

Matematika dasar

Problem set 1.3

Verify that the following are identities

$$a. (1 + \sin z)(1 - \sin z) = \frac{1}{\sec^2 z}$$

$$(1 + \sin z)(1 - \sin z) = \cos^2 z$$

$$\cos^2 z = \frac{1}{\sec^2 z}$$

$$b. (\sec t - 1)(\sec t + 1) = \tan^2 t$$

$$(\sec t - 1)(\sec t + 1) = \sec^2 t - 1 = \tan^2 t$$

$$c. \sec t - \sin t \cdot \tan t = \cos t$$

$$= \frac{1}{\cos t} - \frac{\sin t}{\cos t}$$

$$= \frac{1 - \sin^2 t}{\cos t} = \frac{\cos^2 t}{\cos t} = \cos t$$

$$d. \frac{\sec^2 t - 1}{\sec^2 t} = \sin^2 t$$

$$= \frac{\sec^2 t - 1}{\sec^2 t} = \frac{\tan^2 t}{\sec^2 t} = \frac{\frac{\sin^2 t}{\cos^2 t}}{\frac{1}{\cos^2 t}} = \sin^2 t$$

Verify the following are identities

$$a. \frac{\sin u}{\csc u} + \frac{\cos u}{\sec u} = 1$$

$$= \sin^2 u + \cos^2 u = 1$$

$$c. \sin t (\csc t - \sin t) = \cos^2 t$$

$$= \sin t \left(\frac{1}{\sin t} - \sin t \right)$$

$$= 1 - \sin^2 t = \cos^2 t$$

$$b. (1 - \cos^2 x)(1 + \cot^2 x) = 1$$

$$= (\sin^2 x)(\csc^2 x)$$

$$= \sin^2 x \left(\frac{1}{\sin^2 x} \right) = 1$$

$$d. \frac{1 - \csc^2 t}{\csc^2 t} = \frac{-1}{\sec^2 t}$$

$$= \frac{-\cot^2 t}{\csc^2 t} = \frac{-\cos^2 t}{\frac{1}{\cos^2 t}} = -\cos^2 t$$

$$= \frac{-1}{\sec^2 t}$$

Problem Set 1.9

$$\sin \theta = -\frac{2}{3}$$

$$\cos [2 \sin^{-1}(-\frac{2}{3})] =$$

$$\cos [2\theta] = 1 - 2\sin^2 \theta$$

$$\cos [2 \sin^{-1}(-\frac{2}{3})] = 1 - 2(-\frac{2}{3})^2$$

$$= 1 - 2(\frac{4}{9})$$

$$= 1 - \frac{8}{9}$$

$$= \underline{\underline{\frac{1}{9}}}$$

$$\tan \theta = \frac{1}{3}$$

$$\tan [2 \tan^{-1}(\frac{1}{3})]$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2 \cdot \frac{1}{3}}{1 - (\frac{1}{3})^2} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2/3 \cdot 9}{8} = \frac{6}{8} = \frac{3}{4}$$

$$= \frac{18}{24} = \frac{3}{4}$$

$$\sin [\cos^{-1}(\frac{3}{5}) + \cos^{-1}(\frac{5}{13})]$$

$$\sin [A+B] = \sin A \cdot \cos B + \cos A \sin B$$

$$\cos A = \sqrt{1 - \sin^2(A)} = \sqrt{1 - \frac{9}{25}} = \sqrt{16/25} = 4/5$$

$$\cos B = \sqrt{1 - \sin^2(B)} = \sqrt{1 - \frac{25}{169}} = \sqrt{144/169} = 12/13$$

$$\sin [A+B] = (\frac{3}{5}) \cdot (\frac{12}{13}) + (\frac{4}{5}) \cdot (\frac{5}{13}) = (\frac{36}{65}) + (\frac{20}{65}) = \underline{\underline{\frac{56}{65}}}$$

$$\cos [\cos^{-1}(\frac{4}{5}) + \sin^{-1}(\frac{12}{13})]$$

$$\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \frac{16}{25}} = \sqrt{9/25} = 3/5$$

$$\cos B = \sqrt{1 - \sin^2 B} = \sqrt{1 - \frac{144}{169}} = \sqrt{25/169} = 5/13$$

$$\cos [A+B] = \cos A \cos B - \sin A \sin B$$

$$= (\frac{4}{5}) \cdot (\frac{5}{13}) - (\frac{3}{5}) \cdot (\frac{12}{13})$$

$$= \frac{20}{65} - \frac{36}{65} = \underline{\underline{-\frac{16}{65}}}$$

$$29 \quad \sec(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$$

$$= \frac{\sin(\sin^{-1} x)}{\cos} = \frac{x}{\sqrt{1-x^2}}$$

$$30 \quad \sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$$

$$\sin x = \frac{\tan}{\sec}(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$$

$$31 \quad \cos(2 \sin^{-1} x) = 1 - 2x^2$$

$$\cos 2x =$$

$$\cos^2 x - \sin^2 x = 1 - 2x^2$$

$$\cos^2(\sin^{-1} x) - \sin^2(\sin^{-1} x) = 1 - 2x^2$$

$$32 \quad \tan(2 \tan^{-1} x) = \frac{2x}{1-x^2}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \tan(\tan^{-1} x)}{1 - \tan^2(\tan^{-1} x)} = \frac{2x}{1-x^2}$$