Week 01

1] What is probability

*] Probability is the measure of how likely an event is to happen

*] It gives numerical value (between 0 and 1) to the chance of something occuring
0 → Impossible
1 → Definetly happening

Formula for equally likely outcomes

$$\boxed{P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total no. of possible outcomes}}}$$

Example : Toss a coin
*] Total outcomes = 2 (H, T)
*] Favorab outcome for "getting heads" = 1

$$P(\text{Heads}) = \frac{1}{2} = 0.5$$

2] Complement of probability

*] The complement of a probability means the chance that an event does not happen

$$\boxed{P(E') = 1 - P(E)}$$

3] Sum of probabilities (Disjoint events)

Disjoint events → Two events are disjoint (mutually exclusive) if they cannot happen at the same time.

Ex :- Tossing a dice
*) Event A rolling 2
*) Event B rolling 5

These are disjoint because you can't roll both 2 and 5 in a single toss.

If A and B are disjoint;

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

4) Sum of probabilities (Joint events)

Joint events → Two events are joint if they can occur together

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$P(A)$ = probability of event A
$P(B)$ = probability of event B
$P(A \cap B)$ = probability that both A and B happen together

5] Independence

*) Two events are independent if the outcome of one does not affect the probability of the other.

Product Rule

$$P(A \cap B) = P(A) \cdot P(B)$$

Ex: Coin toses

*) Event A : First coin is heads $\rightarrow P(A) = \frac{1}{2}$

*) Event B : Second coin is Heads $\rightarrow P(B) = \frac{1}{2}$

Both are independent because tossing one coin does not affect the other.

$$P(A \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

6] Birthday problem

30 friend in a party : Probability that every one has a different birthday

1 person $\rightarrow \frac{365}{365} = 1$

2 persons $\rightarrow \frac{364}{365} = 0.997$

3 persons $\rightarrow \frac{363}{365} = 0.992$

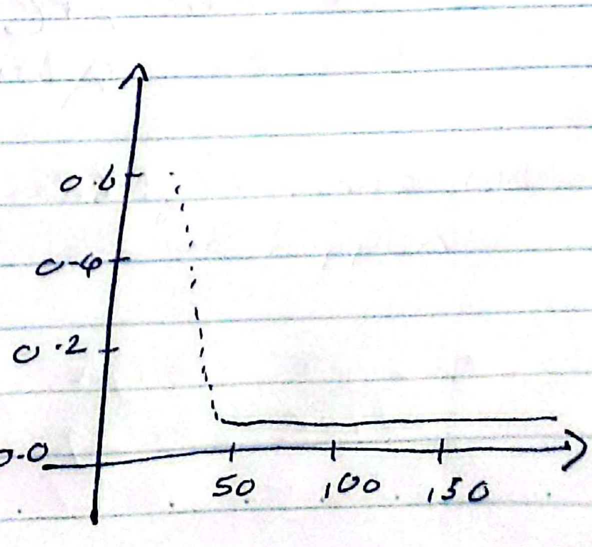
10 persons $= \frac{355}{365} = 0.883$

20 persons $= 0.589$

23 persons $= 0.493$

30 persons $= 0.294$

100 persons $= 0.0000003$



7] Conditional probability

*] Conditional probability of an event A given that another event B has already occured

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{If } P(B) > 0)}$$

•] $P(A|B)$ = probability of A happening given that B happened

•] $P(A \cap B)$ = probability that both A and B happening

•] $P(B)$ = probability of B happening

$$\boxed{P(A \cap B) = P(A|B) \cdot P(B)}$$

*] If A and B are independent then,

$$P(A|B) = P(A)$$

8] Bayes Theorem

Bayes theorem let us update probabilities when we get new information.

$$\boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

$P(A)$ = prior probability of A (our belief about A before seeing B)

$P(B|A)$ = Likelihood (probability of seeing B if A is true)

$P(B)$ = Total probability of B (normalizing factor)

$P(A|B)$ = Posterior probability (updated probability of A after seeing B)

Ex÷ •) 1% people have a certain disease = $P(D) = 0.01$

•) The test is 99% accurate:

•) If a person has the disease, the test is positive 99% = $P(Positive|D) = 0.99$

•] If a person does not have the disease, test is falsely positive 5% of the time

$\rightarrow P(Positive|D') = 0.005$

$$P(D|Positive) = \frac{P(Positive|D) \cdot P(D)}{P(Positive)}$$

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9] Prior and Posterior

Prior ($P(A)$)

*] The prior is your initial belief about an event before you see any new evidence.

*] It is based on background information, previous data or population statistics.

ex:-

Suppose a diseas affect 1% population Before testing anyone, your belief that a random person has the disease is:

$$P(D) = 0.01 \ (\text{Prior})$$

Posterior ($P(A|B)$)

*] The posterior is your updated belief about an event after seeing new evidence.

*] It tells you: given what I observed, how likely is the event now?

ex:- You run a test and it comes back positive Now that your belief that the person has disease updated using bayes theorm

$$P(D \mid \text{Positive}) = \frac{P(\text{Positve} \mid D) \cdot P(D)}{P(\text{positive})} \approx 0.167$$

*] Even though your prior belief was 1% after seeing a positive test, your posterior becomes 16.7%.

10] The naive bayes model

*) Naive bayes is probabilistic classifier based on Baye's' theorm

*) It predicts the probability of a class (label) given some features (evidence)

$$P(C|x) = \frac{P(x|C).P(c)}{P(x)}$$

C = Class label (spam or no spam)
X = features (words in email)
P(C) = prior probability of the class
P(x|C) = likelihood of seeing features X given class C
P(C|x) = posterior probability of class given features

*) It is naive because it assume that all features are independent given the class.

ex ÷ •) Email words : "buy", "cheap", "Offer"
•) Naive bayes assumes each word's presence is independent of others given the email is spam, which is not strictly true in reality
•) This simpification makes the Computation fast and easy, and surprisingly effective.

*) Spam detection, Sentiment analysis, Text classi -fication are the use cases of naive bayes model