



Hamming Codes

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Decoder: Paralelo

$$A = y1 \oplus y7$$

$$B = y2 \oplus y9$$

$$C = y3 \oplus y8$$

$$D = y5 \oplus y10$$

$$E = y6 \oplus y11$$

$$F = A \oplus y11$$

$$P1 = F \oplus B \oplus C \oplus y12$$

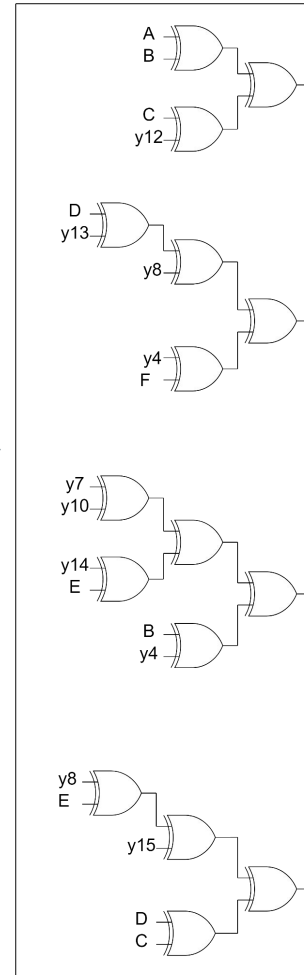
$$P2 = F \oplus y4 \oplus D \oplus y8 \oplus y13$$

$$P3 = B \oplus y4 \oplus E \oplus y7 \oplus y10 \oplus y14$$

$$P4 = C \oplus D \oplus E \oplus y9 \oplus y15$$

Figure 1: Parity Bits

y (1,2,...,15)



P (1,2,3,4)



Decoder: Paralelo

Table 1: Tabela de Descodificação

| P1 | P2 | P3 | P4 | y1 | y2 | y3 | y4 | y5 | y6 | y7 | y8 | y9 | y10 | y11 |
|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|-----|
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

$$A = P1.P2$$

$$B = \overline{P1}.P2$$

$$C = P1.\overline{P2}$$

$$D = \overline{P3}.P4$$

$$E = \overline{P3}.\overline{P4}$$

$$F = P3.\overline{P4}$$

$$y1 = A.\overline{P3}.\overline{P4}$$

$$y2 = C.F$$

$$y3 = C.E$$

$$y4 = B.F$$

$$y5 = B.E$$

$$y6 = \overline{P1}.\overline{P2}.D$$

$$y7 = A.F$$

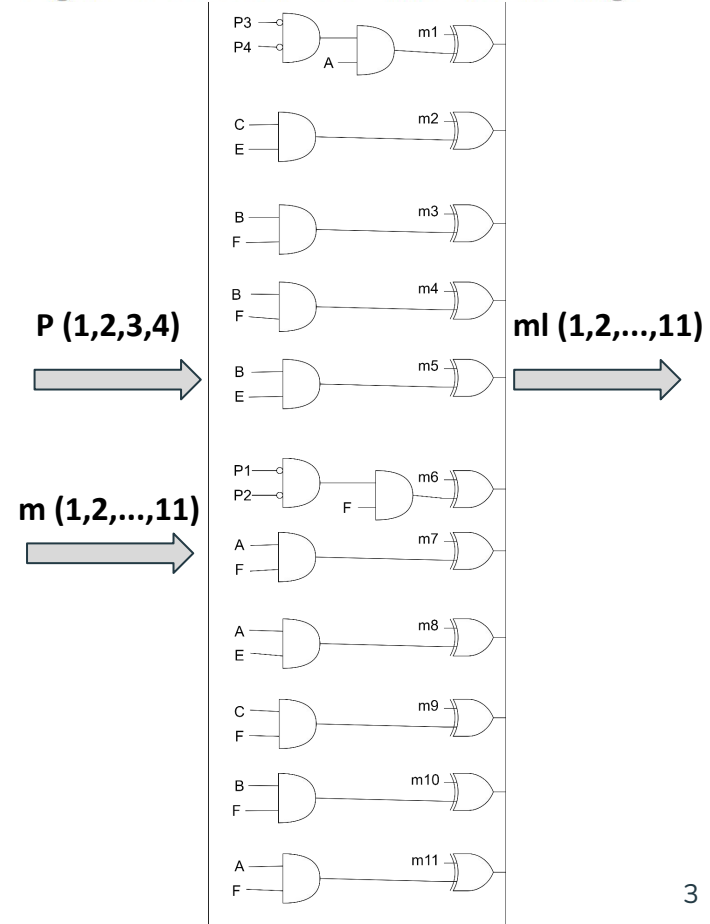
$$y8 = A.E$$

$$y9 = C.D$$

$$y10 = B.D$$

$$y11 = A.D$$

Figure 2: Decoder Xor distorted message



Encoder: Bit Serial

$$X_i = m_i, \text{ para } i = 1, 2, \dots, 11$$

$$X_{12} = m_1 \oplus m_2 \oplus m_3 \oplus m_7 \oplus m_8 \oplus m_9 \oplus m_{11}$$

$$X_{13} = m_1 \oplus m_4 \oplus m_5 \oplus m_7 \oplus m_8 \oplus m_{10} \oplus m_{11}$$

$$X_{14} = m_2 \oplus m_4 \oplus m_6 \oplus m_7 \oplus m_9 \oplus m_{10} \oplus m_{11}$$

$$X_{15} = m_3 \oplus m_5 \oplus m_6 \oplus m_8 \oplus m_9 \oplus m_{10} \oplus m_{11}$$

$$X_i = m_i, \text{ para } i = 1, 2, \dots, 11$$

$$X_{12} = m_1.1 \oplus m_2.1 \oplus m_3.1 \oplus m_4.0 \oplus m_5.0 \oplus m_6.0 \oplus m_7.1 \oplus m_8.1 \oplus m_9.1 \oplus m_{10}.0 \oplus m_{11}.1$$

$$X_{13} = m_1.1 \oplus m_2.0 \oplus m_3.0 \oplus m_4.1 \oplus m_5.1 \oplus m_6.0 \oplus m_7.1 \oplus m_8.1 \oplus m_9.0 \oplus m_{10}.1 \oplus m_{11}.1$$

$$X_{14} = m_1.0 \oplus m_2.1 \oplus m_3.0 \oplus m_4.1 \oplus m_5.0 \oplus m_6.1 \oplus m_7.1 \oplus m_8.0 \oplus m_9.1 \oplus m_{10}.1 \oplus m_{11}.1$$

$$X_{15} = m_1.0 \oplus m_2.0 \oplus m_3.1 \oplus m_4.0 \oplus m_5.1 \oplus m_6.1 \oplus m_7.0 \oplus m_8.1 \oplus m_9.1 \oplus m_{10}.1 \oplus m_{11}.1$$

Table 2: Tabela de Estados

| Counter | a | b | c | d | SPar | SOut | Reset |
|---------|---|---|---|---|------|------|-------|
| 00000 | 1 | 1 | 0 | 0 | XX | 0 | 1 |
| 00001 | 1 | 0 | 1 | 0 | XX | 0 | 1 |
| 00010 | 1 | 0 | 0 | 1 | XX | 0 | 1 |
| 00011 | 0 | 1 | 1 | 0 | XX | 0 | 1 |
| 00100 | 0 | 1 | 0 | 1 | XX | 0 | 1 |
| 00101 | 0 | 0 | 1 | 1 | XX | 0 | 1 |
| 00110 | 1 | 1 | 1 | 0 | XX | 0 | 1 |
| 00111 | 1 | 1 | 0 | 1 | XX | 0 | 1 |
| 01000 | 1 | 0 | 1 | 1 | XX | 0 | 1 |
| 01001 | 0 | 1 | 1 | 1 | XX | 0 | 1 |
| 01010 | 1 | 1 | 1 | 1 | XX | 0 | 1 |
| 01011 | X | X | X | X | 00 | 1 | 1 |
| 01100 | X | X | X | X | 01 | 1 | 1 |
| 01101 | X | X | X | X | 10 | 1 | 1 |
| 01110 | X | X | X | X | 11 | 1 | 1 |
| 01111 | 0 | 0 | 0 | 0 | XX | X | 1 |
| 10000 | 0 | 0 | 0 | 0 | XX | X | 0 |

Encoder: Bit Serial

Figure 3: Encoder Bit Serial Architecture

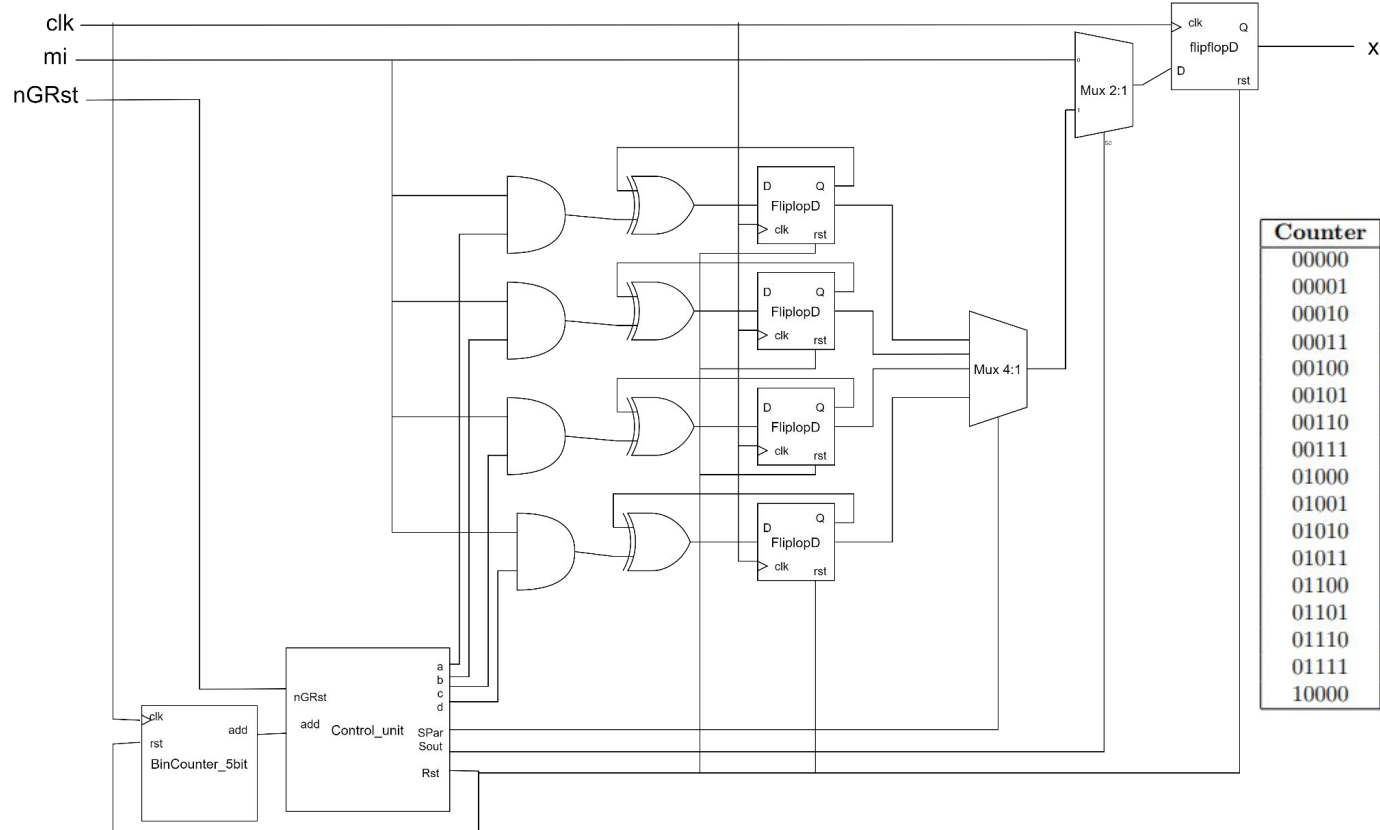


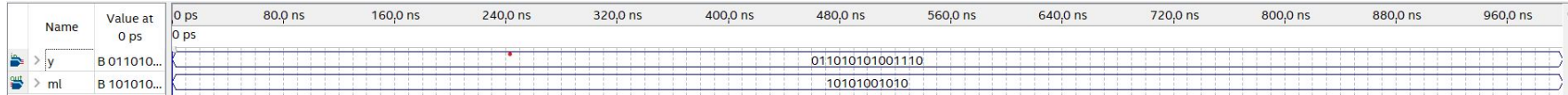
Table 2: Tabela de Estados

| Counter | a | b | c | d | SPar | SOut | Reset |
|---------|---|---|---|---|------|------|-------|
| 00000 | 1 | 1 | 0 | 0 | XX | 0 | 1 |
| 00001 | 1 | 0 | 1 | 0 | XX | 0 | 1 |
| 00010 | 1 | 0 | 0 | 1 | XX | 0 | 1 |
| 00011 | 0 | 1 | 1 | 0 | XX | 0 | 1 |
| 00100 | 0 | 1 | 0 | 1 | XX | 0 | 1 |
| 00101 | 0 | 0 | 1 | 1 | XX | 0 | 1 |
| 00110 | 1 | 1 | 1 | 0 | XX | 0 | 1 |
| 00111 | 1 | 1 | 0 | 1 | XX | 0 | 1 |
| 01000 | 1 | 0 | 1 | 1 | XX | 0 | 1 |
| 01001 | 0 | 1 | 1 | 1 | XX | 0 | 1 |
| 01010 | 1 | 1 | 1 | 1 | XX | 0 | 1 |
| 01011 | X | X | X | X | 00 | 1 | 1 |
| 01100 | X | X | X | X | 01 | 1 | 1 |
| 01101 | X | X | X | X | 10 | 1 | 1 |
| 01110 | X | X | X | X | 11 | 1 | 1 |
| 01111 | 0 | 0 | 0 | 0 | XX | X | 1 |
| 10000 | 0 | 0 | 0 | 0 | XX | X | 0 |

Simulações

Decoder

Figure 4: Decoder Simulation



Encoder

Figure 5: Encoder Simulation

