### Part I: Basic Probability and Statistics

1.

- a. The sample space for this experiment is infinite. The probability of landing on heads on the  $i^{th}$  flip is  $\frac{1}{2^i}$ , because there must be a sequence of i-1 flips that lands on tails and then a flip that lands on heads. Assuming a fair coin, the probability of the coin landing on heads and tails are the same,  $\frac{1}{2}$ .
- b. The set of outcomes that belong to this event are the set of all sequences where tails is flipped an odd number times and then a head is flipped. The probability of this event occurring is:

$$\sum_{1}^{\infty} \frac{1}{2^{2n-1}} * \frac{1}{2} = \sum_{1}^{\infty} \frac{1}{2^{2n}} = \frac{1}{3}$$

2.

$$P(E) = P(Sum = 3) + P(Sum = 5) + P(Sum = 7) + P(Sum = 9) + P(Sum = 11)$$

$$= \frac{2}{36} + \frac{4}{36} + \frac{6}{36} + \frac{4}{36} + \frac{2}{36} = \frac{18}{36}$$

$$P(F) = 1 - P(\neg F) = 1 - \left(\frac{5}{6}\right)^2 = \frac{11}{36}$$

$$P(G) = P(Sum = 5) = \frac{4}{36}$$

a. The set of  $E \cap F$  contains  $\{(1,2), (1,4), (1,6), (2,1), (4,1), (6,1)\}$ 

$$P(E \cap F) = \frac{6}{36}$$

- b. The set of EUF contains  $P(E \cup F) = P(E) + P(F) P(E \cap F) = \frac{18}{36} + \frac{11}{36} \frac{6}{36} = \frac{23}{36}$
- c. The set of F $\cap$ G contains  $\{(1,4), (4,1)\}$

$$P(E \cap F) = \frac{2}{36}$$

d. The set of E $\cap$ ¬F contains {(2,3), (2,5), (3,2), (3,4), (3,6), (4,3), (4,5), (5,2),(5,4),(5,6),(6,3),(6,5)}

$$P(E \cap F) = \frac{12}{36}$$

e. The set of  $E \cap F \cap G$  contains  $\{(1,4), (4,1)\}$ 

$$P(E \cap F) = \frac{2}{36}$$

3.

a. 
$$P(E) = P(d_1) * P(d_2) + P(d_2) * P(d_3) + P(d_1) * P(d_3) - P(d_1) * P(d_2) * P(d_3) = 0.01 * 0.03 + 0.03 * 0.05 + 0.01 + 0.05 - 0.01 * 0.03 * 0.05 = 0.061785$$

b. 
$$P(F) = P(d_1) + P(d_2) * P(d_3) - P(d_1) * P(d_2) * P(d_3) = 0.01 + 0.03 * 0.05 - 0.01 * 0.03 * 0.05 = 0.011485$$

c. 
$$P(F|d_3) = P(d_1) + P(d_2) - P(d_1) * P(d_2) = 0.01 + 0.03 - 0.01 * 0.03 = 0.0397$$

- 4.  $P(Female) = 0.52, P(CS Major) = 0.05, P(Female \cap CS Major) = 0.0055$ 

  - a.  $P(Female | CS \ Major) = \frac{P(Female \cap CS \ Major)}{P(CS \ Major)} = \frac{0.0055}{0.05} = 0.11 = 11\%$ b.  $P(CS \ Major | Female) = \frac{P(Female \cap CS \ Major)}{P(Female)} = \frac{0.0055}{0.52} = 0.01058 = 1.058\%$
  - $P(Female \cap CS Major) = P(Female | CS Major) * P(CS Major) = 0.15 * 0.05 =$ 0.0075

$$P(CS \ Major | Female) = \frac{P(Female \cap CS \ Major)}{P(Female)} = \frac{0.0075}{0.57} = 0.01316 = 1.316\%$$

- 5.
- a. Let  $P(Z) = P(F \cup G)$ . Because E, F, and G are all independent of each other, we know that Z and E are independent. Thus, we know that  $P(E \cap Z) = P(E) * P(Z)$  and we can replace P(Z) with the original value to get  $P(E \cap (F \cup G)) = P(E) * P(F \cup G)$ .
- b. It is given that A and B are independent, so we know that  $P(A \cap B) = P(A) * P(B)$ To find  $P(\neg A \cap B)$ , we are looking for the set of events that occur in B and not in A, which can be expressed as  $P(\neg A \cap B) = P(B) - P(A \cap B) = P(B) - P(A) * P(B)$  by independence. This same relationship can be expressed as

$$P(\neg A \cap B) = P(B)(1 - P(A)) = P(B) * P(\neg A)$$
  
thus  $\neg A$  and B are independent.

c. P(X = 0) = 0.25, P(X = 1) = 0.5, P(X = 2) = 0.25, P(Y = 0) = 0.5, P(Y = 1) = 0.5The events are not independent. If the events were truly independent, then the probability that X and Y both being 1 should be:

$$P(X = 1 \cap Y = 1) = P(X = 1) * P(Y = 1) = 0.5 * 0.5 = 0.25$$

This is not the case in practice. If X=1, then there is exactly one head flipped and one tail, and as such there is not way to have two flips of the same result. Thus, the actual probability  $P(X = 1 \cap Y = 1) = 0$  and the property of independence does not hold for these events.

- 6.
- a.  $E(X_n) = \sum_{1}^{n} 1 * P(heads) 1 * P(tails) = \sum_{1}^{n} 1 * p 1 * (1 p) = \sum_{1}^{n} 2p 1 = \sum_{1}^{n} 2p 1$ n \* (2p-1)b.  $VAR(X_n) = \frac{1}{n} \sum_{i=1}^{n} (x(i) - E(x_n))^2 =$

b. 
$$VAR(X_n) = \frac{1}{n} \sum_{i=1}^{n} (x(i) - E(x_n))^2 =$$

$$\frac{1}{n}\sum_{i=1}^{n}(1*p-1*(1-p)-n*(2p-1))^{2}=\frac{1}{n}\sum_{i=1}^{n}(p-1+p-n(2p-1))^{2}=\frac{1}{n}\sum_{i=1}^{n}(p-1+$$

$$\frac{1}{n}\sum_{i=1}^{n} ((2p-1) - n(2p-1))^{2} = \frac{1}{n}\sum_{i=1}^{n} ((n-1) * (2p-1))^{2} =$$

$$\frac{1}{n} \sum_{i=1}^{n} (n^2 - 2n + 1) * (2p - 1)^2 = \frac{1}{n} (n - 1)^2 * n * (2p - 1)^2 = (n - 1)^2 * (2p - 1)^2$$

c. 
$$E(X_3) = 3 * (2p-1), VAR(X_3) = (3-1)^2 * (2p-1)^2 = 4 * (2p-1)^2$$

- 7.  $E(X) = \sum_{1}^{25} 2 * 0.8 + 0 * 0.2 + \sum_{1}^{35} 1 * 0.75 + 0 * 0.25 + \sum_{1}^{5} 3 * 0.65 + 0 * 0.35 = 40 + 26.25 + 9.75 = 76$  points for the base case  $E(X) = \sum_{1}^{25} 2 * 0.8 + (-0.5) * 0.2 + \sum_{1}^{35} 1 * 0.75 + 0 * 0.25 + \sum_{1}^{5} 3 * 0.65 + 0 * 0.35 = 37.5 + 26.25 + 9.75 = 73.5$  points for the case where a wrong True/False answer incurs a 0.5-point penalty
- 8. If the random variables X and Y are defined on the same sample space S, then they can be said to be jointly distributed over some function  $f_{x,y}(X,Y)$ , which means the expectation can be represented as:

$$E(X + Y) = \sum_{i} \sum_{j} (x_i + y_j) * f_{x,y}(X, Y) =$$

$$\sum_{i} \sum_{j} x_i * f_{x,y}(X, Y) + \sum_{i} \sum_{j} y_j * f_{x,y}(X, Y)$$

By summing over the full set for a variable, we can find the distribution function for the other variable. Thus, we get the following function

$$\sum_{i} \sum_{j} x_{i} * f_{x,y}(X,Y) + \sum_{i} \sum_{j} y_{j} * f_{x,y}(X,Y) =$$

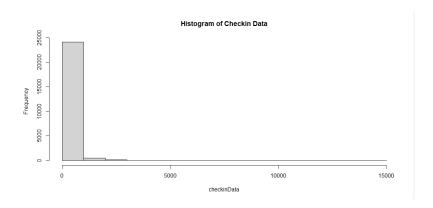
$$\sum_{i} x_{i} * f_{x}(X) + \sum_{j} y_{j} * f_{y}(Y) = E(X) + E(Y)$$

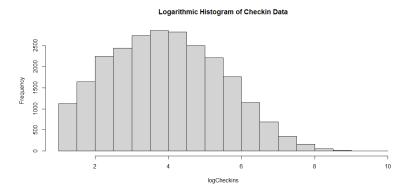
# Part II: R

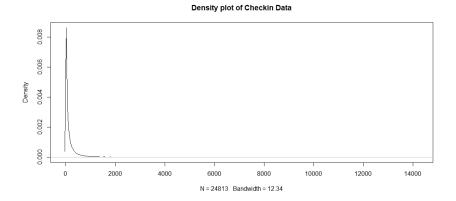
3.

Class :character Cl	name ngth:24813 ass :Character de :Character	fullAddress Length:24813 Class :charact Mode :charact		cter
Class :character 1s Mode :character Me Me 3r	n. :32.88 Mi t Qu.:33.54 1s dian :36.03 Me an :37.53 Me d Qu.:40.41 3r	longitude n. :-115.370 t Qu.:-114.977 dian :-111.924 an :-97.298 d Qu.: -80.807 x. : 8.549	Median :3.500 Mean :3.544	reviewCount Min. : 3.00 1st Qu.: 8.00 Median : 18.00 Mean : 49.03 3rd Qu.: 48.00 Max. :4578.00
Min. : 3 Mode 1st Qu.: 16 FALSE	:logical Lengt :3580 Class	h:24813 L :character C	categories ength:24813 lass :character ode :character	alcohol Length:24813 Class :character Mode :character
Class :character Cl	attire ngth:24813 ass :character de :character	priceRange Min. :1.000 1st Qu.:1.000 Median :2.000 Mean :1.631 3rd Qu.:2.000 Max. :4.000		ambience Length:24813 Class :character Mode :character
Length:24813 Le Class:character Cl	etaryRestriction ngth:24813 ass :character de :character	NA'S :903 s waiterservice Mode :logical FALSE:6208 TRUE :10351 NA'S :8254	smoking Length:24813 Class :charact Mode :charact	
Mode :logical Lengt FALSE:6503 Class	h:24813 Mo :character FA :character TR	de:logical M LSE:2054 F UE:17078 T	oodForKids ode :logical :ALSE:506 :RUE :1283 :A's :23024	
> names(d) [1] "business_ [4] "city" [7] "longitude [10] "checkins" [13] "categorie [16] "attire" [19] "ambience" [22] "waiterSer [25] "caters" [28] "goodForKi	"s" "s "o" "s" "a "p "p vice" "s	ame" tate" tars" pen" lcohol" riceRange" arking" moking" ecommendedFor	"outdoors	e" ount" 'hoods" '/el" ''' Restrictions" Seating"

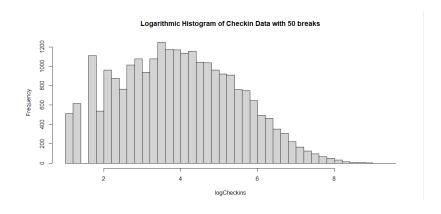
A.



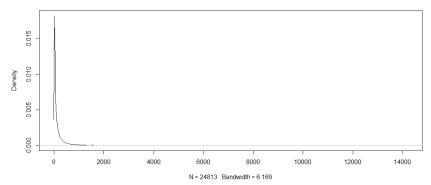




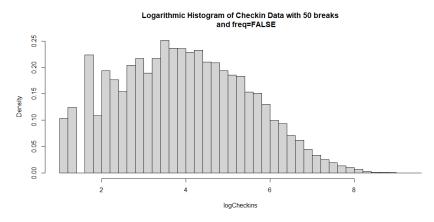
All three graphs depict the data from the "checkins" column of the yelp data. The histogram and density plot show that the distribution of data is tightly concentrated in the regions below 2000. The major difference between these two graphs is that the histogram is discrete and models the explicit values of the data whereas the density plot is continuous and provides information on the specific value with the highest density in the data. The logarithmic histogram provides a clean bell curve of the data which is useful as it makes it easier to approximate the data using normal distributions.



Density plot of Checkin Data with adjust=0.5



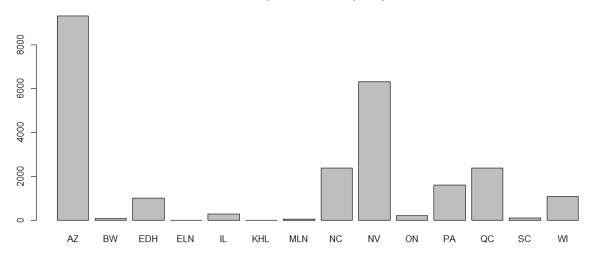
The logarithmic histogram has an increased number of bars across the data which gives a higher resolution look at the data over the domain. The density plot looks very similar to the previous version, but the peak density seems to have almost doubled from 0.008 to 0.015.



When freq=FALSE, the histogram changes from displaying the frequency with which a number appears in the data to displaying the density of each number in the data (i.e. the percentage of results that fall in the bar range).

C.





D.

I personally feel like the logarithmic histogram with 50 bars is the most useful because of its similarity to normal distributions makes it easier to model and the existence of the small spike and empty bar on the left act as outliers that can provide useful information.

5.

A.

Data	
<b>0</b> d	24813 obs. of 30 variables
Values	
alcoholInts	int [1:24813] 3 1 1 1 1 1 1 3 2 1
alcoholLevels	Factor w/ 4 levels "full_bar", "beer_and_wine",: 3 1 1 1 1 1 1 3 2 1
noiseInts	int [1:24813] 4 3 1 1 3 5 2 2 2 2
noiseLevels	Factor w/ 5 levels "quiet", "average",: 4 3 1 1 3 5 2 2 2 2

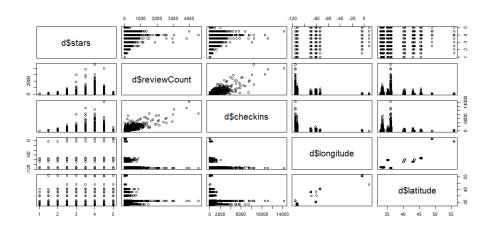
В.

3rd Qu.:5.000 Max. :5.000

#### > quantile(d\$reviewCount) 0% 25% 50% 75% 100% 18 48 4578 fullAddress city business id name Length: 6960 Lenath:6960 Lenath:6960 Length:6960 Class :character Class :character Class :character Class :character Mode :character Mode :character Mode :character Mode :character latitude state longitude stars reviewCount Length:6960 Min. :32.88 Min. :-115.352 Min. :1.000 Min. :3.000 1st Qu.:33.58 1st Qu.:-112.264 1st Qu.:3.000 Class :character 1st Qu.:4.000 Mode :character Median :36.08 Median :-111.823 Median :3.500 Median :5.000 Mean :38.30 Mean : -94.056 Mean :3.418 Mean :5.247 3rd Qu.: -79.998 3rd Qu.:43.07 3rd Qu.:7.000 3rd Ou.:4.000 Max. :55.99 Max. : 8.485 Max. :5.000 Max. :8.000 categories checkins neighborhoods alcohol open Min. : 3.00 1st Qu.: 7.00 Mode :logical Length:6960 Length:6960 Length:6960 FALSE:887 Class :character class :character class :character Median : 13.00 TRUE :6073 Mode :character Mode :character Mode :character Mean : 24.78 3rd Qu.: 29.00 Max. :694.00 noiseLevel priceRange delivery ambience attire Mode :logical Length:6960 Length:6960 Min. :1.000 Length:6960 Class :character Class :character 1st Qu.:1.000 FALSE:2899 Class :character Mode :character Mode :character Median :1.000 TRUE :693 Mode :character Mean :1.546 NA's :3368 3rd Qu.:2.000 :4.000 Max. :825 NA's parking dietaryRestrictions waiterService smoking outdoorSeating Length: 6960 Mode :logical Length:6960 Length:6960 Mode :logical Class :character Class :character FALSE:1323 Class :character FALSE:2672 Mode :character TRUE :1729 Mode :character TRUE :1370 Mode :character NA's :3908 NA's :2918 caters recommendedFor goodForGroups goodForKids alcoholInts Mode :logical Length:6960 Mode :logical Mode :logical Min. :1.000 1st Qu.:3.000 FALSE:1040 class :character FALSE:704 FALSE:15 TRUE :620 Mode :character TRUE :3471 TRUE :31 Median :3.000 NA's :5300 NA's :2785 Mean :2.684 NA's :6914 3rd Qu.:3.000 Max. :3.000 noiseInts Min. :1.000 1st Qu.:2.000 Median :5.000 Mean :3.736

The number of entries for each attribute decreased from 24813 data points in the original summary to 6960 data points in the 1<sup>st</sup> quantile summary. This makes sense as the quantile dataset should contain slightly more than 25% of the datapoints of the original dataset. The distribution of the numbers are overall similar, except for the reviewCount and checkins attributes, which were markedly decreased between the original and quantile dataset.

A.



The strongest visual relationship is between review count and checkins, which makes sense as both are likely correlated with overall popularity of the restaurant. Additionally, both review count and checkins were roughly correlated with the number of stars on a restaurant, with both peaking for restaurants with 4 stars. Additionally, latitude and longitude seem to have a roughly positive correlation.

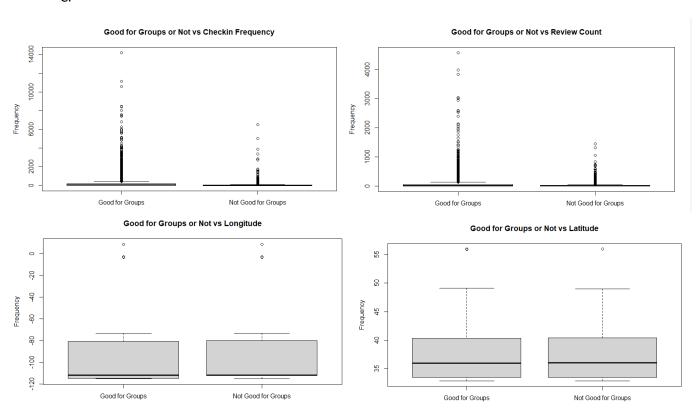
В.

Pair-wise Correlation Table

Stars		_		
0.0107050	Review Count			
0.0944007	0.8274936	Checkins		_
0.1174446	-0.1294142	-0.1789531	Longitude	
0.1211631	-0.09850094	-0.1526046	0.8811018	Latitude

The pair of attributes with the strongest positive correlation are Longitude and Latitude, with the pair Review count and Checkins being a close second. This matches the visual assessment that was performed earlier. The pair of attributes with the strongest negative correlation was

C.



Based on visual inspection, the graph that looks like it has the strongest association for the 'goodforgroup' attribute is the latitude graph. This is most likely due to its lack of outliers which makes the box plot much easier to read. If we look at ranges, the strongest correlation would most likely be in the checkin frequency plot. This makes some sense as people in groups are more likely to check in at a restaurant as they communicate with friends about their location.

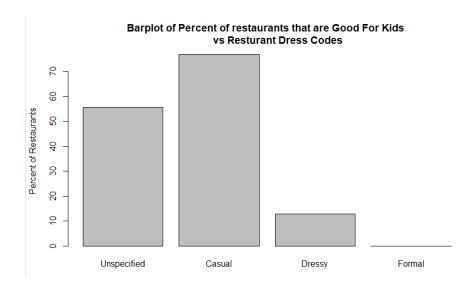
## Interquartile Ranges for Attributes when 'goodforgroups'=TRUE and FALSE

Attributes	Good for Groups IQR	Not Good for Groups IQR		
Checkins	> quantile(good\$checkins) 0% 25% 50% 75% 100%	> quantile(notGood\$checkins) 0% 25% 50% 75% 100%		
	3 19 59 181 14203	3 11 25 66 6485		
	IQR =162	IQR=55		
Review	> quantile(good\$reviewCount)	> quantile(notGood\$reviewCount)		
Count	0% 25% 50% 75% 100% 3 10 24 61 4578	0% 25% 50% 75% 100% 3 7 13 30 1453		
	IQR=51	IQR=23		
Longitude	> quantile(good\$longitude) 0% 25% 50% 75% 100% -115.36973 -115.04307 -111.92574 -80.82606 8.54856	> quantile(notGood\$longitude) 0% 25% 50% 75% 100% -115.328981 -112.152874 -111.840497 -80.018910 8.410954		
	IQR=34.22	IQR=32.13		
Latitude	> quantile(good\$latitude) 0% 25% 50% 75% 100% 32.87687 33.53849 36.02708 40.36092 55.99042	> quantile(notGood\$latitude) 0% 25% 50% 75% 100% 32.87918 33.51192 36.04116 40.45204 55.97743		
	IQN-0.022	IQR=6.94		

The interquartile ranges support my visual analysis of the checkins box plot, but not my analysis of the latitude box plot. Additionally it would seem that there is some correlation between the 'goodforgroups' attribute and the review count.

## Hypothesis 1:

a.



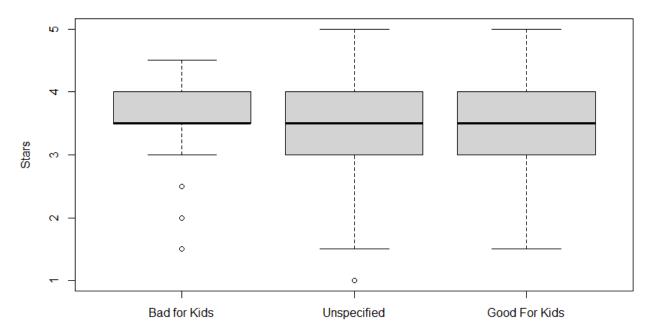
b. Both variables are discrete. There isn't really a good way to directly compare discrete variables, so I modified the data to find the percentage of restaurants that were good for kids with a given dress code.

A Hypothesis for this data set might be "Restaurants with fancier dress codes are less likely to be considered child-friendly because they are targeting more mature clientele." This hypothesis outlines a causal relationship as it proposes a mechanism by which the variables are correlated

## Hypothesis 2:

a.

## Stars vs Kid-Friendly Restaurants



b. Both variables are discrete. Stars has 9 numeric values which means that it could be used to find distribution averages by using a boxplot.

A Hypothesis for this relationship might be, "Reviews that specified a restaurant was not child friendly had a narrower distribution than reviews that specified a restaurant was child friendly or did not specify." This would be a descriptive hypothesis because it is just describing some nature of the relationship and posits neither a relationship, nor a causal link between the variables.

Code

#read data from yelp.csv and store in variable 'd'

#3.a print a summary of the data using summary()

```
summary(d)
#3.b print names of the columns using names()
names(d)
#4A.a plot a histogram of checkin data
checkinData <- d$checkins
hist(checkinData, main="Histogram of Checkin Data")
#4A.b plot a logarithmic histogram of checkin data
logCheckins <- log(d$checkins)</pre>
hist(logCheckins, main="Logarithmic Histogram of Checkin Data")
#4A.c create a density plot of checkin data
checkinDensity <- density(checkinData)</pre>
plot(checkinDensity, main = "Density plot of Checkin Data")
#4B.a plot a logarithmic histogram of checkin data with breaks=50
hist(logCheckins, main = "Logarithmic Histogram of Checkin Data with 50 breaks",
  breaks=50)
#4B.b create a density plot of checkin data with adjust = 0.5
checkinDensity <- density(checkinData, adjust=0.5)</pre>
plot(checkinDensity, main = "Density plot of Checkin Data with adjust=0.5")
#4B.d plot a logarithmic histogram of checkin data with breaks=50
# and freq = FALSE
hist(logCheckins, main = "Logarithmic Histogram of Checkin Data with 50 breaks
   and freq=FALSE",
  breaks=50, freq=FALSE)
```

```
#4C create a frequency barplot for the state attribute
stateData <- table(d$state)</pre>
stateNames <- names(stateData)</pre>
barplot(stateData, main = "Barplot of State Frequency", names.arg = stateNames)
#5A.a transform alcohol and noiseLevel to ordered numeric features
# by using factor()
alcoholLevels <-factor(d$alcohol,
             levels=c("full_bar", "beer_and_wine", "none", ""))
alcoholInts <- as.integer(alcoholLevels)
noiseLevels <- factor(d$noiseLevel,
            levels=c("quiet", "average", "loud", "very_loud", ""))
noiseInts <- as.integer(noiseLevels)</pre>
#5A.b append new columns to the original table
d <- cbind(d, alcoholInts)</pre>
d <- cbind(d, noiseInts)</pre>
#5B.a compute quantiles for the reviewCount attribute
quantile(d$reviewCount)
#5B.b create a subset that is the first quantile of reviewCount
firstQuantile <- subset(d, d$reviewCount <=8)
#5B.c create a summary of the first quantile and compare
summary(firstQuantile)
#6A.a create a scatterplot with the following attributes
  stars, reviewCount, checkins, longitude, and latitude
```

```
scatterData <- data.frame(d$stars, d$reviewCount, d$checkins,
             d$longitude, d$latitude)
pairs(~ d$stars + d$reviewCount + d$checkins + d$longitude +
    d$latitude, data= scatterData)
#6B.a use cor() to calculate the pairwise correlation for all pairs
# I couldn't get a cleaner implementation to work
cor(d$stars, d$reviewCount)
cor(d$stars, d$checkins)
cor(d$stars, d$longitude)
cor(d$stars, d$latitude)
cor(d$reviewCount, d$checkins)
cor(d$reviewCount, d$longitude)
cor(d$reviewCount, d$latitude)
cor(d$checkins, d$longitude)
cor(d$checkins, d$latitude)
cor(d$longitude, d$latitude)
#6C.a use boxplot() to model checkins, reviewcount, longitude, and latitude vs.
  the 'goodforgroups' attribute
good<- subset(d, d$goodForGroups==TRUE)</pre>
notGood <- subset(d, d$goodForGroups==FALSE)
boxplot(good$checkins, notGood$checkins,
    main = "Good for Groups or Not vs Checkin Frequency",
    names = c("Good for Groups","Not Good for Groups"),
    ylab = "Frequency")
boxplot(good$reviewCount, notGood$reviewCount,
    main = "Good for Groups or Not vs Review Count",
    names = c("Good for Groups","Not Good for Groups"),
```

```
ylab = "Frequency")
boxplot(good$longitude, notGood$longitude,
    main = "Good for Groups or Not vs Longitude",
    names = c("Good for Groups","Not Good for Groups"),
    ylab = "Frequency")
boxplot(good$latitude, notGood$latitude,
    main = "Good for Groups or Not vs Latitude",
    names = c("Good for Groups","Not Good for Groups"),
    ylab = "Frequency")
#6C.c Checking interquartile ranges for checkins, reviewCount, Longitude, and
# Latitude for 'goodforgroups'=TRUE and FALSE
quantile(good$checkins)
quantile(notGood$checkins)
quantile(good$reviewCount)
quantile(notGood$reviewCount)
quantile(good$longitude)
quantile(notGood$longitude)
quantile(good$latitude)
quantile(notGood$latitude)
#7A compare attire with presence of goodForKids after excluding null options
dKids <- subset(d, d$goodForKids!="")
uAttire <- subset(dKids, dKids$attire=="")
uAttireKids <- subset(uAttire, uAttire$goodForKids==TRUE)
uProb <- 100*as.double(nrow(uAttireKids))/as.double(nrow(uAttire))
cAttire <- subset(dKids, dKids$attire=="casual")
```

```
cAttireKids <- subset(cAttire, cAttire$goodForKids==TRUE)
cProb <- 100*as.double(nrow(cAttireKids))/as.double(nrow(cAttire))
dAttire <- subset(dKids, dKids$attire=="dressy")
dAttireKids <- subset(dAttire, dAttire$goodForKids==TRUE)
dProb <- 100*as.double(nrow(dAttireKids))/as.double(nrow(dAttire))
fAttire <- subset(dKids, dKids$attire=="formal")
fAttireKids <- subset(fAttire, fAttire$goodForKids==TRUE)
fProb <- 100*as.double(nrow(fAttireKids))/as.double(nrow(fAttire))</pre>
data <- table(c(uProb, cProb, dProb, fProb))
dataNames <- c("Unspecified", "Casual", "Dressy", "Formal")
barplot(c(uProb, cProb, dProb, fProb),
    main = "Barplot of Percent of restaurants that are Good For Kids
    vs Resturant Dress Codes",
    names.arg = dataNames,
    ylab = "Percent of Restaurants")
#7B compare attire with reviewCount
badKid <- subset(d, d$goodForKids==FALSE)</pre>
unspecifiedKid <- subset(d, is.na(d$goodForKids))</pre>
goodKid <- subset(d, d$goodForKids==TRUE)</pre>
boxplot(badKid$stars, unspecifiedKid$stars, goodKid$stars,
    main = "Stars vs Kid-Friendly Restaurants",
    names = c("Bad for Kids","Unspecified", "Good For Kids"),
    ylab = "Stars")
```