MODULE 20

Financial Exponents

This module delves into the concepts of exponents, square roots, and scientific notation.

We provide a comprehensive exploration of these mathematical tools, illustrating their practical applications in financial contexts, by demystifying exponents and analyzing how they contribute to compound growth.

The module continues with square roots, unveiling their significance in calculating interest rates, before we venture into the realm of scientific notation to help students interpret large and tiny financial figures.

POWERED BY

Rapunzl

Rapunzl Investments LLC
http://www.rapunzlinvestments.com

Key Takeaways From This Module

Guiding Questions

- How do exponents help in understanding compound interest and its impact on investments over time?
- How can square roots be applied to assess financial risk and volatility in the stock market?
- How is scientific notation used to simplify and understand large financial figures?
- What is the relationship between compound growth and exponential functions?
- How can understanding exponents and scientific notation aid in making more informed financial decisions?
- What are practical examples where you can apply the concept of square roots in personal financial planning?

Enduring Understandings

- Exponents are fundamental in understanding compound interest, enabling students to appreciate how investments grow over time.
- The use of square roots in finance helps in understanding the variability and risk of investment returns.
- Scientific notation is a valuable tool for managing and interpreting very large or very small financial figures, enhancing clarity in financial analysis.
- Understanding compound growth through exponential functions is crucial in planning for long-term financial goals, such as retirement or education funding.

Recapping Percentages & Interest Rates

Let's remember that percentages represent a fraction of 100 represented by a whole number, P, as the numerator, and 100 as the denominator.

This is commonly used in financial contexts to express proportions or interest rates.

For Example: A bank offers a 5% annual interest rate on a savings account, meaning a $100 deposit earns $5 after one year. How would you calculate the amount of interest a $1250 deposit would earn?

$$\frac{5}{100} \times \$1,250 = \frac{6250}{100} = \$62.50$$

Remember, understanding percentages is crucial for calculating interest rates and discounts when you're shopping. When you borrow money, do you think you want a high or a low interest rate?

Exponents In Theory

An exponent is a mathematical operation that represents repeated multiplication of a base number by itself.

Exponents are written as a small number to the right and above the base number, which is called the power and indicates the number of times the base should be multiplied by itself.

$$2^2 = 2 \times 2 = 4$$

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$9^4 = 9 \times 9 \times 9 \times 9 = 6{,}561$$

Exponents In Finance

Exponents have many practical applications, such as calculating compound interest, expressing large or small numbers in a concise manner, and representing values in scientific notation.

Below are 2 formulas that rely upon exponents. The first formula is for calculating compound growth and the second formula is specifically for determining the value of an investment.

$$AB^X = Y \quad \text{OR} \quad P(1+R)^T = A$$

A: Initial Amount
B: Growth Factor
X: Power
Y: Ending Amount

P: Initial Amount
R: Growth Rate As Decimal
T: Number Of Time Periods
A: Ending Amount

An Example Of Exponential Growth

How much would $100 be worth if you had invested it in the S&P 500 30 years ago?

The S&P 500 is an index that tracks the performance of the 500 largest publicly traded companies in the US. Over the past 30 years, it has historically grown at 10% per year.

P: $100 **R:** 10% = 0.10 **T:** 30

$$P(1+R)^T =$$

$$\$100 \times (1+0.10)^{30} =$$

$$\$100 \times 1.1^{30} =$$

$$\$100 \times 17.45 =$$

$1,745

Our Formula

$$P(1+R)^T = A$$

P: Initial Amount
R: Growth Rate As Decimal
T: Number Of Time Periods
A: Ending Amount

Remember How Exponents Work:

$$2^4 = 2 \times 2 \times 2 \times 2$$

$$8^3 = 8 \times 8 \times 8$$

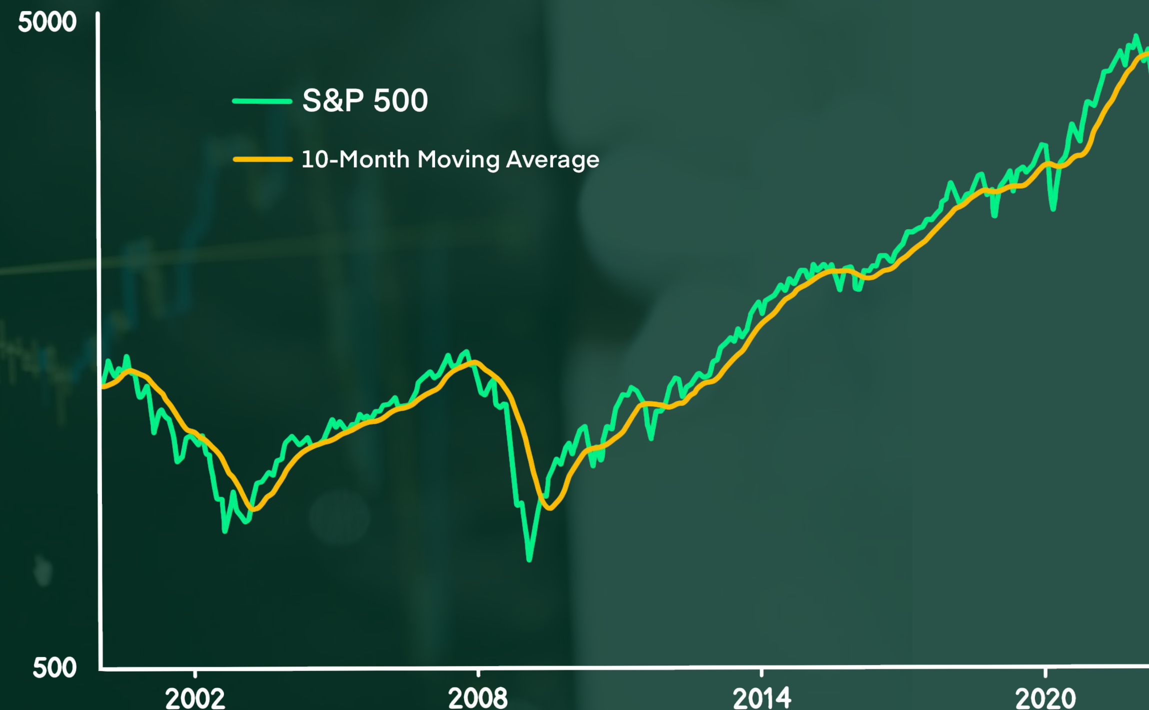
The S&P 500's Historical Performance

In 1926, the value the S&P 500 was 16. In 2021, it was over 4,000.

That's the real power of exponential growth...

If you had invested $100 for 30 years and gained 10%, you'd have over $1,700!

$P(1+R)^T$ =
$\$1000 \times (1.1)^{30}$ =
$10,744



How Can We Reverse Exponents?

Take The Root!

$$\sqrt{4} = \sqrt{2^2} = 2$$

The square root finds a number which, when multiplied by itself, gives the original number. For example, the square root of 4 is 2, since 2 x 2 = 4.

Understanding The Square Root

When a number has a square root, we call it a perfect square. Remember multiplication tables? There's an easy way to find a few squares that come up often in life.

	1	2	3	4	5	6	7	8	9	10
1	**1**	2	3	4	5	6	7	8	9	10
2	2	**4**	6	8	10	12	14	16	18	20
3	3	6	**9**	12	15	18	21	24	27	30
4	4	8	12	**16**	20	24	28	32	36	40
5	5	10	15	20	**25**	30	35	40	45	50
6	6	12	18	24	30	**36**	42	48	54	60
7	7	14	21	28	35	42	**49**	56	63	70
8	8	16	24	32	40	48	56	**64**	72	80
9	9	18	27	36	45	54	63	72	**81**	90
10	10	20	30	40	50	60	70	80	90	**100**

$1^{1} = 1$

$2^{2} = 4$

$3^{3} = 9$

$4^{4} = 16$

$5^{5} = 25$

$6^{6} = 36$

$7^{7} = 49$

$8^{8} = 64$

$9^{9} = 81$

$10^{10} = 100$

Estimating Irrational Numbers

A lot of numbers are not perfect squares, but they do have square roots!

Before understanding these square roots, we first need to understand **Rational** and **Irrational Numbers.**

Rational numbers are fractions or ratios of integers, while irrational numbers cannot be expressed as a simple fraction.

We cannot precisely write an irrational number's value, only provide an approximation.

In most cases, these estimates are fine. Similar to budget estimations where precise values aren't always necessary, but a close approximation is sufficient for planning.

$$\sqrt{2} \approxeq 1.414213562...$$

$$\sqrt{3} \approxeq 1.73205080...$$

$$\sqrt{5} \approxeq 2.23606797...$$

PRACTICE

Square Root Estimation

Use the **Square Roots Estimate Activity**
To Practice Calculating Exponents & Irrational Squares

Exponential Growth & Decay

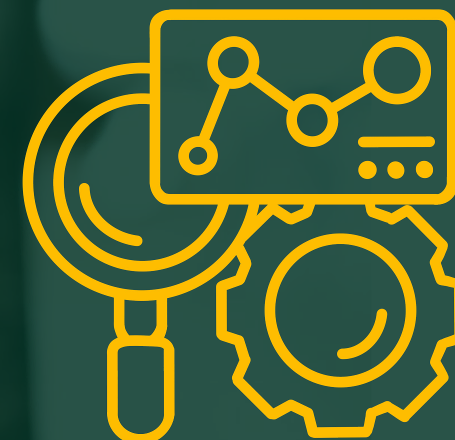
Exponential growth and decay are terms used to describe how a quantity changes by an amount that is relative to its current size. Decay is when the growth factor is less than 1.

Imagine If You Received $500,000,
But Spent 8% Per Year

Year 1: $460,000 **($40,000)**
Year 2: $423,200 **($36,800)**
Year 3: $389, 344 **($33,656)**
Year 4: $358,196 **($30,148)**
...

Notice that the amount that is spent each year decreases because the 8% is of a lesser amount.

Exponential decay is the opposite of exponential growth where the first decline will be the largest; whereas the first step in exponential growth is the smallest.

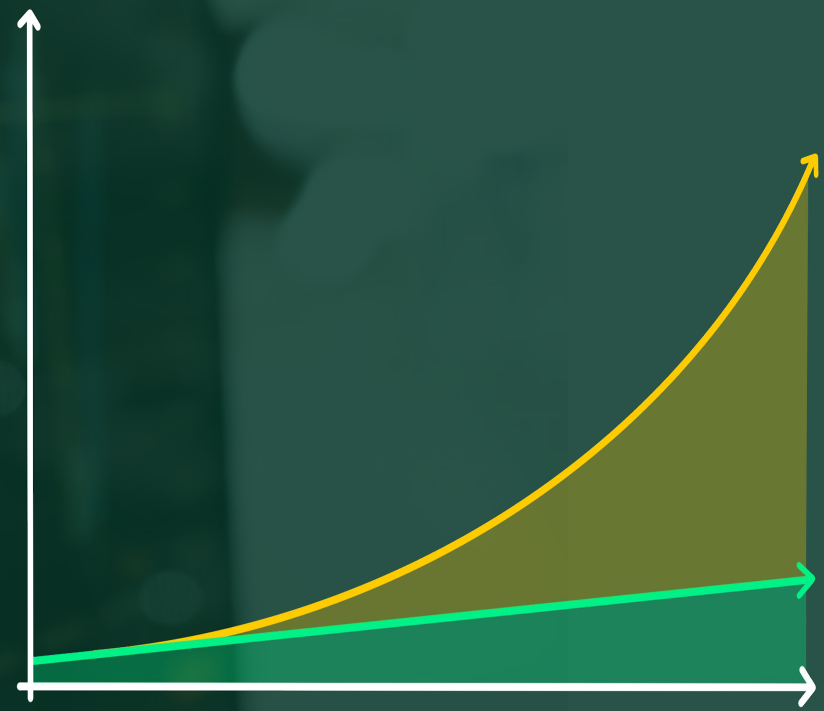


Case Study: Compound Versus Simple

Which Option Would You Chose?

Option A - Simple Growth
$100 growing at a fixed 20% by receiving $20 each month.

Option B - Compound Growth
$100 growing at 5% compound interest each month, so you'll receive $5 the first month.



After 5 years...

Option A
$1,300

Option B
$1,868

Introducing Scientific Notation

Scientific notation is a way of expressing very large or very small numbers, typically as a product of a number between 1 and 10 and a power of 10, like the examples below:

150,000,000 KM =
1.5×10^{8}
The approximate distance from the Earth to the Sun

$31,100,000,000,000 =
$\$3.11 \times 10^{13}$
Market Value of Facebook, Apple, Amazon, Netflix & Google in 2020

0.0024 CM =
2.4×10^{-3}
The diameter or width of a single grain of sand

Scientific notation is critical for simplifying calculations with extremely large or small values, and is frequently used by scientists and financial professionals to represent large figures like those below.

Can you convert a large number such as Apple's Q3 Earnings in 2022, where they reported earning $80,100,000,000 into scientific notation? How might this notation be useful in understanding financial reports? Is it easier to read the number above, or $80.1 Billion?

Put It Into Practice!

For each of the items below, think of the corresponding scientific notation equivalent. Make a note of each answer in your notebook, and be prepared to share with the class.

Item	Numerical Value
Distance from Earth to the Sun	149,600,000 kilometers
Annual Global CO2 Emissions	36,000,000,000 tonnes
2023 Population of the World	7,900,000,000 people
Mass of a Proton	0.00000000000000000000001672 grams
Estimated 2023 World GDP	$84,000,000,000,000
Age of the Universe	13,800,000,000 years

Calculating A Country's GDP

GDP (Gross Domestic Product) is a measure of a country's economic output. Let's practice using Scientific Notation to calculate GDP!

The country of Econoland has three major sectors contributing to its GDP: agriculture, manufacturing, and services with the following revenues:

Agriculture:	5.0×10^{9}
Manufacturing:	1.2×10^{10}
Services:	3.8×10^{10}
Foreign Investment:	2.5×10^{9}
Government Spending:	1.0×10^{10}
Debt Interest Payments:	3.0×10^{9}

Formula For Calculating GDP

Sector Revenues

+

Foreign Investment

+

Government Spending

-

Debt Interest Payments

=

GDP

Calculating A Country's GDP (Pt 2)

Summing the Sectors' Revenues

$(5.0 \times 10^{9}) + (1.2 \times 10^{10}) + (3.8 \times 10^{10})$ ⇒ *Setup the equation*

$(5.0 \times 10^{9}) + (12 \times 10^{9}) + (38 \times 10^{9})$ ⇒ *Rewrite all values in terms of 10^9*

$(5.0 + 12 + 38) \times 10^{9}$ ⇒ *Group similar terms*

55×10^{9} ⇒ *Handle the addition*

5.5×10^{10} ⇒ *Rewrite the expression*

Add Foreign Investments and Government Spending

$(5.5 \times 10^{10}) + (2.5 \times 10^{9}) + (1.0 \times 10^{10})$ ⇒ *Repeat the same procedure above*

$(2.5 \times 10^{9}) + (55 \times 10^{9}) + (10 \times 10^{9})$ ⇒ *Rewrite all values in terms of 10^9*

$(2.5+55+10) \times 10^{9} = 67.5 \times 10^{9} = 6.75 \times 10^{10}$ ⇒ *Group similar terms & rewrite the expression*

Subtract the Debt Interest Payment

$(6.75 \times 10^{10}) - (3.0 \times 10^{9})$ ⇒ *Repeat the same procedure above*

$(67.5 \times 10^{9}) - (3.0 \times 10^{9})$ ⇒ *Rewrite all values in terms of 10^9*

$(67.5 - 3.0) \times 10^{9} = 64.5 \times 10^{9} = 6.45 \times 10^{10}$ ⇒ *Group similar terms & rewrite the expression*

Total GDP:

$6.45 \times 10^{10} = 645{,}000{,}000 = 645$ Million

Try Calculating GDP Yourself!

Now that we understand how to calculate addition and subtraction using Scientific Notation, let's put it to the test!

The country of Moneyland has 5 major sectors contributing to its GDP: technology, agriculture, manufacturing, healthcare, and services with the following revenues:

Agriculture:	5.0×10^9
Agriculture:	5.0×10^9
Manufacturing:	1.2×10^{10}
Agriculture:	5.0×10^9
Services:	3.8×10^{10}
Foreign Investment:	2.5×10^9
Government Spending:	1.0×10^{10}
Debt Interest Payments:	3.0×10^9

Formula For Calculating GDP

Sector Revenues

+

Foreign Investment

+

Government Spending

-

Debt Interest Payments

=

GDP

Key Takeaways From This Module

CORE & FUNDAMENTALS

- Understanding how values increase or decrease exponentially over time, particularly in the context of compound interest.
- Grasping the use of square roots in finance, such as calculating the standard deviation, to understand investment risk.
- Learning how to use scientific notation to simplify complex financial calculations involving large numbers.

APPLIED KNOWLEDGE

- Exponential growth helps analyze the potential future value of investments and the impact of compound interest.
- You can use square roots to calculate and understand the standard deviation of investment portfolios, aiding in risk assessment.
- Scientific notation can help interpret and compare large financial data, such as in reports of multinational corporations or national economic indicators.

RELEVANCE FOR YOU

- Understanding exponents, square roots, and scientific notation provides essential skills to analyze and interpret complex financial data.
- Mastery of these mathematical concepts is crucial in various financial careers, such as banking, investment analysis, and accounting, providing a solid foundation for students aspiring to enter these fields.