NCERT-9.3.11

EE24BTECH11023 - RASAGNA

Question: Find the solution of the following differential equation:

$$(x+y)\frac{dy}{dx} = 1$$

Solution: From the question,

$$\frac{dy}{dx} = \frac{1}{x+y} \tag{0.1}$$

let,

$$x + y = v \tag{0.2}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1 \tag{0.3}$$

Substituting into the equation (0.1), we get:

$$\frac{dy}{dx} = \frac{1}{v} - 1 \tag{0.4}$$

$$\int (1 - \frac{1}{1+v})dv = \int dx \tag{0.5}$$

$$v - \ln|x+y| = x + c \tag{0.6}$$

putting v=x+y,

$$y - \ln|1 + x + y| = c \tag{0.7}$$

let us assume the initial condition to be $y = 0$ and $x = 1$
substituting in equation (0.7),we get the value of c,

$$\therefore c = -ln2 \tag{0.8}$$

the final required equation is,

$$x = 2e^{y} - (1+y) \tag{0.9}$$

Logic for writing the code(method of finite difference)
The slope of a tangent at a point (x_0, y_0) on the curve is:

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} \tag{0.10}$$

By moving infinitesimally small distance(h) along the tangent , we get another point (x_1, y_1) .the value of (x_1, y_1)

$$x_1 = x_0 + h \tag{0.11}$$

$$y_1 = y_0 + \frac{dy}{dx}h \tag{0.12}$$

Here,

$$\frac{dy}{dx} = \frac{1}{x_0 + y_0} \tag{0.13}$$

On substituting the value of $\frac{dy}{dx}$ in equation (0.15) we get,

$$y_1 = y_0 + \frac{1}{x_0 + y_0}h \tag{0.14}$$

Similarly we can obtain n number of points where

$$x_n = x_{n-1} + h \tag{0.15}$$

$$y_n = y_{n-1} + \frac{1}{x_{n-1} + y_{n-1}}h \tag{0.16}$$

Together these points form the curve representing one of the general solutions of the given Differential Equation. The plot is generated by choosing a known point (x_0, y_0) which satisfies the equation.The value of h is taken to be very small.We generate a large number of points and then plot them.