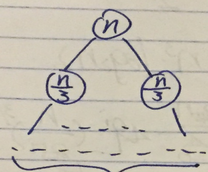
Yunfan Li

CS325-001

Practice 4

2.

a.



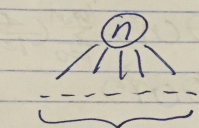
2^i problems : each 1

$$\sum_{i=0}^{\log_3 n} 2^i = 2^0 + 2^1 + \cdots 2^{\log_3 n}$$

$$= \Theta(2^{\log_3 n})$$

$$= \Theta(n^{\log_3 2})$$

b.

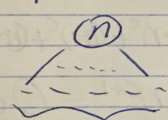


5^i problems: each $n/4^i$

$$\sum_{i=0}^{\log_4 n} (5/4)^i n = n \cdot \sum_{i=0}^{\log_4 n} (5/4)^i$$

$$= n \cdot \Theta((\tfrac{5}{4})^{\log_4 n}) = \Theta(n^{\log_4 5})$$

c.

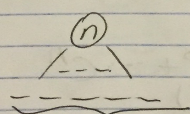


7^i problems: each $n/7^i$

$$\sum_{i=0}^{\log_7 n} (7^i \cdot \frac{n}{7^i}) = \Theta(n) \cdot \sum_{i=0}^{\log_7 n} (1)$$

$$= \Theta(n \cdot \log_7 n)$$

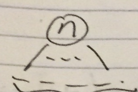
d.



9^i problems : each $(\frac{n}{3^i})^2$

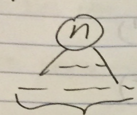
$$\sum_{i=0}^{\log_3 n} (9^i \cdot (\frac{n}{3^i})^2) = \Theta(n^2) \cdot \sum_{i=0}^{\log_3 n} (1)$$

$$= \Theta(n^2 \cdot \log_3 n)$$

e. 

8^i problems: each $(\frac{n}{2^i})^3$

$$\sum_{i=0}^{\log_2 n}(8^i\cdot(\frac{n}{2^i})^3) = n^3\cdot\sum_{i=0}^{\log_2 n}(1)$$
$$= \Theta(n^3\cdot\log_2 n).$$

f. 

49^i problems: each $(\frac{n}{25^i})^{3/2}\cdot\log\frac{n}{25^i}$

$$\sum_{i=0}^{\log_{25} n} 49^i\cdot(\frac{n}{25^i})^{3/2}\cdot\log\frac{n}{25^i}$$
$$= \Theta(n^{3/2})\sum_{i=0}^{\log_{25} n}\cdot(\frac{n}{25^i})^{3/2}\cdot\log\frac{n}{25^i}\cdot 49^i$$
$$= \Theta(n^{3/2})\cdot\sum_{i=0}^{\log_{25} n}(\frac{49}{125})^i\cdot\log\frac{n}{25^i}$$
$$= \Theta(n^{3/2}).$$

g. $T(n) = T(n-1)+2 = T(n-2)+4 = \cdots = 2(n-1)\cdot\Theta(1) = \Theta(n)$

h. $T(n) = T(n-1)+n^c = T(n-2)+(n-1)^c = \cdots = n^c+(n-1)^c+(n-2)^c+\cdots+1^c$

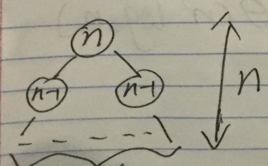
$n^c+(n-1)^c+\cdots 1^c < n^c+n^c+\cdots n^c = n^{c+1} = O(n^{c+1})$

$n^c+(n-1)^c+\cdots 1^c > (\frac{n}{2})^c+\cdots+n^c > (\frac{n}{2})^c+\cdots(\frac{n}{2})^c = (\frac{n}{2})^{c+1} = \Omega(n^{c+1})$

$\therefore \Theta = \Theta(n^{c+1})$

i. $T(n) = T(n-1)+c^n = \cdots = c+c^2+\cdots c^n$

$\because c>1 \therefore \Theta(T(n)) = \Theta(c^n)$

j. 

2^i: each 1

$$\sum_{i=0}^{n} 2^i\Theta(1) = \Theta(2^n).$$

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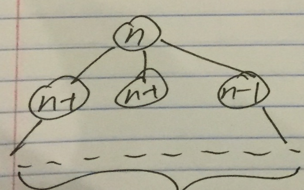
3.

a. It wouldn't. Because if $n=4$, $\lceil 2n/3 \rceil = 3$, $\lfloor 2n/3 \rfloor = 2$;
If we use $\lceil 2n/3 \rceil$, it will sort $A[0 \cdots 2]$ and $A[1 \cdots 3]$
then recursively do this, and there is a overlap part
can ensure it is correct.
If we use $\lfloor 2n/3 \rfloor$, we won't have overlap part so it
become two individually parts, it will wrong.

b. Numbers of recurrence $= \log_2 n$.

c. $T(n) = 3T(n-1) + 1$

$$\sum_{i=0}^{n} 3^i \Theta(1) = \Theta(3^n).$$



3^i problems: each 1