

# NCERT-10.3.3.3

EE24BTECH11023 - RASAGNA

## Question

Form the pair of linear equations for the following problems and find their solution by substitution method. The coach of a cricket team buys 7 bats and 6 balls for ₹3800. Later, she buys 3 bats and 5 balls for ₹1750. Find the cost of each bat and each ball

## Theoretical Solution

Let the cost of each bat be  $x$  (in rupees) and the cost of each ball be  $y$  (in rupees).

From the problem, we can form the following equations:

$$7x + 6y = 3800 \quad (0.1)$$

$$3x + 5y = 1750 \quad (0.2)$$

Solve one equation for one variable

From equation (0.2)

$$3x + 5y = 1750 \quad (0.3)$$

$$3x = 1750 - 5y \quad (0.4)$$

$$x = \frac{1750 - 5y}{3} \quad (0.5)$$

Substitute  $x$  into equation (0.1)

$$7x + 6y = 3800 \quad (0.6)$$

$$7\left(\frac{1750 - 5y}{3}\right) + 6y = 3800 \quad (0.7)$$

$$\frac{7(1750 - 5y)}{3} + 6y = 3800 \quad (0.8)$$

$$\frac{12250 - 35y}{3} + 6y = 3800 \quad (0.9)$$

Simplifying further,

$$12250 - 35y + 18y = 11400 \quad (0.10)$$

$$-17y = -850 \quad (0.11)$$

$$\therefore y = 50 \quad (0.12)$$

Now, Substitute  $y = 50$  into equation (0.5);

$$x = \frac{1750 - 5(50)}{3} \quad (0.13)$$

$$x = \frac{1500}{3} \quad (0.14)$$

$$\therefore x = 500 \quad (0.15)$$

## Computational Solution

### LU FACTORIZATION SOLUTION

We are solving the following system of linear equations:

$$7x + 6y = 3800 \quad (0.16)$$

$$3x + 5y = 1750 \quad (0.17)$$

In matrix form:

$$\begin{bmatrix} 7 & 6 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3800 \\ 1750 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 7 & 6 \\ 3 & 5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3800 \\ 1750 \end{bmatrix}$$

The LU decomposition expresses  $A$  as:

$$A = L \cdot U \quad (0.18)$$

Where,

$$L = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} \quad (0.19)$$

Compute  $L$  and  $U$

$$u_{11} = 7, \quad u_{12} = 6 \quad (0.20)$$

$$l_{21} = \frac{A[2, 1]}{u_{11}} = \frac{3}{7} \quad (0.21)$$

$$u_{22} = A[2, 2] - l_{21} \cdot u_{12} = 5 - \frac{3}{7} \cdot 6 = \frac{17}{7} \quad (0.22)$$

Thus:

$$L = \begin{bmatrix} 1 & 0 \\ \frac{3}{7} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 7 & 6 \\ 0 & \frac{17}{7} \end{bmatrix}$$

Solve  $L \cdot \mathbf{y} = \mathbf{b}$

$$\begin{bmatrix} 1 & 0 \\ \frac{3}{7} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3800 \\ 1750 \end{bmatrix} \quad (0.23)$$

Solving,

$$\Rightarrow y_1 = 3800 \quad (0.24)$$

$$\Rightarrow \frac{3}{7} \cdot 3800 + y_2 = 1750 \quad (0.25)$$

$$y_2 = 1750 - \frac{3}{7} \cdot 3800 \quad (0.26)$$

$$y_2 = 1750 - \frac{11400}{7} = \frac{850}{7} \quad (0.27)$$

Thus,

$$\mathbf{y} = \begin{bmatrix} 3800 \\ \frac{850}{7} \end{bmatrix} \quad (0.28)$$

Solve  $U \cdot \mathbf{x} = \mathbf{y}$

$$\begin{bmatrix} 7 & 6 \\ 0 & \frac{17}{7} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3800 \\ \frac{850}{7} \end{bmatrix} \quad (0.29)$$

Solving,

$$\Rightarrow \frac{17}{7} \cdot y = \frac{850}{7} \quad (0.30)$$

$$y = \frac{850}{7} \cdot \frac{7}{17} = 50 \quad (0.31)$$

$$\Rightarrow 7x + 6y = 3800 \quad (0.32)$$

$$7x + 6 \cdot 50 = 3800 \quad (0.33)$$

$$7x = 3800 - 300 = 3500 \quad (0.34)$$

$$x = \frac{3500}{7} = 500 \quad (0.35)$$

Final computed Solution

Cost of one bat (x): 500.00

Cost of one ball (y): 50.00

