_c^2}{\gamma^2-1} + 1}$
- $\xi_f = 2\xi_c\sqrt{\frac{\gamma^2-1}{4\gamma^2\xi_c^2+\gamma^2-1}}$
- $\beta = \frac{1}{g_0^2}\frac{4\gamma^2\xi_c^2}{4\gamma^2\xi_c^2+\gamma^2}$

The norm wanted to minimized: $\frac{G}{1+GC} =$

$$\frac{g_0\left(s^2+2\xi_z\omega_z s+\omega_z^2\right)}{s^4+2\xi_f\omega_f s^3+\omega_p^2+\omega_f^2-g_0 g_f s^2+2\xi_f\omega_f\omega_p^2 s+\omega_f^2\omega_p^2-g_0 g_f \omega_z}$$

Considering $s = j\omega$ the value of $\left|\frac{G}{1+GC}\right|$ can be extracted

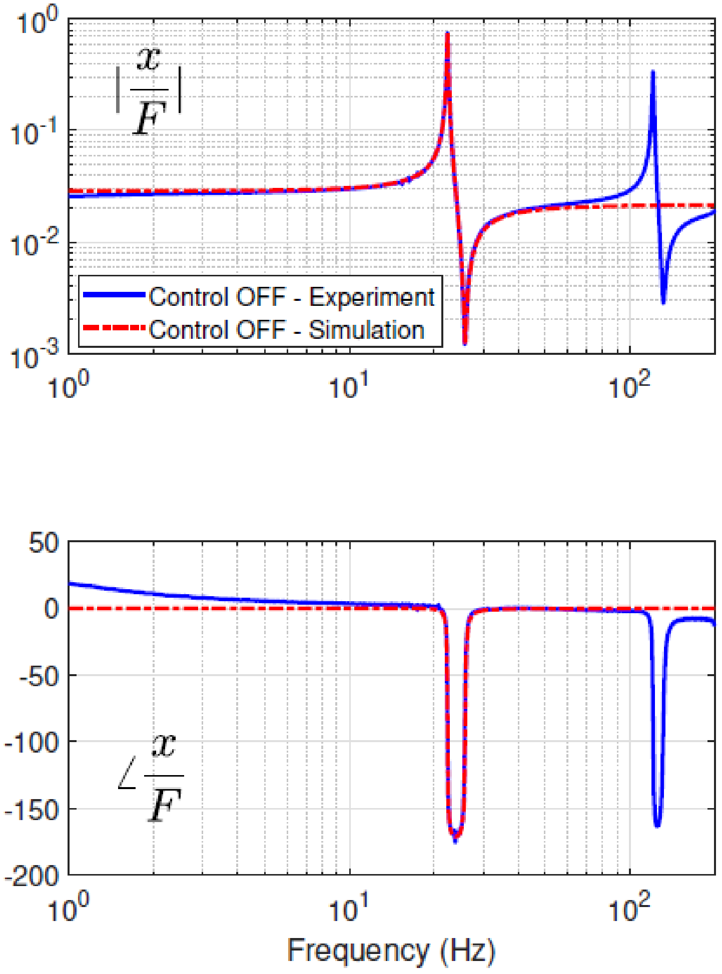


Using H_2 and H_∞ optimization method For Finding an optimal value of closed loop damping

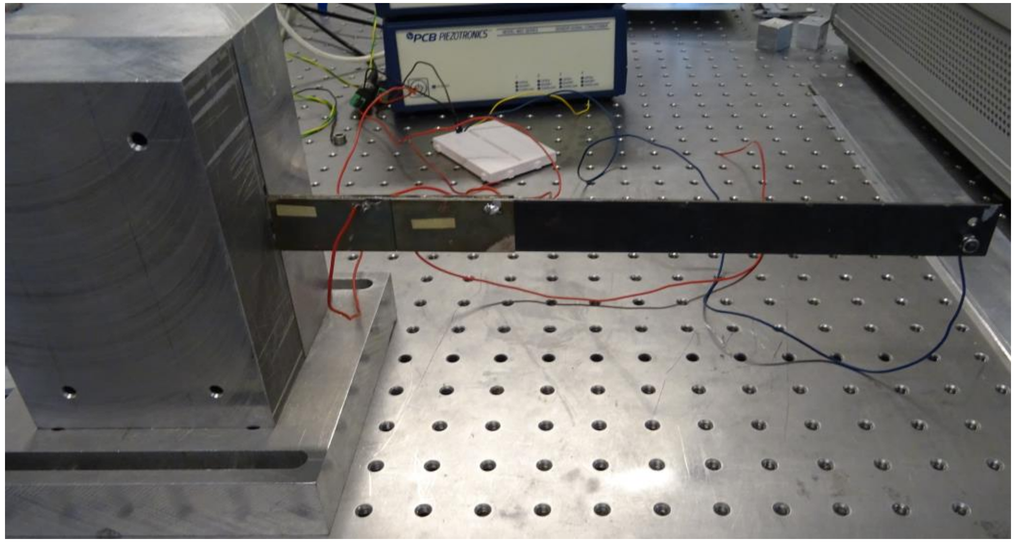
$$H_2 = \int_{\omega_1}^{\omega_2} \left|\frac{G}{1+GC}\right| d\omega$$

$$H_\infty = \min\left(\left|\frac{G}{1+GC}\right|\right)\Big|_{@w_p}$$

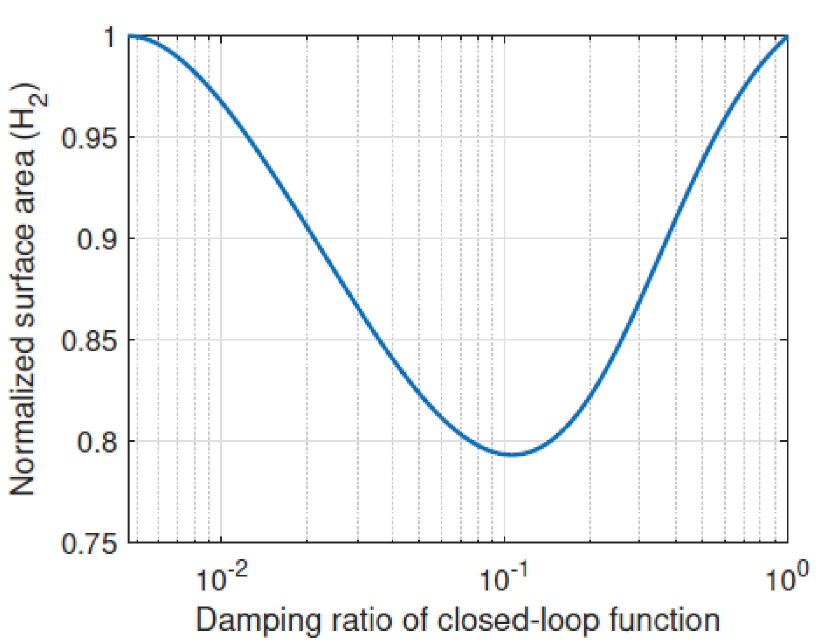
Minimizing the cost function with respect to values of closed loop damping (from zero to 1)



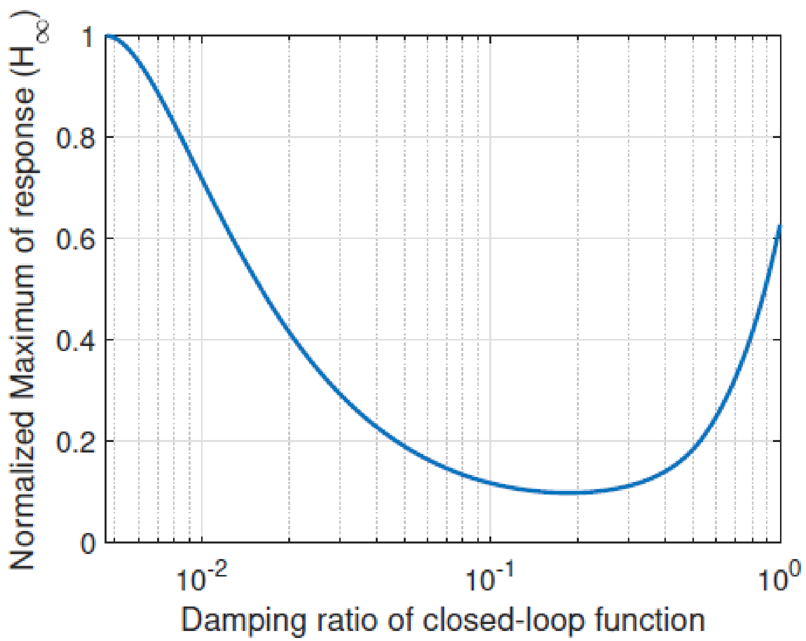
$|\frac{x}{F}|$
Control OFF - Experiment
Control OFF - Simulation
$\angle\frac{x}{F}$
Frequency (Hz)



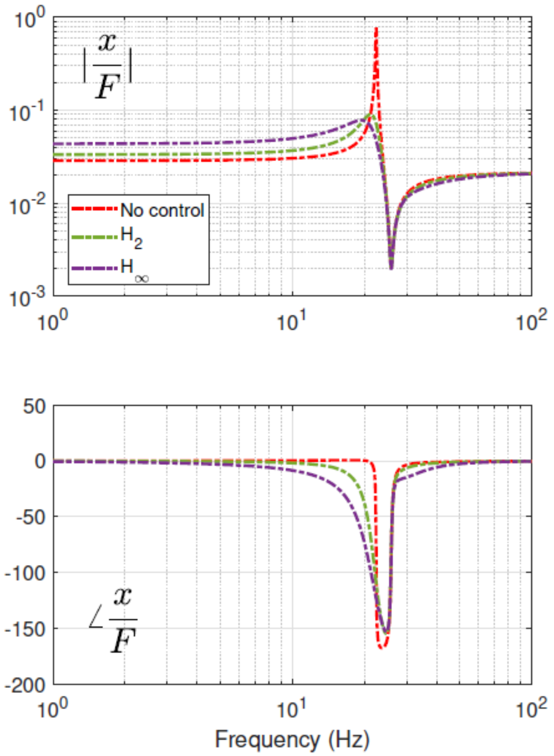
PCB PIEZOTRONICS



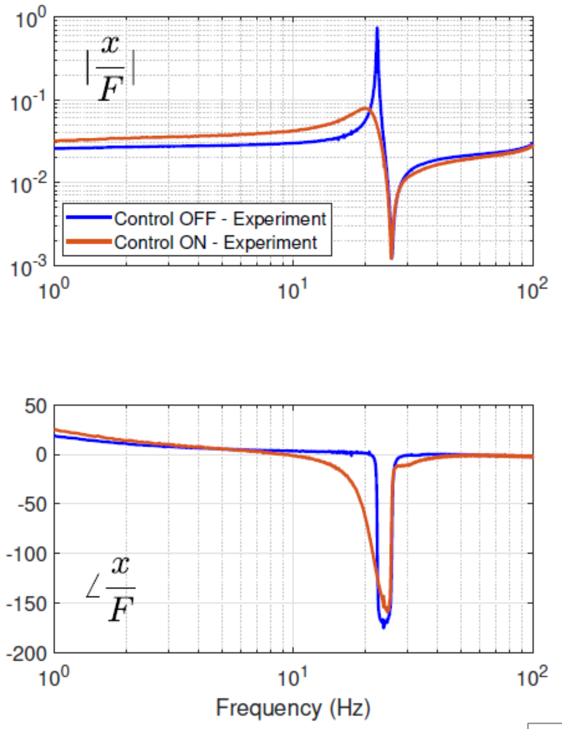
Normalized surface area (H_2)
Damping ratio of closed-loop function



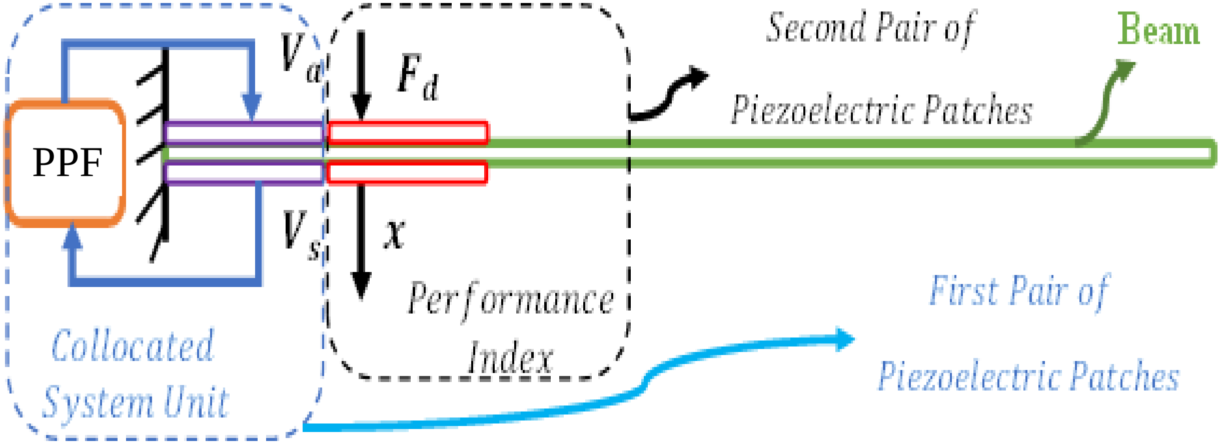
Normalized Maximum of response (H_∞)
Damping ratio of closed-loop function



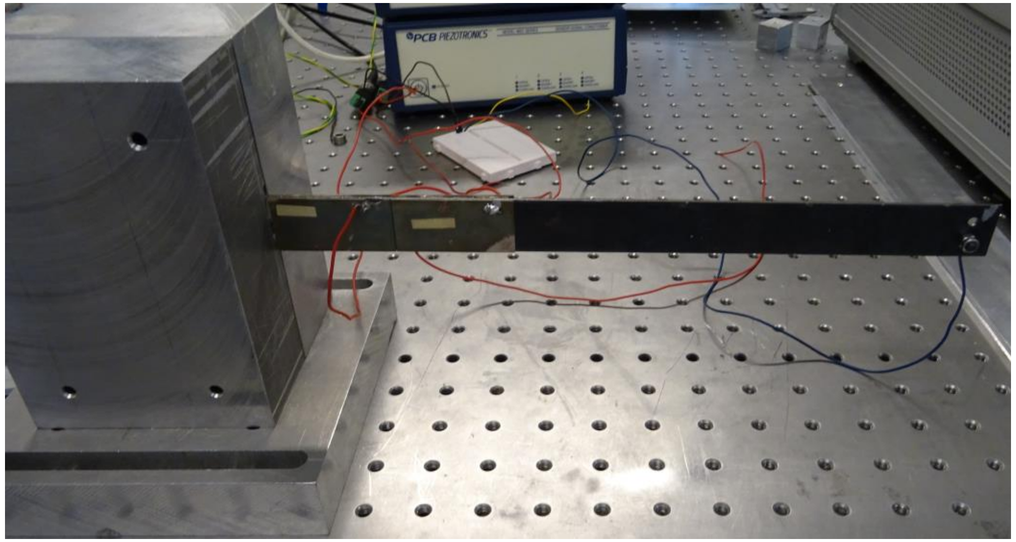
$|\frac{x}{F}|$
No control
H_2
H_∞
$\angle\frac{x}{F}$
Frequency (Hz)



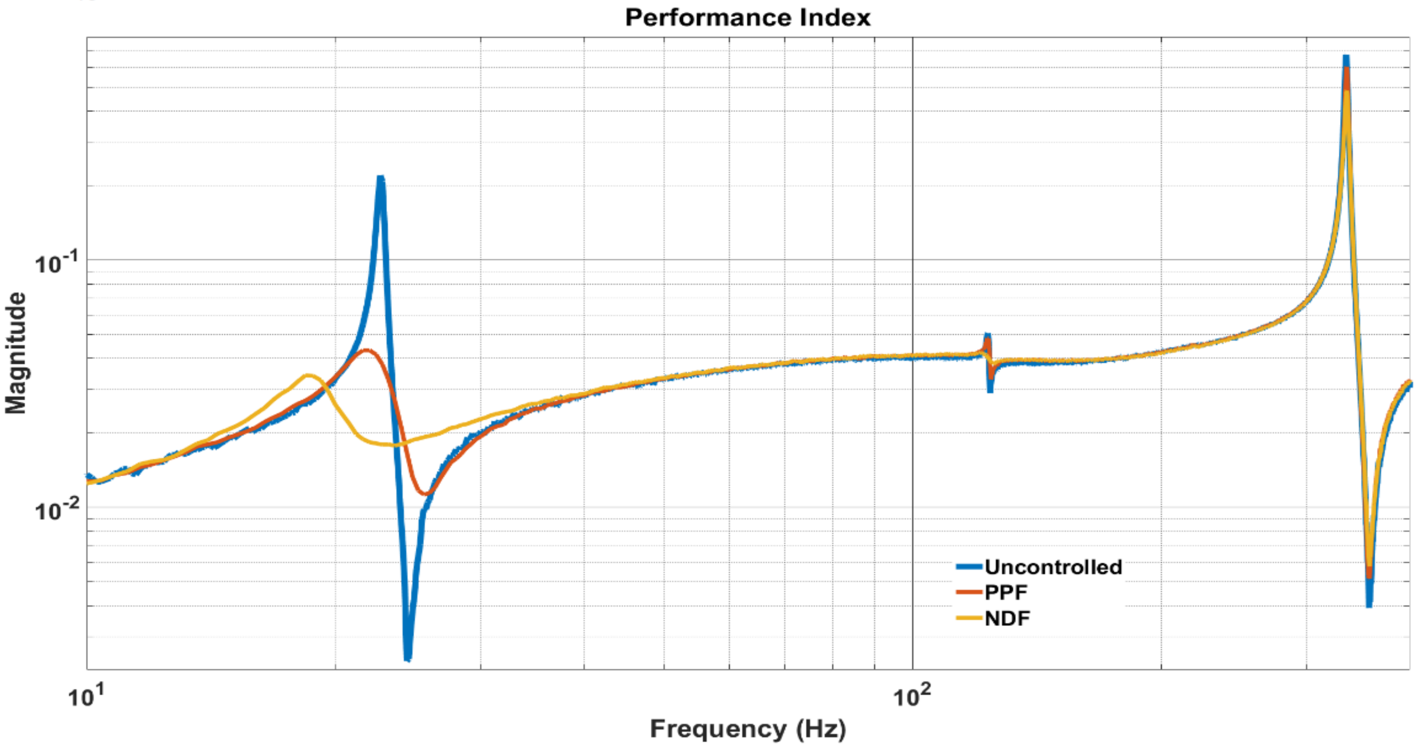
$|\frac{x}{F}|$
Control OFF - Experiment
Control ON - Experiment
$\angle\frac{x}{F}$
Frequency (Hz)



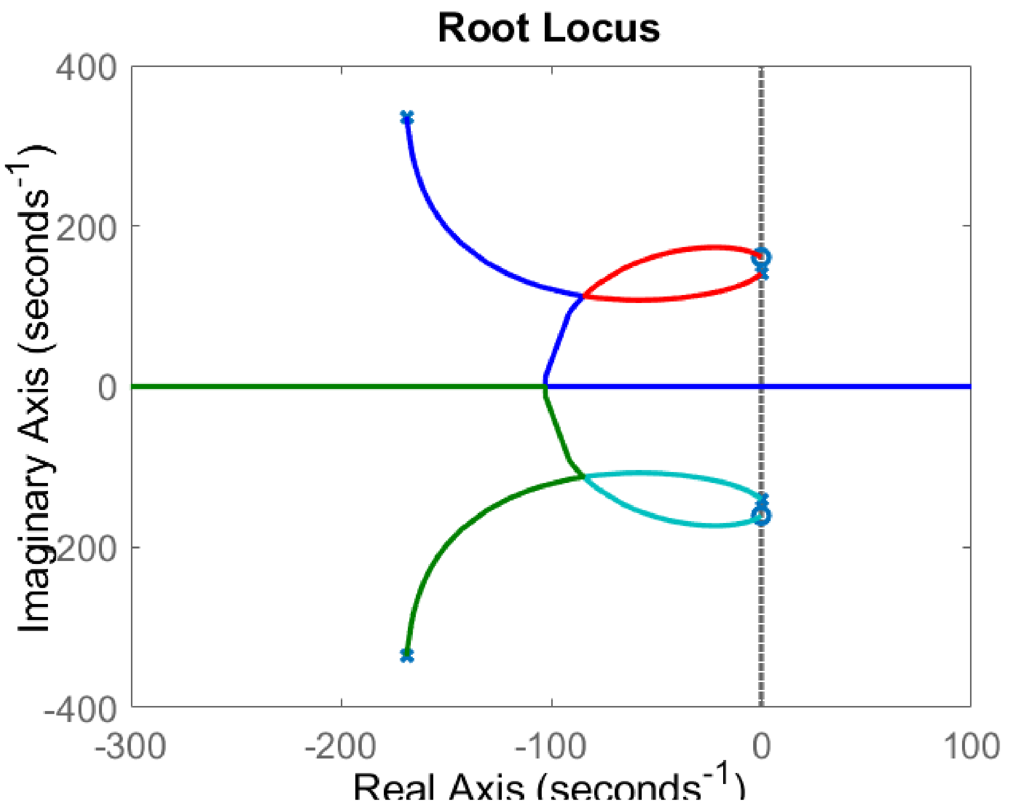
PPF
V_a
F_d
V_s
x
Second Pair of
Piezoelectric Patches
Beam
Performance
Index
Collocated
System Unit
First Pair of
Piezoelectric Patches

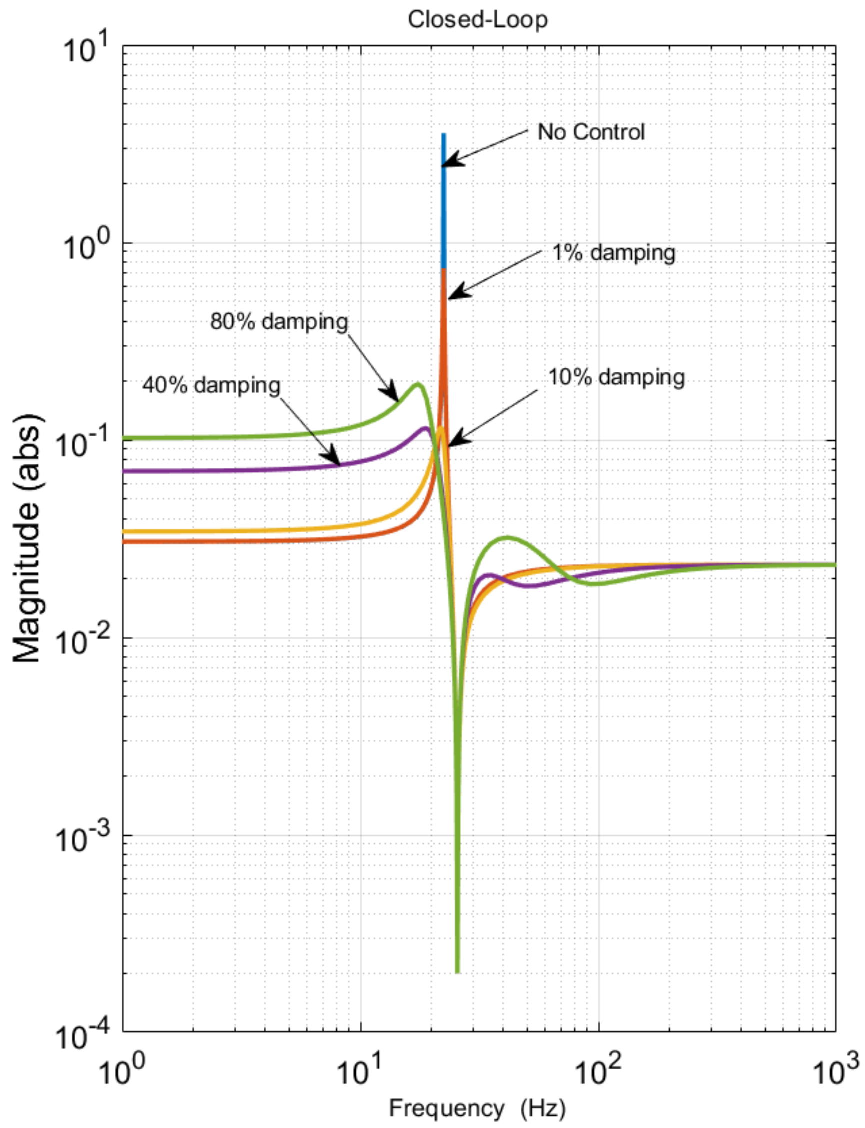


PCB PIEZOTRONICS



Performance Index
Magnitude
Frequency (Hz)
Uncontrolled
PPF
NDF





By increasing the value of the feedback gain, one pole of the closed-loop system crosses the imaginary axis and makes the system unstable.

The other trade-off in the system is that the more damping is added to the system, the more amplification of the static response the closed-loop system has.

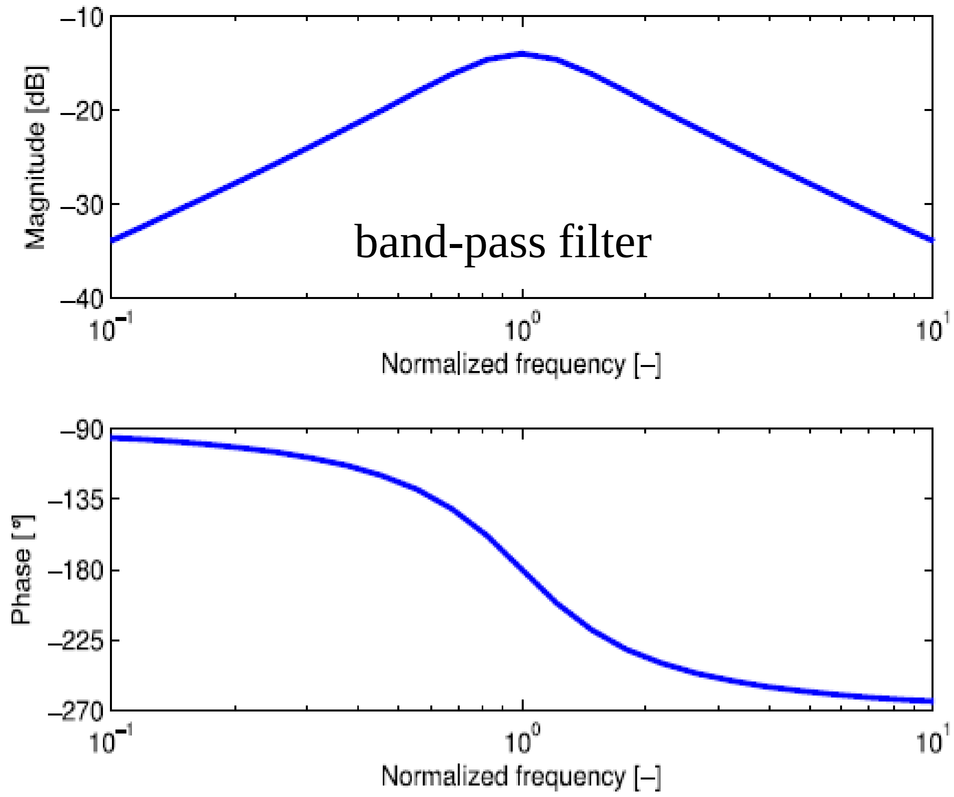
PPF Controller

- Maximum damping happens when the two loops in Root-Locus are intersecting
- For each value of closed-loop damping, there is one controller which causes merged poles of closed-loop and subsequently maximum damping
- More damping requires higher control frequency! This results in some problems
- The amplification at low frequency
- Coupling with the next modes (for continuous structure) The control frequency should be tuned far enough from the next resonance frequency
- Although the formula has been developed for undamped primary system, they can be used for lightly damped structure as well
- The biggest issue with this new method is that the presented method is only applicable for pole before zero pattern!

UNIVERSITÉ LIBRE DE BRUXELLES

Why NDF?

1- The NDF compensator is designed to work as a band-pass filter, cutting off the control action far from the natural frequencies associated with the controlled modes and reducing the so-called spillover effect.



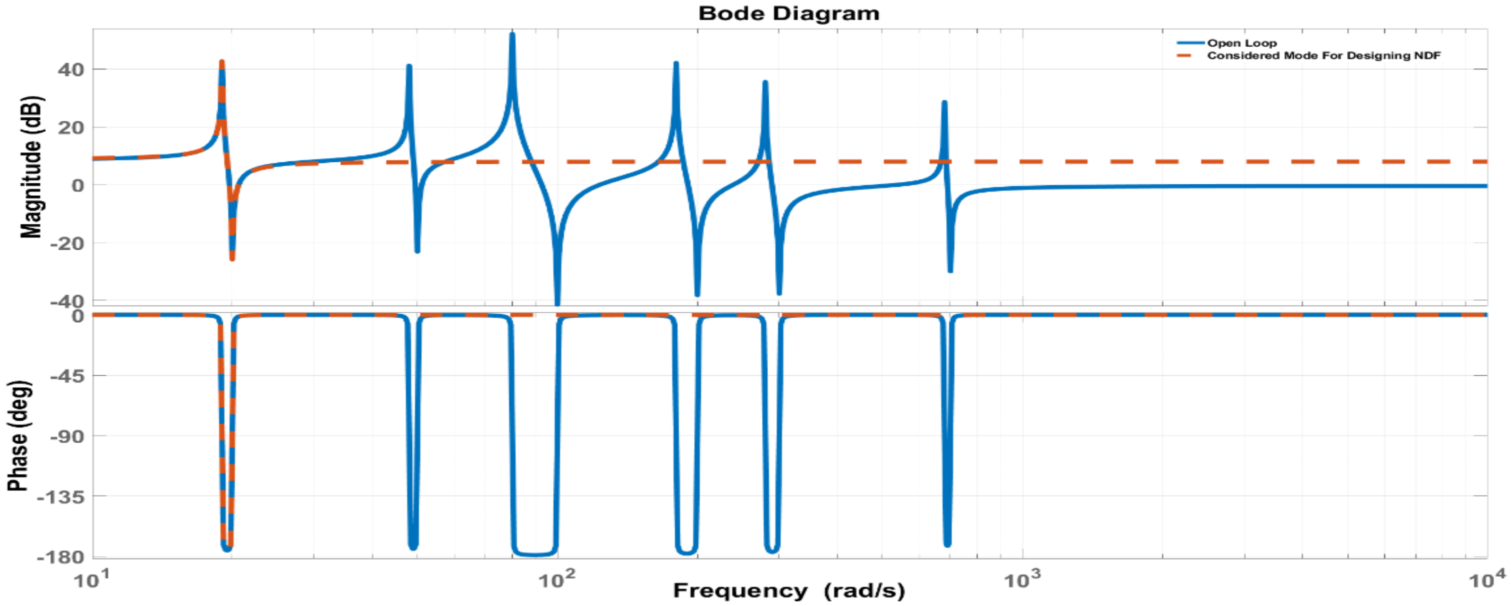
2- **Comparison with PPF:**
As a low-pass filter, PPF is very sensitive to low-frequency disturbances.
To overcome this shortcoming of PPF controller, NDF controller, which acts as a bandpass filter and can effectively control the lower and higher frequency disturbances, has been developed recently.

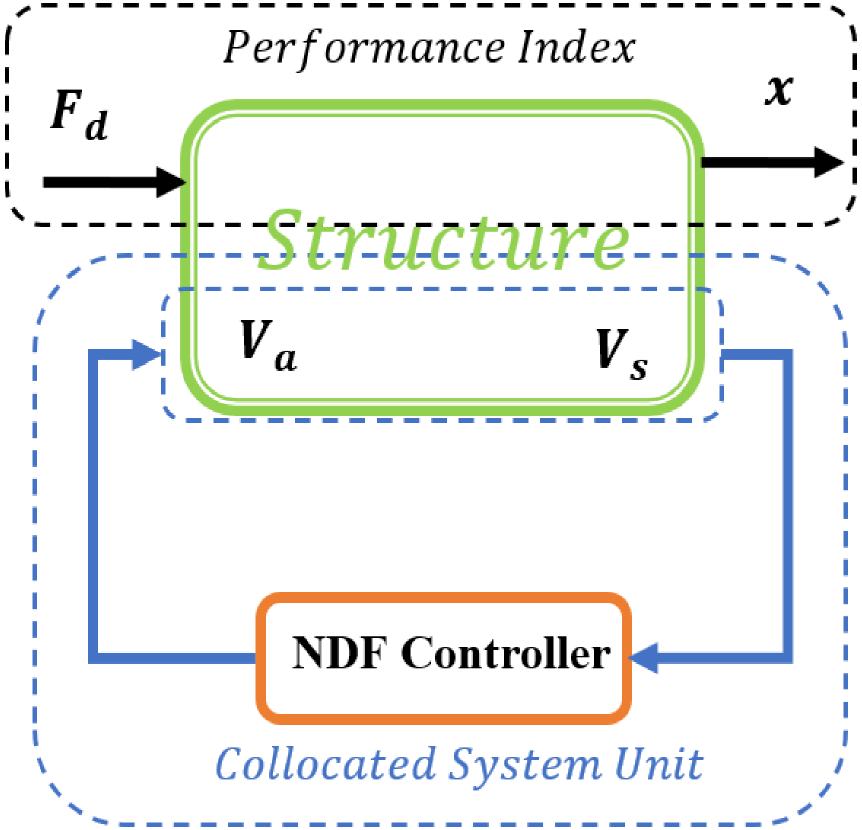
3- Negative derivative feedback (NDF) proves particularly robust against spillover since modal velocity is fed back through a band-pass filter so that undesired effects can be limited both at high and low frequencies.

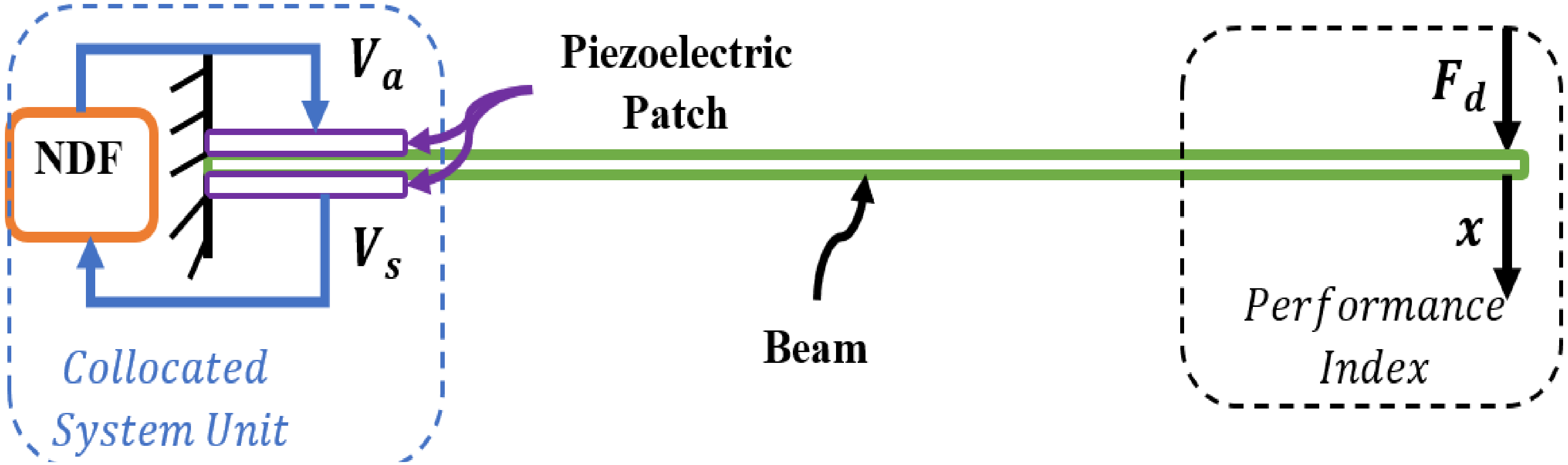
$$NDF = -\frac{\alpha\beta s}{s^2 + 2\xi\alpha s + \alpha^2}$$

4- This type of Controller is a ideal controller for low damped structure vibration reduction

Negative derivative feedback controller (NDF)







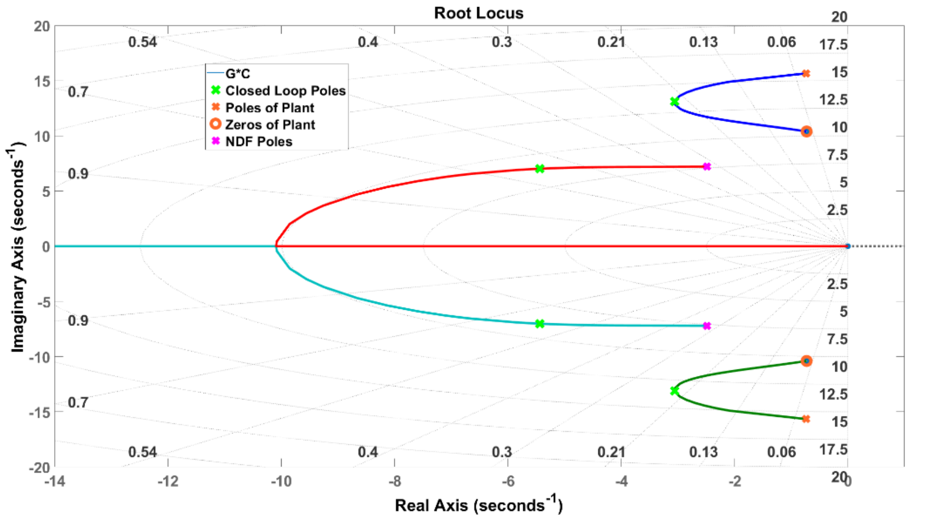
$$G(s) = \frac{V_s}{V_a} = g_0 \frac{s^2 + 2\xi_z \omega_z s + \omega_z^2}{s^2 + 2\xi_p \omega_p s + \omega_p^2}$$

$$C(s) = -\frac{K_c \omega_c s}{s^2 + 2\xi_c \omega_c s + \omega_c^2}$$

NDF For Collocated Systems

ULB UNIVERSITÉ LIBRE DE BRUXELLES

The characteristic equation of the system will be:

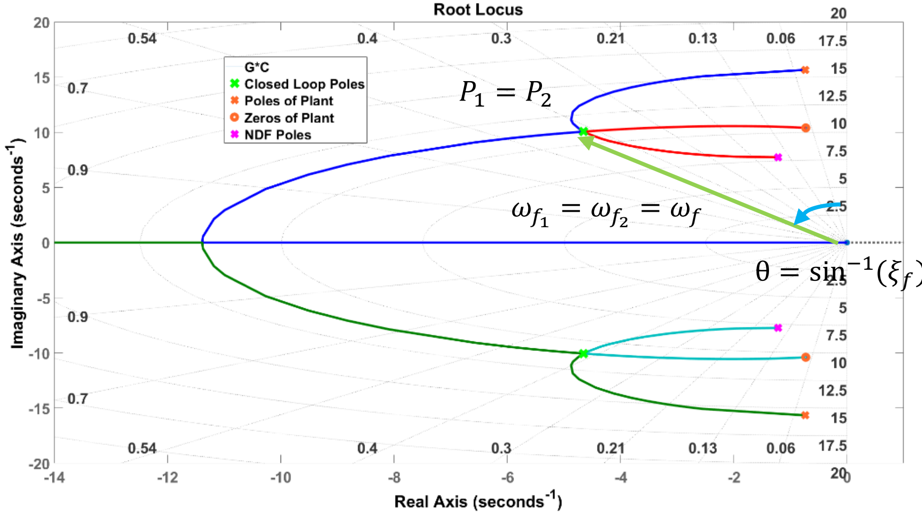


$$s^4 + (2\xi_p\omega_p + 2\xi_c\omega_c + k_c\omega_c g_0)s^3 + (\omega_p^2 + \omega_c^2 + 4\xi_p\xi_c\omega_p\omega_c + 2\xi_z\omega_z K_c\omega_c g_0)s^2 + (2\xi_p\omega_p\omega_c^2 + 2\xi_c\omega_c\omega_p^2 + g_0\omega_z^2 K_c\omega_c)s + \omega_p^2\omega_c^2 = 0$$

Considering Maximum damping Method:

$$(s^2 + 2\xi_f\omega_f s + \omega_f^2)^2 =$$

$$s^4 + 4\xi_f\omega_f s^3 + (4\xi_f^2\omega_f^2 + 2\omega_f^2)s^2 + 4\xi_f\omega_f^3 s + \omega_f^4 = 0$$



Considering:

$$\alpha = \frac{\omega_c}{\omega_p}$$

$$\gamma = \frac{\omega_z}{\omega_p}$$

$$\begin{cases} \gamma < 1 & \text{zero before pole} \\ \gamma > 1 & \text{zero after pole} \end{cases}$$

Negative derivative feedback controller works for both of zero before pole and zero after pole!

The pattern dose not matter!

$$
\begin{cases}
2\xi_p\omega_p + 2\xi_c\omega_c + k_c\omega_c g_0 = 4\xi_f\omega_f & (1)\\
\omega_p^2 + \omega_c^2 + 4\xi_p\xi_c\omega_p\omega_c + 2\xi_z\omega_z K_c\omega_c g_0 = 4\xi_f^2\omega_f^2 + 2\omega_f^2 & (2)\\
2\xi_p\omega_p\omega_c^2 + 2\xi_c\omega_c\omega_p^2 + g_0\omega_z^2 K_c\omega_c = 4\xi_f\omega_f^3 & (3)\\
\omega_p^2\omega_c^2 = \omega_f^4 & (4)
\end{cases}
$$

$$
(2)\begin{cases}
\alpha < 1 \quad \text{then } \alpha = \left(\xi_{eq} + 1\right) - \sqrt{\xi_{eq}{}^2 + 2\xi_{eq}}\\
\alpha > 1 \quad \text{then } \alpha = \left(\xi_{eq} + 1\right) + \sqrt{\xi_{eq}{}^2 + 2\xi_{eq}}
\end{cases}
$$

$\alpha = \dfrac{\omega_c}{\omega_0}$

(Cutoff frequency)

$\xi = \xi_c$

(Damping Ratio)

K_c

(Gain)

Based on Maximum Damping Method

$\boldsymbol{\alpha}$	$(\xi_{eq} + 1) \pm \sqrt{\xi_{eq}{}^2 + 2\xi_{eq}}$ $\left(\xi_{eq} = 2\xi_f^2 - 2\xi_p\xi_c - \xi_z\gamma K_c g_0\right)$
$\boldsymbol{\xi_c}$	$\dfrac{\left(2\xi_f\sqrt{\alpha} - \xi_p\right)(1-\gamma^2) - (1-\alpha)\left[2\xi_f\sqrt{\alpha} - \xi_p(1+\alpha)\right]}{\alpha(1-\gamma^2)}$
$\boldsymbol{K_c}$	$\dfrac{2}{g_0}\dfrac{(1-\alpha)\left[2\xi_f\sqrt{\alpha} - \xi_p(1+\alpha)\right]}{\alpha(1-\gamma^2)}$

The norm wanted to minimized: $\frac{G}{1+GC} =$

$$\frac{g_0\left(s^2+2\xi_z\omega_z s+\omega_z^2\right)}{s^4+\left(2\xi_p\omega_p+2\xi_c\omega_c+k_c\omega_c g_0\right)s^3+\left(\omega_p^2+\omega_c^2+4\xi_p\xi_c\omega_p\omega_c+2\xi_z\omega_z K_c\omega_c g_0\right)s^2+\left(2\xi_p\omega_p\omega_c^2+2\xi_c\omega_c\omega_p^2+g_0\omega_z^2K_c\omega_c\right)s+\omega_p^2\omega_c^2}$$

Considering $s = j\omega$ we have

$$\left|\frac{G}{1+GC}\right| =$$

$$\sqrt{\frac{\left(\omega_z^2-\omega^2\right)^2+(2\xi_z\omega_z\omega)^2}{\left(\omega^4-\left(\omega_p^2+\omega_c^2+2\xi_z\omega_c K_c\omega_c g_0\right)\omega^2+\omega_p^2\omega_c^2\right)^2+\left(-\left(2\xi_p\omega_p+2\xi_c\omega_c+g_0K_c\omega_c\right)\omega^3+\left(2\xi_p\omega_p\omega_c^2+2\xi_c\omega_c\omega_p^2+g_0\omega_z^2K_c'omega_c\right)w\right)^2}}$$

Considering H_2 and H_∞ optimization method

$$H_2 = \int_0^\infty \sqrt{\frac{(\omega_z^2-\omega^2)^2+(2\xi_z\omega_z\omega)^2}{\left(\omega^4-\left(\omega_p^2+\omega_c^2+2\xi_z\omega_c K_c\omega_c g_0\right)\omega^2+\omega_p^2\omega_c^2\right)^2+\left(-\left(2\xi_p\omega_p+2\xi_c\omega_c+g_0K_c\omega_c\right)\omega^3+\left(2\xi_p\omega_p\omega_c^2+2\xi_c\omega_c\omega_p^2+g_0\omega_z^2K_c\omega_c\right)w\right)^2}}\,d\omega$$

$$H_\infty = \min\left(\sqrt{\frac{(\omega_z^2-\omega^2)^2+(2\xi_z\omega_z\omega)^2}{\left(\omega^4-\left(\omega_p^2+\omega_c^2+2\xi_z\omega_c K_c\omega_c g_0\right)\omega^2+\omega_p^2\omega_c^2\right)^2+\left(-\left(2\xi_p\omega_p+2\xi_c\omega_c+g_0K_c\omega_c\right)\omega^3+\left(2\xi_p\omega_p\omega_c^2+2\xi_c\omega_c\omega_p^2+g_0\omega_z^2K_c\omega_c\right)w\right)^2}}\right)\Bigg|_{@w_p}$$

Minimizing the cost function with respect to values of closed loop damping (from zero to 1)

UNIVERSITÉ LIBRE DE BRUXELLES

Stability of the System for various patterns

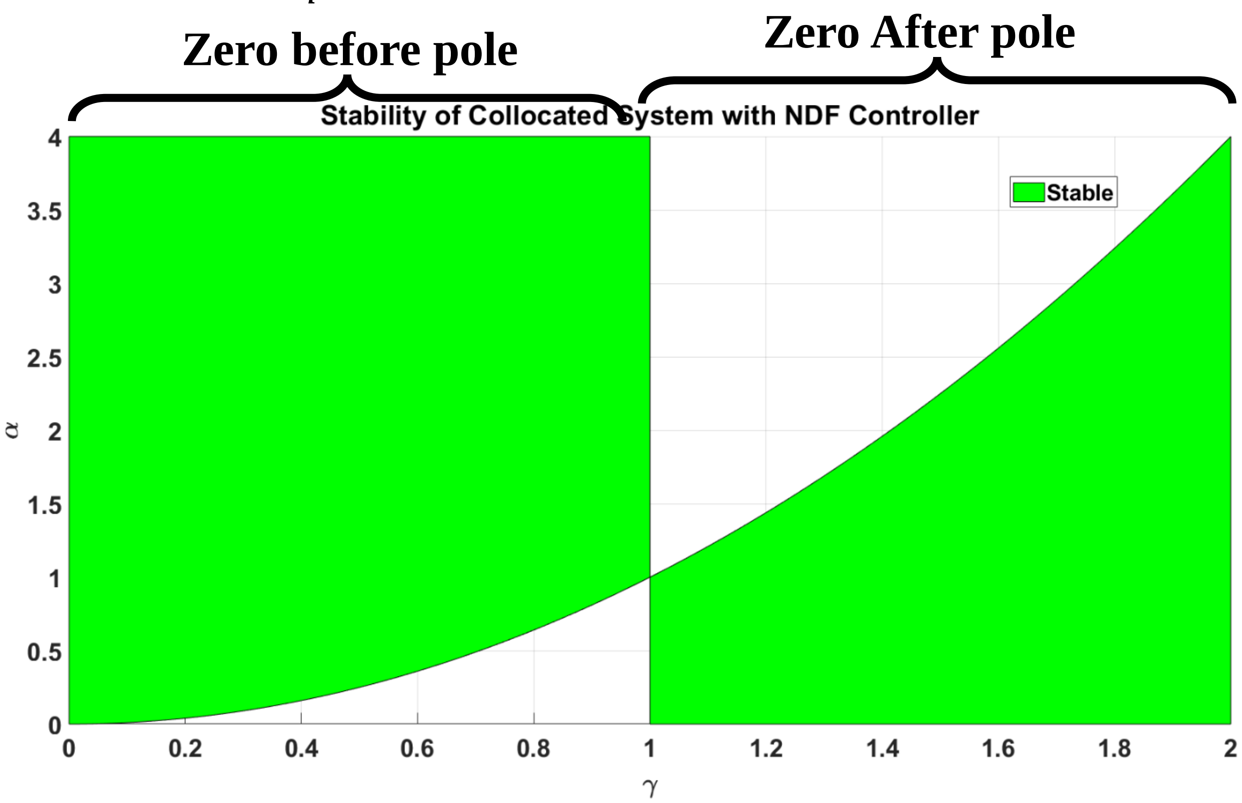
$$s^4 + (2\xi_p\omega_p + 2\xi_c\omega_c + k_c\omega_c g_0)s^3 + (\omega_p^2 + \omega_c^2 + 4\xi_p\xi_c\omega_p\omega_c + 2\xi_z\omega_z K_c\omega_c g_0)s^2 + (2\xi_p\omega_p\omega_c^2 + 2\xi_c\omega_c\omega_p^2 + g_0\omega_z^2 K_c\omega_c)s + \omega_p^2\omega_c^2 = 0$$

Since the equations become very long in this case, a more simple method is considered.

It is considered that the damping of pole and zero to become zero. $\xi_p = 0 \ , \xi_z = 0$

$\boldsymbol{\alpha}$	$(2\xi_f^2 + 1) \pm 2\xi_f\sqrt{{\xi_f}^2 + 1}$
$\boldsymbol{\xi_c}$	$\dfrac{2\xi_f(\alpha - \gamma^2)}{\sqrt{\alpha}(1 - \gamma^2)}$
$\boldsymbol{K_c}$	$\dfrac{4\xi_f(1 - \alpha)}{g_0\sqrt{\alpha}(1 - \gamma^2)}$

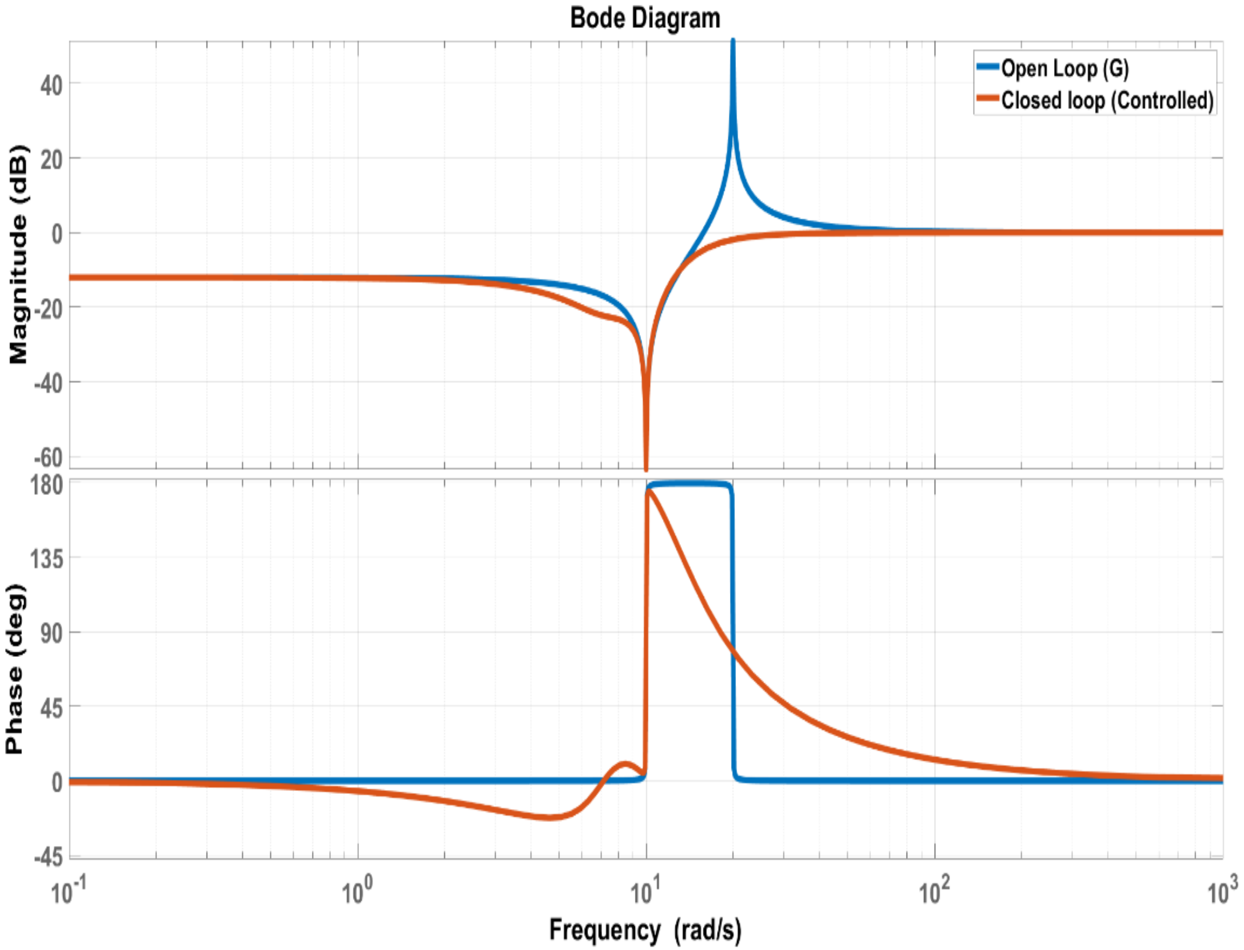
$$\begin{cases} \gamma < 1 & \text{then } \ \alpha > \gamma^2 \\ \gamma > 1 & \text{then } \ \alpha < \gamma^2 \end{cases}$$

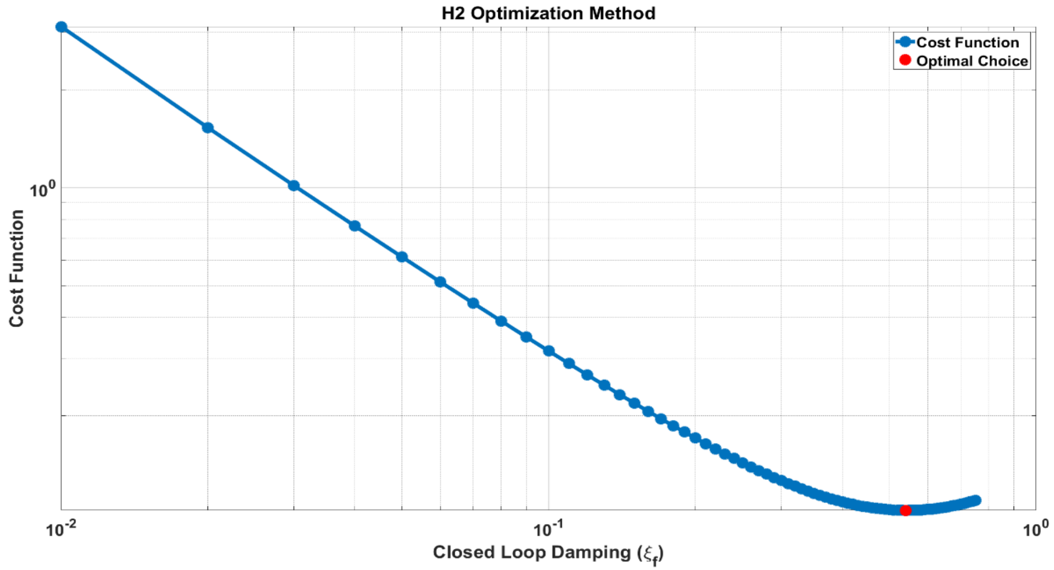


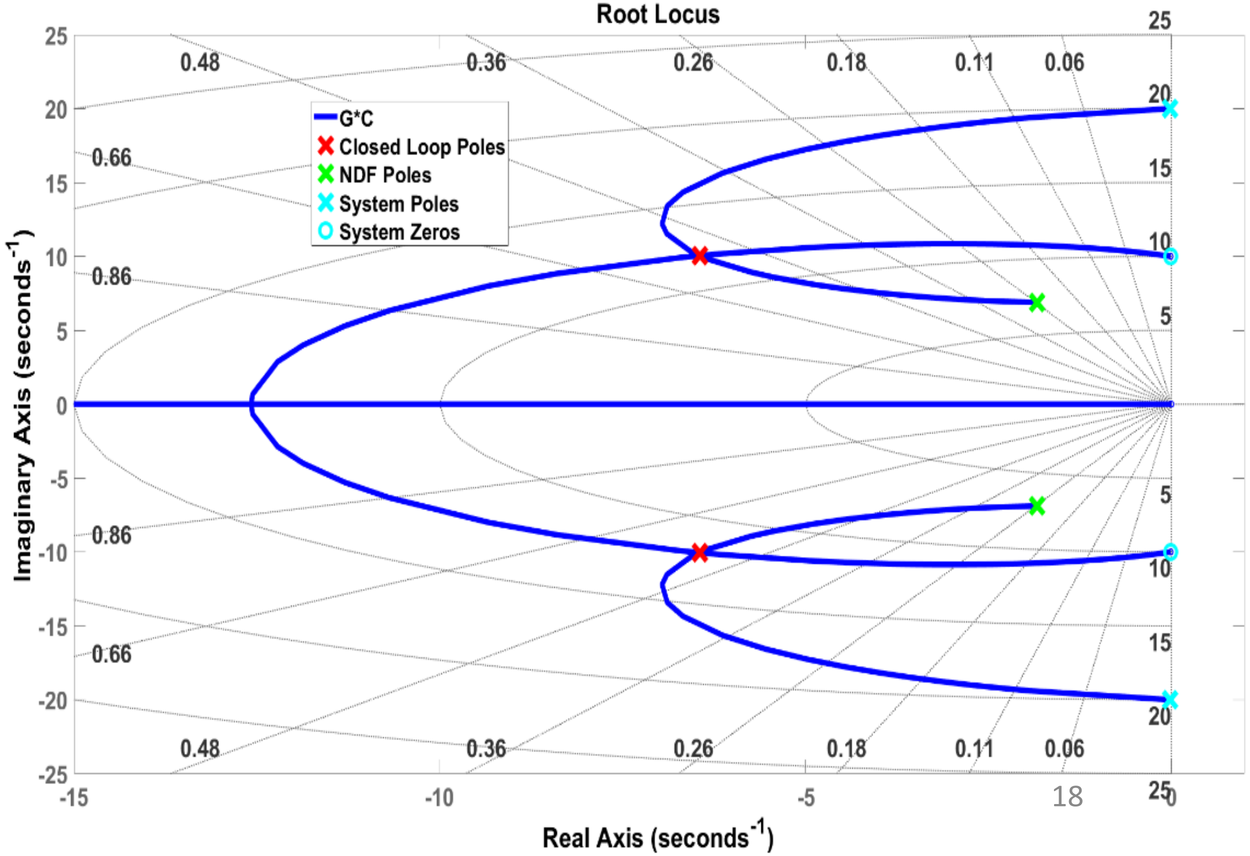
Zero Before Pole ($\lambda > 1$)

Example 1:

$$G(s) = \frac{s^2 + 0.02 * s + 100}{s^2 + 0.04 * s + 400}$$



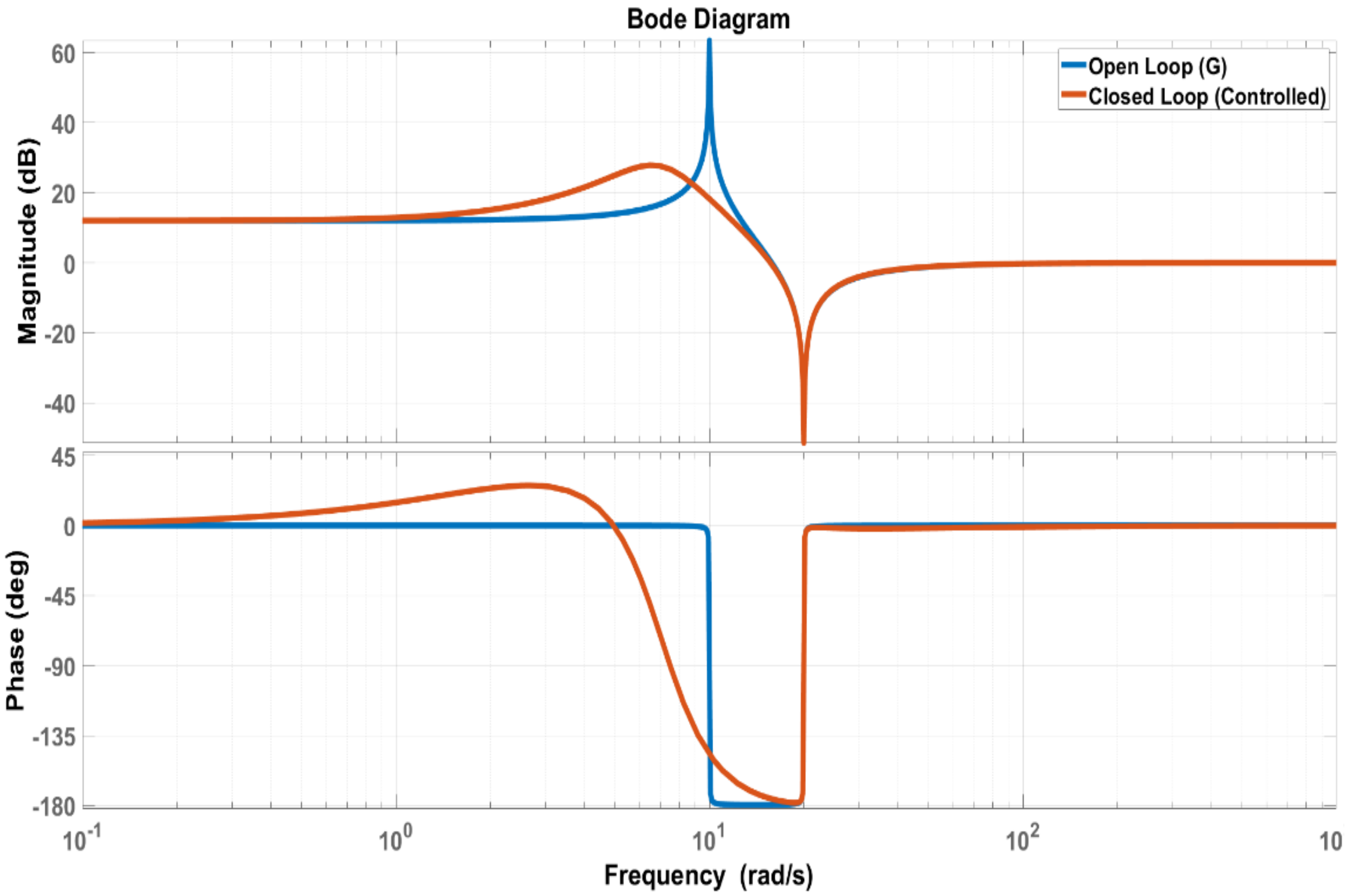


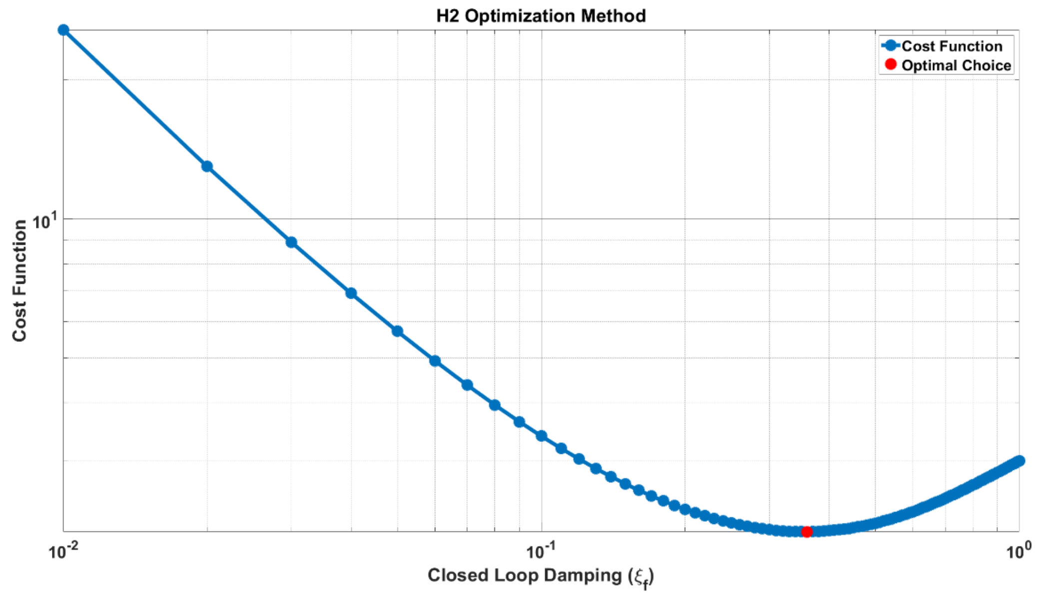


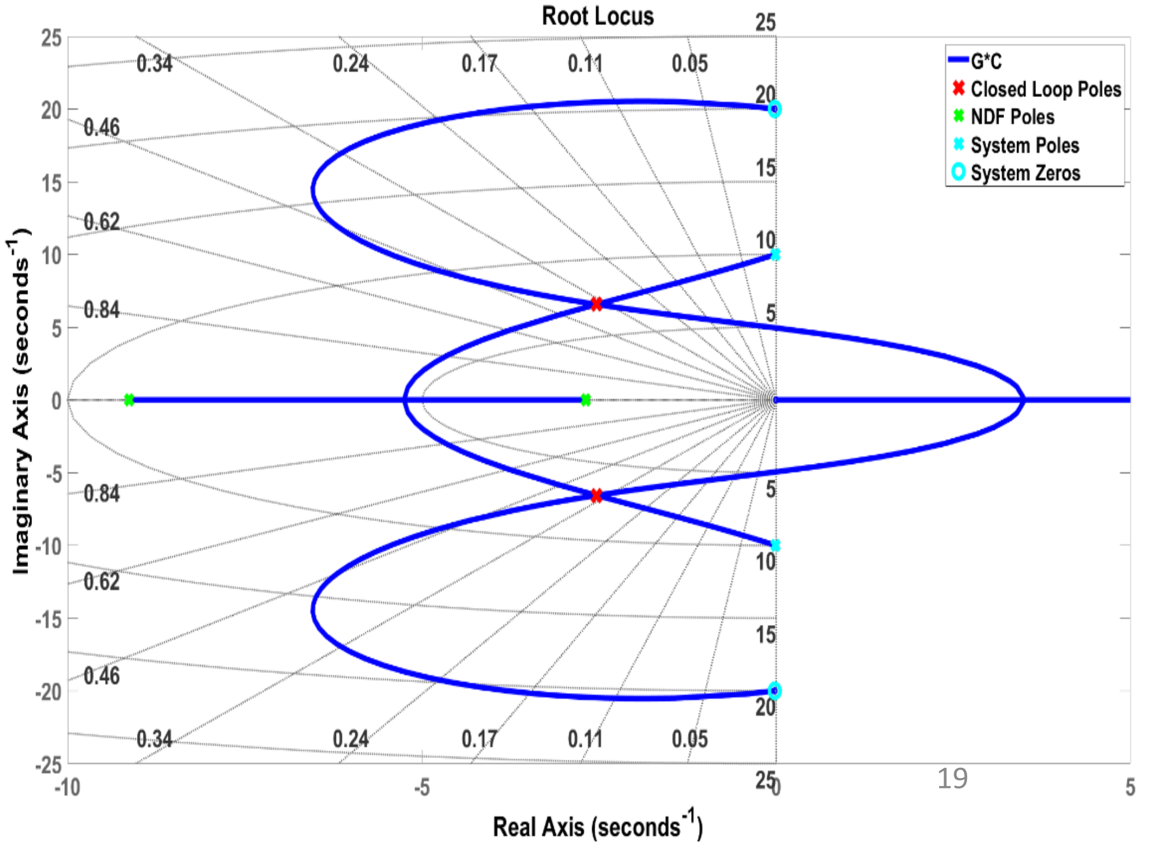
Zero After Pole ($\lambda < 1$)

Example 2:

$$G(s) = \frac{s^2 + 0.04 * s + 400}{s^2 + 0.02 * s + 100}$$

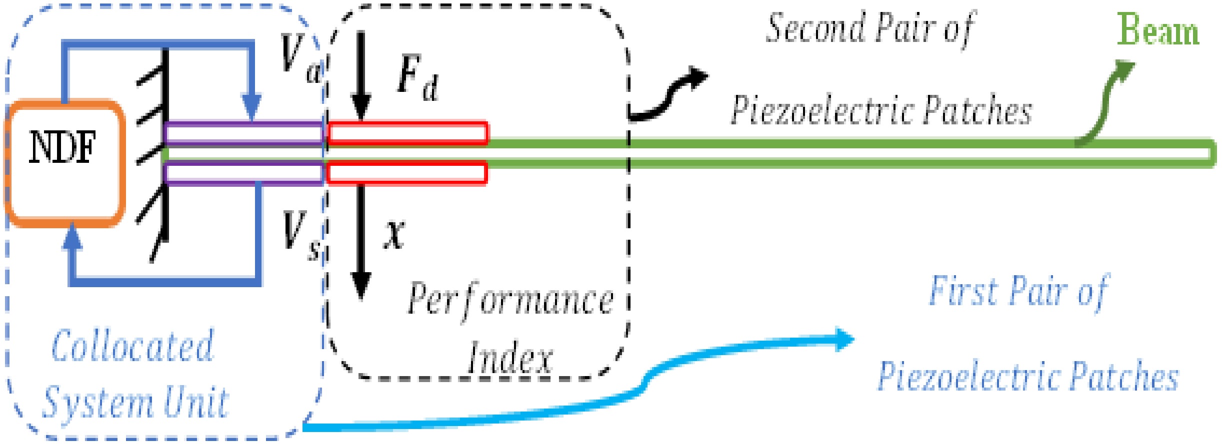


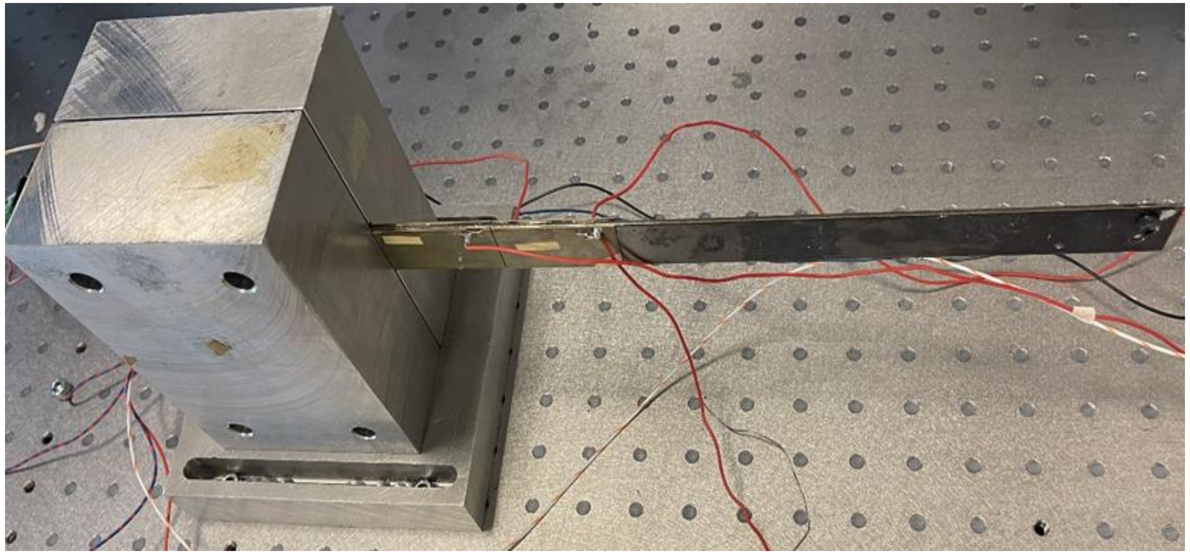


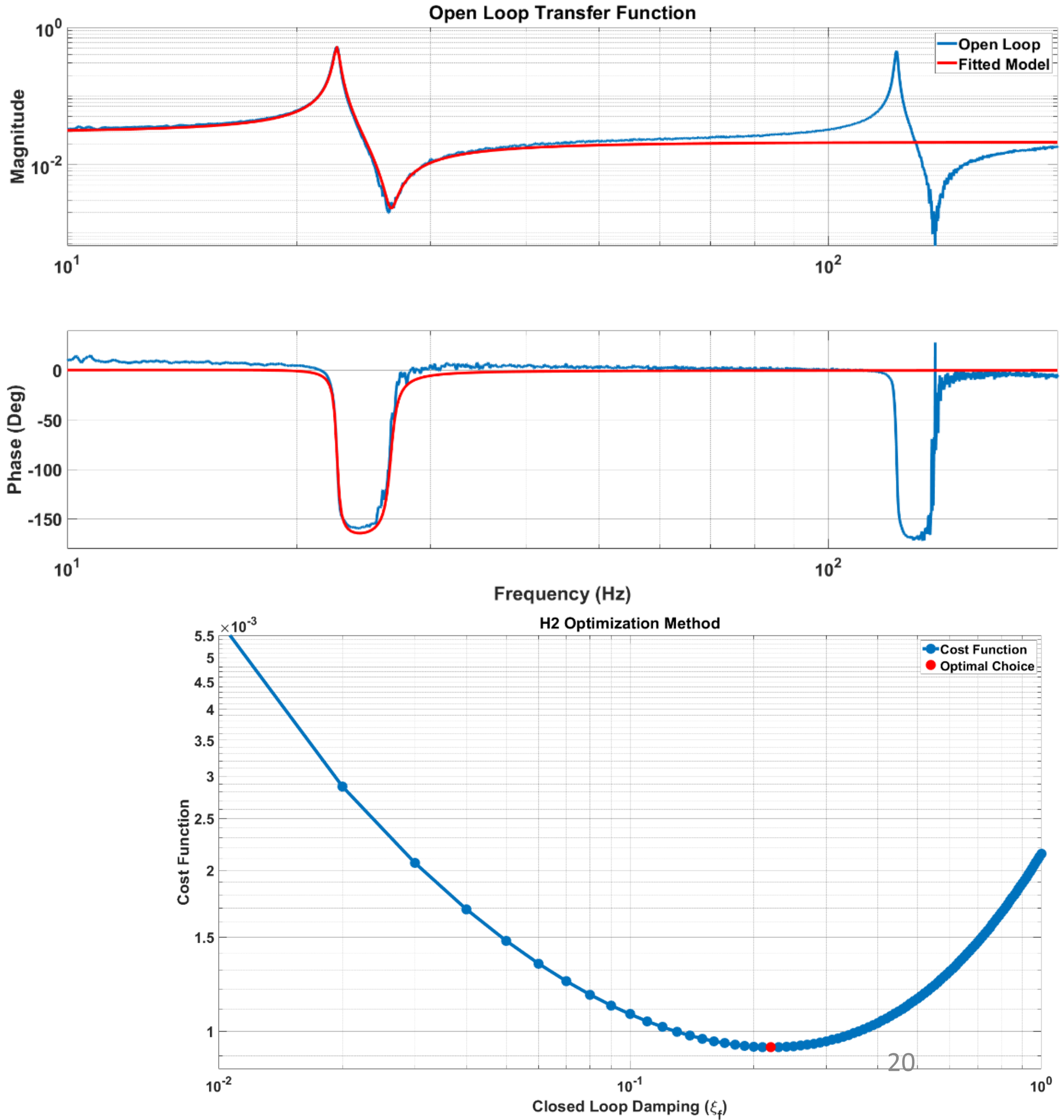


Experimental Verifications

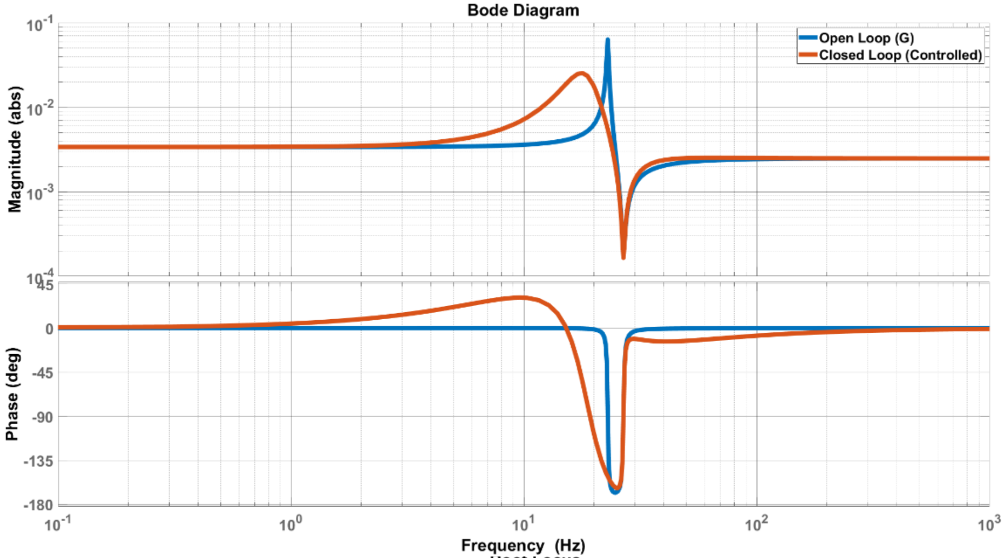
Beam

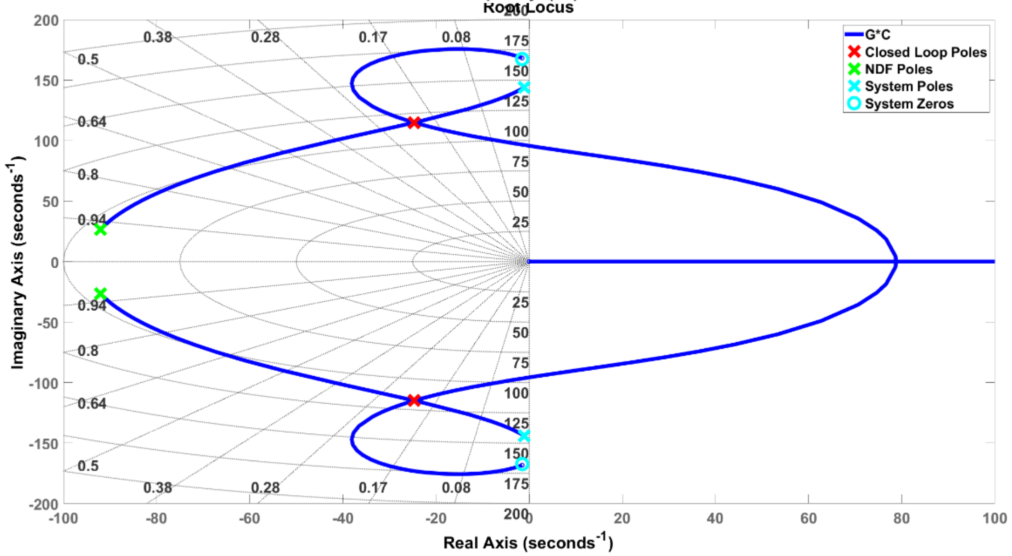


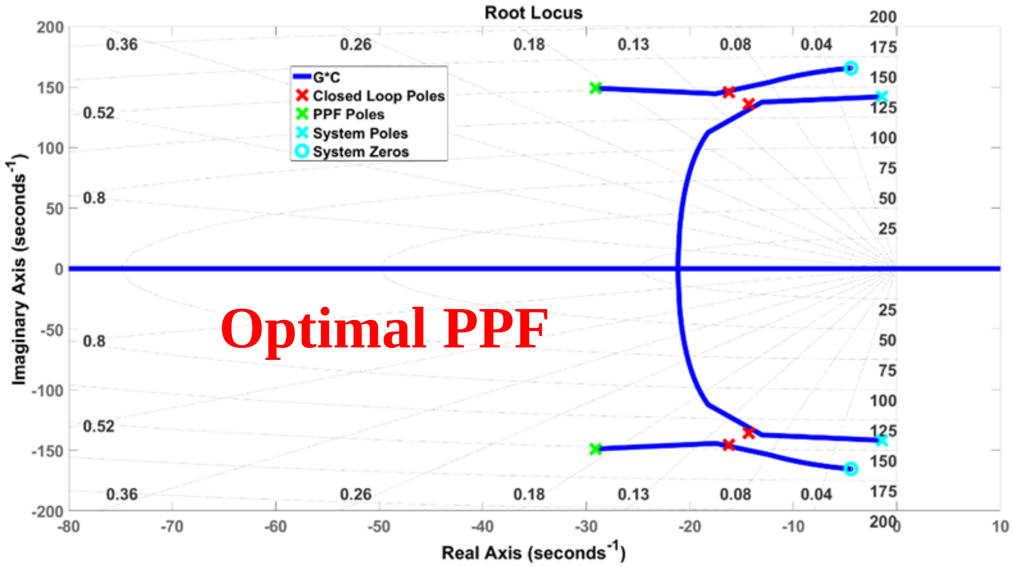




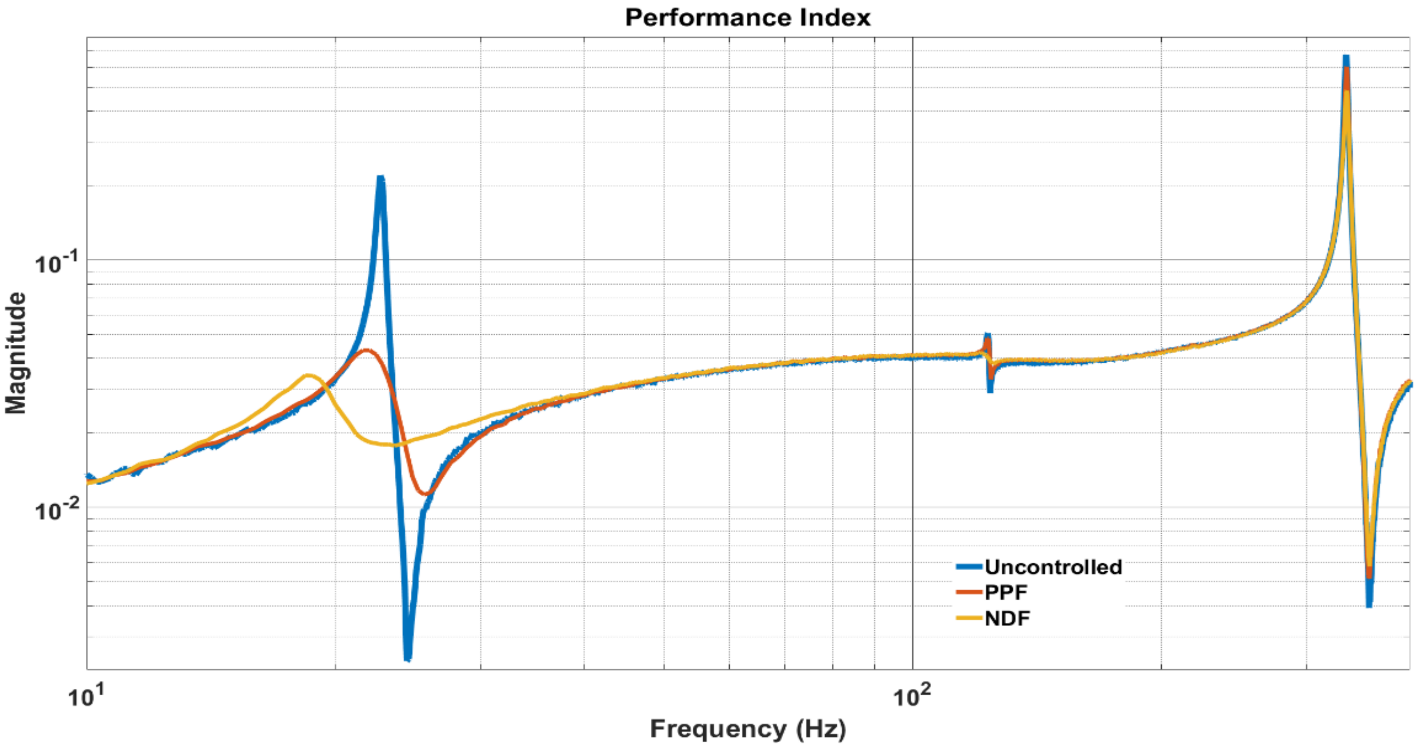
Optimal NDF





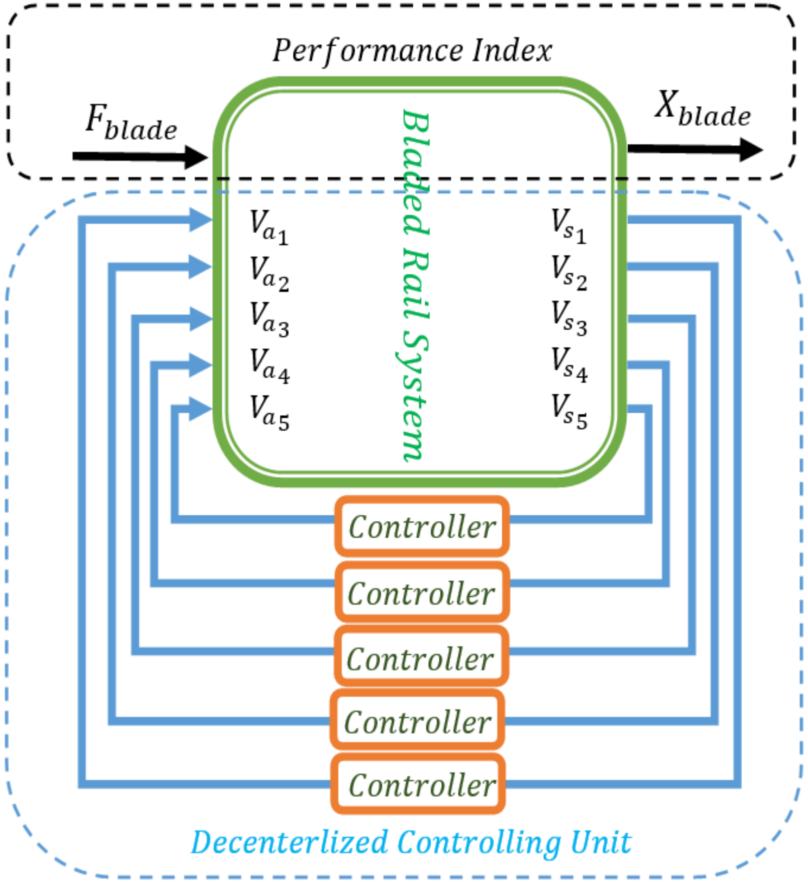


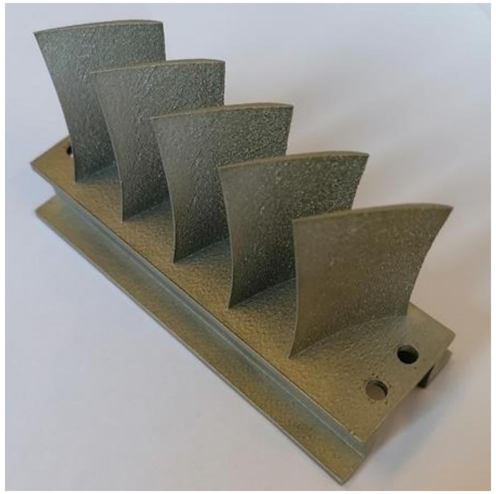
Optimal PPF

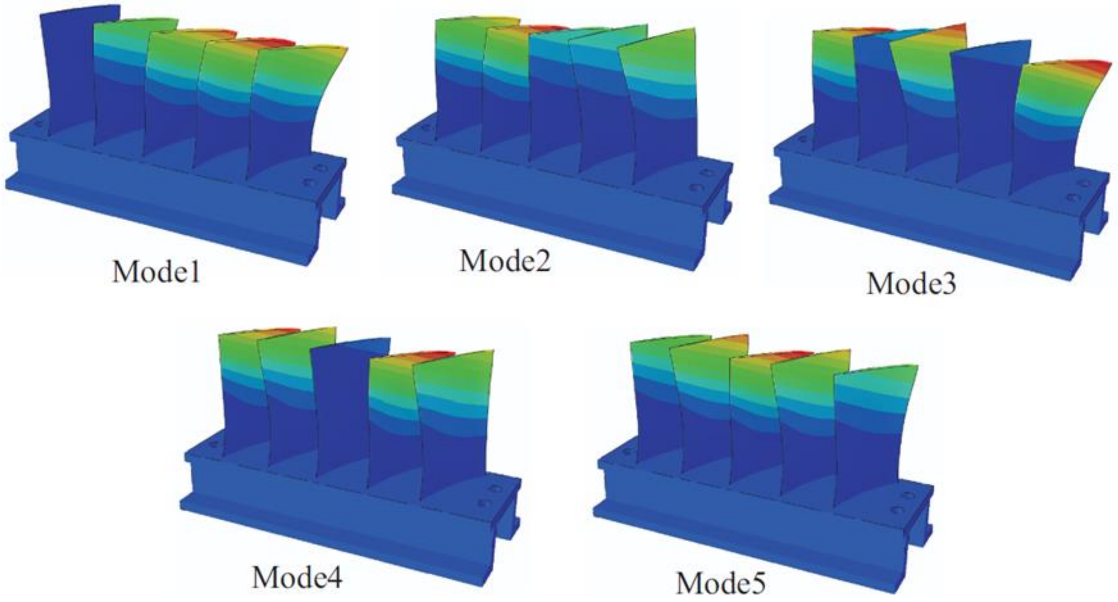


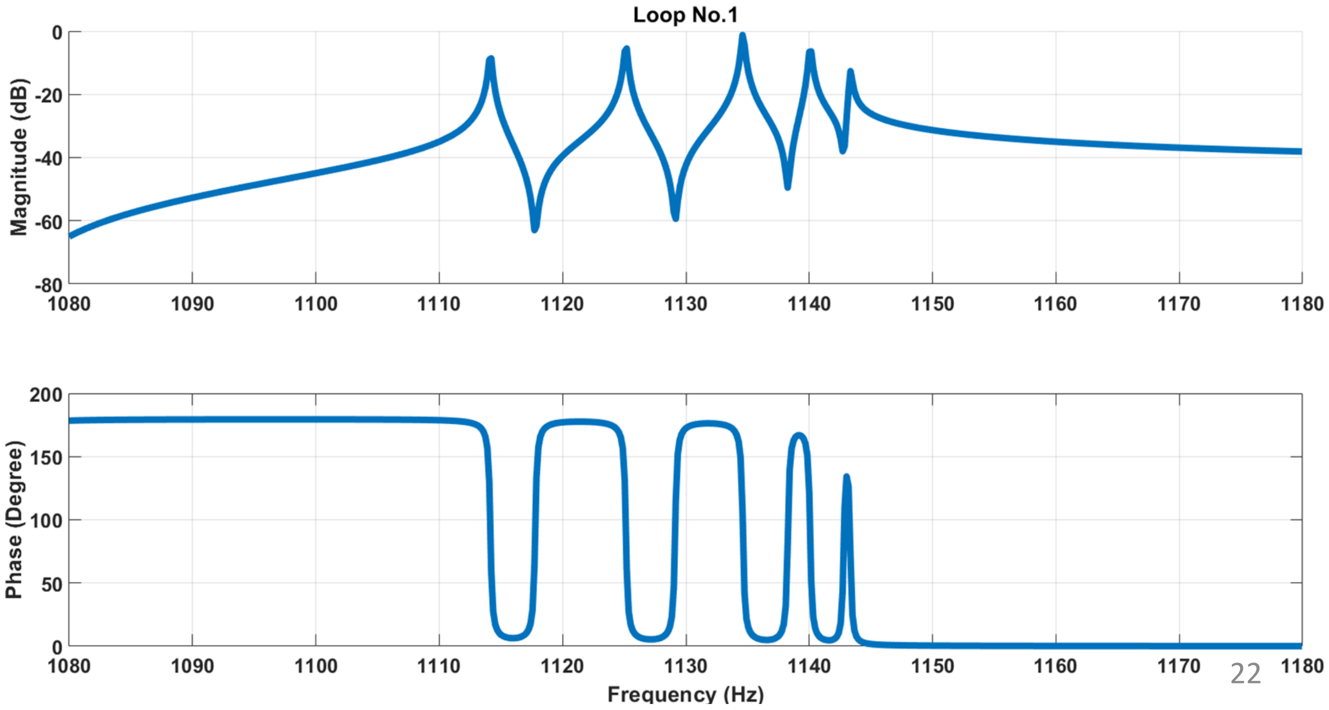
NDF For Collocated Systems

Bladed Rail

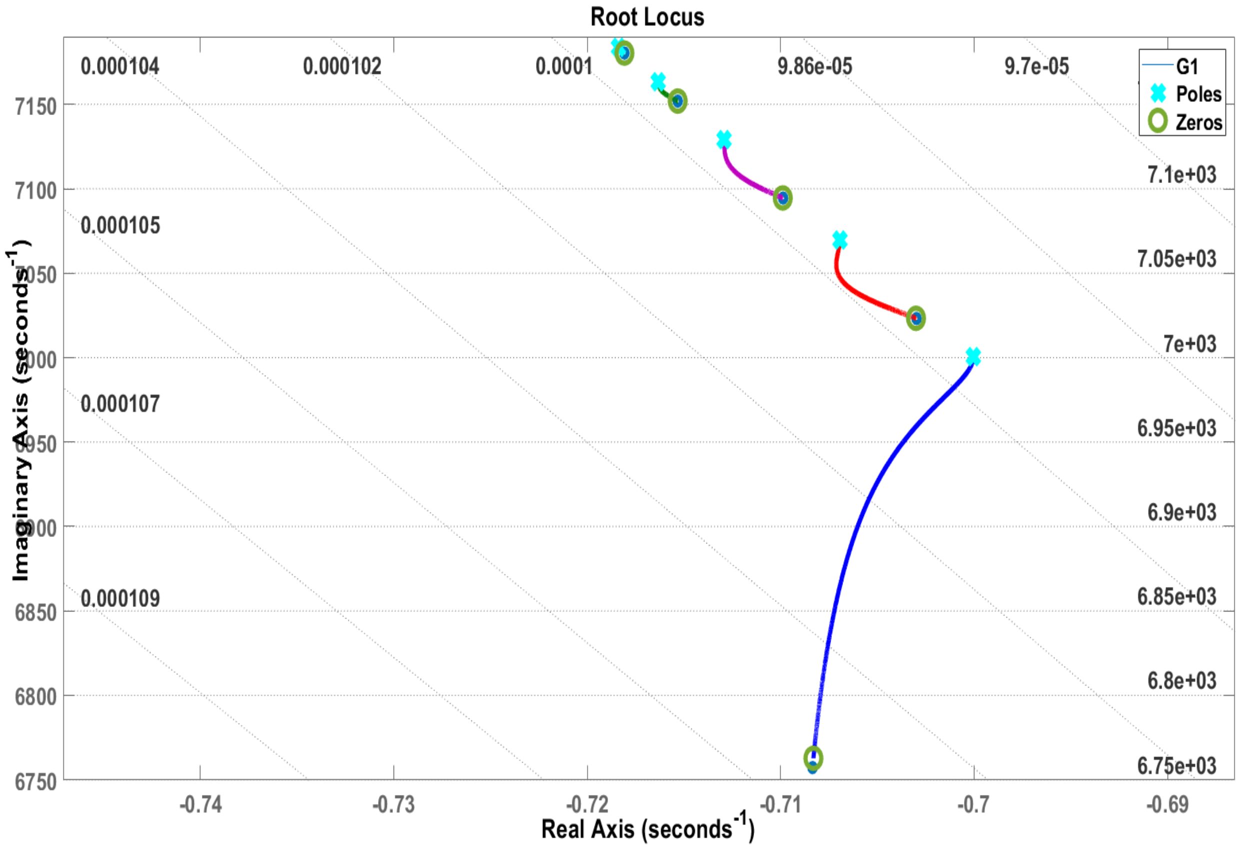




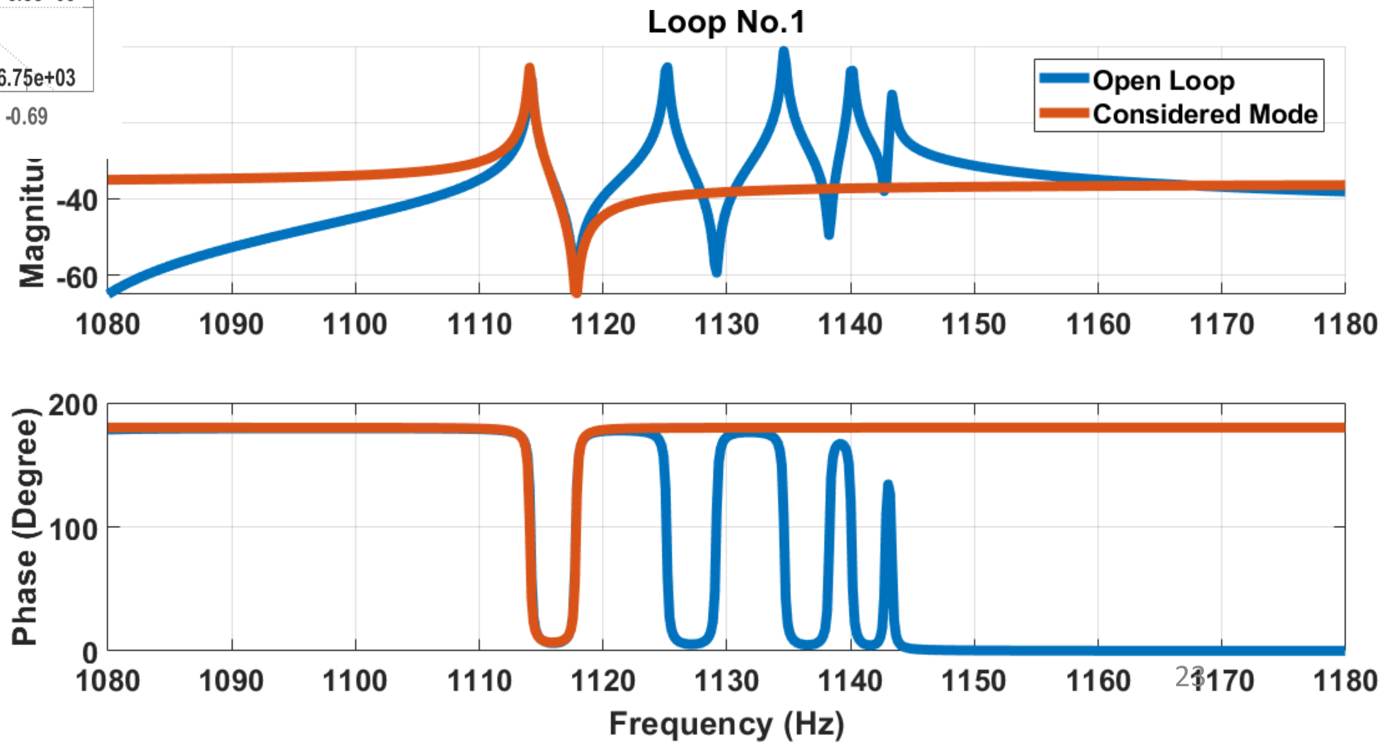




NDF For Bladed Rail

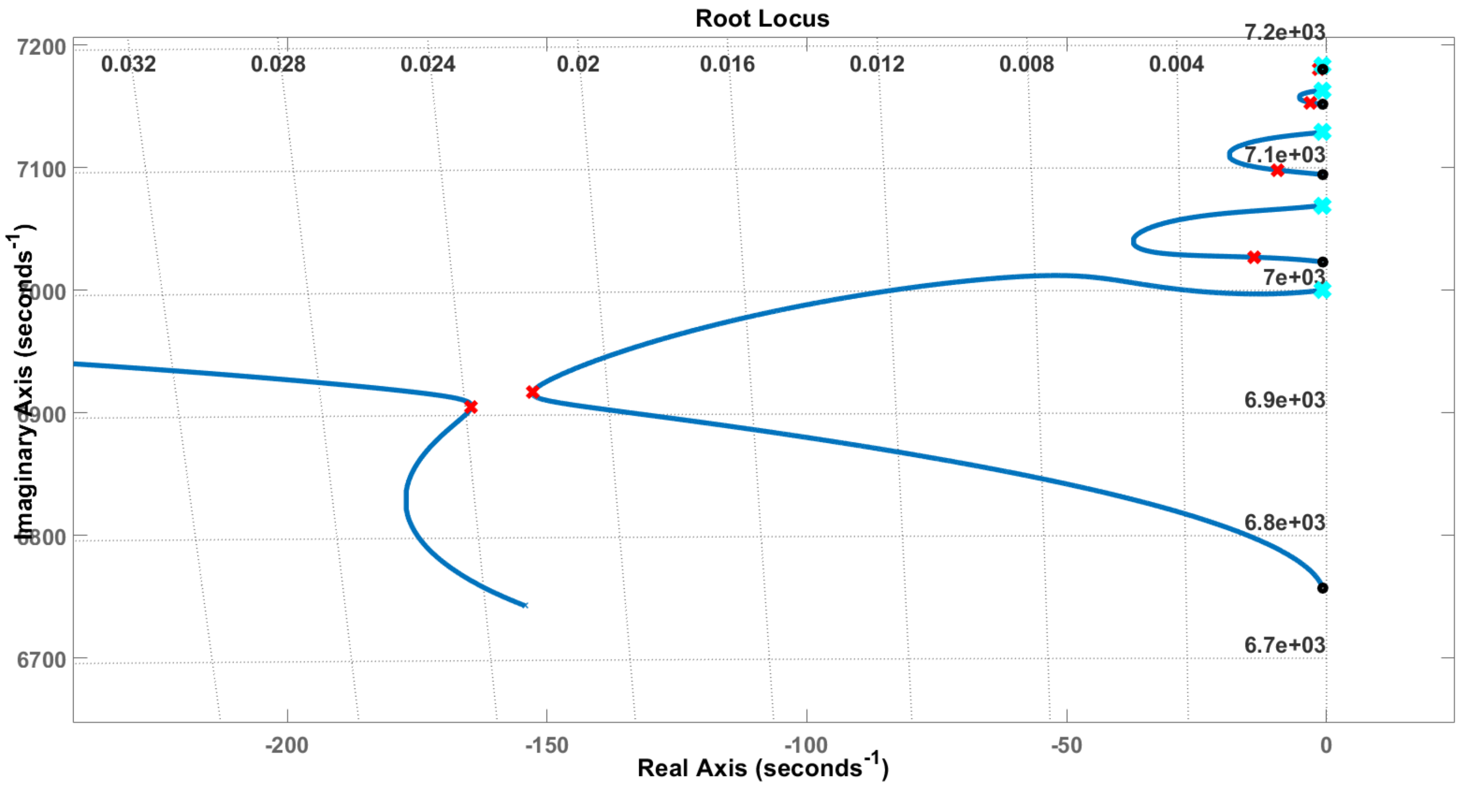


Distances between zero and pole decreases in higher modes!

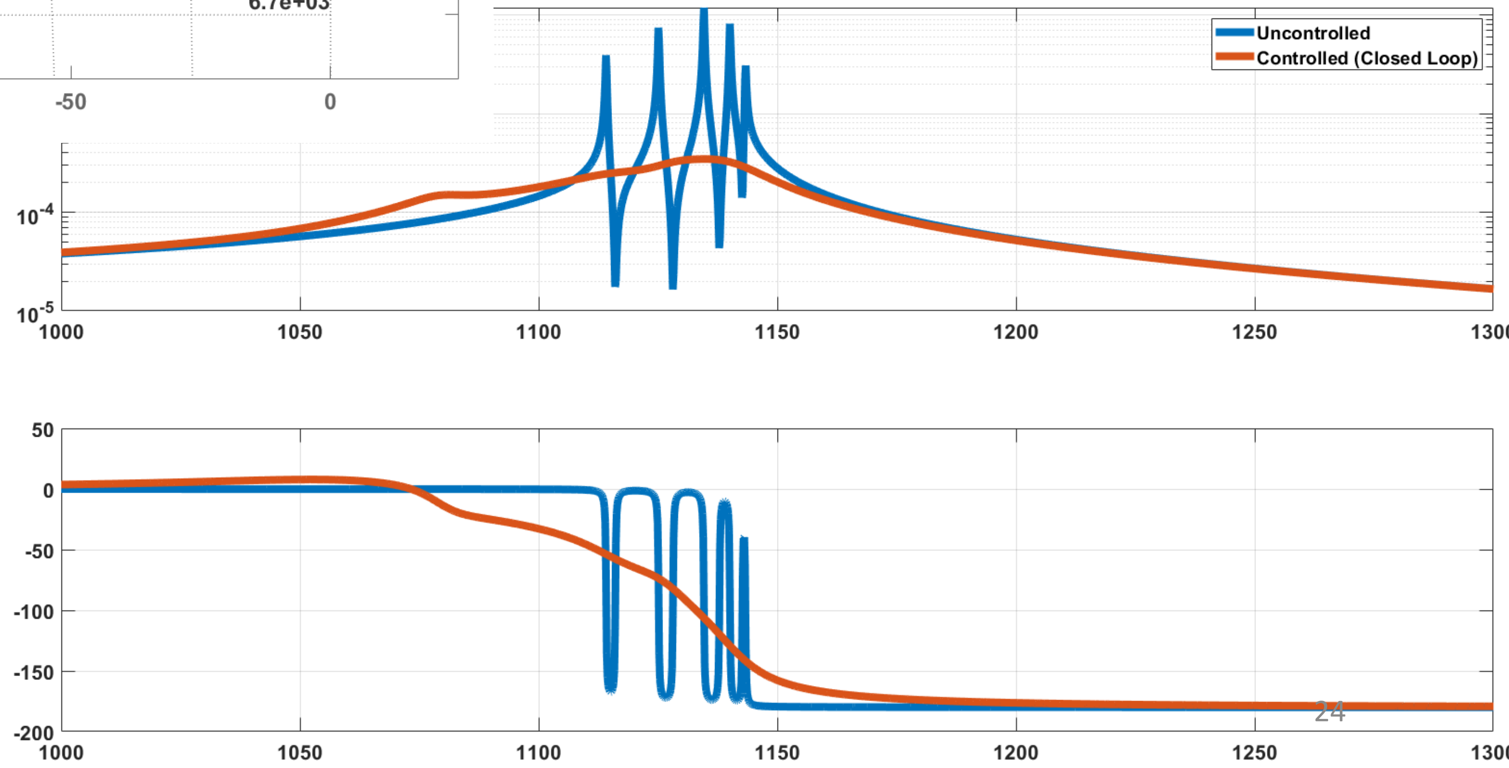


So highest damping can be occurred only in the first Mode!

NDF For Bladed Rail



First Mode damping Increased From 10^{-4} to 0.024!

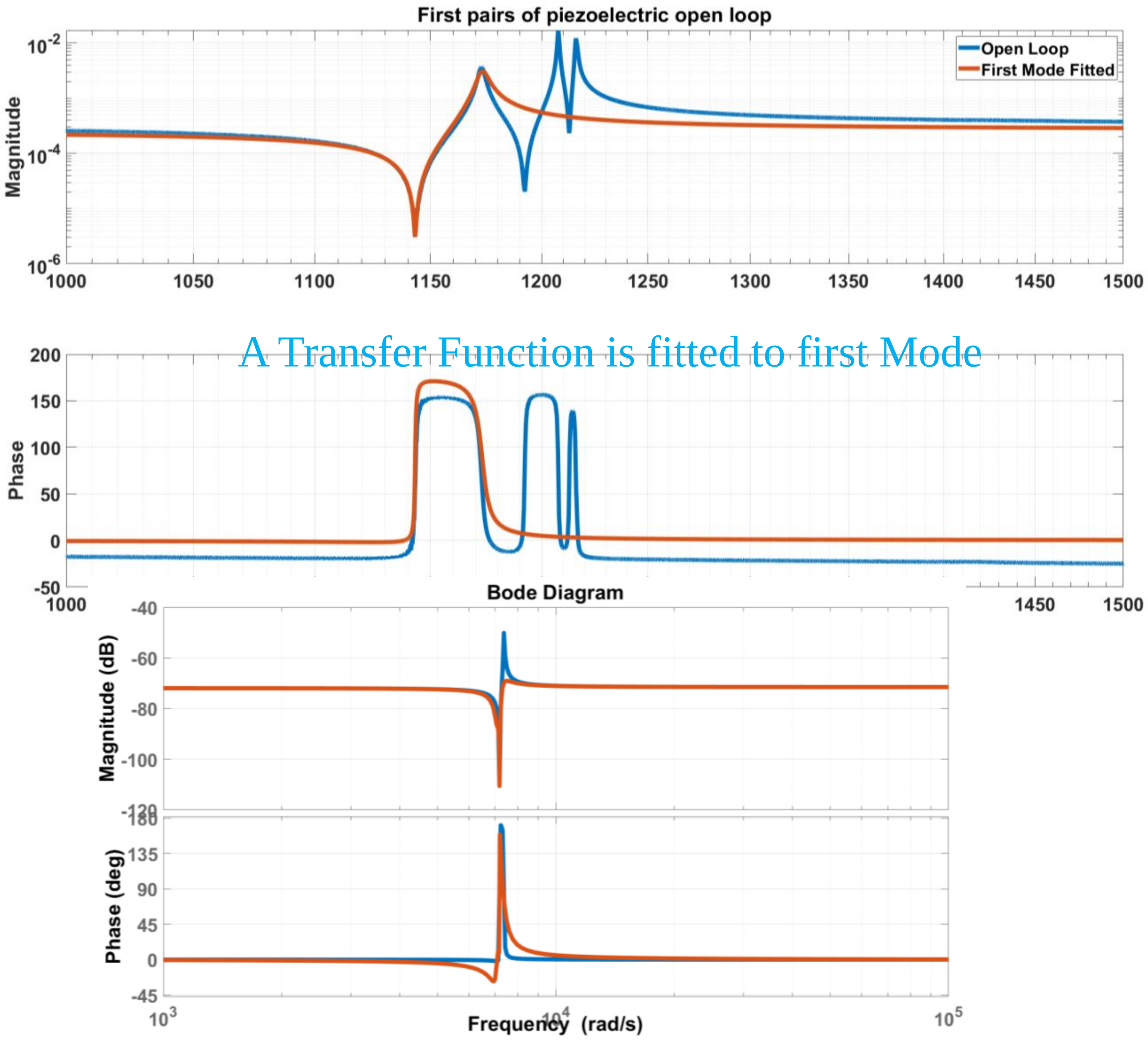


Performance Index shows all the Family mode is damped!

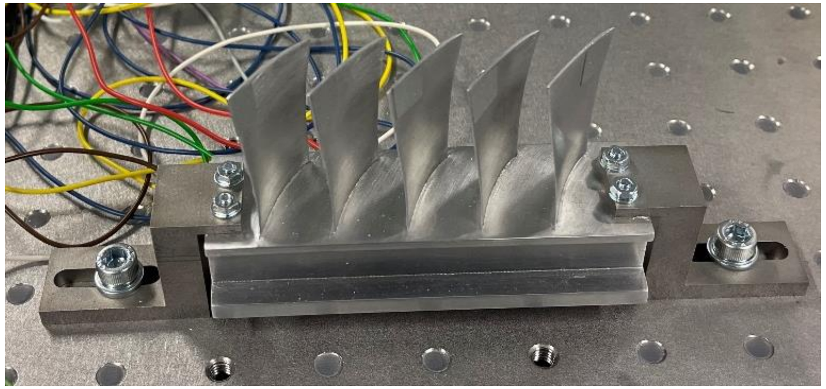
NDF For Bladed Rail

Experimental Test: First Bladed Rail

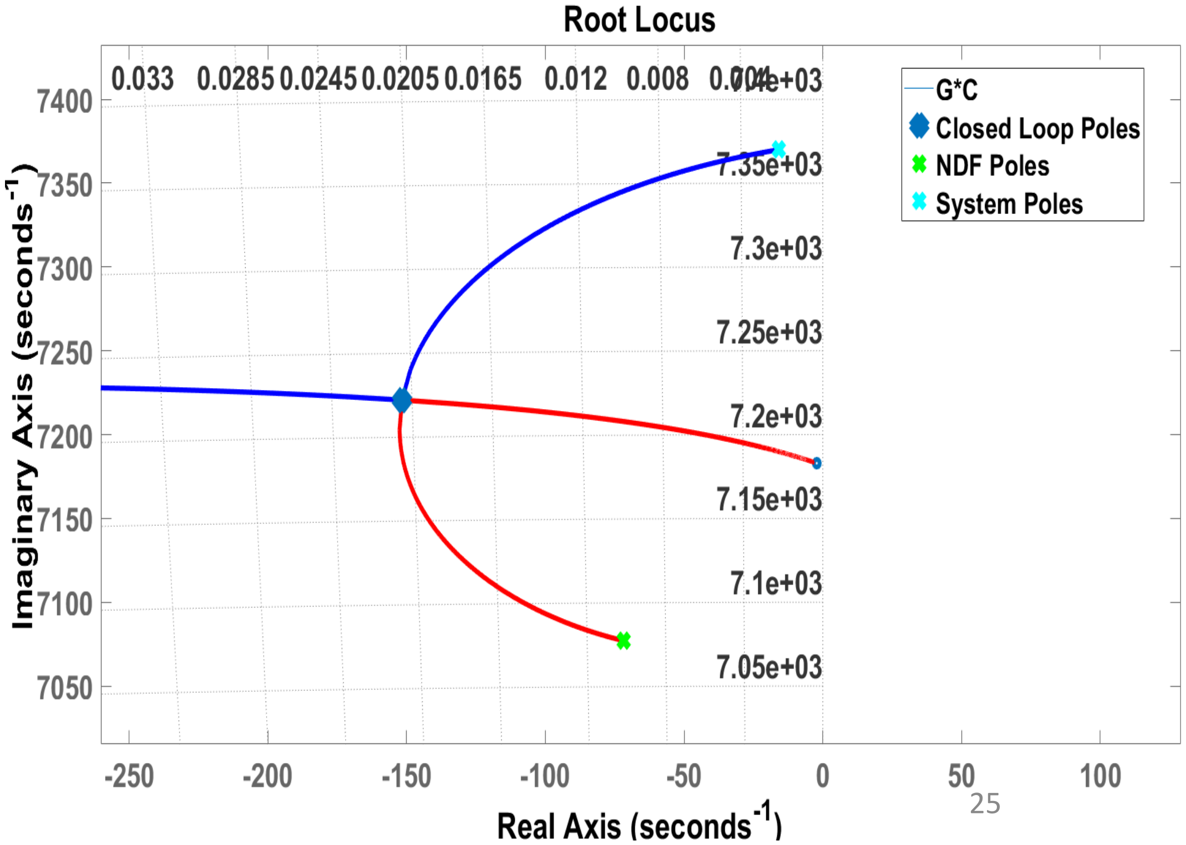
Zero Before Pole Pattern



A Transfer Function is fitted to first Mode

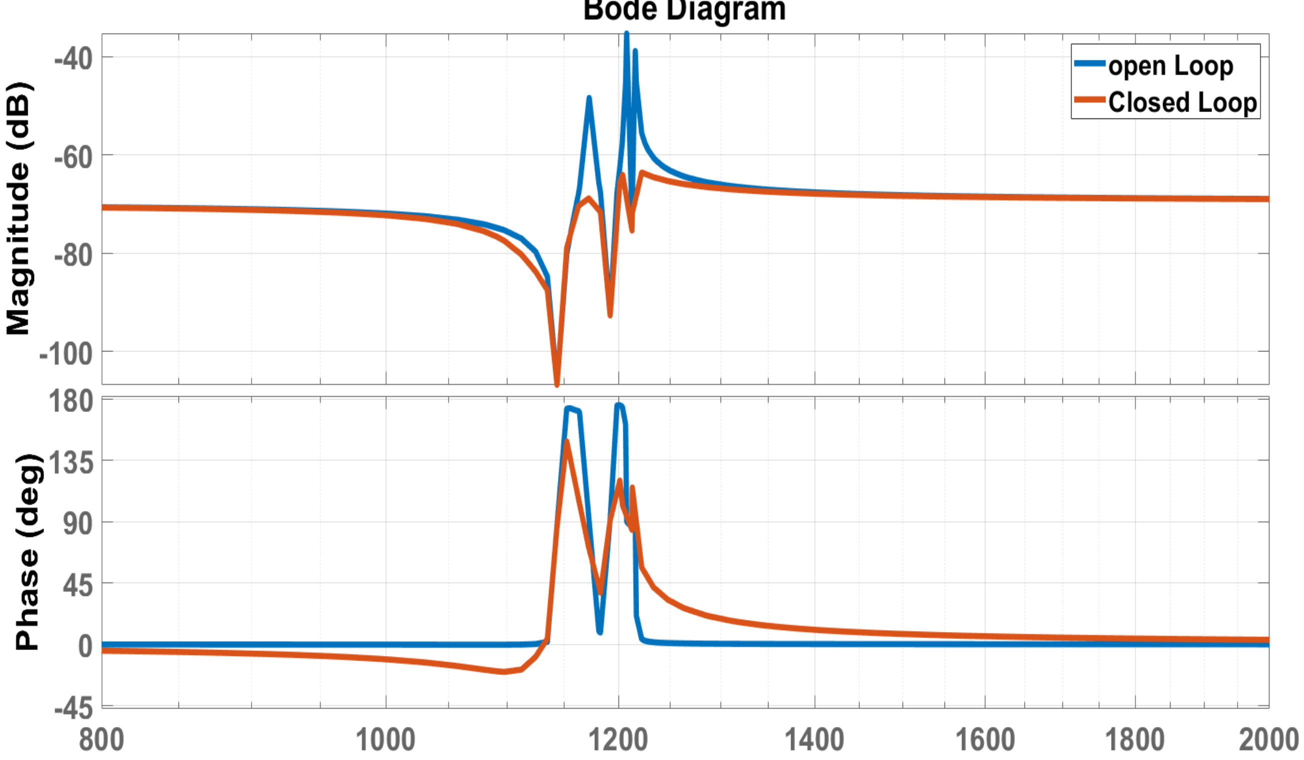


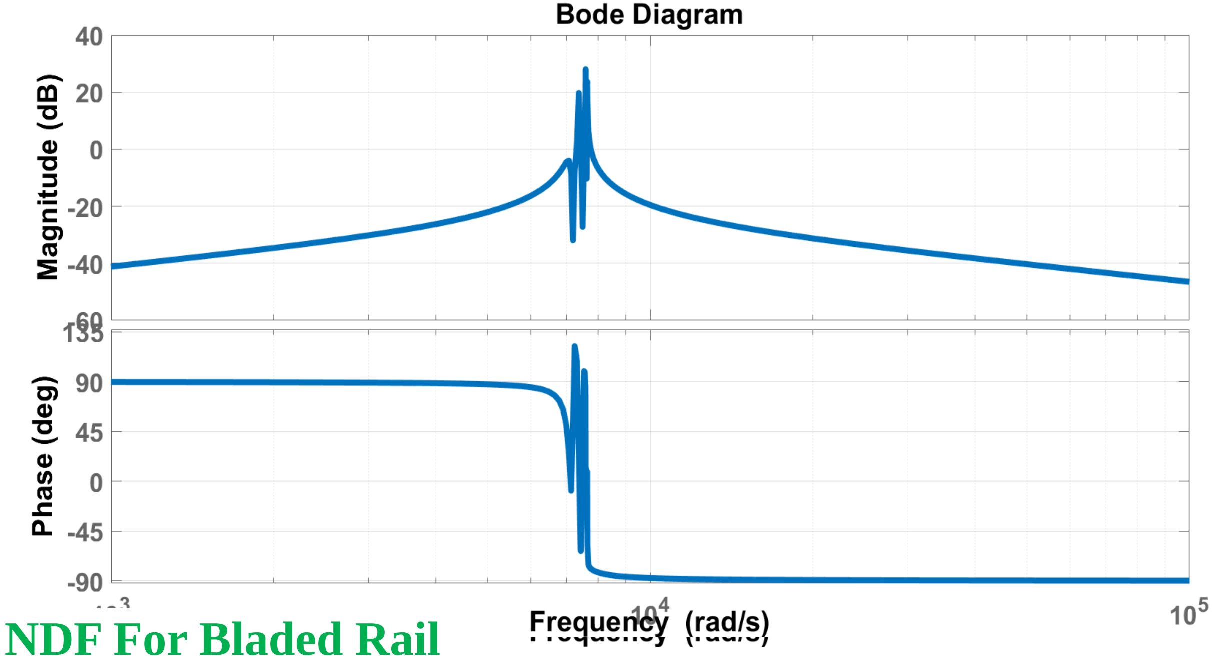
NDF Design



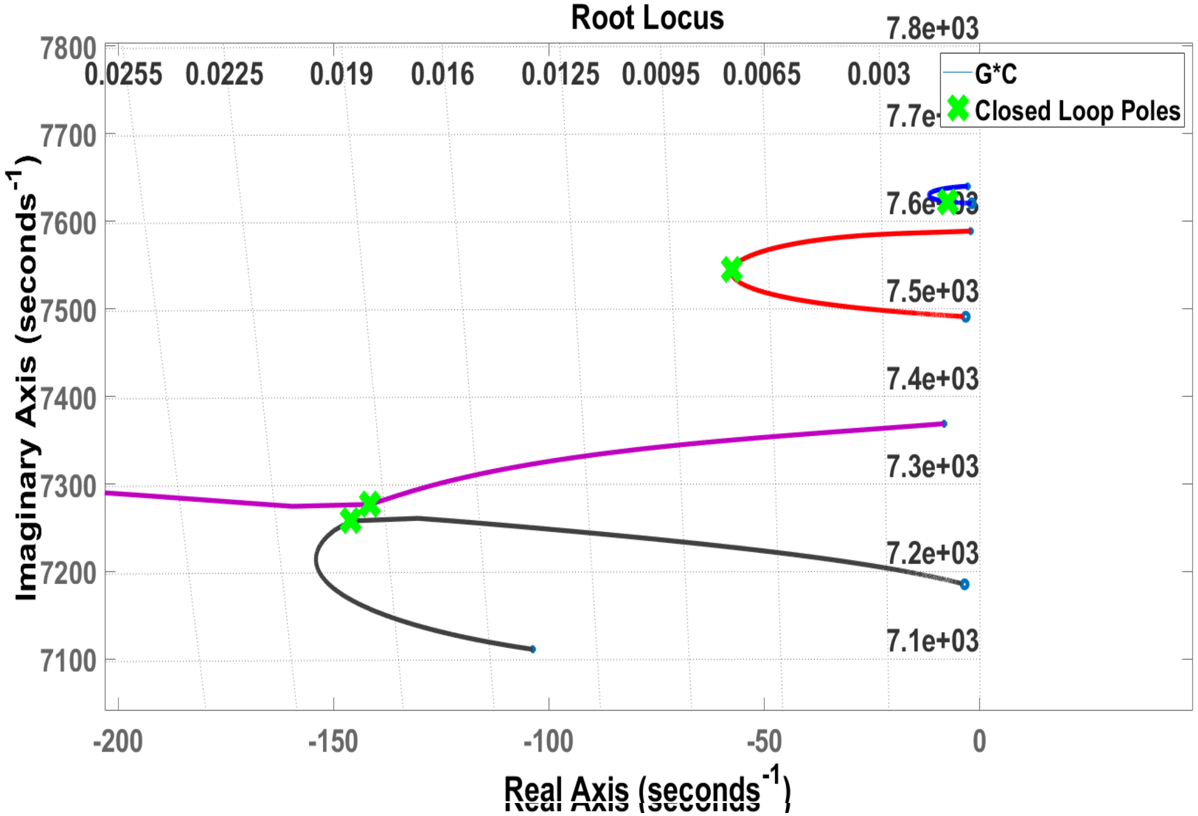
NDF For Bladed Rail

NDF Design



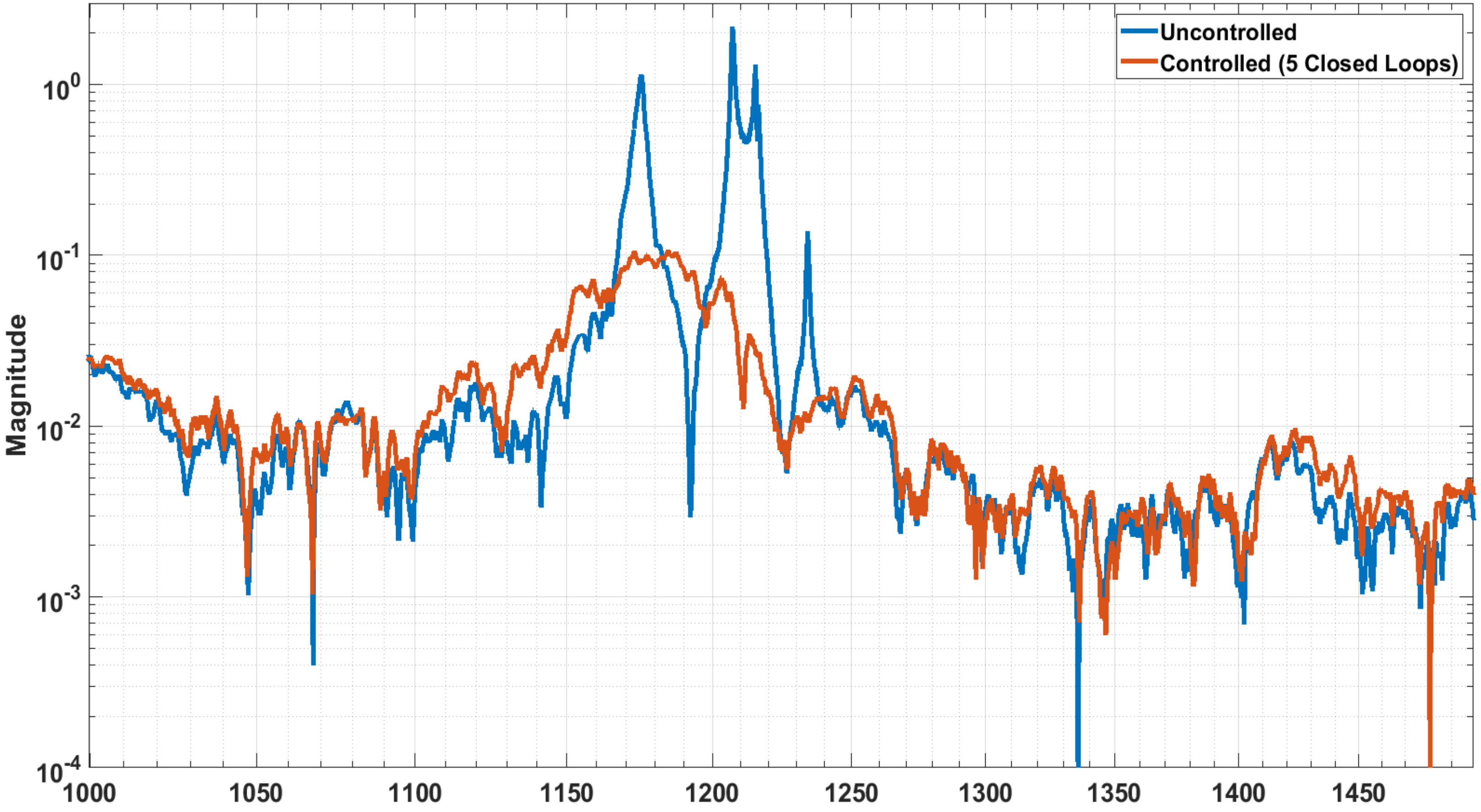


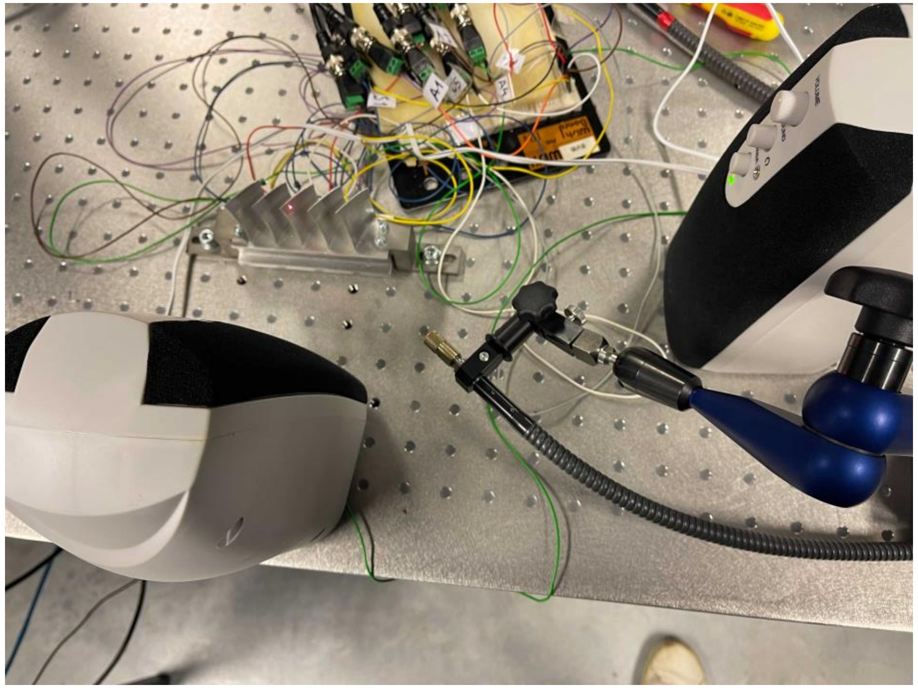
NDF For Bladed Rail



Precision Mechatronics Laboratory

Test Result

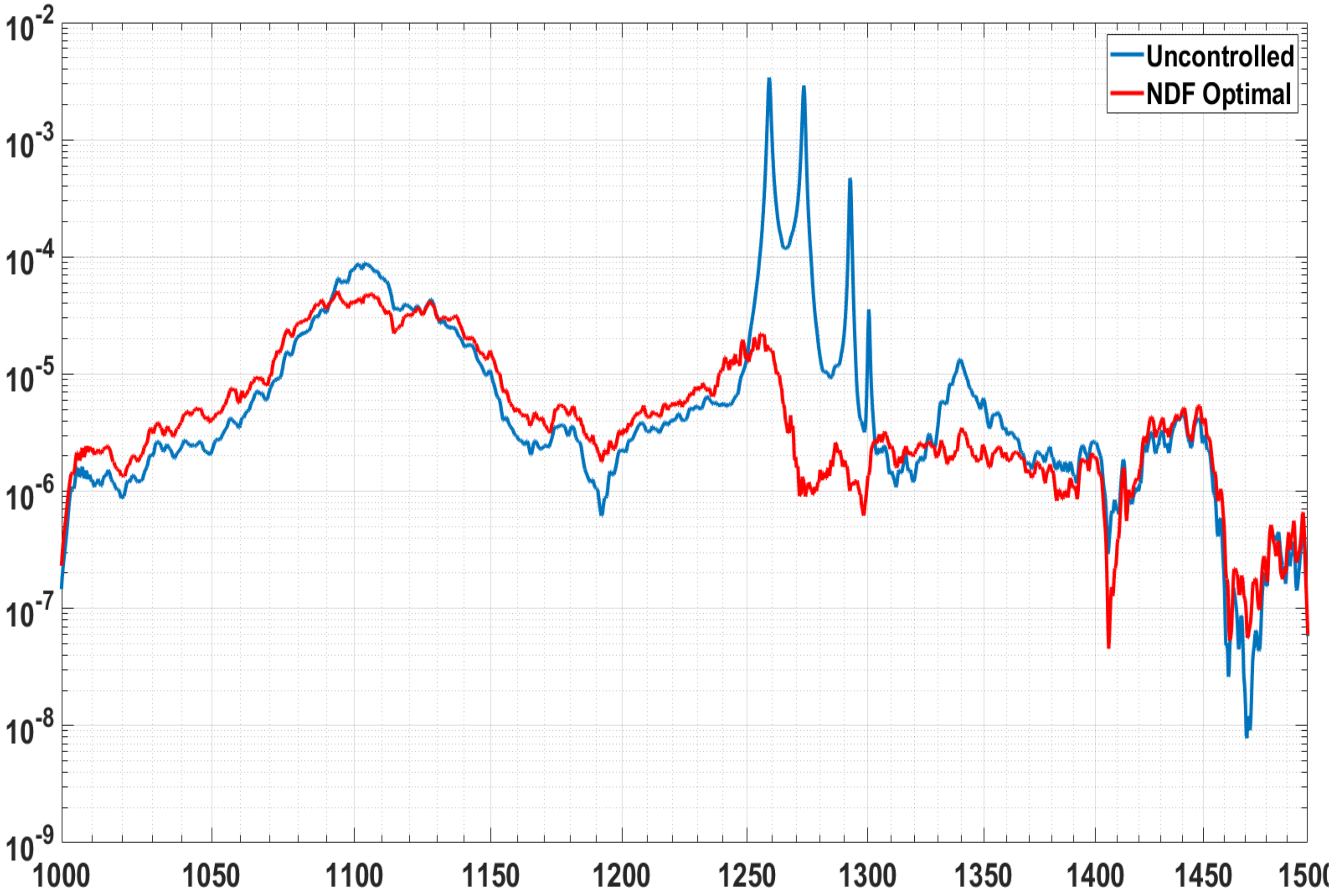


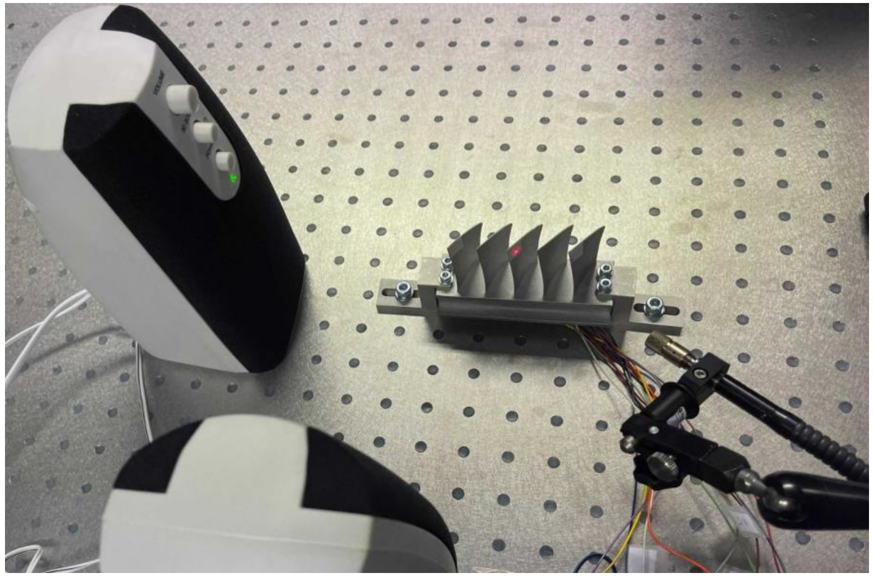


Experimental Test: Second Bladed Rail

Zero After Pole Pattern

The Same Procedure has been done.



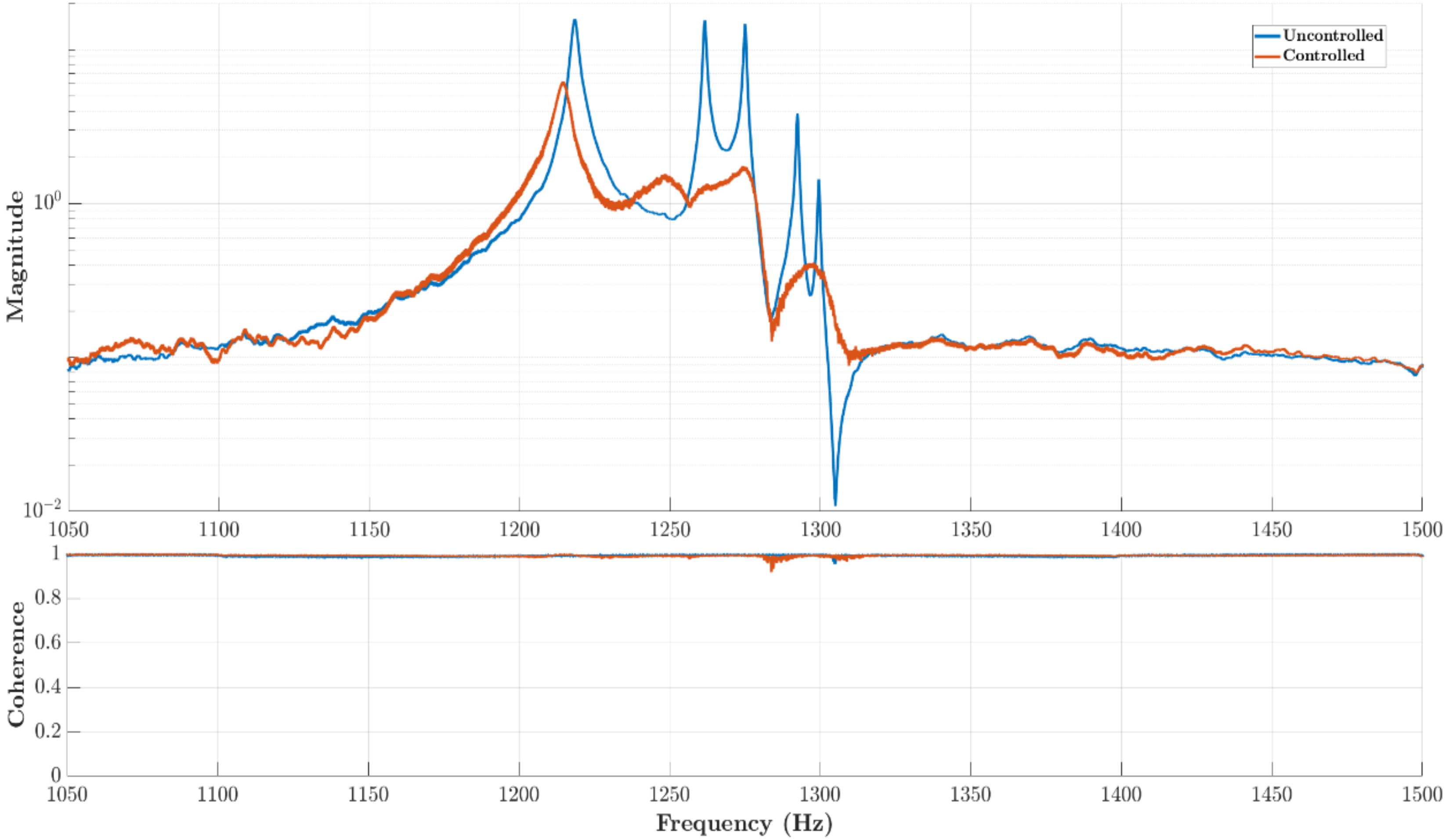


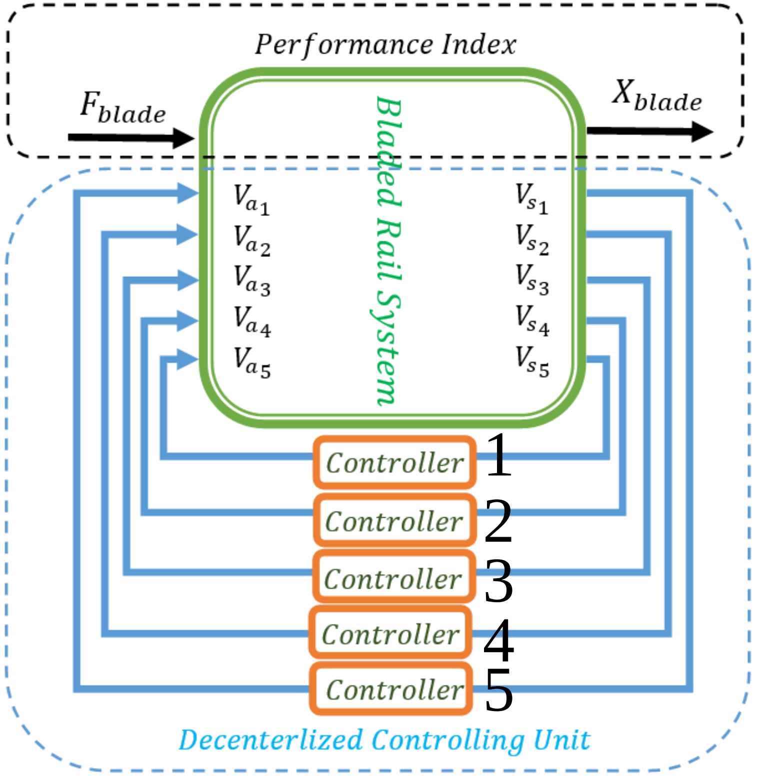
PPF Controller

Controllers are not the same!!!!

$$C(s) = +\frac{g_f \omega_f^2}{s^2 + 2\xi_f \omega_f + \omega_f^2}$$

$\alpha = \sqrt{4\eta^2 + 1}$
$\beta = \dfrac{4\eta^2}{4\eta^2 + 1}$
$\xi_f = \dfrac{2\eta}{\sqrt{4\eta^2 + 1}}$





UNIVERSITÉ LIBRE DE BRUXELLES

Thank You