

## Week 01

### 01] Neural networks

\* A neural network (NN) is a type of machine learning model inspired by how the human brain works. It's made up of layers of small computing units called **neurons**, which process data and learn complex relationships.

\* A typical neural network has 3 parts:

1) **Input layer** = takes in features (like price, day of week, weather)

2) **Hidden layers** = process the input through mathematical operations and learn patterns

3) **Output layer** = gives the final prediction (like predicted demand, price or classification)

\* Each neuron performs:

$$z = w_1x_1 + w_2x_2 + \dots + w_nx_n + b$$

and passes it through a **activation function** like (sigmoid) to introduce non-linearity

\* During the training, the model learns weights and biases that minimize prediction error.

Ex: Demand prediction

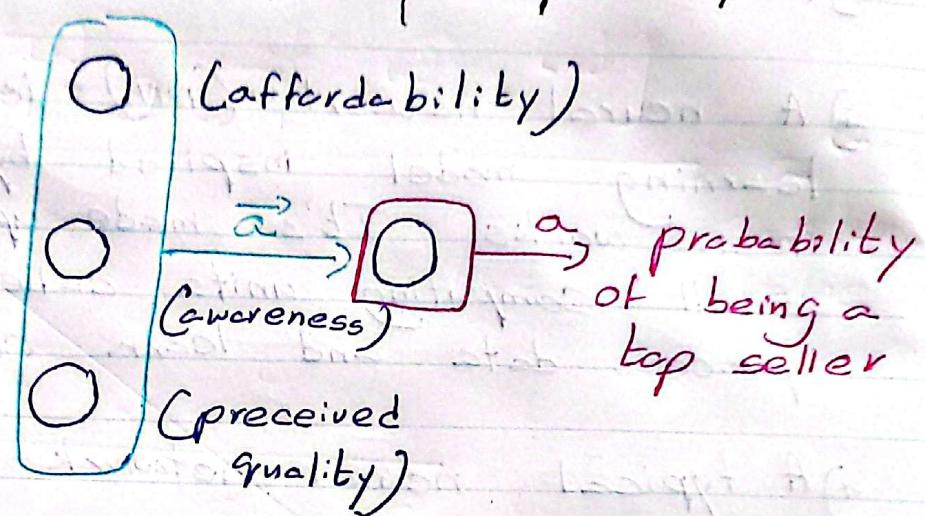
Input layer }      Hidden layer }      Output layer

price

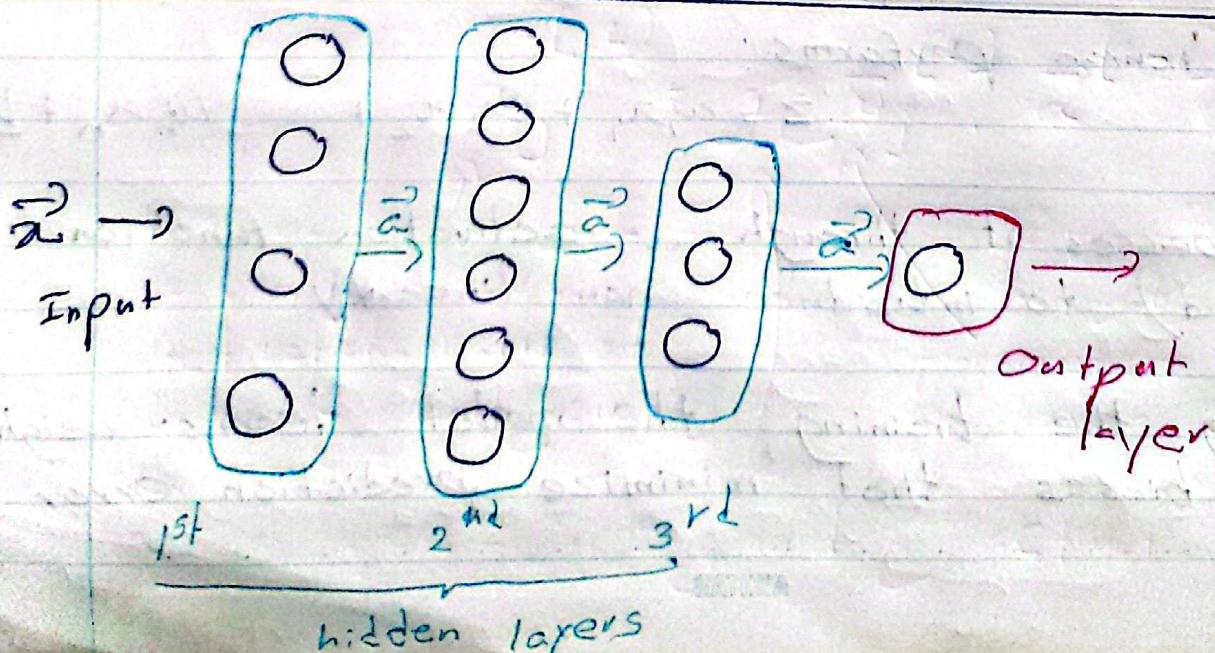
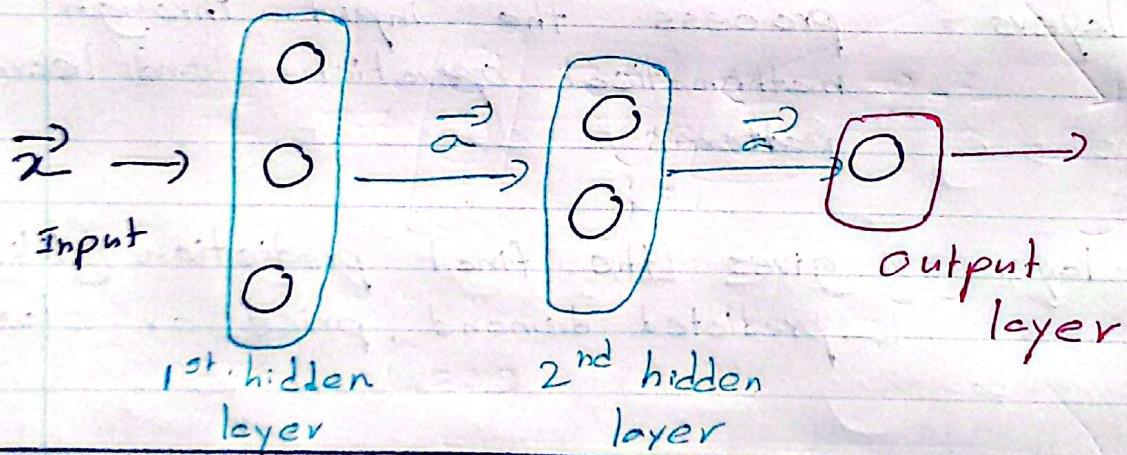
shipping cost

marketing

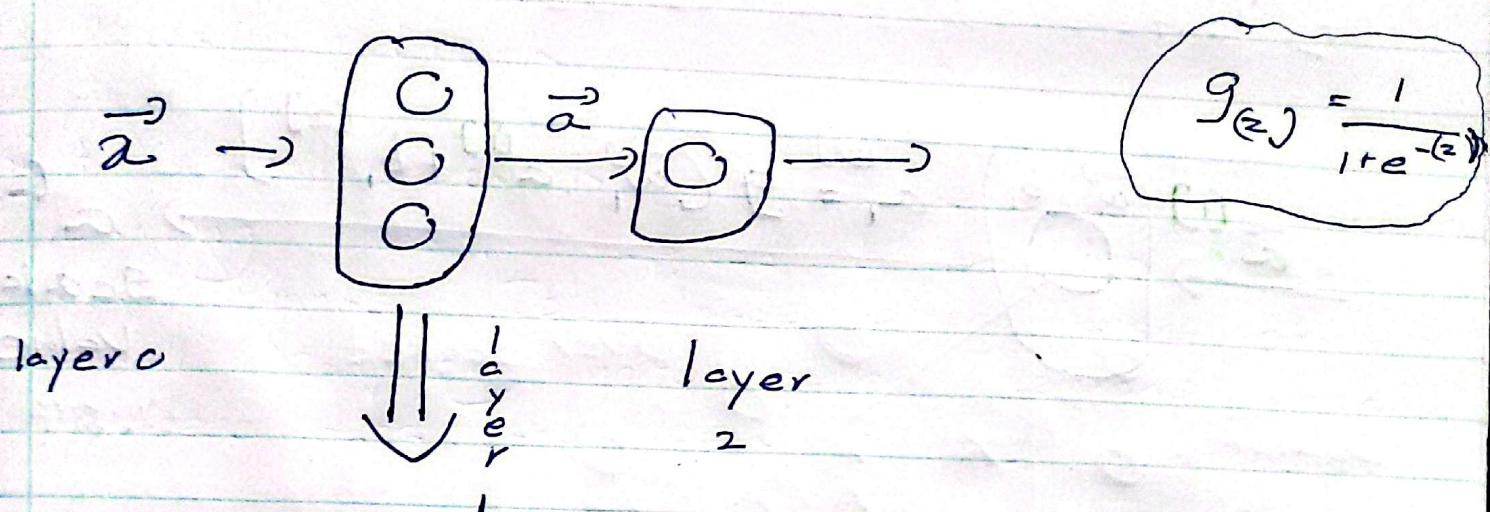
material



Multiple hidden layers



## 02) Neural Network layer



$$\vec{x} \rightarrow \begin{array}{c} \vec{\omega}_1^{(1)}, b_1^{(1)} \\ \vec{\omega}_2^{(1)}, b_2^{(1)} \\ \vec{\omega}_3^{(1)}, b_3^{(1)} \end{array} \quad a_1^{(1)} = g(\vec{\omega}_1^{(1)} \cdot \vec{x} + b_1^{(1)}) \text{ 0.3}$$

$$a_2^{(1)} = g(\vec{\omega}_2^{(1)} \cdot \vec{x} + b_2^{(1)}) \text{ 0.7}$$

$$a_3^{(1)} = g(\vec{\omega}_3^{(1)} \cdot \vec{x} + b_3^{(1)}) \text{ 0.2}$$

$(1), (2) \dots \Rightarrow$  notation for layer numbering

$1, 2, 3 \dots \Rightarrow$  notation for neuron numbering

layer 1  $\rightarrow$  output  $\vec{a}$

$\begin{cases} 0.3 \\ 0.7 \\ 0.2 \end{cases}$  Vector  
 of  
 activation  
 values from  
 layer 1

D) layer 2  $\Rightarrow$

$$a_1^{(2)} = g(\vec{w}_1 \cdot a^{(1)} + b_1^{(2)})$$

Scalar Value  
C. 8/4

D) prediction

(predict category 1 or 0/  
yes or no)

$$\rightarrow a^{(2)}$$

is  $a^{(2)} \geq 0.5$   
(threshold)

yes  $\hat{g} = 1$

No  $\hat{g} = 0$

General Case

$$a_j^{(l)} = g(\vec{w}_j \cdot a^{(l-1)} + b_j^{(l)})$$

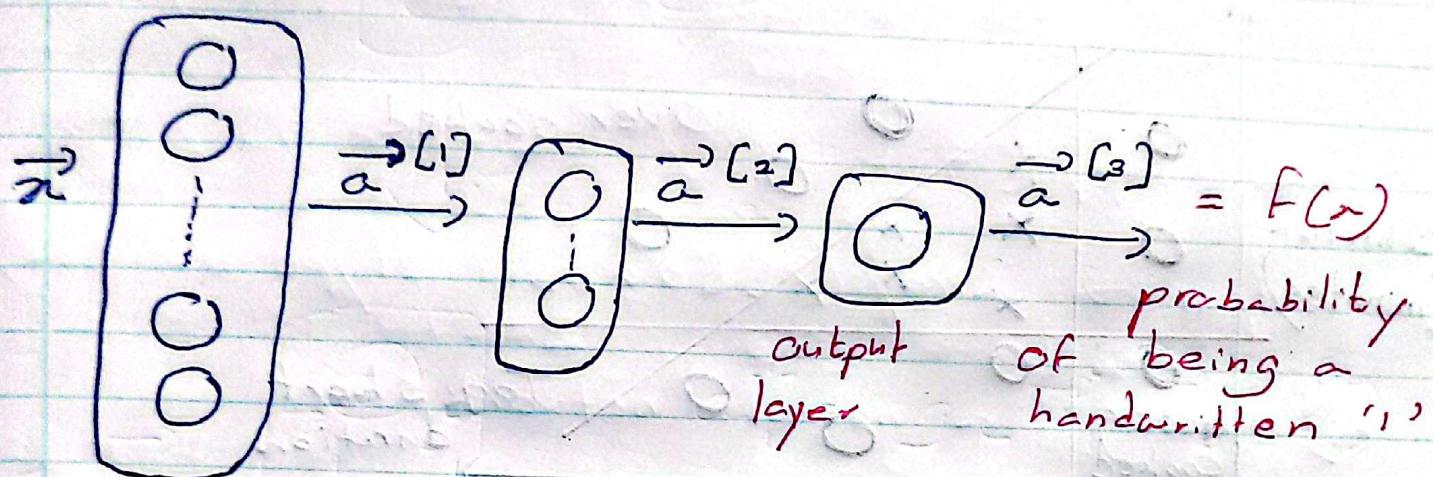
Activation value of  
layer  $l$ , unit (neuron)  $j$

parameters  $w$  and  $b$   
of layer  $l$ , unit  $j$

Sigmoid ("activation function")

### 03] Inference (Making predictions)

Forward propagation



25 units  
layer 1

15 units  
layer 2

1 unit  
layer 3

$$\vec{a}^{[3]} = g\left(\vec{w}_1^{[3]} \cdot \vec{a}^{[2]} + b_1^{[3]}\right)$$

is  $a^{[3]} \geq 0.5$ ?

yes

$$\hat{y} = 1$$

(Image is digit 1)

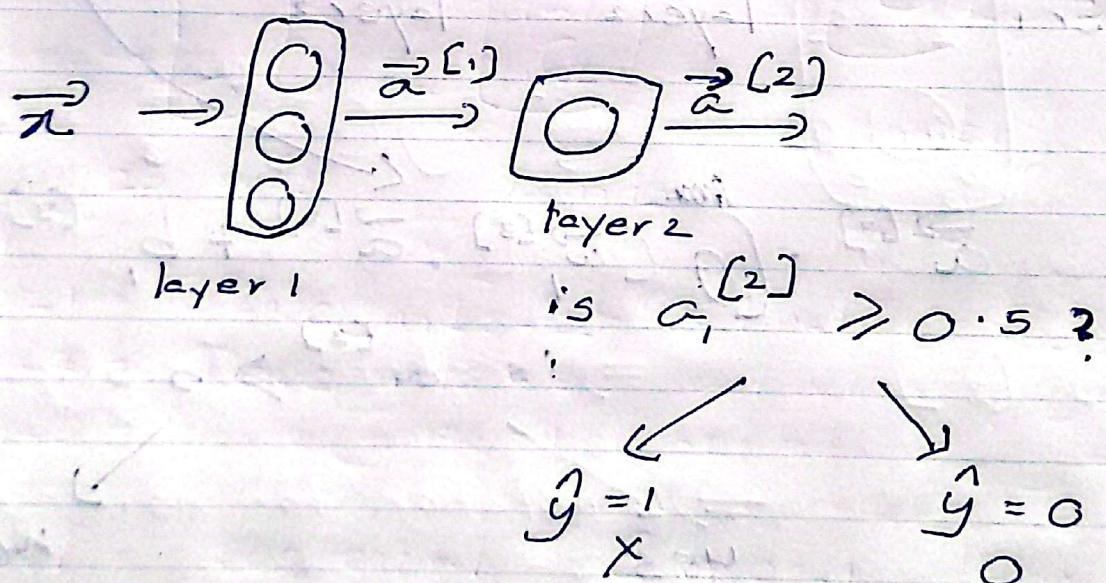
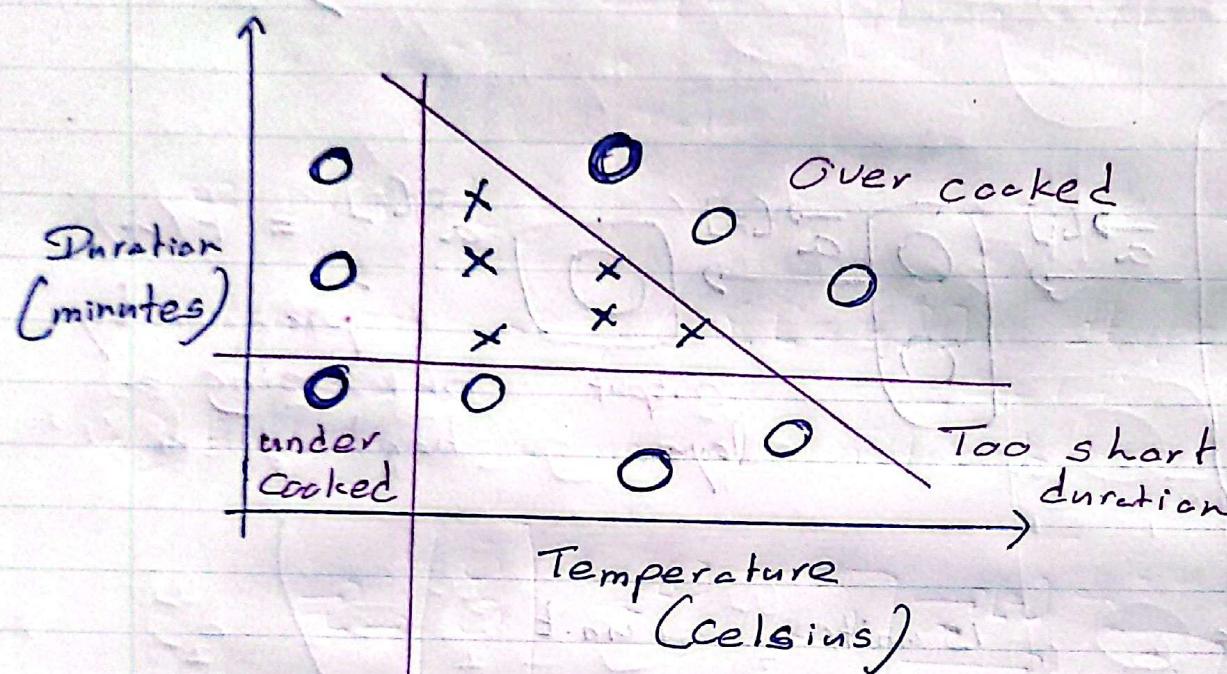
no

$$\hat{y} = 0$$

(Image isn't digit 1)

## 04] Inference in Code

Coffee roasting prediction



$x = \text{np.array}([[200.0, 17.0]])$

$\text{layer\_1} = \text{Dense}(\text{units} = 3, \text{activation} = \text{'sigmoid'})$

$\text{a1} = \text{layer\_1}(x)$

$\text{layer\_2} = \text{Dense}(\text{units} = 1, \text{activation} = \text{'sigmoid'})$

$\text{a2} = \text{layer\_2}(\text{a1})$

```

if  $a_2 > 0.5$  :
     $y_{\text{hat}} = 1$ 
else :
     $y_{\text{hat}} = 0$ 

```

## 05) Data in Tensorflow

Numpy arrays

$$x = \text{np.array}([200, 17]) \rightarrow [200, 17] \quad 1 \times 2$$

$$x = \text{np.array}([200], [17]) \rightarrow \begin{bmatrix} 200 \\ 17 \end{bmatrix} \quad 2 \times 1$$

"2D Arrays"

$$x = \text{np.array}([200, 17]) \quad \text{"1D vector"}$$

Activation vector

$$\begin{aligned}
 x &= \text{np.array}([200.0, 17.0]) \\
 \text{layer\_1} &= \text{Dense}(\text{units}=3, \text{activation}=\text{'Sigmoid'}) \\
 a_1 &= \text{layer\_1}(x)
 \end{aligned}$$

feature vector

$$\begin{aligned}
 \text{output} &\rightarrow \text{tf.Tensor}([0.2, 0.7, 0.3]), \text{shape}(1, 3) \\
 &\quad (\text{1x3 matrix}) \quad \text{dtype} = \text{float32}
 \end{aligned}$$

$a_1$ , numpy

$$\text{output} \rightarrow \text{array}([0.2, 0.7, 0.3]), \text{dtype} = \text{float32}$$

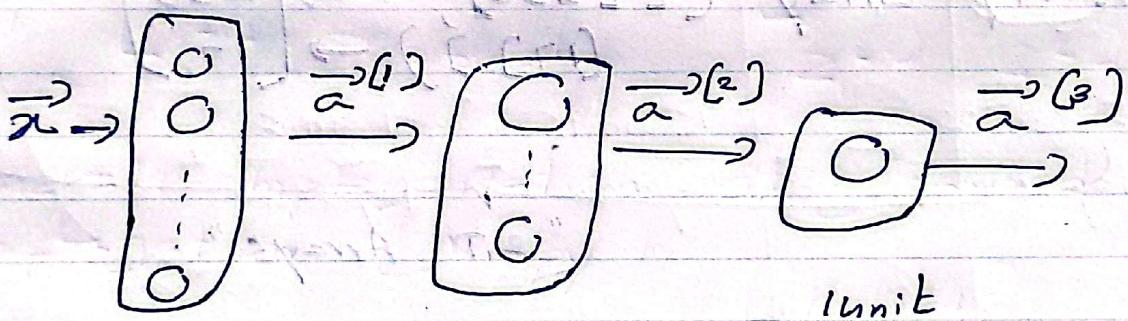
$\text{layer\_2} = \text{Dense}(\text{units}=1, \text{activation}=\text{'sigmoid'})$   
 $a_2 = \text{layer\_2}(a_1)$

Output  $\rightarrow$  tf. ~~is~~ Tensor([[[0.8]]], shape(1, 1),  
                   $\downarrow$   
                  1x1 matrix  
                  scalar  
                  dtype = float32)

$a_2.\text{numpy}()$

Output  $\rightarrow$  array([[[0.8]]], dtype=float32)

## Q6) Building a neural network



$\text{layer\_1} = \text{Dense}(\text{units}=25, \text{activation}=\text{'sigmoid'})$

$\text{layer\_2} = \text{Dense}(\text{units}=15, \text{activation}=\text{'sigmoid'})$

$\text{layer\_3} = \text{Dense}(\text{units}=1, \text{activation}=\text{'sigmoid'})$

$\text{model} = \text{sequential}([\text{layer\_1}, \text{layer\_2}, \text{layer\_3}])$

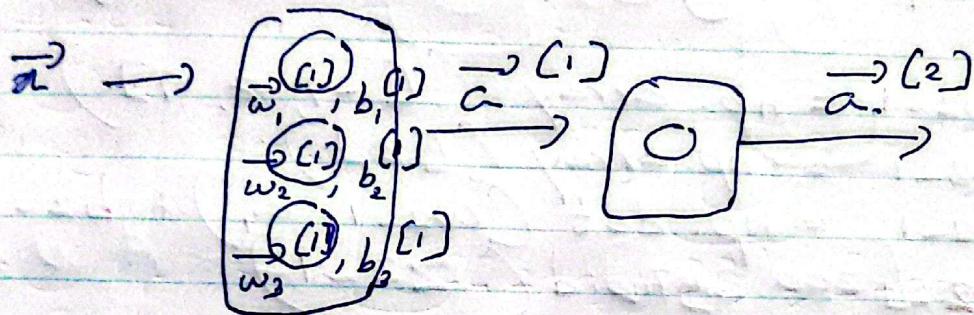
$\text{model}.\text{compile}(\dots)$

$x = \text{np.array}([[[0 \dots, 245, \dots, 17], [0 \dots, 200, \dots, 184]]])$

$y = \text{np.array}([1, 0])$

model. fit( $x, y$ )  
 model. predict( $x_{\text{new}}$ )

07] Forward propagation in a single layer



$$x = \text{np.array}([200, 17])$$

$$a_1^{[1]} = g(\vec{w}_1^{[1]} \cdot \vec{x} + b_1^{[1]})$$

$$w_{1-1} = \text{np.array}([1, 2])$$

$$b_{1-1} = \text{np.array}([-1])$$

$$z_{1-1} = \text{np.dot}(w_{1-1}, x) + b_{1-1}$$

$$a_{1-1} = \text{sigmoid}(z_{1-1})$$

$$a_2^{[1]} = g(\vec{w}_2^{[1]} \cdot \vec{x} + b_2^{[1]})$$

$$w_{1-2} = \text{np.array}([-3, 4])$$

$$b_{1-2} = \text{np.array}([1])$$

$$z_{1-2} = \text{np.dot}(w_{1-2}, x) + b_{1-2}$$

$$a_{1-2} = \text{sigmoid}(z_{1-2})$$

$$a_3^{[1]} = g(\vec{w}_3^{[1]} \cdot \vec{x} + b_3^{[1]})$$

$$a_{1-3} = \text{sigmoid}(z_{1-3})$$

$$a_1 = \text{np.array}([a_{1-1}, a_{1-2}, a_{1-3}])$$

Second layer (Output layer)

$$a_1^{[2]} = g(\vec{\omega}_1^{[2]} \cdot \vec{a}^{[1]} + b_1^{[2]})$$

$$\omega_{2-1} = \text{np.array}([-1, 8, 9])$$

$$b_{2-1} = \text{np.array}([3])$$

$$z_{2-1} = \text{np.dot}(\omega_{2-1}, a_1) + b_{2-1}$$

$$a_{2-1} = \text{Sigmoid}(z_{2-1})$$

$$\omega_1^{(2)} = \omega_{2-1}$$

Q8) General implementation of forward propagation (NumPy)

$$\vec{\omega}_1^{[1]} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \vec{\omega}_2^{[1]} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad \vec{\omega}_3^{[1]} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$$

$$\omega = \text{np.array}([[[1, -3, 5], [2, 4, -6]]])_{2 \times 3}$$

$$b_1^{[1]} = -1 \quad b_2^{[1]} = 1 \quad b_3^{[1]} = 2$$

$$b = \text{np.array}([-1, 1, 2])$$

$$a_{\text{in}} = \underbrace{\text{np.array}([-2, 4])}_{\vec{a} = \vec{a}^{[0]}}$$

```

def dense(a-in, w, b):
    (g)units = w.shape[1]
    a-out = np.zeros([units]) [0,0,0]
    for j in range(units): 0, 1, 2
        w = w[:,j]
        z = np.dot(w, a-in) + b[j]
        aout[j] = g(z)
    return a-out

```

```

def sequential(x):
    a1 = dense(x, w1, b1)
    a2 = dense(a1, w2, b2)
    a3 = dense(a2, w3, b3)
    a4 = dense(a3, w4, b4)
    f-x = a4
    return f-x

```

} 4 layers

Capital W refers to matrix

Instead of this for more efficient

all 2d arrays

```

def dense(A-in, w, B)
    Z = np.matmul(A-in, w) + B
    A-out = g(Z)
    return A-out

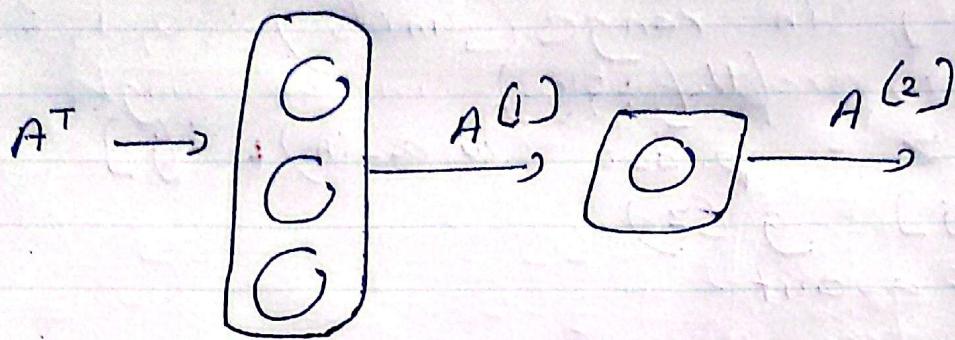
```

Vectorized

matrix multiplication

[1, 0, 1]

## 09] Vectorized Code



$$A^T = \begin{bmatrix} 200 & 17 \end{bmatrix} \quad \left. \right\} z = A^T w + b$$

$$w = \begin{bmatrix} 1 & -3 & 5 \\ -2 & 4 & -6 \end{bmatrix} \quad \left. \right\} \begin{bmatrix} 165 & -531 & 900 \\ z_1^{(1)} & z_2^{(1)} & z_3^{(1)} \end{bmatrix}$$

$$b = \begin{bmatrix} -1 & 1 & 2 \end{bmatrix} \quad \left. \right\} A = g(z) \\ \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

( $A$  Transpose)

$$AT = \text{np.array}([[[200, 17]])$$

$$w = \text{np.array}([[[1, -3, 5], [-2, 4, -6]]])$$

$$b = \text{np.array}([[[-1, 1, 2]]])$$

def dense(AT, w, b):

$$z = \text{np.matmul}(AT, w) + b$$

$$a-out = g(z)$$

return a-out

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$