Yuntan Li (Voldy)
CS325-001

Practice Assignment 1

1. $\because c(\log n+1)^3 \approx c(\log n)^3$ and $1000c(\log n)^3 \approx c(\log n)^3$
$\therefore c(\log n+1)^3 \approx 1000c(\log n)^3$

$\because 7^{2n} = 49^n$ & $2^{7n} = 128^n$ $\quad \therefore 2^{7n} > 7^{2n}$

$\because$ According to log table we know that $\log_3 7 = 1.77$
& $1/2 = 0.5$
$\therefore n^{1/2} < n^{\log_3 7}$

$\because 2^{\log_2 n} = n \quad \therefore n\cdot\log n > n \quad \therefore n\cdot\log n > 2^{\log_2 n}$

We assume that $2^{\log_2 n} < 5^{\log_3 n}$
So if we give $\log_3$ to ~~eac~~ every side of equiation above
$\Rightarrow \log_3(2^{\log_3 n}) < \log_3(5^{\log_3 n})$
$(\because 2^{\log_2 n} = n)$
$\Rightarrow \log_3 n < \log_3 n \cdot \log_3 5$

In ~~alt~~ algorithm, the constant can be ignored.
$\therefore 2^{\log_2 n} \approx 5^{\log_3 n}$

So according to ~~any~~ analysis from above we know that:

① $1000c(\log n)^3 \approx c(\log n+1)^3$

② $2^{7n} > 7^{2n}$

③ $n^{1/2} < n^{\log_3 7}$

④ $n\cdot\log n > 2^{\log_2 n} \approx 5^{\log_3 n}$

Yunfan Li (Voldy)
CS325-001

if we choose $n \cdot \log n$ and $n^{\log_3 7}$

we assume that $n \cdot \log n > n^{\log_3 7}$

and we add log to each side of equations above :

$$\log(n \cdot \log n) > \log(n^{\log_3 7})$$

$$\Rightarrow \log(n \cdot \log n) > \log_3 7 \cdot \log n$$

and it is true that $n \cdot \log n > n$

$\therefore n \cdot \log n > n^{\log_3 7}$

$\therefore n \cdot \log n > n^{\log_3 7} > 2^{\log_2 n} \approx 5^{\log_3 n} > n^{1/2}$

if we choose $n^{1/2}$ and $1000(\log n)^3$.

we assume $n^{1/2} > 1000(\log n)^3$, and add log to each side:

$$\frac{1}{2}\log n > \log(1000(\log n)^3) \approx \log((\log n)^3)$$

$$\Rightarrow \frac{1}{2}\log n > 3\log(\log n)$$

It is true that $n > \log n$ $\therefore n^{1/2} > 1000(\log n)^3$

$\therefore n \cdot \log n > n^{\log_3 7} > 2^{\log_2 n} \approx 5^{\log_3 n} > n^{1/2} > 1000(\log n)^3 \approx (\log n + 1)^3$

$\because 2^n > n \cdot \log n$ $\therefore \cancel{7}2^{7n} > 7^{2n} > 2^n > n \cdot \log n$

$\therefore$ The Result is :

$(\log n + 1)^3 \approx 1000(\log n)^3 < n^{1/2} < 5^{\log_3 n} \approx 2^{\log_2 n} < n^{\log_3 7} < n \cdot \log n < 7^{2n} < 2^{7n}$

Yuntan Li (Voldy)
CS325-001

2.

(a) $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \lim_{n\to\infty}\frac{3n+6}{10000n-500} \approx \lim_{n\to\infty}\frac{3n}{10000n} = \frac{3}{10000}$ is a constant

$\therefore f = \Theta(g)$

(b) $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \lim_{n\to\infty}\frac{n^{1/2}}{n^{2/3}} = \lim_{n\to\infty} n^{-1/6} = \lim_{n\to\infty}\frac{1}{n^{6}} = 0$

$\therefore f = O(g)$

(c) $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \lim_{n\to\infty}\frac{\log(7n)}{\log(n)} = \lim_{n\to\infty}\frac{\log 7+\log n}{\log n} = \lim_{n\to\infty}(1+\frac{\log 7}{\log n}) = 1$ is a constant

$\therefore f = \Theta(g)$

(d) $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \lim_{n\to\infty}\frac{n^{3/2}}{n\cdot\log n} = \lim_{n\to\infty}\frac{n^{1/2}}{\log n} = \lim_{n\to\infty}\frac{(n^{1/2})'}{(\log n)'} = \lim_{n\to\infty}\frac{\frac{1}{2}n^{-\frac{1}{2}}}{\frac{1}{n\ln 10}} \Rightarrow$

$= \lim_{n\to\infty}\frac{1}{2}n^{1/2}\cdot\ln 10 = \lim_{n\to\infty}\frac{\ln 10}{2}\cdot\sqrt{n} \to \infty$

$\therefore f = \Omega(g)$

(e) $\lim_{n\to\infty}\frac{f(n)}{g(n)} = \lim_{n\to\infty}\frac{n^{1/2}}{(\log n)^3} = \lim_{n\to\infty}\frac{(n^{1/2})'}{((\log n)^3)'} = \lim_{n\to\infty}\frac{\frac{1}{2}n^{-\frac{1}{2}}}{3(\log n)^2\cdot\frac{1}{n\cdot\ln 10}} = \lim_{n\to\infty}\frac{\ln 10}{6}\cdot\frac{n^{1/2}}{(\log n)^2} \Rightarrow$

$= \lim_{n\to\infty}\frac{\ln 10}{6}\cdot\frac{(n^{1/2})'}{((\log n)^2)'} = \lim_{n\to\infty}\frac{\ln 10}{6}\cdot\frac{\ln 10}{4}\cdot\frac{n^{1/2}}{\log n} \approx \lim_{n\to\infty}\frac{(n^{1/2})'}{(\log n)'} = \lim_{n\to\infty}\frac{\ln 10}{2}\sqrt{n} \Rightarrow$

$\therefore f = \Omega(g)$.

(f) $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n 2^n}{3^n} = \lim_{n\to\infty} n \cdot (\frac{2}{3})^n \to \infty$

$\because$ ~~f~~ $f = \Omega(g)$

3. $\because g(n) = \begin{cases} \frac{1-c^{n+1}}{1-c} & c \neq 1 \\ (n+1) & c=1 \end{cases}$

(a) assume $f(n) = 1$

$\therefore \lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{\frac{1-c^{n+1}}{1-c}}{1} = \lim_{n\to\infty} \frac{1-c^{n+1}}{1-c}$

$\because 0<c<1$ $\therefore$ when $n\to\infty$ we know that $c^{n+1} \to 0$

$\therefore \lim_{n\to\infty} \frac{g(n)}{f(n)} \to \frac{1}{1-c}$ is a constant

$\therefore$ ~~f~~ $\theta(g(n)) = \theta(1)$

(b) assume $f(n) = n$

$\therefore \lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} (1+\frac{1}{n}) \to 1$ is a constant

$\therefore \theta(g(n)) = \theta(n)$

(c) assume $f(n) = c^n$

$\therefore \lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{\frac{1-c^{n+1}}{1-c}}{c^n} = \lim_{n\to\infty} \frac{1-c^{n+1}}{c^n(1-c)} = \lim_{n\to\infty} \frac{c^{n+1}-1}{c^n(c-1)} \approx \lim_{n\to\infty} \frac{c^{n+1}}{c^n(c-1)}$

$\to \frac{c}{c-1}$ is a positive constant

$\therefore \theta(g(n)) = \theta(c^n)$

Yunfan Li (Voldy)
CS325-001

4. We assume that $f(n) = \log(n!)$ and $g(n) = n \cdot \log n$

$\therefore f(n) = \frac{\ln(n!)}{\ln 10}$ and according to Stirling's approximation

we know that $\ln(n!) = n\ln(n) - n + O(\ln(n)) \approx n\ln(n) - n$

$\therefore f(n) \approx \frac{n \cdot \ln n - n}{\ln 10}$

$\therefore \lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n \cdot \ln n - n}{\ln 10 \cdot n \cdot \log n} = \lim_{n\to\infty} \frac{n \cdot \ln n - n}{\ln 10 \cdot n \cdot \frac{\ln n}{\ln 10}} = \lim_{n\to\infty} \frac{n \cdot \ln n - n}{n \cdot \ln n}$

$= \lim_{n\to\infty} (1 - \frac{1}{\ln n}) \to 1$ is a constant

$\therefore \log(n!) = \Theta(n \cdot \log n)$.