NCERT-9.3.11

EE24BTECH11023 - RASAGNA

Question:
Find the solution of the following differential equation:

$$(x^3 + x^2 + x + 1)\frac{d}{dx}(y) = 2x^2 + x$$

Given that $x = 0$ when $y = 1$.

Solution: From the question, after simplification:

$$\frac{dy}{dx} = \frac{2x^2 + x}{(1 + x)(1 + x^2)}$$

Let,

$$\frac{2x^2 + x}{(1 + x)(1 + x^2)} = \frac{A}{1 + x} + \frac{Bx + C}{1 + x^2}$$

When $x = 0$:

$$A + C = 0 \quad \text{(eq 1)}$$

When $x = 1$:

$$A + B + C = \frac{3}{2} \quad \text{(eq 2)}$$

When $x = \frac{-1}{2}$:

$$5A - B + 2C = 0 \quad \text{(eq 3)}$$

Solving these equations,

$$A = \frac{1}{2}, \quad B = \frac{3}{2}, \quad C = -\frac{1}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2(1 + x)} + \frac{3x - 1}{2(1 + x^2)}$$

Integrating both sides:

$$y = \int \left(\frac{1}{2(1 + x)} + \frac{3x}{2(1 + x^2)} - \frac{1}{2(1 + x^2)} \right) dx$$

The solution is,

$$y = \frac{1}{2}\ln(1 + x) + \frac{3}{4}\ln(1 + x^2) - \frac{1}{2}\tan^{-1}(x) + C$$

Given $x = 0$ when $y = 1$, solve for C;

$$C = 1 - \left(\frac{1}{2}\ln(1 + 0) + \frac{3}{4}\ln(1 + 0^2) - \frac{1}{2}\tan^{-1}(0) \right)$$

$$\therefore C = 1$$

Thus, the final required equation is,

$$y = \frac{1}{2}\ln(1+x) + \frac{3}{4}\ln(1+x^2) - \frac{1}{2}\tan^{-1}(x) + 1$$

Logic for writing the code

The slope of a tangent at a point (x_0, y_0) on the curve is:

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x_0+h) - f(x_0)}{h}$$

By moving infinitesimally small distance(h) along the tangent , we get another point (x_1, y_1) .the value of (x_1, y_1)

$$x_1 = x_0 + h$$

$$y_1 = y_0 + h\frac{dy}{dx}$$

Here, $\left.\frac{dy}{dx}\right|_{(x_0,y_0)}$ is the slope at point (x_0, y_0). By substituting the values, we obtain (x_1, y_1). Similarly,

$$x_2 = x_1 + h$$

$$y_2 = y_1 + h\frac{dx}{dy}$$

here, $\left.\frac{dy}{dx}\right|_{(x_1,y_1)}$ is the slope at point (x_1, y_1). Similarly we can obtain n number of points where

$$x_n = x_{n-1} + h$$

and

$$y_n = y_{n-1} + h\left.\frac{dy}{dx}\right|_{(x_{n-1},y_{n-1})}$$

Together these points form the curve representing one of the general solutions of the given Differential Equation. The plot is generated by choosing a known point (x_0, y_0) which satisfies the equation.The value of h is taken to be very small.We generate a large number of points and then plot them.

PLOTTING THE GRAPH