Secure and Dependable

HW 1

Question 3

a) initial Availability $= \frac{MTBF}{MTBF + MTTR} = \frac{2000}{2000+100} = \frac{2000}{2100} = 0{,}9524$

=> initial availability is $0{,}9524 \cdot 100 = 95{,}24\%$

b) New Availability $= \frac{MTBF}{MTBF + MTTR} = \frac{2000}{2000+50} = \frac{2000}{2050} = 0{,}9756$

=> new availability is $0{,}9756 \cdot 100 = 97{,}56\%$

c) Improvement $= \frac{\text{New Availability} - \text{Initial Availability}}{\text{New Availability}} = \frac{0{,}9756 - 0{,}9524}{0{,}9524} = 0{,}0244$

=> improvement percentage is $0{,}0244 \cdot 100 = 2{,}44\%$

Question 2

a) $R_i(t)$ -> reliability of i at time t; k-out-of-n system

$\binom{n}{i}$ -> the number of i element which can be taken from n

$$R(t) = \sum_{i=k}^{n} \binom{n}{i} [R_i(t)]^i [1 - R_i(t)]^{n-i}$$

b) if in the special case where all components have same Reability = p

$$R(t) = \sum_{i=k}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

c) $n = 8$; $k = \{1, 2, \dots, 8\}$; $p = 0{,}8$

$$R(t) = \sum_{i=k}^{8} \binom{8}{i} \cdot 0{,}8^i \cdot 0{,}2^{(8-i)}$$

I $k = 1$ => $R(t) = \sum_{i=1}^{8} \binom{8}{i} \cdot (0{,}8)^i (0{,}2)^{8-i}$ => the reliability will be close to 100%
at least one component work

II $k = 8$ (all components work)

=> $R(t) = (0{,}8)^8 \simeq 0{,}1678$ -> the reability will be 16,78%

Question 1

a) R_2, R_3, R_4 are in series : $R_{234} = R_2 \cdot R_3 \cdot R_4$

R_6, R_7 are in parallel : $R_{67} = 1 - (1-R_6)\cdot(1-R_7)$

R_5, R_{67} are in series : $R_{567} = R_5 \cdot R_{67} = R_5 \cdot (1-[(1-R_6)(1-R_7)]$

R_{234}, R_{567} are in parallel : $R_{2345567} = 1 - [(1-R_{234})\cdot(1-R_{567})]$

$= 1 - (1 - R_2 \cdot R_3 \cdot R_4) - [1 - (R_5 \cdot (1 - (1-R_6)(1-R_7))$

$R_{2345567}$, R_1, R_8 are in series :

$R_{12345678} = R_1 \cdot R_8 \cdot [1 - (1 - R_2 R_3 R_4) \cdot [1 - R_5 \cdot (1 - R_7 - R_6 - R_6 R_7)]$

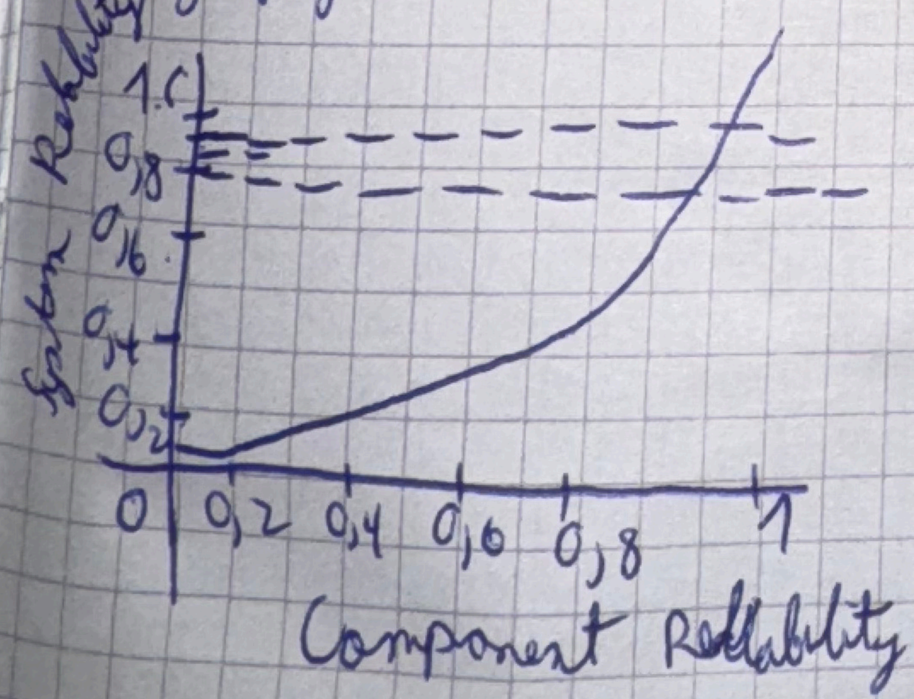
$= R_1 R_8 \cdot [1 - (1 - R_2 R_3 R_4) \cdot (1 - R_5 - R_5 R_7 - R_5 R_6 - R_5 R_6 R_7)]$

$= R_1 R_8 \cdot (1 - (1 - R_5 - R_5 R_7 - R_5 R_6 - R_5 R_6 R_7) -$

$R_1, R_8, R_{2345567}$ are series :

$R_{12345678} = R_8 \cdot R_1 \cdot [1 - (1 - R_2 \cdot R_3 \cdot R_4) \cdot (1 - R_5 \cdot (1 - (1-R_6)(1-R_7))]$

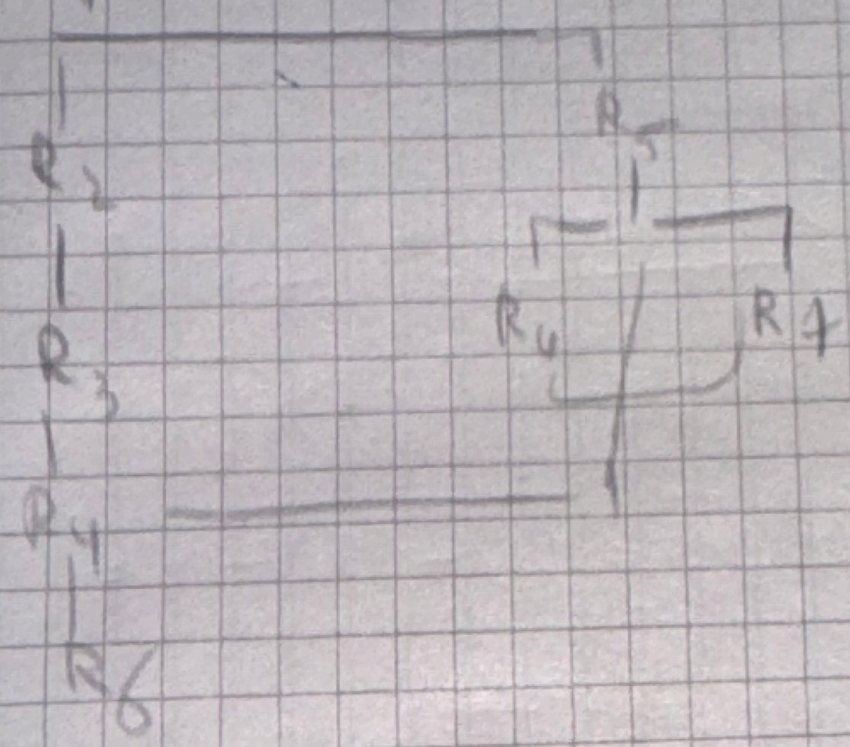
b) We will use a $p = 0,8$ as a reference, after making the plot using python this is how it looked:



=> for $p < 0.8$, the system reliability is less than the component reliability

for $p > 0,8$, the system reliability becomes better than the component reliability

c) We should replace R_1 or R_8 since they are in series and do not impact it



$R_{total}(p=0,8) = 0,8 \cdot [1-(1-(0,8)^3) \cdot (1-p \cdot (1-0,8)^2))] \cdot 0,8$

$R_{total} = 0,8 \cdot (1-0.512)(1-0,768) \cdot 0,8$

$R_{total} = 0,576$

=> original system reability: 57,60%

For $R_1 = 0.95 \Rightarrow R_{total} = 0,95 \cdot 0,886784 \cdot 0,8 = 0,67396$

=> new system reability: 64,13 %

to calculate improvement:

$$\frac{R_{new} - R_{original}}{R_{original}} = \frac{0,10636}{0,5676}$$ => percentage improvement is 18,75%

We