Queuing Theory Project

Modeling and Simulation CSE 422

Content Table

1. Introduction

This project is on Queue Theory. It is a mathematical design for efficiently providing services from servers to its customers. It also assists businesses to balance the price of an additional service and the cost of customers who spend too much time waiting. It offers data-oriented systematical design for the customers and the service provider. Ensure that the resources are used efficiently along with the satisfaction of the customers. "Queueing theory is the mathematical study of the phenomenon of waiting in lines, which is concerned with the probabilistic performance of a system at that point in time" (ScienceDirect Topics, n.d.). [1]

The history of queueing theory is connected with the name of a Danish engineer Agner Krarup Erlang (1878 - 1929). [2] The queuing theory was first published in 1909 by Erlang who studied telephone traffic problems. In the paper, he demonstrated that Poisson distribution is observed in a telephone traffic. The queueing theory has been applied in the modelling of urban or road traffic, elevator traffic control, airport operations, air traffic control, crowd dynamics, emergency egress analysis, railway, telephone and internet traffic (Larson and Odoni, 1981; Newell, 1982). [3]

2. Theory

2.1 Uniform Random Variable

A random variable is U is said to be a uniform distribution if on a given interval all the random variable's events are equally likely to be occur. Here the interval [0, 1]. This uniform distribution is denoted as -

$$U \sim Ų(0, 1)$$

Probability density function (PDF) of a uniform distribution,

$$f_U(u) = \begin{cases} 1, 0 \leq u \leq 1, & 0, \text{otherwise.} \end{cases}$$

Cumulative distribution function (CDF) of a uniform distribution,

$$FU(u) = P(U \leq u) = u, 0 \leq u \leq 1.$$

2.2 M/M/1 Queueing System

M/M/1 queue is the foundational model in queueing theory. An M/M/1 queue is a single-server model that and it's queueing characterized by:

- Poisson arrivals,
- Exponentially distributed service times,
- One server,
- Infinite queue capacity.

Let, λ denote the arrival rate and μ denote the service rate.

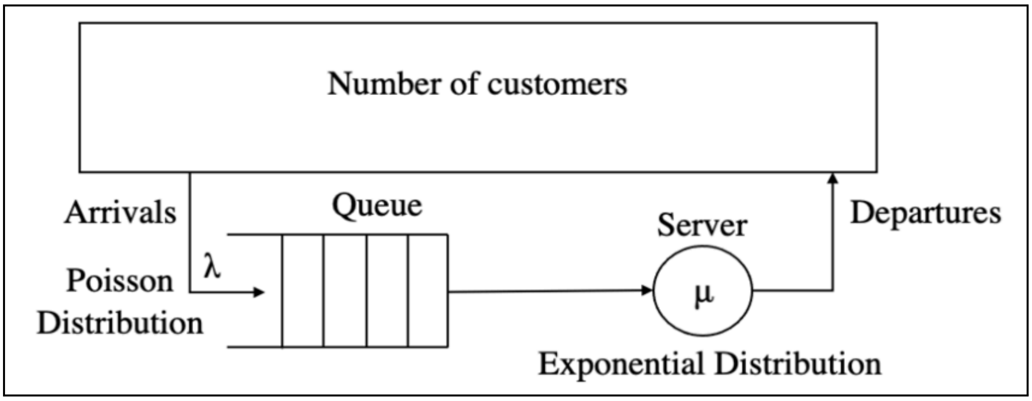
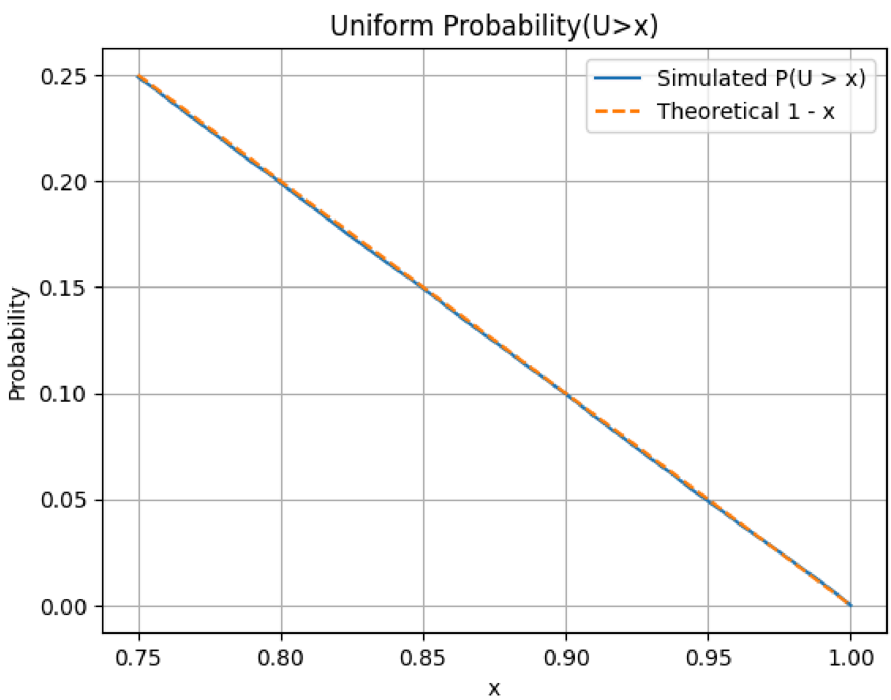


Fig 2.1 : The-M-M-1-queuing-system
Source: Adapted from "A comparison of last mile alternatives for instant delivery of groceries," *Friedrich and Chen, (2021).* [4]

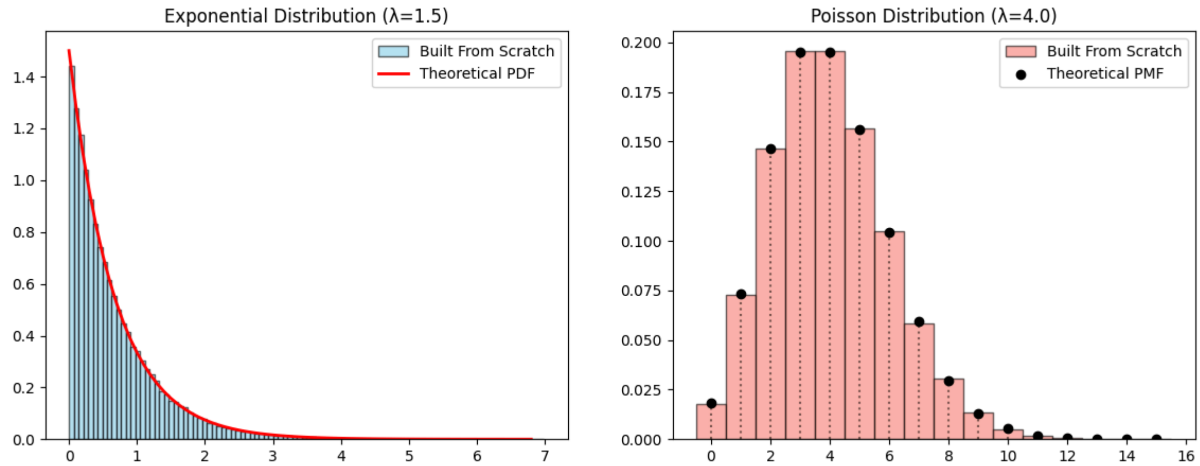
For system stability, the condition $\rho < 1$ must be satisfied. To model the randomness and arrival/service processes, the given study also uses basic probability distributions: Uniform, Exponential, and Poisson. The distributions are the theoretical foundation of simulating and analyzing the M/M/1 queueing system and the steady-state behavior of the system with the various traffic conditions.

3. Simulation

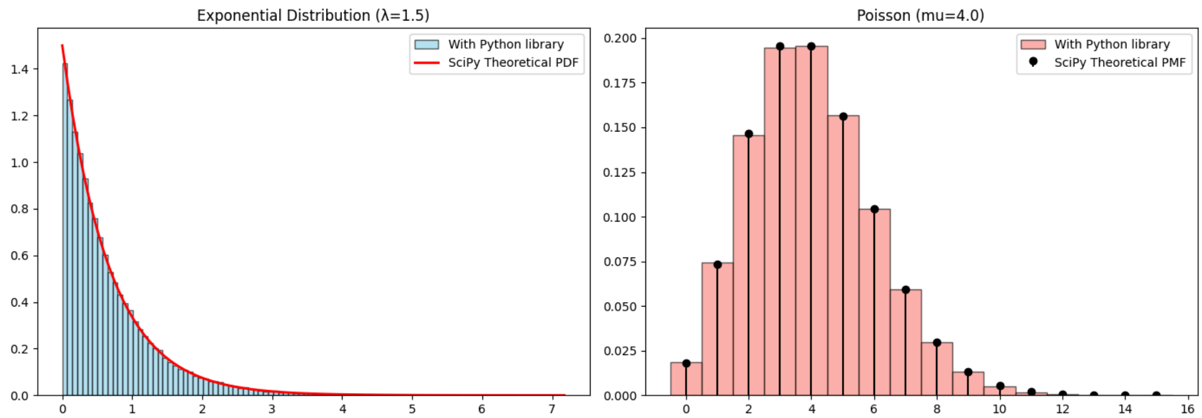


Q2 A.1 : PDF - Own function to generate Exponential (X) and Poisson Random Variables (Z)

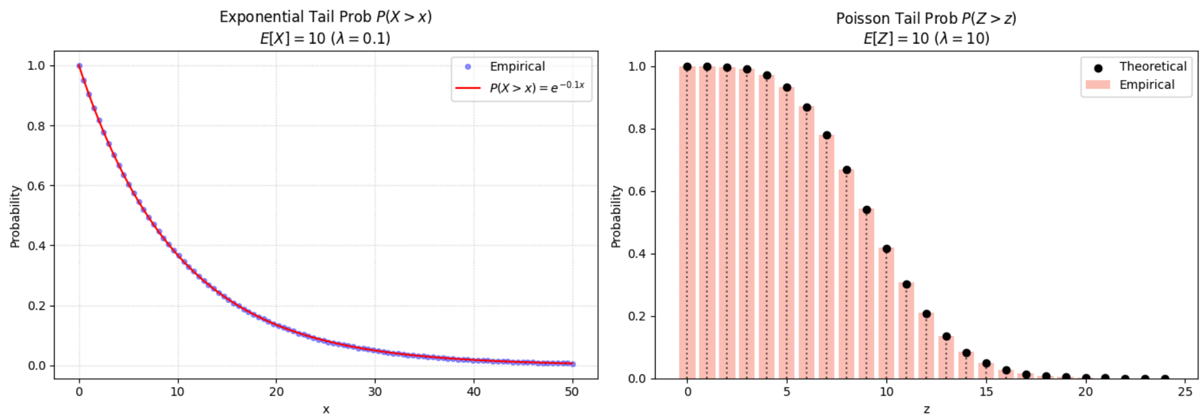
Q1 : $Uniform(0,1)$ & $Plot\ P(U > x)\ for\ x \in (0.75, 1)$



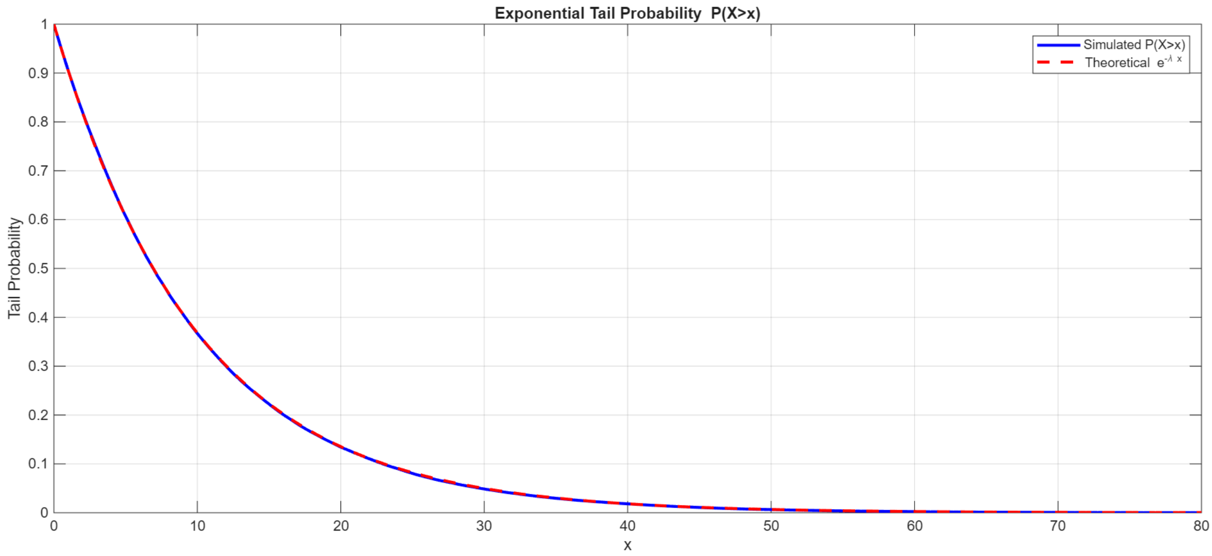
Q2 A.2 : PDF - Using Python Library (`scipy.stats import expon, poisson`)
Exponential (X) and Poisson Random Variables (Z) for cross check



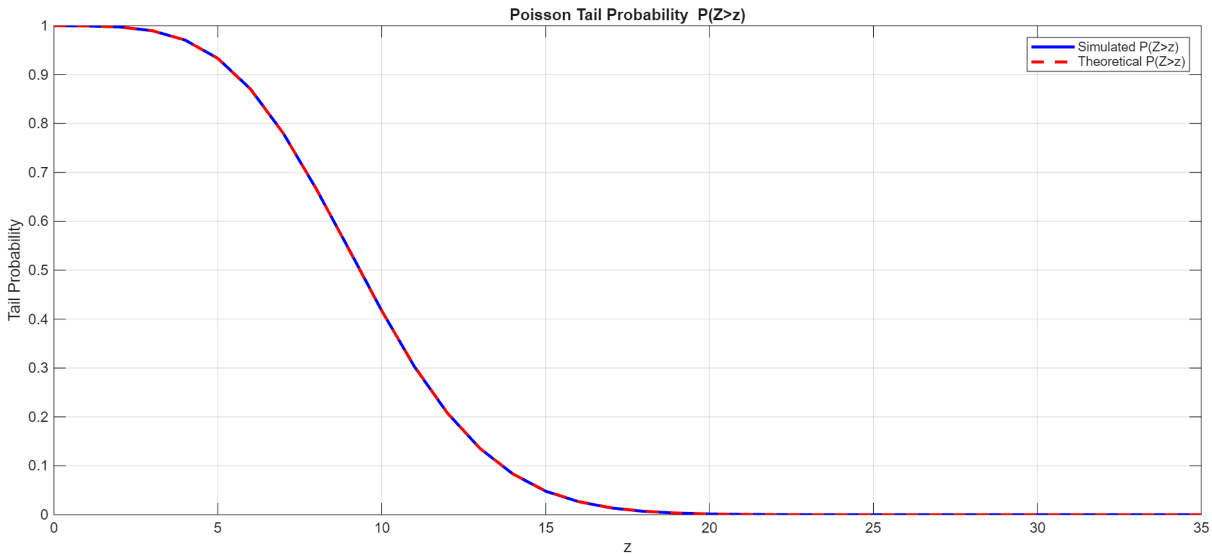
Q2 B.1 : Python - E[X] = 10 and E[Z] = 10, and plot P(X > x) and P(Z > z)



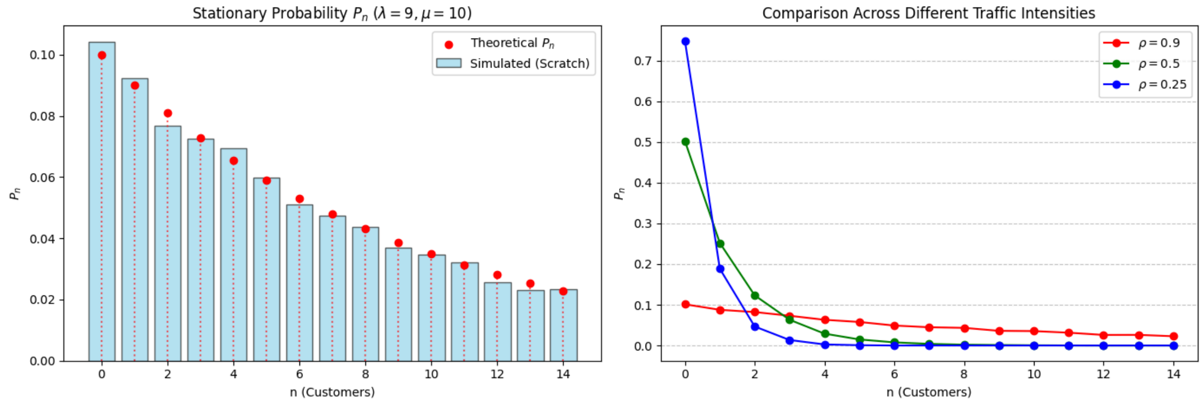
Q2 B.2 : MATLAB Verifying - E[X] = 10 & plot P(X > x)



Q2 B.3 : MATLAB Verifying - E[Z] = 10 and and P(Z > z)



Q3 : M/M/1 Queue Stationary Probability & Traffic Intensity (By using)



Generated without Python built-in libraries using Q1 & Q2 logic

4. Graph Analysis

The outputs of the simulation offer a illustration of the behavior and stability. M/M/1 queue are determined by the (ρ) traffic intensity. In 1st plot, with $\rho = 0.9$ we have a high-load system where geometric distribution that decays slow. Since the arrival rate (λ) is almost equivalent to the service rate (μ), there exists a high likelihood of having several customers in the system. This leads to a fat-tailed distribution. The close correspondence, between the simulated bar heights and the theoretical red markers, justifies the code on-the-fly implementation of the Inverse Transform Method and the manual tracking technique of the departure times.

The second plot emphasizes that the system is sensitive to the changes in the intensity of the traffic. The highest point of the probability distribution moves abruptly to towards 0.25. Under light load conditions the system is idle 75% of the time and the likelihood of over two customers is insignificant. This presents a basic trade-off between queueing theory that the low traffic level guarantees a very responsive system with a short wait time, but means a low server utilization. On the other hand, approaching 1, not only does the utilization of a server increase, but also the queue length and the corresponding stationary probabilities of larger grow exponentially, ultimately resulting in system instability in case of arrivals larger than service capacity.

5. Conclusion

The theoretical behavior of an M/M/1 queue, which is the most important determinant of the system performance, is validated by the simulation results. The high area of ρ gives a heavy loaded queue with high delays, and the low value of ρ gives short queues with an increase in the idle time. The fact that simulated and analytical stationary probabilities are very close confirms the implementation of the model and the queueing theory. On the whole, it is necessary to keep λ < μ to be stable and not allow the queue length to grow uncontrollably.

9. References

[1] Queueing theory. (n.d.). *Queueing Theory.* Retrieved December 25, 2025, from https://www.sciencedirect.com/topics/engineering/queueing-theory

[2] Eiselt, H.A., Sandblom, C.L. (2010). *Waiting Line Models. In: Operations Research.* Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-10326-1_12

[3] Queueing theory. (n.d.). *ScienceDirect Topics*. Retrieved December 25, 2025, from https://www.sciencedirect.com/topics/engineering/queueing-theory#chapters-articles

[4] Chen, L.-S. (2021, July). *A comparison of last mile alternatives for instant delivery of groceries.* (Master's thesis, Kühne Logistics University)