NCERT-10.4.4.1.2

EE24BTECH11023 - RASAGNA

Question: Find the nature of the roots of the following quadratic equation. If the real roots exist,find them:

$$3x^2 - 4\sqrt{3}x + 4 = 0$$

Theoritical Solution: The standard quadratic equation is:

$$ax^2 + bx + c = 0 \tag{0.1}$$

Here, $a = 3, b = -4\sqrt{3}, c = 4$. Computing the value of determinant,

$$\triangle = b^2 - 4ac \tag{0.2}$$

$$\triangle = (-4\sqrt{3})^2 - 4(3)(4) \tag{0.3}$$

$$\therefore \triangle = 0 \tag{0.4}$$

$\therefore$ The given equation has two equal and real roots.
The roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{0.5}$$

Substitute the values of a, b, and c:

$$x = \frac{-(-4\sqrt{3}) \pm \sqrt{(-4\sqrt{3})^2 - 4(3)(4)}}{2(3)} \tag{0.6}$$

$$= \frac{2}{\sqrt{3}} \tag{0.7}$$

Thus, the roots of the equation are:

$$x_1 = x_2 = \frac{2}{\sqrt{3}} \tag{0.8}$$

Computational Solution
We use fixed point iteration method to find the roots,
From the equation we can reformulate it as

$$g(x) = \frac{\sqrt{4\sqrt{3}x - 4}}{\sqrt{3}} \tag{0.9}$$

Now let us assume the initial guess of x as $x_0 = 2$ Use the iterative formula,

$$x_{n+1} = g(x_n) \tag{0.10}$$

Repeat the iterations until the sequence x_{n+1} converges to a fixed point. Convergence is often checked using A predefined tolerance ϵ where $|x_{n+1} - x_n| < \epsilon$ or maximum number of iterations. The result obtained by computational approach is :
Root 1 = 1.154701 + 0.000000i
Root 2 = 1.154701 - 0.000000i