Preface

In 1966 I began teaching a third year undergraduate course in the geometric theory of differential equations. This had previously been given by my friend and colleague Stewart Robertson (now of Southampton University). We both felt that no modern text book really covered the course, and we decided to collaborate in writing one. We had in mind something very simple, with plenty of pictures and examples and with clean proofs of some nice geometric results like the Poincaré–Bendixson theorem, the Poincaré–Hopf theorem and Liapunov's direct method. Unfortunately, over the years, this book stubbornly refused to materialize in a publishable form. I am afraid that I was mainly responsible for this. I became increasingly interested in detailed proofs and in presenting a coherent development of the basic theory, and, as a result, we lost momentum. Eventually two really excellent introductions (Arnold [1] and Hirsch and Smale [1]) appeared, and it is to these that one would now turn for an undergraduate course book. The point of this piece of history is to emphasize the very considerable contribution that Professor Robertson has made to the present book, for this has developed out of our original project. I am very happy to have the opportunity of thanking him both for this and also for his help and encouragement in my early years at Liverpool.

The book that has finally appeared is, I suppose, mainly for postgraduates, although, naturally, I should like to foist parts of it upon undergraduates as well. I hope that it will be useful in filling the gap that still exists between the above-mentioned text books and the research literature. In the first six chapters, I have given a rather doctrinaire introduction to the subject, influenced by the quest for generic behaviour that has dominated research in recent years. I have tried to give rigorous proofs and to sort out answers to questions that crop up naturally in the course of the development. On the other hand, in Chapter 7, which deals with some aspects of the rich flowering of the subject that has taken place in the last twenty-odd years, I have gone in for informal sketches of the proofs of selected theorems. Of course, the choice of results surveyed is very much a function of my own interests and, particularly, competence. This explains, for example, my failure to say anything much about ergodic theory or Hamiltonian systems.

I have tried to make the book reasonably self-contained. I have presupposed a grounding in several-variable differential calculus and a certain amount of elementary point set topology. Very occasionally results from algebraic topology are quoted, but they are of the sort that one happily takes on trust. Otherwise, the basic material (or, at least, enough of it to get by with) is contained in various slag heaps, labelled Appendix, that appear at the end of chapters and at the end of the book. For example, there is a long appendix on the theory of smooth manifolds, since one of the aims of the book is to help students to make the transition to the global theory on manifolds. The appendix establishes the point of view taken in the book and assembles all the relevant apparatus. Its later pages are an attempt to alleviate the condition of the student who shares my congenital inability to grasp the concept of affine connection. To make room for such luxuries, I have, with regret, omitted some attractive topics from the book. In particular, the large body of theory special to two dimensions is already well treated in text books, and I did not feel that I could contribute anything new. Similarly, there is not much emphasis on modelling applications of the theory, except in the introduction. I feel more guilty about ducking transversality theory, and this is, in part, due to a lack of steam. However, after a gestation period that would turn an Alpine black salamander green with envy, it must now be time to stand and deliver.

When working my way into the subject, I found that the books by Coddington and Levinson [1], Hurewicz [1], Lefschetz [1], Nemitskiĭ and Stepanov [1] and, at a later stage, Abraham [1] and Abraham and Robbin [1] were especially helpful. I should like to express my gratitude to my colleague at Liverpool, Bill Newns, who at an early stage read several of the chapters with great care and insight. I am also indebted to Plinio Moreira, who found many errors in a more recent version of the text, and to Andy du Plessis for helpful comments on several points. Finally, a special thank-you to Jean Owen, who typed the whole manuscript beautifully and is still as friendly as ever.