MODULE 13

Financial Algebra

This module provides a solid foundation in mathematical concepts and skills necessary for understanding and making financial decisions.

Topics covered in this module include interest rates, loans, compound growth, stocks and bonds, and net present value. We explore linear and exponential functions and work through case studies that apply mathematical concepts to practical life scenarios.

POWERED BY

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Key Takeaways From This Module

Guiding Questions

- How do interest rates affect the cost of loans and investments, and how can we calculate the interest paid or earned on a financial product?
- How can we calculate the net present value of different financial decisions, and how can we use this information to make informed choices?
- How can we use algebraic equations and functions to model and solve financial problems in the real world?
- How can we apply the principles of financial algebra to real-world scenarios?

Enduring Understandings

- Financial decisions involve weighing the costs and benefits of different options, and algebraic equations and functions can be used to model and compare these options.
- Interest rates play a crucial role in the cost of loans and investments.
- The net present value of a financial decision takes into account the time value of money and is a critical factor in making informed financial decisions.
- Financial algebra can be applied to real-world scenarios, such as buying a house, starting a business, or planning for retirement.

Understanding Fractions

A fraction represents a part of a whole or a group and is written as a number on top (numerator) and a number on bottom (denominator) separated by a horizontal line.

- The numerator represents how many parts you have, while the denominator represents how many equal parts the whole or group is divided into.
- For example, if you have a pizza and you divide it into 8 equal slices, each slice is one-eighth of the pizza. So the fraction representing one slice is 1/8.

Numerator → 25

Denominator → 100

$$\frac{25}{100} = 0.25$$

Simplifying Improper Fractions

Simplifying a fraction expresses it in its simplest form, and we achieve this when dividing the numerator and denominator by the same number to simplify the fraction.

We do this by finding the greatest common factor (GCF) of the numerator and denominator. The GCF is the largest number that divides both numbers evenly. Divide both the numerator and denominator by the GCF to get the simplified or reduced fraction.

GCF 8

$$\frac{40}{100} = \frac{4 \text{ x } 10}{10 \text{ x } 10} = \frac{2 \text{ x } 2}{2 \text{ x } 5} = \frac{2}{5}$$

GCF 50

$$\frac{50}{250} = \frac{5 \text{ x } 10}{25 \text{ x } 10} = \frac{5 \text{ x } 1}{5 \text{ x } 5} = \frac{1}{5}$$

Multiplying Fractions

To multiply fractions, you need to multiply the numerators (top numbers) together and the denominators (bottom numbers) together.

After multiplying the numbers, you should then simplify the fraction into its simplest form. The example below is already in its simplest form because 2 and 7 do not share any common factors, nor do 5 and 11.

$$\frac{2}{11} \times \frac{7}{5} = \frac{2 \times 7}{11 \times 5} = \frac{14}{55}$$

Fractions In Practice

1. A group of friends shared a pizza that was divided into 12 equal slices. If each person ate 1/4 of a slice, how many friends were there in the group?

2. A store is having a sale where everything is 20% off. If a shirt originally costs $25, what is the sale price of the shirt?

3. A fish tank is filled with 1/3 of its capacity with water. If 5 more liters of water are added, the tank is then filled to 2/3 of its capacity. What is the total capacity of the fish tank?

4. A marathon is 26.2 miles long. If a runner has completed 3/4 of the marathon, how many miles has the runner completed?

5. A group of students ran a race. If 3/4 of the students finished the race, and there were 24 students who did not finish, how many students were in the race?

Now It’s Time For

PERCENTAGES

$$\frac{1}{4} = \frac{25}{100} = 25\%$$

Percentages Are Just Fractions Over 100

When you go into a store and a sale says 50% off, that is the same thing as saying that prices are half off.

With what we know about equivalent factors:

$$^{1}/_{2} = {}^{1*50}/_{2*50} = {}^{50}/_{100} = 50\%$$

This is true for every percentage. 40% can be expressed as 40/100, 4/10, 2/5, or any other equivalent fraction. 25% can expressed as 25/100, 5/20, 1/4, or other equivalents.

We can also multiply percentages, just like fractions:

$$\$250 \times 50\% = \$250 \times {}^{50}/_{100} = \$250 \times {}^{1}/_{2} = {}^{\$250}/_{2} = \$125$$

THESE ARE ALL EQUAL!

75%

$$\frac{3}{4} \quad \frac{75}{100}$$

Case Study: Income After Tax

Assume that a company hired an employee with a gross annual salary of $50,000. The employee's tax rate is 25% of their gross income.

1. After taxes, what is the employee's net annual income?

2. If the employee wants to save 10% of their net income each month, how much money will they save each month?

3. Assuming that the employee deposits their savings into a savings account with an annual interest rate of 2.5%, what is the total interest earned on their savings after one year?

4. If the employee wants to increase their savings rate to 20% of their net income, what percentage of their gross income would they need to save to achieve this new savings goal?

Interest Rates In Theory

An interest rate is the amount of money that a borrower must pay to a lender in order to borrow money or the amount of money that a lender pays to a borrower for depositing money.

- Interest rates are typically expressed as an annual percentage rate (APR).
- The interest rate can be fixed or variable, depending on the terms of the loan or investment.
- The higher the interest rate, the more expensive it is to borrow money and the more profitable it is to invest money.
- Interest rates are influenced by various factors, including inflation, economic growth, and monetary policy.
- Interest rates play a crucial role in financial planning, as they impact the cost of borrowing for loans such as mortgages.

Interest Rates In Practice

1. If you deposit $1000 into a savings account that earns 5% interest per year, how much money will you have in the account after 3 years?

2. You borrow $5000 from a bank at an annual interest rate of 8%. If you take 4 years to repay the loan, how much total interest will you pay?

3. Sarah invests $2000 in a mutual fund that pays an annual interest rate of 7%. If she keeps the money in the fund for 5 years, how much money will she have at the end of the period?

4. You want to save $5000 to buy a new car in 2 years. If you can earn 3% interest on your savings, how much money do you need to deposit today to reach your goal?

5. You have a credit card with an annual interest rate of 18%. If you make a $1000 purchase on the card and do not pay it off for 6 months, how much interest will you owe at the end of the period?

Case Study: Capital Gains

Samantha bought a share of a company's stock for $100. After holding onto it for three years, the stock price increased to $150, and Samantha decided to sell her share.

1. What is Samantha's profit from this investment?
2. A profit on an investment that has taken longer than 6 months is subject to a lower, capital gains tax, instead of paying ordinary income tax. Assuming Samantha's profits are subject to a capital gains tax rate of 20%, how much capital gains tax would she owe on the investment?
3. If Samantha's initial investment was $10,000, what percentage return did she earn on her investment after taking into account the capital gains tax?
4. Why is it important to not trade too frequently, with what you have learned about capital gains taxes?

Exponents In Theory

An exponent is a mathematical operation that represents repeated multiplication of a base number by itself.

Exponents are written as a small number to the right and above the base number, which is called the power and indicates the number of times the base should be multiplied by itself.

$$2^2 = 2 \times 2 = 4$$

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

$$8^3 = 8 \times 8 \times 8 = 512$$

$$9^4 = 9 \times 9 \times 9 \times 9 = 6{,}561$$

Exponents In Finance

Exponents have many practical applications, such as calculating compound interest, expressing large or small numbers in a concise manner, and representing values in scientific notation.

Below are 2 formulas that rely upon exponents. The first formula is for calculating compound growth and the second formula is specifically for determining the value of an investment.

$$AB^X = Y \quad \text{OR} \quad P(1+R)^T = A$$

A: Initial Amount
B: Growth Factor
X: Power
Y: Ending Amount

P: Initial Amount
R: Growth Rate As Decimal
T: Number Of Time Periods
A: Ending Amount

An Example Of Exponential Growth

How much would $100 be worth if you had invested it in the S&P 500 30 years ago?

The S&P 500 is an index that tracks the performance of the 500 largest publicly traded companies in the US. Over the past 30 years, it has historically grown at 10% per year.

P: $100 **R:** 10% = 0.10 **T:** 30

$P(1+R)^T =$

$\$100 \times (1+0.10)^{30} =$

$\$100 \times 1.1^{30} =$

$\$100 \times 17.45 =$

$1,745

Our Formula

$$P(1+R)^T = A$$

P: Initial Amount
R: Growth Rate As Decimal
T: Number Of Time Periods
A: Ending Amount

Remember How Exponents Work:

$$2^4 = 2 \times 2 \times 2 \times 2$$

$$8^3 = 8 \times 8 \times 8$$

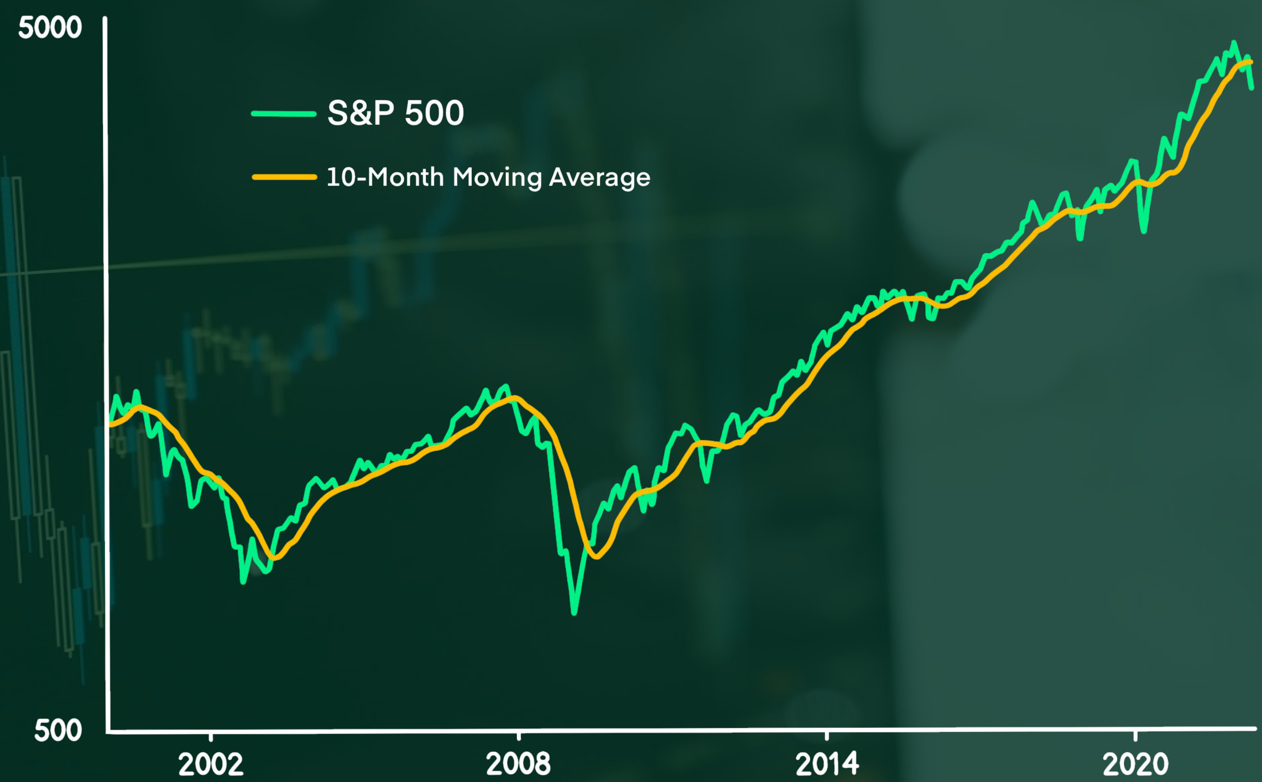
The S&P 500's Historical Performance

In 1926, the value the S&P 500 was 16. In 2021, it was over 4,000.

That's the real power of exponential growth...

If you had invested $100 for 30 years and gained 10%, you'd have over $1,700!

$P(1+R)^T$ =
$\$1000 \times (1.1)^{30}$ =
$10,744



Exponential Growth & Decay

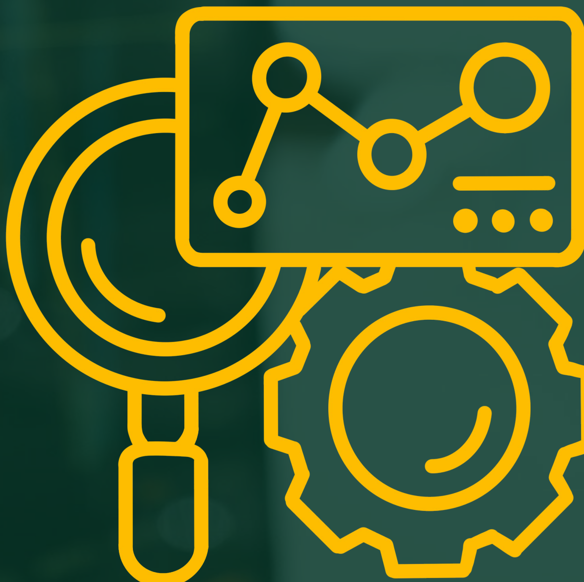
Exponential growth and decay are terms used to describe how a quantity changes by an amount that is relative to its current size. Decay is when the growth factor is less than 1.

Imagine If You Received $500,000, But Spent 8% Per Year

Year 1: $460,000 **($40,000)**
Year 2: $423,200 **($36,800)**
Year 3: $389, 344 **($33,656)**
Year 4: $358,196 **($30,148)**
...

Notice that the amount that is spent each year decreases because the 8% is of a lesser amount.

Exponential decay is the opposite of exponential growth where the first decline will be the largest; whereas the first step in exponential growth is the smallest.



Case Study: Compound Versus Simple

Which Option Would You Chose?

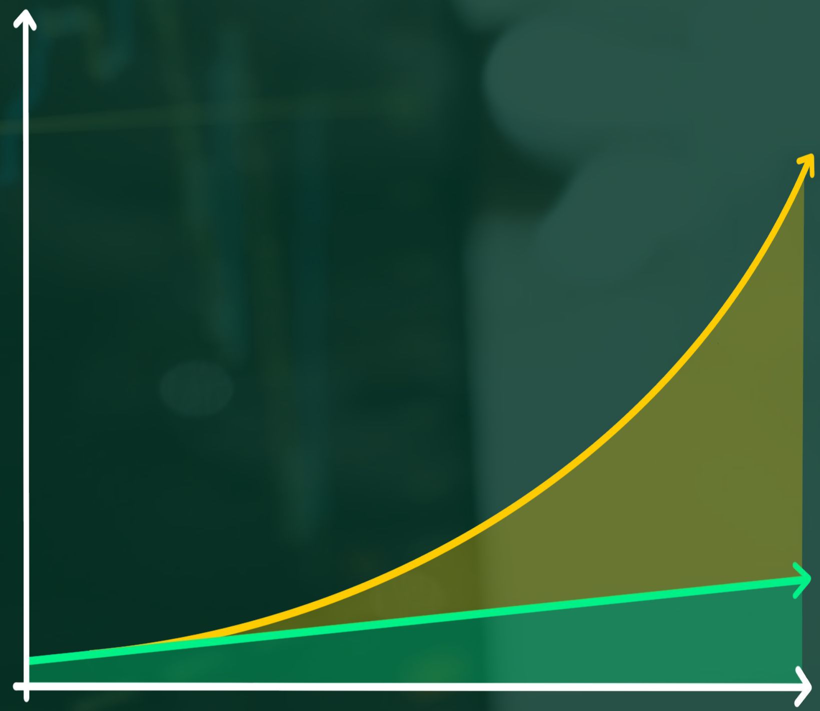
Option A - Simple Growth
$100 growing at a fixed 20% by receiving $20 each month.

Option B - Compound Growth
$100 growing at 5% compound interest each month, so you'll receive $5 the first month.

After 5 years...

Option A
$1,300

Option B
$1,868



Key Takeaways From This Module

CORE & FUNDAMENTALS

- How to represent a value as a fraction or a percentage.
- Percentages are equivalent to fractions with a denominator of 100, and they are interchangeable.
- Exponents allow us to calculate exponential growth over different periods of time.

APPLIED KNOWLEDGE

- You can multiply percentages with your income in order to calculate your payroll taxes.
- Holding an investment for over 6 months means that you only have to pay capital gains taxes instead of ordinary income taxes.
- The S&P 500 has, on average, exhibited exponential growth of 10% over the past 40 years.

RELEVANCE FOR YOU

- The best investment opportunities offer a high rate of return by leveraging the power of compound and exponential growth.
- You can use percentages and fractions interchangeably to help calculate the value of things when they are on sale.
- Capital gains taxes are much lower than income taxes, which is why investing is a long-term game.