Supply Chain Network Operations Management of a Blood Banking System with Cost and Risk Minimization

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November 2010; April 2011; July 2011
Computational Management Science (2012), **9(2)**: 205-231.

Abstract: Blood service operations are a key component of the healthcare system all over the world and yet the modeling and the analysis of such systems from a complete supply chain network optimization perspective have been lacking due to their associated unique challenges.

In this paper, we develop a generalized network optimization model for the complex supply chain of human blood, which is a life-saving, perishable product. In particular, we consider a regionalized blood banking system consisting of collection sites, testing and processing facilities, storage facilities, distribution centers, as well as points of demand, which, typically, include hospitals. Our multicriteria system-optimization approach on generalized networks with arc multipliers captures many of the critical issues associated with blood supply chains such as the determination of the optimal allocations, and the induced supply-side risk, as well as the induced cost of discarding the waste, while satisfying the uncertain demands as closely as possible.

The framework that we present is also applicable, with appropriate modifications, to the optimization of other supply chains of perishable products.

Keywords: Supply chains, perishable products, blood banking, healthcare, medical waste, network optimization, cost minimization, risk minimization, multicriteria decision-making, generalized networks, variational inequalities

1. Introduction

In today's world, supply chains are more complex than ever before. Consumers' demand for new products as well as the still-critical economic situation require that companies, as well as organizations, be more innovative while also becoming more cost-effective in the procurement and production of their products and services as well as in their delivery. However, despite numerous significant achievements, the discipline of supply chain management (SCM) is still incapable of satisfactorily addressing many practical, real-world challenges (Georgiadis, Vlachos, and Iakovou (2005)).

Supply chains for time-sensitive products and, in particular, for perishable products, pose specific and unique challenges. By definition, a *perishable product* has a limited lifetime during which it can be used, after which it should be discarded (Federgruen, Prastacos, and Zipkin (1986)). Examples of perishable goods include food and food products, medicines and vaccines, cut flowers, etc. (see, e.g., Hsu, Hung, and Li (2007), Zanoni and Zavanella (2007), Osvald and Stirn (2008), and Ahumada and Villalobos (2009)). Clearly, not all perishable products are alike and, notably, in some cases, such as that of medicines and vaccines, the quality of a product, or lack thereof, may result in a matter of "life or death" for its consumers.

In this paper, we focus on a specific perishable product – that of human blood – and the optimization of a blood banking network system. Nahmias (1982) claimed that: "The interest among researchers in perishable inventory problems has been sparked primarily by problems of blood bank management. Some of the possible reasons for this interest might be that blood bank research has been supported by public funds." Prastacos (1984), subsequently, provided a review, to that date, of blood inventory management, from both theoretical and practical perspectives. Whether or not Nahmias' statement is still valid – considering all the recent concerns about the safety of perishable products, blood bank management from a supply chain network perspective merits a fresh and updated approach. This topic is especially timely today, since it has been reported that the number of disasters and the number of people affected by disasters has been growing over the past decade and blood is certainly a life-saving product (cf. Nagurney and Qiang (2009)).

Blood service operations are a key component of the healthcare system all over the world. According to the American Red Cross, over 39,000 donations are needed everyday in the United States, alone, and the blood supply is frequently reported to be just 2 days away from running out. Of 1,700 hospitals participating in a survey in 2007, a total of 492 reported cancellations of elective surgeries on one or more days due to blood shortages. While for

many hospitals, the reported number of blood-related delays was not significant, hospitals with as many days of surgical delays as 50 or even 120 have been observed. Furthermore, in 2006, the national estimate for the number of units of whole blood and all components outdated by blood centers and hospitals was 1,276,000 out of 15,688,000 units (Whitaker et al. (2007)). Considering also the ever-increasing hospital cost of a unit of red blood cells with a 6.4% increase from 2005 to 2007 further highlights the criticality of this perishable, life-saving product. In the US, this criticality has become more of an issue in the Northeastern and Southwestern states since this cost is meaningfully higher compared to that of the Southeastern and Central states. Moreover, hospitals were responsible for approximately 90% of the outdates, with this volume of medical waste imposing discarding costs to the already financially-stressed hospitals (The New York Times (2010)).

Several authors have applied integer optimization models such as facility location, set covering, allocation, and routing to address the optimization / design of supply chains of blood or other perishable critical products (see Jacobs, Silan, and Clemson (1996), Pierskalla (2004), Yang (2006), Sahin, Sural, and Meral (2007), Sivakumar, Ganesh, and Parthiban (2008), Cetin and Sarul (2009), and Ghandforoush and Sen (2010)). In addition, inventory management methods (cf. Cohen and Pierskalla (1979), Karaesmen, Scheller-Wolf, and Deniz (2011), and the references therein), Markov models (Boppana and Chalasani (2007)) as well as simulation techniques (Rytila and Spens (2006), Katsaliaki and Brailsford (2007), and Mustafee, Katsaliaki, and Brailsford (2009)) have been used to handle blood banking systems. Yegul (2007), in his dissertation, which has extensive references on the subject of blood supply chains, also utilized simulation for a blood supply chain with a focus on Turkey. He noted that there were few studies which consider multiple echelons (as the model in our paper does). Haijema, van der Wal, and van Dijk (2007) used a Markov dynamic programming and simulation approach with data from a Dutch blood bank (see also the dissertation of Haijema (2008)).

In this paper, we develop a multicriteria *system-optimization* framework for a regionalized blood supply chain network, such as that of the American Red Cross. In the US, the American Red Cross (ARC) is the major supplier of blood products to hospitals and medical centers satisfying over 45% of the demand for blood components nationally (Walker (2010)). A system-optimization approach is believed to be mandated for critical supplies (Nagurney, Yu, and Qiang (2011)) in that the demand for such products must be satisfied as closely as possible at minimal total cost. The use of a profit maximization criterion, as in Nagurney (2010a, c), is not appropriate for an organization such as the American Red Cross, due to its non-profit status.

Unlike many other supply chain models that assume a fixed lifetime for a perishable good (see, e.g., Hwang and Hahn (2000), Omosigho (2002), and Zhou and Yang (2003)), our system-optimization approach for supply chain network management, as we shall demonstrate, captures the perishability/waste that occurs over the relevant links associated with various activities of the supply chain, similar to the spatial price equilibrium model in Nagurney and Aronson (1989). However, in contrast to the latter model, here we also take into account the discarding cost of the waste over the relevant links as well as the discarding cost of outdated product at the demand points due to the possible excess supply delivered. Furthermore, we capture in the model the uncertainty of the demand and the associated shortage penalties at the demand points. System-optimization models have been developed to capture various issues of supply chain management including that of mergers and acquisition as well as network design (cf. Nagurney (2009, 2010b, c), Nagurney and Woolley (2010), Nagurney, Yu, and Qiang (2011), and Nagurney and Nagurney (2010)).

This paper is organized as follows. In Section 2, we describe, in detail, the structure of a regionalized blood banking system. We develop the supply chain network model for the blood banking system problem, and establish that the multicriteria optimization problem is equivalent to a variational inequality problem, with nice features for computations. We also present simple numerical examples and conduct sensitivity analysis. Our model has several novel features:

1. it captures perishability of this life-saving product through the use of arc multipliers;

2. it contains discarding costs associated with waste/disposal;

3. it handles uncertainty associated with demand points;

4. it assesses costs associated with shortages/surpluses at the demand points, and

5. it quantifies the supply-side risk associated with procurement.

In Section 3, we propose an algorithm which, when applied, yields the optimal level of blood product flows. We then apply the algorithm to compute the solution to a larger-scale numerical example using data motivated by a real-world application in order to further illustrate the modeling and computational framework for blood banking supply chain network management. In Section 4, we summarize the results and present our conclusions.

2. The Supply Chain Network Model of a Regionalized Blood Banking System

In this Section, we develop the supply chain network model for regionalized blood banks. It is necessary to mention that our model for blood banking management is applicable to many perishable products, with minor modifications, but with the same foundations. Also, it is worth noting that although the structure of the network for a blood banking system, or the way that the modules of the supply chain are called, may be slightly different from country to country, or from one region to another, our network framework is sufficiently general to address any blood supply chain network.

2.1 The Components of a Regionalized Blood Banking System

In most parts of the world, blood banking operations systems conduct procurement and distribution in a regionalized manner. In other words, there exists a *Regional Blood Center* in each geographic area which is in charge of coordination and administration of its lower-level units. Nevertheless, despite advances in storage and distribution technologies, hospitals may need to acquire blood products from suppliers that are located in other regions, sometimes even hundreds of miles away.

In the US, for example, the regional divisions of the American Red Cross oversee the entire operation of their corresponding regions. Other suppliers of blood are hospitals - typically the larger ones with blood collection programs - which, however, account for less than 5% of the market share (Whitaker et al. (2007)). There also exist private blood suppliers across the country.

Since 1960, the Red Cross has been reimbursed by the hospitals for the costs associated with providing blood to hospital patients. The Red Cross does not charge for the blood itself; it only recovers the costs associated with the recruitment and screening of potential donors, the collection of blood by trained staff, the processing and testing of each unit of blood in state-of-the-art laboratories, and the labeling, storage, and distribution of blood components.

Figure 1 depicts a network topology of a regionalized blood banking system as for the ARC in the US. In this network, the top level (origin) node represents the ARC regional division. Every other node in the network denotes a component/facility in the system. A path connecting the origin node to a destination node, corresponding to a demand point, consists of a sequence of directed links which correspond to supply chain network activities that ensure that the blood is collected, processed, and, ultimately, distributed to the demand point. We assume that in the supply chain network topology there exists at least one path

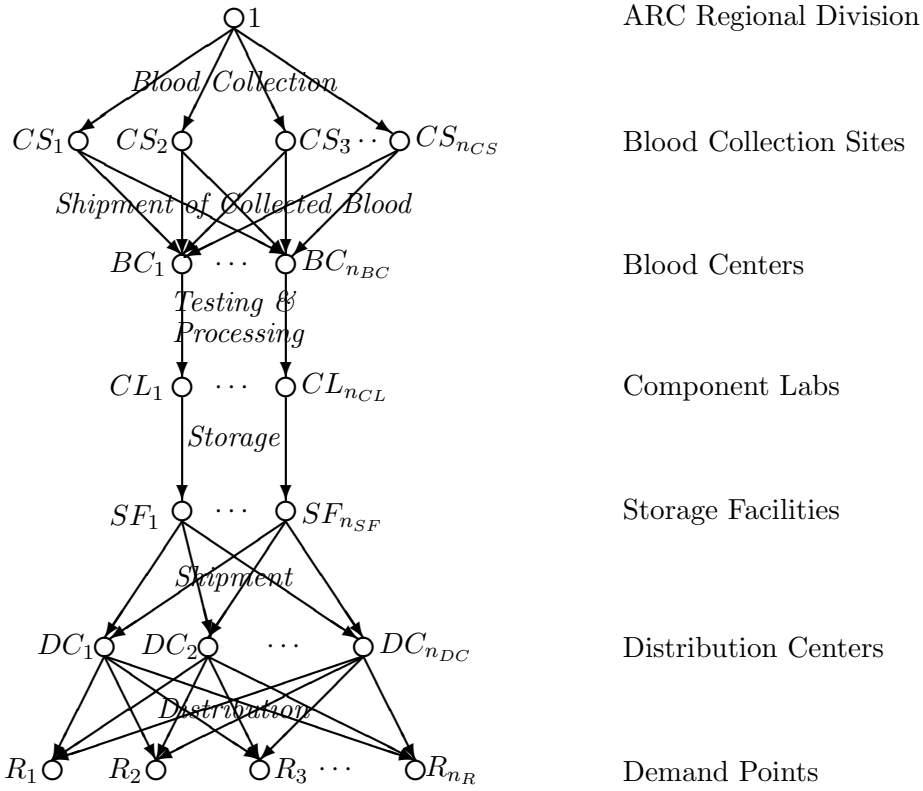

Figure 1: Supply Chain Network Topology for a Regionalized Blood Bank

joining node 1 with each destination node. This assumption guarantees that the demand at each demand point will be met as closely as possible, given that we will be considering uncertain demand for blood at each demand point. The solution of the model yields the optimal flows of blood at minimum total cost and risk, as we shall demonstrate.

In the network in Figure 1, we assume that the division is considering n_C *blood collection sites* constituting the second tier of the network. Many of these collection sites are mobile or temporary locations while others are permanent sites. In the case of drastic shortages; i.e., natural or man-made disasters, the regional divisions are likely to need to import blood products from other regions or even other countries, an aspect that is excluded from this model. In the current topology, the first set of the links connecting the origin node to the second tier corresponds to the process of "blood collection." These collection sites are denoted by: $CS_1, CS_2, \ldots, CS_{n_{CS}}$.

The next set of nodes, located in the third tier, are the *blood centers*. There exist n_{BC} of these facilities in one region, denoted by $BC_1, BC_2, \ldots, BC_{n_{BC}}$, to which the whole blood (WB) is shipped after being collected at the collection sites. Thus, the next set of links connecting tiers two and three of the network topology represents the "shipment of collected blood."

The fourth tier of the network is composed of processing facilities, commonly referred to as *component labs*. The number of these facilities in one region is assumed to be n_{CL}. These facilities are denoted by $CL_1, \ldots, CL_{n_{CL}}$, respectively, and are typically located within the blood center locations. At these labs, the collected blood is separated into parts, i.e., red blood cells and plasma, since most recipients need only a specific component for transfusions. Every unit of donated whole blood - 450 to 500 milliliters on average - can provide one unit of red blood cells (RBC) and one unit of plasma. What we refer to as the flow of product is the amount of whole blood (WB) on the first three sets of links. Likewise, the flow on the links thereafter denotes the number of units of red blood cells (RBC) processed at the component labs which are, ultimately, delivered to the hospitals.

We exclude plasma and other side derivatives from our model for several reasons. First of all, although plasma can be derived from donated whole blood, in practice, plasma is mainly produced in a different process called *apheresis*. Apheresis is a blood donation method where the blood is passed through an apparatus that separates out one particular constituent - plasma - and returns the remainder - red blood cells - to the donor. Secondly, plasma can be stored frozen for an extended period of time, typically one year, which is not comparable to the approximately 5 week lifetime of red blood cells. Most important of all, whole blood and red blood cells account for the major part of donations and transfusions rather than plasma and other components (Whitaker et al. (2007)).

The safety of the blood supply is considered to be the most important issue in blood services. In the US, federal law mandates that every single unit of donated blood be tested before being transfused, regardless of the number of the times one donor has donated blood in the past. The *National Testing Laboratories* of the American Red Cross are in charge of this vital task, testing blood for multiple infectious disease markers, including but not limited to HIV, hepatitis, and the West Nile Virus (Redcrossblood.org (2010)). These facilities are owned and operated by the ARC, and require heavy investments for specialized equipment. Presently, only 5 testing labs are operating across the US, and these labs are shared among 36 blood regions. Only a small sample of every donated blood unit is sent to the testing labs, overnight, and these samples are discarded regardless of the results of the tests. Due to the high perishability of many of the blood products, the two processes of testing and separating take place concurrently yet sometimes hundreds of miles away. If the result of a test for a specific unit of donated blood at the testing lab turns out to be positive, the remainder of that unit will be later discarded at the corresponding storage facility. In our model, the set of the links connecting the component labs to storage facilities corresponds to "testing and processing," and the cost on these links represents the operational cost of

testing and processing combined. The fraction of the flow lost during or as a result of the testing process is also taken care of in our model.

The fifth set of nodes denotes the short-term *storage facilities*. There are n_{SF} of such nodes in the network, denoted by $SF_1, SF_2, \ldots, SF_{n_{SF}}$, which are usually located in the same place as the component labs. The links connecting the upper level nodes to the storage facilities denote the procedure of "storage" of the tested and processed blood before it is shipped to be distributed.

The next set of nodes in the network represents the *distribution centers*, denoted by $DC_1, DC_2, \ldots, DC_{n_{DC}}$, where n_{DC} is the total number of such facilities in the region. Distribution centers act as transshipment nodes, and are in charge of facilitating the distribution of blood to the ultimate destinations. The links connecting the storage tier to the distribution centers are of "shipment" link type.

The last set of links joining the bottom two tiers of the network are "distribution" links, ending in n_R *demand points*. Hospitals and surgical medical centers are the predominant users of blood. The actual but uncertain demands of the demand points $R_1, R_2, \ldots, R_{n_R}$ are denoted by: $d_{R_1}, d_{R_2}, \ldots, d_{R_{n_R}}$, respectively.

It is necessary to mention that specific components of the system may physically coincide with some others; however, this network topology is process-based rather than location-based, which is compatible with our blood banking problem. Moreover, as mentioned earlier, in general cases of perishable product supply chains, these facilities may be located far apart which can be nicely addressed using our presented model.

The supply chain network topology is represented by $G = [N, L]$, where N and L denote the sets of nodes and links, respectively. The ultimate solution of the complete model will yield the optimal flow on the various links of the network.

2.2 The Formulation

Our formulation is of a single-period type, where the time horizon spans the various activities of procurements, processing, and distribution. Since whole blood is highly perishable, all modules of the blood supply chain network tend to avoid long term storage (except for plasma, which is excluded from our model). Hence, the assumption of a single-period time horizon is realistic with the focus of this paper being on operations management, rather than on inventory management. Nevertheless, our model takes into account the potential shortage associated with the uncertain demand at the demand points, that is, the lost "sales."

In addition, the surplus penalty can address additional relevant costs, whether in terms of excess supply or even if short-term inventory holding cost is included.

Associated with each link of the network is a unit operational cost function that reflects the cost of operating the particular supply chain activity, that is, the collection of blood at blood drive sites, the shipment of collected blood, the testing and processing, the storage, and the distribution. We denote these links by a, b, etc. The unit operational cost on link a is denoted by c_a and is a function of flow on that link, f_a. The *total* operational cost on link a is denoted by $\hat{c}_a$, and is constructed as:

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \qquad \forall a \in L. \tag{1}$$

The link total cost functions are assumed to be convex and continuously differentiable.

The origin/destination (O/D) nodes consist of the pairs of nodes $(1,\ R_k);\ 1, \ldots, n_R$, where $\mathcal{P}_k$ denotes the set of paths, which represent alternative associated possible supply chain network processes, joining $(1,\ R_k)$. $\mathcal{P}$ denotes the set of all paths joining node 1 to the destination nodes, and $n_{\mathcal{P}}$ denotes the number of paths.

Let v_k denote the *projected demand* for blood at the demand point $R_k; k = 1, \ldots, n_R$. We assume that the demand at each demand point is uncertain with a known probability distribution. Recall that d_k denotes the actual demand at demand point $R_k; k = 1, \ldots, n_R$, and is a random variable with probability density function given by $\mathcal{F}_k(t)$. Let P_k be the probability distribution function of d_k, that is, $P_k(D_k) = Prob(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(t)d(t)$. Therefore,

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \tag{2}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R, \tag{3}$$

where Δ_k^- and Δ_k^+ represent the shortage and surplus of blood at demand point R_k, respectively.

The expected values of the shortage (Δ_k^-) and the surplus (Δ_k^+) are given by:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)d(t), \qquad k = 1, \ldots, n_R, \tag{4}$$

$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - t)\mathcal{F}_k(t)d(t), \qquad k = 1, \ldots, n_R. \tag{5}$$

Due to the vitalness of the availability of blood at the demand points, a relatively large penalty of λ_k^- is associated with the shortage of a unit of blood at demand point R_k, where

λ_k^- corresponds to the social cost of a death or a severe injury of a patient, due to a blood shortage. Also, since blood is extremely perishable and will be outdated if not used over a certain period after being delivered, the outdating penalty of λ_k^+ is assigned to the unit of a possible supply surplus. Note that, in our formulation, this surplus penalty is charged to the organization even though the ARC is not directly responsible for the outdated blood at the hospitals once it is delivered to them. This is because human blood is scarce, and the ARC aims to minimize the amount of outdated blood at demand points, which actually dominates the amount of blood waste during the other activities of blood banking within the ARC network (Rios (2010)). Hence, λ_k^+, in the case of blood (as for other perishable products), includes the cost of short-term inventory holding (cold storage), and, possibly, the discarding cost of the outdated product. It is necessary to mention that having excessive supplies outdated at the demand points not only imposes a discarding cost on the already financially stressed healthcare institutions such as hospitals, but also leads to further environmental damage. Similar examples of penalty costs, due to excessive supplies, as well as to shortages, can be found in the literature (see, e.g., Dong, Zhang, and Nagurney (2004), and Nagurney, Yu, and Qiang (2011)). These penalties can be assessed by the authority who is contracting with the organization to deliver the blood.

Thus, the expected total penalty at demand point $k; k = 1, \ldots, n_R$, is:

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+). \tag{6}$$

Nevertheless, the demand points (such as hospitals) are not the only modules of the blood supply chain in which the perishability of the collected blood occurs. Throughout the processes of blood collection, shipment, testing and processing, storage, and distribution, a fraction of the collected blood may deteriorate, be lost, or be wasted. The fraction of the lost product depends on the type of the activity since various processes of collection, testing, storage, etc., lead to different amounts of waste. This fraction, in general, can also be different among the various facilities at the same tier of the network, depending upon the technology used, the efficiency of the staff personnel, and so forth.

Hence, we associate with every link a in the network, a multiplier α_a, which, for all activities of the blood supply chain, lies in the range of (0,1] where $\alpha_a = 1$ means that $\alpha_a \times 100\%$ of the initial flow on link a reaches the successor node of that link, reflecting that there is no waste/loss on link a. The average percentage of loss due to the testing process was reported to be 1.7% (Sullivan et al. (2007)); consequently, the corresponding multiplier, α_a, would be equal to $1 - 0.017 = 0.983$.

As mentioned earlier, f_a denotes the (initial) flow on link a. Let f'_a denote the final flow

on that link; i.e., the flow that reaches the successor node of the link. Therefore,

$$f'_a = \alpha_a f_a, \qquad \forall a \in L. \tag{7}$$

Thus, the waste/loss on link a, denoted by w_a, is equal to:

$$w_a = f_a - f'_a = (1 - \alpha_a) f_a, \qquad \forall a \in L. \tag{8}$$

The organization such as ARC is responsible for discarding this waste which is potentially hazardous. Contractors are typically employed to remove and dispose of the waste. The corresponding discarding cost, y_a, is a function of the waste, w_a, which is charged to the organization:

$$y_a(w_a) = y_a(f_a - f'_a) = y_a\big((1 - \alpha_a) f_a\big), \qquad \forall a \in L. \tag{9a}$$

Since α_a is constant, and known a priori, a new total discarding cost function, $\hat{z}_a$, can be defined accordingly, which is a function of the flow, f_a, and is assumed to be convex and continuously differentiable:

$$\hat{z}_a = \hat{z}_a(f_a), \qquad \forall a \in L. \tag{9b}$$

Also, let x_p represent the (initial) flow of blood (or a general perishable product) on path p joining the origin node with a destination node. The path flows must be nonnegative, that is,

$$x_p \geq 0, \qquad \forall p \in \mathcal{P}, \tag{10}$$

since the product will be collected, shipped, etc., in nonnegative quantities.

Let μ_p denote the multiplier corresponding to the throughput on path p, which is defined as the product of all link multipliers on links comprising that path, that is,

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \qquad \forall p \in \mathcal{P}. \tag{11}$$

The projected demand at demand point R_k, v_k, is the sum of all the final flows on paths joining $(1, R_k)$:

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p \mu_p, \qquad k = 1, \ldots, n_R. \tag{12}$$

Indeed, although the amount of blood that originates on a path p is x_p, the amount (due to perishability) that actually arrives at the destination of this path is $x_p \mu_p$.

We also define the multiplier, α_{ap}, which is the product of the multipliers of the links on path p that precede link a in that path. This multiplier can be expressed as:

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a'<a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \tag{13}$$

where $\{a' < a\}$ denotes the set of the links preceding link a in path p, δ_{ap} is defined as equal to 1 if link a is contained in path p, and 0, otherwise, and $\emptyset$ denotes the null set. Hence, α_{ap} is equal to the product of all link multipliers preceding link a in path p. If link a is not contained in path p, then α_{ap} is set to zero. If a belongs to the first set of links, the blood collection links, this multiplier is equal to 1. The relationship between the link flow, f_a, and the path flows is as follows:

$$f_a = \sum_{p \in \mathcal{P}} x_p \, \alpha_{ap}, \qquad \forall a \in L. \tag{14}$$

Similar examples of multipliers corresponding to the loss/waste on links or paths can be found in the literature (see, e.g., Nagurney and Aronson (1989)).

We group the path flows into the vector x. Also, the link flows, and the projected demands are grouped into the respective vectors f and v.

The total cost minimization objective faced by the organization includes the total cost of operating the various links, the total discarding cost of waste/loss over the links, and the expected total blood supply shortage cost as well as the total discarding cost of outdated blood at the demand points. This optimization problem can be expressed as:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L} \hat{z}_a(f_a) + \sum_{k=1}^{n_R} \left(\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+) \right) \tag{15}$$

subject to: constraints (10), (12), and (14).

As mentioned earlier, the minimization of total costs is not the only objective of suppliers of perishable goods. One of the most significant challenges for the ARC, for example, is to capture the risk associated with different activities in the blood supply chain network. Unlike the demand which can be projected according to the historical data, albeit with some uncertainty involved, the amount of donated blood at the collection sites has been observed to be highly stochastic. Even though the ARC encourages blood donors to make appointments beforehand, donors may miss their appointments due to inclement weather situations, traffic, personal issues, etc.

Interestingly, disasters, such as the 2010 earthquake in Haiti, may stimulate people's sympathy and dramatically increase the number of blood donors. As in Nagurney et al.

(2005), we introduce a total risk function $\hat{r}_a$ corresponding to link a for every blood collection link. This function is assumed to be convex and continuously differentiable, and a function of the flow, that is, the amount of collected blood, on its corresponding link. The organization attempts to minimize the total risk over all links connecting the first two tiers of the network, denoted by $L_1 \subset L$. The remainder of the links in the network, i.e., the shipment of collected blood, the processing, the storage, shipment, and the distribution links, comprise the set L_1^C. The subset L_1 and its complement L_1^C partition the entire set of links L, that is, $L_1 \cup L_1^C = L$.

Thus, the risk minimization objective function for the organization can be expressed as:

$$\text{Minimize} \quad \sum_{a \in L_1} \hat{r}_a(f_a), \tag{16}$$

where $\hat{r}_a = \hat{r}_a(f_a)$ is the total risk function on link a.

The supply chain network optimization problem for a regionalized blood banking system can be expressed as a multicriteria decision-making problem. The organization seeks to determine the optimal levels of blood processed on each supply chain network link subject to the minimization of the total cost (operational and discarding) as well as the minimization of the total supply risk. We associate with the total supply risk objective, (16), a risk aversion factor θ, which is assigned by the decision-maker. Thus, the multicriteria optimization problem is:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L} \hat{z}_a(f_a) + \sum_{k=1}^{n_R} \left(\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)\right) + \theta \sum_{a \in L_1} \hat{r}_a(f_a) \tag{17}$$

subject to: constraints (10), (12), and (14).

The above optimization problem is in terms of link flows. It can also be expressed in terms of path flows:

$$\text{Minimize} \quad \sum_{p \in \mathcal{P}} \left(\hat{C}_p(x) + \hat{Z}_p(x)\right) + \sum_{k=1}^{n_R} \left(\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)\right) + \theta \sum_{p \in \mathcal{P}} \hat{R}_p(x) \tag{18}$$

subject to: constraints (10) and (12), where the total operational cost, $\hat{C}_p(x)$, the total discarding cost, $\hat{Z}_p(x)$, and the total risk, $\hat{R}_p(x)$, corresponding to path p are, respectively, derived from $C_p(x)$, $Z_p(x)$, and $R_p(x)$ as follows:

$$\hat{C}_p(x) = x_p \times C_p(x), \qquad \forall p \in \mathcal{P}, \tag{19a}$$

$$\hat{Z}_p(x) = x_p \times Z_p(x), \qquad \forall p \in \mathcal{P}, \tag{19b}$$

$$\hat{R}_p(x) = x_p \times R_p(x), \qquad \forall p \in \mathcal{P}, \tag{19c}$$

with the unit cost functions on path p, i.e., $C_p(x), Z_p(x)$, and $R_p(x)$, in turn,as below:

$$C_p(x) \equiv \sum_{a \in L} c_a(f_a)\alpha_{ap}, \qquad \forall p \in \mathcal{P}, \tag{20a}$$

$$Z_p(x) \equiv \sum_{a \in L} z_a(f_a)\alpha_{ap}, \qquad \forall p \in \mathcal{P}, \tag{20b}$$

$$R_p(x) \equiv \sum_{a \in L_1} r_a(f_a)\alpha_{ap}, \qquad \forall p \in \mathcal{P}. \tag{20c}$$

Next, we present some preliminaries that enable us to express the partial derivatives of the expected total shortage and discarding costs of outdated blood at the demand points solely in terms of path flow variables. Observe that, for each O/D pair w_k:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = \frac{\partial E(\Delta_k^-)}{\partial v_k} \times \frac{\partial v_k}{\partial x_p}, \qquad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R. \tag{21}$$

By Leibniz's integral rule, we have:

$$\frac{\partial E(\Delta_k^-)}{\partial v_k} = \frac{\partial}{\partial v_k}\left(\int_{v_k}^{\infty}(t - v_k)\mathcal{F}_k(t)d(t)\right) = \int_{v_k}^{\infty}\frac{\partial}{\partial v_k}(t - v_k)\mathcal{F}_k(t)d(t)$$

$$= P_k(v_k) - 1, \qquad k = 1, \ldots, n_R. \tag{22a}$$

Therefore,

$$\frac{\partial E(\Delta_k^-)}{\partial v_k} = P_k\left(\sum_{p \in \mathcal{P}_k} x_p\mu_p\right) - 1, \qquad k = 1, \ldots, n_R. \tag{22b}$$

On the other hand, we have:

$$\frac{\partial v_k}{\partial x_p} = \frac{\partial}{\partial x_p}\sum_{p \in \mathcal{P}_k} x_p\mu_p = \mu_p, \qquad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R. \tag{23}$$

The above is correct since the μ_p's are constant values. Therefore, from (22b) and (23), we conclude that

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = \mu_p\left[P_k\left(\sum_{p \in \mathcal{P}_k} x_p\mu_p\right) - 1\right], \qquad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R. \tag{24}$$

Similarly, for the surplus, we have:

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = \frac{\partial E(\Delta_k^+)}{\partial v_k} \times \frac{\partial v_k}{\partial x_p}, \qquad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R, \tag{25}$$

$$\frac{\partial E(\Delta_k^+)}{\partial v_k} = \frac{\partial}{\partial v_k}\left(\int_0^{v_k}(v_k - t)\mathcal{F}_k(t)d(t)\right) = \int_0^{v_k}\frac{\partial}{\partial v_k}(v_k - t)\mathcal{F}_k(t)d(t)$$

$$= P_k(v_k), \qquad k = 1, \ldots, n_R. \tag{26a}$$

Thus,

$$\frac{\partial E(\Delta_k^+)}{\partial v_k} = P_k \left(\sum_{p \in \mathcal{P}_k} x_p \mu_p \right), \qquad k = 1, \ldots, n_R. \tag{26b}$$

From (26b) and (23) we have:

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = \mu_p P_k \left(\sum_{p \in \mathcal{P}_k} x_p \mu_p \right), \qquad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R. \tag{27}$$

Let K denote the feasible set such that:

$$K \equiv \{x | x \in R_+^{n_{\mathcal{P}}}\}. \tag{28}$$

Before deriving the variational inequality formulation of the problem, we establish a lemma that formalizes the construction of the partial derivatives of the path total operational cost, the total discarding cost, and the total risk with respect to a path flow.

Lemma 1

The partial derivatives of the total operational cost, the total discarding cost, and the total risk with respect to the corresponding path flow are, respectively, given by:

$$\frac{\partial (\sum_{q \in \mathcal{P}} \hat{C}_q(x))}{\partial x_p} = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}, \tag{29a}$$

$$\frac{\partial (\sum_{q \in \mathcal{P}} \hat{Z}_q(x))}{\partial x_p} = \sum_{a \in L} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}, \tag{29b}$$

$$\frac{\partial (\sum_{q \in \mathcal{P}} \hat{R}_q(x))}{\partial x_p} = \sum_{a \in L_1} \frac{\partial \hat{r}_a(f_a)}{\partial f_a} \alpha_{ap}, \quad \forall p \in \mathcal{P}. \tag{29c}$$

Proof: We establish the equivalence for (29a); the equivalences (29b) and (29c) can be obtained in a similar fashion. The partial derivative of the total operational cost with respect to the flow on path p is first defined as:

$$\frac{\partial (\sum_{q \in \mathcal{P}} \hat{C}_q)}{\partial x_p} = \sum_{q \in \mathcal{P}} \frac{\partial \hat{C}_q}{\partial x_p}, \qquad \forall p \in \mathcal{P}, \tag{30a}$$

which, based on the total path cost (19a), can be rewritten as:

$$\frac{\partial(\sum_{q\in\mathcal{P}}\hat{C}_q)}{\partial x_p}=\sum_{q\in\mathcal{P}}\frac{\partial(C_q x_q)}{\partial x_p}=C_p+\sum_{q\in\mathcal{P}}x_q\frac{\partial C_q}{\partial x_p},\qquad\forall p\in\mathcal{P}.\tag{30b}$$

According to the definition of $C_q(x)$ in (20a), we have:

$$\frac{\partial C_q}{\partial x_p}=\frac{\partial\sum_{a\in L}c_a\alpha_{aq}}{\partial x_p}=\sum_{a\in L}\frac{\partial c_a}{\partial x_p}\alpha_{aq}=\sum_{a\in L}\frac{\partial c_a}{\partial f_a}\frac{\partial f_a}{\partial x_p}\alpha_{aq},\quad\forall p,q\in\mathcal{P}.\tag{31}$$

On the other hand, by referring to (14), yields:

$$\frac{\partial f_a}{\partial x_p}=\alpha_{ap},\qquad\forall a\in L,\forall p\in\mathcal{P}.\tag{32}$$

From (31) and (32), we obtain:

$$\frac{\partial C_q}{\partial x_p}=\sum_{a\in L}\frac{\partial c_a}{\partial f_a}\alpha_{ap}\alpha_{aq},\qquad\forall p,q\in\mathcal{P}.\tag{33}$$

Substituting (33) into (30b) yields:

$$\frac{\partial(\sum_{q\in\mathcal{P}}\hat{C}_q)}{\partial x_p}=C_p+\sum_{q\in\mathcal{P}}x_q\sum_{a\in L}\frac{\partial c_a}{\partial f_a}\alpha_{ap}\alpha_{aq}=C_p+\sum_{a\in L}\sum_{q\in\mathcal{P}}x_q\frac{\partial c_a}{\partial f_a}\alpha_{ap}\alpha_{aq}.$$

Thus,

$$\frac{\partial(\sum_{q\in\mathcal{P}}\hat{C}_q)}{\partial x_p}=C_p+\sum_{a\in L}\frac{\partial c_a}{\partial f_a}\alpha_{ap}\sum_{q\in\mathcal{P}}x_q\alpha_{aq},\qquad\forall p\in\mathcal{P}.\tag{34}$$

By substituting proper equivalences from (14) and (20a) into (34), we have:

$$\frac{\partial(\sum_{q\in\mathcal{P}}\hat{C}_q)}{\partial x_p}=\sum_{a\in L}c_a\alpha_{ap}+\sum_{a\in L}\frac{\partial c_a}{\partial f_a}\alpha_{ap}f_a=\sum_{a\in L}(c_a+\frac{\partial c_a}{\partial f_a}f_a)\alpha_{ap},\qquad\forall p\in\mathcal{P}.\tag{35}$$

On the other hand, from (1):

$$\frac{\partial\hat{c}_a}{\partial f_a}=c_a+\frac{\partial c_a}{\partial f_a}f_a,\qquad\forall a\in L.\tag{36}$$

From (35) and (36), we conclude that:

$$\frac{\partial(\sum_{q\in\mathcal{P}}\hat{C}_q)}{\partial x_p}=\sum_{a\in L}\frac{\partial\hat{c}_a(f_a)}{\partial f_a}\alpha_{ap},\qquad\forall p\in\mathcal{P}.\tag{37}$$

Thus, (29a) has been established. □

We now derive the variational inequality formulation of the blood supply chain network optimization problem in terms of path flows and link flows.

Theorem 1

The vector x^* is an optimal solution to the multicriteria optimization problem (18), subject to (10) and (12), if and only if it is a solution to the variational inequality problem: determine the vector of optimal path flows $x^* \in K$, such that:

$$\sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial (\sum_{q \in \mathcal{P}} \hat{C}_q(x^*))}{\partial x_p} + \frac{\partial (\sum_{q \in \mathcal{P}} \hat{Z}_q(x^*))}{\partial x_p} + \lambda_k^+ \mu_p P_k \left(\sum_{p \in \mathcal{P}_k} x_p^* \mu_p \right) - \lambda_k^- \mu_p \left(1 - P_k \left(\sum_{p \in \mathcal{P}_k} x_p^* \mu_p \right) \right) \right.$$

$$\left. + \theta \frac{\partial (\sum_{q \in \mathcal{P}} \hat{R}_q(x^*))}{\partial x_p} \right] \times [x_p - x_p^*] \geq 0, \qquad \forall x \in K. \tag{38}$$

The variational inequality (38), in turn, can be rewritten in terms of link flows as: determine the vector of optimal link flows, and the vector of optimal projected demands $(f^*, v^*) \in K^1$, such that:

$$\sum_{a \in L_1} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \theta \frac{\partial \hat{r}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*] + \sum_{a \in L_1^C} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{k=1}^{n_R} \left[\lambda_k^+ P_k(v_k^*) - \lambda_k^- (1 - P_k(v_k^*)) \right] \times [v_k - v_k^*] \geq 0, \qquad \forall (f, v) \in K^1, \tag{39}$$

where K^1 denotes the feasible set as defined below:

$$K^1 \equiv \{(f, v) | \exists x \geq 0, \ (10), (12), \ \textit{and} \ (14) \ \textit{hold}\}.$$

Proof: First, we prove the result for path flows (cf. (38)).

The convexity of $\hat{C}_p$, $\hat{Z}_p$, and $\hat{R}_p$ for all paths p holds since $\hat{c}_a$, $\hat{z}_a$, and $\hat{r}_a$ were assumed to be convex for all links a. We need to verify that $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$ is also convex. We have:

$$\frac{\partial^2}{\partial {x_p}^2} \left[\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+) \right] = \lambda_k^- \frac{\partial^2 E(\Delta_k^-)}{\partial {x_p}^2} + \lambda_k^+ \frac{\partial^2 E(\Delta_k^+)}{\partial {x_p}^2}, \qquad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R. \tag{40a}$$

Substituting the first order derivatives from (24) and (27) into (40a) yields:

$$\frac{\partial^2}{\partial {x_p}^2}\left[\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)\right] = \lambda_k^- \frac{\partial}{\partial x_p}\mu_p \left[P_k\left(\sum_{p\in\mathcal{P}_k} x_p\mu_p\right) - 1\right] + \lambda_k^+ \frac{\partial}{\partial x_p}\mu_p P_k\left(\sum_{p\in\mathcal{P}_k} x_p\mu_p\right)$$

$$= (\lambda_k^- + \lambda_k^+)(\mu_p)^2 \mathcal{F}_k\left(\sum_{p\in\mathcal{P}_k} x_p\mu_p\right) > 0, \qquad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R. \tag{40b}$$

The above inequality holds provided that $(\lambda_k^- + \lambda_k^+)$, i.e., the sum of shortage and surplus penalties, is assumed to be positive. Hence, $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$, and, as a consequence, the multicriteria objective function in (18) is also convex.

Since the objective function (18) is convex and the feasible set K is closed and convex, the variational inequality (38) follows from the standard theory of variational inequalities (see Nagurney (1999)).

As for the proof of the variational inequality (39), now that (38) is established, we can apply Lemma 1. Also, from (12) and (14), we can rewrite the formulation in terms of link flows and projected demands rather than path flows. Thus, the second part of Theorem 1, that is, the variational inequality in link flows (39), holds. □

Note that variational inequality (38) can be put into standard form (see Nagurney (1999)) as follows: determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \qquad \forall X \in \mathcal{K}, \tag{41}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous and the feasible set $\mathcal{K}$ being closed and convex.

Indeed, if we define the feasible set $\mathcal{K} \equiv K$, and the column vector $X \equiv x$, and $F(X)$, such that:

$$F(X) \equiv \left[\frac{\partial \sum_{q\in\mathcal{P}} \hat{C}_q(x)}{\partial x_p} + \frac{\partial \sum_{q\in\mathcal{P}} \hat{Z}_q(x)}{\partial x_p} + \lambda_k^+ \mu_p P_k\left(\sum_{p\in\mathcal{P}_k} x_p\mu_p\right)\right.$$

$$\left. -\lambda_k^- \mu_p \left(1 - P_k\left(\sum_{p\in\mathcal{P}_k} x_p\mu_p\right)\right) + \theta \frac{\partial \sum_{q\in\mathcal{P}} \hat{R}_q(x)}{\partial x_p}; \quad p \in \mathcal{P}_k; k = 1, \ldots, n_R\right], \tag{42}$$

then the variational inequality (38) can be reexpressed in standard form (41).

We will utilize variational inequality (38) in path flows for our computations since our proposed computational procedure will yield closed form expressions at each iteration. Once we have solved problem (38), by using (14), which relates the links flows to the path flows, we can obtain the solution f^* that minimizes the total cost as well as the total supply risk (cf. (17)) associated with the optimization of the supply chain network of a regionalized blood banking system.

2.3 Illustrative Blood Supply Chain Network Numerical Examples

In order to further illustrate the above model, we now present several simple examples.

Consider the blood supply chain network topology in Figure 2 in which the organization has a single blood collection site, a single blood center, one component lab, one storage facility, a single distribution center and is to serve a single demand point. The links are labeled as in Figure 2, that is, a, b, c, d, e, and f.

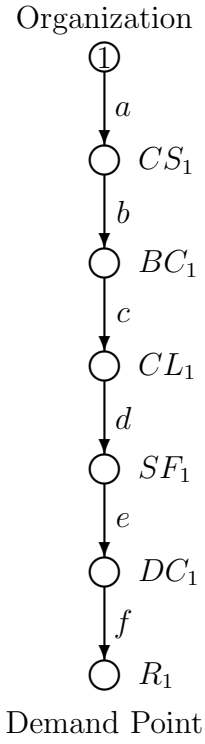

Figure 2: Supply Chain Network Topology for Numerical Examples 1 and 2

Example 1

The total cost functions on the links were:

$$\hat{c}_a(f_a) = f_a^2 + 6f_a, \quad \hat{c}_b(f_b) = 2f_b^2 + 7f_b, \quad \hat{c}_c(f_c) = f_c^2 + 11f_c, \quad \hat{c}_d(f_d) = 3f_d^2 + 11f_d,$$

$$\hat{c}_e(f_e) = f_e^2 + 2f_e, \quad \hat{c}_f(f_f) = f_f^2 + f_f.$$

We assumed that there was no waste so that $\alpha_a = 1$ for all links in Figure 2. Hence, all the functions $\hat{z}_a$ were set equal to 0 for all the links $a, \ldots, f$.

The total risk cost function on the blood collection link a was: $\hat{r}_a = 2f_a^2$, and the risk aversion factor, θ, was assumed to be 1.

There is only a single path p_1 which was defined as: $p_1 = (a, b, c, d, e, f)$ with $\mu_{p_1} = 1$.

We assumed that the demand for the product followed a uniform distribution on the interval $[0, 5]$ so that: $P_1(x_{p_1}) = \frac{x_{p_1}}{5}$.

The penalties were: $\lambda_1^- = 100, \quad \lambda_1^+ = 0$.

Substitution of the values of $\lambda_1^+, \lambda_1^-, \mu_{p_1}$, and θ into (38), yields:

$$\left[\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} - 100(1 - P_1(x_{p_1}^*)) + \frac{\partial \hat{R}_{p_1}(x^*)}{\partial x_{p_1}}\right] \times [x_{p_1} - x_{p_1}^*] \geq 0, \qquad \forall x \in K. \tag{43}$$

Under the assumption that $x_{p_1}^* > 0$, the left-hand side of inequality (43) must be equal to zero, that is:

$$\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} - 100(1 - P_1(x_{p_1}^*)) + \frac{\partial \hat{R}_{p_1}(x^*)}{\partial x_{p_1}} = 0. \tag{44}$$

It follows from Lemma 1 that:

$$\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} = \frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}\alpha_{ap_1} + \frac{\partial \hat{c}_b(f_b^*)}{\partial f_b}\alpha_{bp_1} + \frac{\partial \hat{c}_c(f_c^*)}{\partial f_c}\alpha_{cp_1} + \frac{\partial \hat{c}_d(f_d^*)}{\partial f_d}\alpha_{dp_1} + \frac{\partial \hat{c}_e(f_e^*)}{\partial f_e}\alpha_{ep_1} + \frac{\partial \hat{c}_f(f_f^*)}{\partial f_f}\alpha_{fp_1}. \tag{45}$$

Since $\alpha_{ap_1} = \alpha_{bp_1} = \alpha_{cp_1} = \alpha_{dp_1} = \alpha_{ep_1} = \alpha_{fp_1} = 1$, and $f_a^* = f_b^* = f_c^* = f_d^* = f_e^* = f_f^* = x_{p_1}^*$, with substitution into (45), gives us:

$$\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} = (2f_a^* + 6) + (4f_b^* + 7) + (2f_c^* + 11) + (6f_d^* + 11) + (2f_e^* + 2) + (2f_f^* + 1) = 18x_{p_1}^* + 38. \tag{46}$$

Similarly:

$$\frac{\partial \hat{R}_{p_1}(x^*)}{\partial x_{p_1}} = \frac{\partial \hat{r}_a(f_a^*)}{\partial f_a}\alpha_{ap_1} = 4f_a^* = 4x_{p_1}^*. \tag{47}$$

Therefore, using the above relationships, (44) may be reexpressed as:

$$18x_{p_1}^* + 38 - 100(1 - \frac{x_{p_1}^*}{5}) + 4x_{p_1}^* = 0, \tag{48}$$

whose solution yields the optimal path flow: $x^*_{p_1} = 1.48$, and the corresponding optimal link flow pattern: $f^*_a = f^*_b = f^*_c = f^*_d = f^*_e = f^*_f = 1.48$. Following (12), the projected demand is equal to: $v^*_1 = x^*_{p_1} = 1.48$.

Example 2

Example 2 had the same data as Example 1 except that now there was a loss associated with the testing and processing link with $\alpha_c = .8$. Hence, we now set (cf. (9a, b)) $\hat{z}_c = .5f_c^2$ and $\mu_{p_1} = \alpha_c = .8$.

Similar to the solution procedure used for Example 1, from variational inequality formulation (38), under the assumption that $x^*_{p_1} > 0$, the following equation must hold for Example 2:

$$\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} + \frac{\partial \hat{Z}_{p_1}(x^*)}{\partial x_{p_1}} - 100 \times 0.8(1 - P_1(0.8 \times x^*_{p_1})) + \frac{\partial \hat{R}_{p_1}(x^*)}{\partial x_{p_1}} = 0. \tag{49}$$

Since in this example, $\alpha_{ap_1} = \alpha_{bp_1} = \alpha_{cp_1} = 1$, $\alpha_{dp_1} = \alpha_{ep_1} = \alpha_{fp_1} = 0.8$, $f^*_a = f^*_b = f^*_c = x^*_{p_1}$, and $f^*_d = f^*_e = f^*_f = 0.8x^*_{p_1}$, therefore:

$$\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} = (2f^*_a + 6) + (4f^*_b + 7) + (2f^*_c + 11) + 0.8(6f^*_d + 11) + 0.8(2f^*_e + 2) + 0.8(2f^*_f + 1)$$

$$= 8x^*_{p_1} + 24 + 0.8(10 \times 0.8x^*_{p_1} + 14) = 14.4x^*_{p_1} + 35.2. \tag{50}$$

Also,

$$\frac{\partial \hat{Z}_{p_1}(x^*)}{\partial x_{p_1}} = \frac{\partial \hat{z}_c(f^*_c)}{\partial f_c}\alpha_{cp_1} = f^*_c = x^*_{p_1}. \tag{51}$$

The partial derivative of the total risk function was equal to that of Example 1:

$$\frac{\partial \hat{R}_{p_1}(x^*)}{\partial x_{p_1}} = \frac{\partial \hat{r}_a(f^*_a)}{\partial f_a}\alpha_{ap_1} = 4f^*_a = 4x^*_{p_1}. \tag{52}$$

However, now we have

$$P_1(0.8x^*_{p_1}) = \frac{0.8x^*_{p_1}}{5}. \tag{53}$$

Therefore, the following equation needs to be solved:

$$14.4x^*_{p_1} + 35.2 + x^*_{p_1} - 80(1 - \frac{0.8x^*_{p_1}}{5}) + 4x^*_{p_1} = 0. \tag{54}$$

The new optimal path flow solution was: $x^*_{p_1} = 1.39$, which corresponds to the optimal link flow pattern: $f^*_a = f^*_b = f^*_c = 1.39$, and $f^*_d = f^*_e = f^*_f = 1.11$. The projected

demand was: $v_1^* = x_{p_1}^* \mu_{p_1} = 1.11$. Comparing the results of Examples 1 and 2 reveals the fact that when perishability is taken into consideration, with $\alpha_c = .8$ and the above data, the organization chooses to produce/ship slightly smaller quantities so as to minimize the discarding cost of the waste, despite the shortage penalty of λ_1^-.

Note that when $\lambda_1^- = 200$, the optimal path flow solution becomes: $x_{p_1}^* = 2.77$, and the corresponding optimal link flow pattern: $f_a^* = f_b^* = f_c^* = 2.77$, and $f_d^* = f_e^* = f_f^* = 2.22$, with a projected demand of: $v_1^* = x_{p_1}^* \mu_{p_1} = 2.22$.

In fact, using equation (49), with λ_1^- substituted for 100, we can derive the optimal path flow $x_{p_1}^*$ as a function of λ_1^-, that is:

$$x_{p_1}^* = \frac{100(\lambda_1^- - 44)}{16\lambda_1^- + 2425}. \tag{55}$$

Therefore, an appropriate increase in the unit shortage penalty cost λ_1^- results in the organization processing larger volumes of the blood product, even exceeding the optimal flow in Example 1, which makes sense intuitively. Furthermore. for $\lambda_1^- \leq 44$, the organization acquires and, hence, processes and distributes, zero units of the blood product.

Sensitivity Analysis

We conducted additional sensitivity analysis by varying the loss associated with the testing and processing activity, α_c, and the unit shortage penalty cost, λ_1^-, respectively. The computed optimal path flows $x_{p_1}^*$ and the optimal values of the objective function (OF) (18) are reported in Table 1.

It is interesting to note from Table 1 that, under a specific unit penalty cost, the path flow will be reduced when the loss of testing and processing, $(1 - \alpha_c)$, increases within some specific range. For instance, when λ_1^- is 200 and the loss, $1-\alpha_c$, increases from 0.4 to 0.8, the optimal path flow decreases from 2.83 to 0.88. However, the optimal value of the objective function always keeps on increasing. Table 1 also illustrates, under the same loss associated with testing and processing, the optimal path flow rises with an increase in the unit shortage penalty cost, λ_1^-.

Furthermore, the optimal path flow in Example 2, assuming a nonnegative value, as a function of the link multiplier and the shortage penalty *simultaneously* can be expressed as follows:

$$x_{p_1}^* = \frac{5\alpha_c(\lambda_1^- - 14) - 120}{\alpha_c^2(\lambda_1^- + 50) + 65}. \tag{56}$$

Table 1: Computed Optimal Path Flows $x^*_{p_1}$ and Optimal Values of the Objective Function as α_c and λ_1^- Vary

λ_1^- \ α_c		.2	.4	.6	.8	1
100	$x^*_{p_1}$	0.00	0.58	1.16	1.39	1.48
	OF	250.00	246.96	234.00	218.83	204.24
200	$x^*_{p_1}$	0.88	2.40	2.83	2.77	2.61
	OF	494.19	439.52	376.23	326.94	288.35
300	$x^*_{p_1}$	2.10	3.74	3.86	3.54	3.20
	OF	715.12	581.15	464.85	387.17	331.44
400	$x^*_{p_1}$	3.20	4.76	4.57	4.03	3.55
	OF	914.75	689.71	525.36	425.56	357.63
500	$x^*_{p_1}$	4.20	5.57	5.09	4.37	3.79
	OF	1096.03	775.55	569.30	452.16	375.23
1000	$x^*_{p_1}$	8.09	7.95	6.41	5.19	4.33
	OF	1799.11	1027.94	681.89	515.88	415.67
2000	$x^*_{p_1}$	12.69	9.80	7.27	5.68	4.65
	OF	2631.32	1224.45	755.64	554.47	439.05
3000	$x^*_{p_1}$	15.33	10.58	7.60	5.86	4.76
	OF	3107.51	1307.25	783.73	568.57	447.39
4000	$x^*_{p_1}$	17.03	11.01	7.77	5.96	4.82
	OF	3415.88	1352.89	798.54	575.88	451.68

3. The Algorithm and an Additional Numerical Example

In this Section, we recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Its realization for the solution of the blood bank supply chain management problem governed by variational inequality (38) (see also (41)) induces subproblems that can be solved explicitly and in closed form.

Specifically, at an iteration τ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \tag{57}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (41).

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \rightarrow 0$, as $\tau \rightarrow \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996) and the references therein. Applications of this Euler method to the solution of oligopolistic supply chain network design problems can be found in Nagurney (2010a).

Explicit Formulae for the Euler Method Applied to the Blood Supply Chain Network Variational Inequality (38)

The elegance of this procedure for the computation of solutions to the blood supply chain network operations management problem modeled in Section 2 can be seen in the following explicit formulae. In particular, (57) for the blood supply chain network management problem governed by variational inequality problem (38) yields the following closed form expressions for the blood product path flows:

$$x_p^{\tau+1} = \max\{0, x_p^{\tau} + a_{\tau}(\lambda_k^- \mu_p (1 - P_k(\sum_{p \in \mathcal{P}_k} x_p^{\tau} \mu_p)) - \lambda_k^+ \mu_p P_k(\sum_{p \in \mathcal{P}_k} x_p^{\tau} \mu_p)$$

$$- \frac{\partial \sum_{q \in \mathcal{P}} \hat{C}_q(x^{\tau})}{\partial x_p} - \frac{\partial \sum_{q \in \mathcal{P}} \hat{Z}_q(x^{\tau})}{\partial x_p} - \theta \frac{\partial \sum_{q \in \mathcal{P}} \hat{R}_q(x^{\tau})}{\partial x_p})\}, \forall p \in \mathcal{P}_{w_k}; k = 1, \ldots, n_R. \tag{58}$$

This closed form expression was applied to calculate the updated product flow during the steps of the Euler Method for our blood banking optimization problem.

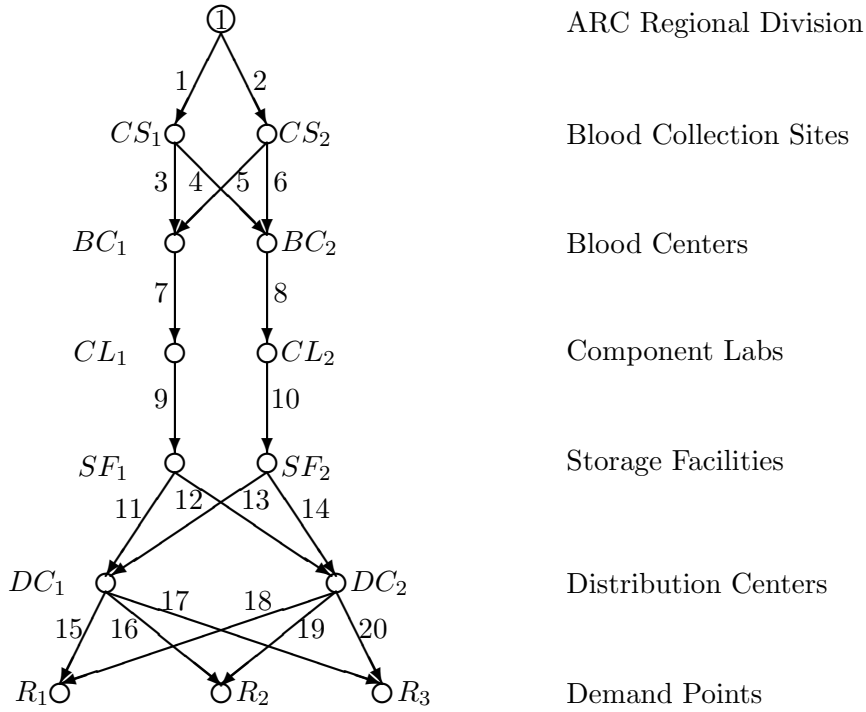

Figure 3: Supply Chain Network Topology for Numerical Example 3

Example 3

We now apply the Euler method to compute solutions to a larger-scale numerical blood supply chain network problem. The numerical example consisted of two blood collection sites, two blood centers, two component labs, two storage facilities, two distribution centers, and three demand points, as depicted in Figure 3.

We assumed that R_1 was a small surgical center while R_2 and R_3 were large hospitals with higher demand for red blood cells. The demands at these demand points followed uniform probability distribution on the intervals [5,10], [40,50], and [25,40], respectively. Hence,

$$P_1\big(\sum_{p\in\mathcal{P}_1}\mu_p x_p\big) = \frac{\sum_{p\in\mathcal{P}_1}\mu_p x_p - 5}{5},\ P_2\big(\sum_{p\in\mathcal{P}_2}\mu_p x_p\big) = \frac{\sum_{p\in\mathcal{P}_2}\mu_p x_p - 40}{10},$$

$$P_3\big(\sum_{p\in\mathcal{P}_3}\mu_p x_p\big) = \frac{\sum_{p\in\mathcal{P}_3}\mu_p x_p - 25}{15},$$

where The first O/D pair of nodes is $(1, R_1)$, the second is $(1, R_2)$, and the third is $(1, R_3)$.

The shortage and outdating penalties for each of the three demand points were:

$$\lambda_1^- = 2200, \quad \lambda_1^+ = 50,$$

$$\lambda_2^- = 3000, \quad \lambda_2^+ = 60,$$
$$\lambda_3^- = 3000, \quad \lambda_3^+ = 50.$$

The total risk functions corresponding to the blood collection links were:

$$\hat{r}_1(f_1) = 2f_1^2, \quad \text{and} \quad \hat{r}_2(f_2) = 1.5f_2^2,$$

and the risk aversion factor, θ, was 0.7.

The multipliers corresponding to the links, the total cost functions, and the total discarding cost functions were as reported in Table 2. These numbers have been selected based on the average historical data for the American Red Cross Northeast Division Blood Services (Rios (2010)).

The Euler method (cf. (58)) for the solution of variational inequality (38) was implemented in Matlab. A Microsoft Windows System with a Dell PC at the University of Massachusetts Amherst was used for all the computations. We set the sequence $a_\tau = .1(1, \frac{1}{2}, \frac{1}{2}, \cdots)$, and the convergence tolerance was $\epsilon = 10^{-6}$, that is, the absolute value of the difference between each path flow in two successive iterations was less than or equal to this ϵ. The algorithm was initialized by setting the projected demand at each demand point and all other variables equal to zero. Table 2 also provides the computed optimal solutions.

Thus, under the given demand probability distributions for our three demand points, the amounts of optimal product flow on each link were as reported above. These numbers clearly demonstrate the effect of the waste throughout the network, and are subject to the mentioned link multipliers as well as the various costs associated with those links, as formerly stated.

The computed amounts of projected demand for each of the three demand points were:

$$v_1^* = 6.06, \quad v_2^* = 44.05, \quad \text{and} \quad v_3^* = 30.99.$$

It is interesting to note that, between the two blood collection links, although link 1 has a higher waste/loss rate, and higher total risk and discarding costs, it has a higher optimal flow of blood product as compared to link 2 due to its lower total cost function. Furthermore, for the small surgical center, R_1, the value of projected demand, $v_1^* = 6.06$, is closer to the lower bound of its uniform probability distribution due to the relatively smaller shortage penalty cost. In contrast, the values of projected demand for the larger hospitals, R_2 and R_3, are closer to the respective upper bounds of their uniform distributions.

Table 2: Total Cost and Total Discarding Cost Functions and Solution for Numerical Example 3

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.97	$6f_1^2 + 15f_1$	$.8f_1^2$	54.72
2	.99	$9f_2^2 + 11f_2$	$.7f_2^2$	43.90
3	1.00	$.7f_3^2 + f_3$	$.6f_3^2$	30.13
4	.99	$1.2f_4^2 + f_4$	$.8f_4^2$	22.42
5	1.00	$f_5^2 + 3f_5$	$.6f_5^2$	19.57
6	1.00	$.8f_6^2 + 2f_6$	$.8f_6^2$	23.46
7	.92	$2.5f_7^2 + 2f_7$	$.5f_7^2$	49.39
8	.96	$3f_8^2 + 5f_8$	$.8f_8^2$	42.00
9	.98	$.8f_9^2 + 6f_9$	$.4f_9^2$	43.63
10	1.00	$.5f_{10}^2 + 3f_{10}$	$.7f_{10}^2$	39.51
11	1.00	$.3f_{11}^2 + f_{11}$	$.3f_{11}^2$	29.68
12	1.00	$.5f_{12}^2 + 2f_{12}$	$.4f_{12}^2$	13.08
13	1.00	$.4f_{13}^2 + 2f_{13}$	$.3f_{13}^2$	26.20
14	1.00	$.6f_{14}^2 + f_{14}$	$.4f_{14}^2$	13.31
15	1.00	$1.3f_{15}^2 + 3f_{15}$	$.7f_{15}^2$	5.78
16	1.00	$.8f_{16}^2 + 2f_{16}$	$.4f_{16}^2$	25.78
17	.98	$.5f_{17}^2 + 3f_{17}$	$.5f_{17}^2$	24.32
18	1.00	$.7f_{18}^2 + 2f_{18}$	$.7f_{18}^2$	.29
19	1.00	$.6f_{19}^2 + 4f_{19}$	$.4f_{19}^2$	18.28
20	.98	$1.1f_{20}^2 + 5f_{20}$	$.5f_{20}^2$	7.29

4. Summary and Conclusions

In this paper, we developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of a perishable product – that of human blood.

The original contributions in the paper, include the blood supply chain network model operations management model, which has the notable features that it captures perishability of this life-saving product through the use of arc multipliers; it contains discarding costs associated with waste/disposal; it handles uncertainty associated with demand points; it assesses costs associated with shortages/surpluses at the demand points, and it also quantifies the supply-side risk associated with procurement.

For the sake of generality, and the establishment of the foundations that will enable further extensions and applications, we used a variational inequality approach for model formulation and solution. We illustrated the model through transparent numerical examples, which

vividly demonstrate the flexibility and generality of our supply chain network optimization model.

The framework developed here can be applied, with appropriate adaptation, to other perishable products, such as medicines and vaccines, as well as to agricultural products, including food.

Acknowledgments

This research was supported by the John F. Smith Memorial Fund at the Isenberg School of Management. This support is gratefully acknowledged.

The authors acknowledge Mr. Len Walker, the Director of Business Development for the American Red Cross Blood Services in the greater Boston area, who shared valuable information on the subject. They also thank Dr. Jorge Rios, the Medical Director for the American Red Cross Northeast Division Blood Services, who graciously agreed to be interviewed, and who provided enlightening thoughts on the model.

The first author also acknowledges Professor Georg Pflug of the University of Vienna, the organizer of the International Conference on Computational Management Science that took place in Vienna, Austria, July 28-30, 2010 for many interesting conversations and lectures.

Comments and suggestions made by reviewers of earlier versions of this paper were most appreciated.

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