Week 02

1] Partial derivatives

*] A partial derivative is a type of derivative used when dealing with functions of multiple variables ($f(x,y)$, $g(a,b,c)$)

$$f_{(x,y)} = x^2 + y^2$$

Task → Find partial derivative of with respect to x and y

Step 1 : Treat all other variables as a constant. In our case x and y

Step 2 : Differentiate the function using normal rules of differentiation

$$\frac{\partial f}{\partial x} = 2x \quad (y \text{ constant})$$

$$\frac{\partial f}{\partial y} = 2y \quad (x \text{ constant})$$

ex ÷ $f(x,y) = 3x^2y^3$

$\frac{\partial f}{\partial y} = \underbrace{3x^2}_{const.} 3y^2 = 9x^2y^2$

$\frac{\partial f}{\partial x} = \underbrace{3y^3}_{const.} 2x = 6xy^3$

2] Gradient

$$\boxed{\nabla f = \begin{matrix} \frac{\partial f}{\partial x} \\ \\ \frac{\partial f}{\partial y} \end{matrix}}$$

$f_{(x,y)} = x^2 + y^2$

$\text{Gradient} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$

3] Gradient and maxima / minima

One variable

$f(x) = x^2$

*) Minimum is when slope = 0

$f'(x) = 0$

$f'(x) \Rightarrow 2x = 0$

$x = 0$ //

Two or More variables

$$f_{(x,y)} = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

$\Rightarrow$ Minimum is when both slopes $= 0$

$$2x = 0 \qquad 2y = 0$$

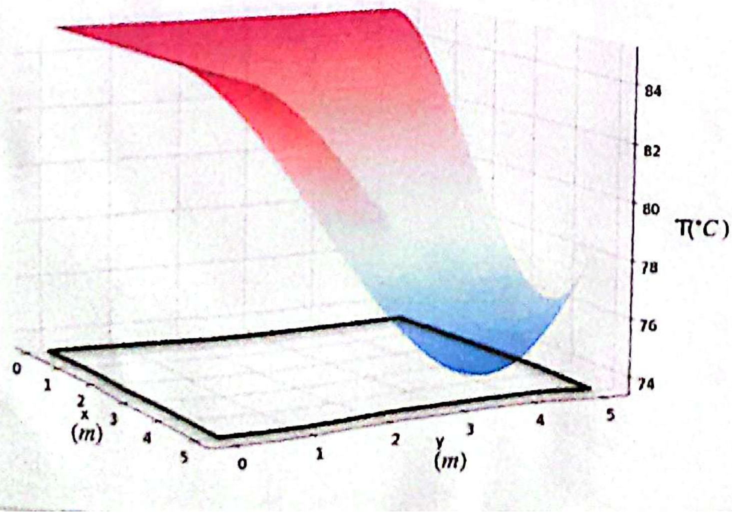
$$(x, y) = (0, 0)$$

4] Optimization with gradients

An example

Motivation for Optimization in Two Variables

$$T = f(x, y) = 85 - \frac{1}{90}x^2(x-6)y^2(y-6)$$



$$\frac{\partial f}{\partial x} = -\frac{1}{90}\underbrace{\boxed{x}}_{x=0}\underbrace{\boxed{(3x-12)}}_{x=4}\underbrace{\boxed{y^2}}_{y=0}\underbrace{\boxed{(y-6)}}_{y=6} = 0$$

$$\frac{\partial f}{\partial y} = -\frac{1}{90}\underbrace{\boxed{x^2}}_{x=0}\underbrace{\boxed{(x-6)}}_{x=6}\underbrace{\boxed{y}}_{y=0}\underbrace{\boxed{(3y-12)}}_{y=4} = 0$$

Motivation for Optimization in Two Variables

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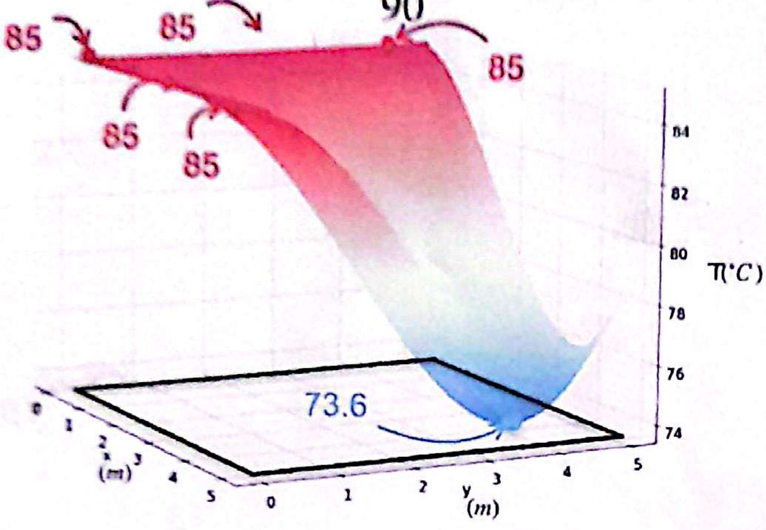


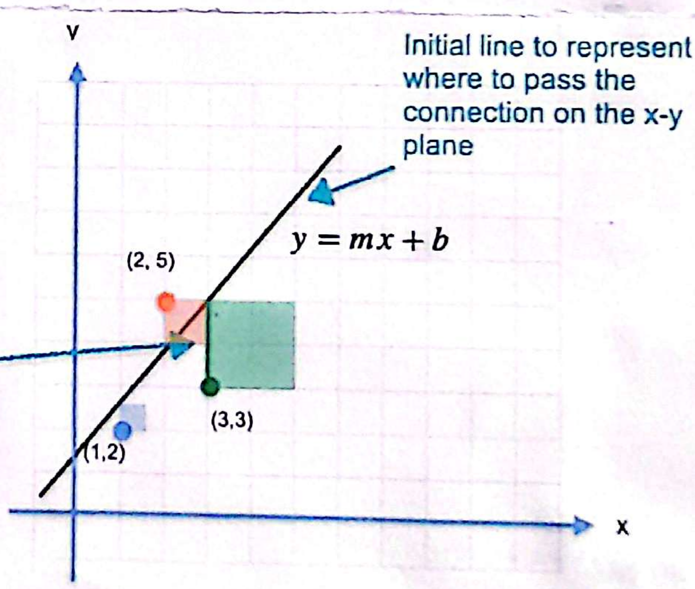
Candidate points for the minima

$x = 0$, $y = 0$	Maxima
$x = 0, y = 0$; $x = 0, y = 4$	Maxima
$x = 0, y = 6$	Outside
$x = 4, y = 0$	Maxima
$x = 4, y = 4$	Minimum
$x = 6, y = 0$; $x = 6, y = 6$	Outside

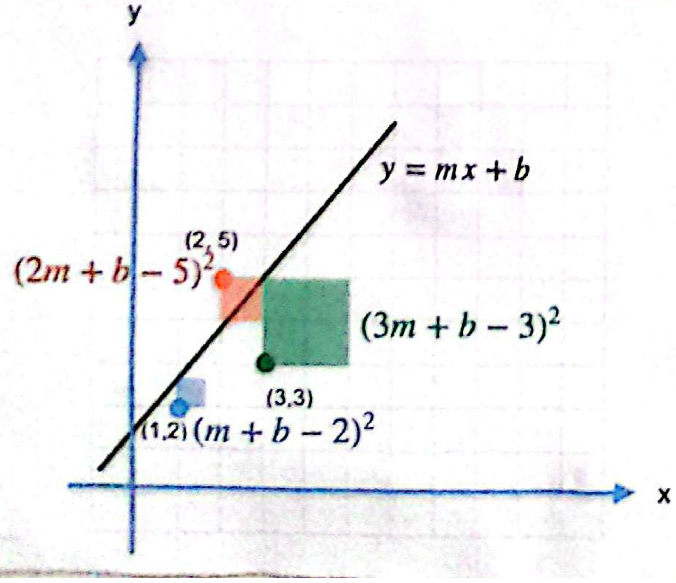
Analytical method

Location on the x,y plane (1, 2) (2, 5) (3,3)

The cost of connecting connection to the powerline



Goal: Find m, b such that you minimize sum of squares cost



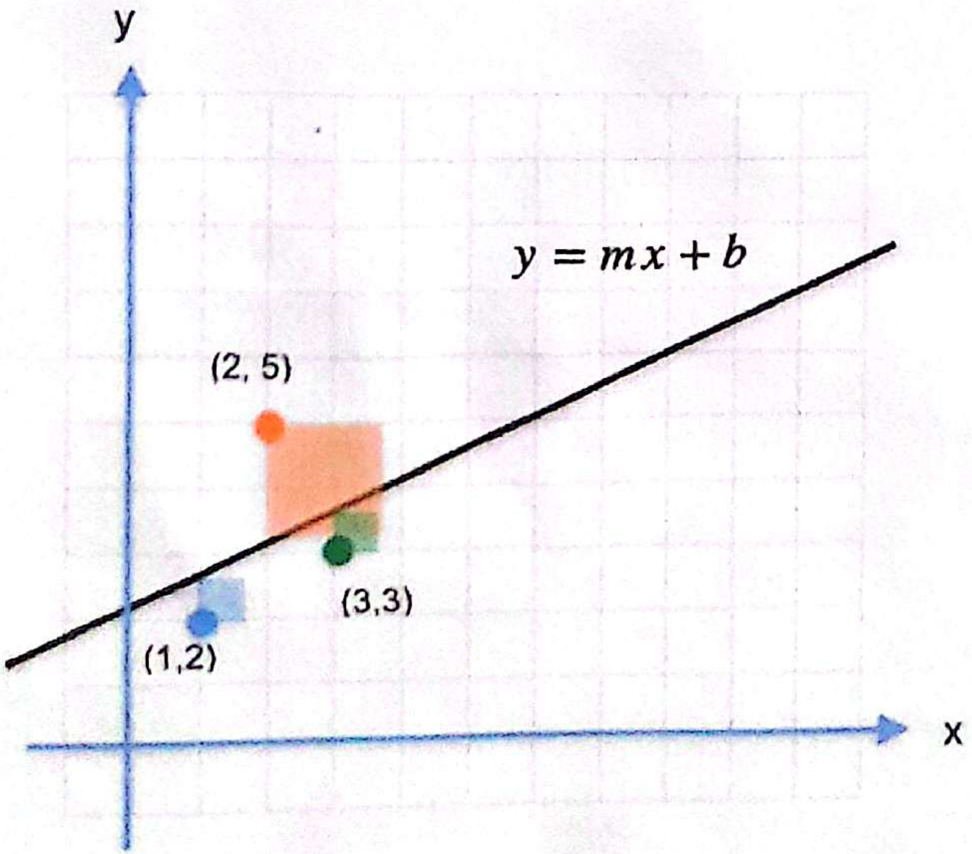
Goal: Minimize sum of squares cost

$$(m + b - 2)^2 + (2m + b - 5)^2 + (3m + b - 3)^2$$

$$m^2 + b^2 + 4 + 2mb - 4m - 4b$$
$$+4m^2 + b^2 + 25 + 4mb - 20m - 10b$$
$$+9m^2 + b^2 + 9 + 6mb - 18m - 6b$$

$$E(m, b) = 14m^2 + 3b^2 + 38 + 12mb - 42m - 20b$$

Linear Regression: Optimal Solution



$$m = \frac{1}{2}$$

$$b = \frac{7}{3}$$

$$E(m = \frac{1}{2}, b = \frac{7}{3}) \approx 4.167$$