



DeepLearning.AI

## Math for Machine Learning

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# Probability and Statistics

# W1 Lesson 1

# Introduction to Probability



DeepLearning.AI

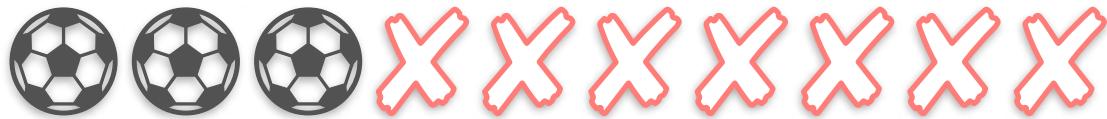
## Introduction to probability

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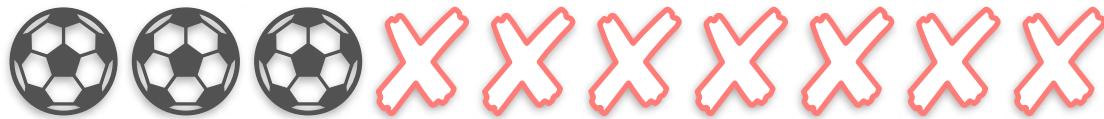
# What is Probability?

# Introduction to Probability

# Introduction to Probability

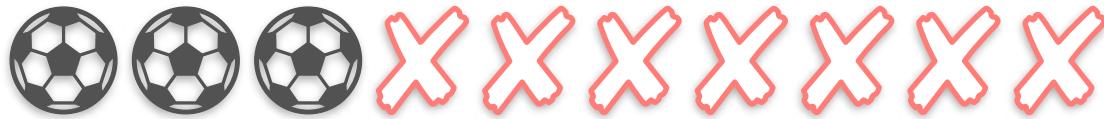


# Introduction to Probability



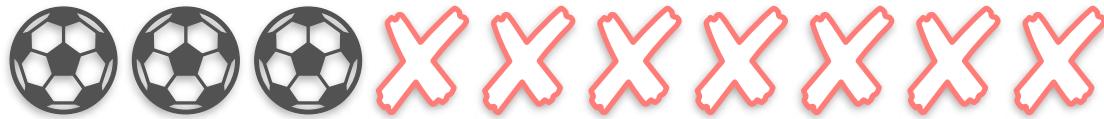
Find the probability that a child picked at random plays soccer.

# Introduction to Probability



Find the probability that a child picked at random plays soccer.

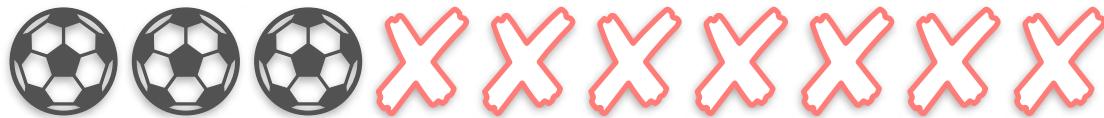
# Introduction to Probability



Find the probability that a child picked at random plays soccer.

**The probability that a child picked at random plays soccer.**

# Introduction to Probability

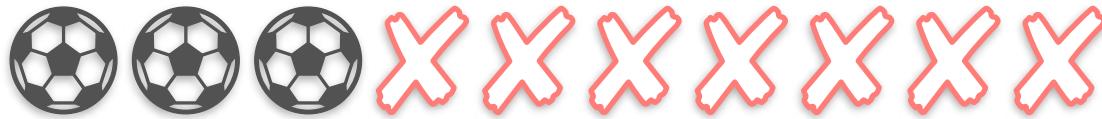


Find the probability that a child picked at random plays soccer.

**The probability that a child picked at random plays soccer.**

$$P(\text{soccer})$$

# Introduction to Probability



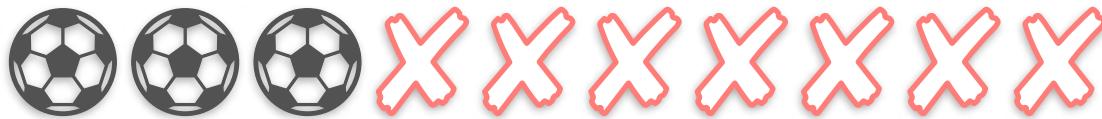
Find the probability that a child picked at random plays soccer.

**The probability that a child picked at random plays soccer.**

$$P(\text{soccer})$$

A teal curved arrow points from the text "The probability that a child picked at random plays soccer." down to the mathematical expression  $P(\text{soccer})$ .

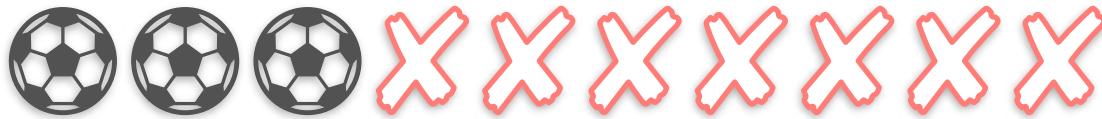
# Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer})$$

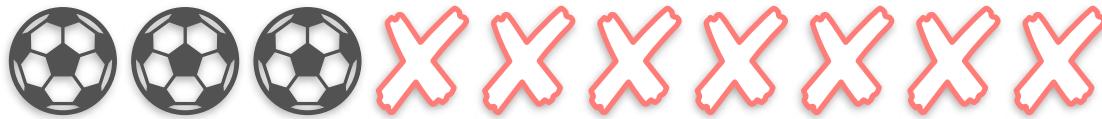
# Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}}$$

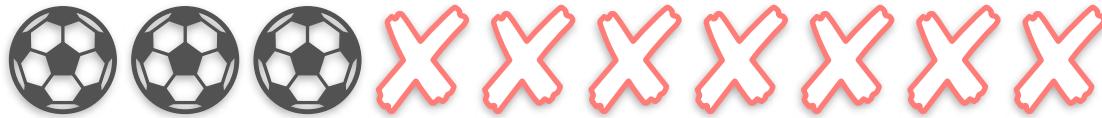
# Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \underline{\hspace{2cm}}$$

# Introduction to Probability

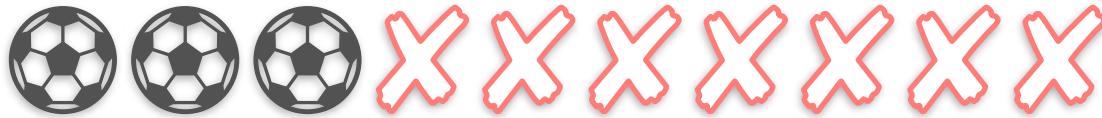


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10}$$



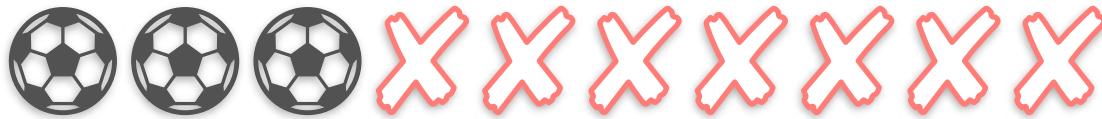
# Introduction to Probability



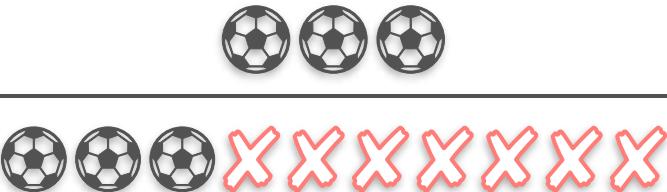
Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{soccer balls}}{\text{total items}}$$
The equation shows the probability of picking a soccer ball as a fraction. The numerator is represented by three black soccer balls. The denominator is represented by a horizontal line above which are three black soccer balls, and below which is a sequence of three black soccer balls followed by seven red 'X' marks.

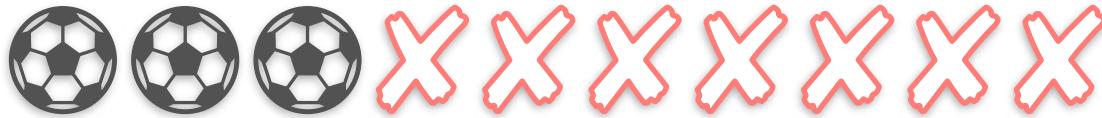
# Introduction to Probability



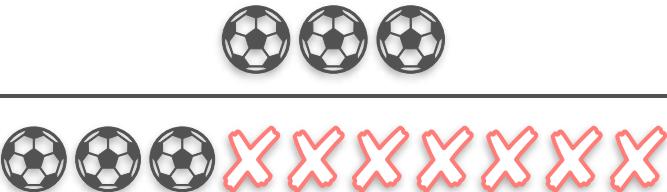
Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10}$$
The fraction is displayed with a horizontal line separating the numerator from the denominator. The numerator consists of three black and white soccer balls. The denominator consists of ten items, specifically three black and white soccer balls and seven red 'X' marks.

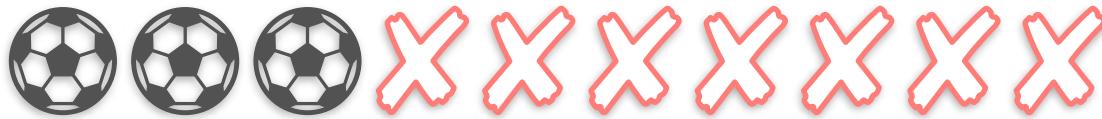
# Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10} = 0.3$$
A mathematical equation illustrating the calculation of probability. The fraction  $\frac{\text{soccer}}{\text{total}}$  is shown. The numerator, "soccer", is represented by three black soccer ball icons positioned above the fraction line. The denominator, "total", is represented by a row of ten items below the fraction line, identical to the one shown at the top of the slide. To the right of the fraction, the result  $= \frac{3}{10} = 0.3$  is displayed.

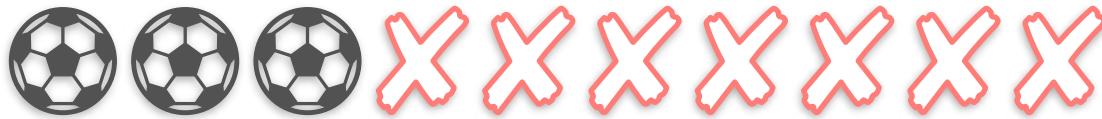
# Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{3}{10} = 0.3$$
The equation shows the probability of picking a soccer ball (soccer) over the total number of items (total). The numerator is represented by a teal box containing the first three items from the sequence above. The denominator is represented by a teal box containing the first three items from the sequence below. Both sequences consist of three soccer balls followed by seven red 'X' marks.

# Introduction to Probability



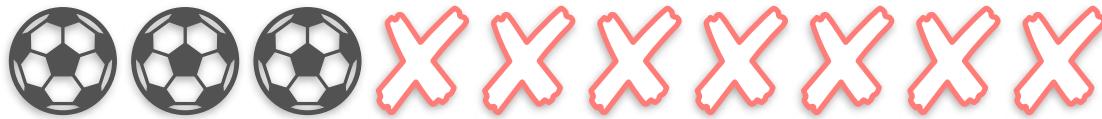
Find the probability that a child picked at random plays soccer.

Event

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Number of soccer balls}}{\text{Total number of items}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of the probability of picking a soccer ball. It shows a sequence of 10 items: 3 soccer balls and 7 'X' marks. A teal box highlights the first three items (soccer balls). Below the sequence, another teal box highlights the first three items (soccer balls), corresponding to the numerator in the probability formula.

# Introduction to Probability

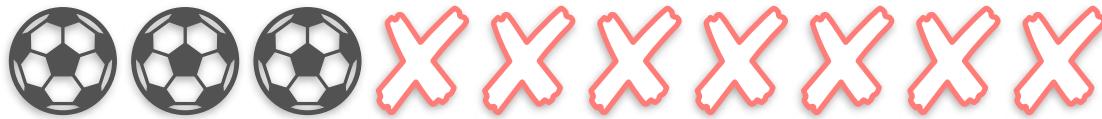


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Total}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A teal box labeled "Event" highlights the 3 soccer balls. A horizontal line separates the event from the total population. Below the line, the 3 soccer balls are followed by the 7 red 'X' marks, representing the total population of 10 children.

# Introduction to Probability

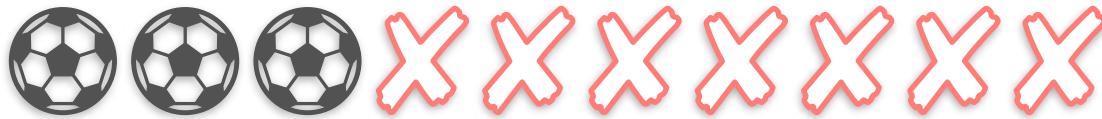


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Total Population}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A teal bracket labeled "Event" encloses the three soccer balls. Another teal bracket labeled "Total Population" encloses all ten items (three soccer balls and seven 'X's). An arrow points from the word "Event" to the top of the bracket enclosing the soccer balls.

# Introduction to Probability

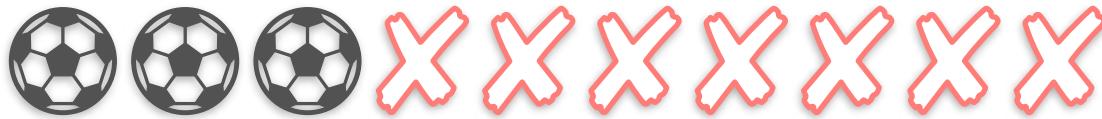


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Sample space}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A teal bracket labeled "Event" encloses the three soccer balls. Another teal bracket labeled "Sample space" encloses all ten items (three soccer balls and seven 'X' marks).

# Introduction to Probability



Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Sample space}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A horizontal bar represents the 'Sample space' containing 10 items: 3 soccer balls and 7 red 'X' marks. Above the bar, a smaller box labeled 'Event' contains the 3 soccer balls. Arrows point from the labels 'Event' and 'Sample space' to their respective parts of the diagram.

# Introduction to Probability: Venn Diagram

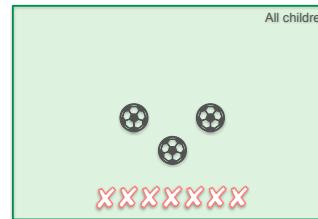


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# Introduction to Probability: Venn Diagram



# Introduction to Probability: Venn Diagram

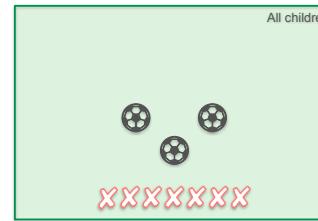
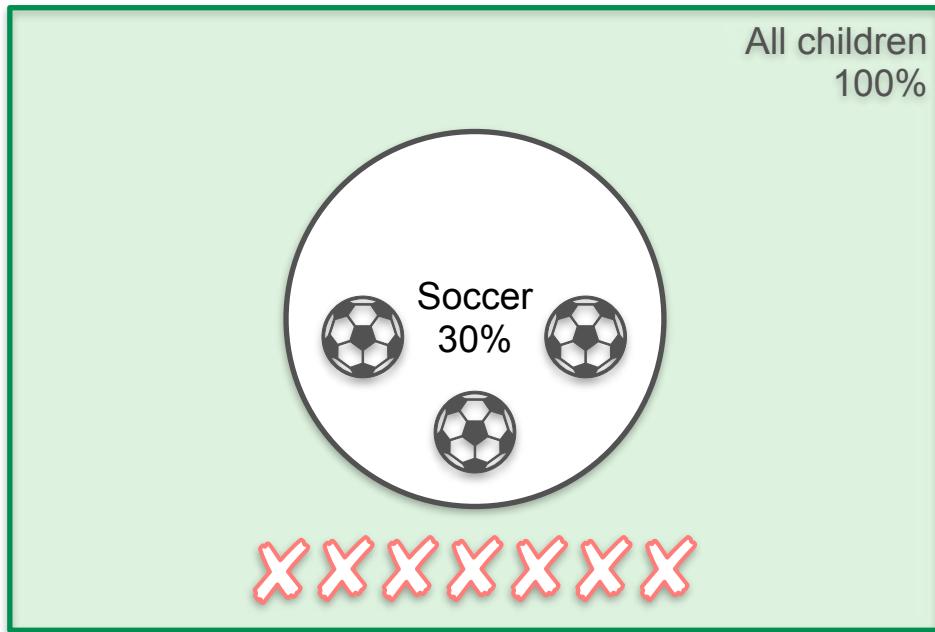


# Introduction to Probability: Venn Diagram



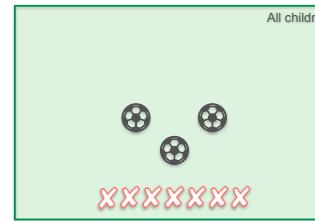
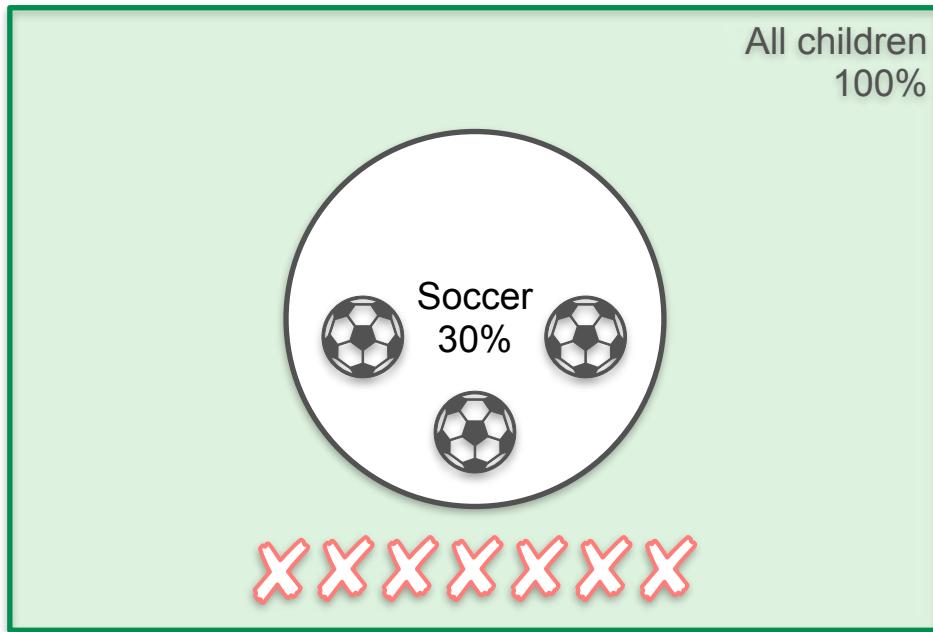
Sample Space

# Introduction to Probability: Venn Diagram



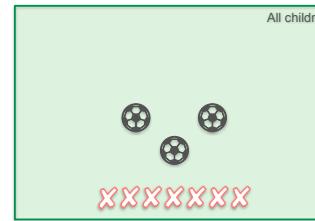
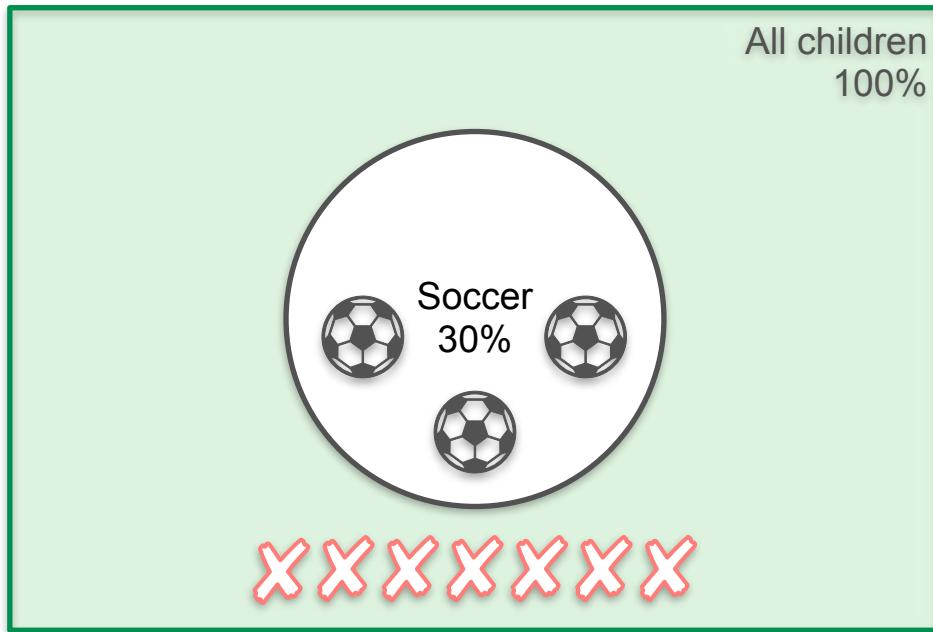
Sample Space

# Introduction to Probability: Venn Diagram

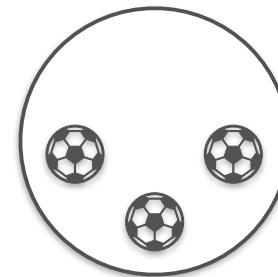


Sample Space

# Introduction to Probability: Venn Diagram



Sample Space



Event

# Introduction to Probability: Coin Example 1



# Introduction to Probability: Coin Example 1



# Introduction to Probability: Coin Example 1



**Experiment**

# Introduction to Probability: Coin Example 1



## Experiment

Probability of landing on heads

# Introduction to Probability: Coin Example 1



## Experiment

Probability of landing on heads

$$P(\text{heads})$$

# Introduction to Probability: Coin Example 1

# Introduction to Probability: Coin Example 1



# Introduction to Probability: Coin Example 1



50%

# Introduction to Probability: Coin Example 1



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# Introduction to Probability: Coin Example 1



$$P(\text{heads}) = \underline{\hspace{2cm}}$$

# Introduction to Probability: Coin Example 1



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$$P(\text{heads}) = \frac{\text{ }}{\text{ }}$$

# Introduction to Probability: Coin Example 1



$$P(\text{heads}) = \frac{\text{H}}{\text{H} \quad \text{T}}$$

# Introduction to Probability: Coin Example 1



$$P(\text{heads}) = \frac{\text{Number of heads}}{\text{Total number of outcomes}} = \frac{1}{2} = 0.5$$
A fraction is shown with 'P(heads)' as the numerator. The numerator is represented by a single gold coin with 'H' (heads) facing up. The denominator is represented by two gold coins, one with 'H' (heads) and one with 'T' (tails) facing up. A horizontal line separates the numerator from the denominator.

# Introduction to Probability: Coin Example 2



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# Introduction to Probability: Coin Example 2



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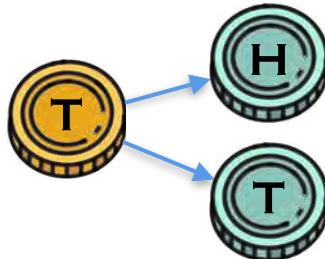
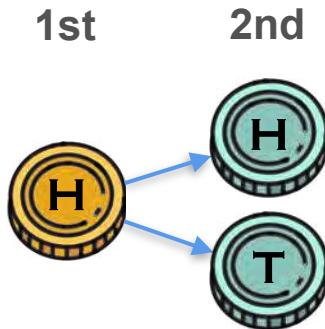
1st



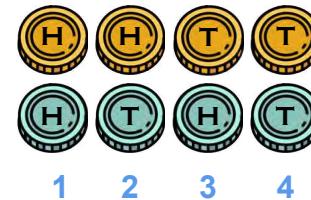
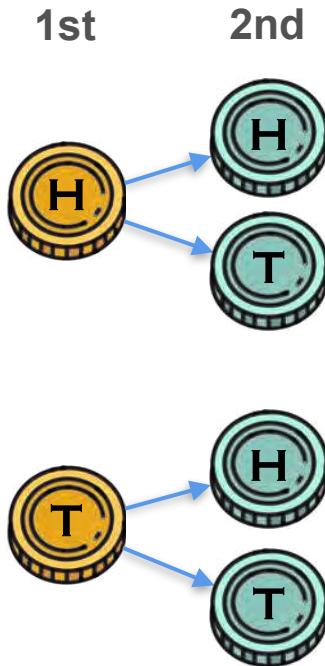
# Introduction to Probability: Coin Example 2



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# Introduction to Probability: Coin Example 2

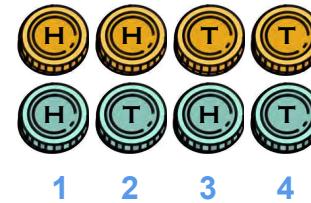
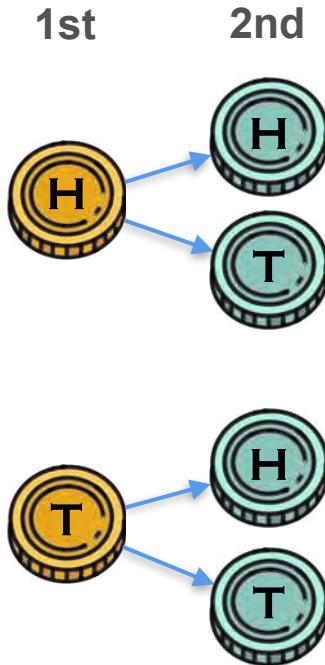


# Introduction to Probability: Coin Example 2



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What is the probability of landing on heads twice?



# Introduction to Probability: Coin Example 2



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# Introduction to Probability: Coin Example 2



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$$P(HH) = \underline{\hspace{2cm}}$$



# Introduction to Probability: Coin Example 2



50%    50%

$$P(HH) = \underline{\hspace{2cm}}$$



# Introduction to Probability: Coin Example 2



50%    50%



$$P(HH) = \underline{\hspace{2cm}}$$



# Introduction to Probability: Coin Example 2



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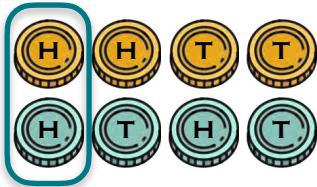
$$P(HH) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}}$$

The equation shows the probability of getting two heads (HH) as a fraction. The numerator is a stack of two coins, one yellow (H) on top of one blue (H). The denominator is a 2x4 grid of eight coins, with four yellow (H) and four blue (T) coins distributed across the four columns.

# Introduction to Probability: Coin Example 2



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$$P(HH) = \frac{1}{4} = 0.25$$
A 2x4 grid of eight coins representing all possible outcomes of two coin flips. The top row contains two yellow coins with 'H' and two yellow coins with 'T'. The bottom row contains two teal coins with 'H' and two teal coins with 'T'. This visualizes the sample space for the event of getting two heads (HH).

# Introduction to Probability: Coin Example 3



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# Introduction to Probability: Coin Example 3



50%   50%

1st

# Introduction to Probability: Coin Example 3



50%    50%

1st



# Introduction to Probability: Coin Example 3



50%    50%

1st              2nd

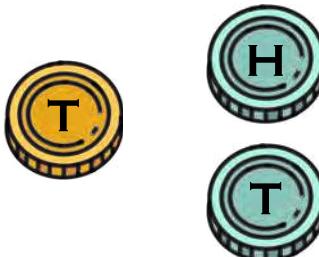
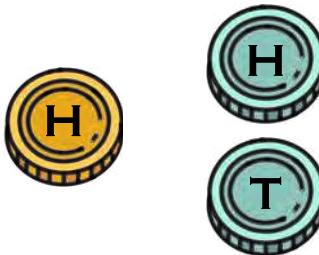


# Introduction to Probability: Coin Example 3



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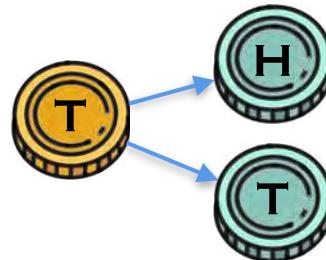
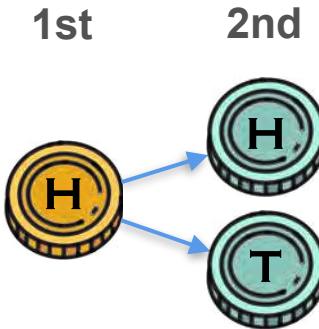
1st            2nd



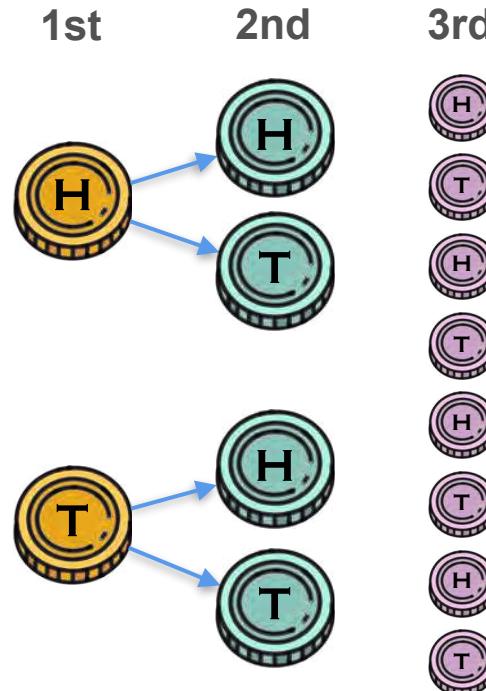
# Introduction to Probability: Coin Example 3



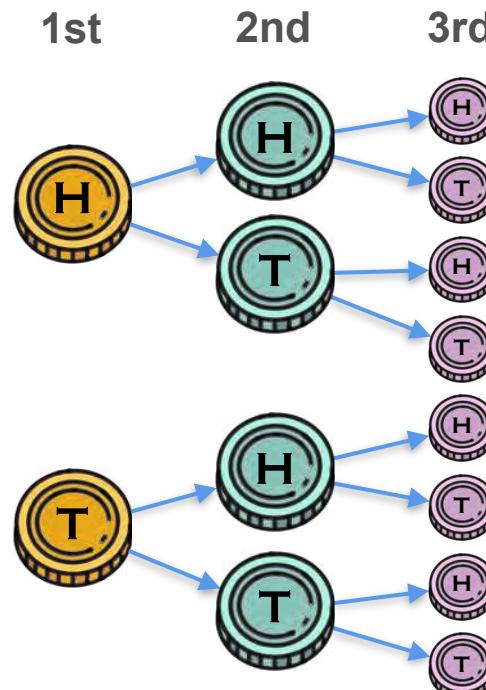
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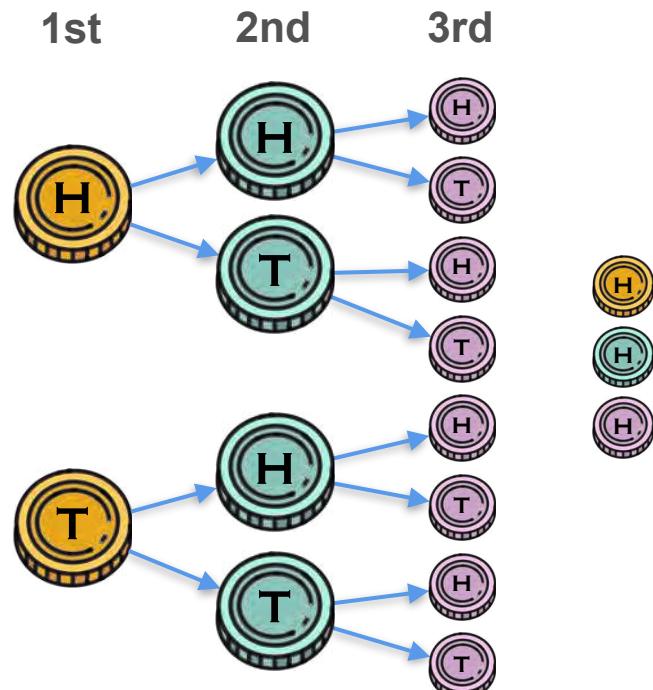
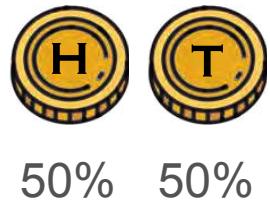
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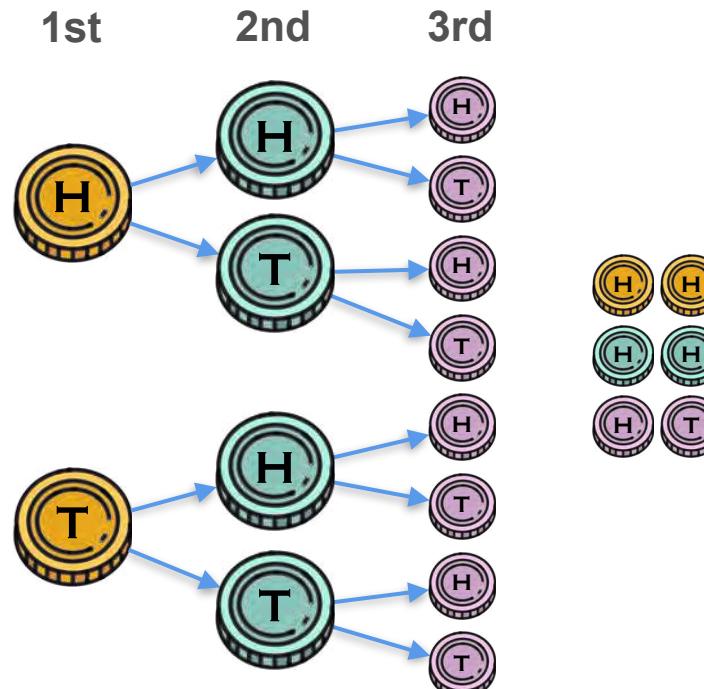
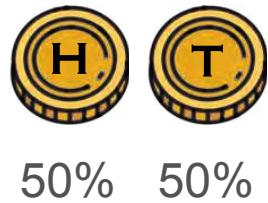
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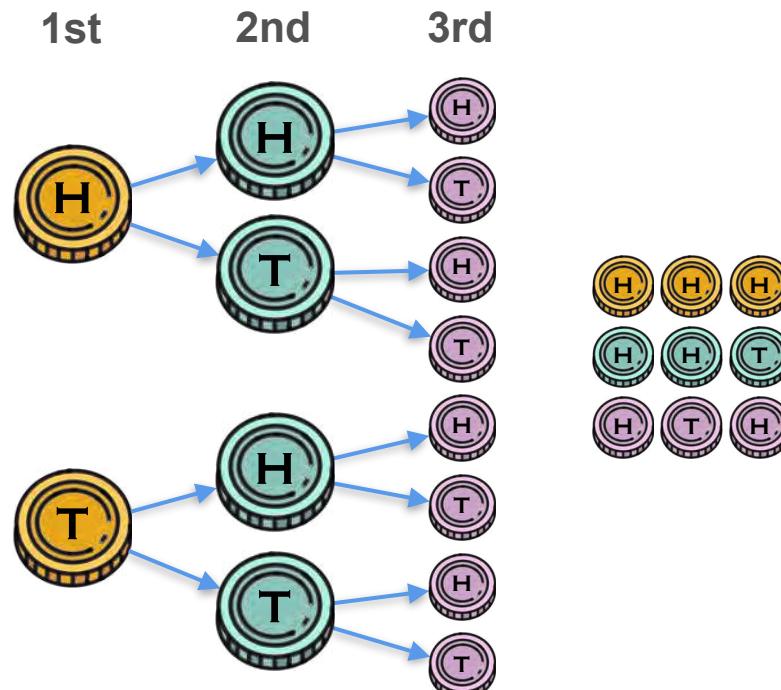
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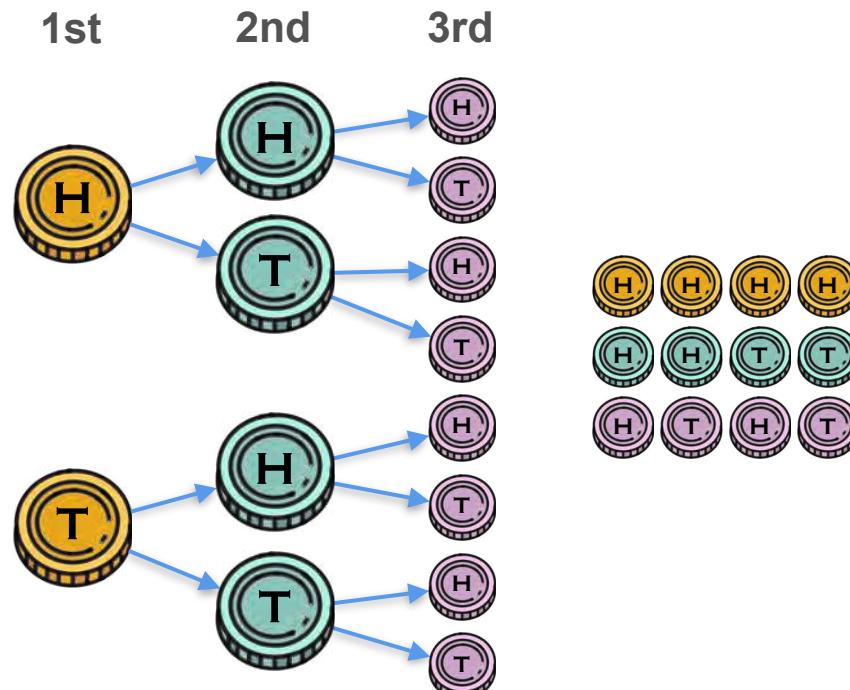
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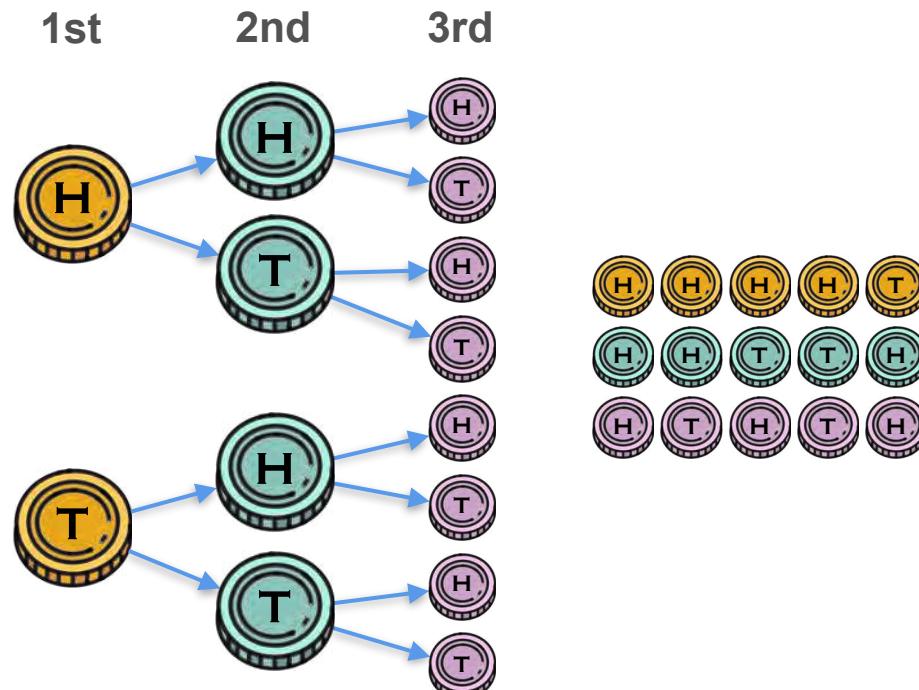
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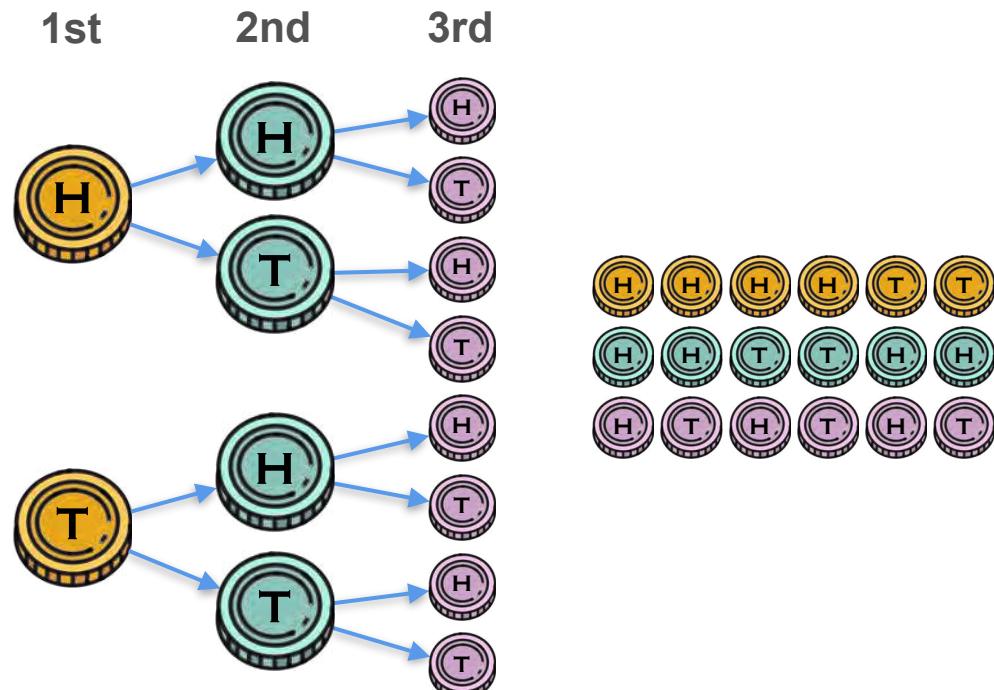
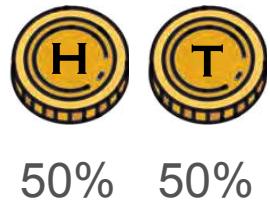
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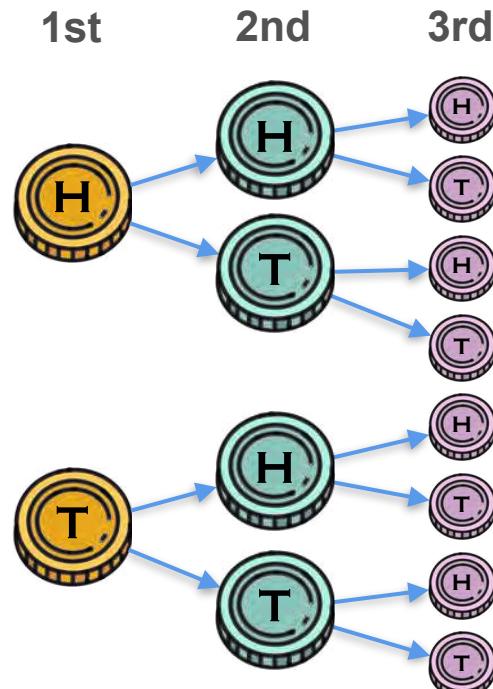
# Introduction to Probability: Coin Example 3



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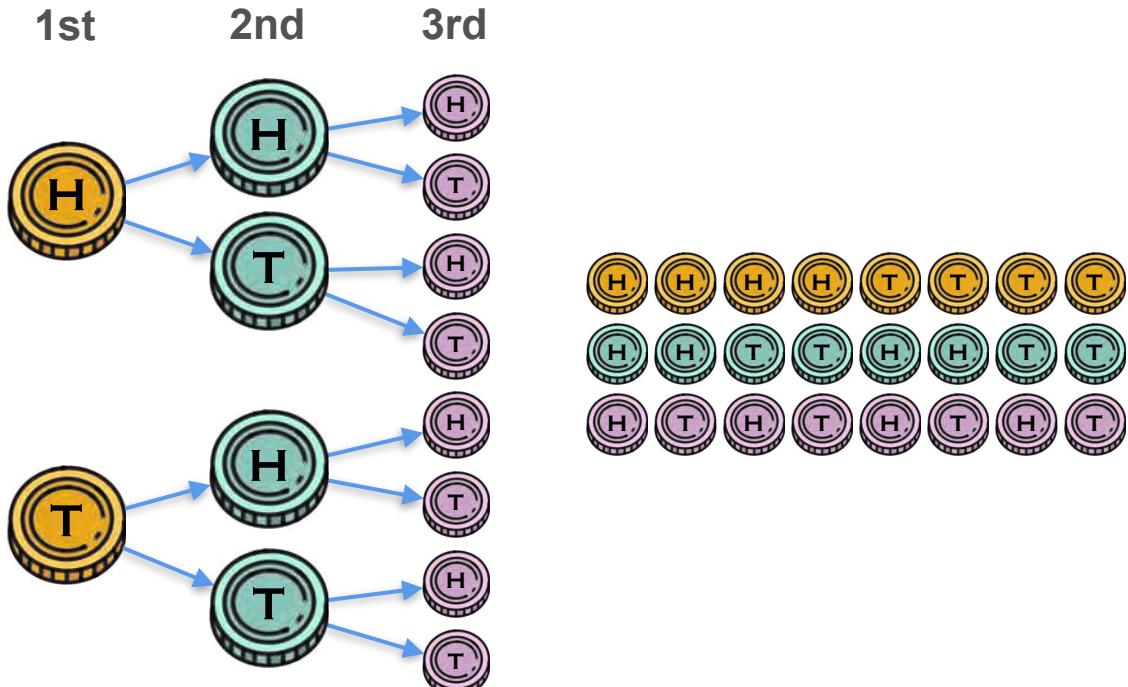
# Introduction to Probability: Coin Example 3



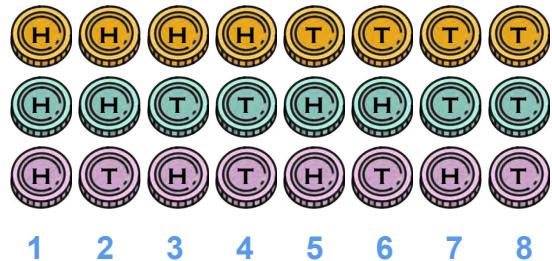
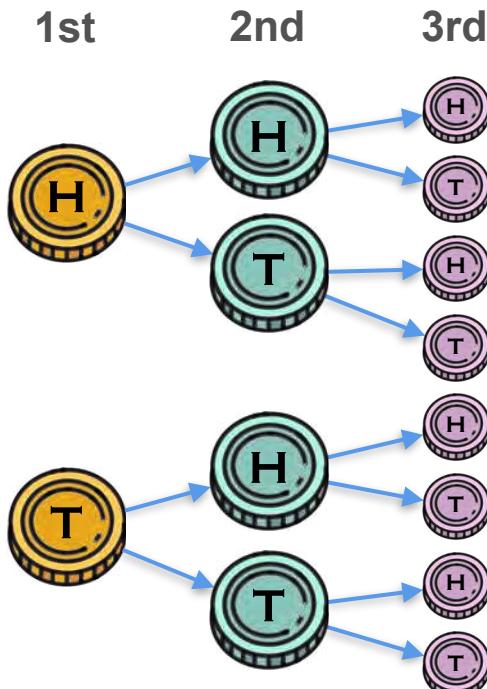
# Introduction to Probability: Coin Example 3



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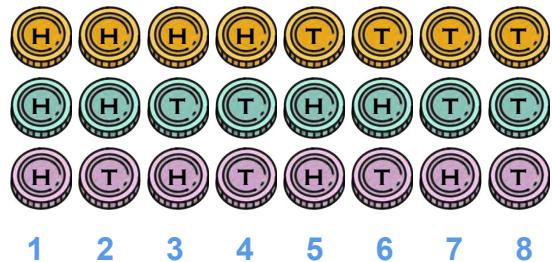
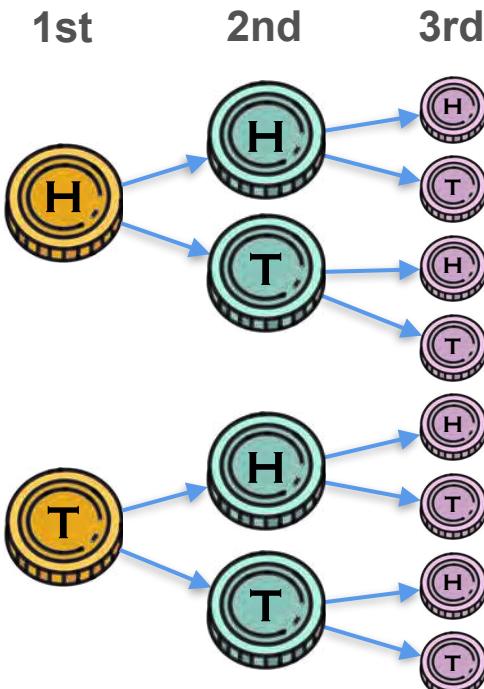
# Introduction to Probability: Coin Example 3



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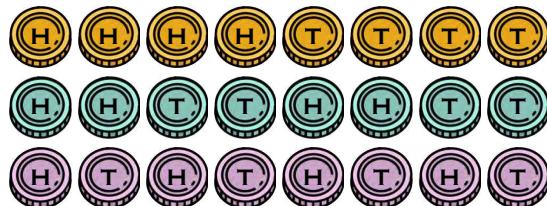
What is the probability of landing on heads 3 times?



# Introduction to Probability: Coin Example 3



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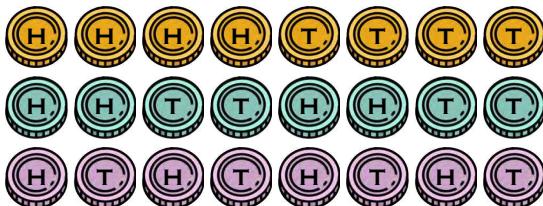


# Introduction to Probability: Coin Example 3



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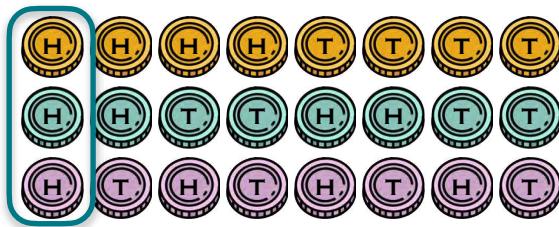
$$\mathbf{P}(HHH) = \underline{\hspace{2cm}}$$



# Introduction to Probability: Coin Example 3



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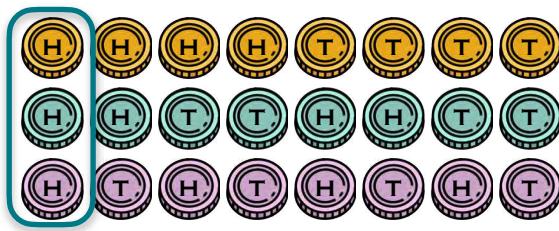


$$\mathbf{P}(HHH) = \underline{\hspace{2cm}}$$

# Introduction to Probability: Coin Example 3



50% 50%



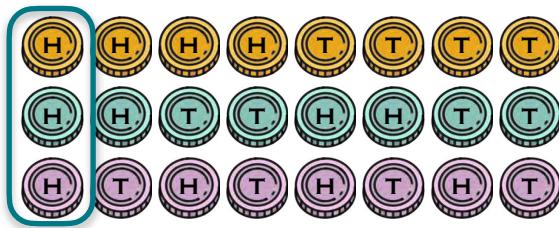
$$\mathbf{P}(HHH) = \underline{\hspace{2cm}}$$



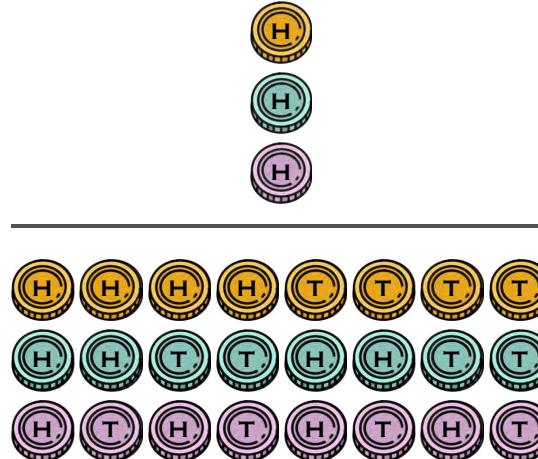
# Introduction to Probability: Coin Example 3



50%    50%



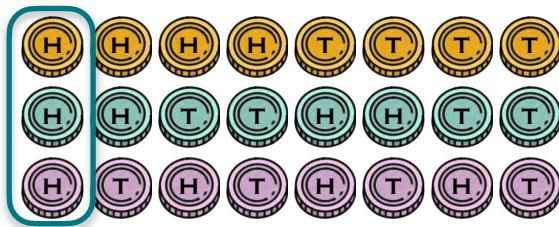
$$P(HHH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



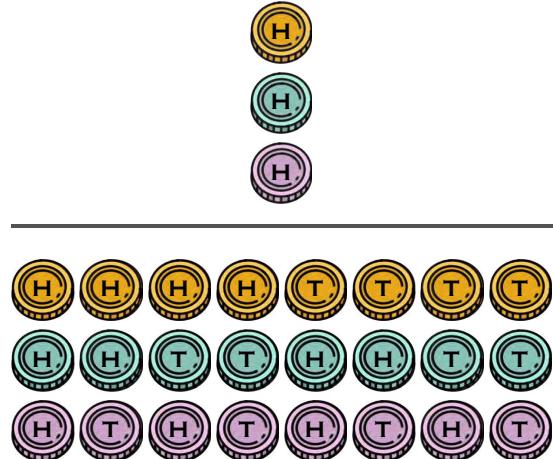
# Introduction to Probability: Coin Example 3



50% 50%



$$P(HHH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



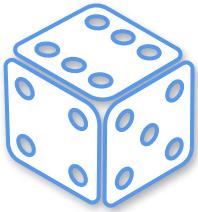
$$= \frac{1}{8} = 0.125$$

# Introduction to Probability: Dice Example 1

# Introduction to Probability: Dice Example 1

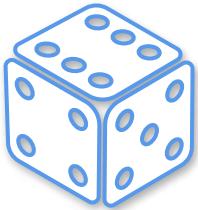


# Introduction to Probability: Dice Example 1



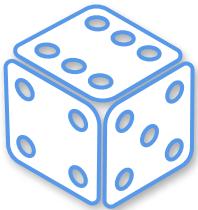
What is the probability of obtaining 6?

# Introduction to Probability: Dice Example 1



What is the probability of obtaining 6?

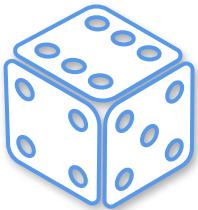
# Introduction to Probability: Dice Example 1



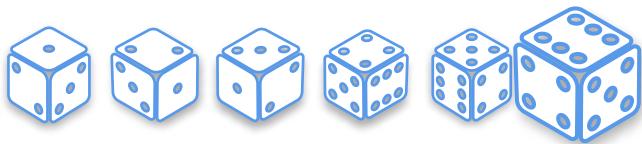
What is the probability of obtaining 6?



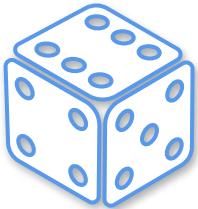
# Introduction to Probability: Dice Example 1



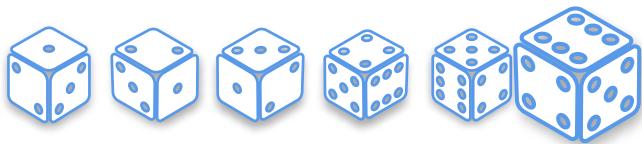
What is the probability of obtaining 6?



# Introduction to Probability: Dice Example 1

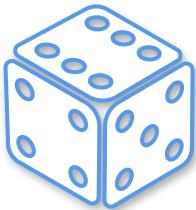


What is the probability of obtaining 6?

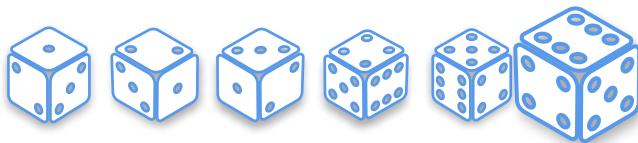


$$P(6) = \underline{\hspace{10em}}$$

# Introduction to Probability: Dice Example 1



What is the probability of obtaining 6?



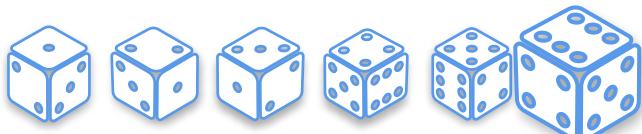
$$P(6) = \underline{\hspace{2cm}}$$



# Introduction to Probability: Dice Example 1



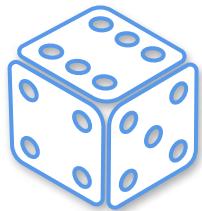
What is the probability of obtaining 6?



$$P(6) = \frac{\text{Number of 6s}}{\text{Total number of dice}}$$

The fraction is shown with a horizontal line separating the numerator from the denominator. Above the numerator is a single die showing a 6. Below the denominator is a row of seven dice, with the first six showing 1 through 5 dots and the last one showing 6 dots.

# Introduction to Probability: Dice Example 1



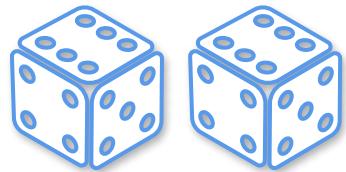
What is the probability of obtaining 6?

$$P(6) = \frac{1}{6}$$

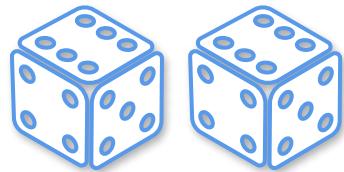
The equation illustrates the probability of rolling a 6 on a single die. The numerator is represented by a single die showing a 6, and the denominator is represented by a horizontal line with seven dice underneath it, where the first six dice show faces other than 6, and the seventh die shows a 6.

# Introduction to Probability: Dice Example 2

# Introduction to Probability: Dice Example 2

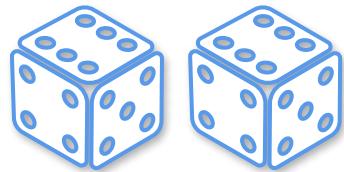


# Introduction to Probability: Dice Example 2



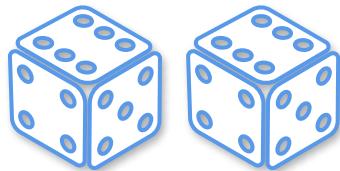
What is the probability of obtaining 6,6?

# Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

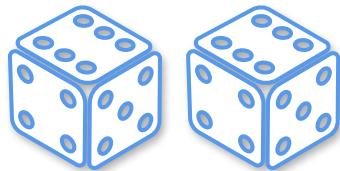
# Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?



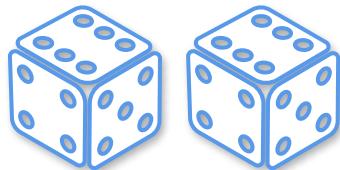
# Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

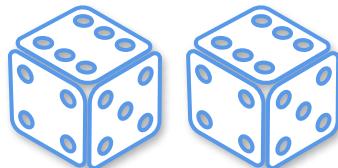
# Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

# Introduction to Probability: Dice Example 2

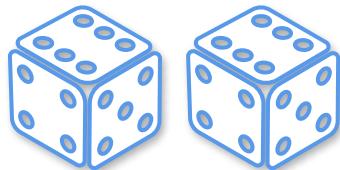


What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(6,6) = \underline{\hspace{10em}}$$

# Introduction to Probability: Dice Example 2

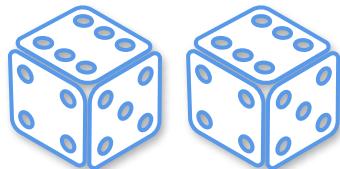


What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(6,6) = \frac{1}{36}$$

# Introduction to Probability: Dice Example 2



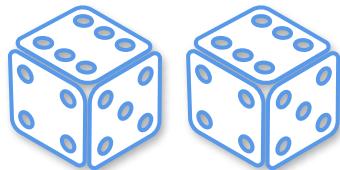
What is the probability of obtaining 6,6?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(6,6) = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(6,6) = \frac{1}{36} = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6



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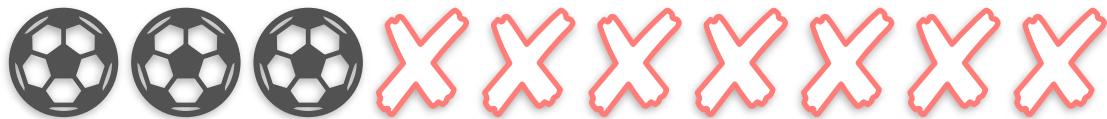
## Introduction to probability

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## Complement of Probability

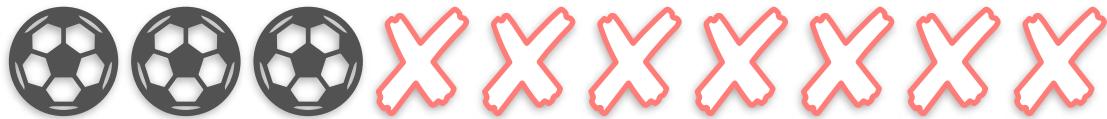
# Complement of Probability

# Complement of Probability



30%

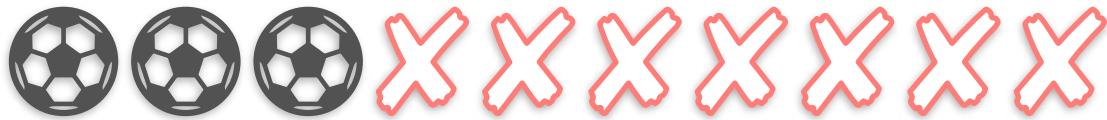
# Complement of Probability



30%

What is the probability of a child NOT playing soccer?

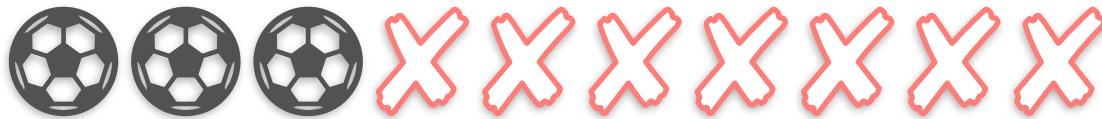
# Complement of Probability



30%

What is the probability of a child NOT playing soccer?

# Complement of Probability

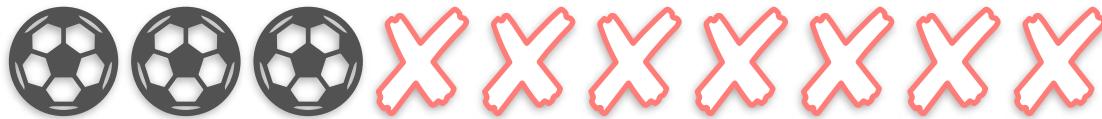


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}}$$

# Complement of Probability

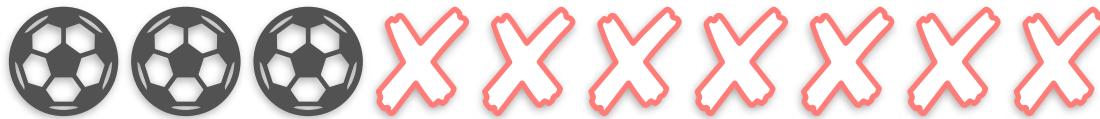


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \underline{\hspace{2cm}}$$

# Complement of Probability

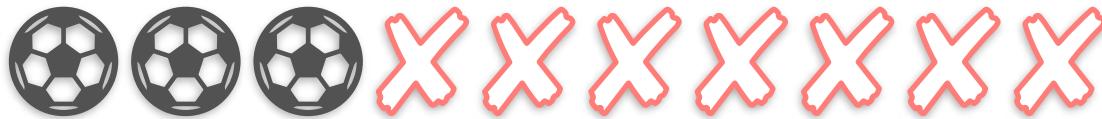


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXX}}{\text{XXXXXX}}$$

# Complement of Probability

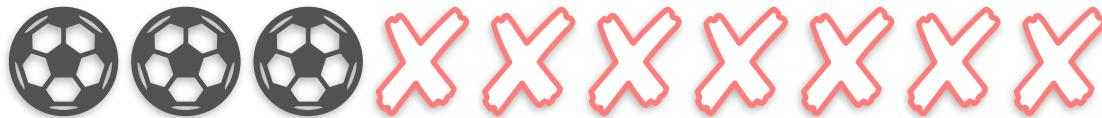


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXXX}}{\text{XXXXXXX}}$$

# Complement of Probability

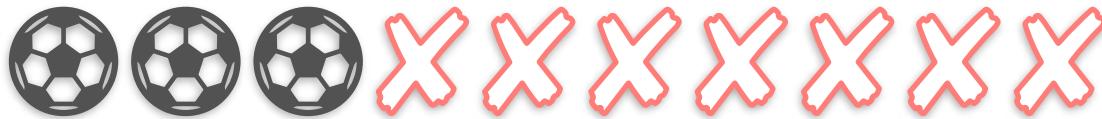


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXXX}}{\text{XXXXXXX}} = \frac{7}{10}$$

# Complement of Probability



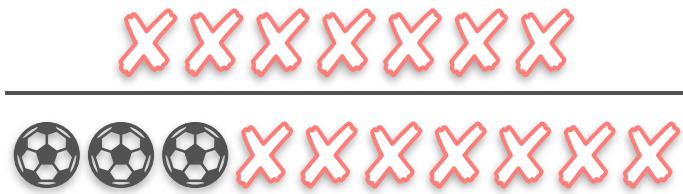
30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXXX}}{\text{XXXXXXX}} = \frac{7}{10} = 0.7$$

# Complement of Probability

# Complement of Probability



$P(\text{not soccer})$

0.7

# Complement of Probability

XXXXXX

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●●●●XXXXXX

$P(\text{not soccer})$

0.7

●●●

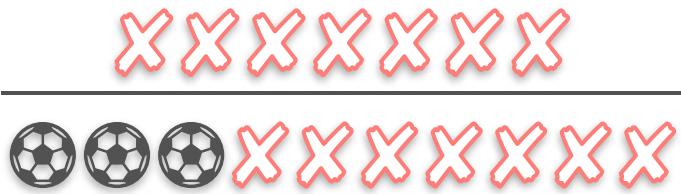
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●●●●XXXXXX

$P(\text{soccer})$

0.3

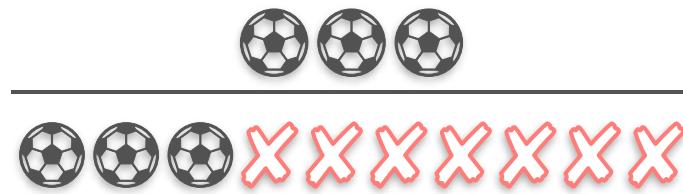
# Complement of Probability



$P(\text{not soccer})$

0.7

+

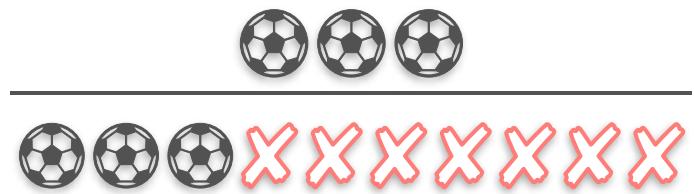
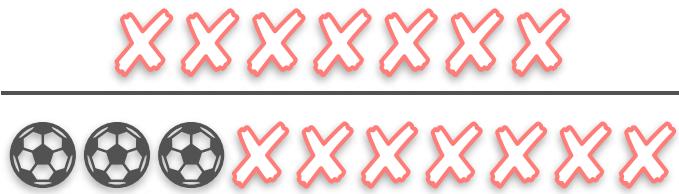


$P(\text{soccer})$

0.3

= 1

# Complement of Probability



$P(\text{not soccer})$

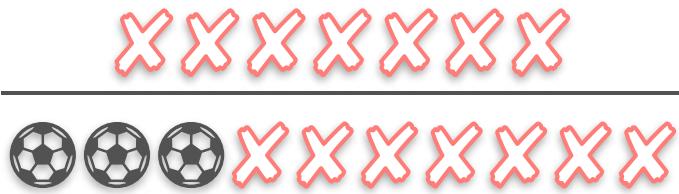
0.7

= 1

$P(\text{soccer})$

0.3

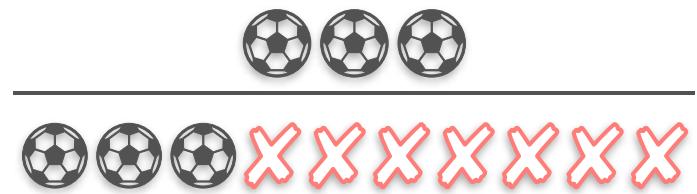
# Complement of Probability



$P(\text{not soccer})$

0.7

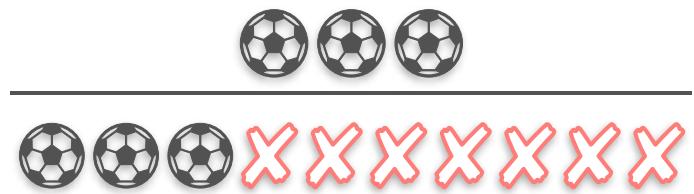
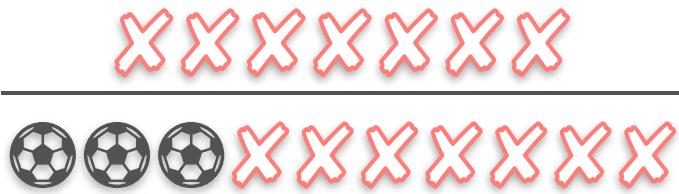
$$= 1 -$$



$P(\text{soccer})$

0.3

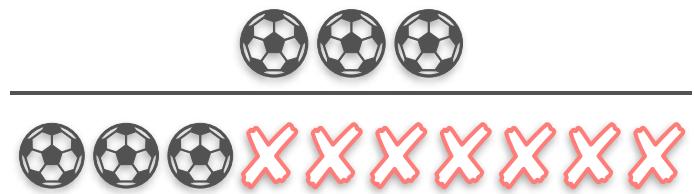
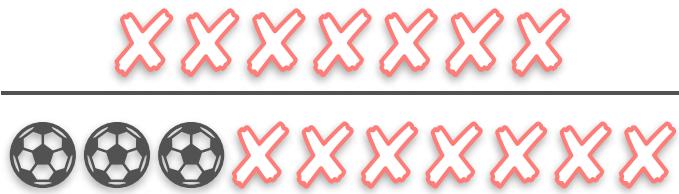
# Complement of Probability



$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$0.7 = 1 - 0.3$$

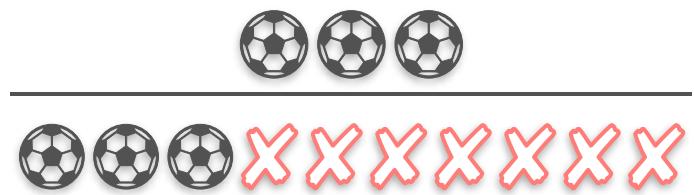
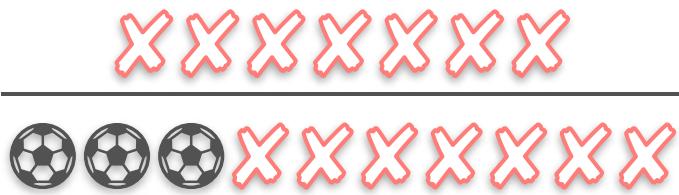
# Complement of Probability



$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$0.7 = 1 - 0.3$$

# Complement of Probability



$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$0.7 = 1 - 0.3$$

Complement Rule

# Complement of Probability

$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

Complement Rule

# Complement of Probability

$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$P(A') = 1 - P(A)$$

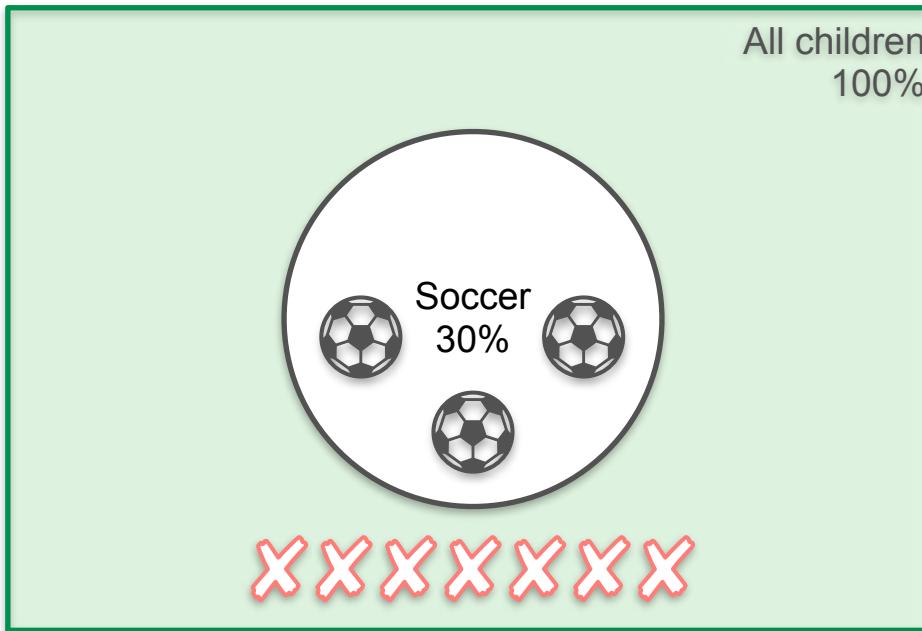
Complement Rule

# Complement of Probability: Venn Diagram

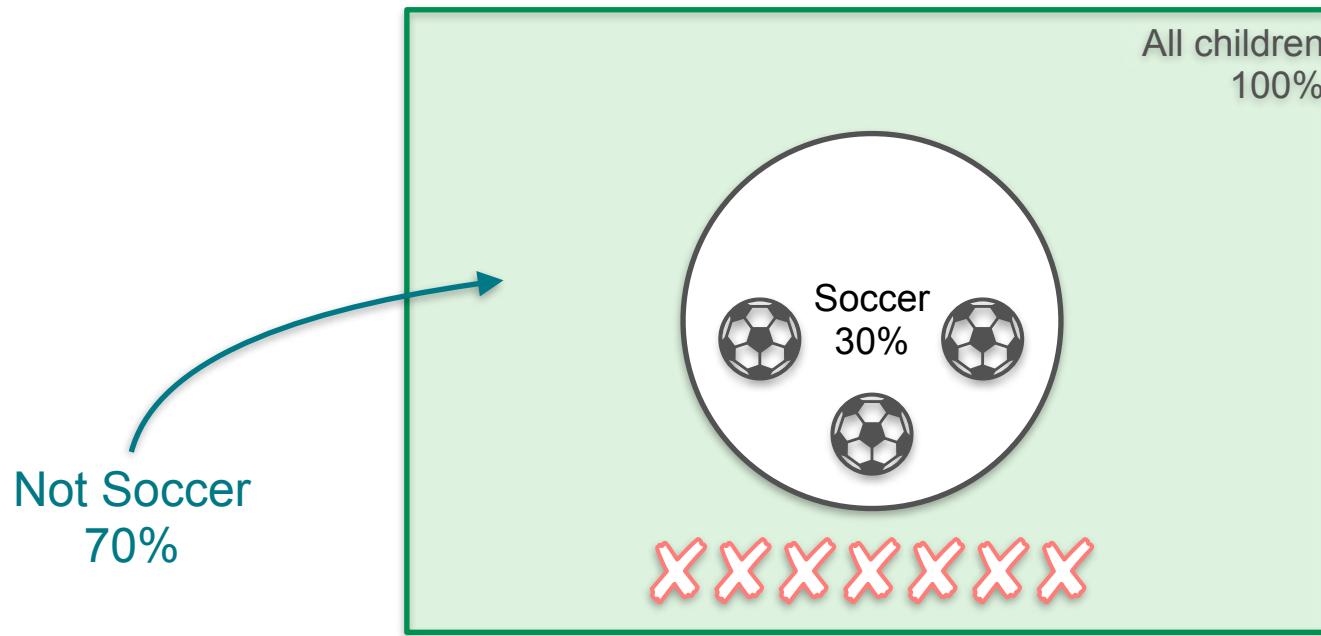
# Complement of Probability: Venn Diagram



# Complement of Probability: Venn Diagram



# Complement of Probability: Venn Diagram



# Complement of Probability: Coin Example 1



# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH)$$

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) =$$

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1$$

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

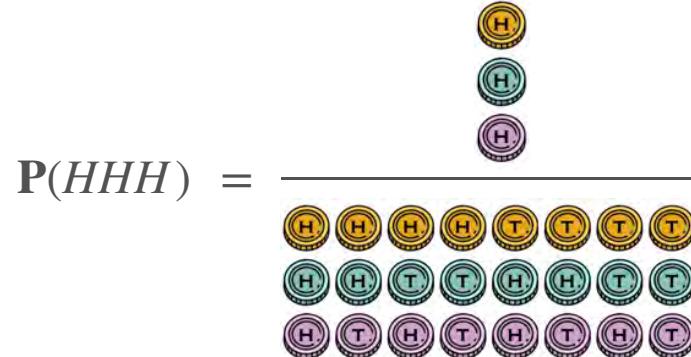
$$\mathbf{P}(HHH) =$$

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

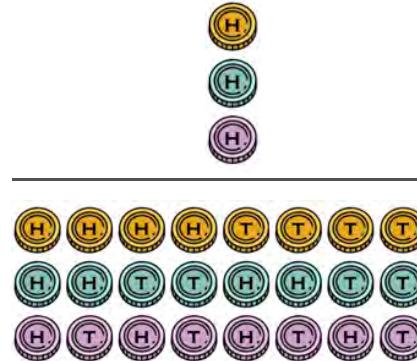


# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$



# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{\text{_____}}{\text{_____}}$$

The denominator consists of three rows of 8 coins each, totaling 24 coins. The top row has all heads (H). The middle row has 7 heads and 1 tail (T). The bottom row has 4 heads and 4 tails (T).

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

# Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

# Complement of Probability: Coin Example 1

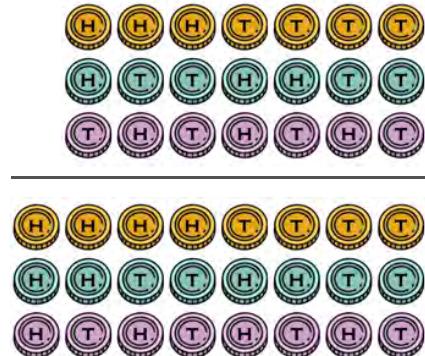


What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$



# Complement of Probability: Dice Example 1

# Complement of Probability: Dice Example 1



# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?

# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?

# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$

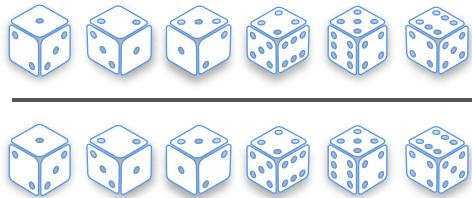
# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) = \frac{\text{Number of outcomes not 6}}{\text{Total number of outcomes}}$$



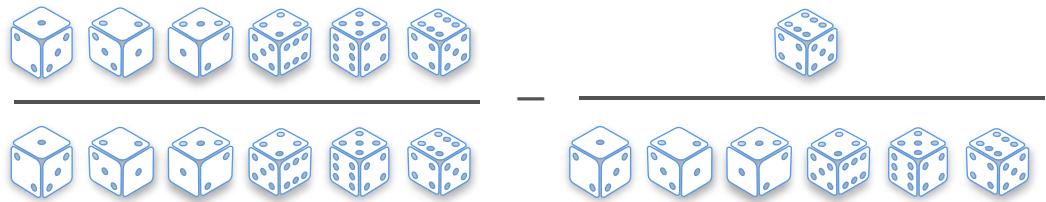
# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



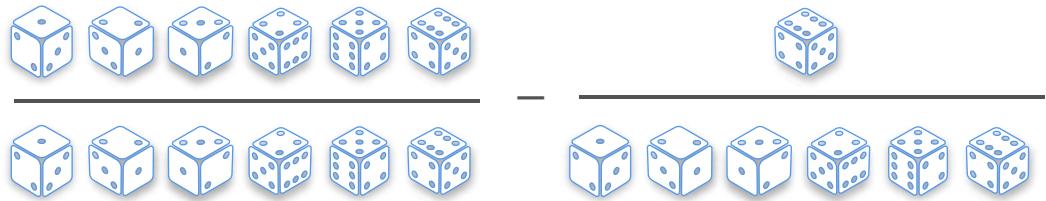
# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



=

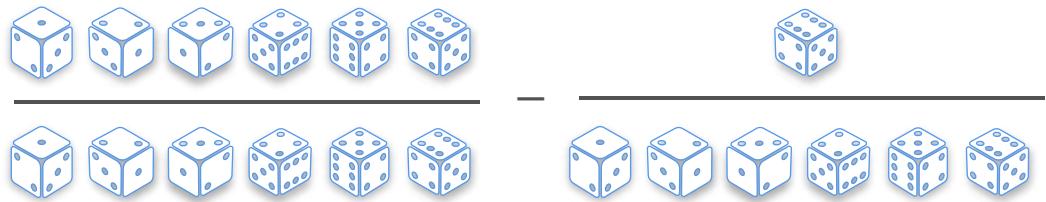
# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



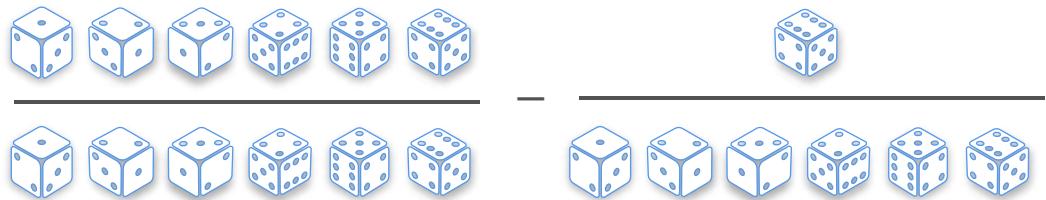
# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



$$= \frac{\text{---}}{\text{---}}$$

# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$

$$\frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}} - \frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$
$$= \frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$

# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$

$$\frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}} - \frac{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{cccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$
$$= \frac{\begin{array}{ccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \\ \hline \end{array}}{\begin{array}{ccccc} \text{dice} & \text{dice} & \text{dice} & \text{dice} & \text{dice} \end{array}}$$

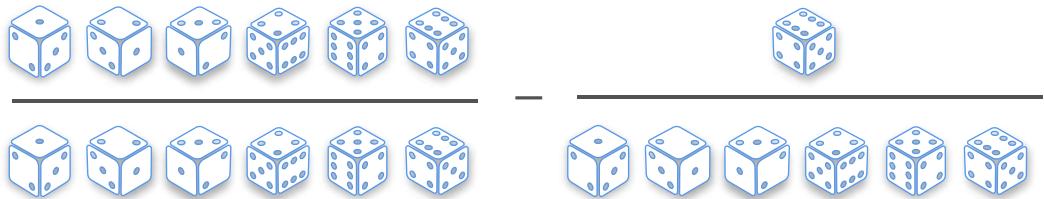
# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



$$= \frac{\text{dice showing 1 through 5}}{\text{dice showing 1 through 6}}$$

=

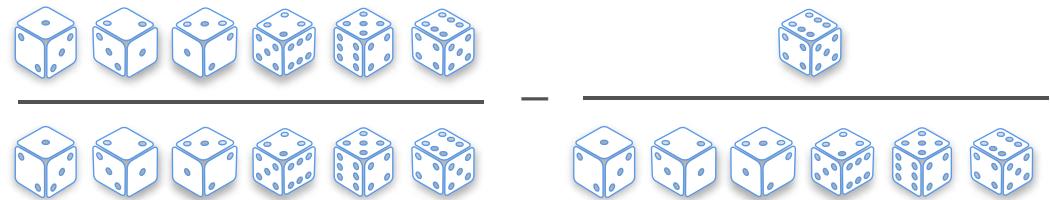
# Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



$$\begin{aligned} &= \frac{\text{Number of outcomes not 6}}{\text{Total number of outcomes}} \\ &= \frac{5}{6} \end{aligned}$$



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# Introduction to probability

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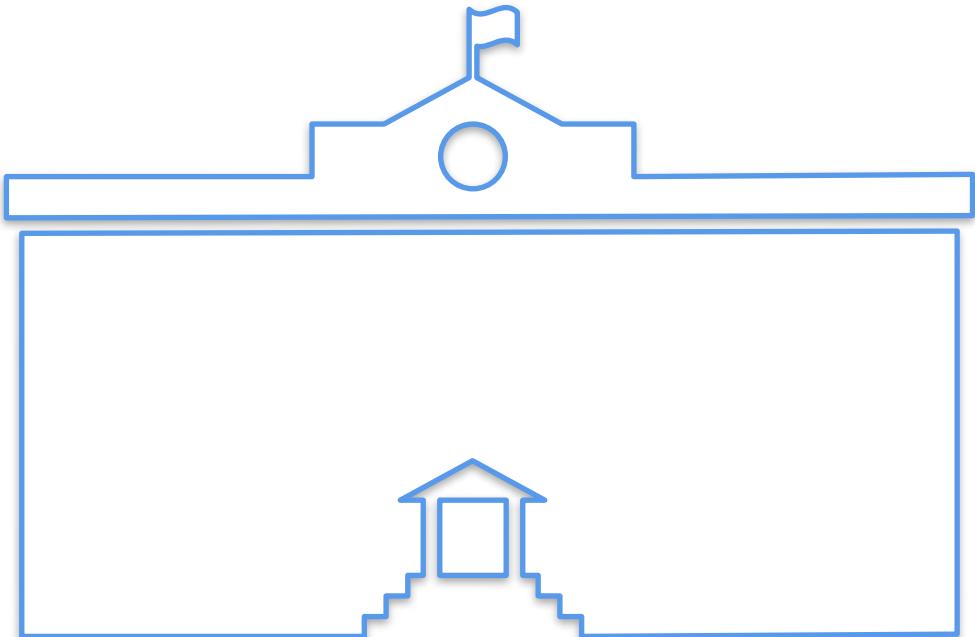
## Sum of Probabilities

# Sum of Probabilities: Quiz 1

# Sum of Probabilities: Quiz 1

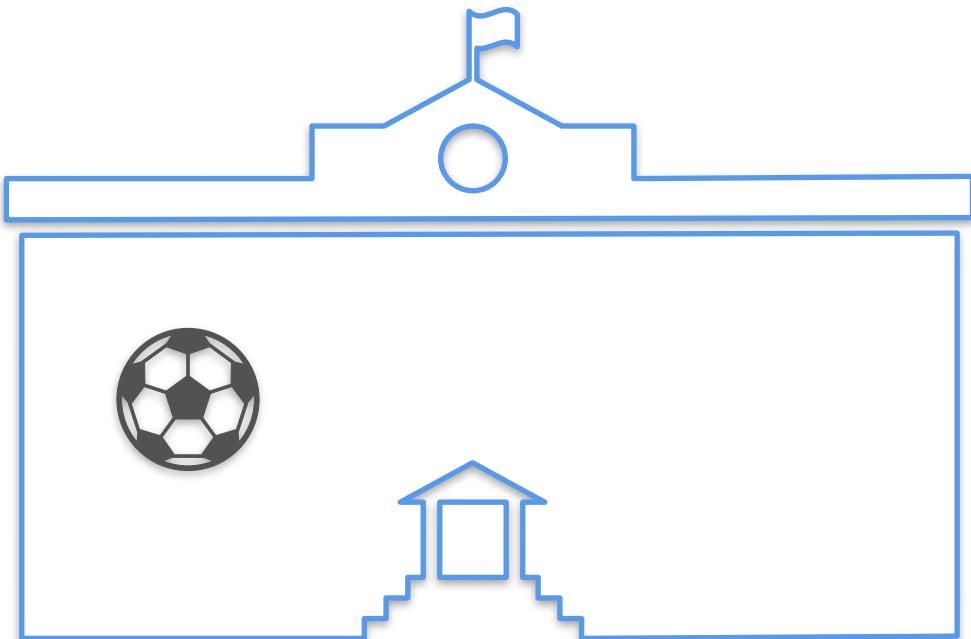
At a school, kids can only play one sport.

# Sum of Probabilities: Quiz 1



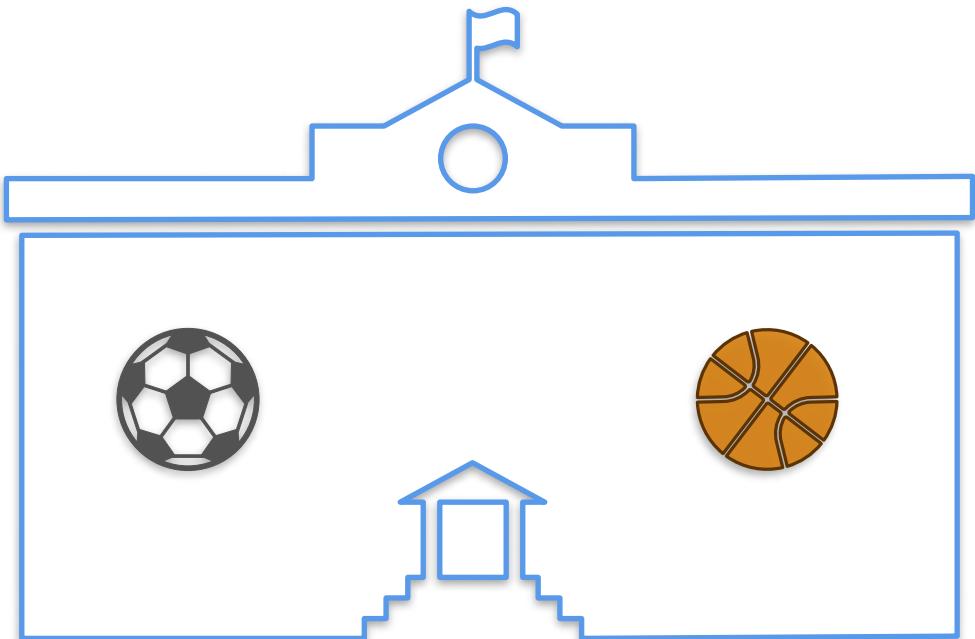
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# Sum of Probabilities: Quiz 1



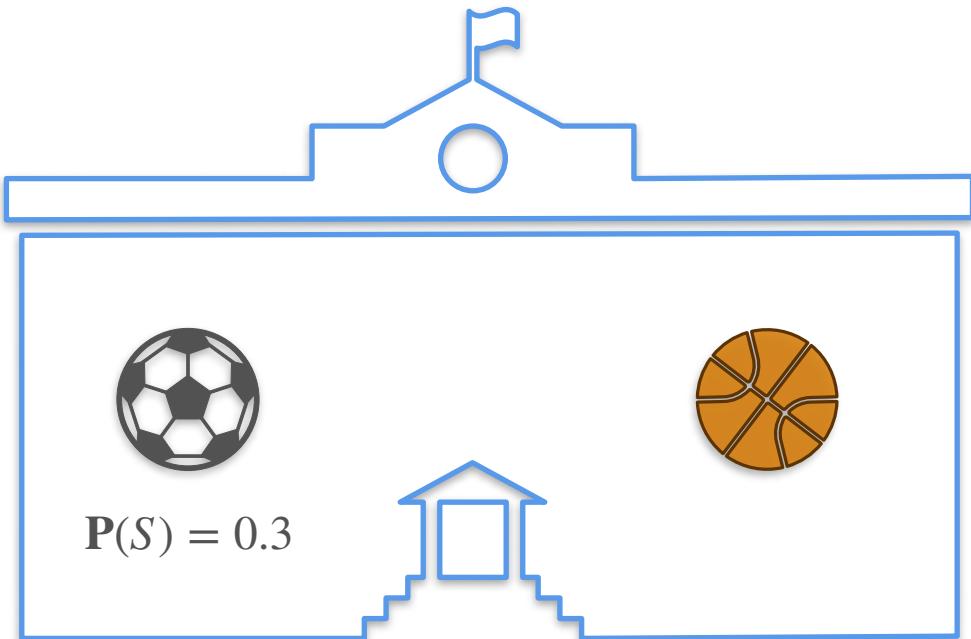
At a school, kids can only play one sport.

# Sum of Probabilities: Quiz 1



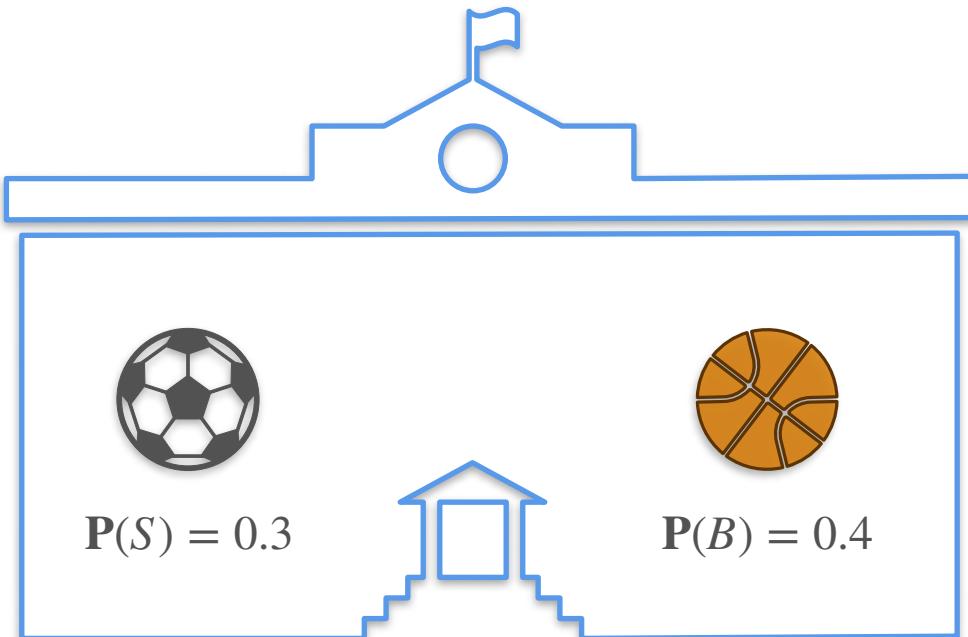
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# Sum of Probabilities: Quiz 1



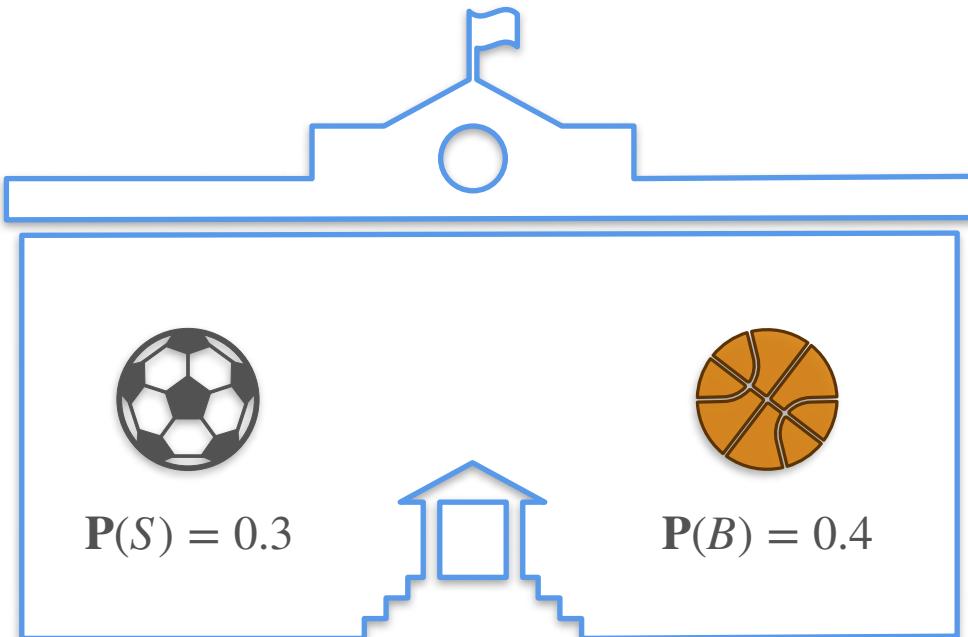
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# Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

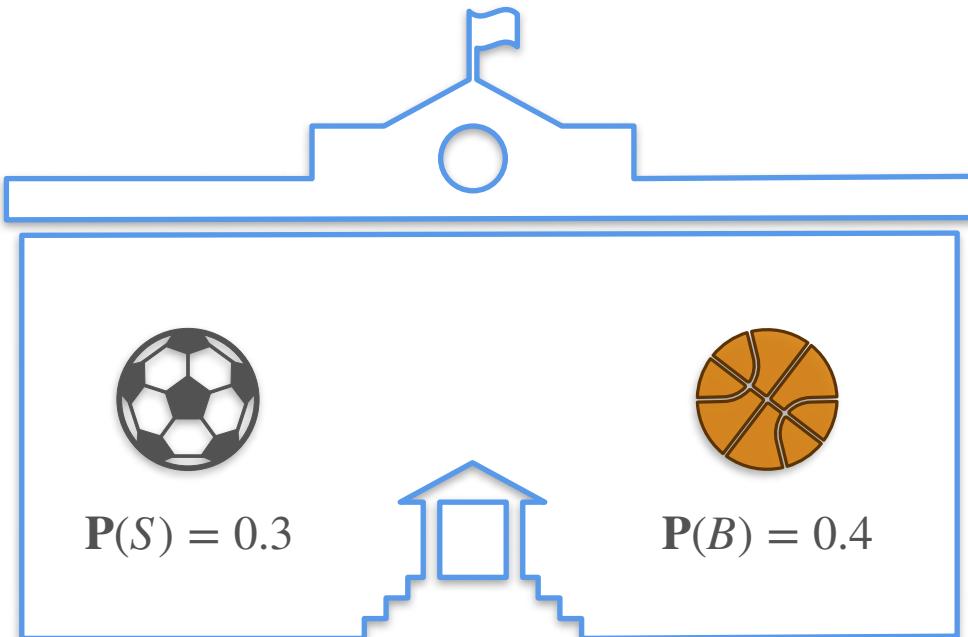
# Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

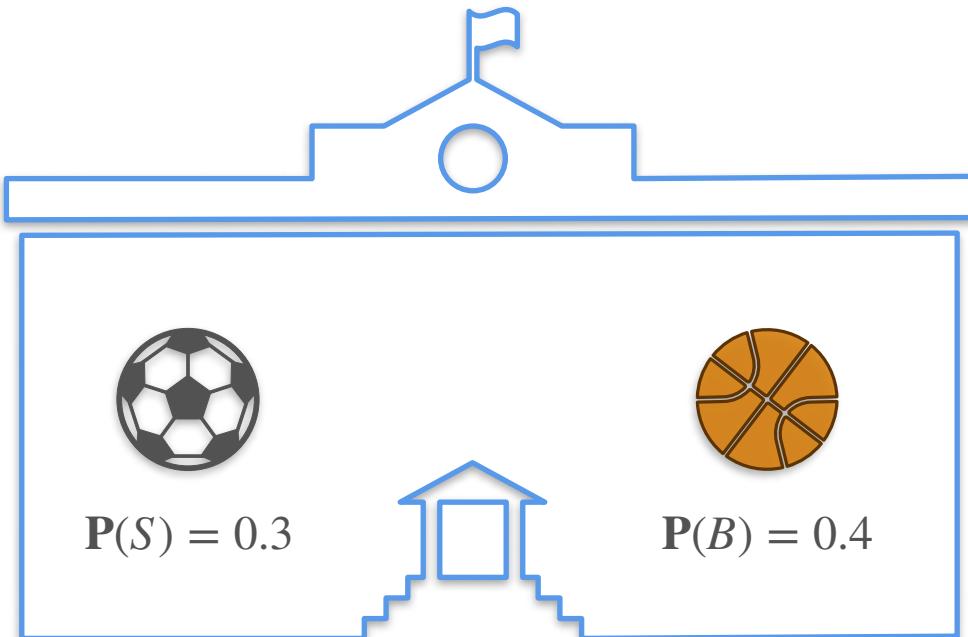
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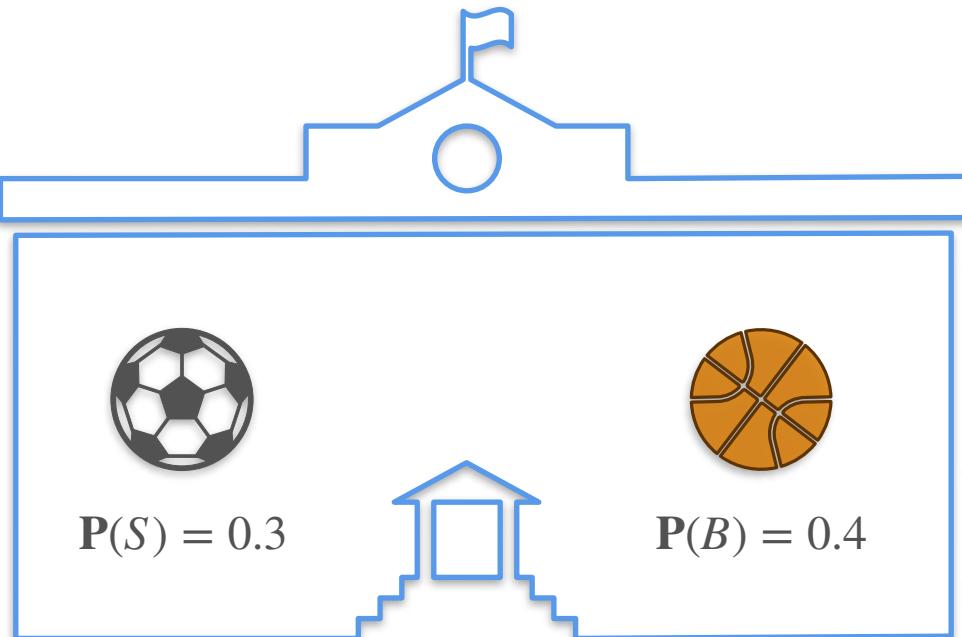
# Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

# Sum of Probabilities: Quiz 1



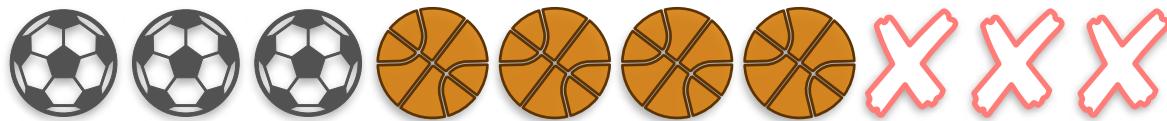
At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

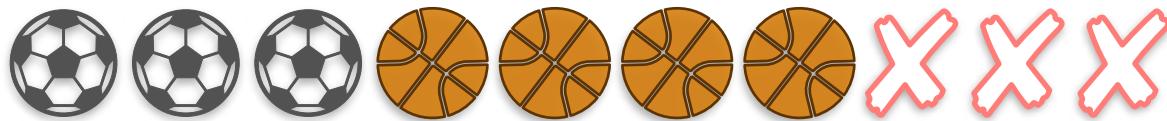
Hint: What if there were only 10 kids?

# Sum of Probabilities: Quiz 1 Solution

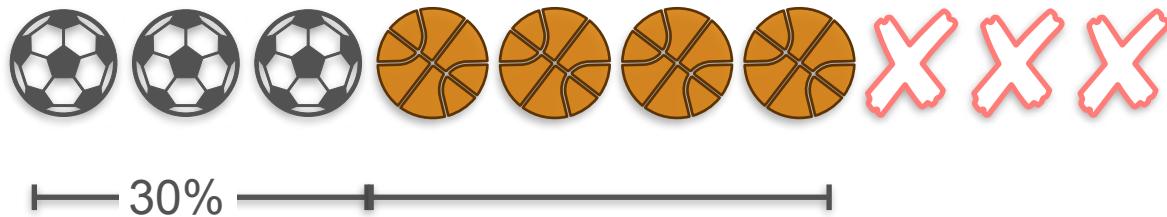
# Sum of Probabilities: Quiz 1 Solution



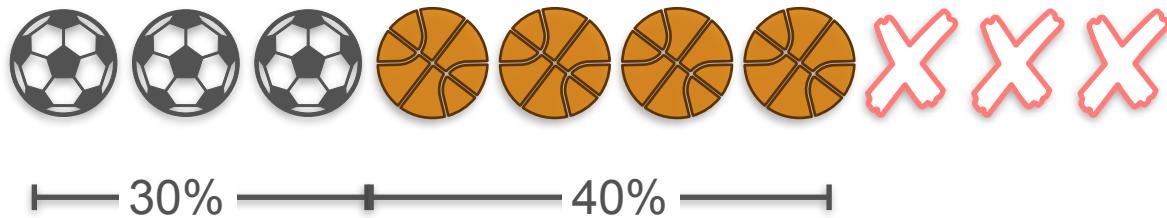
# Sum of Probabilities: Quiz 1 Solution



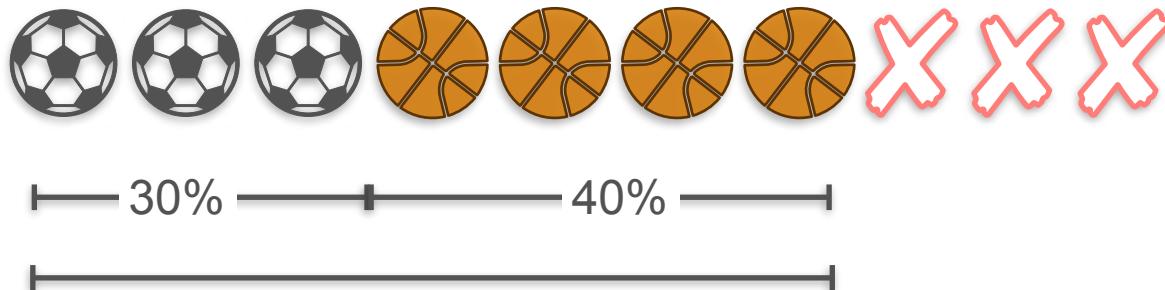
# Sum of Probabilities: Quiz 1 Solution



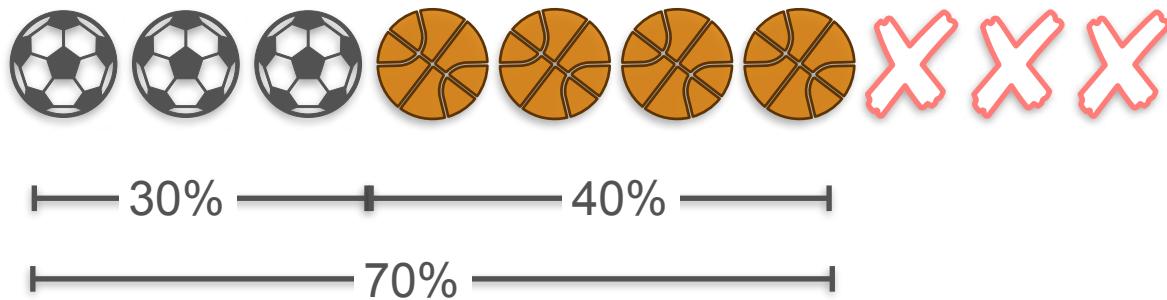
# Sum of Probabilities: Quiz 1 Solution



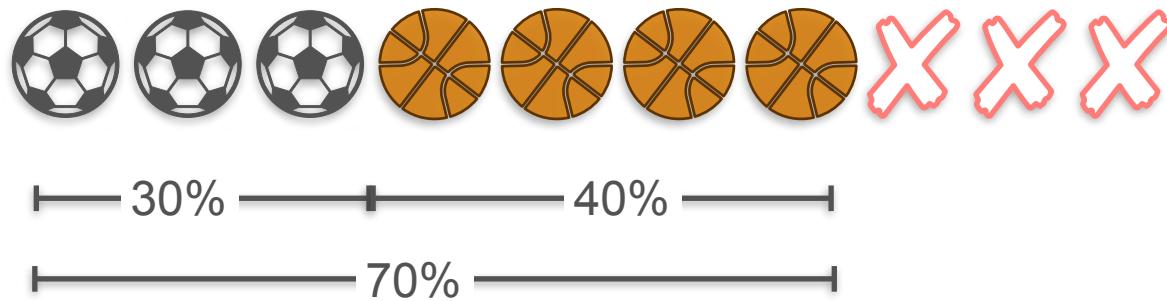
# Sum of Probabilities: Quiz 1 Solution



# Sum of Probabilities: Quiz 1 Solution

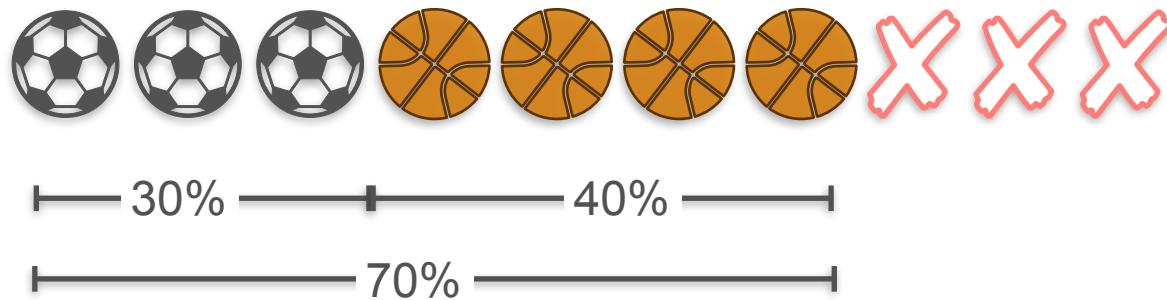


# Sum of Probabilities: Quiz 1 Solution



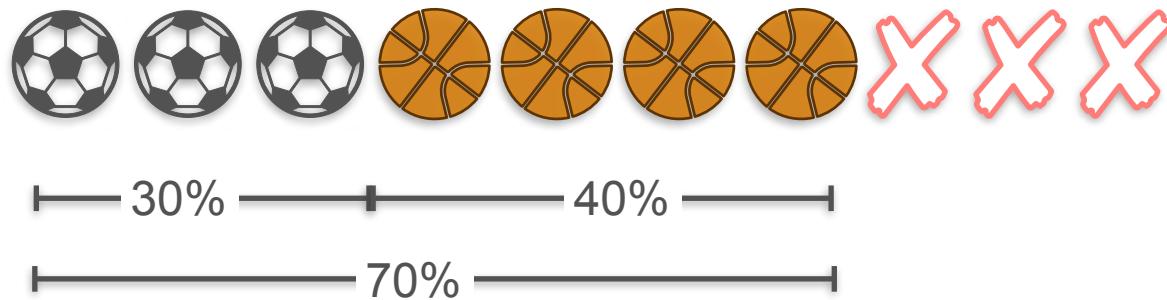
$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}}$$

# Sum of Probabilities: Quiz 1 Solution



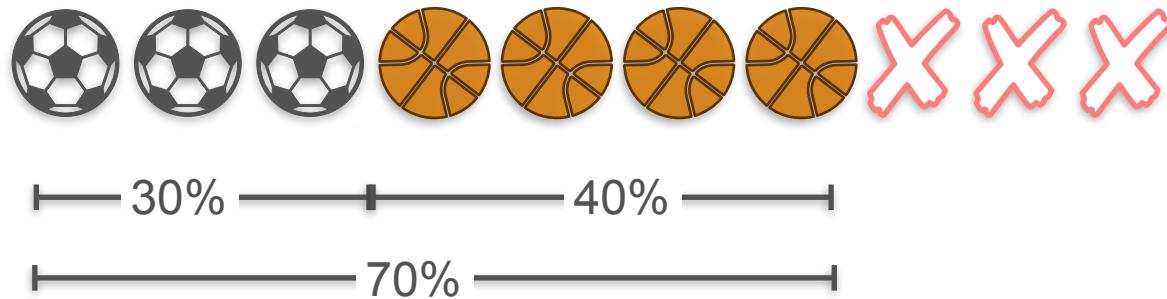
$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3 + 4}{10}$$

# Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3 + 4}{10} = 0.7$$

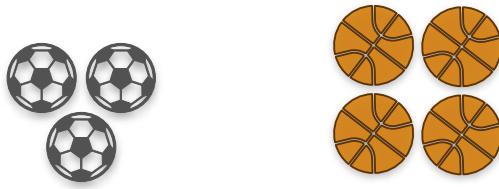
# Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3 + 4}{10} = 0.7$$

$$P(\text{soccer or basketball}) = P(\text{soccer}) + P(\text{basketball})$$

# Sum of Probabilities: Quiz 1 Solution



XXX

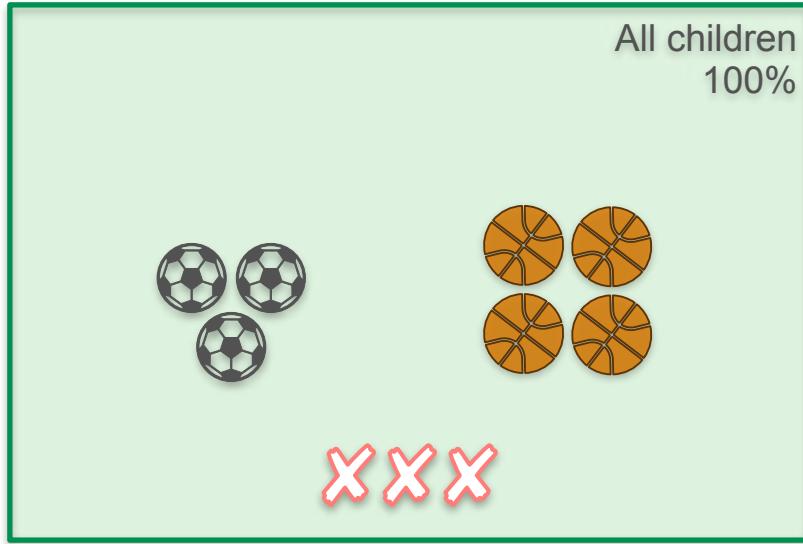
# Sum of Probabilities: Quiz 1 Solution



XXX

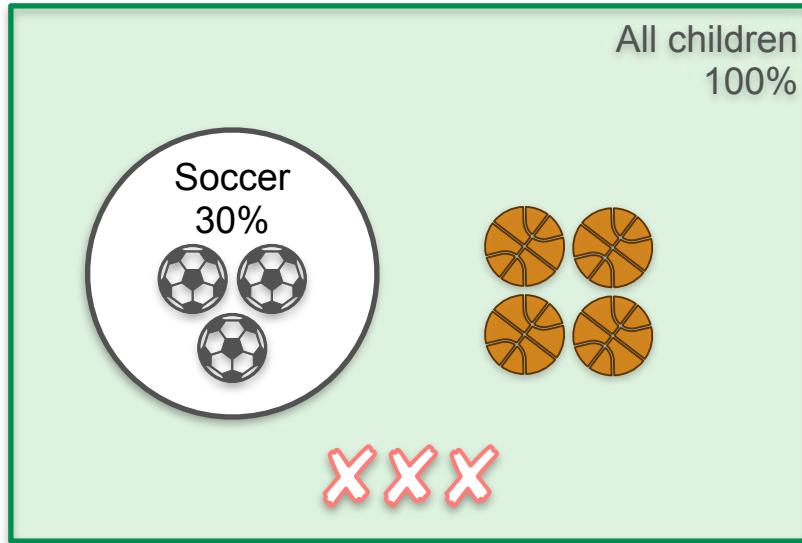
$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

# Sum of Probabilities: Quiz 1 Solution



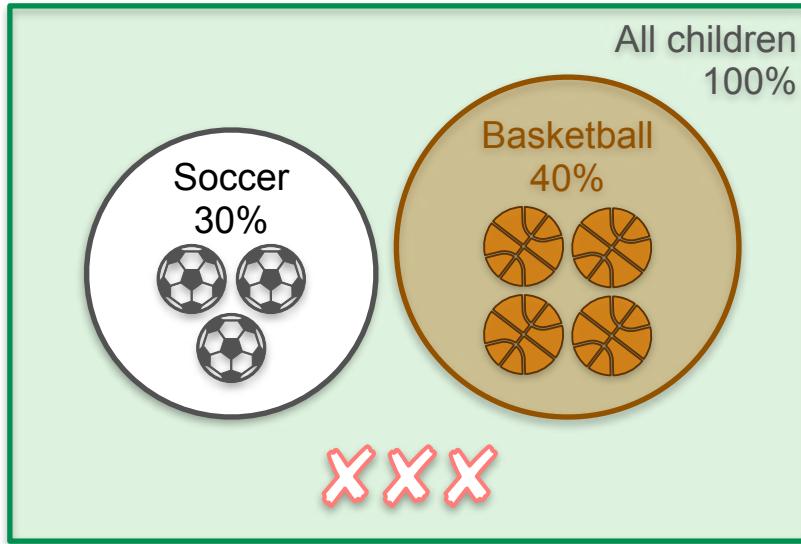
$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

# Sum of Probabilities: Quiz 1 Solution



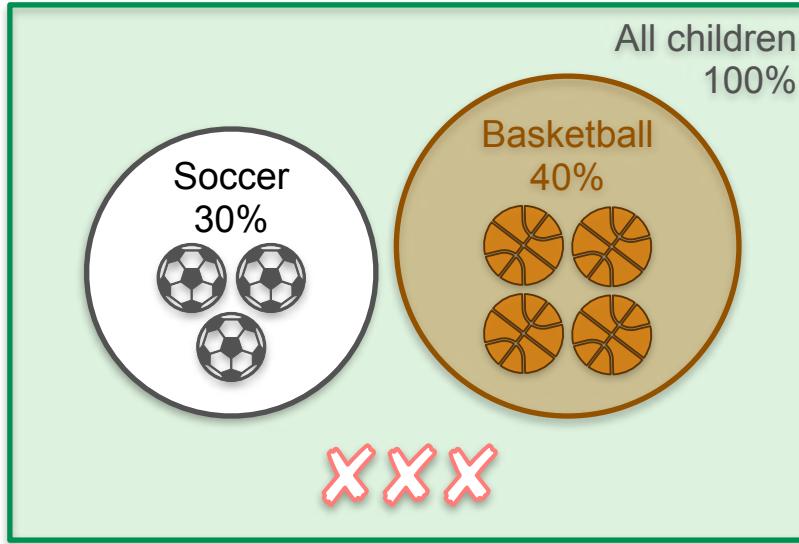
$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

# Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

# Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

$$P(A \cup B) = P(A) + P(B)$$

# Sum of Probabilities: Dice Example 1

# Sum of Probabilities: Dice Example 1



# Sum of Probabilities: Dice Example 1



What is the probability of obtaining  
an even number or a 5?

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining  
an even number or a 5?

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A

B

# Sum of Probabilities: Dice Example 1



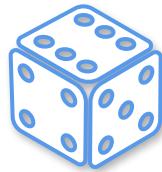
What is the probability of obtaining an even number or a 5?

A



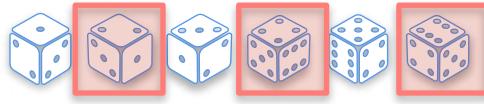
B

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



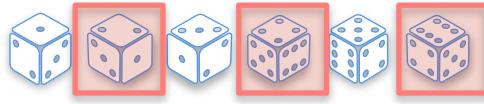
B

# Sum of Probabilities: Dice Example 1



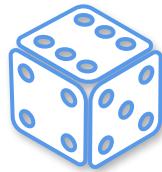
What is the probability of obtaining an even number or a 5?

A



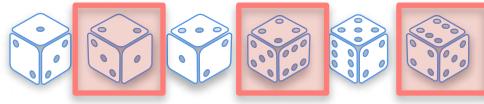
B

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



B

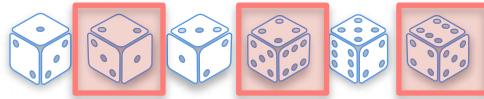


# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

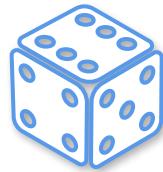
A



B

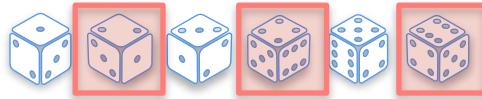


# Sum of Probabilities: Dice Example 1

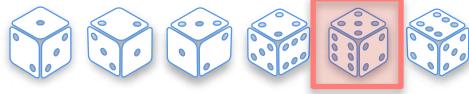


What is the probability of obtaining an even number or a 5?

A



B

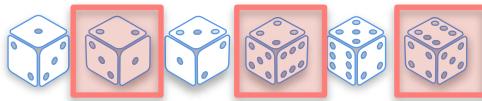


# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



B



+

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



B



+



# Sum of Probabilities: Dice Example 1

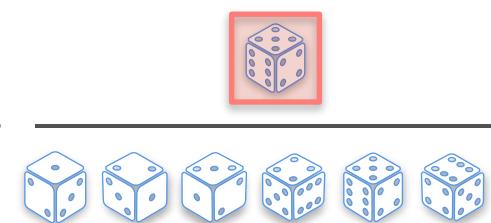
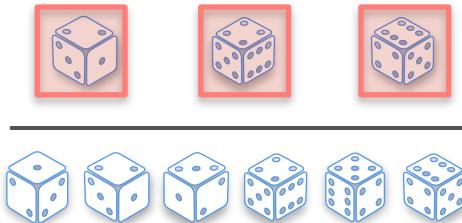


What is the probability of obtaining an even number or a 5?

A



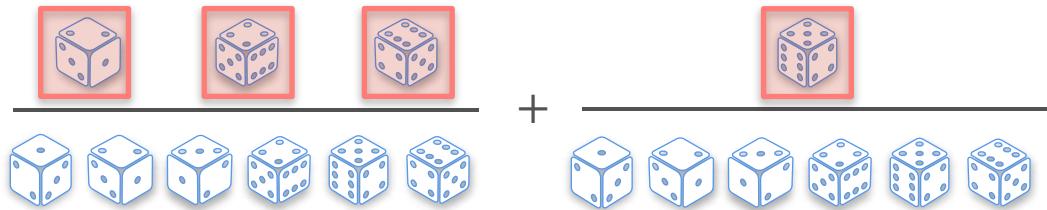
B



# Sum of Probabilities: Dice Example 1



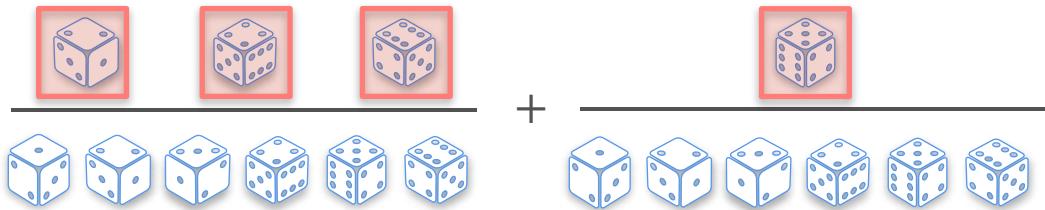
What is the probability of obtaining an even number or a 5?



# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

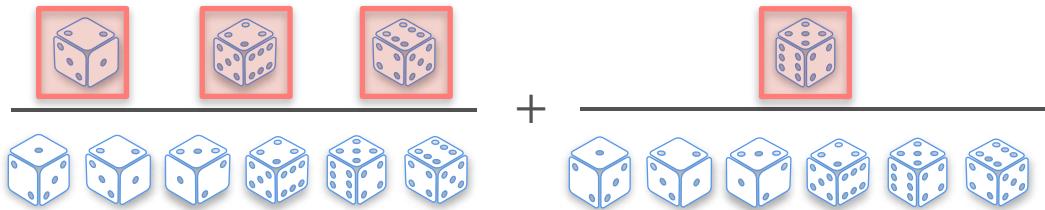


$$P(\text{even number or } 5) =$$

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

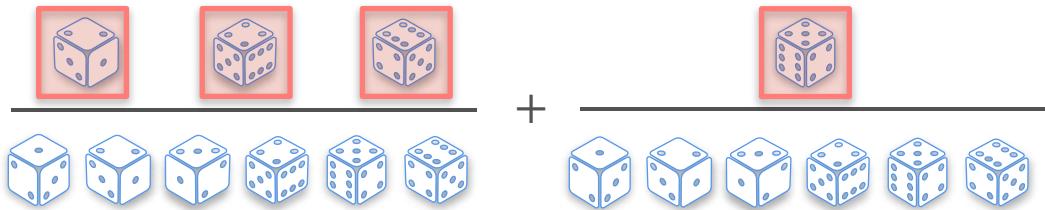


$$P(\text{even number or } 5) = P(\text{even number})$$

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

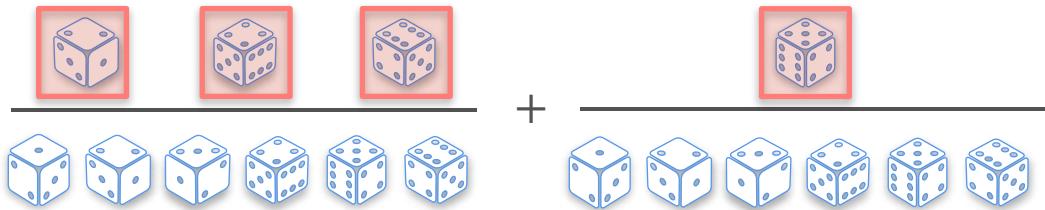


$$P(\text{even number or 5}) = P(\text{even number}) + P(5)$$

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

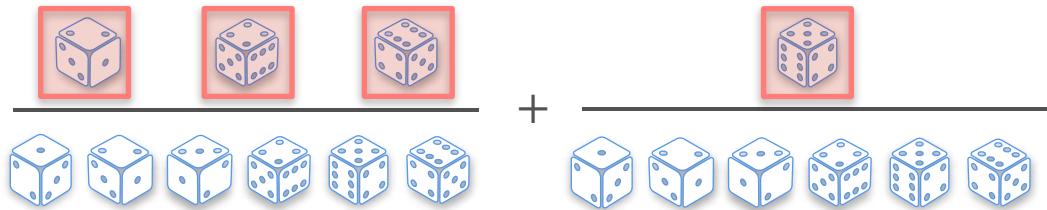


$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6}$$

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

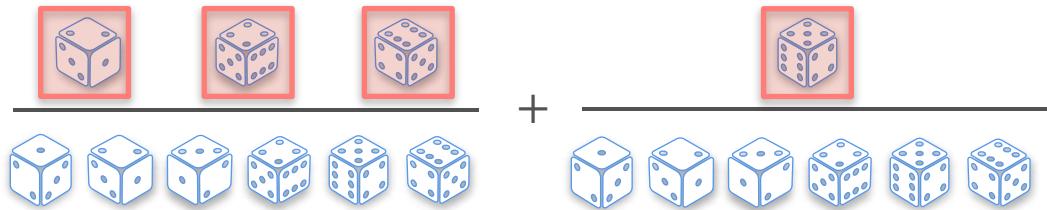


$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6} + \frac{1}{6}$$

# Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

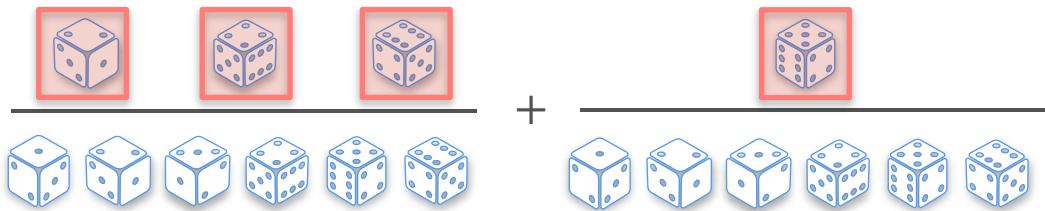


$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

# Sum of Probabilities: Dice Example 1



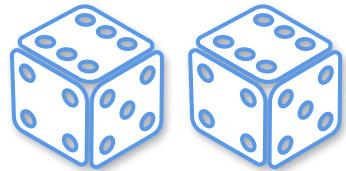
What is the probability of obtaining an even number or a 5?



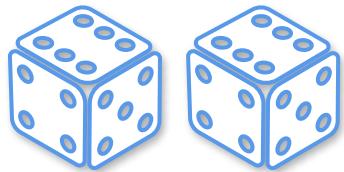
$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

# Sum of Probabilities: Dice Example 2

# Sum of Probabilities: Dice Example 2



# Sum of Probabilities: Dice Example 2



What is the probability of obtaining a sum of 7 or a sum of 10?

# Sum of Probabilities: Dice Example 2

# Sum of Probabilities: Dice Example 2

*A*

*B*

# Sum of Probabilities: Dice Example 2

*A*

sum of 7

*B*

sum of 10

# Sum of Probabilities: Dice Example 2

A

sum of 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6
1,6						

B

sum of 10

# Sum of Probabilities: Dice Example 2

A

sum of 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6

B

sum of 10

# Sum of Probabilities: Dice Example 2

A

sum of 7

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

sum of 10

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities: Dice Example 2

*A* or *B*

sum of 7 or sum of 10

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities: Dice Example 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

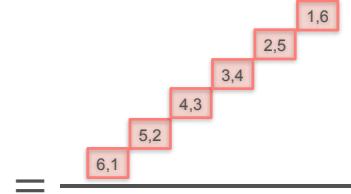
P(sum of 7 or sum of 10)

= \_\_\_\_\_

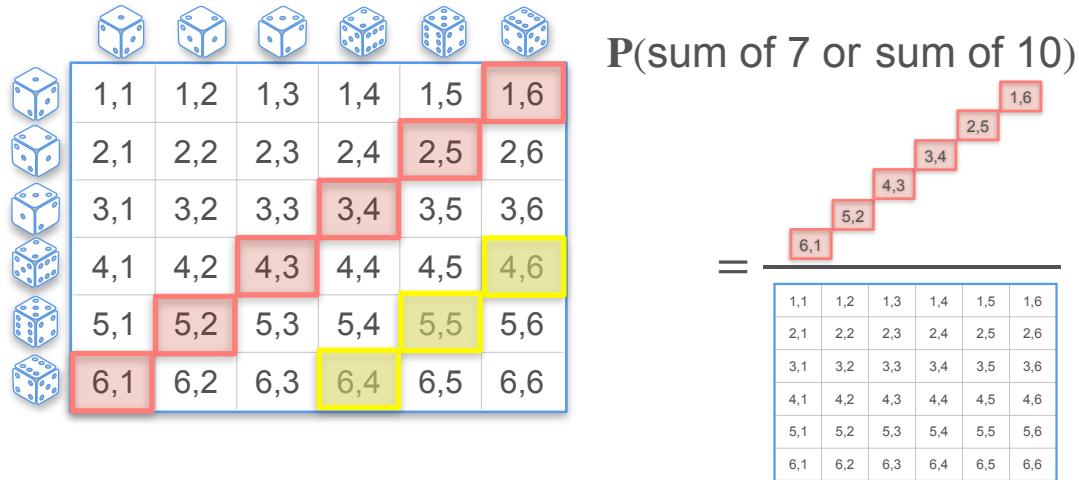
# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)



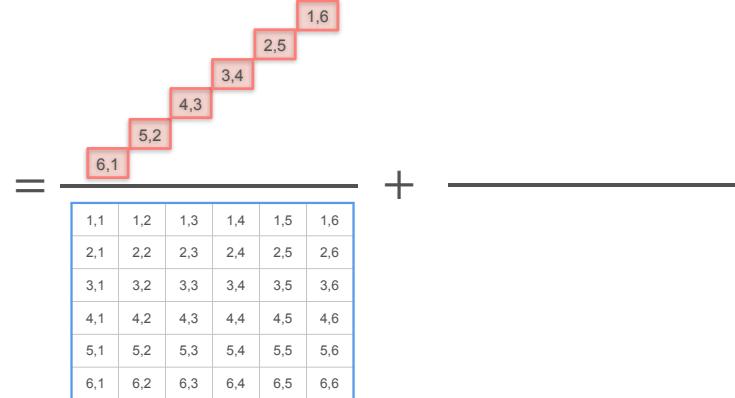
# Sum of Probabilities: Dice Example 2



# Sum of Probabilities: Dice Example 2



**P(sum of 7 or sum of 10)**



# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

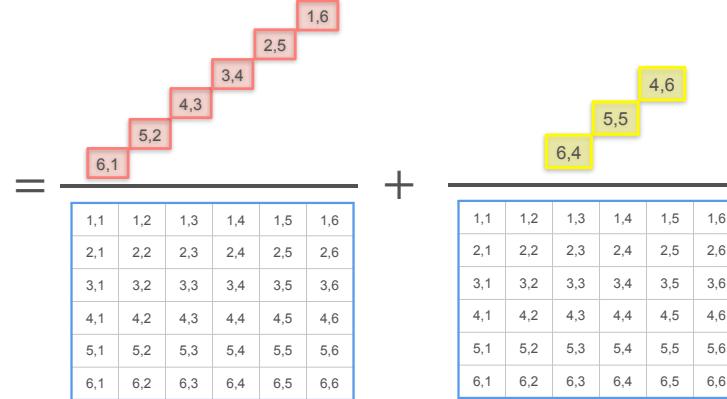
P(sum of 7 or sum of 10)



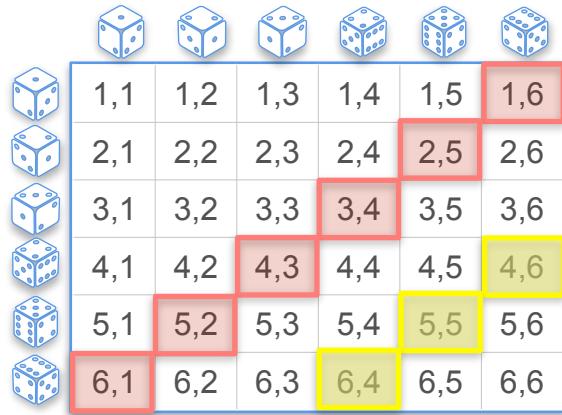
# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)



# Sum of Probabilities: Dice Example 2



**P(sum of 7 or sum of 10)**

$$\begin{array}{c}
 \begin{array}{c}
 = - \frac{1}{36} \left[ \begin{array}{cccccc}
 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\
 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\
 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\
 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\
 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\
 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6
 \end{array} \right] + \begin{array}{cccccc}
 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\
 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\
 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\
 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\
 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\
 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6
 \end{array} \right] \\
 \end{array}
 \end{array}$$

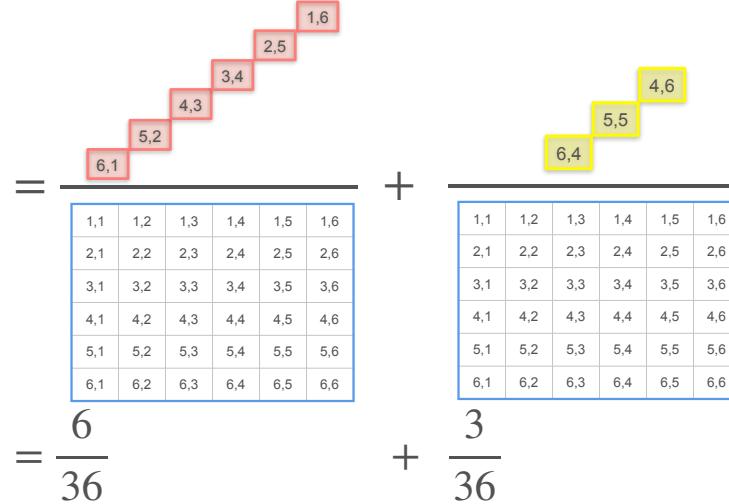
# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

$$\begin{aligned} &= \frac{\text{Number of outcomes for sum 7}}{\text{Total number of outcomes}} + \frac{\text{Number of outcomes for sum 10}}{\text{Total number of outcomes}} \\ &= \frac{6}{36} + \frac{3}{36} \end{aligned}$$

The diagram illustrates the calculation of the probability of rolling a sum of 7 or 10 with two six-sided dice. It shows the sample space as a 6x6 grid of outcomes. Outcomes are highlighted in red for a sum of 7 and yellow for a sum of 10. The total number of outcomes is 36. The outcomes for a sum of 7 are (1,6), (2,5), (3,4), (4,3), (5,2), and (6,1). The outcomes for a sum of 10 are (4,6), (5,5), (6,4), and (6,6).



# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

$$\begin{aligned} &= \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} \end{aligned}$$

# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

$$\begin{aligned} &= \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,6 \\ 2,5 \\ 3,4 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} = \frac{1}{4} \end{aligned}$$

# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

$$\begin{aligned} &= \boxed{P(\text{sum of 7})} + \frac{\begin{array}{|c|c|c|c|c|c|c|}\hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ \hline 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ \hline 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ \hline 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ \hline 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ \hline 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \\ \hline \end{array}}{36} \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} = \frac{1}{4} \end{aligned}$$

# Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

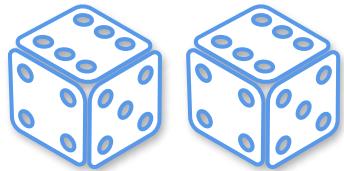
P(sum of 7 or sum of 10)

$$\begin{aligned} &= \boxed{P(\text{sum of 7})} + \boxed{P(\text{sum of 10})} \\ &= \frac{6}{36} + \frac{3}{36} \\ &= \frac{9}{36} = \frac{1}{4} \end{aligned}$$

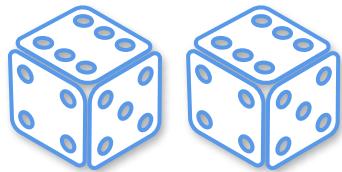
The diagram illustrates the calculation of the probability of rolling a sum of 7 or 10. It shows a 6x6 grid of dice outcomes. Outcomes where the sum is 7 are highlighted in red boxes (e.g., (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)). Outcomes where the sum is 10 are highlighted in yellow boxes (e.g., (4,6), (5,5), (6,4)). The total number of favorable outcomes is 9, while the total number of possible outcomes is 36.

# Sum of Probabilities: Dice Example 3

# Sum of Probabilities: Dice Example 3



# Sum of Probabilities: Dice Example 3



What is the probability of obtaining  
a difference of 2 or a difference of 1?

# Sum of Probabilities: Dice Example 3

# Sum of Probabilities: Dice Example 3

*A*

diff = 2

*B*

diff = 1

# Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	1,1	1,2	1,3	1,4	1,5	1,6
2,2	2,1	2,2	2,3	2,4	2,5	2,6
2,3	3,1	3,2	3,3	3,4	3,5	3,6
2,4	4,1	4,2	4,3	4,4	4,5	4,6
2,5	5,1	5,2	5,3	5,4	5,5	5,6
2,6	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

# Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

# Sum of Probabilities: Dice Example 3

A

diff = 2

		dice					
		1,1	1,2	1,3	1,4	1,5	1,6
		2,1	2,2	2,3	2,4	2,5	2,6
		3,1	3,2	3,3	3,4	3,5	3,6
		4,1	4,2	4,3	4,4	4,5	4,6
		5,1	5,2	5,3	5,4	5,5	5,6
		6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

		dice					
		1,1	1,2	1,3	1,4	1,5	1,6
		2,1	2,2	2,3	2,4	2,5	2,6
		3,1	3,2	3,3	3,4	3,5	3,6
		4,1	4,2	4,3	4,4	4,5	4,6
		5,1	5,2	5,3	5,4	5,5	5,6
		6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities: Dice Example 3

*A* or *B*

diff = 2 or diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Dice icons are placed along the top row and left column of the grid.

# Sum of Probabilities: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

# Sum of Probabilities: Dice Example 3

					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

# Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

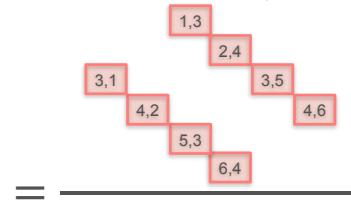
$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

= \_\_\_\_\_

# Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

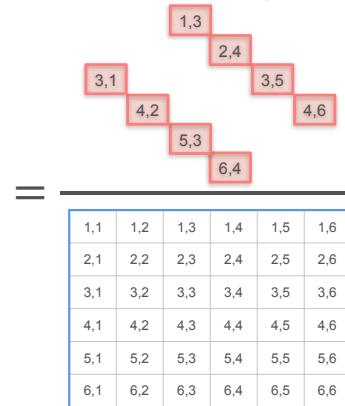
$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$



# Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$



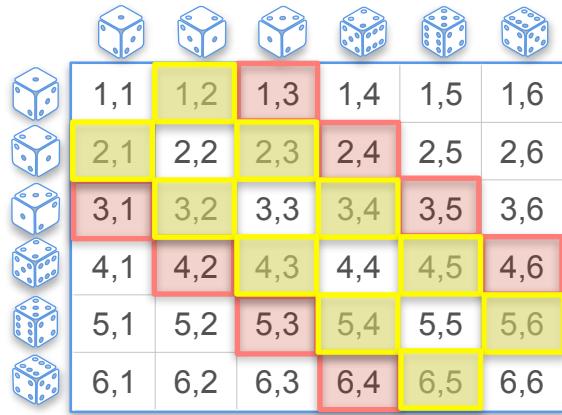
# Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \boxed{\begin{array}{ccccccc} & & 1,3 & & 2,4 & & \\ & & 3,1 & 4,2 & & 5,3 & 6,4 \\ & & & & 5,3 & & \\ & & & & & 6,4 & \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}} + \boxed{\begin{array}{ccccccc} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{array}}$$

# Sum of Probabilities: Dice Example 3



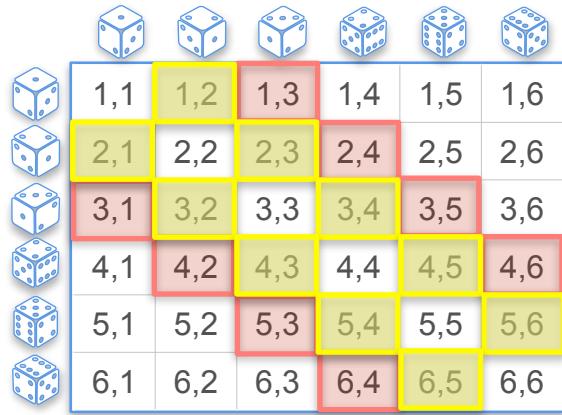
$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \begin{array}{c} \text{red boxes: } (1,3), (2,4), (3,5), (4,6), (5,3), (6,4) \\ \text{yellow boxes: } (1,2), (2,3), (3,2), (4,3), (5,4), (6,5) \\ \text{blue boxes: } (1,1), (2,1), (3,1), (4,1), (5,1), (6,1) \\ \text{orange boxes: } (1,6), (2,6), (3,6), (4,6), (5,6), (6,6) \end{array} + \begin{array}{c} \text{red boxes: } (1,3), (2,4), (3,5), (4,6) \\ \text{yellow boxes: } (1,2), (2,3), (3,4), (4,5), (5,6), (6,5) \\ \text{blue boxes: } (1,1), (2,1), (3,1), (4,1), (5,1), (6,1) \end{array}$$

$=$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities: Dice Example 3

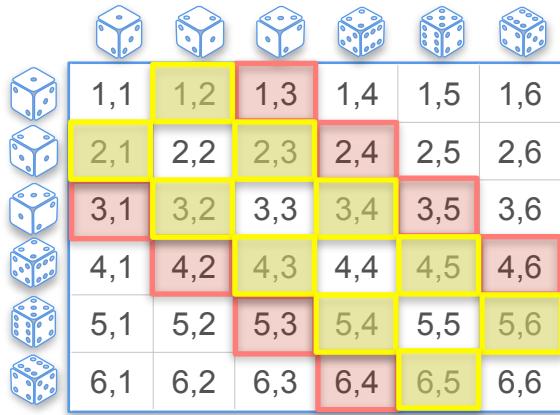


$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \underbrace{\begin{array}{cccccc} & & 1,3 & & & \\ & & 2,4 & & & \\ & 3,1 & & 3,5 & & \\ & 4,2 & & 4,6 & & \\ & 5,3 & & & & \\ & 6,4 & & & & \end{array}}_{+} + \underbrace{\begin{array}{cccccc} 1,2 & & & & & \\ 2,1 & & 2,3 & & & \\ 3,2 & & & 3,4 & & \\ 4,3 & & & 4,5 & & \\ 5,4 & & & 5,6 & & \\ 6,5 & & & & & \end{array}}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

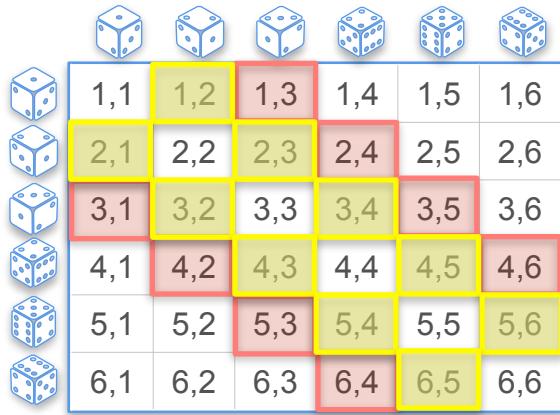
# Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{c} 1,3 \\ 2,4 \\ 3,5 \\ 4,6 \\ 5,3 \\ 6,4 \end{array}}{36} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{36}$$
$$= \frac{8}{36}$$

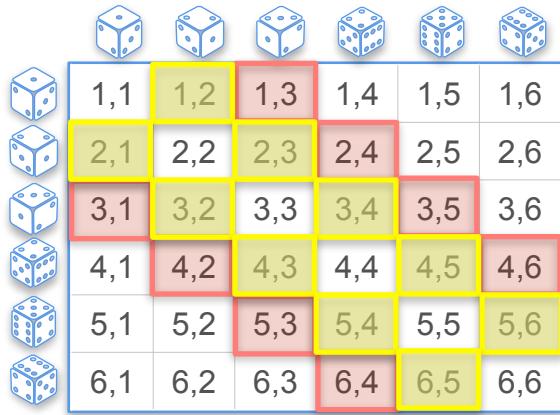
# Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{c} 1,3 \\ 2,4 \\ 3,5 \\ 4,6 \\ 5,3 \\ 6,4 \end{array}}{36} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{36}$$
$$= \frac{8}{36} + \frac{10}{36}$$

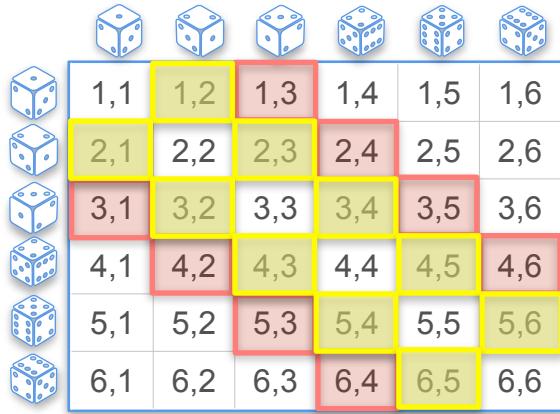
# Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{c} 1,3 \\ 2,4 \\ 3,5 \\ 4,6 \\ 5,3 \\ 6,4 \end{array}}{36} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{36} \\ &= \frac{8}{36} + \frac{10}{36} \\ &= \frac{18}{36} \end{aligned}$$

# Sum of Probabilities: Dice Example 3

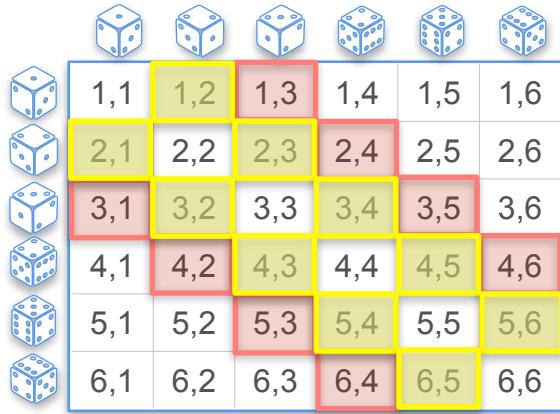


$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= P(\text{diff} = 2) + \\ &\quad \begin{array}{c} \text{---} \\ \begin{matrix} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,4 \\ 6,5 \end{matrix} \end{array} \\ &= \frac{8}{36} + \frac{10}{36} \\ &= \frac{18}{36} \end{aligned}$$

The diagram illustrates the probability calculation. It shows the total sample space of 36 outcomes as a 6x6 grid. The outcomes where the difference is 2 are highlighted in yellow. The outcomes where the difference is 1 are highlighted in red. The intersection of these two sets (outcomes where the difference is both 1 and 2) is shown in white, indicating they are counted in both sets.

# Sum of Probabilities: Dice Example 3



$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= P(\text{diff} = 2) + P(\text{diff} = 1) \\ &= \frac{8}{36} + \frac{10}{36} \\ &= \frac{18}{36} \end{aligned}$$

The diagram illustrates the calculation of the probability of the difference between the two dice being 1 or 2. It shows the sample space of 36 outcomes and highlights the outcomes that satisfy the condition: (1,3), (1,2), (2,1), (2,3), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (6,1), (6,2), (6,3), (1,5), (1,6), (2,4), (2,5), (3,4), (3,5), (4,4), (4,5), (5,4), (5,5), (6,4), (6,5), (1,3), (2,4), (3,5), (4,6), (1,2), (2,3), (3,4), (4,5), (5,6), (2,1), (3,2), (4,3), (5,4), (3,1), (4,2), (5,3), (6,4), (4,1), (5,2), (6,3), (5,1), (6,2), (6,6).



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# Introduction to probability

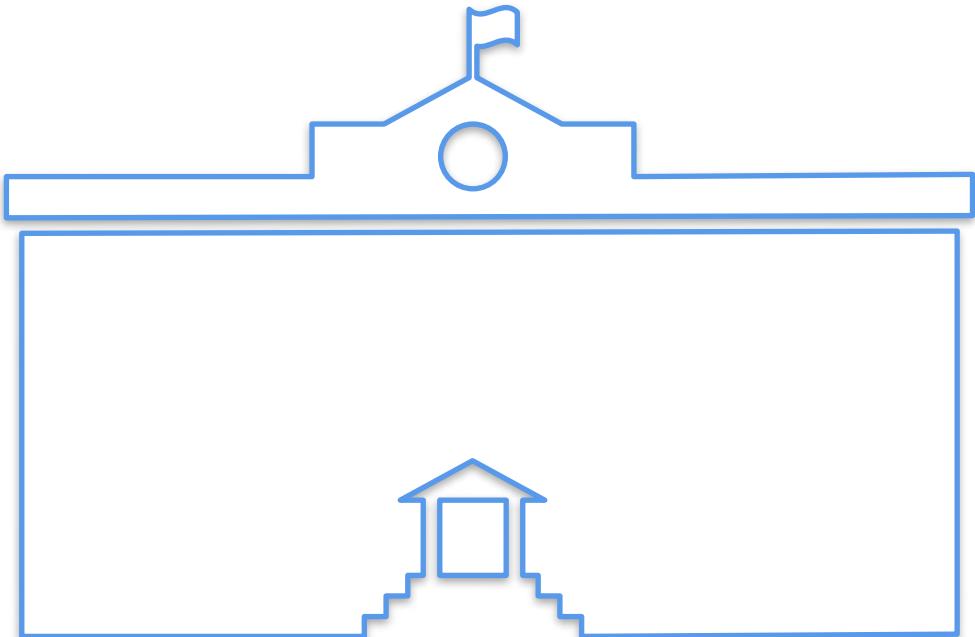
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**Sum of Probabilities  
(Joint Events)**

# Sum of Probabilities (Joint Events): Quiz 1

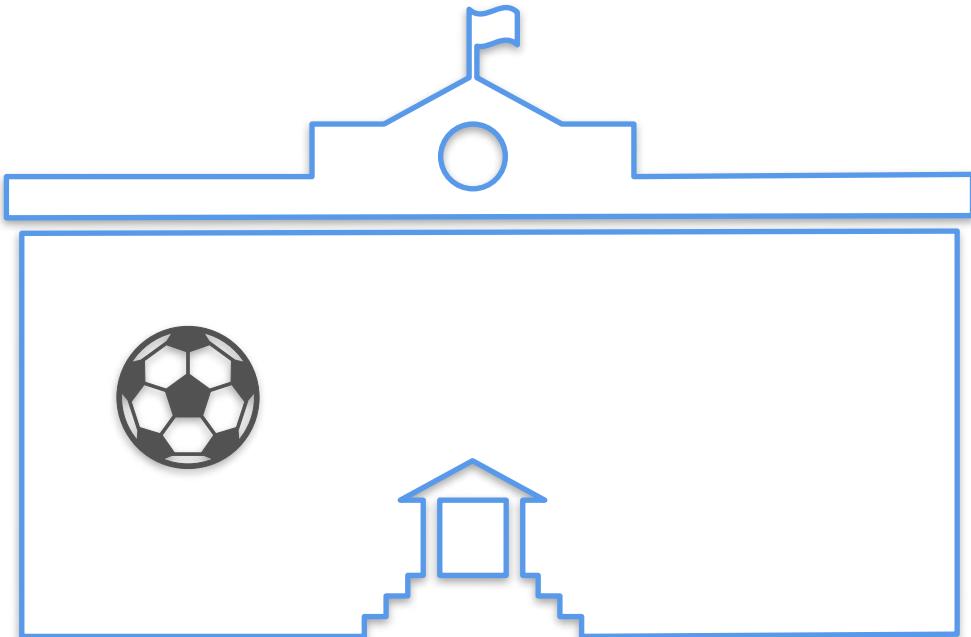
At a school, kids can play as many sports as they want.

# Sum of Probabilities (Joint Events): Quiz 1



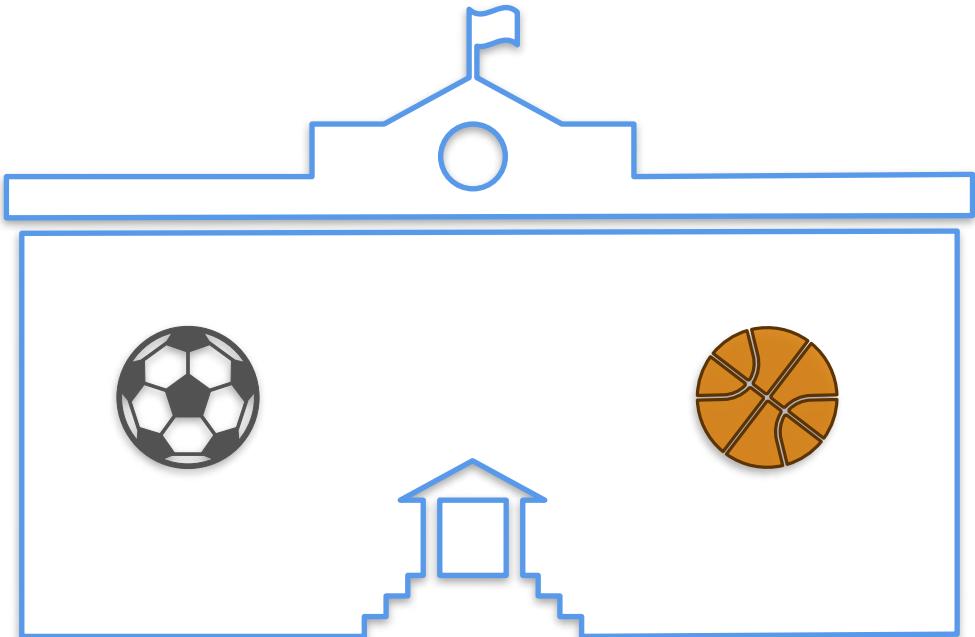
At a school, kids can play as many sports as they want.

# Sum of Probabilities (Joint Events): Quiz 1



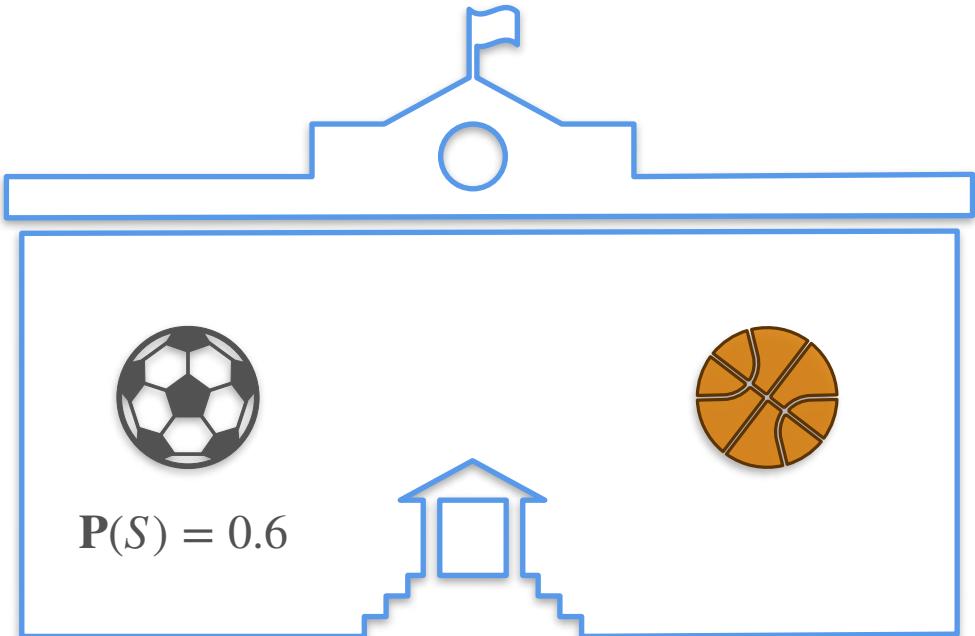
At a school, kids can play as many sports as they want.

# Sum of Probabilities (Joint Events): Quiz 1



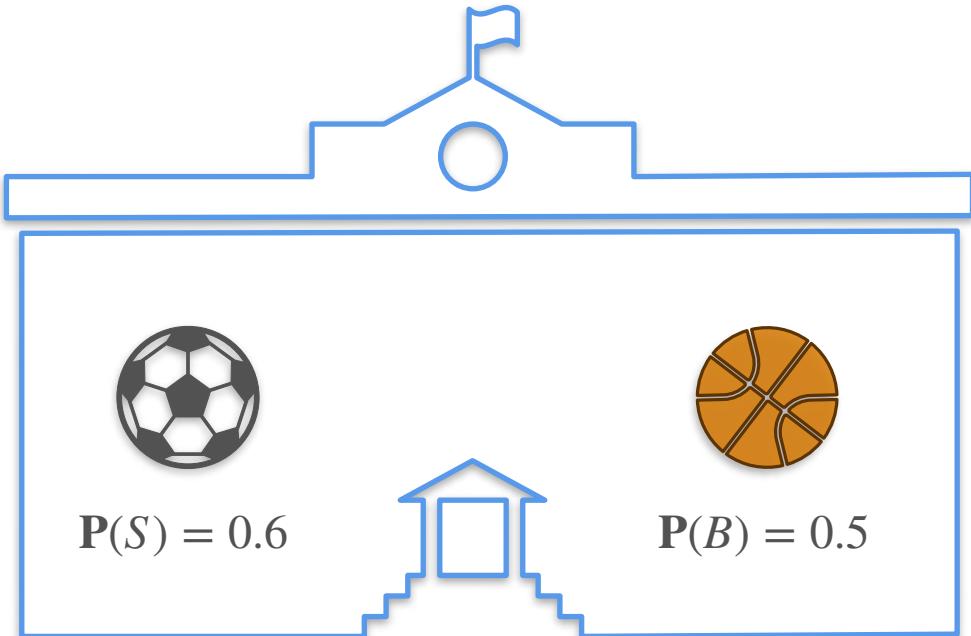
At a school, kids can play as many sports as they want.

# Sum of Probabilities (Joint Events): Quiz 1



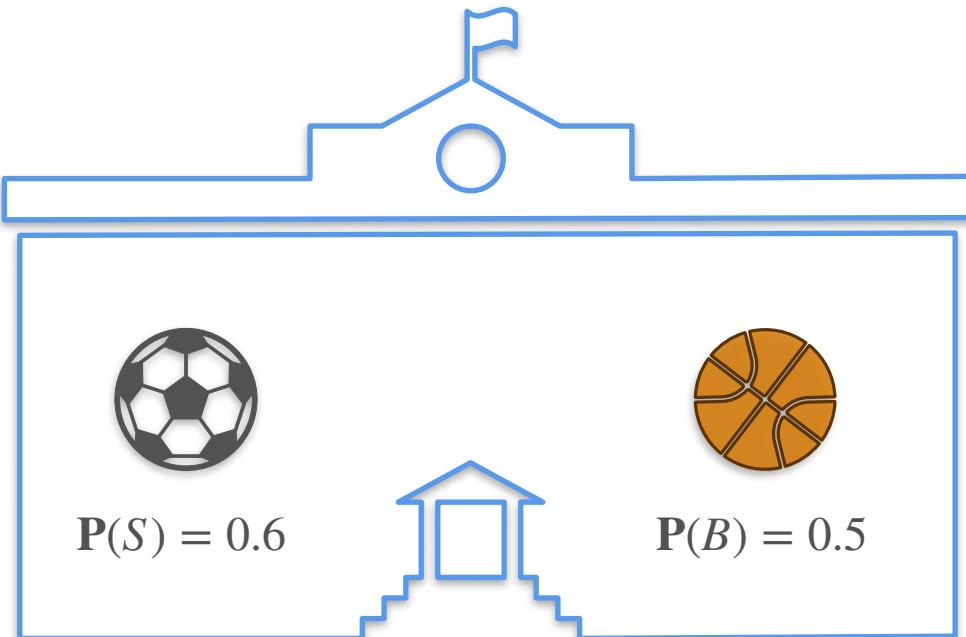
At a school, kids can play as many sports as they want.

# Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

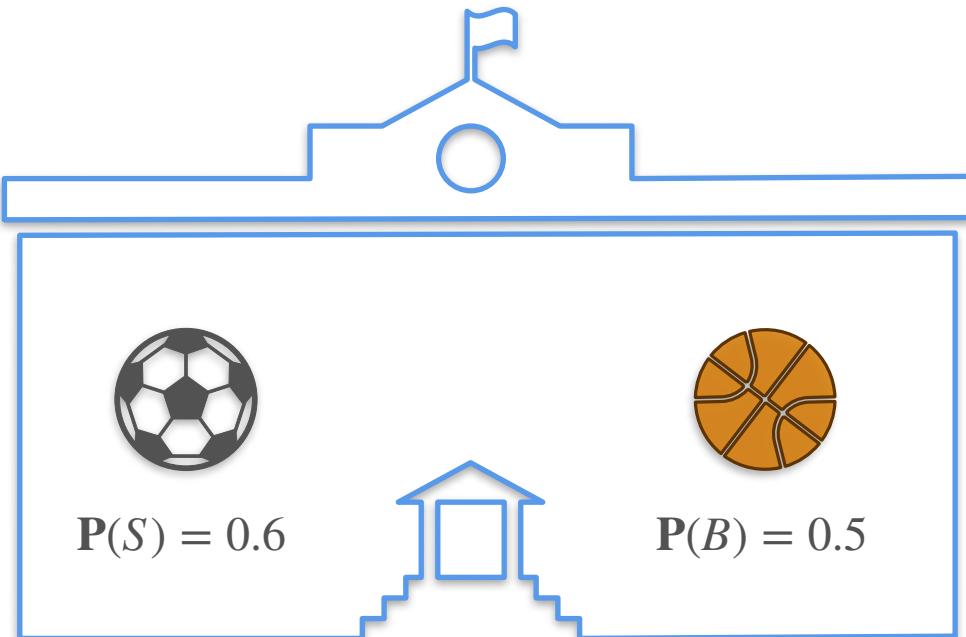
# Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

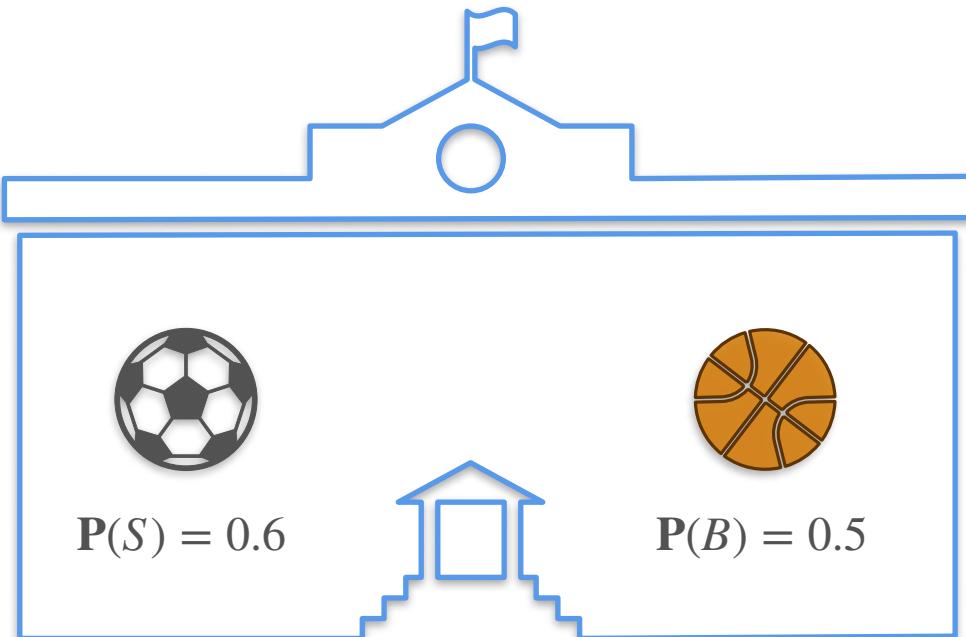
# Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

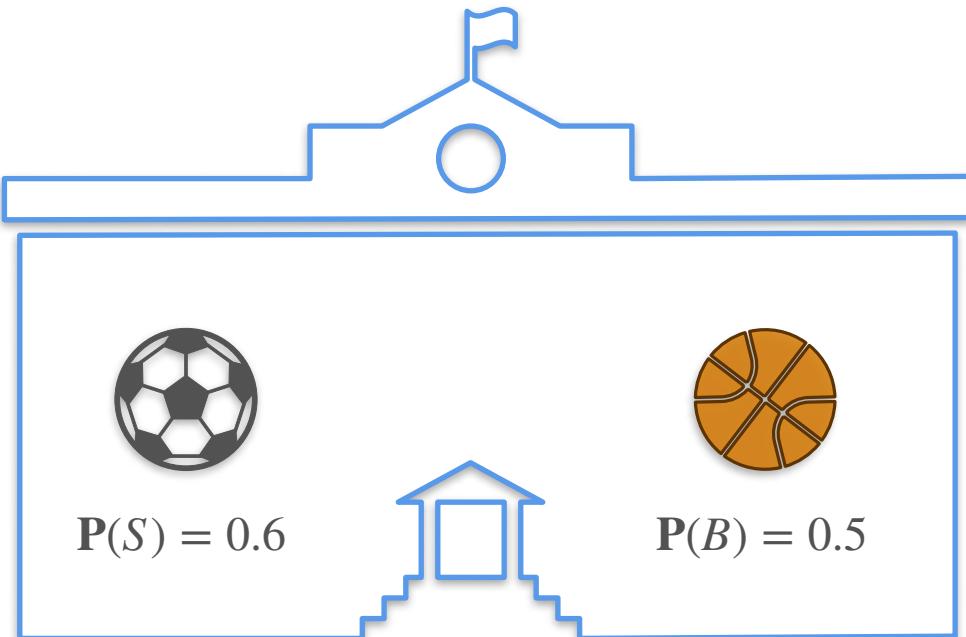
# Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

# Sum of Probabilities (Joint Events): Quiz 1



At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

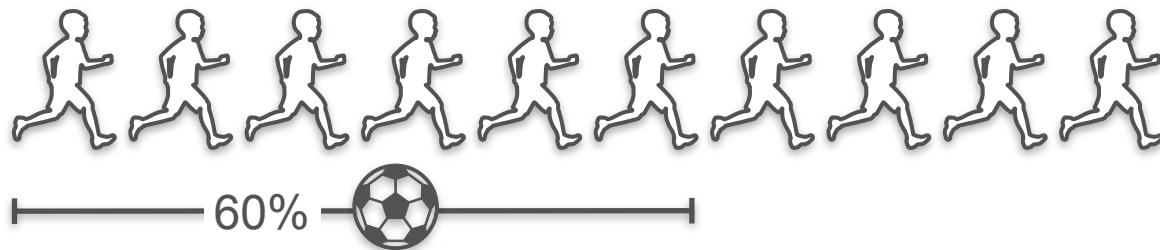
Hint: What if there were only 10 kids?

# Sum of Probabilities (Joint Events): Quiz 1 Solution

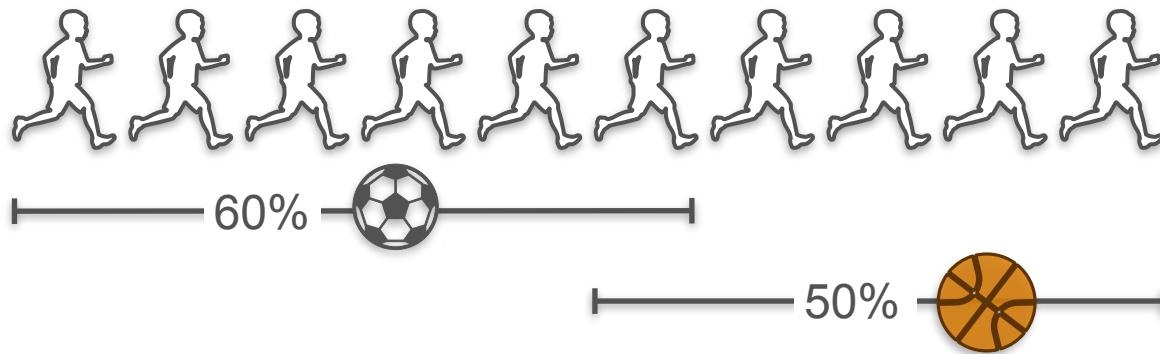
# Sum of Probabilities (Joint Events): Quiz 1 Solution



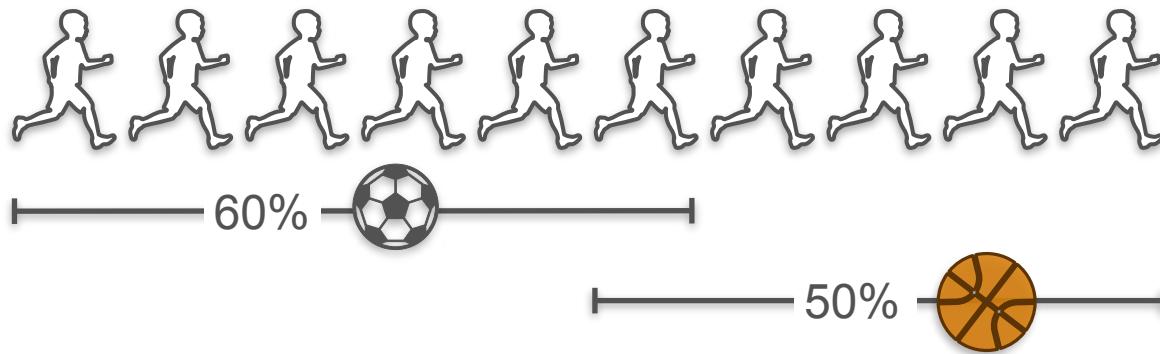
# Sum of Probabilities (Joint Events): Quiz 1 Solution



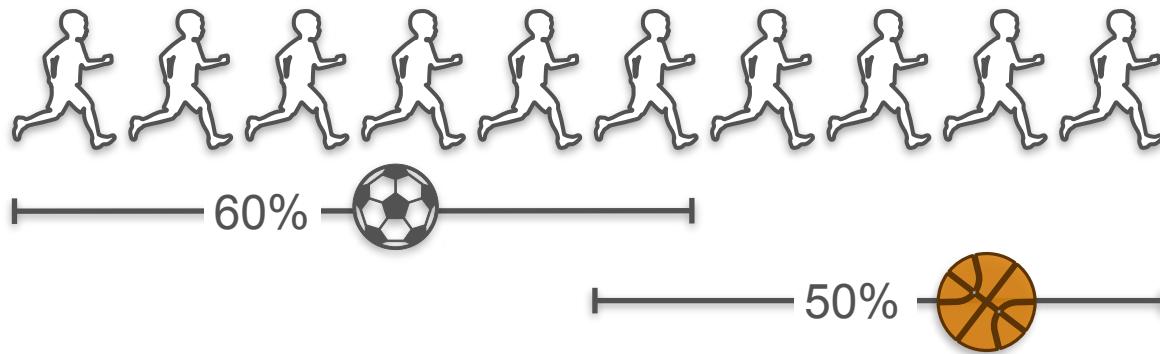
# Sum of Probabilities (Joint Events): Quiz 1 Solution



# Sum of Probabilities (Joint Events): Quiz 1 Solution

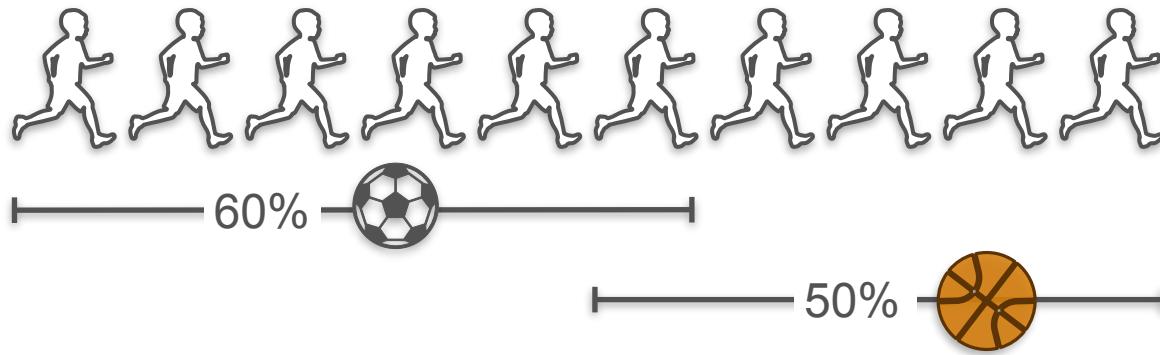


# Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = ?$$

# Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = ?$$

We don't know how many children play multiple sports

# Sum of Probabilities (Joint Events): Quiz 1 Solution

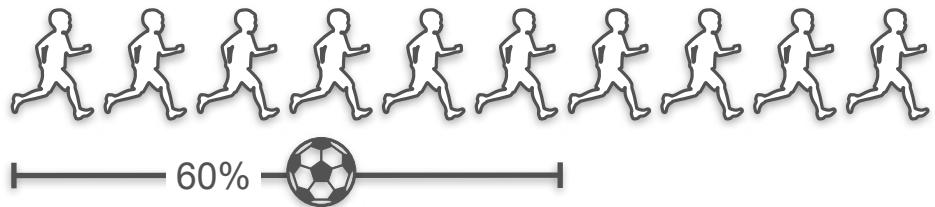
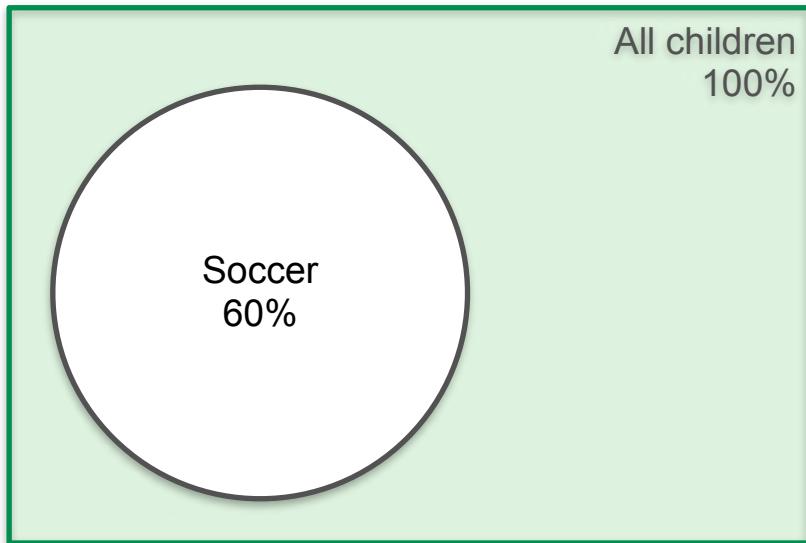


# Sum of Probabilities (Joint Events): Quiz 1 Solution

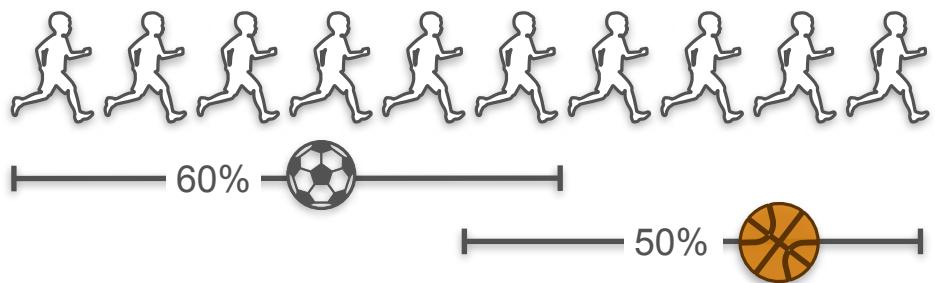
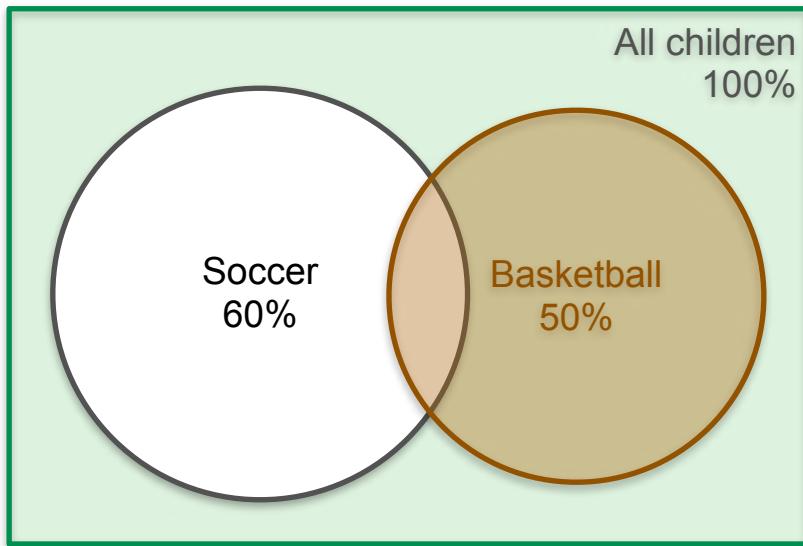
All children  
100%



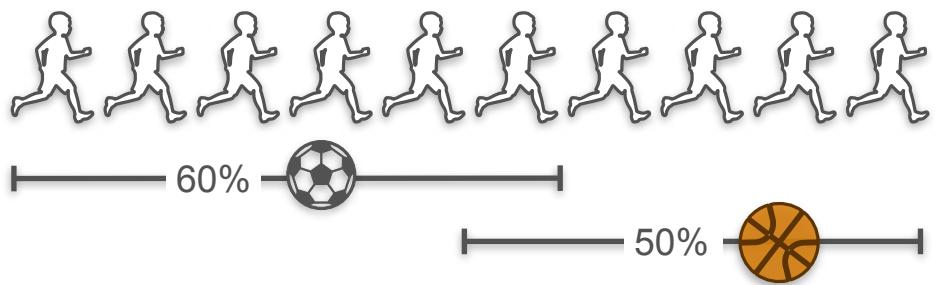
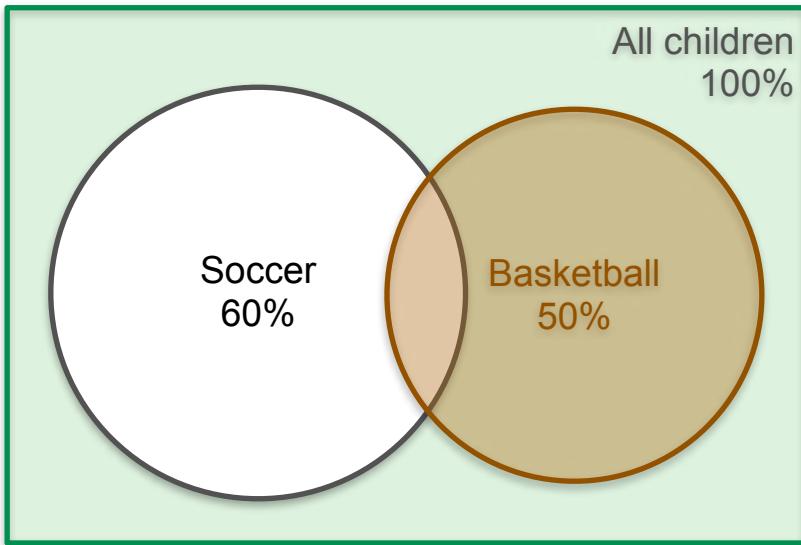
# Sum of Probabilities (Joint Events): Quiz 1 Solution



# Sum of Probabilities (Joint Events): Quiz 1 Solution

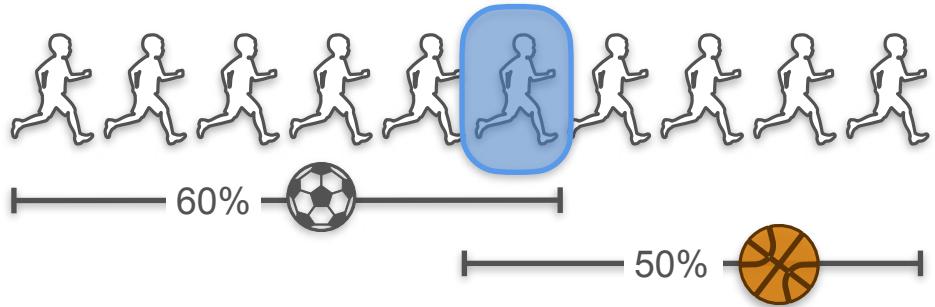
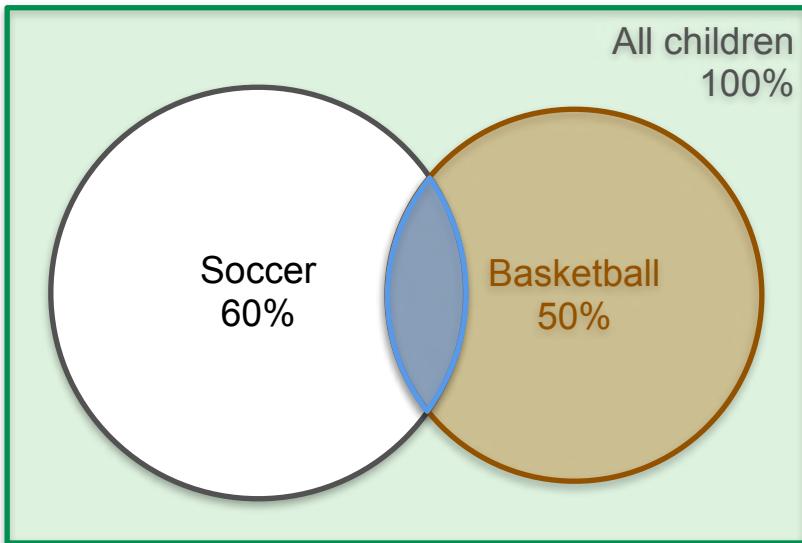


# Sum of Probabilities (Joint Events): Quiz 1 Solution



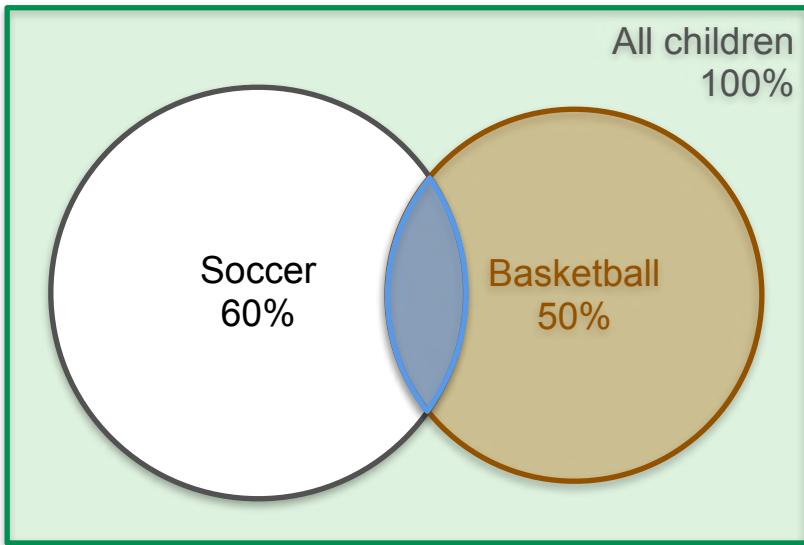
$$P(S \cup B) = P(S) + P(B)$$

# Sum of Probabilities (Joint Events): Quiz 1 Solution

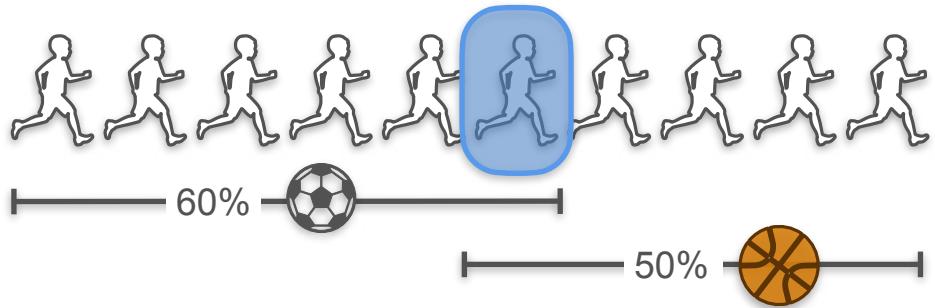


$$P(S \cup B) = P(S) + P(B)$$

# Sum of Probabilities (Joint Events): Quiz 1 Solution

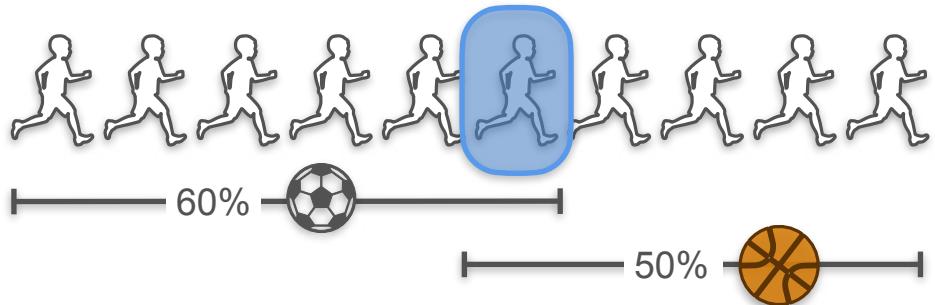
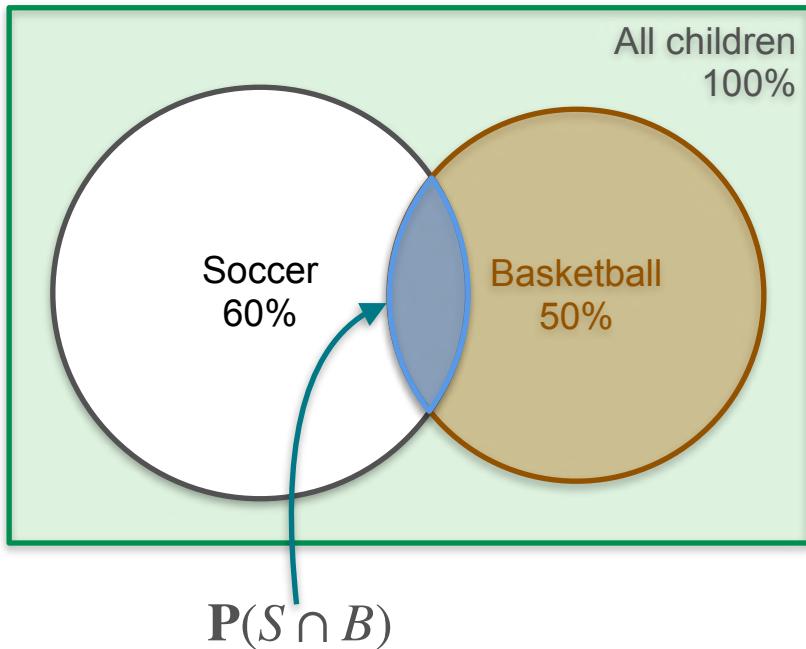


$$\mathbf{P}(S \cap B)$$



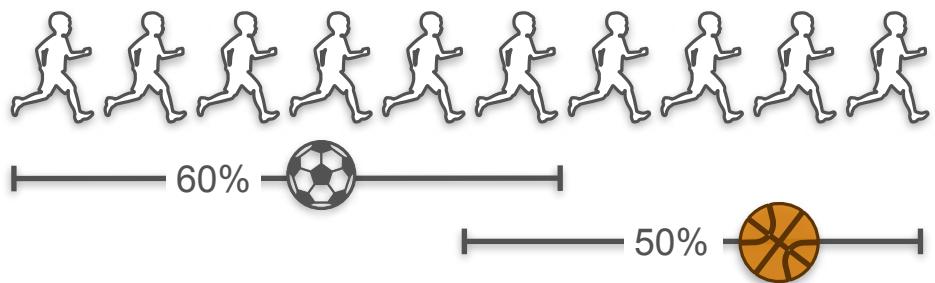
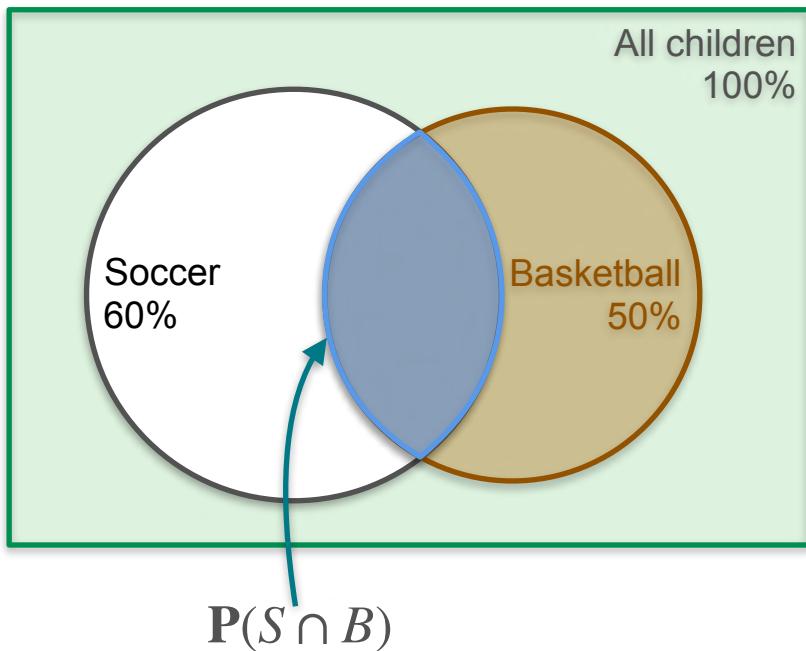
$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

# Sum of Probabilities (Joint Events): Quiz 1 Solution



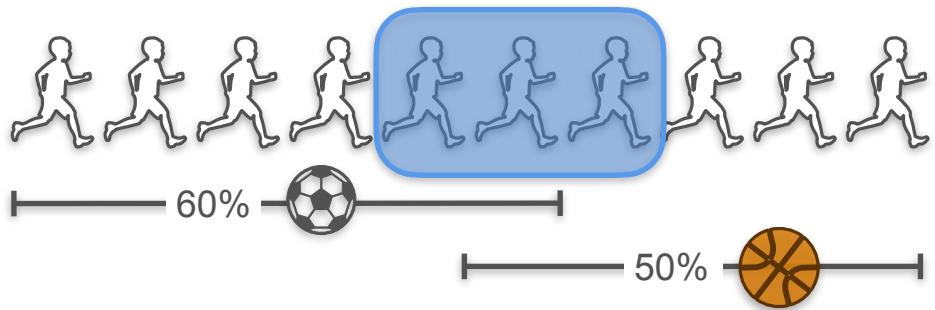
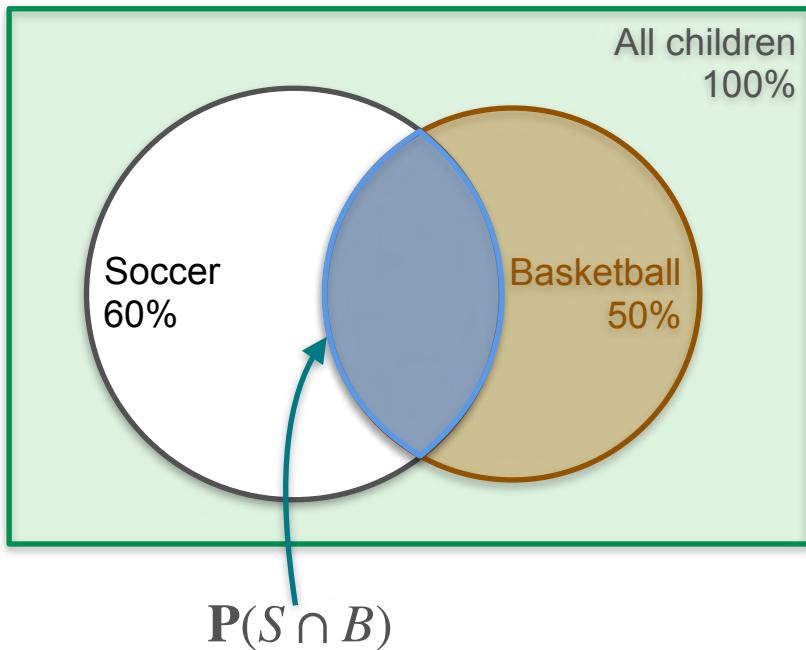
$$P(S \cup B) = P(S) + P(B)$$

# Sum of Probabilities (Joint Events): Quiz 1 Solution



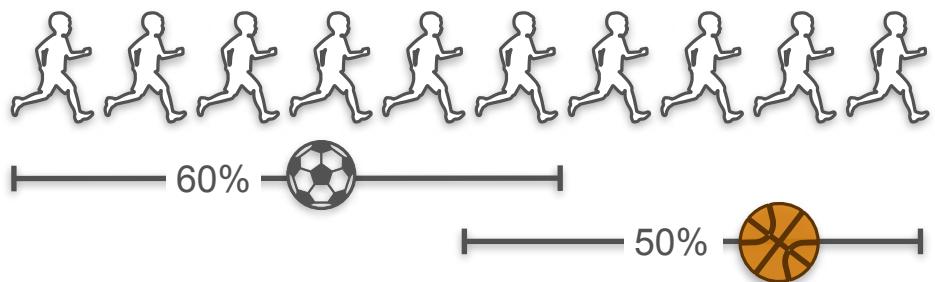
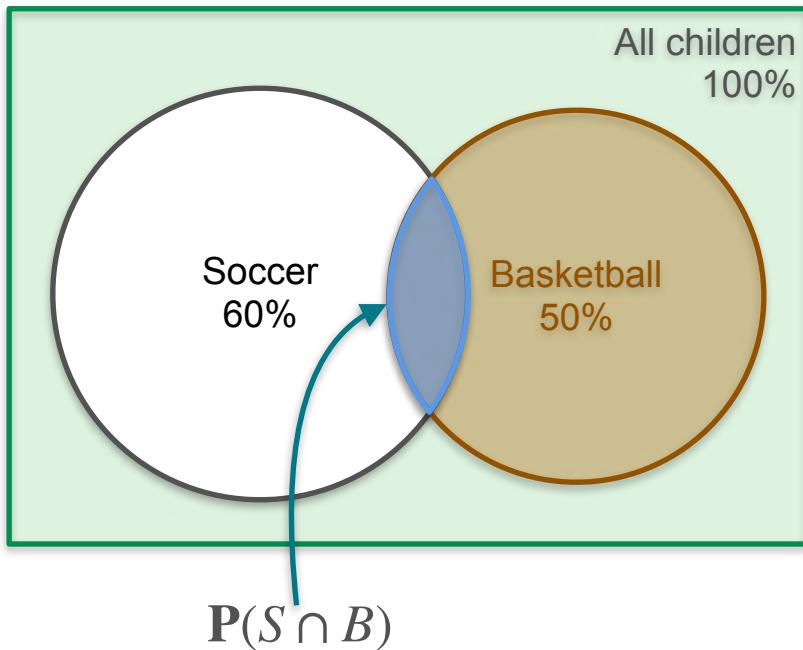
$$P(S \cup B) = P(S) + P(B)$$

# Sum of Probabilities (Joint Events): Quiz 1 Solution



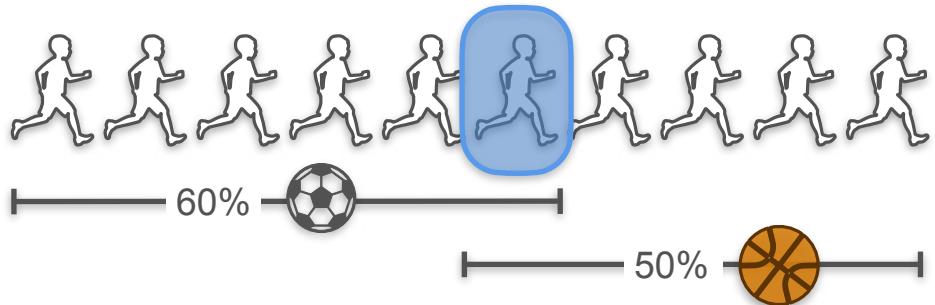
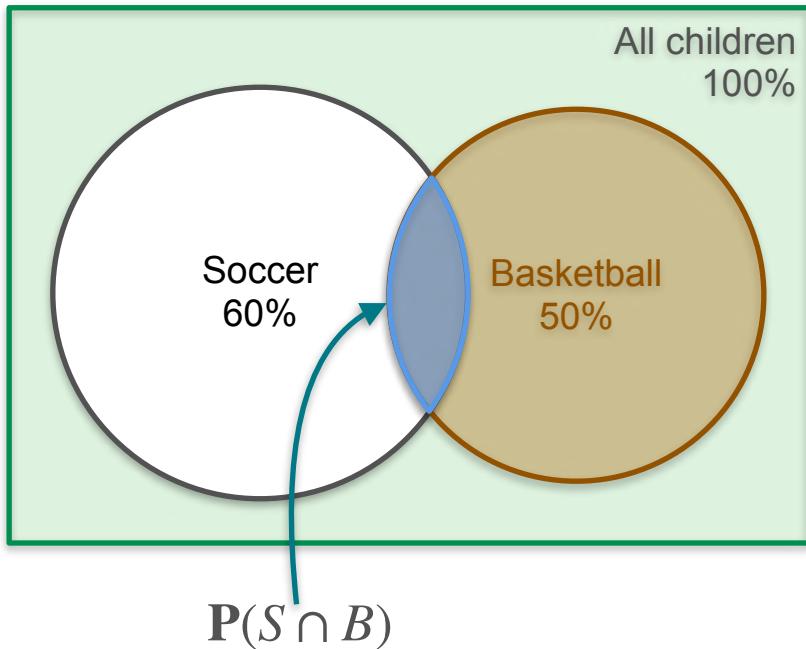
$$P(S \cup B) = P(S) + P(B)$$

# Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(S \cup B) = P(S) + P(B)$$

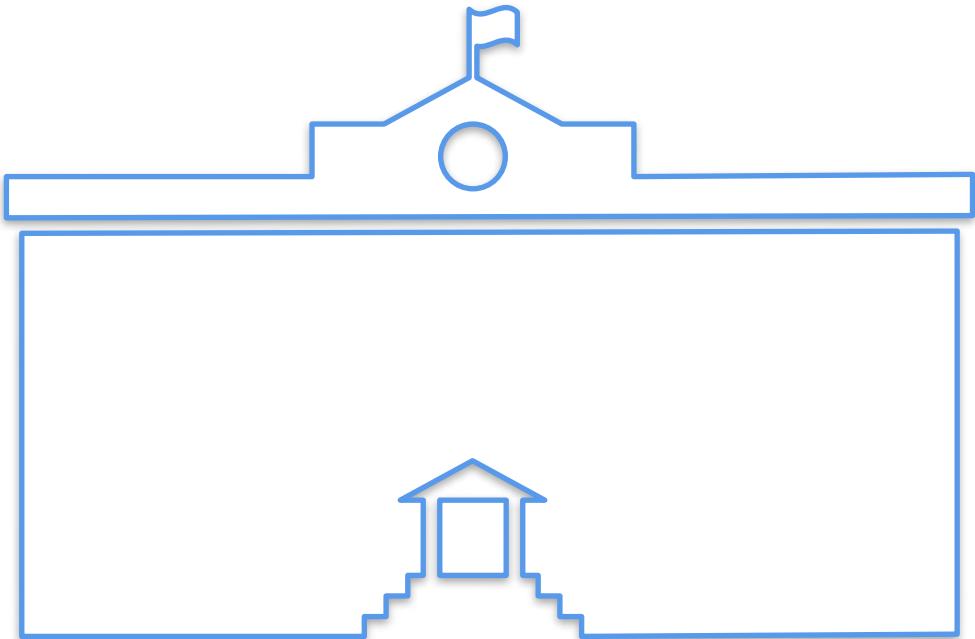
# Sum of Probabilities (Joint Events): Quiz 1 Solution



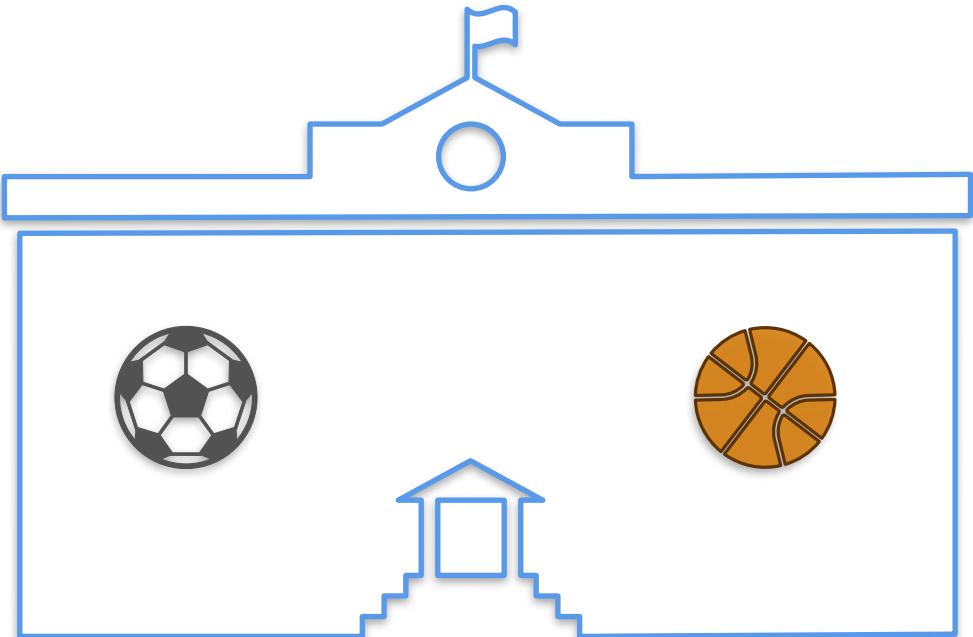
$$P(S \cup B) = P(S) + P(B)$$

# Sum of Probabilities (Joint Events): Quiz 2

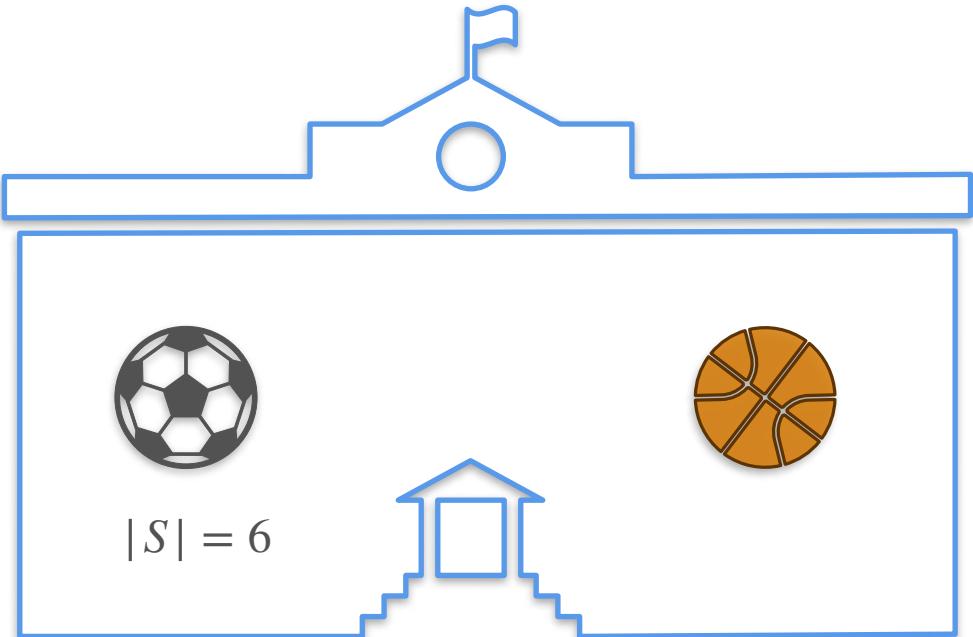
# Sum of Probabilities (Joint Events): Quiz 2



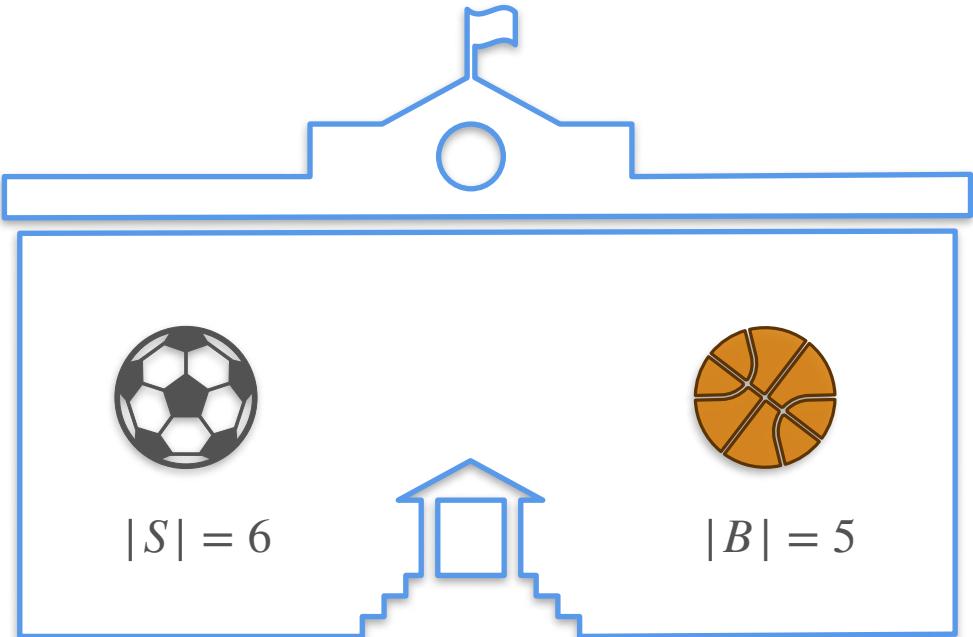
# Sum of Probabilities (Joint Events): Quiz 2



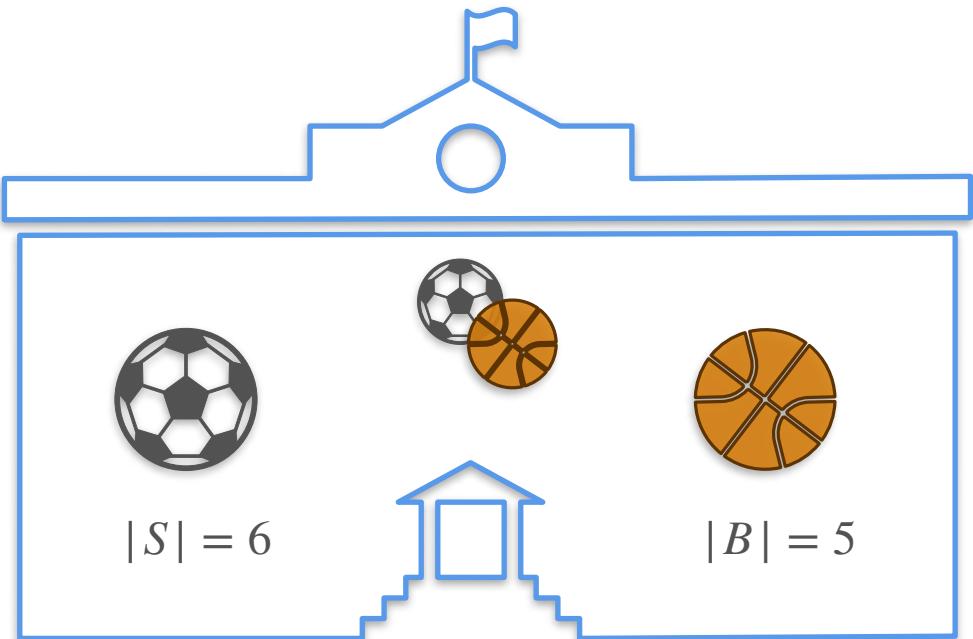
# Sum of Probabilities (Joint Events): Quiz 2



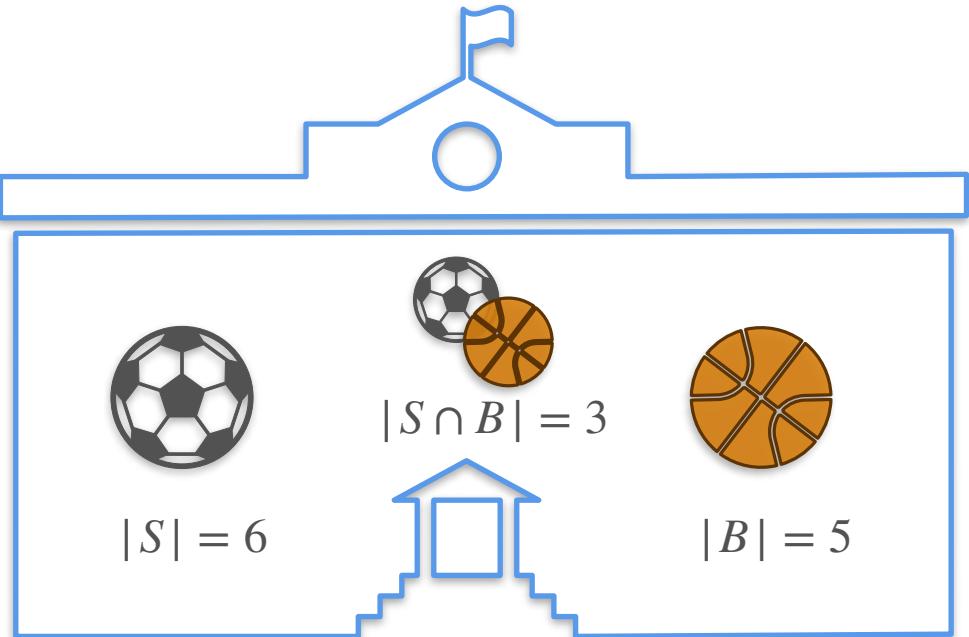
# Sum of Probabilities (Joint Events): Quiz 2



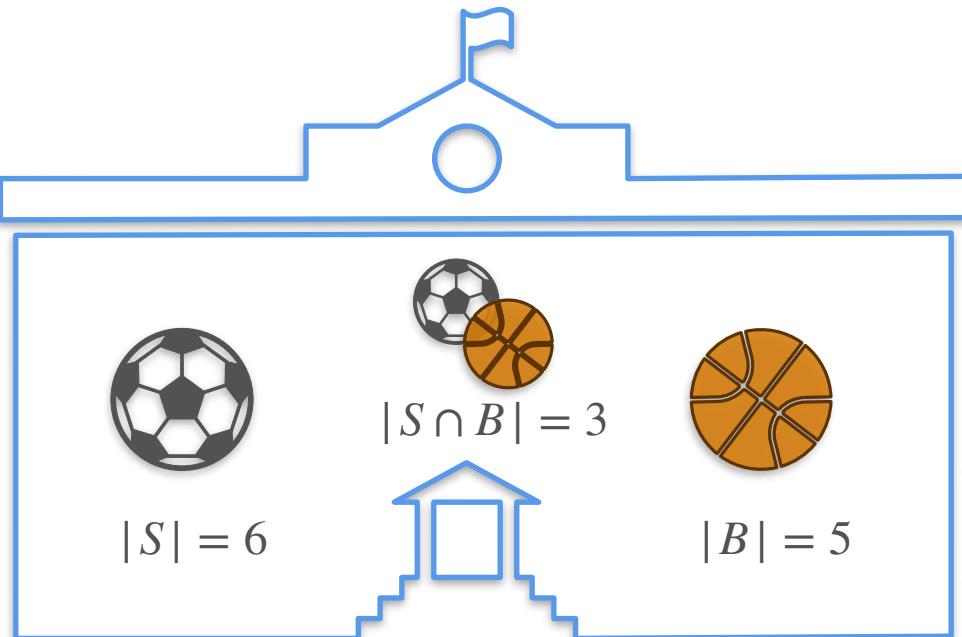
# Sum of Probabilities (Joint Events): Quiz 2



# Sum of Probabilities (Joint Events): Quiz 2

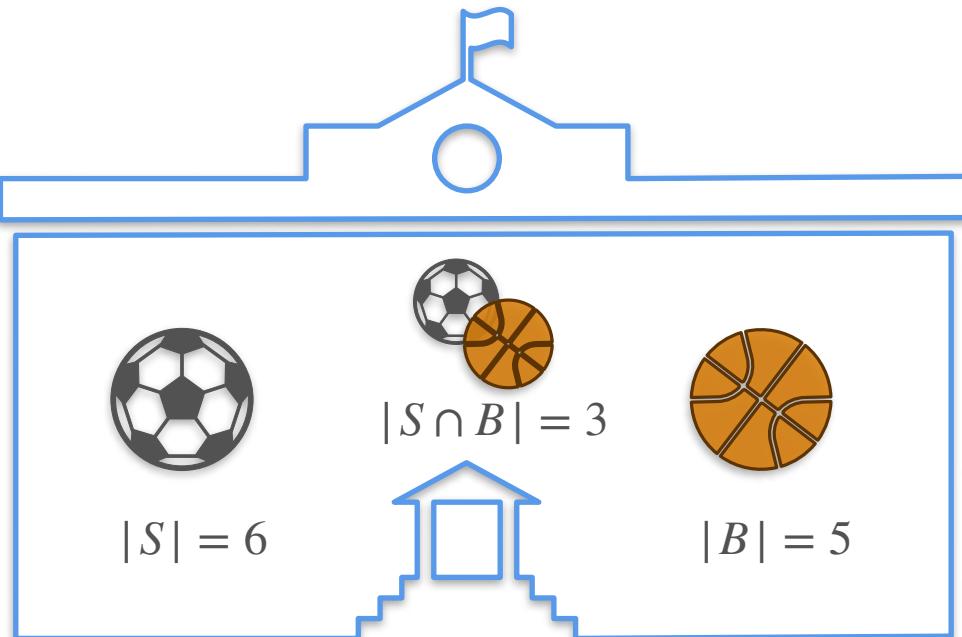


# Sum of Probabilities (Joint Events): Quiz 2



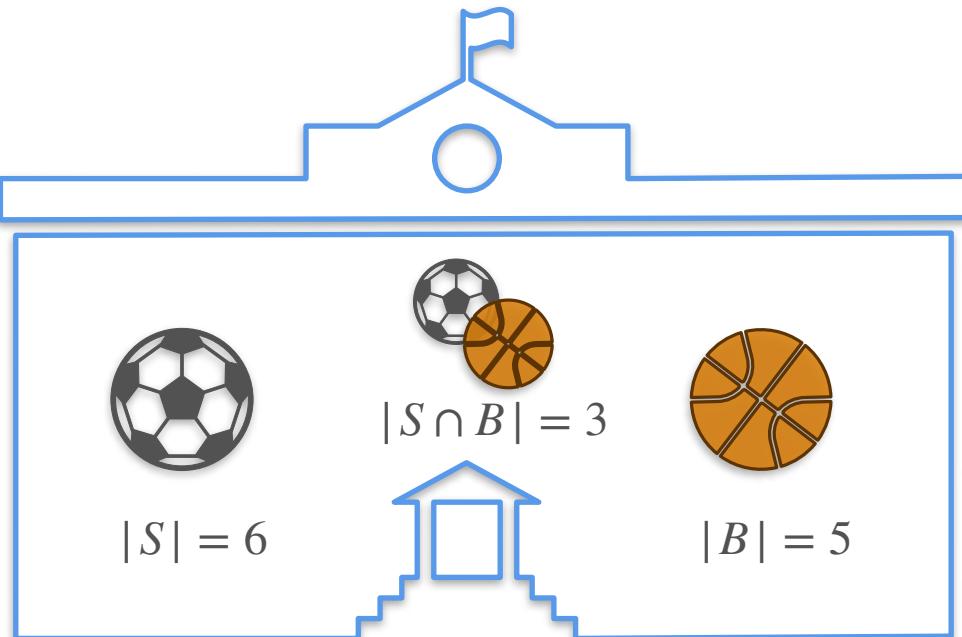
How many kids play soccer or basketball?

# Sum of Probabilities (Joint Events): Quiz 2



How many kids play soccer or basketball?

# Sum of Probabilities (Joint Events): Quiz 2



How many kids play soccer or basketball?

Hint: What if there were only 10 kids?

# Sum of Probabilities (Joint Events): Quiz 2 Solution

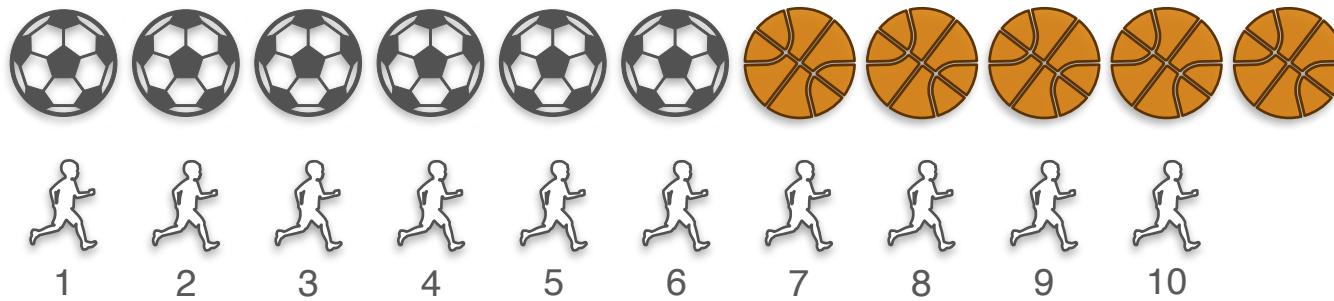
# Sum of Probabilities (Joint Events): Quiz 2 Solution



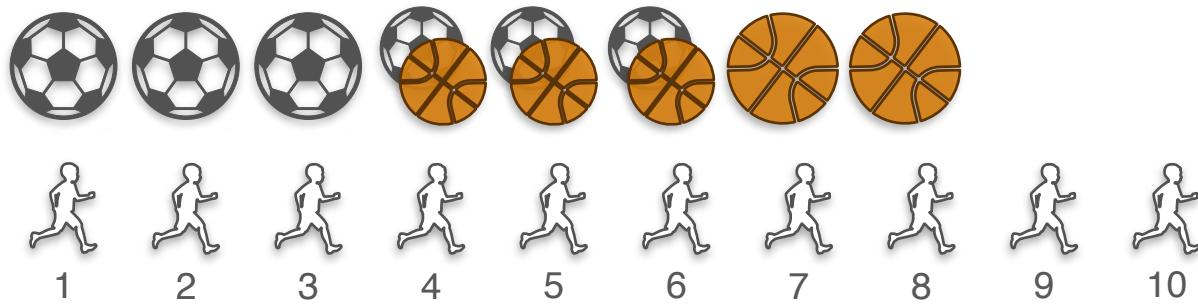
# Sum of Probabilities (Joint Events): Quiz 2 Solution



# Sum of Probabilities (Joint Events): Quiz 2 Solution



# Sum of Probabilities (Joint Events): Quiz 2 Solution



# Sum of Probabilities (Joint Events): Quiz 2 Solution



1



2



3



4



5



6



7



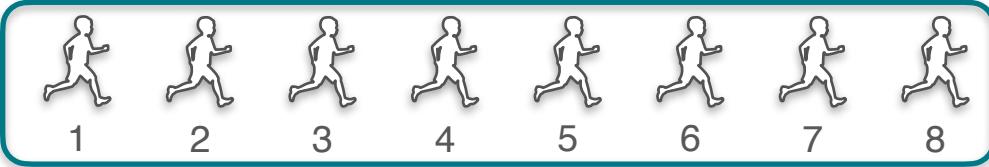
8



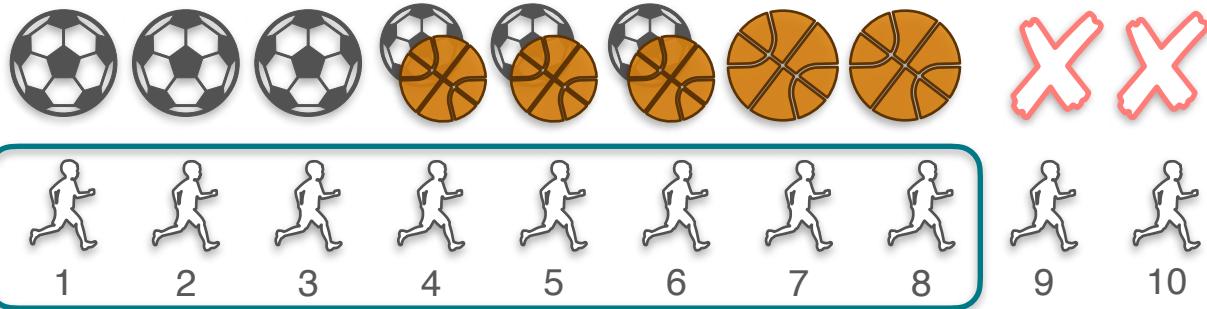
9



10



# Sum of Probabilities (Joint Events): Quiz 2 Solution

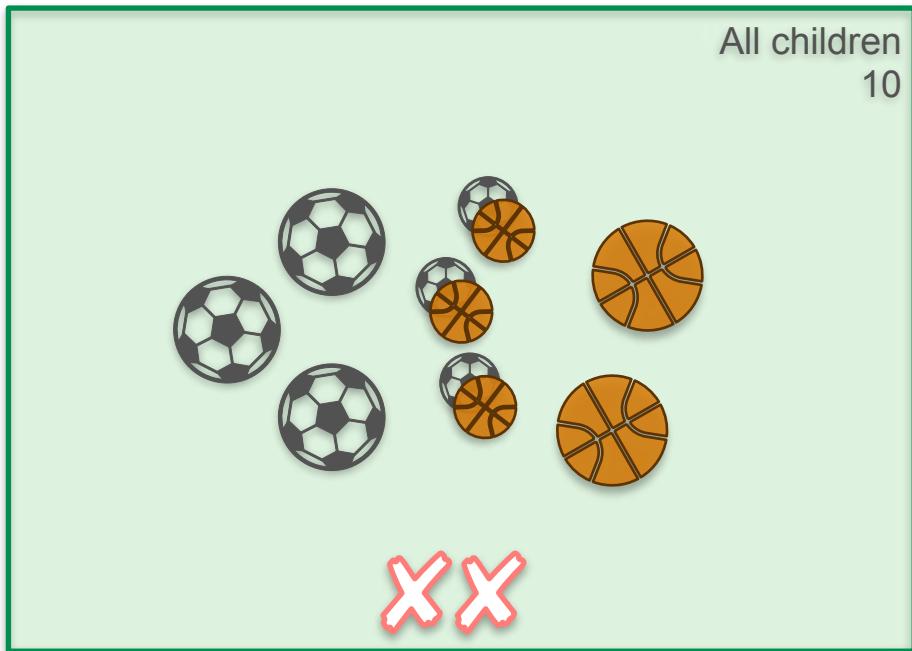


# Sum of Probabilities (Joint Events): Venn Diagram

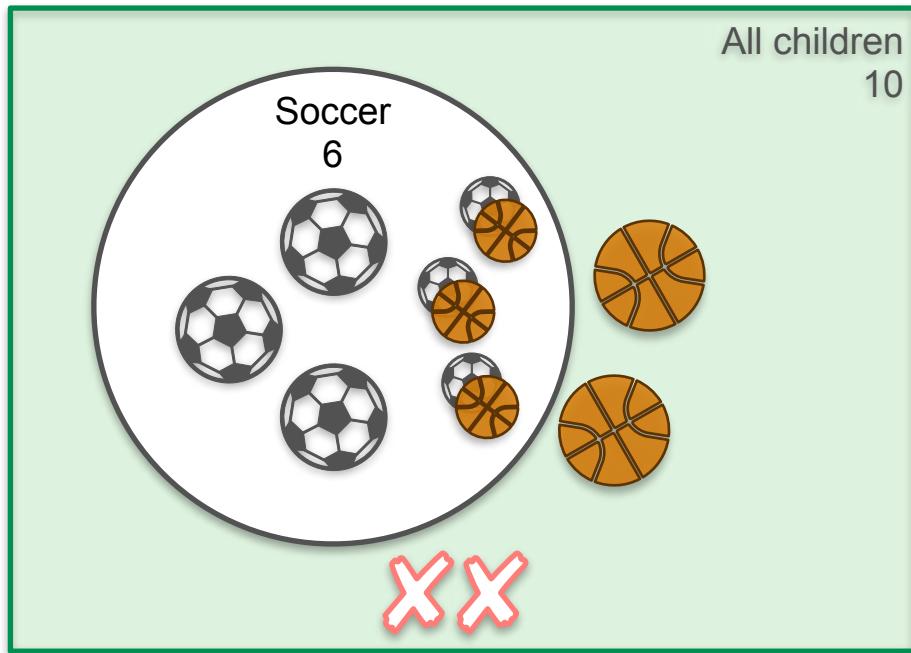


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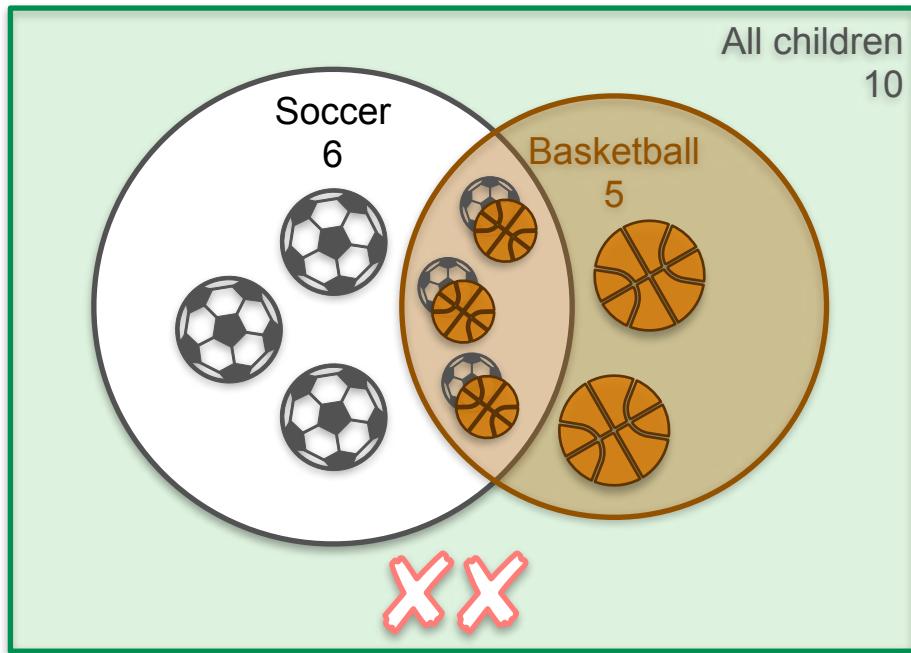
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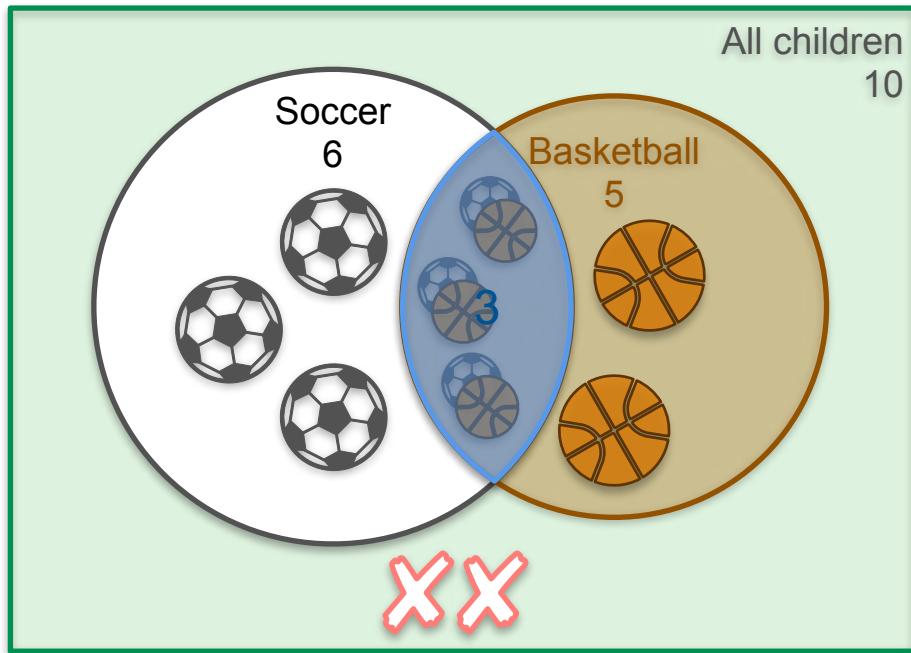
# Sum of Probabilities (Joint Events): Venn Diagram



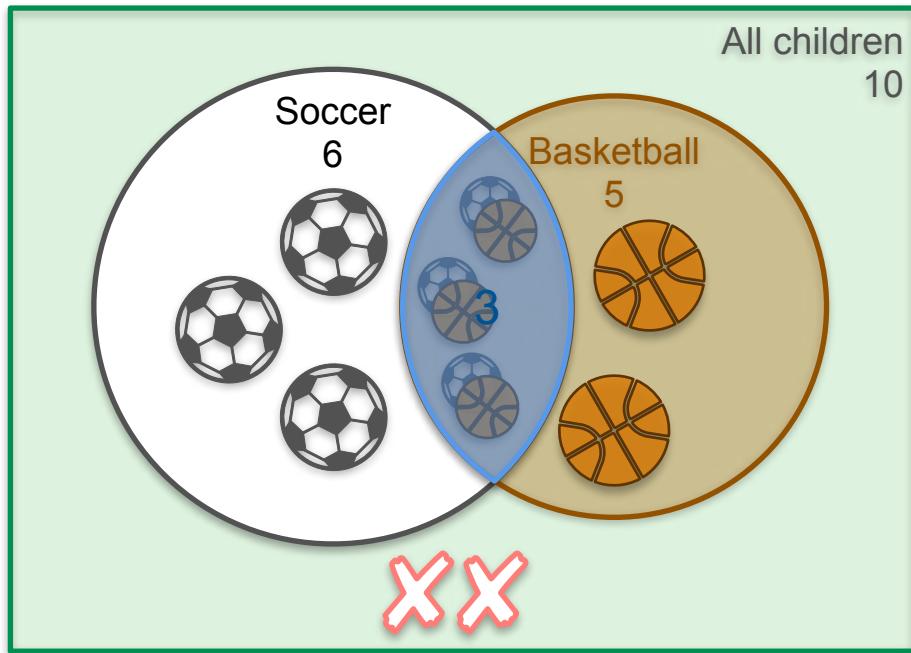
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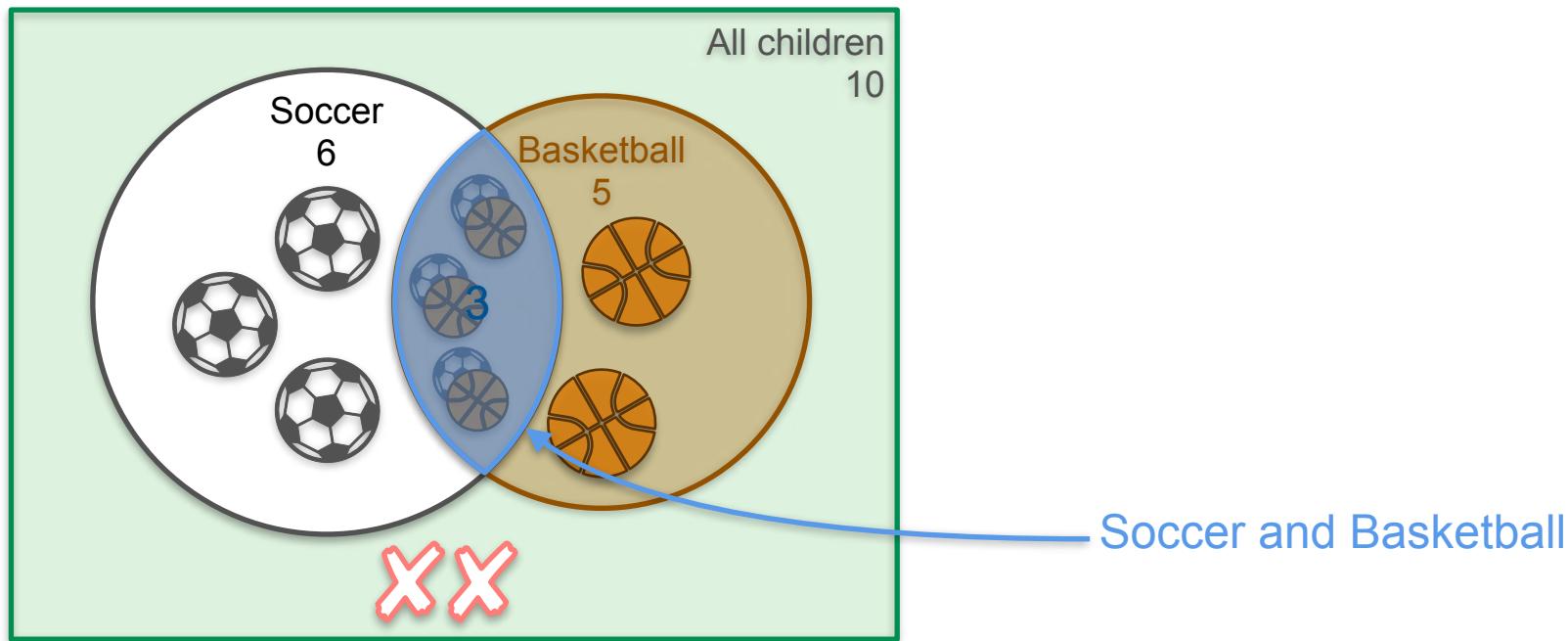


# Sum of Probabilities (Joint Events): Venn Diagram

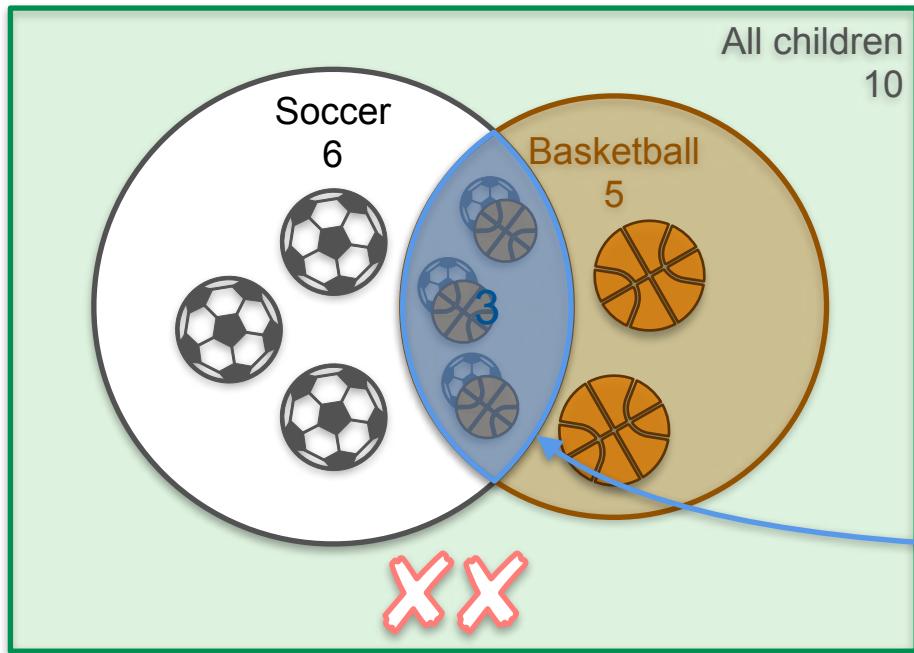


Soccer and Basketball

# Sum of Probabilities (Joint Events): Venn Diagram



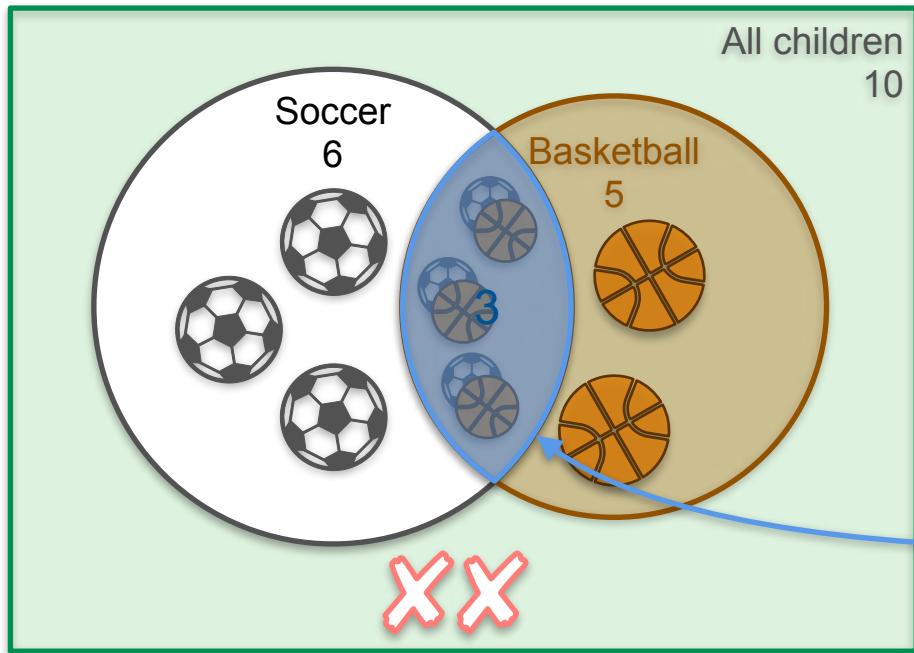
# Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| =$$

Soccer and Basketball

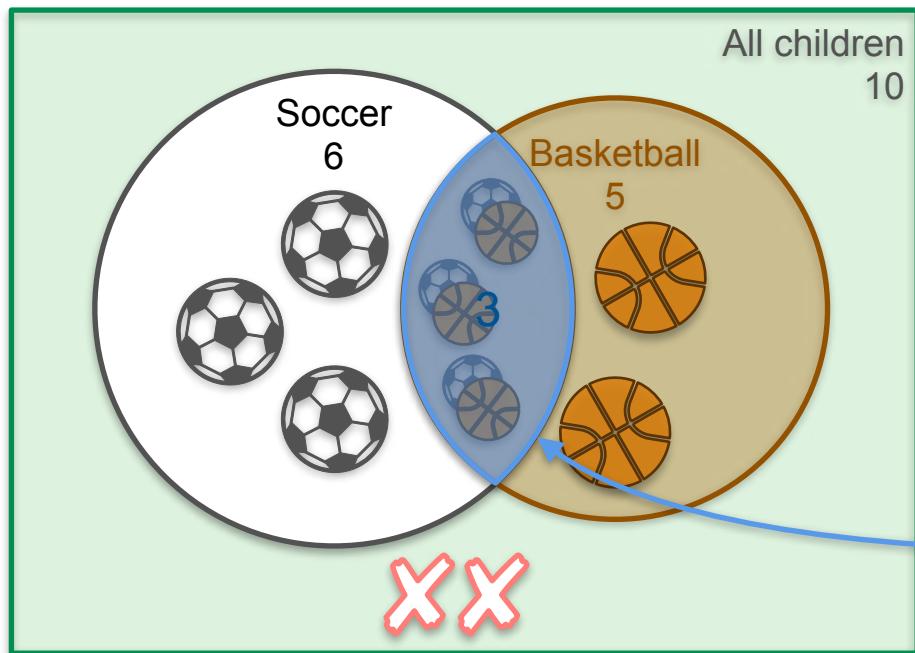
# Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S|$$

Soccer and Basketball

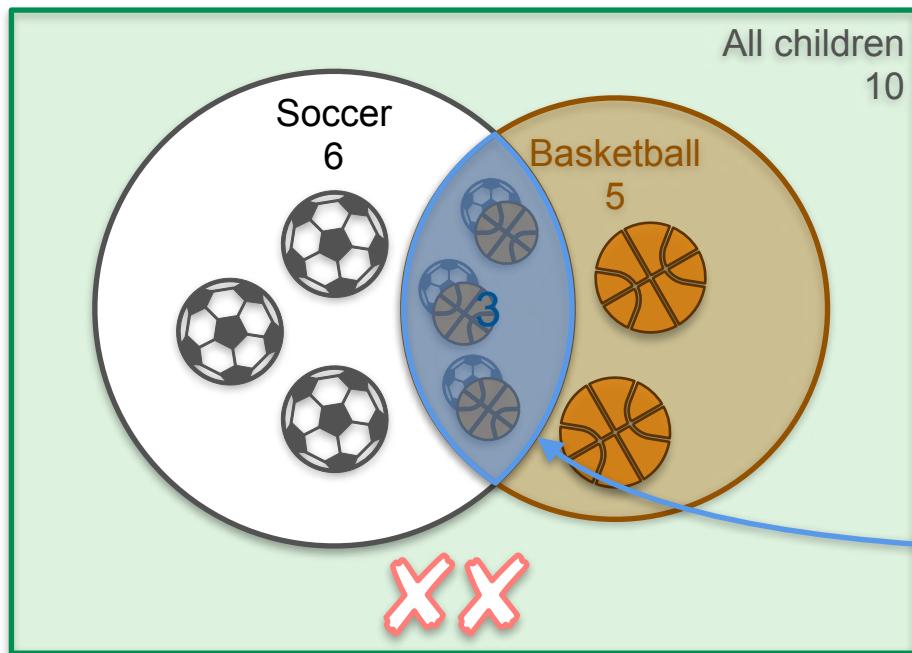
# Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S|$$

Soccer and Basketball

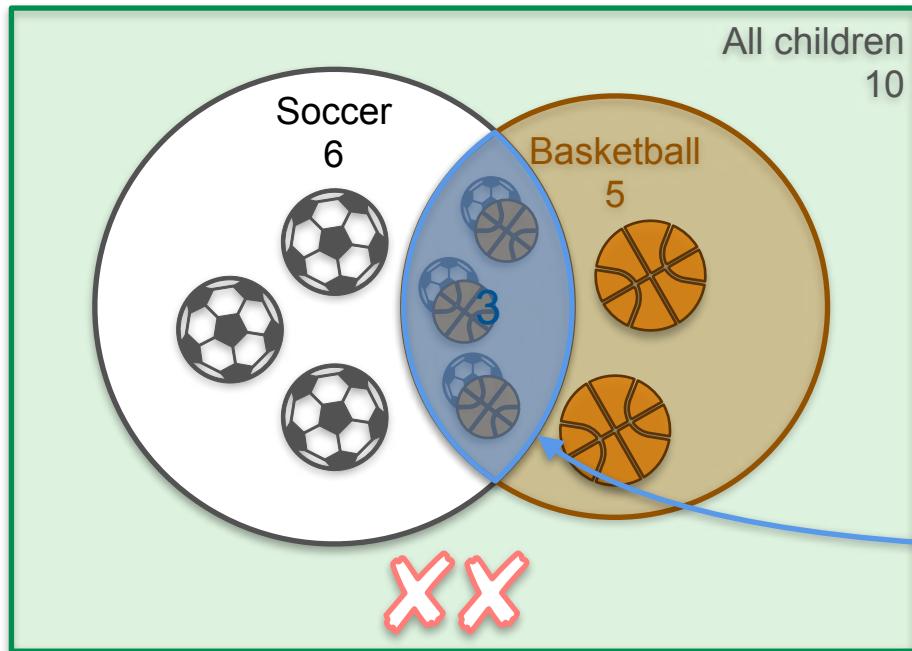
# Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B|$$

Soccer and Basketball

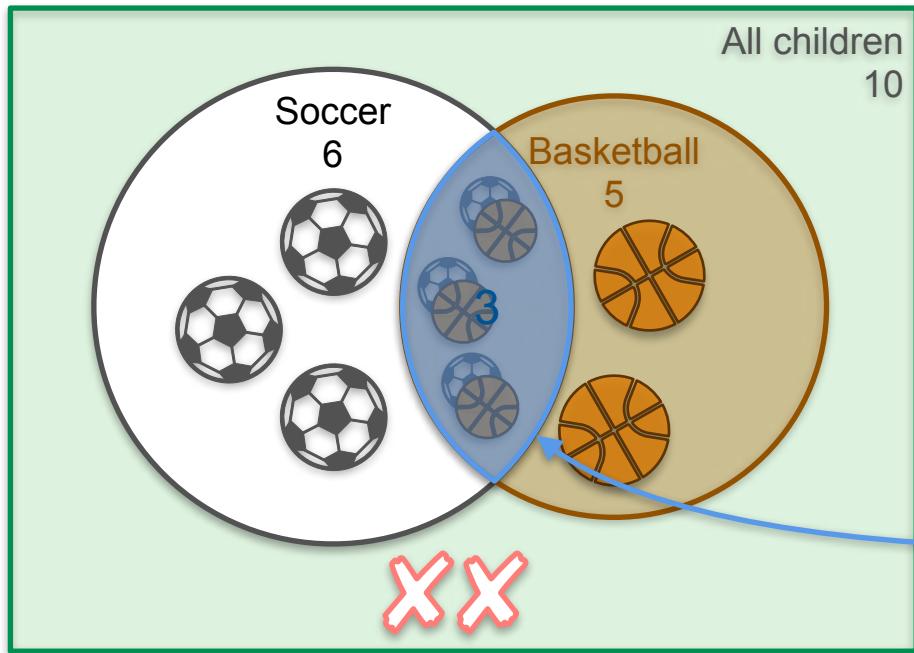
# Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B|$$

Soccer and Basketball

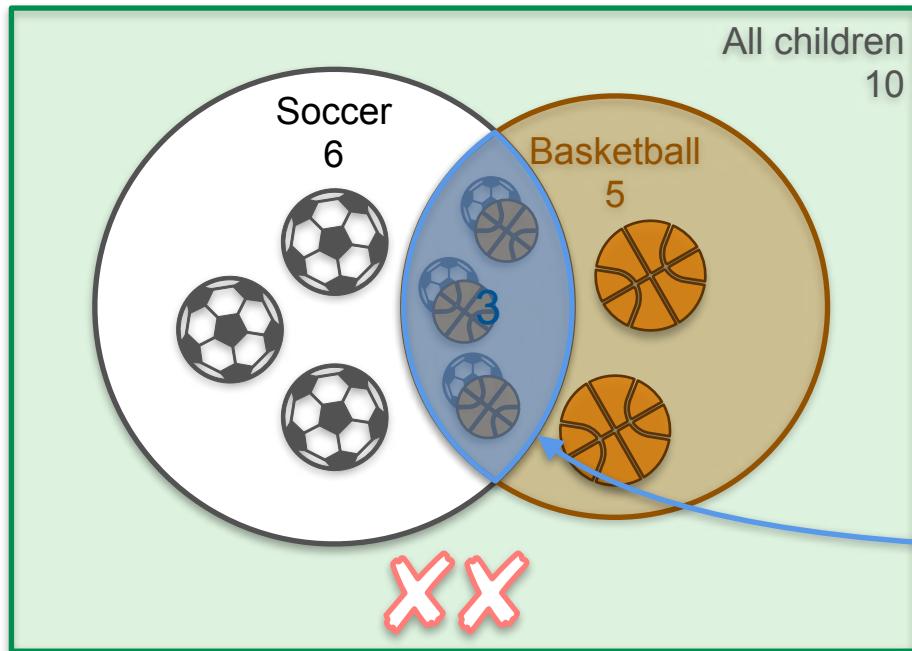
# Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B| - |S \cap B|$$

Soccer and Basketball

# Sum of Probabilities (Joint Events): Venn Diagram

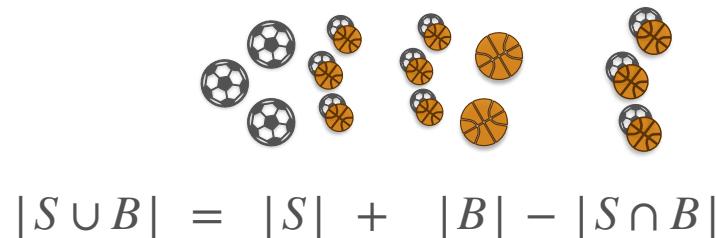
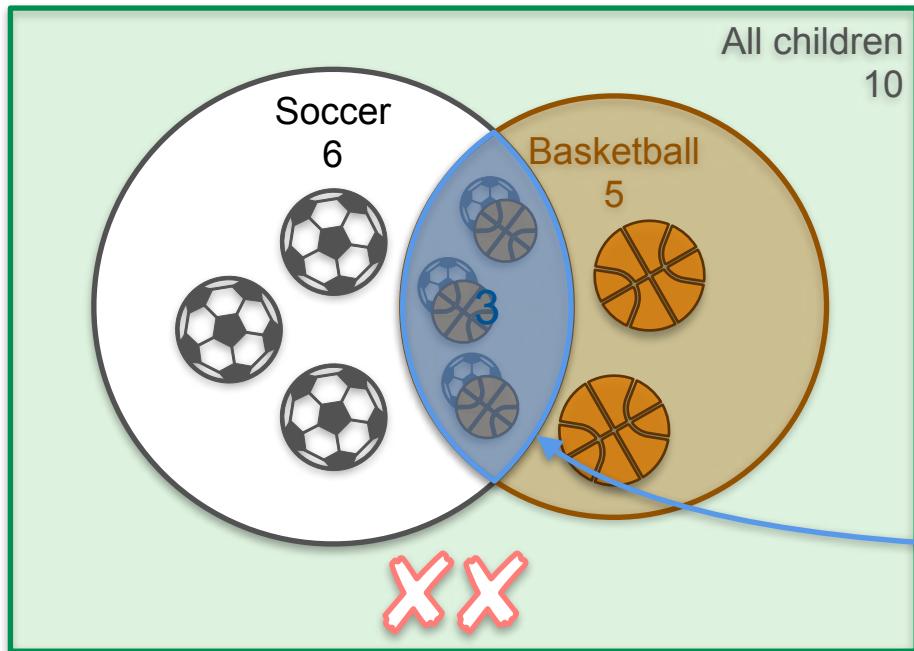


$$|S \cup B| = |S| + |B| - |S \cap B|$$

Soccer and Basketball

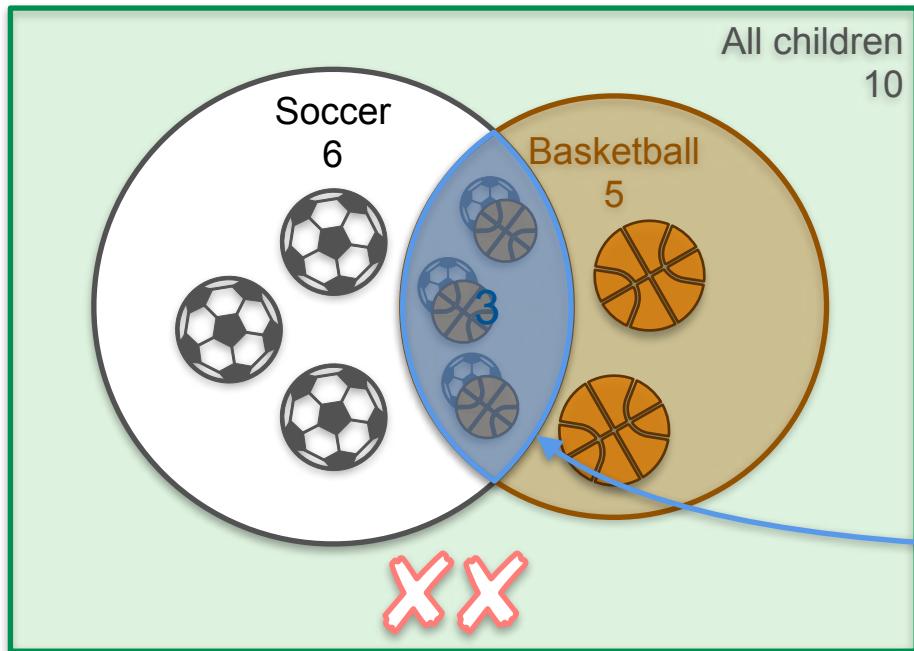


# Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B| - |S \cap B| \\ =$$

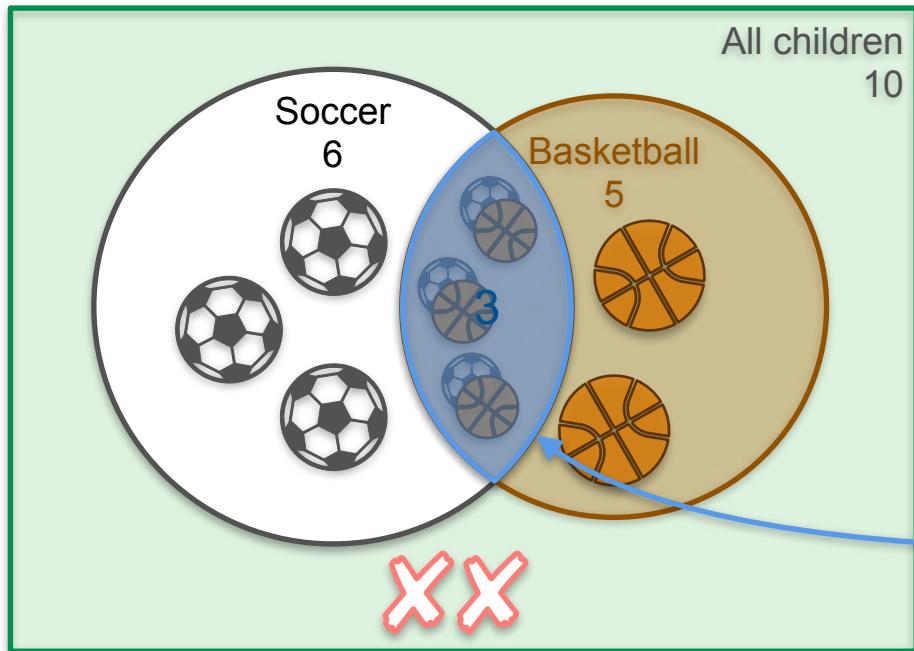
# Sum of Probabilities (Joint Events): Venn Diagram



$$\begin{aligned}|S \cup B| &= |S| + |B| - |S \cap B| \\&= 6 + 5 - 3\end{aligned}$$

Soccer and Basketball

# Sum of Probabilities (Joint Events): Venn Diagram

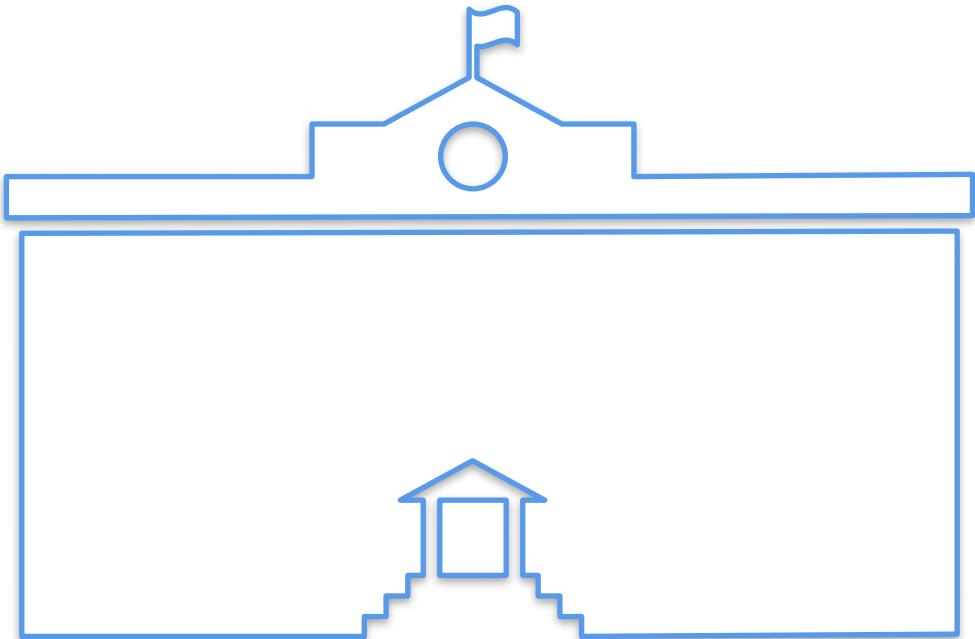


$$\begin{aligned}|S \cup B| &= |S| + |B| - |S \cap B| \\&= 6 + 5 - 3 \\&= 8\end{aligned}$$

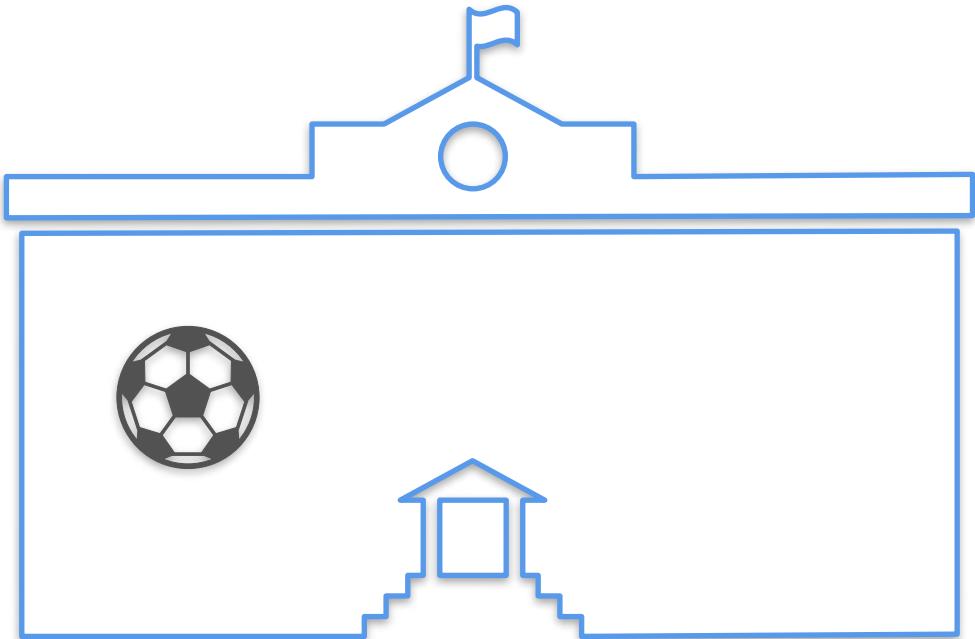
Soccer and Basketball

# Sum of Probabilities (Joint Events): Quiz 3

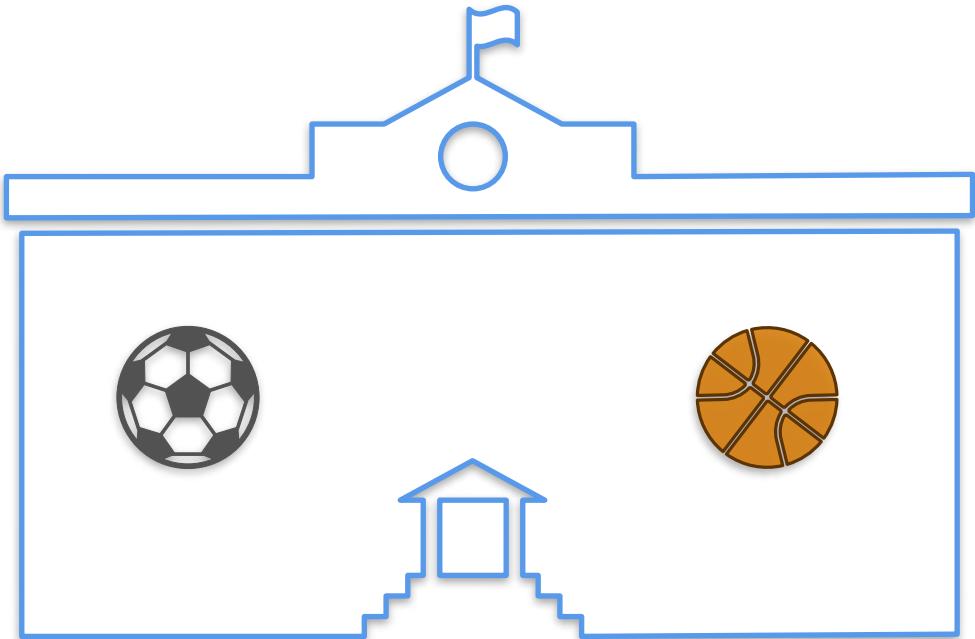
# Sum of Probabilities (Joint Events): Quiz 3



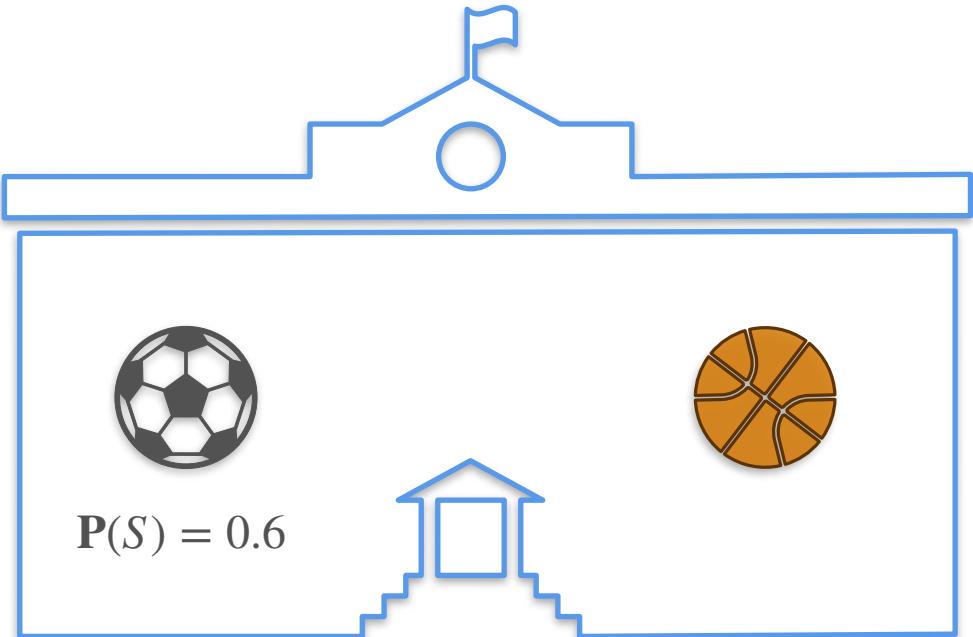
# Sum of Probabilities (Joint Events): Quiz 3



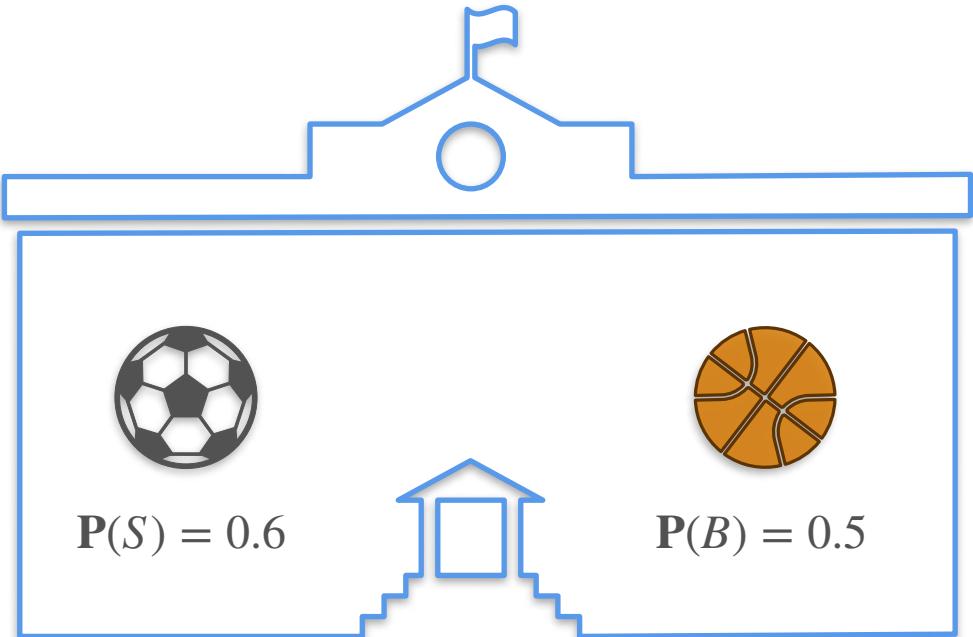
# Sum of Probabilities (Joint Events): Quiz 3



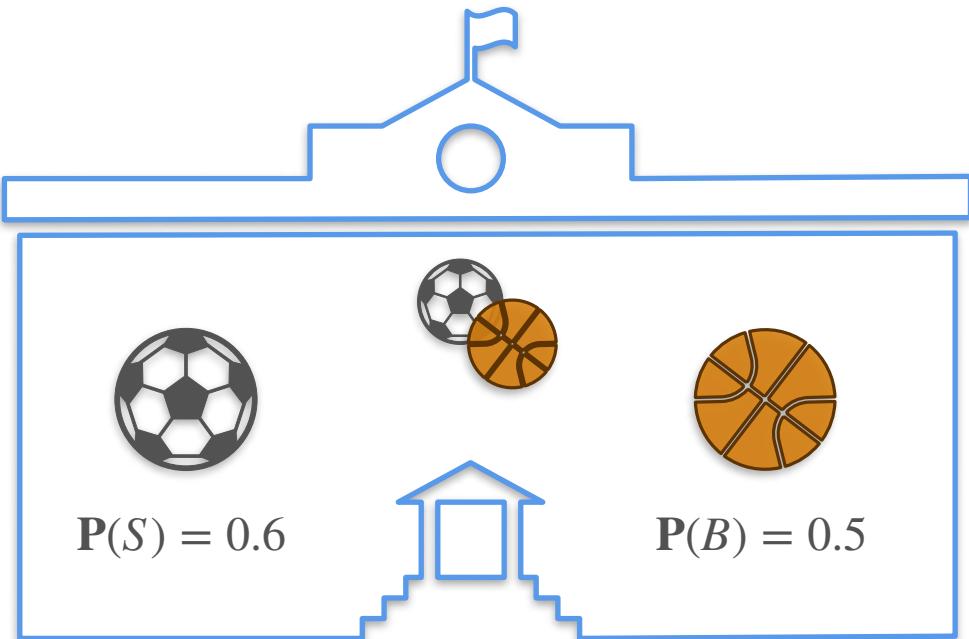
# Sum of Probabilities (Joint Events): Quiz 3



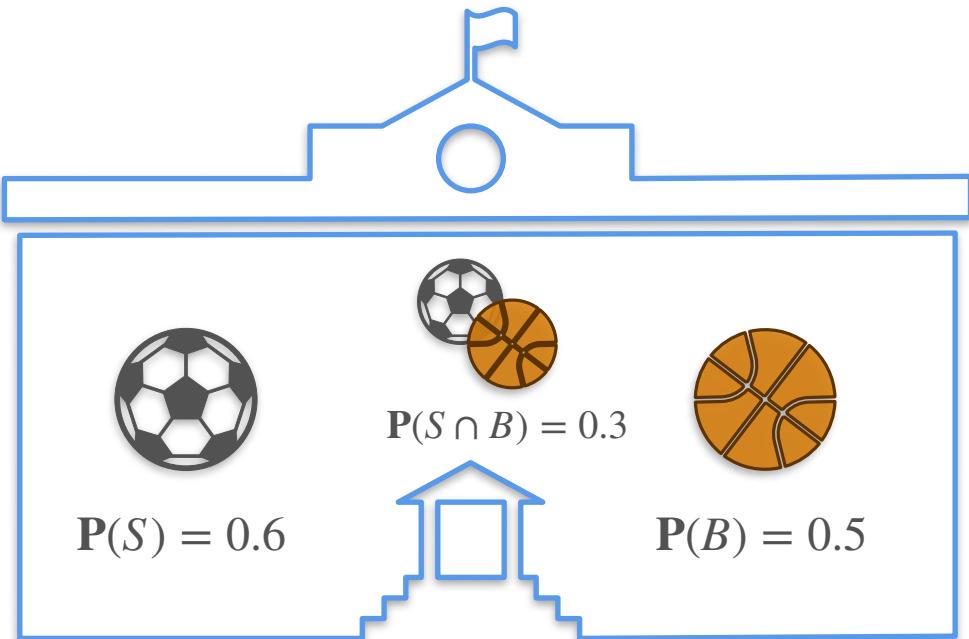
# Sum of Probabilities (Joint Events): Quiz 3



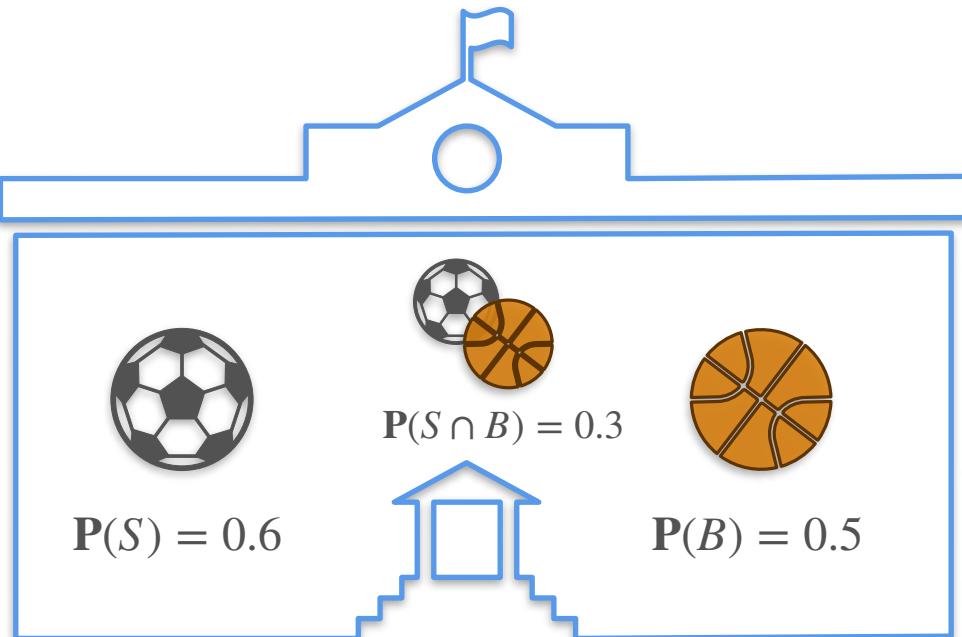
# Sum of Probabilities (Joint Events): Quiz 3



# Sum of Probabilities (Joint Events): Quiz 3



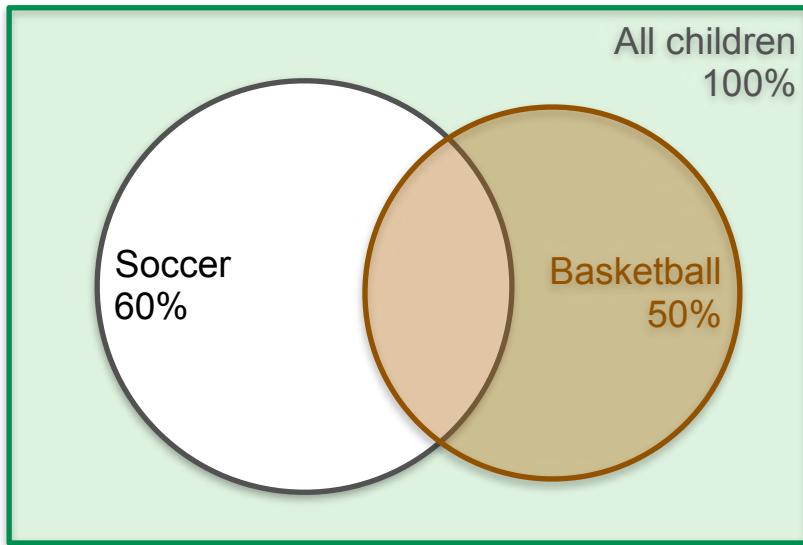
# Sum of Probabilities (Joint Events): Quiz 3



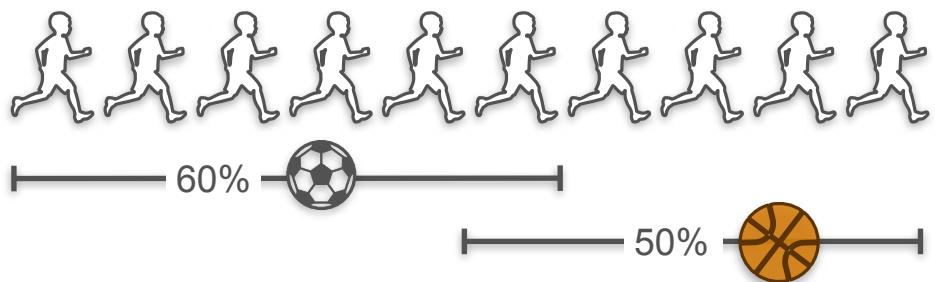
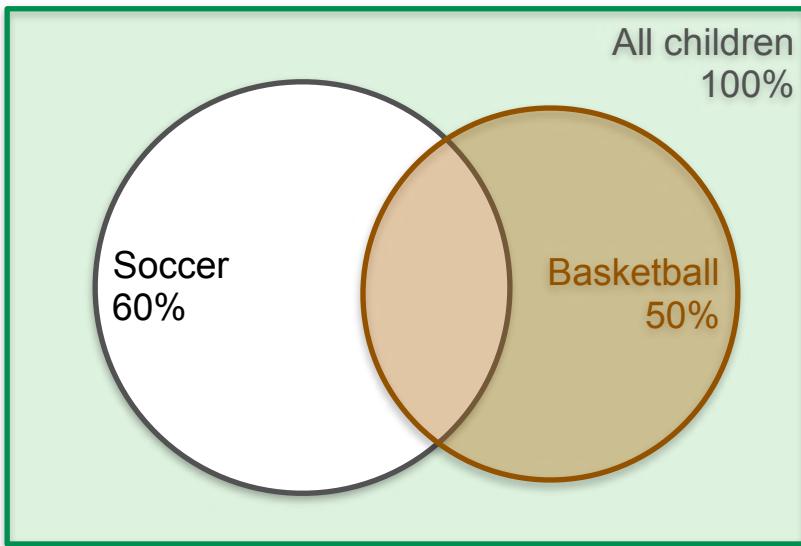
What is the probability that a child plays soccer or basketball?

# Sum of Probabilities (Joint Events): Quiz 3 Solution

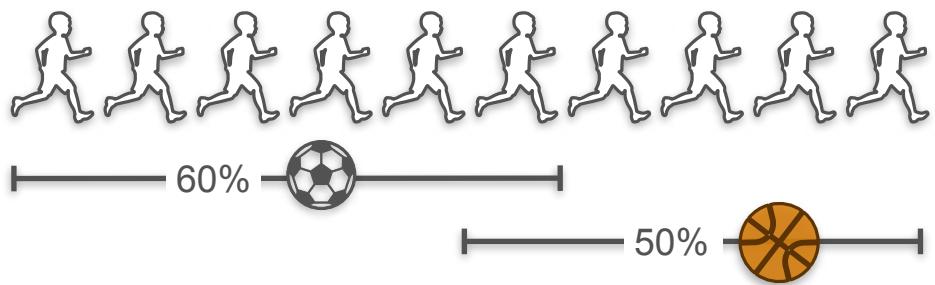
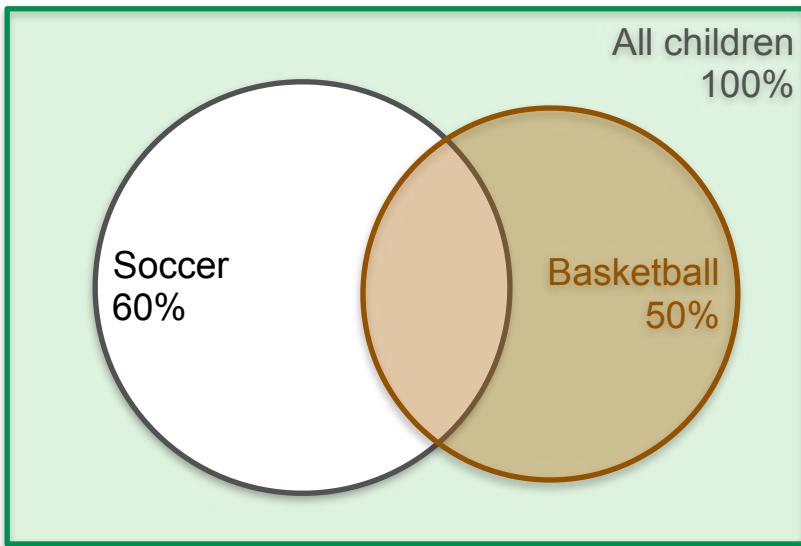
# Sum of Probabilities (Joint Events): Quiz 3 Solution



# Sum of Probabilities (Joint Events): Quiz 3 Solution

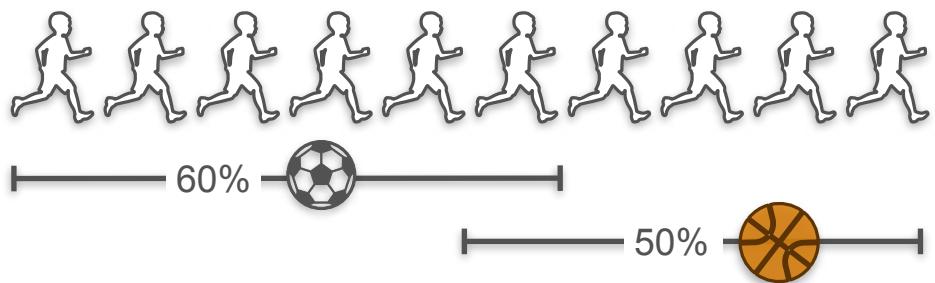
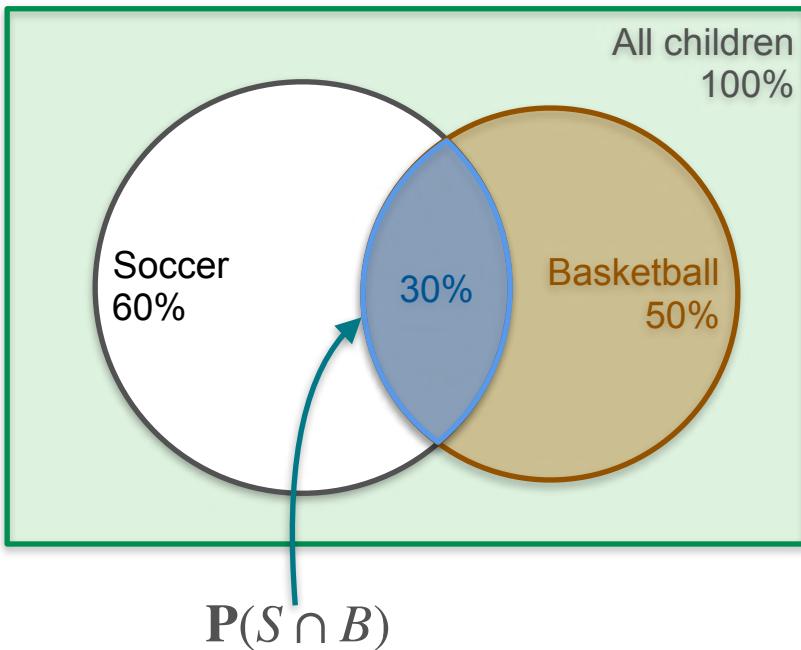


# Sum of Probabilities (Joint Events): Quiz 3 Solution



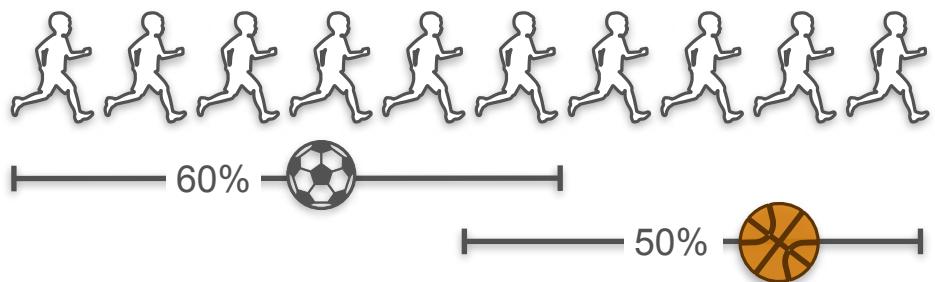
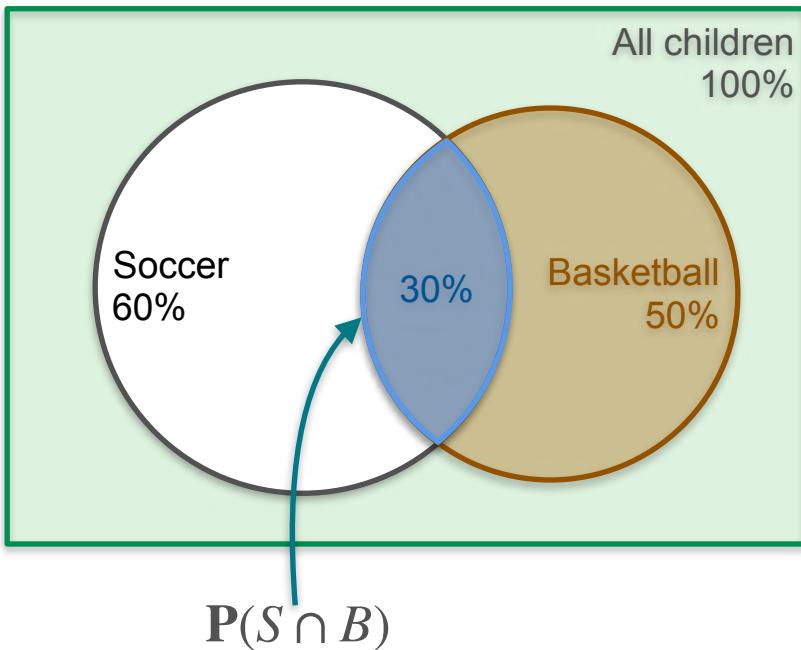
$$P(S \cup B) = P(S) + P(B)$$

# Sum of Probabilities (Joint Events): Quiz 3 Solution



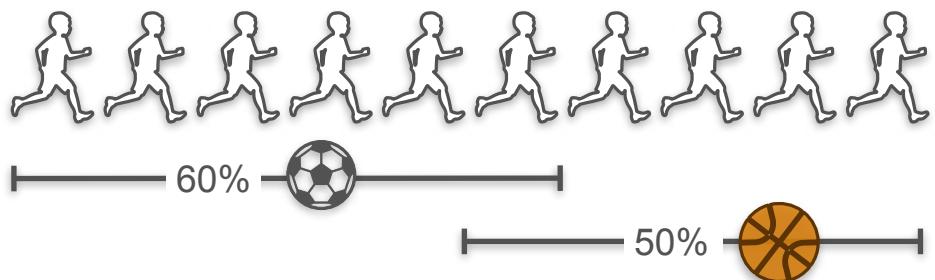
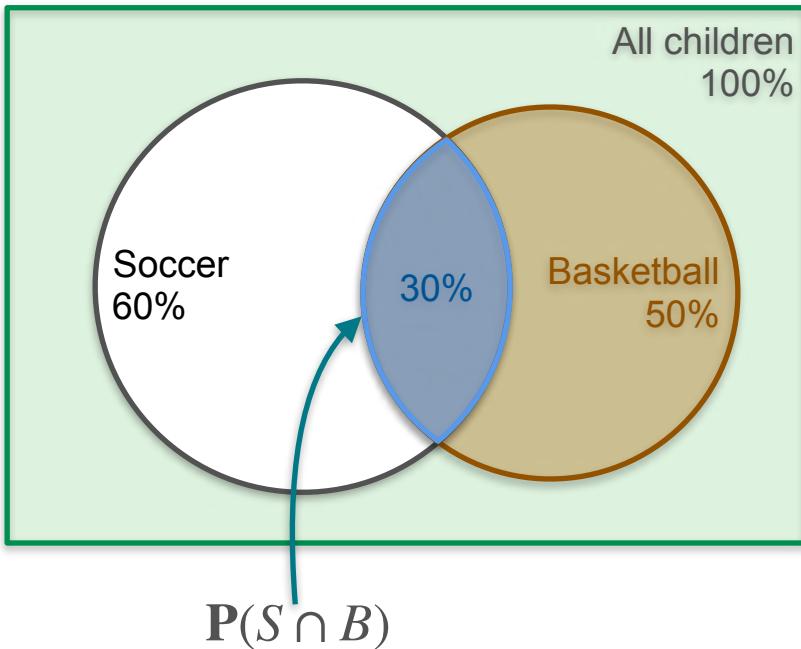
$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

# Sum of Probabilities (Joint Events): Quiz 3 Solution



$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B)$$

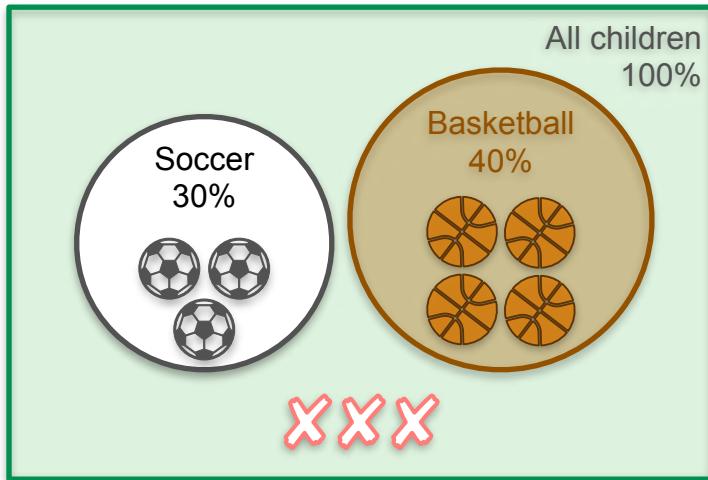
# Sum of Probabilities (Joint Events): Quiz 3 Solution



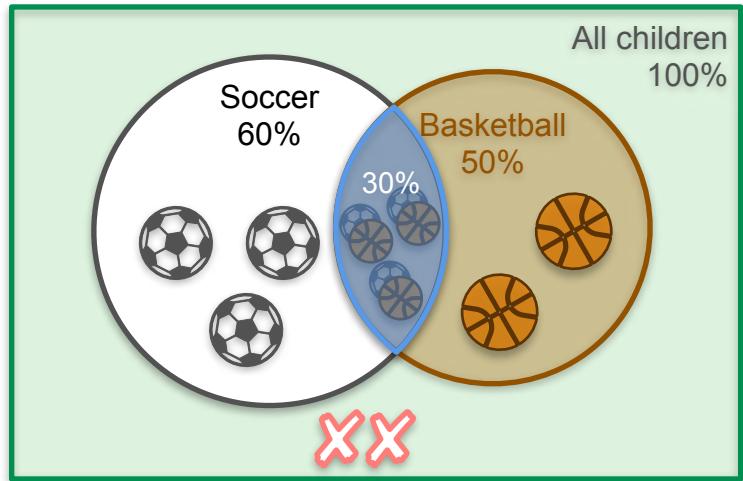
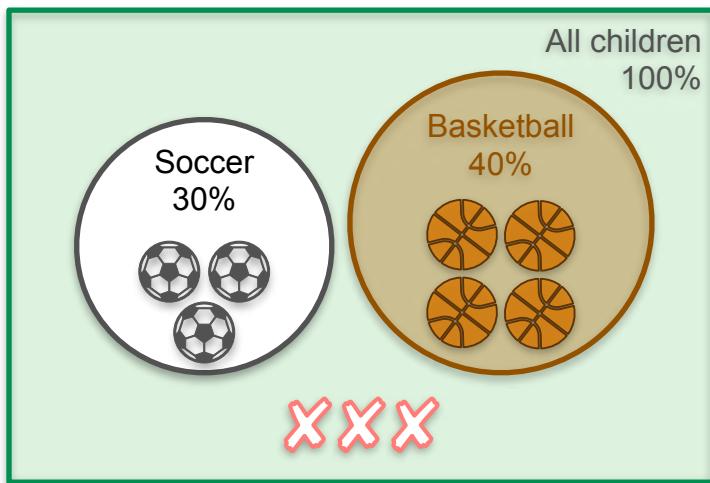
$$\begin{aligned}\mathbf{P}(S \cup B) &= \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B) \\ &= 0.6 + 0.5 - 0.3 \\ &= 0.8\end{aligned}$$

# Disjoint Events Vs Joint Events

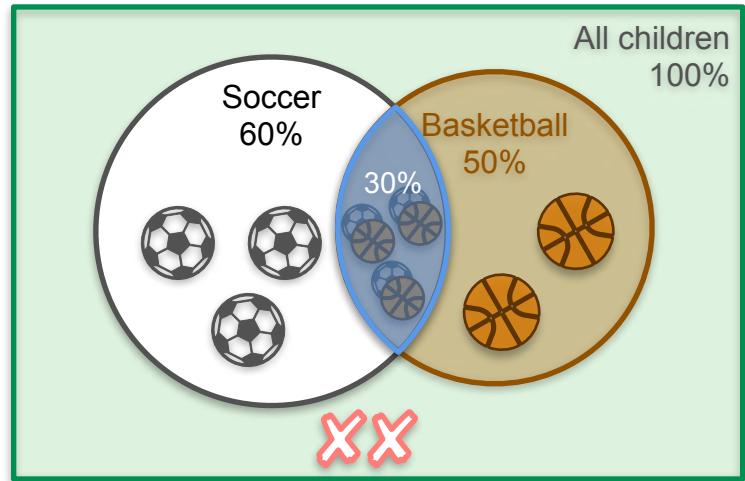
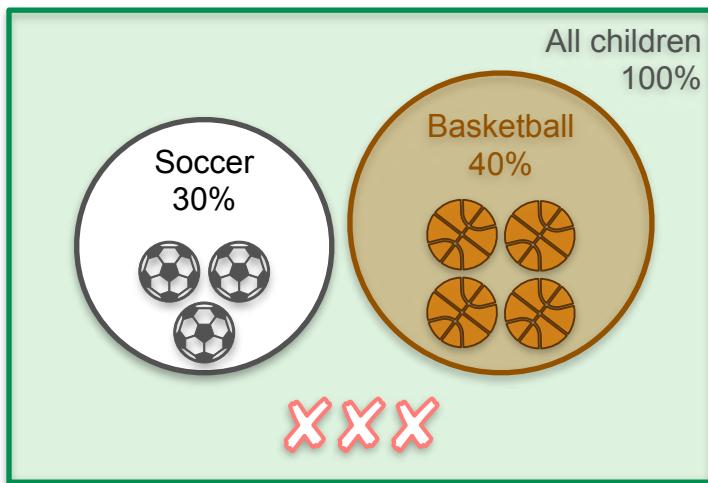
# Disjoint Events Vs Joint Events



# Disjoint Events Vs Joint Events

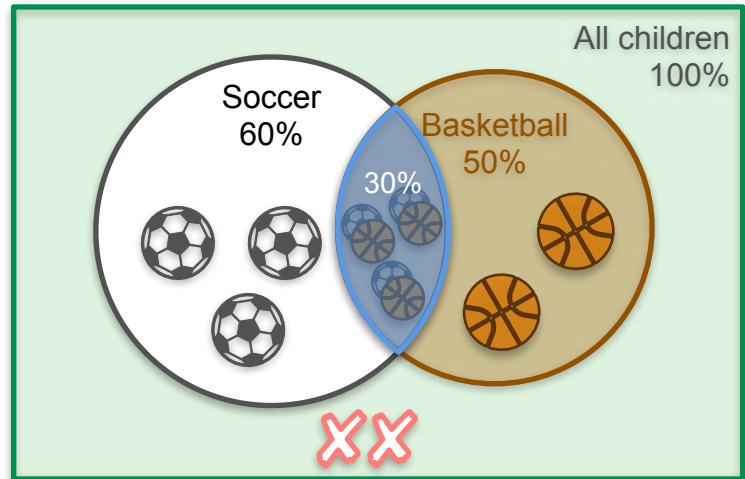
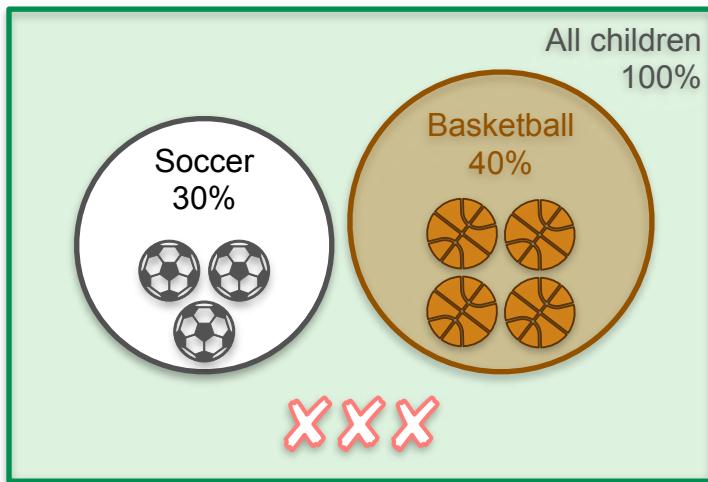


# Disjoint Events Vs Joint Events



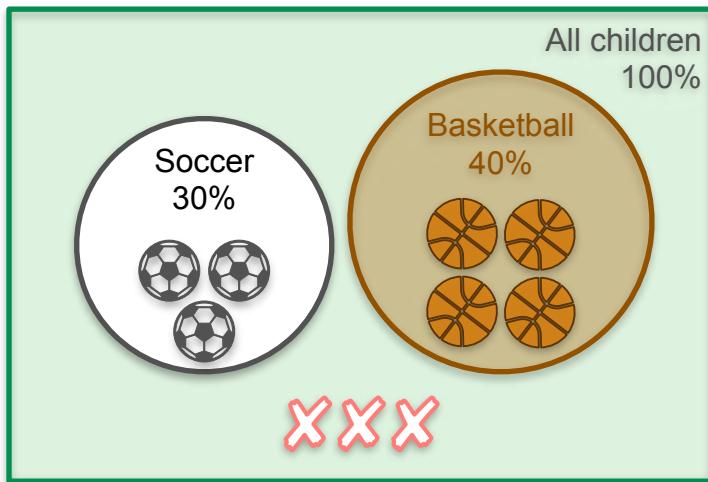
# Disjoint Events Vs Joint Events

Disjoint

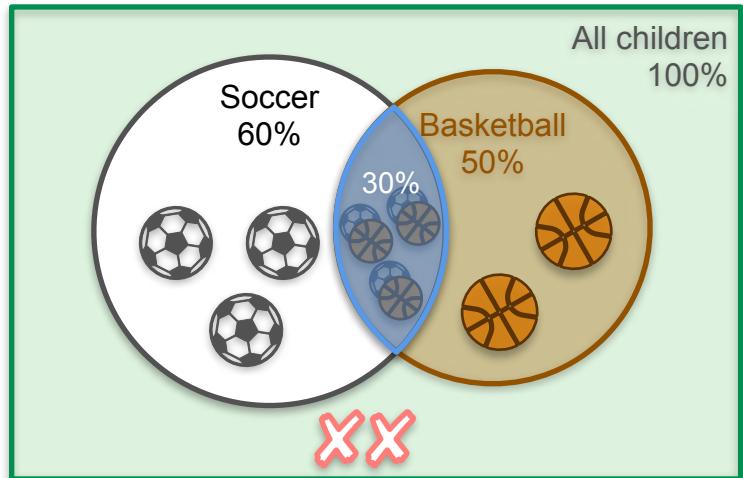


# Disjoint Events Vs Joint Events

Disjoint

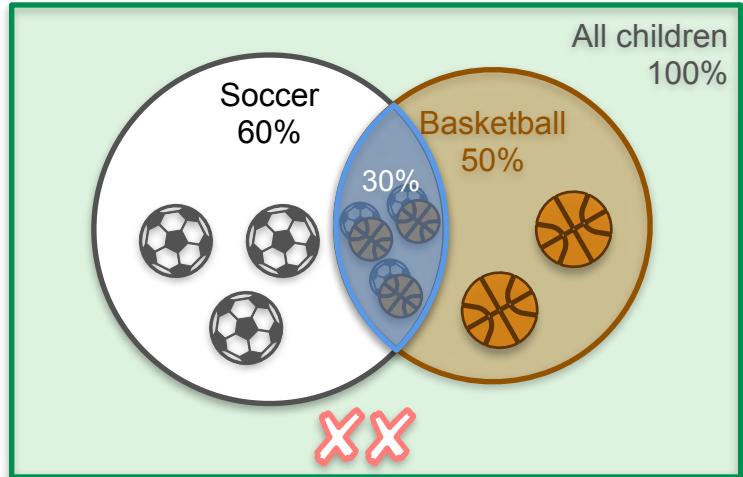
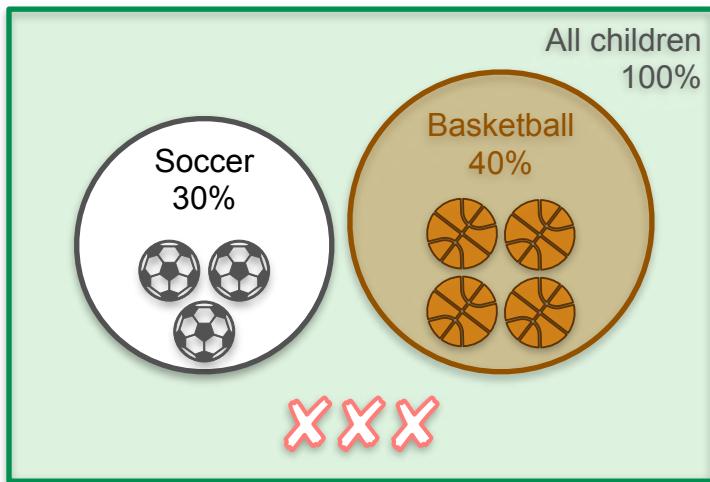


Mutually exclusive



# Disjoint Events Vs Joint Events

Disjoint

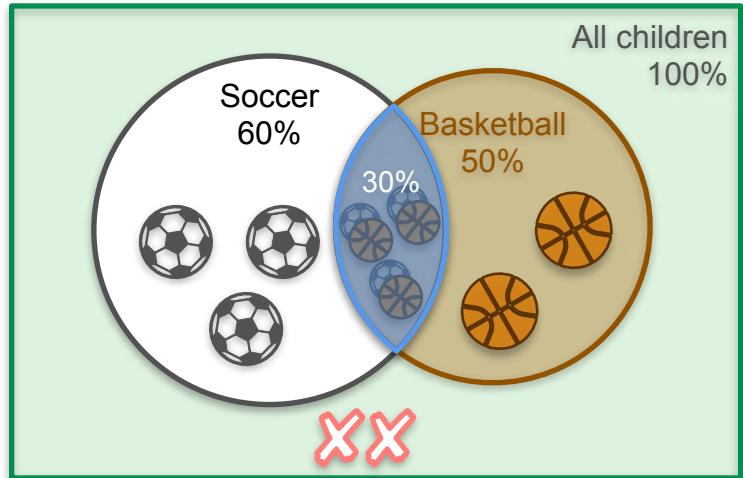
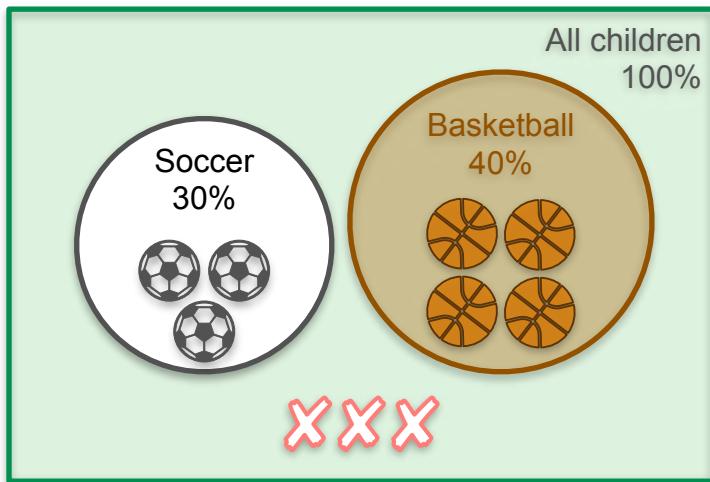


Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

# Disjoint Events Vs Joint Events

Disjoint

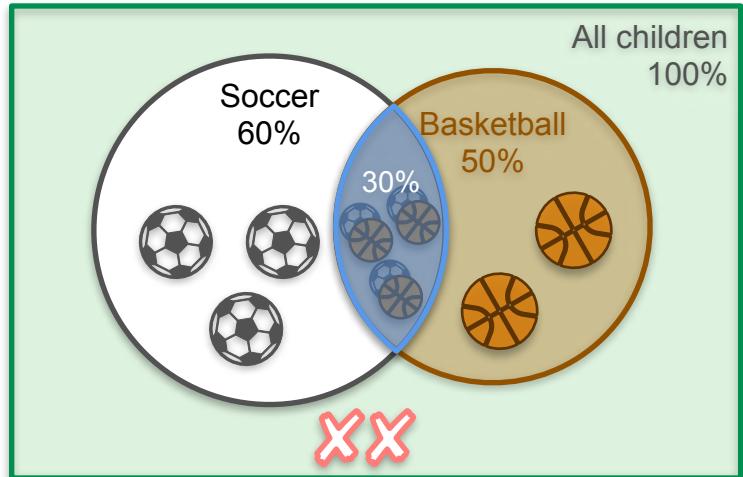
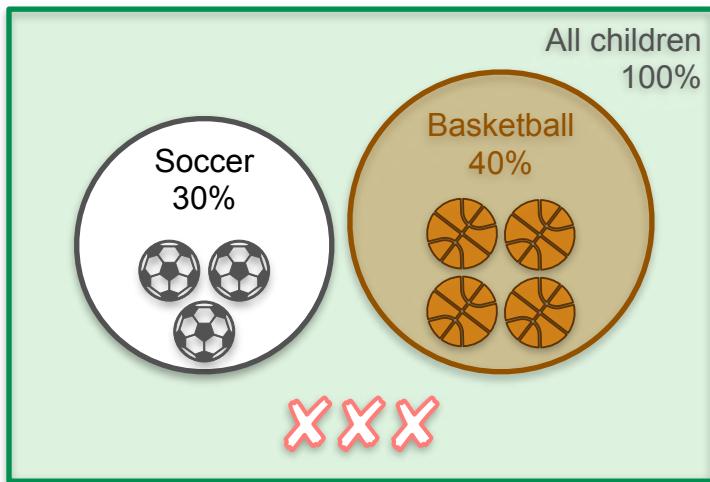


Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

# Disjoint Events Vs Joint Events

Disjoint

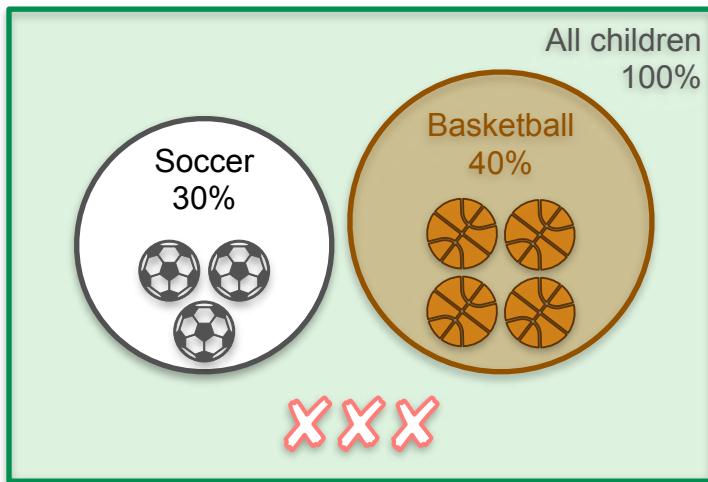


Mutually exclusive

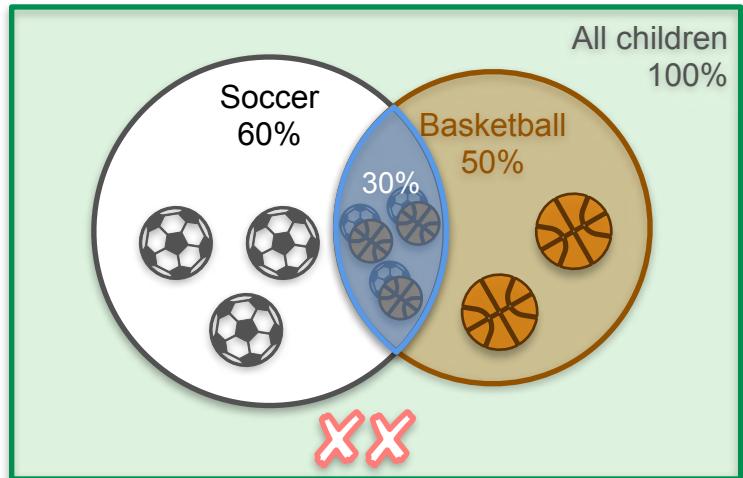
$$P(S \cup B) = P(S) + P(B)$$

# Disjoint Events Vs Joint Events

Disjoint



Joint

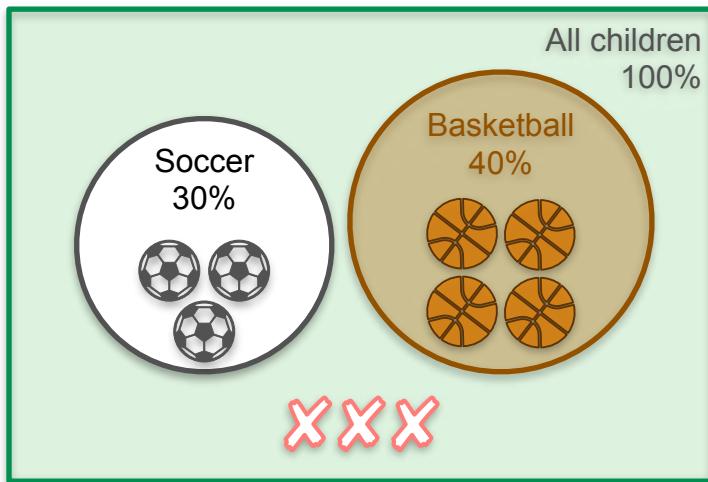


Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

# Disjoint Events Vs Joint Events

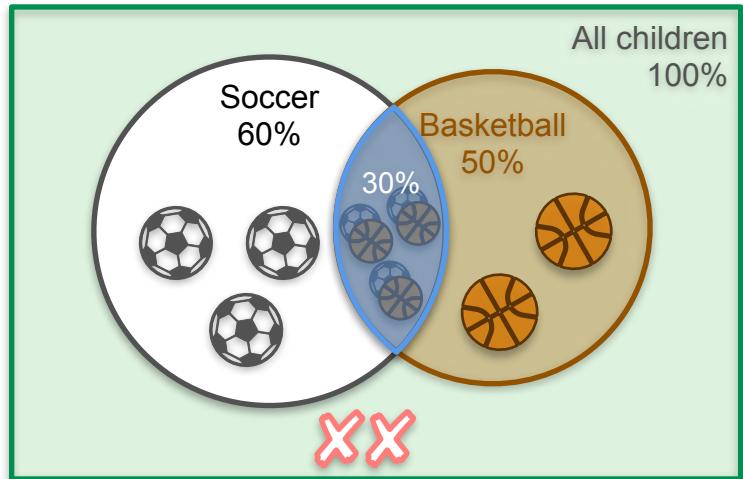
Disjoint



Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

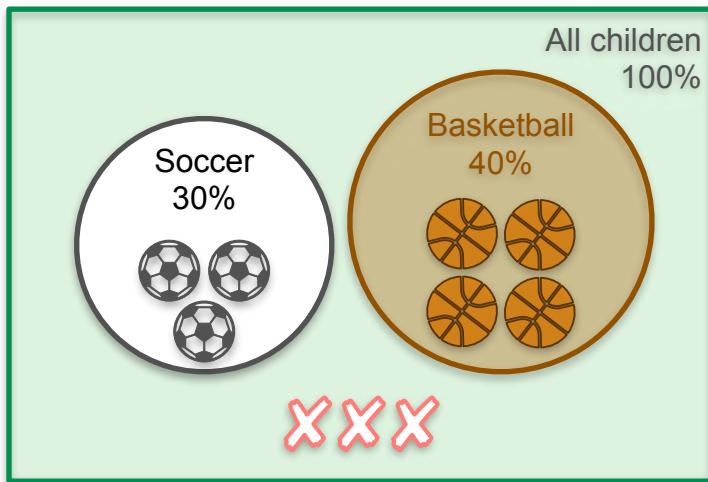
Joint



Non-mutually exclusive

# Disjoint Events Vs Joint Events

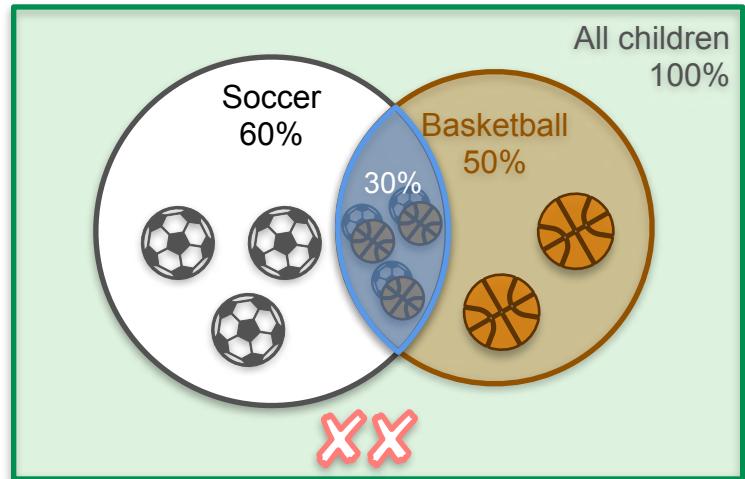
Disjoint



Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

Joint

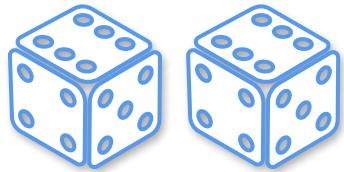


Non-mutually exclusive

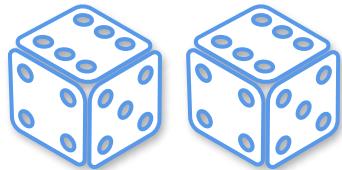
$$P(S \cup B) = P(S) + P(B) - P(S \cap B)$$

# Sum of Probabilities (Joint Events): Dice Example 1

# Sum of Probabilities (Joint Events): Dice Example 1



# Sum of Probabilities (Joint Events): Dice Example 1



What is the probability of obtaining a sum of 7 or a difference of 1?

# Sum of Probabilities (Joint Events): Dice Example 1

# Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

# Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities (Joint Events): Dice Example 1

A

sum = 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities (Joint Events): Dice Example 1

A or B

sum = 7 or diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities (Joint Events): Dice Example 1

A or B

sum = 7 or diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,2	2,1	2,2	2,3	2,4	2,5	2,6
2,3	3,1	3,2	3,3	3,4	3,5	3,6
3,4	4,1	4,2	4,3	4,4	4,5	4,6
4,5	5,1	5,2	5,3	5,4	5,5	5,6
5,6	6,1	6,2	6,3	6,4	6,5	6,6

sum = 7 and diff = 1

# Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

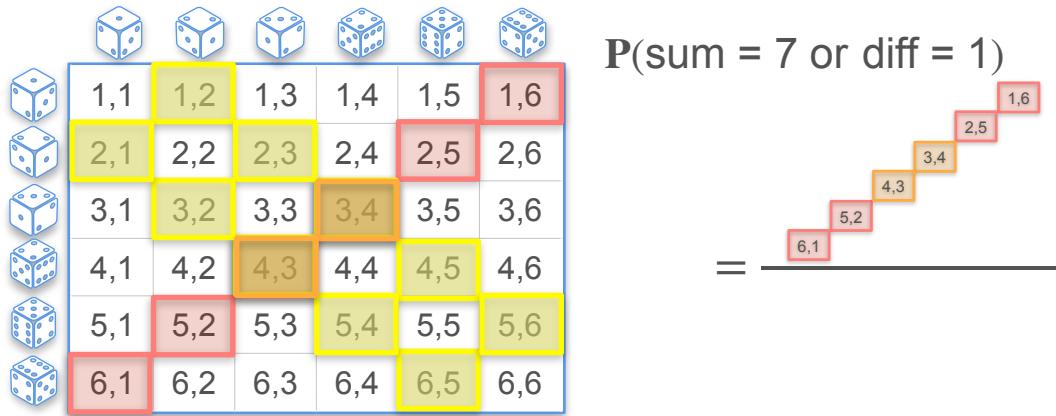
# Sum of Probabilities (Joint Events): Dice Example 1

					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

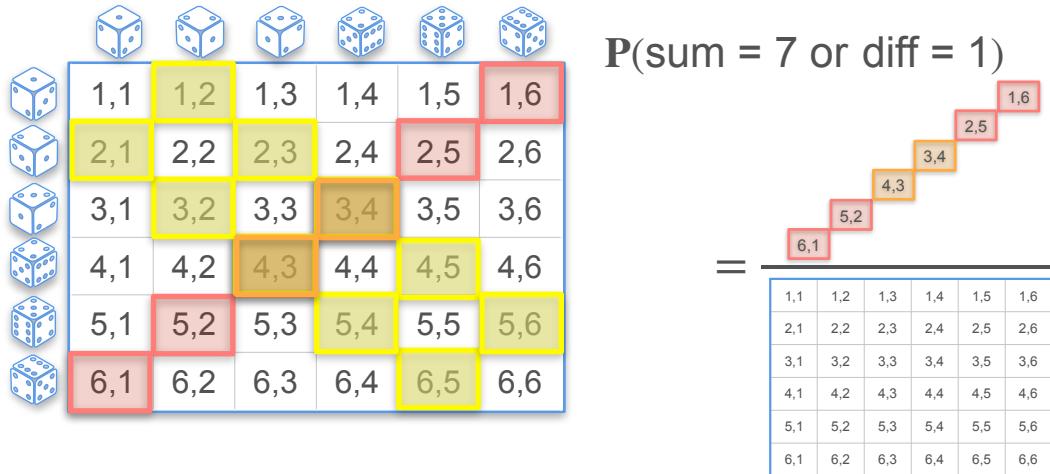
P(sum = 7 or diff = 1)

= \_\_\_\_\_

# Sum of Probabilities (Joint Events): Dice Example 1



# Sum of Probabilities (Joint Events): Dice Example 1



# Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

$$= \frac{\begin{array}{ccccccc} & & & & & 1,6 \\ & & & & & 2,5 \\ & & & & & 3,4 \\ & & & & & 4,3 \\ & & & & & 5,2 \\ & & & & & 6,1 \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \underline{\hspace{10em}}$$

# Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{ccccccc} & 1,2 & & & & & \\ & 2,1 & 2,3 & & & & \\ & 3,2 & 3,3 & 3,4 & & & \\ & 4,3 & 4,4 & 4,5 & 4,6 & & \\ & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ & 6,1 & & & & & \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}{36} + \frac{\begin{array}{ccccccc} & 1,2 & & & & & \\ & 2,1 & & 2,3 & & & \\ & 3,2 & & 3,4 & & & \\ & 4,3 & & 4,5 & & & \\ & 5,4 & & 5,6 & & & \\ & 6,5 & & & & & \\ \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}{36}$$

# Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}$$

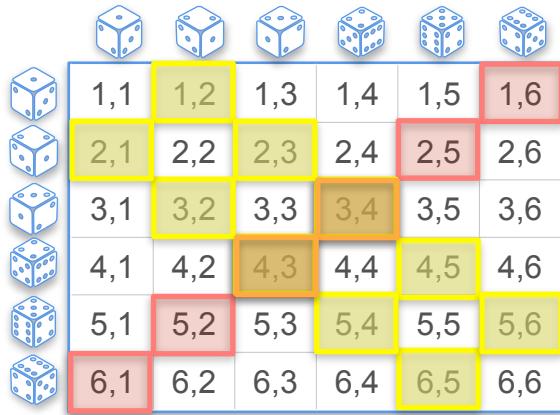
# Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{ccccccc} & 1,2 & & & & & \\ & 2,1 & 2,3 & & & & \\ & 3,2 & 3,3 & 3,4 & & & \\ & 4,3 & 4,4 & 4,5 & 4,6 & & \\ & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ & 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}{36} + \frac{\begin{array}{ccccccc} & 1,2 & & & & & \\ & 2,1 & & 2,3 & & & \\ & 3,2 & 3,4 & & & & \\ & 4,3 & 4,5 & & & & \\ & 5,4 & 5,6 & & & & \\ & 6,5 & & & & & \end{array}}{36} - \frac{\begin{array}{ccccccc} & 1,2 & & & & & \\ & 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ & 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ & 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ & 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ & 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}{36}$$

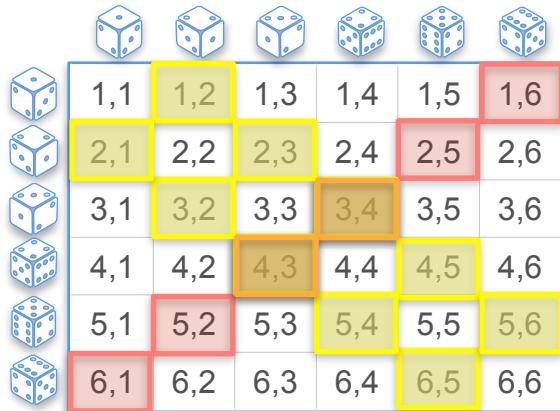
# Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,2 \\ 6,1 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,3 \\ 3,4 \\ 4,5 \\ 5,6 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}} - \frac{\begin{array}{c} 3,4 \\ 4,3 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 & \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 & \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 & \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 & \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 & \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 & \end{array}}$$

# Sum of Probabilities (Joint Events): Dice Example 1



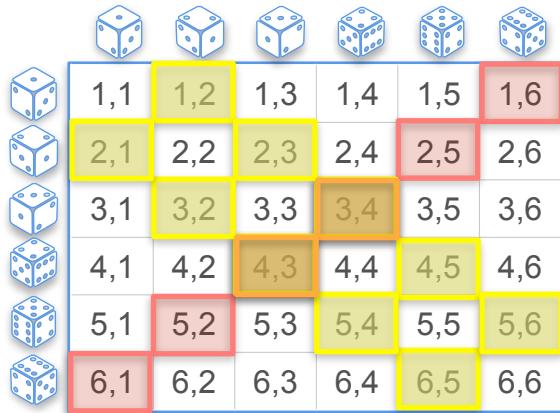
$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{array}{c} \text{Diagram showing the decomposition of the joint event into two separate events: } \\ \text{sum} = 7 \text{ and } \text{diff} = 1. \end{array}$$

The diagram illustrates the decomposition of the joint event into two separate events:  $\text{sum} = 7$  and  $\text{diff} = 1$ . The first term shows the sum = 7 outcomes: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). The second term shows the difference = 1 outcomes: (1,2), (2,1), (3,2), (4,1), (5,2), (6,1). The third term, which would normally be subtracted, is shown as a single row of zeros.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

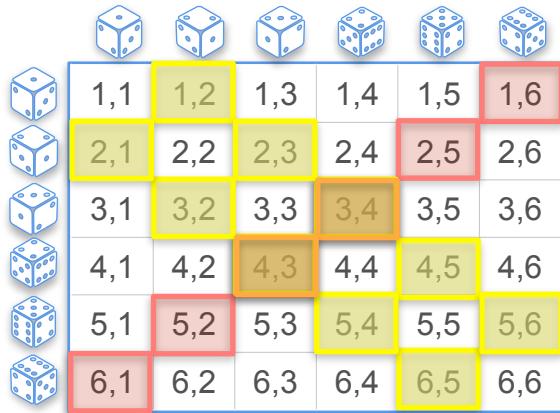
# Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,4 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,4 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} - \frac{\begin{array}{c} 3,4 \\ 4,3 \\ 5,4 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} \\ &= \frac{6}{36} \end{aligned}$$

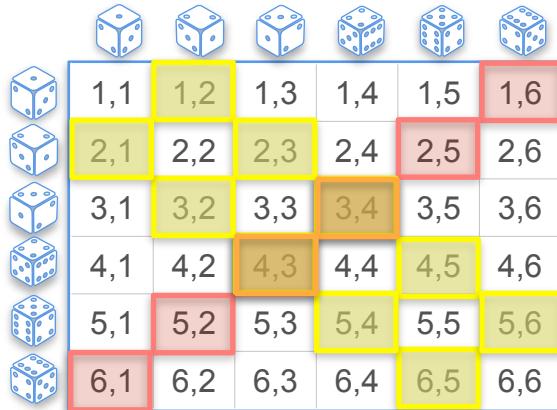
# Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned} &= \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,4 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\begin{array}{c} 1,2 \\ 2,1 \\ 3,2 \\ 4,3 \\ 5,4 \\ 6,5 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} - \frac{\begin{array}{c} 3,4 \\ 4,3 \\ 5,4 \end{array}}{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} \\ &= \frac{6}{36} + \frac{10}{36} \end{aligned}$$

# Sum of Probabilities (Joint Events): Dice Example 1

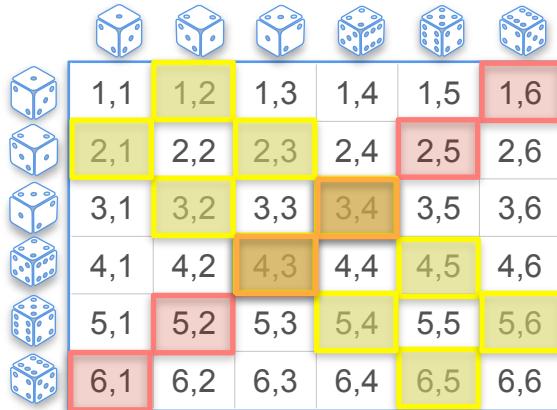


$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$\begin{aligned}
 &= \frac{\begin{array}{ccccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}{36} + \frac{\begin{array}{ccccccc} 1,2 & & & & & & \\ 2,1 & 2,3 & & & & & \\ 3,2 & 3,4 & & & & & \\ 4,3 & 4,5 & & & & & \\ 5,4 & 5,6 & & & & & \\ 6,5 & & & & & & \end{array}}{36} - \frac{\begin{array}{ccccccc} 3,4 & & & & & & \\ 4,3 & & & & & & \\ 5,2 & & & & & & \\ 6,1 & & & & & & \end{array}}{36}
 \end{aligned}$$

$$= \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$$

# Sum of Probabilities (Joint Events): Dice Example 1

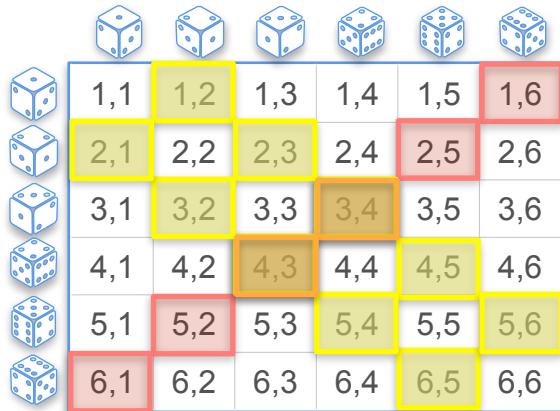


P(sum = 7 or diff = 1)

$$\begin{aligned}
 &= \frac{\text{Count of } (\text{sum} = 7) + \text{Count of } (\text{diff} = 1) - \text{Count of } (\text{sum} = 7 \text{ and } \text{diff} = 1)}{36} \\
 &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\
 &= \frac{14}{36}
 \end{aligned}$$

The diagram illustrates the calculation of the probability of joint events. It shows three separate sets of outcomes highlighted by colored boxes (yellow, red, orange) and their intersections. The first set (yellow) represents outcomes where the sum is 7. The second set (red) represents outcomes where the difference is 1. The third set (orange) represents outcomes where both conditions are met simultaneously. The counts of these sets are used to calculate the probability as a fraction of the total 36 possible outcomes.

# Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= P(\text{sum} = 7) + \frac{\text{Number of outcomes for diff=1}}{36} - \frac{\text{Number of outcomes for both}}{36}$$

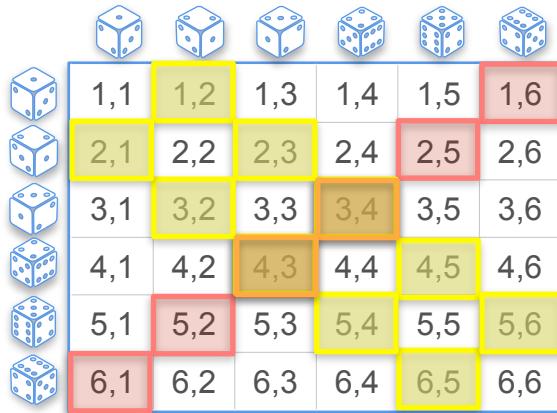
The diagram shows three sets of outcomes highlighted with colored boxes:

- Yellow boxes (sum=7):** 1,6; 2,5; 3,4; 4,3; 5,2; 6,1
- Orange boxes (diff=1):** 1,2; 2,1; 3,2; 4,1; 5,3; 6,2
- Red boxes (both):** 1,5; 2,4; 3,5; 4,5; 5,4; 6,4

The final result is calculated as follows:

$$\begin{aligned} &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\ &= \frac{14}{36} \end{aligned}$$

# Sum of Probabilities (Joint Events): Dice Example 1



$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= P(\text{sum} = 7) + P(\text{diff} = 1) - \frac{2}{36}$$

The equation shows the calculation of the probability of the union of two events. It consists of three parts: the probability of the first event (sum = 7), the probability of the second event (difference = 1), and the negative probability of their intersection (both sum = 7 and difference = 1).

The first part,  $P(\text{sum} = 7)$ , is calculated from a 6x6 grid of all possible outcomes (2,1 to 6,6). The outcomes where the sum is 7 are highlighted in yellow. There are 6 such outcomes, so the probability is  $\frac{6}{36}$ .

The second part,  $P(\text{diff} = 1)$ , is calculated from the same 6x6 grid. The outcomes where the difference is 1 are highlighted in red. There are 6 such outcomes, so the probability is  $\frac{6}{36}$ .

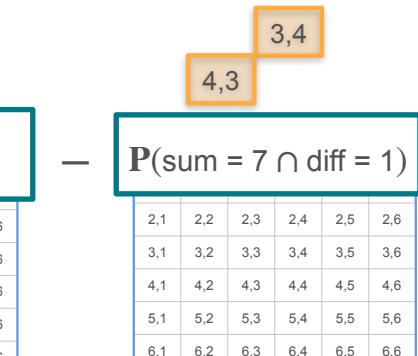
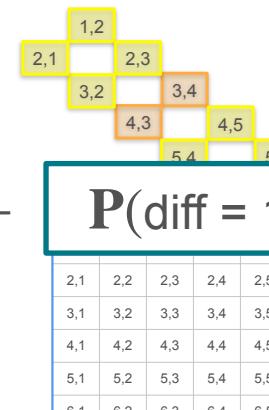
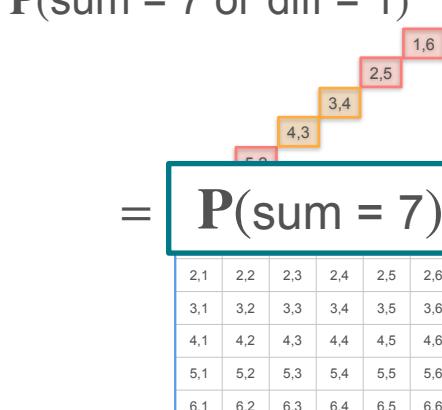
The third part,  $-\frac{2}{36}$ , represents the negative probability of the intersection of the two events. The intersection consists of the outcomes where both the sum is 7 and the difference is 1. These outcomes are the cells (1, 2), (2, 1), (3, 2), (4, 1), (5, 2), and (6, 1), which are highlighted in both yellow and red. There are 2 such outcomes, so the probability is  $\frac{2}{36}$ .

# Sum of Probabilities (Joint Events): Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

$$\begin{aligned}
 &= \boxed{\mathbf{P(\text{sum} = 7)}} + \boxed{\mathbf{P(\text{diff} = 1)}} - \boxed{\mathbf{P(\text{sum} = 7} \cap \text{diff} = 1)} \\
 &= \frac{6}{36} + \frac{10}{36} - \frac{2}{36} \\
 &= \frac{14}{36}
 \end{aligned}$$





DeepLearning.AI

# Introduction to probability

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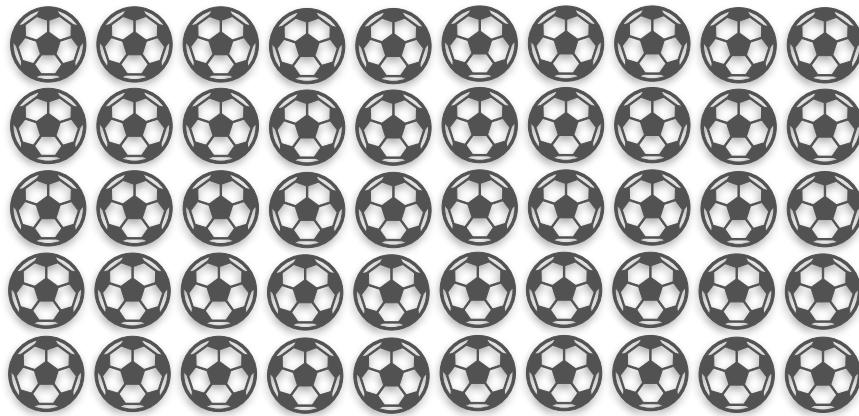
## Independence

# Independence: Quiz 1

# Independence: Quiz 1

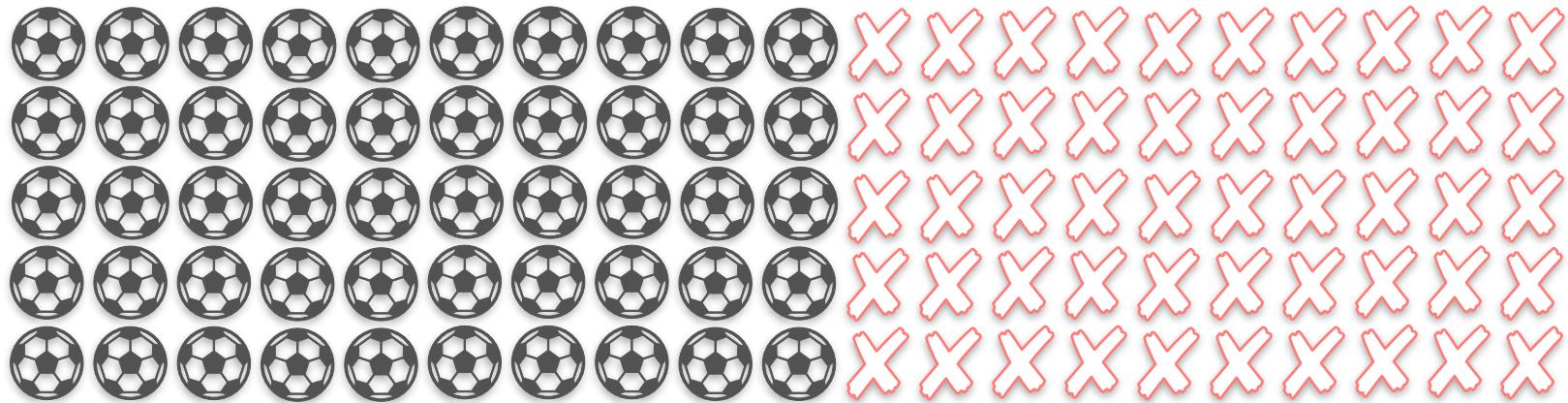
**100 kids**

# Independence: Quiz 1



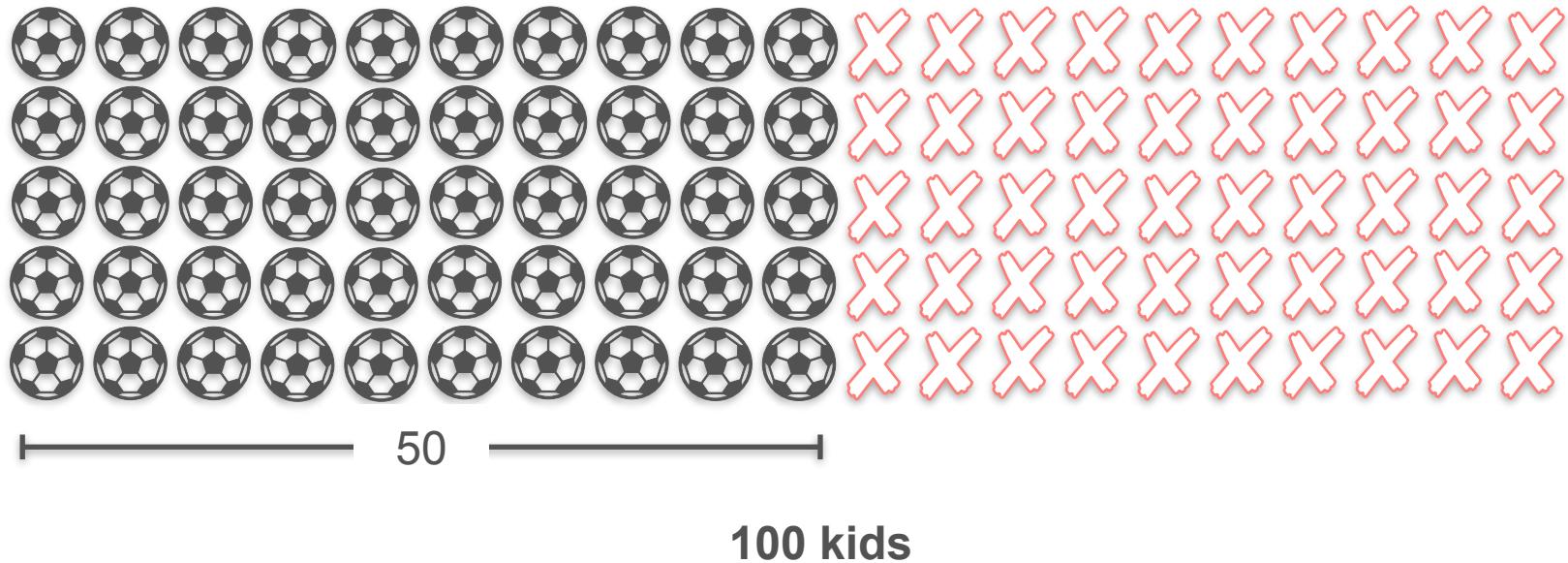
**100 kids**

# Independence: Quiz 1

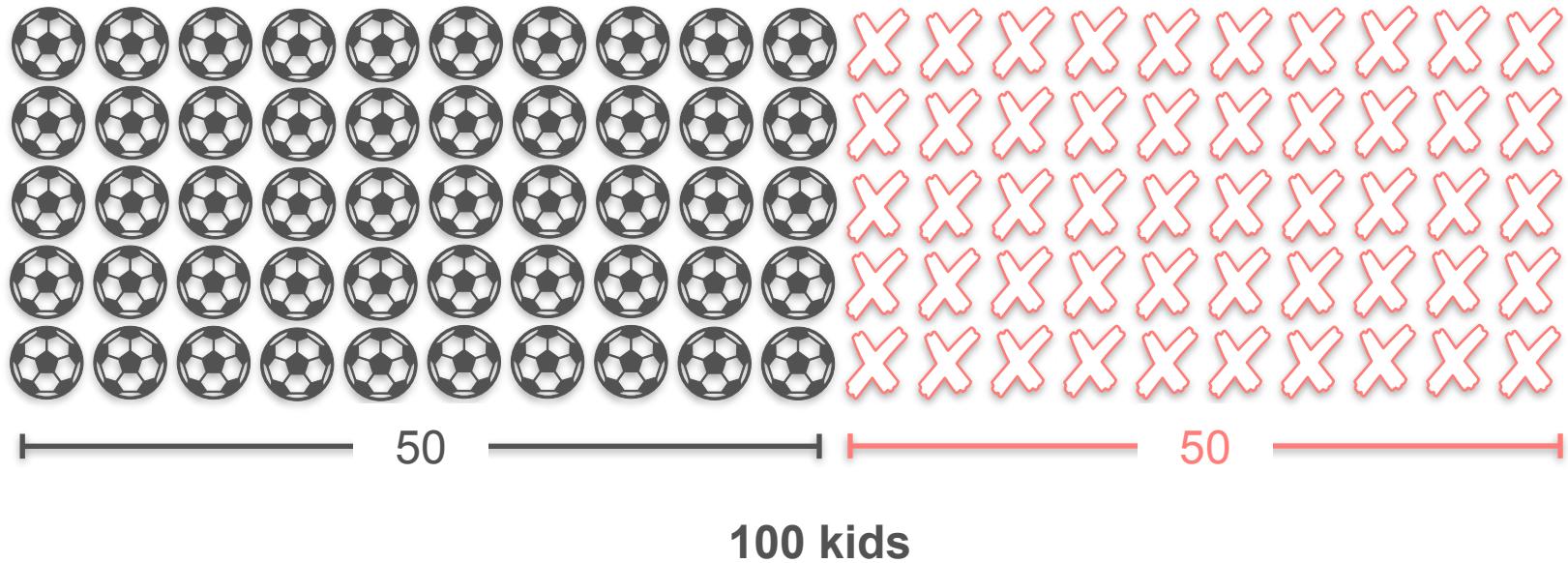


100 kids

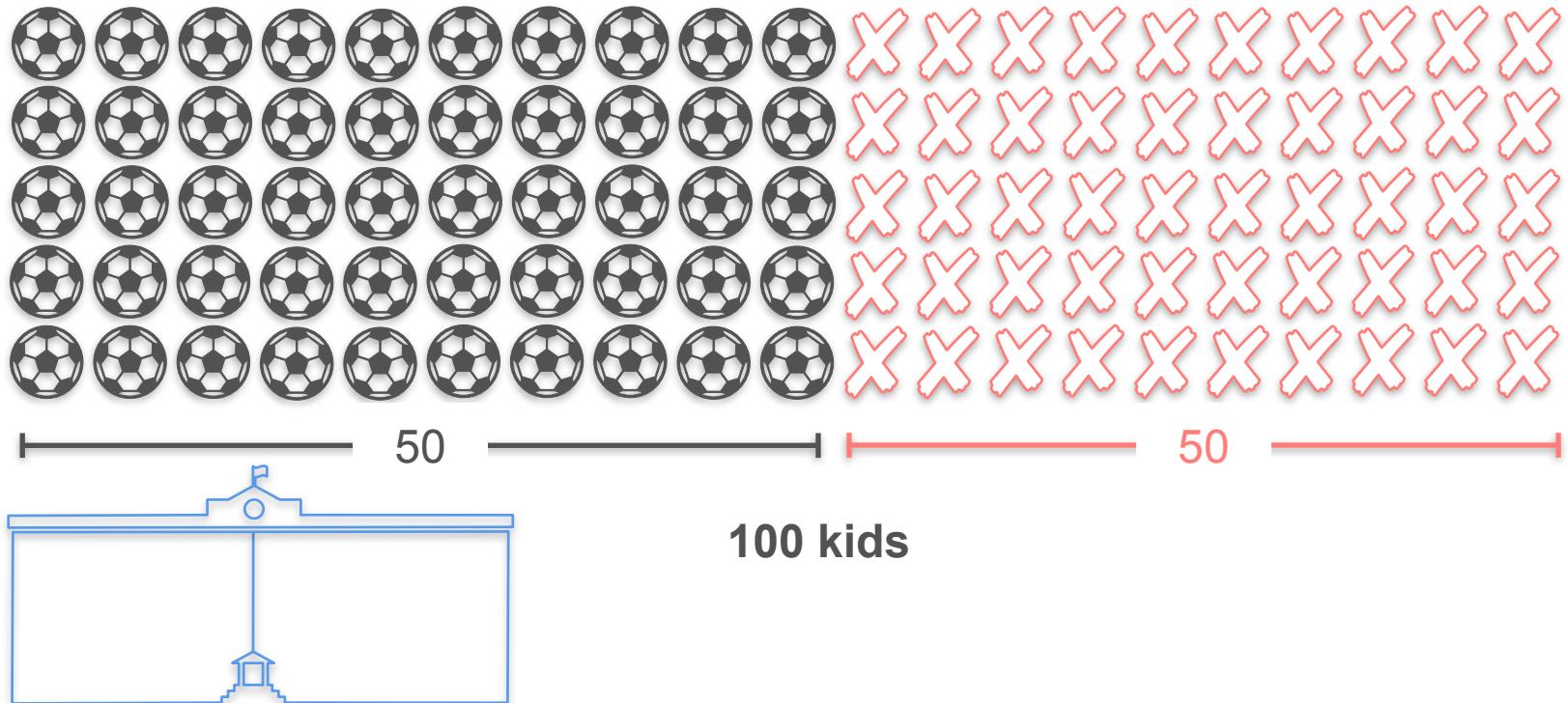
# Independence: Quiz 1



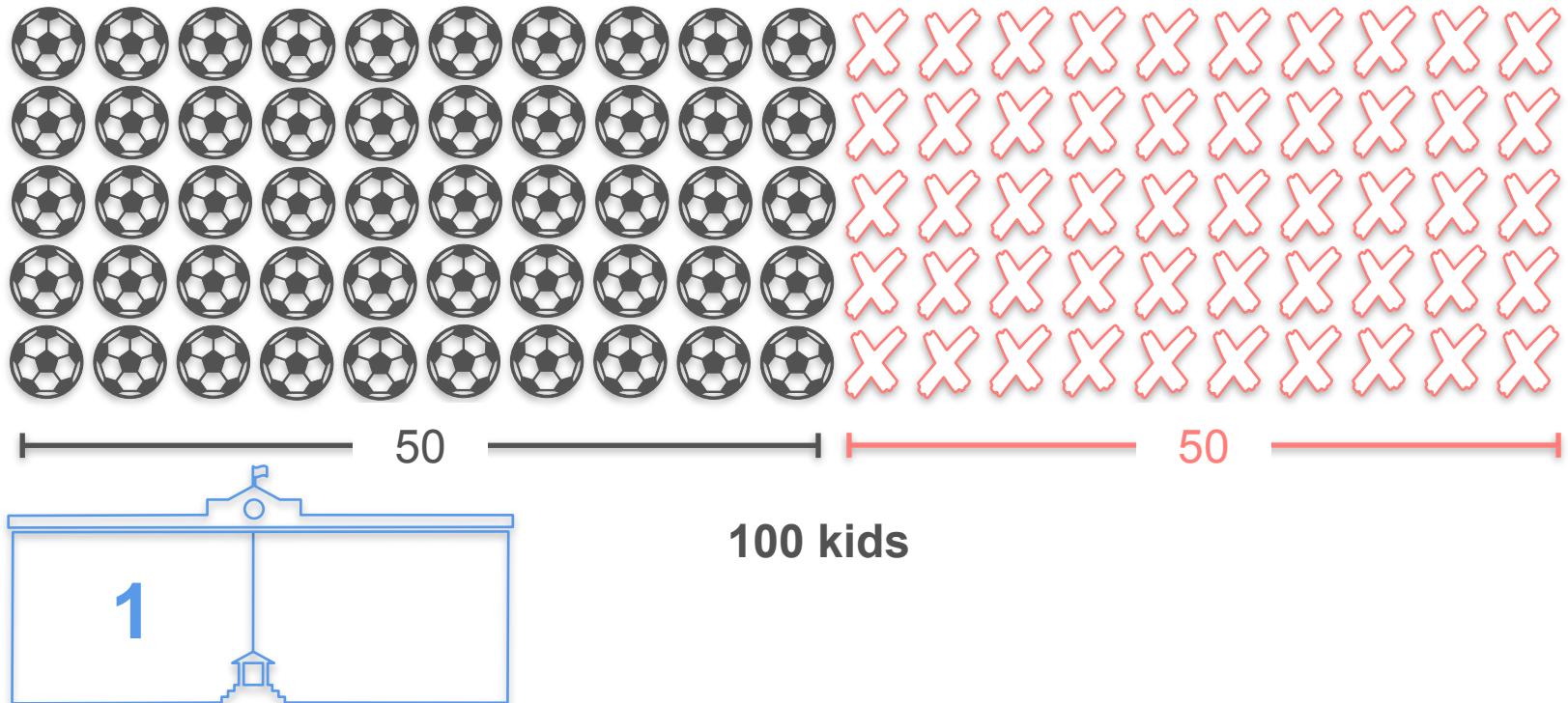
# Independence: Quiz 1



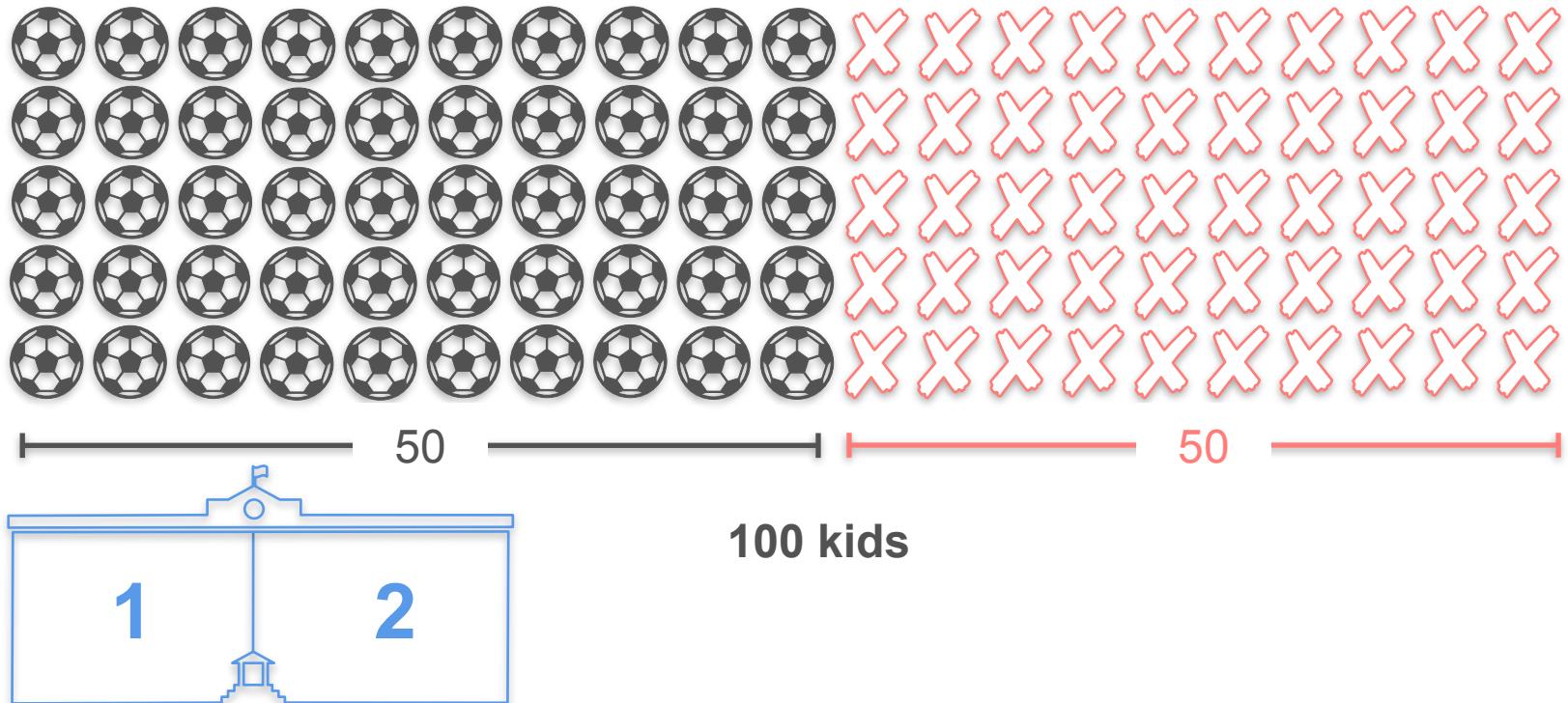
# Independence: Quiz 1



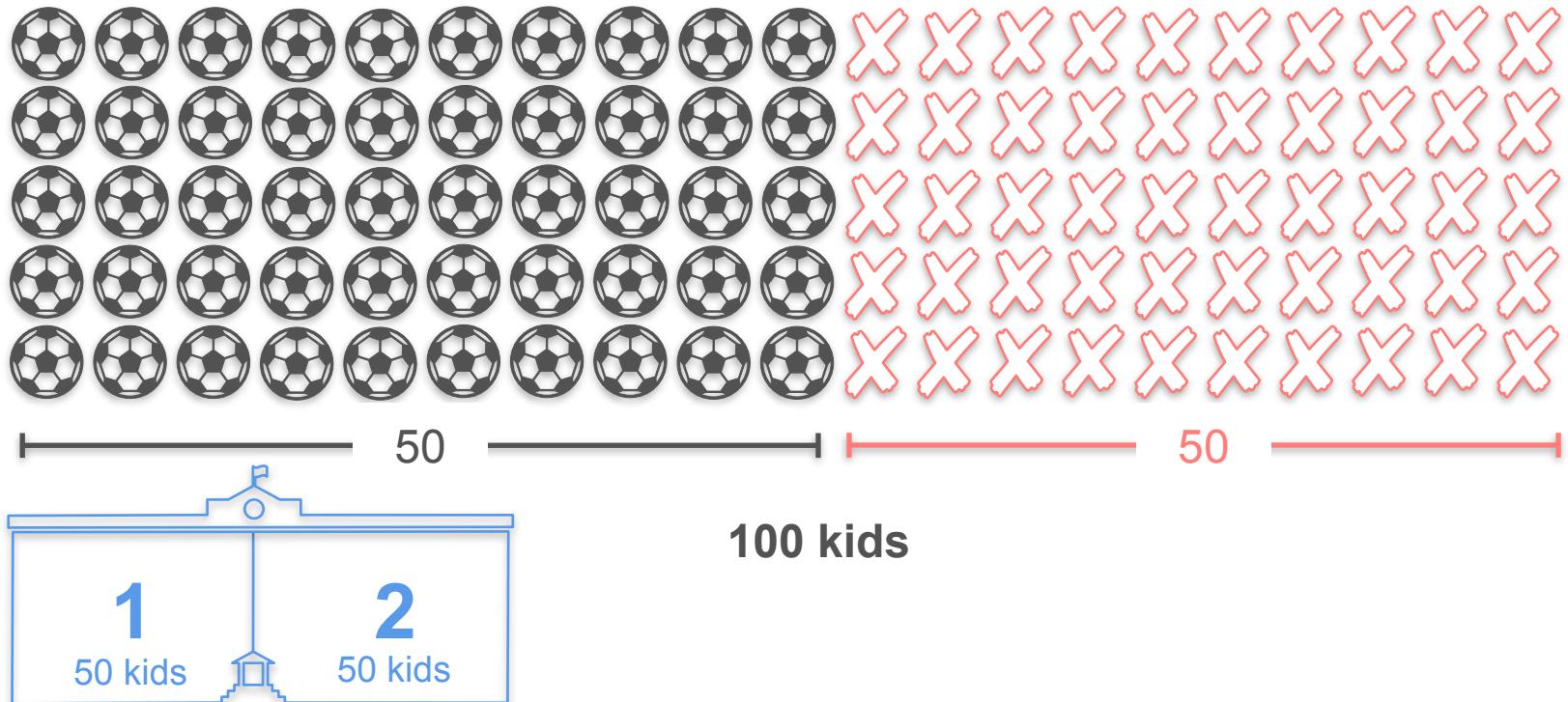
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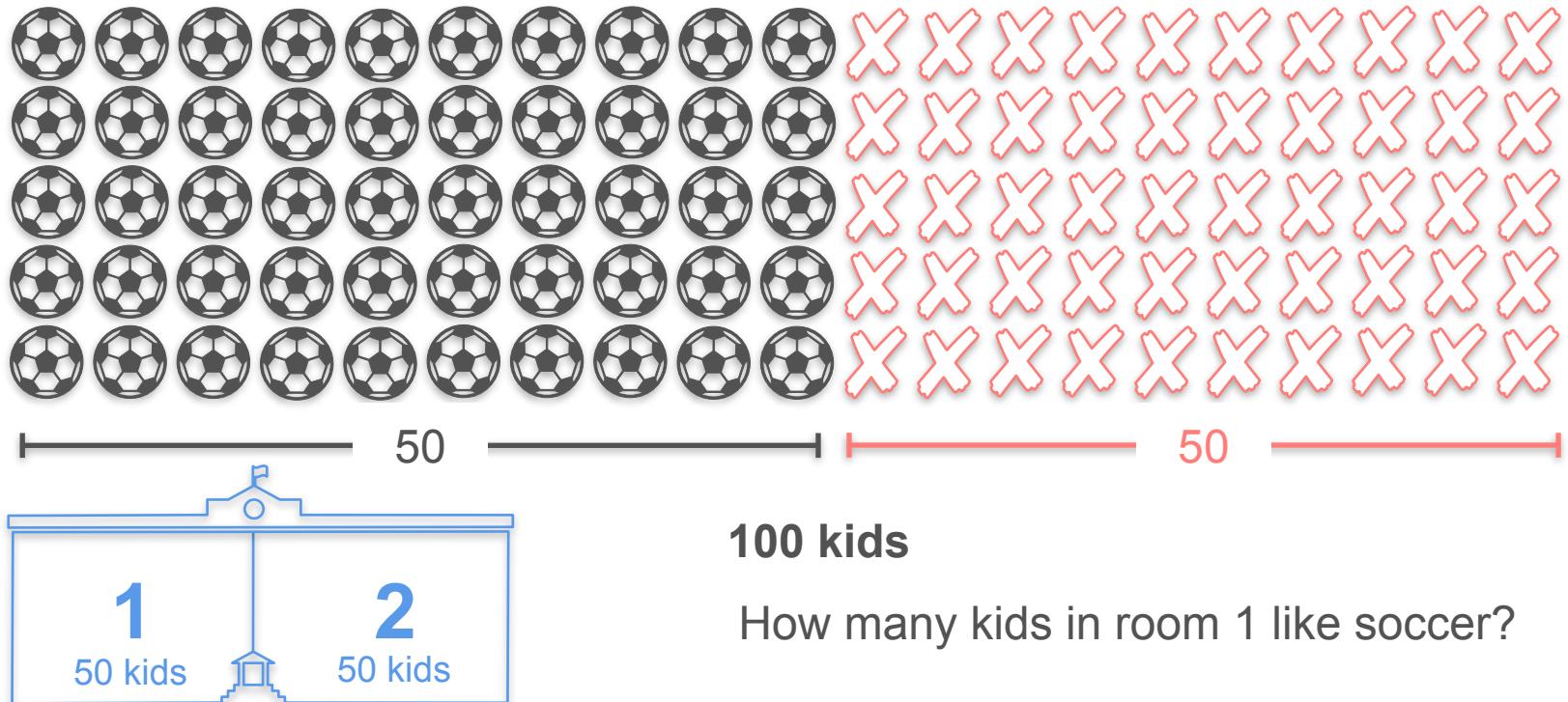
# Independence: Quiz 1



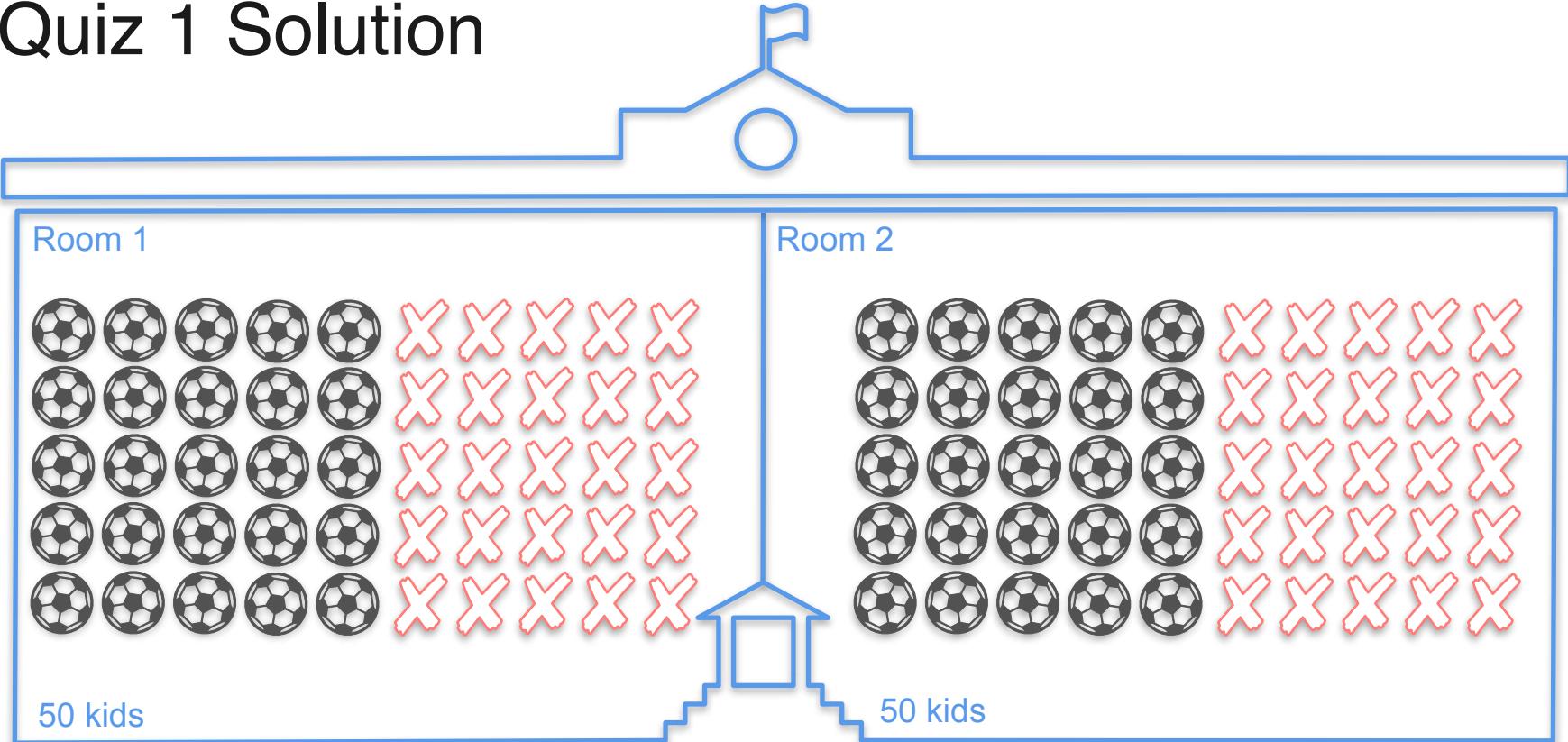
# Independence: Quiz 1



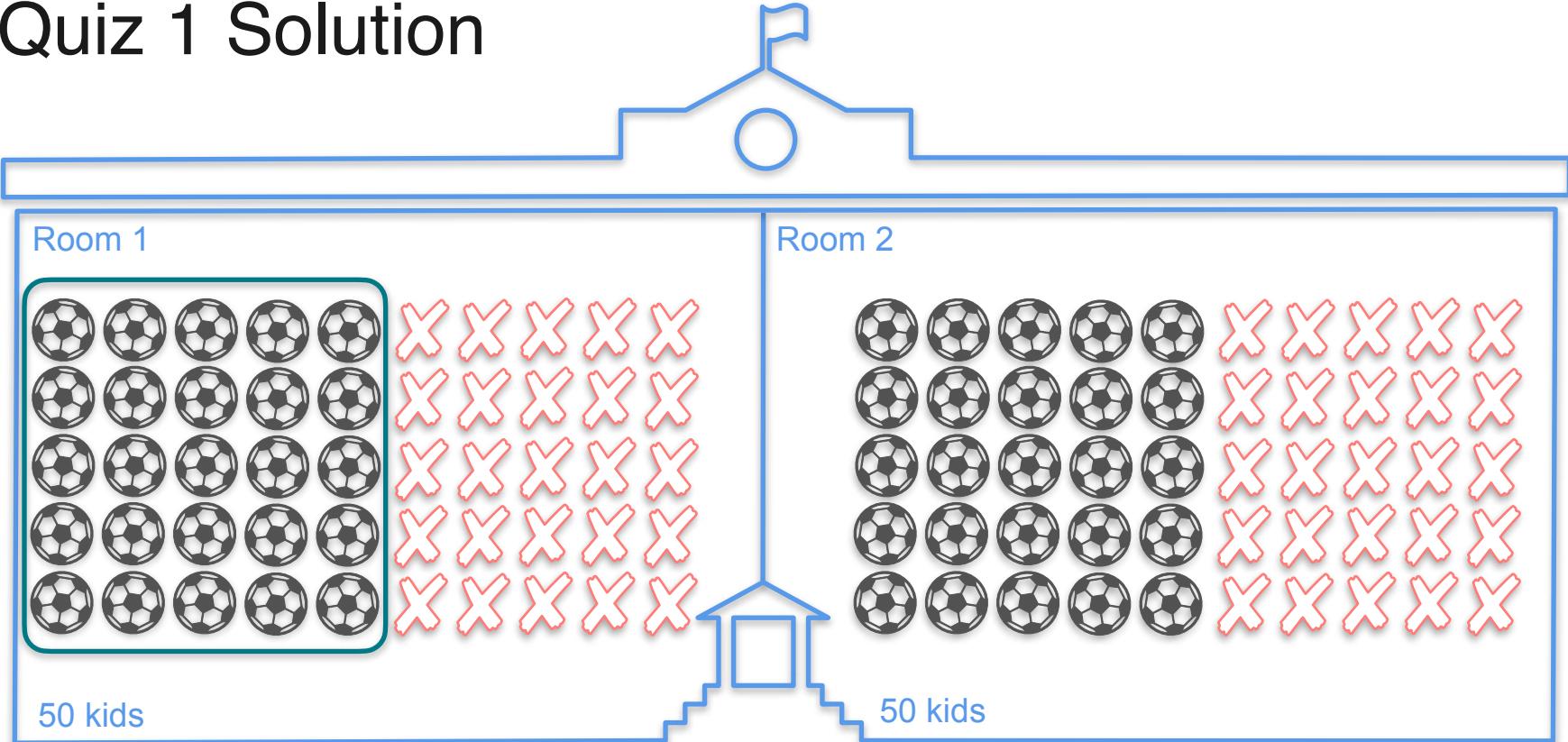
# Independence: Quiz 1



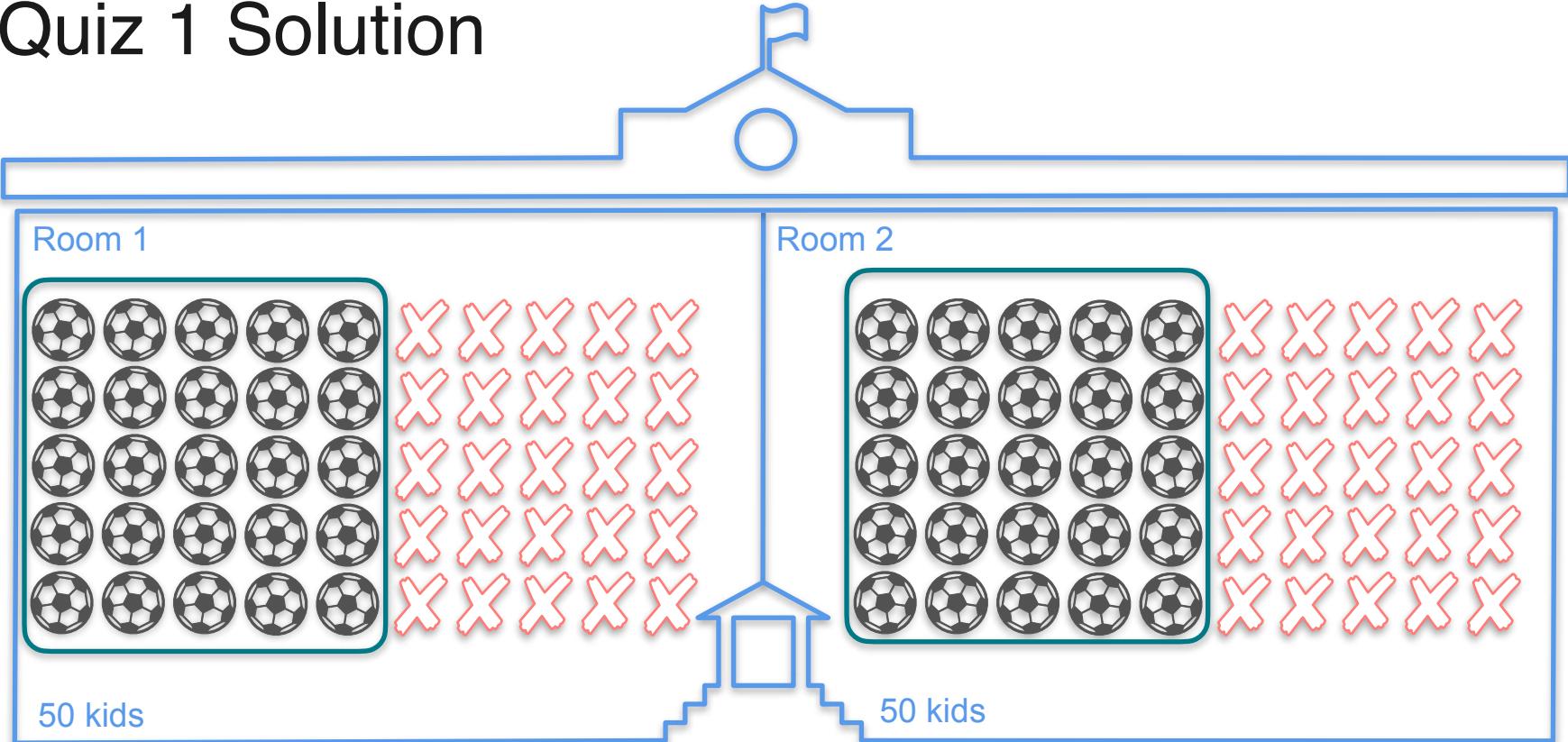
# Quiz 1 Solution



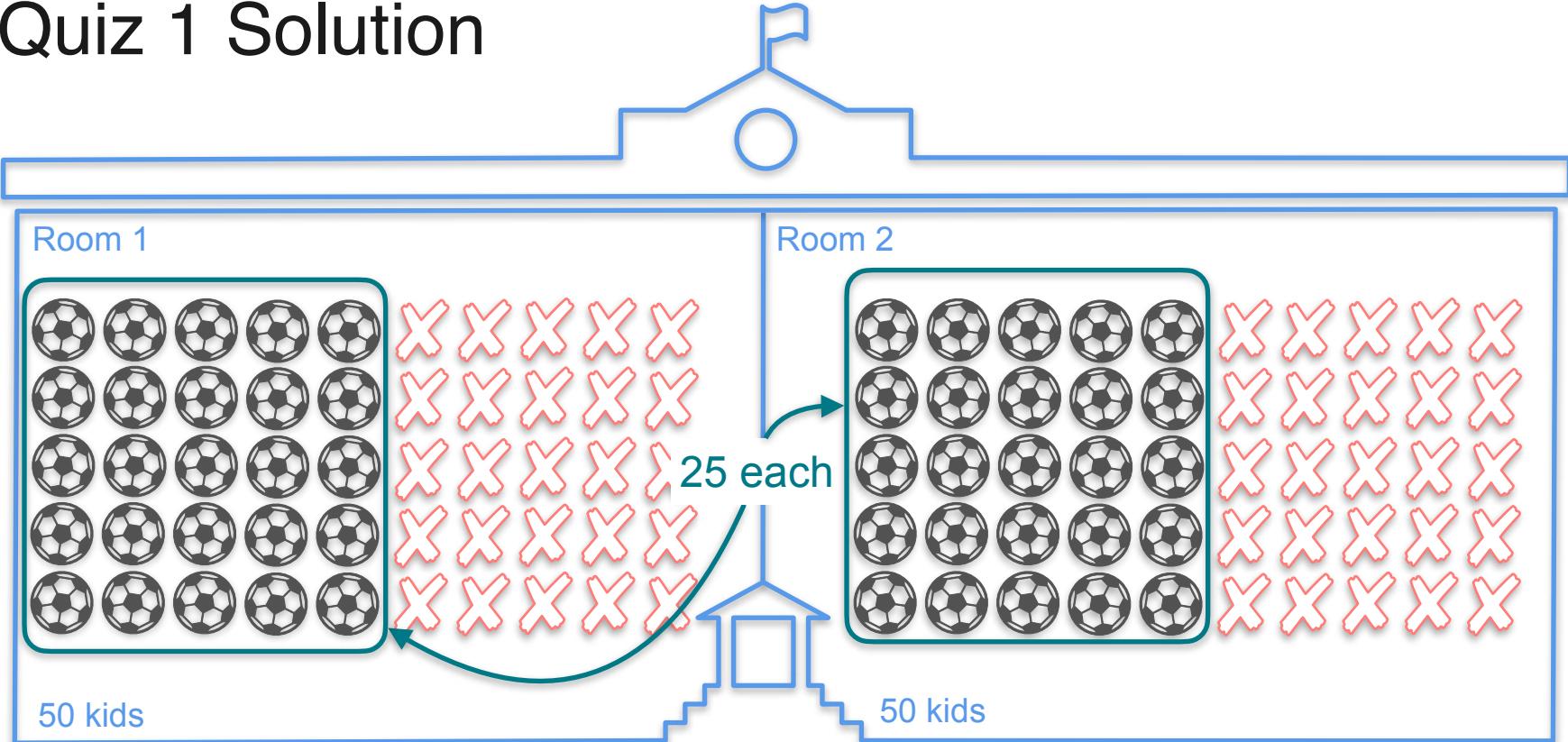
# Quiz 1 Solution



# Quiz 1 Solution

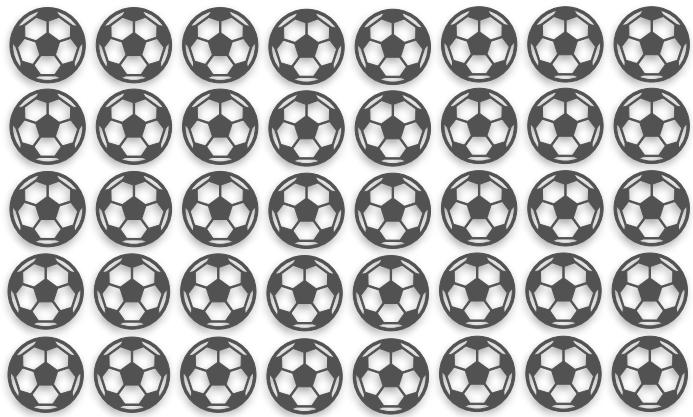


# Quiz 1 Solution

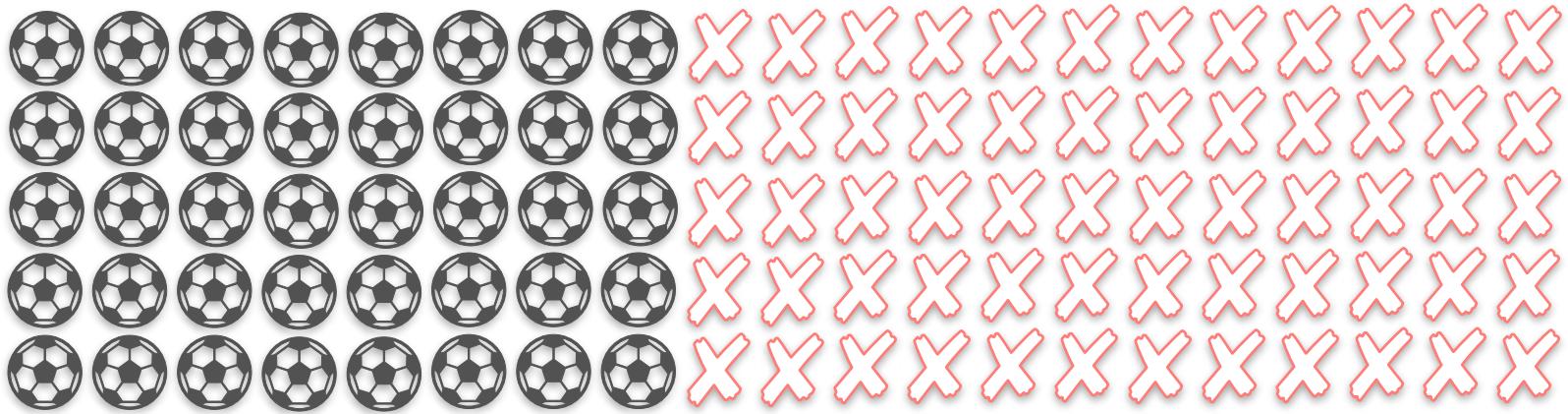


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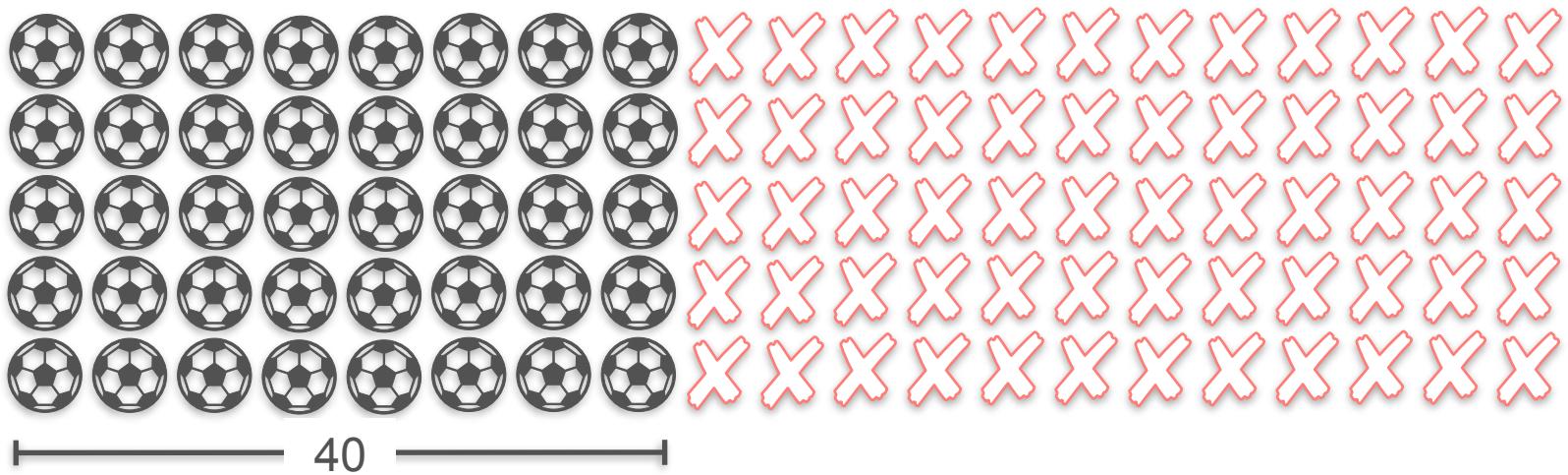
# Independence: Quiz 2



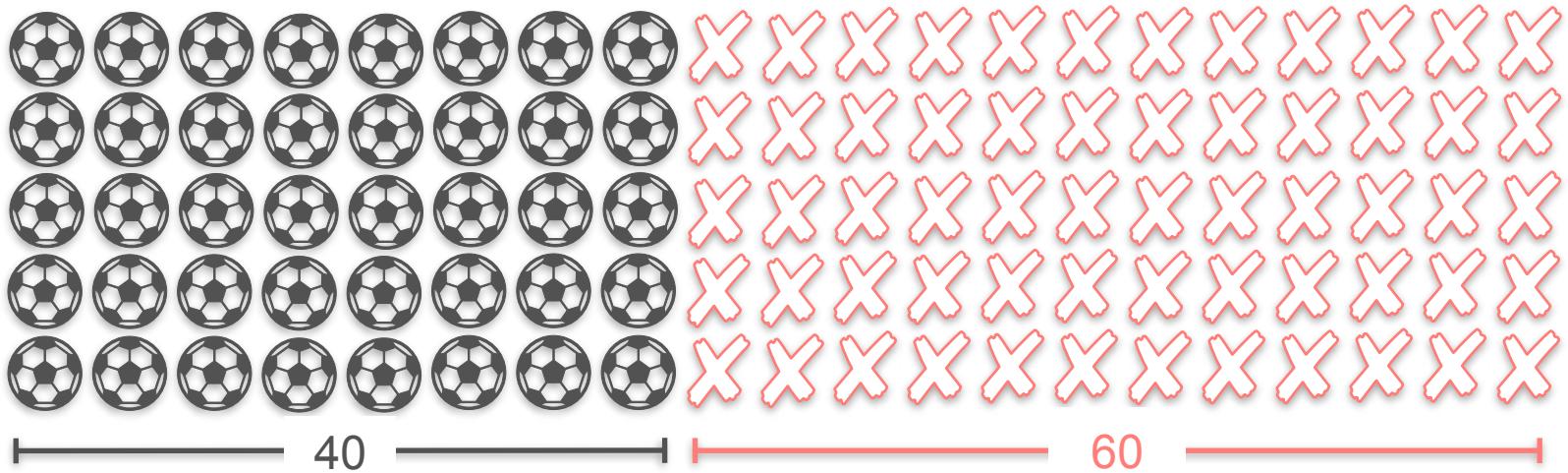
# Independence: Quiz 2



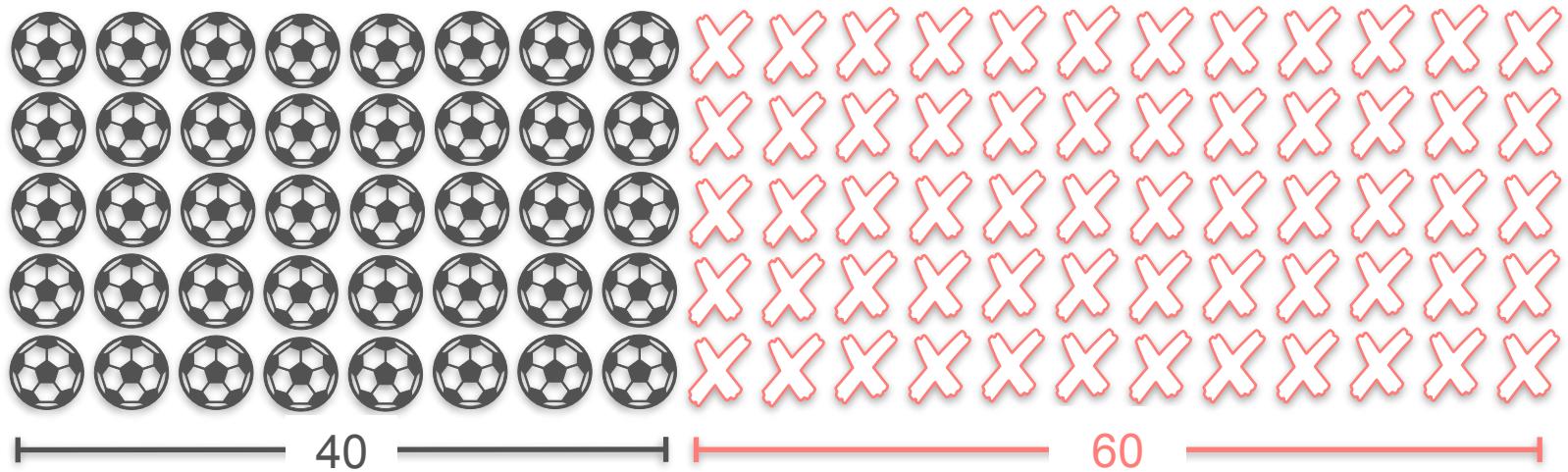
# Independence: Quiz 2



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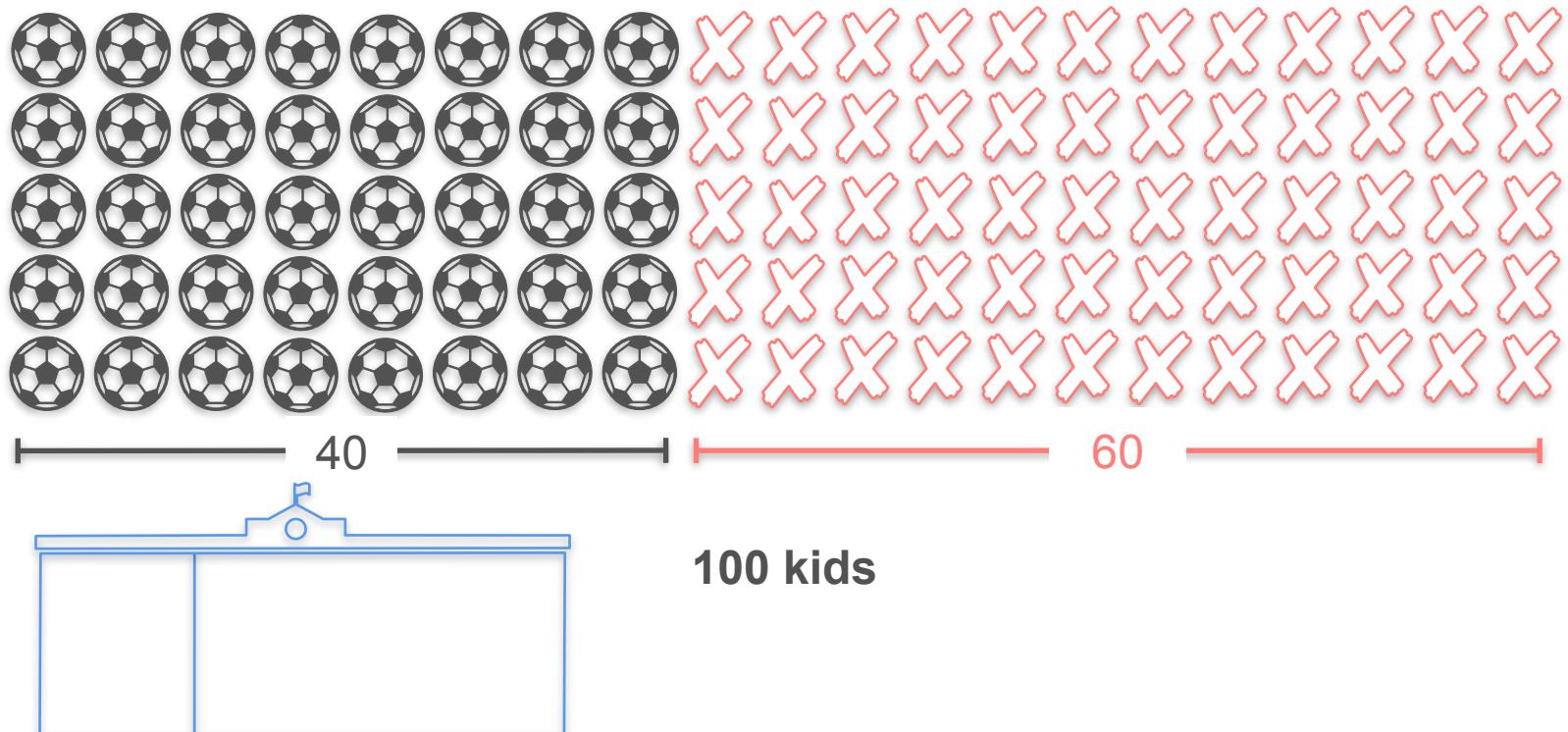


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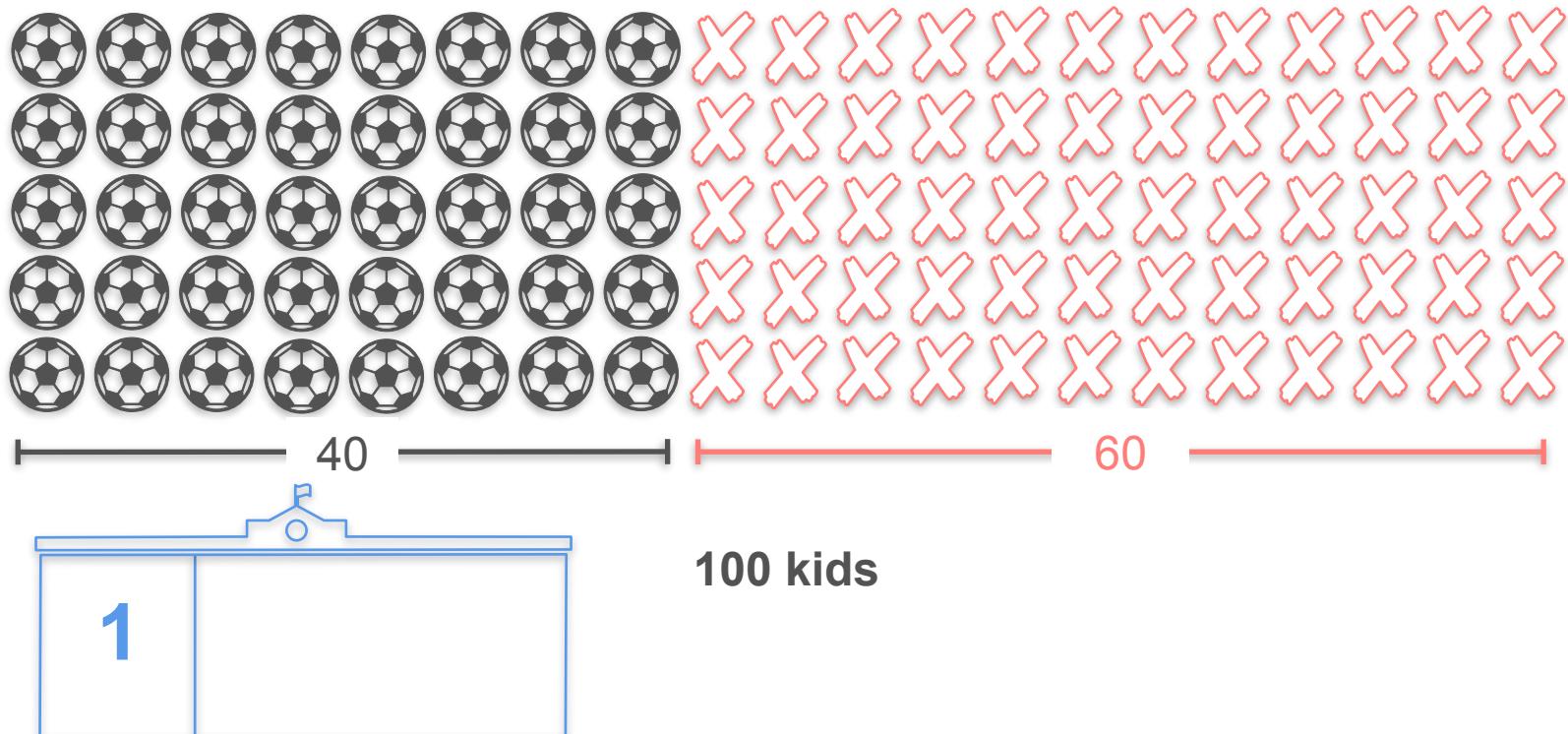


**100 kids**

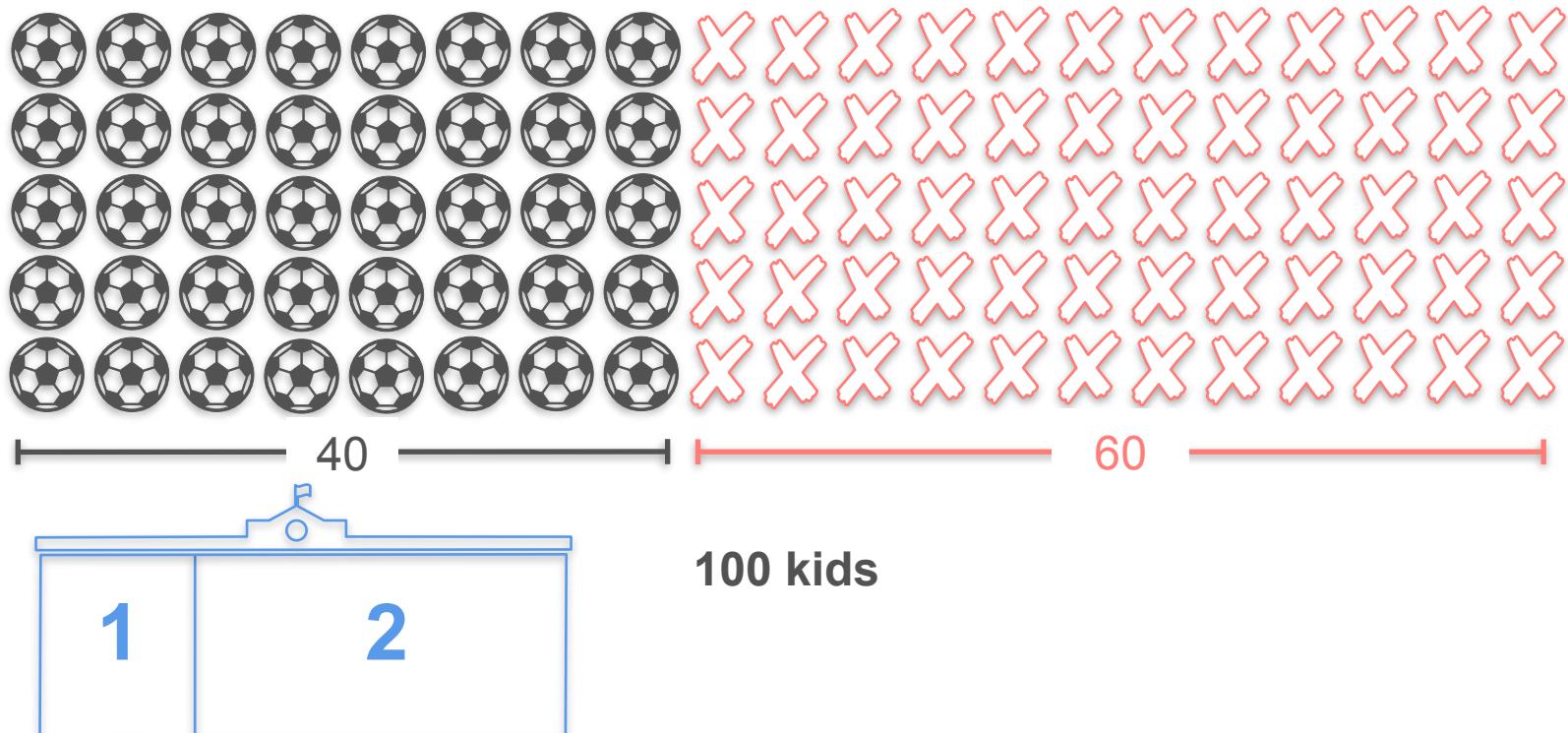
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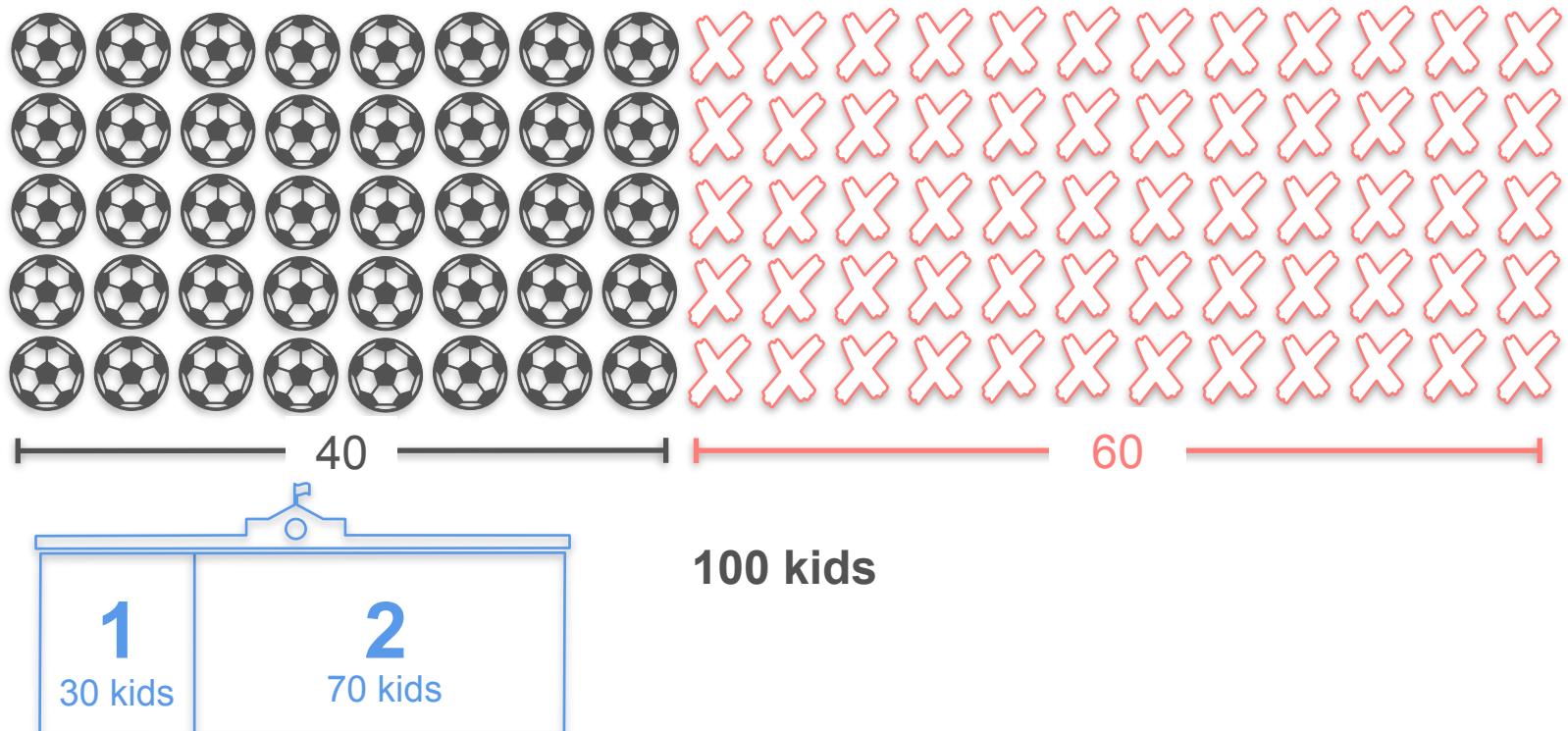
# Independence: Quiz 2



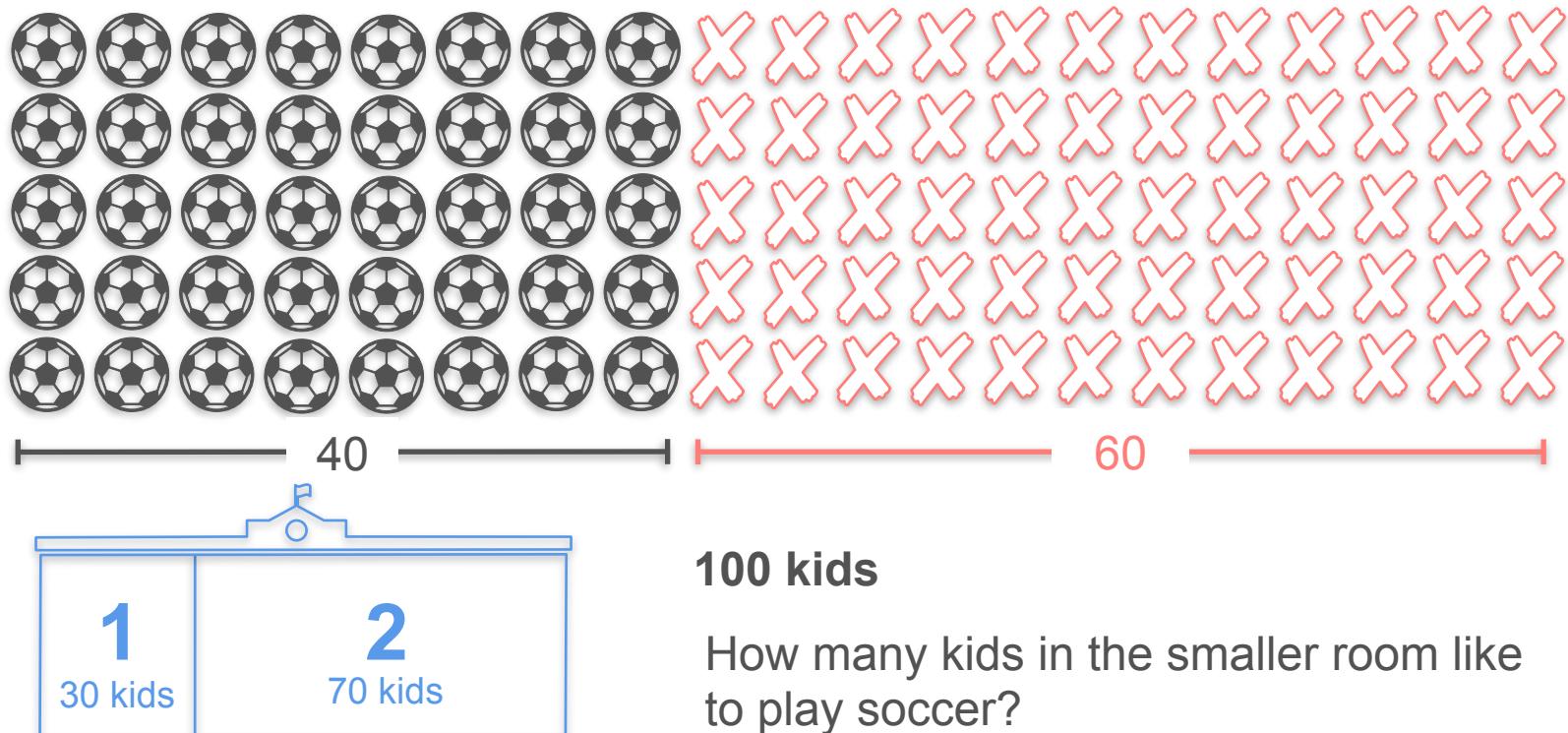
# Independence: Quiz 2



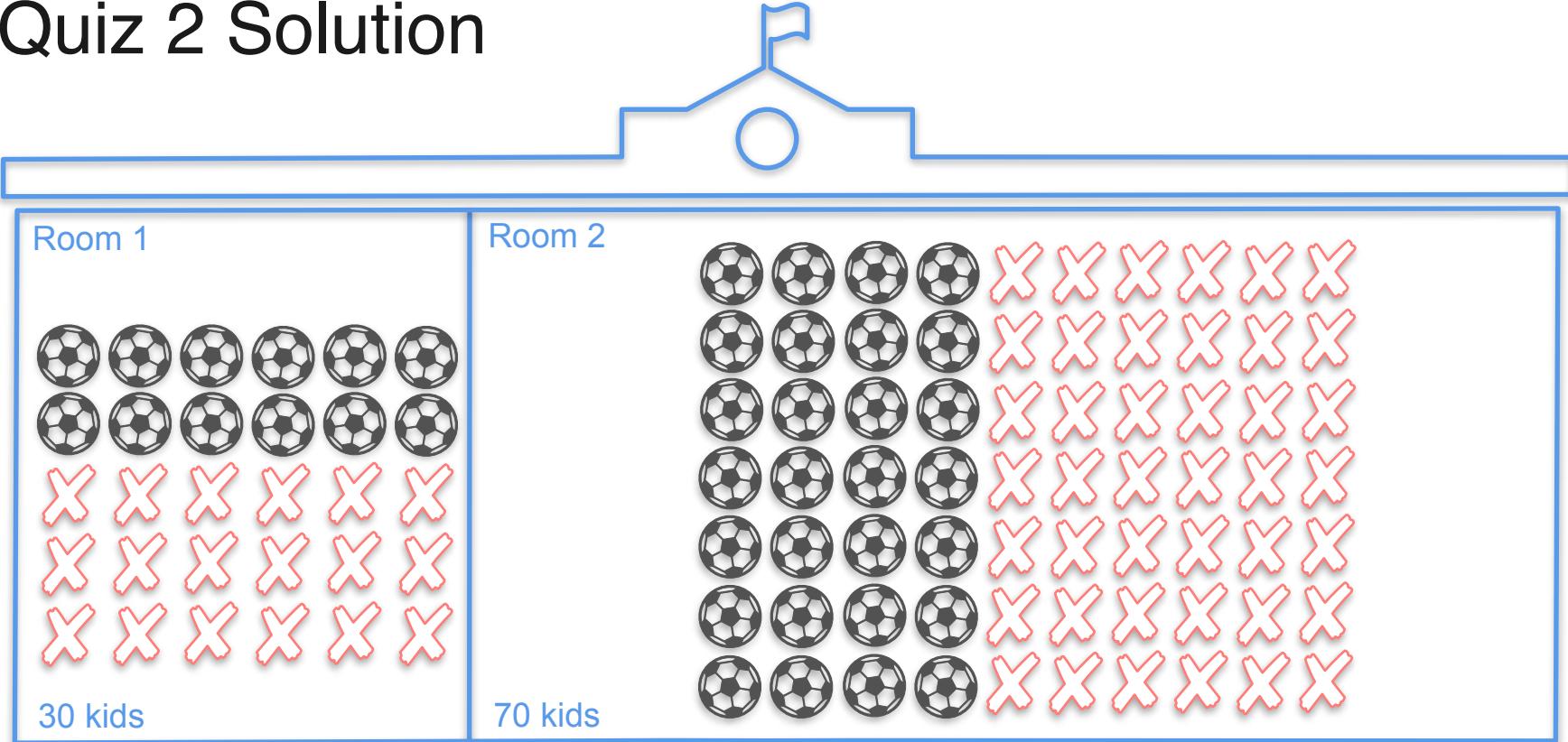
# Independence: Quiz 2



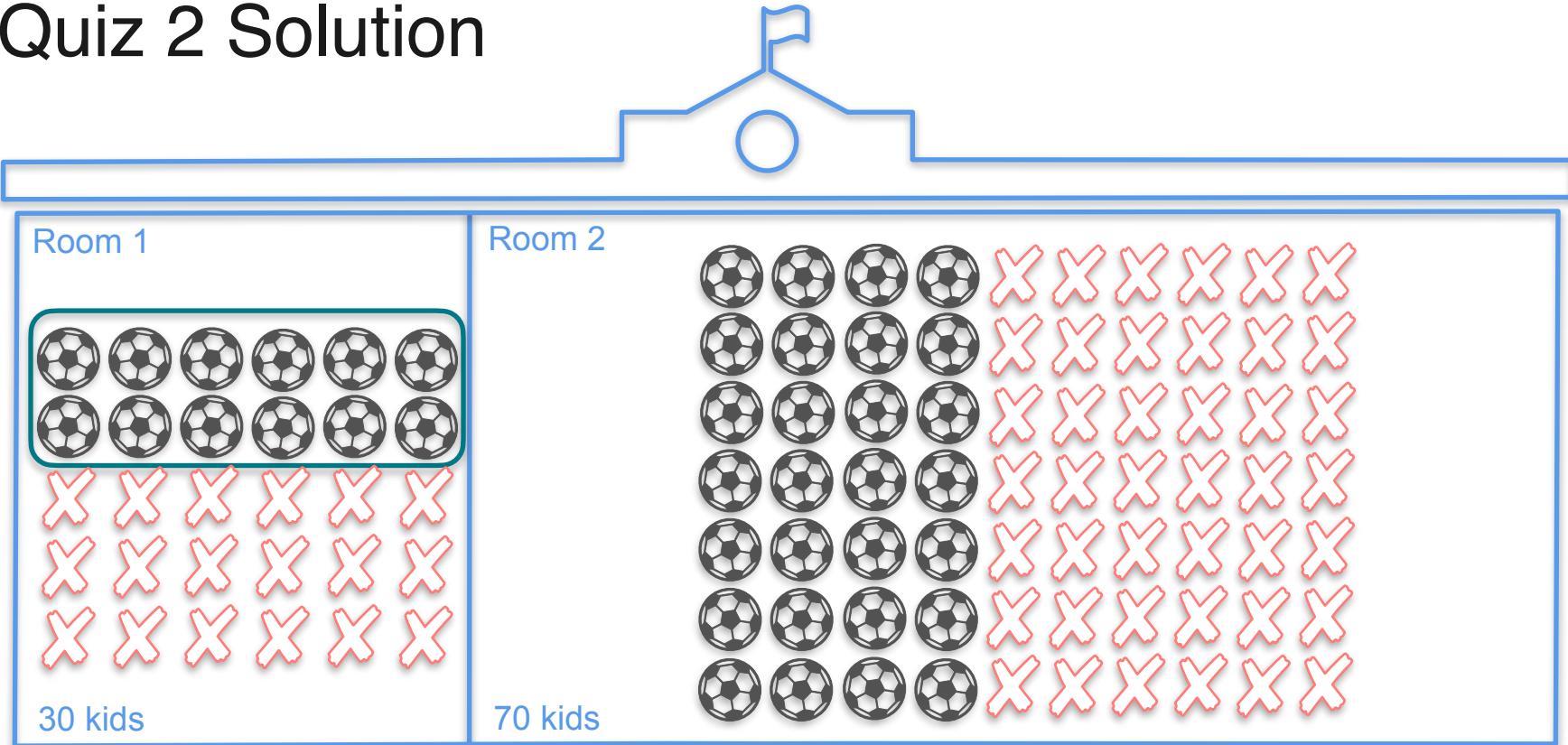
# Independence: Quiz 2



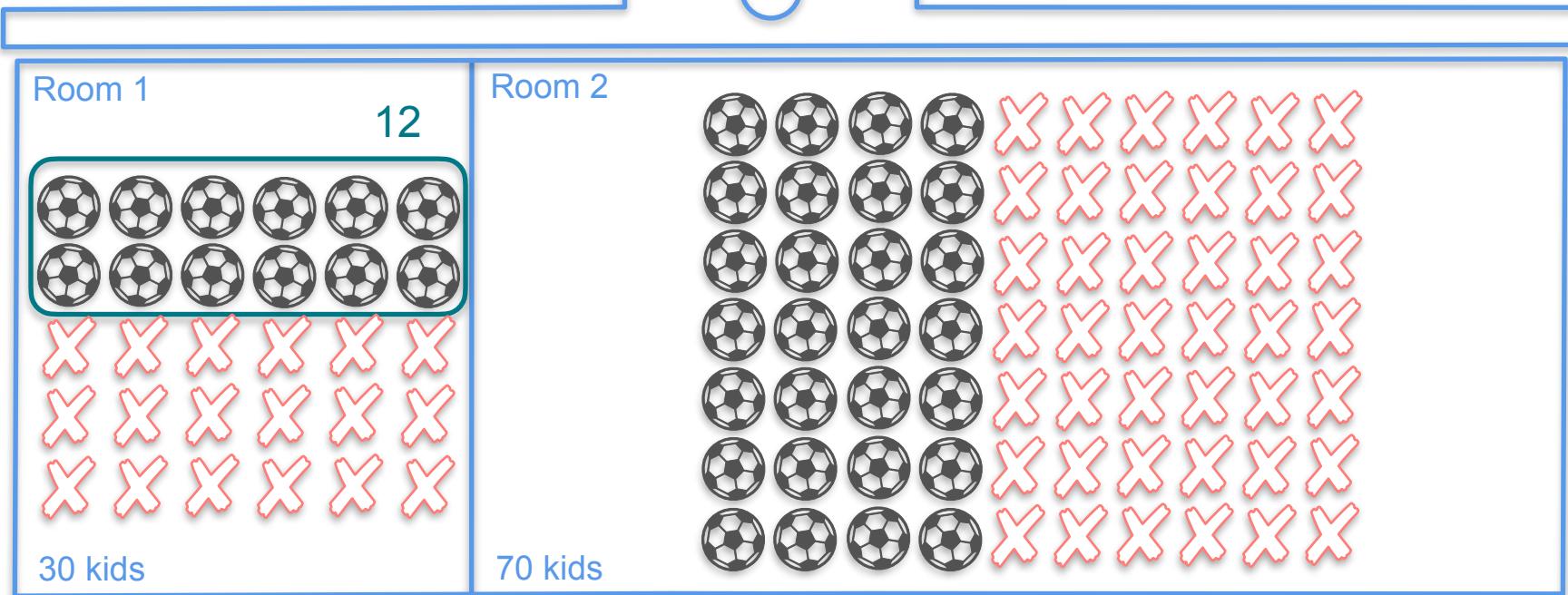
# Quiz 2 Solution



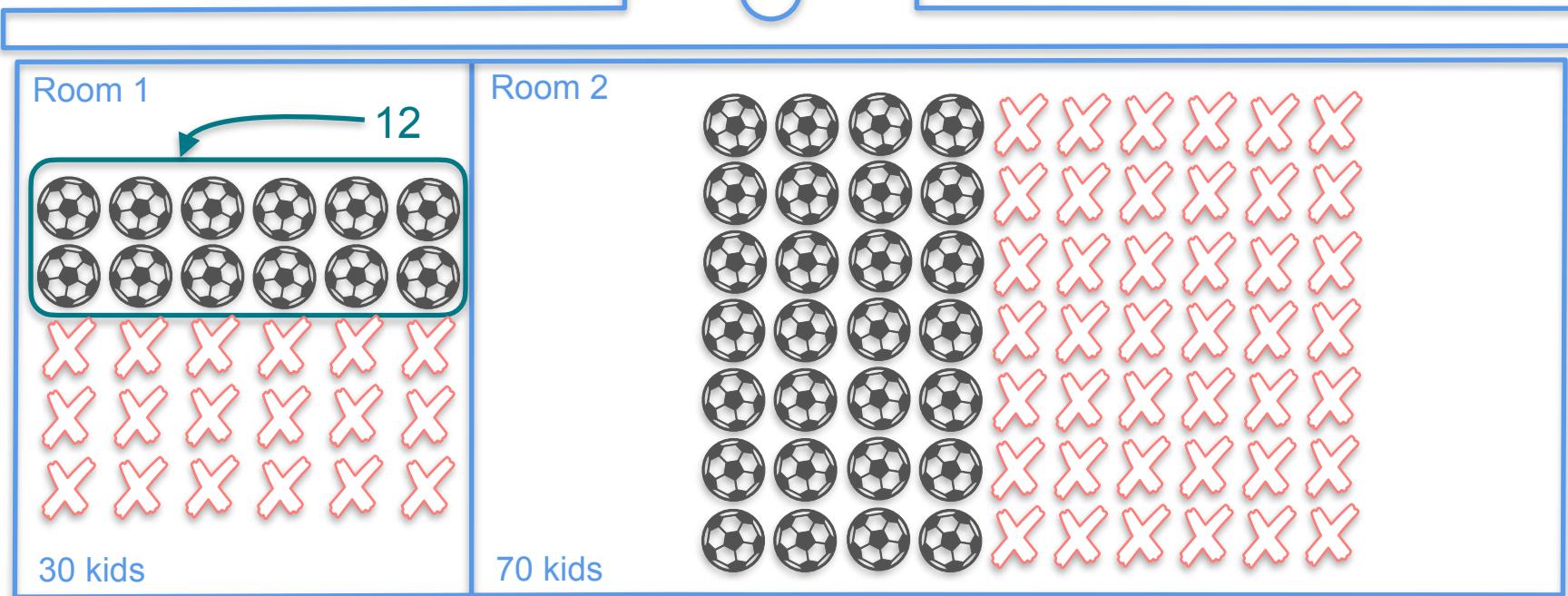
# Quiz 2 Solution



# Quiz 2 Solution

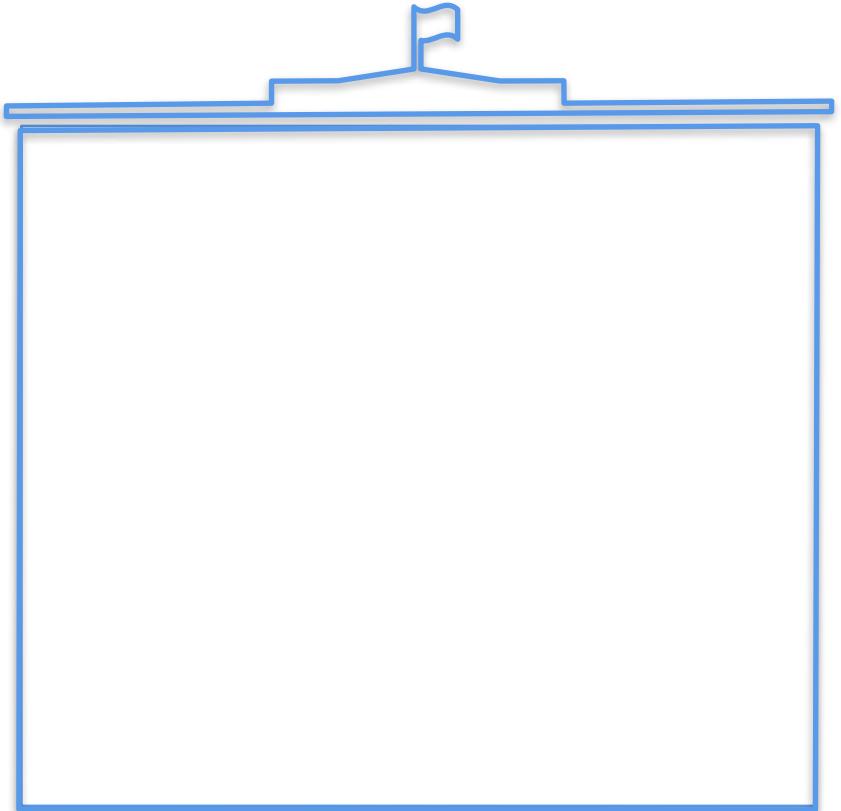


# Quiz 2 Solution



# Independent Events

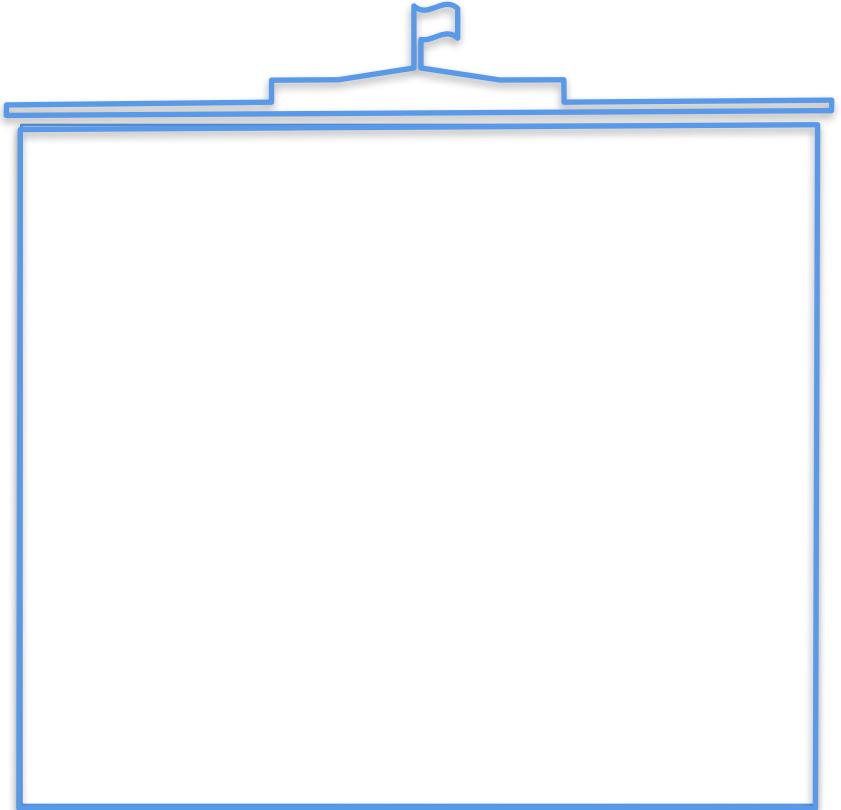
# Independent Events



# Independent Events



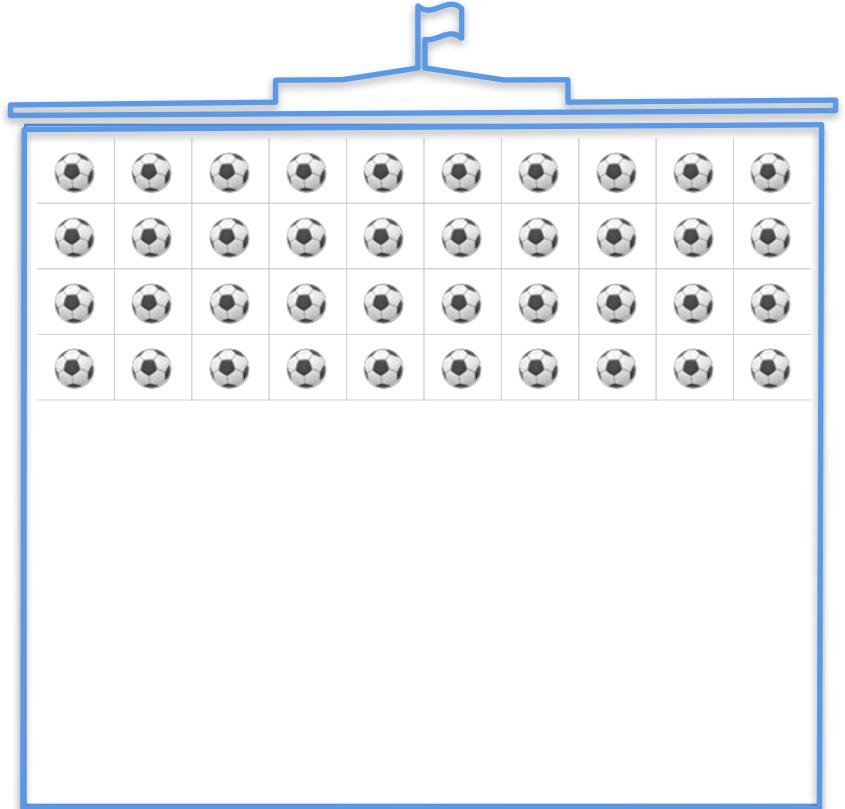
$$\mathbf{P}(S) = 0.4$$



# Independent Events



$$\mathbf{P}(S) = 0.4$$



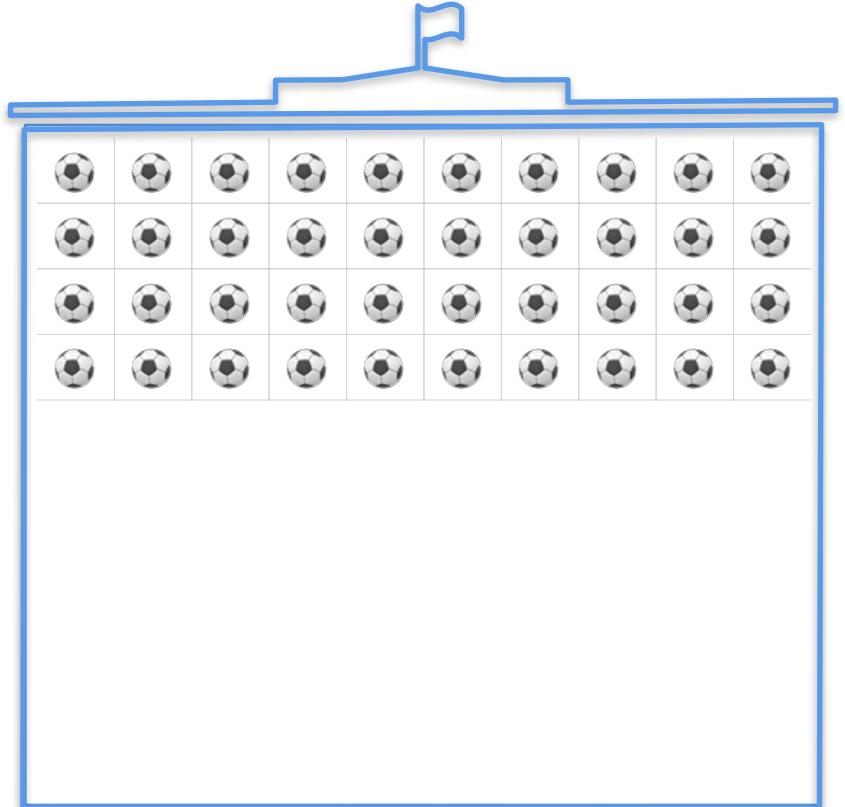
# Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



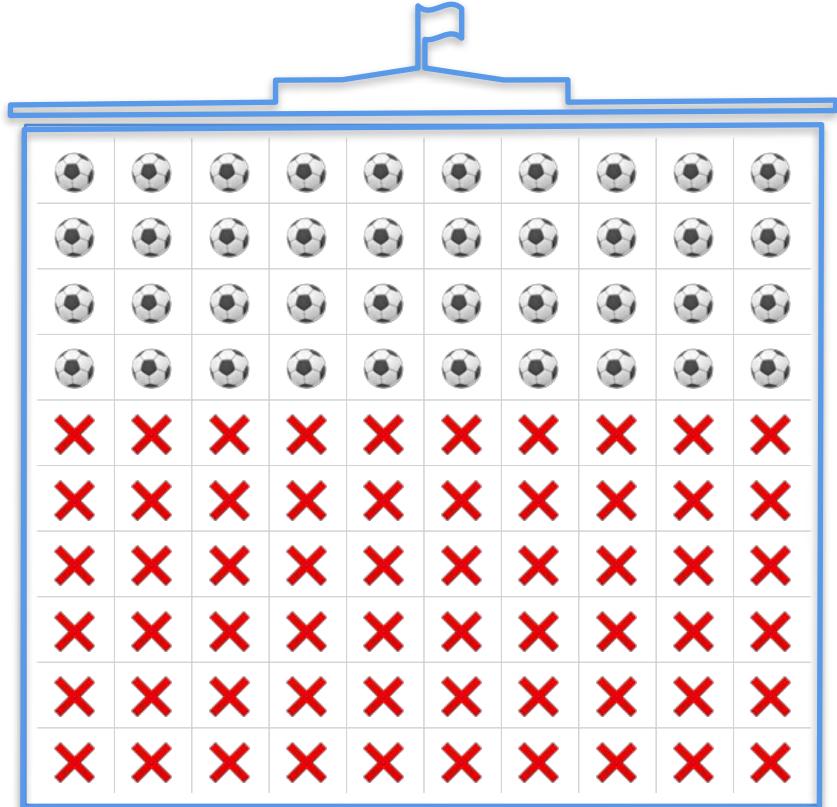
# Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



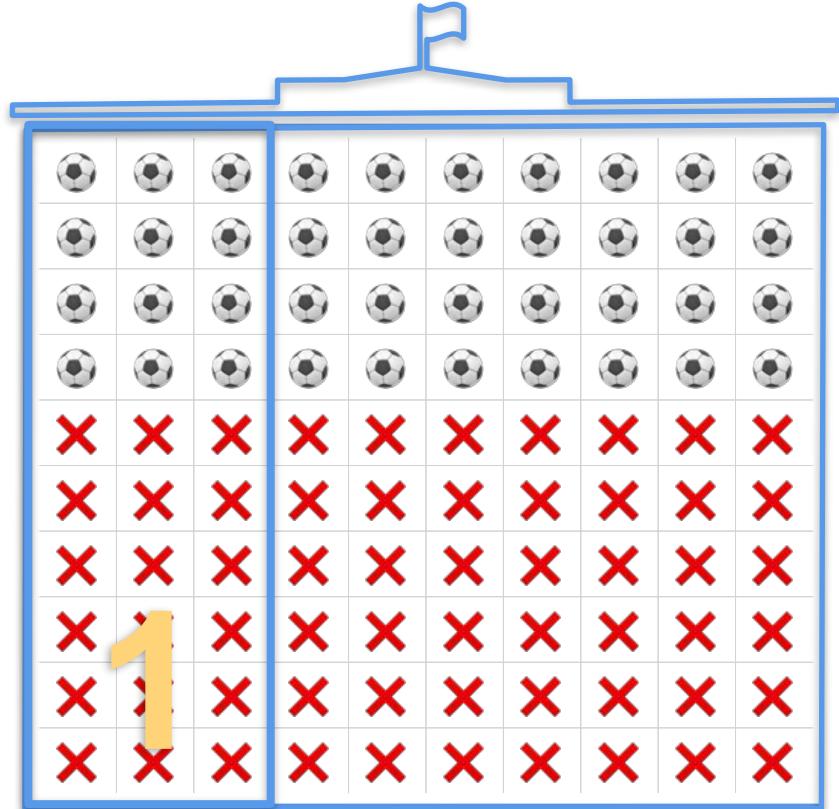
# Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



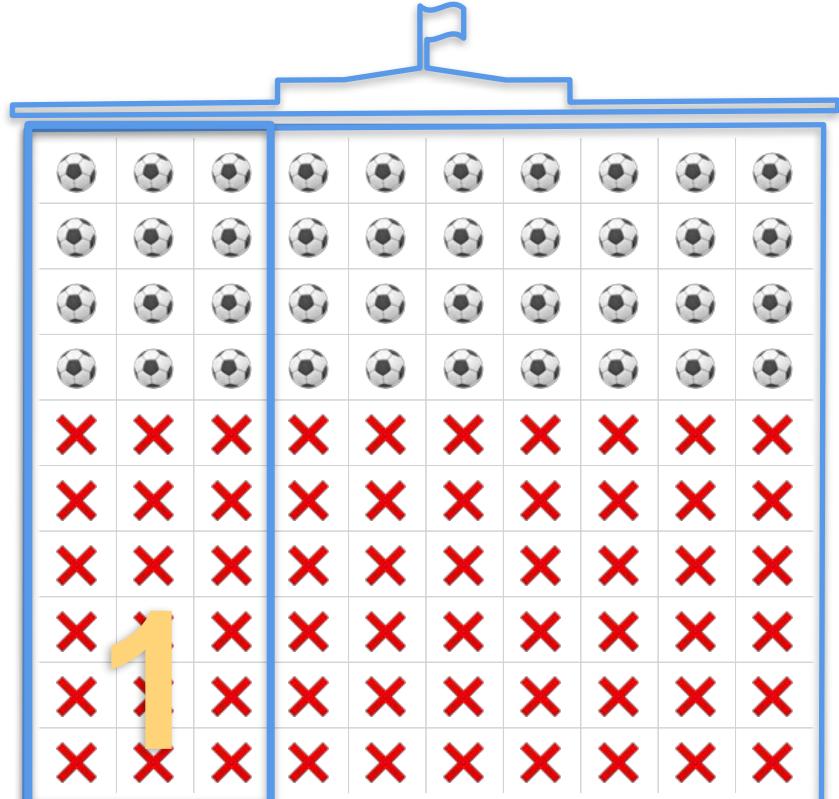
# Independent Events



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



$$P(R_1) = 0.3$$

# Independent Events

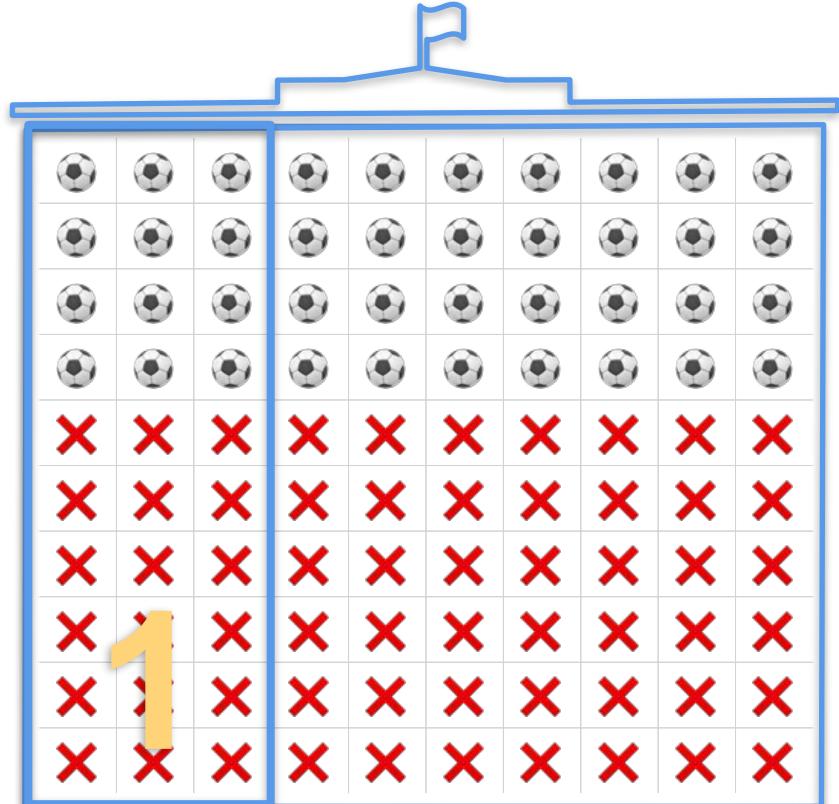
$P(\text{Soccer and Room 1})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



$$P(R_1) = 0.3$$

# Independent Events

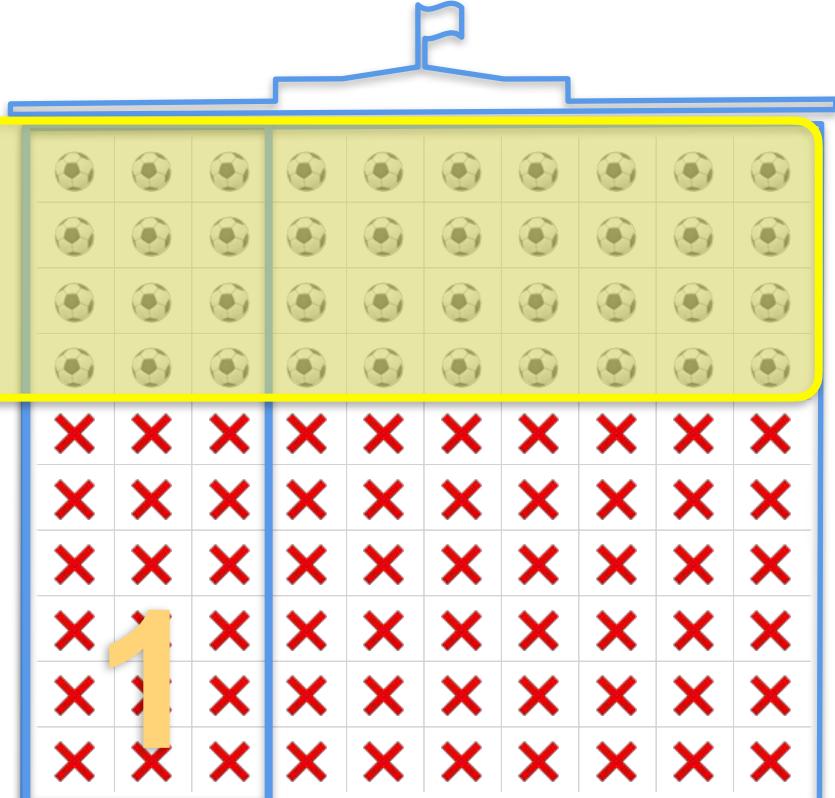
$P(\text{Soccer and Room 1})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Independent Events

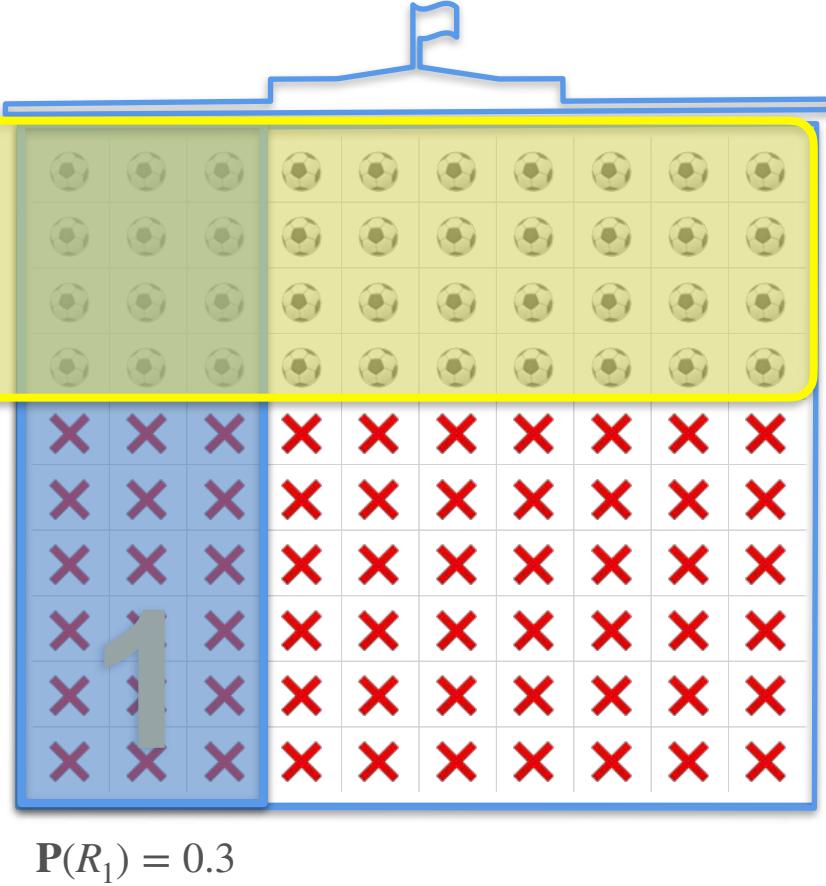
$P(\text{Soccer and Room 1})$



$$P(S) = 0.4$$

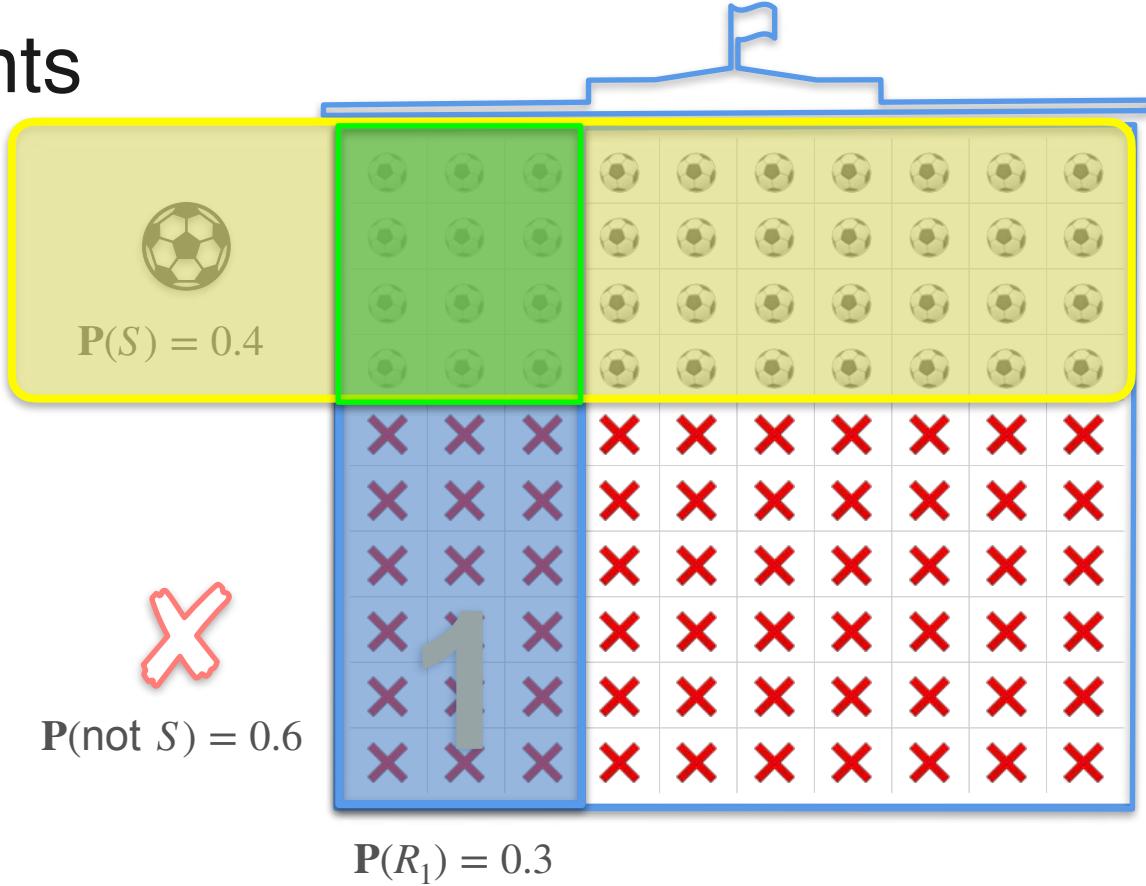


$$P(\text{not } S) = 0.6$$



# Independent Events

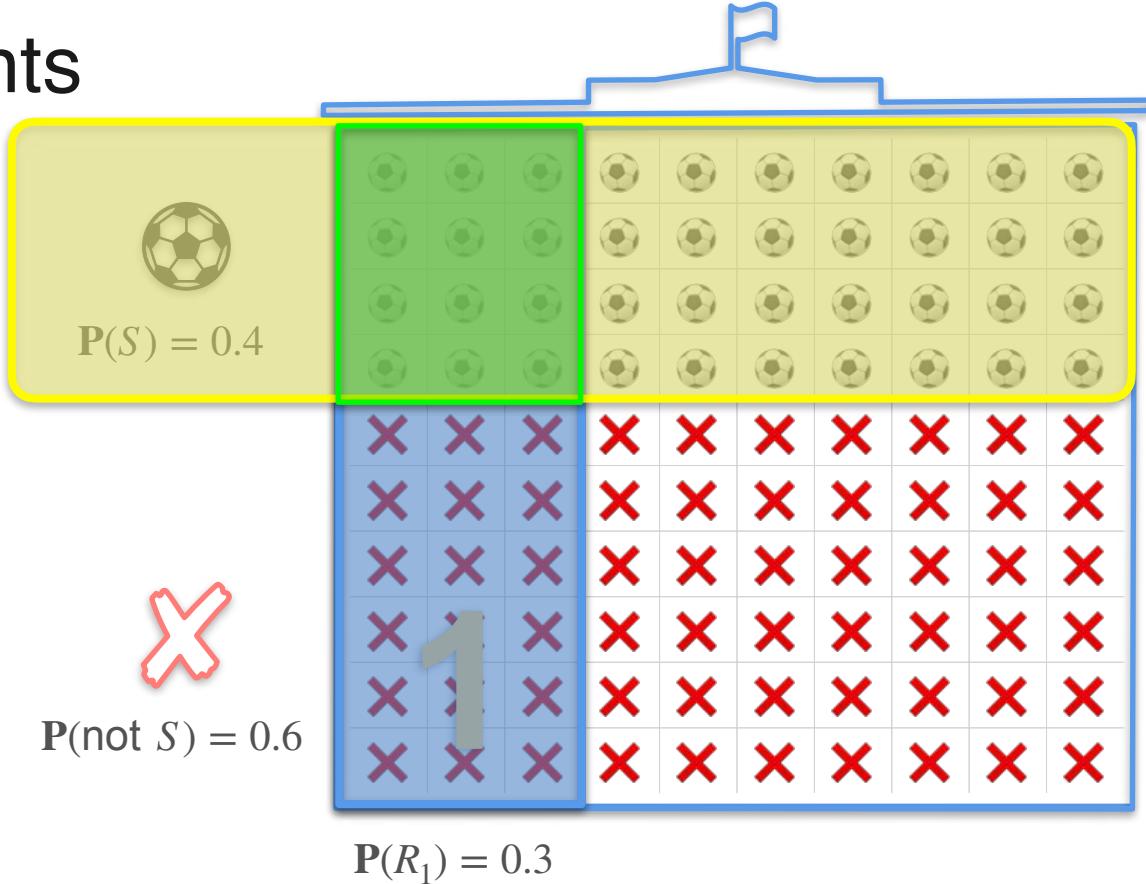
$P(\text{Soccer and Room 1})$



# Independent Events

$P(\text{Soccer and Room 1})$

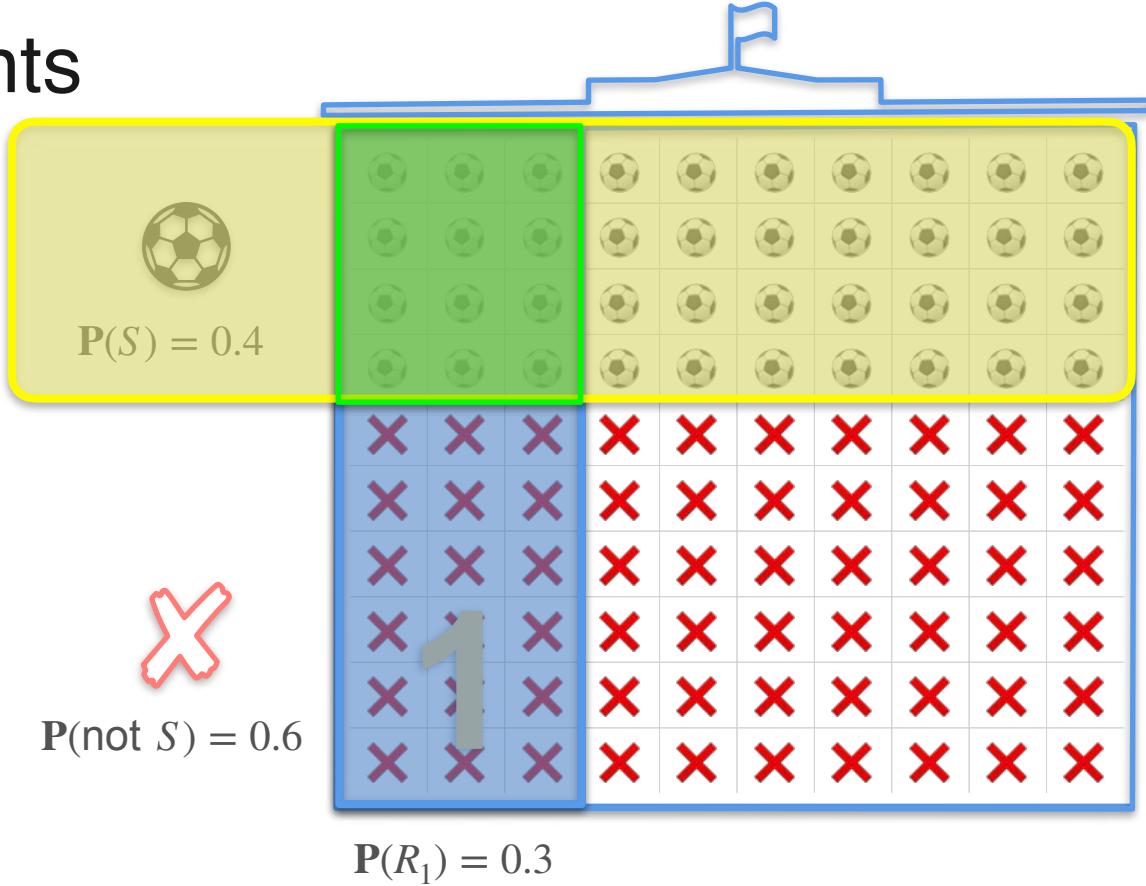
$P(S \cap R_1) =$



# Independent Events

$P(\text{Soccer and Room 1})$

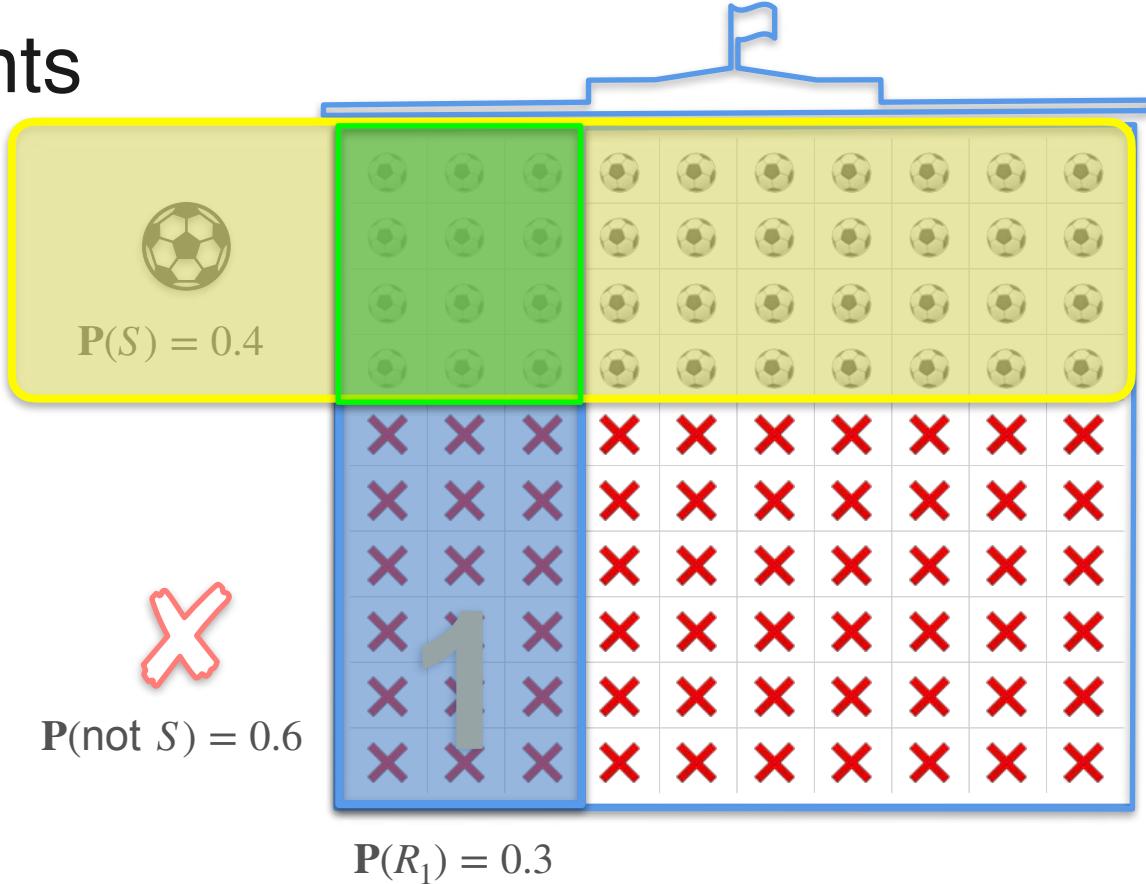
$$P(S \cap R_1) = P(S)$$



# Independent Events

$P(\text{Soccer and Room 1})$

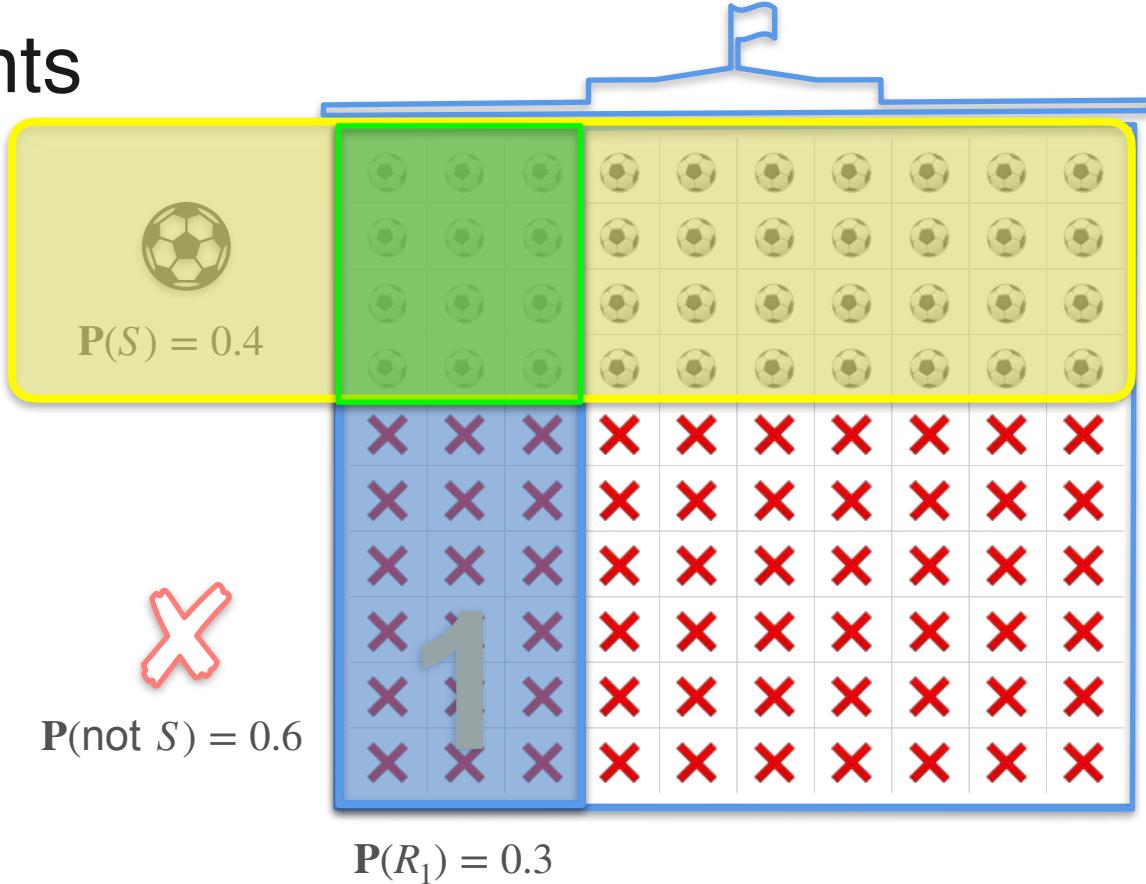
$$P(S \cap R_1) = P(S)$$



# Independent Events

$P(\text{Soccer and Room 1})$

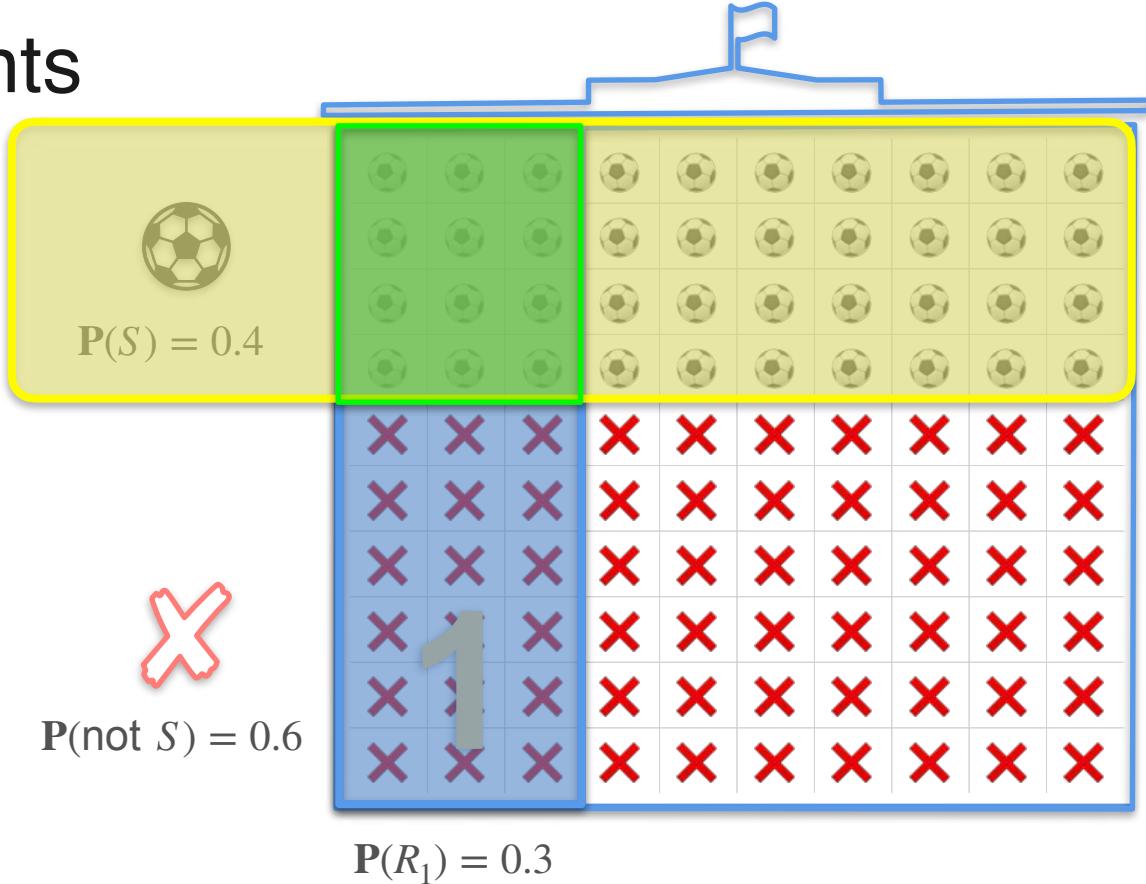
$P(S \cap R_1) = P(S) \bullet$



# Independent Events

$P(\text{Soccer and Room 1})$

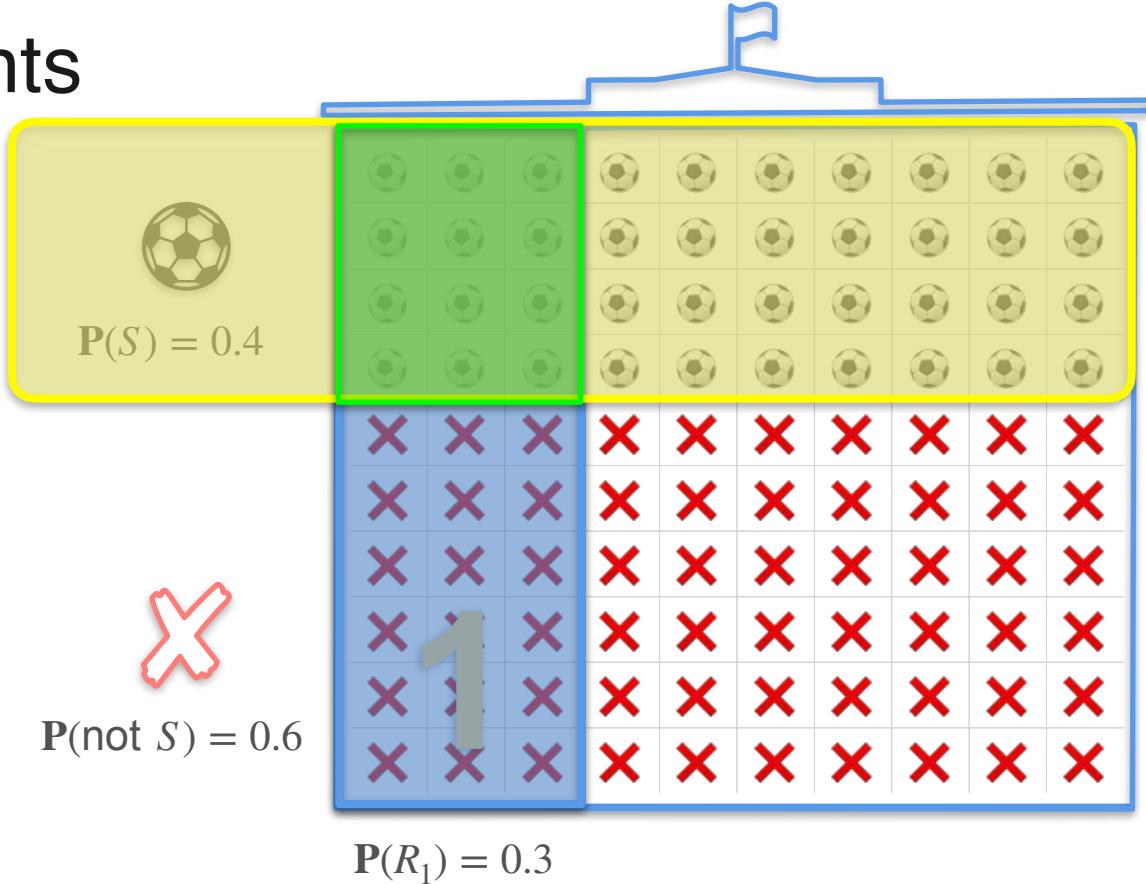
$$P(S \cap R_1) = P(S) \bullet P(R_1)$$



# Independent Events

$P(\text{Soccer and Room 1})$

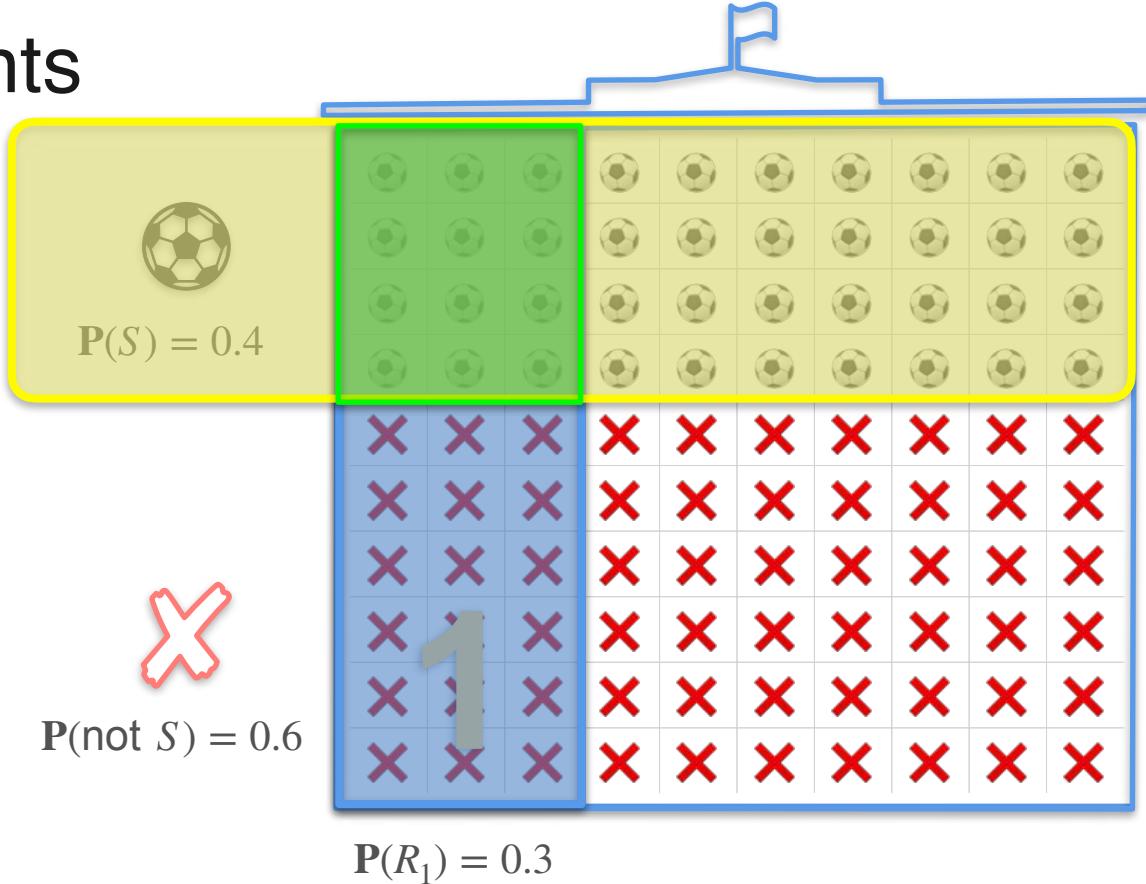
$$P(S \cap R_1) = P(S) \bullet P(R_1)$$



# Independent Events

$P(\text{Soccer and Room 1})$

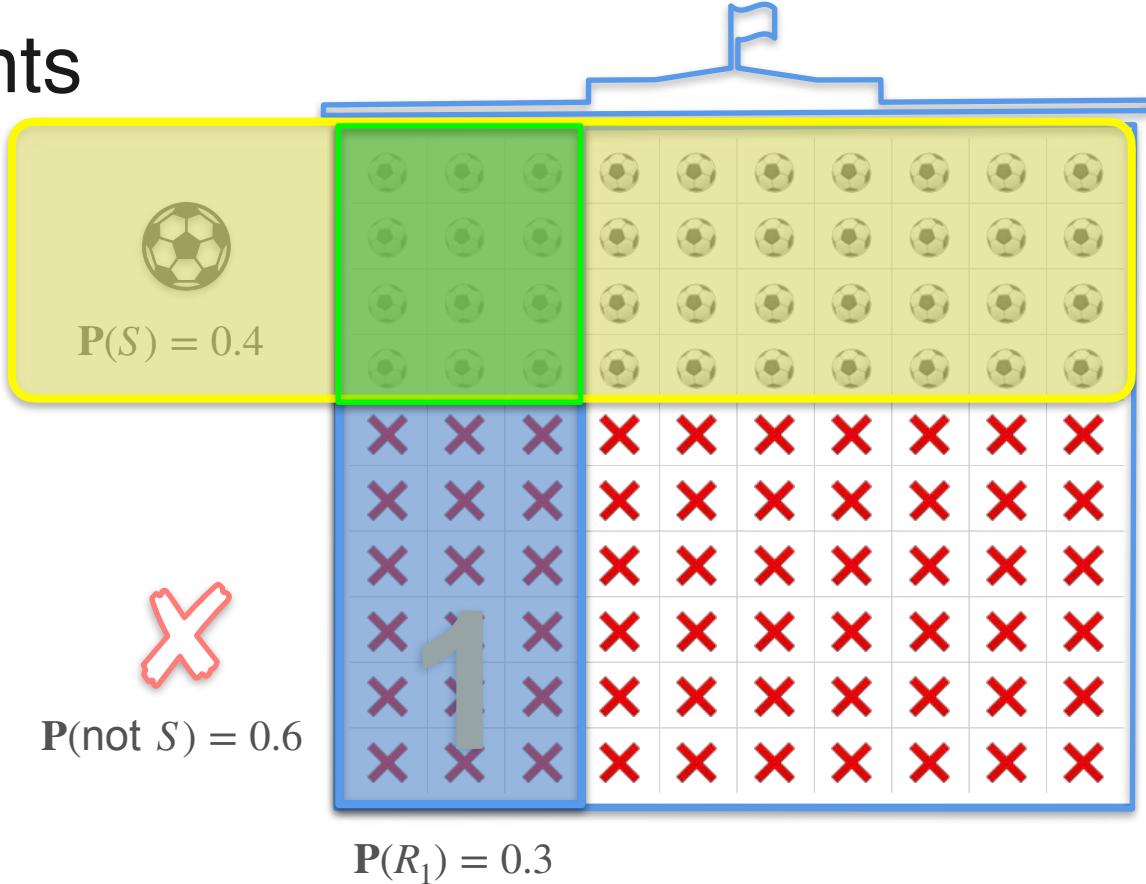
$$\begin{aligned} P(S \cap R_1) &= P(S) \bullet P(R_1) \\ &= 0.4 \end{aligned}$$



# Independent Events

$P(\text{Soccer and Room 1})$

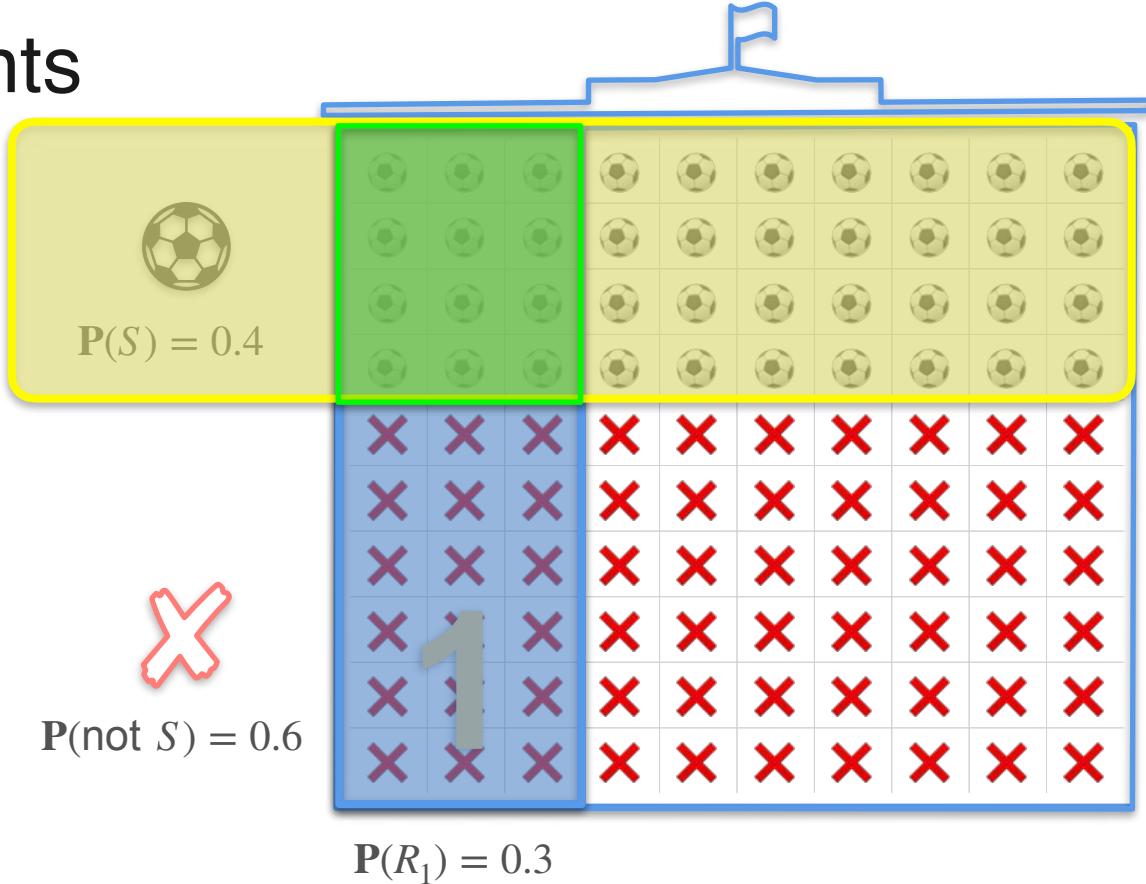
$$\begin{aligned} P(S \cap R_1) &= P(S) \bullet P(R_1) \\ &= 0.4 \bullet \end{aligned}$$



# Independent Events

$P(\text{Soccer and Room 1})$

$$\begin{aligned} P(S \cap R_1) &= P(S) \bullet P(R_1) \\ &= 0.4 \bullet 0.3 \end{aligned}$$



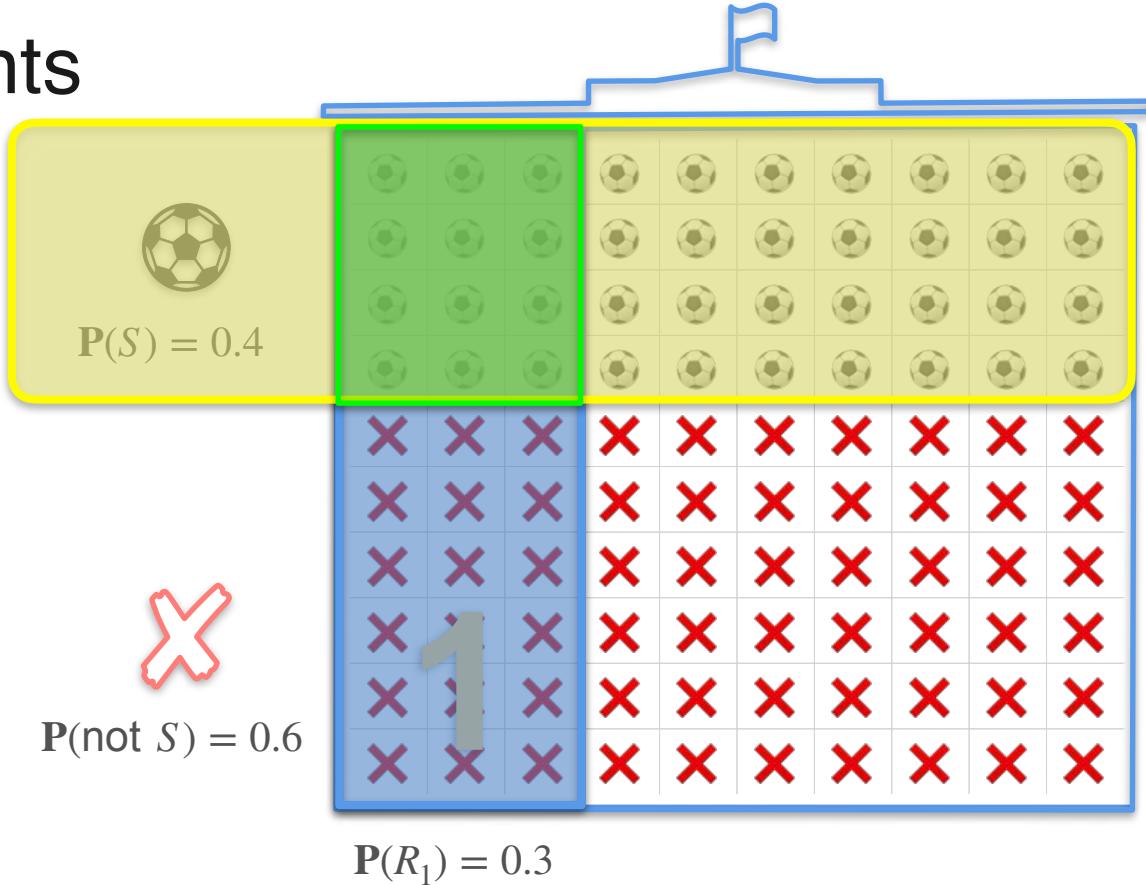
# Independent Events

$P(\text{Soccer and Room 1})$

$$P(S \cap R_1) = P(S) \bullet P(R_1)$$

$$= 0.4 \bullet 0.3$$

$$= 0.12$$



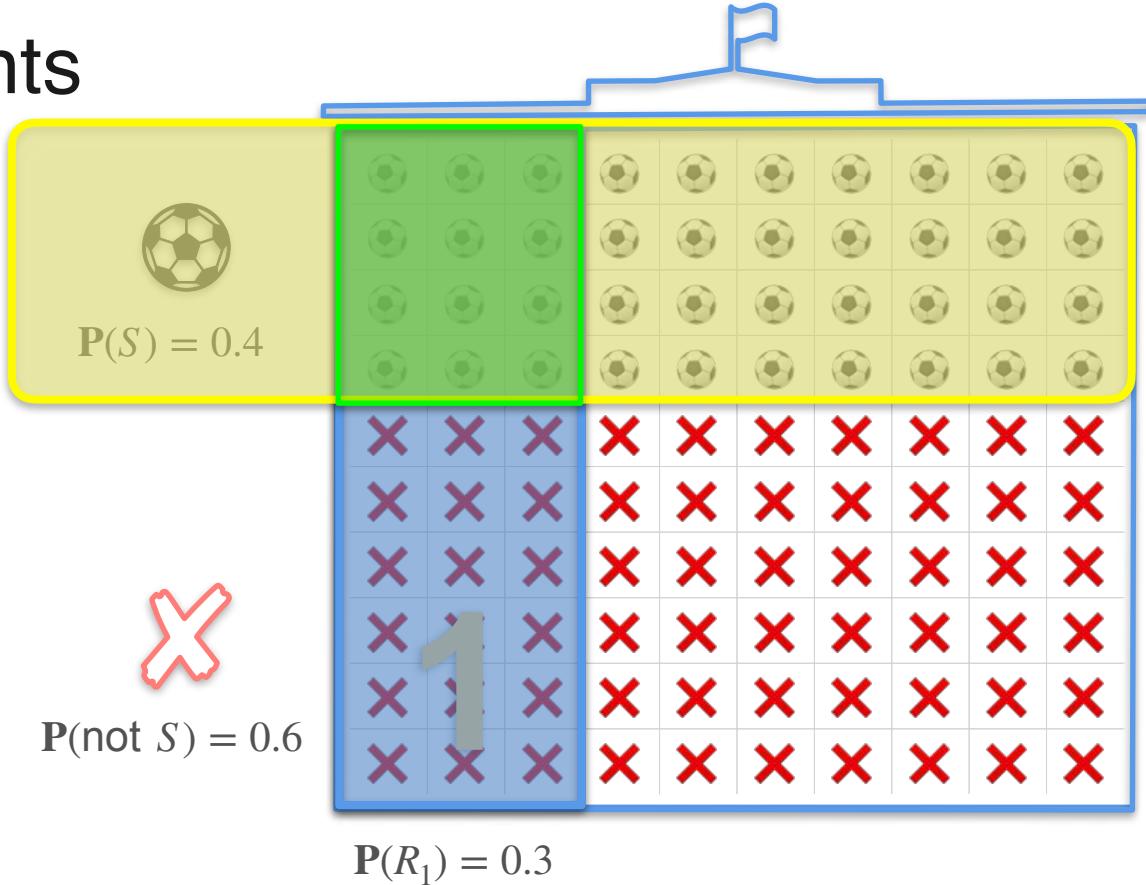
# Independent Events

$P(\text{Soccer and Room 1})$

$$P(S \cap R_1) = P(S) \bullet P(R_1)$$

$$= 0.4 \bullet 0.3$$

$$= 0.12$$



# Product Rule (for Independent Events)

# Product Rule (for Independent Events)

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

# Independent Events: Coin Example 1



50% 50%

# Independent Events: Coin Example 1



50% 50%



# Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?



# Independent Events: Coin Example 1



What is the probability of landing on heads five times?



# Independent Events: Coin Example 1



What is the probability of landing on heads five times?

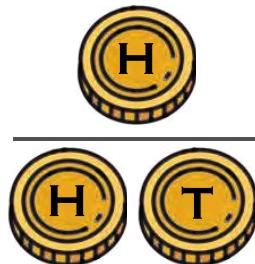


# Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

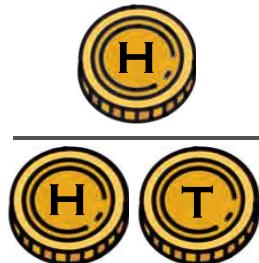


# Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?



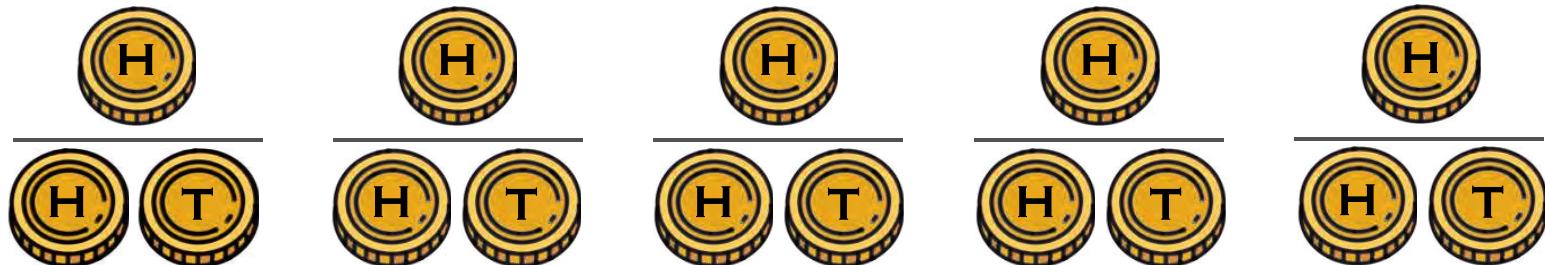
$$\frac{1}{2}$$

# Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?



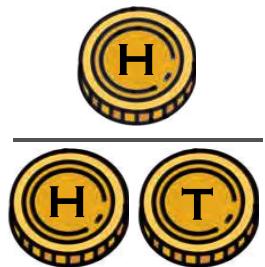
$$\frac{1}{2}$$

# Independent Events: Coin Example 1

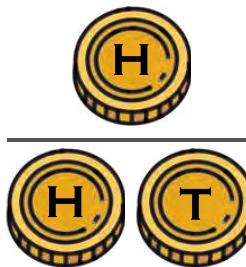


50% 50%

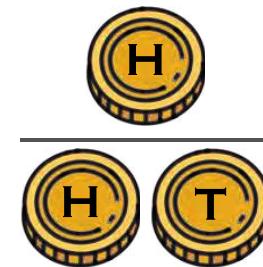
What is the probability of landing on heads five times?



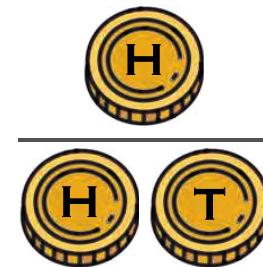
$$\frac{1}{2}$$



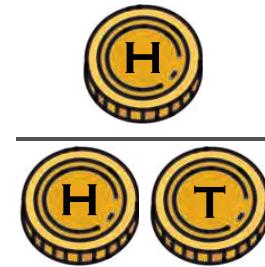
$$\frac{1}{2}$$



$$\frac{1}{2}$$



$$\frac{1}{2}$$



$$\frac{1}{2}$$

# Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

$P(5 \text{ heads}) =$

$$\frac{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}}{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}} + \frac{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}}{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}} + \frac{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}}{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}} + \frac{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}}{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}} + \frac{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}}{\begin{array}{c} \text{H} \\ \hline \text{H} \quad \text{T} \end{array}}$$
$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

# Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) = \frac{\text{Diagram of 5 heads}}{\text{Diagram of 5 heads} + \text{Diagram of 5 tails}}$$

The equation shows the probability of getting 5 heads in a row. It is represented as a fraction where the numerator is a sequence of 5 coins all showing heads (H), and the denominator is a sequence of 5 coins showing either heads or tails (H or T). Below the fraction, there are two rows of numbers:  $\frac{1}{2}$  under each head in the numerator, and  $\frac{1}{2}$  under each head and tail in the denominator.

The diagram consists of two rows of five gold coins each. The top row shows all five coins with 'H' (heads) facing up. The bottom row shows the first four coins with 'H' (heads) facing up, and the fifth coin with 'T' (tails) facing up. Between the two rows, there are five horizontal lines with black dots at their intersections, representing the multiplication of probabilities for each coin flip.

# Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

$P(5 \text{ heads}) =$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$
A sequence of five coin tosses is shown. Each toss is represented by a horizontal line with two circles. The top circle contains 'H' (heads) and the bottom circle contains 'T' (tails). The first toss shows H-T. Subsequent tosses show H-H, H-T, H-T, and H-T. Between each pair of circles is a small black dot, and between each pair of lines is a larger black dot, representing the multiplication of probabilities.

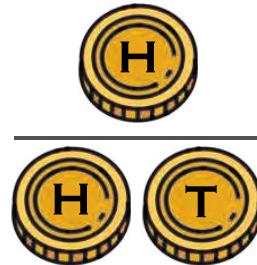
# Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) =$$



$$\frac{1}{2}$$

# Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) = \left( \frac{1}{2} \right)^5$$
A diagram showing three gold-colored coins. Two coins are at the bottom, both showing heads (H). A third coin is positioned above them, also showing heads (H).

$$\frac{1}{2}$$

# Independent Events: Coin Example 2



50% 50%

What is the probability of landing 5n heads five times?

$$P(5 \text{ heads}) = \left( \frac{\text{Diagram of 5 coins showing all heads}}{\text{Diagram of 5 coins showing mixed heads and tails}} \right)$$
A diagram illustrating the probability calculation. It shows five coins stacked vertically. The top coin is heads (H). Below it, the second coin is heads (H), the third is tails (T), the fourth is heads (H), and the fifth is tails (T). This visualizes the concept of independent events where each coin flip is a separate trial.

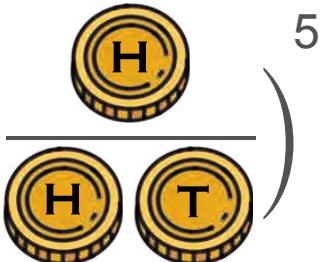
$$\frac{1}{2}$$

# Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) = \left( \frac{\text{Heads}}{\text{Heads} + \text{Tails}} \right)^5$$
A diagram showing three gold-colored coins. The top coin shows 'H' (heads). The bottom-left coin shows 'H' (heads). The bottom-right coin shows 'T' (tails).

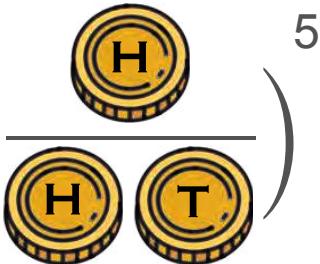
$$\frac{1}{2}$$

# Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) = \left( \frac{\text{Heads}}{\text{Heads} + \text{Tails}} \right)^5$$
A diagram showing three gold-colored coins. The top coin shows 'H' (heads). The bottom-left coin shows 'H' (heads). The bottom-right coin shows 'T' (tails).

$$\left( \frac{1}{2} \right)^5$$

# Independent Events: Coin Example 2



50% 50%

What is the probability of landing on heads five times?

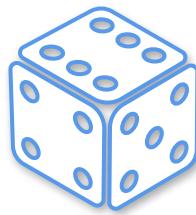
$$P(5 \text{ heads}) = \left( \frac{\text{Diagram of 5 heads}}{\text{Diagram of 2 heads and 1 tail}} \right)^5$$

The fraction in the equation compares two scenarios. The numerator is a stack of five coins, all showing heads ('H'). The denominator is a stack of three coins: the top one shows heads ('H'), the bottom-left shows heads ('H'), and the bottom-right shows tails ('T').

$$\left( \frac{1}{2} \right)^5 = \frac{1}{32}$$

# Independent Events: Dice Example 1

# Independent Events: Dice Example 1

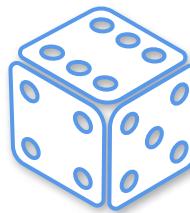


# Independent Events: Dice Example 1



$$\mathbf{P}(6) = \underline{\hspace{2cm}}$$

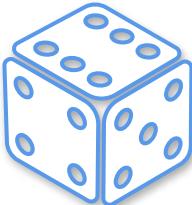
# Independent Events: Dice Example 1



$$P(6) = \underline{\hspace{2cm}}$$

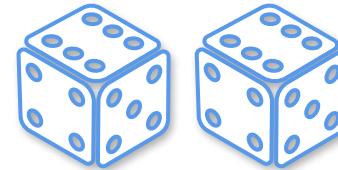


# Independent Events: Dice Example 1

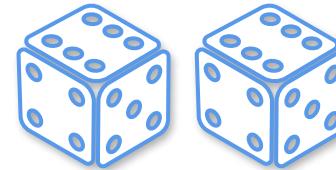
$$P(6) = \frac{1}{6}$$


# Independent Events: Dice Example 1

# Independent Events: Dice Example 1



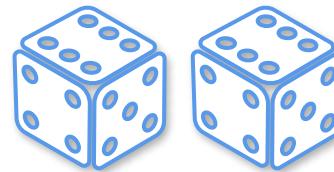
# Independent Events: Dice Example 1



2 dice

# Independent Events: Dice Example 1

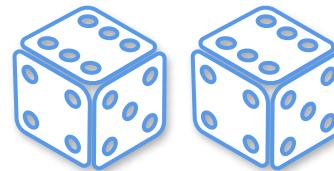
	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6



2 dice

# Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
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5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

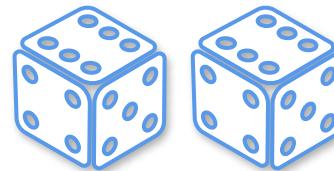


2 dice

$$P(6,6) = \underline{\hspace{2cm}}$$

# Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6

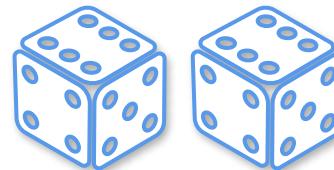


2 dice

$$P(6,6) = \underline{\hspace{2cm}}$$

# Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6



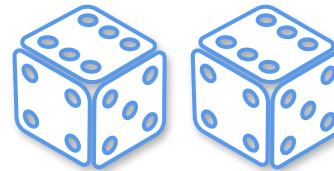
2 dice

6,6

$$P(6,6) = \underline{\hspace{2cm}}$$

# Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6



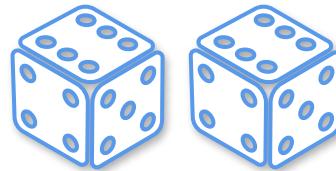
2 dice

$$P(6,6) = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6



2 dice

$$P(6,6) = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Independent Events: Dice Example 1

2 dice

						
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6



# Independent Events: Dice Example 1

						
1,1	1,2	1,3	1,4	1,5	1,6	
2,1	2,2	2,3	2,4	2,5	2,6	
3,1	3,2	3,3	3,4	3,5	3,6	
4,1	4,2	4,3	4,4	4,5	4,6	
5,1	5,2	5,3	5,4	5,5	5,6	
6,1	6,2	6,3	6,4	6,5		

$$P(6,6) =$$

2 dice



# Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6
$\frac{1}{6}$						

$$P(6,6) =$$



2 dice

# Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
1,2	3,1	3,2	3,3	3,4	3,5	3,6
1,3	4,1	4,2	4,3	4,4	4,5	4,6
1,4	5,1	5,2	5,3	5,4	5,5	5,6
1,5	6,1	6,2	6,3	6,4	6,5	6,6
1,6						

2 dice

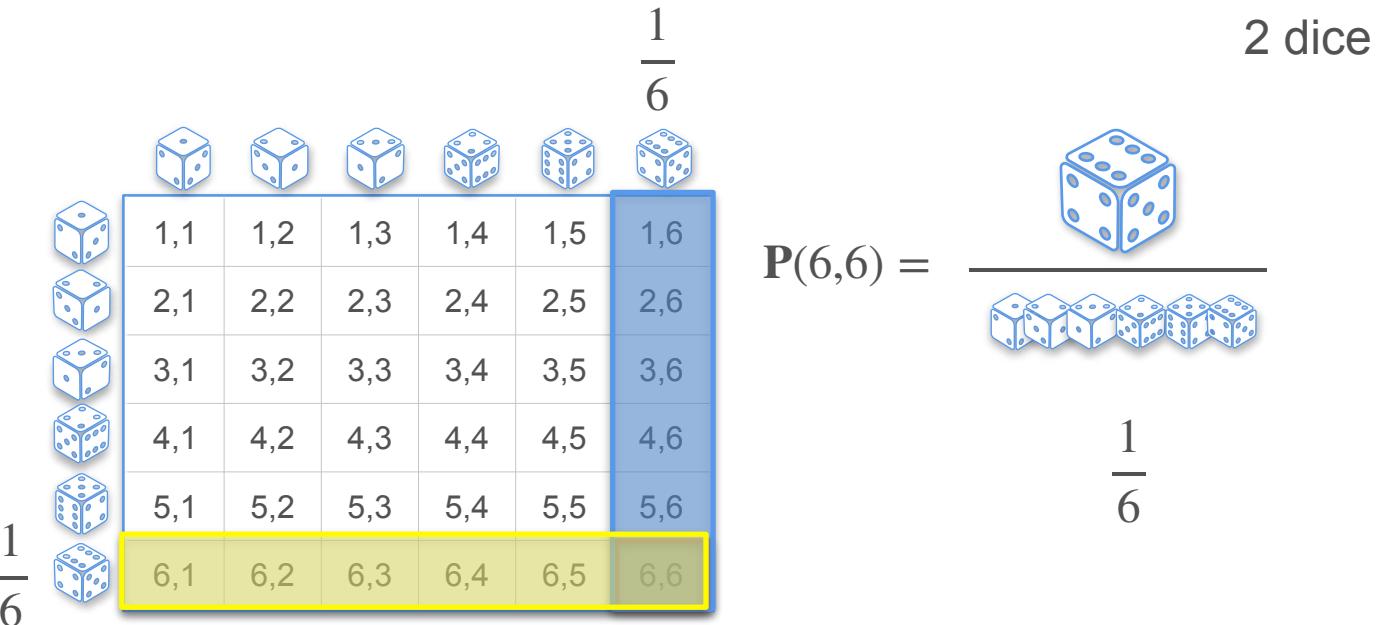
$$P(6,6) =$$



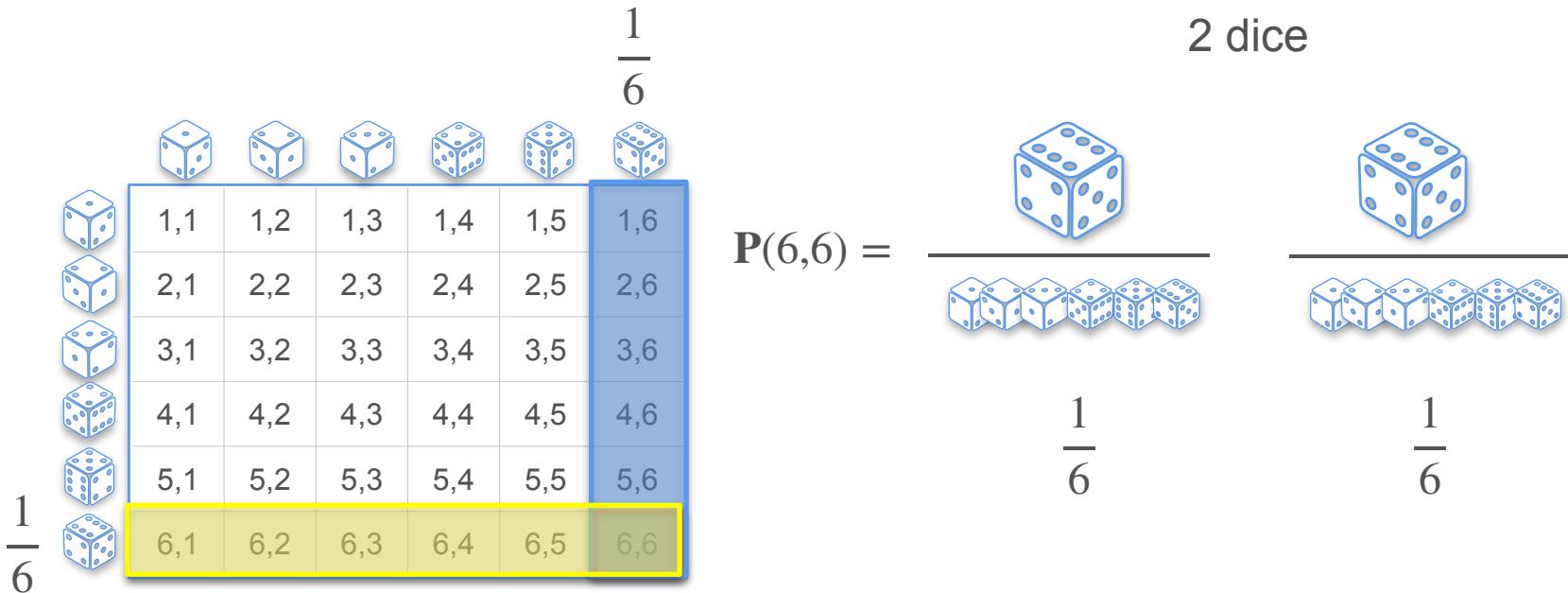
$$\frac{1}{6}$$

$$\frac{1}{6}$$

# Independent Events: Dice Example 1



# Independent Events: Dice Example 1



# Independent Events: Dice Example 1

					$\frac{1}{6}$	
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

2 dice

$P(6,6) = \frac{1}{6} \quad \frac{1}{6}$

# Independent Events: Dice Example 1

					$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5
1,1	2,1	2,2	2,3	2,4	2,5
1,2	3,1	3,2	3,3	3,4	3,5
1,3	4,1	4,2	4,3	4,4	4,5
1,4	5,1	5,2	5,3	5,4	5,5
1,5	6,1	6,2	6,3	6,4	6,5
1,6	6,6				

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

# Independent Events: Dice Example 1

					$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5
1,1	2,1	2,2	2,3	2,4	2,5
2,1	3,1	3,2	3,3	3,4	3,5
3,1	4,1	4,2	4,3	4,4	4,5
4,1	5,1	5,2	5,3	5,4	5,5
5,1	6,1	6,2	6,3	6,4	6,5
6,1	6,6				

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)$$

The diagram illustrates the probability calculation for rolling two dice. It shows a 6x6 grid of outcomes. The top row and left column are labeled with dice icons. The bottom-right cell, representing the outcome (6,6), is highlighted with a green border. The entire grid is divided into two main sections by a diagonal line from the top-left to the bottom-right. The section above the diagonal is shaded yellow, and the section below it is shaded blue. The total number of outcomes is represented by the product of the number of dice (2) and the number of faces per die (6). The number of favorable outcomes is 1, corresponding to the single cell (6,6).

# Independent Events: Dice Example 1

					$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5
1,1	2,1	2,2	2,3	2,4	2,5
1,2	3,1	3,2	3,3	3,4	3,5
1,3	4,1	4,2	4,3	4,4	4,5
1,4	5,1	5,2	5,3	5,4	5,5
1,5	6,1	6,2	6,3	6,4	6,5
1,6	6,6				

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)$$

The diagram illustrates the probability calculation for rolling two dice. It shows a 6x6 grid of outcomes. The top row and left column are labeled with dice icons. The bottom-right cell, representing the outcome (6,6), is highlighted with a green border. The entire grid is divided into two main sections by a diagonal line from the top-left to the bottom-right. The section above the diagonal is shaded yellow, and the section below it is shaded blue. The total number of outcomes is represented by the product of the number of dice (2) and the number of faces per die (6). The number of favorable outcomes is 1, corresponding to the single cell (6,6).

# Independent Events: Dice Example 1

					$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5
1,1	2,1	2,2	2,3	2,4	2,5
1,2	3,1	3,2	3,3	3,4	3,5
1,3	4,1	4,2	4,3	4,4	4,5
1,4	5,1	5,2	5,3	5,4	5,5
1,5	6,1	6,2	6,3	6,4	6,5
1,6	6,6				

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2$$

The diagram illustrates the probability calculation for rolling two dice. It shows a 6x6 grid of outcomes. The top row and left column are labeled with dice icons. The bottom-right cell, representing the outcome (6,6), is highlighted with a green border. The entire grid is divided into two main sections by a diagonal line from the top-left to the bottom-right. The section above the diagonal is shaded yellow, and the section below it is shaded blue. The total number of outcomes is represented by the product of the number of dice (2) and the number of faces per die (6). The number of favorable outcomes is 1, corresponding to the single cell (6,6).

# Independent Events: Dice Example 1

						$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
$\frac{1}{6}$	6,1	6,2	6,3	6,4	6,5	6,6

2 dice

$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$

The diagram illustrates the concept of independent events using two dice rolls. It shows two separate sets of six dice each, representing the first and second rolls respectively. The rolls are shown as horizontal sequences of dice, with a black dot between them indicating they are independent events.

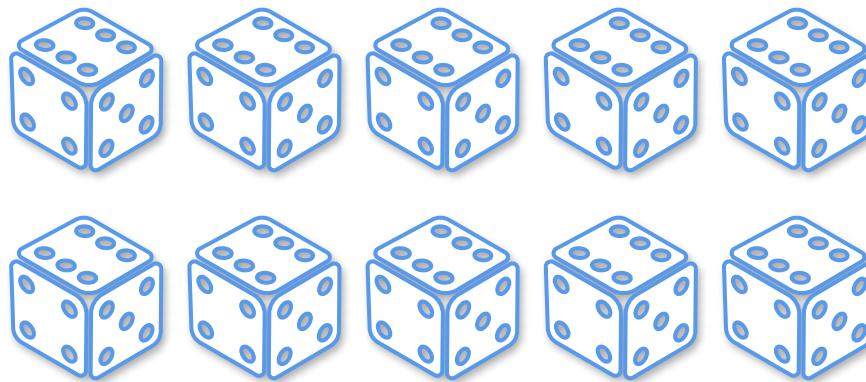
# Independent Events: Dice Example 2

# Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

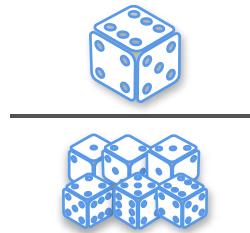
# Independent Events: Dice Example 2

What is the probability of getting 10 sixes?



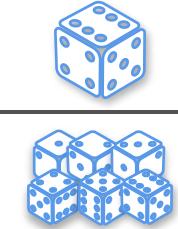
# Independent Events: Dice Example 2

What is the probability of getting 10 sixes?



# Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$P(10 \text{ sixes}) = \frac{\text{one outcome}}{\text{all outcomes}}$$


# Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$P(10 \text{ sixes}) = \left( \frac{\text{one die}}{\text{ten dice}} \right)$$

# Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$P(10 \text{ sixes}) = \left( \frac{\text{one die showing 6}}{\text{one row of 6 dice}} \right)^{10}$$

# Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$\begin{aligned} P(10 \text{ sixes}) &= \left( \frac{\text{one die showing 6}}{\text{one die showing 6}} \right)^{10} \\ &= \left( \frac{1}{6} \right)^{10} \end{aligned}$$



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# Introduction to probability

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## Birthday problem

# 6. The Birthday Problem

- Quiz: You have a party with 30 friends. What do you think is more likely, that there are two with the same birthday, or not?
  - Answer: Same birthday
- Calculate the probability that two people have the same birthday. Show that it's very close to 1.
- Question: How many people do you think there should be for the probability that 2 have the same birthday is 50?
  - Answer: 23
  - Show calculation

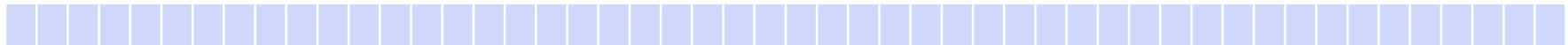
# Quiz

- You have 30 friends at a party. What do you think is more likely:
  - That there exist two people with the same birthday
  - That no two of them have the same birthday
- (Assume the year has 365 days, nobody has a birthday on Feb 29).

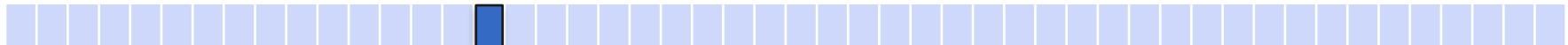
# Quiz

- Answer: It's more likely that 2 people have the same birthday.
- In fact, the probability of no two people having the same birthday is around 0.3.

# Probability That Everyone Has a Different Birthday



# Probability That Everyone Has a Different Birthday



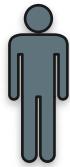
$$\frac{365}{365}$$

# Probability That Everyone Has a Different Birthday



$$\frac{365}{365} \quad \frac{364}{365}$$

# Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

$$\frac{364}{365}$$

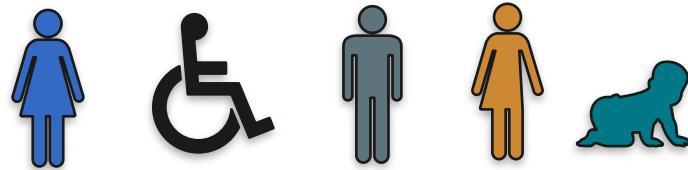
$$\frac{363}{365}$$

# Probability That Everyone Has a Different Birthday



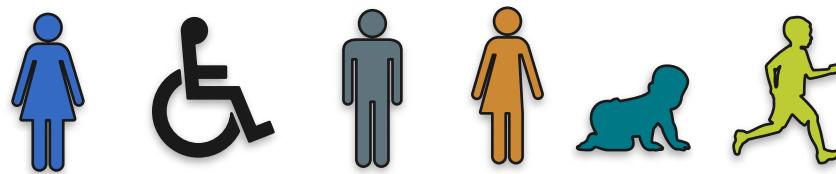
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365}$$

# Probability That Everyone Has a Different Birthday



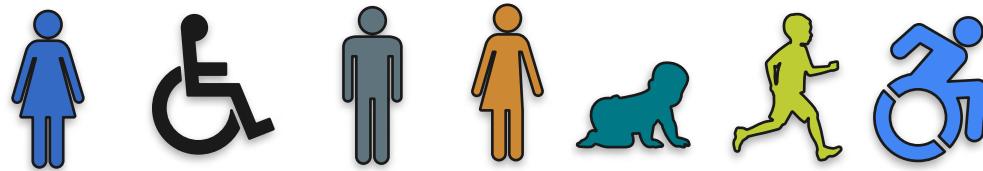
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365}$$

# Probability That Everyone Has a Different Birthday



$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365} \quad \frac{360}{365}$$

# Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

$$\frac{364}{365}$$

$$\frac{363}{365}$$

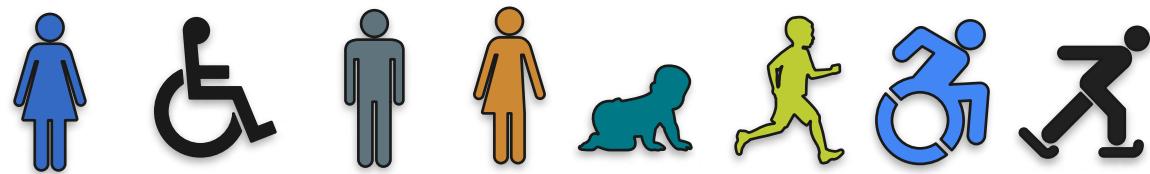
$$\frac{362}{365}$$

$$\frac{361}{365}$$

$$\frac{360}{365}$$

$$\frac{359}{365}$$

# Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

$$\frac{364}{365}$$

$$\frac{363}{365}$$

$$\frac{362}{365}$$

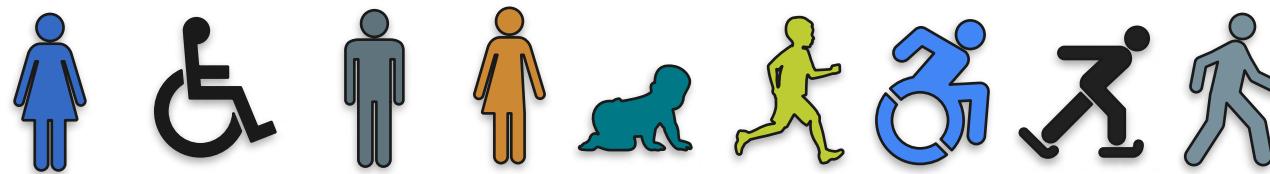
$$\frac{361}{365}$$

$$\frac{360}{365}$$

$$\frac{359}{365}$$

$$\frac{358}{365}$$

# Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

$$\frac{364}{365}$$

$$\frac{363}{365}$$

$$\frac{362}{365}$$

$$\frac{361}{365}$$

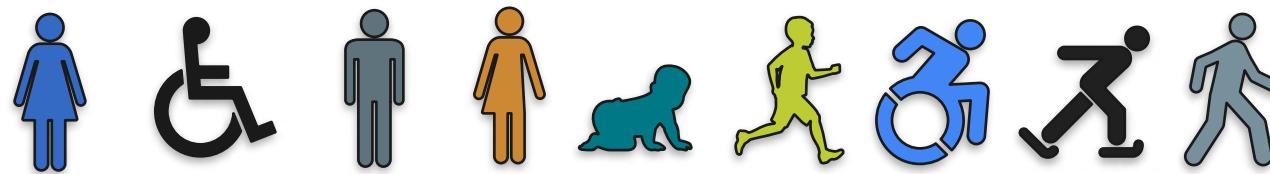
$$\frac{360}{365}$$

$$\frac{359}{365}$$

$$\frac{358}{365}$$

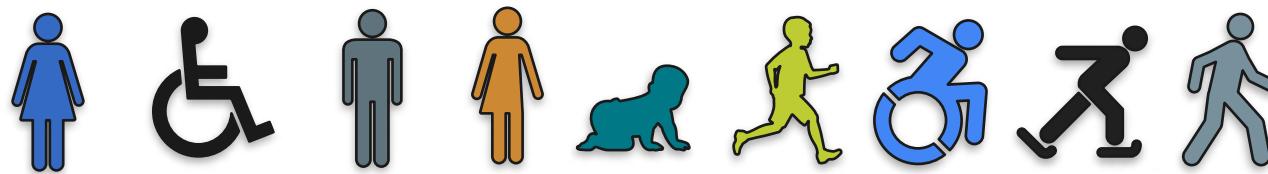
$$\frac{357}{365}$$

# Probability That Everyone Has a Different Birthday



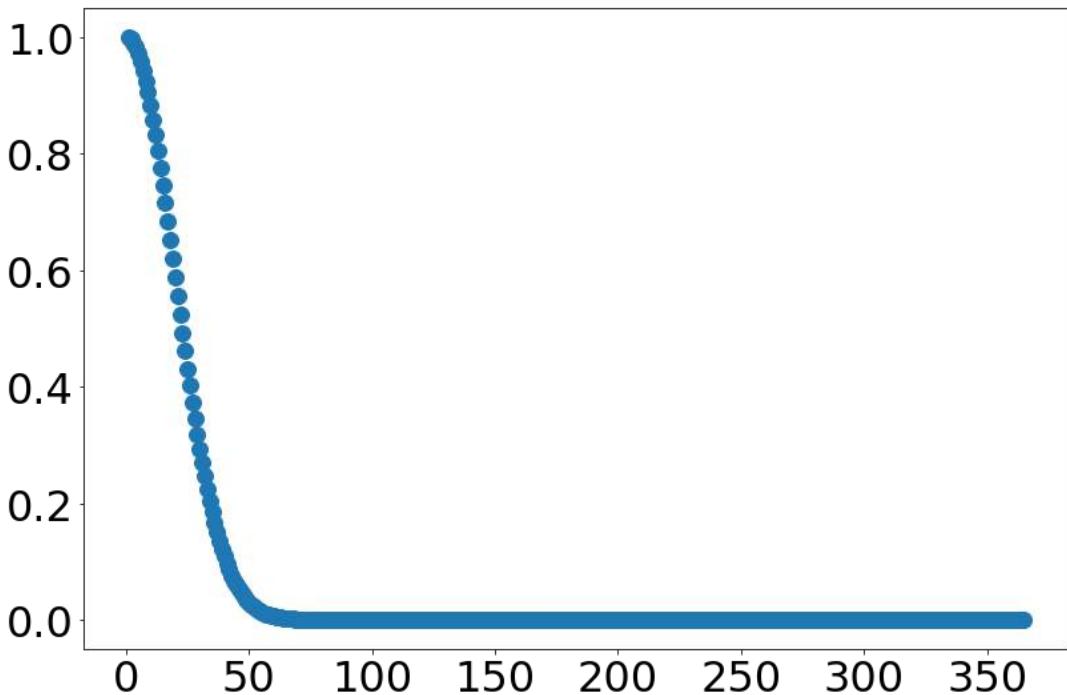
$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} =$$

# Probability That Everyone Has a Different Birthday



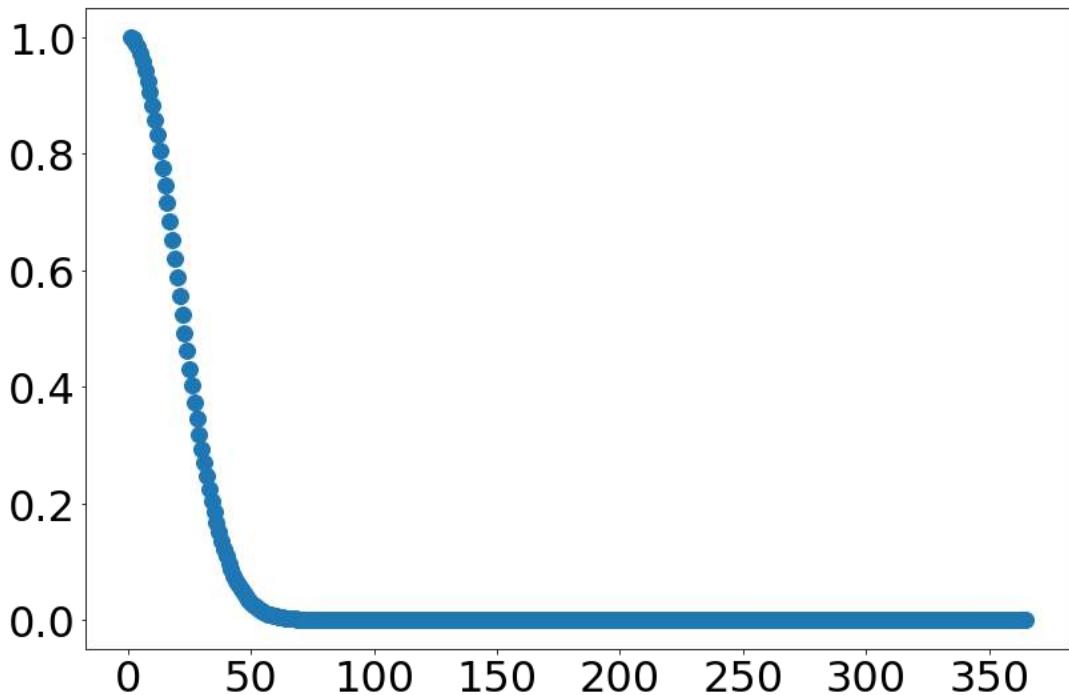
$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} = 0.905$$

# Probability That no Two People Have the Same Birthday



# Probability That no Two People Have the Same Birthday

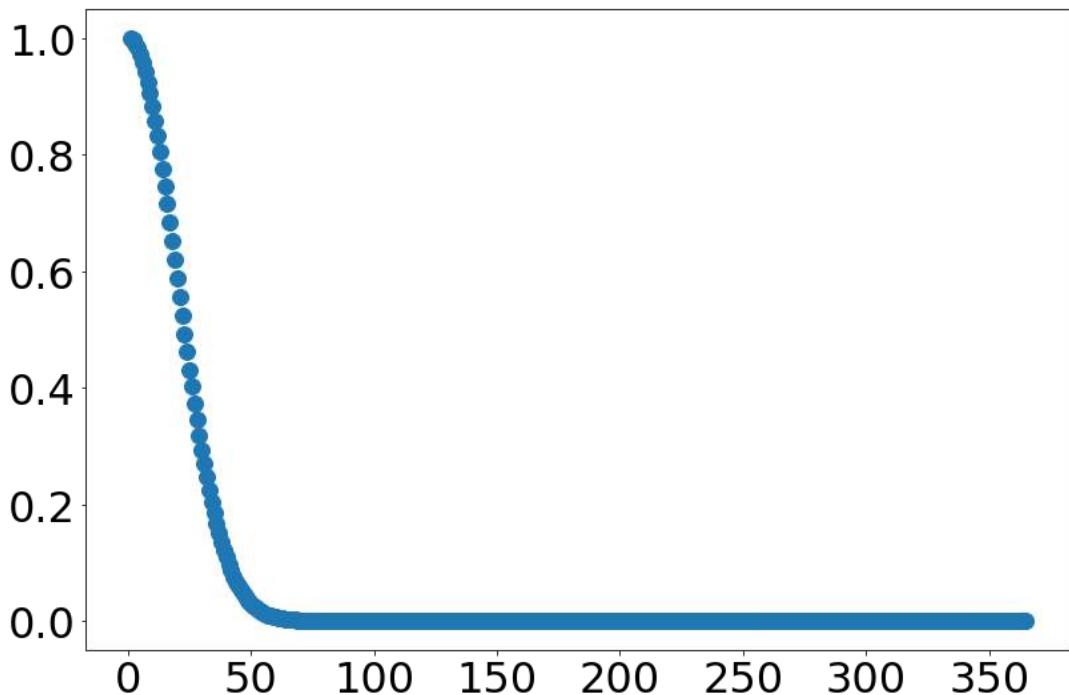
1 person: 1



# Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

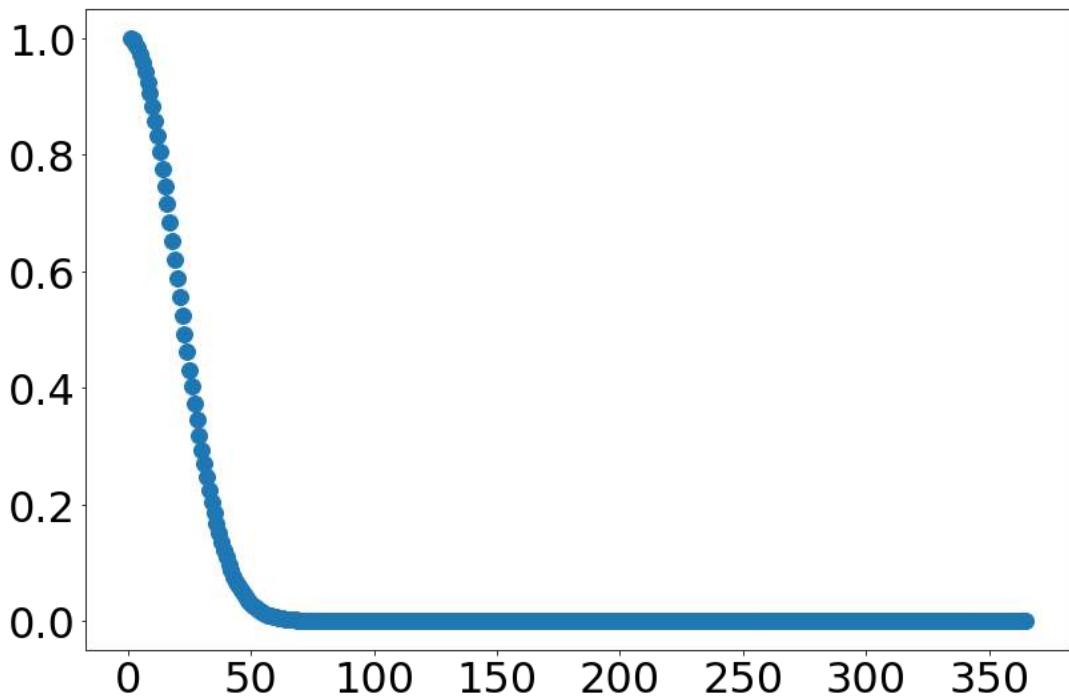


# Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992



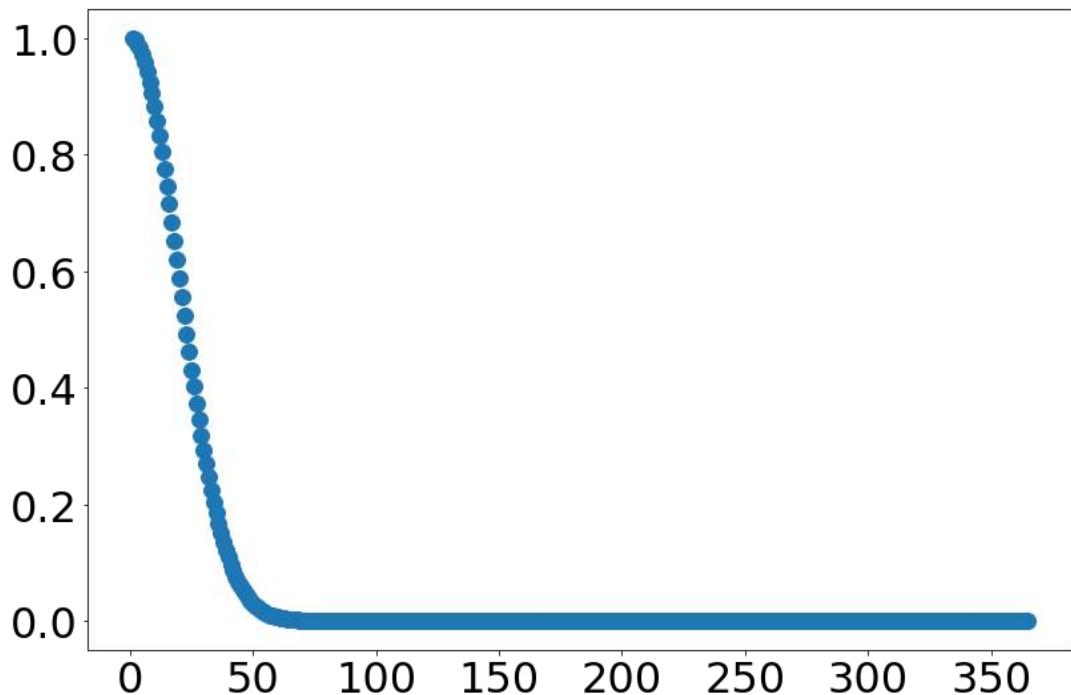
# Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992

4 people: 0.984



# Probability That no Two People Have the Same Birthday

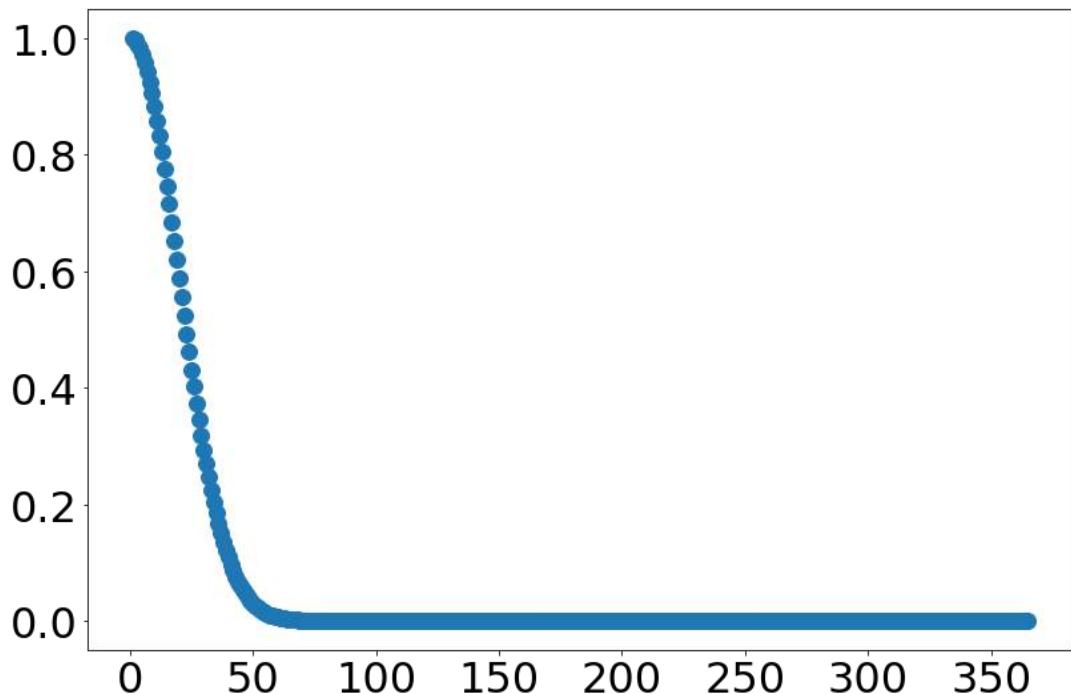
1 person: 1

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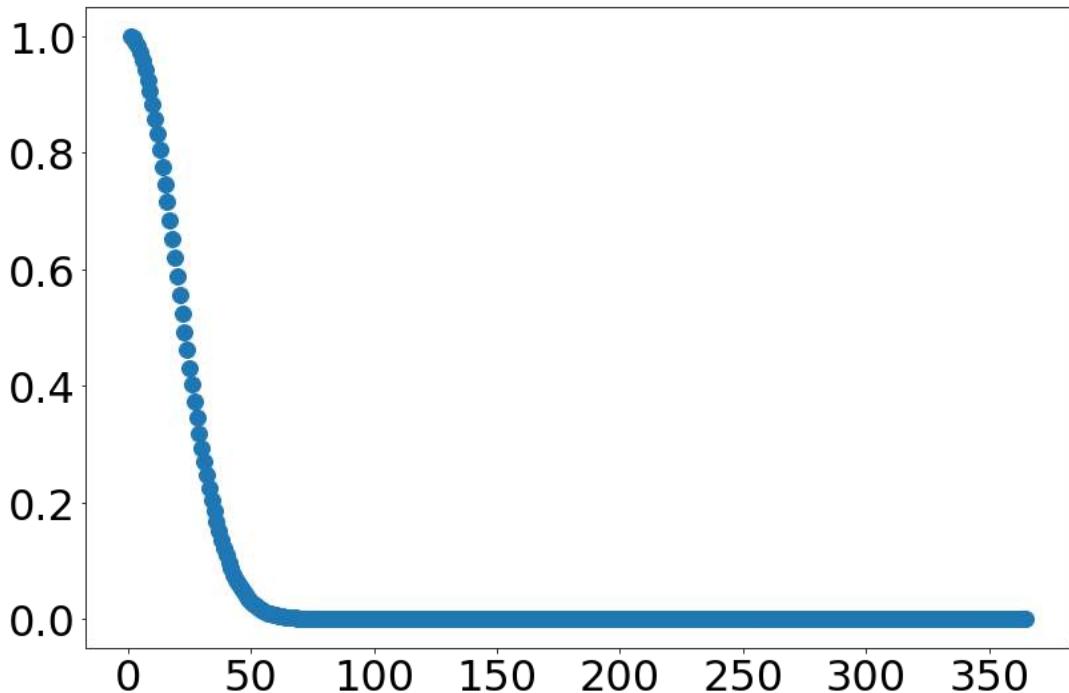
4 people: 0.984

5 people: 0.973



# Probability That no Two People Have the Same Birthday

1 person: 1  
2 people: 0.997  
3 people: 0.992  
4 people: 0.984  
5 people: 0.973  
10 people: 0.883



# Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

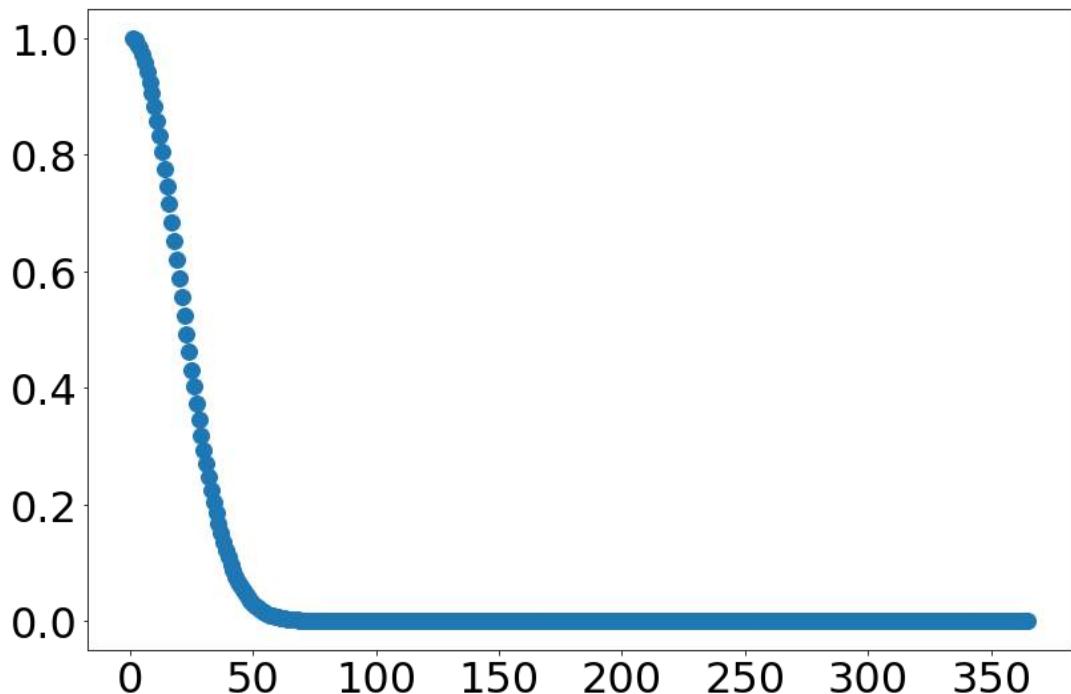
3 people: 0.992

4 people: 0.984

5 people: 0.973

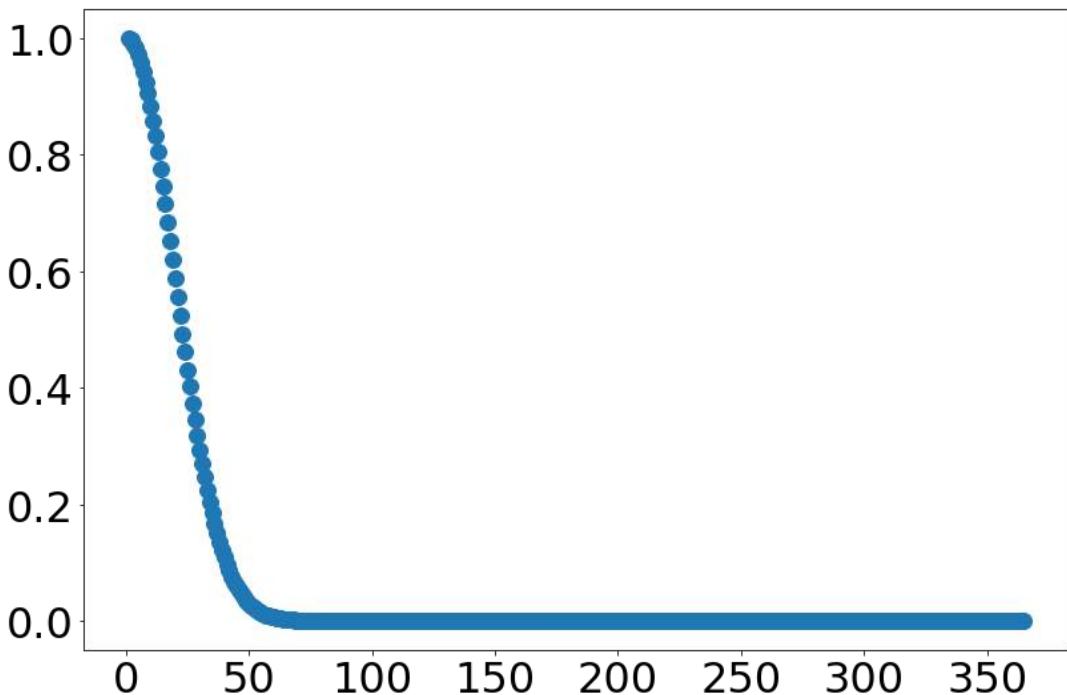
10 people: 0.883

20 people: 0.589



# Probability That no Two People Have the Same Birthday

1 person: 1  
2 people: 0.997  
3 people: 0.992  
4 people: 0.984  
5 people: 0.973  
10 people: 0.883  
20 people: 0.589  
23 people: 0.493



# Probability That no Two People Have the Same Birthday

1 person: 1

2 people: 0.997

3 people: 0.992

4 people: 0.984

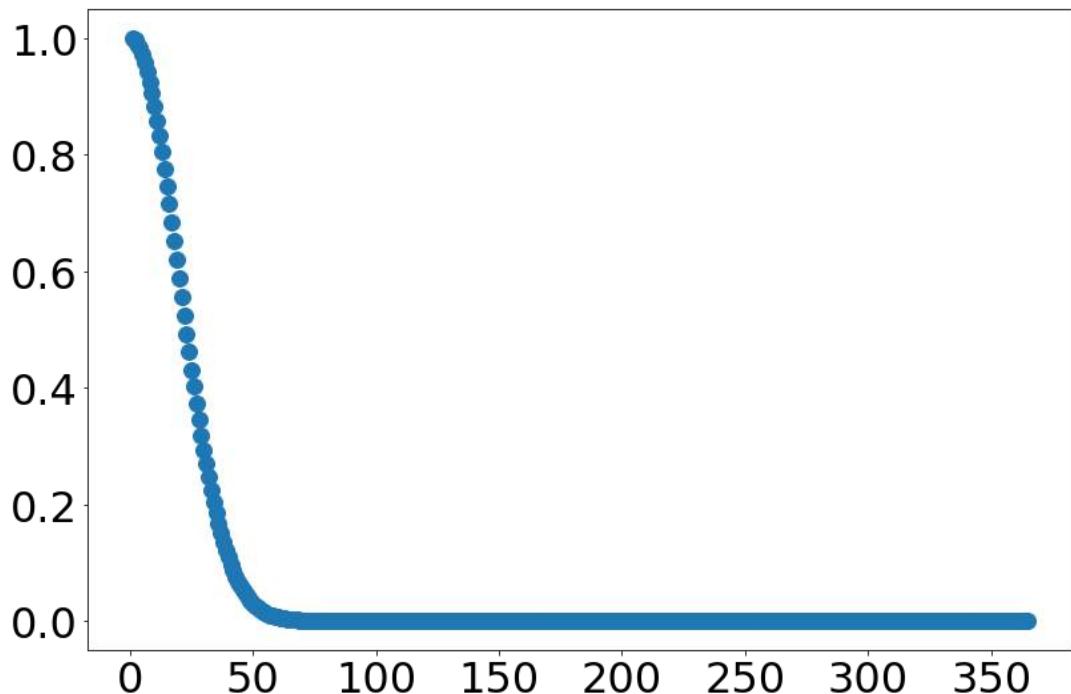
5 people: 0.973

10 people: 0.883

20 people: 0.589

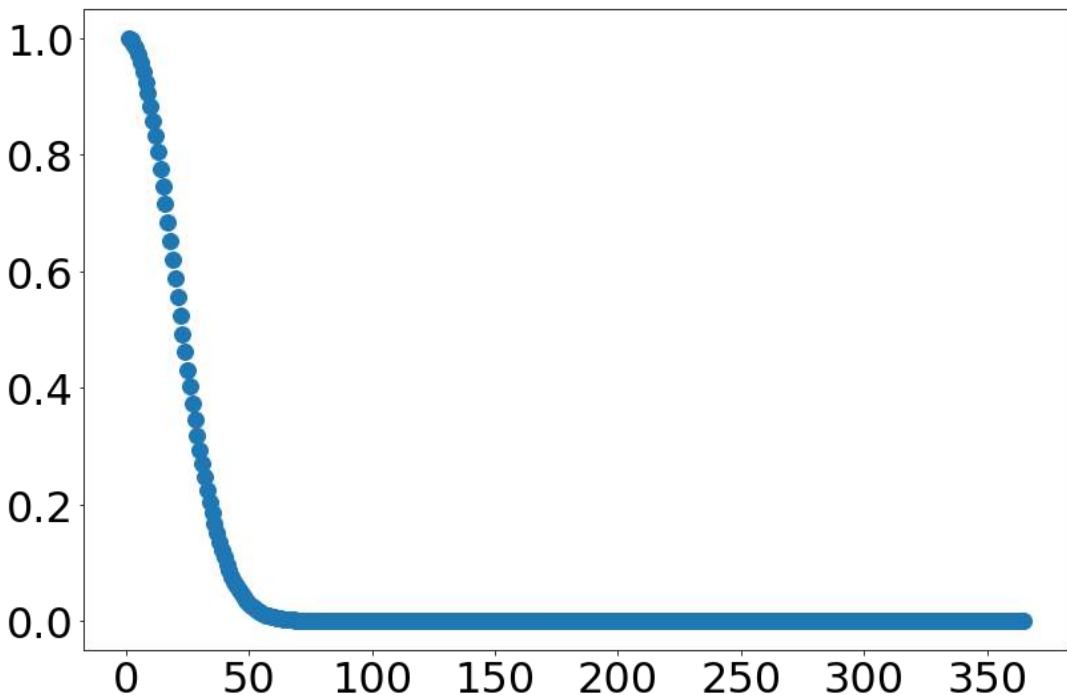
23 people: 0.493

30 people: 0.294



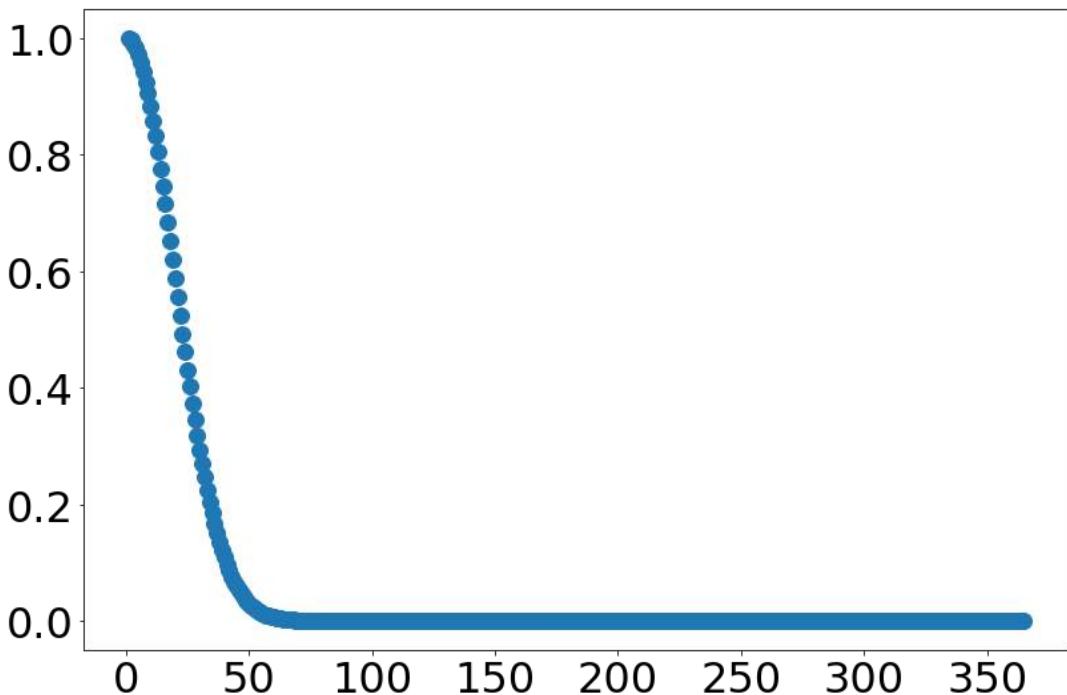
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5 people: 0.973  
10 people: 0.883  
20 people: 0.589  
23 people: 0.493  
30 people: 0.294  
50 people: 0.030



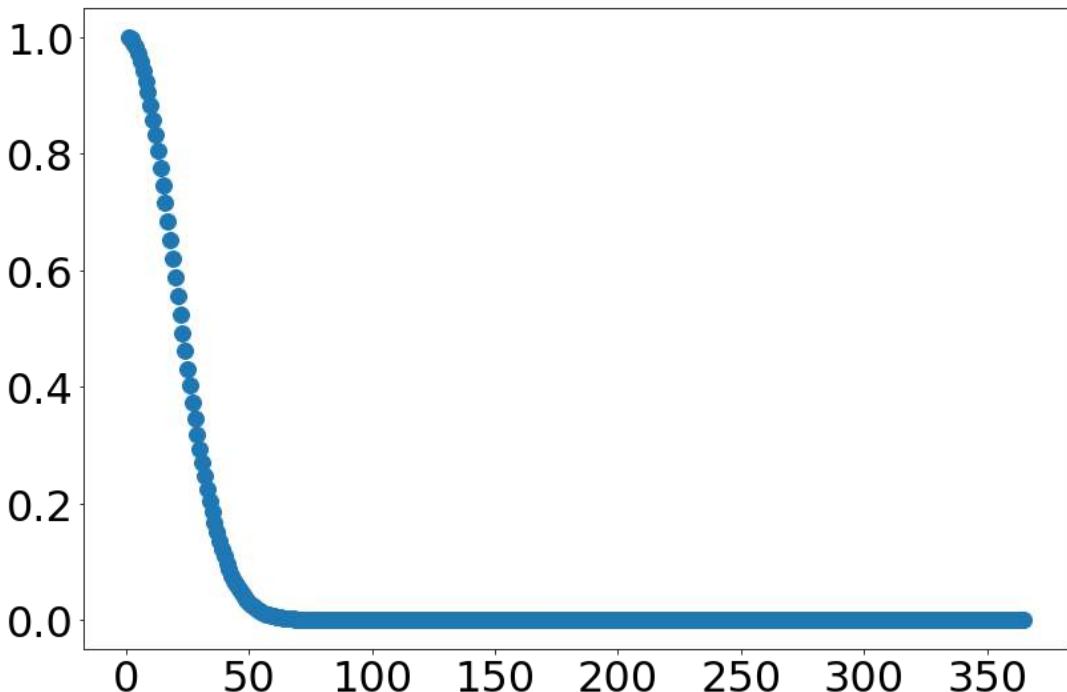
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20 people: 0.589  
23 people: 0.493  
30 people: 0.294  
50 people: 0.030  
100 people: 0.0000003



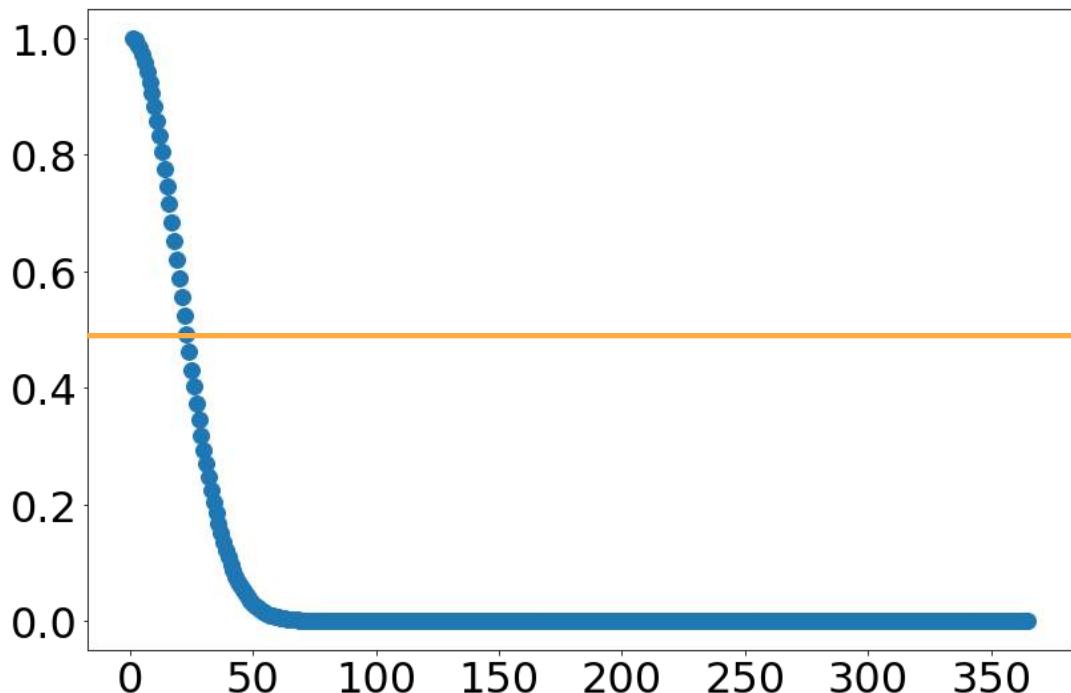
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23 people: 0.493  
30 people: 0.294  
50 people: 0.030  
100 people: 0.0000003  
365 people: 0



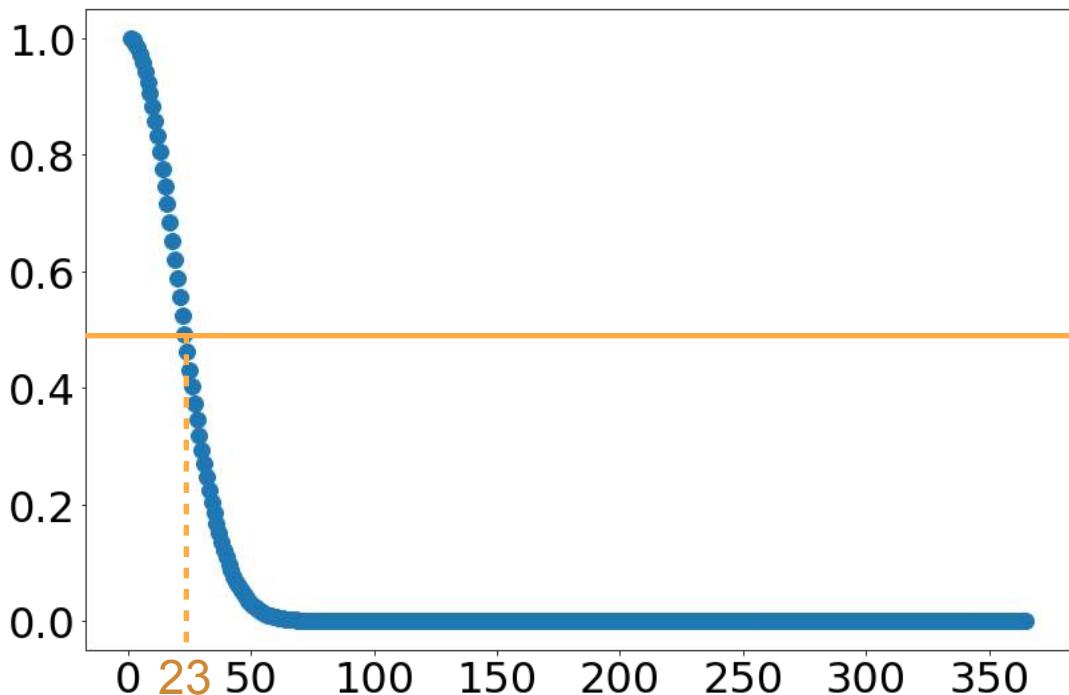
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30 people: 0.294  
50 people: 0.030  
100 people: 0.0000003  
365 people: 0



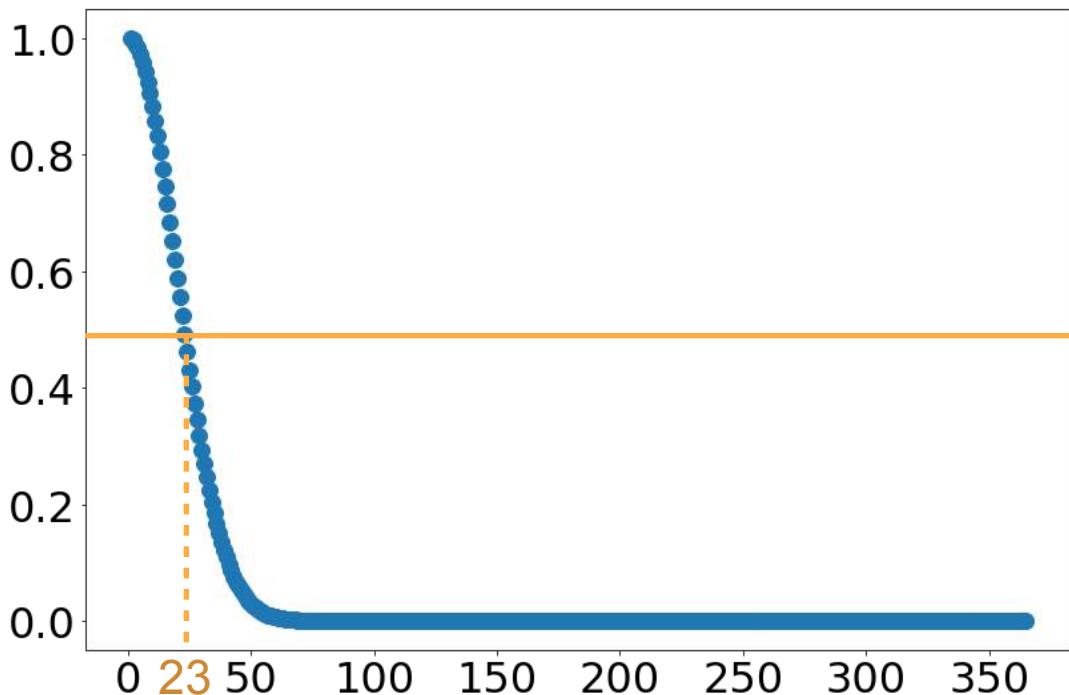
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100 people: 0.0000003  
365 people: 0



# Probability That no Two People Have the Same Birthday

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**23 people: 0.493**  
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50 people: 0.030  
100 people: 0.0000003  
365 people: 0





DeepLearning.AI

# Introduction to probability

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## Conditional probability

# Conditional Probability: Coin Example 1



50%    50%

# Conditional Probability: Coin Example 1



50%    50%

What is the probability of landing on heads twice?

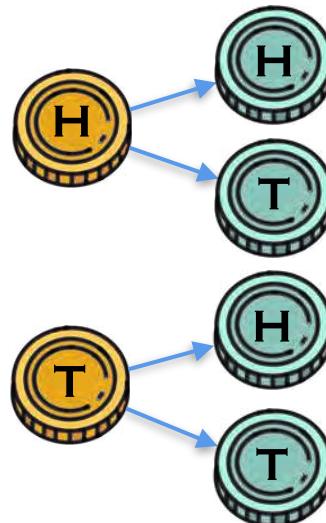
# Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st            2nd



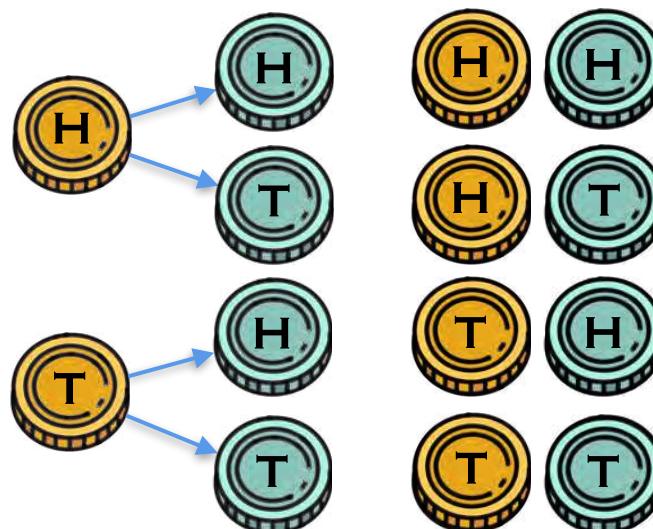
# Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st      2nd



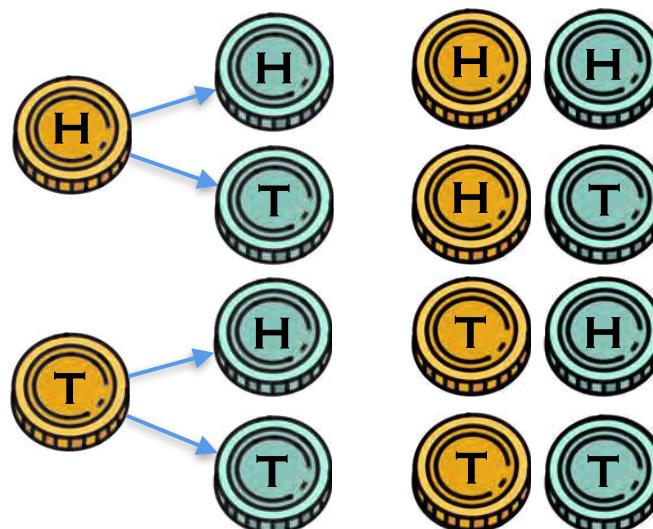
# Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st      2nd



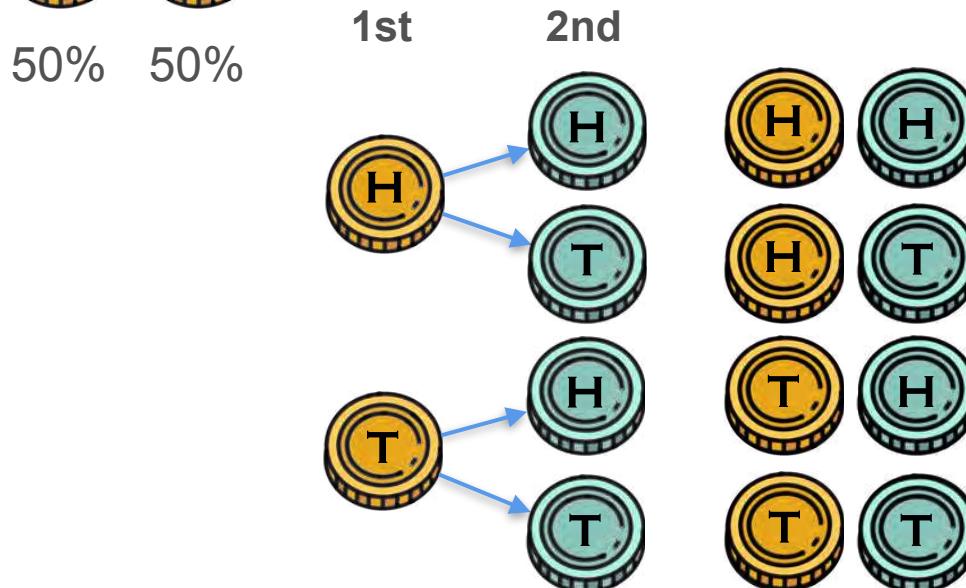
$$P(HH) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}}$$



# Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?



$$P(HH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The numerator shows two coins both showing heads (H). The denominator shows all four possible outcomes: HH, HT, TH, TT.

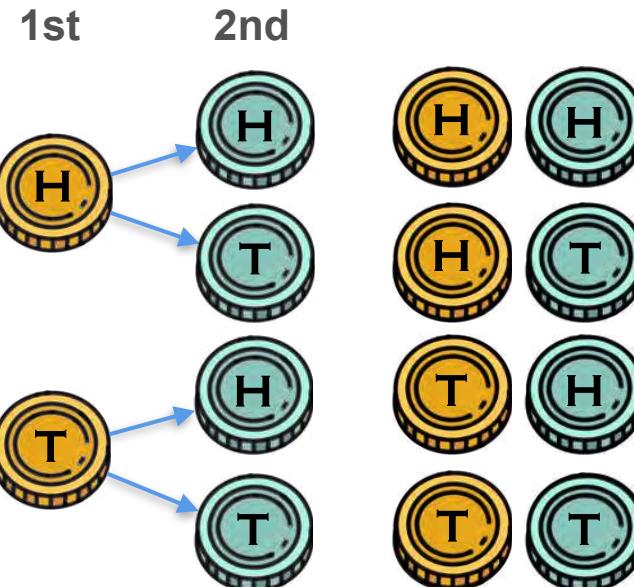
Two yellow coins, both with 'H' in blue, are shown side-by-side. Below them is a horizontal line. To the right of the line is a fraction bar. Below the fraction bar is a 2x2 grid of four coins: top-left yellow with 'H', top-right teal with 'H', bottom-left yellow with 'T', and bottom-right teal with 'T'.  
$$P(HH) = \frac{1}{4}$$

# Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?



$$P(HH) = \frac{1}{4}$$

The diagram shows a 2x2 grid of four coins representing the outcomes of two coin flips. The top-left coin is yellow (H) and teal (H). The top-right coin is yellow (H) and teal (H). The bottom-left coin is yellow (T) and teal (H). The bottom-right coin is yellow (T) and teal (T). The top-left coin is highlighted in yellow, and the top-right coin is highlighted in teal, representing the event of getting heads on both flips. Below the grid, there are four additional coin pairs, each with one yellow coin and one teal coin, representing the total sample space of outcomes.

# Conditional Probability: Coin Example 1

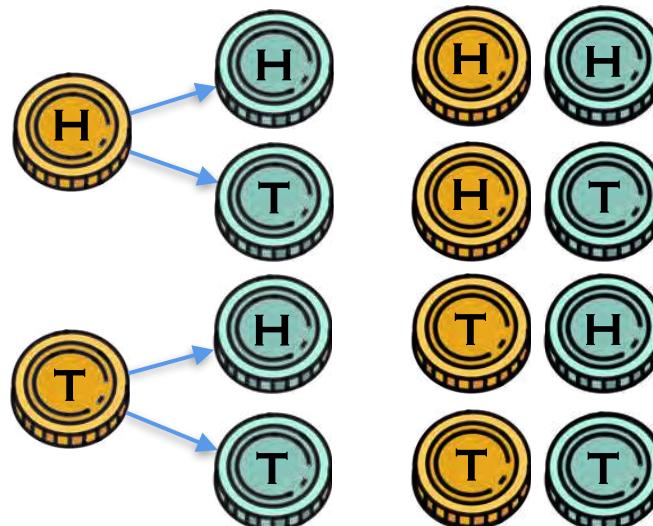


50% 50%

What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



# Conditional Probability: Coin Example 1

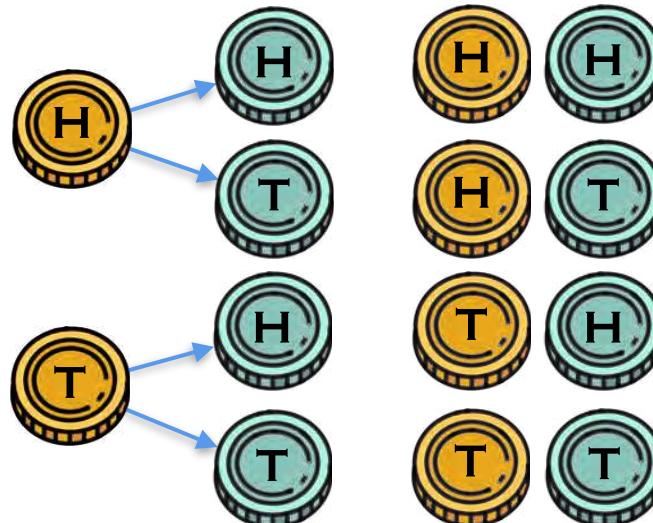


50% 50%

What is the probability of landing on heads twice?

1st      2nd

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# Conditional Probability: Coin Example 1

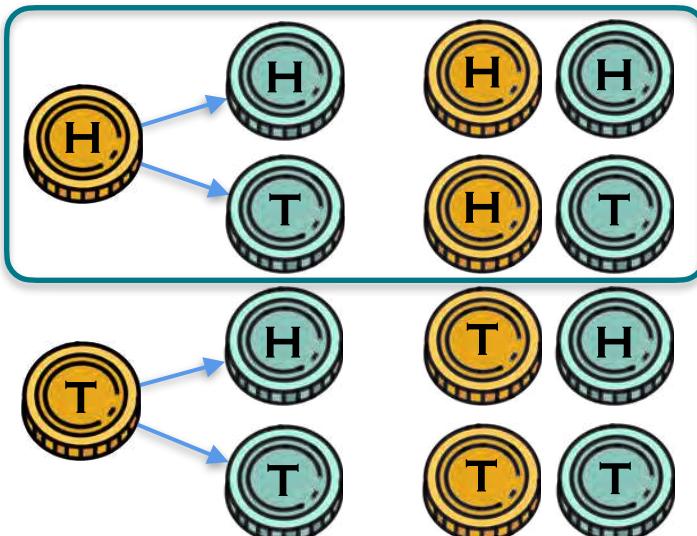


50% 50%

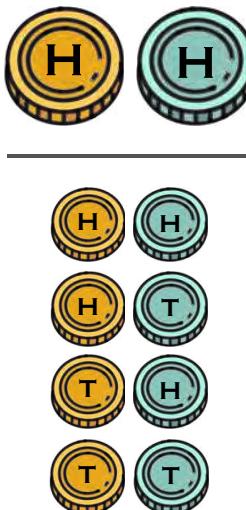
What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is heads



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



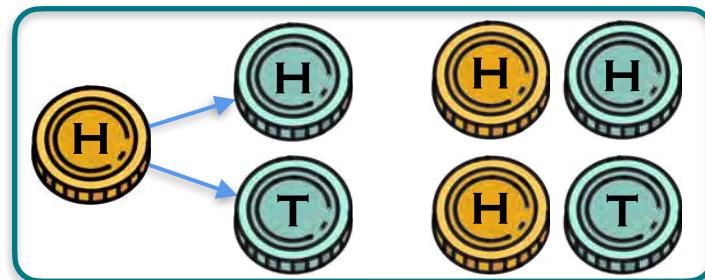
# Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is heads



$$P(HH) = \frac{1}{4}$$



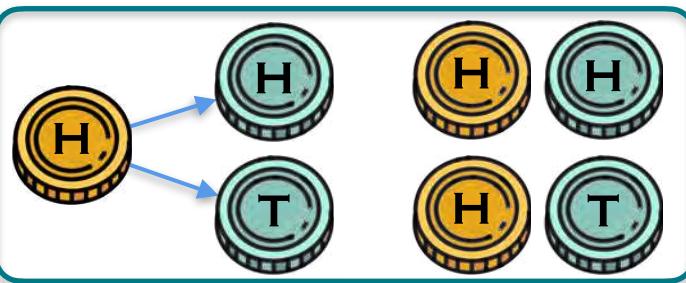
# Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is heads



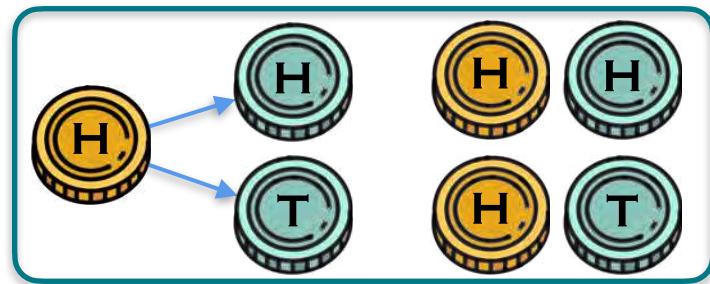
$$P(HH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} = \frac{1}{4}$$
A diagram illustrating the calculation of conditional probability. It shows two rows of four coins each. The top row represents the outcome of two coin flips given that the first flip was heads: the first coin is heads (H) and the second is heads (H). The bottom row represents all possible outcomes for the second flip, given that the first flip was heads: the first coin is heads (H) and the second is heads (H); the first coin is heads (H) and the second is tails (T); the first coin is heads (H) and the second is heads (H); and the first coin is tails (T) and the second is heads (H). This illustrates that there is only one favorable outcome (HH) out of four possible outcomes (HH, HT, HH, TT).

# Conditional Probability: Coin Example 1

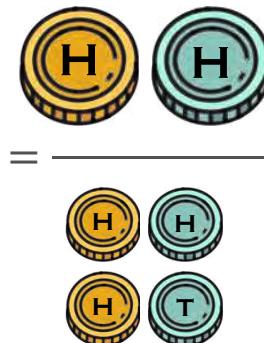


What is the probability of landing on heads twice?

1st            2nd            **GIVEN that the first one is heads**



$$P(HH \mid \text{1st is } H) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes}} = \frac{1}{2}$$

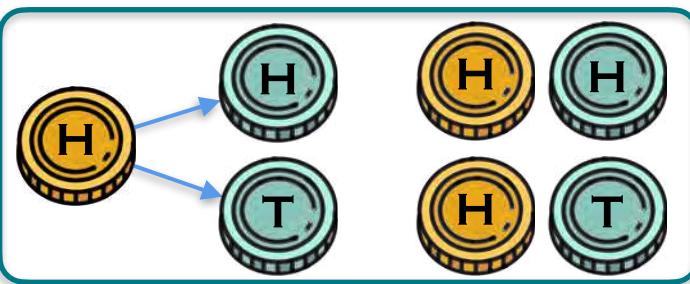


# Conditional Probability: Coin Example 1



What is the probability of landing on heads twice?

1st            2nd            **GIVEN that the first one is heads**



$$P(HH | \text{1st is } H)$$

$$= \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{2}$$

A diagram illustrating the conditional probability of getting two heads given that the first coin is heads. It shows two rows of four coins each. The top row starts with a yellow coin labeled 'H', followed by three light blue coins labeled 'H', 'H', and 'H' respectively. The bottom row starts with a light blue coin labeled 'T', followed by three yellow coins labeled 'T', 'T', and 'T' respectively. The top row is highlighted with a teal border.

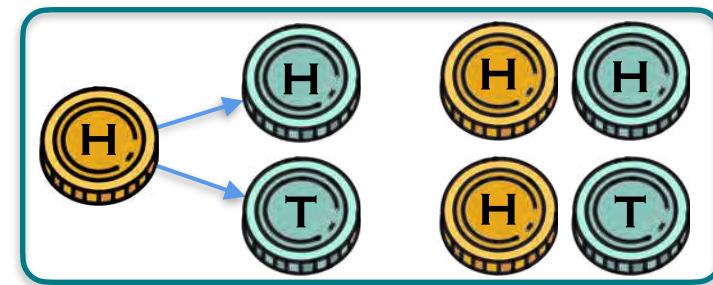
# Conditional Probability: Coin Example 1



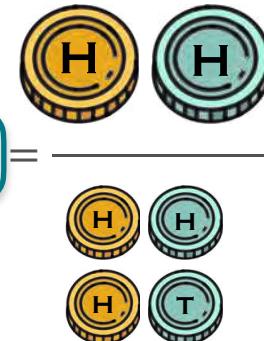
What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is heads



$$P(HH \mid \text{1st is } H) = \frac{\text{Number of HH pairs}}{\text{Total number of outcomes}} = \frac{1}{2}$$



# Conditional Probability: Coin Example 1

What is the probability of landing on heads twice?

**GIVEN** that the first one is heads

$$P(HH | \text{1st is } H)$$

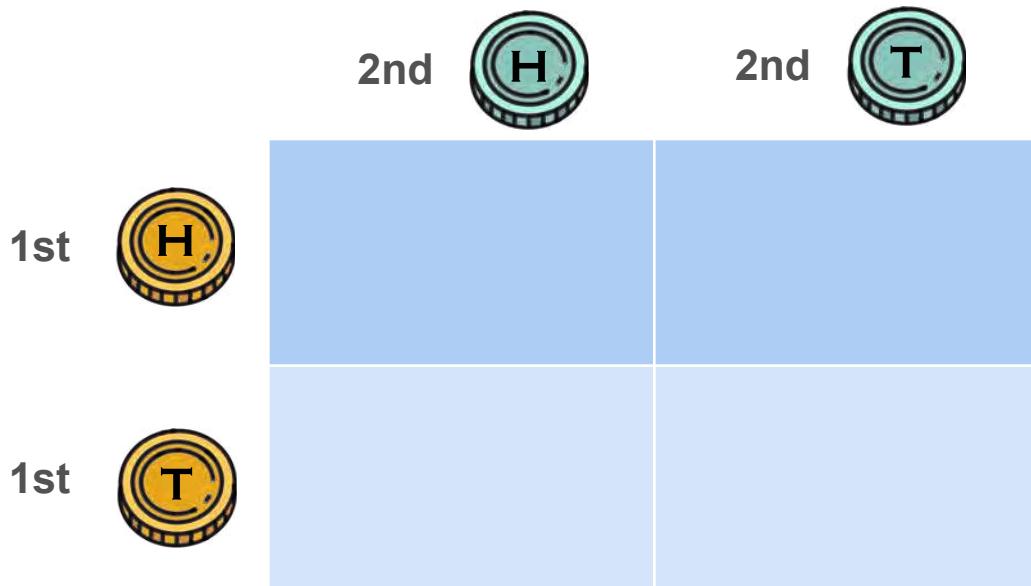


# Conditional Probability: Coin Example 1

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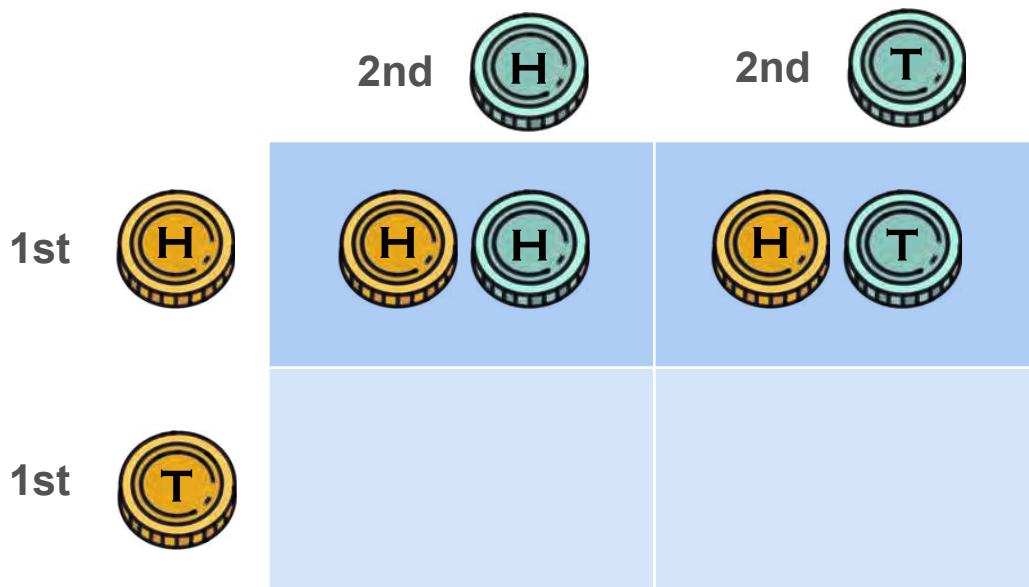
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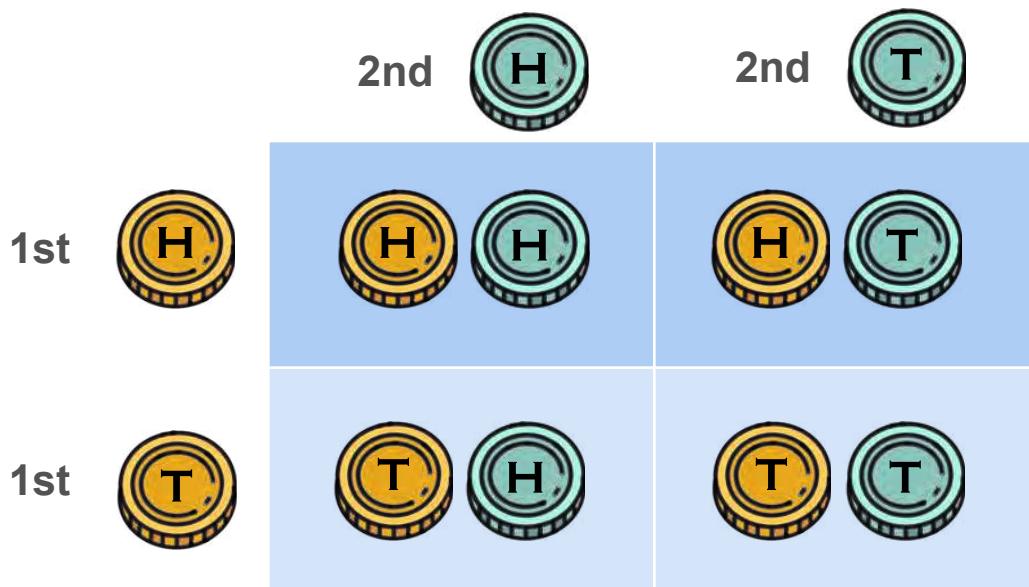
# Conditional Probability: Coin Example 1



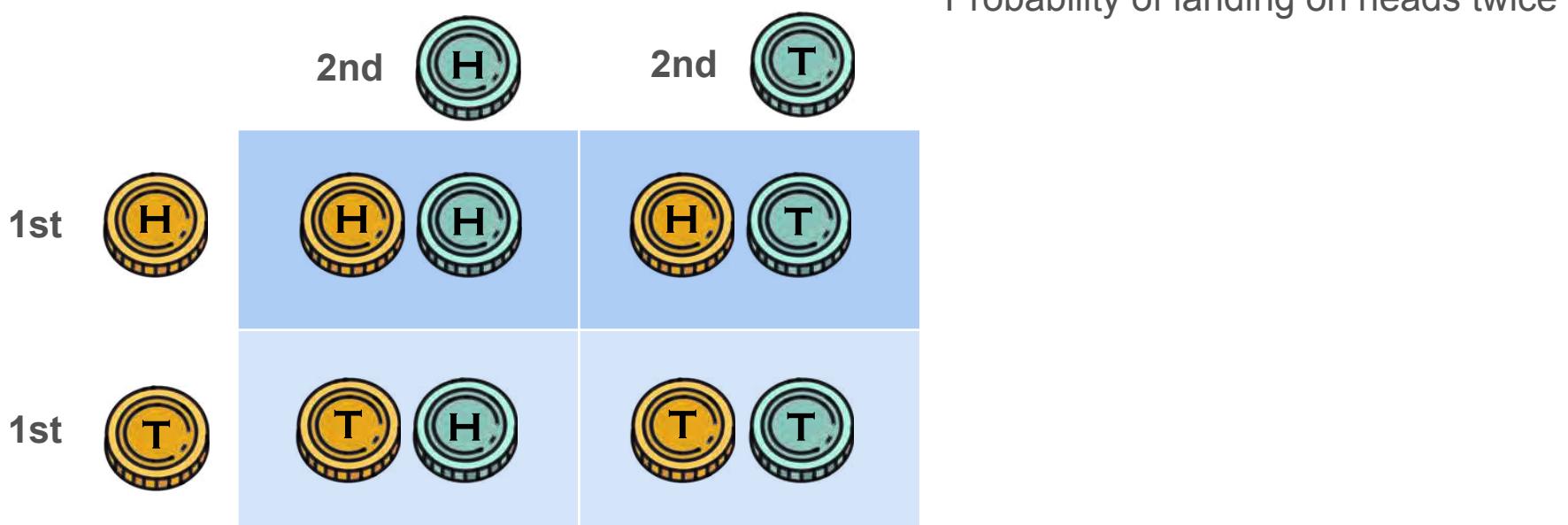
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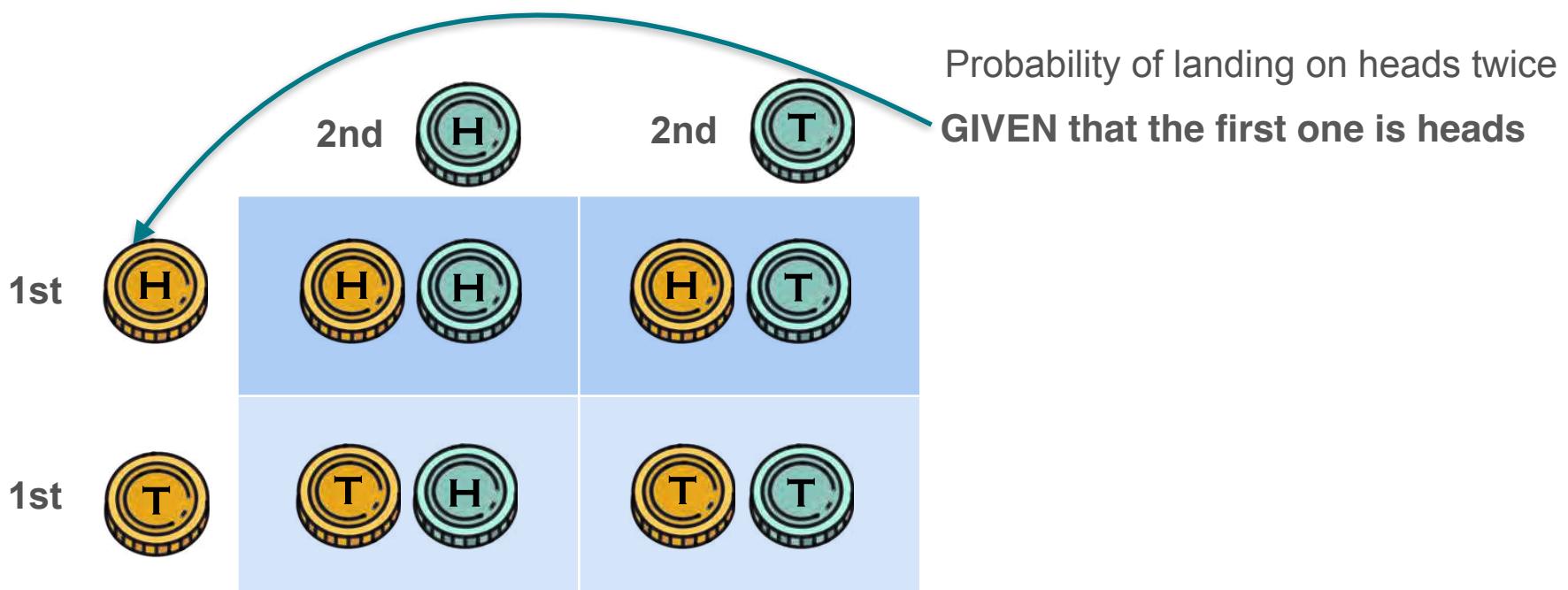
# Conditional Probability: Coin Example 1



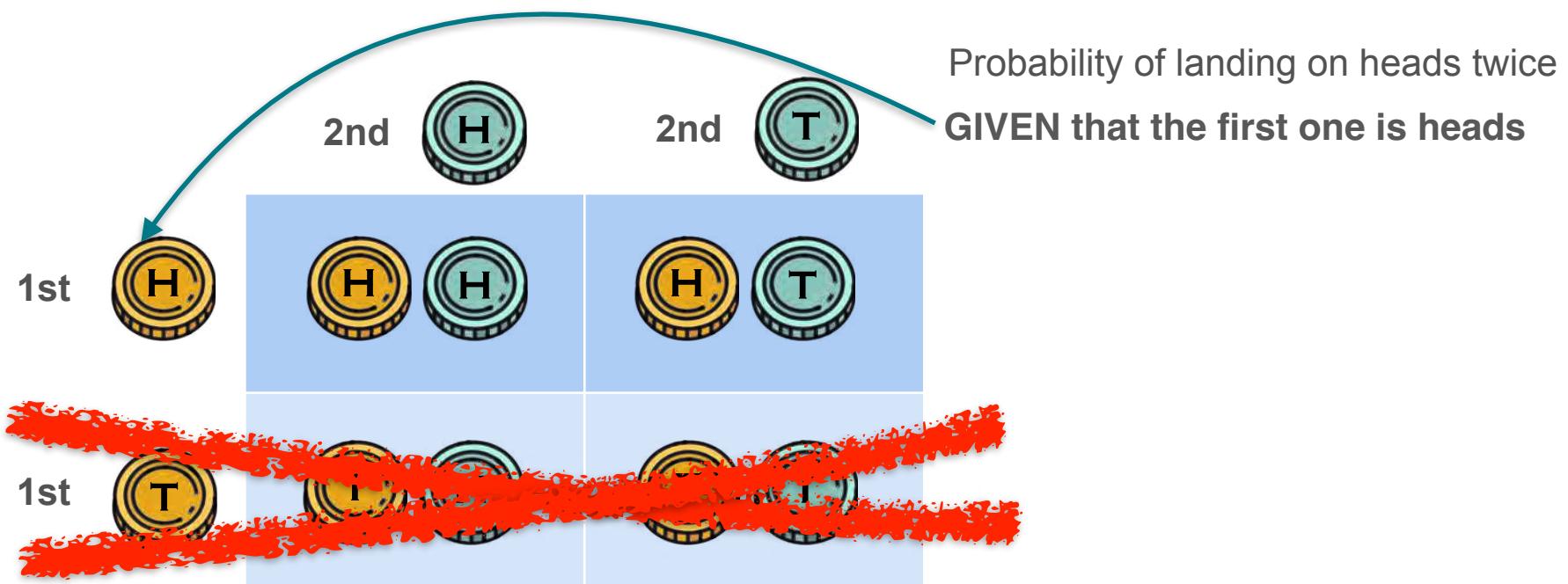
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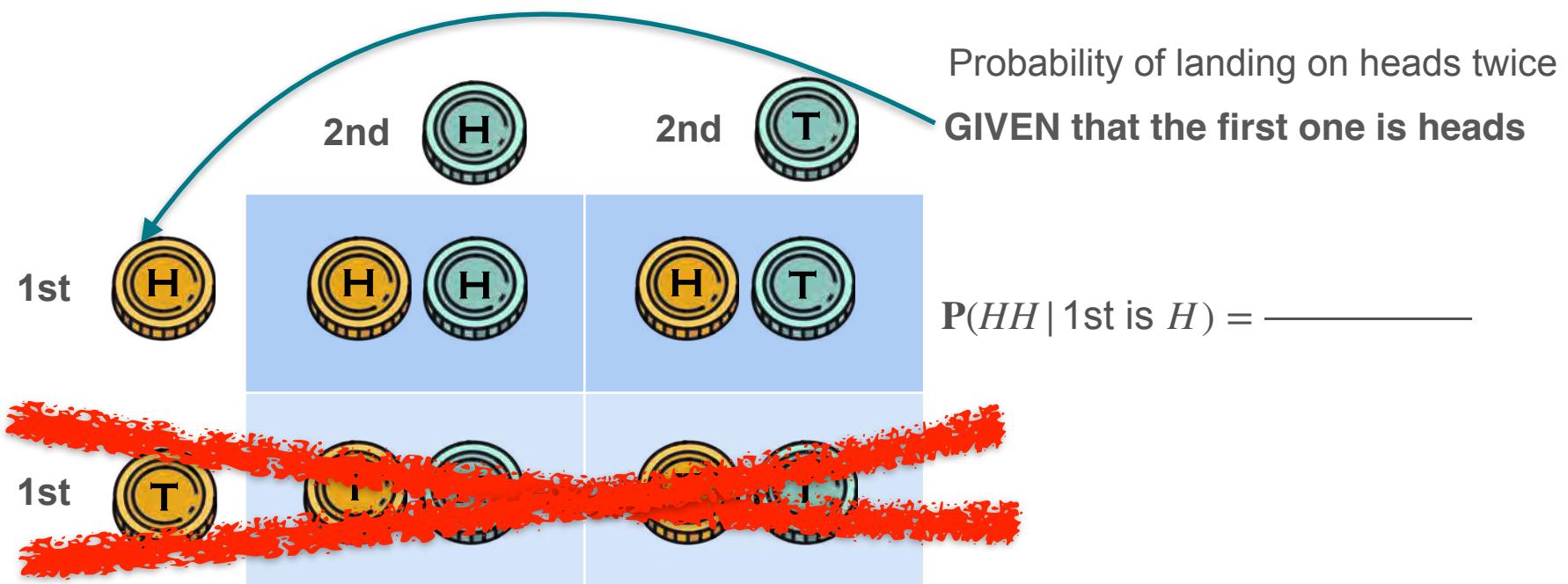
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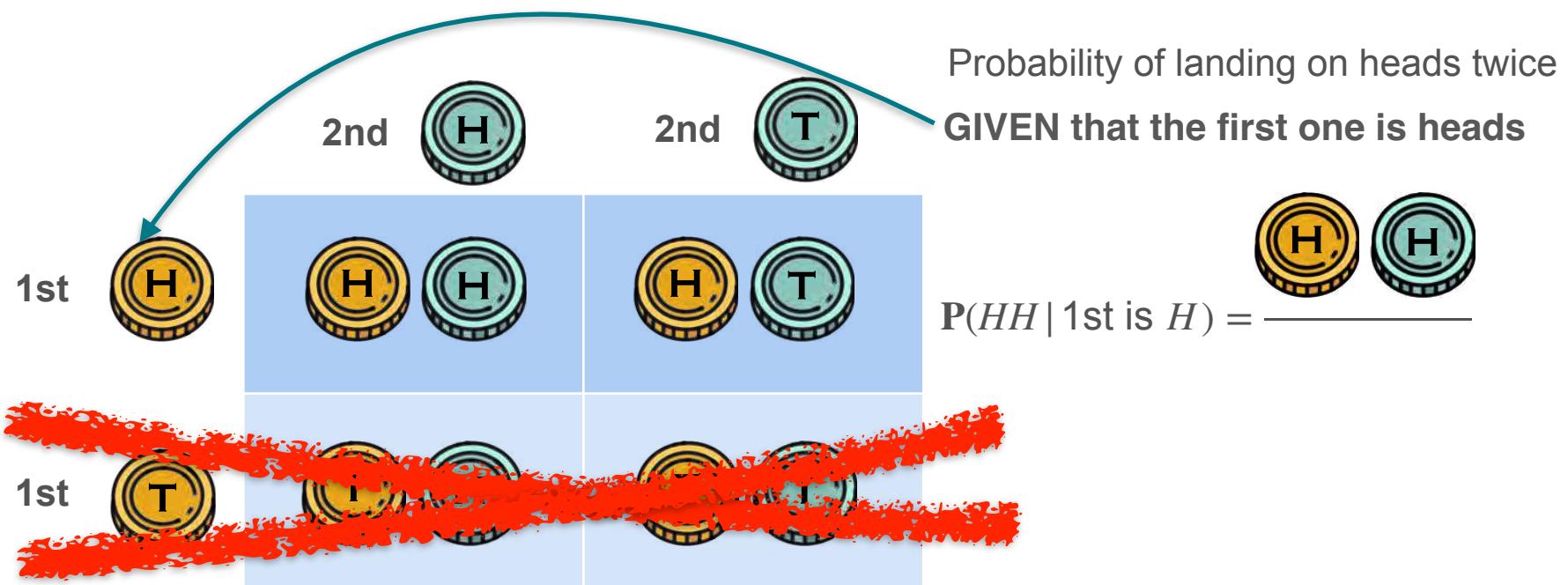
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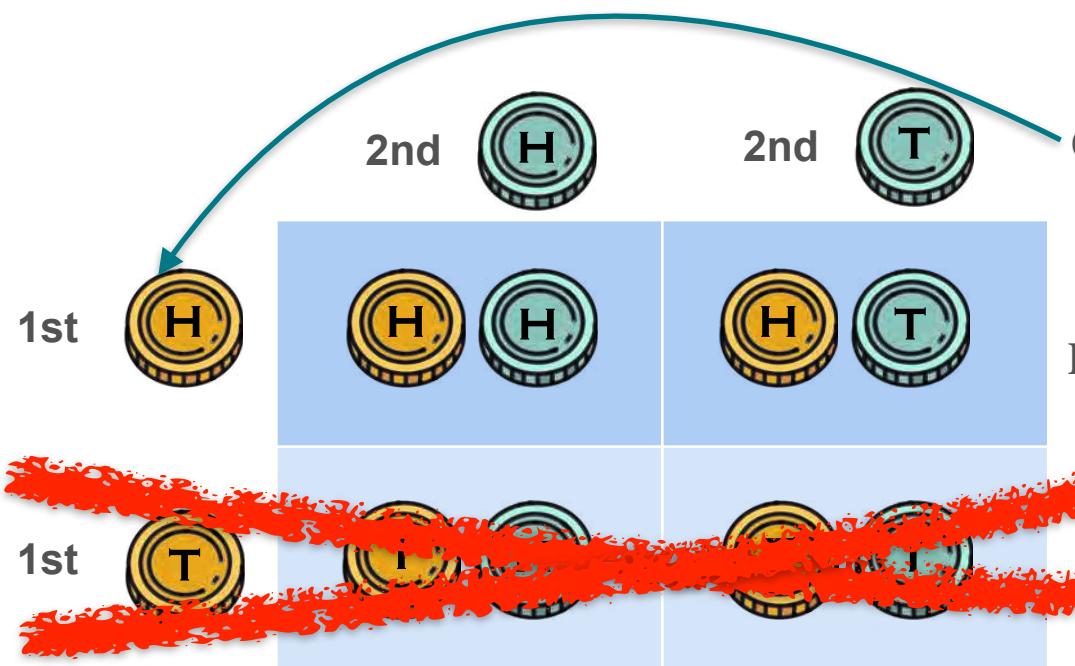
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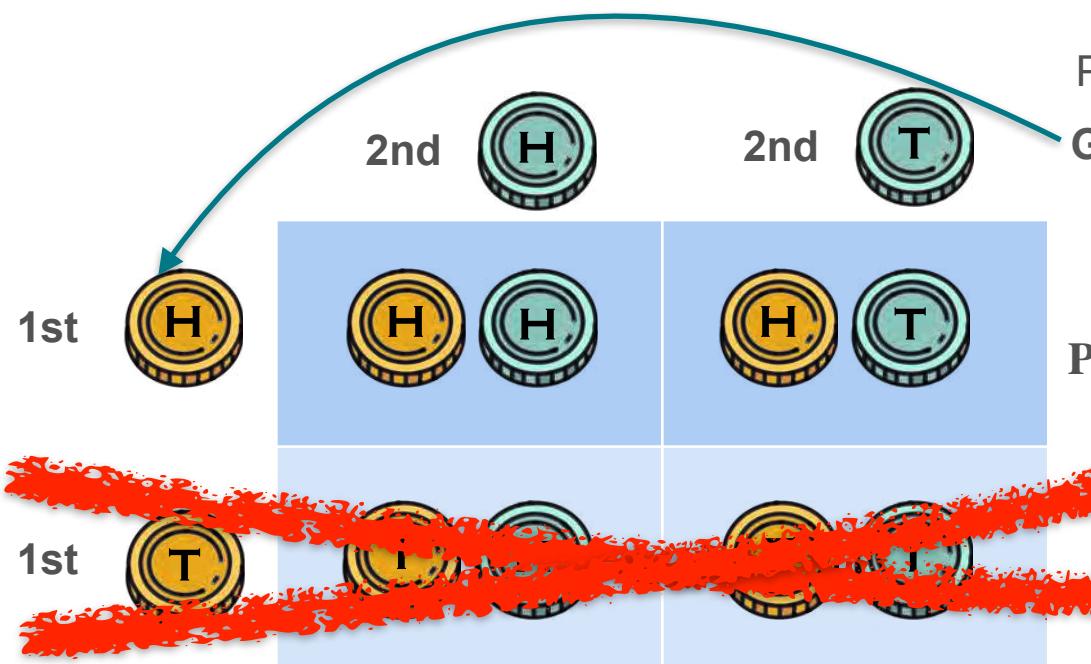


Probability of landing on heads twice  
GIVEN that the first one is heads

$$P(HH \mid \text{1st is } H) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

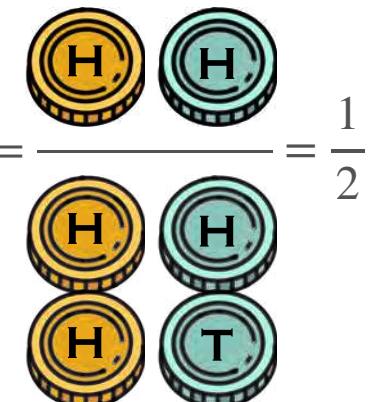
The diagram shows two rows of coins. The top row has two heads (H). The bottom row has one head (H) and one tail (T). A red brush stroke highlights the bottom row.

# Conditional Probability: Coin Example 1



Probability of landing on heads twice  
GIVEN that the first one is heads

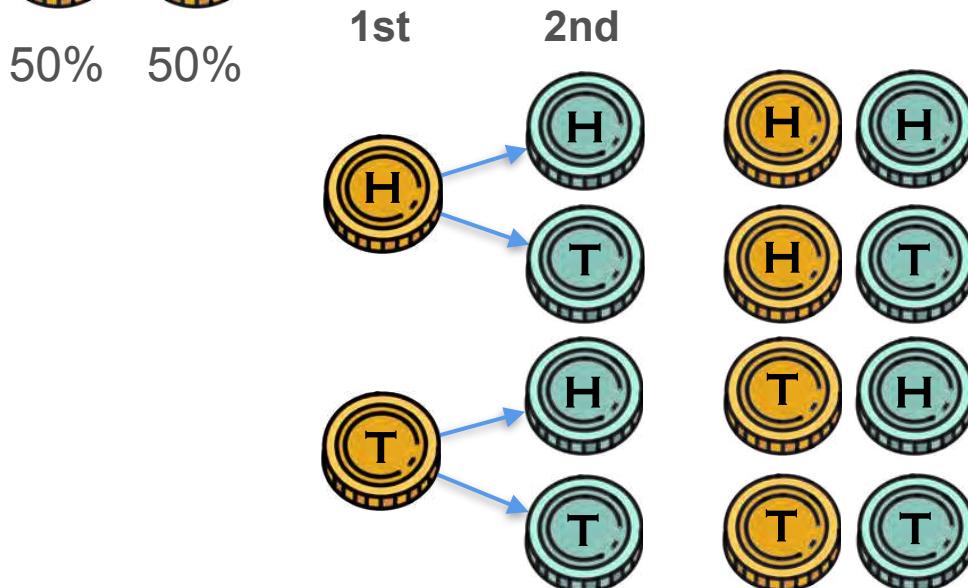
$$P(HH \mid \text{1st is } H) = \frac{1}{2}$$



# Conditional Probability: Coin Example 2



What is the probability of landing on heads twice?



$$P(HH) = \frac{1}{4}$$

The probability of getting heads twice is calculated as follows:

The total number of outcomes is 4 (HH, HT, TH, TT).

The number of favorable outcomes (HH) is 1.

Therefore,  $P(HH) = \frac{1}{4}$ .

A visual representation of the sample space shows a row of four coins where the first two are yellow (H) and the last two are blue (H), representing the outcome HH. Below it is a row of four coins where the first is yellow (H) and the next three are blue (T), representing the outcome HT. The next row shows TH, and the bottom row shows TT.

# Conditional Probability: Coin Example 2

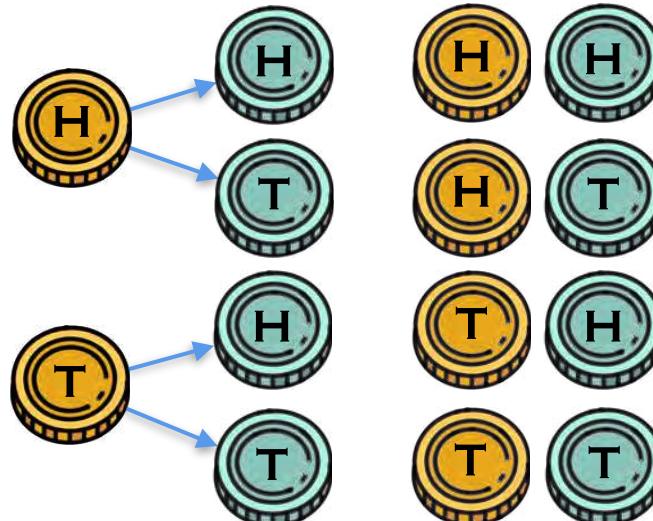


50% 50%

What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



# Conditional Probability: Coin Example 2

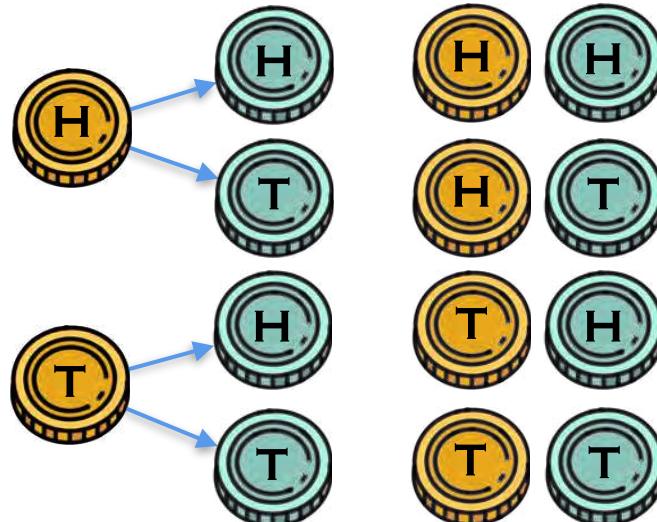


50% 50%

What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



# Conditional Probability: Coin Example 2

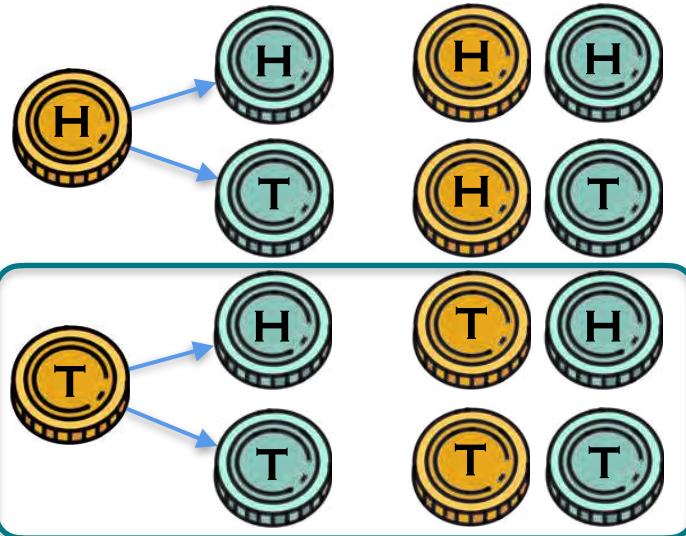


50% 50%

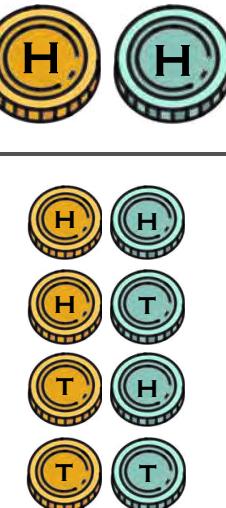
What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



# Conditional Probability: Coin Example 2

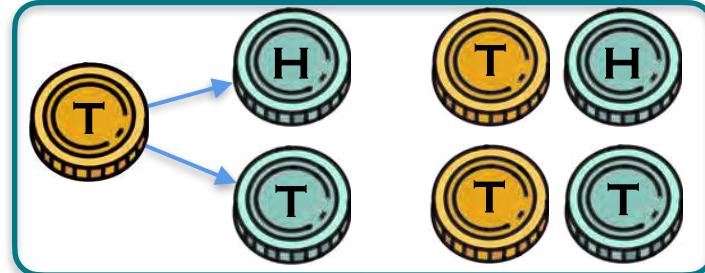


50% 50%

What is the probability of landing on heads twice?

1st            2nd

**GIVEN** that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



# Conditional Probability: Coin Example 2



50% 50%

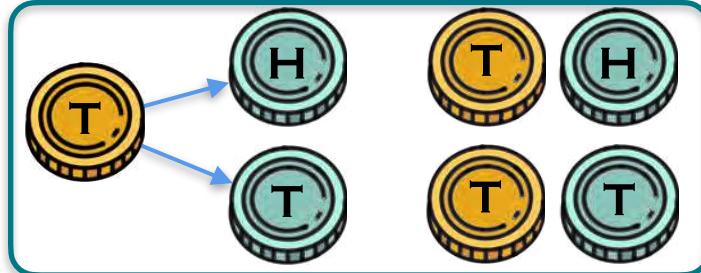
What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



# Conditional Probability: Coin Example 2



50% 50%

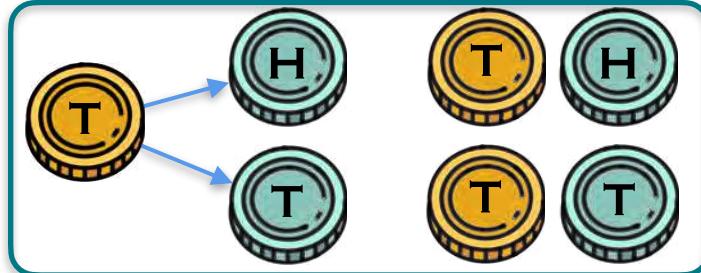
What is the probability of landing on heads twice?

1st            2nd

**GIVEN** that the first one is tails



$$P(HH) = \text{_____} = \frac{1}{4}$$



# Conditional Probability: Coin Example 2

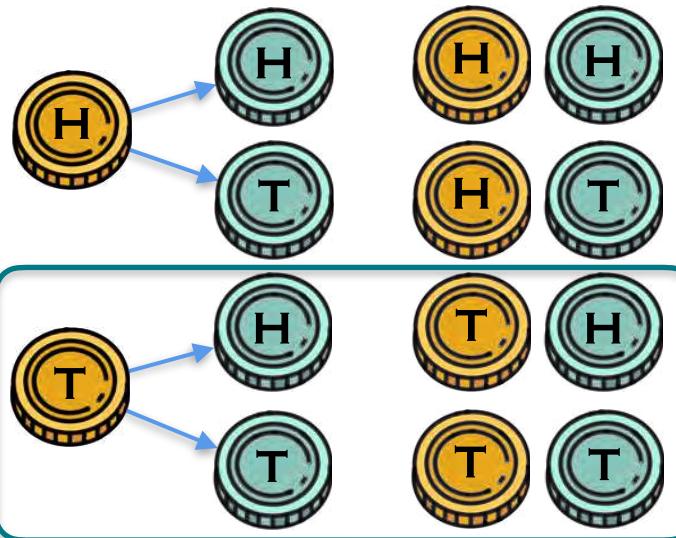


50% 50%

What is the probability of landing on heads twice?

1st      2nd

**GIVEN** that the first one is tails



$$P(HH \mid \text{1st is } T) = \frac{0}{4} = 0$$

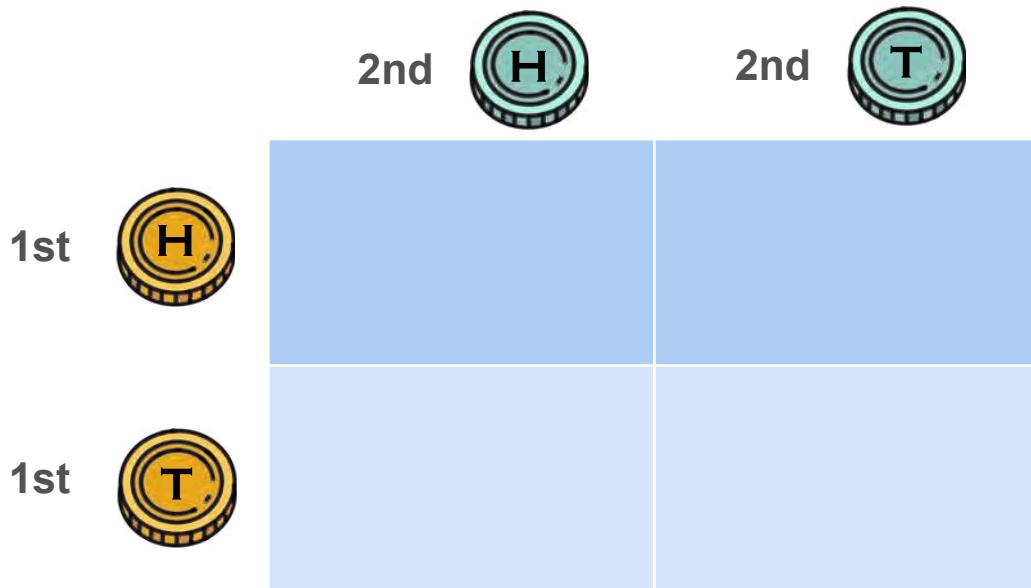


# Conditional Probability: Coin Example 2

# Conditional Probability: Coin Example 2



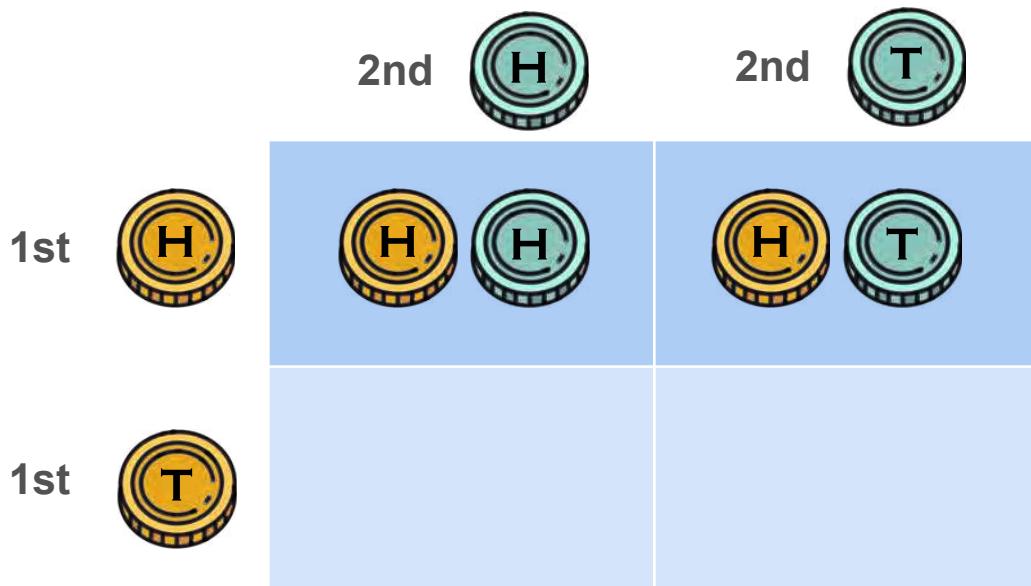
# Conditional Probability: Coin Example 2



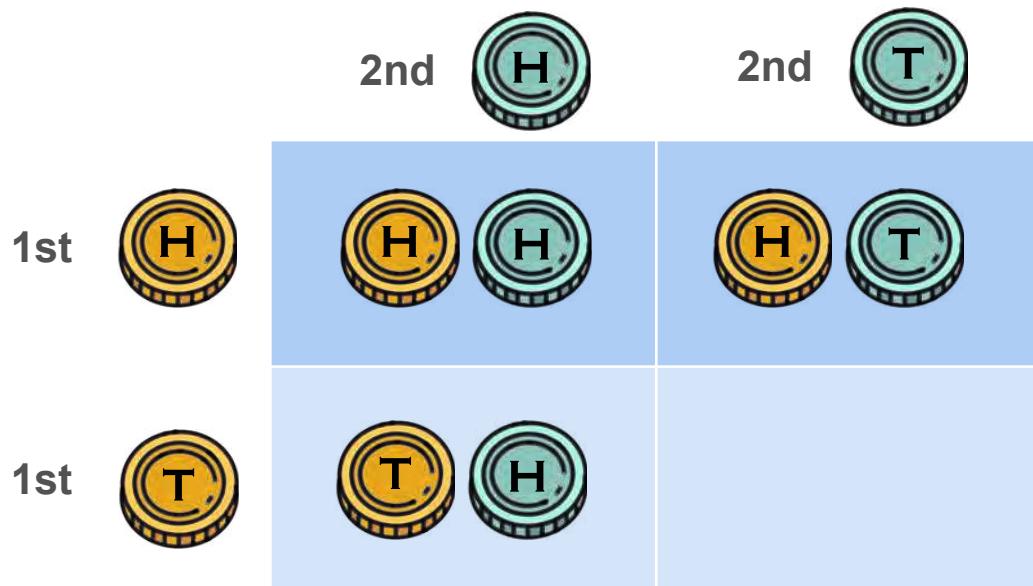
# Conditional Probability: Coin Example 2



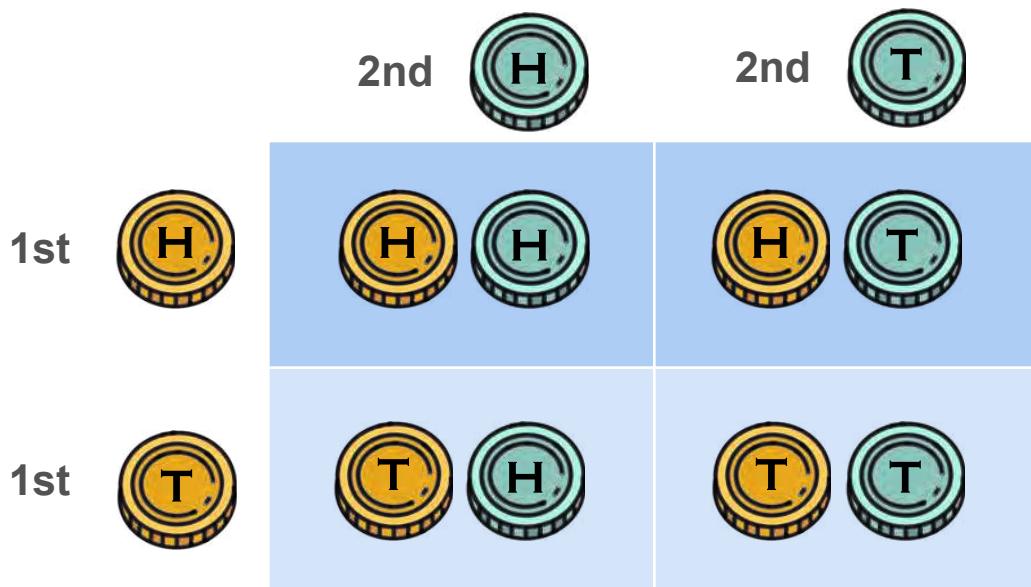
# Conditional Probability: Coin Example 2



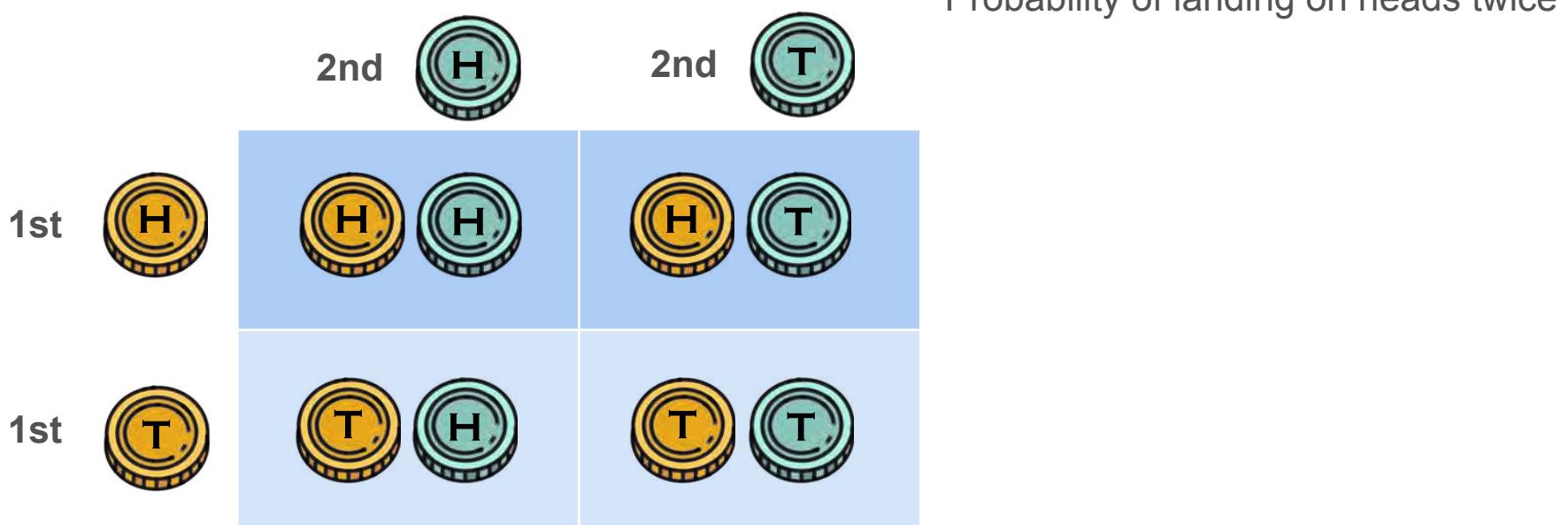
# Conditional Probability: Coin Example 2



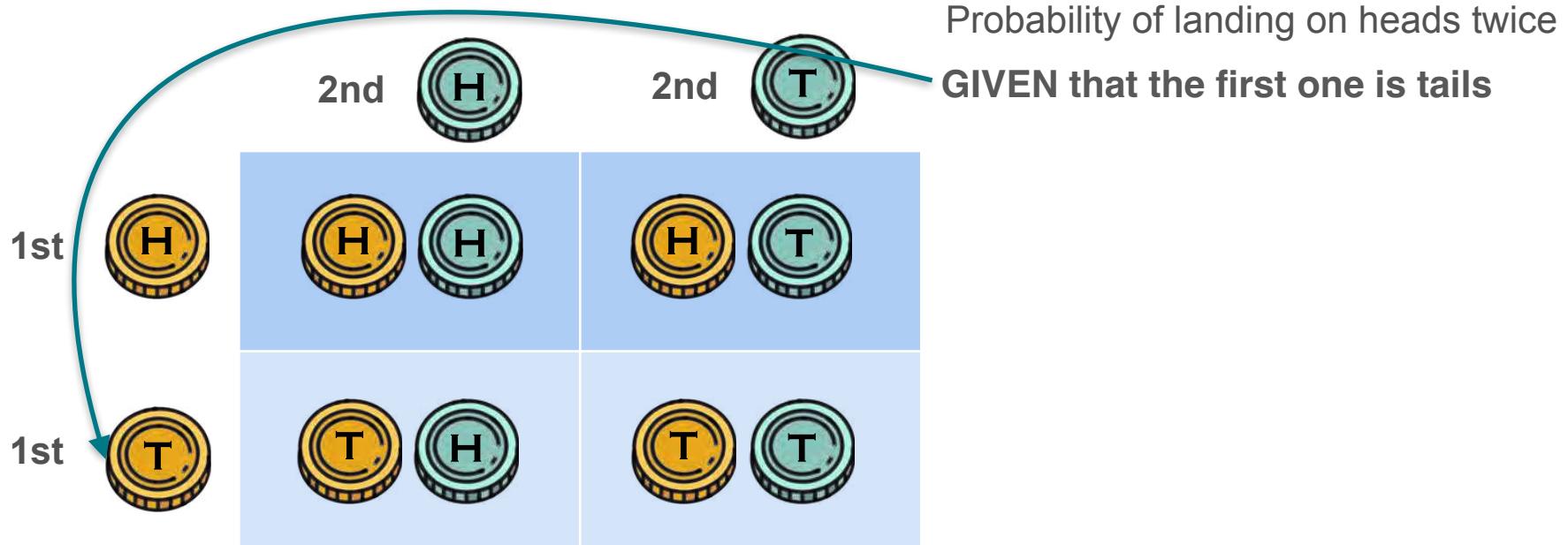
# Conditional Probability: Coin Example 2



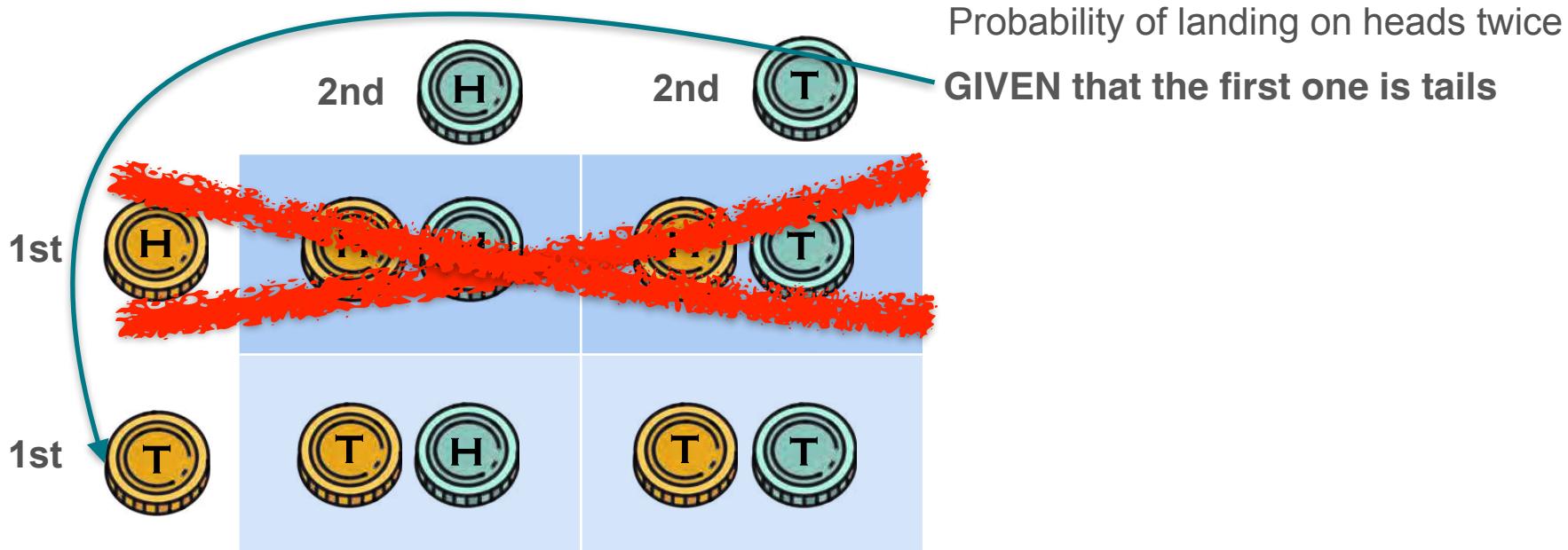
# Conditional Probability: Coin Example 2



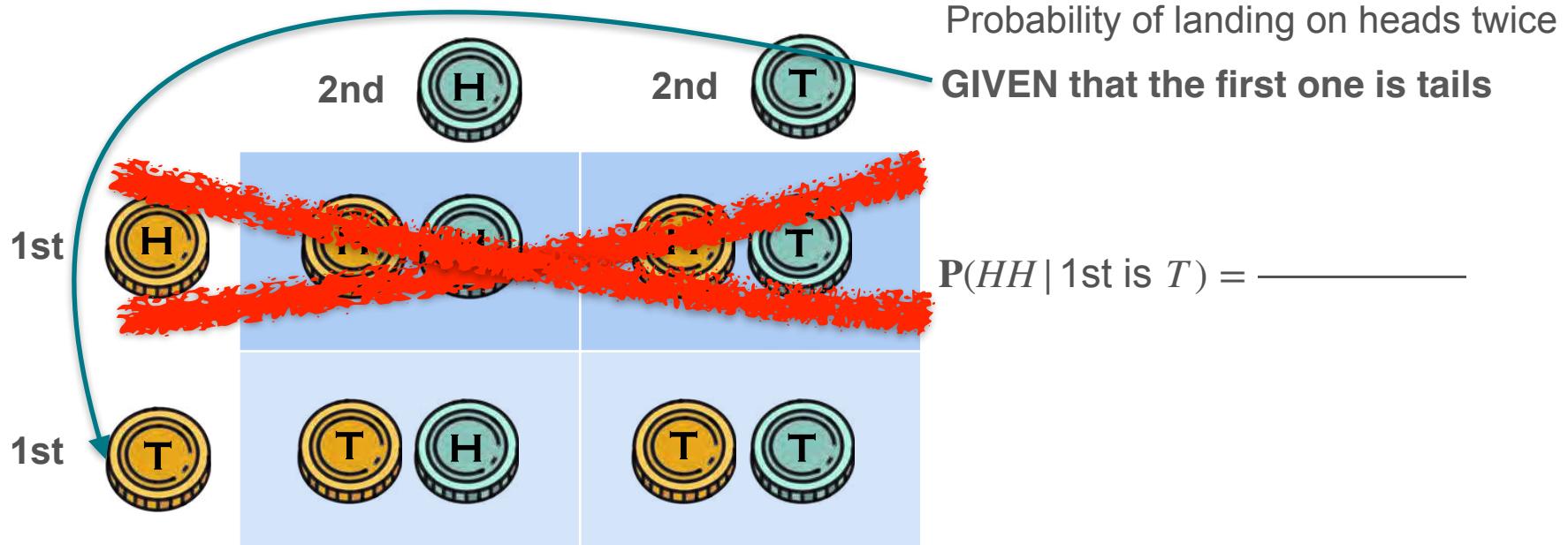
# Conditional Probability: Coin Example 2



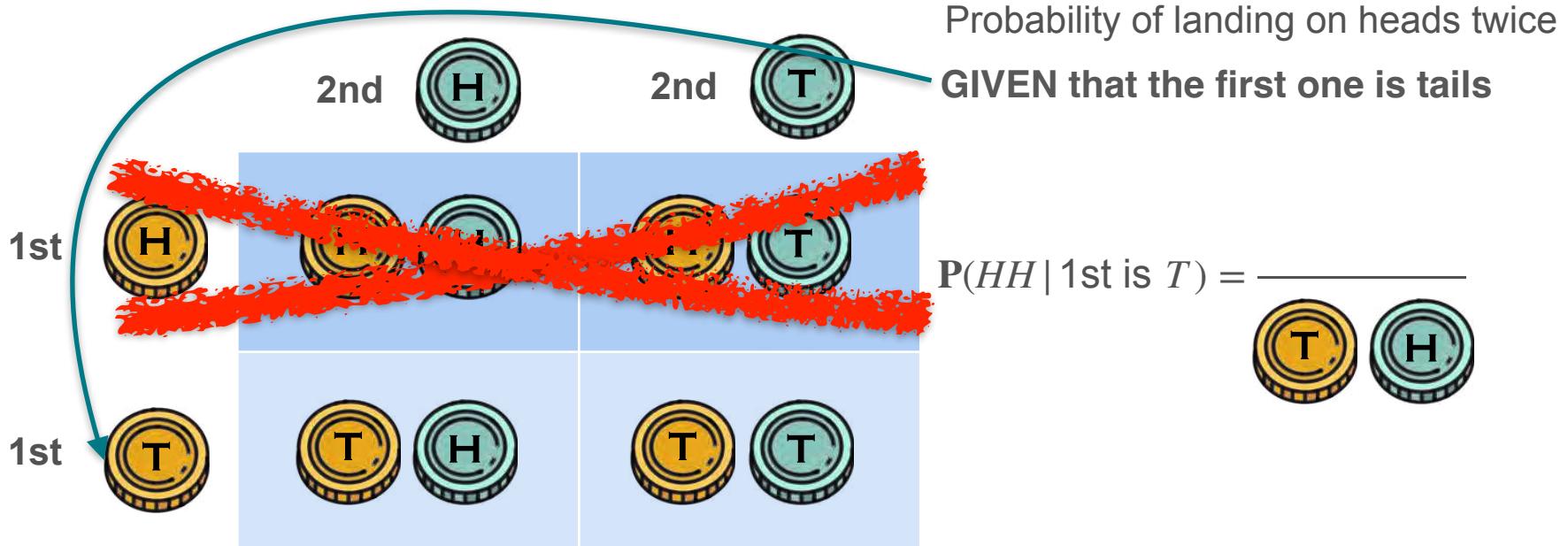
# Conditional Probability: Coin Example 2



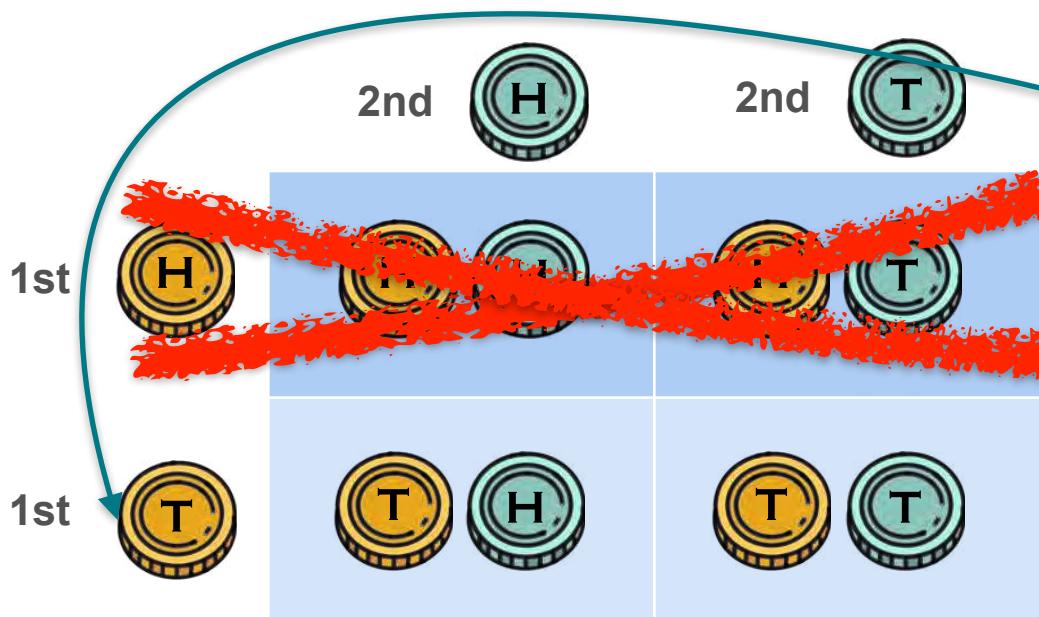
# Conditional Probability: Coin Example 2



# Conditional Probability: Coin Example 2

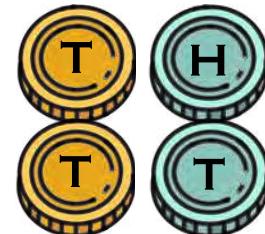


# Conditional Probability: Coin Example 2

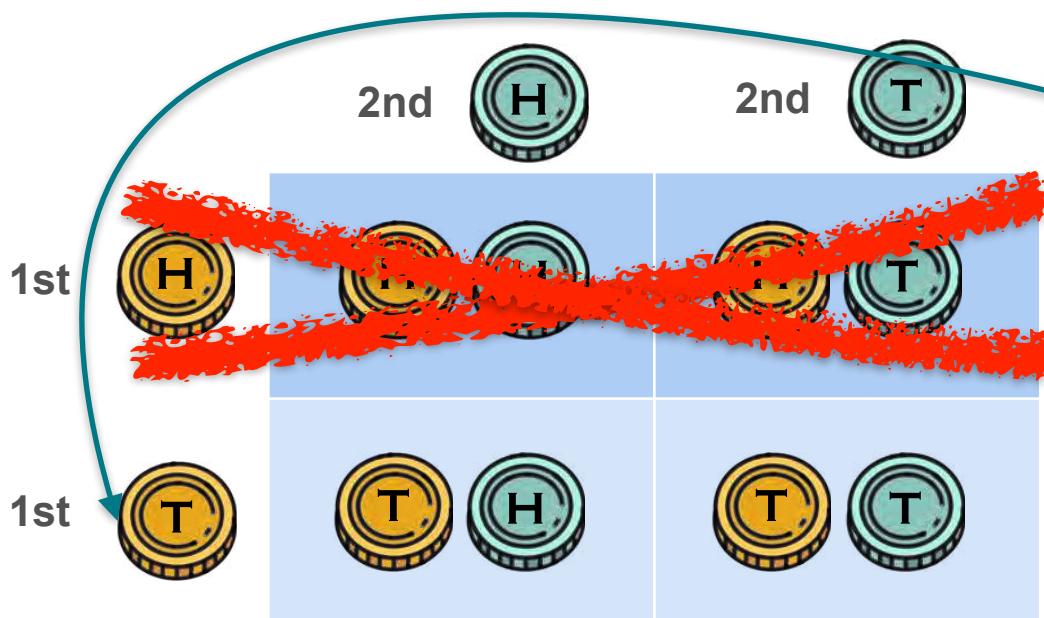


Probability of landing on heads twice  
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



# Conditional Probability: Coin Example 2

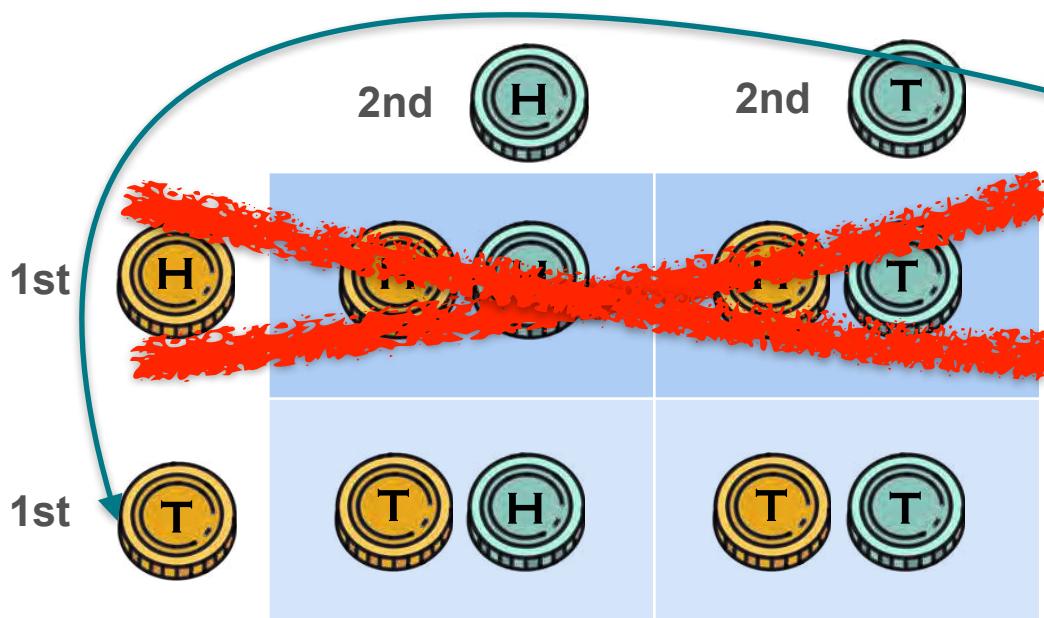


Probability of landing on heads twice  
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of HH outcomes}}{\text{Number of T outcomes}}$$



# Conditional Probability: Coin Example 2

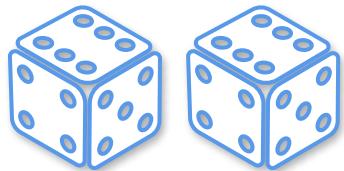


Probability of landing on heads twice  
GIVEN that the first one is tails

$$P(HH | \text{1st is } T) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = 0$$

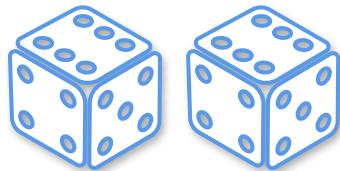


# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

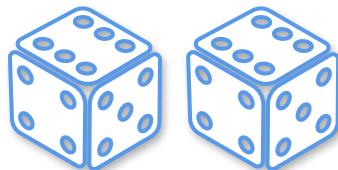
# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

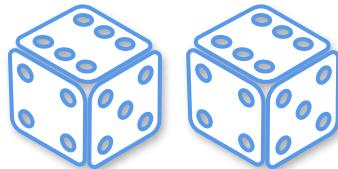
# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 1

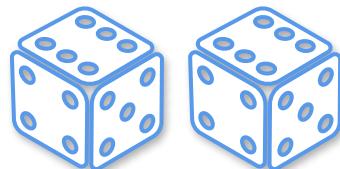


What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \underline{\hspace{10em}}$$

# Conditional Probability: Dice Example 1



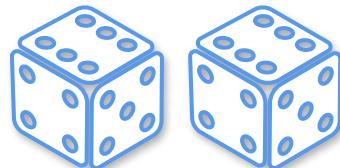
What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. Outcomes where the sum is 10 are highlighted with red boxes: (4,6), (5,5), (6,4), and (6,5). The outcome (5,5) is also highlighted with a red box.

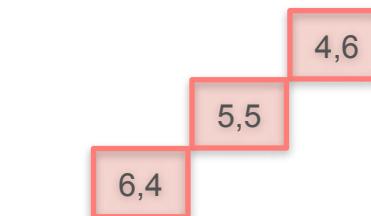
# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

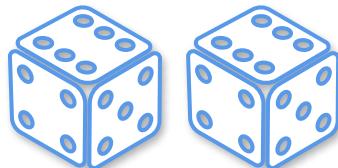
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

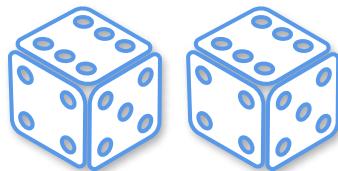
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. The outcomes are represented by small dice icons above each cell. The cells are colored based on their sum: light blue for sums 2 through 9, pink for sum 10, and light green for sums 11 and 12. The three pink cells representing a sum of 10 are highlighted with red boxes and labeled 4,6, 5,5, and 6,4.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

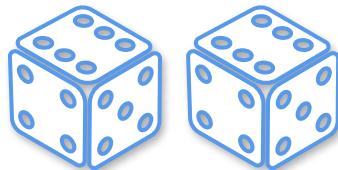
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{12} = \frac{1}{4}$$

The diagram shows a 6x6 grid of outcomes from two dice rolls. The outcomes are labeled with their respective sums. The outcomes that sum to 10 are highlighted with red boxes: (4,6), (5,5), and (6,4). There are 3 such outcomes.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

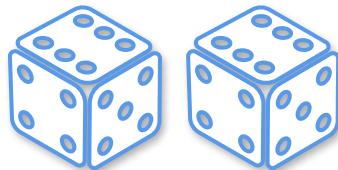
# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 1

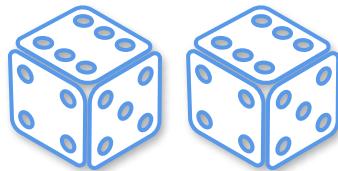


What is the probability that the sum is 10?

**GIVEN** that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 1



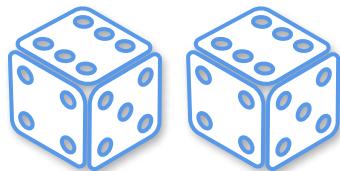
What is the probability that the sum is 10?

**GIVEN** that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

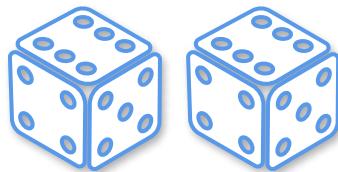
**GIVEN** that the first one is 6

A 6x6 grid of dice outcomes. The columns are labeled with the first die's value (1, 2, 3, 4, 5, 6) and the rows are labeled with the second die's value (1, 2, 3, 4, 5, 6). The outcome (6,4) is highlighted with a red border. The entire grid is surrounded by a light blue border. Above the grid, there is a 6x6 grid of small dice icons, each showing a different combination of faces.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

# Conditional Probability: Dice Example 1



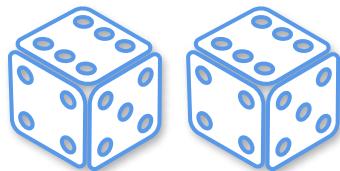
What is the probability that the sum is 10?

**GIVEN** that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1						
2,2						
2,3						
2,4						
2,5						
2,6						
3,1						
3,2						
3,3						
3,4						
3,5						
3,6						
4,1						
4,2						
4,3						
4,4						
4,5						
4,6						
5,1						
5,2						
5,3						
5,4						
5,5						
5,6						
6,1						
6,2						
6,3						
6,4						
6,5						
6,6						

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

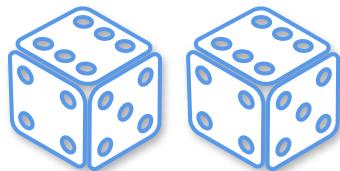
**GIVEN** that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

**GIVEN** that the first one is 6

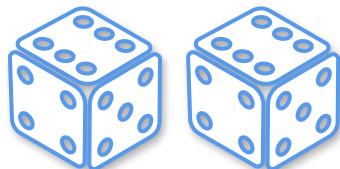
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

6,4

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

# Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

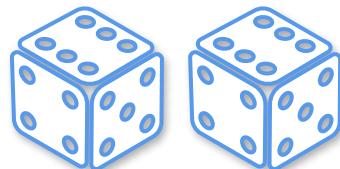
**GIVEN** that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{1}{6}$$

6,4      6,1 6,2 6,3 6,4 6,5 6,6

# Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

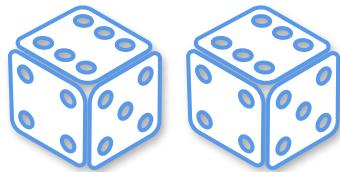
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

A diagram showing a staircase path through a 6x6 grid of dice rolls. The steps are red-bordered boxes. The path starts at (1,1), goes up-right to (2,2), up to (3,3), right to (4,4), up to (5,5), and right to (6,6). The final outcome (6,6) is also highlighted with a red border.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

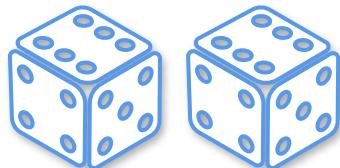
# Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 2

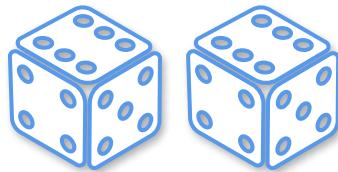


What is the probability that the sum is 10?

**GIVEN** that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 2



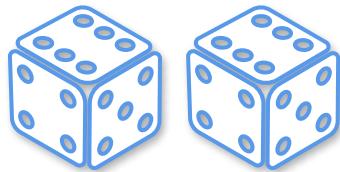
What is the probability that the sum is 10?

**GIVEN** that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

# Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

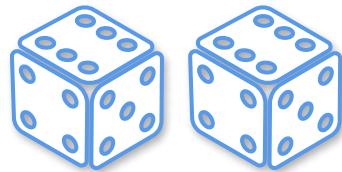
**GIVEN** that the first one is 1

A 6x6 grid representing all possible outcomes of two dice rolls. The columns are labeled with the first die's value (1, 2, 3, 4, 5, 6) and the rows are labeled with the second die's value (1, 2, 3, 4, 5, 6). The first column contains icons of dice. The first row contains the numbers 1, 2, 3, 4, 5, 6. The grid cells contain pairs of numbers representing the outcome of the two dice. The cell at the intersection of the first column and the first row (1,1) contains the number 1,1. The cell at the intersection of the first column and the second row (1,2) contains the number 1,2. The cell at the intersection of the second column and the first row (2,1) contains the number 2,1. The cell at the intersection of the second column and the second row (2,2) contains the number 2,2. The cell at the intersection of the third column and the first row (3,1) contains the number 3,1. The cell at the intersection of the third column and the second row (3,2) contains the number 3,2. The cell at the intersection of the fourth column and the first row (4,1) contains the number 4,1. The cell at the intersection of the fourth column and the second row (4,2) contains the number 4,2. The cell at the intersection of the fifth column and the first row (5,1) contains the number 5,1. Thecell at the intersection of the fifth column and the second row (5,2) contains the number 5,2. The cell at the intersection of the sixth column and the first row (6,1) contains the number 6,1. The cell at the intersection of the sixth column and the second row (6,2) contains the number 6,2. The cell at the intersection of the first column and the third row (1,3) contains the number 1,3. The cell at the intersection of the first column and the fourth row (1,4) contains the number 1,4. The cell at the intersection of the first column and the fifth row (1,5) contains the number 1,5. The cell at the intersection of the first column and the sixth row (1,6) contains the number 1,6. The cell at the intersection of the second column and the third row (2,3) contains the number 2,3. Thecell at the intersection of the second column and the fourth row (2,4) contains the number 2,4. Thecell at the intersection of the second column and the fifth row (2,5) contains the number 2,5. Thecell at the intersection of the second column and the sixth row (2,6) contains the number 2,6. Thecell at the intersection of the third column and the third row (3,3) contains the number 3,3. Thecell at the intersection of the third column and the fourth row (3,4) contains the number 3,4. Thecell at the intersection of the third column and the fifth row (3,5) contains the number 3,5. Thecell at the intersection of the third column and the sixth row (3,6) contains the number 3,6. Thecell at the intersection of the fourth column and the third row (4,3) contains the number 4,3. Thecell at the intersection of the fourth column and the fourth row (4,4) contains the number 4,4. Thecell at the intersection of the fourth column and the fifth row (4,5) contains the number 4,5. Thecell at the intersection of the fourth column and the sixth row (4,6) contains the number 4,6. Thecell at the intersection of the fifth column and the third row (5,3) contains the number 5,3. Thecell at the intersection of the fifth column and the fourth row (5,4) contains the number 5,4. Thecell at the intersection of the fifth column and the fifth row (5,5) contains the number 5,5. Thecell at the intersection of the fifth column and the sixth row (5,6) contains the number 5,6. Thecell at the intersection of the sixth column and the third row (6,3) contains the number 6,3. Thecell at the intersection of the sixth column and the fourth row (6,4) contains the number 6,4. Thecell at the intersection of the sixth column and the fifth row (6,5) contains the number 6,5. Thecell at the intersection of the sixth column and the sixth row (6,6) contains the number 6,6. The cell (1,1) is highlighted with a red border.

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

# Conditional Probability: Dice Example 2



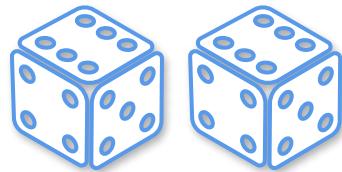
What is the probability that the sum is 10?

**GIVEN** that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

# Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

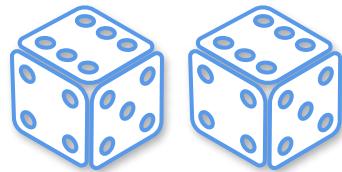
**GIVEN** that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

1,1	1,2	1,3	1,4	1,5	1,6
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# Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

**GIVEN** that the first one is 1

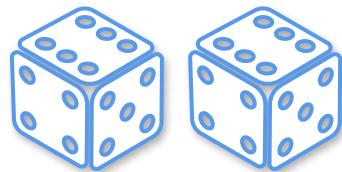
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,2	1,3	1,4	1,5	1,6

1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,2	1,3	1,4	1,5	1,6

# Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

**GIVEN** that the first one is 1

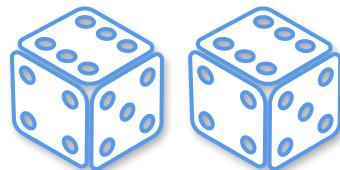
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

1,1	1,2	1,3	1,4	1,5	1,6
0	1,2	1,3	1,4	1,5	1,6

1,1	1,2	1,3	1,4	1,5	1,6
0	1,2	1,3	1,4	1,5	1,6

# Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

**GIVEN** that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

The total number of outcomes is 6 (the last row).

The number of favorable outcomes is 0 (the last row).

$$= 0$$

# Product Rule (for Independent Events)

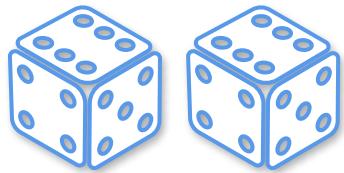
$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

# Product Rule (for Independent Events)

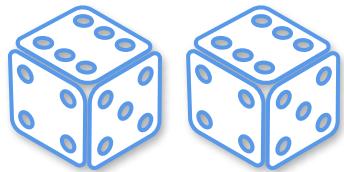
When A and B independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

# Conditional Probability: Dice Example 3

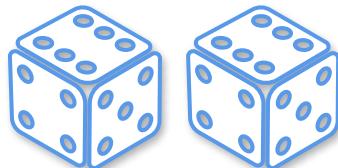


# Conditional Probability: Dice Example 3



What is the probability that  
the first is 6 **AND** the sum = 10?

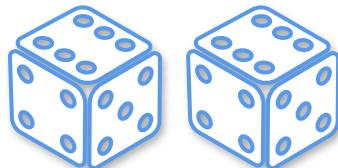
# Conditional Probability: Dice Example 3



What is the probability that  
the first is 6 **AND** the sum = 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

# Conditional Probability: Dice Example 3

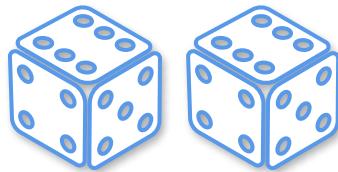


What is the probability that  
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) = \underline{\hspace{10cm}}$$

# Conditional Probability: Dice Example 3



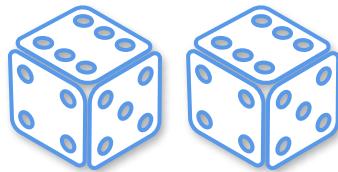
What is the probability that  
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) =$$

6,4

# Conditional Probability: Dice Example 3



What is the probability that the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) =$$

6,4

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{1}{36}$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1						
1,2						
1,3						
1,4						
1,5						
1,6						
2,1	2,1	2,2	2,3	2,4	2,5	2,6
2,2						
2,3						
2,4						
2,5						
2,6						
3,1	3,1	3,2	3,3	3,4	3,5	3,6
3,2						
3,3						
3,4						
3,5						
3,6						
4,1	4,1	4,2	4,3	4,4	4,5	4,6
4,2						
4,3						
4,4						
4,5						
4,6						
5,1	5,1	5,2	5,3	5,4	5,5	5,6
5,2						
5,3						
5,4						
5,5						
5,6						
6,1	6,1	6,2	6,3	6,4	6,5	6,6
6,2						
6,3						
6,4						
6,5						
6,6						

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1						

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{6}{36}$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
<hr/>					
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4

$$\frac{1}{6}$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{1st is } 6 \cap \text{sum} = 10) =$$

$$P(\text{1st is } 6)$$

$$P(\text{sum} = 10 | \text{1st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4
6,1
6,2
6,3
6,4
6,5
6,6

$$\frac{1}{6}$$

$$\frac{1}{6}$$

# Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(1\text{st is } 6 \cap \text{sum} = 10) =$$

$$P(1\text{st is } 6)$$

$$\bullet P(\text{sum} = 10 | 1\text{st } 6)$$

6,1	6,2	6,3	6,4	6,5	6,6
-----	-----	-----	-----	-----	-----

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

6,4
6,1    6,2    6,3    6,4    6,5    6,6

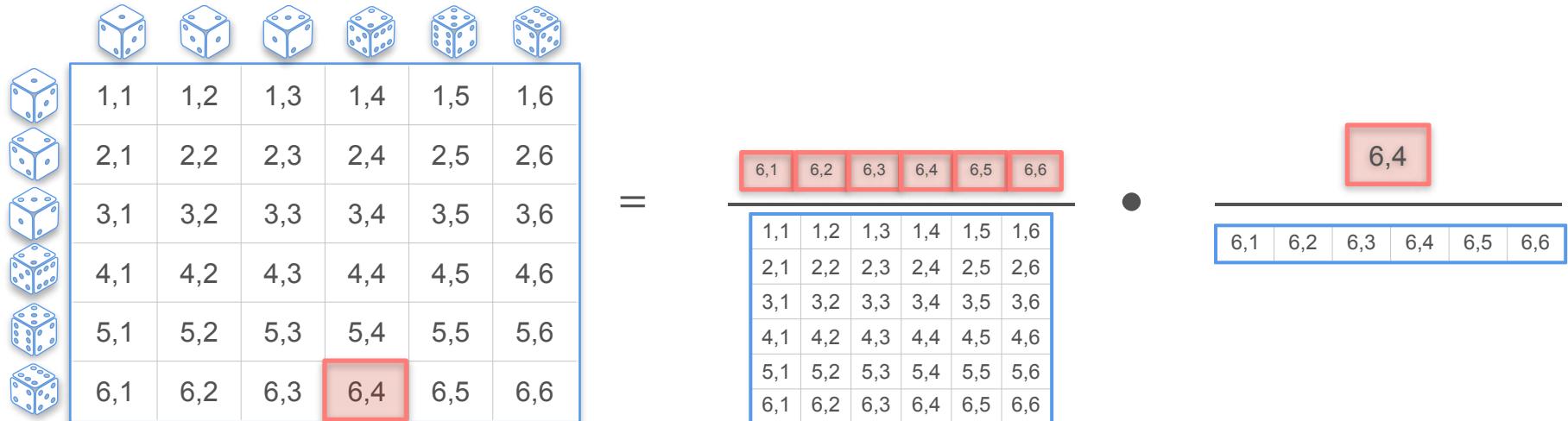
$$\frac{1}{6}$$

•

$$\frac{1}{6}$$

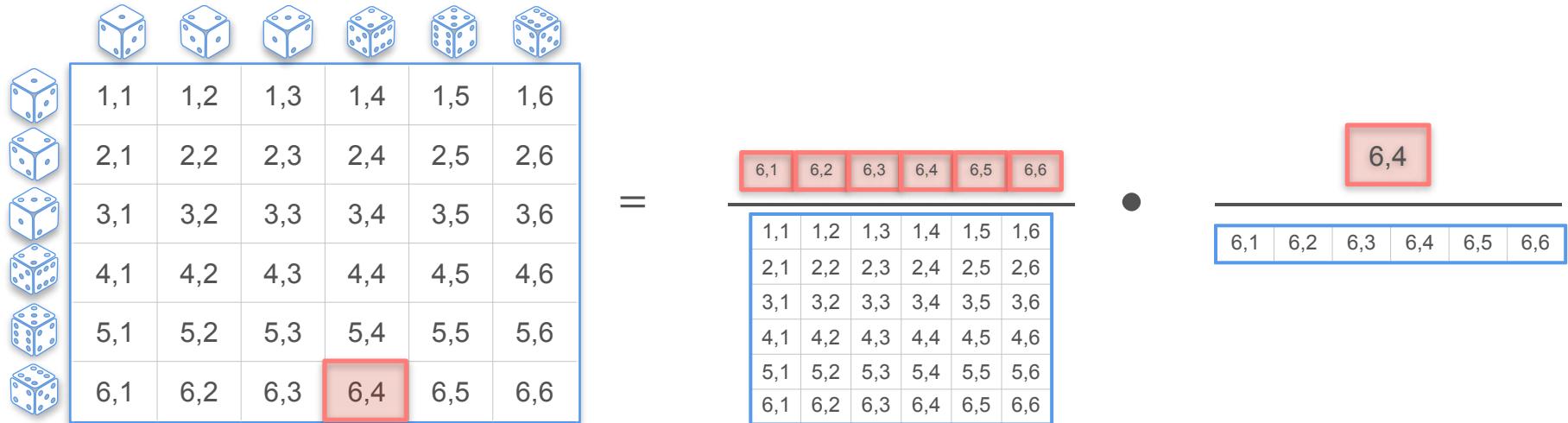
$$= \frac{1}{36}$$

# Conditional Probability: Dice Example 3



$$P(1\text{st is } 6 \cap \text{sum} = 10) = P(1\text{st is } 6) \bullet P(\text{sum} = 10 | 1\text{st } 6)$$

# Conditional Probability: Dice Example 3



$$P(A \cap B) = P(A) \cdot P(B | A)$$

# The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B | A)$$

# The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B | A)$$

When independent

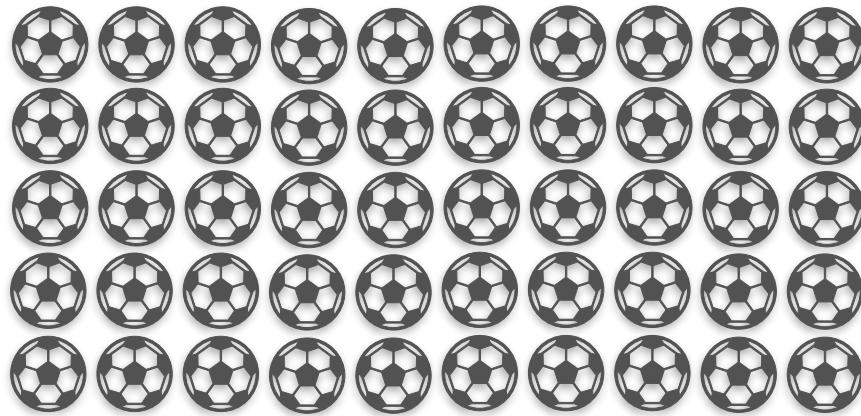
$$\mathbf{P}(B | A) = \mathbf{P}(B)$$

# Quiz 1

# Quiz 1

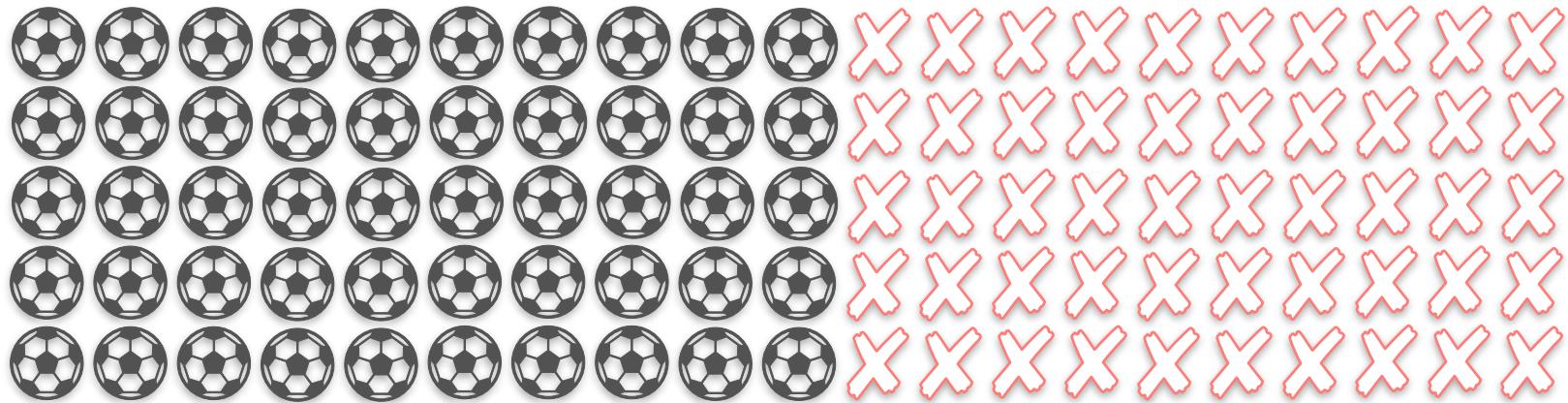
**100 kids**

# Quiz 1



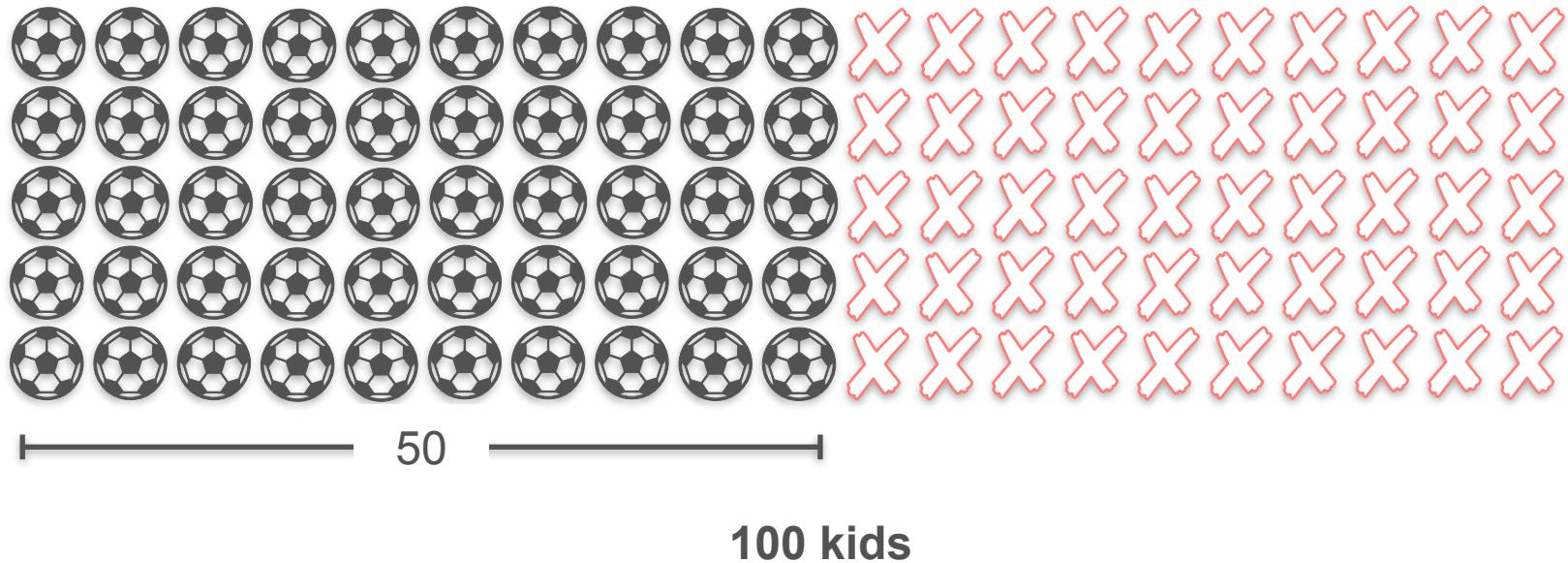
**100 kids**

# Quiz 1

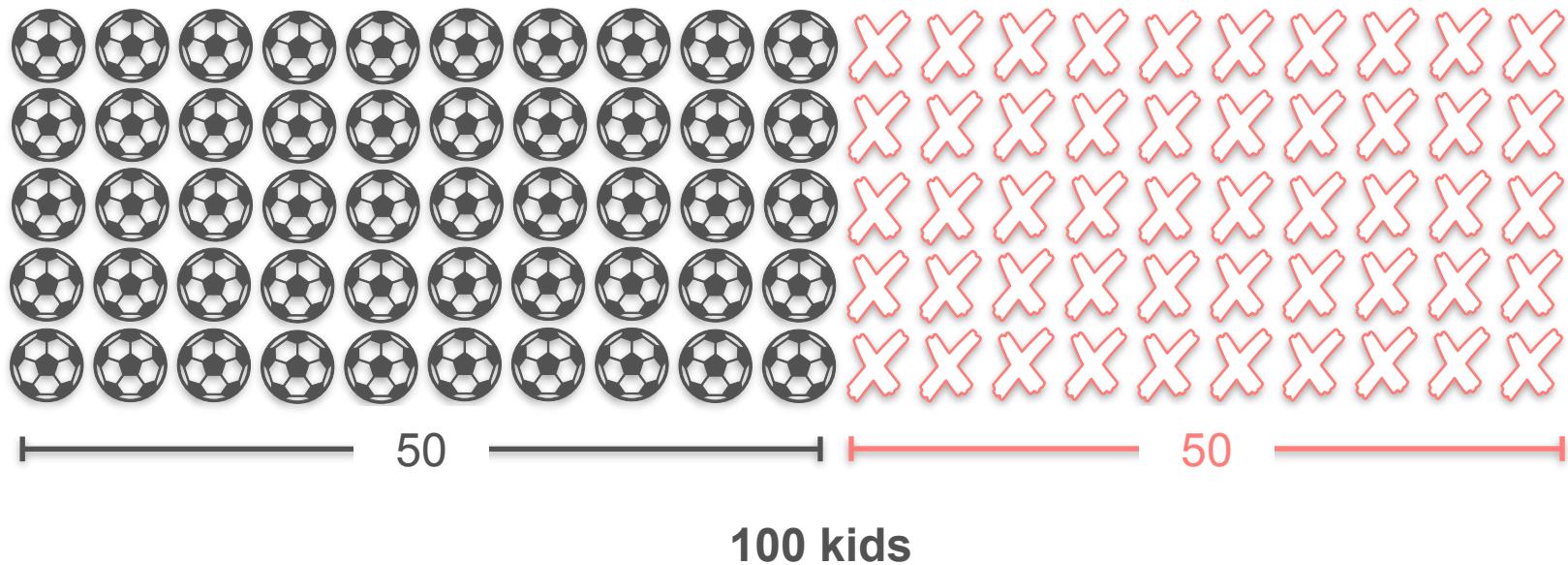


**100 kids**

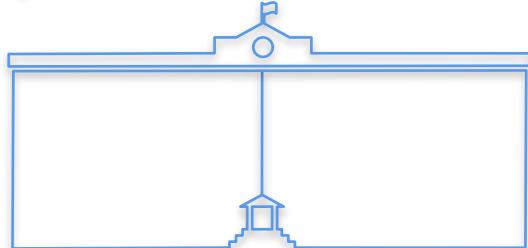
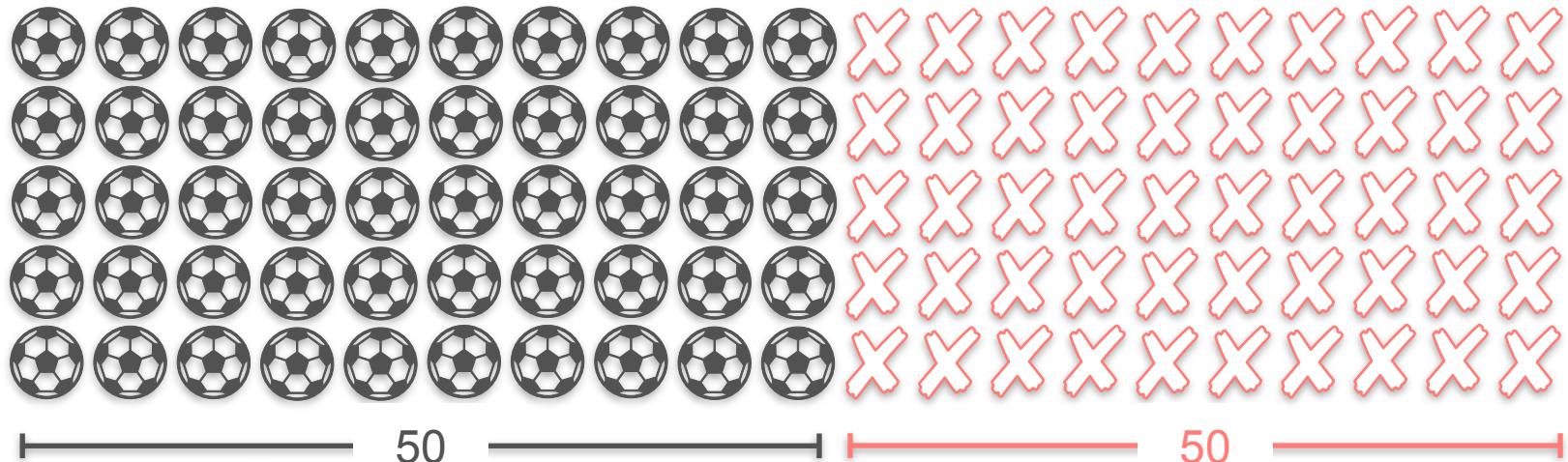
# Quiz 1



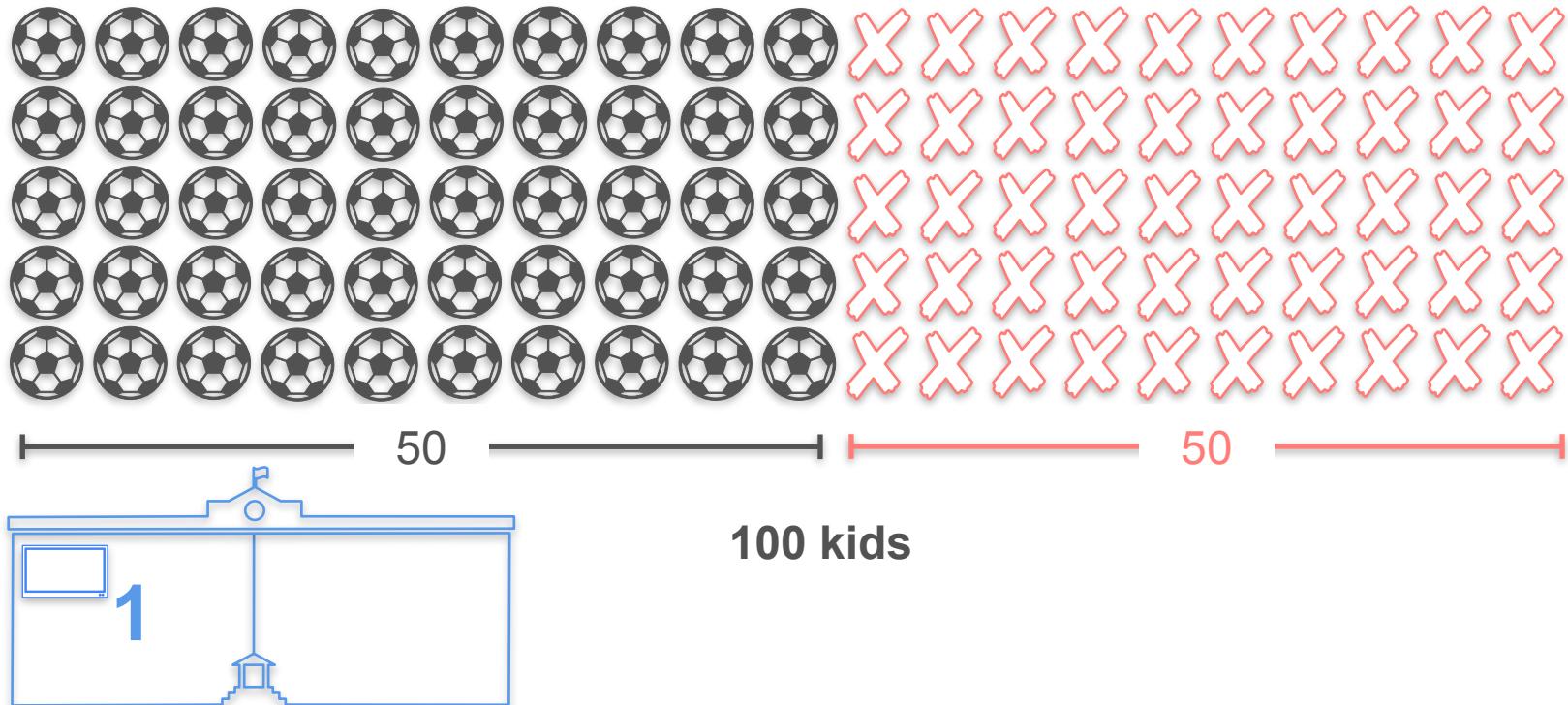
# Quiz 1



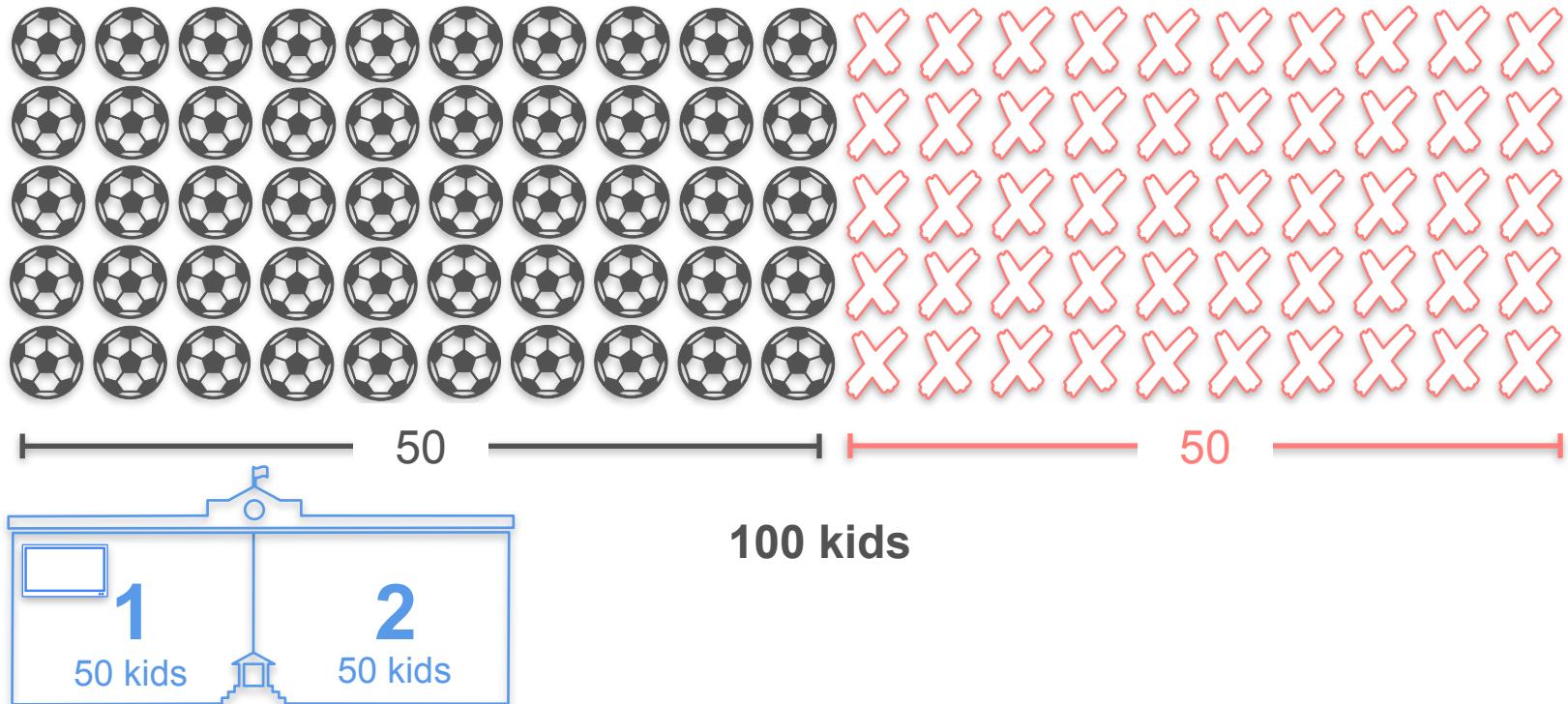
# Quiz 1



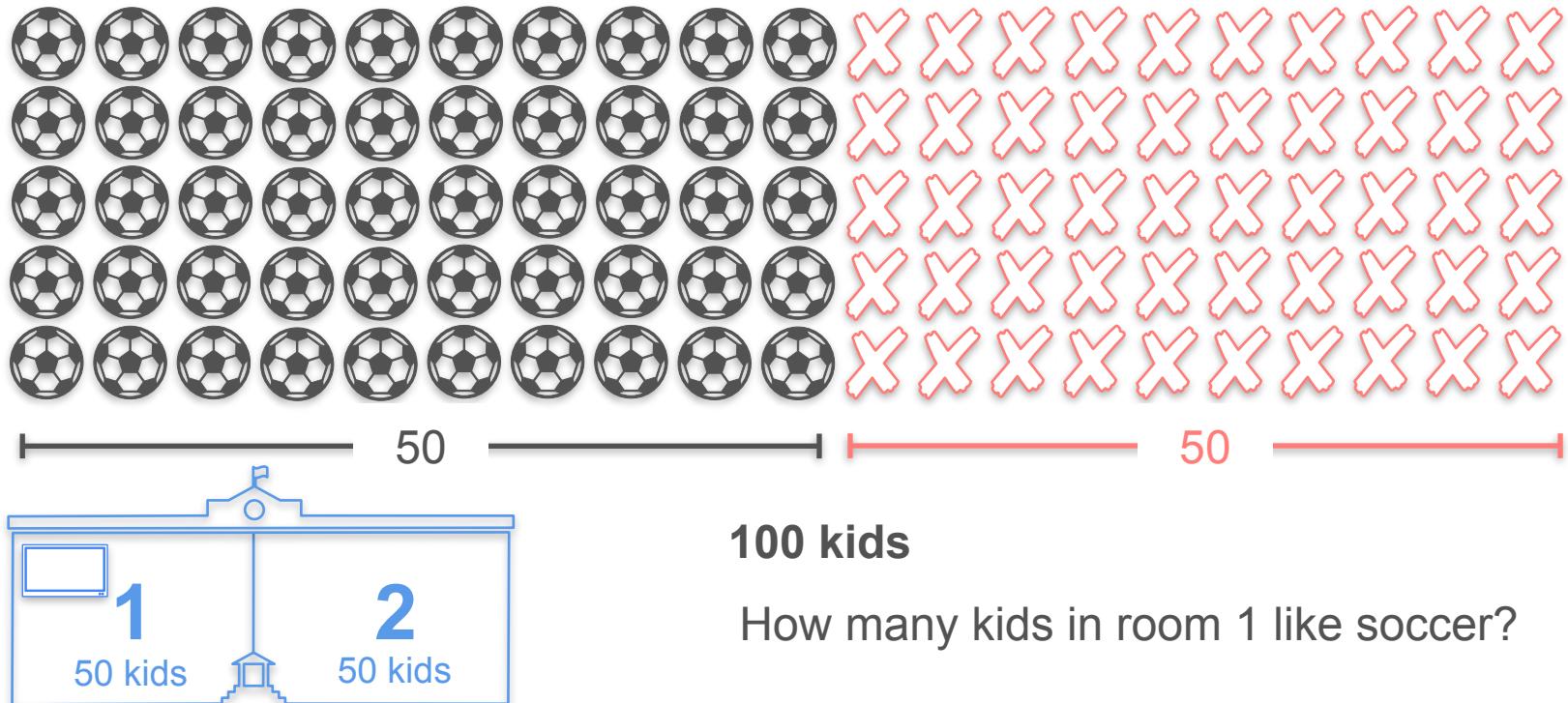
# Quiz 1



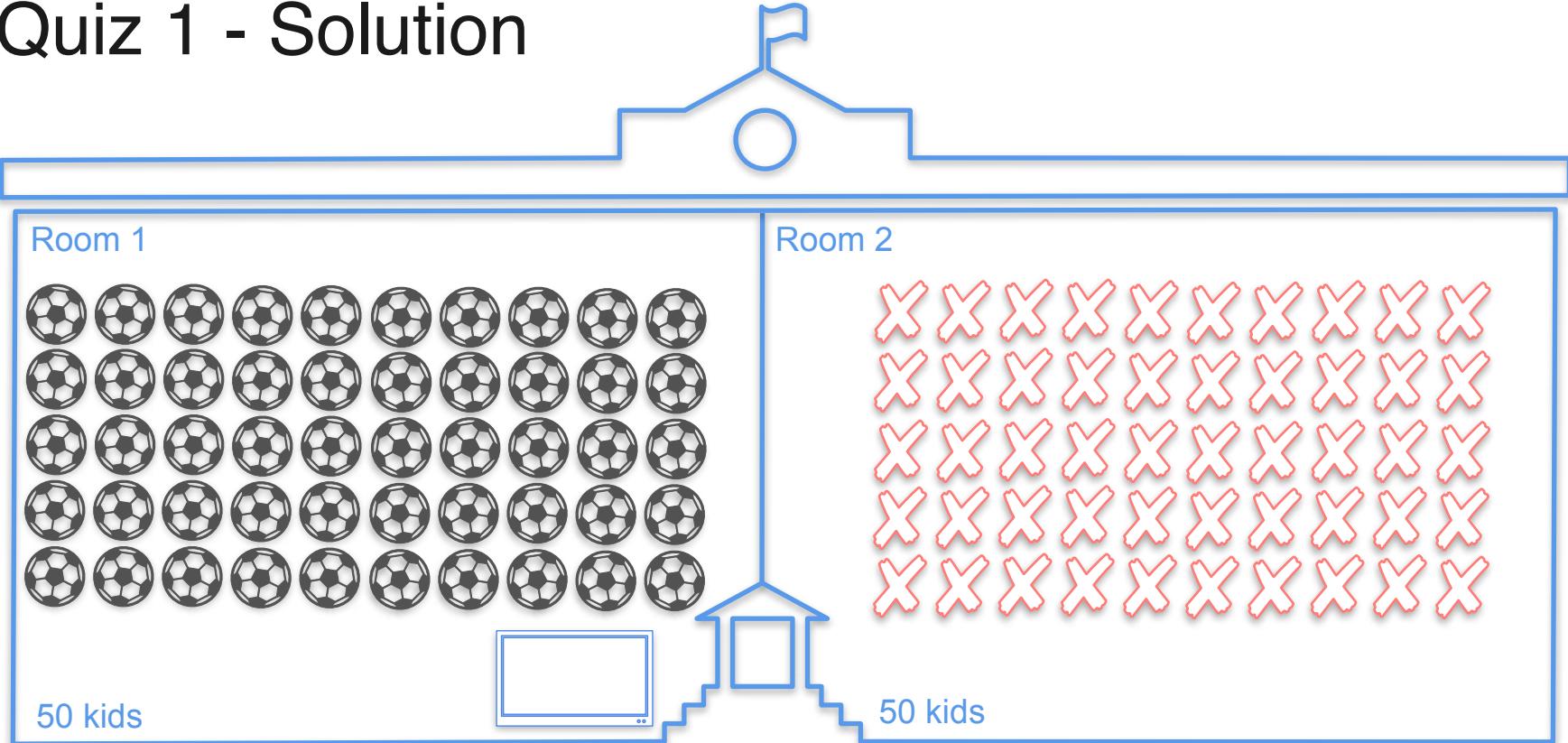
# Quiz 1



# Quiz 1

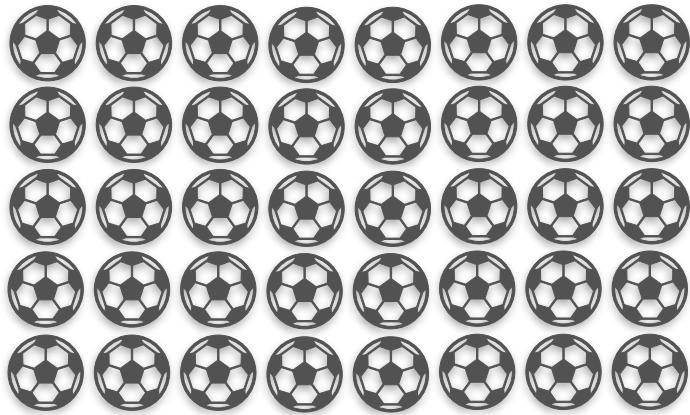


# Quiz 1 - Solution

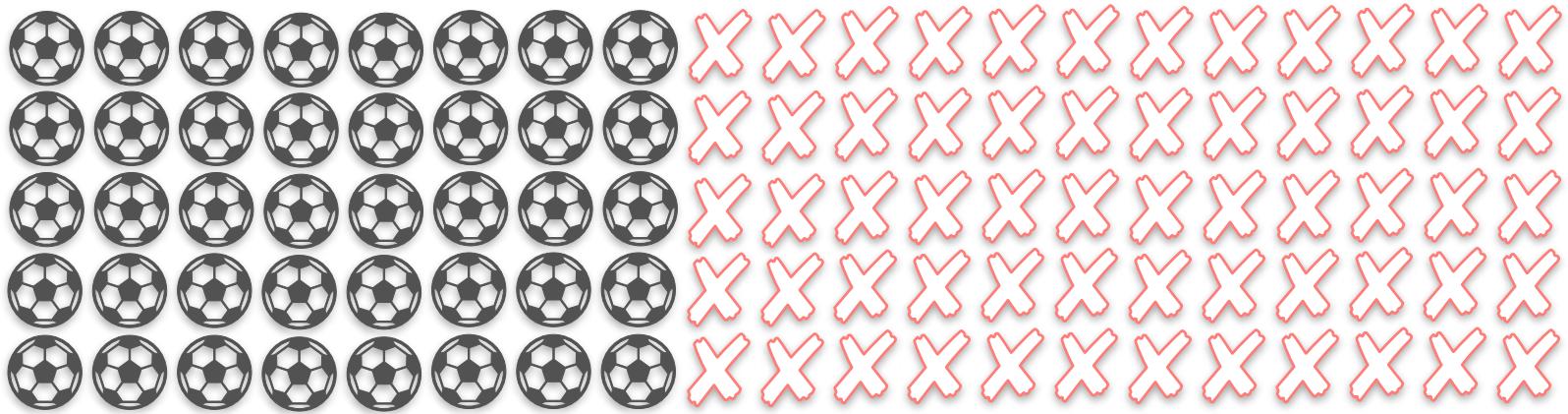


# Quiz 2

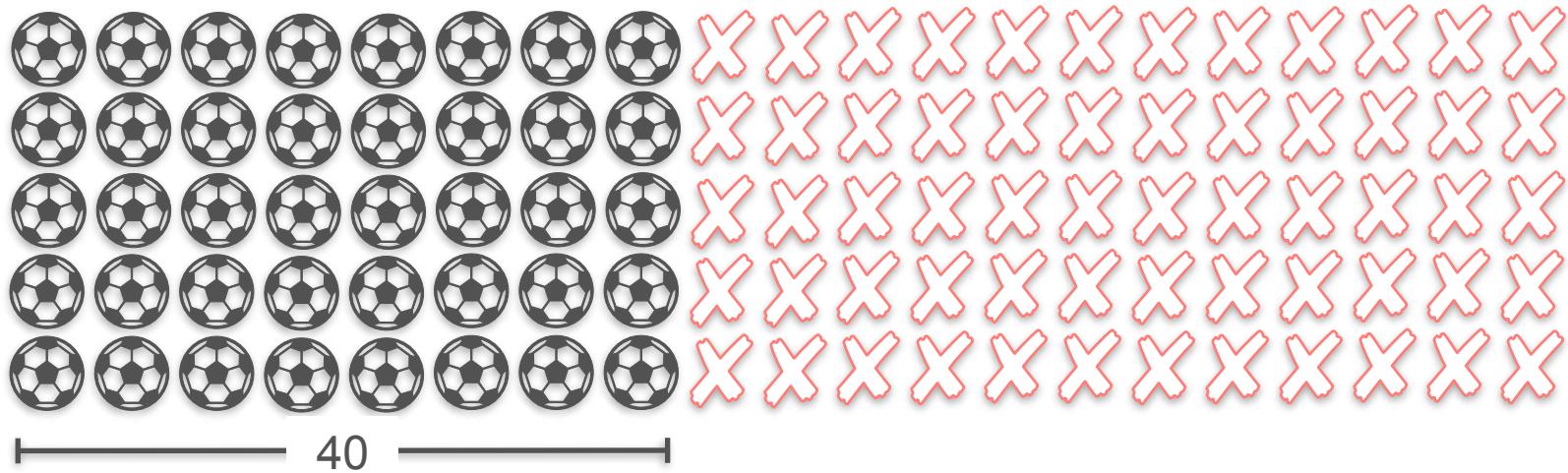
# Quiz 2



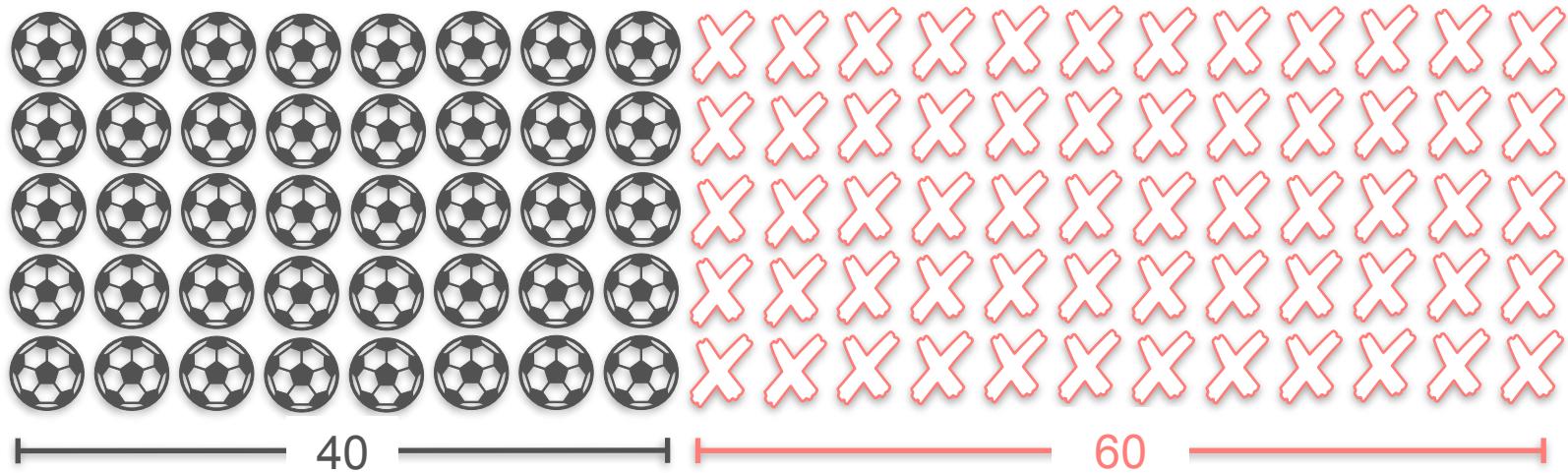
# Quiz 2



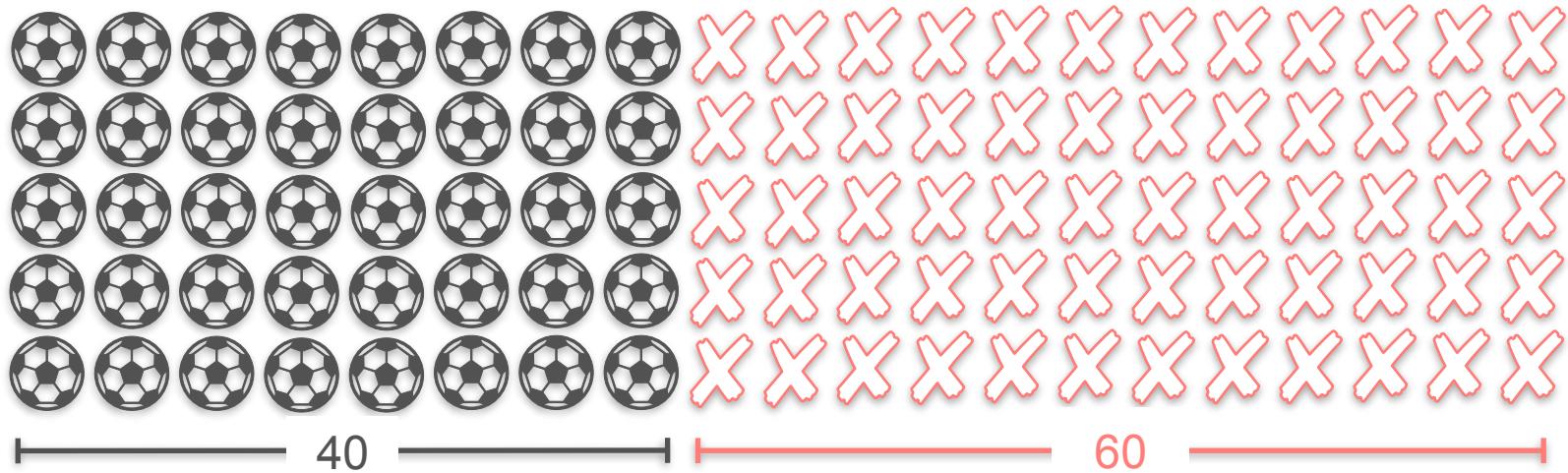
# Quiz 2



## Quiz 2

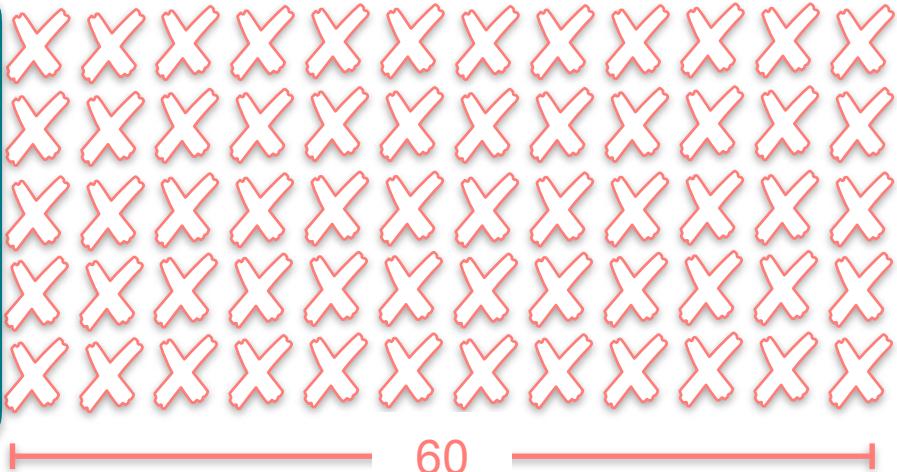
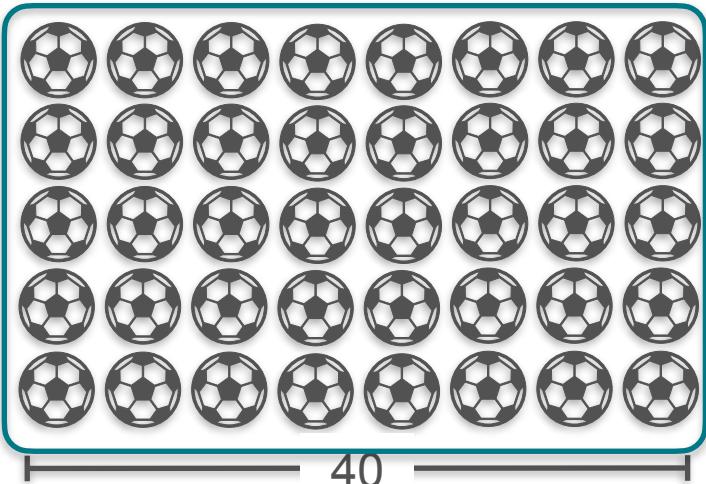


## Quiz 2



100 kids

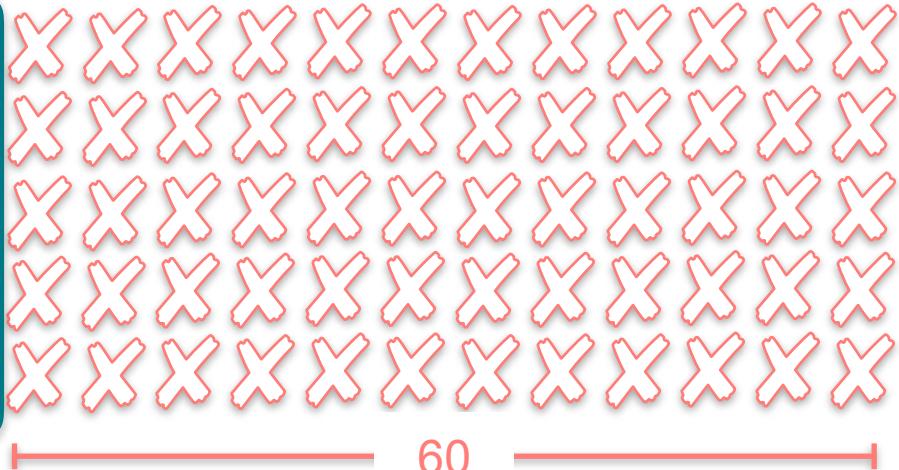
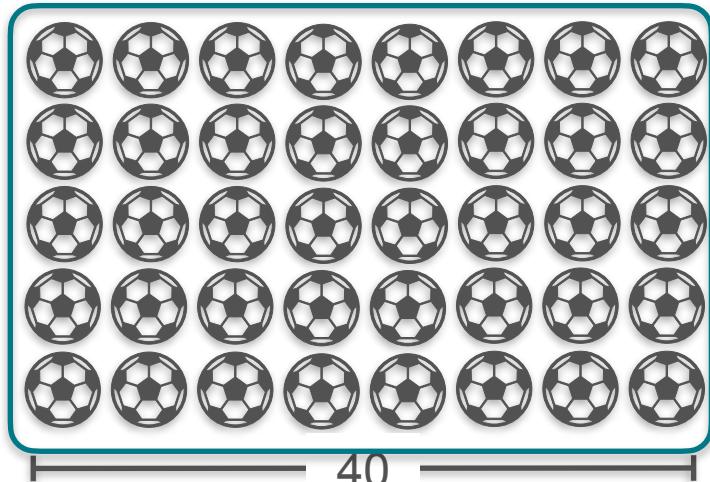
## Quiz 2



80%

100 kids

## Quiz 2

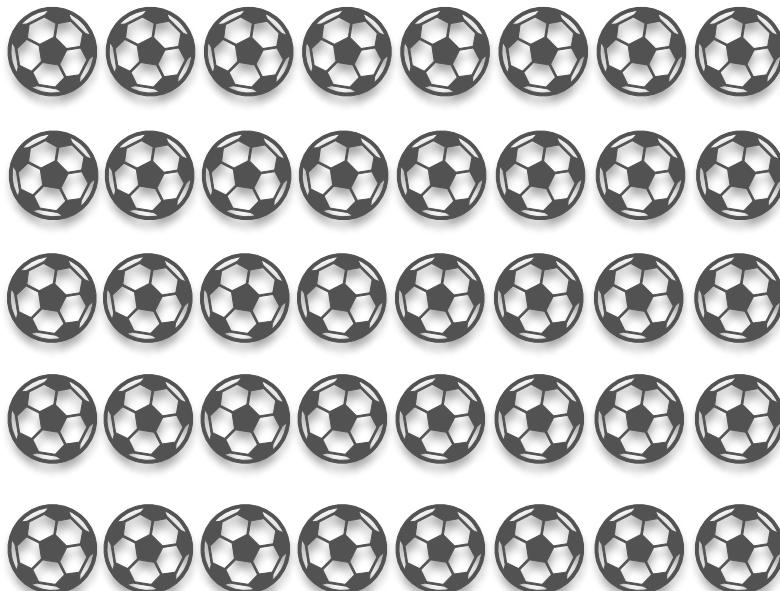


80%

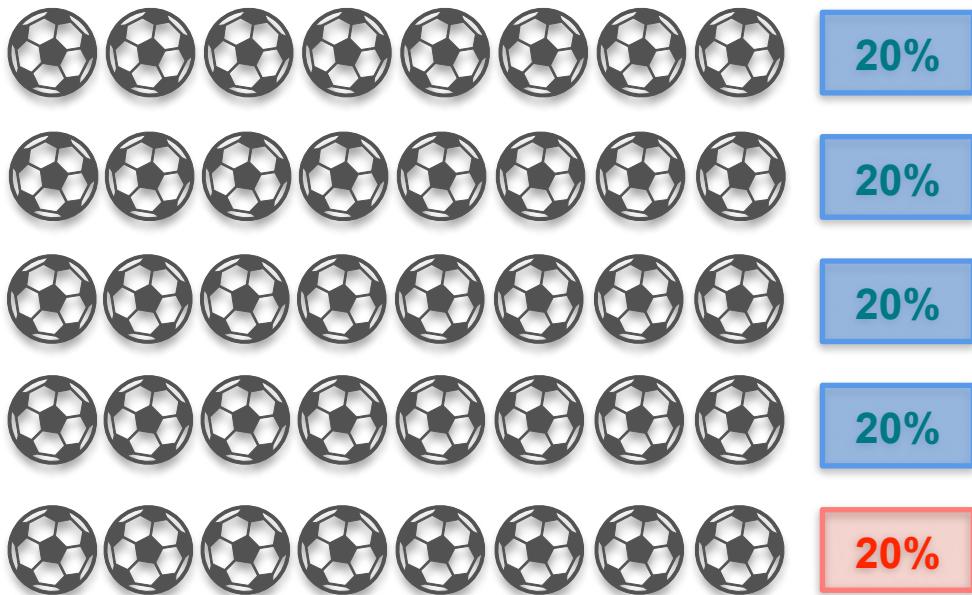
**100 kids**

How many kids play soccer and wear running shoes

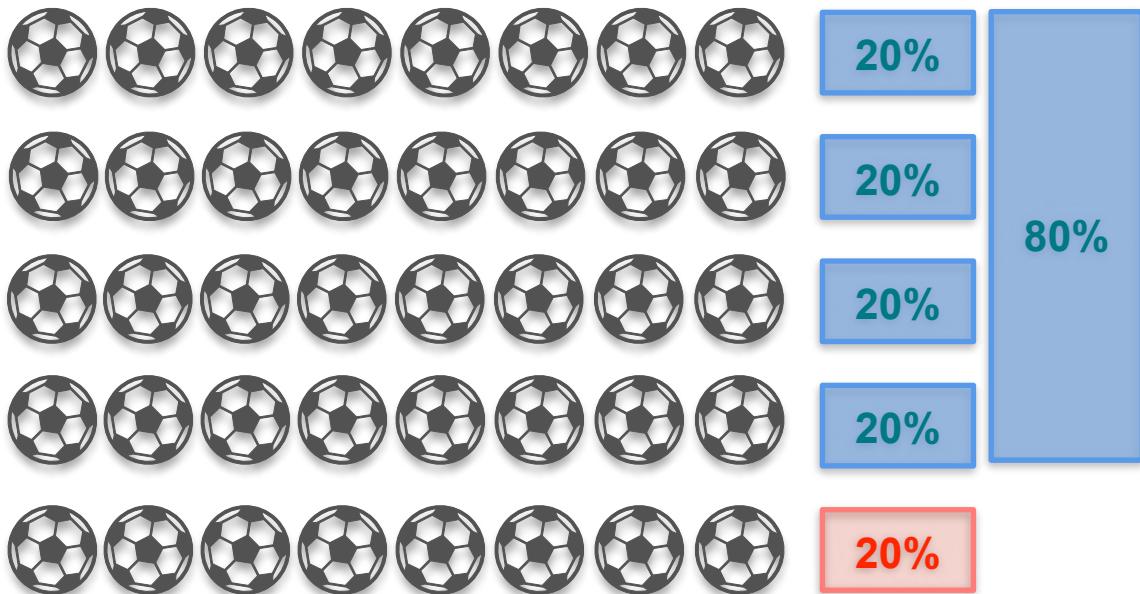
# Quiz 2 - Solution



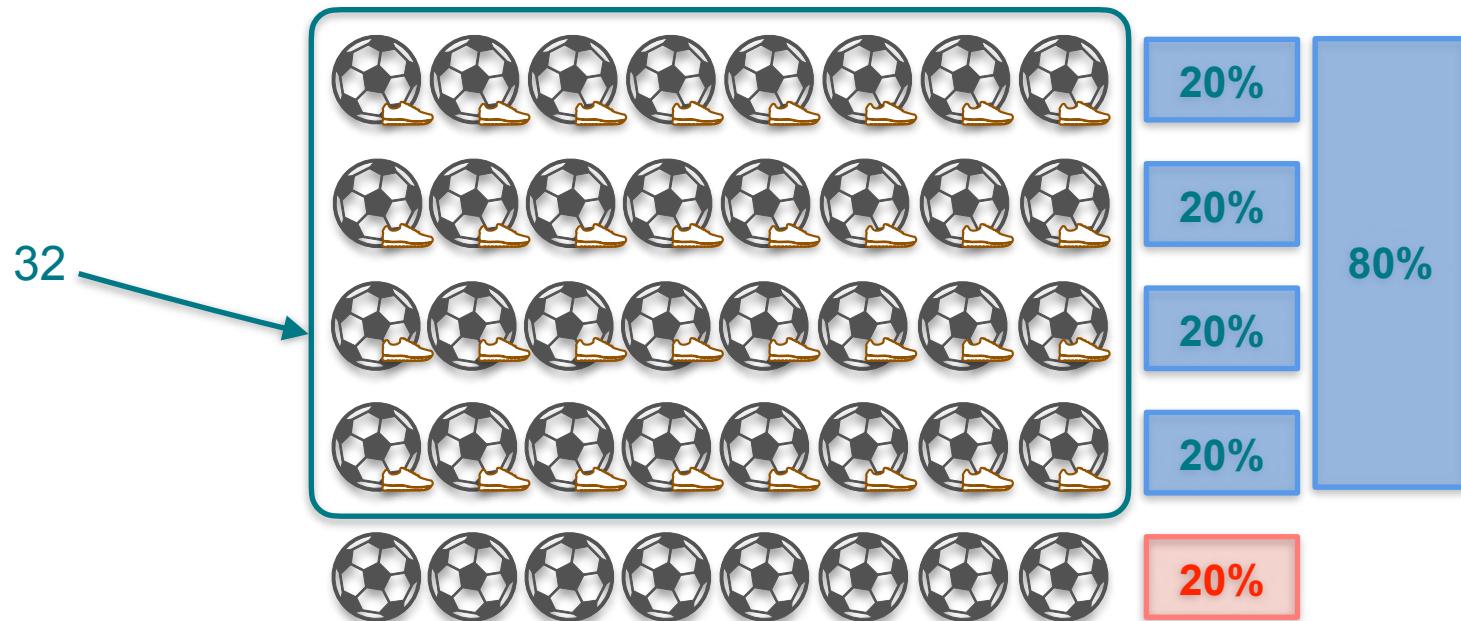
# Quiz 2 - Solution



# Quiz 2 - Solution

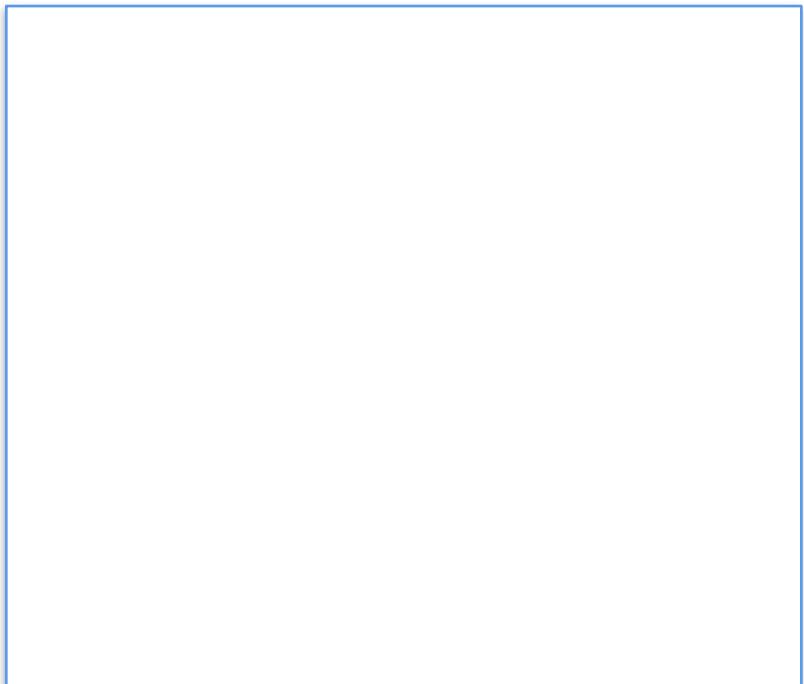


# Quiz 2 - Solution



# Conditional Probability

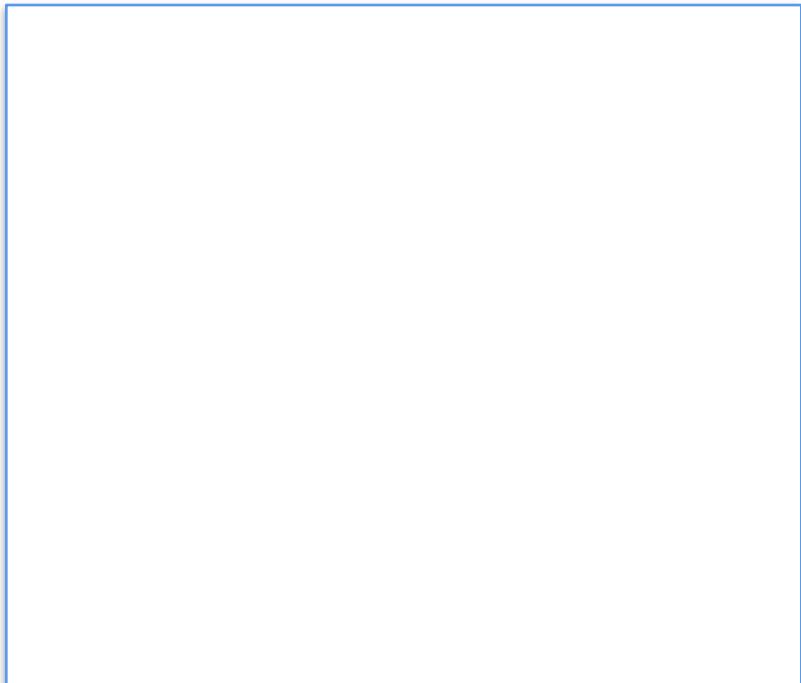
# Conditional Probability



# Conditional Probability



$$P(S) = 0.4$$



# Conditional Probability



$$P(S) = 0.4$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○

# Conditional Probability



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○

# Conditional Probability



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

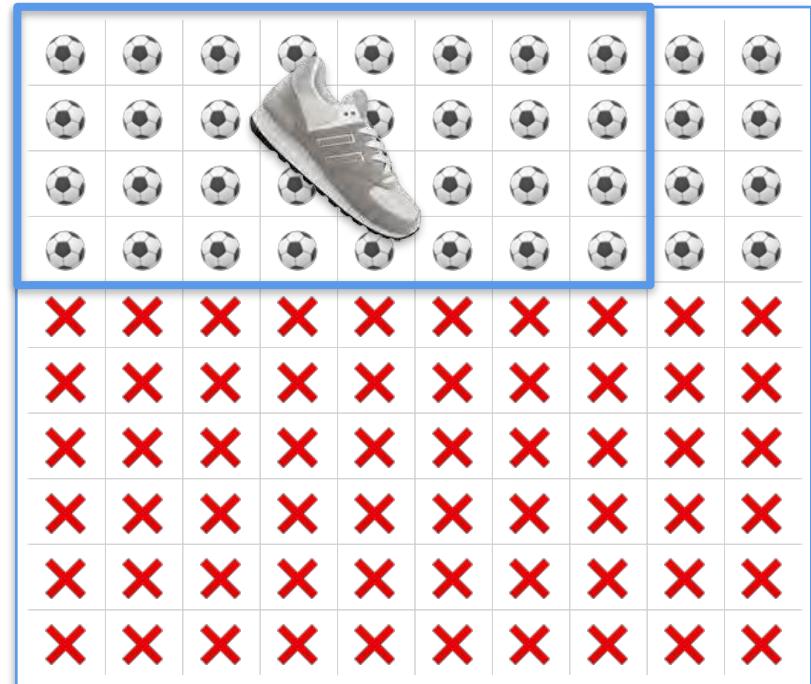
# Conditional Probability



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

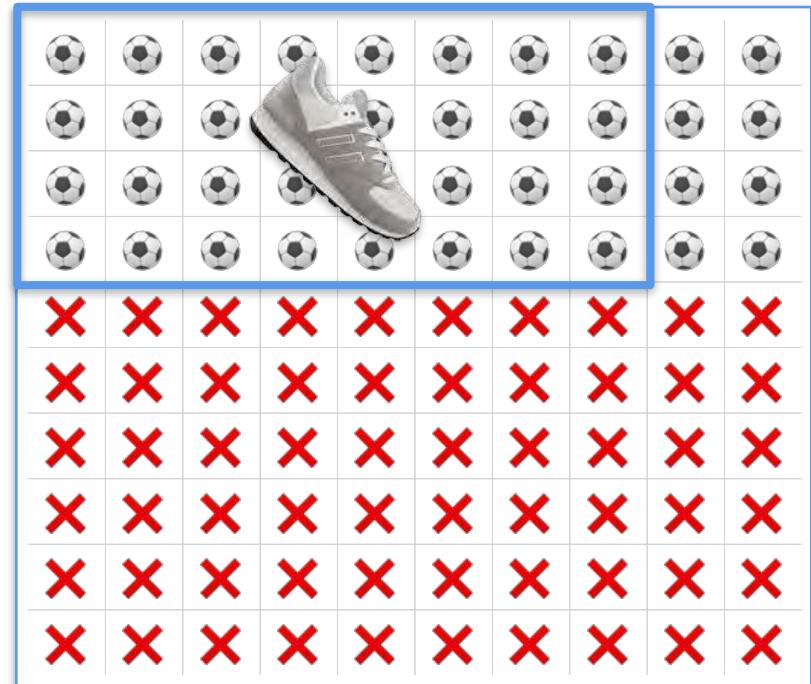


$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$

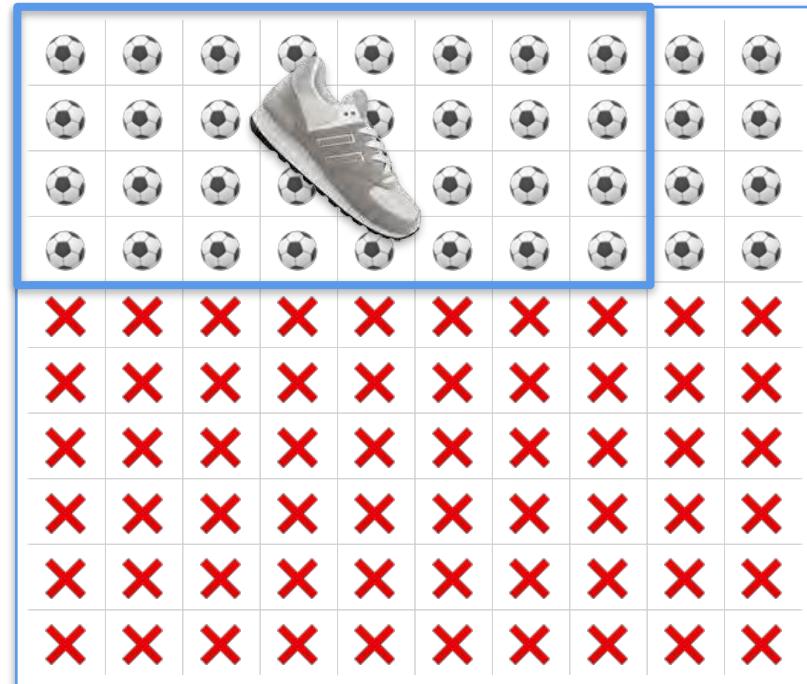


$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

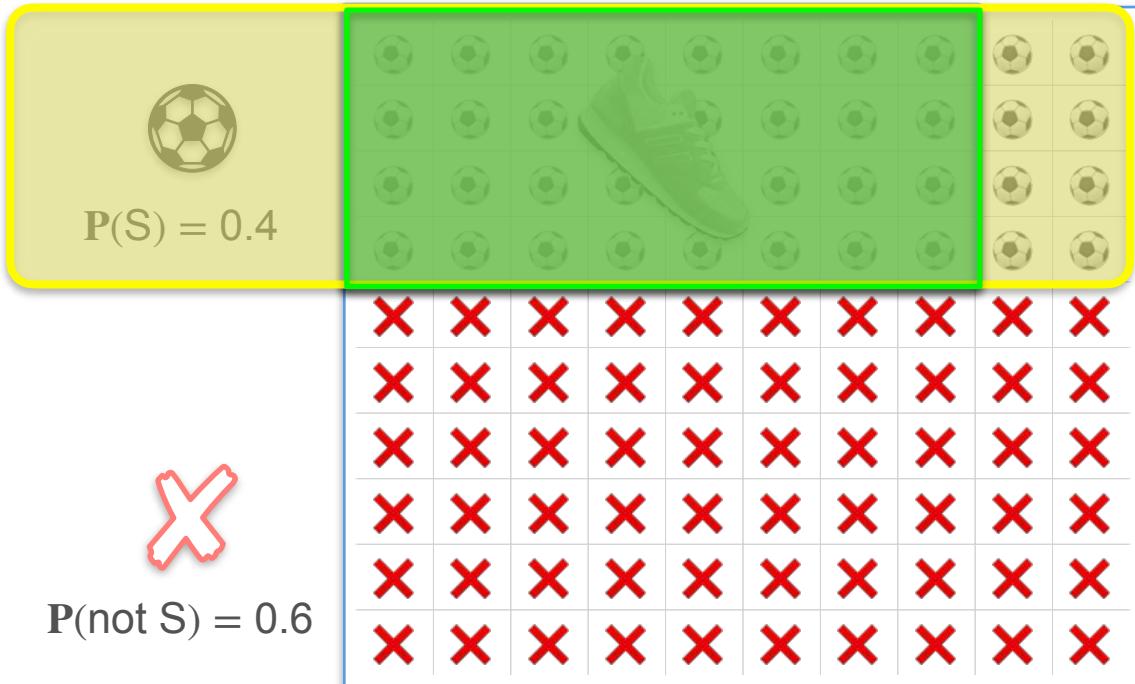
$P(\text{Soccer and Running shoes})$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

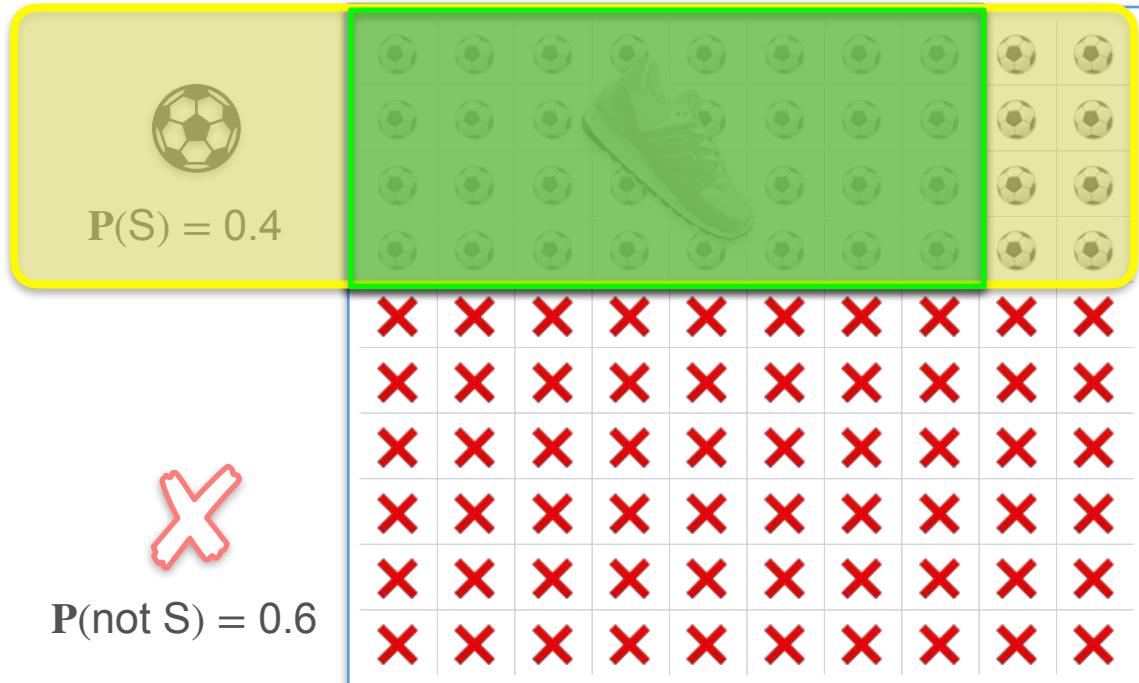
$$P(S \cap R) =$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

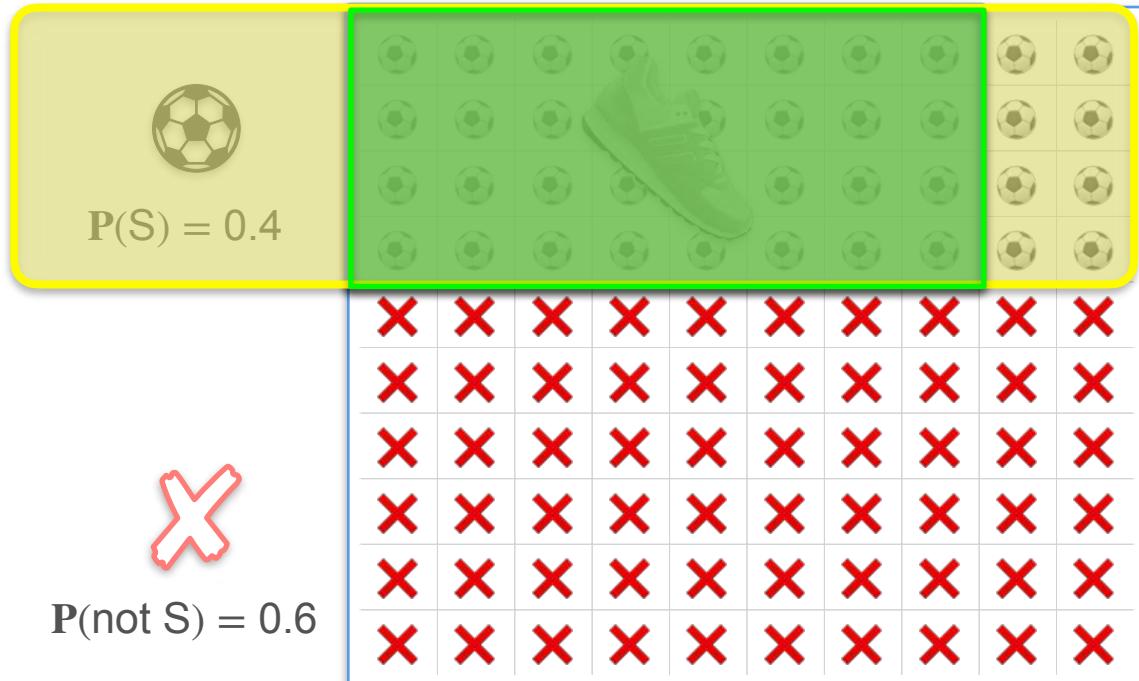
$$P(S \cap R) = P(S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

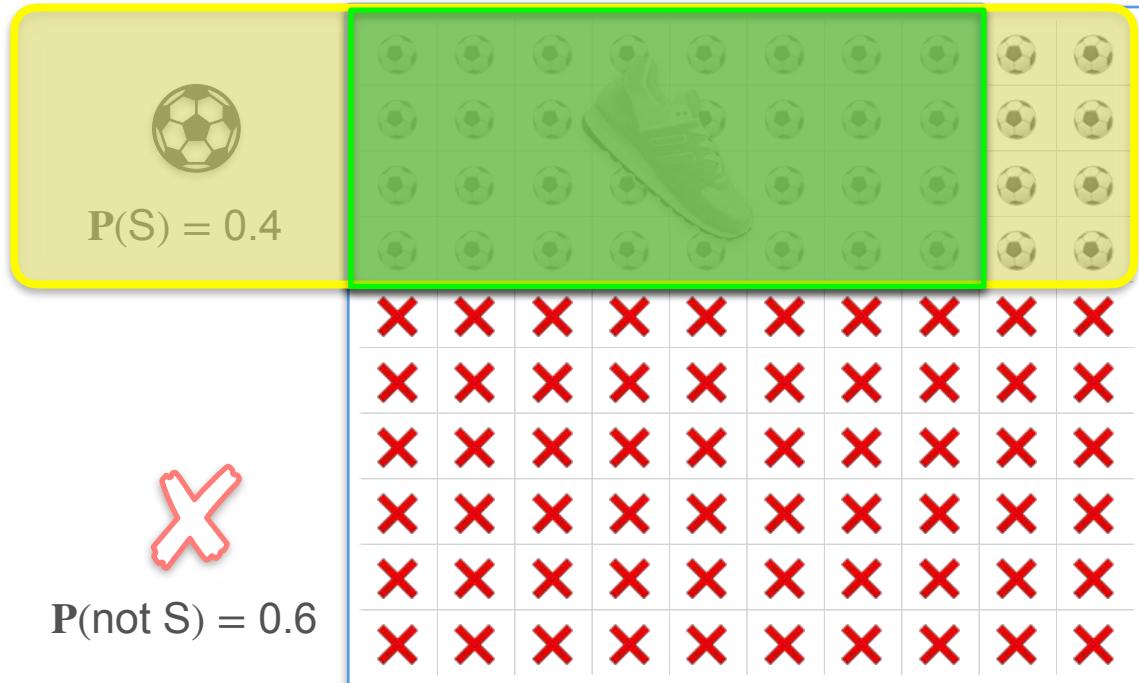
$$P(S \cap R) = P(S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \cdot$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$$P(R | S) = 0.8$$

$P(\text{Soccer and Running shoes})$

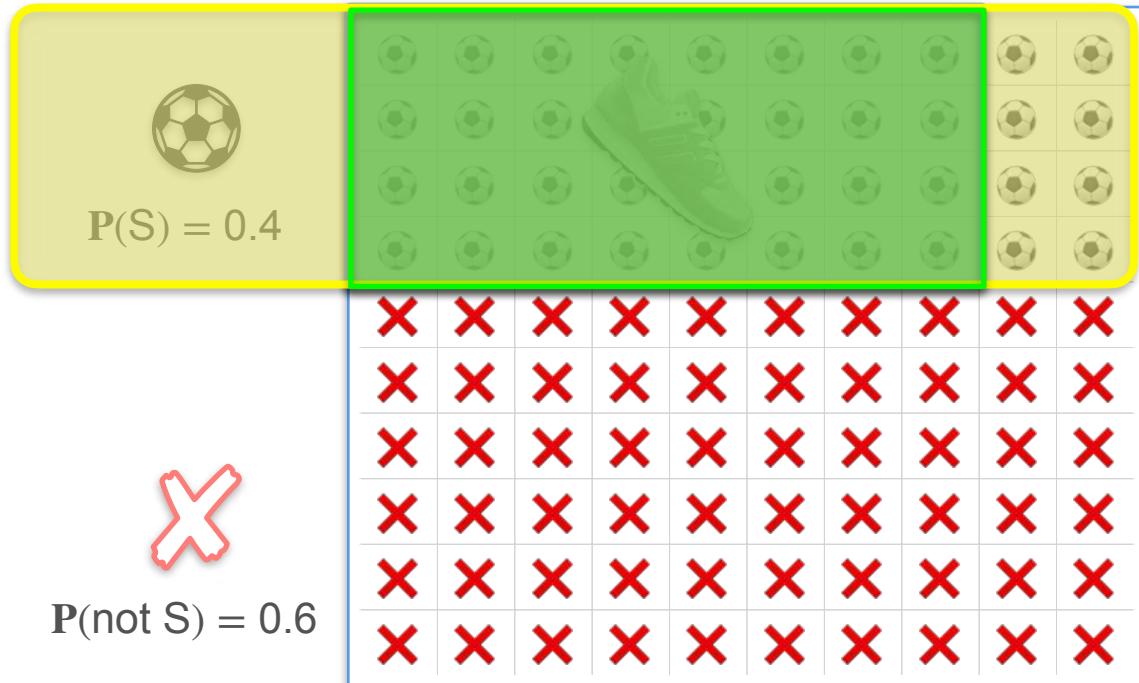
$$P(S \cap R) = P(S) \bullet P(R|S)$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$

$$\begin{aligned} P(S \cap R) &= P(S) \bullet P(R|S) \\ &= 0.4 \end{aligned}$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet$$

$$P(R | S) = 0.8$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

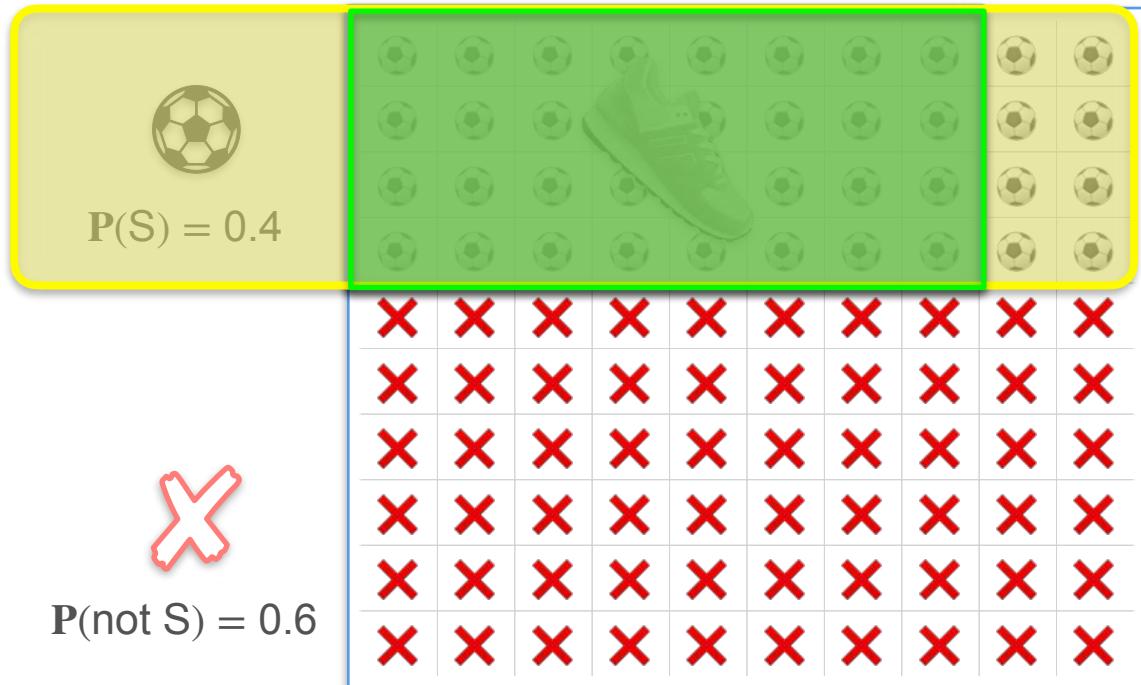


# Conditional Probability

$P(\text{Soccer and Running shoes})$

$$\begin{aligned} P(S \cap R) &= P(S) \bullet P(R|S) \\ &= 0.4 \bullet 0.8 \end{aligned}$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



$$P(S) = 0.4$$

$$P(R | S) = 0.8$$



$$P(\text{not } S) = 0.6$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



$$P(S) = 0.4$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$

$P(S \cap R) = 0.32$



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

# Conditional Probability

$P(\text{Soccer and Running shoes})$

$P(S \cap R) = 0.32$



$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$

○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
○	○	○	○	○	○	○	○	○	○
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗
✗	✗	✗	✗	✗	✗	✗	✗	✗	✗

$$P(R | \text{not } S) = 0.5$$

# Conditional Probability

$P(\text{Soccer and Running shoes})$



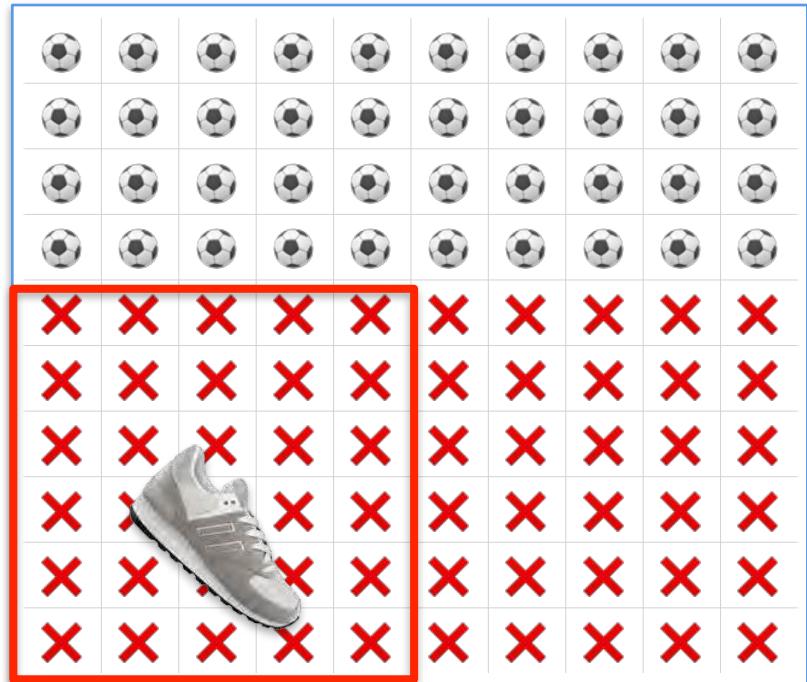
$P(S \cap R) = 0.32$

$P(S) = 0.4$



$P(\text{not } S) = 0.6$

$P(R | S) = 0.8$



$P(R | \text{not } S) = 0.5$

# Conditional Probability

$P(\text{Soccer and Running shoes})$



$P(S \cap R) = 0.32$

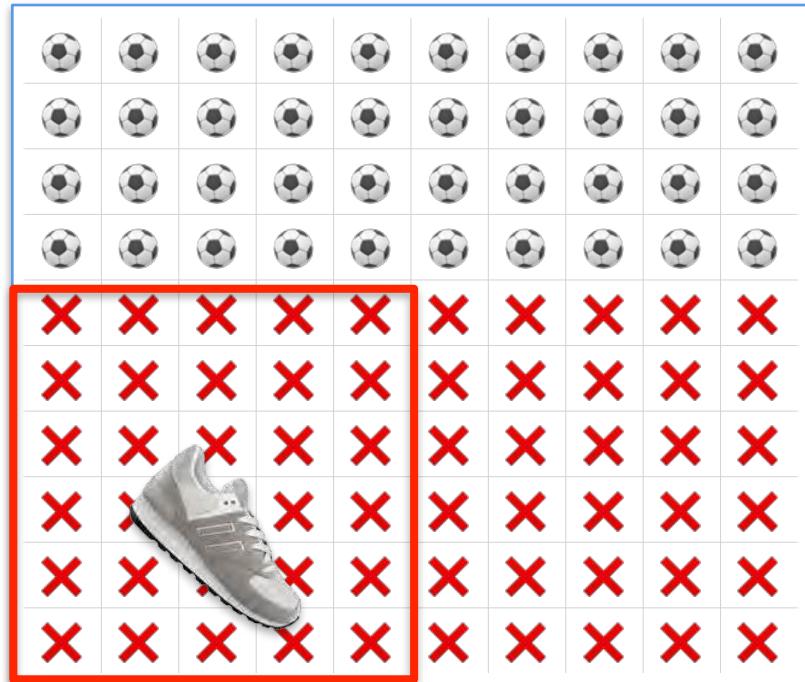
$P(S) = 0.4$

$P(\text{not Soccer and Running shoes})$



$P(\text{not } S) = 0.6$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

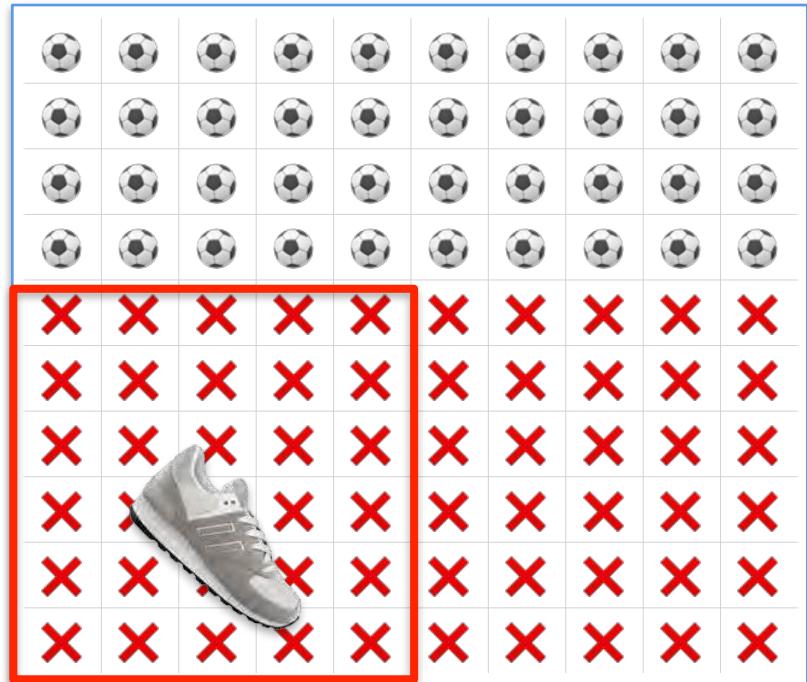
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) =$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

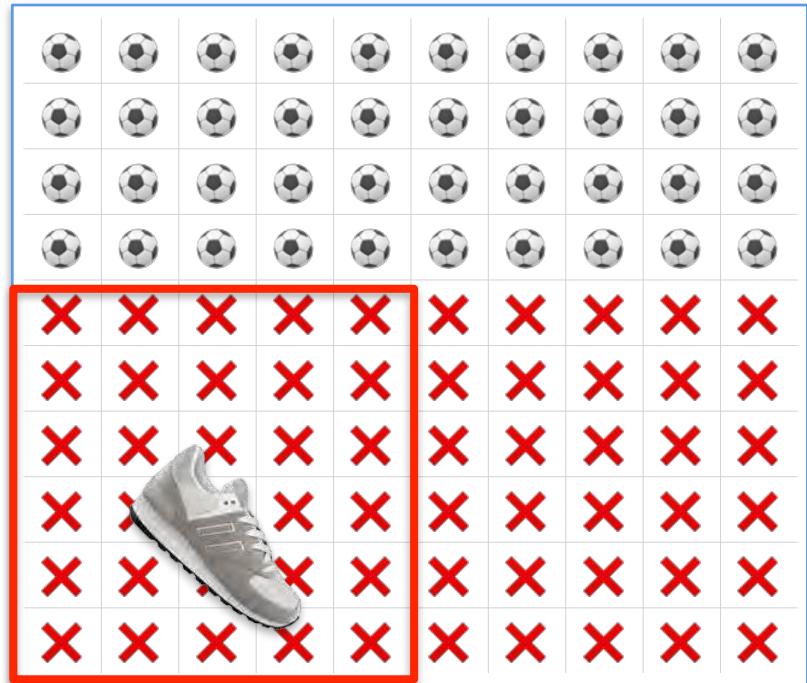
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

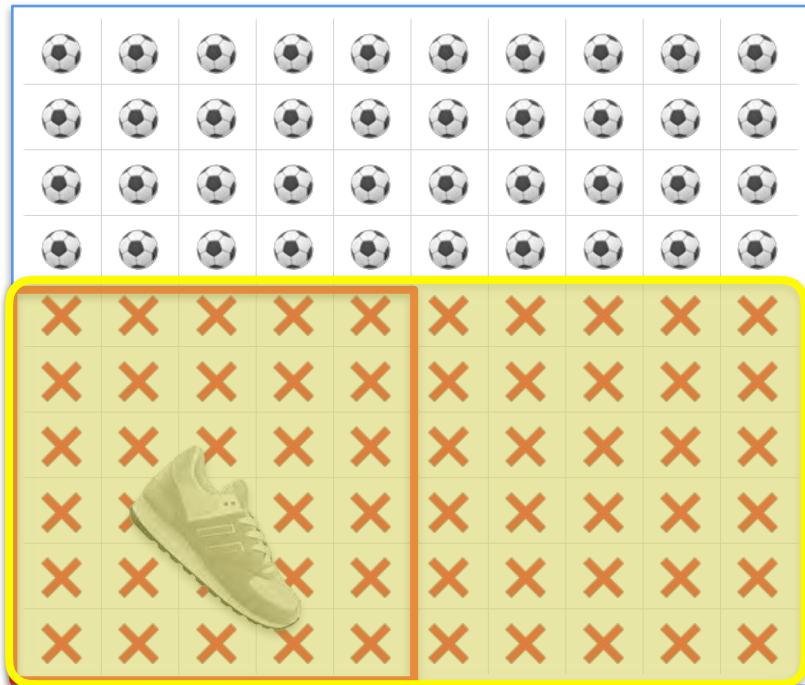
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

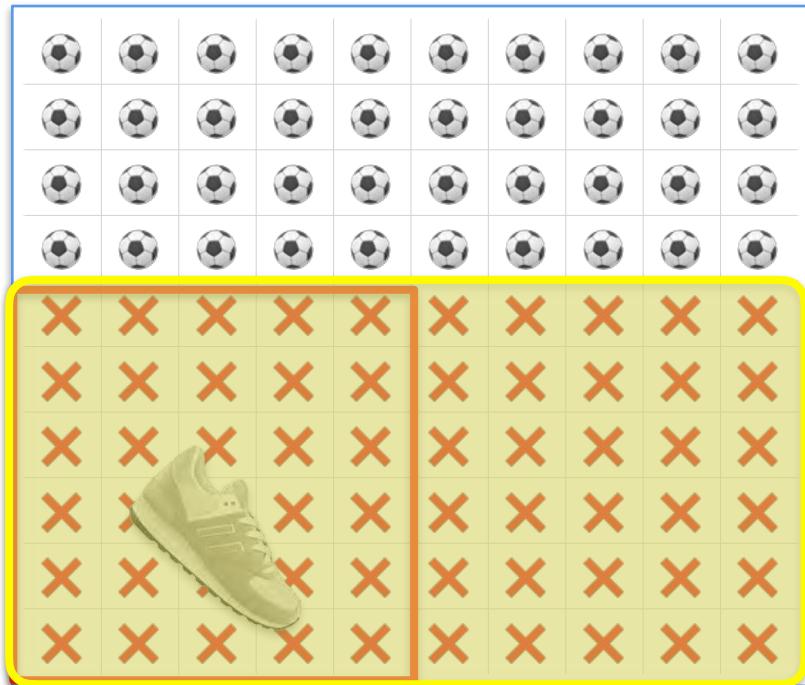
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

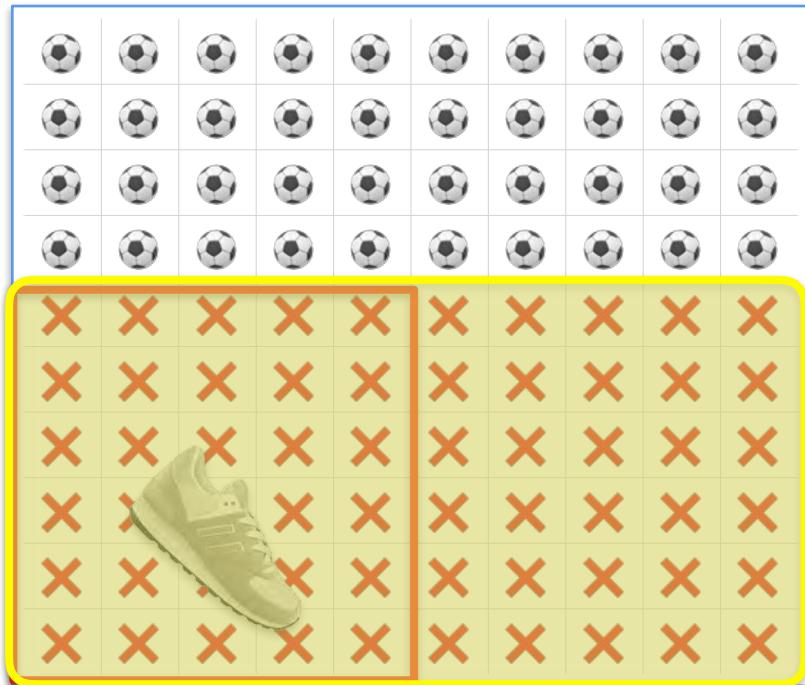
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

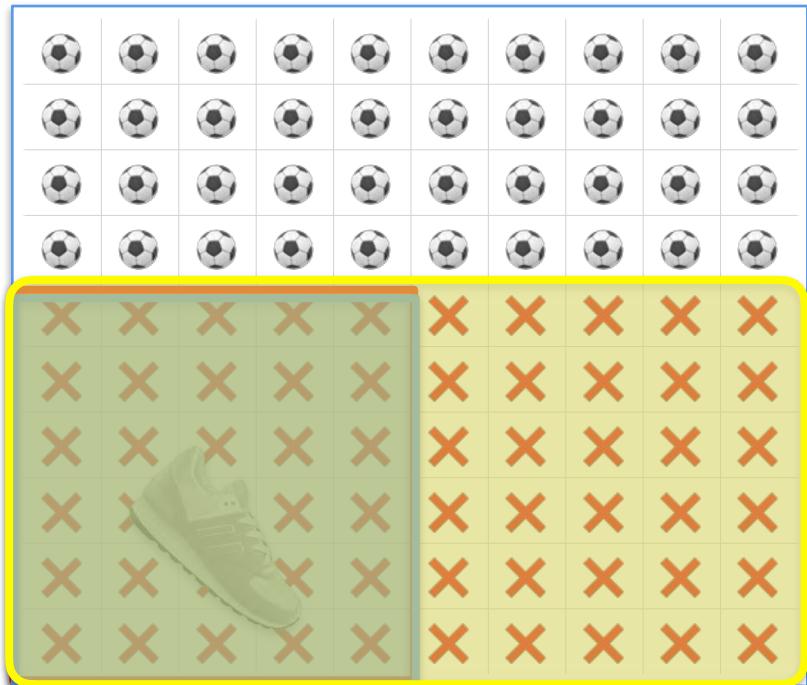
$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

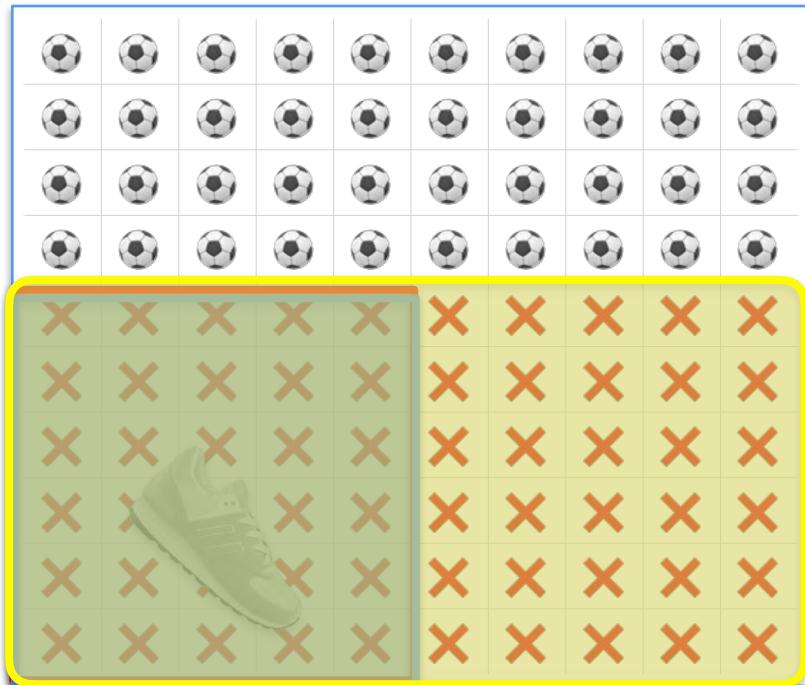
$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

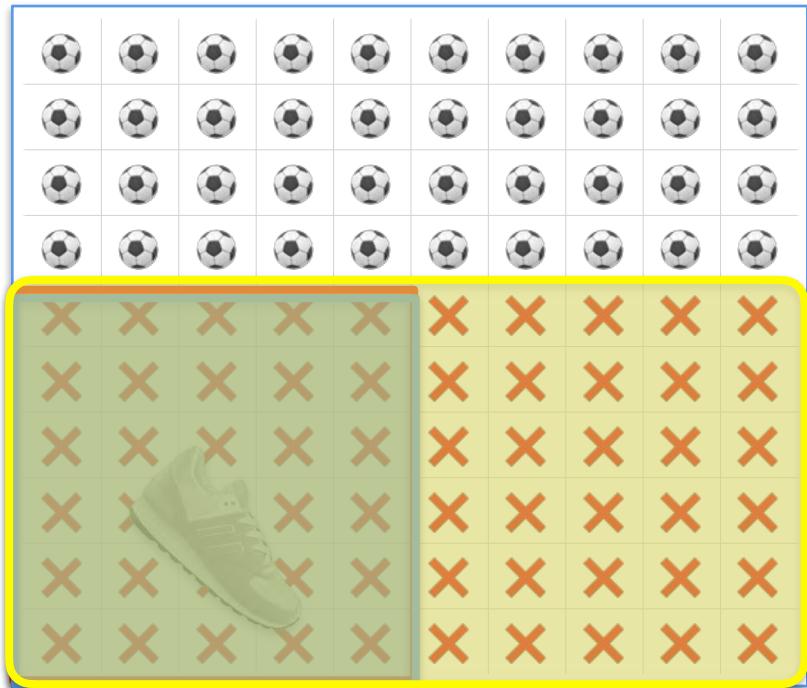
$P(\text{not Soccer and Running shoes})$

$$\begin{aligned} P(\text{not } S \cap R) &= P(\text{not } S) \bullet P(R | \text{not } S) \\ &= 0.6 \bullet \end{aligned}$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

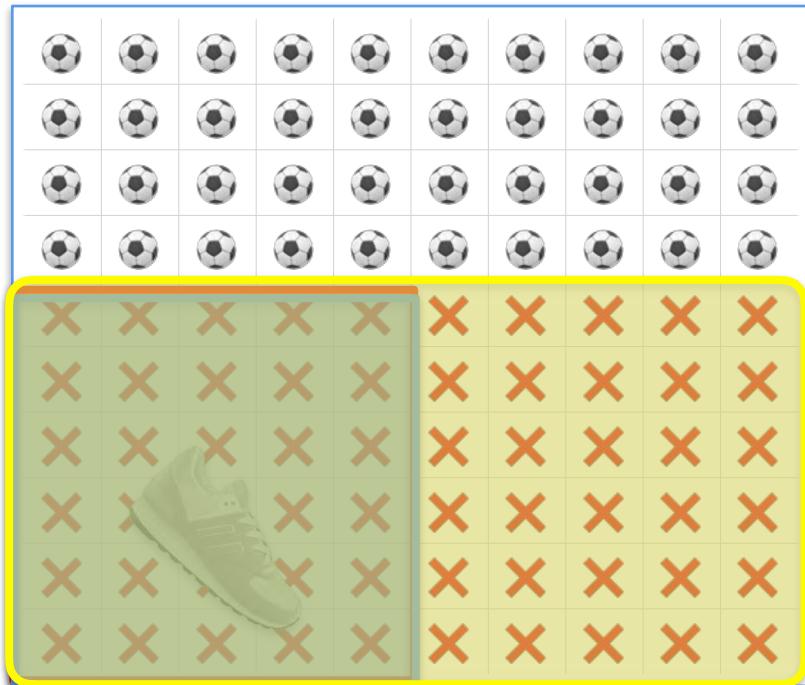
$P(\text{not Soccer and Running shoes})$

$$\begin{aligned} P(\text{not } S \cap R) &= P(\text{not } S) \bullet P(R | \text{not } S) \\ &= 0.6 \bullet 0.5 \end{aligned}$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

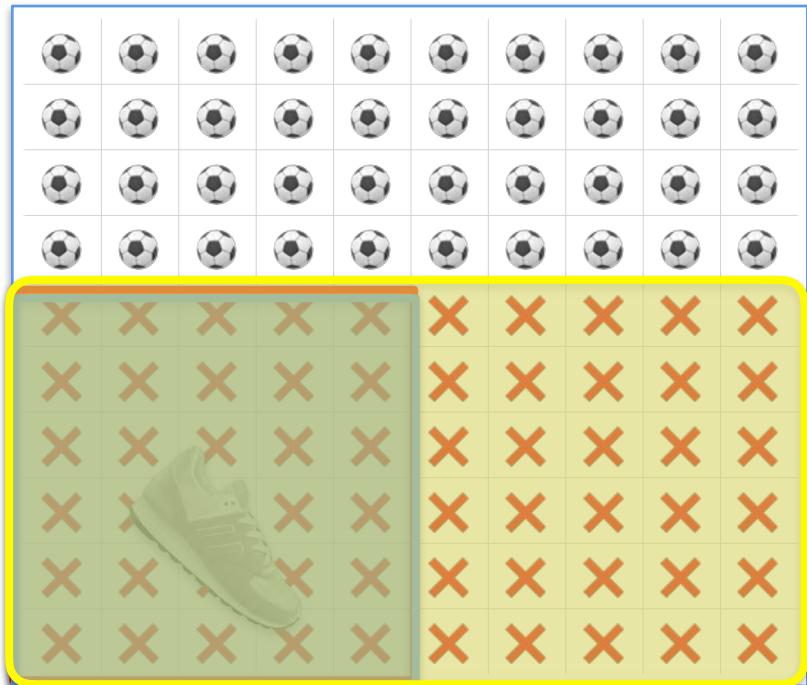
$$= 0.6 \bullet 0.5$$

$$= 0.3$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



# Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6 \bullet 0.5$$

$$= 0.3$$

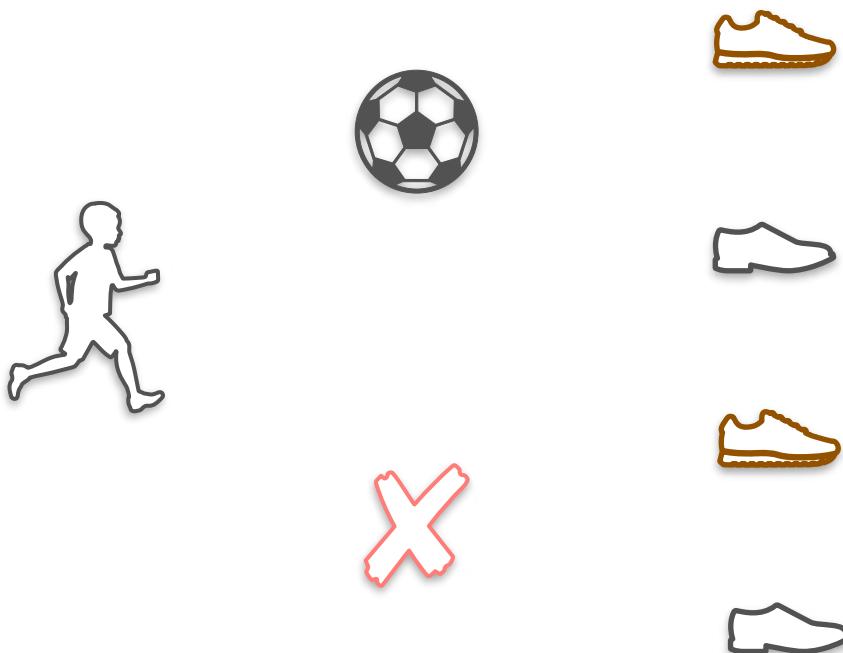


$$P(\text{not } S) = 0.6$$

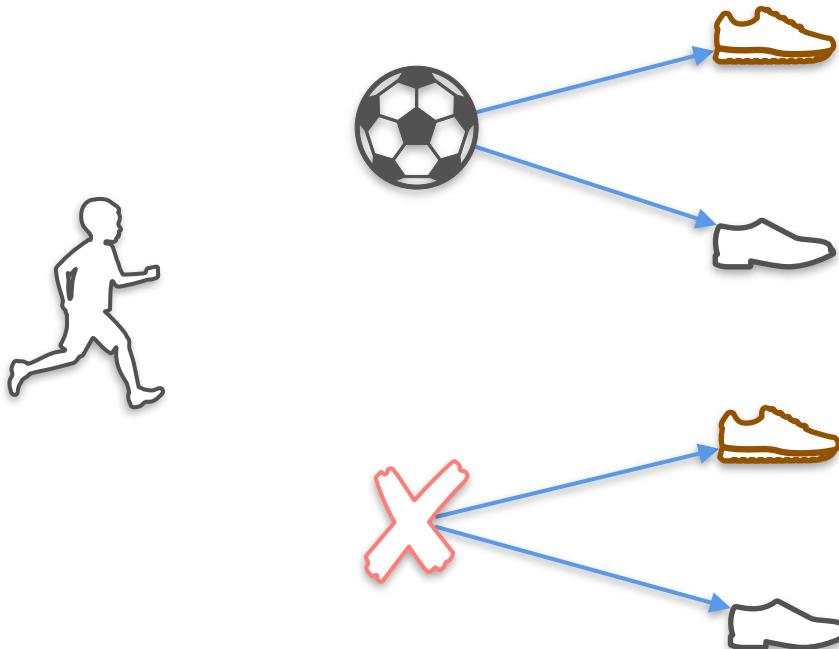
$$P(R | S) = 0.8$$



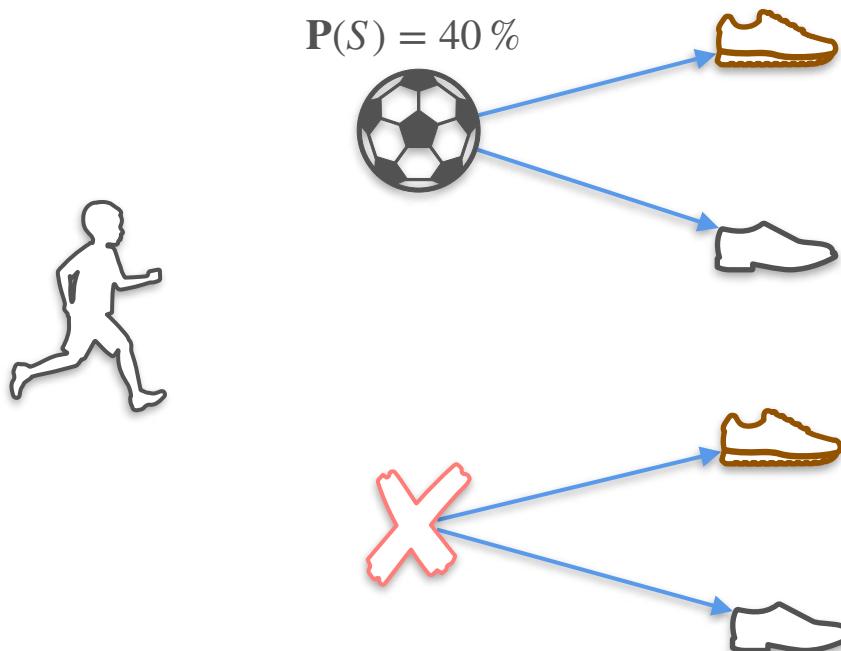
# Conditional Probability



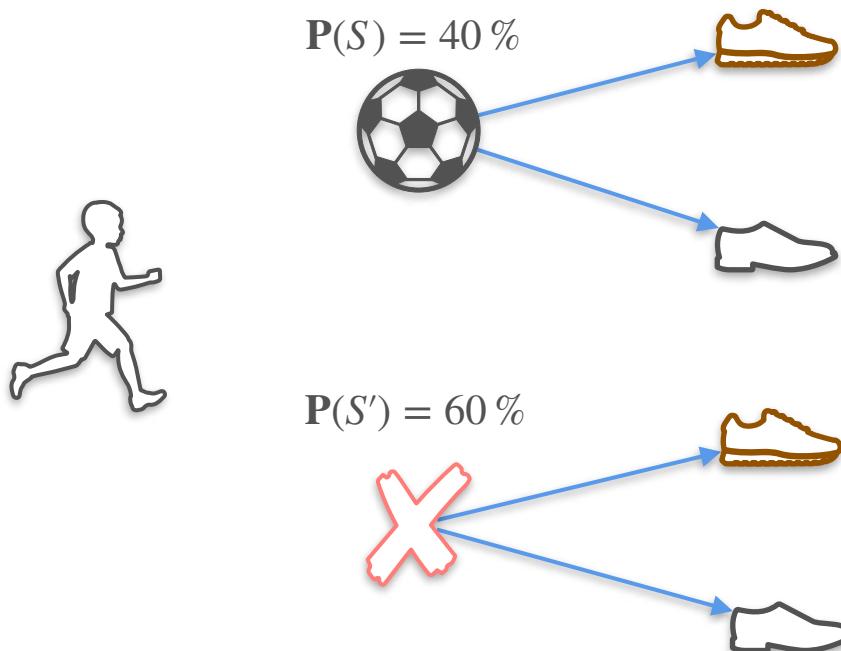
# Conditional Probability



# Conditional Probability

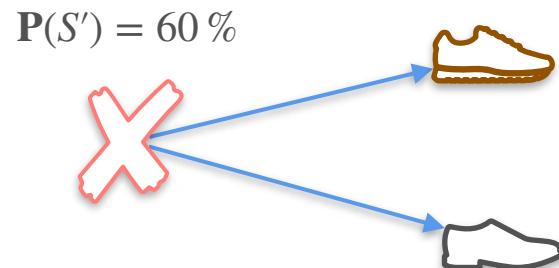
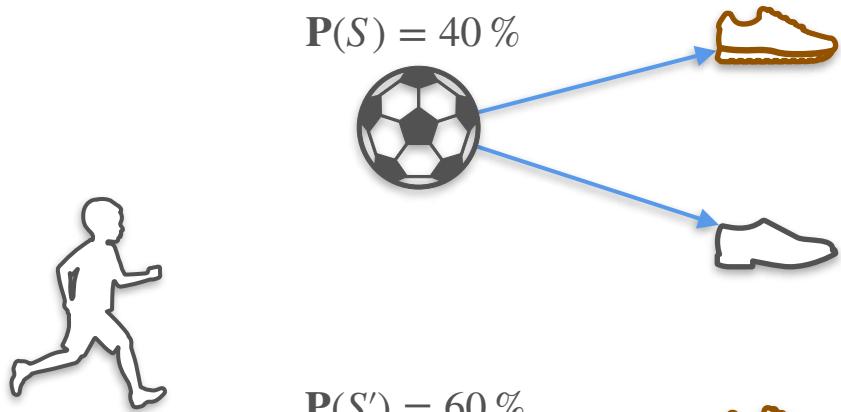


# Conditional Probability

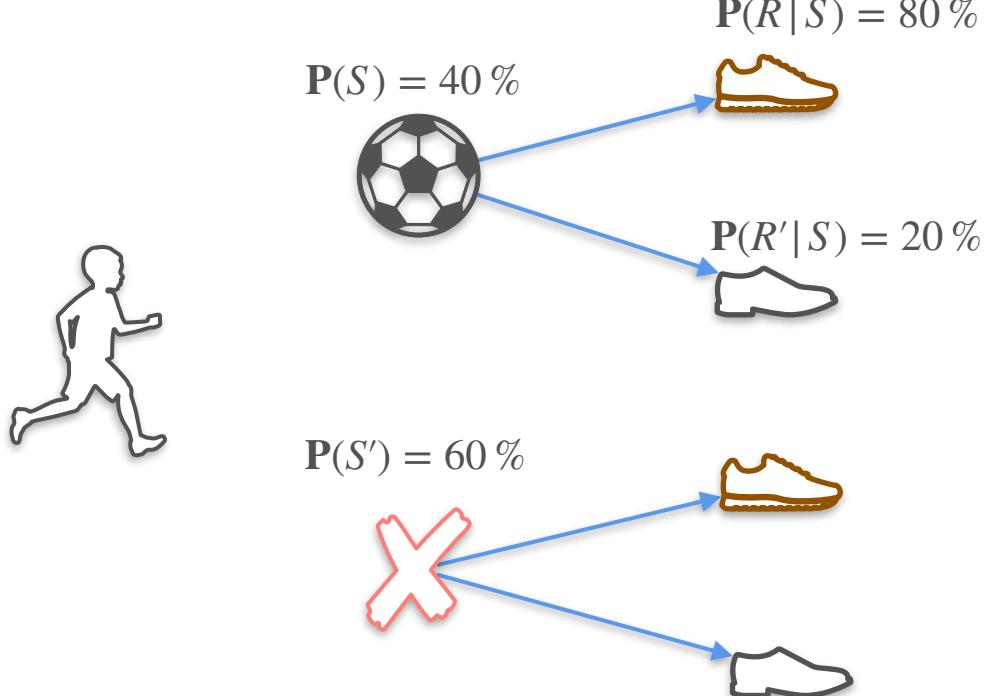


# Conditional Probability

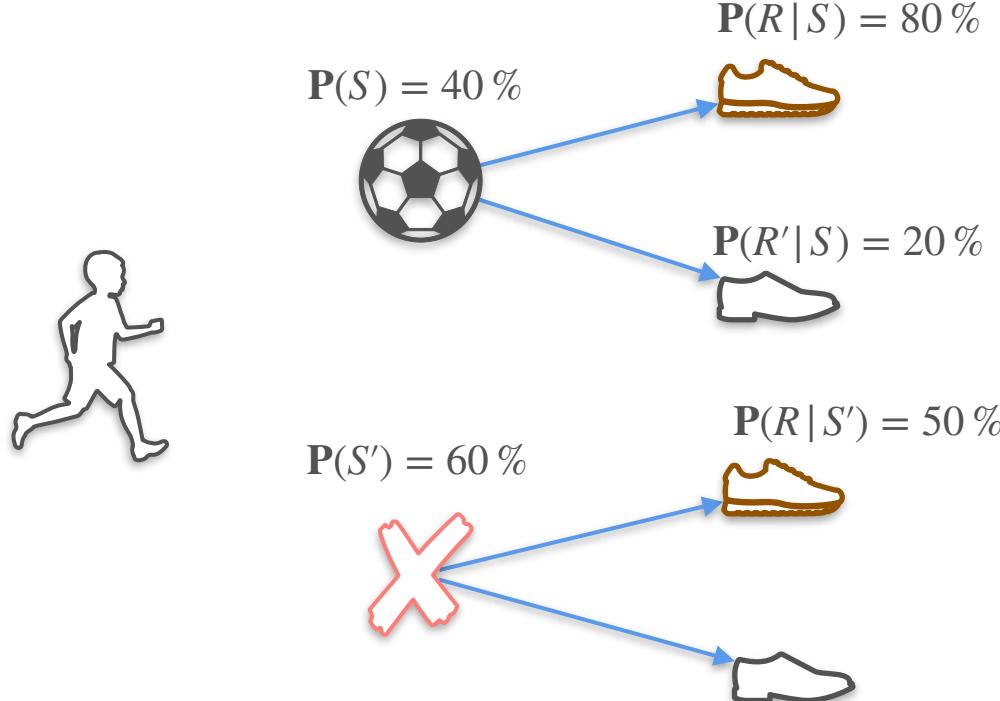
$$P(R | S) = 80 \%$$



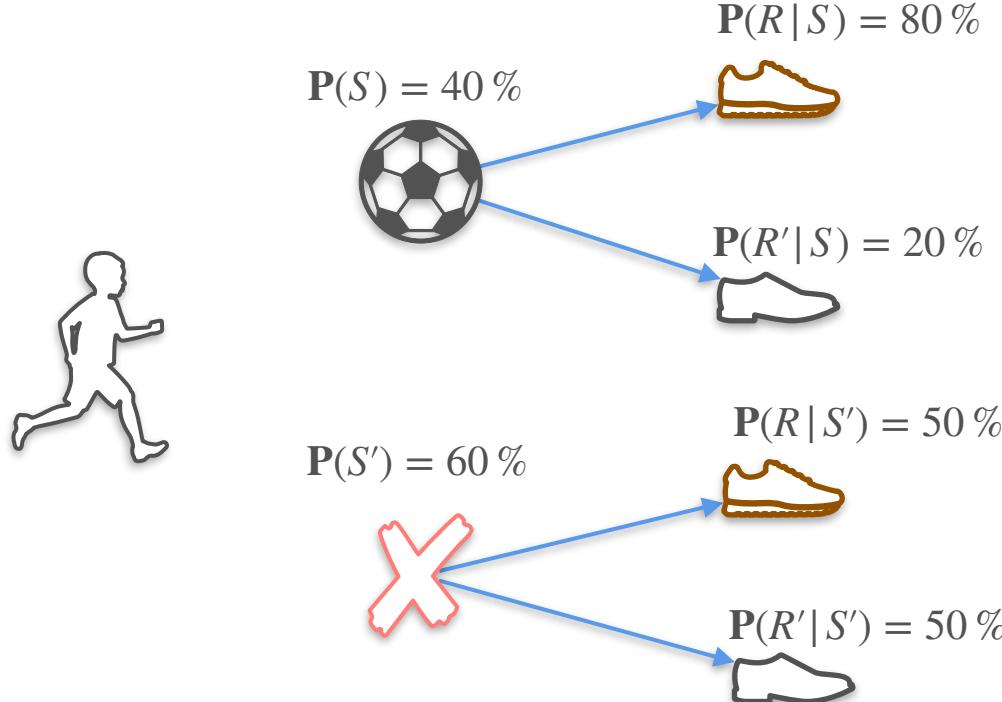
# Conditional Probability



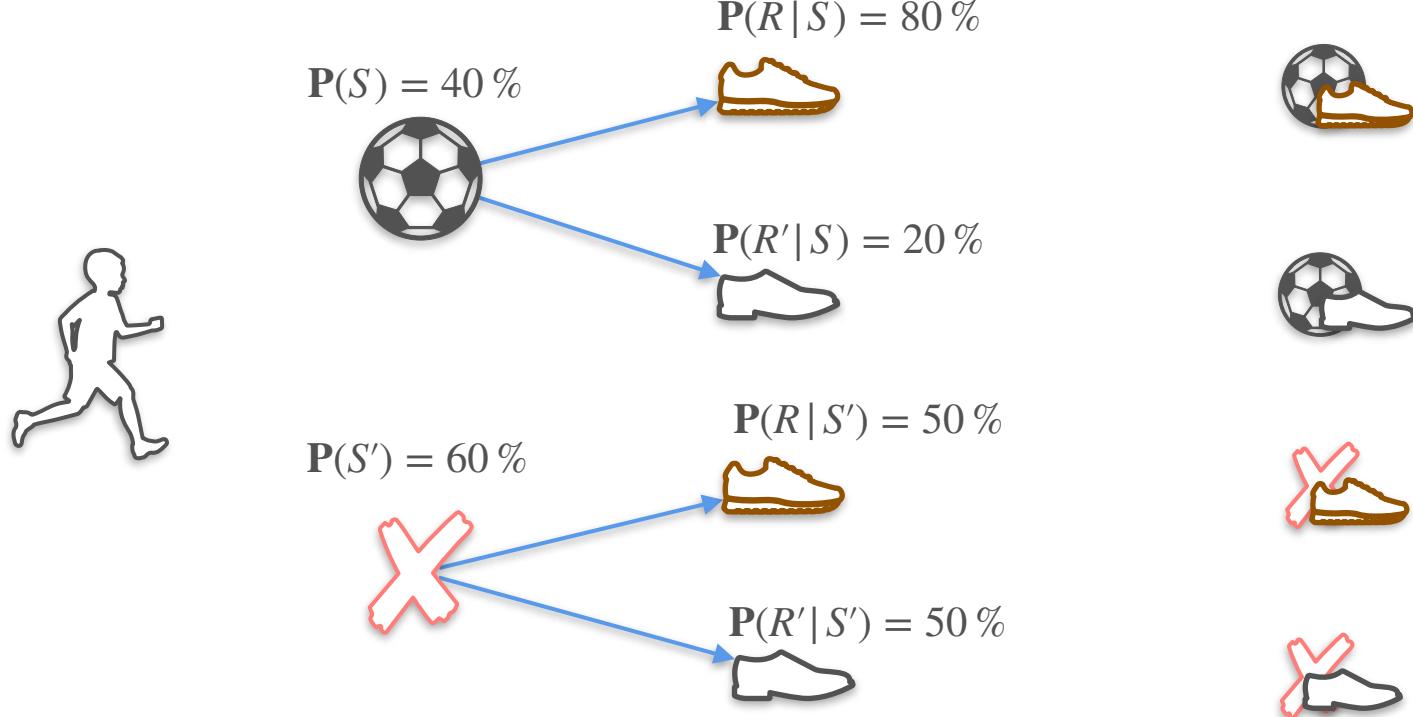
# Conditional Probability



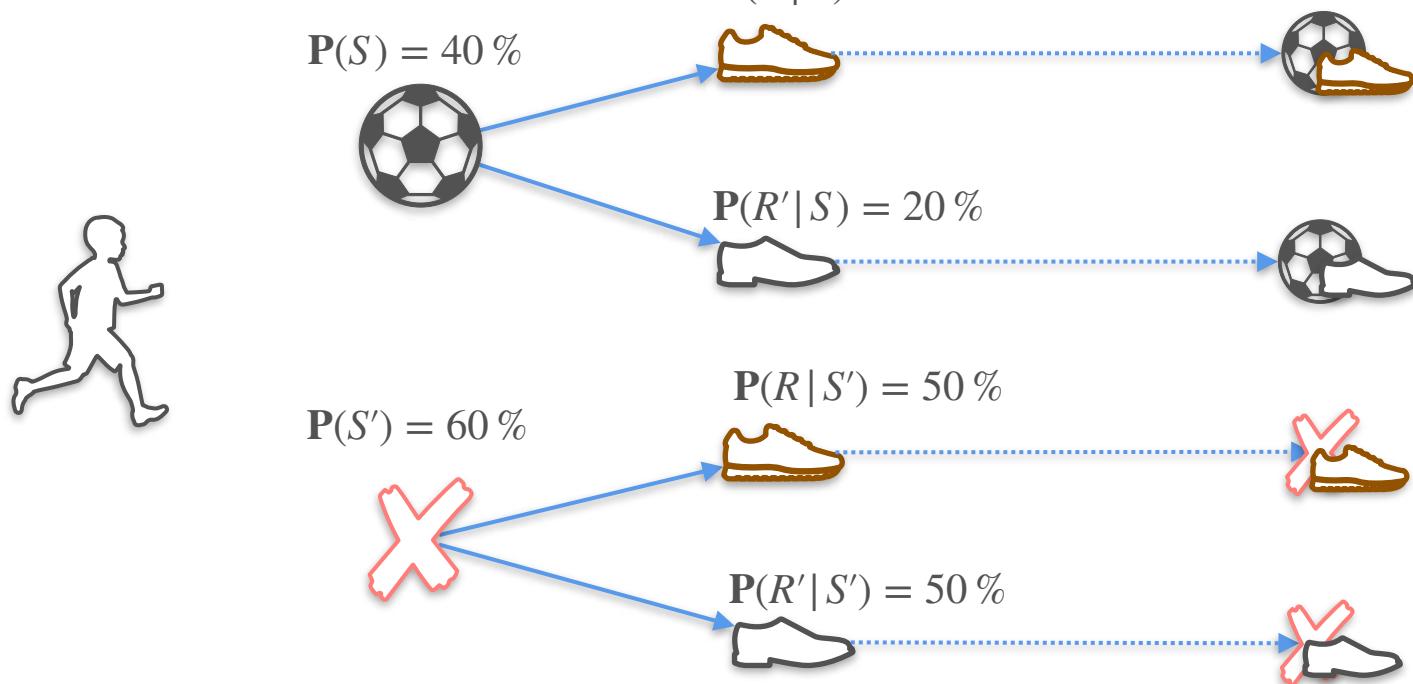
# Conditional Probability



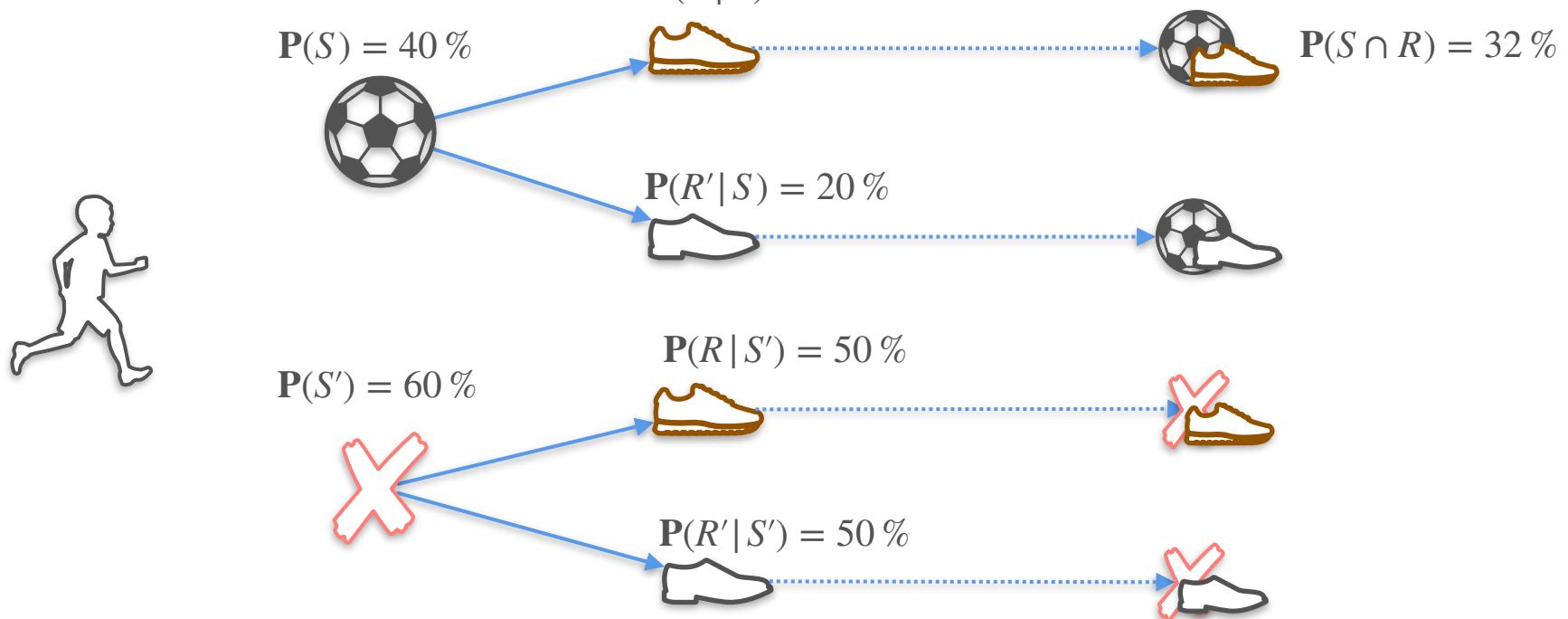
# Conditional Probability



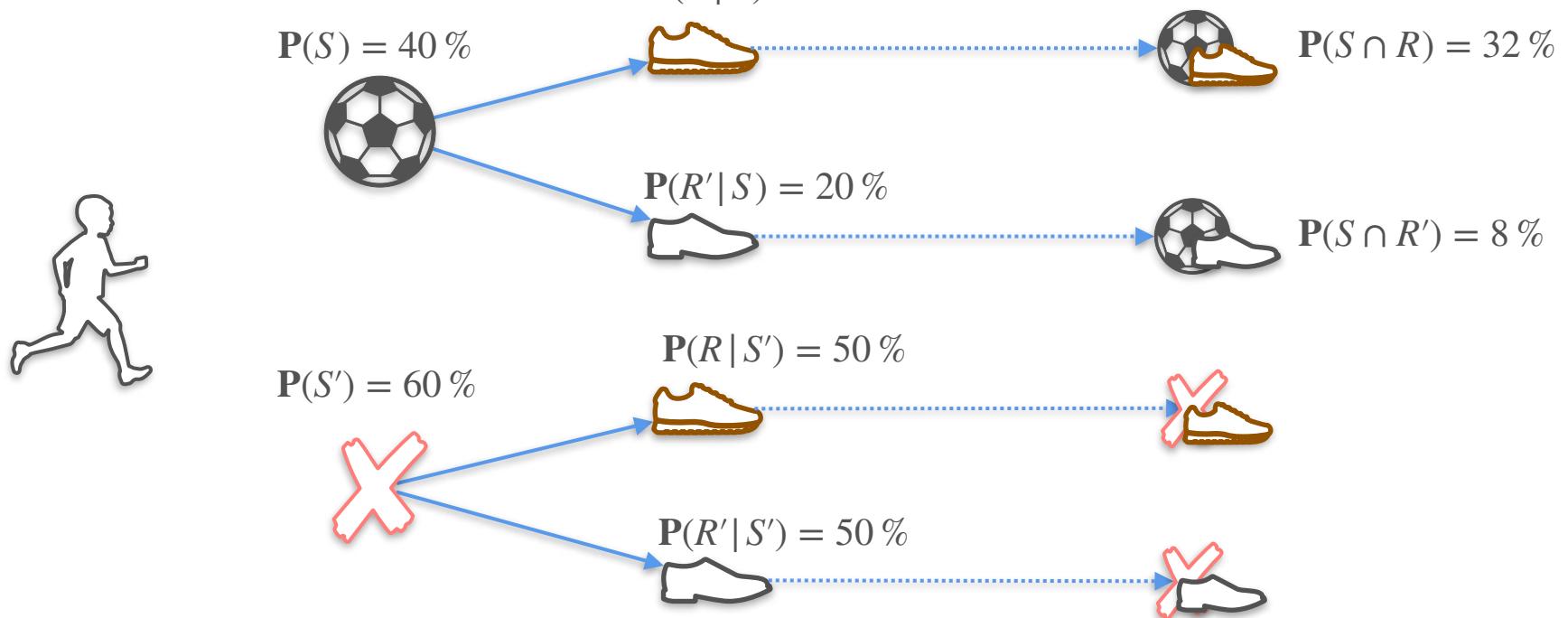
# Conditional Probability



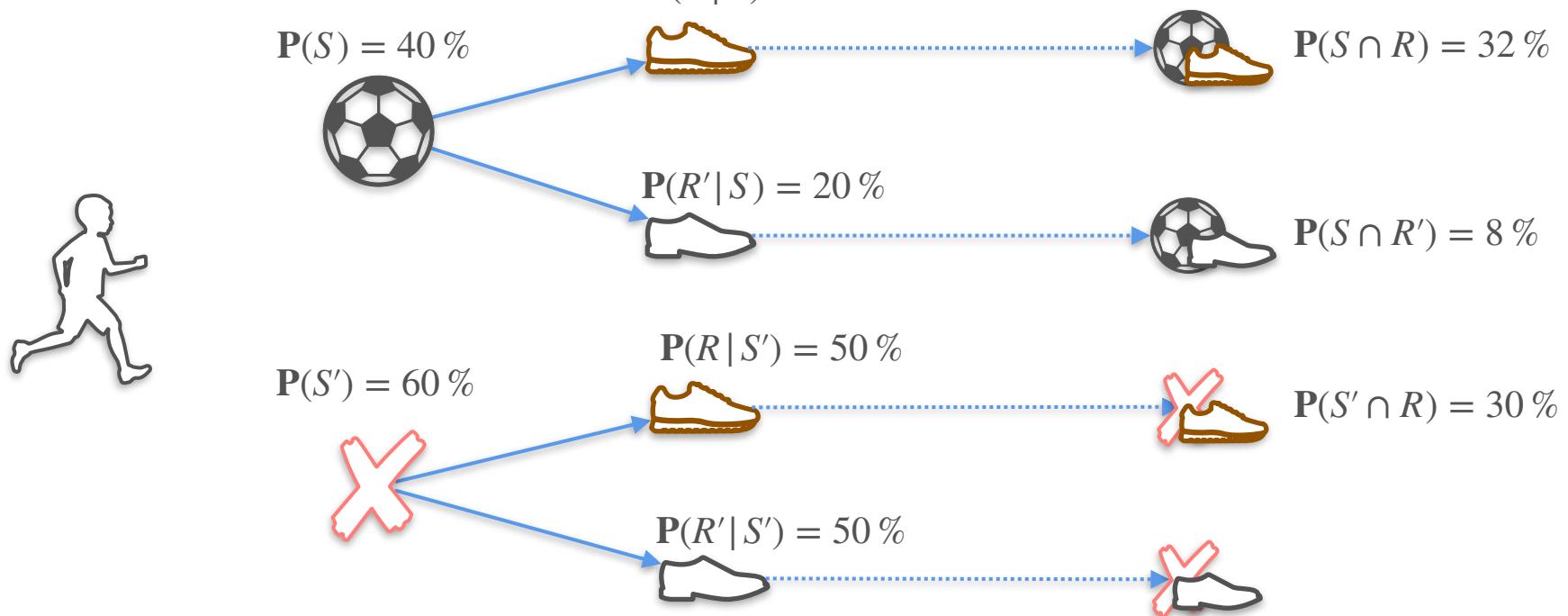
# Conditional Probability



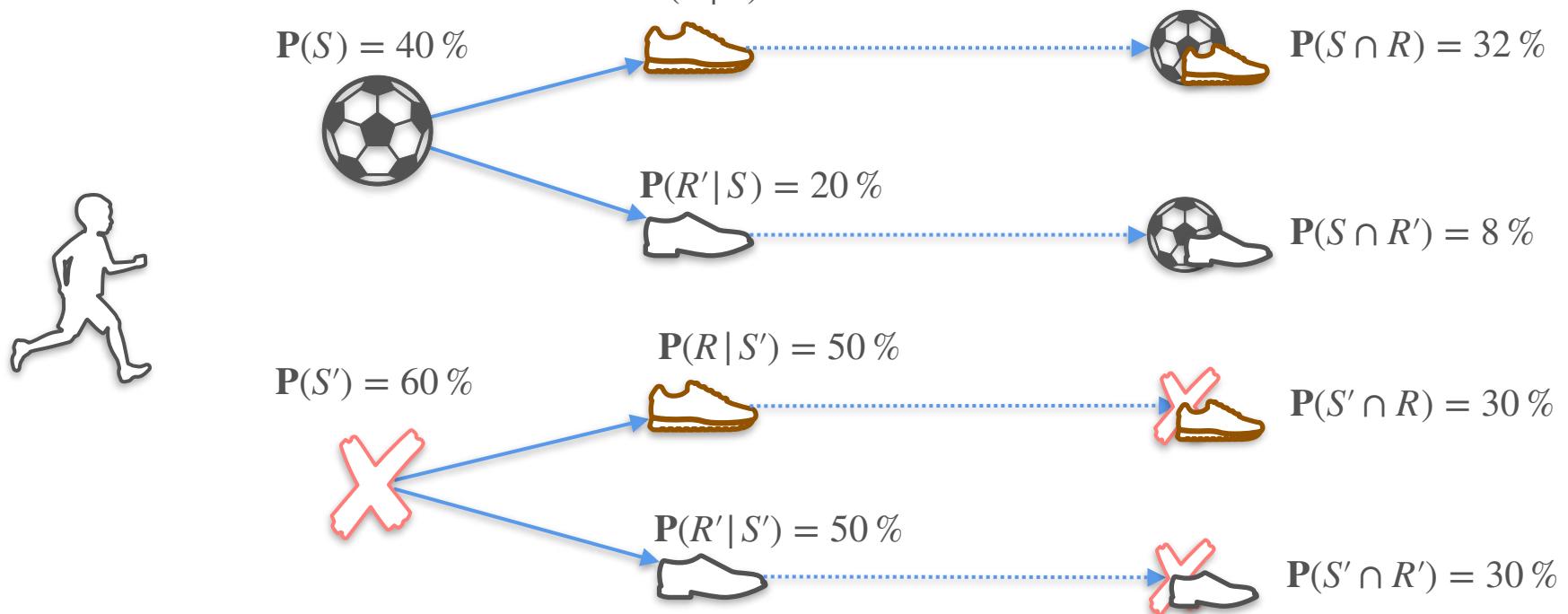
# Conditional Probability



# Conditional Probability



# Conditional Probability

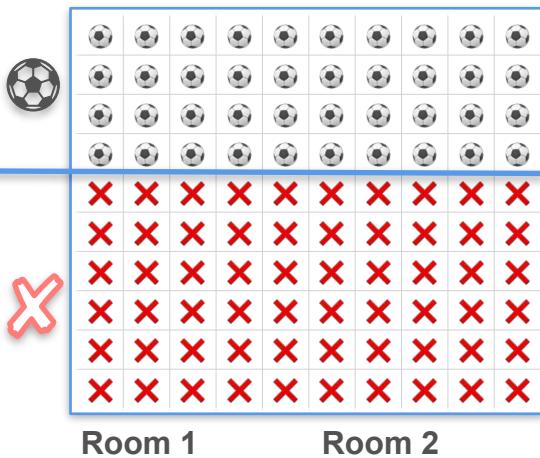


# Independent vs Dependent Events

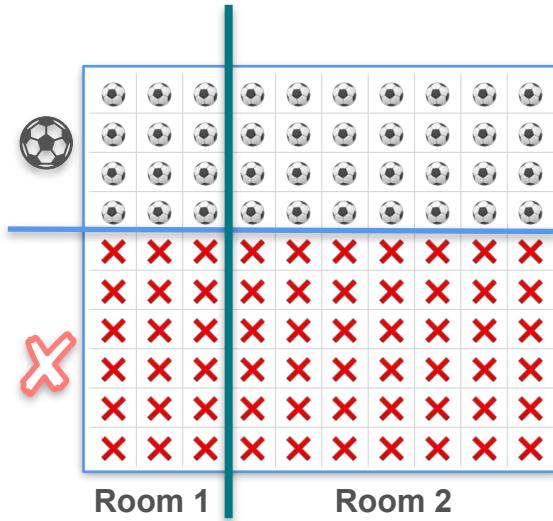
# Independent vs Dependent Events



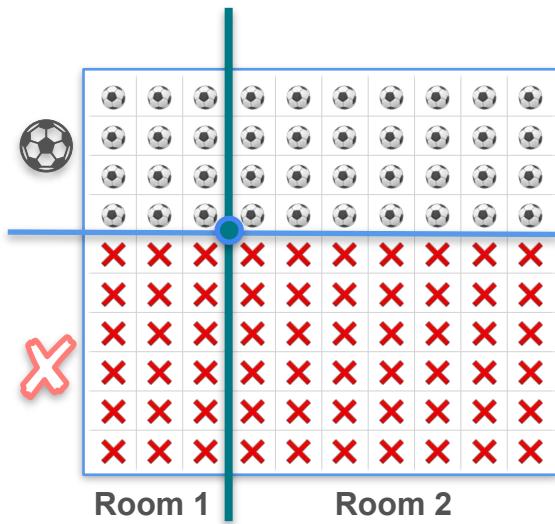
# Independent vs Dependent Events



# Independent vs Dependent Events



# Independent vs Dependent Events



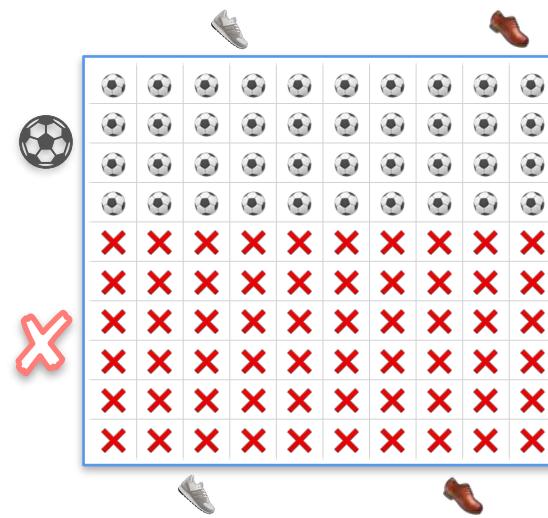
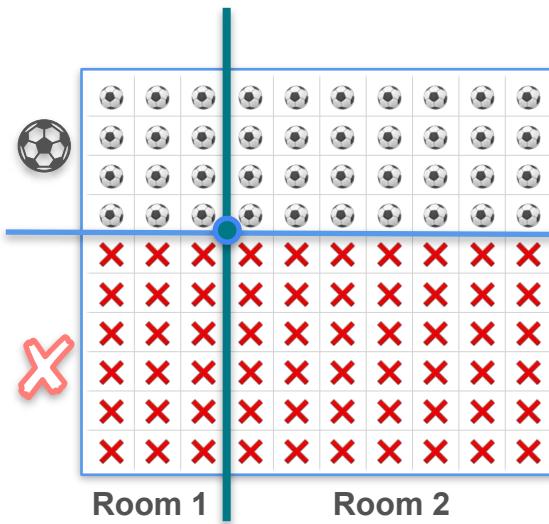
# Independent vs Dependent Events

Independent



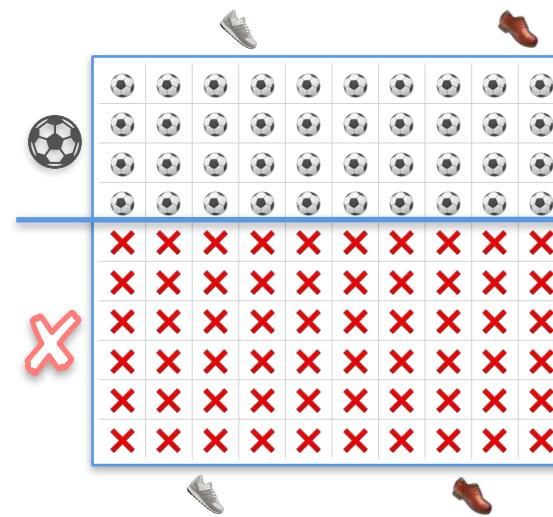
# Independent vs Dependent Events

Independent



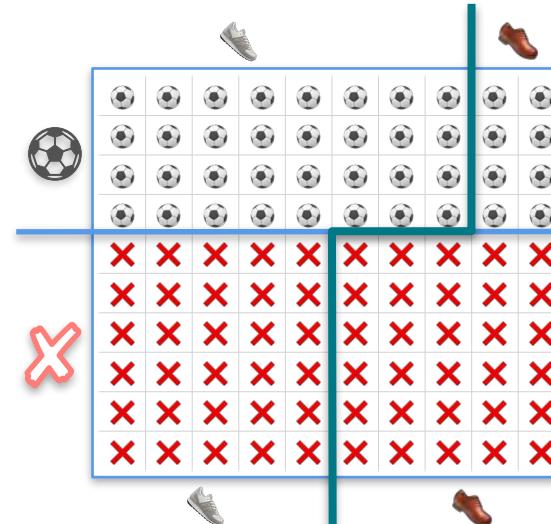
# Independent vs Dependent Events

Independent



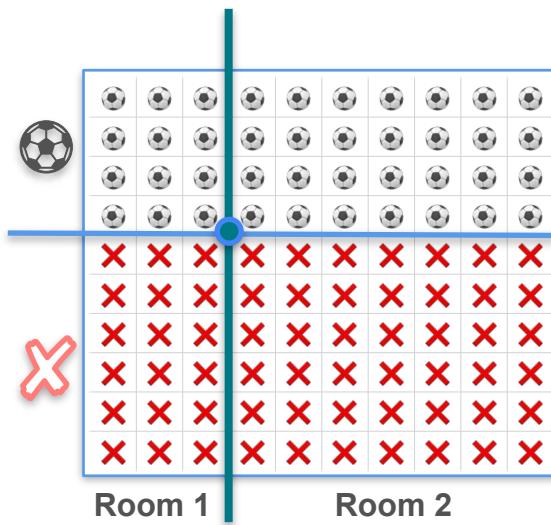
# Independent vs Dependent Events

Independent

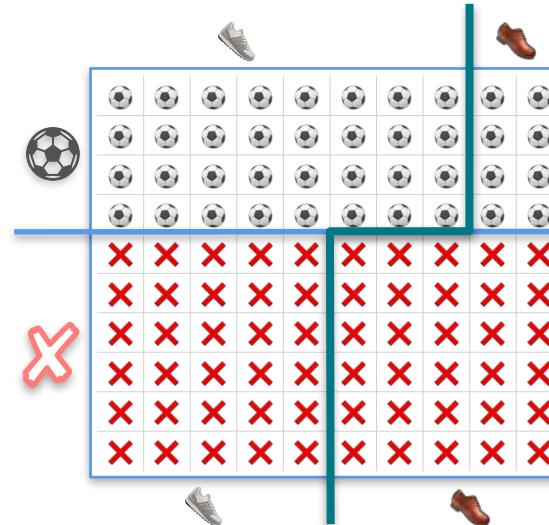


# Independent vs Dependent Events

Independent

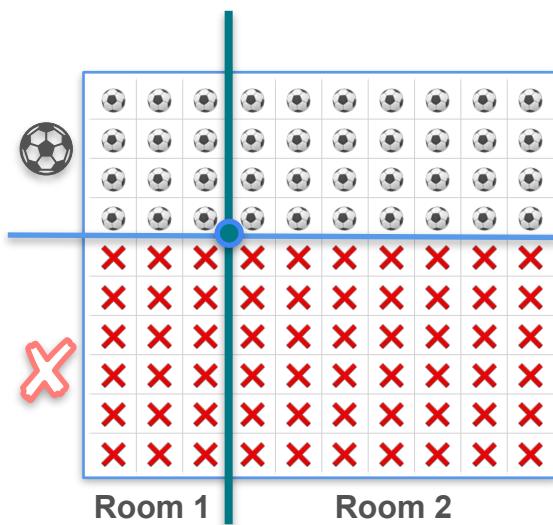


Dependent

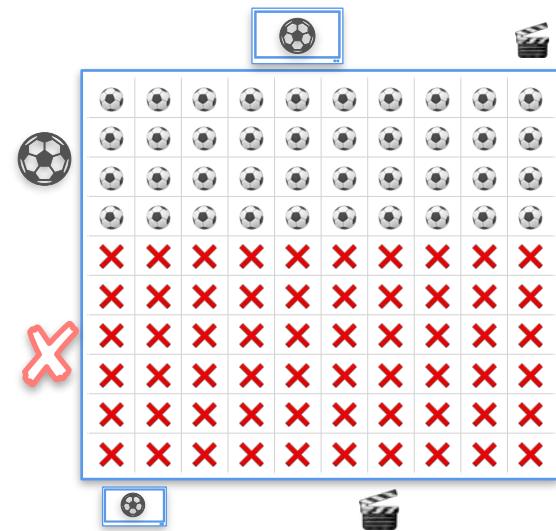
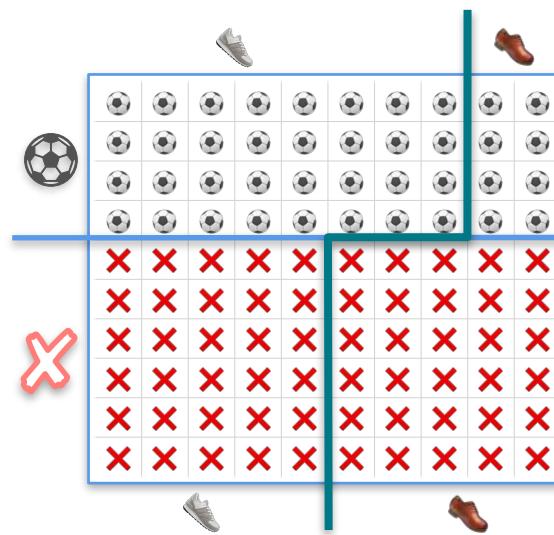


# Independent vs Dependent Events

Independent



Dependent

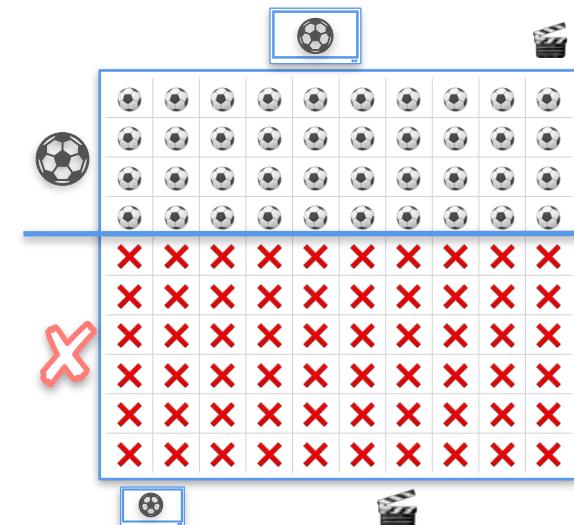
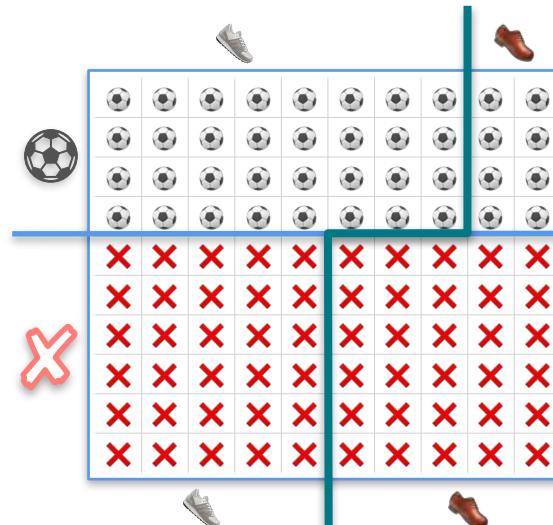


# Independent vs Dependent Events

Independent

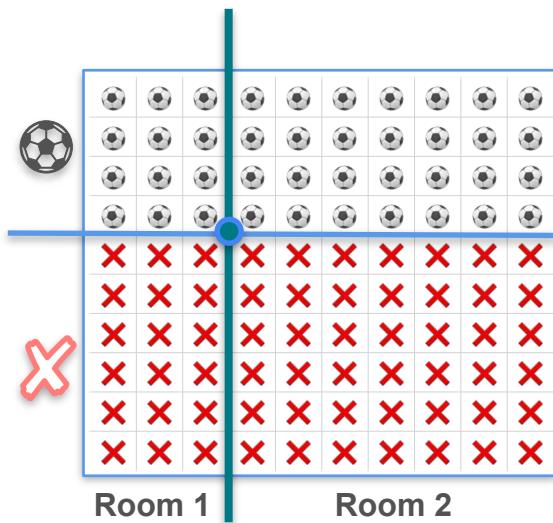


Dependent

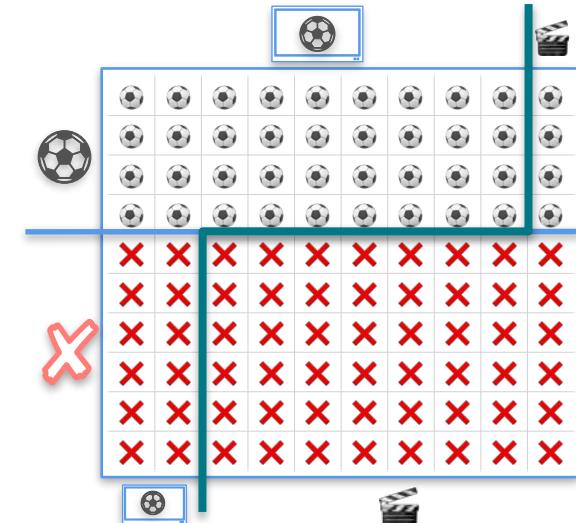
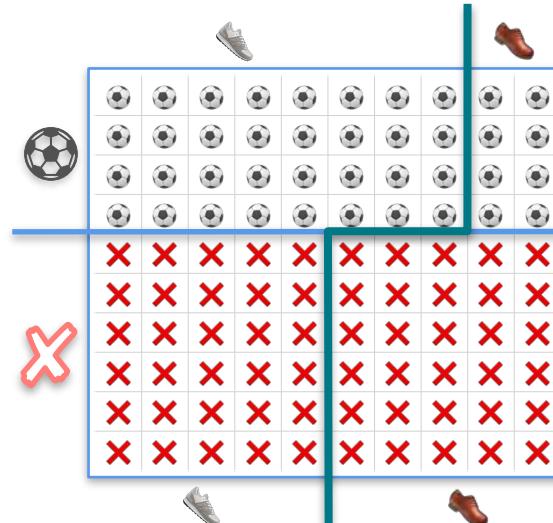


# Independent vs Dependent Events

Independent



Dependent

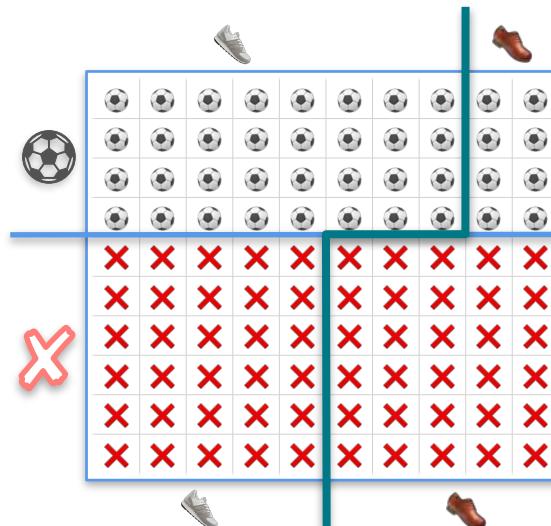


# Independent vs Dependent Events

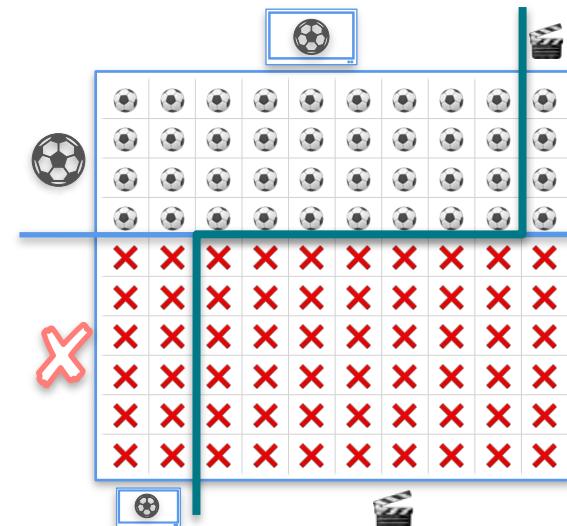
Independent



Dependent



Dependent



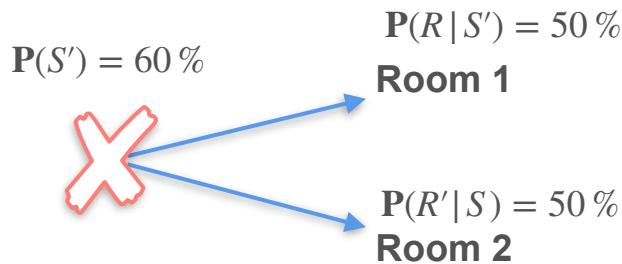
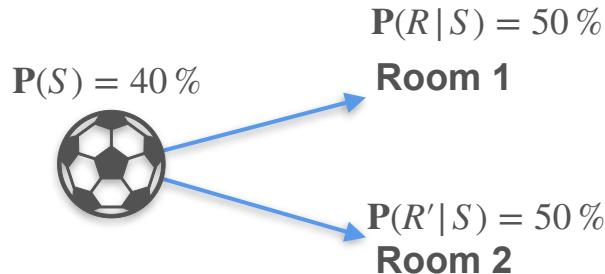
# Conditional Probability

# Conditional Probability

Independent

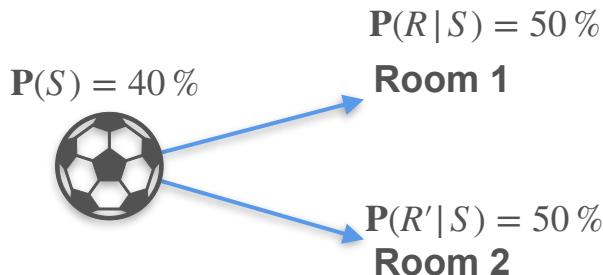
# Conditional Probability

## Independent

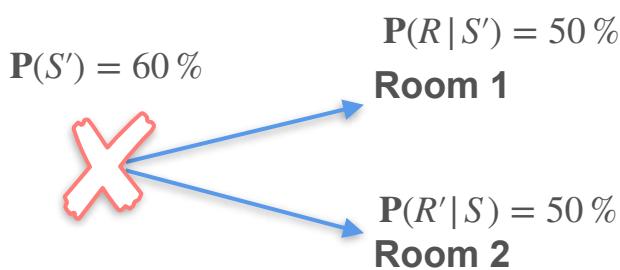


# Conditional Probability

Independent

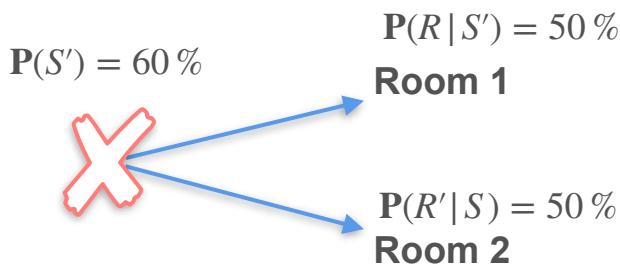
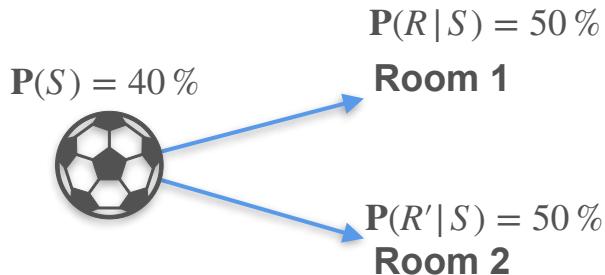


Dependent

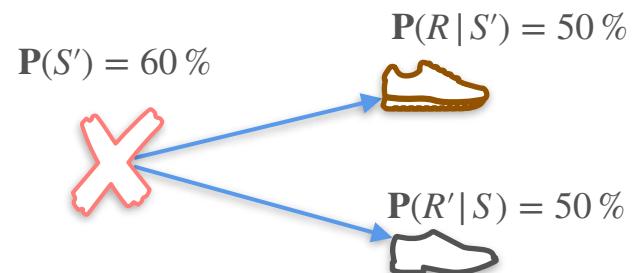
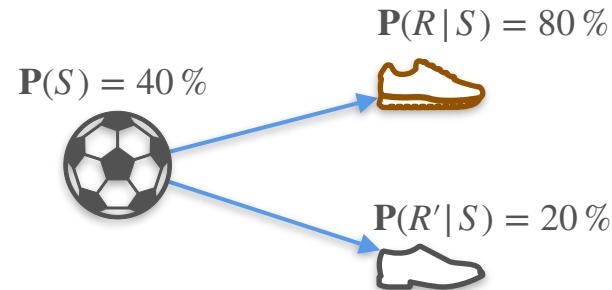


# Conditional Probability

## Independent

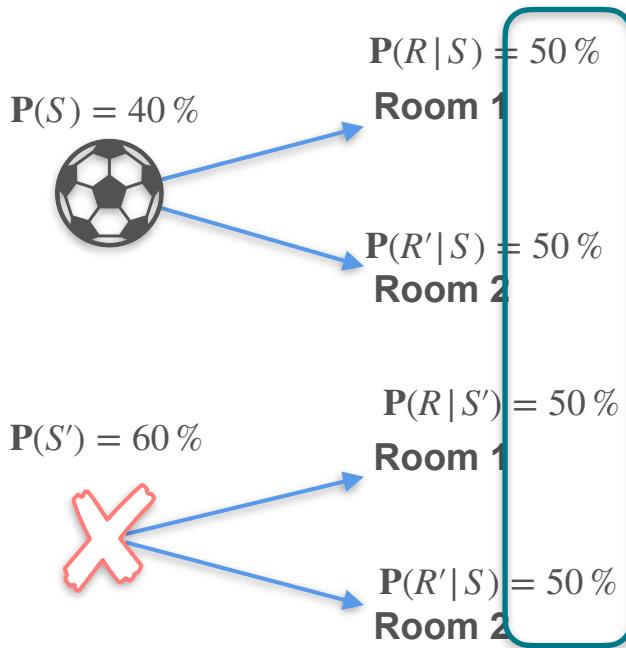


## Dependent

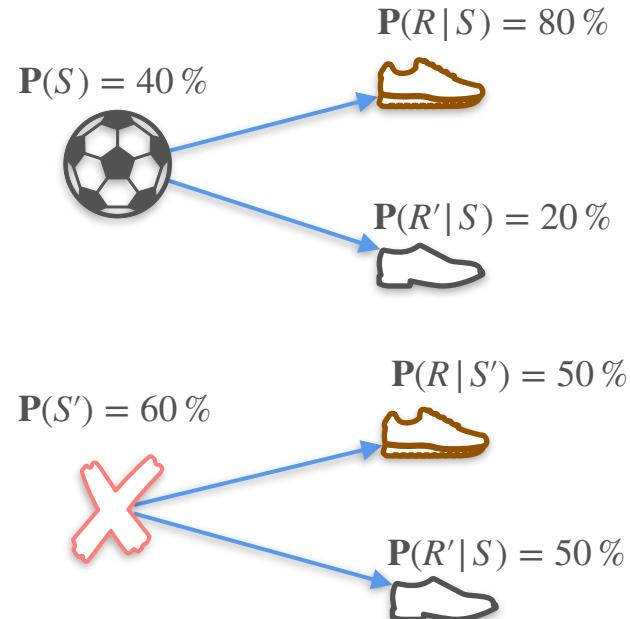


# Conditional Probability

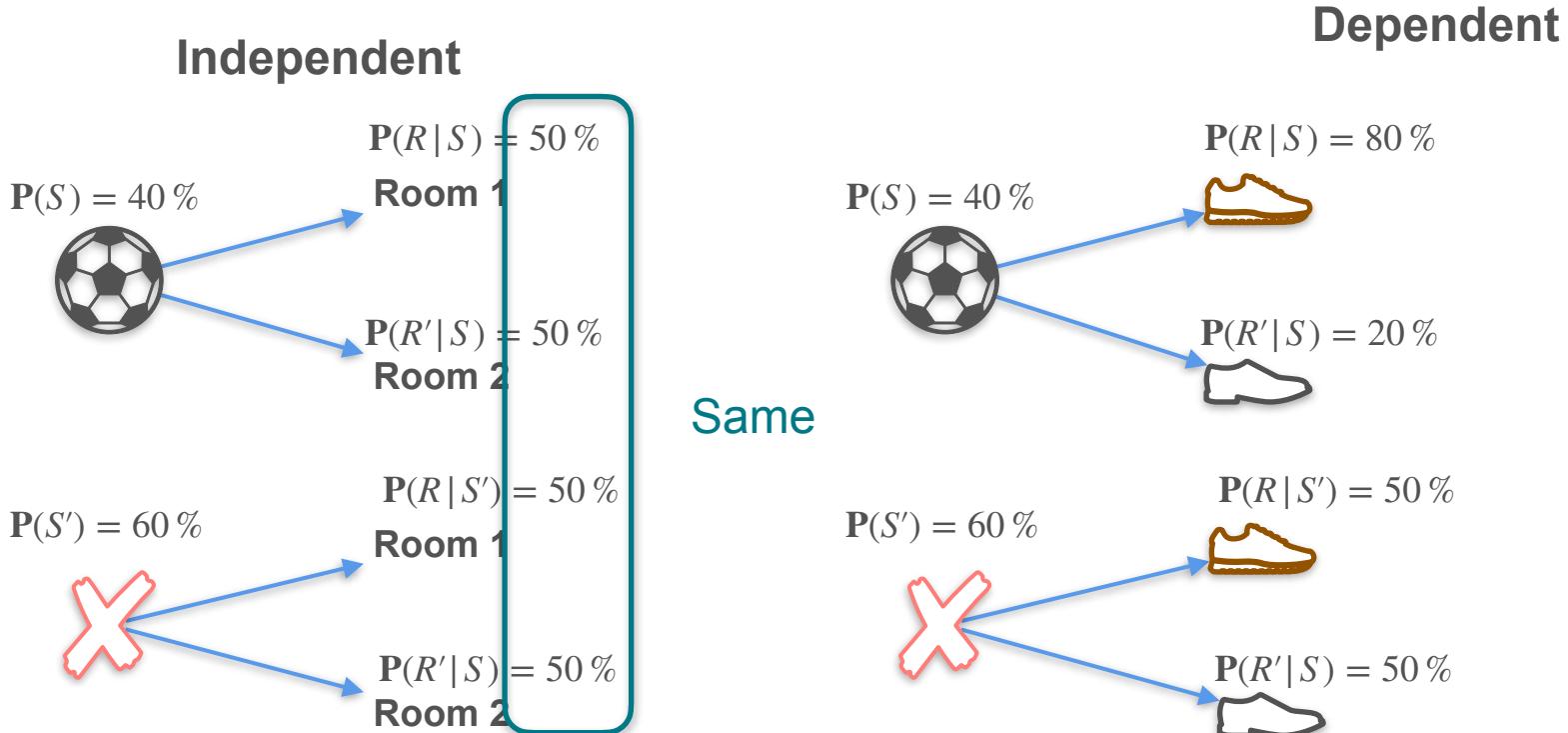
## Independent



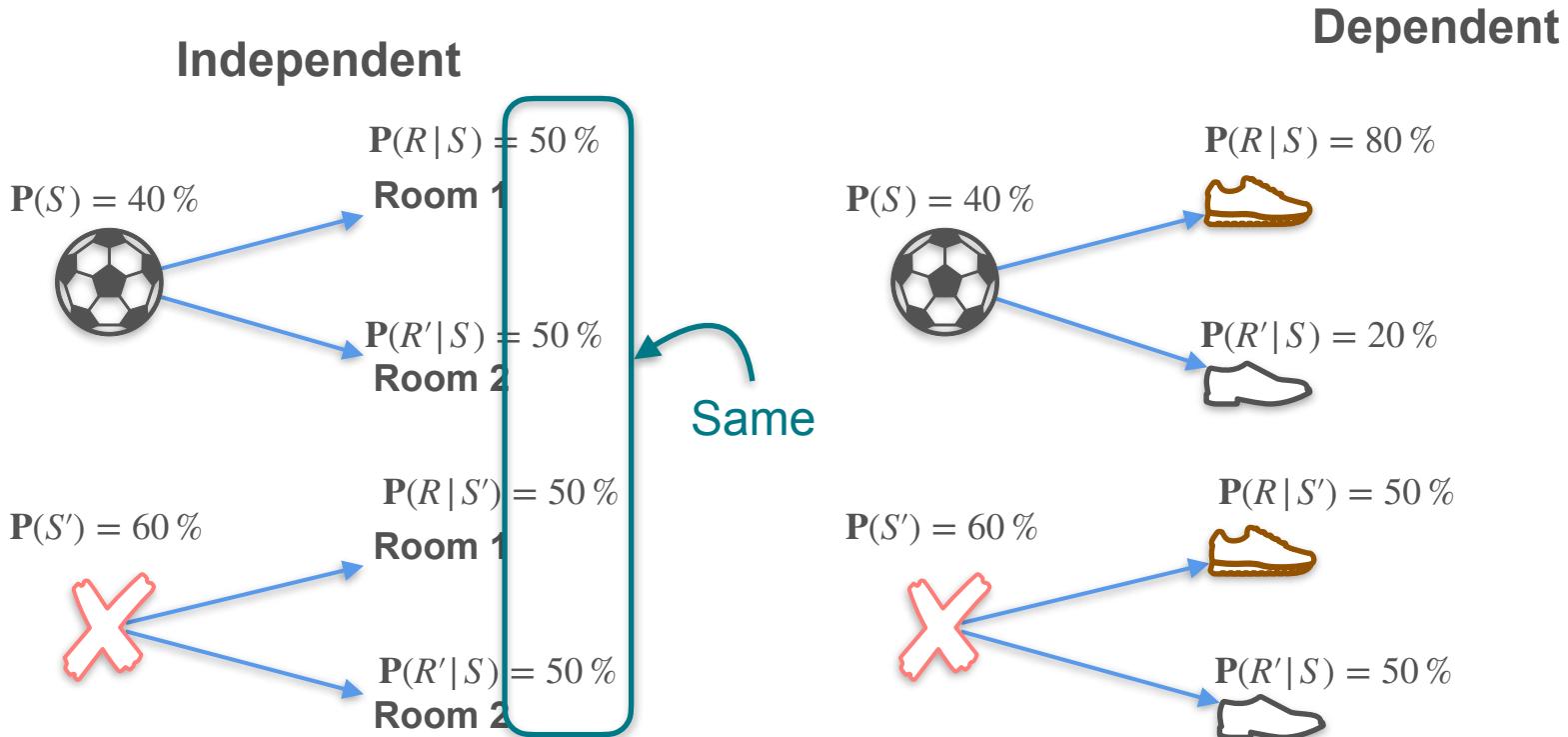
## Dependent



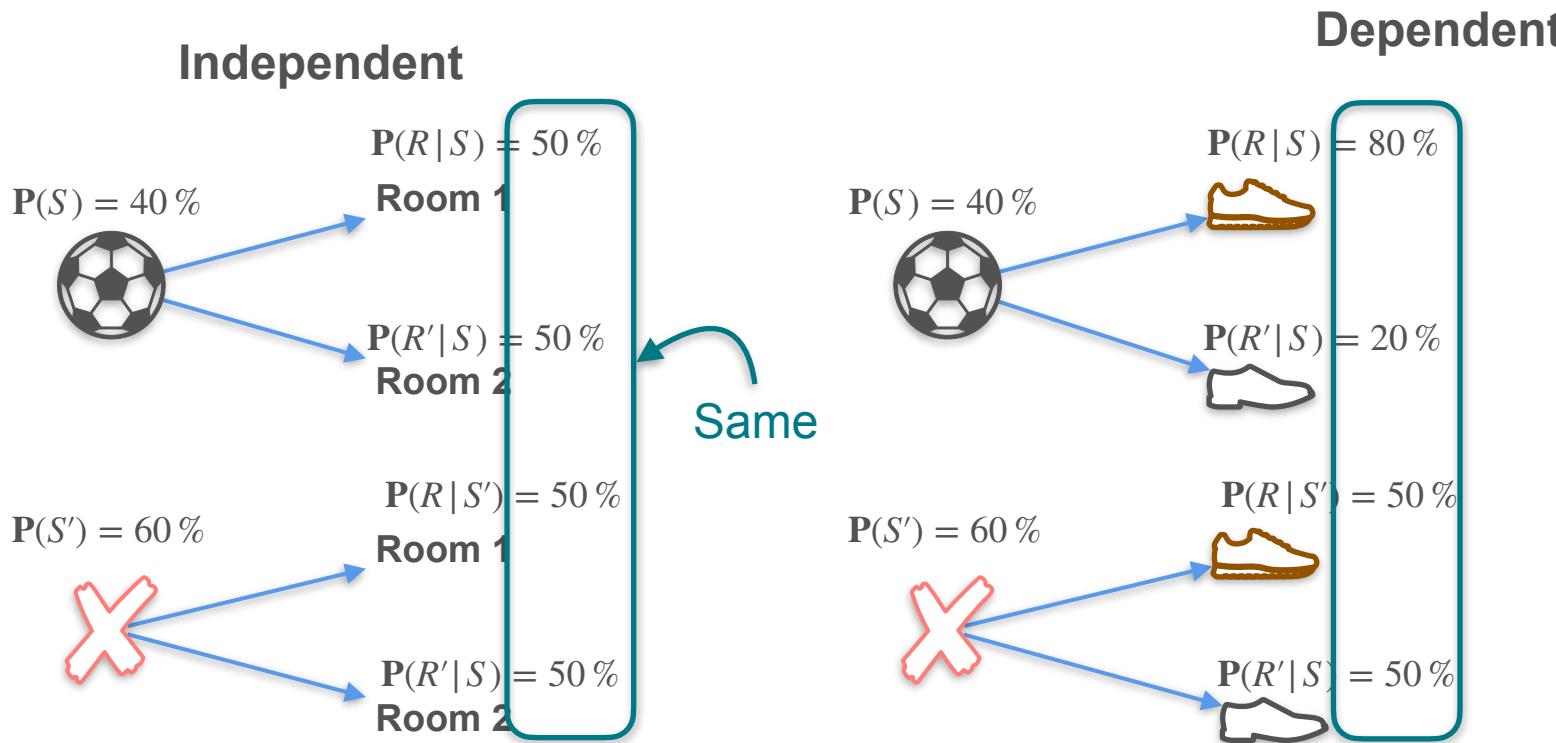
# Conditional Probability



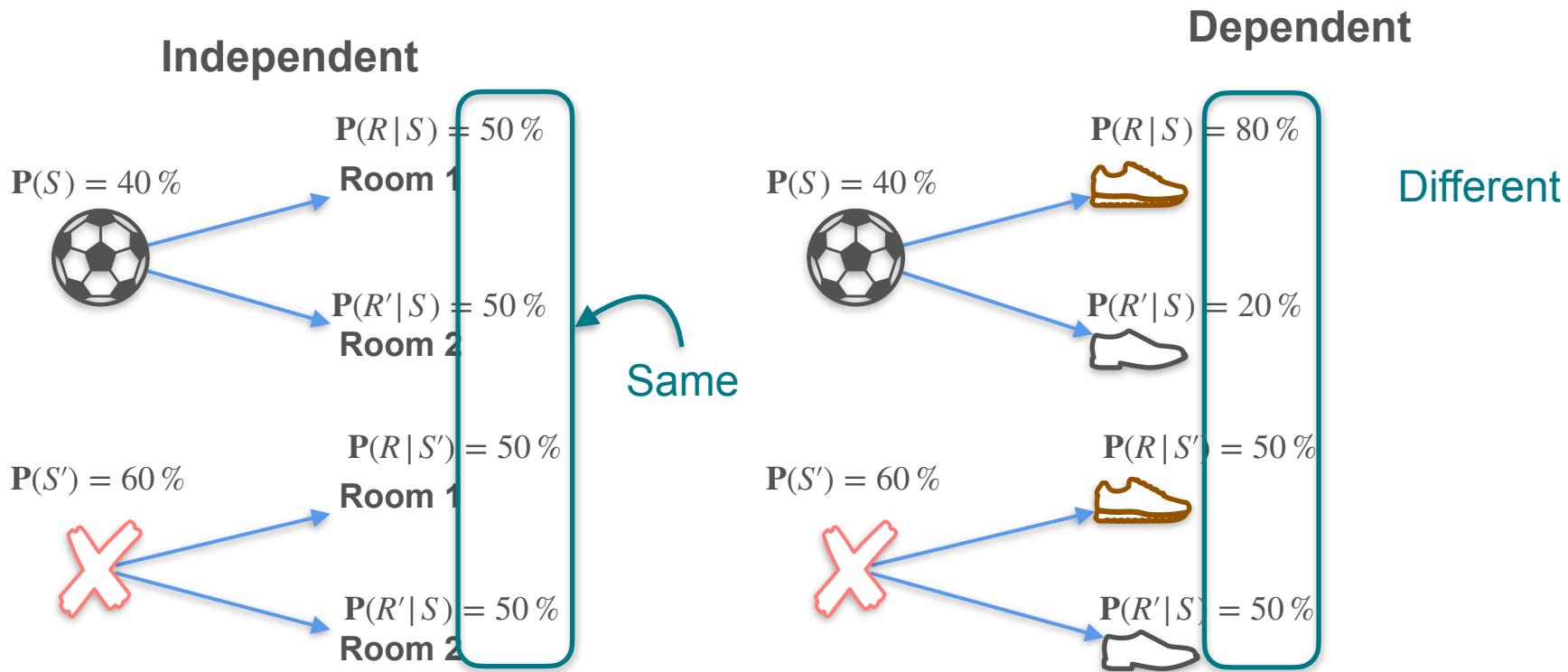
# Conditional Probability



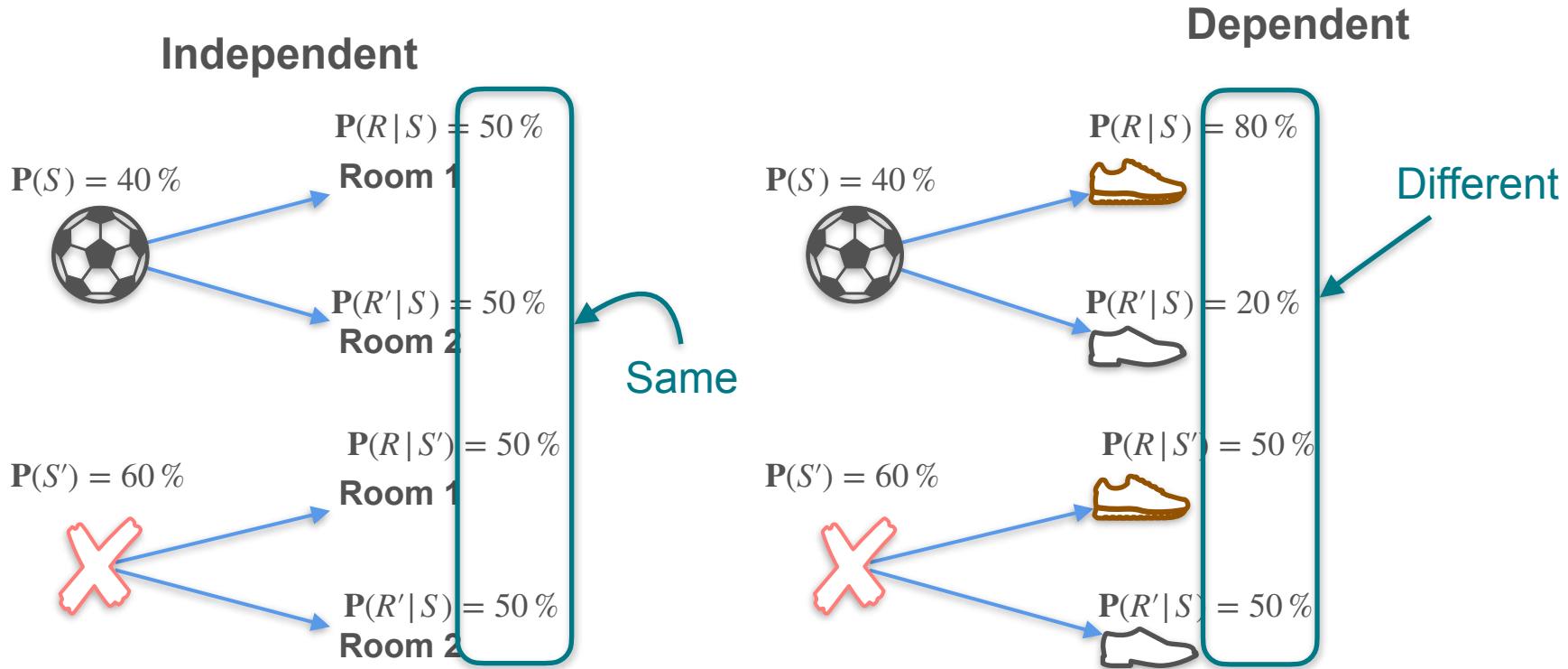
# Conditional Probability



# Conditional Probability



# Conditional Probability





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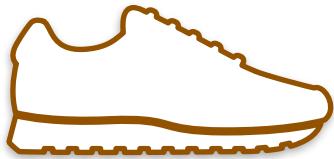
# Introduction to probability

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## Bayes theorem

# Bayes Theorem - Motivation

# Bayes Theorem - Motivation



60%

# Bayes Theorem - Motivation

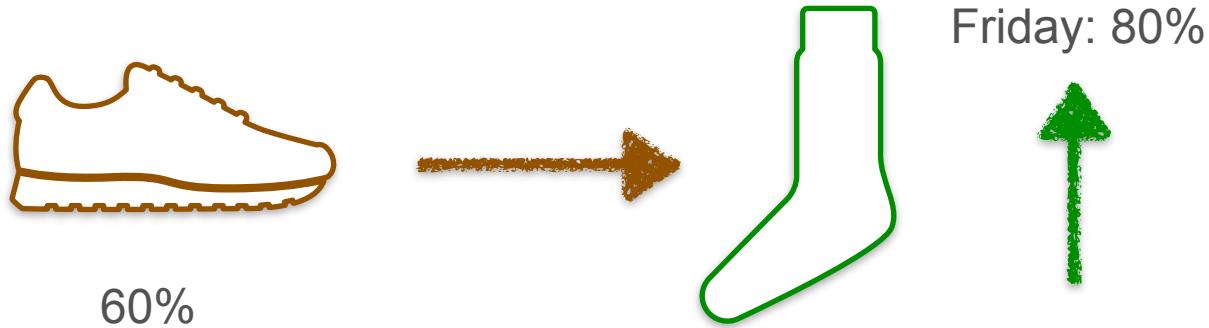


60%

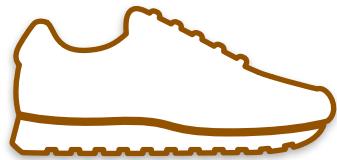
# Bayes Theorem - Motivation



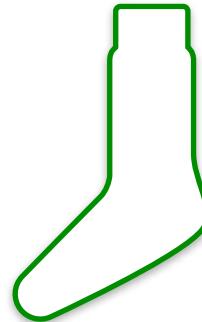
# Bayes Theorem - Motivation



# Bayes Theorem - Motivation



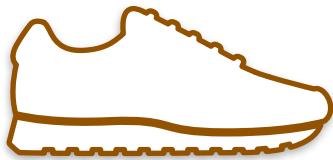
60%



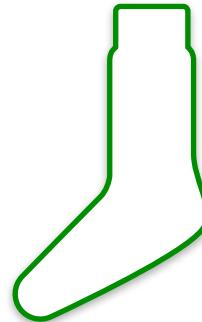
Friday: 80%



# Bayes Theorem - Motivation



60%



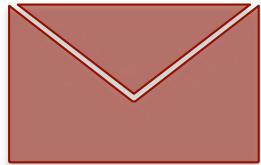
Friday: 80%



What is the probability that a customer will purchase a pair of socks given that they purchased shoes?

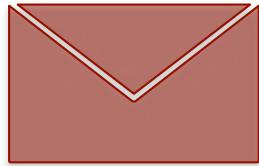
# Bayes Theorem: Motivation

# Bayes Theorem: Motivation



spam

# Bayes Theorem: Motivation

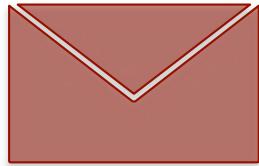


spam



contains  
“lottery”

# Bayes Theorem: Motivation



spam

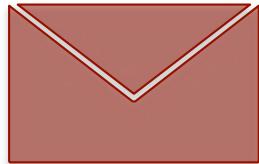


contains  
“lottery”

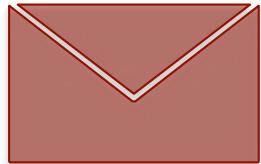
so



# Bayes Theorem: Motivation



# Bayes Theorem: Motivation

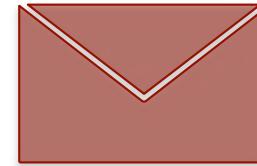


spam

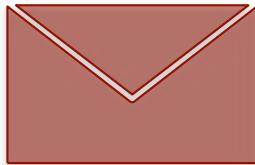


contains  
“lottery”

so

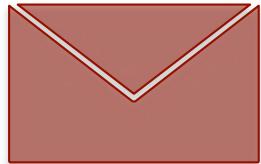


spam



spam

# Bayes Theorem: Motivation

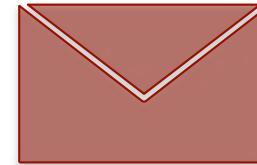


spam

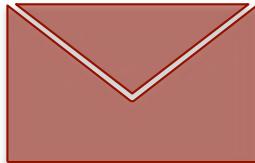


contains  
“lottery”

so



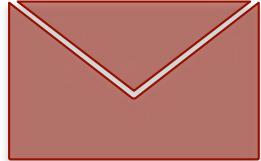
spam



spam

so

# Bayes Theorem: Motivation

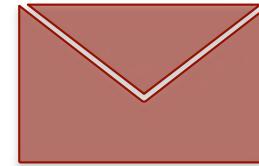


spam

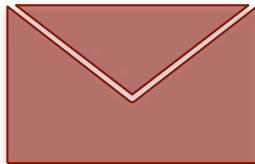


contains  
“lottery”

so



spam



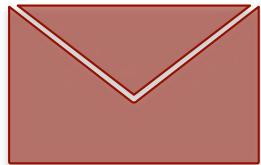
spam

so



contains  
lottery

# Bayes Theorem: Motivation

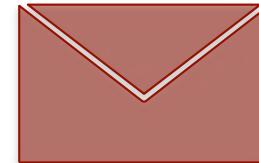


spam

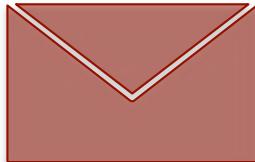


contains  
“lottery”

so



spam



spam

?



contains  
lottery

so

# Bayes Theorem: Intuition

# Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people

# Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people

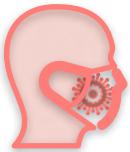


99% Effective

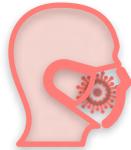
# Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people

Diagnosed Sick



99

Diagnosed Healthy



1

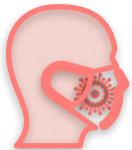


99% Effective

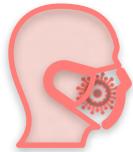
# Bayes Theorem: Intuition



1,000,000 people

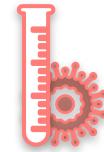


1 / 10,000 people



100 people

Diagnosed Sick



99

Diagnosed Healthy



1



99% Effective



100 people



1

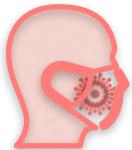


99

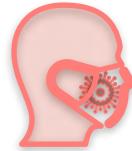
# Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



100 people



1



99

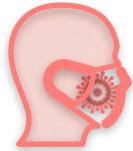
# Bayes Theorem: Intuition



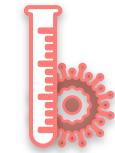
1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



100 people



1



99

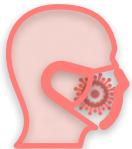


What's the probability that **you are sick**

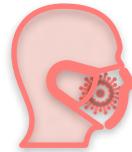
# Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



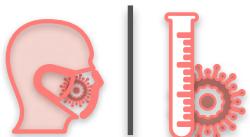
100 people



1



99

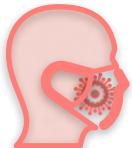


What's the probability that **you are sick**  
**GIVEN that you tested sick?**

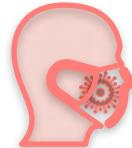
# Bayes Theorem: Intuition



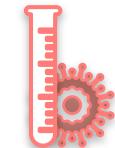
1,000,000 people



1 / 10,000 people



100 people



99



1



99% Effective



Tested Sick



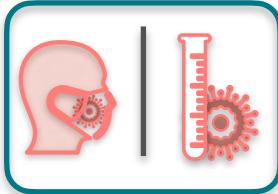
100 people



1

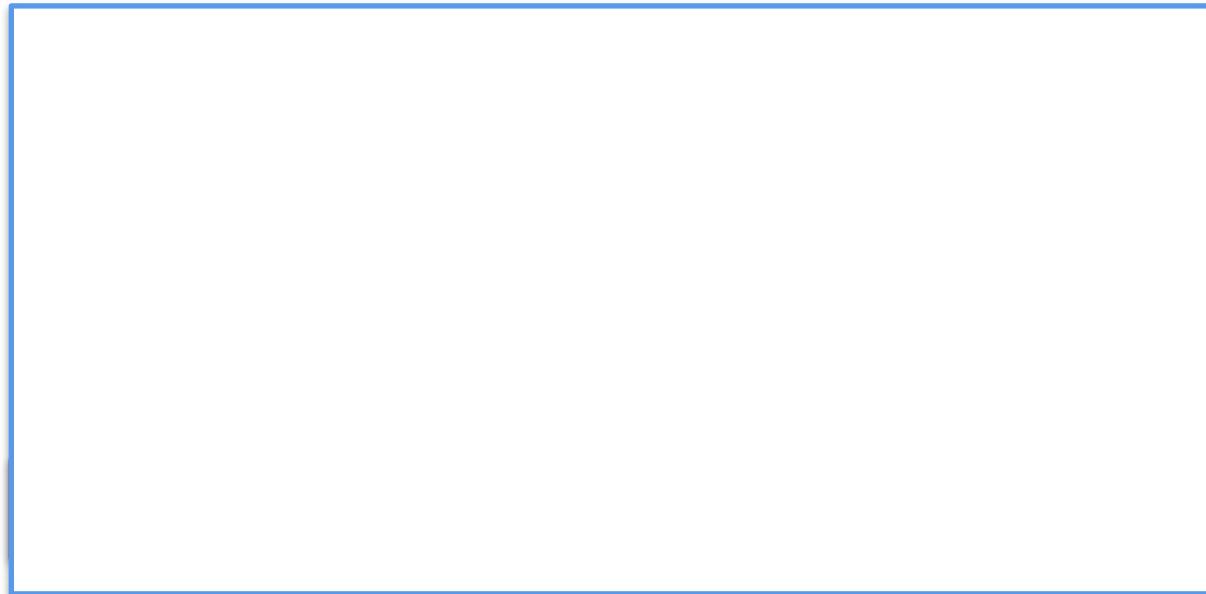


99



What's the probability that **you are sick**  
**GIVEN that you tested sick?**

# Bayes Theorem: Intuition



1,000,000 people

# Bayes Theorem: Intuition



Healthy

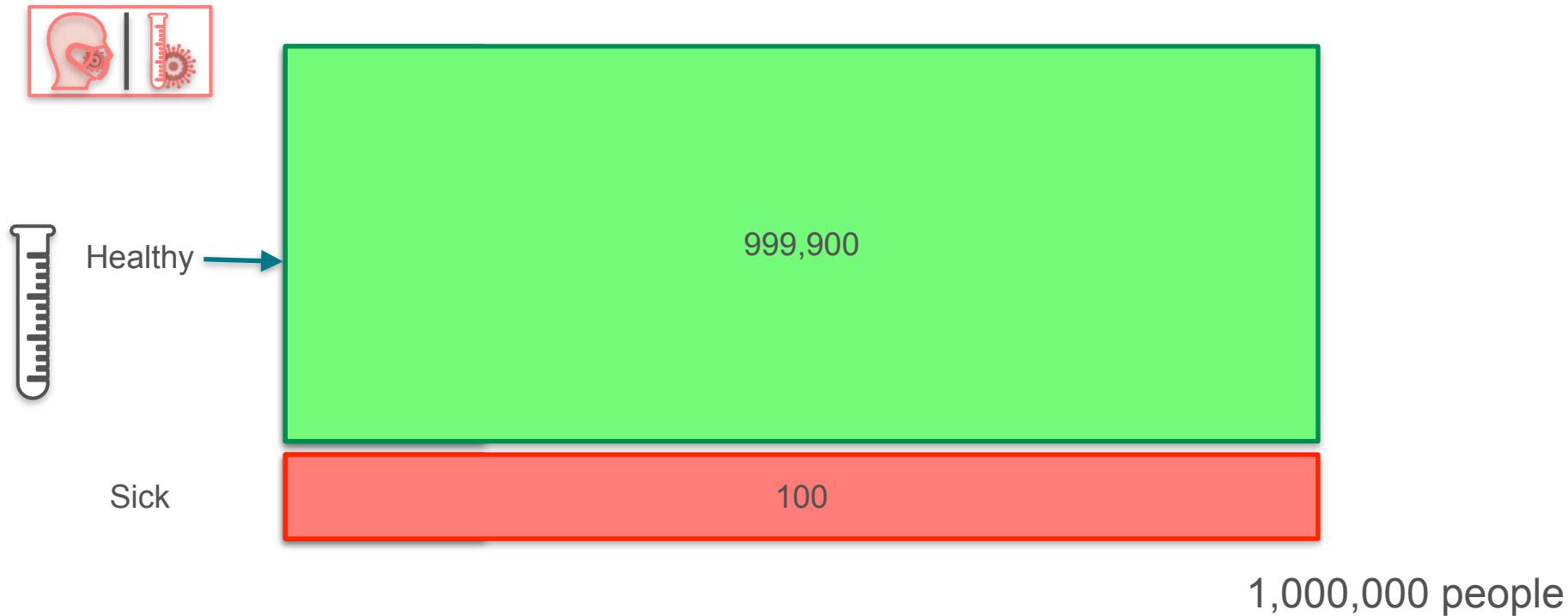
999,900

Sick

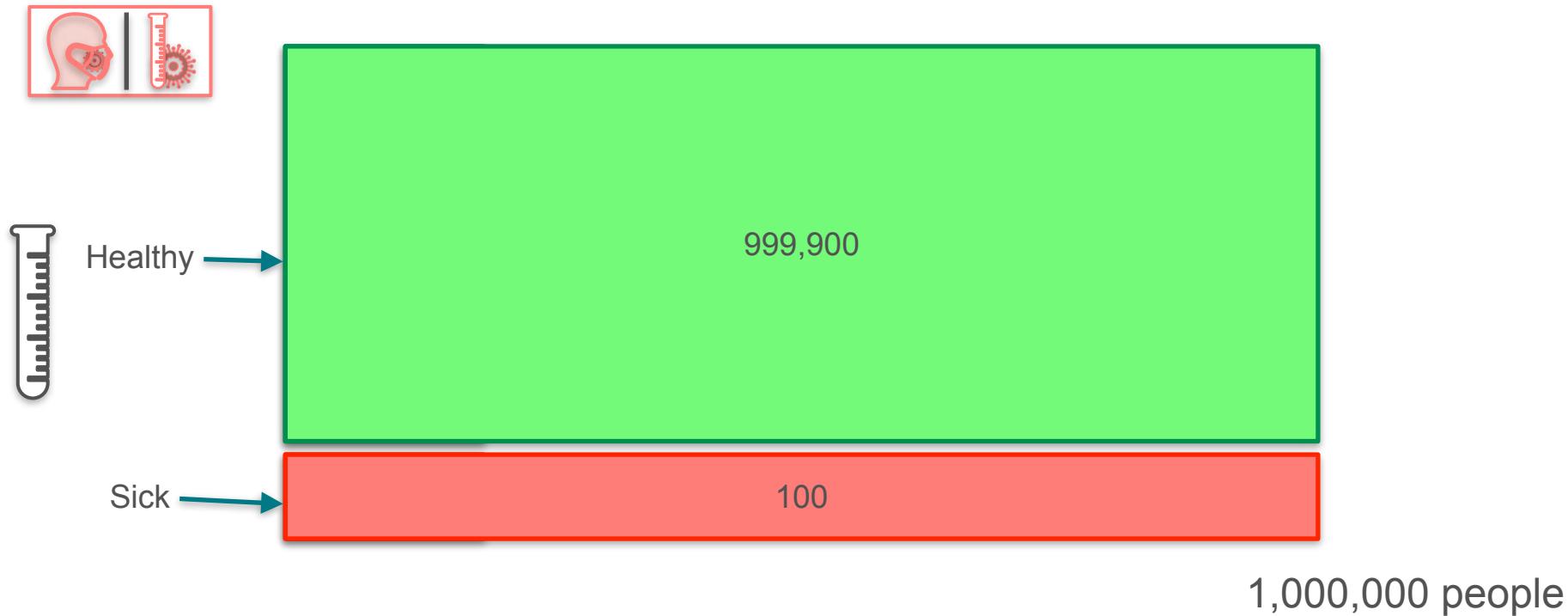
100

1,000,000 people

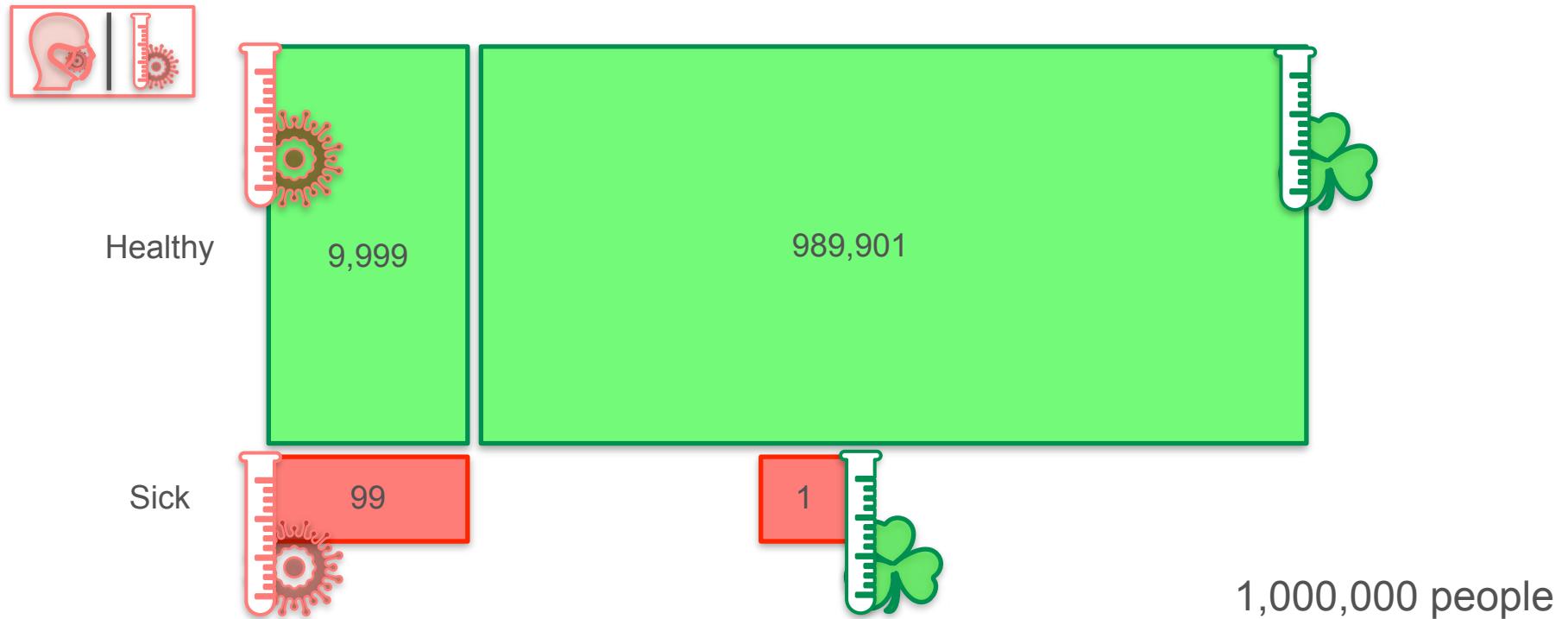
# Bayes Theorem: Intuition



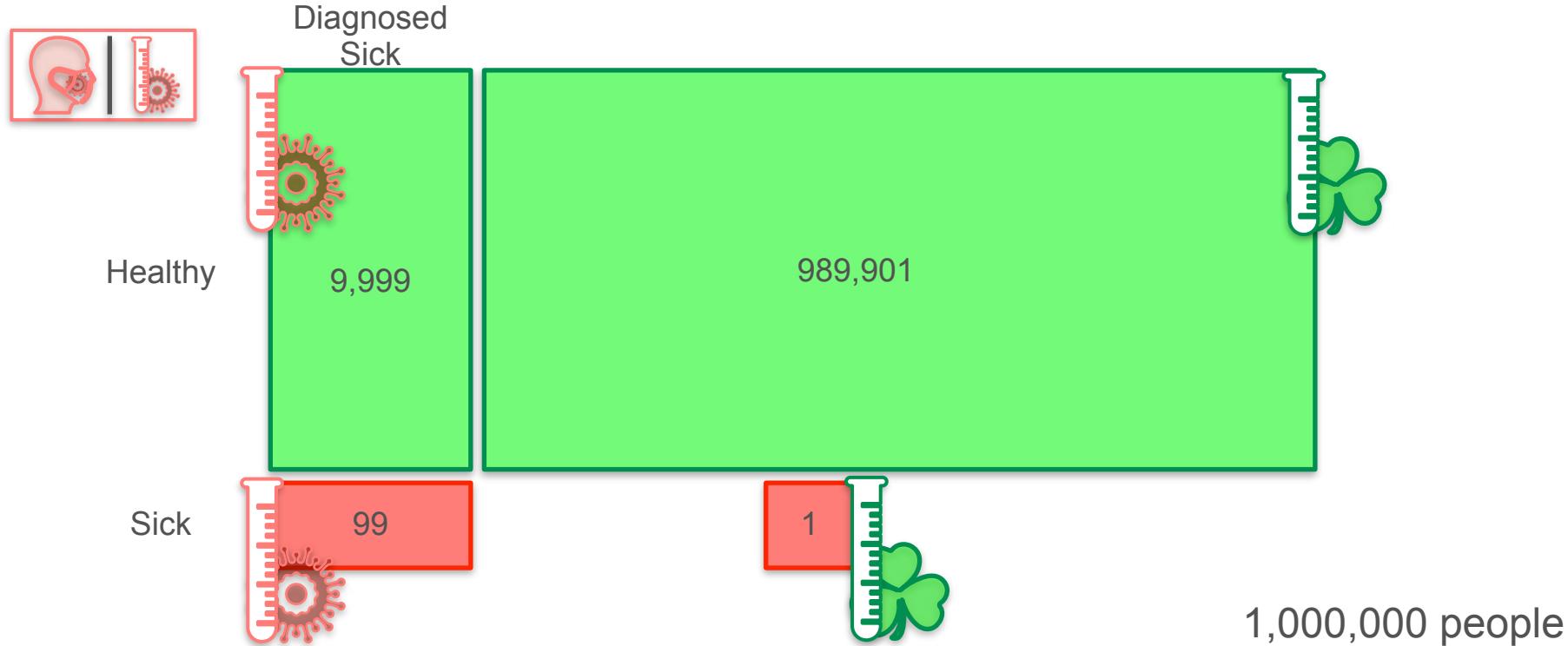
# Bayes Theorem: Intuition



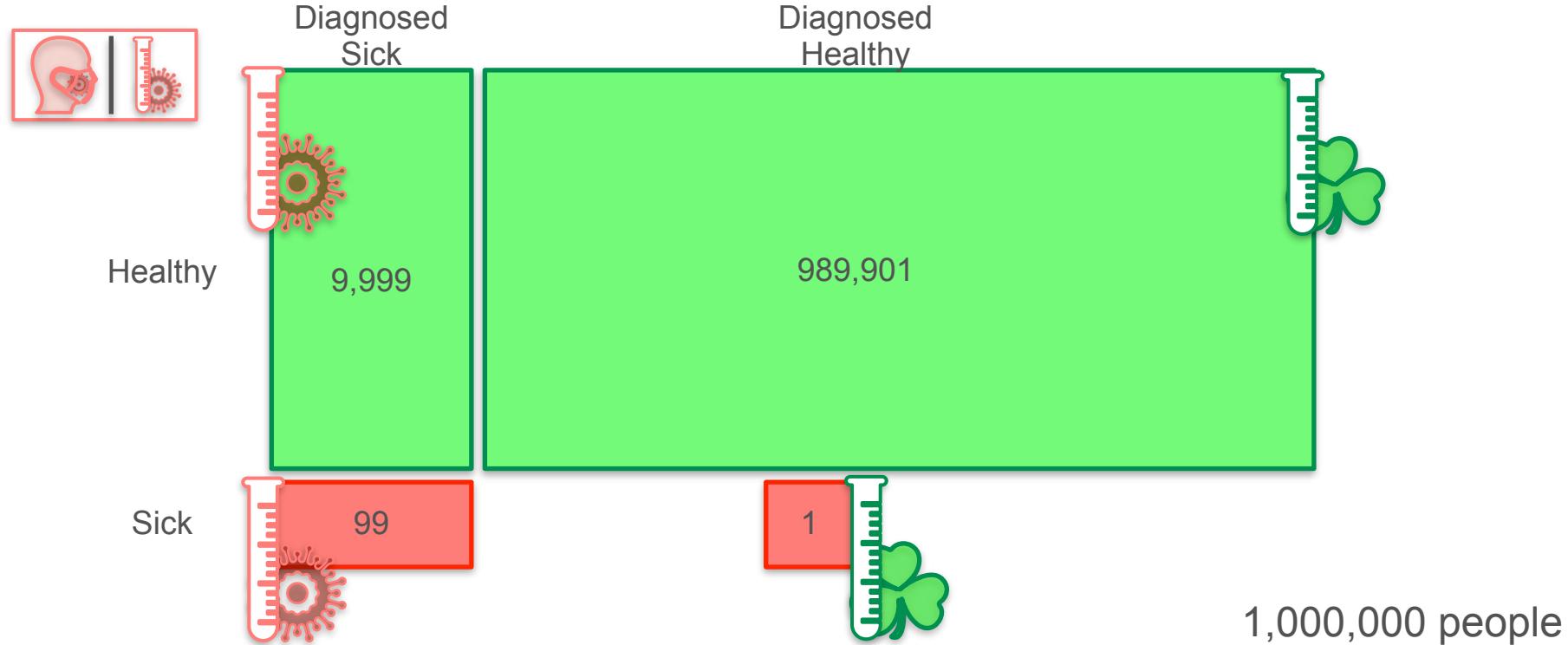
# Bayes Theorem: Intuition



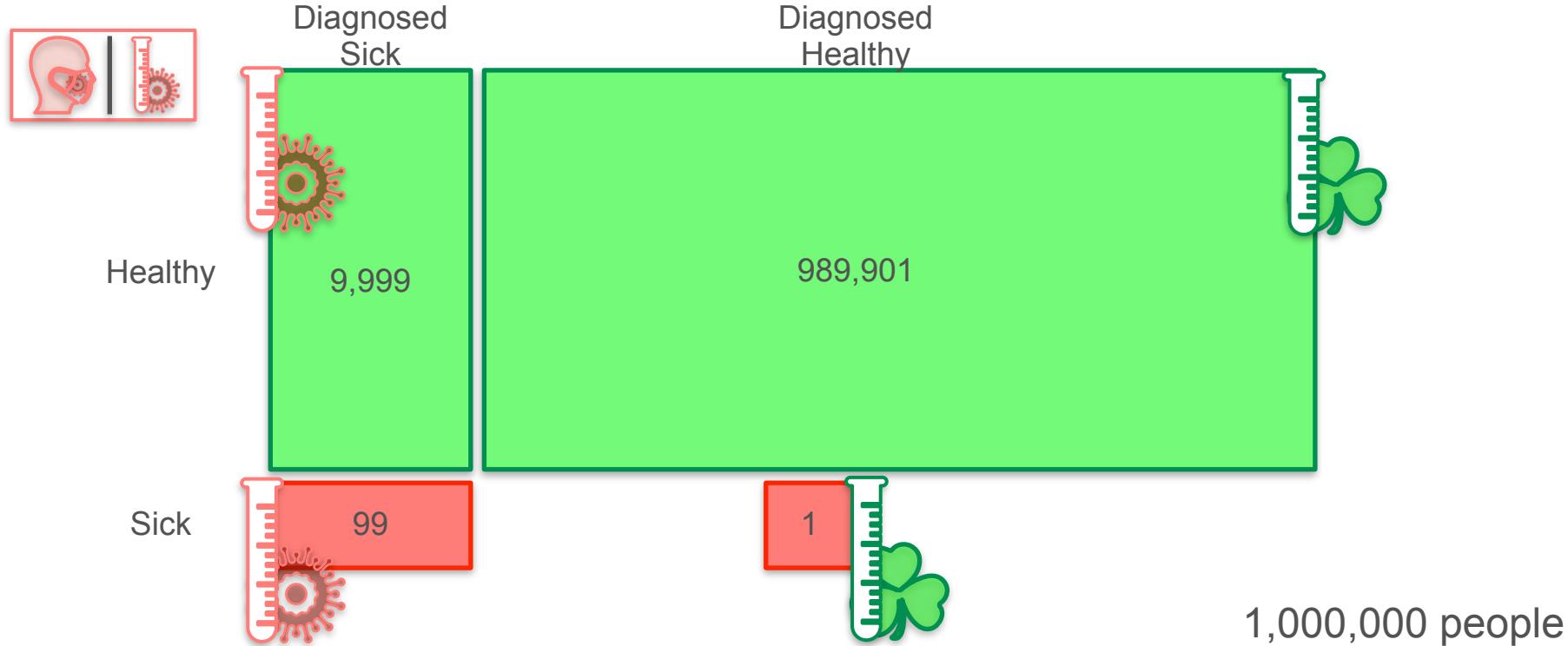
# Bayes Theorem: Intuition



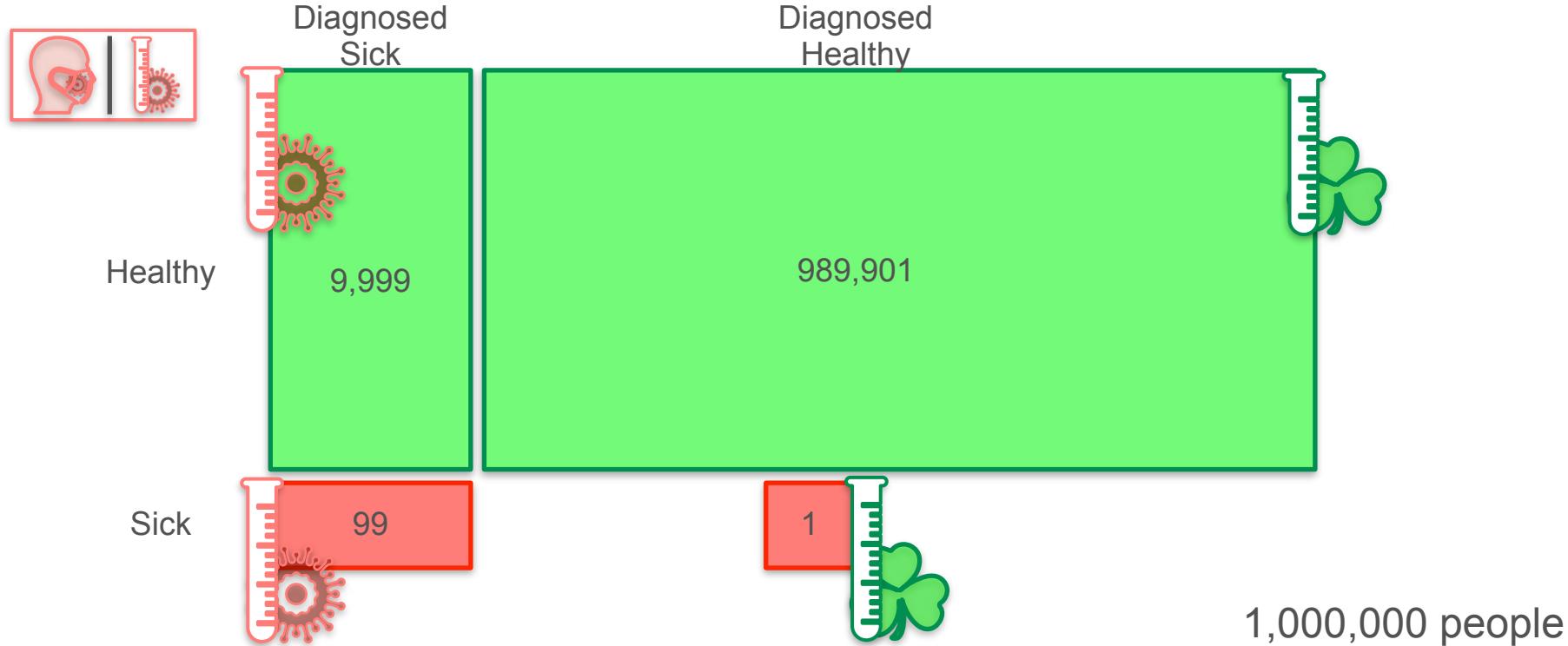
# Bayes Theorem: Intuition



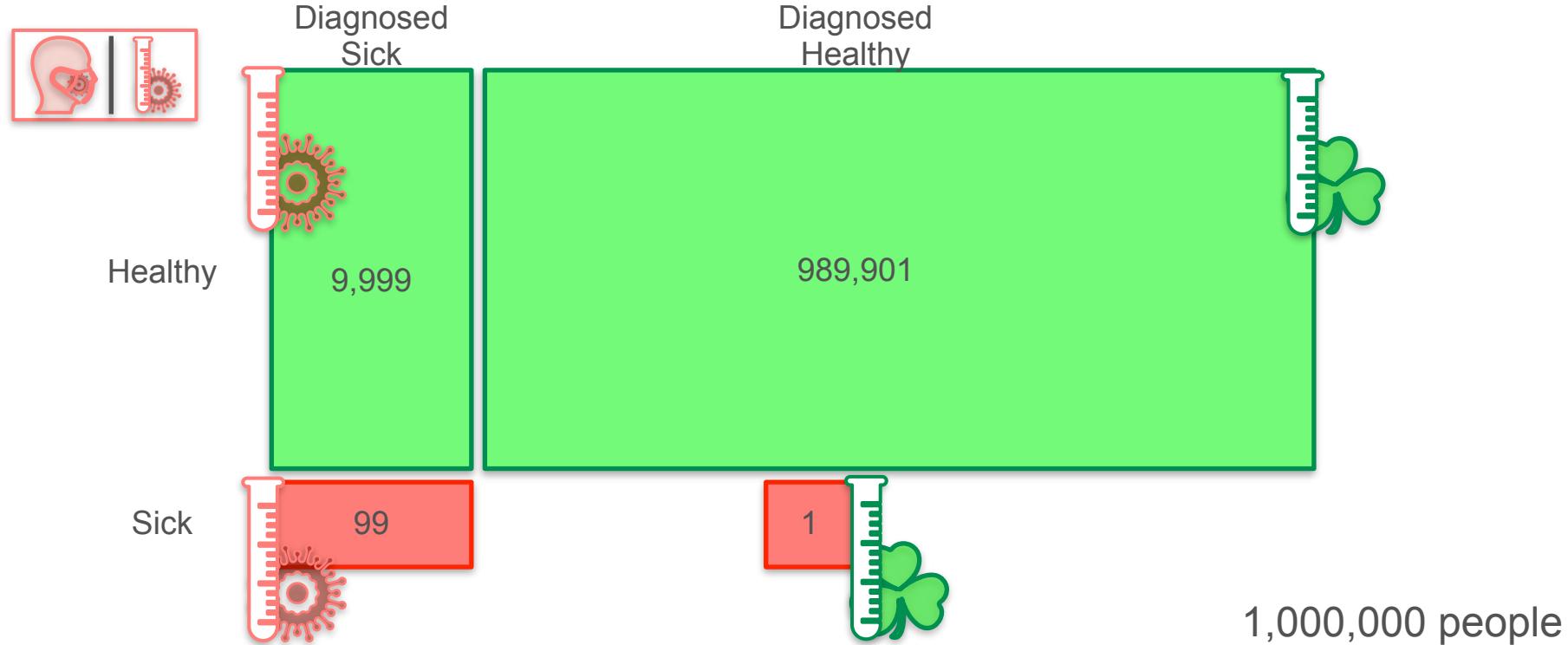
# Bayes Theorem: Intuition



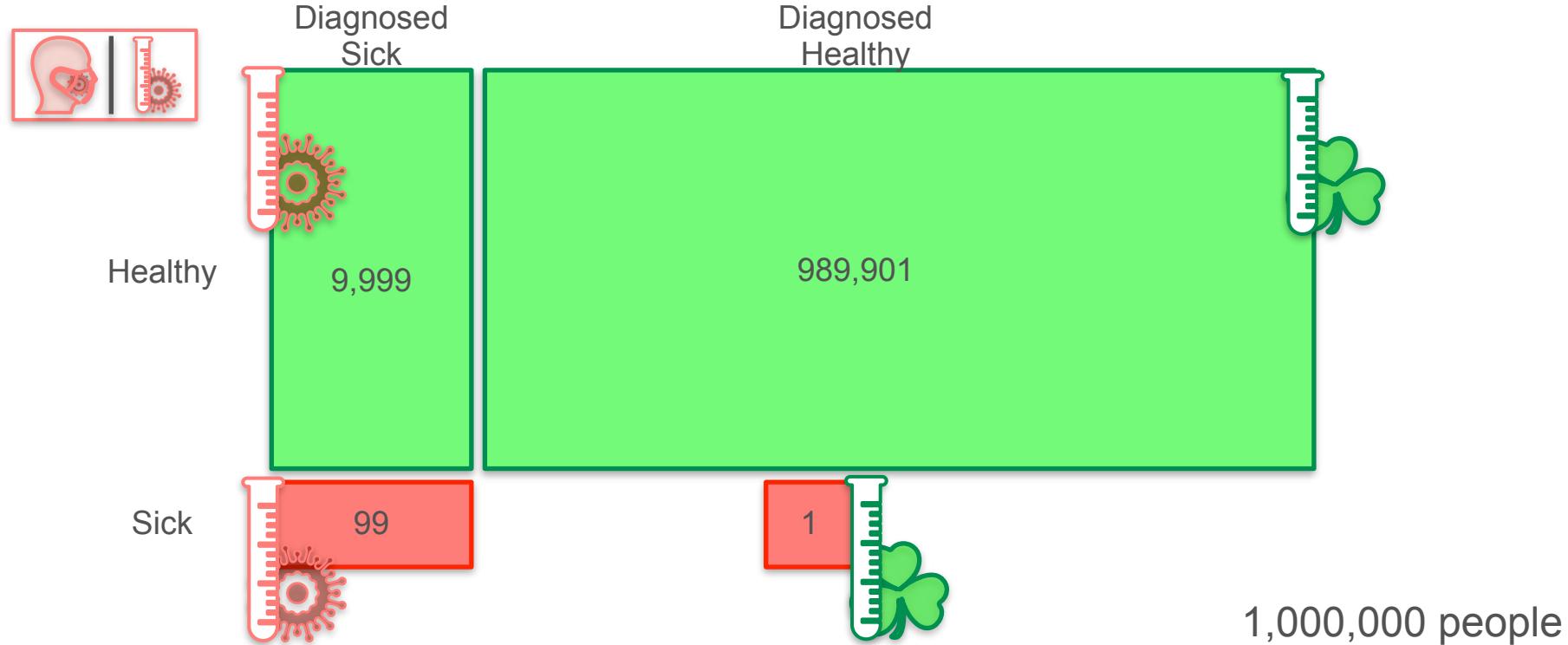
# Bayes Theorem: Intuition



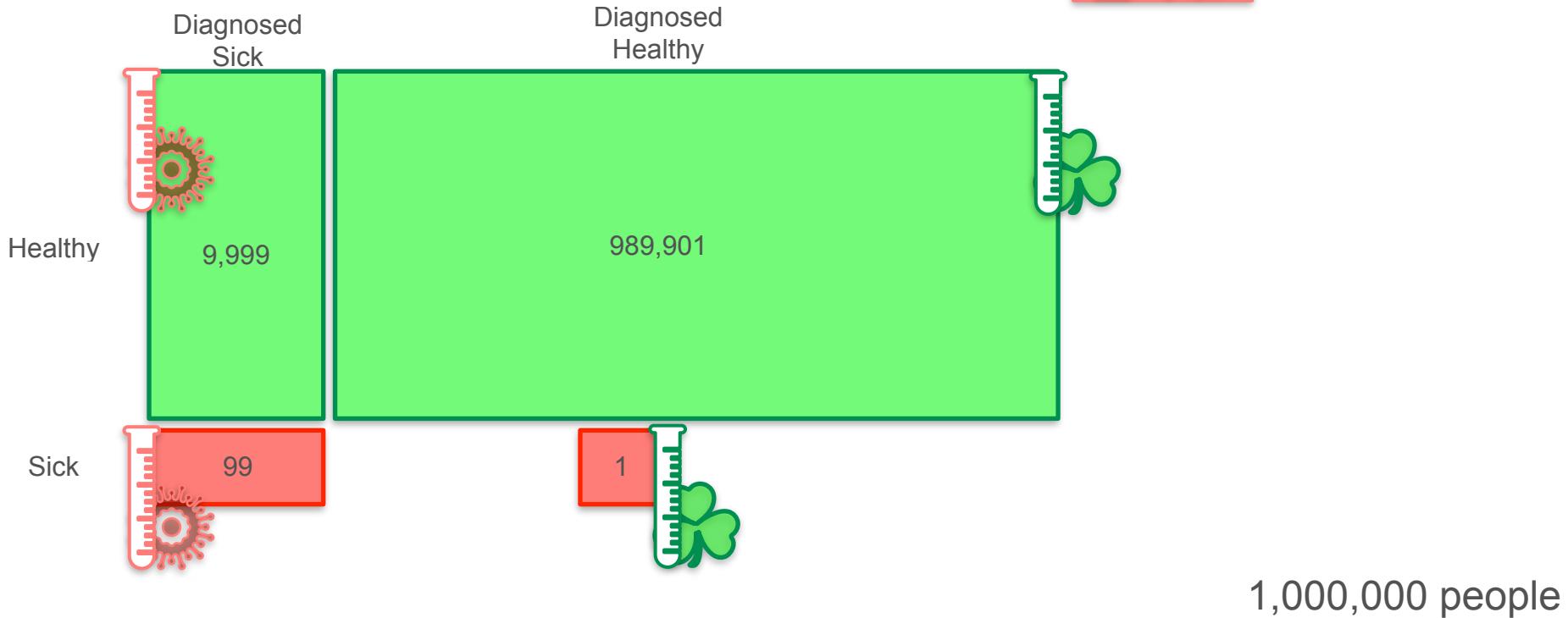
# Bayes Theorem: Intuition



# Bayes Theorem: Intuition



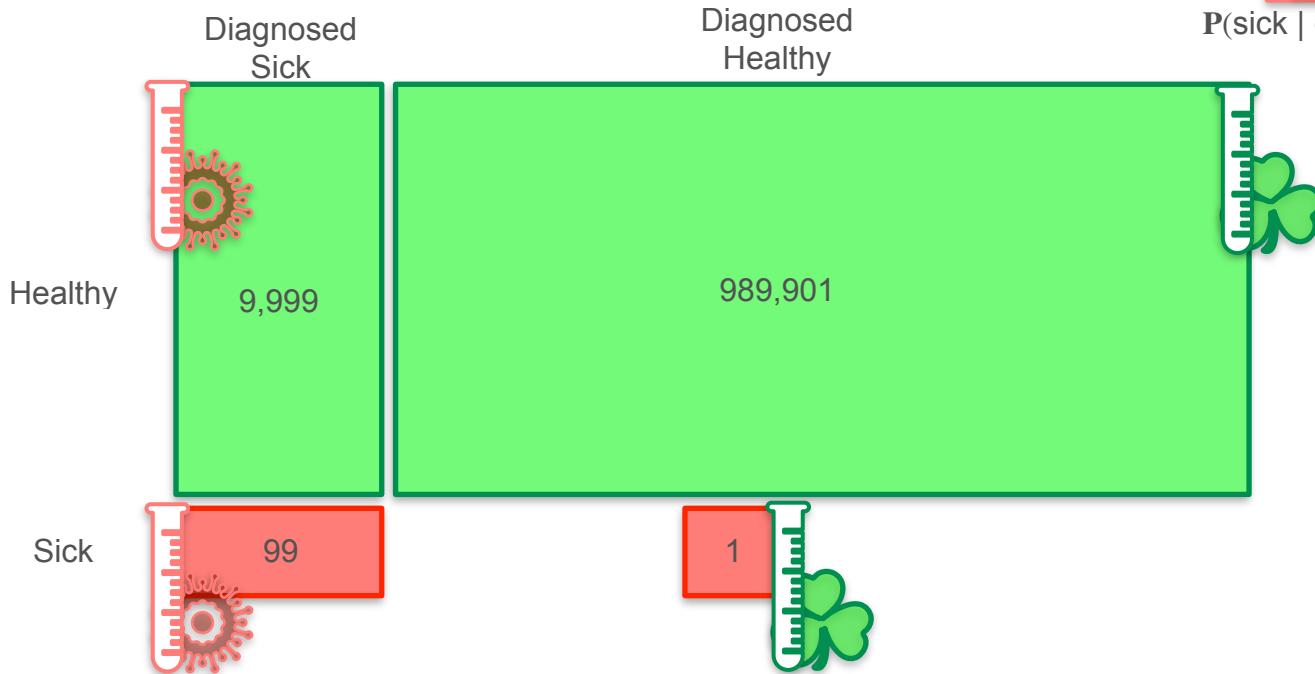
# Bayes Theorem: Intuition



# Bayes Theorem: Intuition



$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$

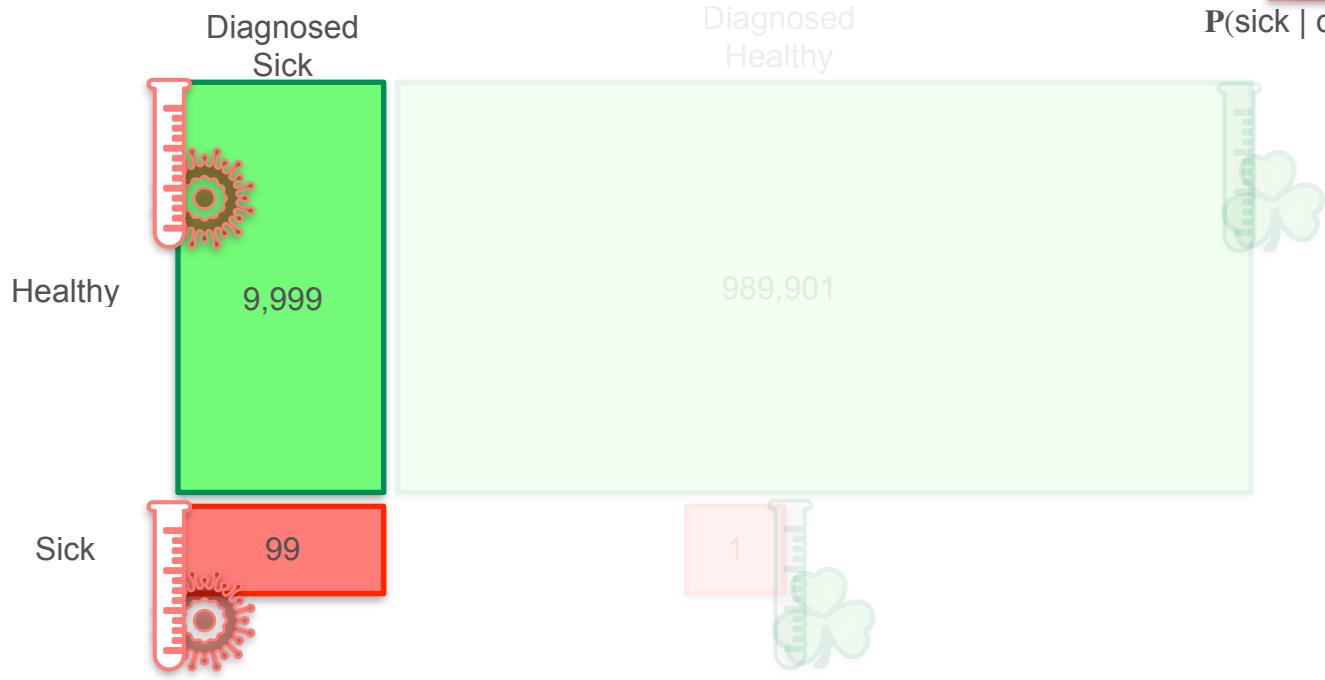


1,000,000 people

# Bayes Theorem: Intuition



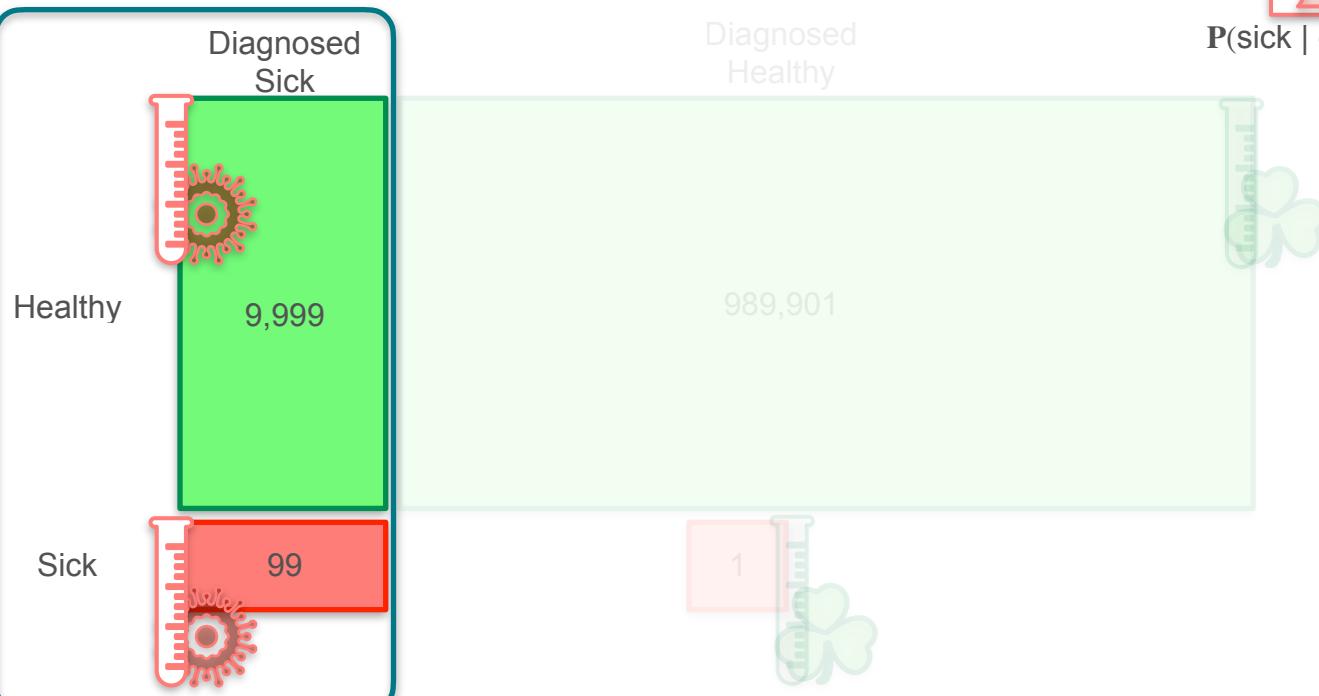
$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$



# Bayes Theorem: Intuition

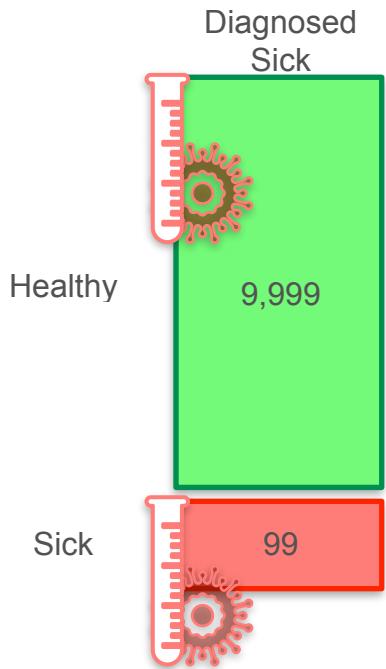


$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$



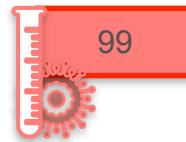
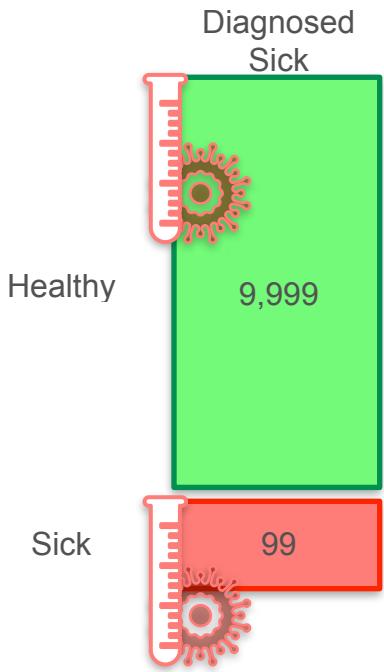
1,000,000 people

# Bayes Theorem: Intuition



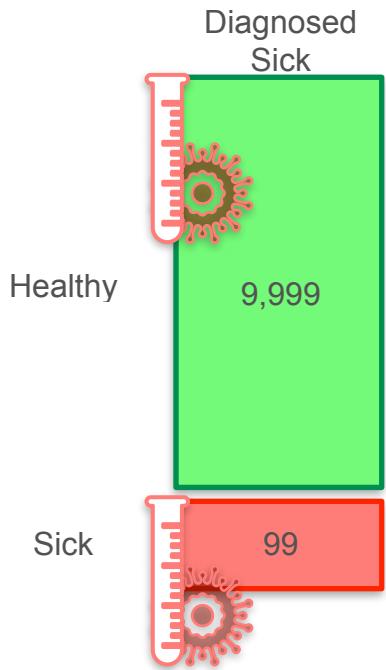
$$P(\text{sick} \mid \text{diagnosed sick}) = \underline{\hspace{2cm}}$$

# Bayes Theorem: Intuition

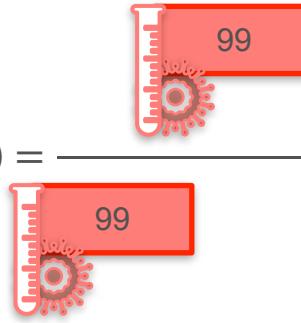


$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$

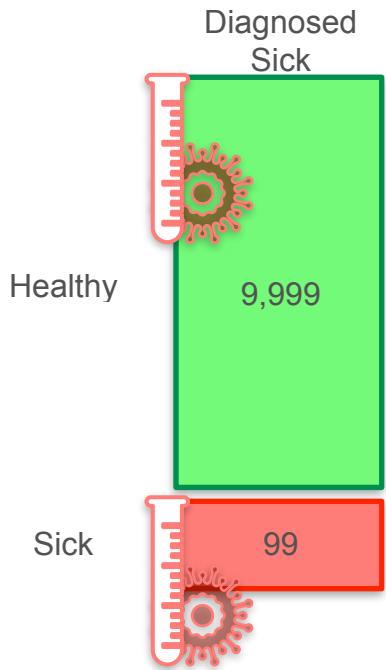
# Bayes Theorem: Intuition



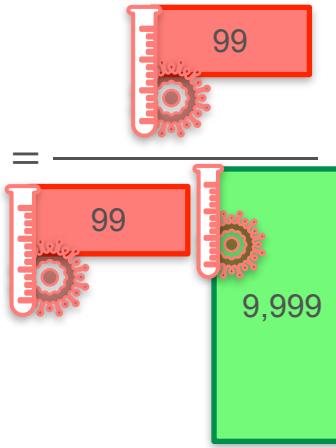
$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$



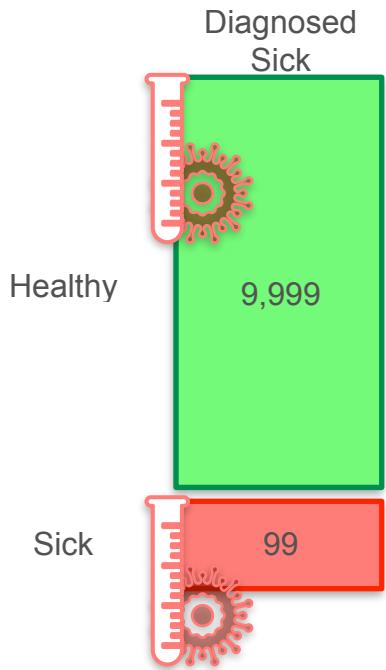
# Bayes Theorem: Intuition



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{Probability of true positive}}{\text{Probability of true positive} + \text{Probability of false positive}}$$

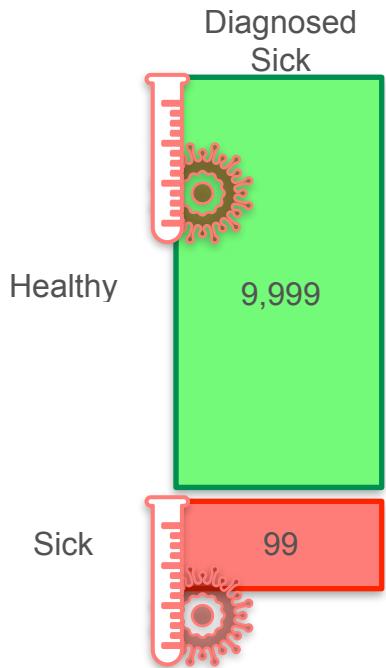


# Bayes Theorem: Intuition

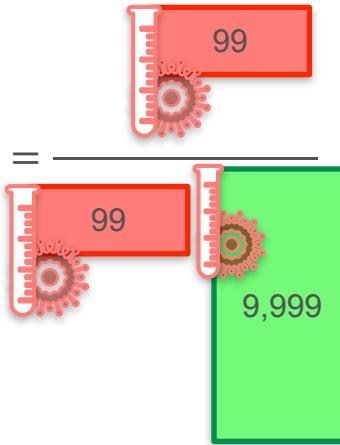


$$\begin{aligned} P(\text{sick} | \text{diagnosed sick}) &= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}} \\ &= \frac{99}{99 + 9,999} \end{aligned}$$

# Bayes Theorem: Intuition

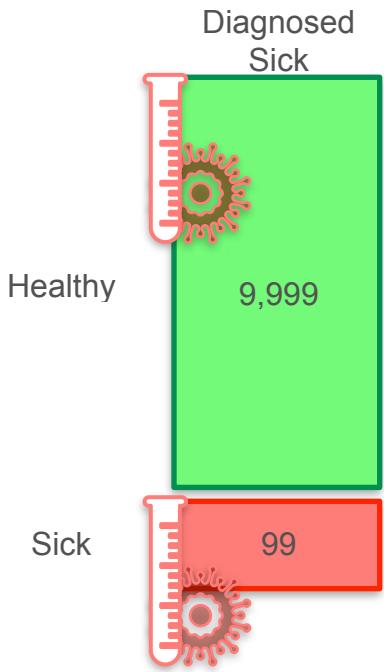


$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}}$$



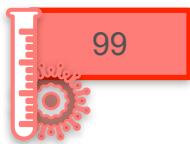
$$= \frac{99}{99 + 9999}$$

# Bayes Theorem: Intuition

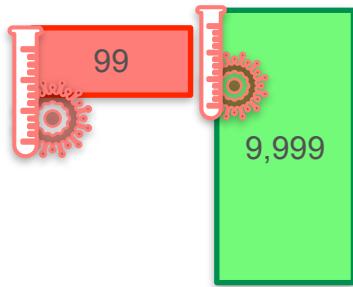


$$\begin{aligned} P(\text{sick} | \text{diagnosed sick}) &= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}} \\ &= \frac{99}{99 + 9999} \\ &= \frac{99}{10098} = 0.0098 \end{aligned}$$

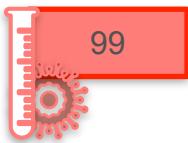
# Bayes Theorem: Intuition



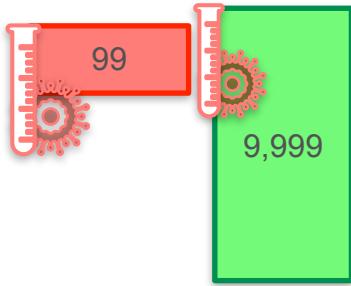
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$



# Bayes Theorem: Intuition



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{_____}}{\text{_____}}$$

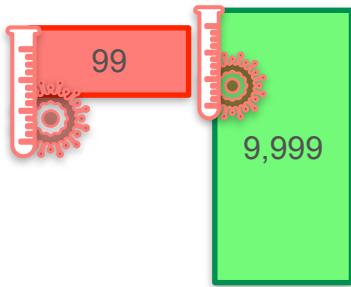


=  $\frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$

# Bayes Theorem: Intuition



$$P(\text{sick} | \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

# Bayes Theorem: Intuition



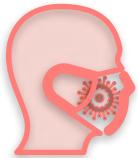
1,000,000  
people

# Bayes Theorem: Intuition

sick = 100



1,000,000  
people

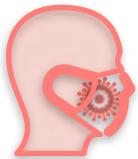


# Bayes Theorem: Intuition

sick = 100



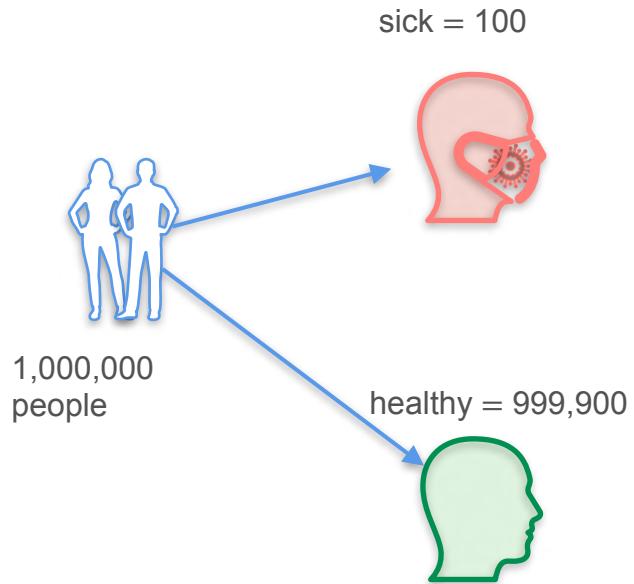
1,000,000  
people



healthy = 999,900

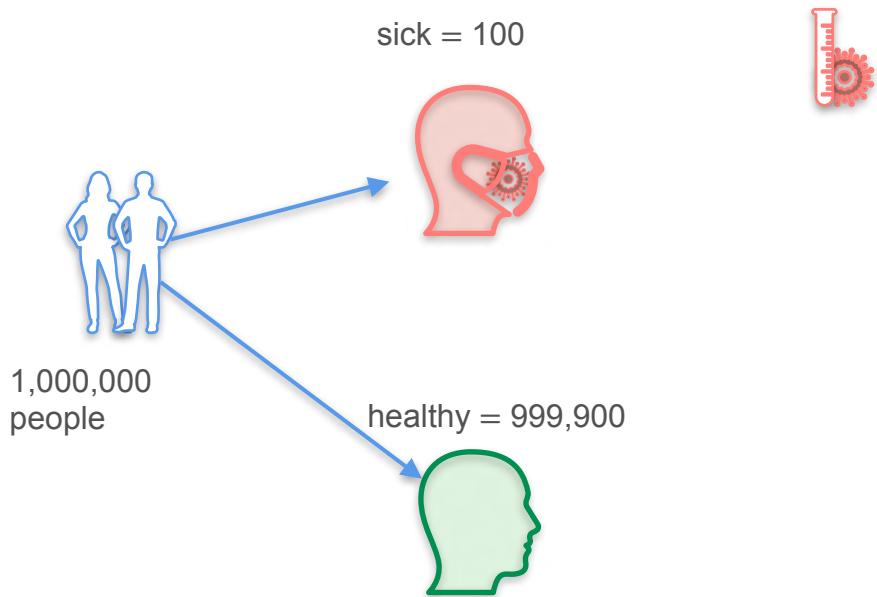


# Bayes Theorem: Intuition

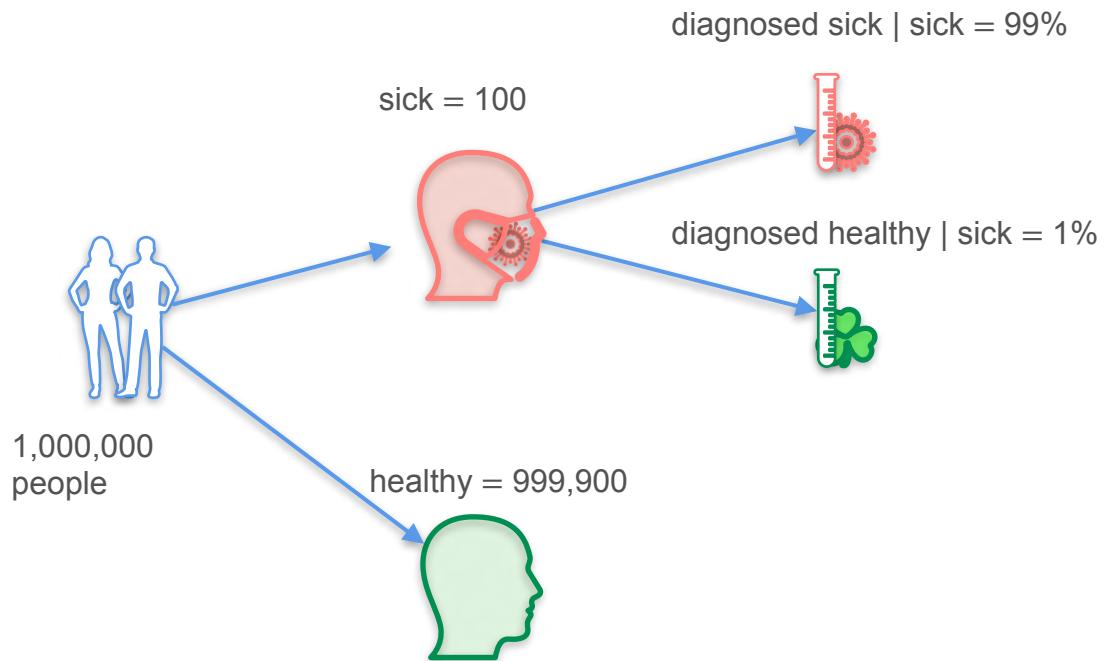


# Bayes Theorem: Intuition

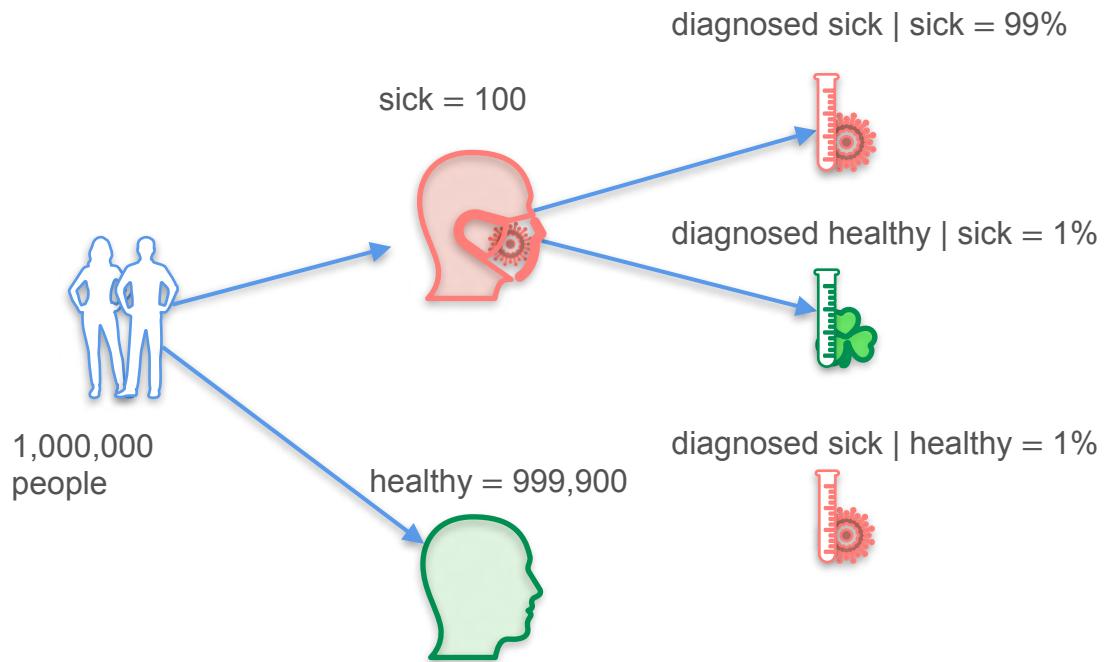
diagnosed sick | sick = 99%



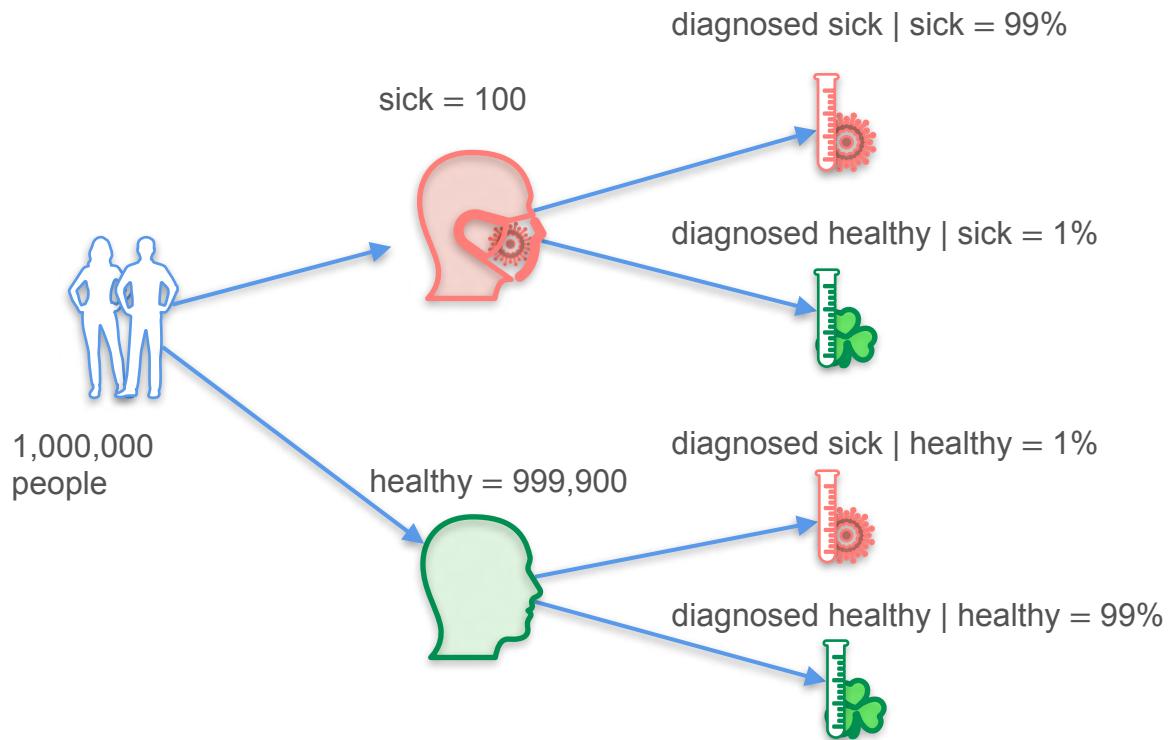
# Bayes Theorem: Intuition



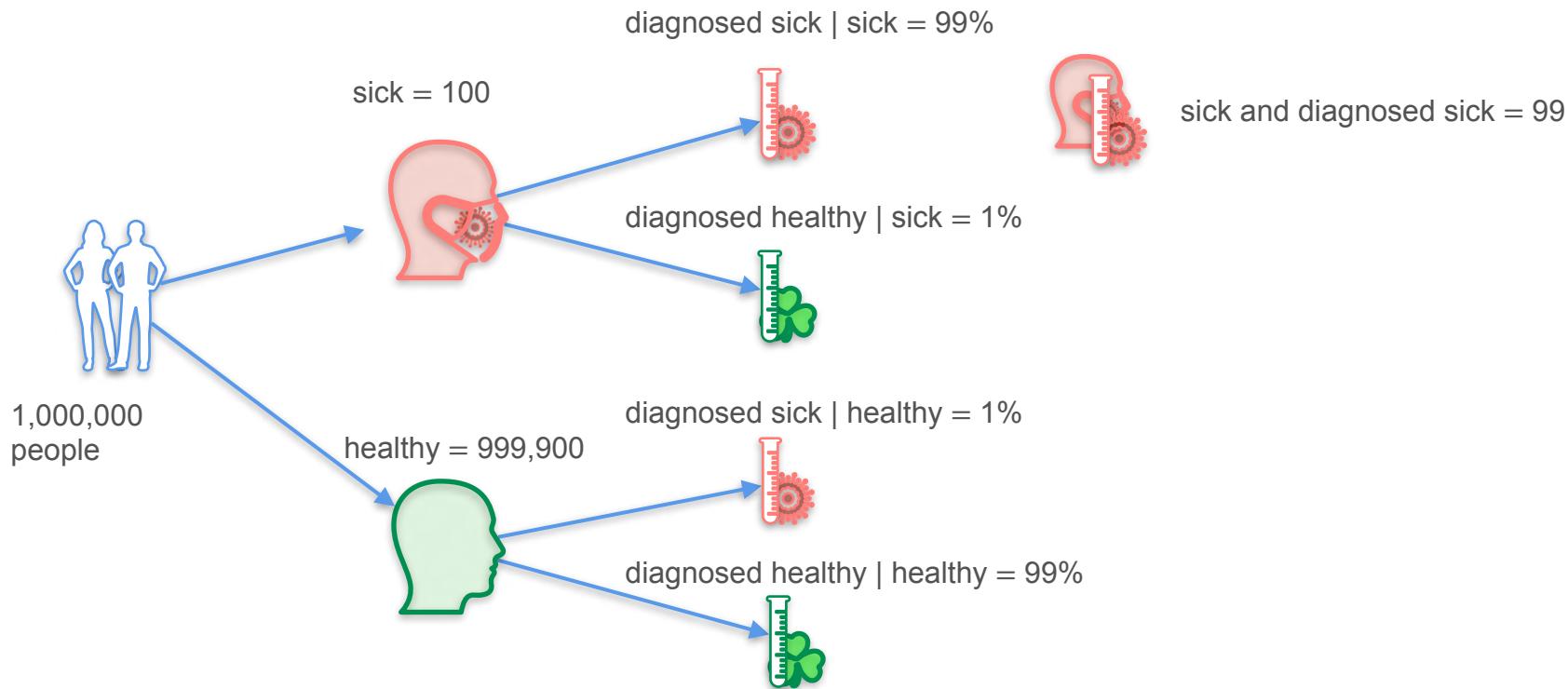
# Bayes Theorem: Intuition



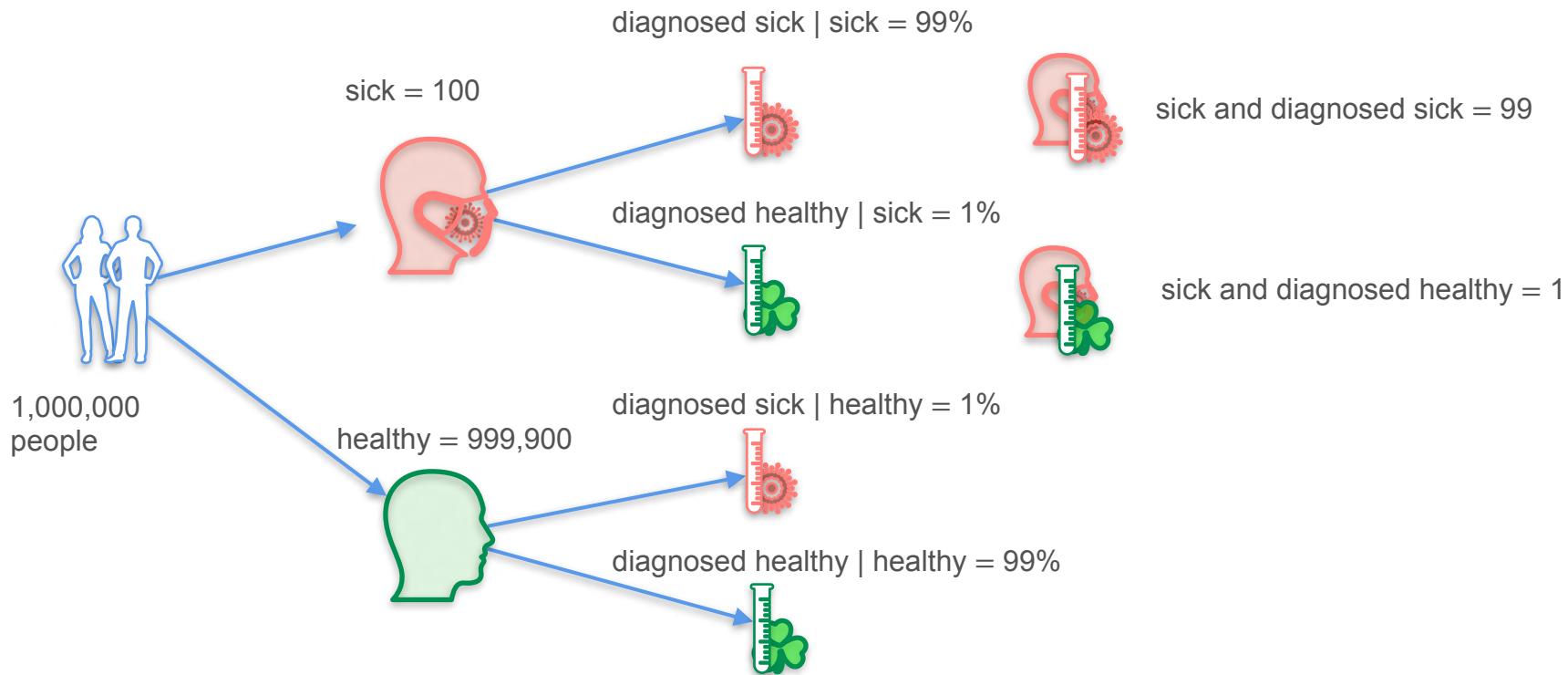
# Bayes Theorem: Intuition



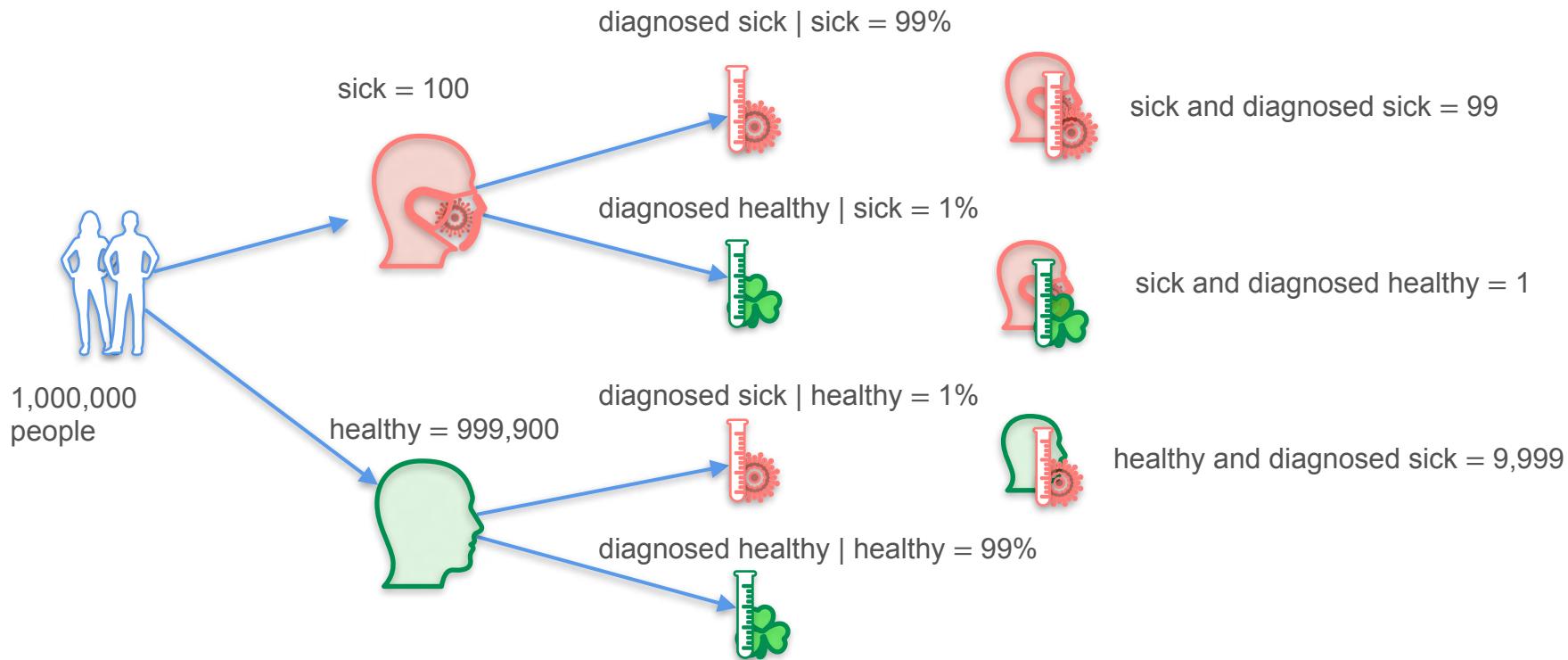
# Bayes Theorem: Intuition



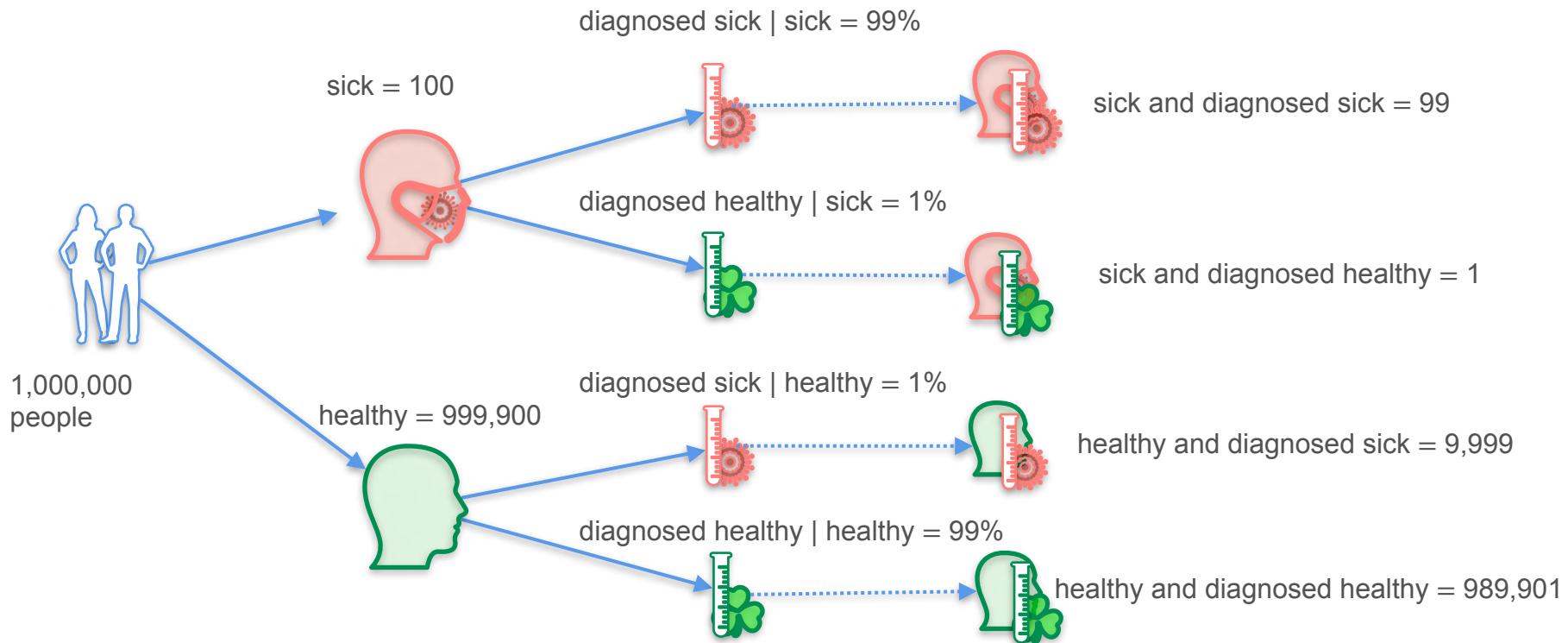
# Bayes Theorem: Intuition



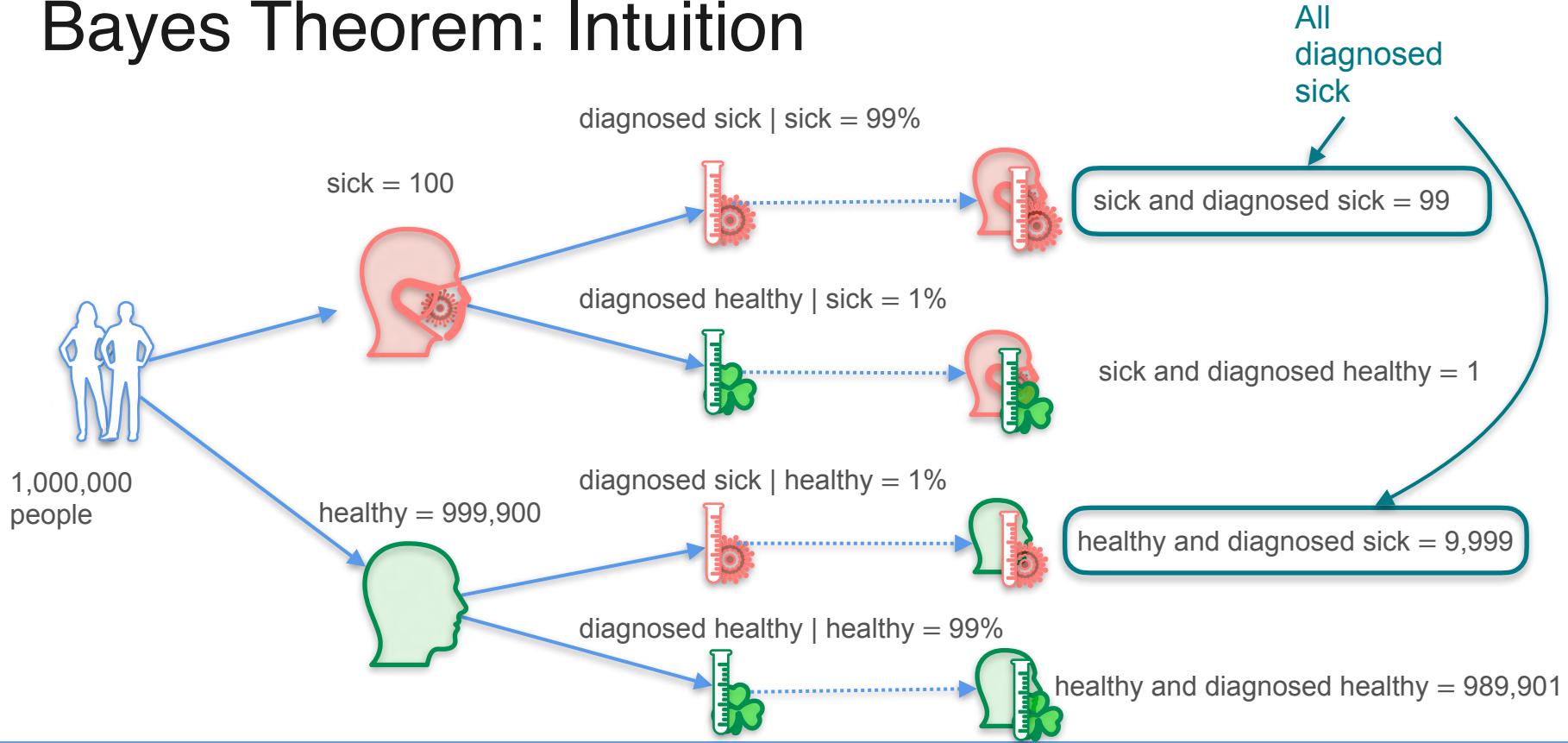
# Bayes Theorem: Intuition



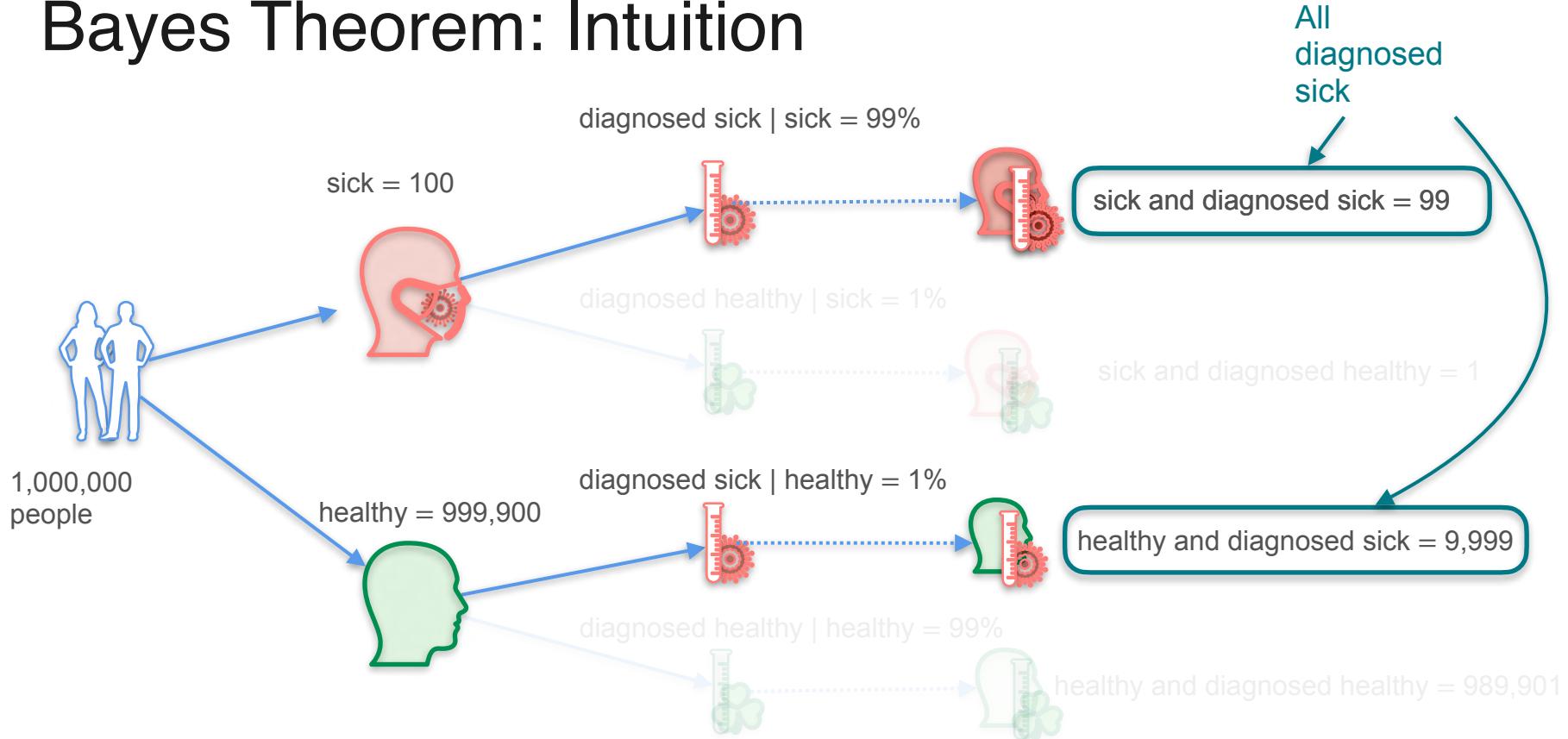
# Bayes Theorem: Intuition



# Bayes Theorem: Intuition

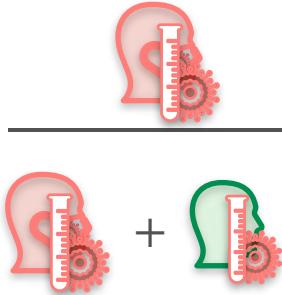


# Bayes Theorem: Intuition



# Bayes Theorem: Intuition

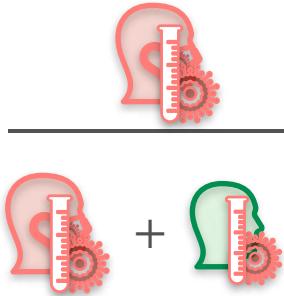
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

# Bayes Theorem: Intuition

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$

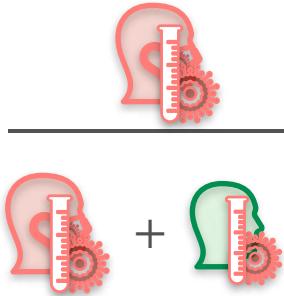


$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098}$$

# Bayes Theorem: Intuition

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

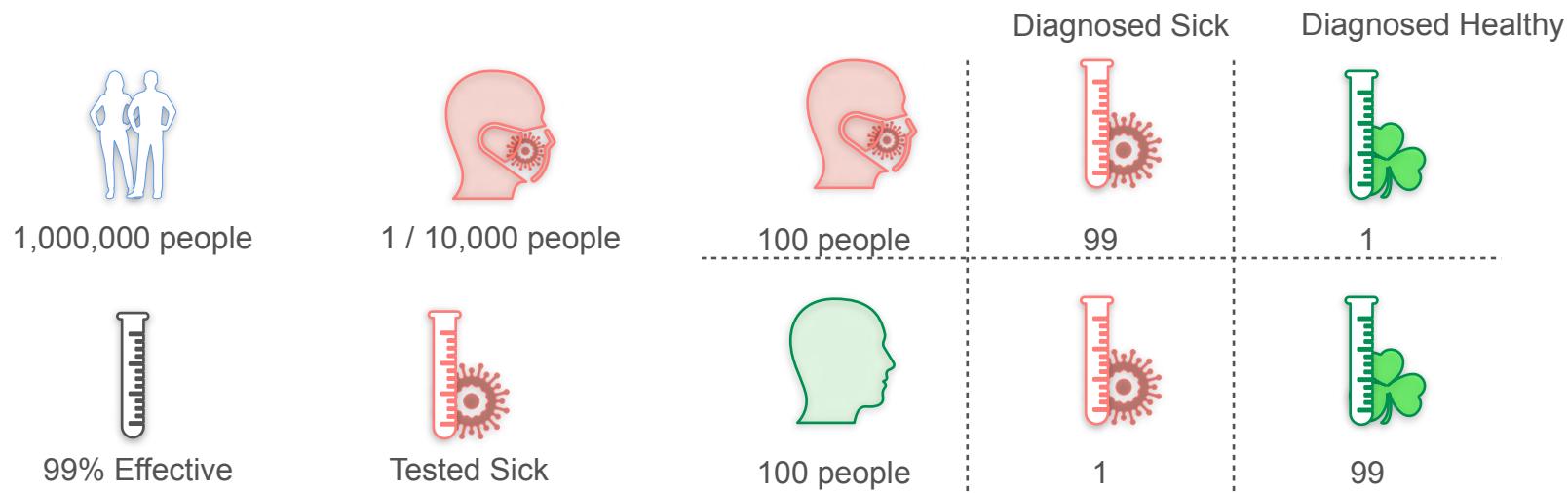
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098} = 0.0098$$

# Bayes Theorem: Formula

# Bayes Theorem: Formula

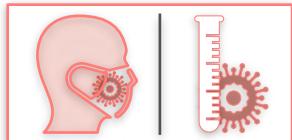
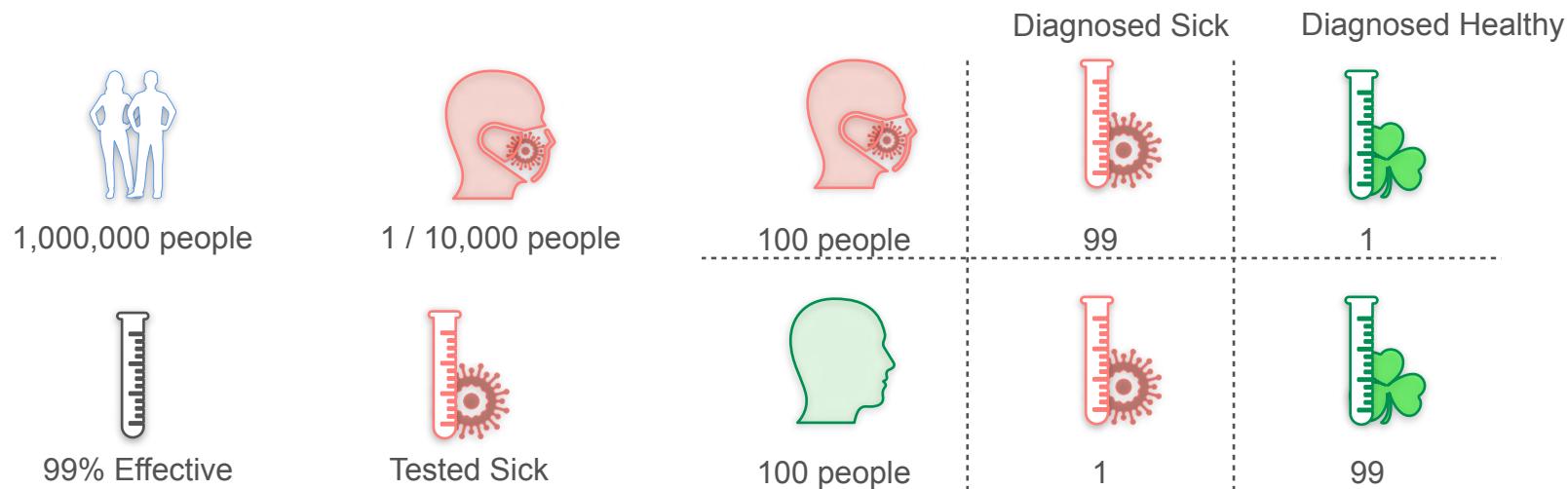
The probability that **you are sick**  
**GIVEN that you tested sick**

# Bayes Theorem: Formula



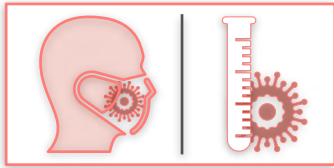
The probability that **you are sick**  
**GIVEN that you tested sick**

# Bayes Theorem: Formula



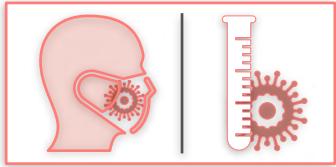
The probability that **you are sick**  
**GIVEN** that you tested sick

# Bayes Theorem: Formula



The probability that **you are sick**  
**GIVEN** that you tested sick

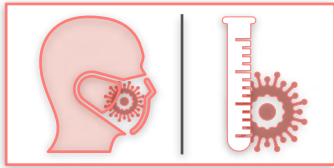
# Bayes Theorem: Formula



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

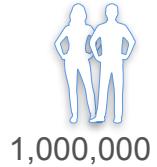
The probability that **you are sick**  
**GIVEN** that you tested sick

# Bayes Theorem: Formula

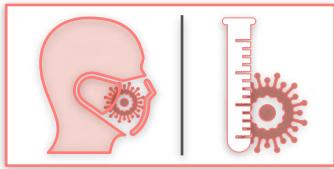


$P(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick**  
**GIVEN** that you tested sick



# Bayes Theorem: Formula



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick**  
**GIVEN** that you tested sick

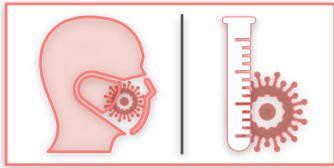


1,000,000



1 / 10,000

# Bayes Theorem: Formula



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

The probability that **you are sick**  
**GIVEN** that you tested sick

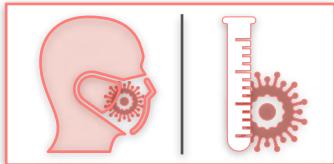


1,000,000



1 / 10,000

# Bayes Theorem: Formula

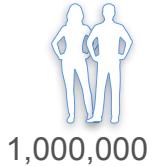


$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

The probability that **you are sick**  
**GIVEN** that you tested sick

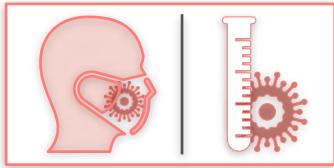


1,000,000



1 / 10,000

# Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

The probability that **you are sick**  
**GIVEN** that you tested sick



1,000,000

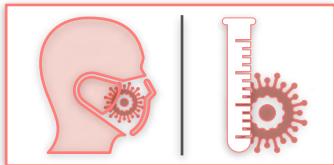


1 / 10,000



99% Effective

# Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} | \text{sick}) = 99\%$

The probability that **you are sick**  
**GIVEN** that you tested sick



1,000,000

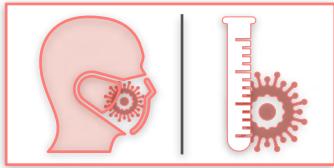


1 / 10,000



99% Effective

# Bayes Theorem: Formula



$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

The probability that **you are sick**  
**GIVEN** that you tested sick



1,000,000

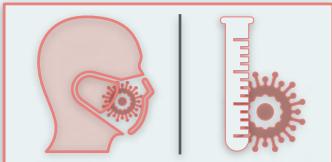


1 / 10,000



99% Effective

# Bayes Theorem: Formula



$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

The probability that **you are sick**  
**GIVEN** that you tested sick



1,000,000

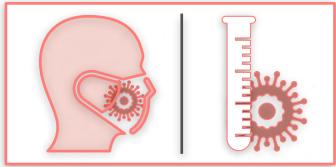


1 / 10,000



99% Effective

# Bayes Theorem: Formula



$P(\text{sick} | \text{diagnosed sick}) = ?$

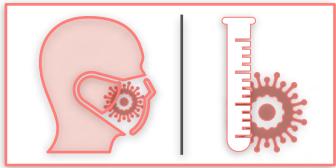
$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} | \text{sick}) = 99\%$

$P(\text{diagnosed sick} | \text{not sick}) = 1\%$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

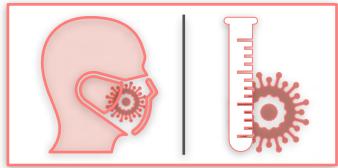
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

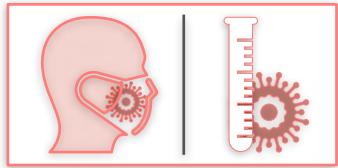
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

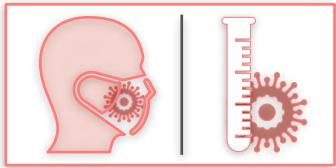
From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

**From Conditional Probability**

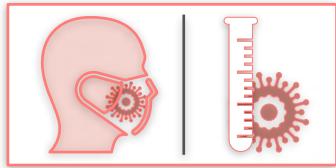
$$P(\text{not sick}) = 99.99\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

**From Conditional Probability**

$$P(\text{not sick}) = 99.99\%$$

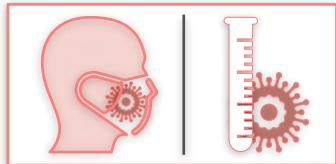
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \underline{\hspace{10em}}$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

**From Conditional Probability**

$$P(\text{not sick}) = 99.99\%$$

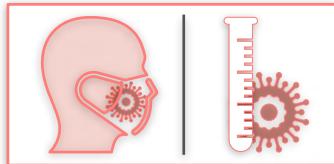
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{}$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

**From Conditional Probability**

$$P(\text{not sick}) = 99.99\%$$

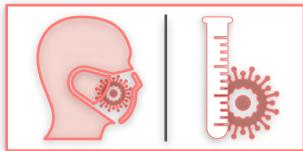
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick} | \text{diagnosed sick}) = ?$$

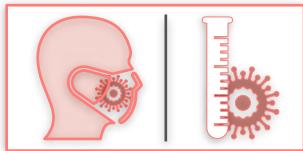
$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

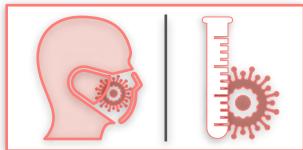
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

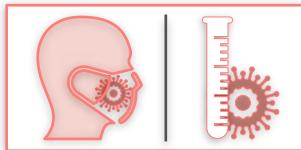
$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick and diagnosed sick}) = ?$$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

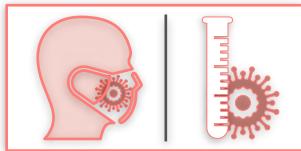
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick and diagnosed sick}) = ?$$

$$P(\text{diagnosed sick}) = ?$$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

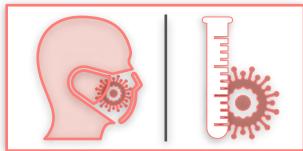
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$P(\text{sick and diagnosed sick}) = ?$

$P(\text{diagnosed sick}) = ?$

**BAYES THEOREM FORMULA CAN HELP**

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

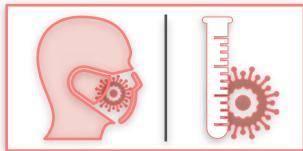
$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

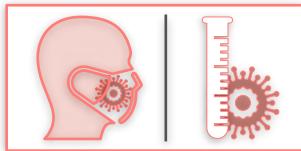
$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

**From Conditional Probability**

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

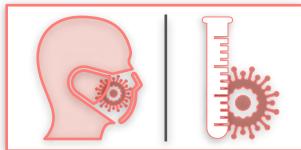
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

**From Conditional Probability**

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

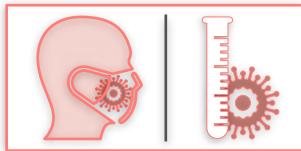
$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

**From Conditional Probability**

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$P(\text{sick and diagnosed sick}) =$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

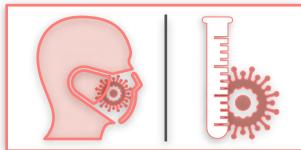
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick}) = ?}{P(\text{diagnosed sick}) = ?}$$

**From Conditional Probability**

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick})$$

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

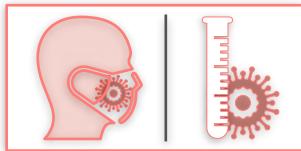
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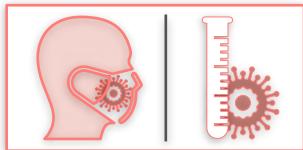
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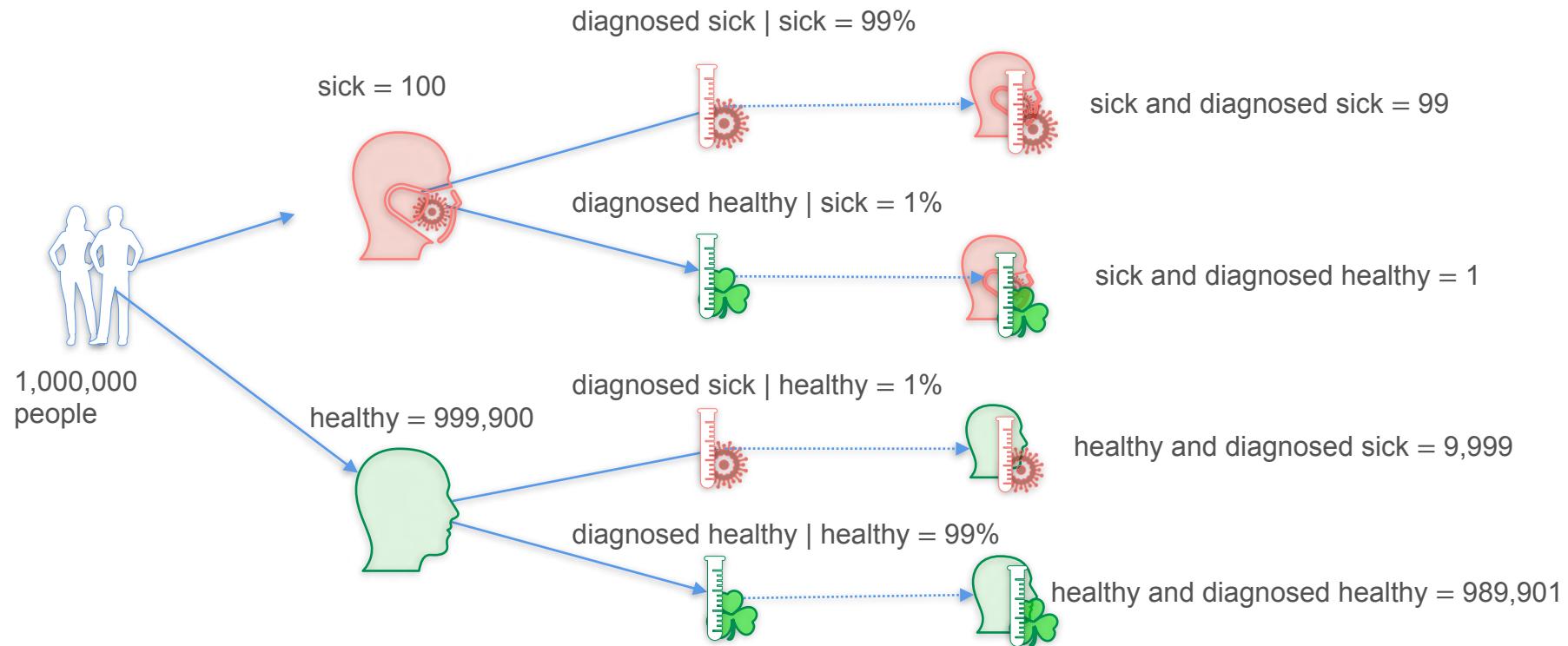
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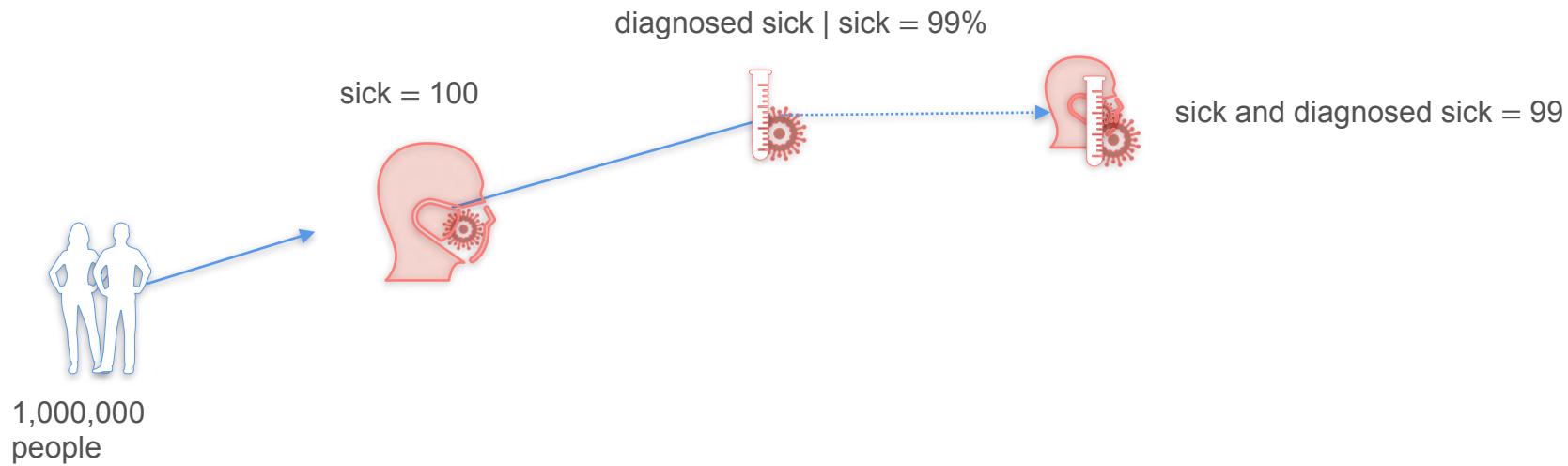
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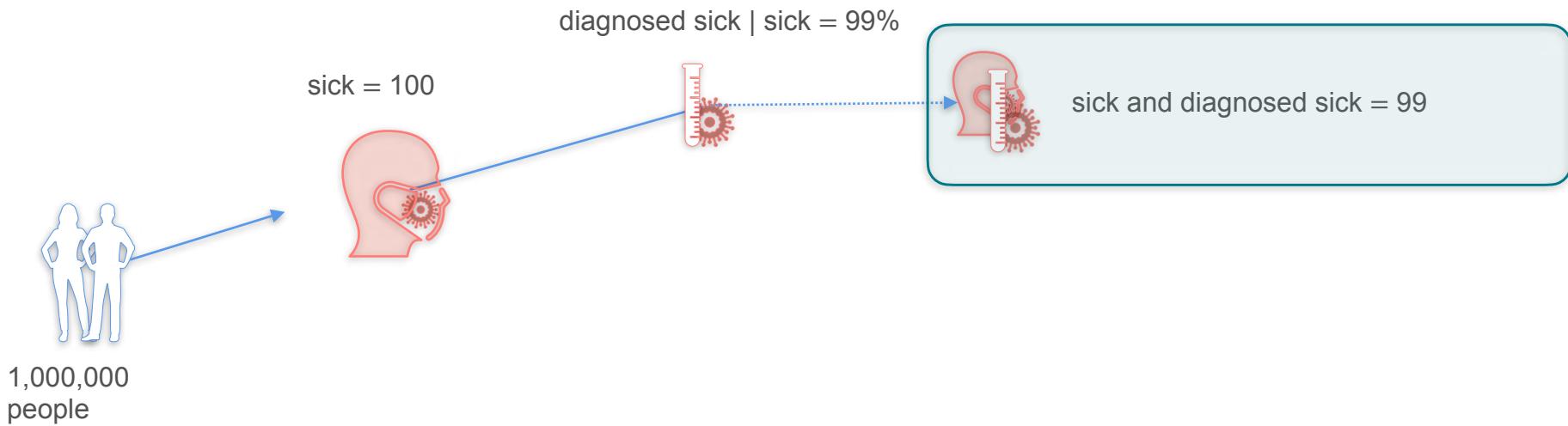
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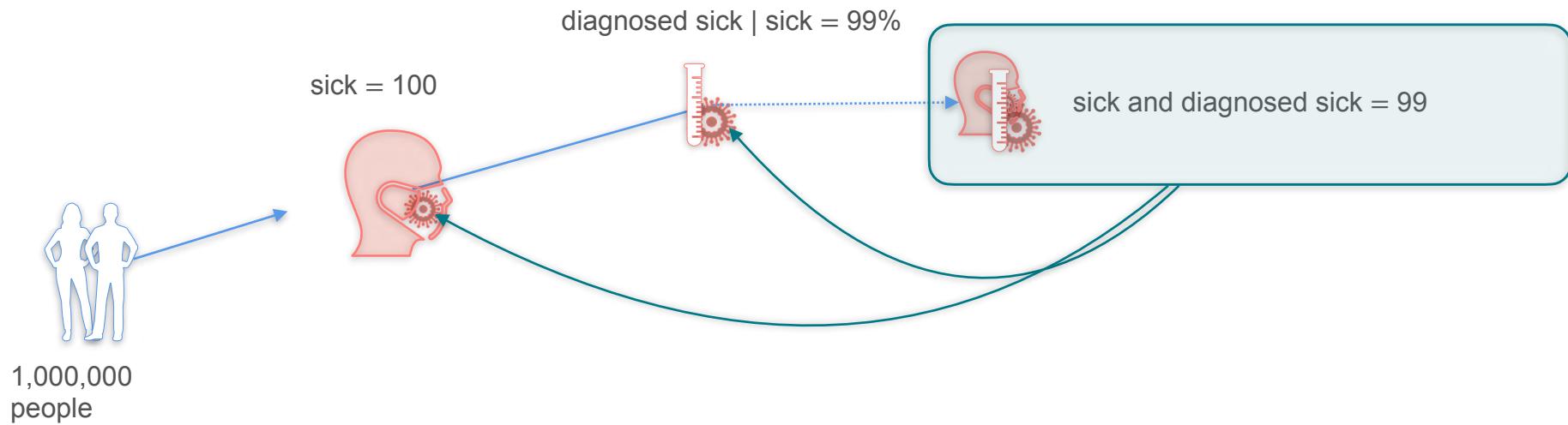
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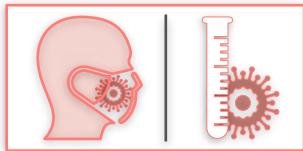
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# Bayes Theorem: Formula



# Bayes Theorem: Formula



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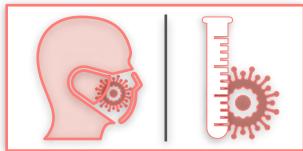
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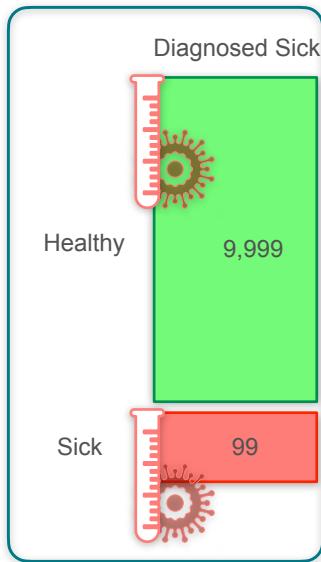
$$P(\text{diagnosed sick}) =$$

# Bayes Theorem: Formula

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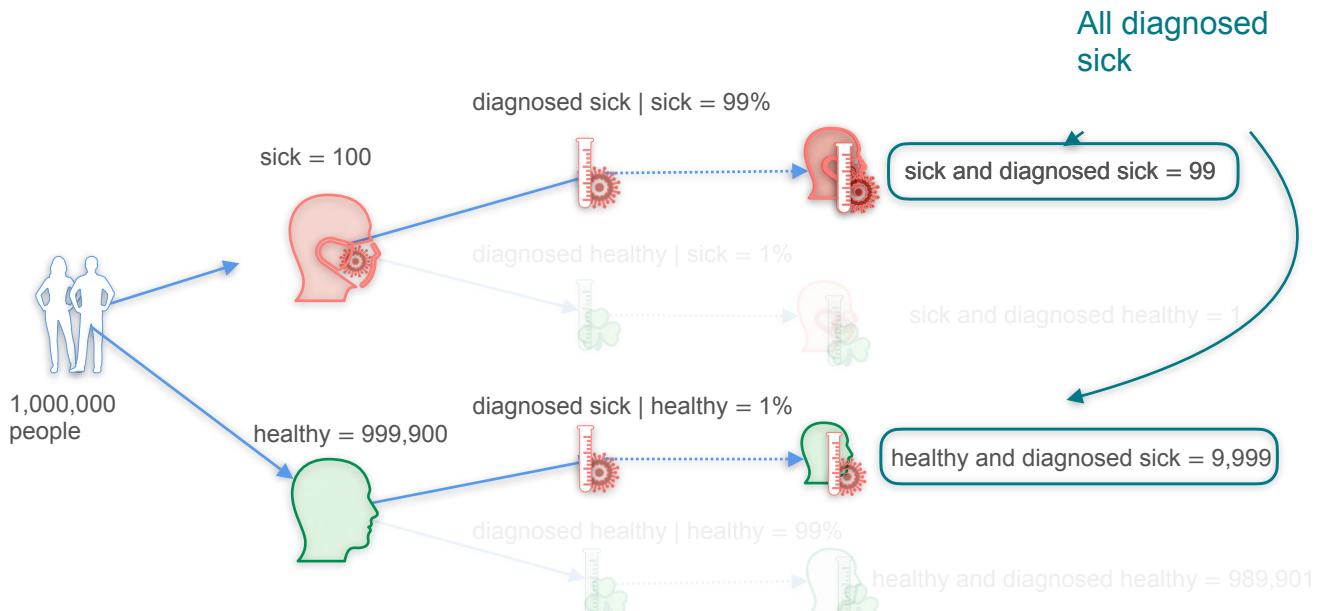
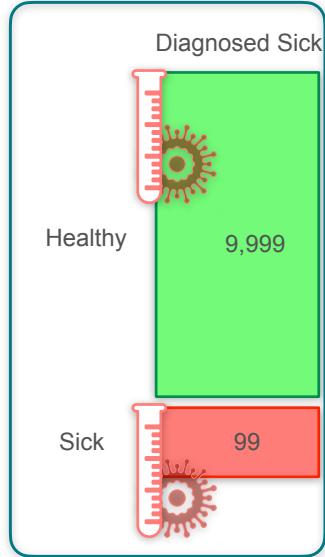
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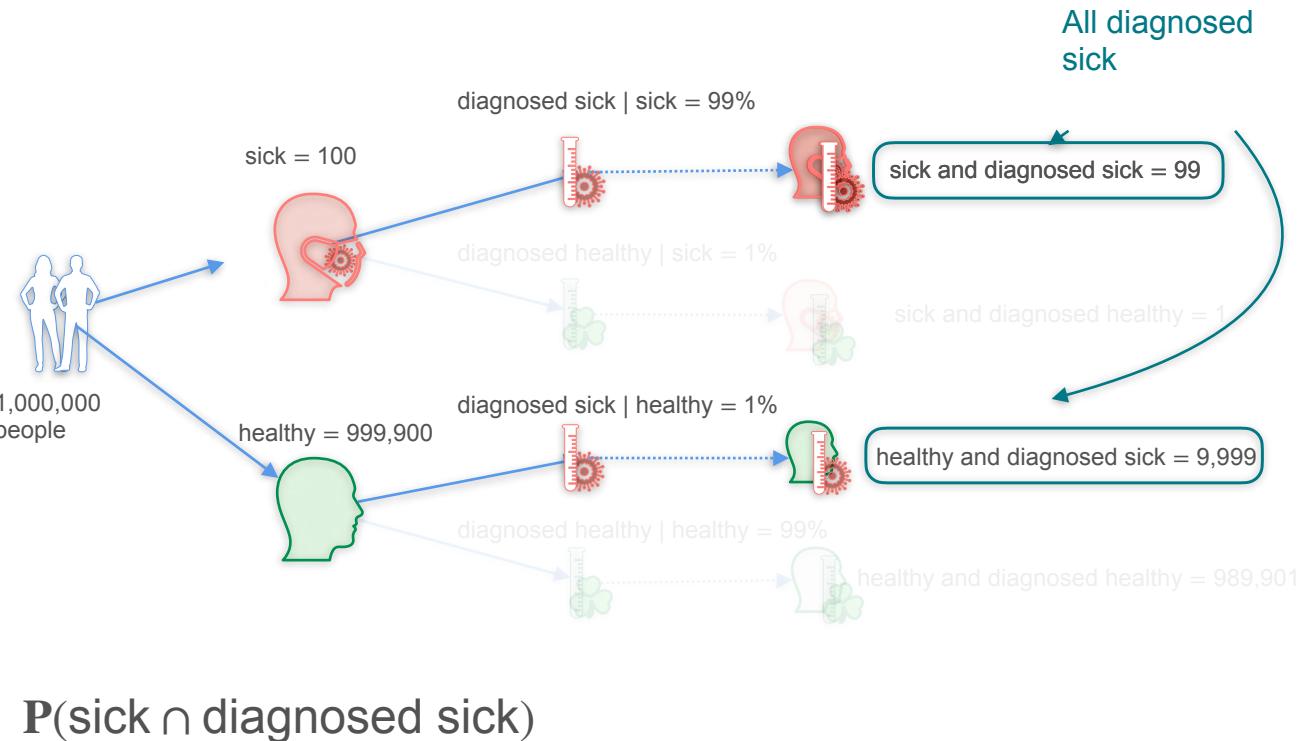
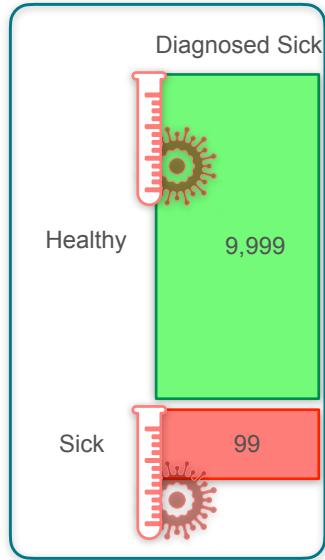
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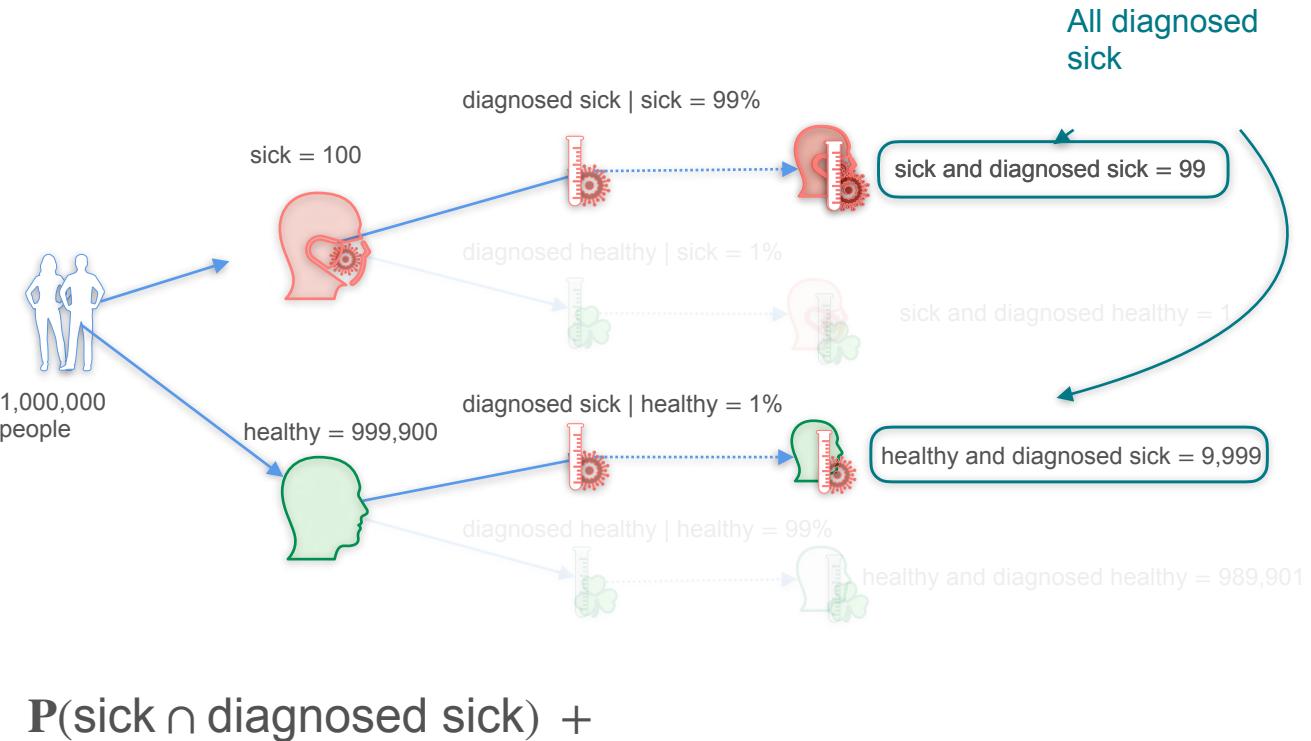
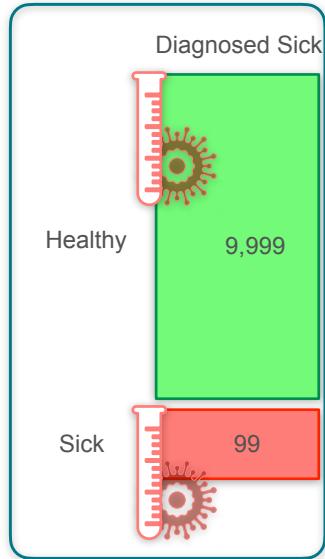
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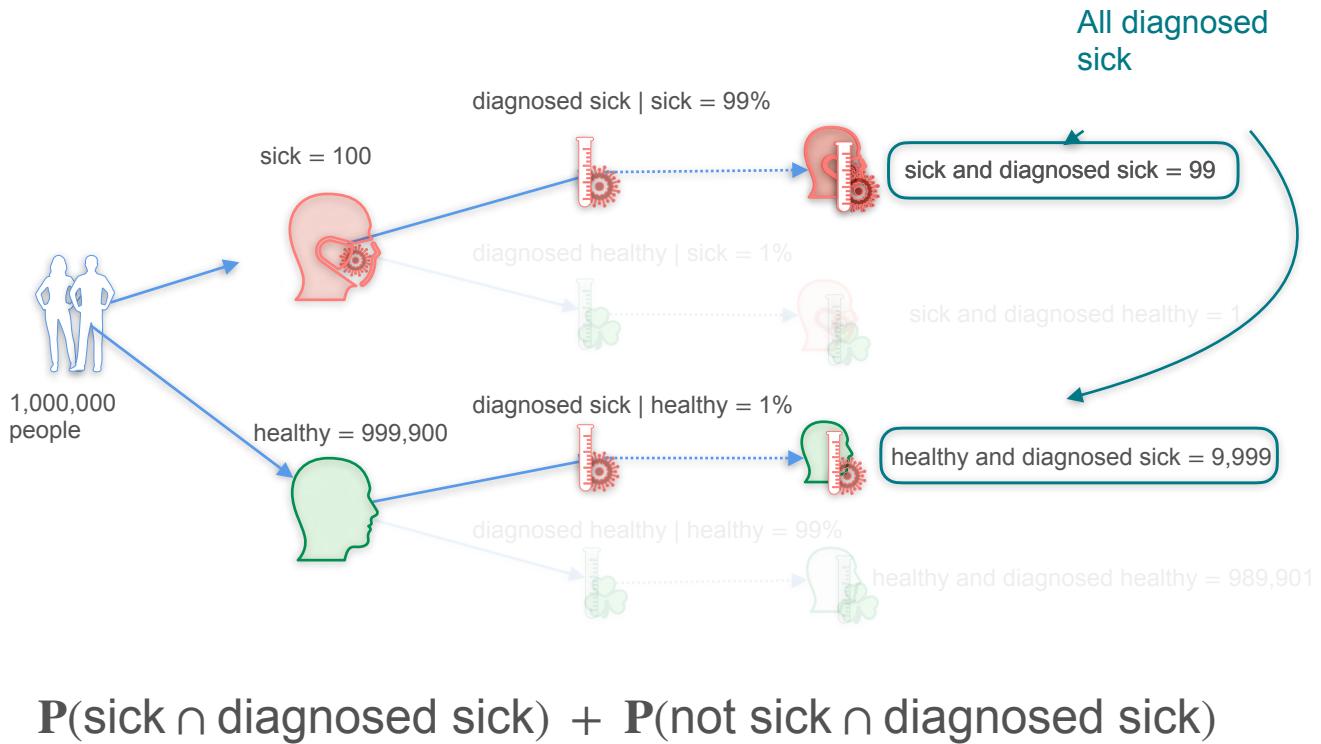
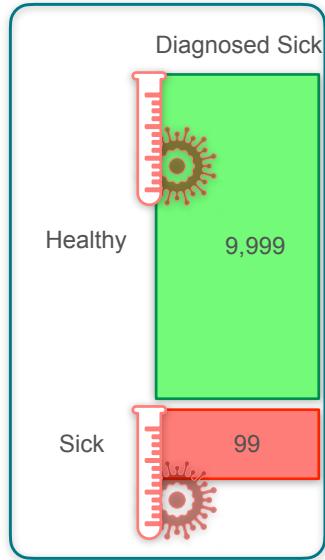
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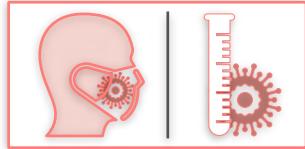


# Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$



# Bayes Theorem: Formula



$A$ : sick

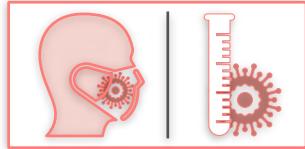
$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

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# Bayes Theorem: Formula



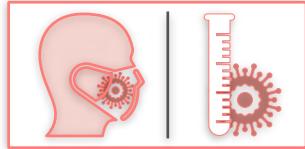
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# Bayes Theorem: Formula



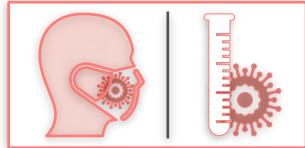
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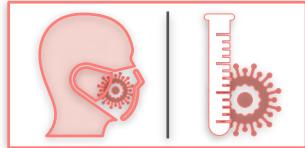
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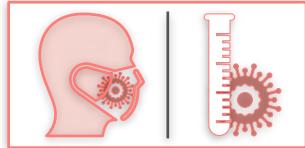
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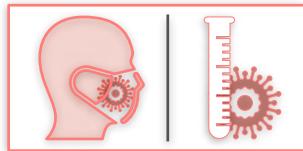
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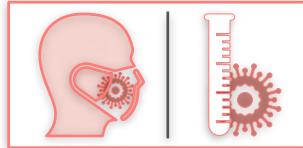
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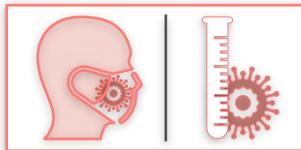
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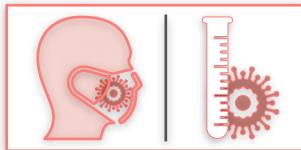
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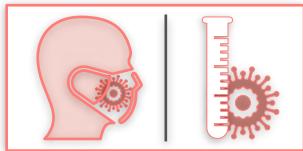
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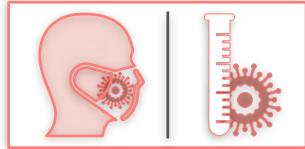
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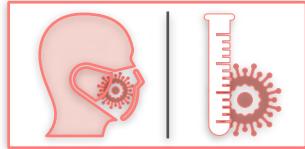
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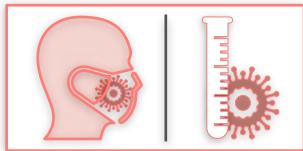
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**BAYES THEOREM  
FORMULA**

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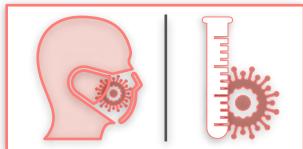
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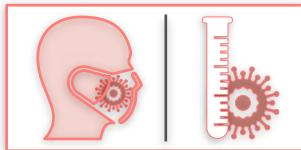
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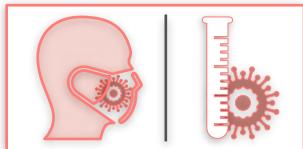
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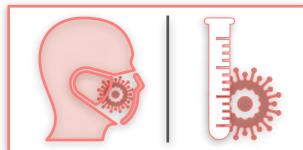
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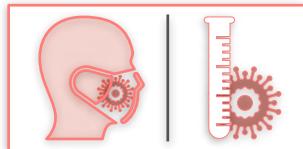
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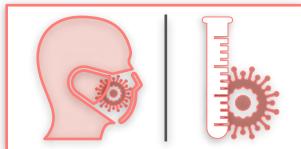
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FORMULA**

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$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

?

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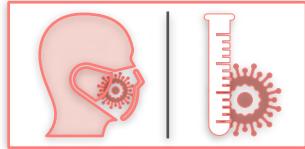
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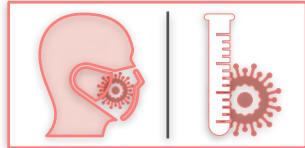
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$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

# Bayes Theorem: Formula



A: sick

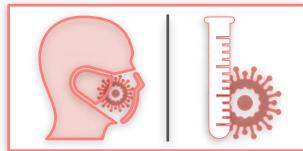
B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 1\%$$

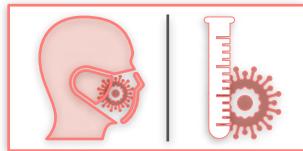
$$P(\text{not sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

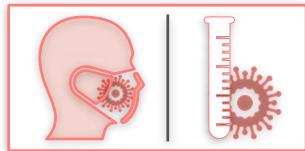
$$P(\text{not sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

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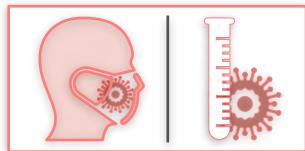
$$P(A') = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

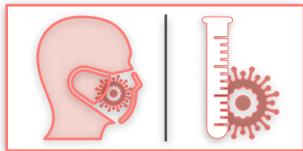
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

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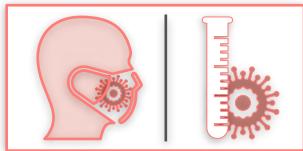
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

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**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

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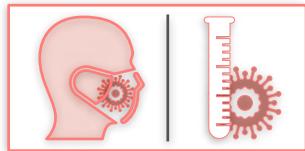
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

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**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

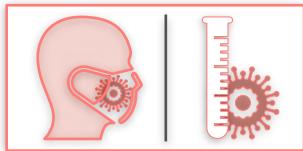
$$P(A') = 99.99\%$$

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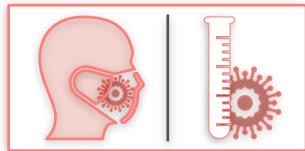
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# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

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$$P(A) = 0.01\%$$

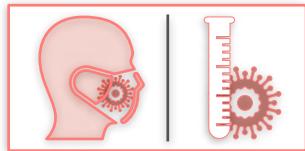
$$P(A') = 99.99\%$$

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$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(A) = 0.01\%$$

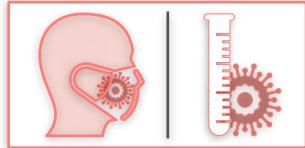
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

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# Bayes Theorem: Formula



$A$ : sick

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$$P(A) = 0.01\%$$

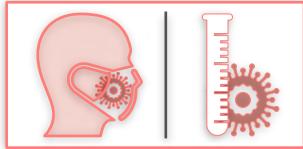
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$\mathbf{P}(A | B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

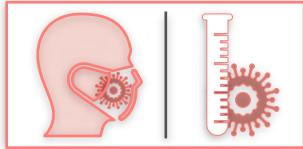
$$\mathbf{P}(A) = 0.01\%$$

$$\mathbf{P}(A') = 99.99\%$$

$$\mathbf{P}(B | A) = 99\%$$

$$\mathbf{P}(B | A') = 1\%$$

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = 0.01\%$$

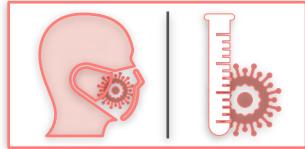
$$P(A') = 99.99\%$$

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$$P(B | A') = 1\%$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



$A$ : sick

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$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

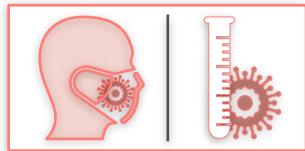
$$P(A') = 99.99\%$$

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**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

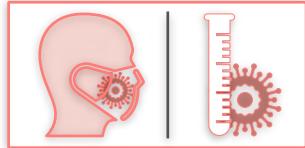
$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

$$P(A | B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Formula



$A$ : sick

$B$ : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

$$P(A | B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

$$P(A | B) = 0.0098$$

**BAYES THEOREM  
FORMULA**

# Bayes Theorem: Spam Example

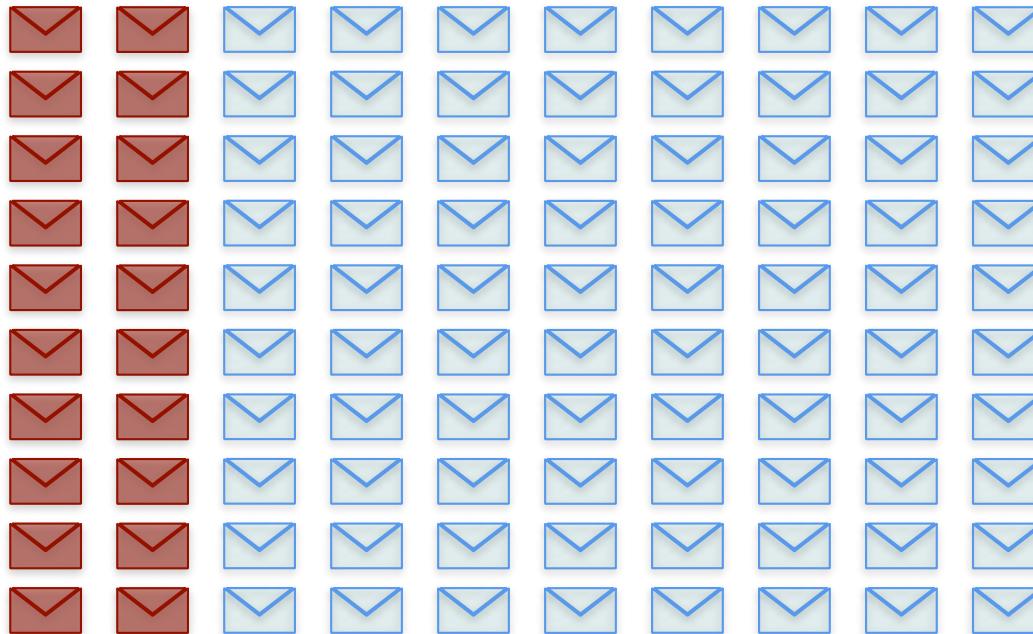
# Bayes Theorem: Spam Example



# Bayes Theorem: Spam Example



# Bayes Theorem: Spam Example



20 spam

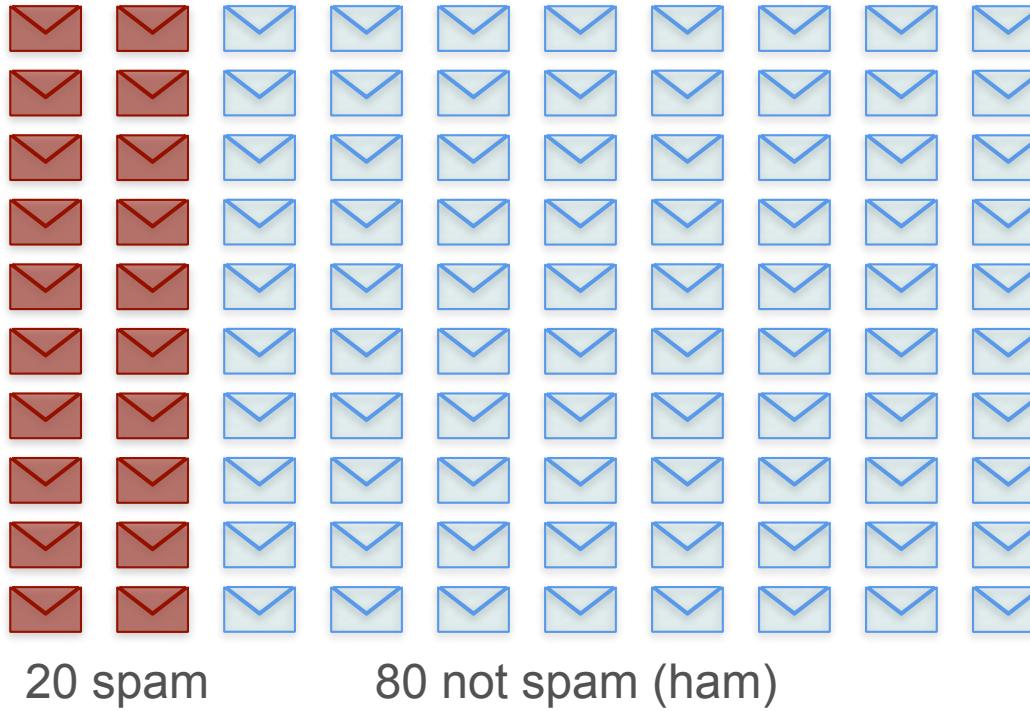
# Bayes Theorem: Spam Example



20 spam

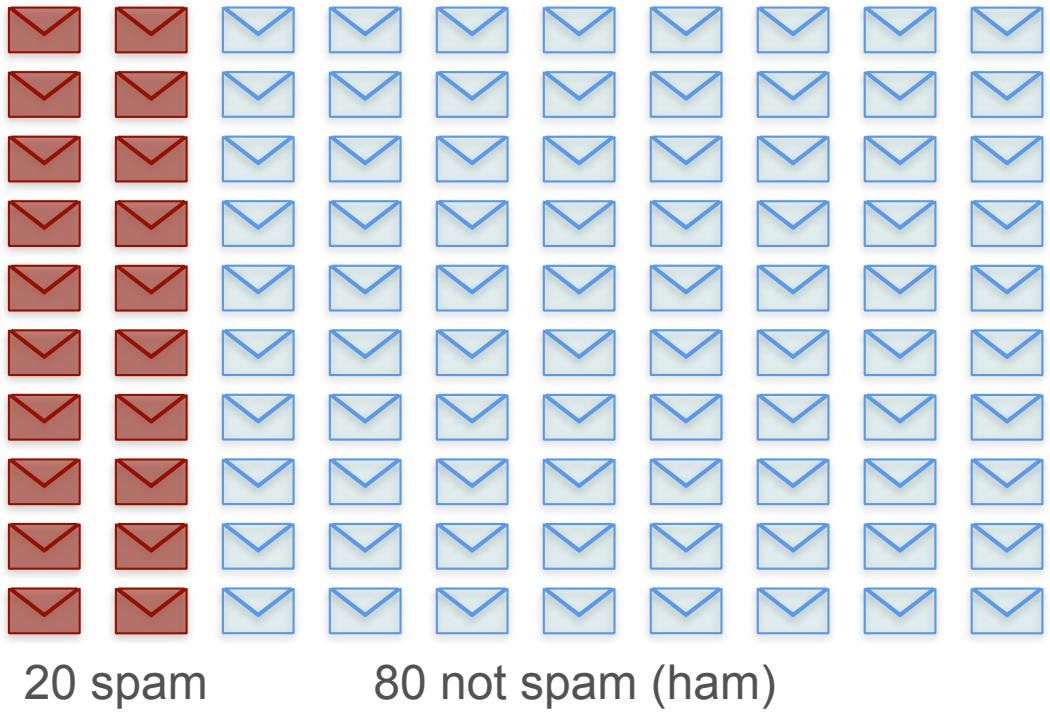
80 not spam (ham)

# Bayes Theorem: Spam Example

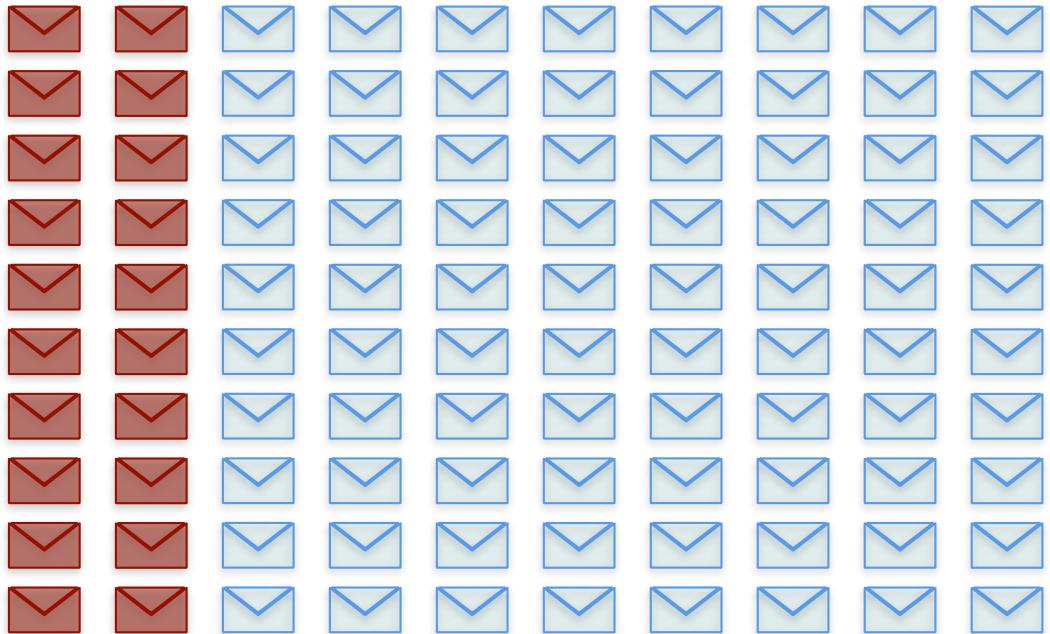


“lottery”

# Bayes Theorem: Spam Example



# Bayes Theorem: Spam Example



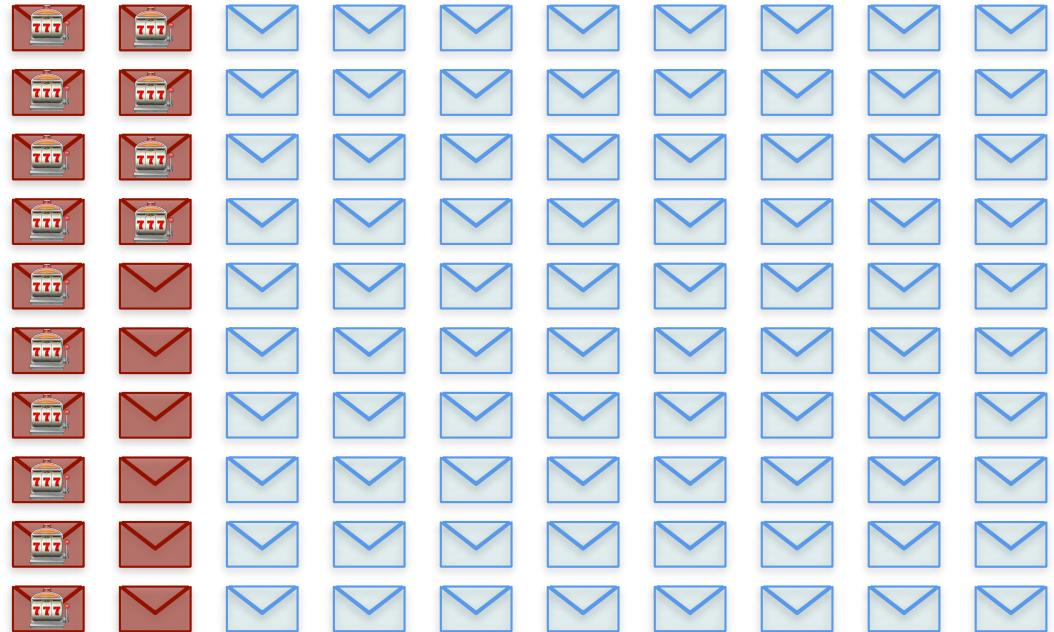
20 spam

80 not spam (ham)

# Bayes Theorem: Spam Example



14



20 spam

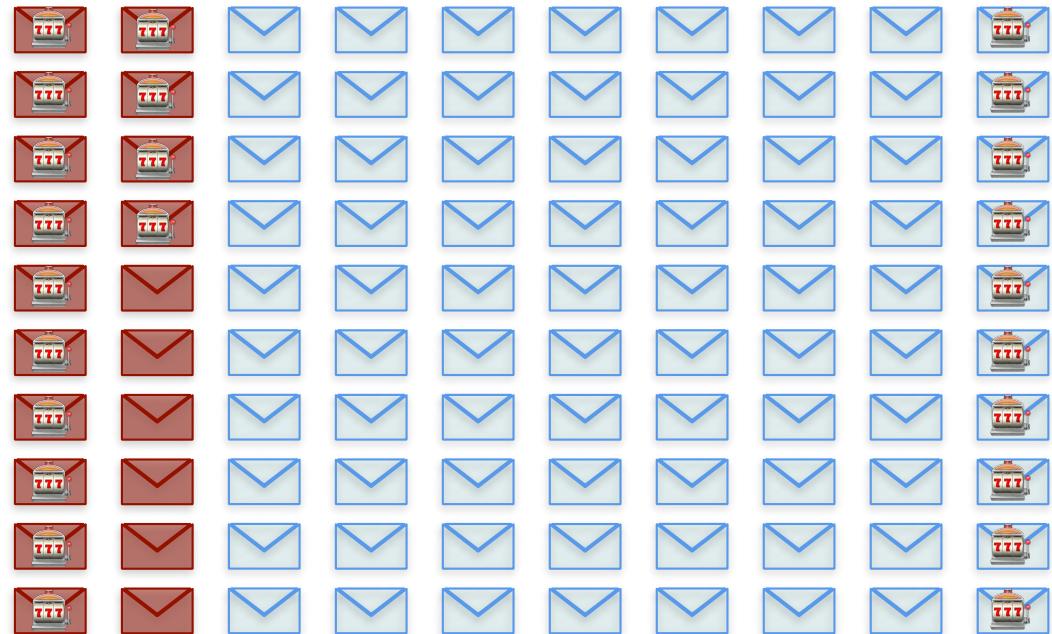
80 not spam (ham)

# Bayes Theorem: Spam Example

10



14

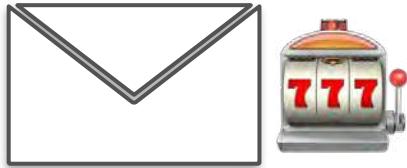


20 spam

80 not spam (ham)

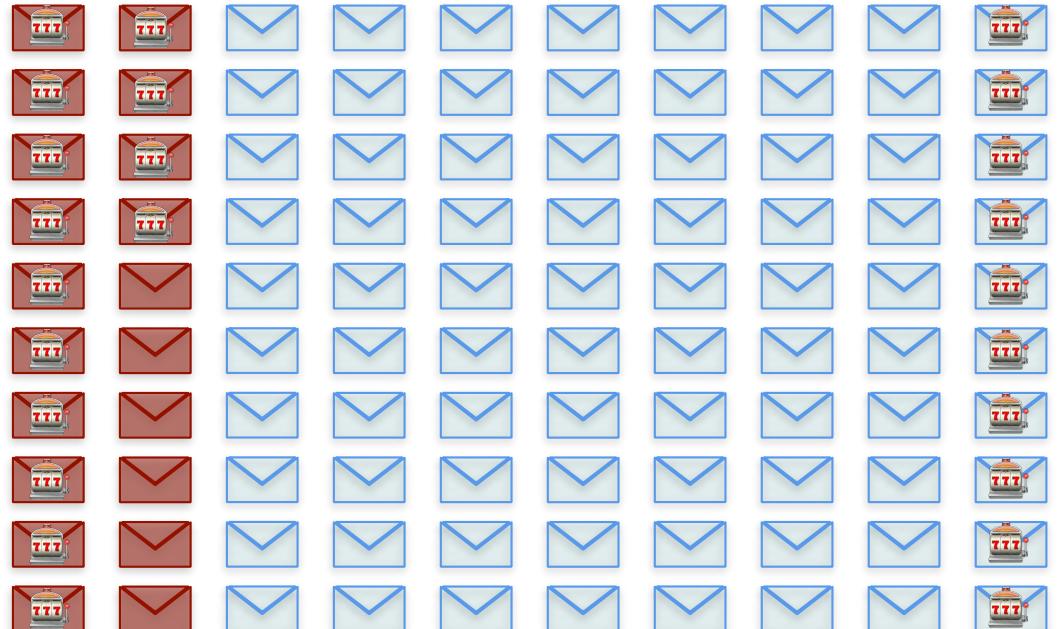
# Bayes Theorem: Spam Example

10



What is the probability that an email containing lottery is a spam?

14

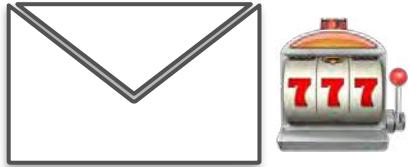


20 spam

80 not spam (ham)

# Bayes Theorem: Spam Example

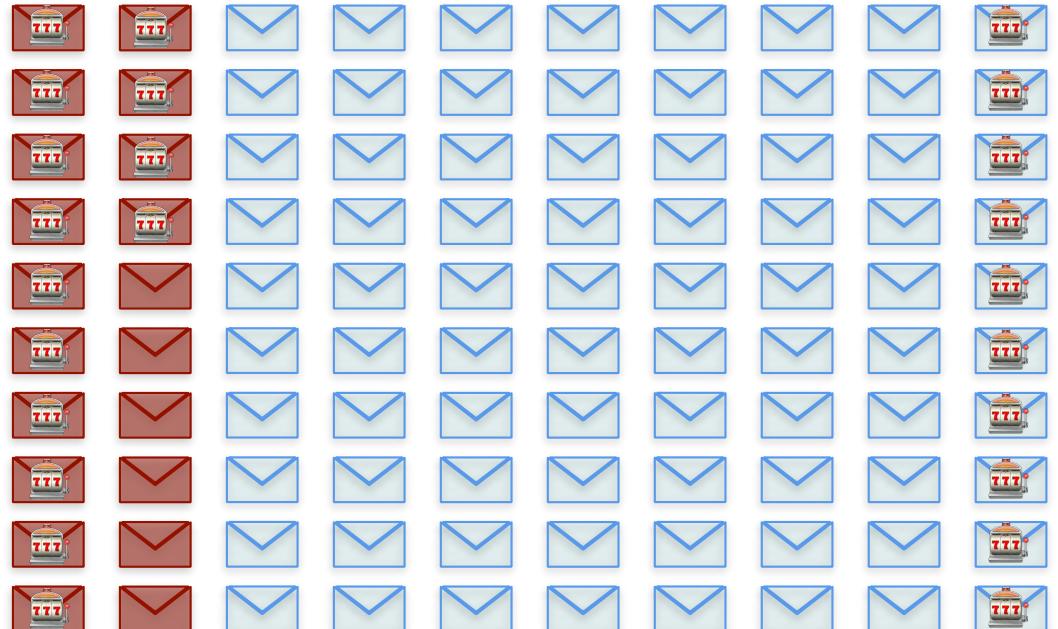
10



What is the probability that an email containing lottery is a spam?

$P(\text{spam} \mid \text{lottery})$

14



20 spam

80 not spam (ham)

# Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

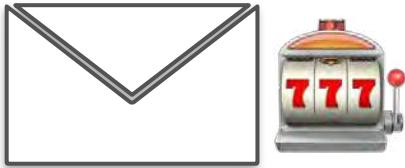


14



10

# Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$



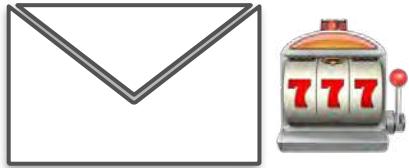
14

24 emails  
containing lottery



10

# Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



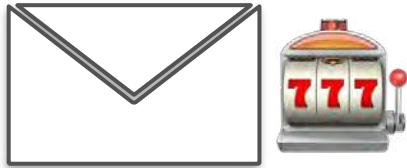
14

24 emails  
containing lottery



10

# Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



24 emails  
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

14

10

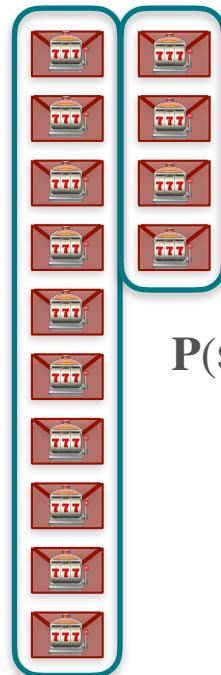


# Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



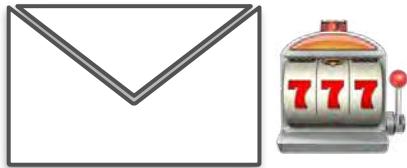
14

24 emails  
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

10

# Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

24 emails  
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

10



# Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

$$= \frac{7}{12} = 0.583$$

10



# Bayes Theorem: Spam Example (Formula Solution)

# Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam} \mid \text{lottery})$$

# Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

# Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

*A: Email is spam*

# Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

$A$ : Email is spam       $B$ : Email contains lottery

# Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

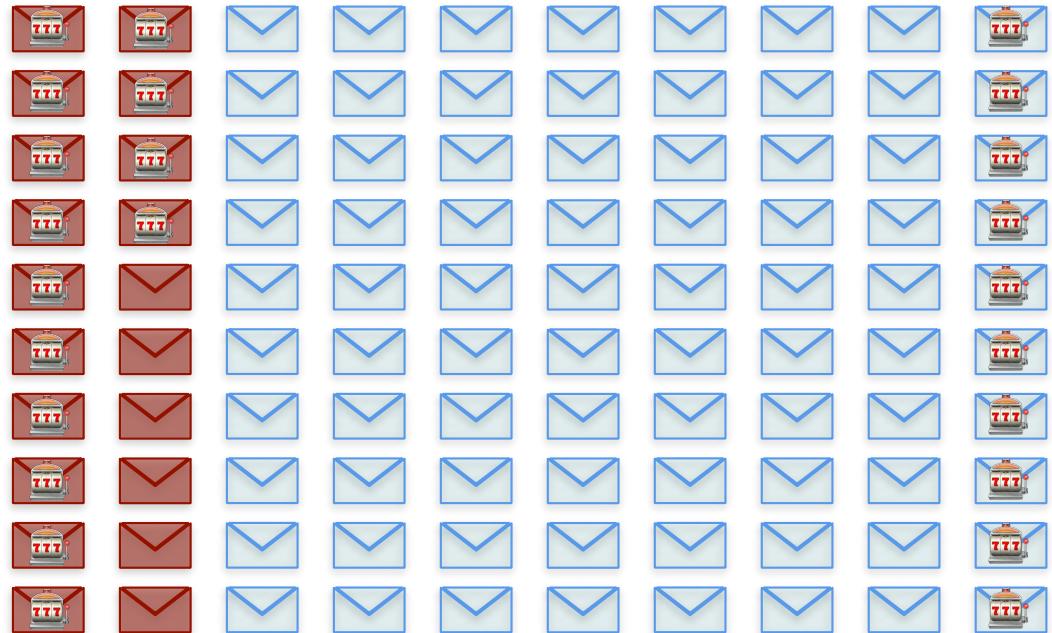
$A$ : Email is spam       $B$ : Email contains lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

# Bayes Theorem: Spam Example (Formula Solution)

10

14



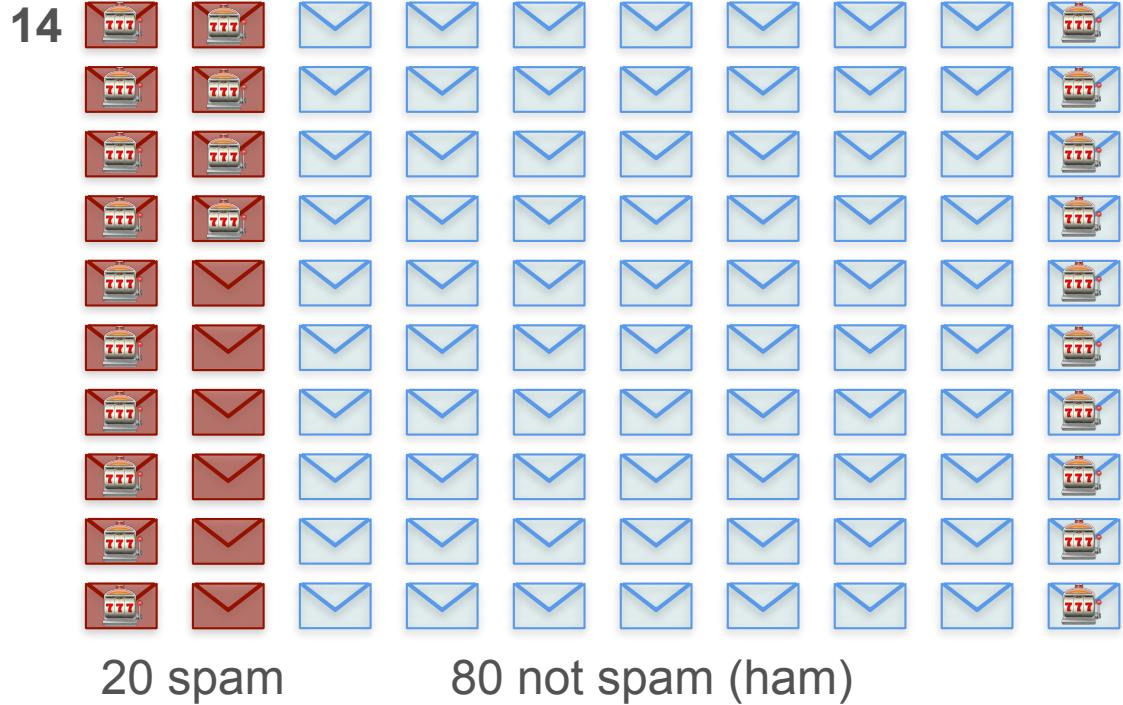
20 spam

80 not spam (ham)

# Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

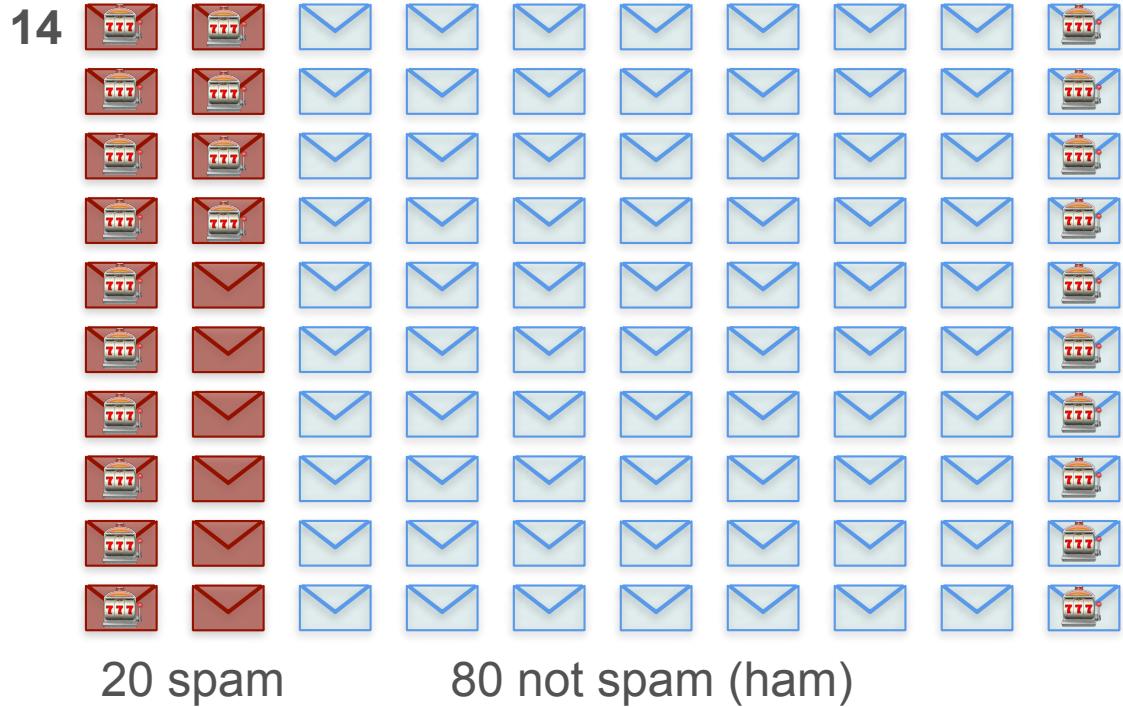


# Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$



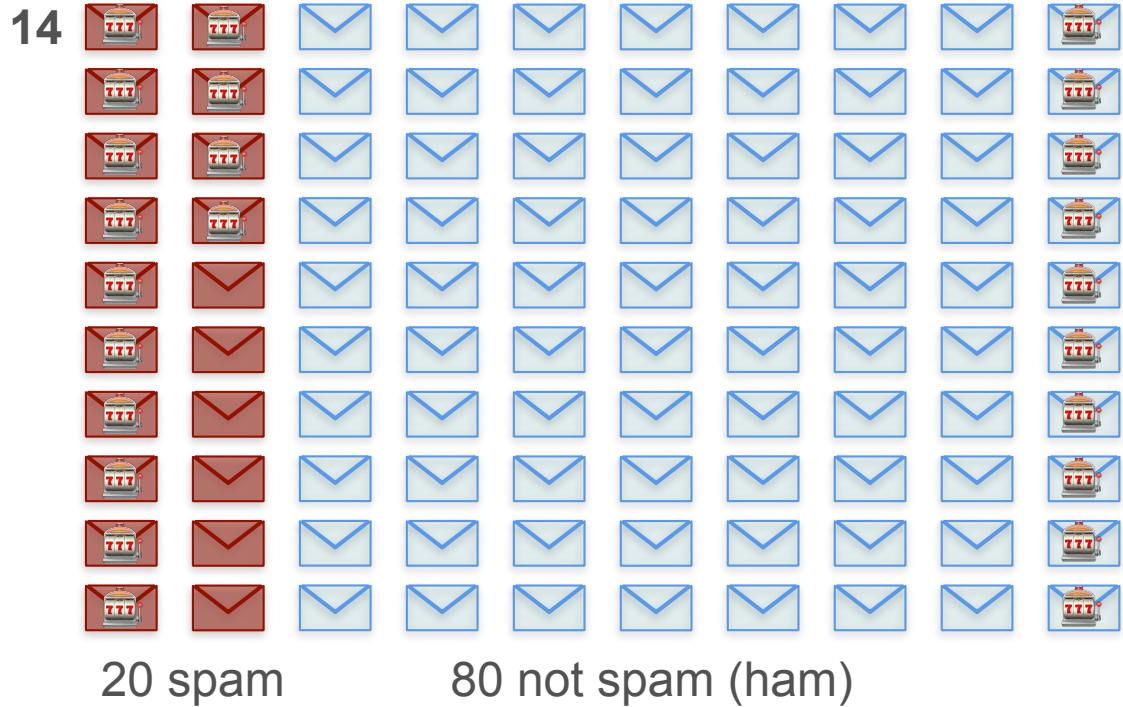
# Bayes Theorem: Spam Example (Formula Solution)

10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



# Bayes Theorem: Spam Example (Formula Solution)

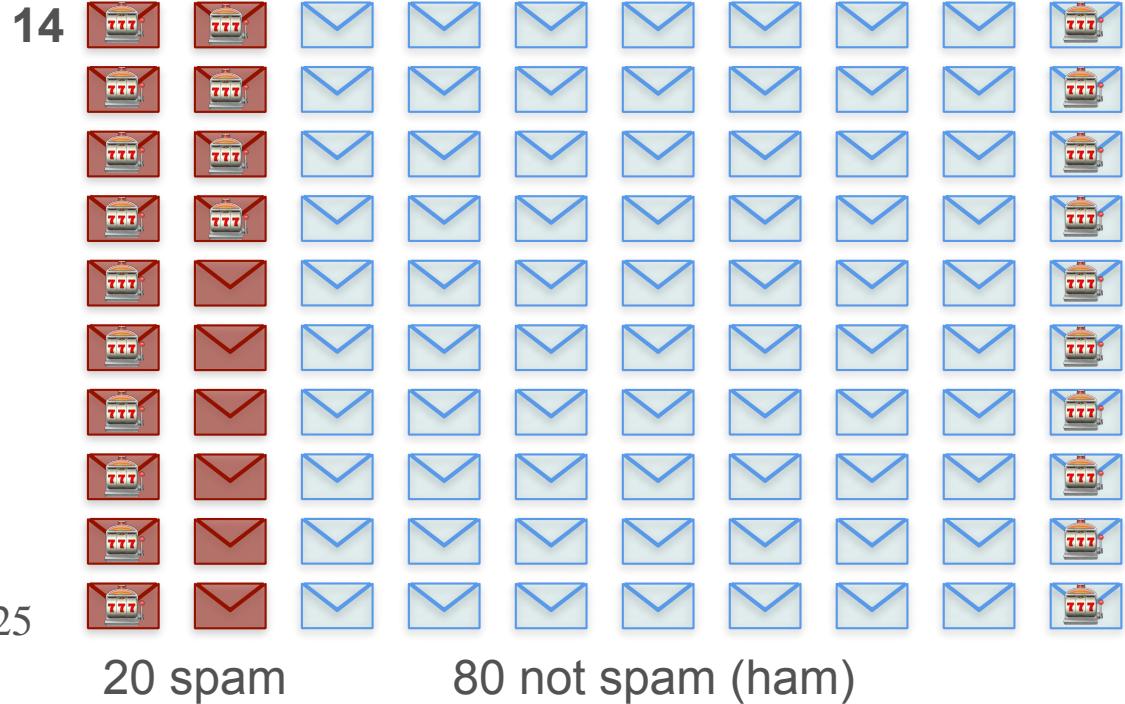
10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} | \text{spam}) = \frac{14}{20} = 0.7$$

$$P(\text{lottery} | \text{not spam}) = \frac{10}{80} = 0.125$$



# Bayes Theorem: Spam Example (Formula Solution)

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$$P(\text{spam}) = 0.2$$

# Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

# Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

# Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

# Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

# Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

$$P(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)}$$

# Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

$$P(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)} = 0.583$$

# Bayes Theorem

# Bayes Theorem

PRIOR

# Bayes Theorem

PRIOR

EVENT

# Bayes Theorem

PRIOR

EVENT

POSTERIOR

# Bayes Theorem

PRIOR

EVENT

POSTERIOR

$$\mathbf{P}(A)$$

# Bayes Theorem

PRIOR

$\mathbf{P}(A)$

EVENT

$E$

POSTERIOR

# Bayes Theorem

PRIOR

$$\mathbf{P}(A)$$

EVENT

$$E$$

POSTERIOR

$$\mathbf{P}(A | E)$$

# Prior and Posterior

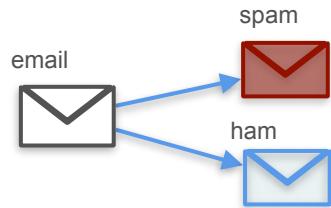
PRIOR

EVENT

POSTERIOR

# Prior and Posterior

PRIOR



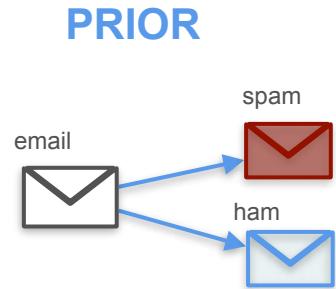
EVENT

POSTERIOR

$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

```
math display="block">P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}
```

# Prior and Posterior



**EVENT**

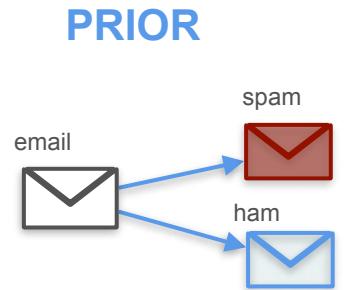


Email contains lottery

**POSTERIOR**

$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

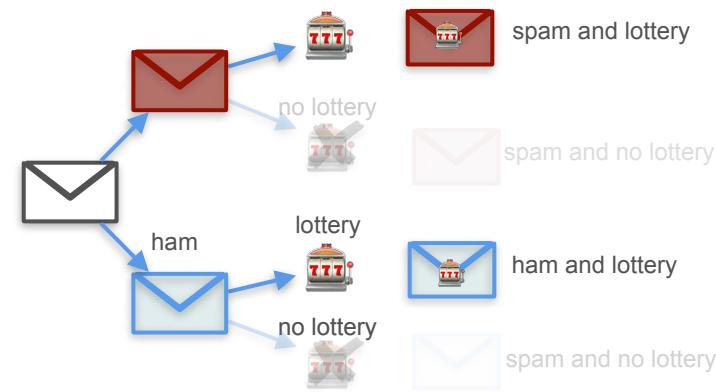
# Prior and Posterior



**EVENT**



Email contains lottery



$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

$$P(\text{spam} | \text{lottery}) = \frac{\text{spam and lottery}}{\text{spam and lottery} + \text{ham and lottery}}$$

# Prior and Posterior

PRIOR

EVENT

POSTERIOR

# Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

# Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

# Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

EVENT

POSTERIOR

$$P(\text{sum} = 10) = \frac{3}{36}$$

# Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



POSTERIOR

1st dice is 6

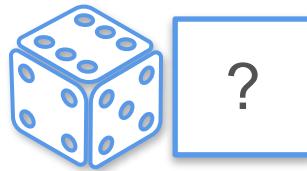
$$P(\text{sum} = 10) = \frac{3}{36}$$

# Prior and Posterior

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

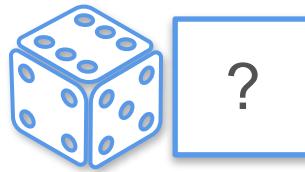
$$P(\text{sum} = 10) = \frac{3}{36}$$

# Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

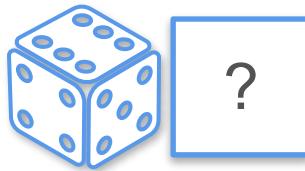
$$P(\text{sum} = 10) = \frac{3}{36}$$

# Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{3}{36}$$

$$P(\text{sum} = 10 | \text{1st is } 6) = \frac{1}{6}$$

# Prior and Posterior

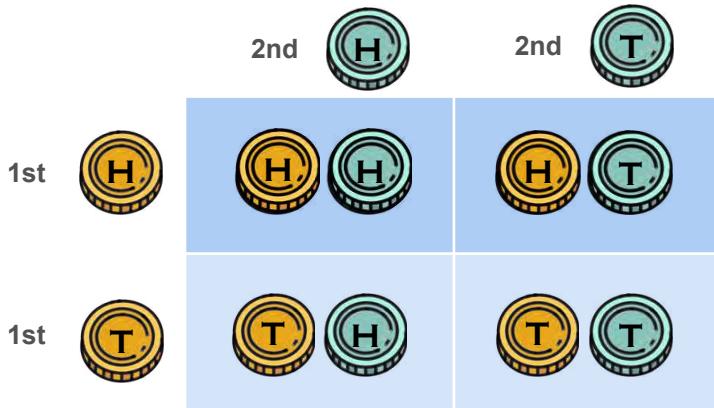
PRIOR

EVENT

POSTERIOR

# Prior and Posterior

PRIOR

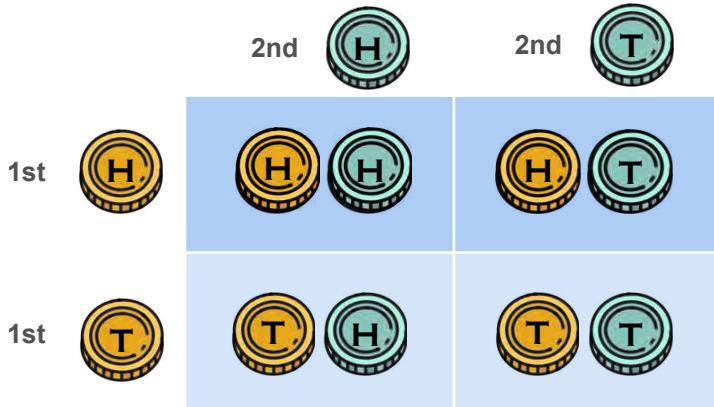


EVENT

POSTERIOR

# Prior and Posterior

PRIOR



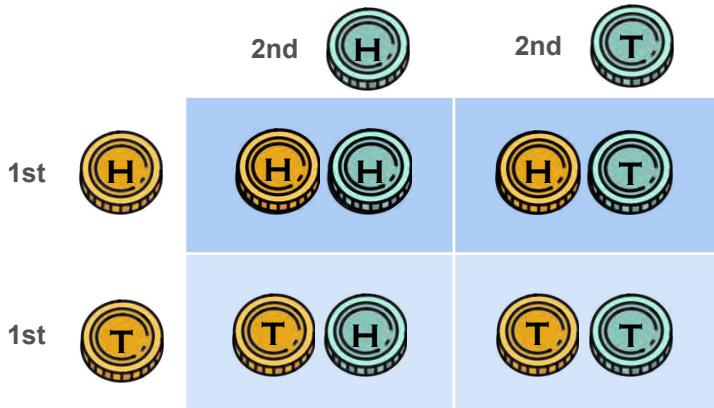
EVENT

POSTERIOR

$$P(HH) = \frac{1}{4}$$

# Prior and Posterior

PRIOR



EVENT



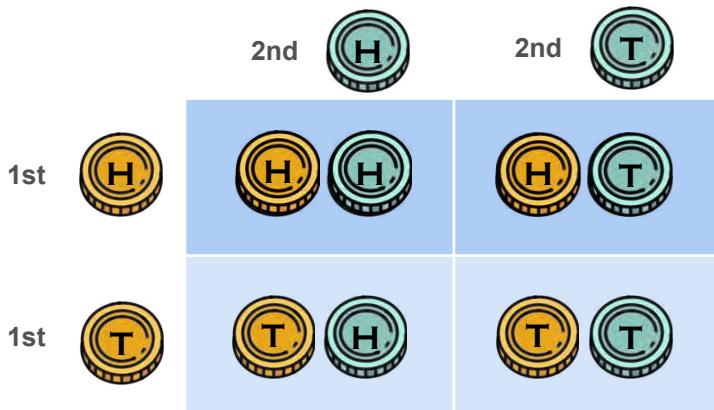
1st coin is  $H$

POSTERIOR

$$P(HH) = \frac{1}{4}$$

# Prior and Posterior

PRIOR



EVENT



1st coin is  $H$

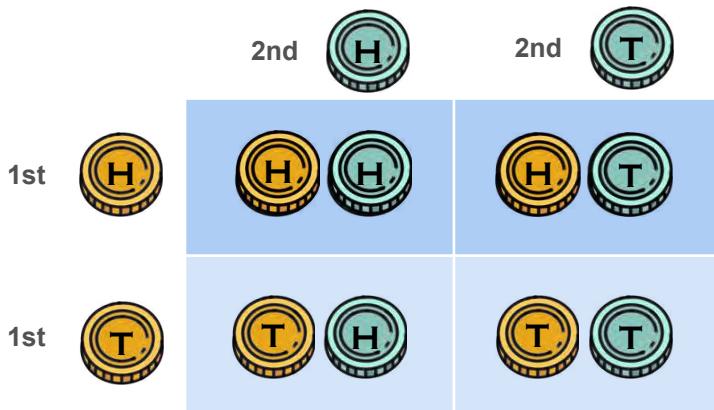
POSTERIOR



$$P(HH) = \frac{1}{4}$$

# Prior and Posterior

PRIOR



$$P(HH) = \frac{1}{4}$$

EVENT



1st coin is  $H$

POSTERIOR



$$P(HH | \text{1st is } H) = \frac{1}{2}$$

# Video 8e: the Naive Bayes Model

# What About 2 Events?

PRIOR

EVENT

POSTERIOR

# What About 2 Events?

PRIOR

EVENT

POSTERIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

# What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'

POSTERIOR

# What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

# What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

# What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$

# What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery}}{\text{Red Envelope with lottery} + \text{Blue Envelope with lottery}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning}}{\text{Red Envelope with winning} + \text{Blue Envelope with winning}}$$



Email contains 'lottery' and 'winning'

# What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$

?

# What About 2 Events?

EVENT



POSTERIOR

Email contains 'lottery' and 'winning'

# What About 2 Events?

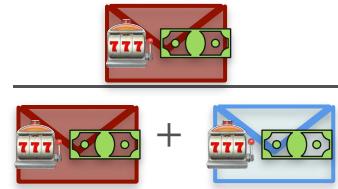
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$



# What About 2 Events?

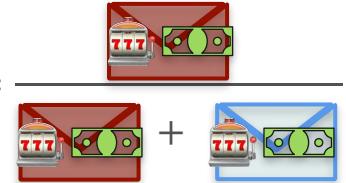
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery \& winning}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{spam} | \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} | \text{ham})}$$

# What About 2 Events?

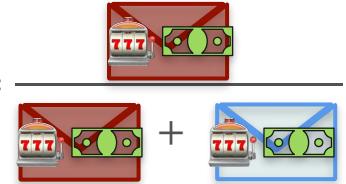
EVENT



Email contains 'lottery' and 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{?}}{\text{?}}$$



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

# What About 2 Events?

EVENT



Email contains 'lottery' and 'winning'

POSTERIOR



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{# Spam emails with 'lottery' and 'winning'}}{\text{# Emails with 'lottery' and 'winning'}}$$

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

# What About More Than 2 Events?

EVENT

POSTERIOR

# What About More Than 2 Events?

EVENT

POSTERIOR

Email contains  $w_1, w_2, \dots, w_{100}$

# What About More Than 2 Events?

EVENT

POSTERIOR

Email contains  $w_1, w_2, \dots, w_{100}$

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

# What About More Than 2 Events?

EVENT

POSTERIOR

Email contains  $w_1, w_2, \dots, w_{100}$

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{P(\text{spam}) P(w_1, \dots, w_{100} | \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

?

# What About More Than 2 Events?

EVENT

POSTERIOR

Email contains  $w_1, w_2, \dots, w_{100}$

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

A red arrow points from the fraction above to the term  $P(w_1, \dots, w_{100} | \text{spam})$ , which is highlighted with a red oval and followed by a question mark.

# What About More Than 2 Events?

EVENT

POSTERIOR

Email contains  $w_1, w_2, \dots, w_{100}$

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

The fraction  $\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Emails with } w_1, \dots, w_{100}}$  is highlighted with a red underline and a red arrow points from it to a question mark  $?$ . The term  $P(w_1, \dots, w_{100} | \text{spam})$  is also highlighted with a red circle.

# Is There a Quicker Way To Estimate the Probability?

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

# Is There a Quicker Way To Estimate the Probability?

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

# Is There a Quicker Way To Estimate the Probability?

## Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

# Is There a Quicker Way To Estimate the Probability?

## Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

$P(A \cap B)$

↓

# Is There a Quicker Way To Estimate the Probability?

## Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

$P(A \cap B) = P(A) \cdot P(B)$

↓

The terms  $P(\text{lottery \& winning} \mid \text{spam})$  and  $P(\text{lottery \& winning} \mid \text{ham})$  are circled in red.

# Is There a Quicker Way To Estimate the Probability?

## Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

# Is There a Quicker Way To Estimate the Probability?

## Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery} \mid \text{ham}) \cdot \mathbf{P}(\text{winning} \mid \text{ham})}$$

# Is There a Quicker Way To Estimate the Probability?

Naive assumption

$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_n \mid \text{ham})}$$

# Is There a Quicker Way To Estimate the Probability?

## Naive assumption

The appearances of the words  $w_1, w_2, \dots, w_n$  are independent

$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_n \mid \text{ham})}$$

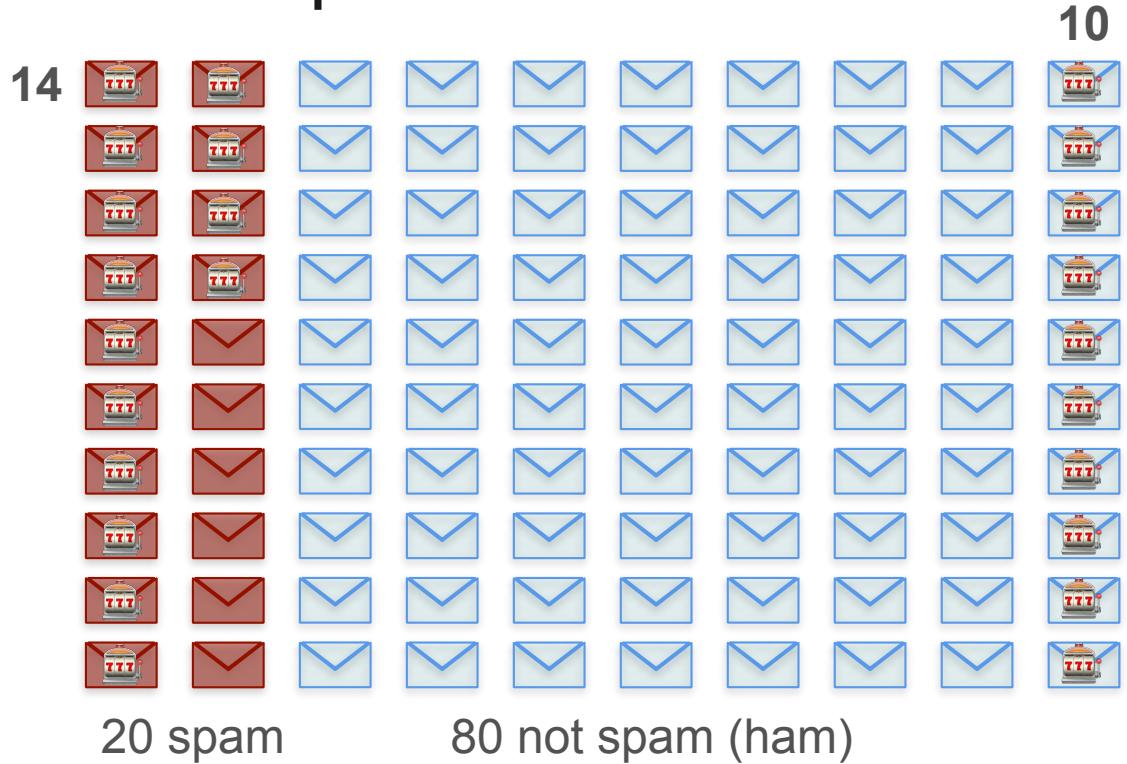
# Is There a Quicker Way To Estimate the Probability?

## Naive assumption

The appearances of the words  $w_1, w_2, \dots, w_n$  are independent

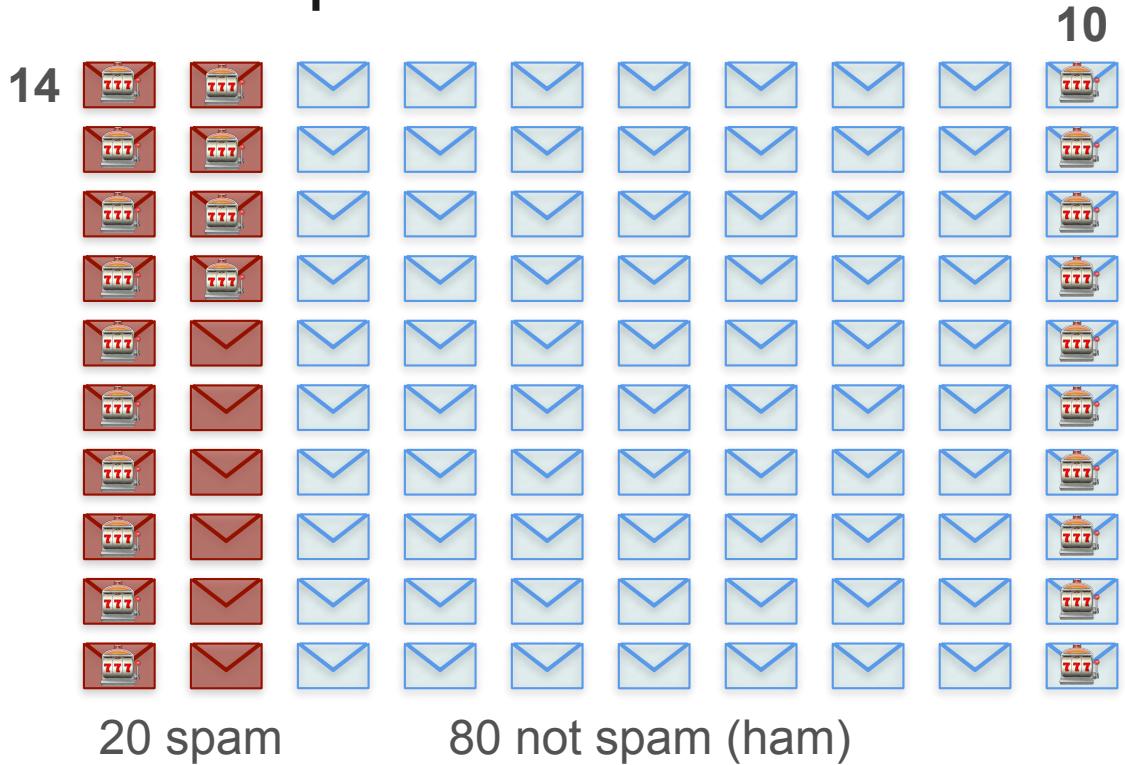
$$P(\text{spam} | w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1 | \text{spam}) \cdots P(w_n | \text{spam})}{P(\text{spam}) \cdot P(w_1 | \text{spam}) \cdots P(w_n | \text{spam}) + P(\text{ham}) \cdot P(w_1 | \text{ham}) \cdots P(w_n | \text{ham})}$$

# Naive Bayes: Spam Example



# Naive Bayes: Spam Example

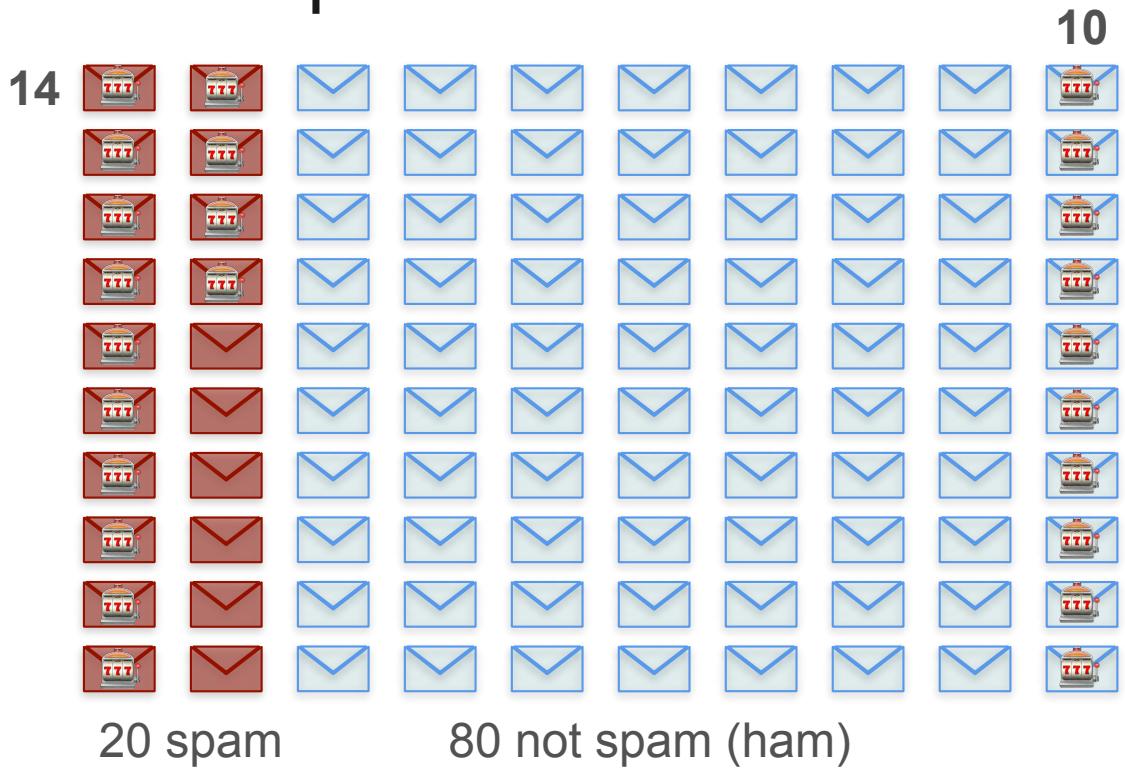
$$P(\text{spam}) = \frac{20}{100} = 0.2$$



# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

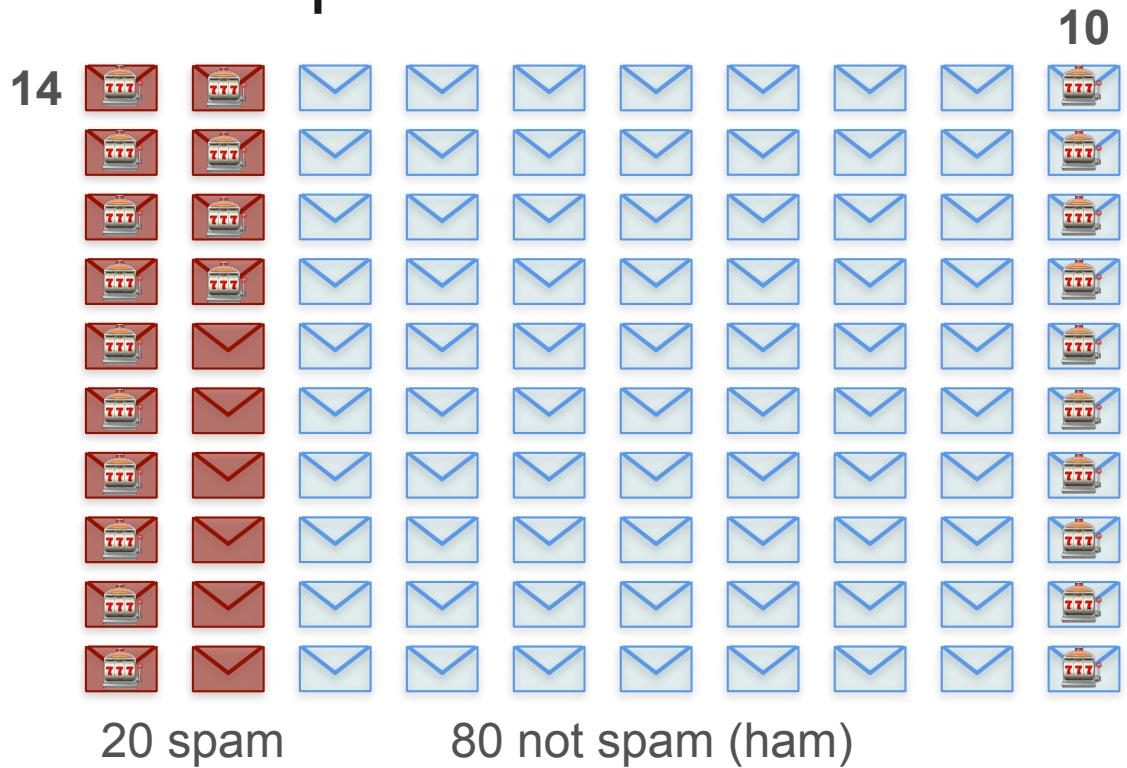


# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



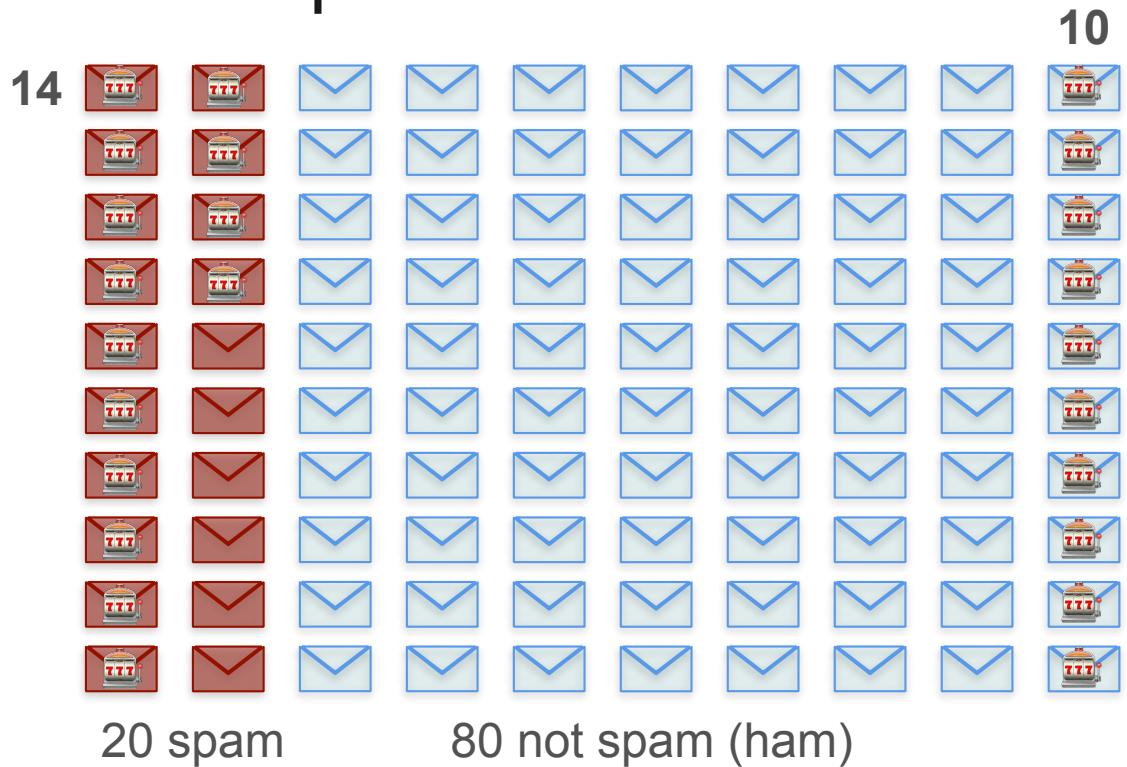
# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} | \text{spam}) = \frac{14}{20} = 0.7$$

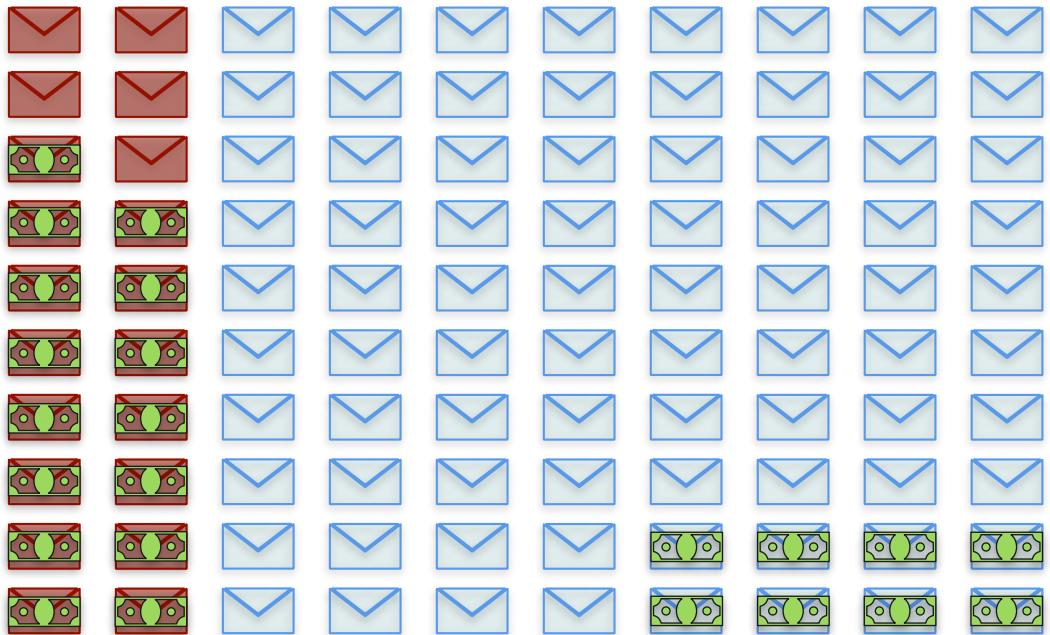
$$P(\text{lottery} | \text{ham}) = \frac{10}{80} = 0.125$$



# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$



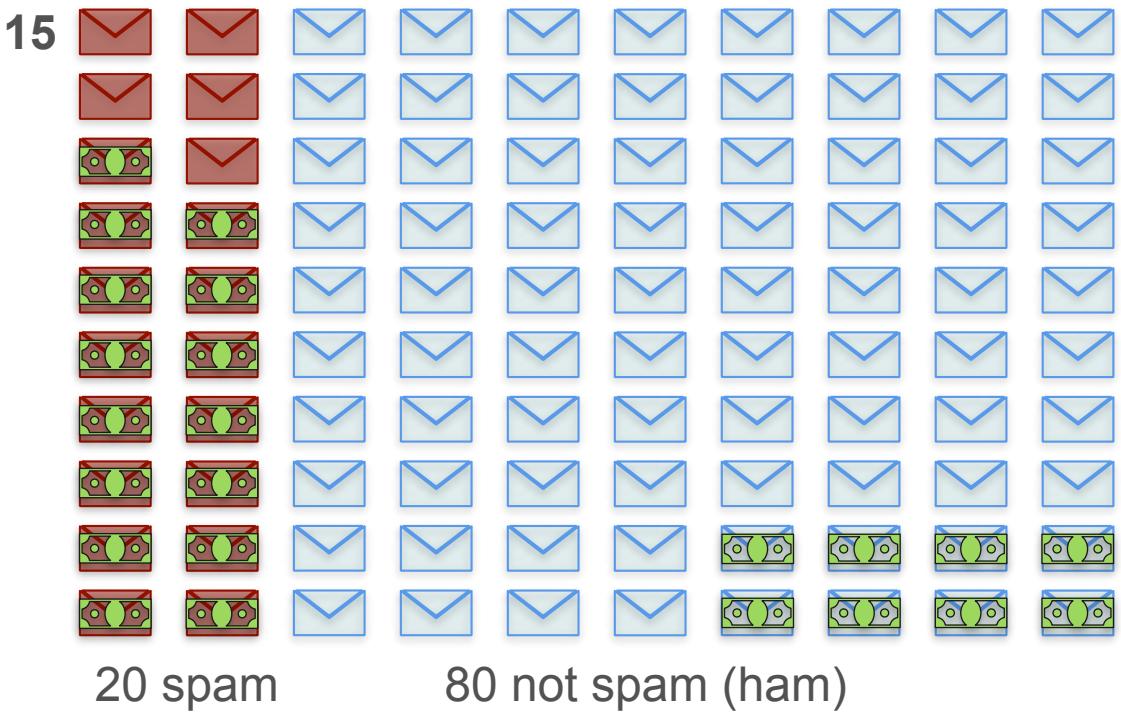
20 spam

80 not spam (ham)

# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

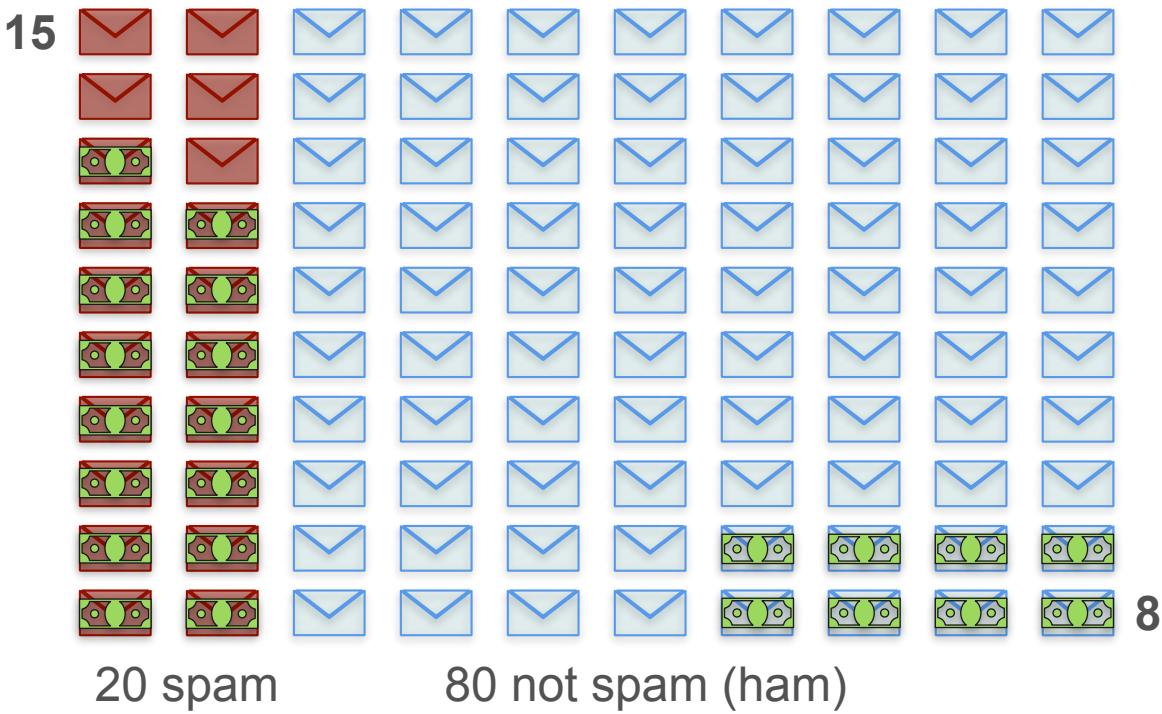
$$P(\text{ham}) = \frac{80}{100} = 0.8$$



# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

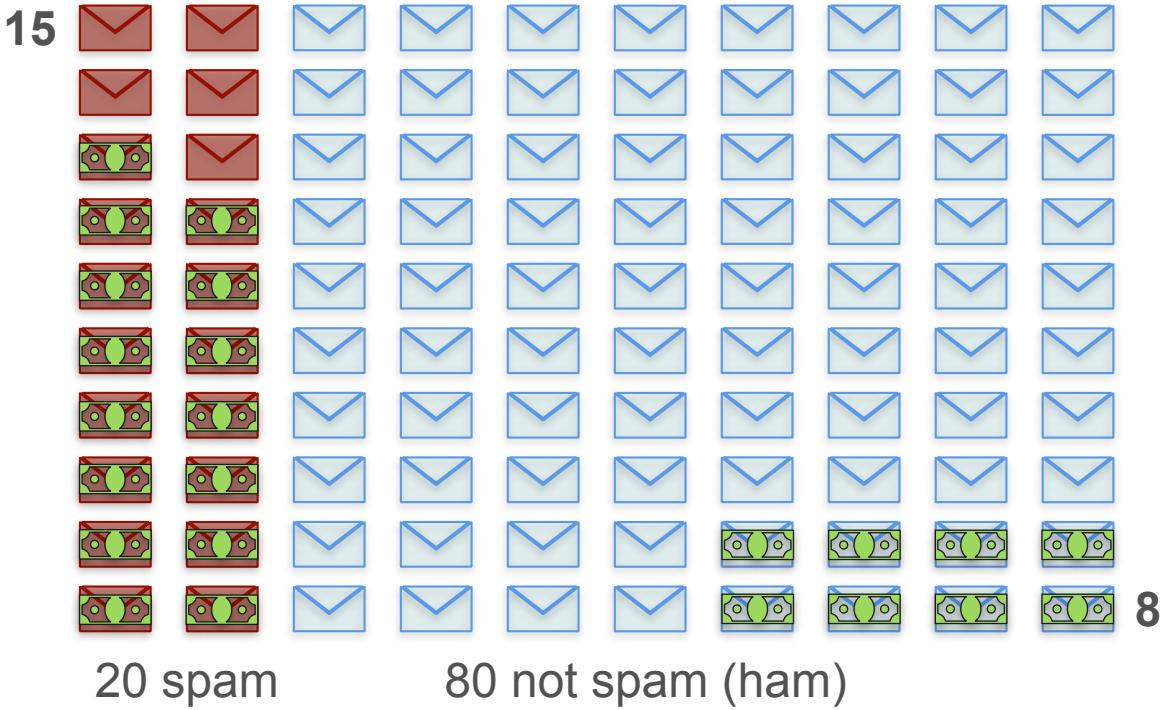


# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$



# Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$

$$P(\text{winning} \mid \text{ham}) = \frac{8}{80} = 0.1$$



# Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

# Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

# Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)}$$

# Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)} = 0.913$$



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## Introduction to probability

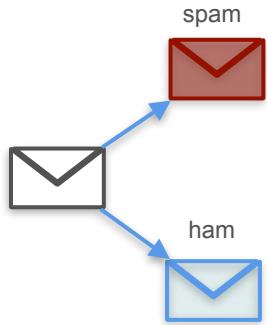
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# Probability in Machine Learning

# Bayes Theorem

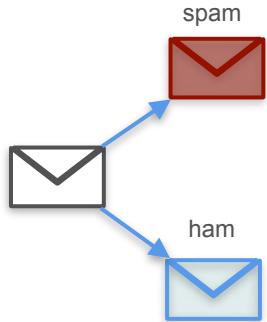
# Bayes Theorem

PRIOR



# Bayes Theorem

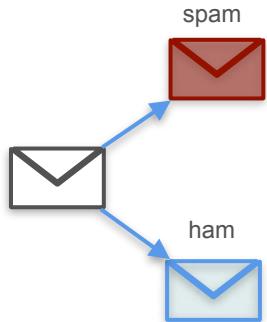
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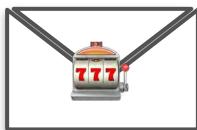
$$P(\text{spam}) = \frac{\text{spam icon}}{\text{spam icon} + \text{ham icon}}$$

# Bayes Theorem

PRIOR



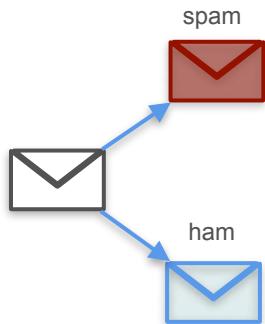
EVENT



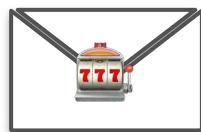
$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

# Bayes Theorem

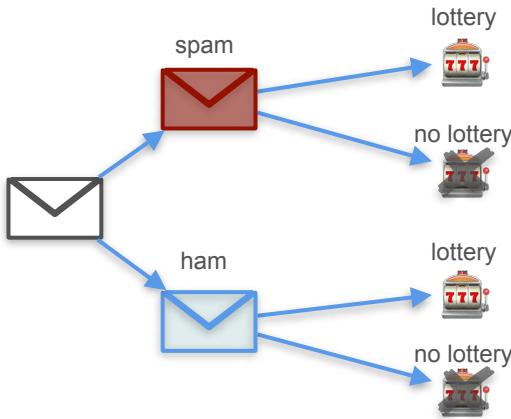
PRIOR



EVENT



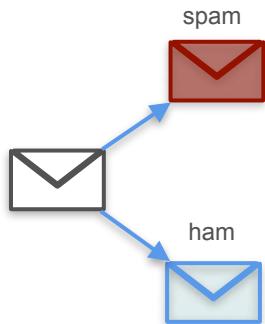
POSTERIOR



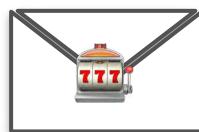
$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

# Bayes Theorem

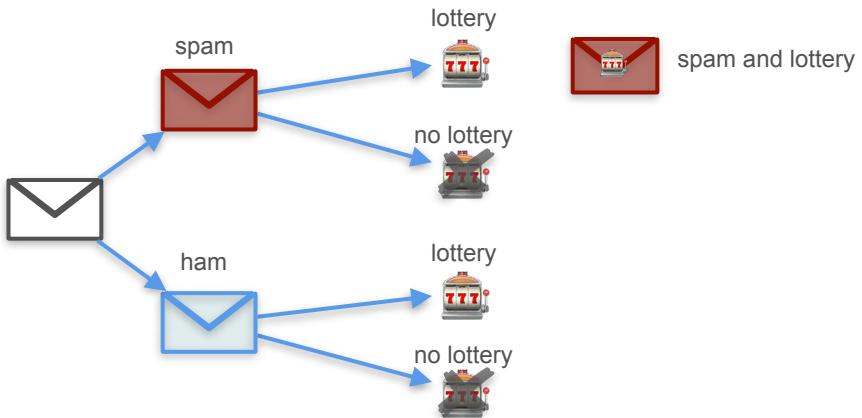
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EVENT



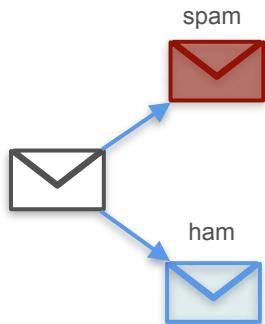
POSTERIOR



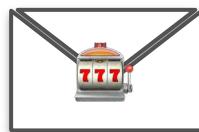
$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

# Bayes Theorem

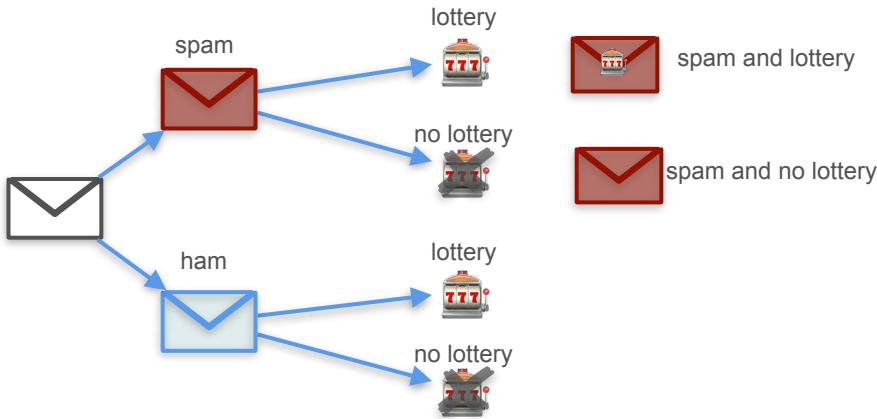
PRIOR



EVENT



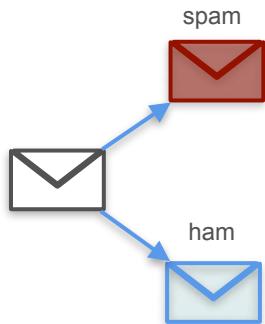
POSTERIOR



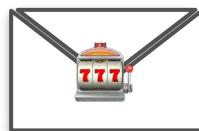
$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

# Bayes Theorem

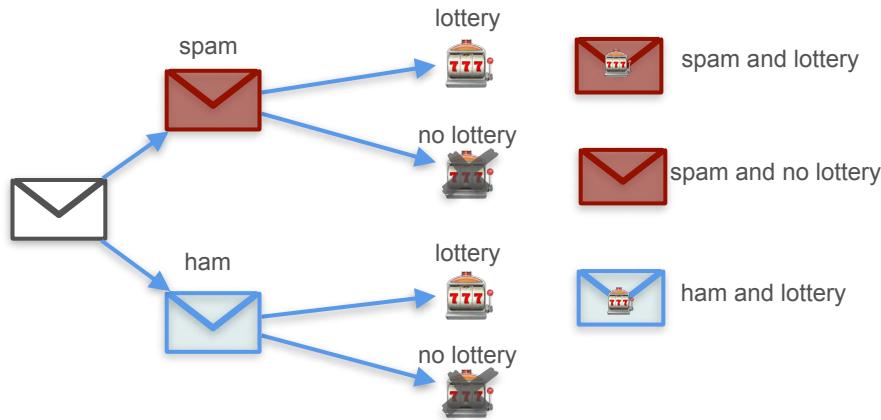
PRIOR



EVENT



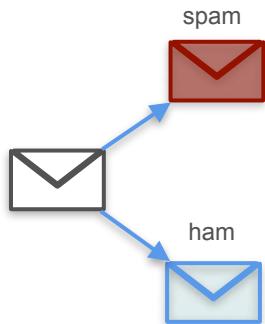
POSTERIOR



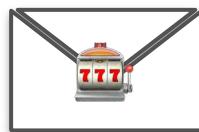
$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

# Bayes Theorem

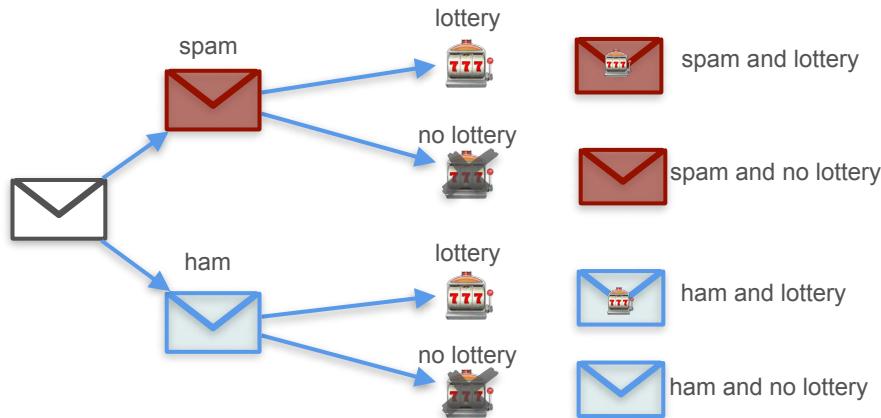
PRIOR



EVENT



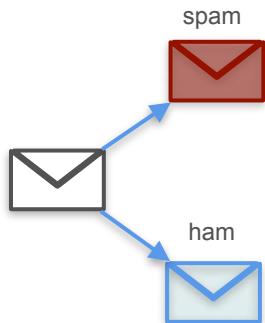
POSTERIOR



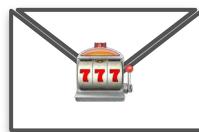
$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

# Bayes Theorem

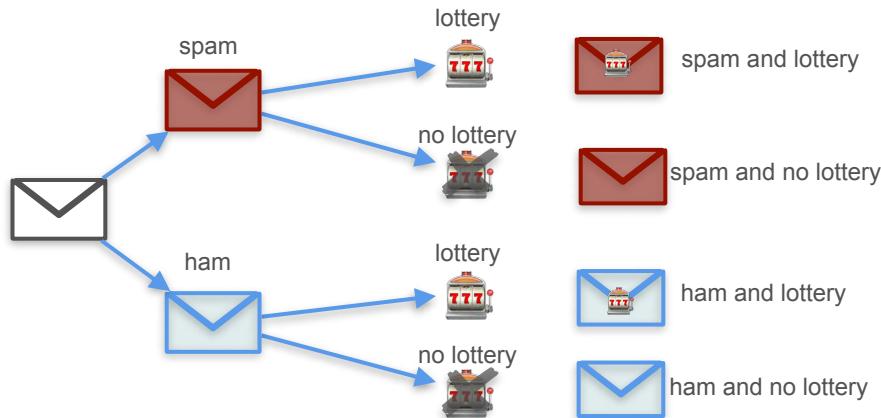
PRIOR



EVENT



POSTERIOR

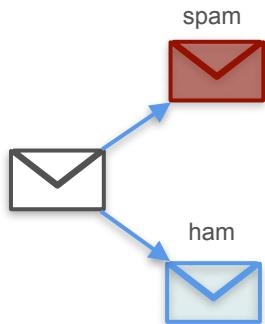


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

$$P(\text{spam} | \text{lottery}) =$$

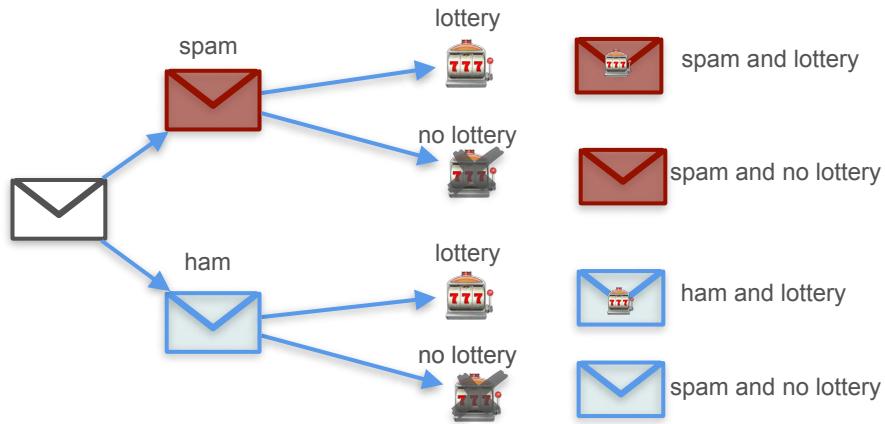
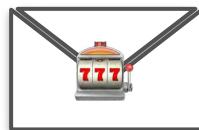
# Bayes Theorem

PRIOR



$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

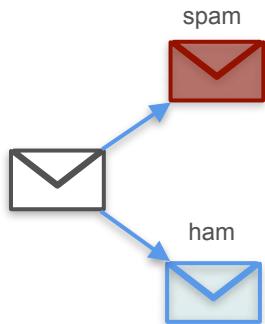
EVENT



$$P(\text{spam} | \text{lottery}) =$$

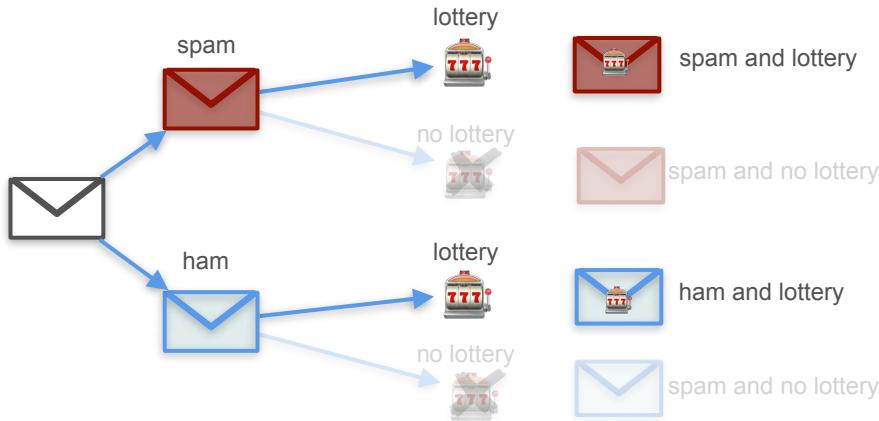
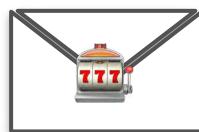
# Bayes Theorem

PRIOR



$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

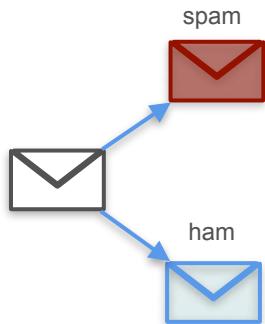
EVENT



$$P(\text{spam} | \text{lottery}) =$$

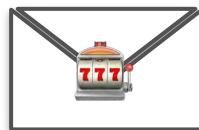
# Bayes Theorem

PRIOR

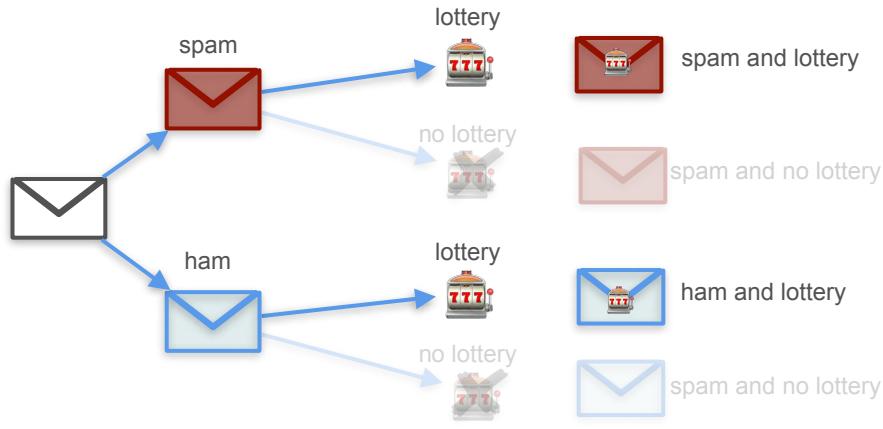


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

EVENT



POSTERIOR



$$P(\text{spam} | \text{lottery}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

# Example Problem

# Example Problem

Image recognition

# Example Problem

Image recognition

- What is the probability that there is a cat in the image



# Example Problem

Image recognition

- What is the probability that there is a cat in the image
- $P(\text{cat} \mid \text{image}) = P(\text{cat} \mid \text{pixel}_1, \text{pixel}_2, \dots, \text{pixel}_n)$



# Example Problem

# Example Problem

Classification

# Example Problem

## Classification

Patient 1		
A	Age	29
G	Gender	Female
H	Height	169 cm
W	Weight	62 kg
S	Smoker	No
...	...	...
B	Heart rate	63
B	Blood pressure	120 90

- Is this patient healthy?

# Example Problem

## Classification

Patient 1		
A	Age	29
G	Gender	Female
H	Height	169 cm
W	Weight	62 kg
S	Smoker	No
...	...	...
B	Heart rate	63
B	Blood pressure	120 90

- Is this patient healthy?
- Calculate  $P(\text{healthy} \mid \text{symptoms and history})$

# Example Problem

# Example Problem

Sentiment analysis

# Example Problem

Sentiment analysis

the first cold shower  
even the monkey seems to want  
a little coat of straw

Matsuo Bashō

# Example Problem

Sentiment analysis

the first cold shower  
even the monkey seems to want  
a little coat of straw

- Is this a happy sentence?

Matsuo Bashō

# Example Problem

Sentiment analysis

the first cold shower  
even the monkey seems to want  
a little coat of straw

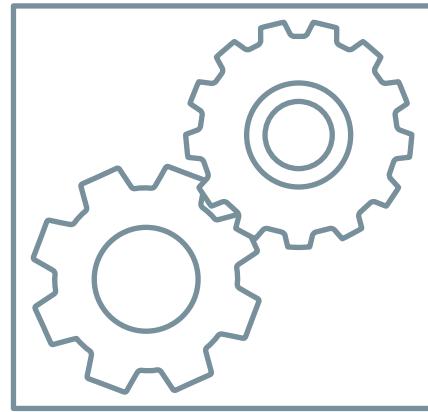
Matsuo Bashō

- Is this a happy sentence?
- Calculate  $P(\text{happy} \mid \text{words in the sentence})$

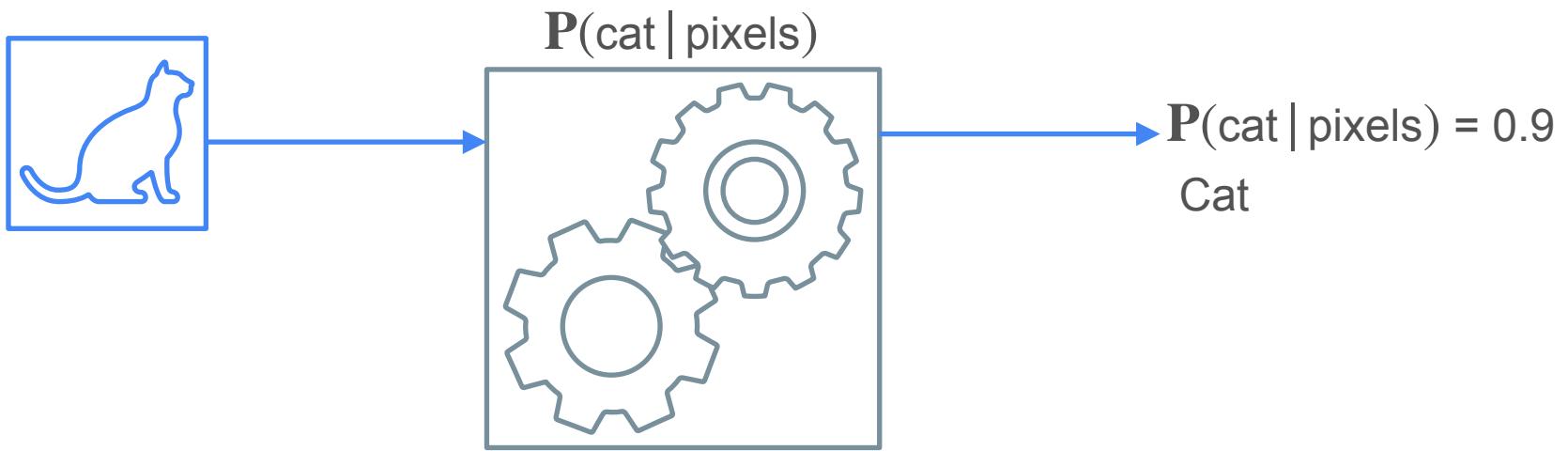
# Example Solution

# Example Solution

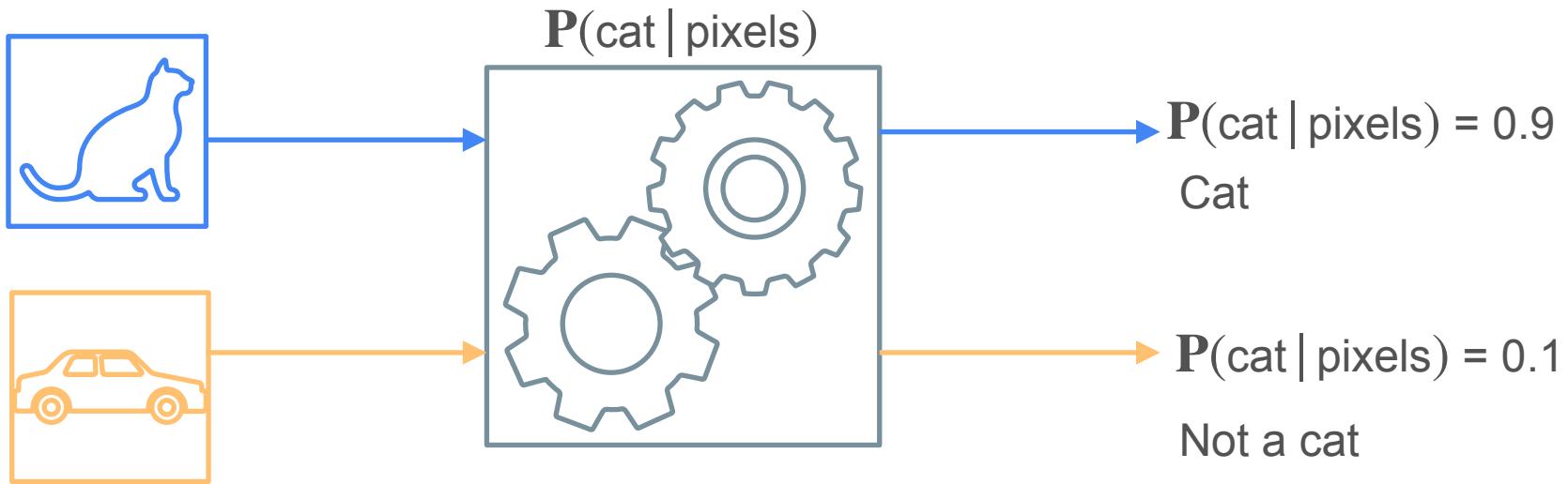
$P(\text{cat} \mid \text{pixels})$



# Example Solution



# Example Solution



# Example Problem: Generative Models

# Example Problem: Generative Models

Face generation

# Example Problem: Generative Models

Face generation



Image generated by a StyleGAN

# Example Problem: Generative Models

Face generation

- Generate a group of pixels such that the resulting image looks like a human face.
- Goal: generate images such that  $P(\text{face} \mid \text{pixels})$  is high.



Image generated by a StyleGAN

# W1 Lesson 2

# Probability Distributions



DeepLearning.AI

# Probability Distributions

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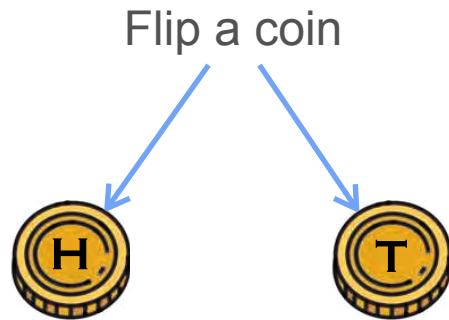
## Random Variables

# From Events to Random Variables

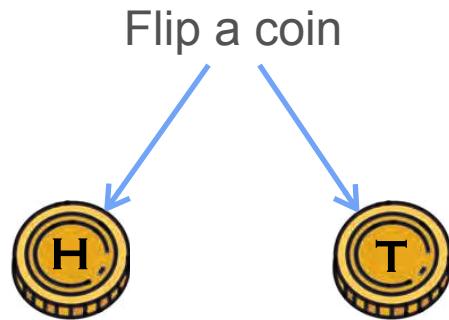
# From Events to Random Variables

Flip a coin

# From Events to Random Variables

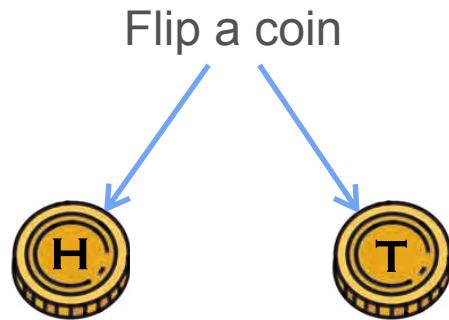


# From Events to Random Variables



$$P(H) = 0.5$$

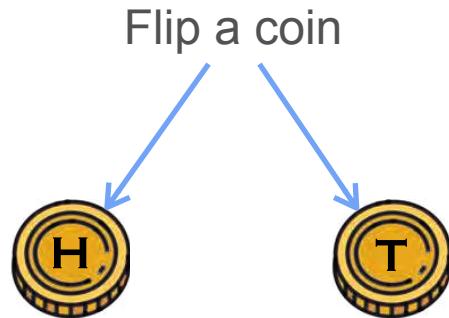
# From Events to Random Variables



$$\mathbf{P}(\textcolor{blue}{H}) = 0.5 \quad \mathbf{P}(\textcolor{blue}{T}) = 0.5$$

# From Events to Random Variables

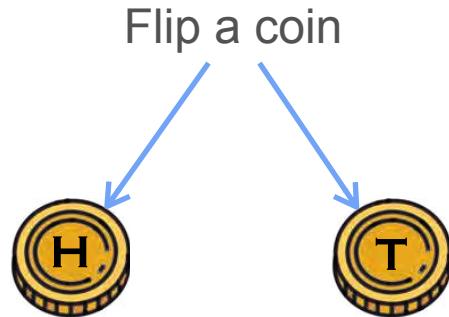
$X$  = Number of heads



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

# From Events to Random Variables

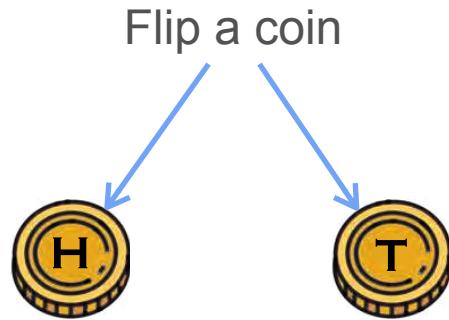
$X$  = Number of heads



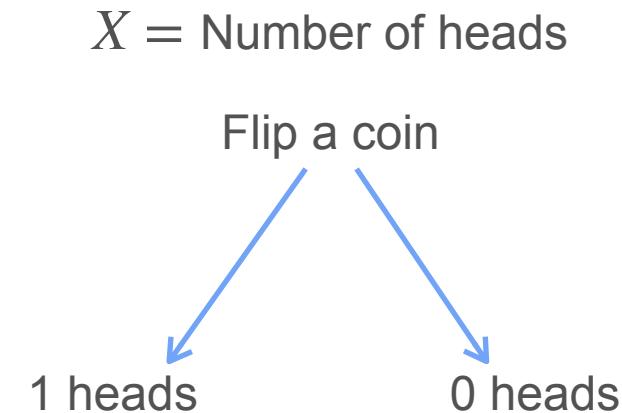
Flip a coin

$$\mathbf{P}(\textcolor{blue}{H}) = 0.5 \quad \mathbf{P}(\textcolor{blue}{T}) = 0.5$$

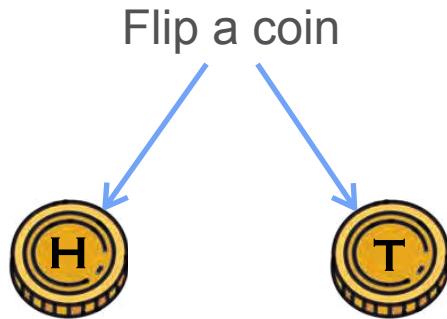
# From Events to Random Variables



$$P(H) = 0.5 \quad P(T) = 0.5$$

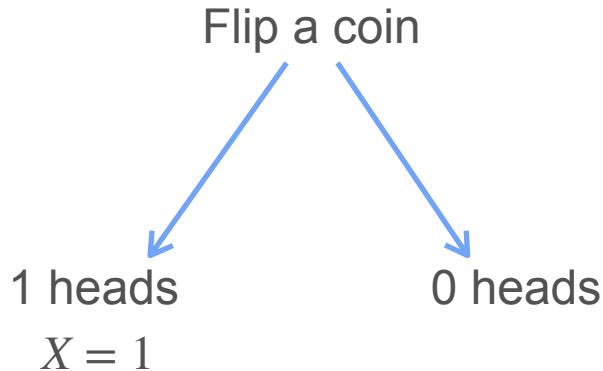


# From Events to Random Variables

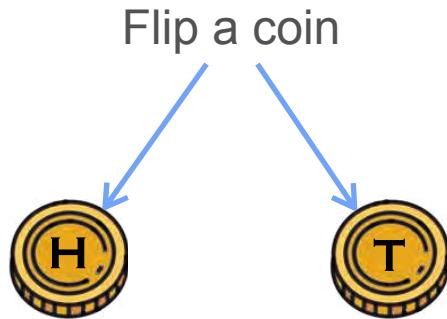


$$P(H) = 0.5 \quad P(T) = 0.5$$

$X = \text{Number of heads}$

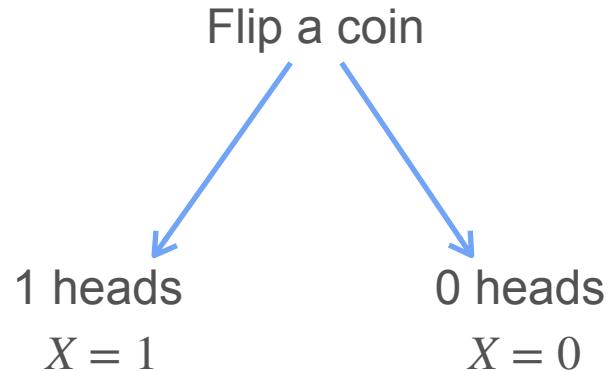


# From Events to Random Variables

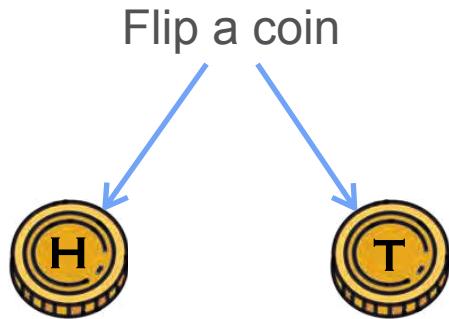


$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$X = \text{Number of heads}$

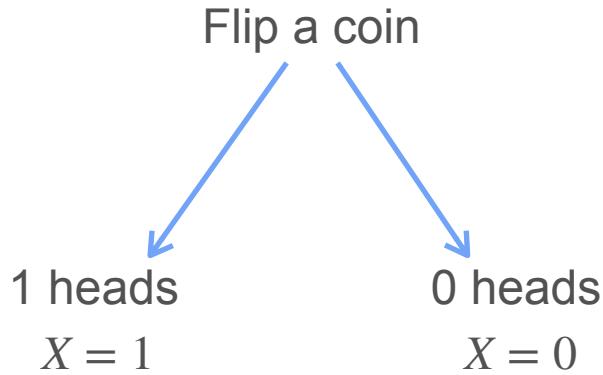


# From Events to Random Variables



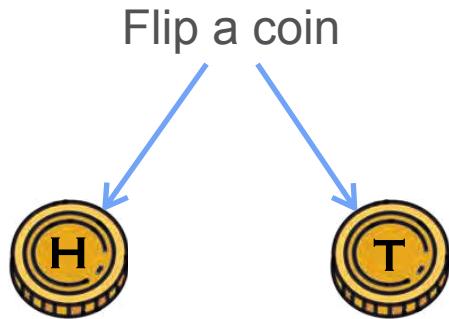
$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

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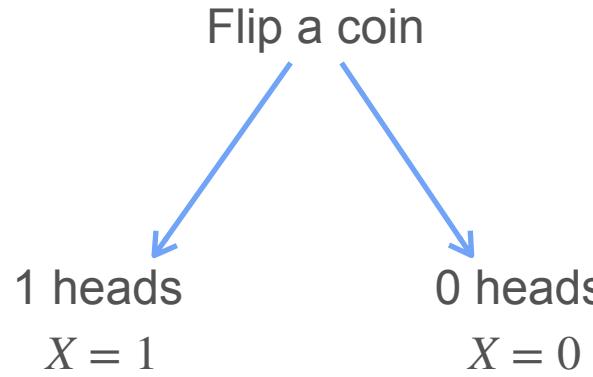
$$\mathbf{P}(X = 1) = 0.5$$

# From Events to Random Variables



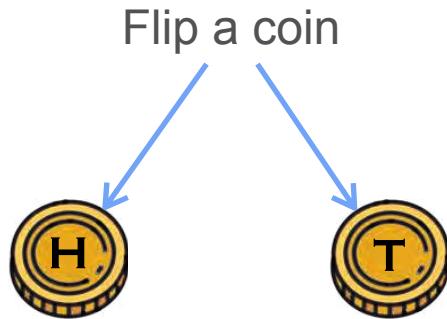
$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$X = \text{Number of heads}$



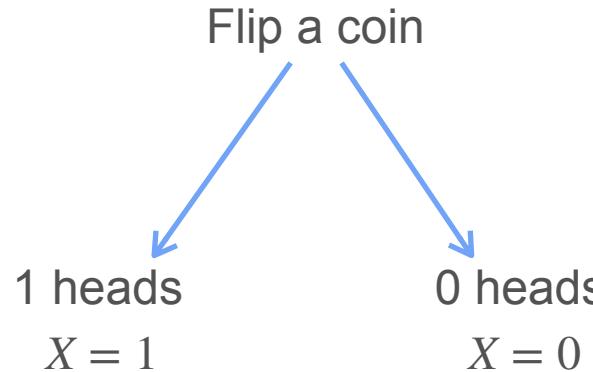
$$\mathbf{P}(X = 1) = 0.5 \quad \mathbf{P}(X = 0) = 0.5$$

# From Events to Random Variables



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$X$  = Number of heads



$$\mathbf{P}(X = 1) = 0.5 \quad \mathbf{P}(X = 0) = 0.5$$

$X$  is a random variable

# From Events to Random Variables

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

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$X$  = Number of heads in 10 coin tosses



$$X = 10$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



$$X = 10$$



$$X = 9$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



$$X = 10$$



$$X = 9$$



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# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$



$$X = 9$$



$$X = 9$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$



$$X = 9$$



$$X = 9$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$



$$X = 9$$

$$0.5^9 0.5$$



$$X = 9$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$



$$X = 9$$

$$0.5^9 0.5$$



$$X = 9$$

$$0.5^9 0.5$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$



$$X = 9$$

$$0.5^9 0.5$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 1)?$$



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# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

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$$\mathbf{P}(X = 0)?$$



$$X = 9$$

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$$\mathbf{P}(X = 1)?$$

...



$$X = 9$$

$$0.5^9 0.5$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 1)?$$

...



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 10)?$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



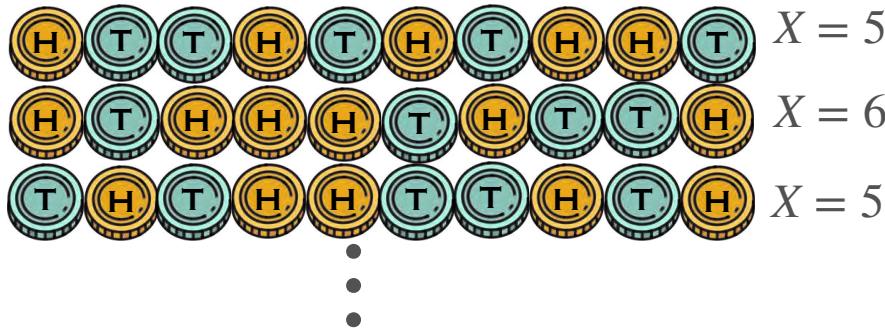
# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



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$X$  = Number of heads in 10 coin tosses



# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



Possible outcomes:



# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	



⋮

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	



⋮

$$P(H) = 0.5$$

# From Events to Random Variables

$X$  = Number of heads in 10 coin tosses



Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	



⋮

$$P(H) = 0.5$$

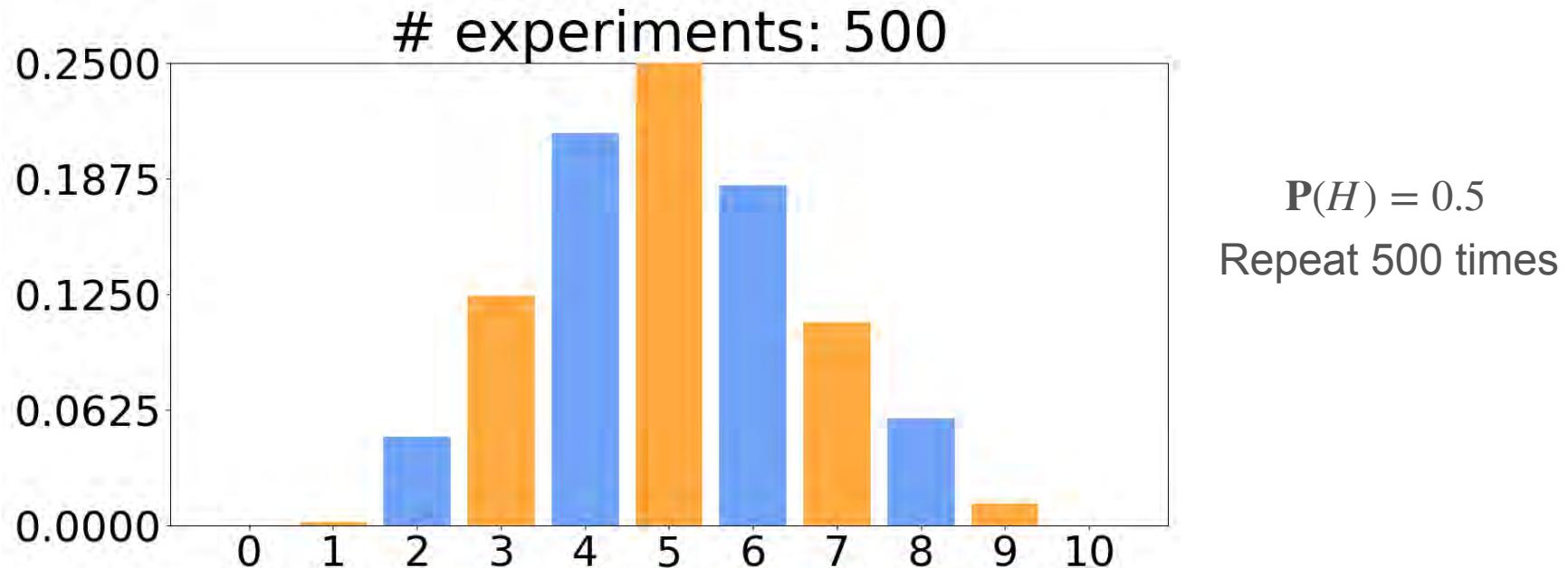
Repeat 500 times

# Flipping a Fair Coin 500 Times

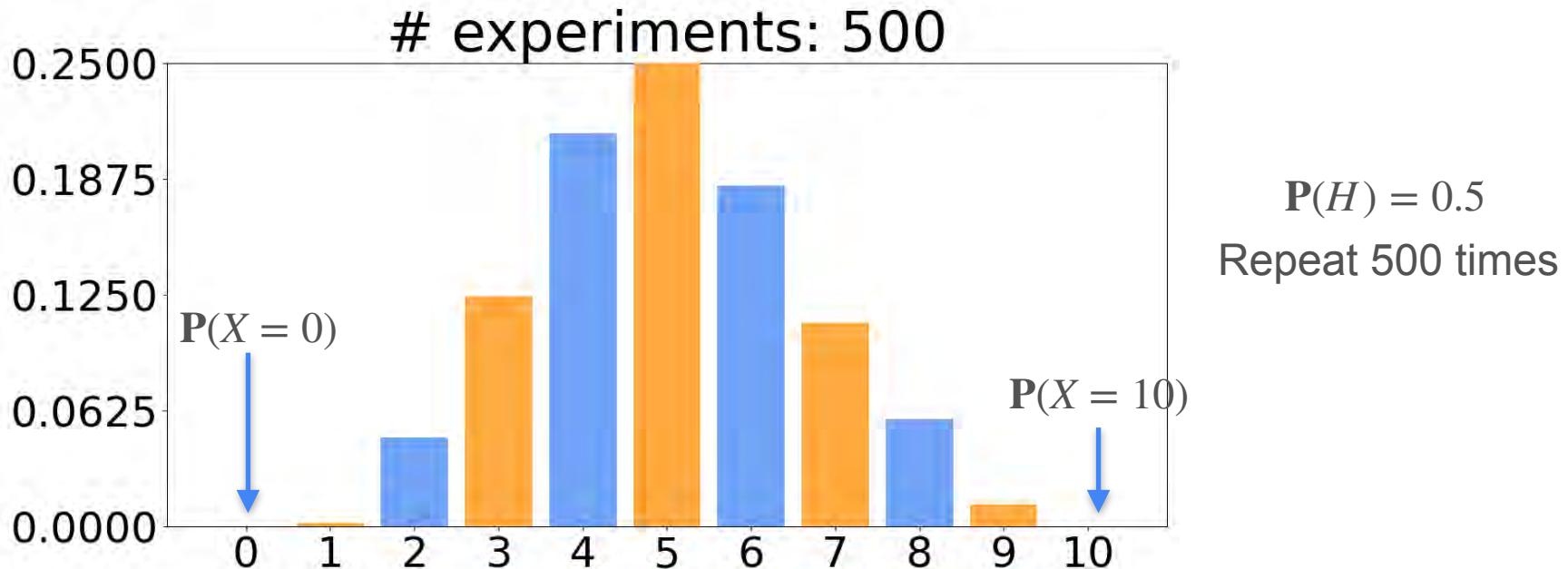
$$\mathbf{P}(H) = 0.5$$

Repeat 500 times

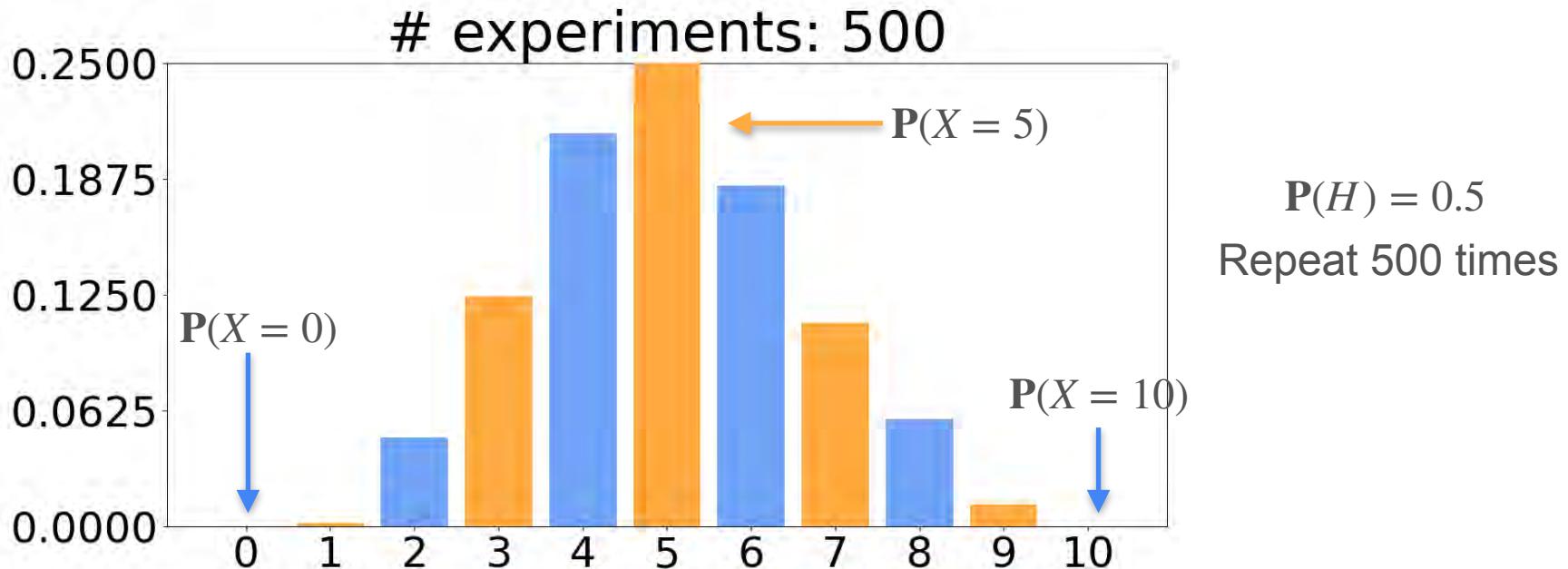
# Flipping a Fair Coin 500 Times



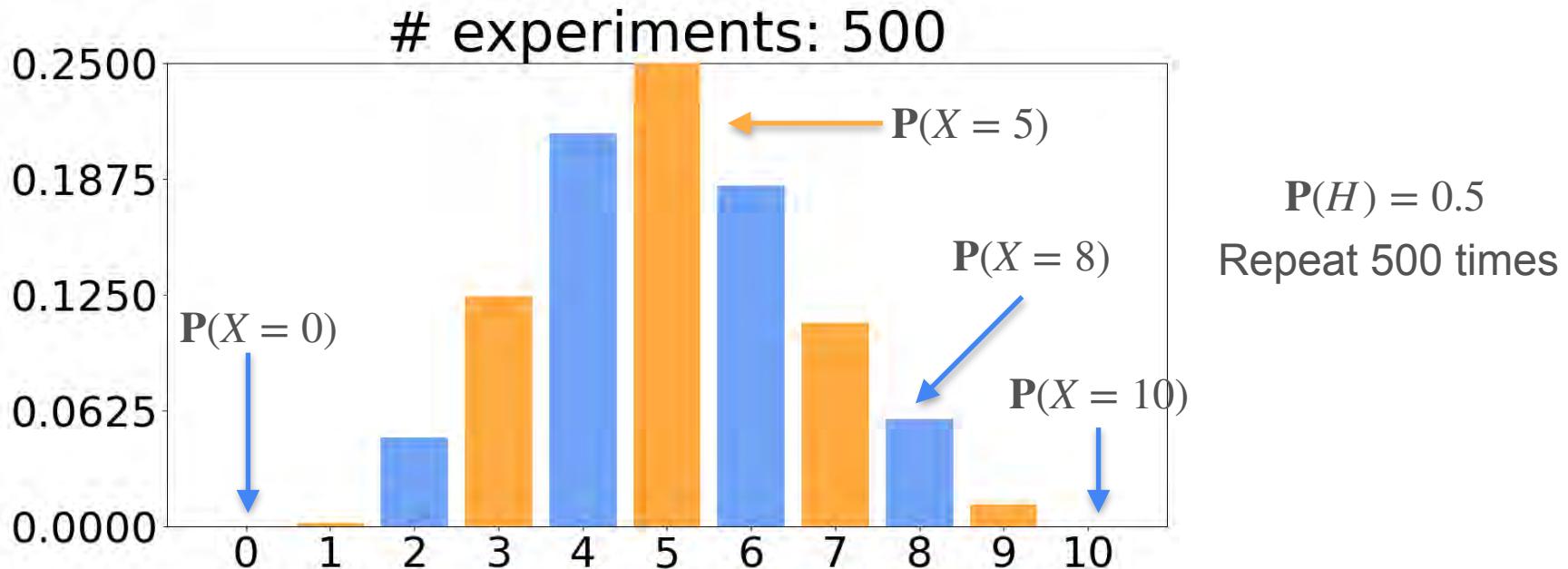
# Flipping a Fair Coin 500 Times



# Flipping a Fair Coin 500 Times



# Flipping a Fair Coin 500 Times



# Why Random Variables?

- Random variables allow you to model the whole experiment at once

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$X$  = Number of heads

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$X$  = Number of heads



$X$  = Number of 1's

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$X$  = Number of sick patients

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?

# Why Random Variables?

- Random variables allow you to model the whole experiment at once



$X$  = Number of heads



$X$  = Number of 1's



$X$  = Number of sick patients

?

$$\mathbf{P}(X = 1) = 0.5$$

# Why Random Variables?

- Random variables allow you to model the whole experiment at once



$X$  = Number of heads



$X$  = Number of 1's



$X$  = Number of sick patients

?

$$P(X = 1) = 0.5$$

$$P(X = -7) = 0.2$$

# Why Random Variables?

- Random variables allow you to model the whole experiment at once



$X$  = Number of heads



$X$  = Number of 1's



$X$  = Number of sick patients



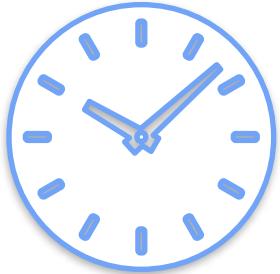
$$\mathbf{P}(X = 1) = 0.5$$

$$\mathbf{P}(X = -7) = 0.2$$

$$\mathbf{P}(X = 3.14159) = 0.3$$

# Other Random Variables

# Other Random Variables

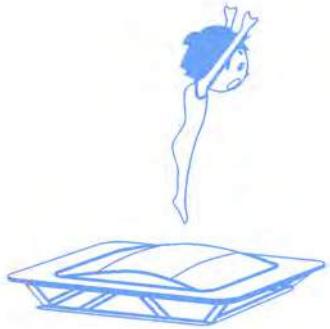


Wait time until the  
next bus arrives

# Other Random Variables

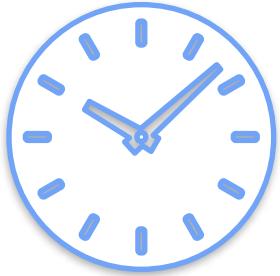


Wait time until the  
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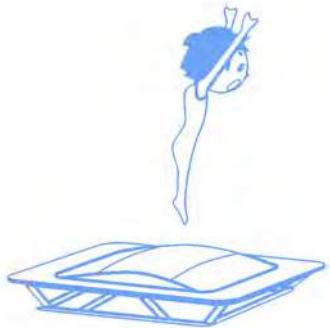


Height of an  
gymnast's jump

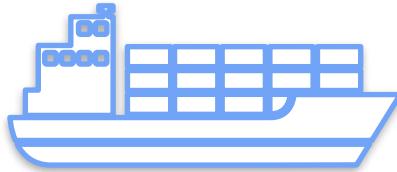
# Other Random Variables



Wait time until the  
next bus arrives

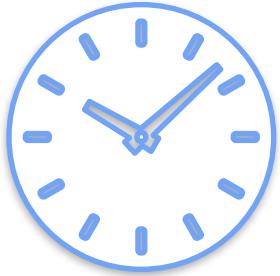


Height of an  
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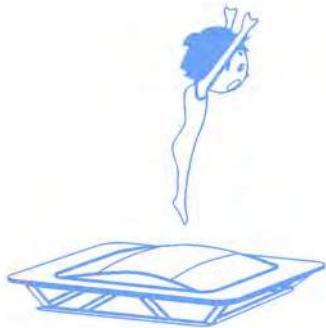


Number of  
defective products  
in a shipment

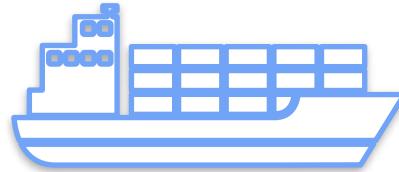
# Other Random Variables



Wait time until the next bus arrives



Height of an gymnast's jump



Number of defective products in a shipment



mm. of rain in November

# Discrete and Continuous Random Variables

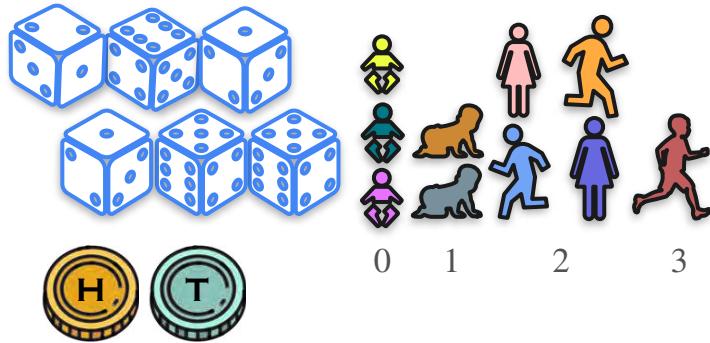
# Discrete and Continuous Random Variables

Discrete random variables

Continuous random variables

# Discrete and Continuous Random Variables

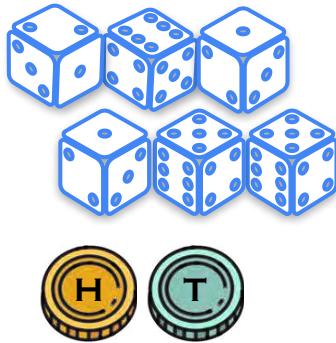
Discrete random variables



Continuous random variables

# Discrete and Continuous Random Variables

## Discrete random variables

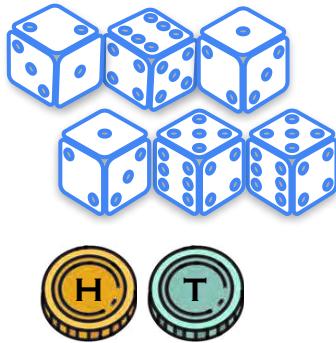


## Continuous random variables



# Discrete and Continuous Random Variables

## Discrete random variables



0 1 2 3

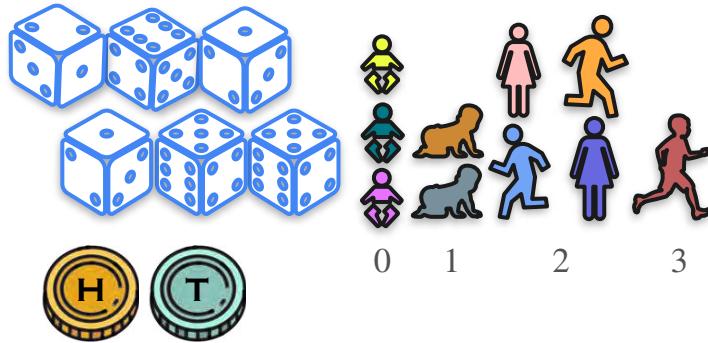
## Continuous random variables



Finite number of values

# Discrete and Continuous Random Variables

## Discrete random variables



Finite number of values

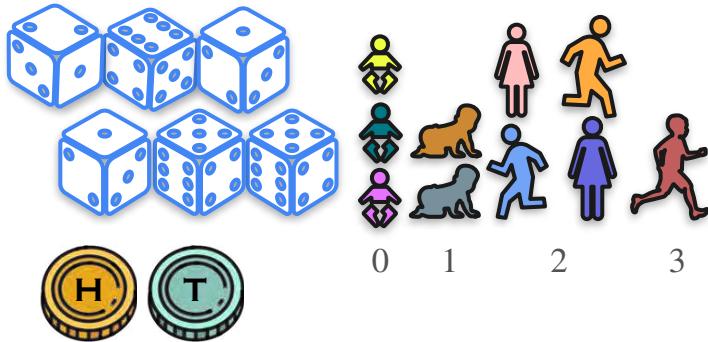
## Continuous random variables



Infinite number of values

# Discrete and Continuous Random Variables

## Discrete random variables



~~Finite number of values~~

(Could be infinite too)

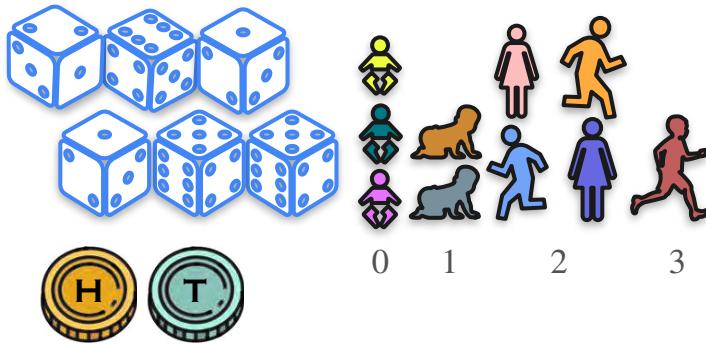
## Continuous random variables



Infinite number of values

# Discrete and Continuous Random Variables

## Discrete random variables



~~Finite number of values~~

(Could be infinite too)

Can take only a **countable**  
number of values

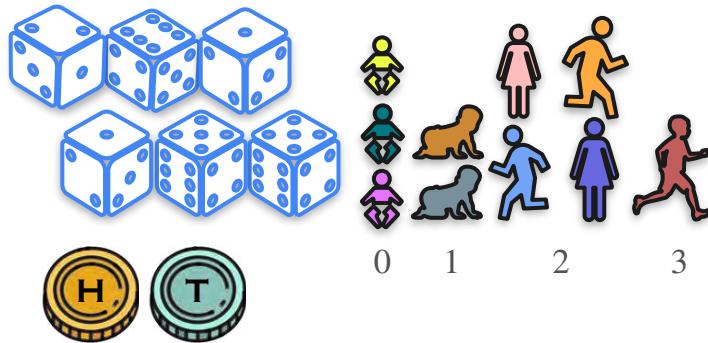
## Continuous random variables



Infinite number of values

# Discrete and Continuous Random Variables

## Discrete random variables



~~Finite number of values~~

(Could be infinite too)

Can take only a **countable** number of values

## Continuous random variables



Infinite number of values

Takes values on an interval

# Random Variable Vs. Deterministic Variable

# Random Variable Vs. Deterministic Variable

Deterministic

Random

# Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2, f(x) = x^2$$

Random

# Random Variable Vs. Deterministic Variable

**Deterministic**

$$x = 2, f(x) = x^2$$

**Random**

$X$  = number of defective items in  
a shipment

# Random Variable Vs. Deterministic Variable

## Deterministic

$$x = 2, f(x) = x^2$$

Fixed outcome

## Random

$X$  = number of defective items in  
a shipment

# Random Variable Vs. Deterministic Variable

## Deterministic

$$x = 2, f(x) = x^2$$

Fixed outcome

## Random

$X$  = number of defective items in  
a shipment

Uncertain outcome



DeepLearning.AI

# Probability Distributions

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## Probability Distributions (Discrete)

# Discrete Distributions: Flip Three Coins

# Discrete Distributions: Flip Three Coins

$X_1$ : number of heads in 3  
coin tosses

# Discrete Distributions: Flip Three Coins

$X_1$ : number of heads in 3 coin tosses



Coin<sup>1</sup> Coin<sup>2</sup> Coin<sup>3</sup>

All three tails ( $X = 0$ )

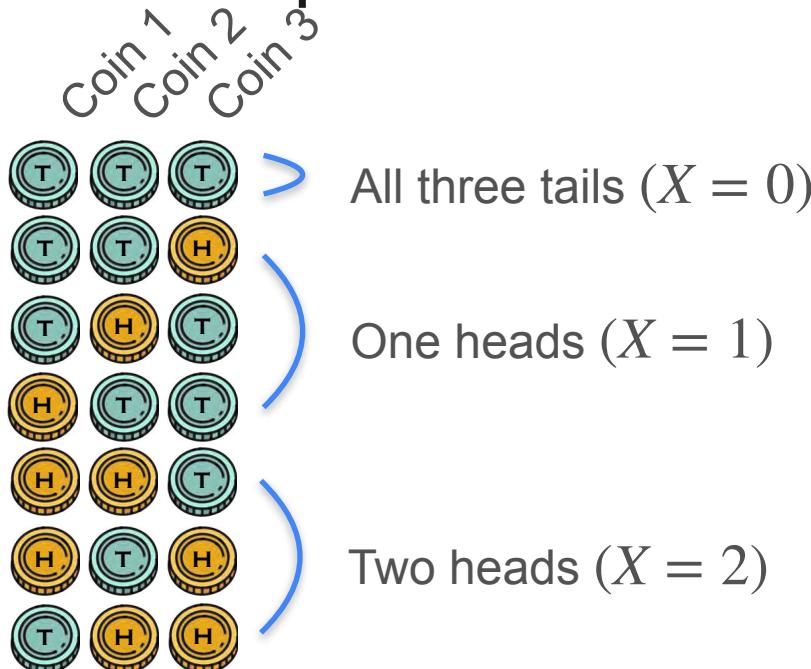
# Discrete Distributions: Flip Three Coins

$X_1$ : number of heads in 3 coin tosses



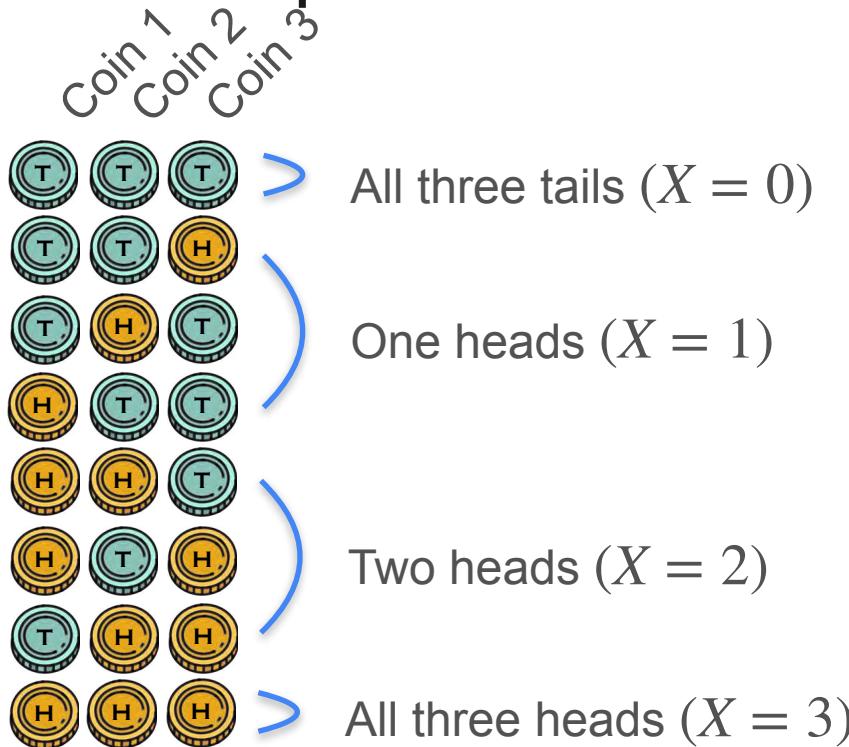
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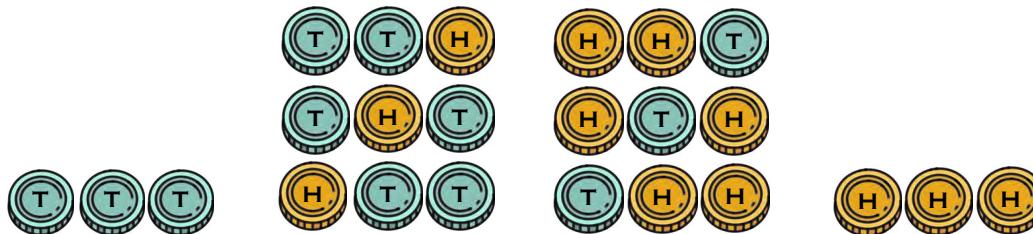
# Discrete Distributions: Flip Three Coins

$X_1$ : number of heads in 3 coin tosses



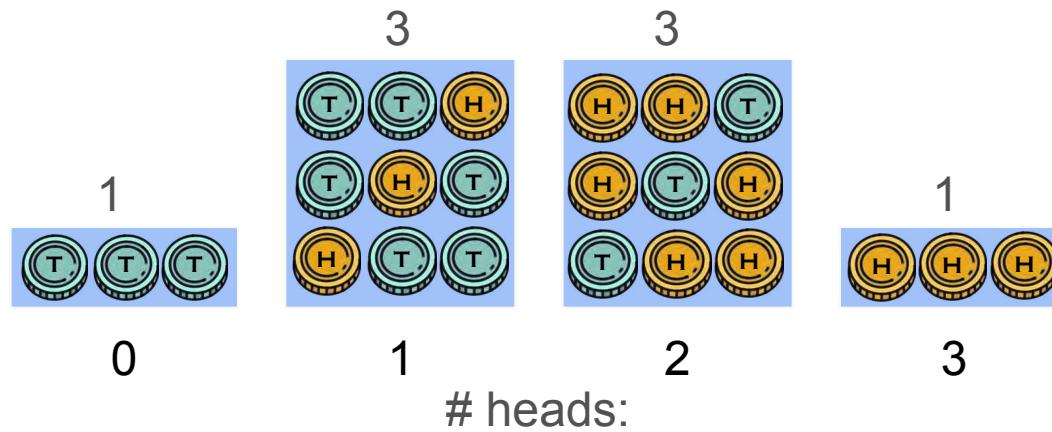
# Discrete Distributions: Flip Three Coins

$X_1$ : number of heads in 3  
coin tosses



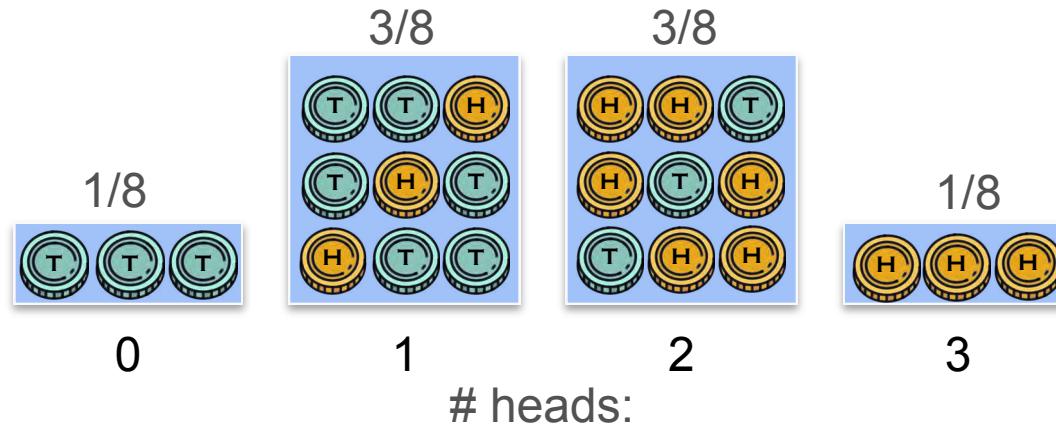
# Discrete Distributions: Flip Three Coins

$X_1$ : number of heads in 3 coin tosses



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$X_1$ : number of heads in 3 coin tosses



# Discrete Distributions: Flip Three Coins

$X_1$ : number of heads in 3 coin tosses



# Discrete Distributions: Flip Four Coins

# Discrete Distributions: Flip Four Coins

$X_2$ : number of heads in 4  
coin tosses

# Discrete Distributions: Flip Four Coins

$X_2$ : number of heads in 4  
coin tosses

0                  1                  2                  3                  4  
# heads:

# Discrete Distributions: Flip Four Coins

$X_2$ : number of heads in 4 coin tosses



0

1

2

3

4

# heads:



# Discrete Distributions: Flip Four Coins

$X_2$ : number of heads in 4 coin tosses

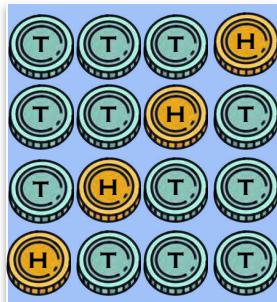


# Discrete Distributions: Flip Four Coins

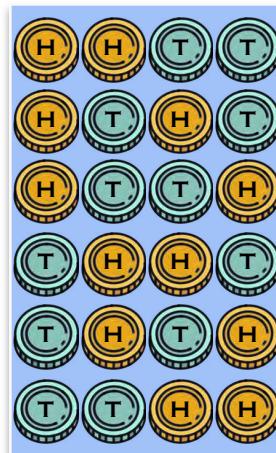
$X_2$ : number of heads in 4 coin tosses



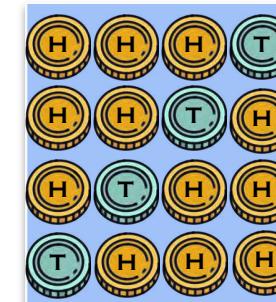
0



1



2



3



4

# heads:

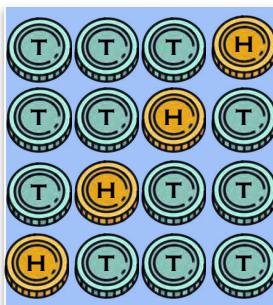
# Discrete Distributions: Flip Four Coins

$X_2$ : number of heads in 4 coin tosses

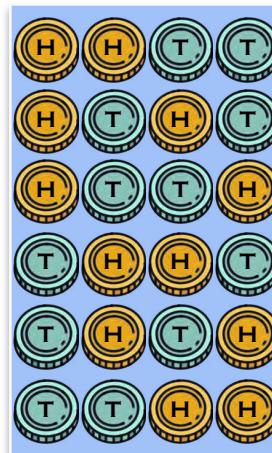


0

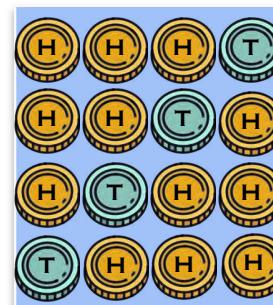
1/16



1



2



3

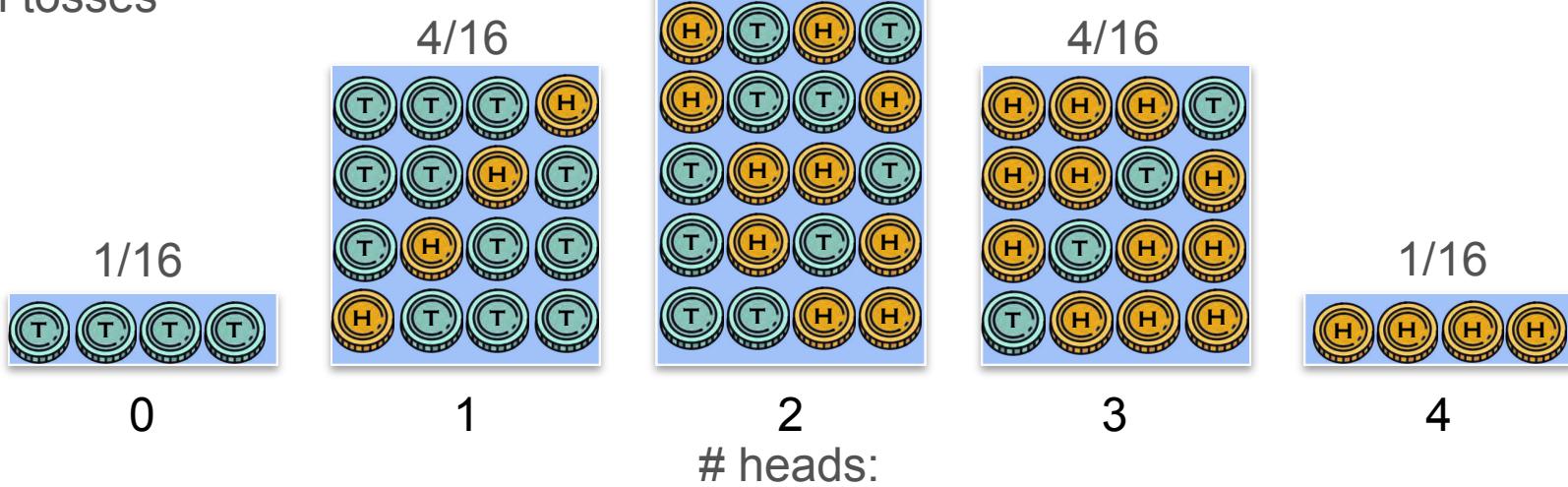


4

# heads:

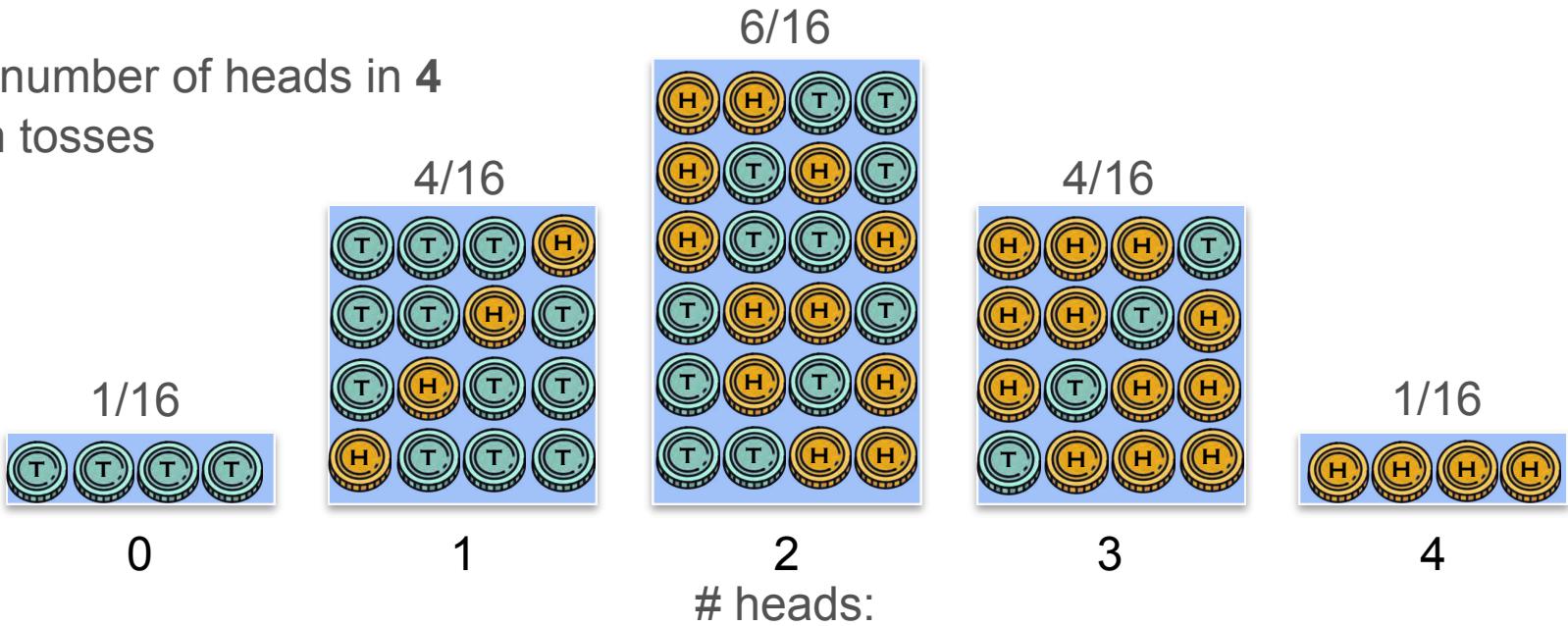
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$X_2$ : number of heads in 4 coin tosses

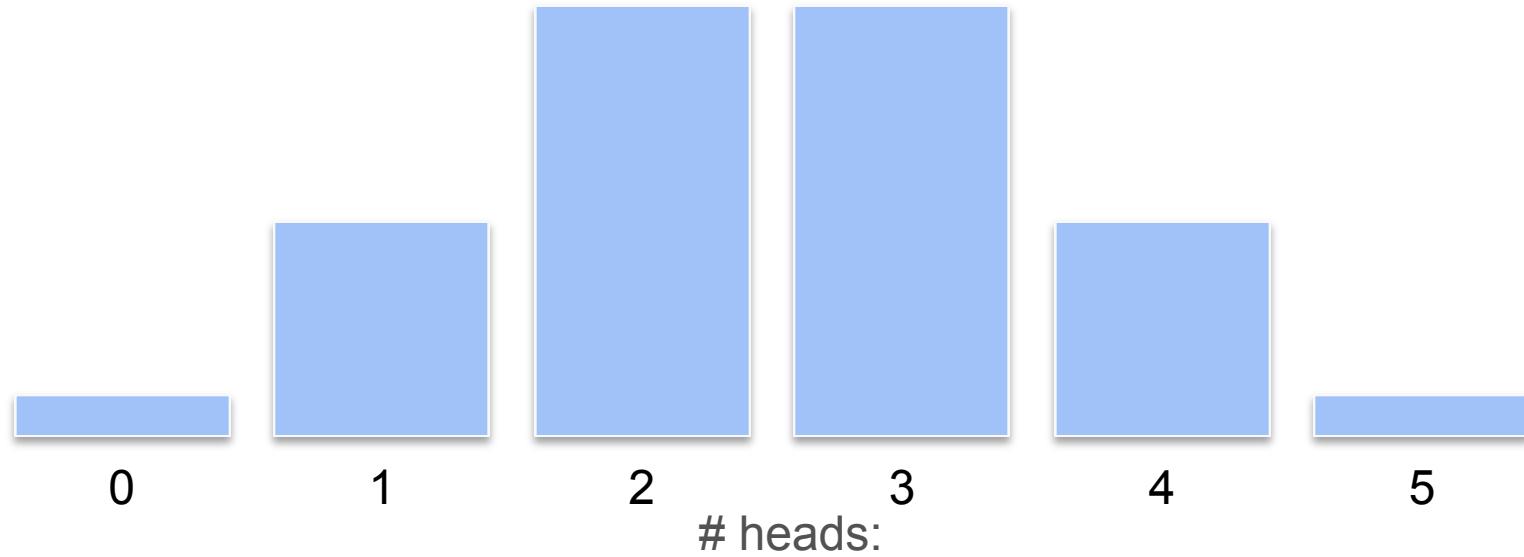


# Discrete Distributions: Flip Four Coins

$X_2$ : number of heads in 4 coin tosses

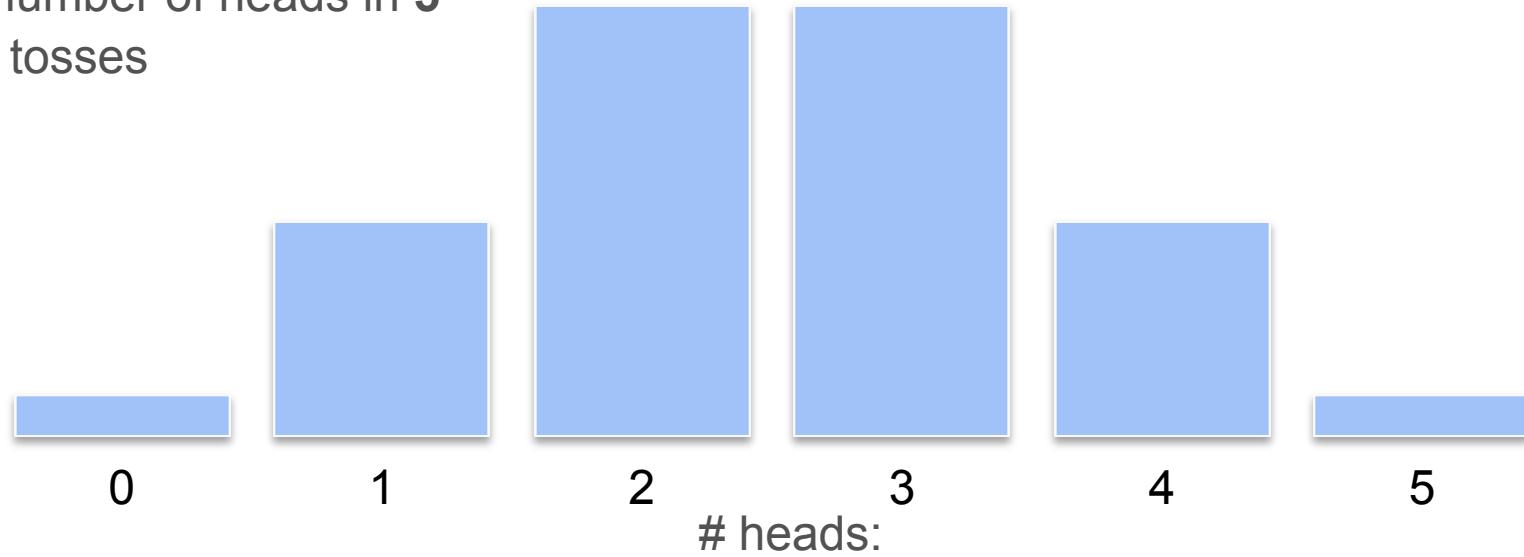


# Discrete Distributions: Flip Five Coins



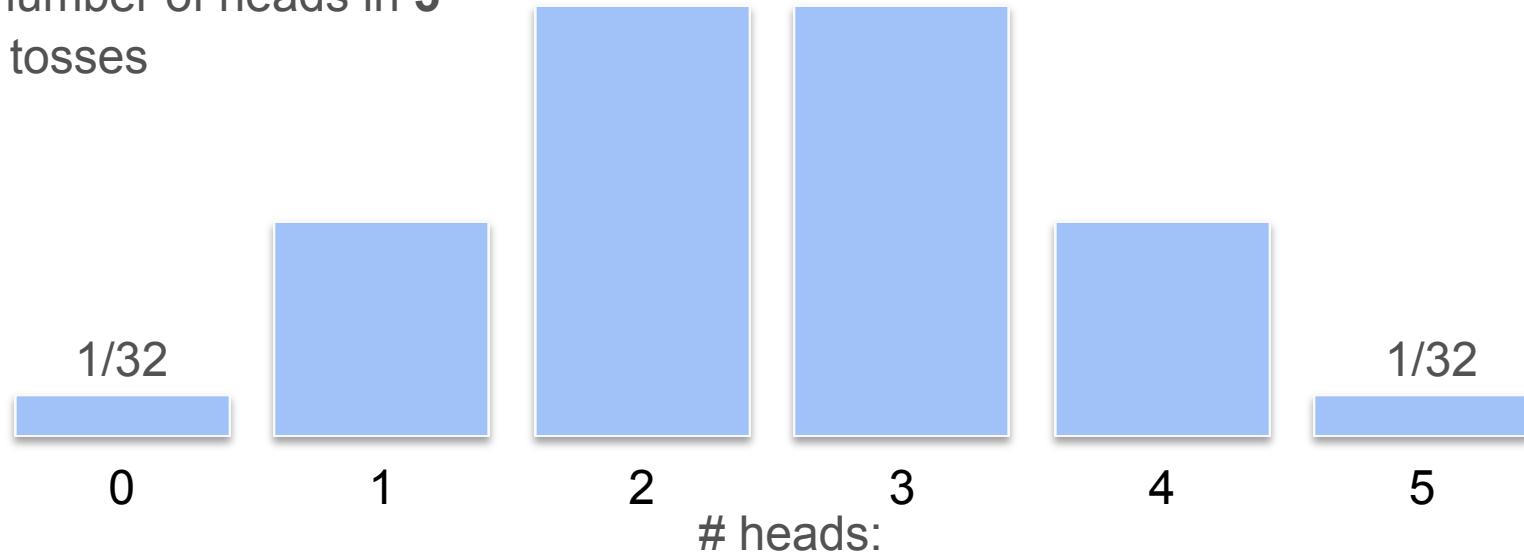
# Discrete Distributions: Flip Five Coins

$X_3$ : number of heads in **5** coin tosses



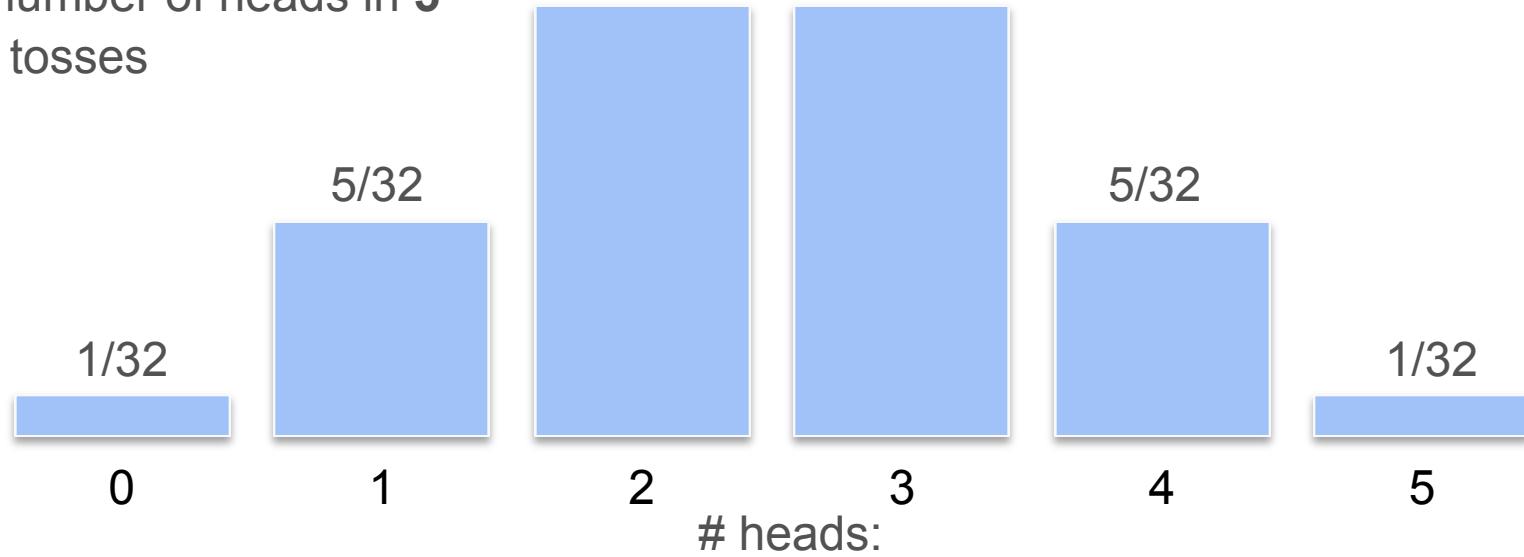
# Discrete Distributions: Flip Five Coins

$X_3$ : number of heads in **5** coin tosses



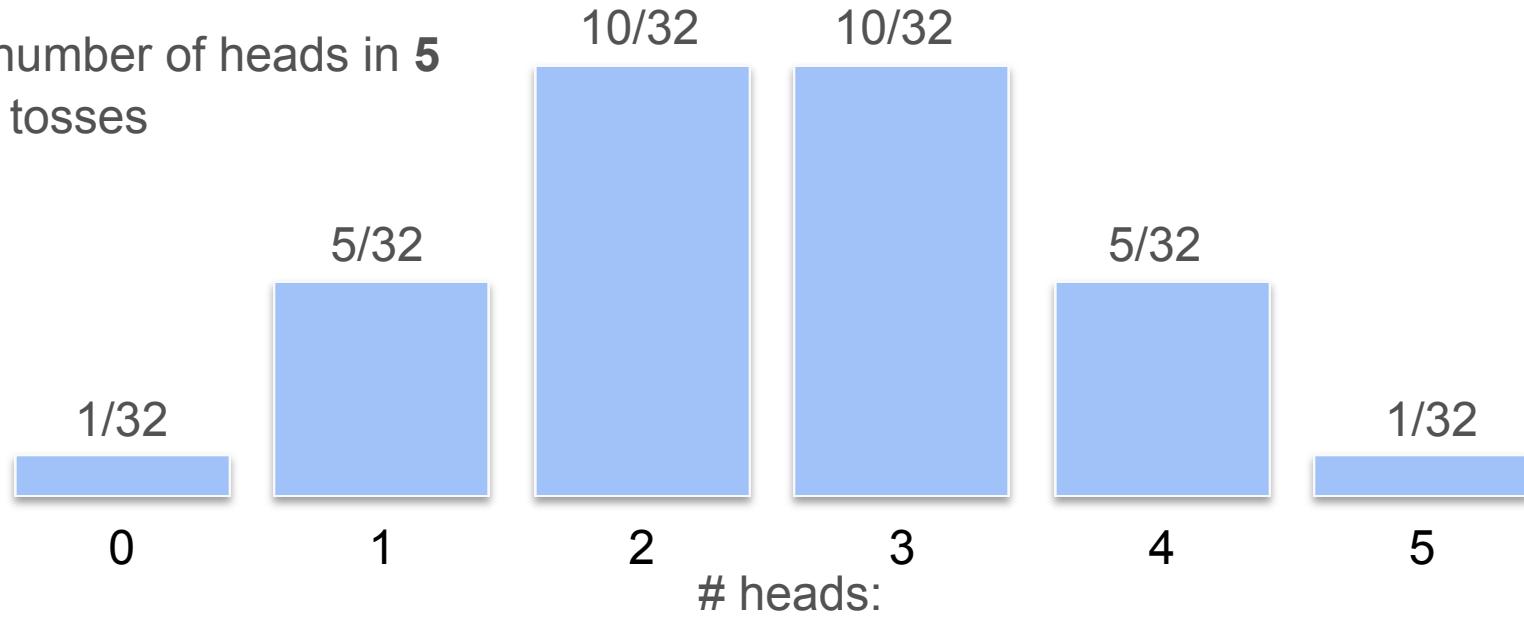
# Discrete Distributions: Flip Five Coins

$X_3$ : number of heads in **5** coin tosses



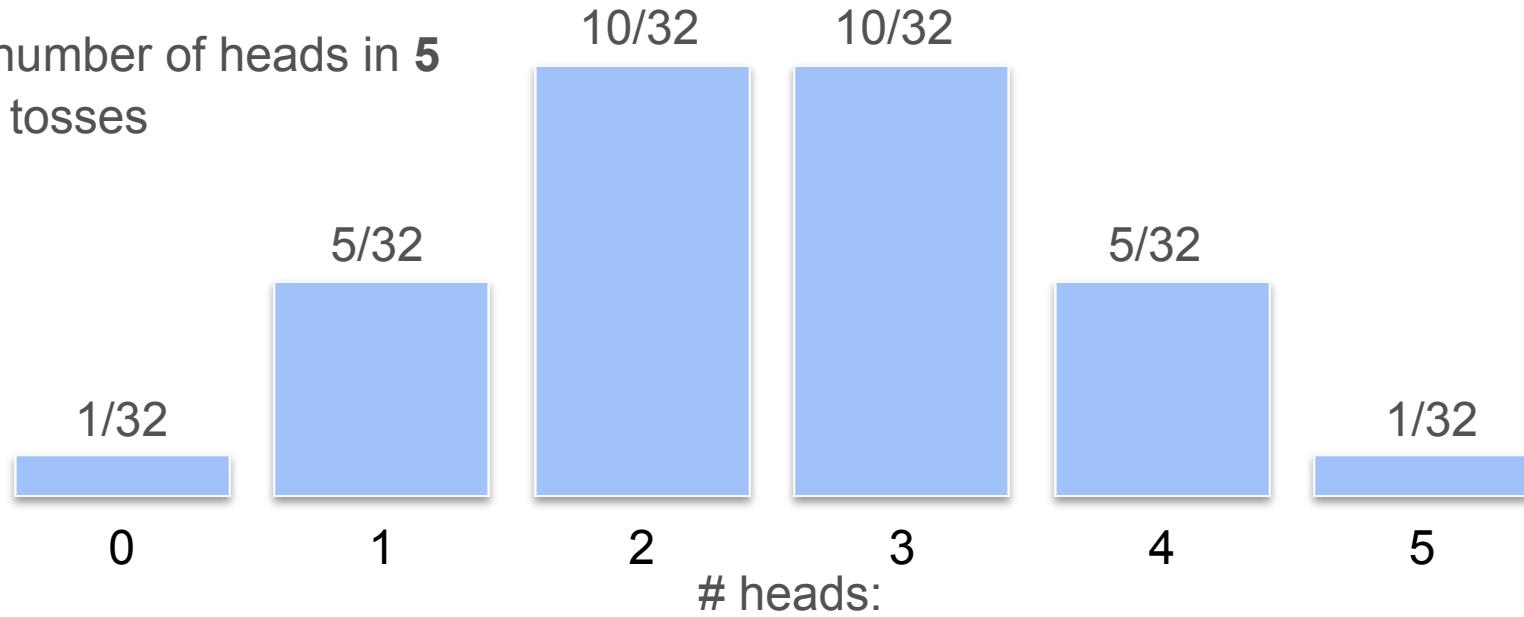
# Discrete Distributions: Flip Five Coins

$X_3$ : number of heads in 5 coin tosses



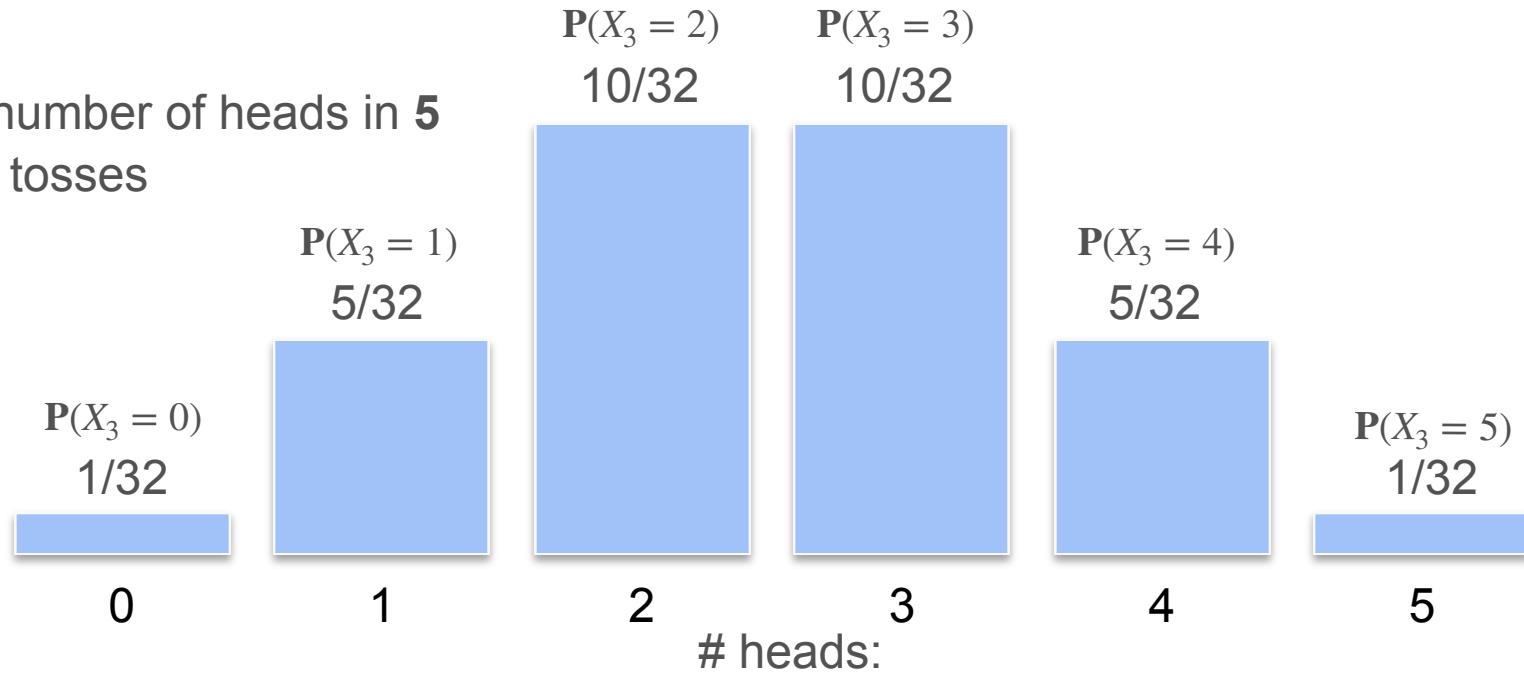
# Discrete Distributions: Flip Five Coins

$X_3$ : number of heads in 5 coin tosses



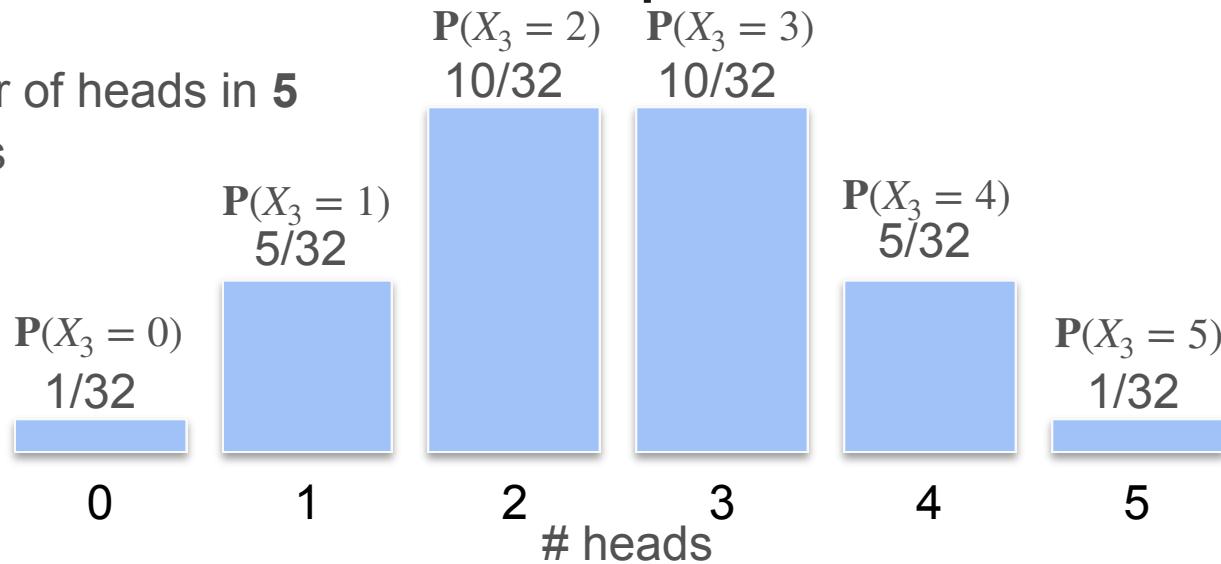
# Discrete Distributions: Flip Five Coins

$X_3$ : number of heads in 5 coin tosses



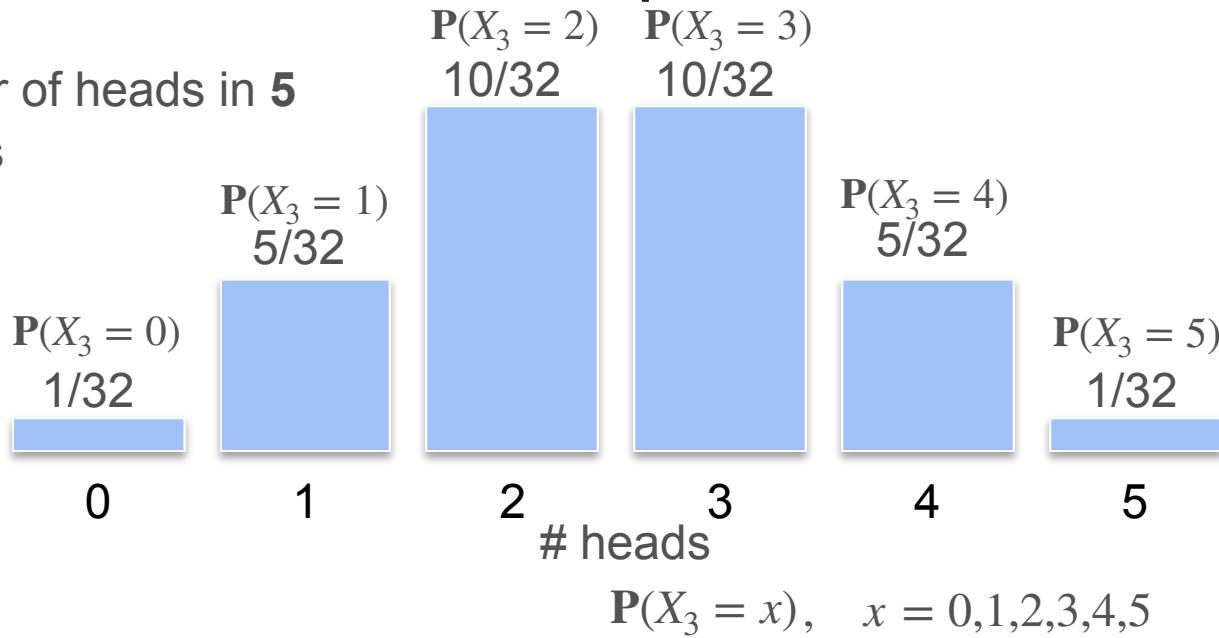
# Discrete Distributions: Flip Five Coins

$X_3$ : number of heads in 5 coin tosses



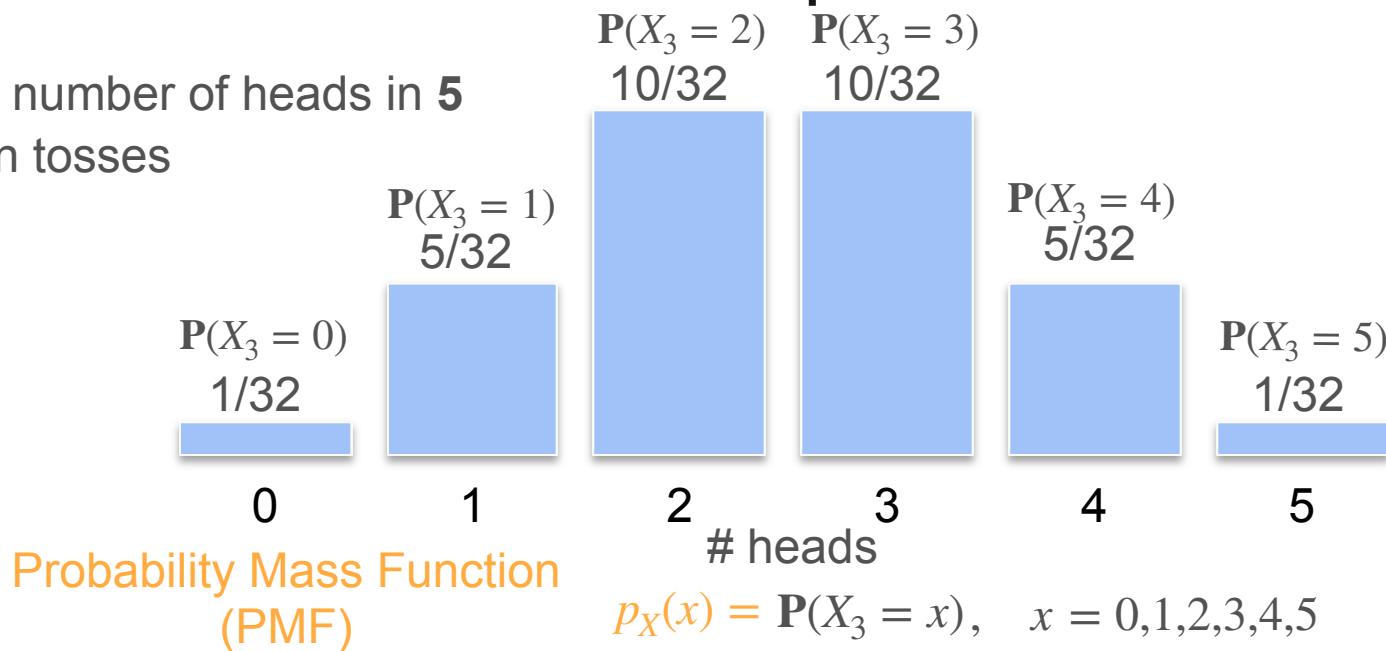
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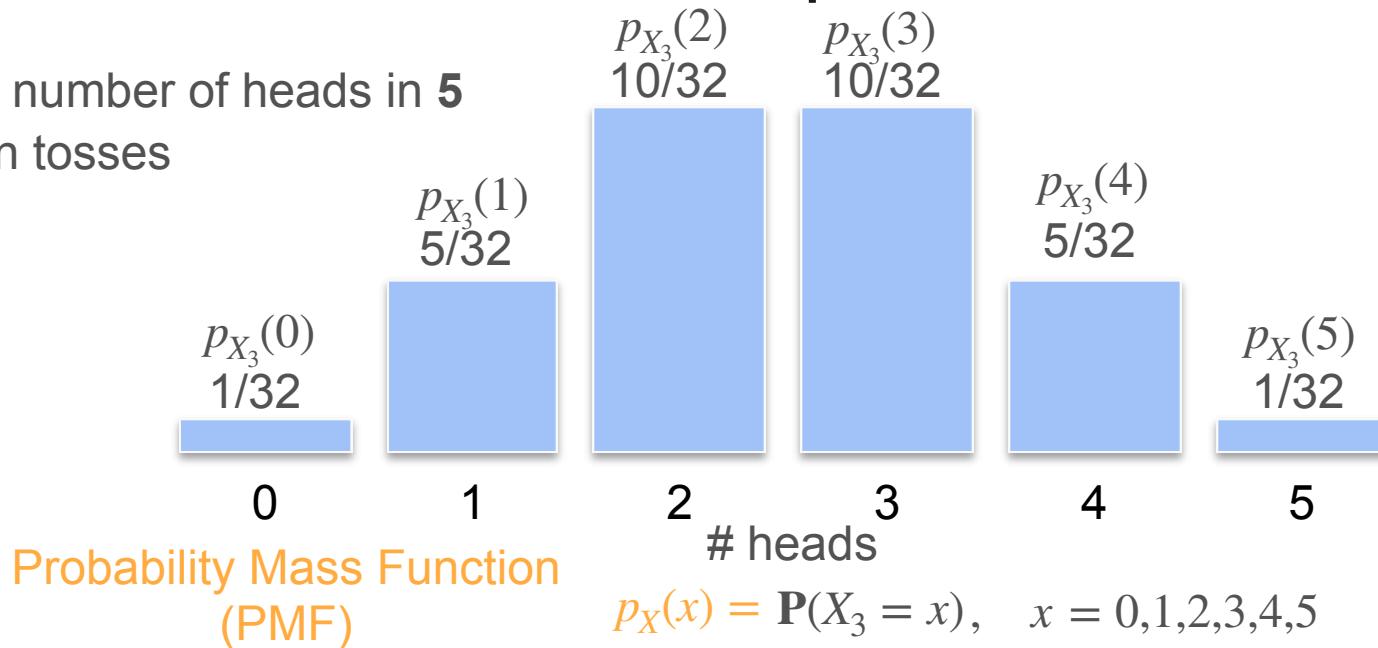
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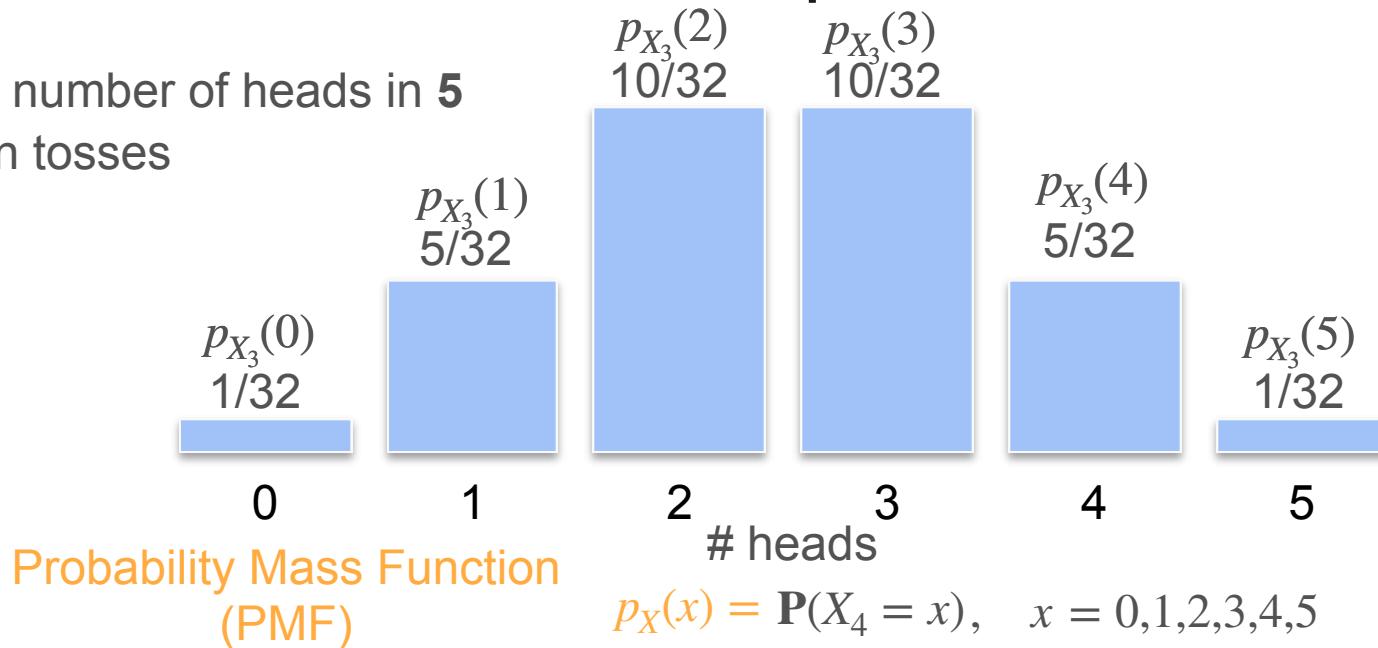
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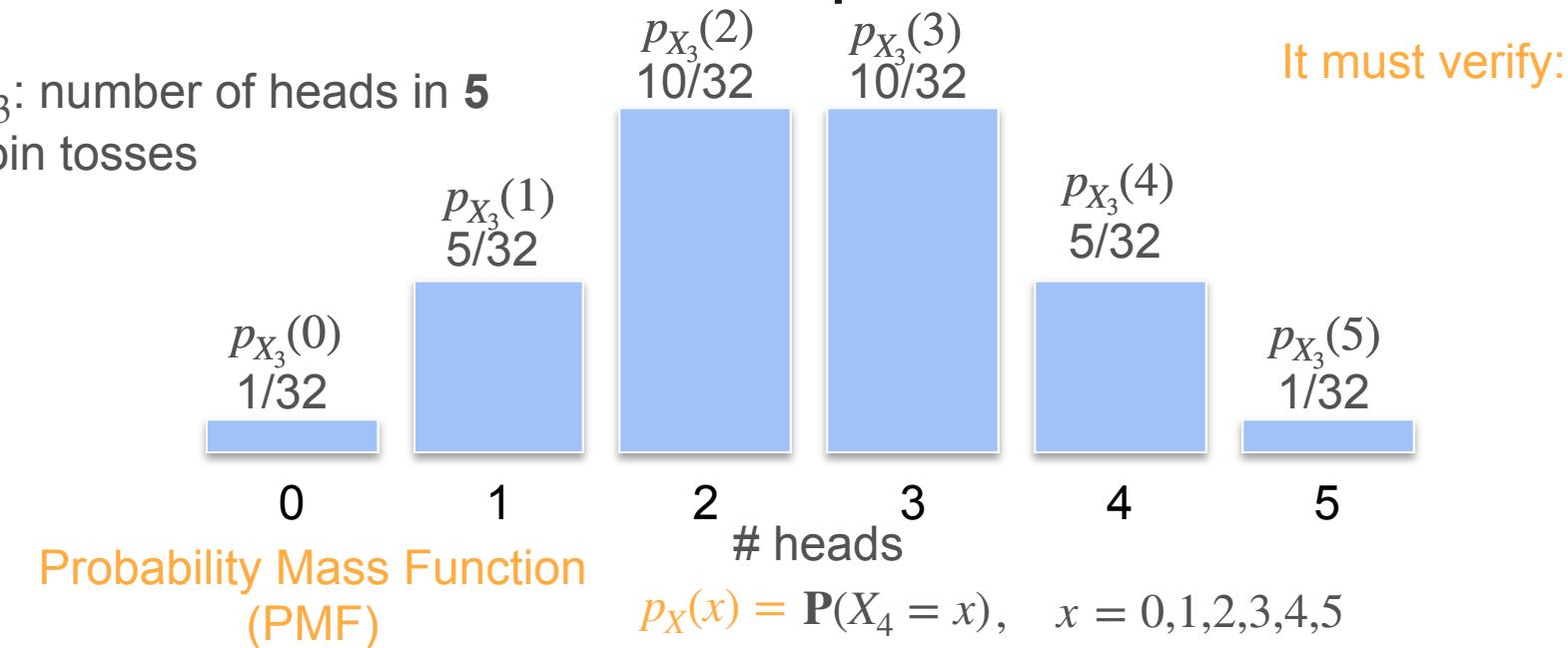
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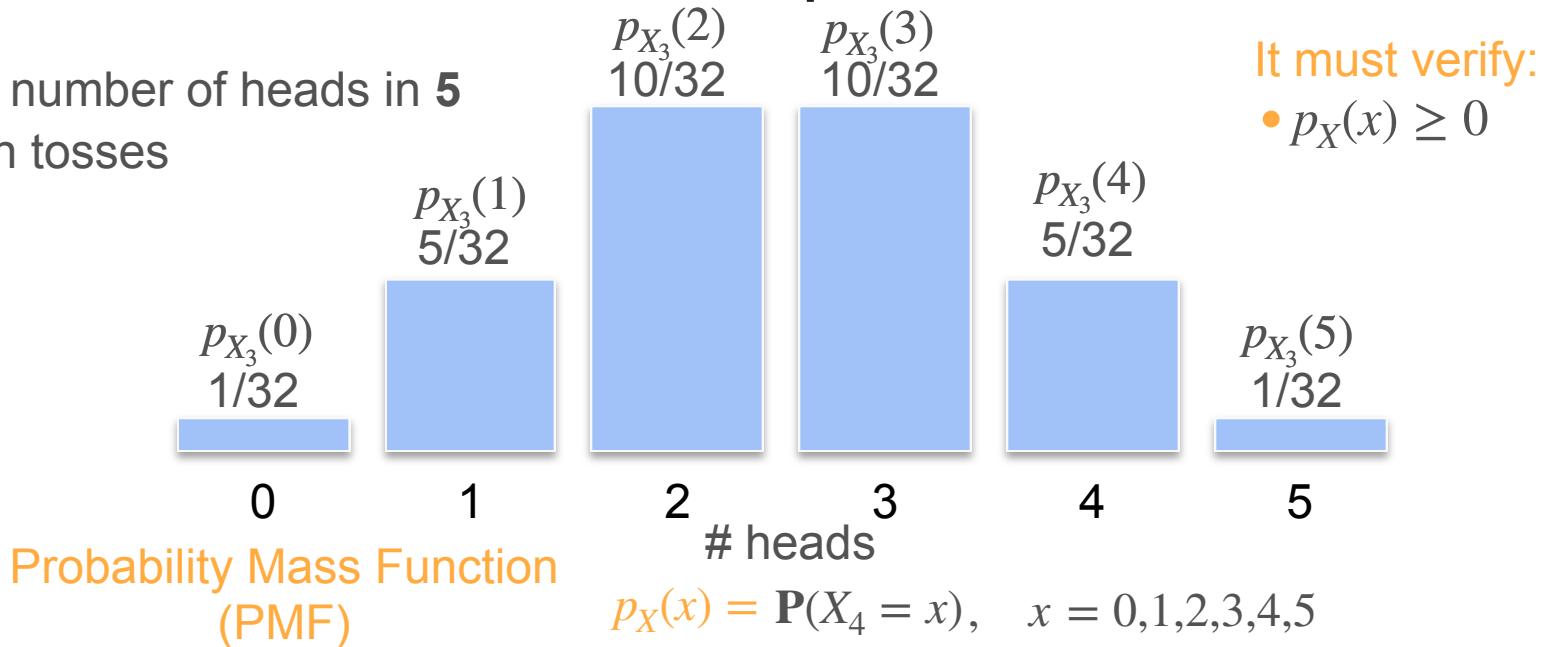
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It must verify:

# Discrete Distributions: Flip Five Coins

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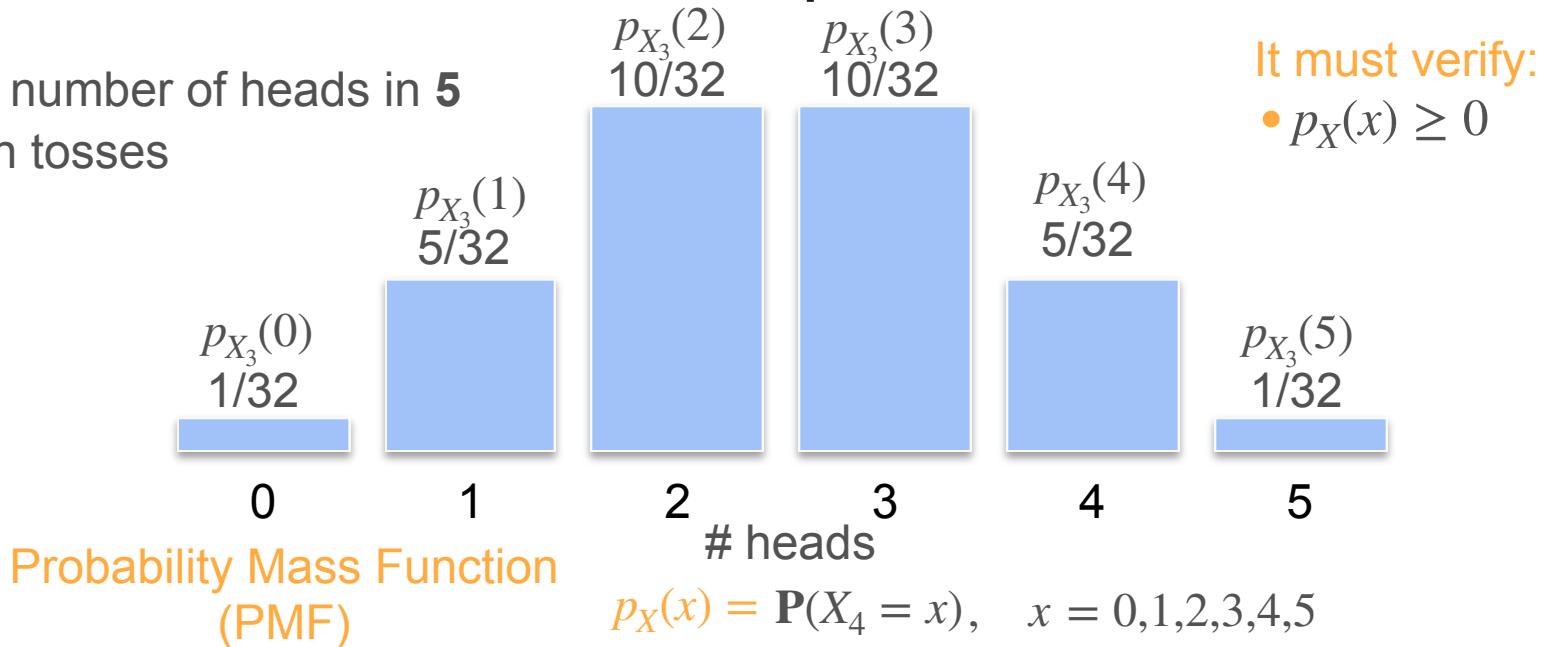


It must verify:

- $p_X(x) \geq 0$

# Discrete Distributions: Flip Five Coins

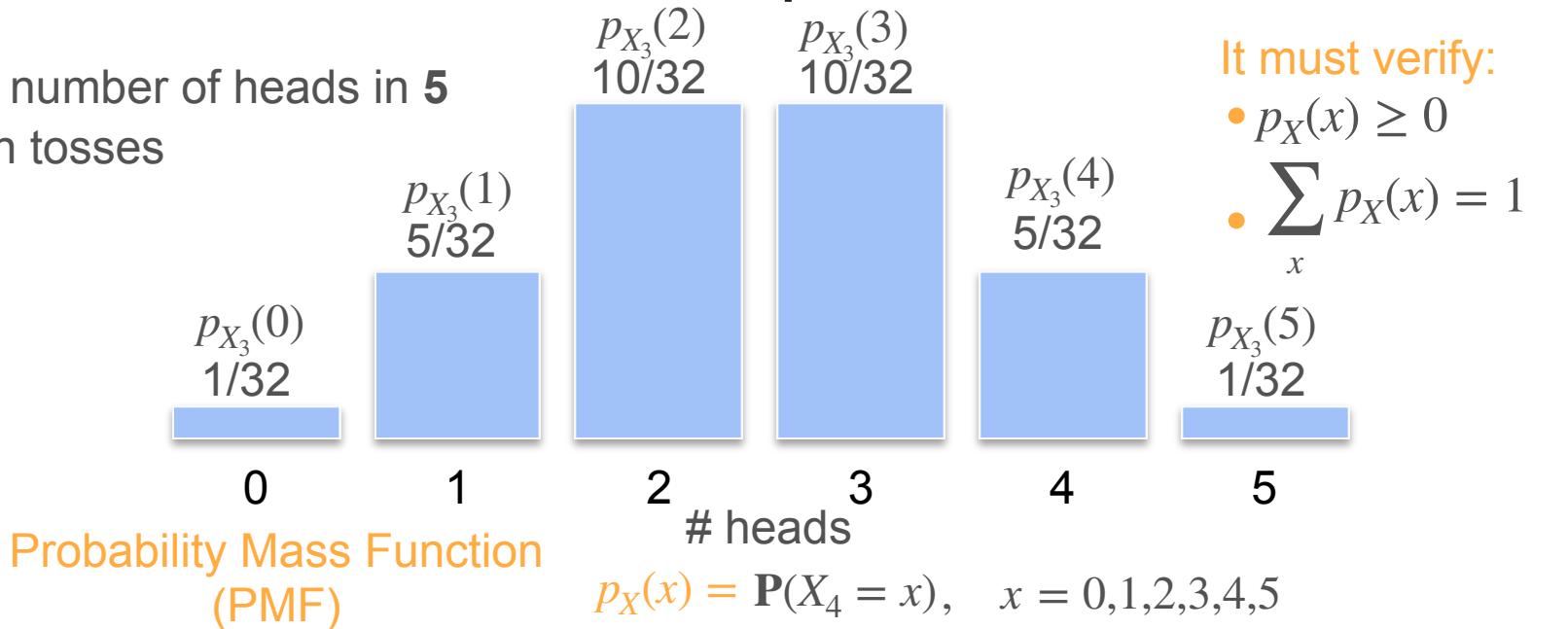
$X_3$ : number of heads in 5 coin tosses



$$p_{X_3}(0) + p_{X_3}(1) + p_{X_3}(2) + p_{X_3}(3) + p_{X_3}(4) + p_{X_3}(5) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = 1$$

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It must verify:

- $p_X(x) \geq 0$

- $\sum_x p_X(x) = 1$

# Binomial Distribution

# Binomial Distribution

$X_1, X_2, X_3, X_4$  are very similar

They all represent **number of heads in  $n$  experiments**

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# Binomial Distribution

$X_1, X_2, X_3, X_4$  are very similar

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The way the probability distributes along the possible outcomes seems to have a similar pattern

Could there be a **single model** to represent all this variables?



Binomial distribution



DeepLearning.AI

# Probability Distributions

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## Binomial Distribution

# Binomial Distribution: Example

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What is the probability that if I flip 5 coins, 2 of them land in heads?

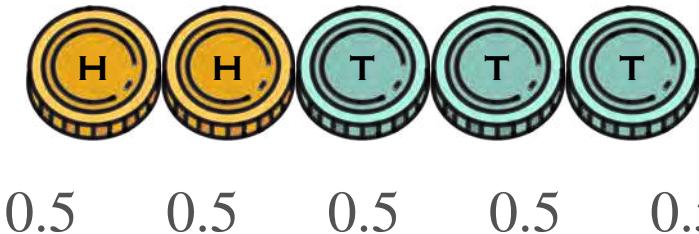
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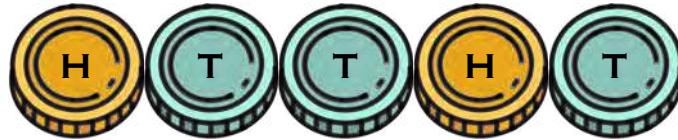
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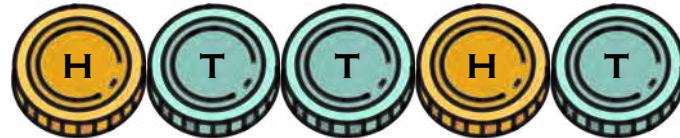


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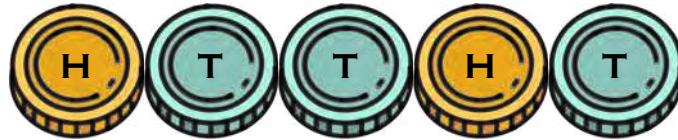
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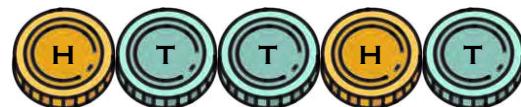


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What is the probability that if I flip 5 coins, 2 of them land in heads?

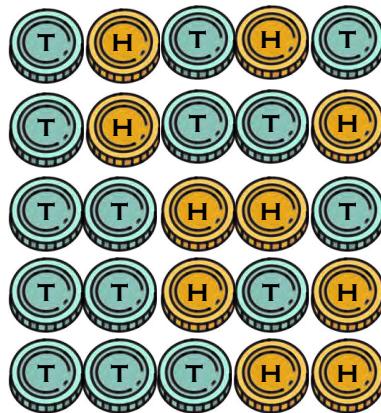


10 ways to have 2 heads in 5 coin tosses



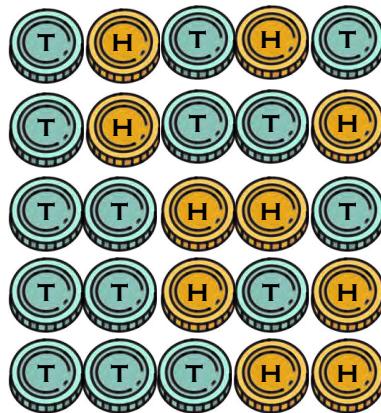
# Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



# Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \underline{\hspace{2cm}}$$

# Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{\text{Number of ways you can order 5 coins}}$$

# Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!}$$

Number of ways you can order 5 coins

Number of H

A diagram showing three coins at the bottom: one yellow coin labeled 'H' and two teal coins labeled 'T'.

# Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!}$$

Number of ways you can order 5 coins

Number of H

The equation  $10 = \frac{5!}{2!}$  is shown with arrows pointing from the text "Number of ways you can order 5 coins" to the 5! term and from the text "Number of H" to the 2! term.

# Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!(5-2)!}$$

Number of ways you can order 5 coins

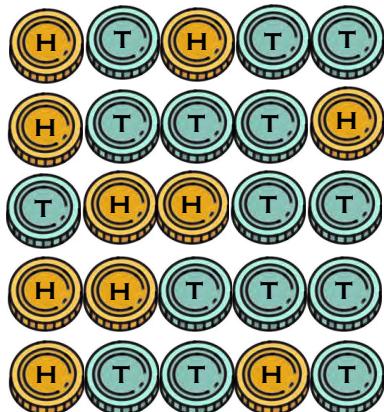
Number of H      Number of T

The equation  $10 = \frac{5!}{2!(5-2)!}$  represents the number of ways to order 5 coins. The term  $5!$  is highlighted with a blue arrow pointing to it. The term  $2!(5-2)!$  is also highlighted with a blue arrow pointing to it. Below the equation, the words "Number of ways you can order 5 coins" are written, with arrows pointing from the terms  $5!$  and  $2!(5-2)!$  to their respective definitions: "Number of H" and "Number of T".



# Binomial Distribution: Example

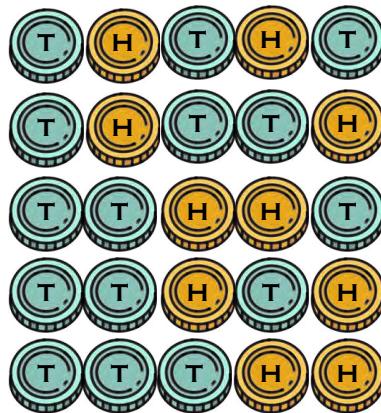
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# Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!(5-2)!} = \binom{5}{2}$$

Binomial coefficient

Number of ways you can get 2 heads in 5 coin tosses

# Binomial Distribution: Binomial Coefficient

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In general:

$\binom{n}{k}$  counts all the combinations for landing  $k$  heads in  $n$  coin tosses

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$$\binom{n}{k} = \binom{n}{n - k}$$

# Binomial Distribution: Binomial Coefficient

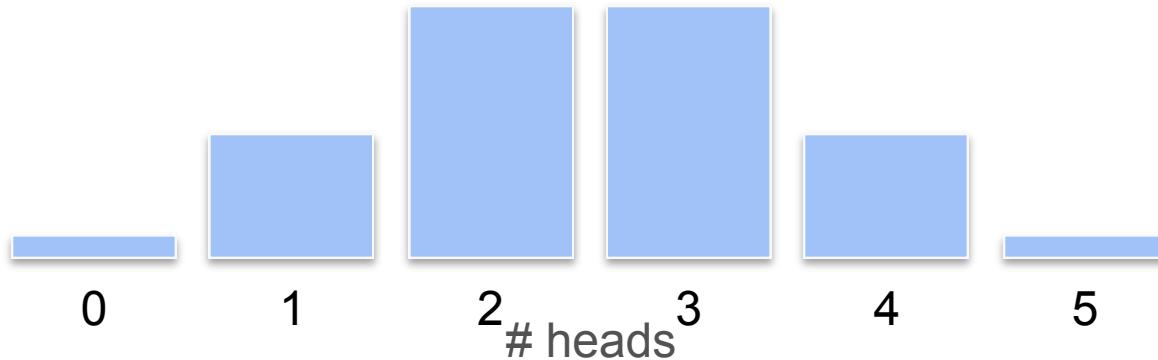
In general:

$\binom{n}{k}$  counts all the combinations for landing  $k$  heads in  $n$  coin tosses

Property:

The PMF with a fair coin is symmetrical

$$\binom{n}{k} = \binom{n}{n-k}$$



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General PMF for  $X$  : number of heads in 5 coin tosses?

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$$p^x$$

↑  
Probability of seeing  $x$  heads

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General PMF for  $X$  : number of heads in 5 coin tosses?

Your coin has  $\mathbf{P}(H) = p$

Event:  $X = x$ :  $x$  heads in 5 tosses

$$p^x (1 - p)^{5-x}$$

Probability of seeing  $x$  heads

Probability of seeing  $5 - x$  tails

The diagram illustrates the binomial probability formula  $p^x (1 - p)^{5-x}$ . It features a blue rectangular box enclosing the two terms  $p^x$  and  $(1 - p)^{5-x}$ . A blue arrow originates from the text "Probability of seeing  $x$  heads" and points to the term  $p^x$ . Another blue arrow originates from the text "Probability of seeing  $5 - x$  tails" and points to the term  $(1 - p)^{5-x}$ .

# Binomial Distribution: Binomial Coefficient

General PMF for  $X$  : number of heads in 5 coin tosses?

Your coin has  $\mathbf{P}(H) = p$

Event:  $X = x$ :  $x$  heads in 5 tosses

$$\binom{5}{x} p^x (1-p)^{5-x}$$

All the possible orders →  $\binom{5}{x}$

Probability of seeing  $x$  heads ↑  $p^x$

Probability of seeing  $5 - x$  tails  $(1-p)^{5-x}$

# Binomial Distribution

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$$\binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

# Binomial Distribution

General PMF for  $X$  : number of heads in 5 coin tosses?

Your coin has  $\mathbf{P}(H) = p$

Event:  $X = x$ :  $x$  heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

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X follows a binomial distribution

$X \sim \text{Binomial}(5, p)$

Number of flips

$\mathbf{P}(H)$

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**$X$  follows a binomial distribution**

$X \sim \text{Binomial}(5, p)$

Number of flips       $\mathbf{P}(H)$

```
graph LR; A["p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, x = 0,1,2,3,4,5"] -- green bracket --> B["X ~ Binomial(5, p)"]; A -- green arrow --> C["X follows a binomial distribution"]; B -- blue arrow --> D["Number of flips"]; B -- blue arrow --> E["P(H)"]
```

# Binomial Distribution

# Binomial Distribution

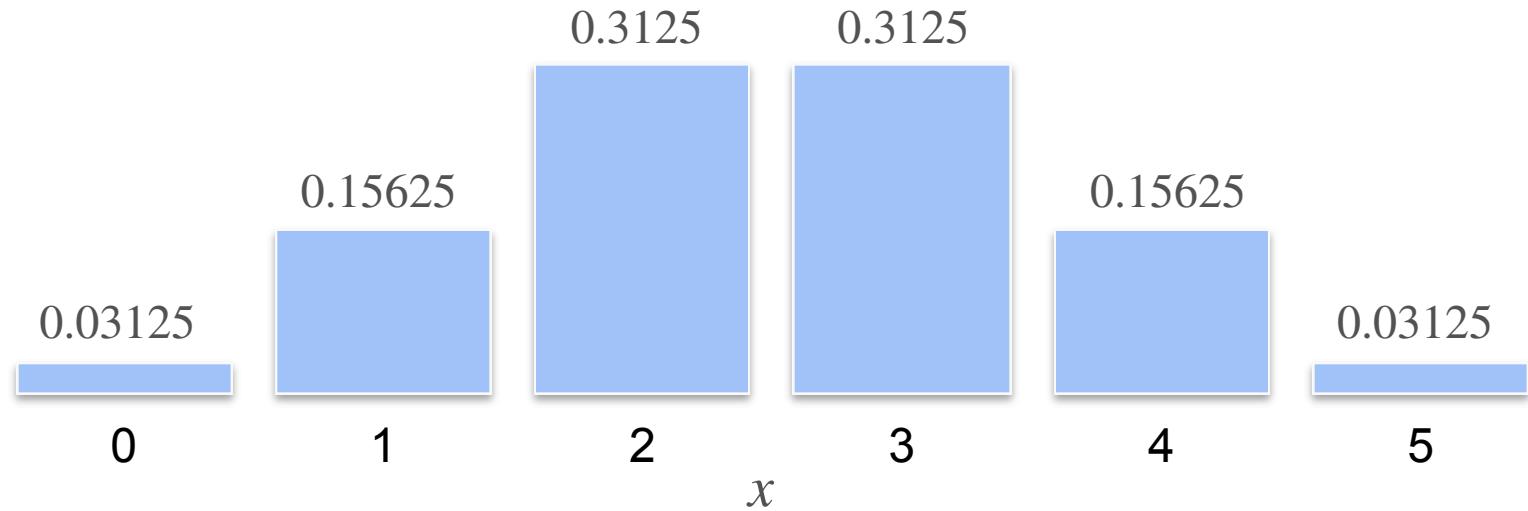
$$\begin{aligned} n &= 5 \\ p &= 0.5 \end{aligned}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

# Binomial Distribution

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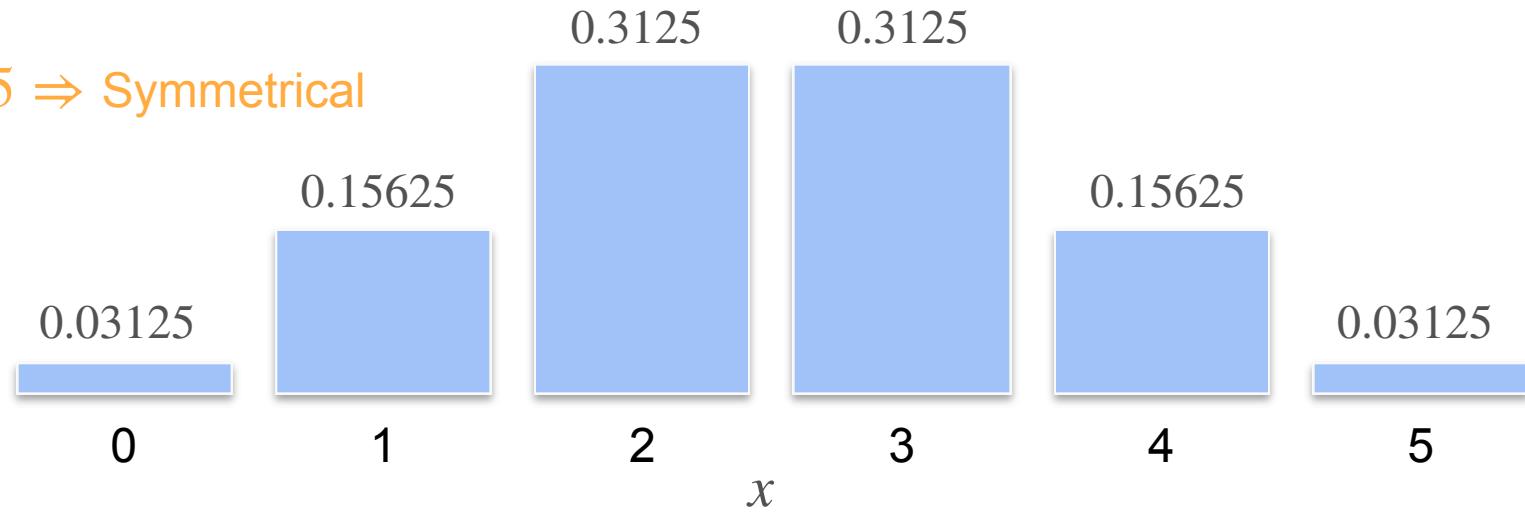


# Binomial Distribution

$$\begin{array}{|l|} \hline n = 5 \\ p = 0.5 \\ \hline \end{array}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

$p = 0.5 \Rightarrow$  Symmetrical



# Binomial Distribution

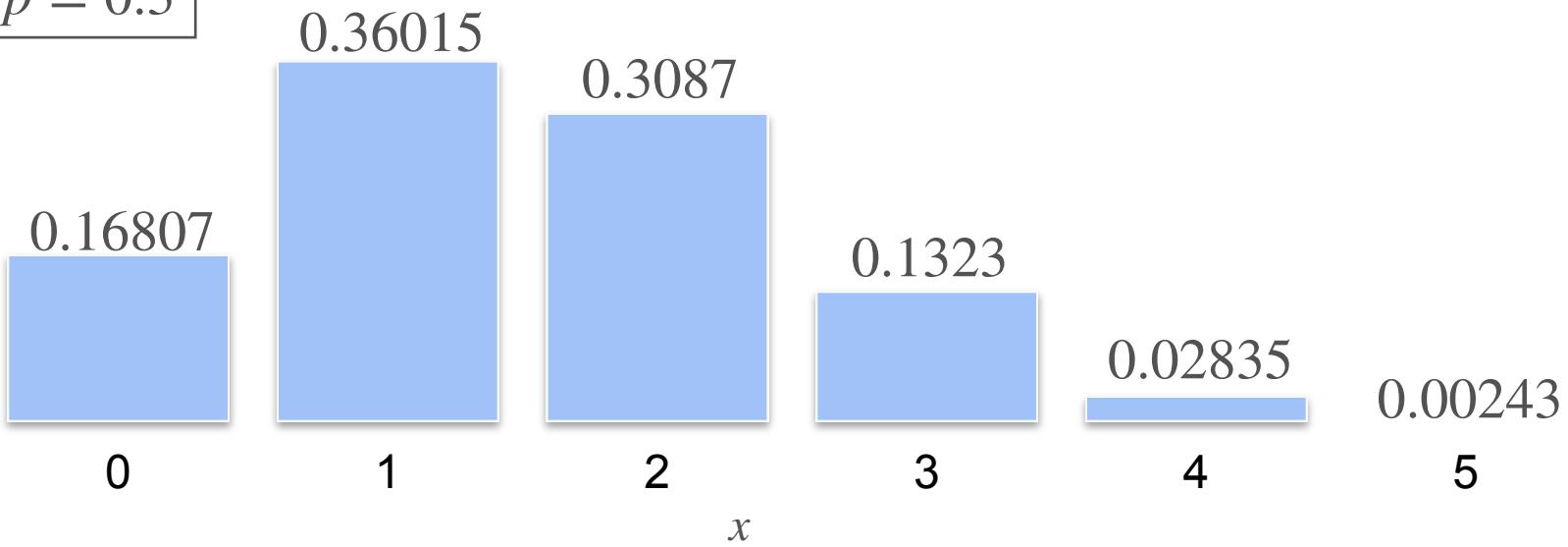
$$\begin{aligned} n &= 5 \\ p &= 0.3 \end{aligned}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$

# Binomial Distribution

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# Binomial Distribution

# Binomial Distribution

General PMF for  $X$  : number of heads in 5 coin tosses?

Your coin has  $\mathbf{P}(H) = p$

Event:  $X = x$ :  $x$  heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

$$X \sim \text{Binomial}(5,p)$$

# Binomial Distribution

General PMF for  $X$  : number of heads in  $n$  coin tosses?

Your coin has  $\mathbf{P}(H) = p$

Event:  $X = x$ :  $x$  heads in  $n$  tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$X \sim \text{Binomial}(n, p)$$

# Binomial Distribution

General PMF for  $X$  : number of heads in  $n$  coin tosses?

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Event:  $X = x$ :  $x$  heads in  $n$  tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$X \sim \text{Binomial}(n, p)$

$n$  and  $p$  are called the **parameters** of the binomial distribution

# Binomial Coefficient

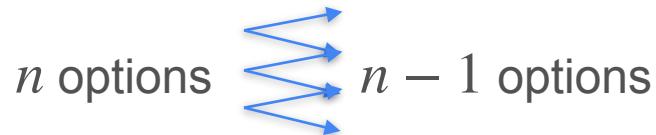
# Binomial Coefficient

Pick 1st number

$n$  options

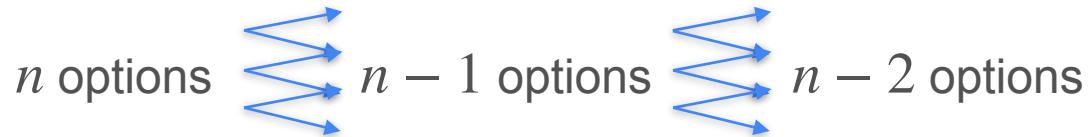
# Binomial Coefficient

Pick 1st number    Pick 2nd number



# Binomial Coefficient

Pick 1st number    Pick 2nd number    Pick 3rd number



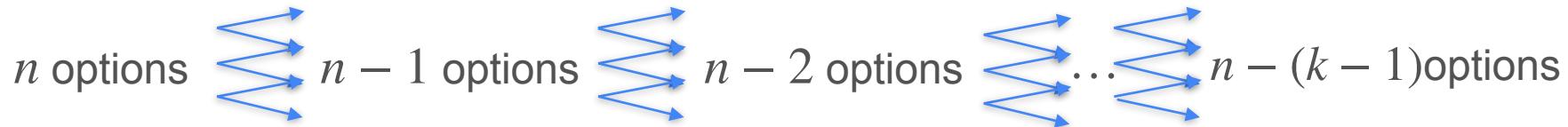
# Binomial Coefficient

Pick 1st number    Pick 2nd number    Pick 3rd number



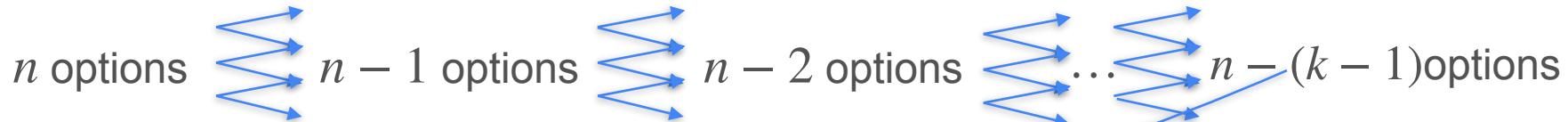
# Binomial Coefficient

Pick 1st number    Pick 2nd number    Pick 3rd number    Pick  $k$ -th number



# Binomial Coefficient

Pick 1st number    Pick 2nd number    Pick 3rd number    Pick  $k$ -th number



Ordered sets of length  $k$

1,2,3,4,...,  $k$

2,1,3,4,...,  $k$

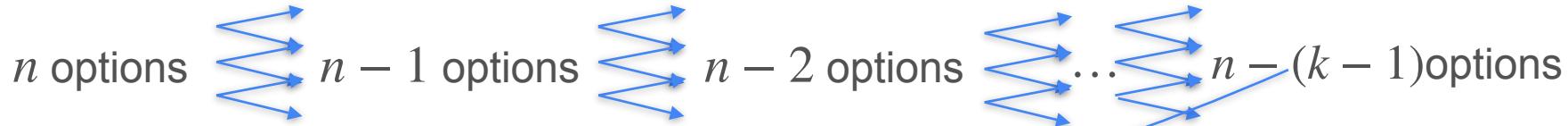
5,1,3, $k$ , ...,2

$n$ ,1,3,4,...,2

...

# Binomial Coefficient

Pick 1st number    Pick 2nd number    Pick 3rd number    Pick  $k$ -th number



Ordered sets of length  $k$

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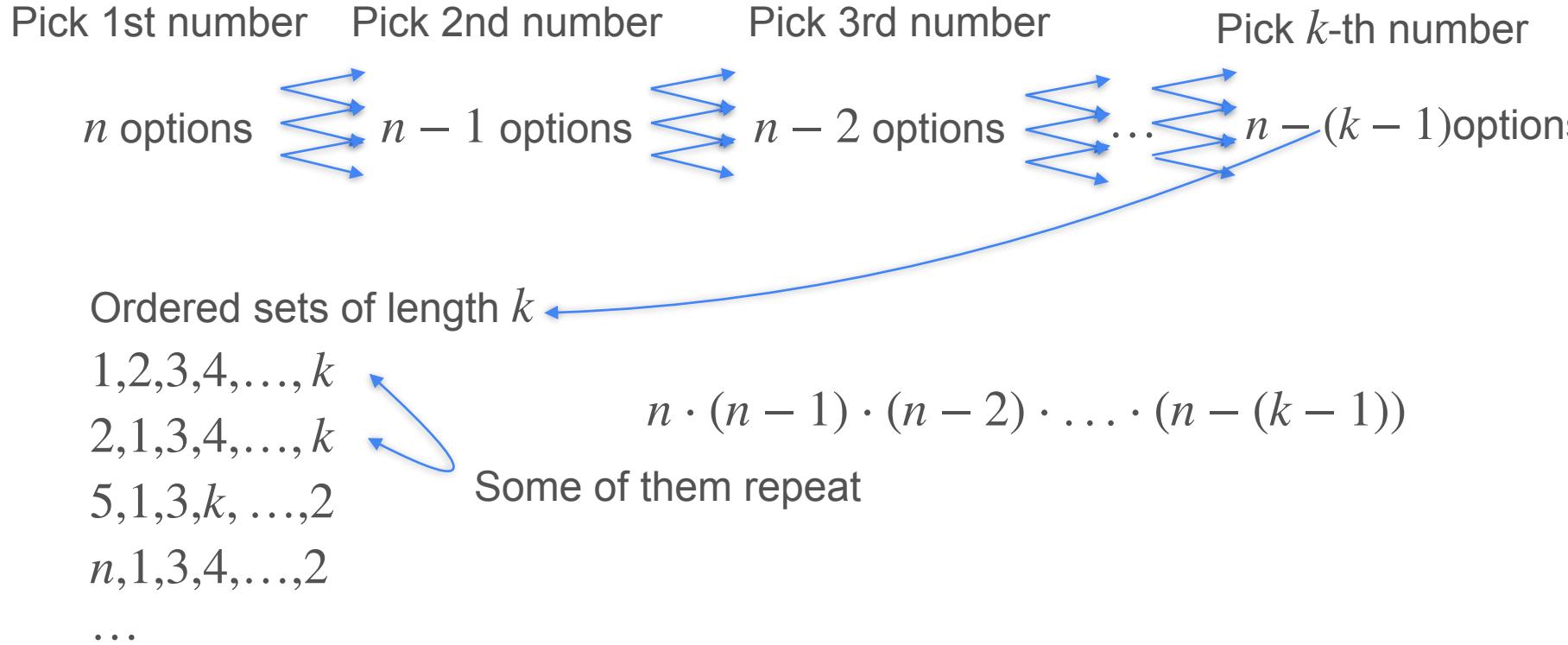
5,1,3, $k$ , ...,2

$n$ ,1,3,4,...,2

...

$$n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))$$

# Binomial Coefficient



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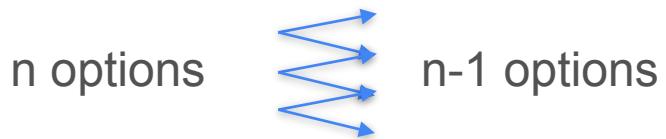
Pick 1st number

n options

# Binomial Coefficient

Pick 1st number

Pick 2nd number



# Binomial Coefficient

Pick 1st number

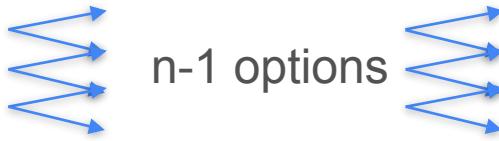
$n$  options

Pick 2nd number

$n-1$  options

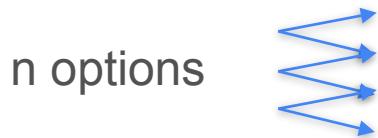
Pick 3rd number

$n-2$  options

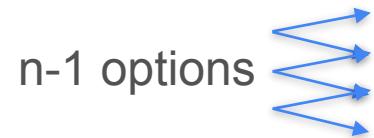


# Binomial Coefficient

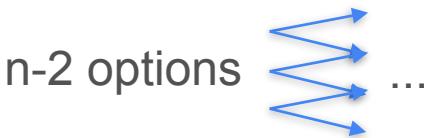
Pick 1st number



Pick 2nd number



Pick 3rd number



...

# Binomial Coefficient



# Binomial Coefficient

Pick 1st number

$n$  options

Pick 2nd number

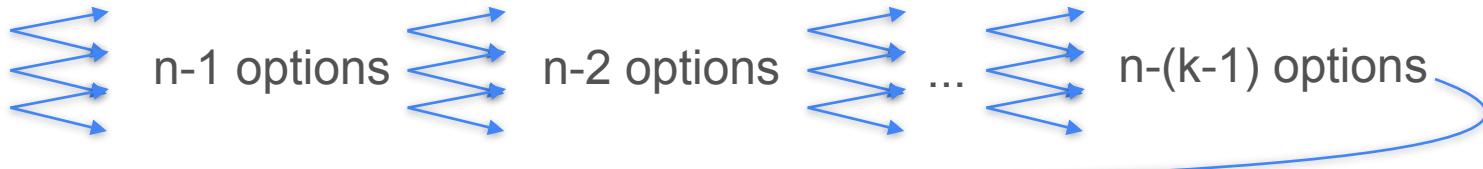
$n-1$  options

Pick 3rd number

$n-2$  options

Pick  $k$ -th number

$n-(k-1)$  options



Unordered sets of length  $k$

1, 2, 3, 4, ...  $k$

2, 1, 3, 4, ...  $k$

5, 1, 3,  $k$ , ... 2

$n$ , 1, 3, 4, ... 2

...

# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

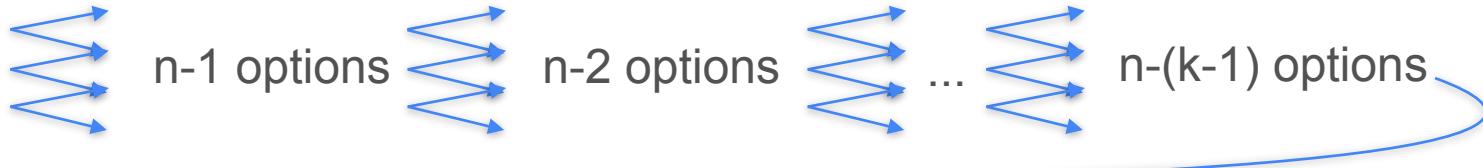
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Unordered sets of length k

1, 2, 3, 4, ... k

$n * (n-1) * (n-2) * \dots * (n-(k-1))$

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

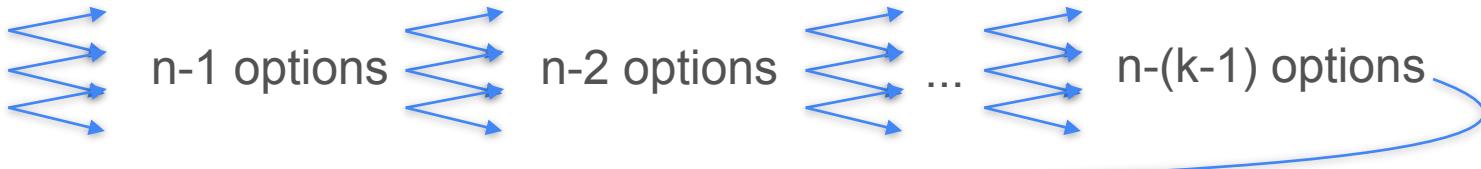
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

$$n * (n-1) * (n-2) * \dots * (n-(k-1))$$

Some of them repeat

...

# Binomial Coefficient

# Binomial Coefficient

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

# Binomial Coefficient

Pick 1st

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

# Binomial Coefficient

Pick 1st

Pick 2nd

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

# Binomial Coefficient

Pick 1st

Pick 2nd

Pick 3rd

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

# Binomial Coefficient

Pick 1st

Pick 2nd

Pick 3rd

Pick 4th

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

# Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options			
1,2,3,4				
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

# Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options		
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

# Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

# Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

# Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4		$4 \cdot 3 \cdot 2 \cdot 1 = 4!$		
1,3,4,2				
...				
4,3,2,1				

# Binomial Coefficient

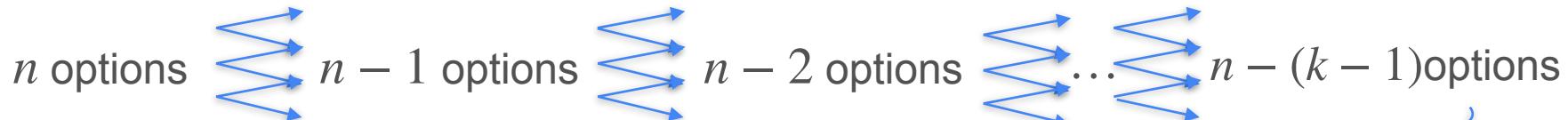
	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4		$4 \cdot 3 \cdot 2 \cdot 1 = 4!$		
1,3,4,2		For five numbers:		
		$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$		
...				
4,3,2,1				

# Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4		$4 \cdot 3 \cdot 2 \cdot 1 = 4!$		
1,3,4,2		For five numbers:		
		$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$		
...				
4,3,2,1		General solution:	$k!$	

# Binomial Coefficient

Pick 1st number    Pick 2nd number    Pick 3rd number    Pick  $k$ -th number



Ordered sets of length  $k$

1,2,3,4,...,  $k$

2,1,3,4,...,  $k$

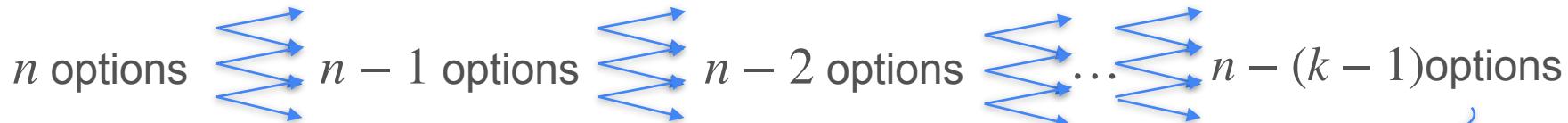
5,1,3, $k$ , ...,2     $n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))$

$n$ ,1,3,4,...,2

...

# Binomial Coefficient

Pick 1st number    Pick 2nd number    Pick 3rd number    Pick  $k$ -th number



Ordered sets of length  $k$

1,2,3,4,...,  $k$

2,1,3,4,...,  $k$

5,1,3, $k$ , ...,2

$n,1,3,4,...,2$

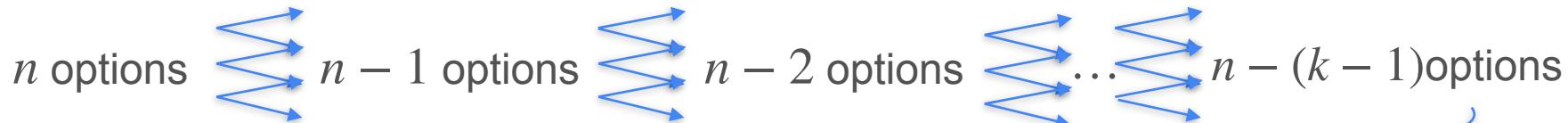
...

→ Unordered sets of length  $k$

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-(k-1))}{k!}$$

# Binomial Coefficient

Pick 1st number      Pick 2nd number      Pick 3rd number      Pick  $k$ -th number



Ordered sets of length  $k$

1,2,3,4,...,  $k$

2,1,3,4,...,  $k$

5,1,3, $k$ , ...,2

$n,1,3,4,...,2$

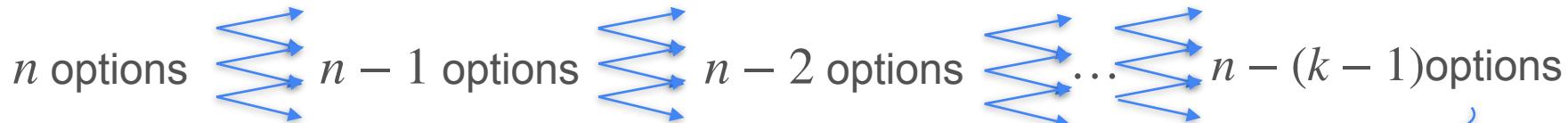
...

→ Unordered sets of length  $k$

$$\frac{n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))}{k!} = \underline{\hspace{1cm}}$$

# Binomial Coefficient

Pick 1st number      Pick 2nd number      Pick 3rd number      Pick  $k$ -th number



Ordered sets of length  $k$

1,2,3,4,...,  $k$

2,1,3,4,...,  $k$

5,1,3, $k$ , ...,2

$n$ ,1,3,4,...,2

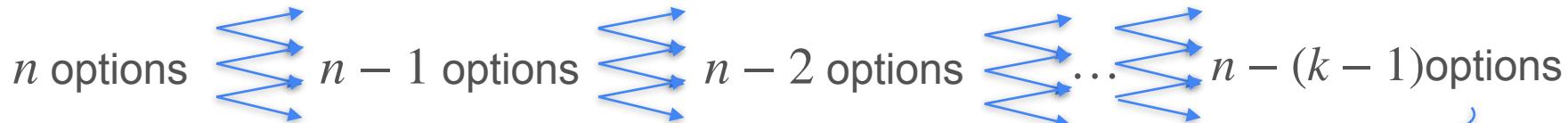
...

→ Unordered sets of length  $k$

$$\frac{n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))}{k!} = \frac{n!}{(n - k)!}$$

# Binomial Coefficient

Pick 1st number      Pick 2nd number      Pick 3rd number      Pick  $k$ -th number



Ordered sets of length  $k$

1,2,3,4,...,  $k$

2,1,3,4,...,  $k$

5,1,3, $k$ , ...,2

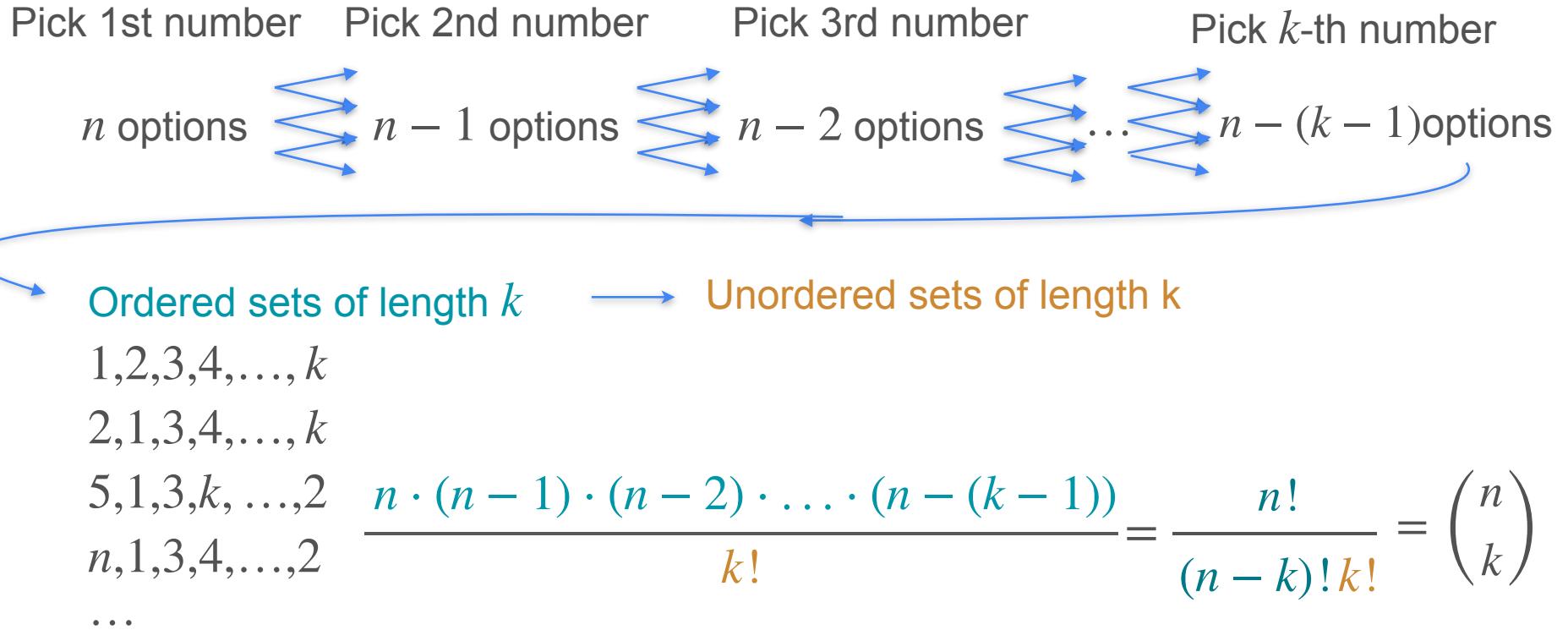
$n$ ,1,3,4,...,2

...

→ Unordered sets of length  $k$

$$\frac{n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot (n - (k - 1))}{k!} = \frac{n!}{(n - k)! k!}$$

# Binomial Coefficient



# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

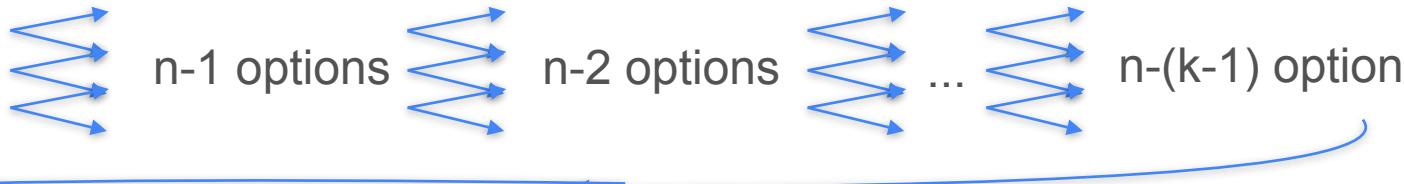
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

$$n * (n-1) * (n-2) * \dots * (n-(k-1))$$

...

# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

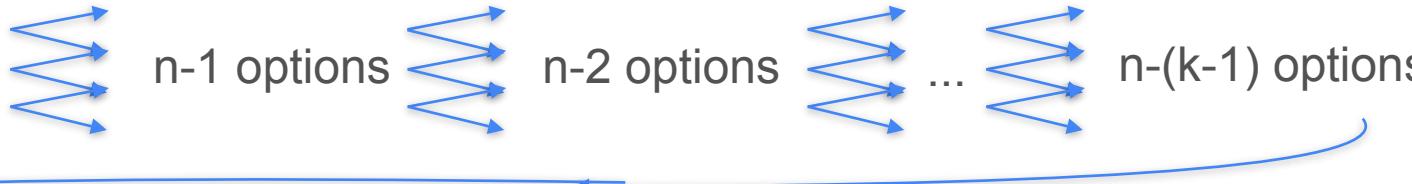
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length  $k$

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

→ Unordered sets of length  $k$

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!}$$

# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

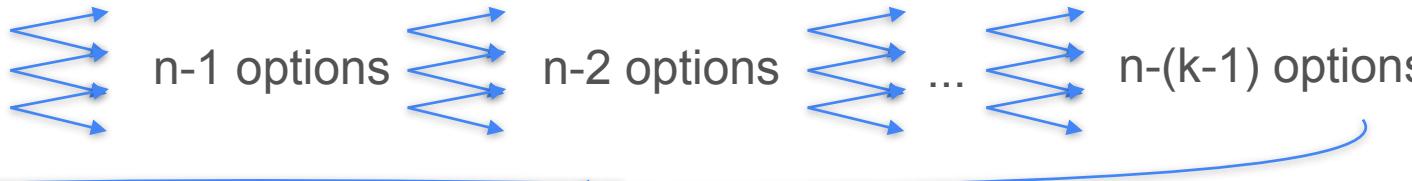
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

→ Unordered sets of length k

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!} =$$

# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

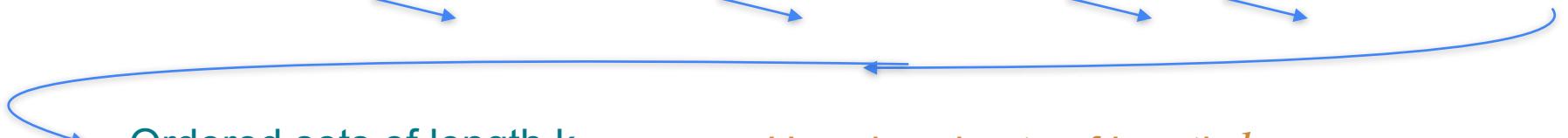
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length  $k$

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

→ Unordered sets of length  $k$

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!} = \frac{n!}{(n-k)!}$$

# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

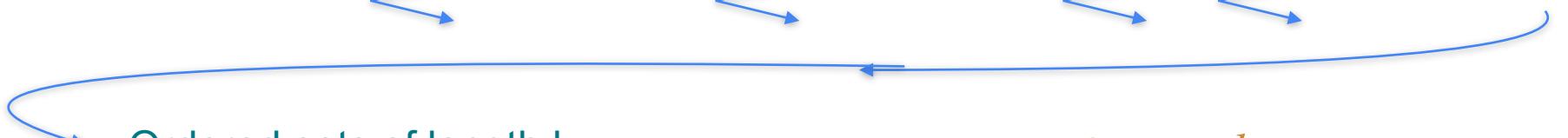
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length  $k$

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

→ Unordered sets of length  $k$

$$n * (n-1) * (n-2) * \dots * (n-(k-1))$$

$$k!$$

$$= \frac{n!}{(n-k)! * k!}$$

# Binomial Coefficient

Pick 1st number

n options

Pick 2nd number

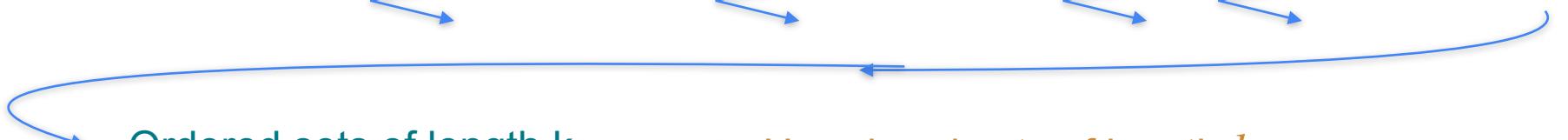
n-1 options

Pick 3rd number

n-2 options

Pick k-th number

n-(k-1) options



Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

→ Unordered sets of length k

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!}$$

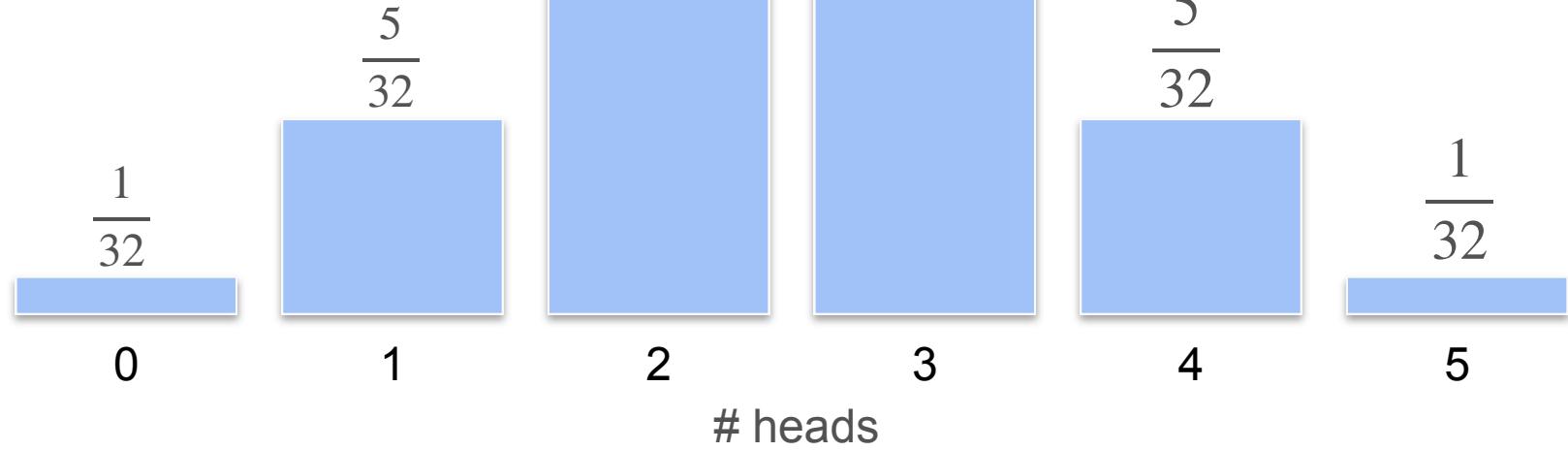
$$= \frac{n!}{(n-k)! * k!} = \binom{n}{k}$$

# Binomial Distribution: Fair Coins

$$\binom{5}{0} = \frac{5!}{0!5!} = 1$$

$$\frac{10}{32}$$

$$\frac{10}{32}$$



# Binomial Distribution: Fair Coins



50 %



50 %



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

# Binomial Distribution: Biased Coins



30 %



70 %



# Binomial Distribution: Biased Coins



30 %



70 %



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$



# Binomial Distribution: Biased Coins



30 %



70 %



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$



$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.01323$$

# Binomial Distribution: Biased Coins

0.3 · 0.3 · 0.3 · 0.3 · 0.3 · 0.3

0.3 · 0.3 · 0.3 · 0.3 · 0.3 · 0.7

0.3 · 0.3 · 0.3 · 0.7 · 0.7

0.3 · 0.3 · 0.7 · 0.7 · 0.7

0.3 · 0.7 · 0.7 · 0.7 · 0.7

0.7 · 0.7 · 0.7 · 0.7 · 0.7

# Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

# Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

# Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$
$$= 0.3^k \cdot 0.7^{n-k}$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

# Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

# Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 = 0.3^4 \cdot 0.7^1$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 = 0.3^3 \cdot 0.7^2$$

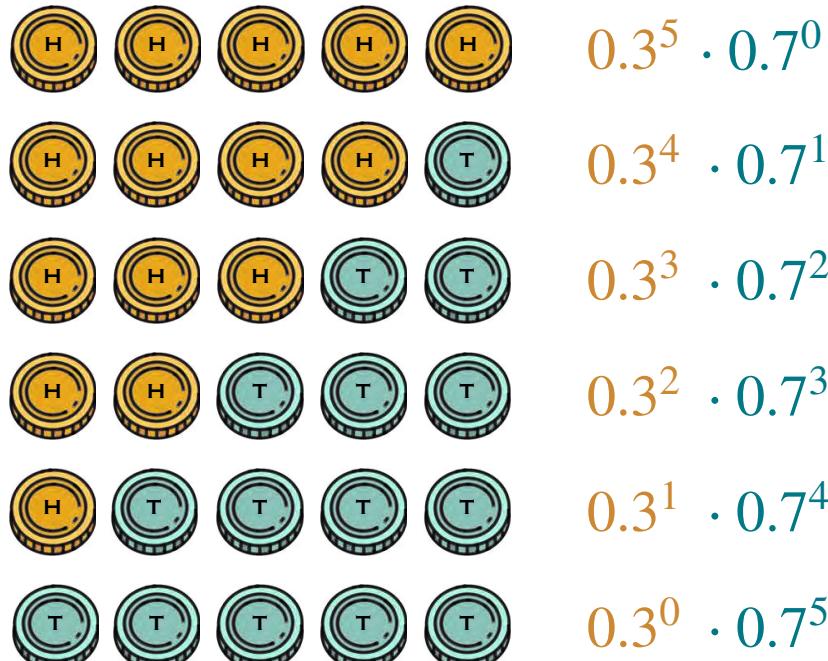

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^2 \cdot 0.7^3$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^1 \cdot 0.7^4$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

# Binomial Distribution: Biased Coins



$$0.3^5 \cdot 0.7^0$$

$$0.3^4 \cdot 0.7^1$$

$$0.3^3 \cdot 0.7^2$$

$$0.3^2 \cdot 0.7^3$$

$$0.3^1 \cdot 0.7^4$$

$$0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

# Binomial Distribution: Biased Coins

	$0.3^5 \cdot 0.7^0$
	$0.3^4 \cdot 0.7^1$
	$0.3^3 \cdot 0.7^2$
	$0.3^2 \cdot 0.7^3$
	$0.3^1 \cdot 0.7^4$
	$0.3^0 \cdot 0.7^5$

$$= 0.3^k \cdot 0.7^{n-k} \rightarrow \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Account for all possible orders of heads and tails

# Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

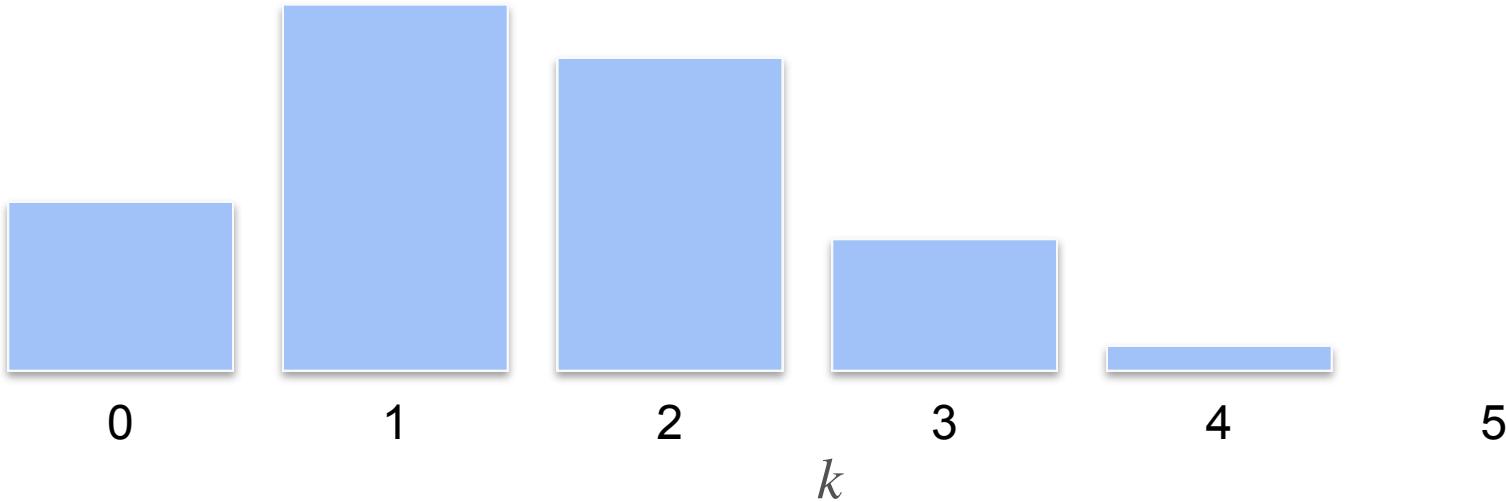
# Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k} \quad n = 5$$

# Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

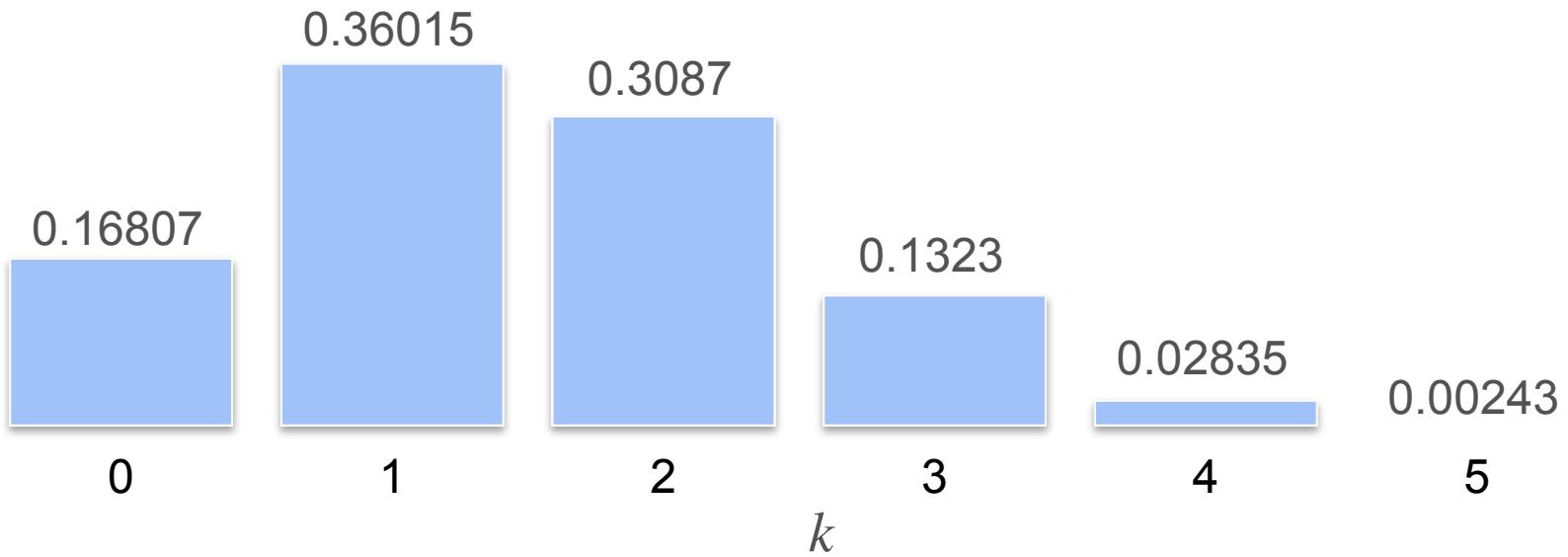
n = 5  
p = 0.3



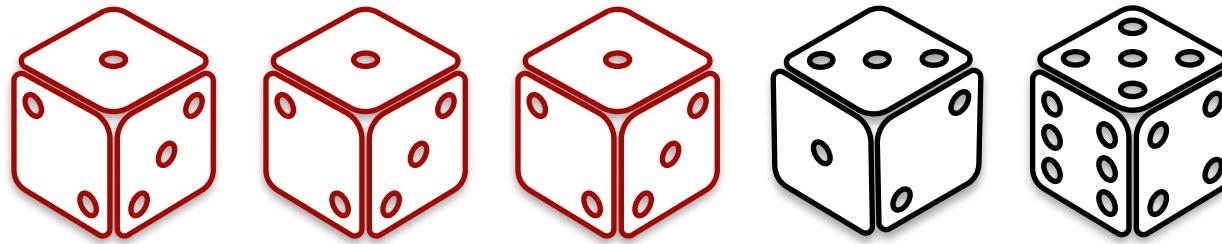
# Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

n = 5  
p = 0.3

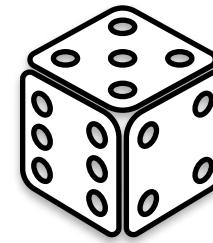
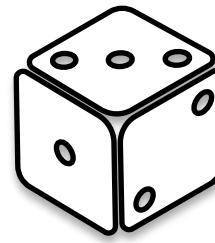
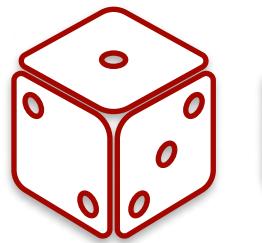


# Binomial Distribution: Quiz

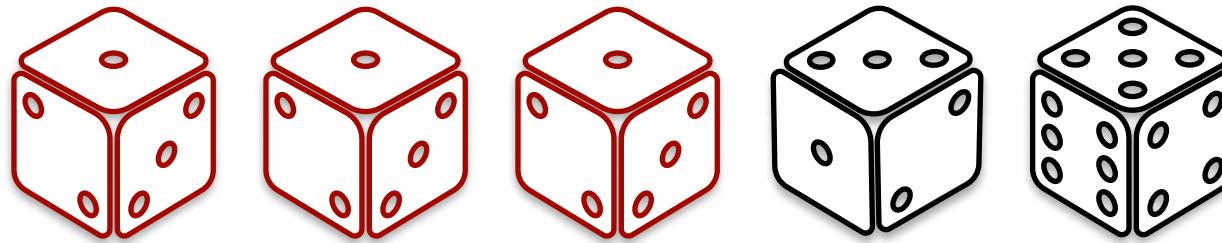


# Binomial Distribution: Quiz

What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?

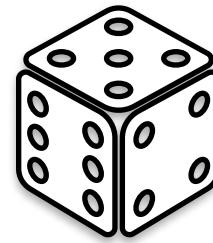
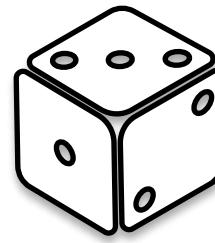


# Binomial Distribution: Quiz

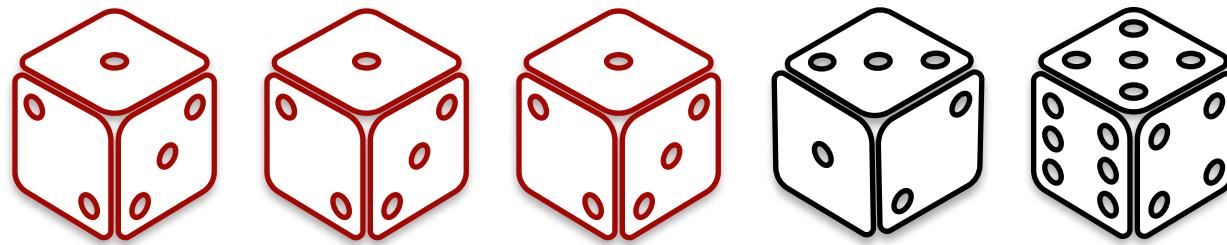


# Binomial Distribution: Quiz

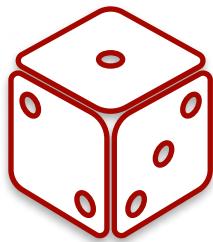
What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?



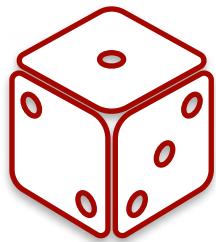
# Binomial Distribution: Dice Is a Biased Coin!



# Binomial Distribution: Dice Is a Biased Coin!



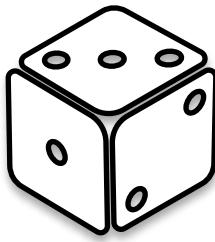
one  
 $p = \frac{1}{6}$



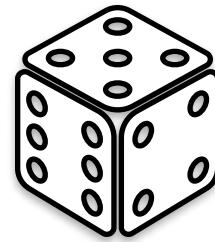
one  
 $p = \frac{1}{6}$



one  
 $p = \frac{1}{6}$

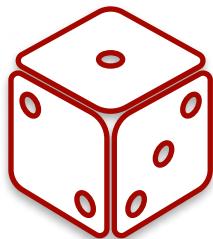


not one  
 $p = \frac{5}{6}$



not one  
 $p = \frac{5}{6}$

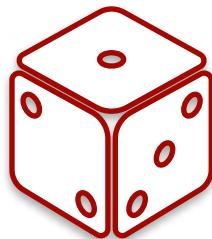
# Binomial Distribution: Dice Is a Biased Coin!



one  
 $p = \frac{1}{6}$



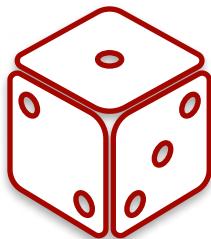
heads  
 $p = \frac{1}{6}$



one  
 $p = \frac{1}{6}$



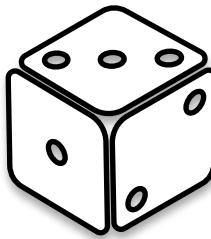
heads  
 $p = \frac{1}{6}$



one  
 $p = \frac{1}{6}$



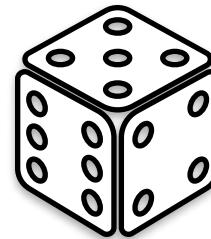
heads  
 $p = \frac{1}{6}$



not one  
 $p = \frac{5}{6}$



not heads  
 $p = \frac{5}{6}$

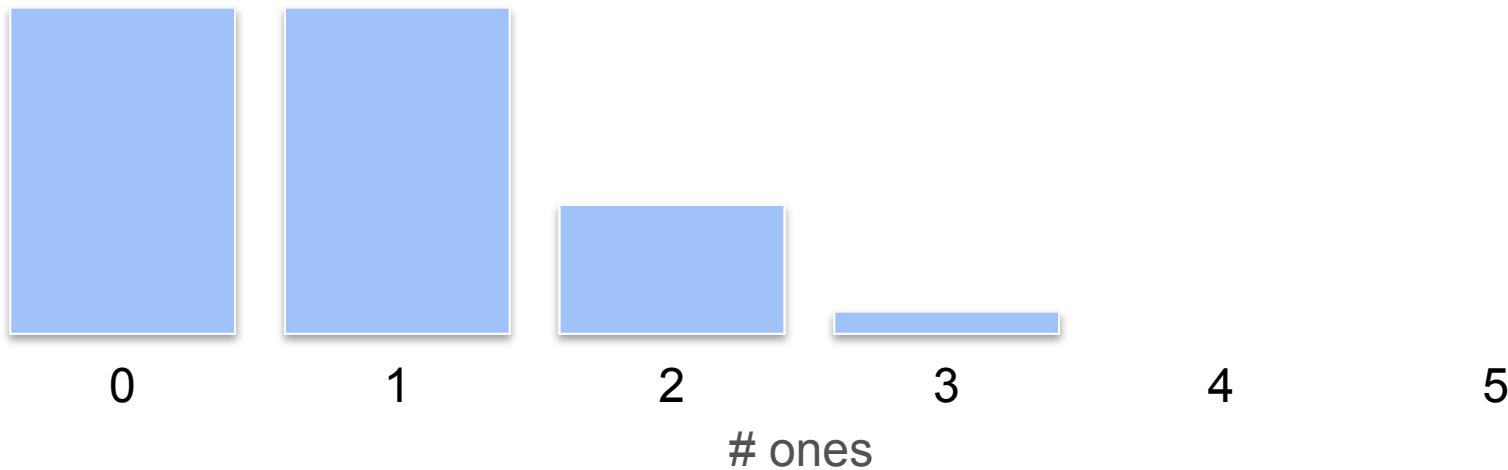


not one  
 $p = \frac{5}{6}$



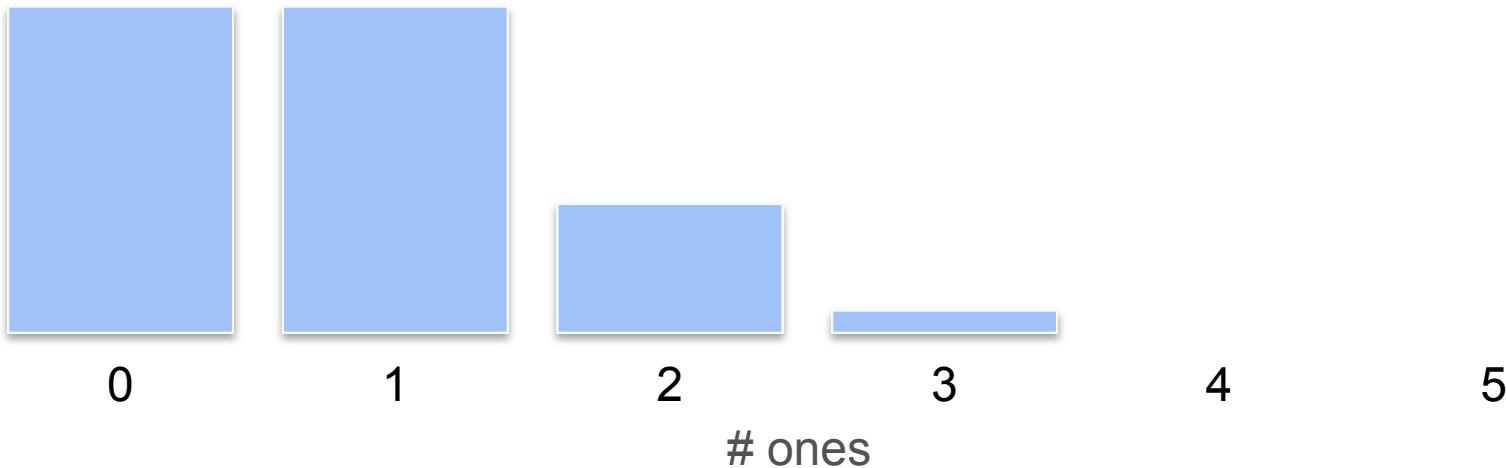
not heads  
 $p = \frac{5}{6}$

# Binomial Distribution: Dice Is a Biased Coin!



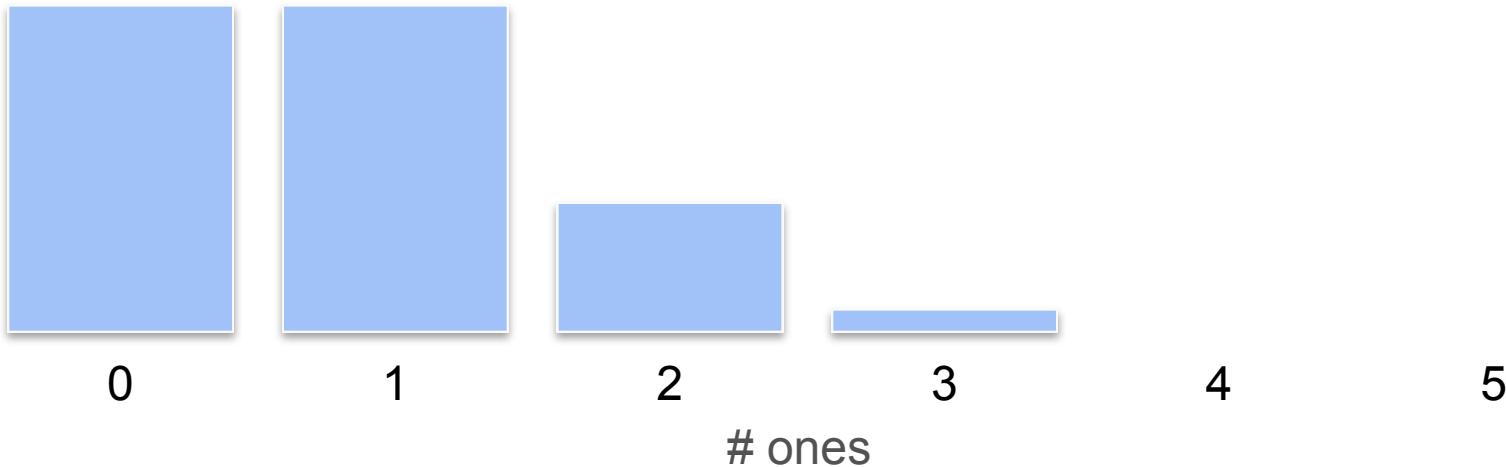
# Binomial Distribution: Dice Is a Biased Coin!

$n = 5$



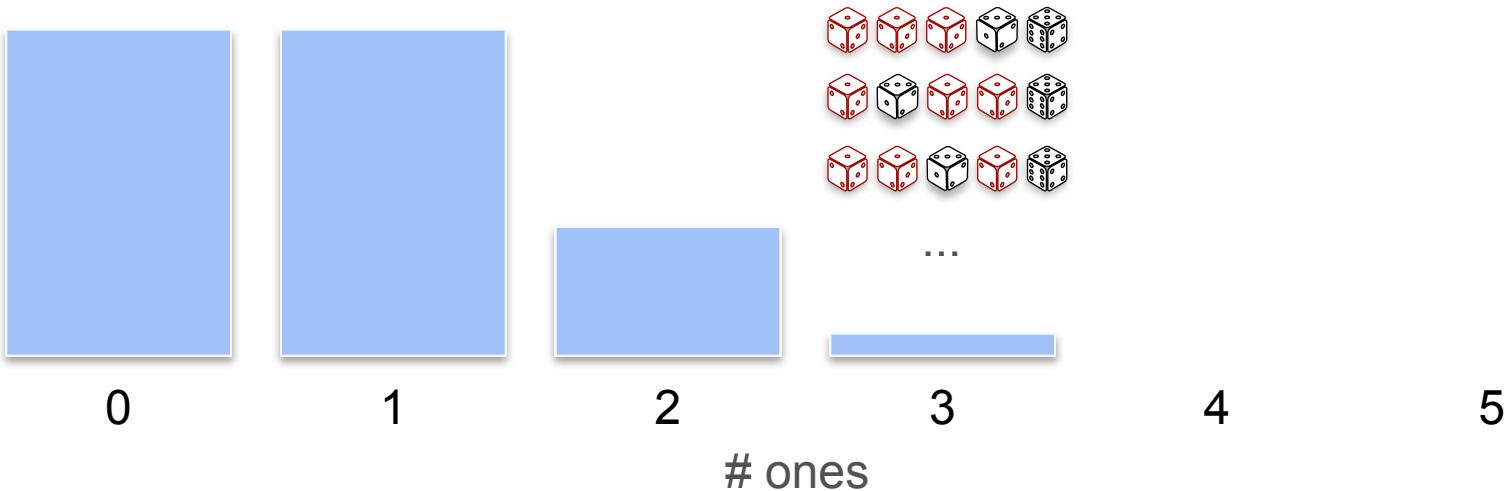
# Binomial Distribution: Dice Is a Biased Coin!

n = 5  
p = 0.1666

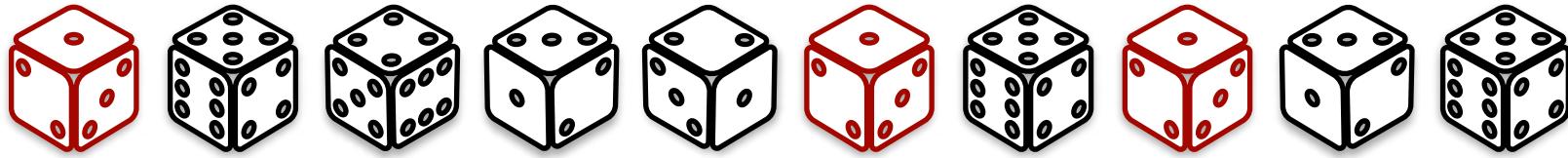


# Binomial Distribution: Dice Is a Biased Coin!

$n = 5$   
 $p = 0.1666$

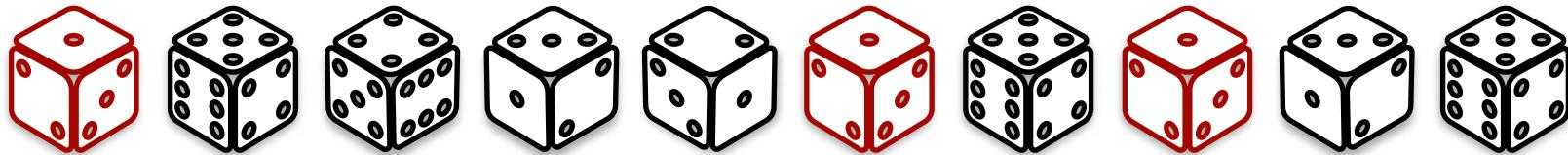


# Binomial Distribution: Quiz

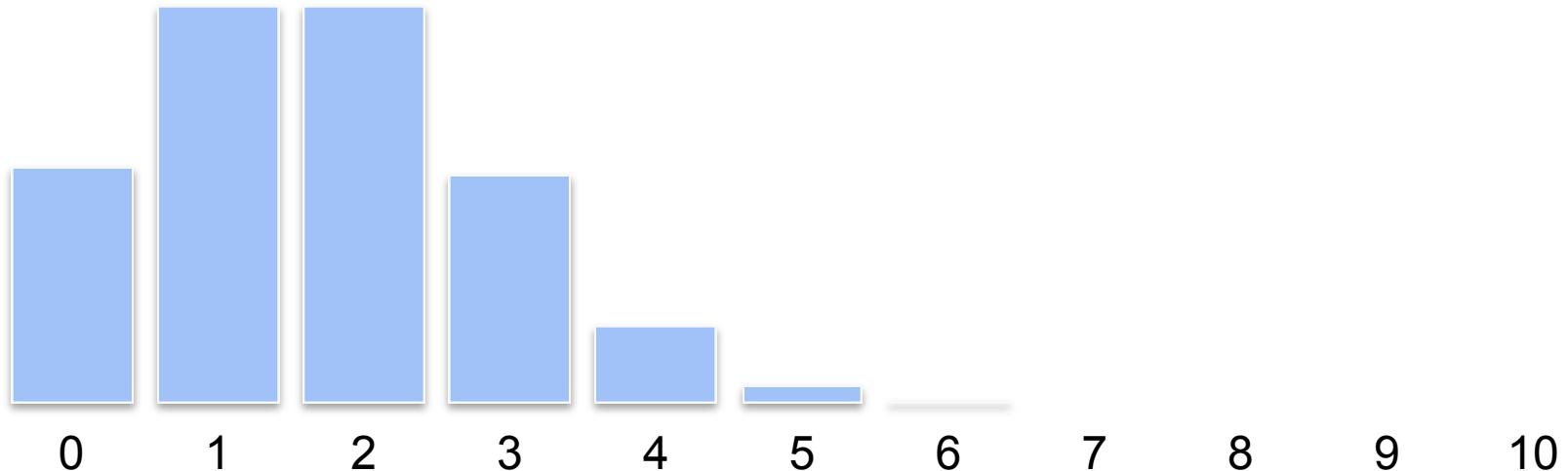


# Binomial Distribution: Quiz

- Quiz: What are the parameters for the following binomial distribution:
  - I roll 10 dice
  - I want to record the number of times I obtain the number 1

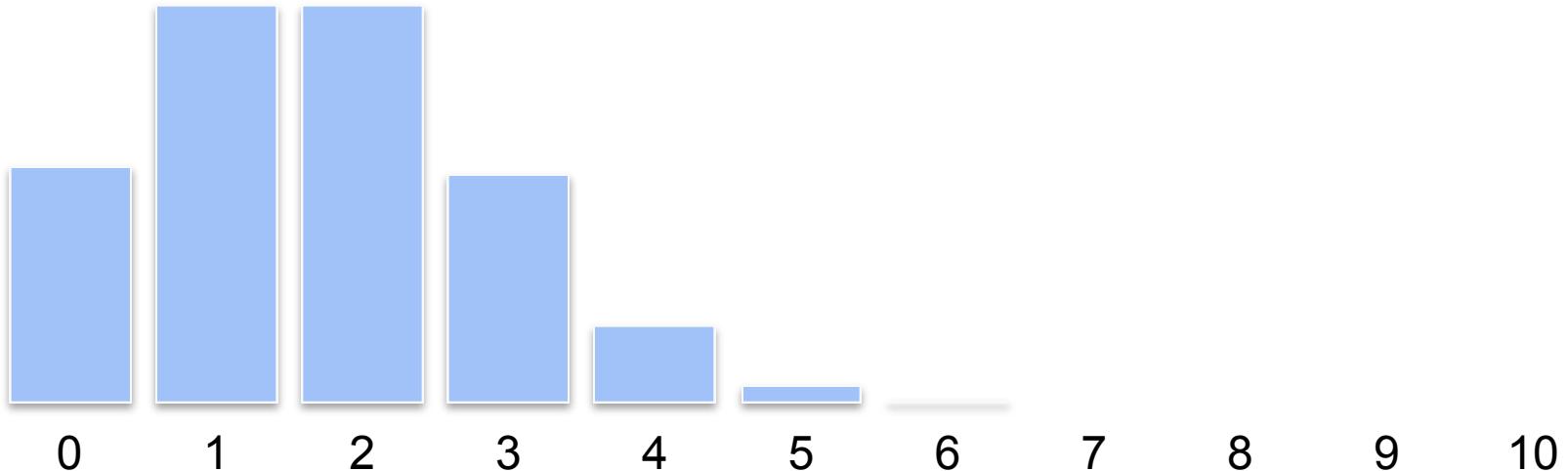


# Binomial Distribution: Quiz



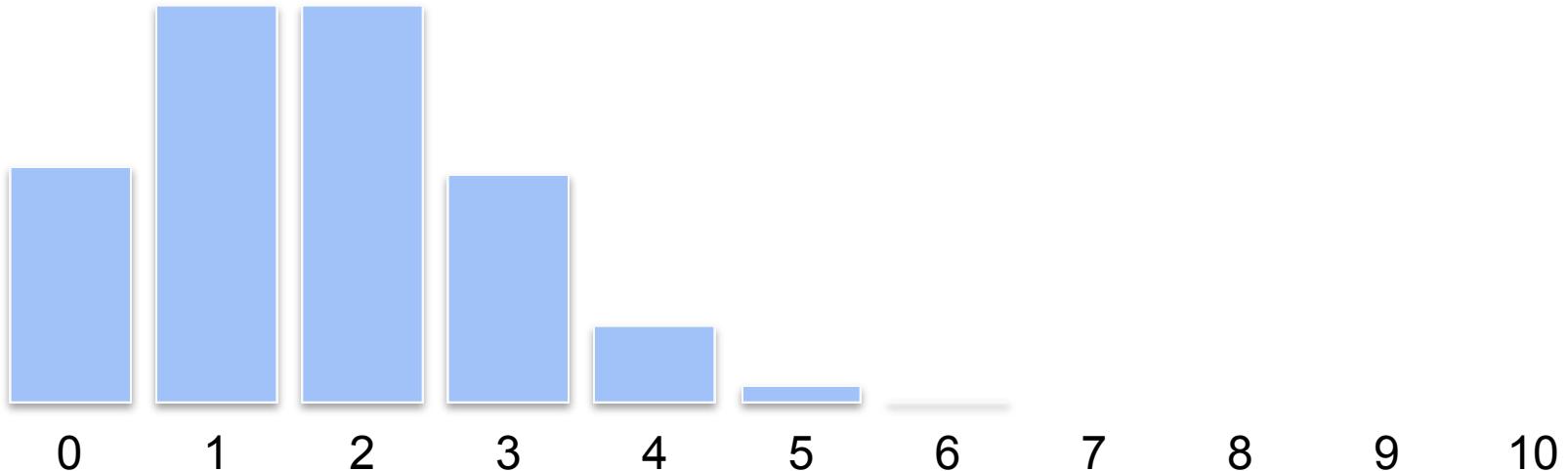
# Binomial Distribution: Quiz

$$n = 10$$



# Binomial Distribution: Quiz

$$\begin{aligned}n &= 10 \\p &= 0.1666\end{aligned}$$



# Bernoulli Distribution

# Bernoulli Distribution

$X$  = Number of heads

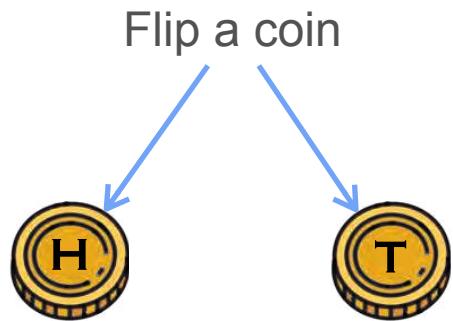
# Bernoulli Distribution

$X$  = Number of heads

Flip a coin

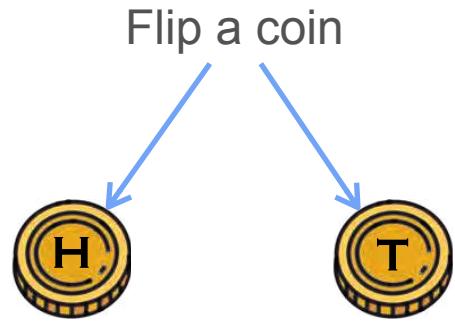
# Bernoulli Distribution

$X$  = Number of heads



# Bernoulli Distribution

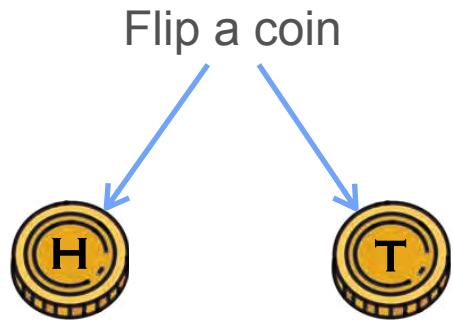
$X$  = Number of heads



$$\mathbf{P}(X = 1) = 0.5$$

# Bernoulli Distribution

$X$  = Number of heads

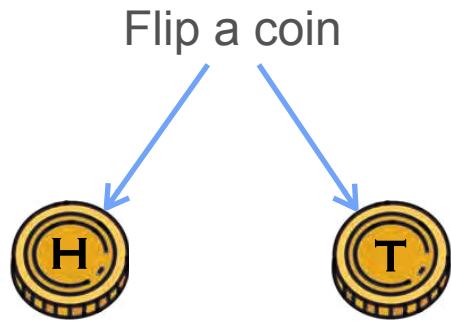


$$\mathbf{P}(X = 1) = 0.5$$

$$\mathbf{P}(X = 0) = 0.5$$

# Bernoulli Distribution

$X$  = Number of heads



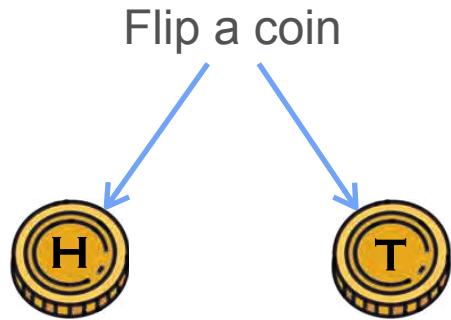
$$\mathbf{P}(X = 1) = 0.5$$

$$\mathbf{P}(X = 0) = 0.5$$

Success

# Bernoulli Distribution

$X$  = Number of heads



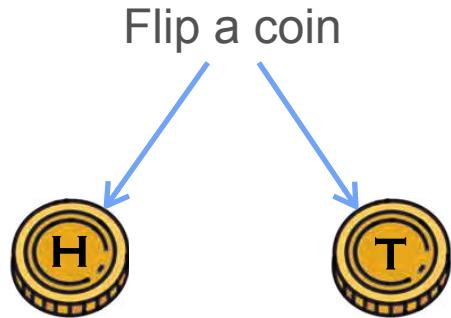
$$\mathbf{P}(X = 1) = 0.5 \quad \mathbf{P}(X = 0) = 0.5$$

Success

Failure

# Bernoulli Distribution

$X$  = Number of heads



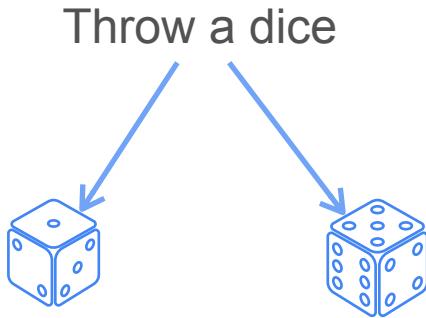
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

$X$  = Number of 1's



$$P(X = 1) = \frac{1}{6}$$

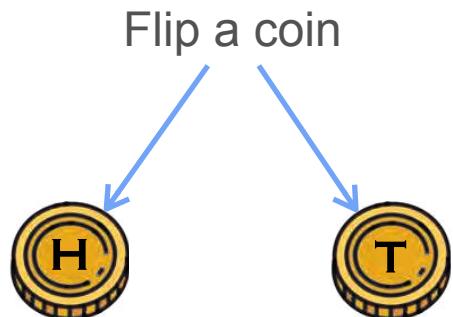
Success

$$P(X = 0) = \frac{5}{6}$$

Failure

# Bernoulli Distribution

$X$  = Number of heads



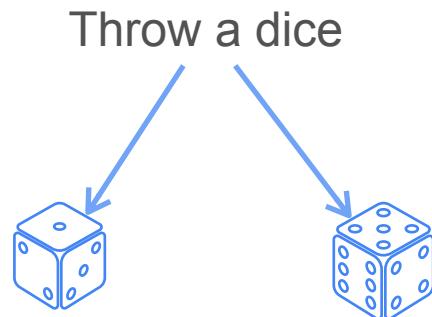
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

$X$  = Number of 1's



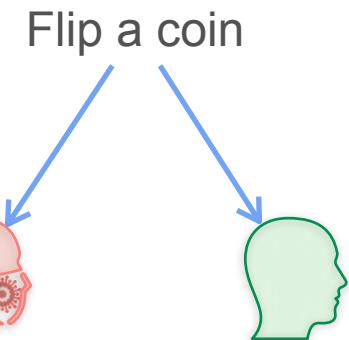
$$P(X = 1) = \frac{1}{6}$$

Success

$$P(X = 0) = \frac{5}{6}$$

Failure

$X$  = Number of sick patients



$$P(X = 1) = p$$

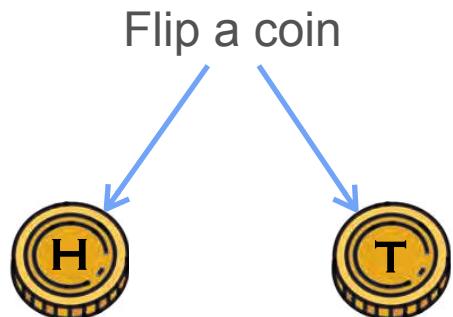
Success

$$P(X = 0) = 1 - p$$

Failure

# Bernoulli Distribution

$X$  = Number of heads



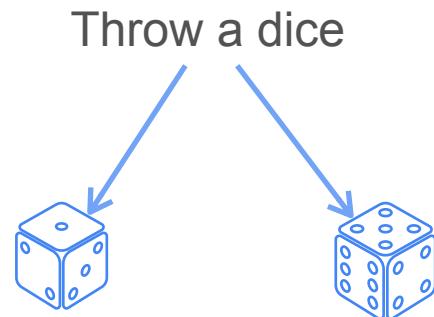
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

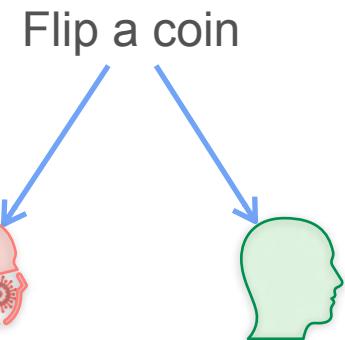
$X$  = Number of 1's



$$P(X = 1) = \frac{1}{6}$$

$$P(X = 0) = \frac{5}{6}$$

$X$  = Number of sick patients



$$P(X = 1) = p$$

Success

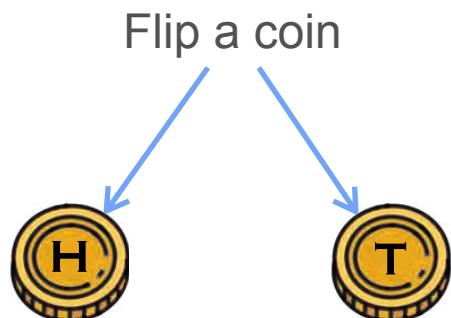
$$P(X = 0) = 1 - p$$

Failure

$$X \sim \text{Bernoulli}(p)$$

# Bernoulli Distribution

$X$  = Number of heads



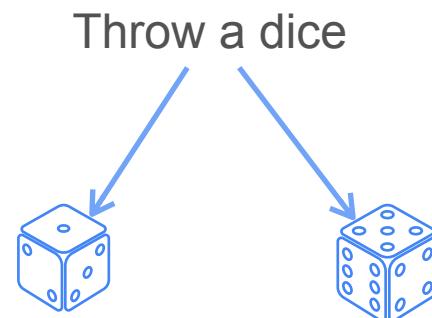
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

$X$  = Number of 1's



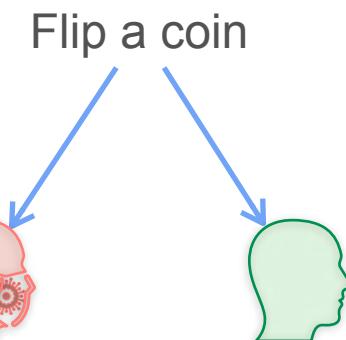
$$P(X = 1) = \frac{1}{6}$$

Success

$$P(X = 0) = \frac{5}{6}$$

Failure

$X$  = Number of sick patients



$$P(X = 1) = p$$

Success

$$P(X = 0) = 1 - p$$

Failure

$$X \sim \text{Bernoulli}(p)$$

$p$  is the parameter of the Bernoulli distribution



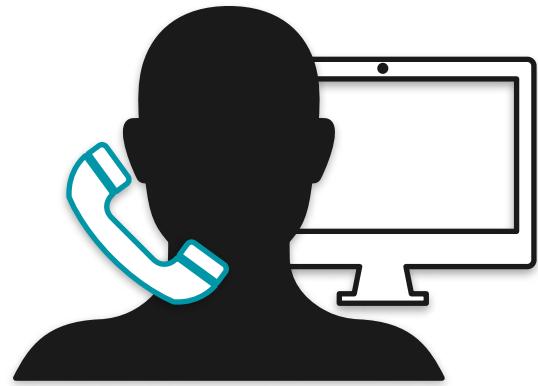
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# Probability Distributions

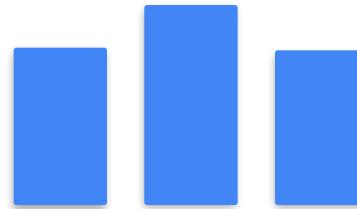
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## Probability Distributions (Continuous)

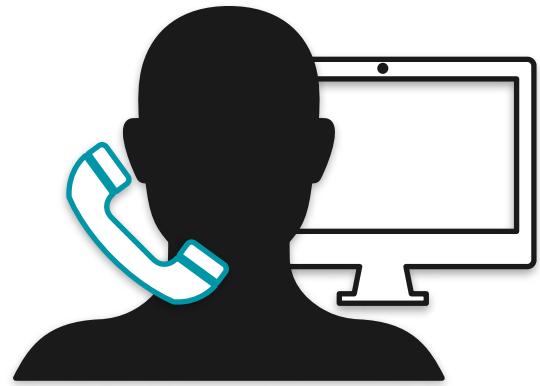
# From Discrete to Continuous Distributions



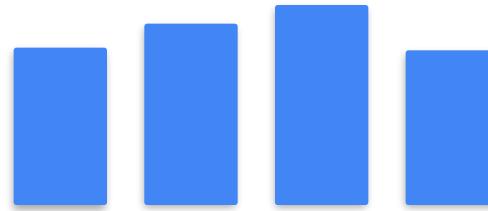
Waiting time: 1 2 3 (min)



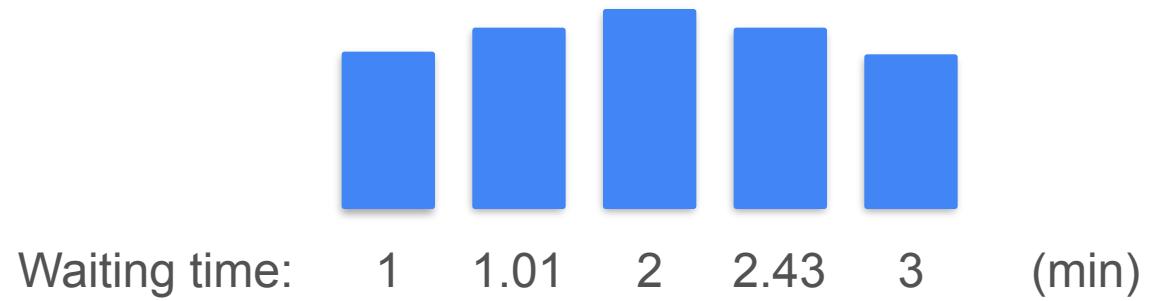
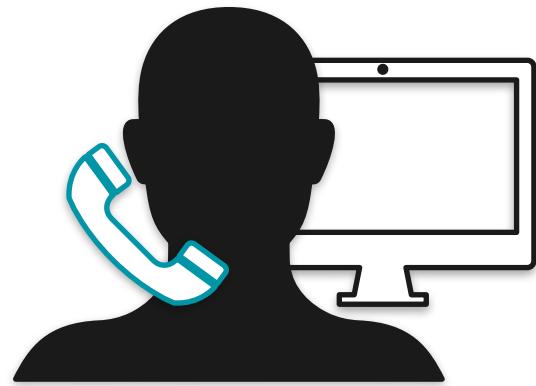
# From Discrete to Continuous Distributions



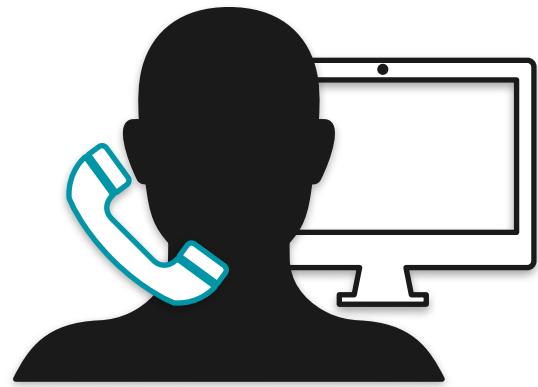
Waiting time: 1 1.01 2 3 (min)



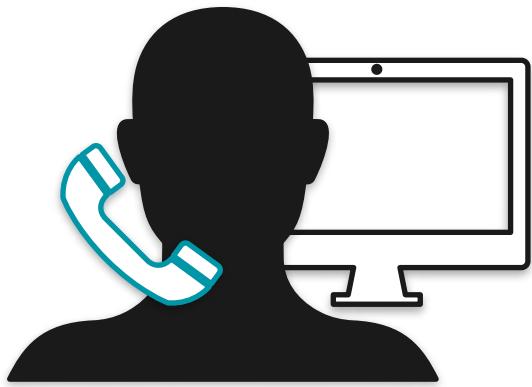
# From Discrete to Continuous Distributions



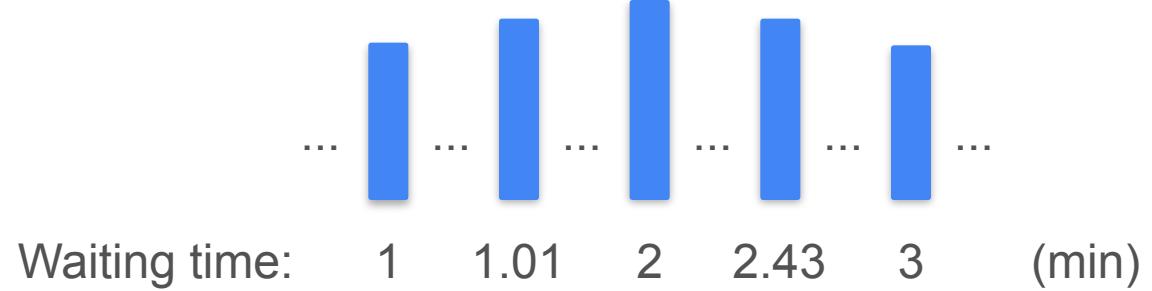
# From Discrete to Continuous Distributions



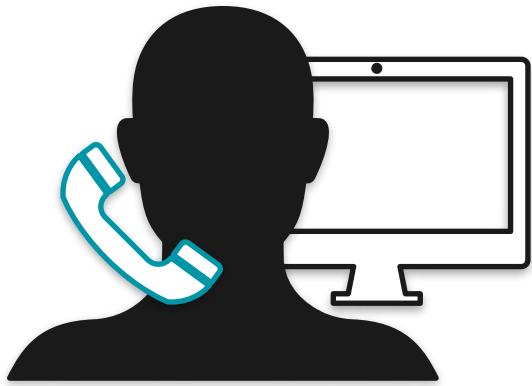
# From Discrete to Continuous Distributions



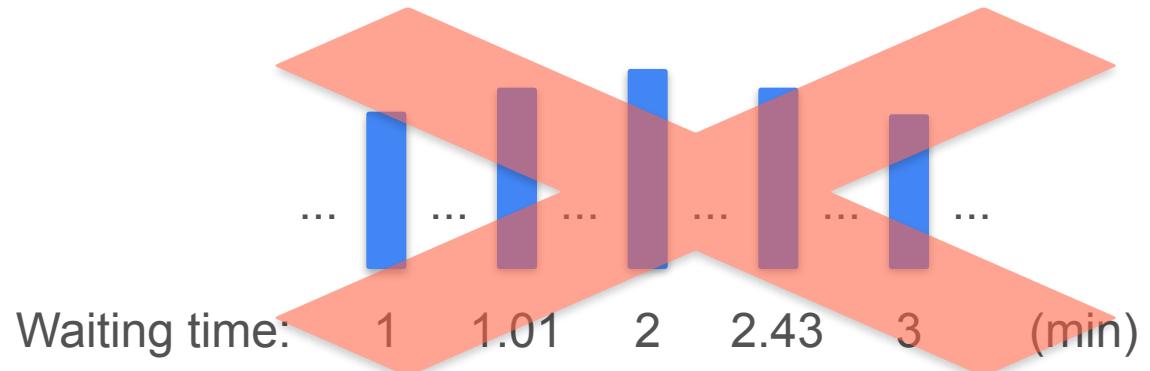
This is a continuous distribution!



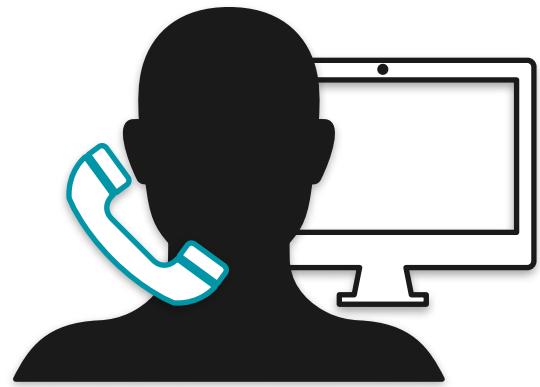
# From Discrete to Continuous Distributions



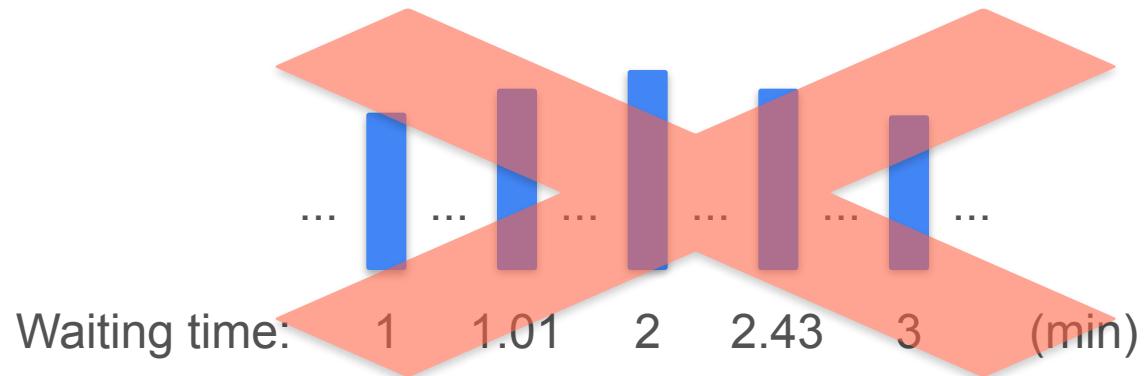
This is a continuous distribution!



# From Discrete to Continuous Distributions

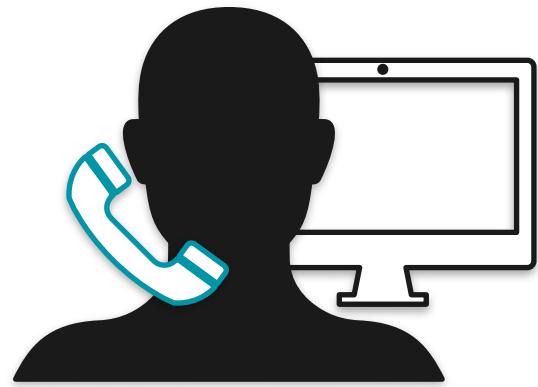


This is a continuous distribution!

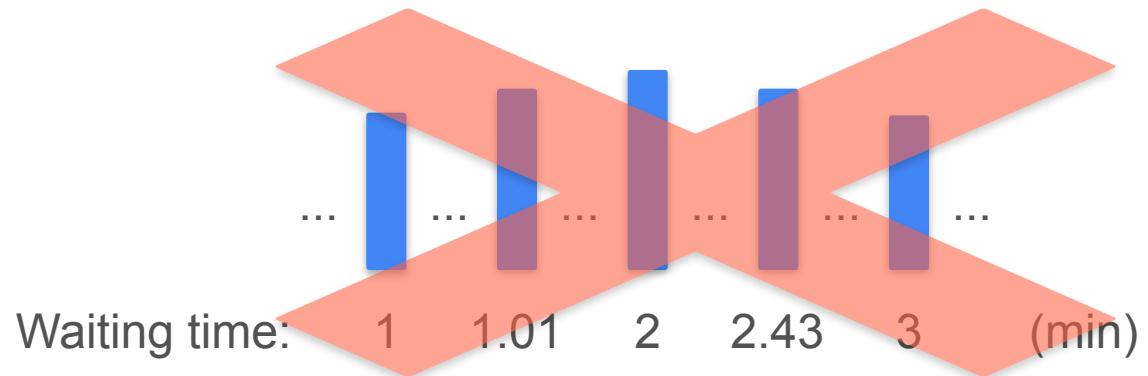


What is the probability that you will wait EXACTLY one minute for the call?

# From Discrete to Continuous Distributions



This is a continuous distribution!



What is the probability that you will wait EXACTLY one minute for the call?

Answer: ZERO

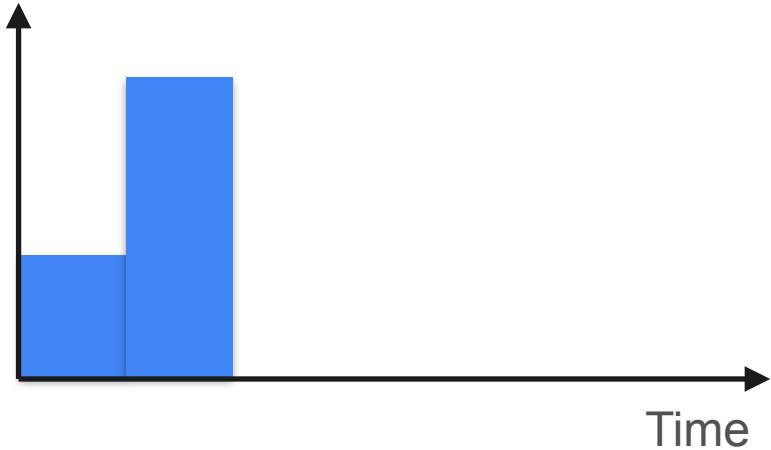
# From Discrete to Continuous Distributions



# From Discrete to Continuous Distributions



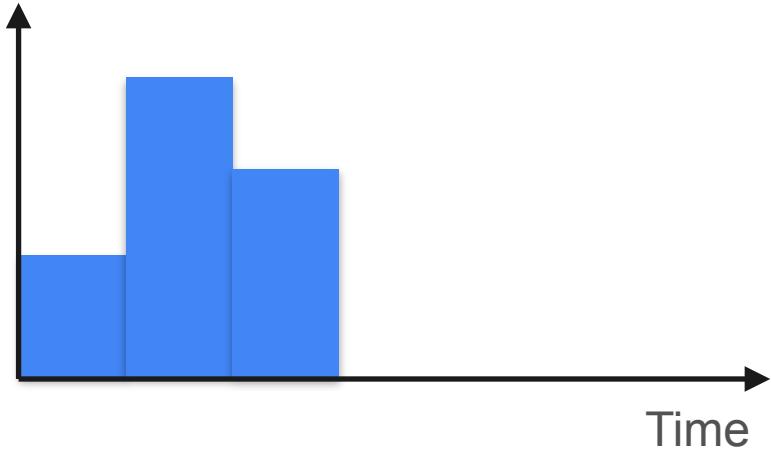
# From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

$P(\text{between 1 and 2 mins})$

# From Discrete to Continuous Distributions

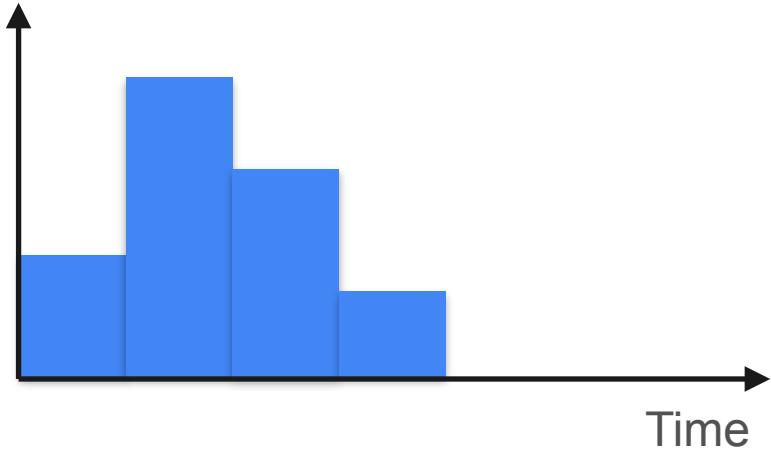


$P(\text{between 0 and 1 mins})$

$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

# From Discrete to Continuous Distributions



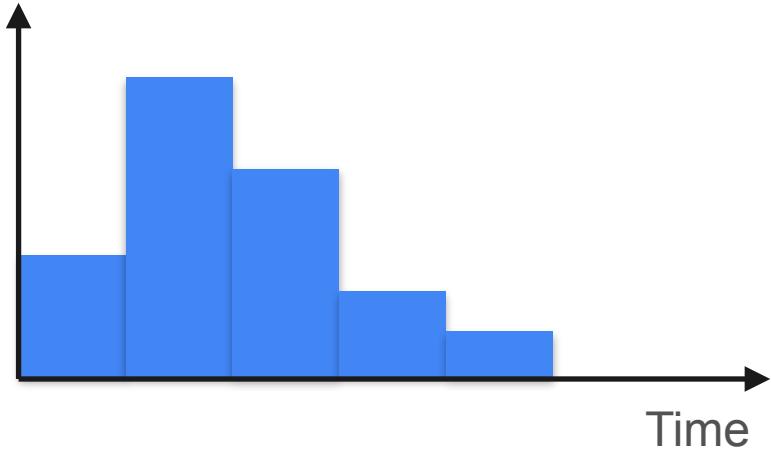
$P(\text{between 0 and 1 mins})$

$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

$P(\text{between 3 and 4 mins})$

# From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

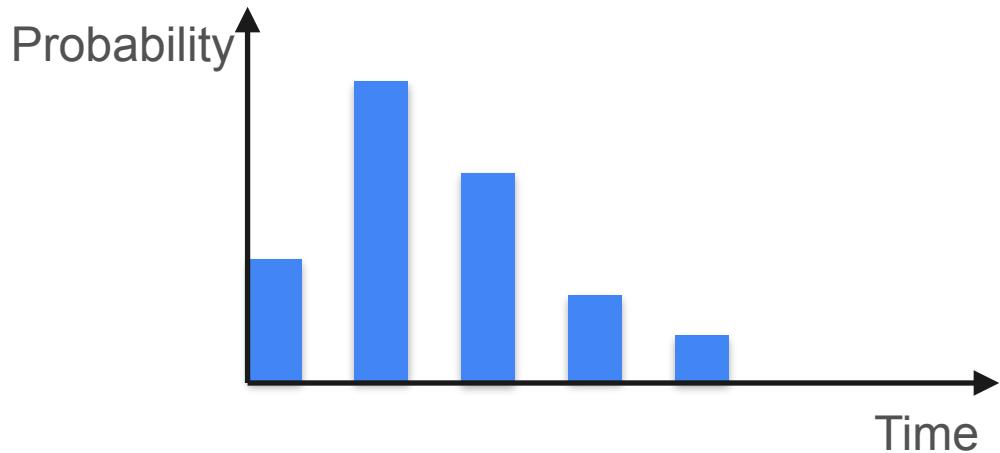
$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

$P(\text{between 3 and 4 mins})$

$P(\text{between 4 and 5 mins})$

# From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

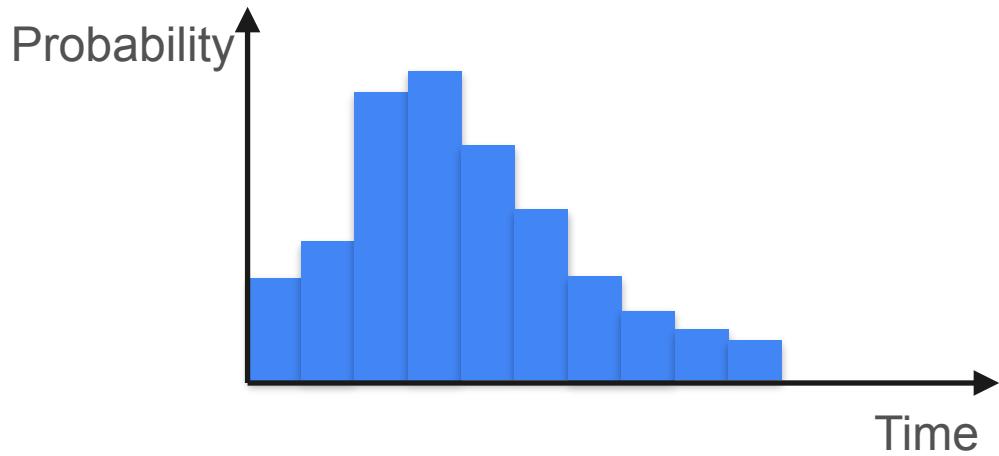
$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

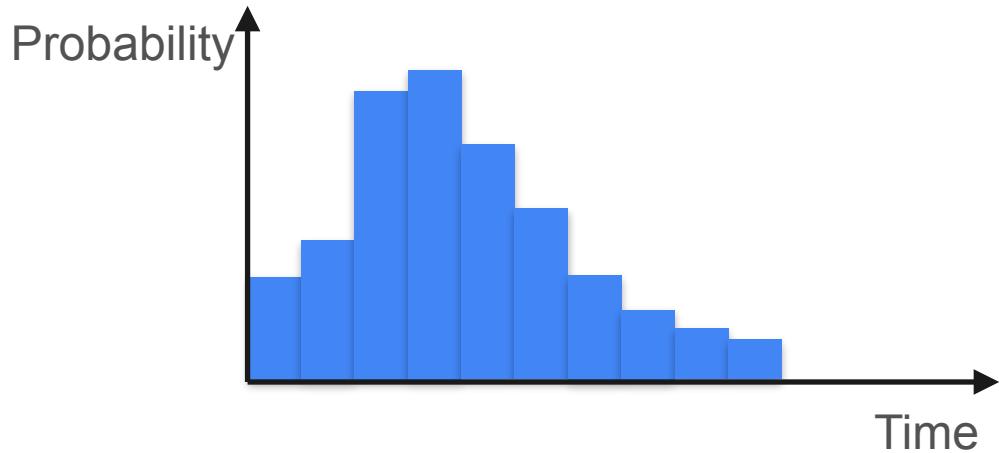
$P(\text{between 3 and 4 mins})$

$P(\text{between 4 and 5 mins})$

# From Discrete to Continuous Distributions

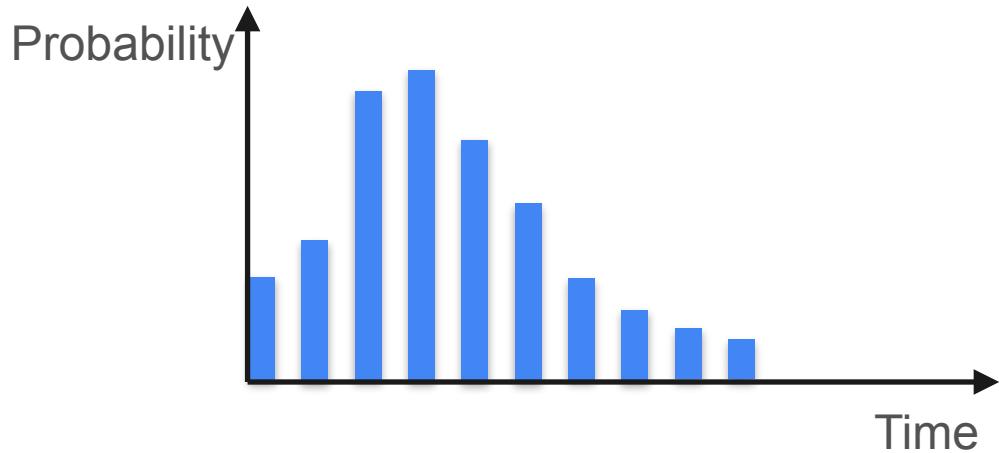


# From Discrete to Continuous Distributions



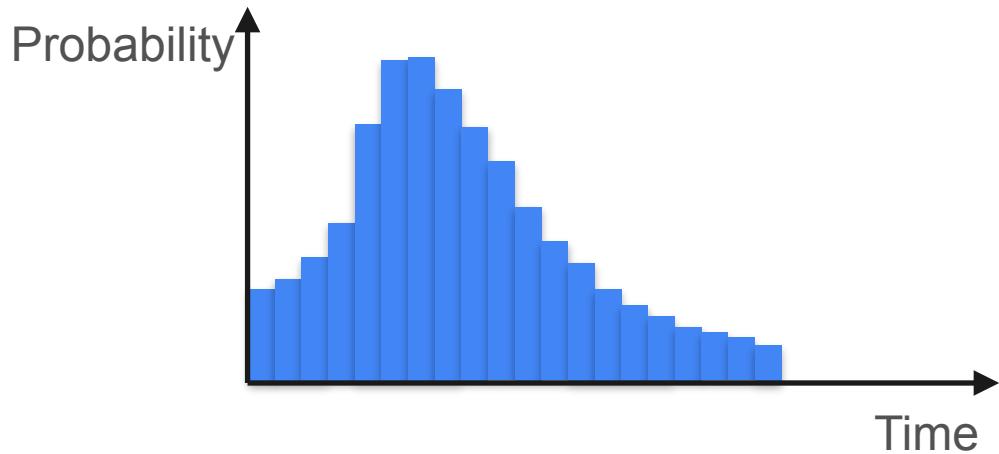
$P(\text{between } 0 \text{ and } 0.5 \text{ mins})$   
 $P(\text{between } 0.5 \text{ and } 1 \text{ mins})$   
 $P(\text{between } 1 \text{ and } 1.5 \text{ mins})$   
⋮  
 $P(\text{between } 3.5 \text{ and } 4 \text{ mins})$   
 $P(\text{between } 4 \text{ and } 4.5 \text{ mins})$   
 $P(\text{between } 4.5 \text{ and } 5 \text{ mins})$

# From Discrete to Continuous Distributions

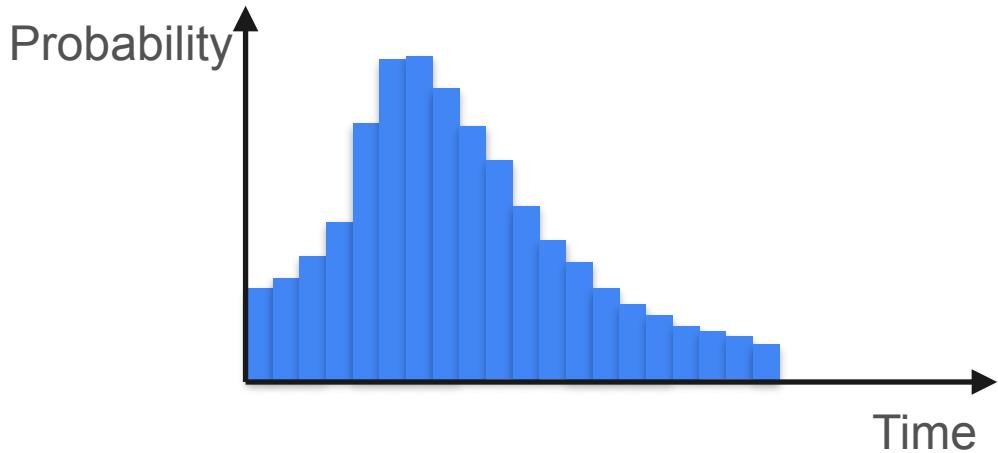


$P(\text{between } 0 \text{ and } 0.5 \text{ mins})$   
 $P(\text{between } 0.5 \text{ and } 1 \text{ mins})$   
 $P(\text{between } 1 \text{ and } 1.5 \text{ mins})$   
⋮  
 $P(\text{between } 3.5 \text{ and } 4 \text{ mins})$   
 $P(\text{between } 4 \text{ and } 4.5 \text{ mins})$   
 $P(\text{between } 4.5 \text{ and } 5 \text{ mins})$

# From Discrete to Continuous Distributions

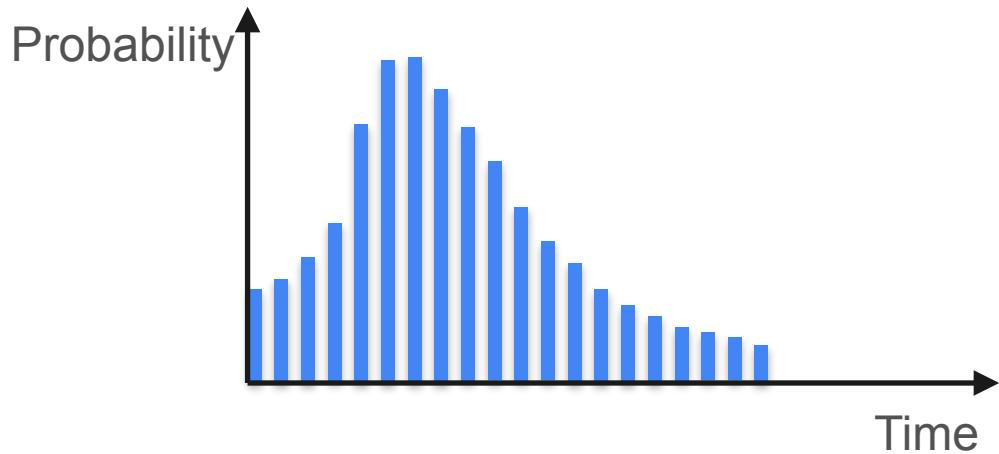


# From Discrete to Continuous Distributions

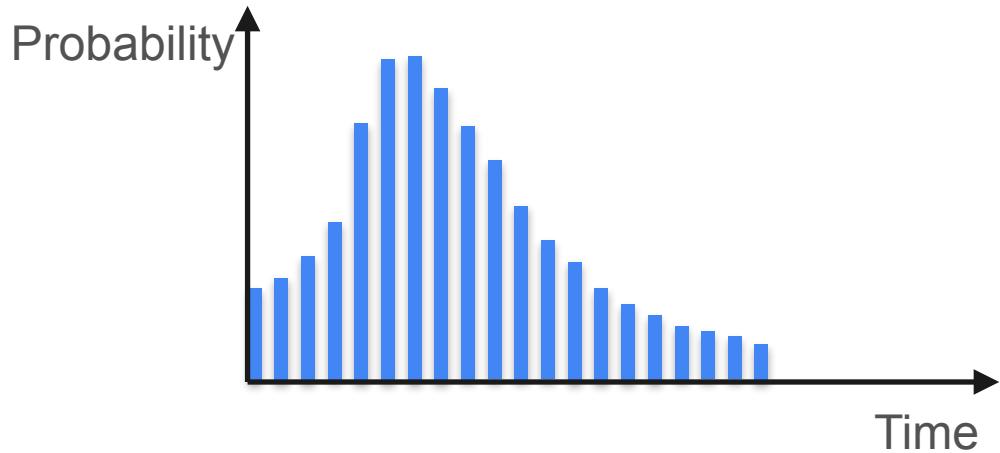


$P(\text{between } 0 \text{ and } 0.25 \text{ mins})$   
 $P(\text{between } 0.25 \text{ and } 0.5 \text{ mins})$   
 $P(\text{between } 0.5 \text{ and } 0.75 \text{ mins})$   
⋮  
 $P(\text{between } 4.25 \text{ and } 4.5 \text{ mins})$   
 $P(\text{between } 4.5 \text{ and } 4.75 \text{ mins})$   
 $P(\text{between } 4.75 \text{ and } 5 \text{ mins})$

# From Discrete to Continuous Distributions

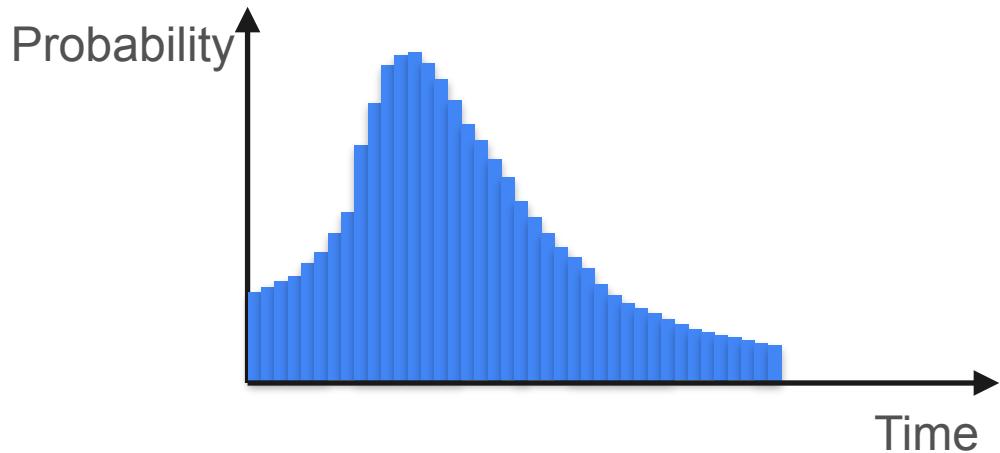


# From Discrete to Continuous Distributions



$P(\text{between } 0 \text{ and } 0.25 \text{ mins})$   
 $P(\text{between } 0.25 \text{ and } 0.5 \text{ mins})$   
 $P(\text{between } 0.5 \text{ and } 0.75 \text{ mins})$   
⋮  
 $P(\text{between } 4.25 \text{ and } 4.5 \text{ mins})$   
 $P(\text{between } 4.5 \text{ and } 4.75 \text{ mins})$   
 $P(\text{between } 4.75 \text{ and } 5 \text{ mins})$

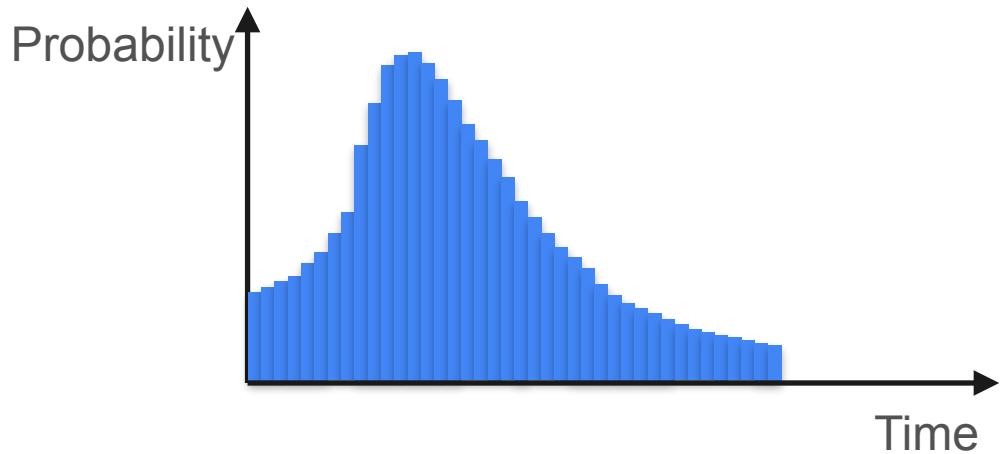
# From Discrete to Continuous Distributions



$P(\text{between } 0 \text{ and } 0.125 \text{ mins})$

$P(\text{between } 4.875 \text{ and } 5 \text{ mins})$

# From Discrete to Continuous Distributions

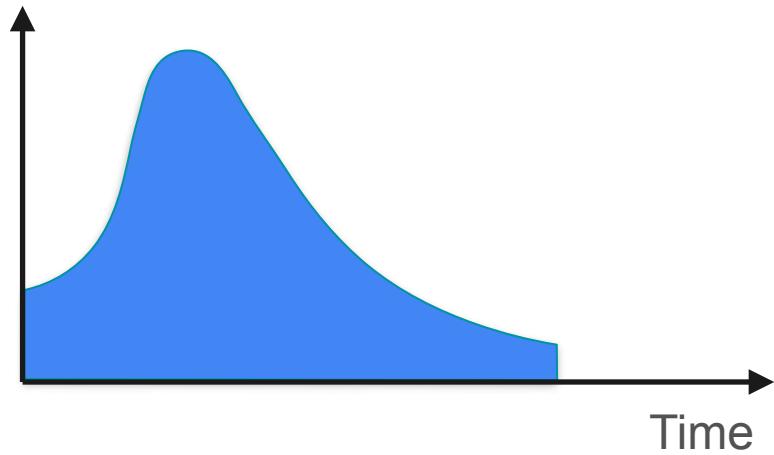


$P(\text{between } 0 \text{ and } 0.125 \text{ mins})$

⋮

$P(\text{between } 4.875 \text{ and } 5 \text{ mins})$

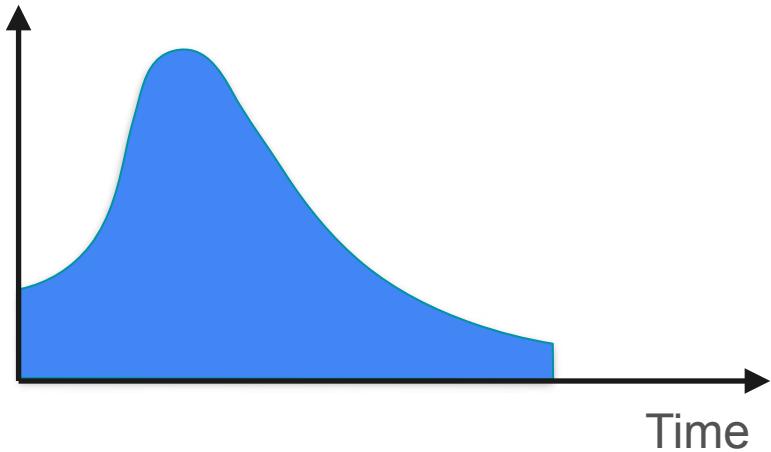
# From Discrete to Continuous Distributions



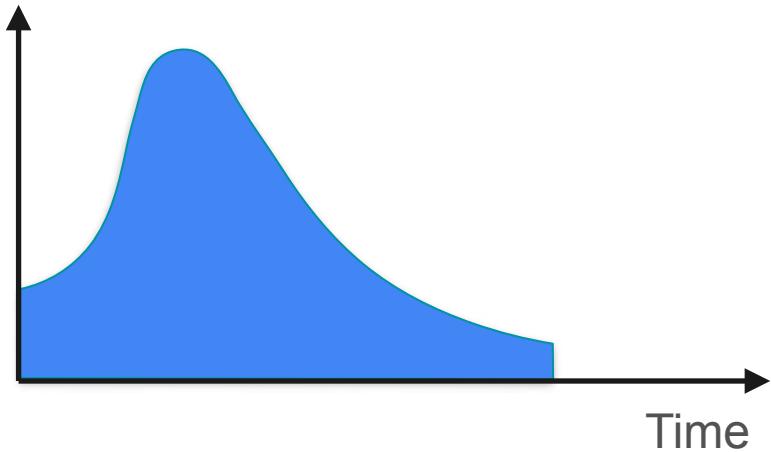
# From Discrete to Continuous Distributions



- Discrete:
  - Sum of heights equals 1

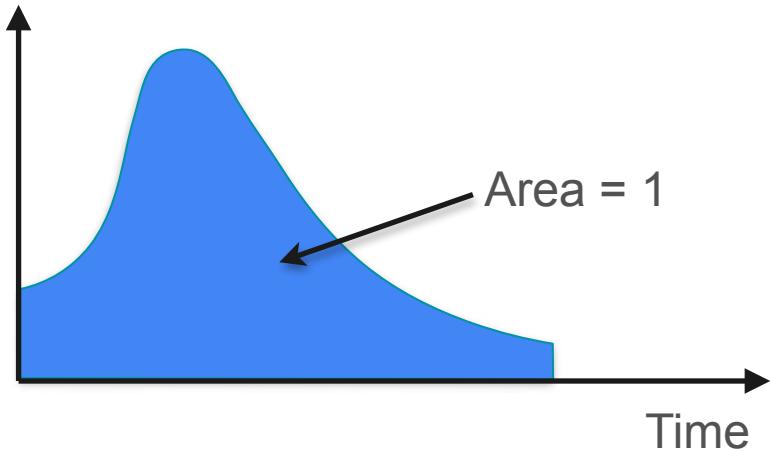


# From Discrete to Continuous Distributions



- Discrete:
  - Sum of heights equals 1
- Continuous:
  - Area under the curve equals 1

# From Discrete to Continuous Distributions



- Discrete:
  - Sum of heights equals 1
- Continuous:
  - Area under the curve equals 1



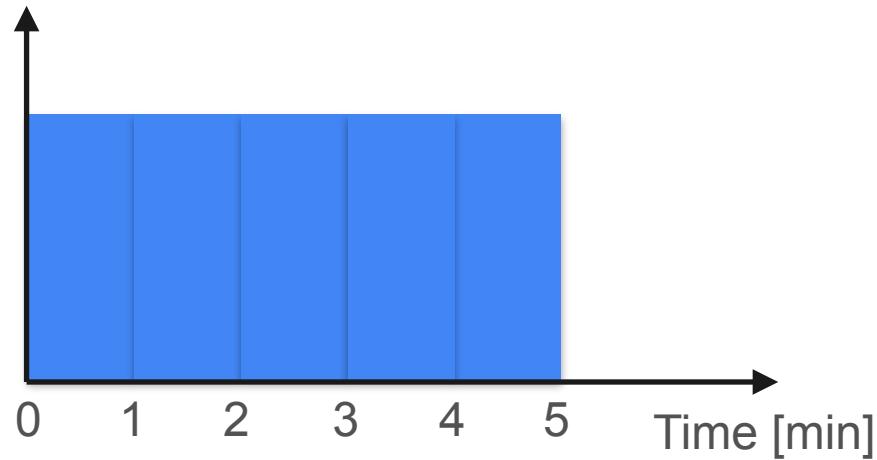
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# Probability Distributions

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## Probability density function

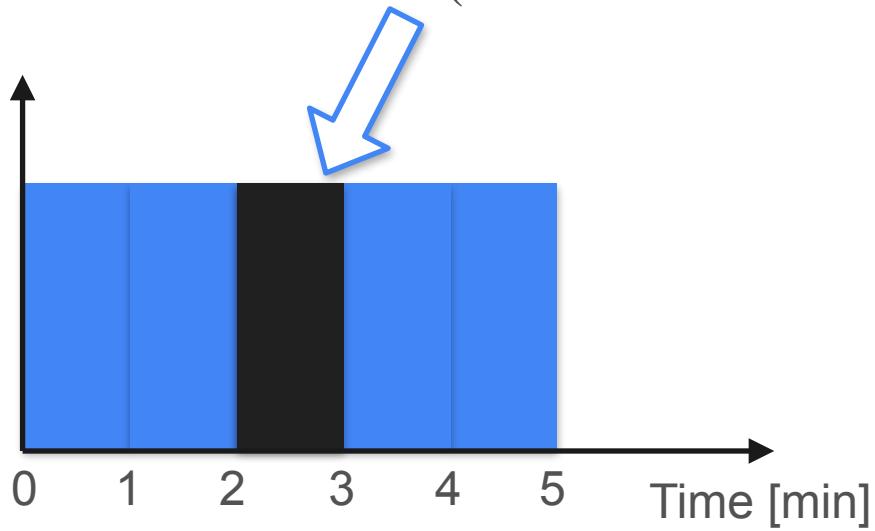
# Probability Density Function



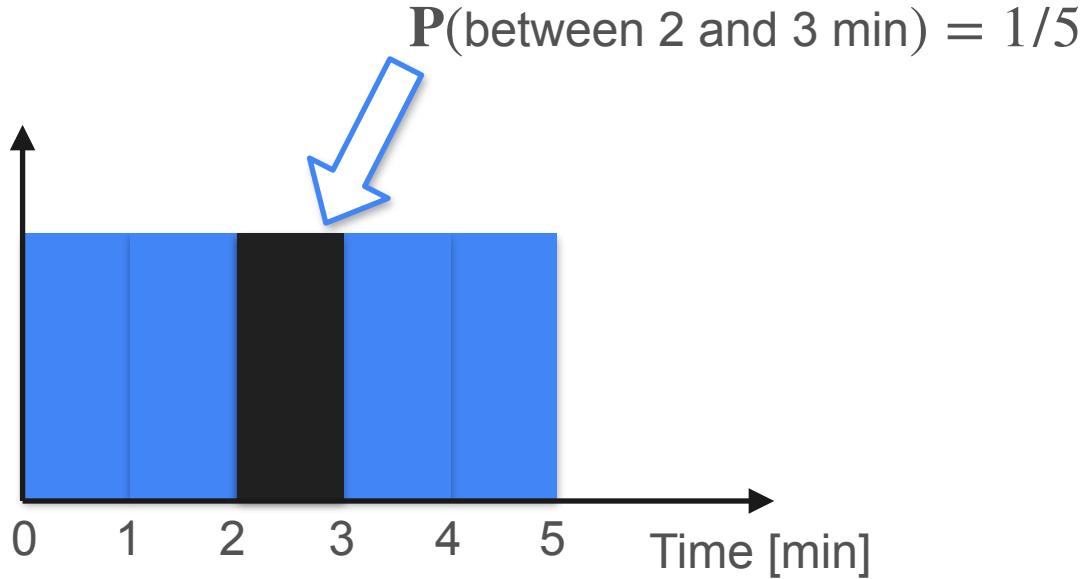
# Probability Density Function



$P(\text{between 2 and 3 min}) = ?$



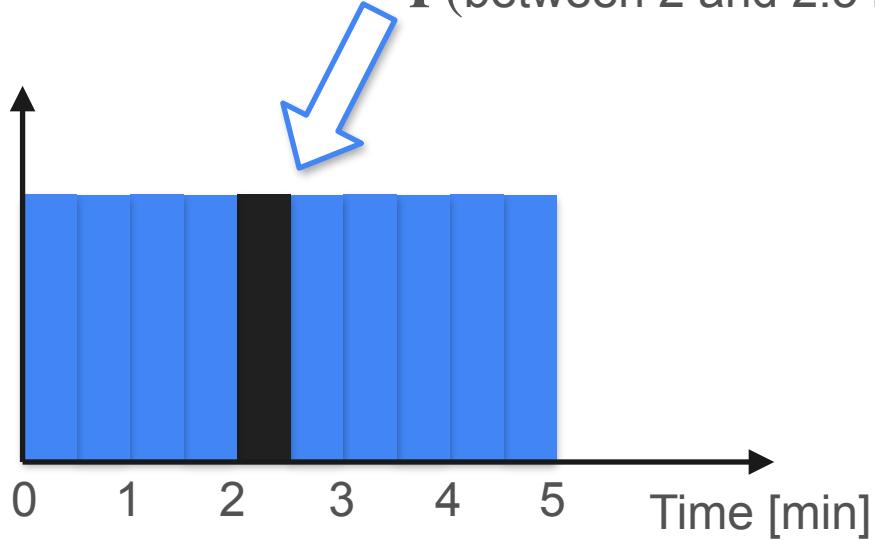
# Probability Density Function



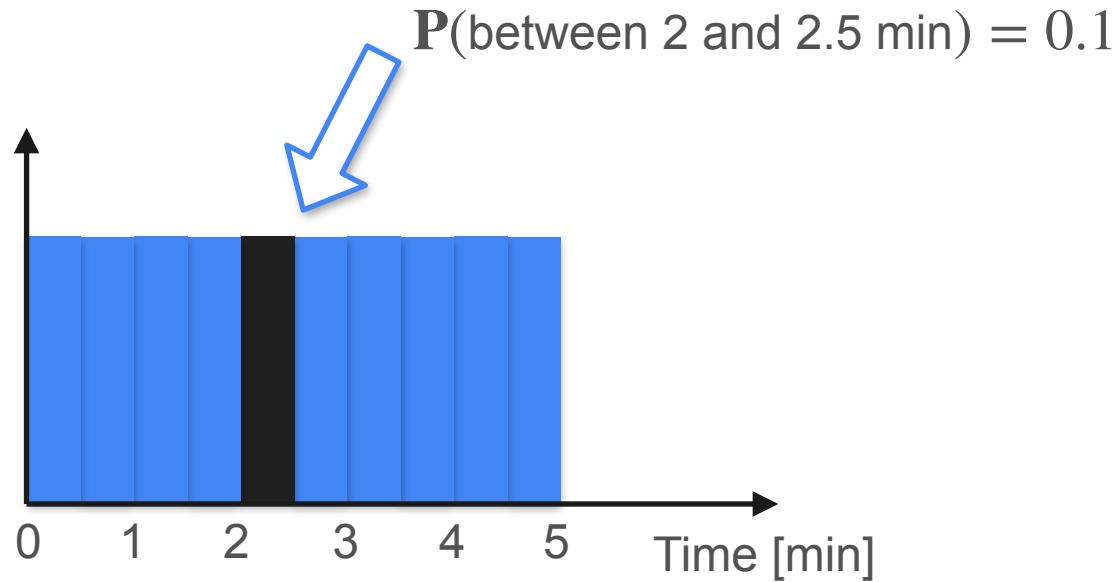
# Probability Density Function



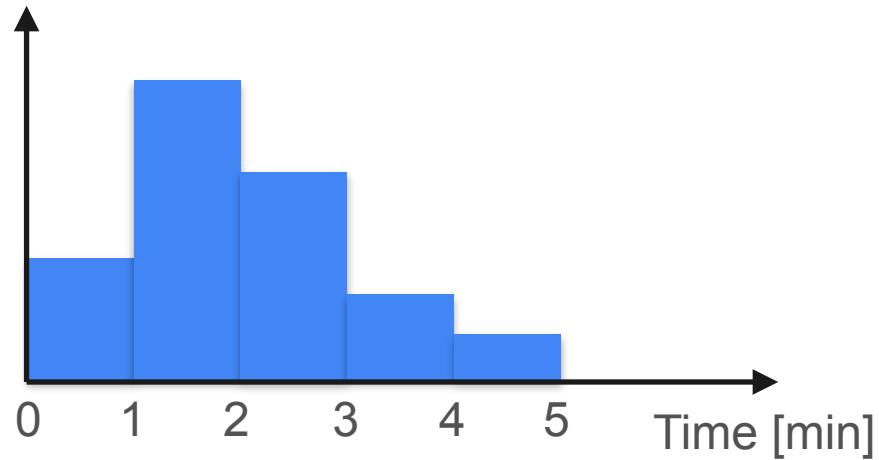
$P(\text{between 2 and 2.5 min}) = ?$



# Probability Density Function



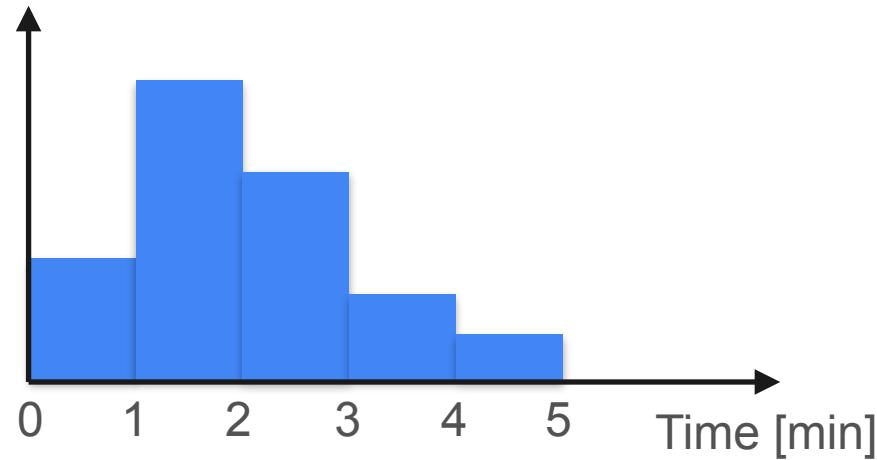
# Probability Density Function



# Probability Density Function



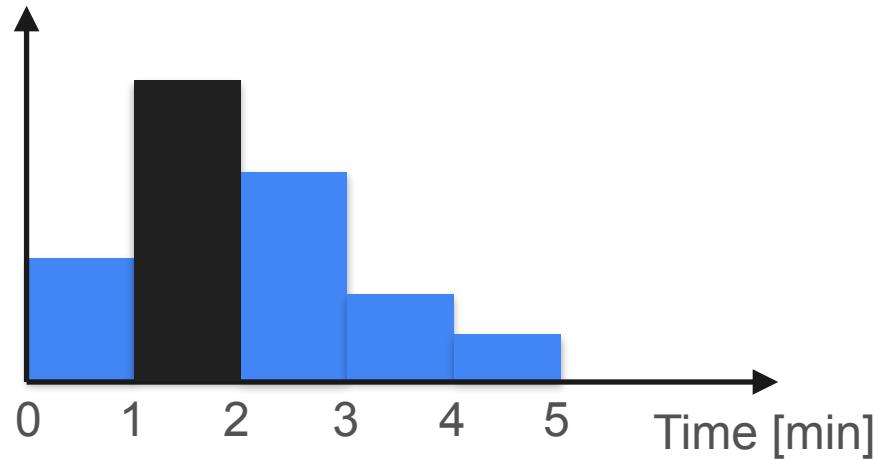
$P(\text{between 1 and 2 min})$



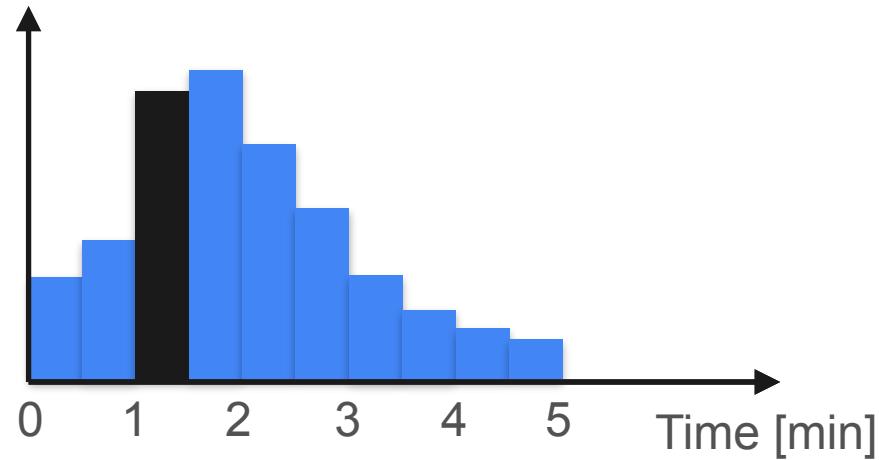
# Probability Density Function



$P(\text{between 1 and 2 min})$



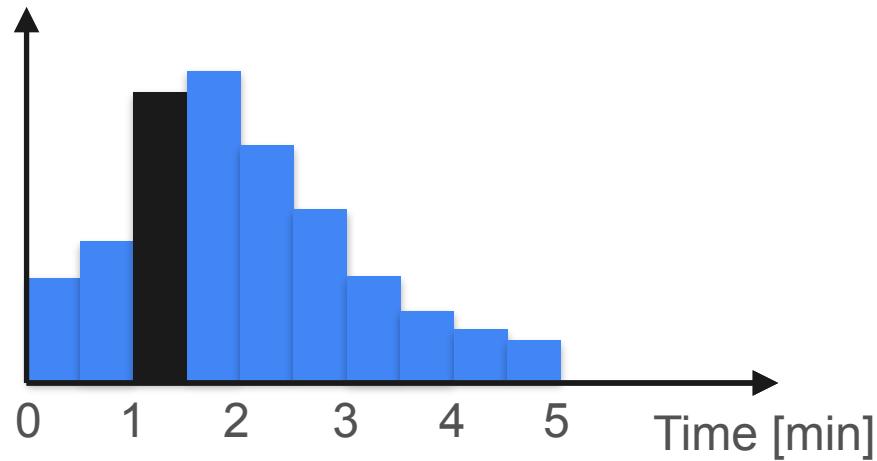
# Probability Density Function



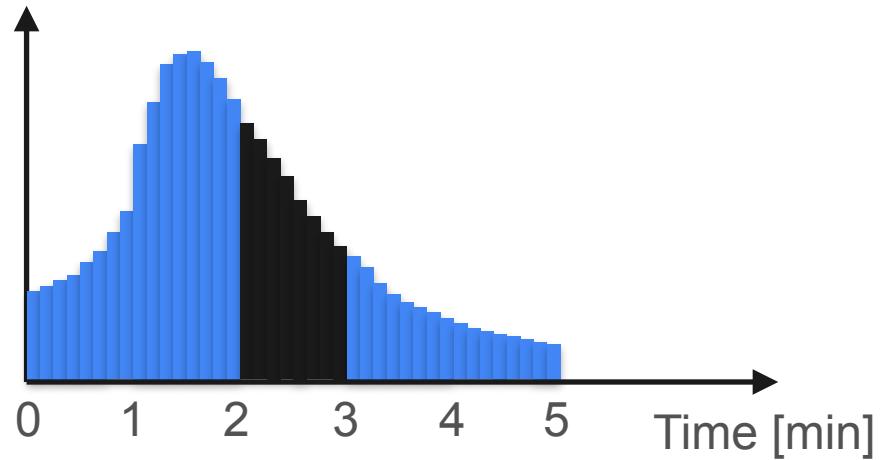
# Probability Density Function



$P(\text{between 1 and 1:30})$



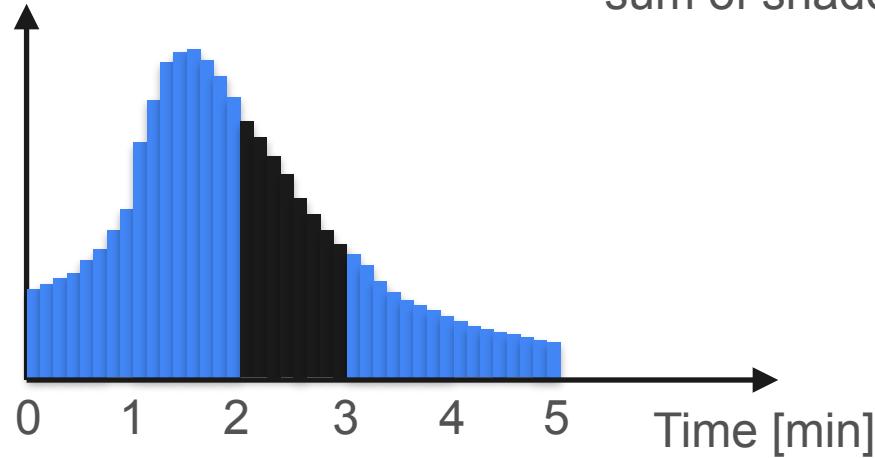
# Probability Density Function



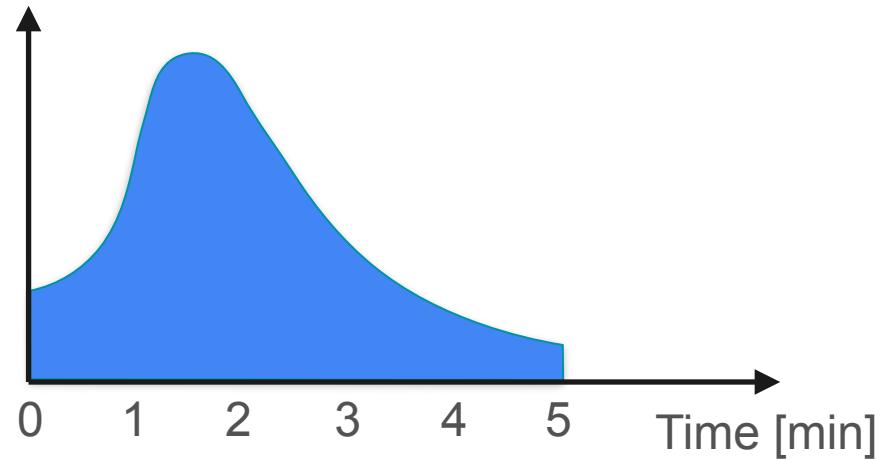
# Probability Density Function



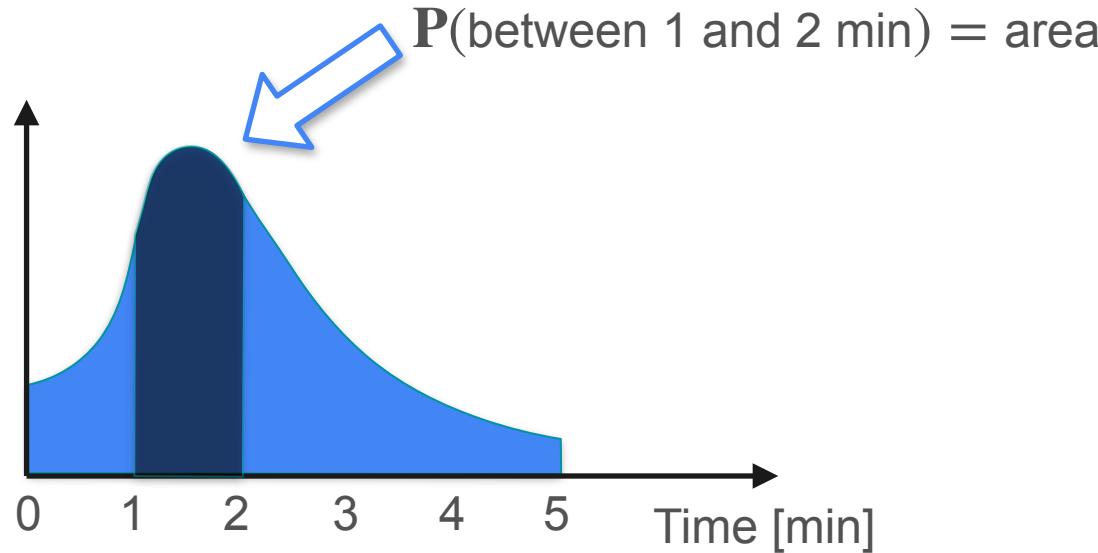
$P(\text{between 2 and 3 min}) =$   
sum of shaded areas



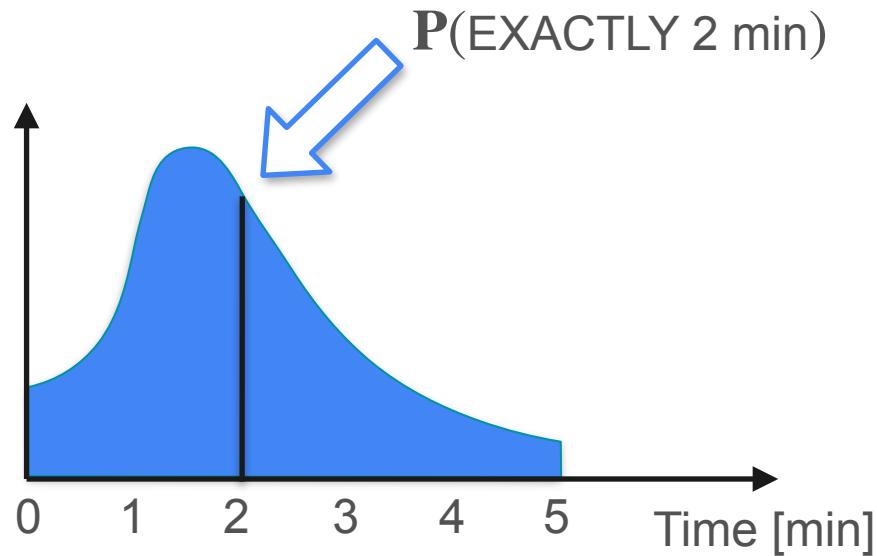
# Probability Density Function



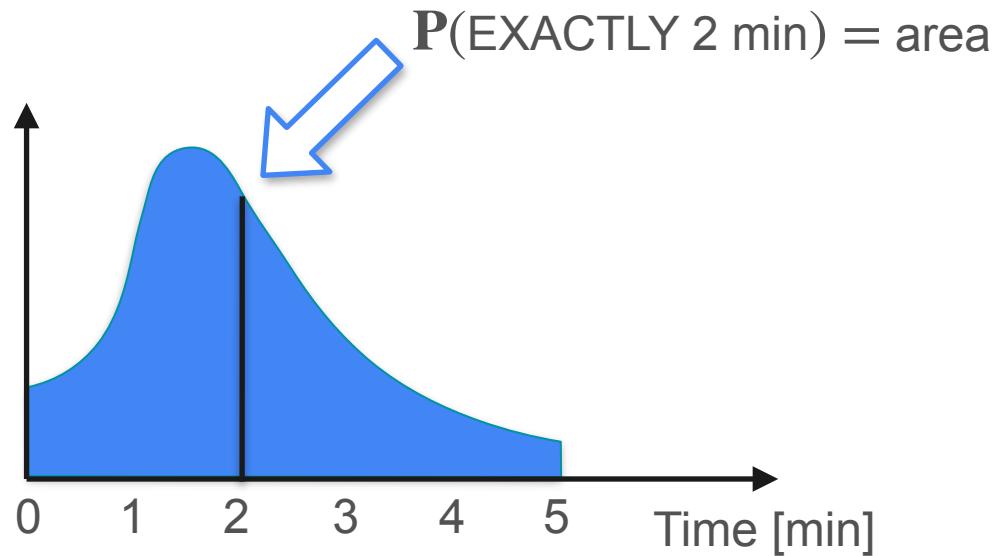
# Probability Density Function



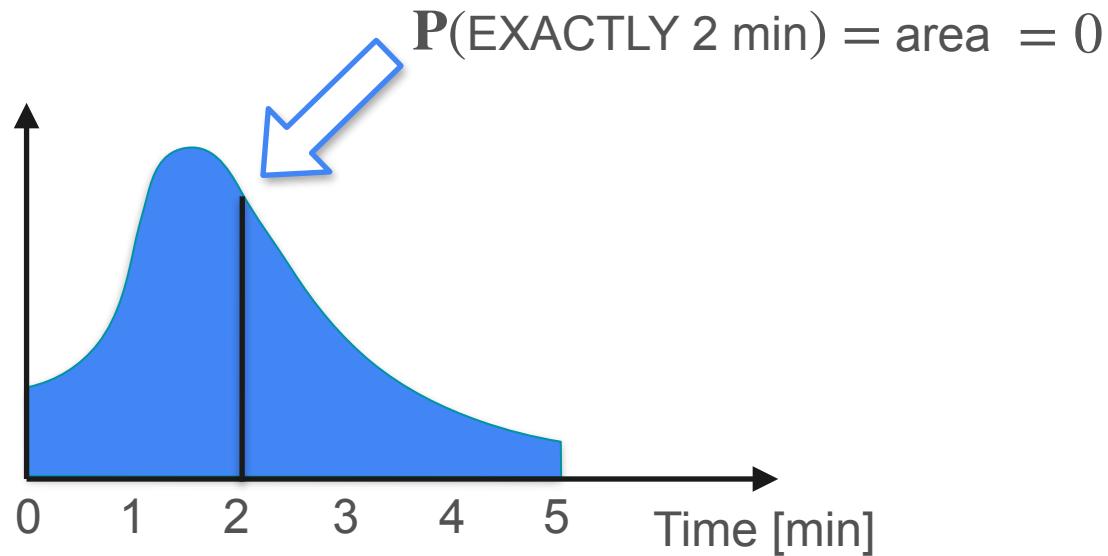
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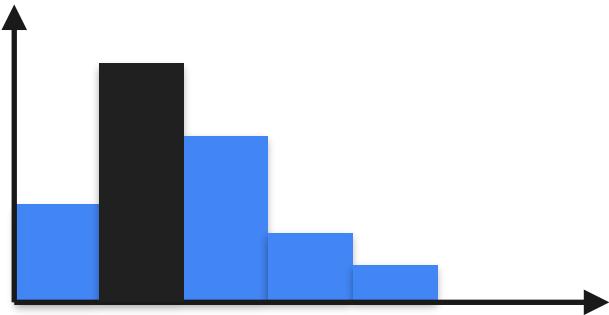


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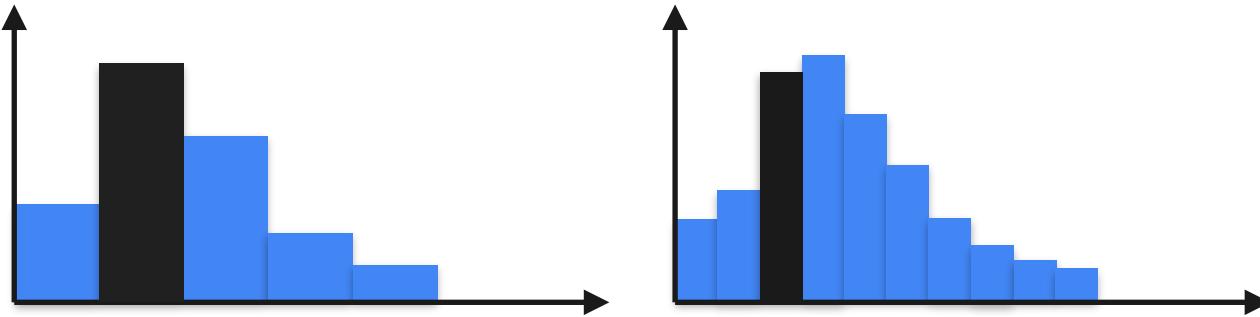


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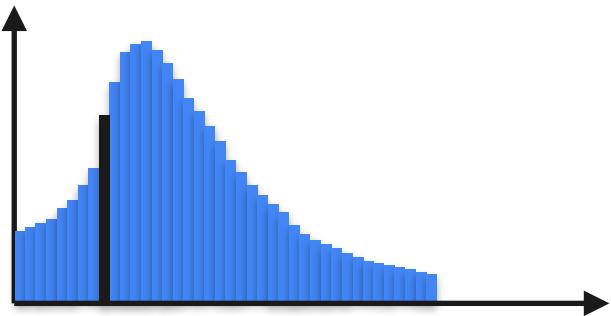
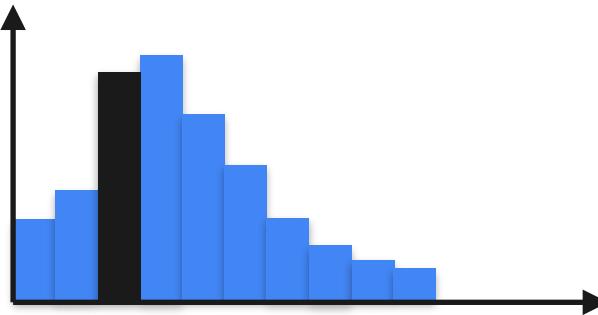
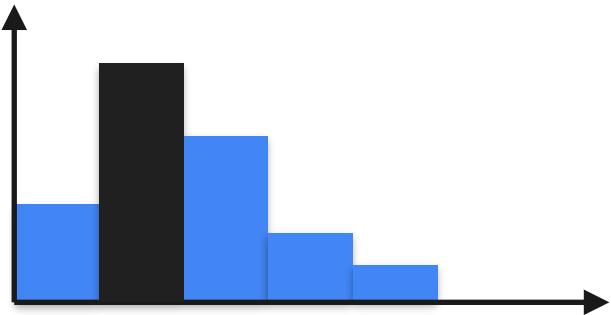
# Probability Density Function



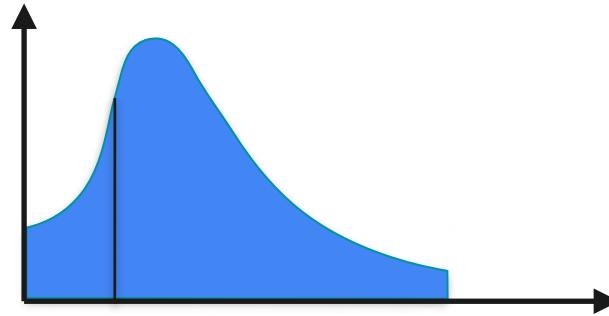
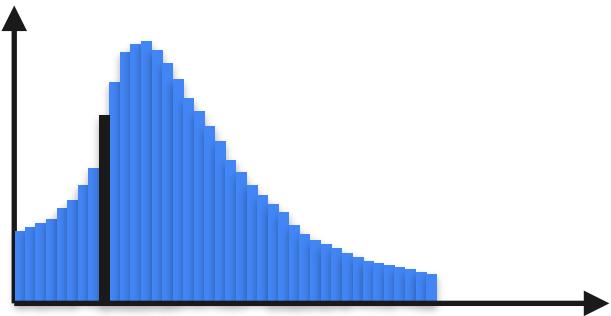
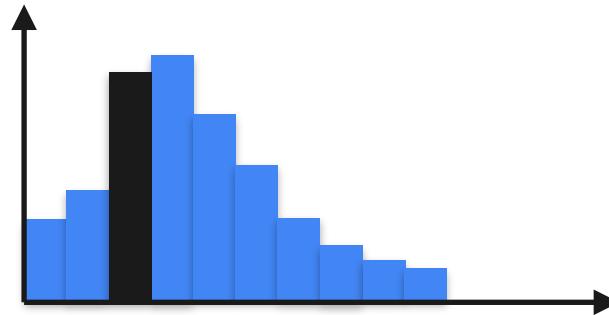
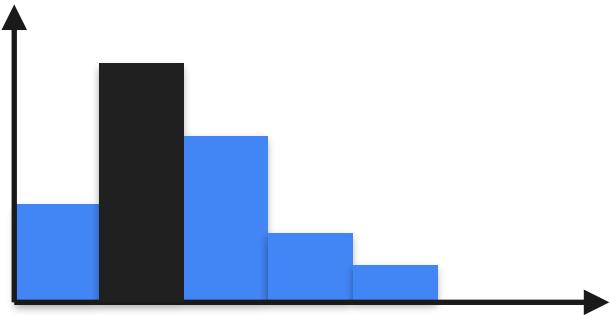
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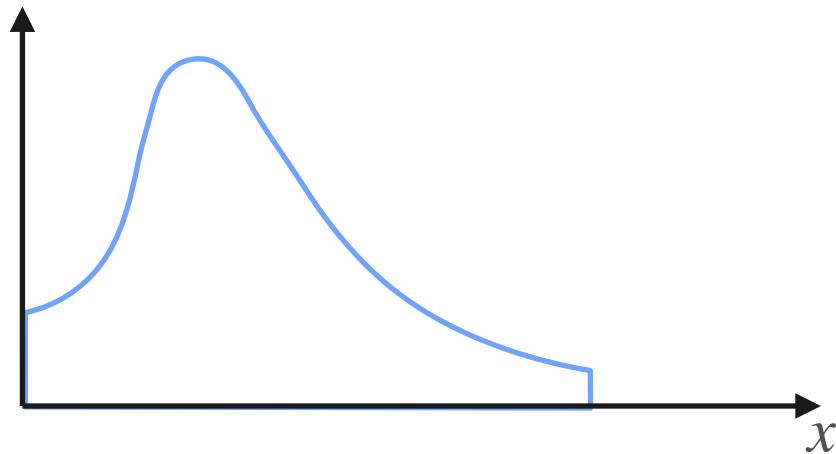


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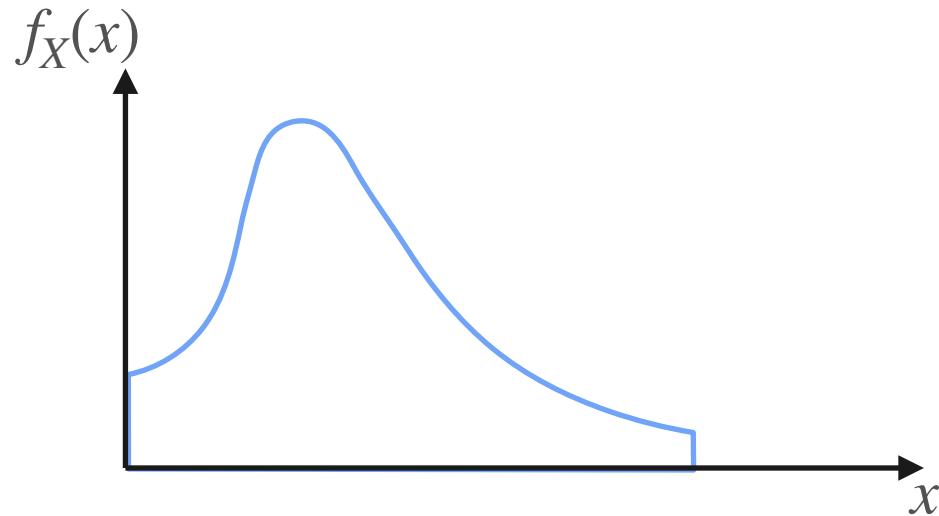


# Probability Density Function: Formal Definition

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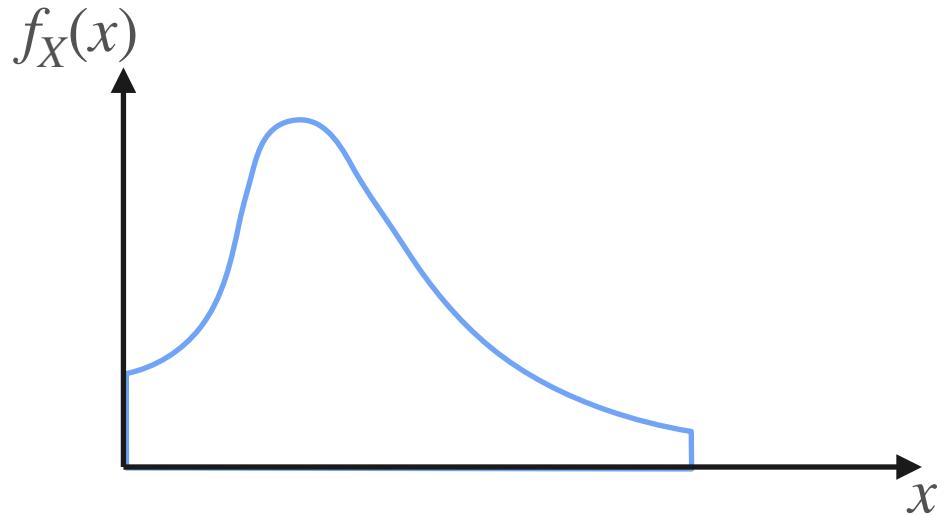


# Probability Density Function: Formal Definition



Probability Density Function (PDF)  
 $f_X(x)$

# Probability Density Function: Formal Definition

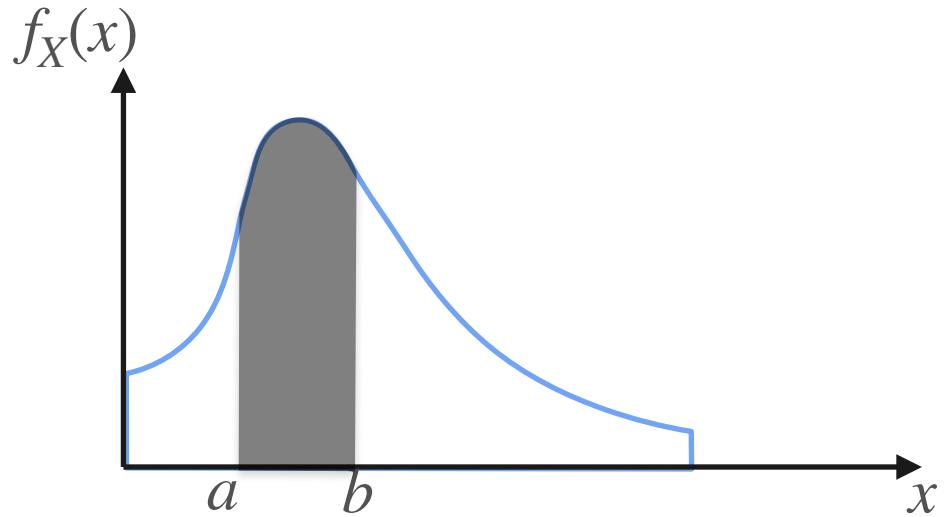


Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.  
**Only defined for continuous variables!**

# Probability Density Function: Formal Definition



Probability Density Function (PDF)

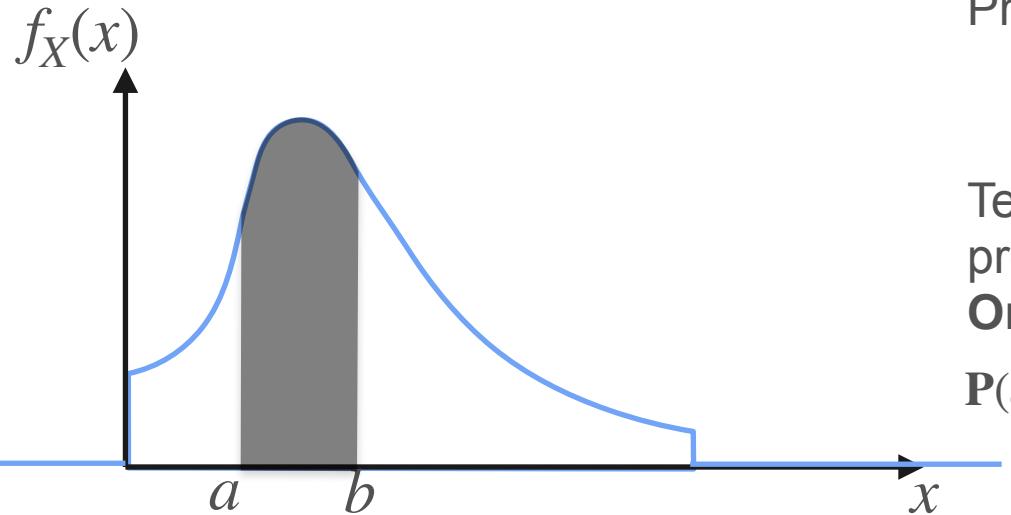
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$P(a < X < b) = \text{area under } f_X(x)$

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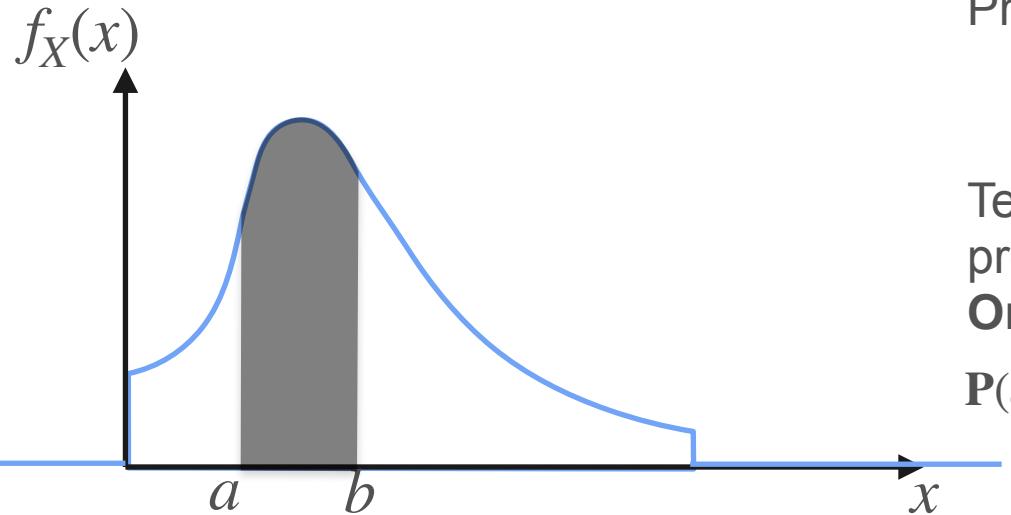
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$f_X(x)$  needs to satisfy:

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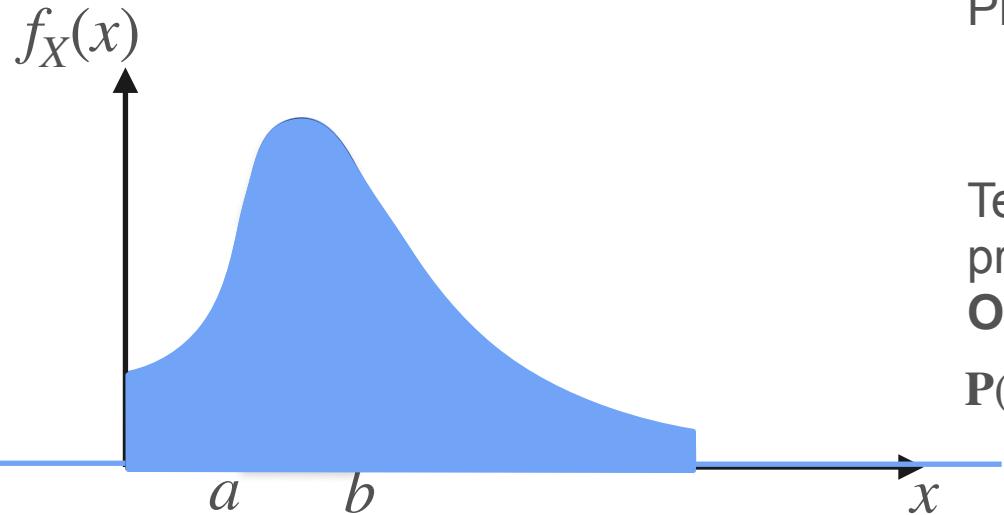
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- $f_X(x) \geq 0$

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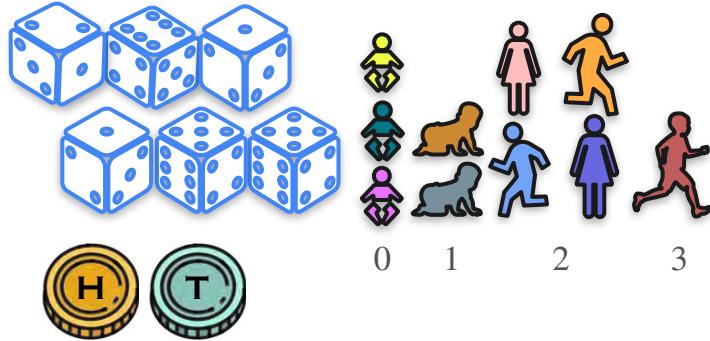
$f_X(x)$  needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$
- Area under  $f_X(x) = 1$

# Discrete and Continuous Random Variables

# Discrete and Continuous Random Variables

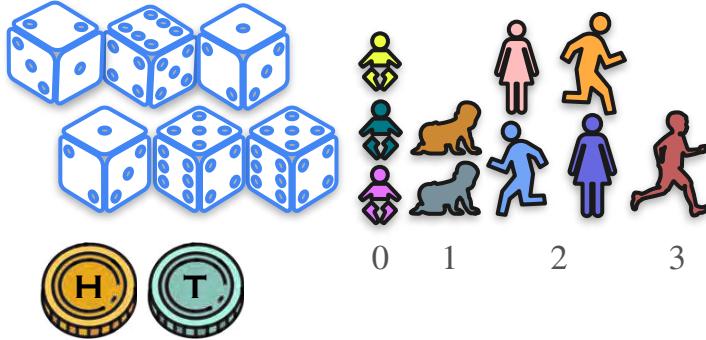
## Discrete random variables



Can take only a **finite** or at most countable number of values

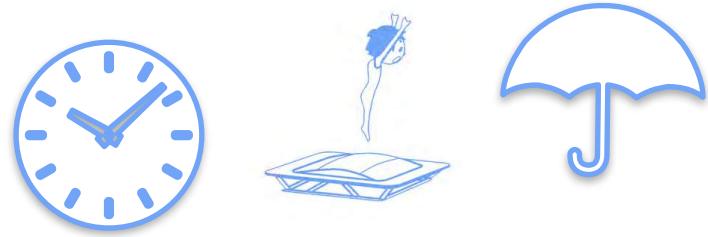
# Discrete and Continuous Random Variables

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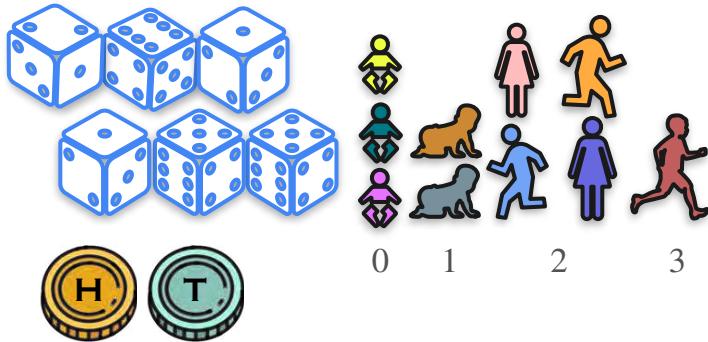
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Takes values on an interval  
(infinite possibilities!)

# Discrete and Continuous Random Variables

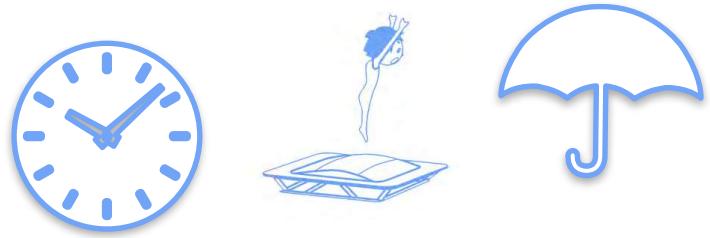
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$$\text{PMF: } p_X(x) = \mathbf{P}(X = x)$$

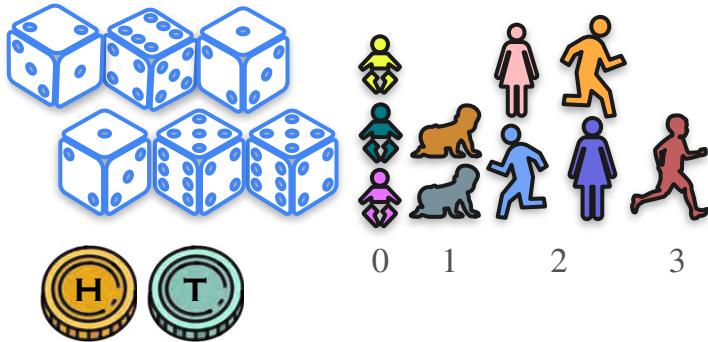
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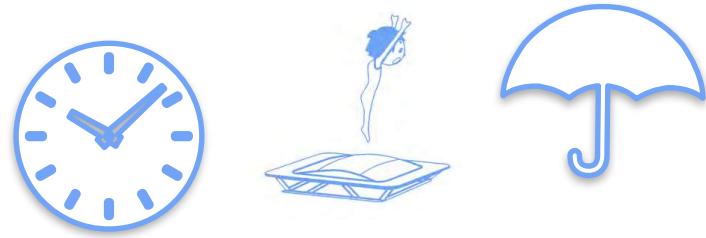
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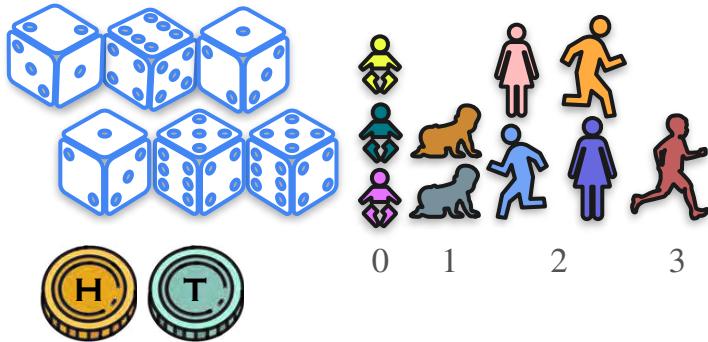


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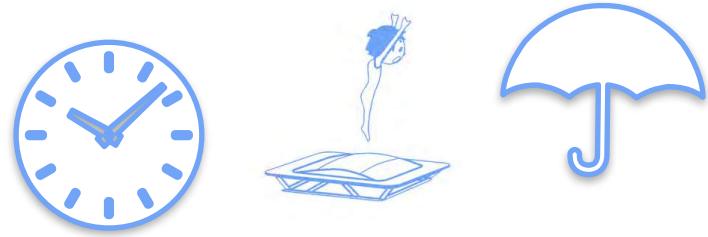
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## Continuous random variables



Takes values on an interval  
(infinite possibilities!)

$$\begin{aligned} \text{PDF: } & f_X(x) \\ \mathbf{P}(X = x) &= 0 \quad \forall x \end{aligned}$$



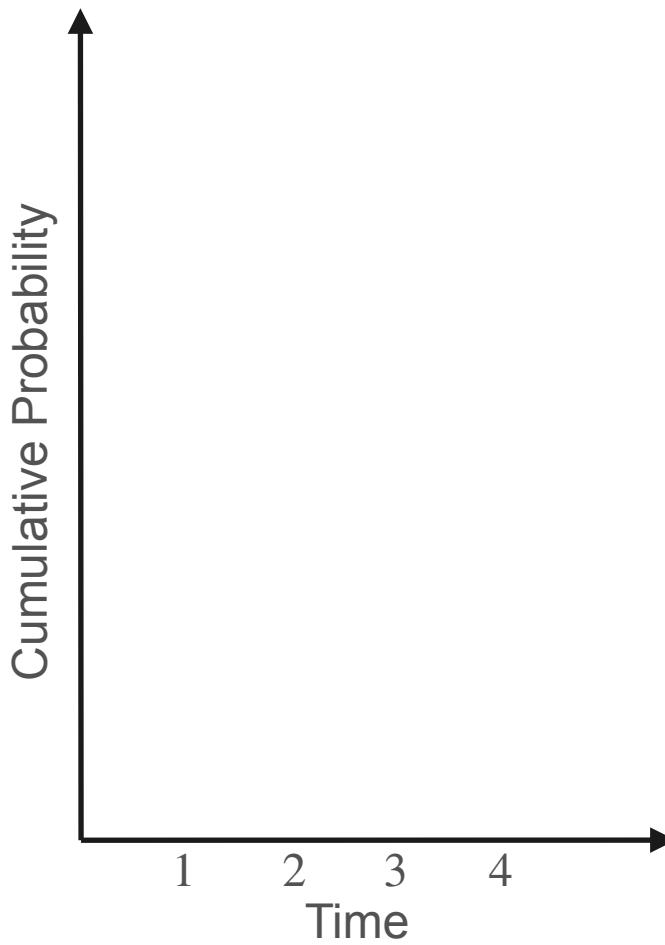
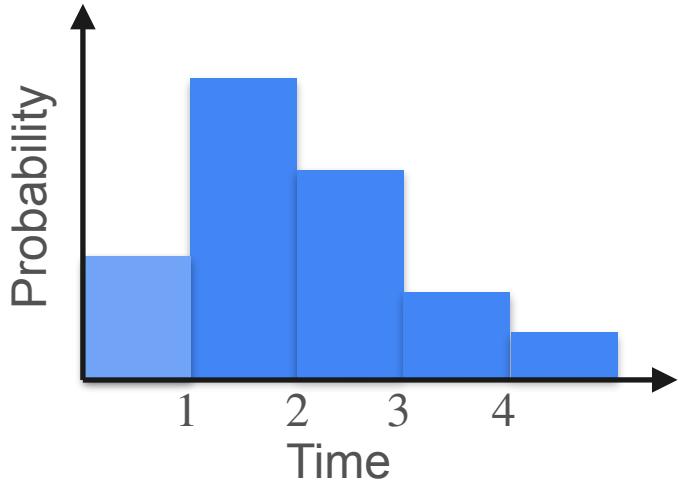
DeepLearning.AI

# Probability Distributions

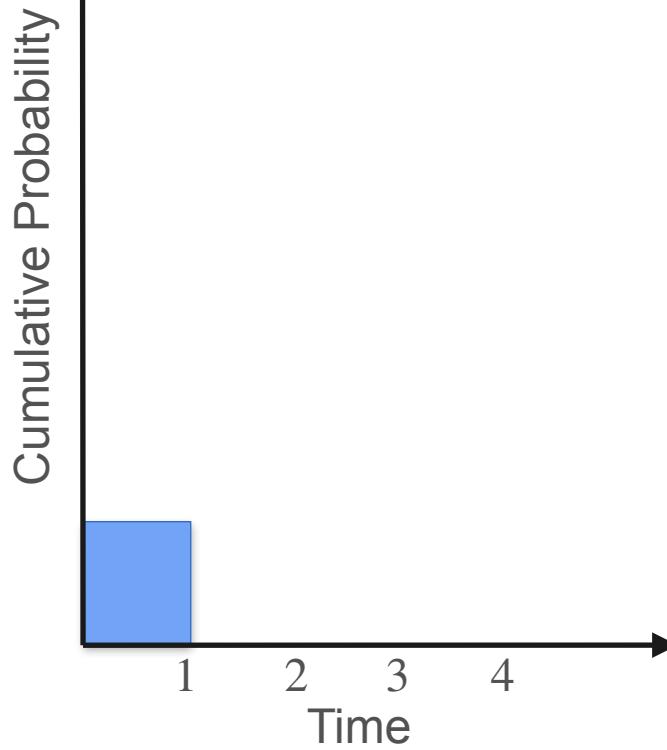
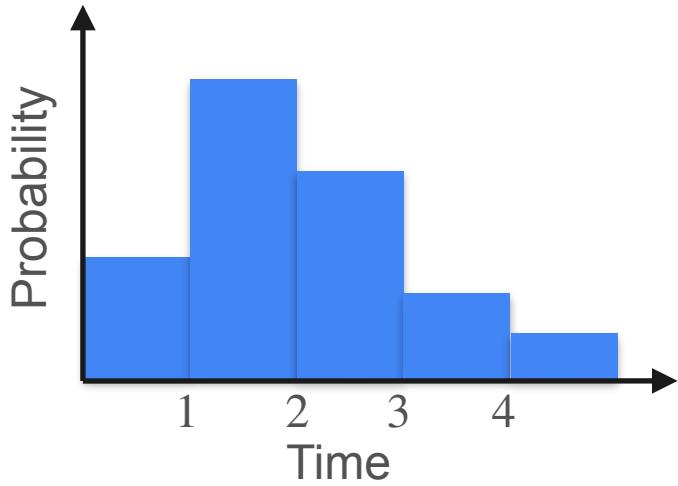
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## Cumulative Distribution Function

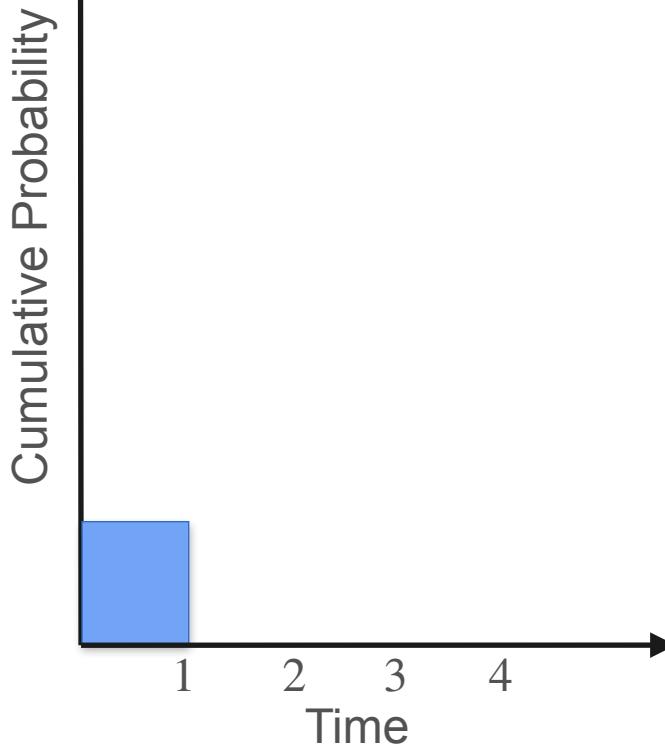
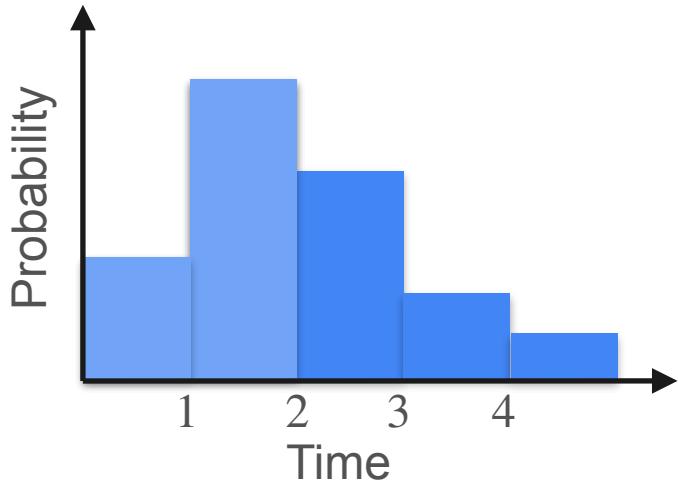
# Cumulative Distribution



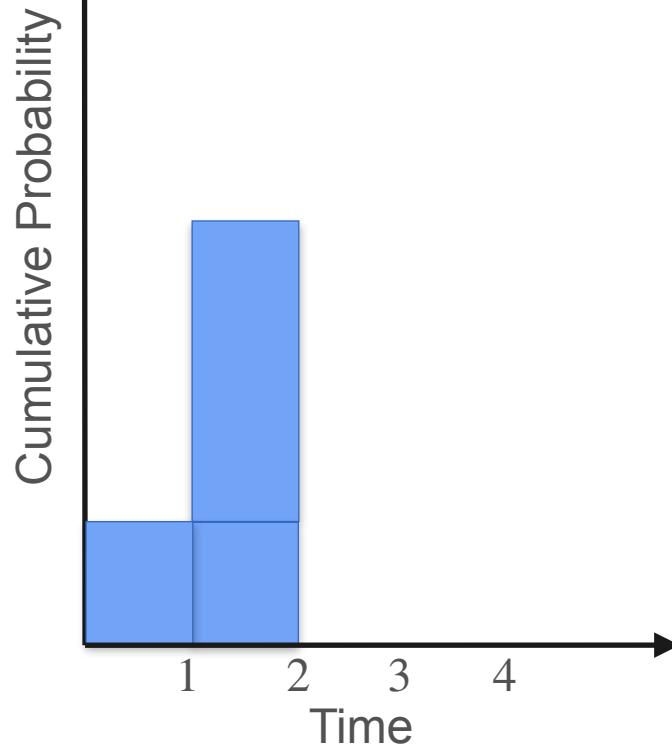
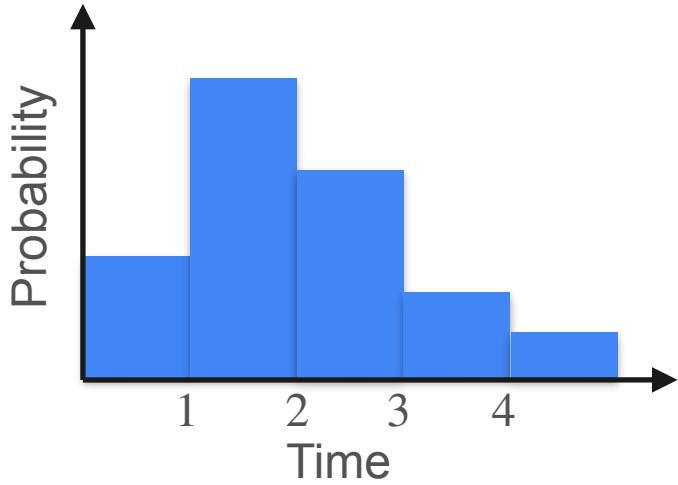
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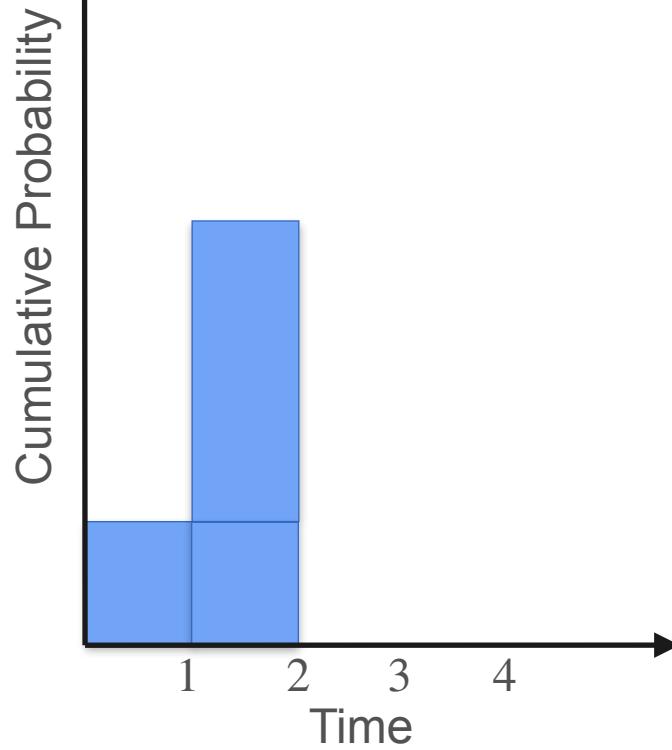
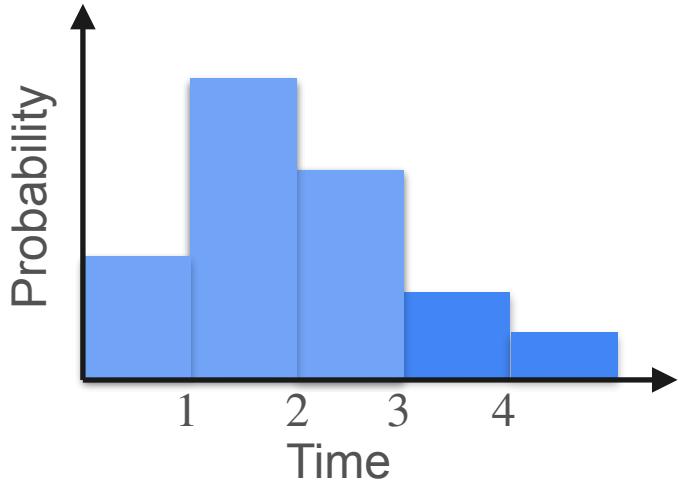
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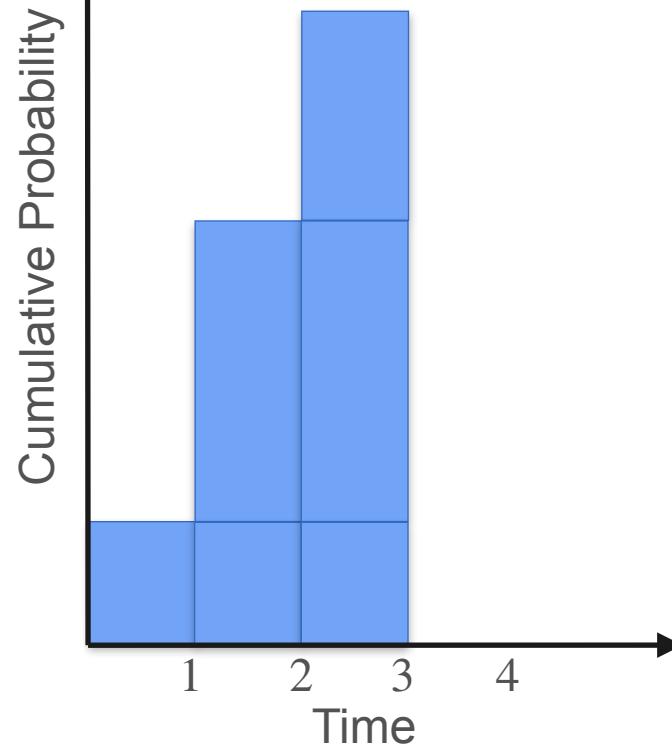
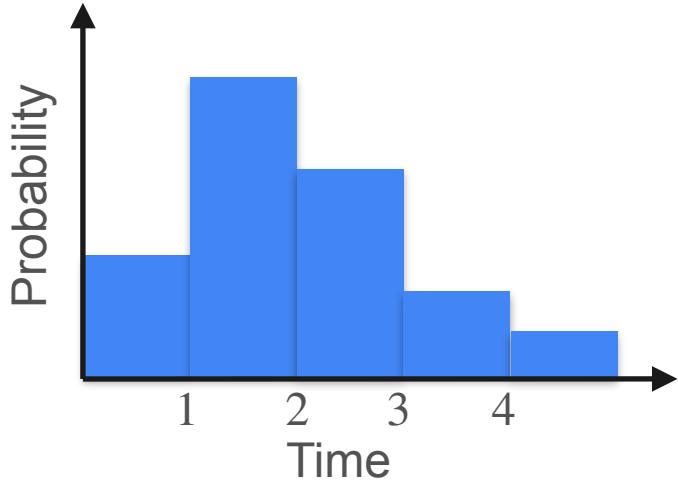
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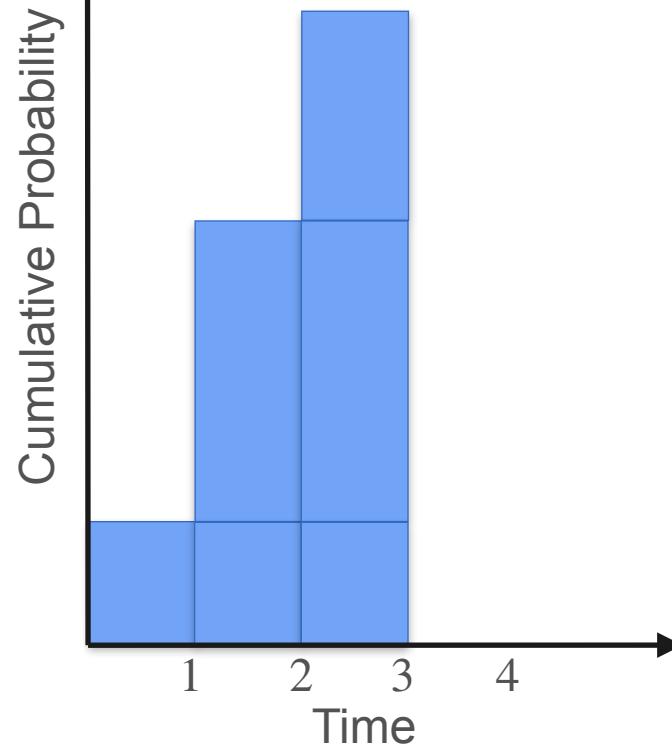
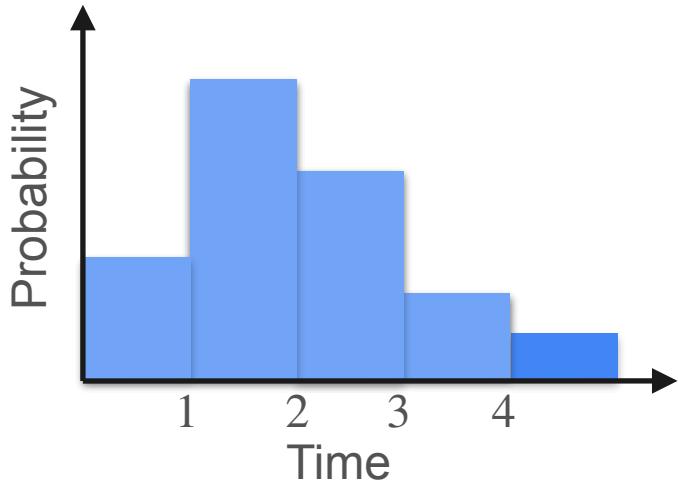
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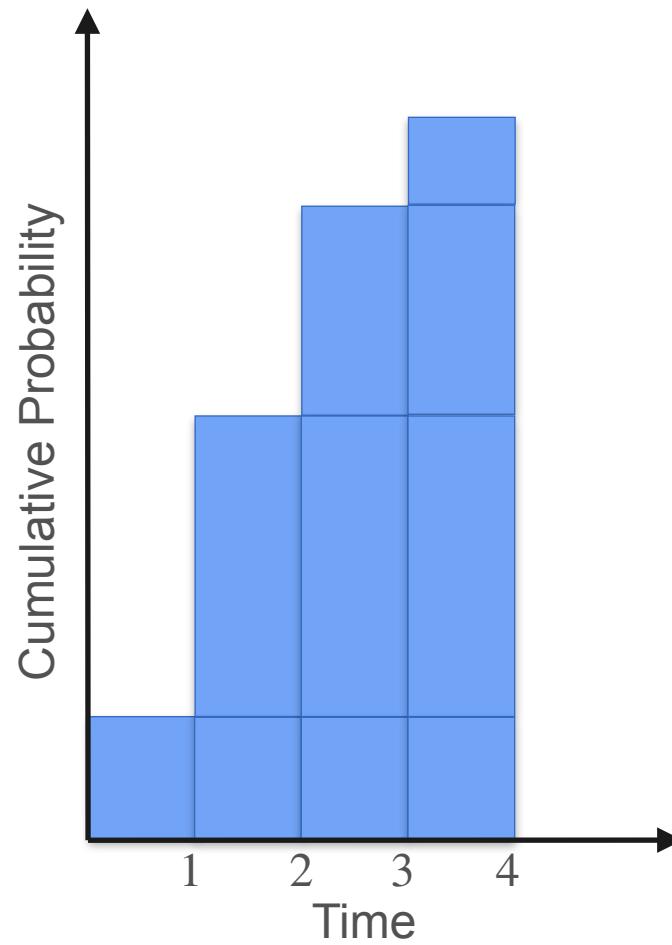
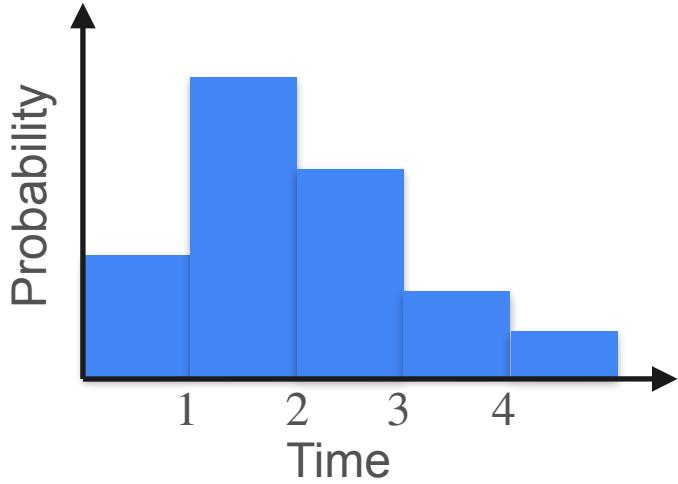
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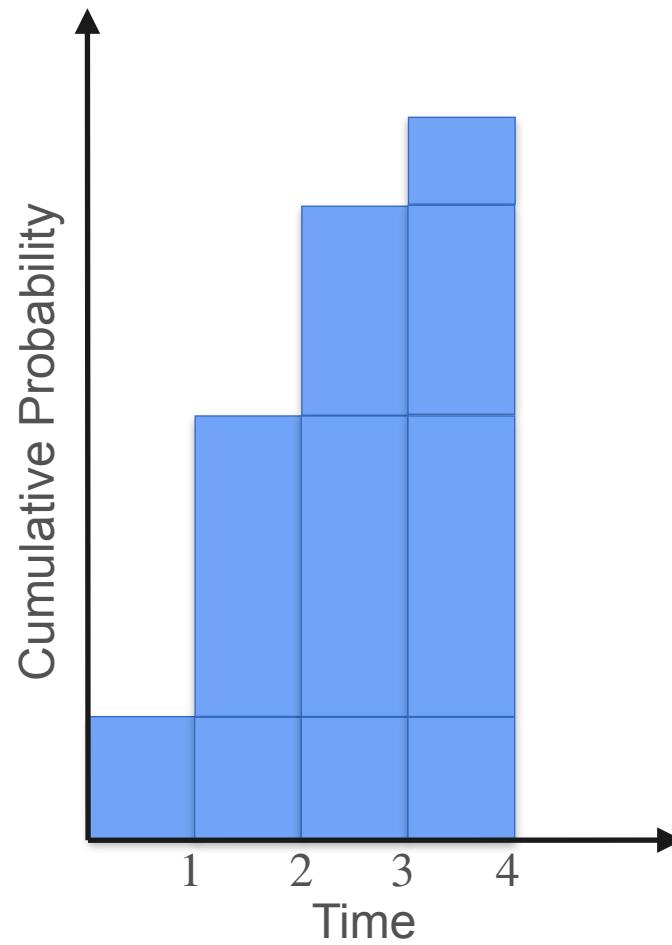
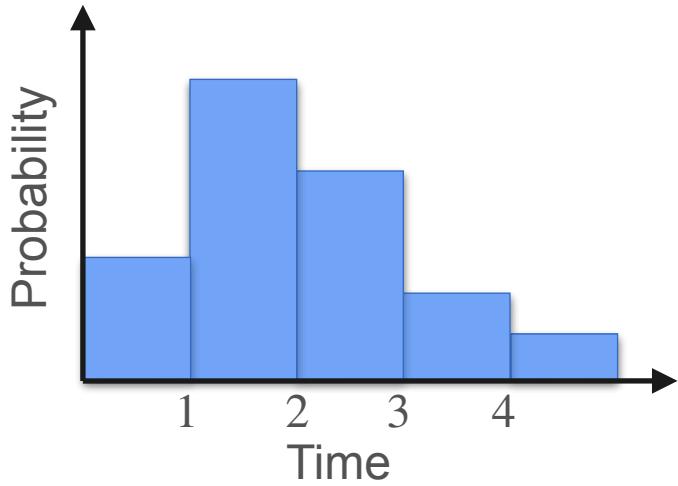
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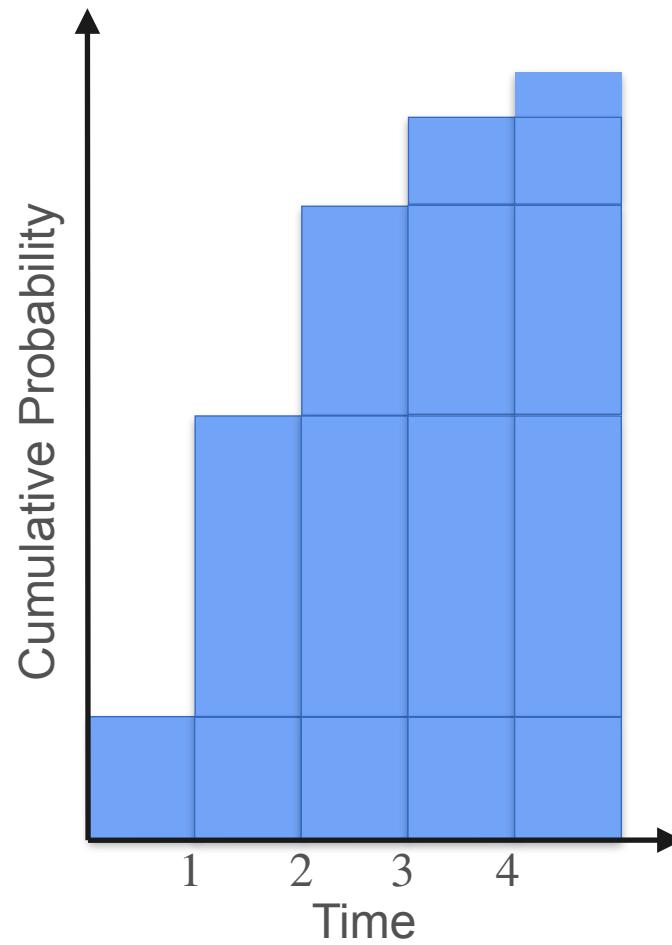
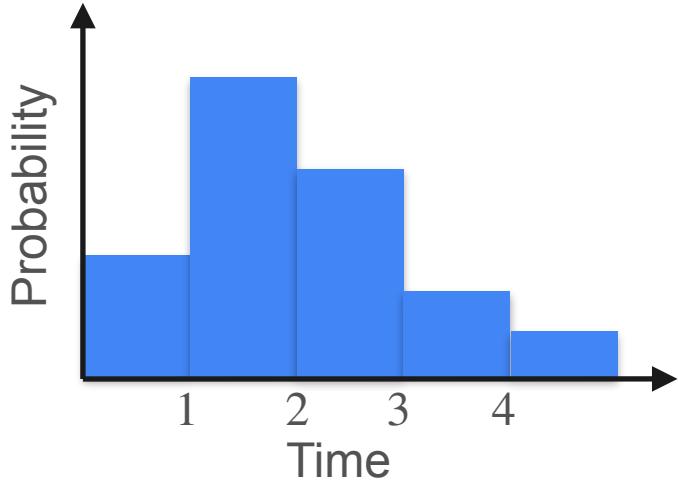
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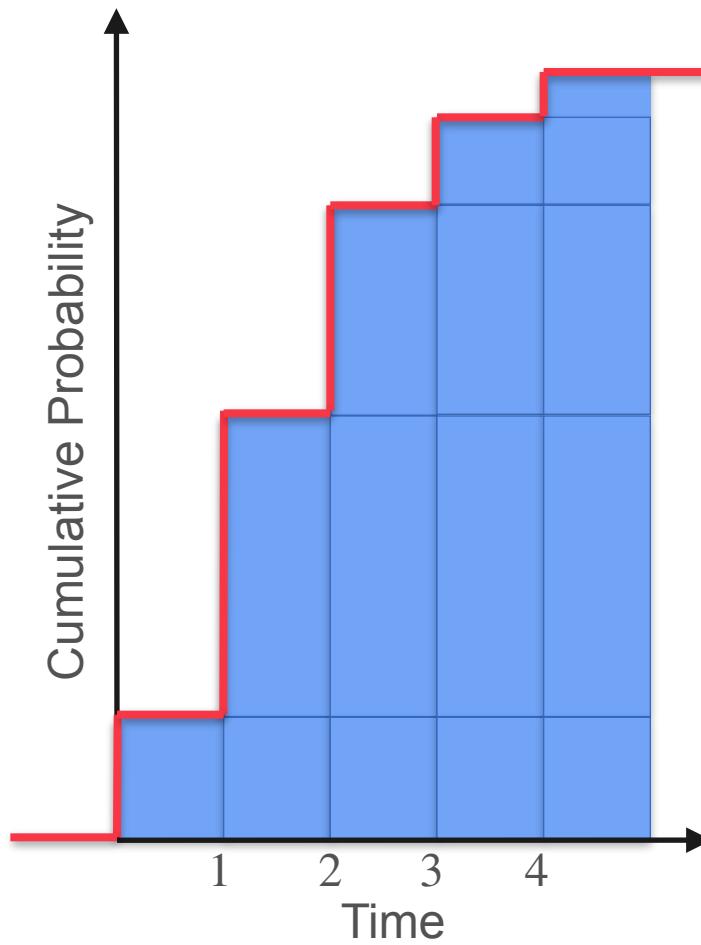
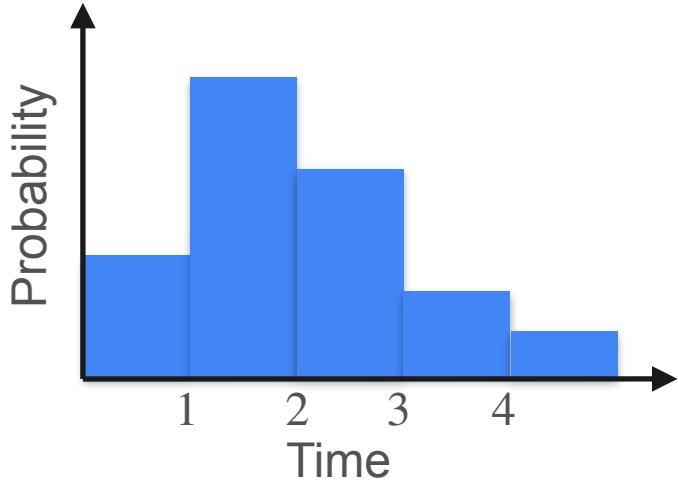
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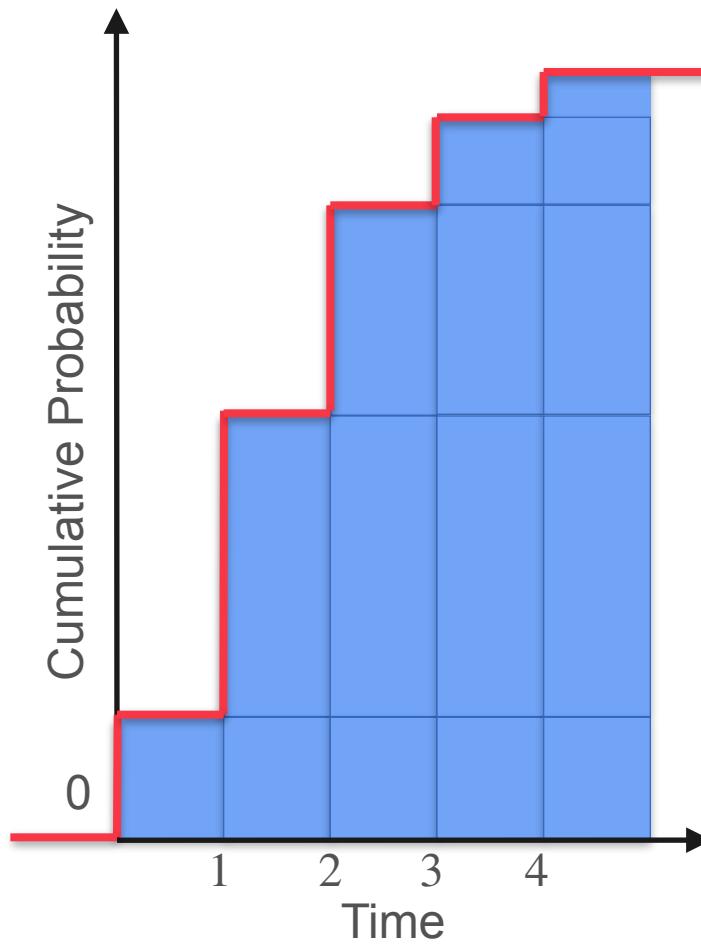
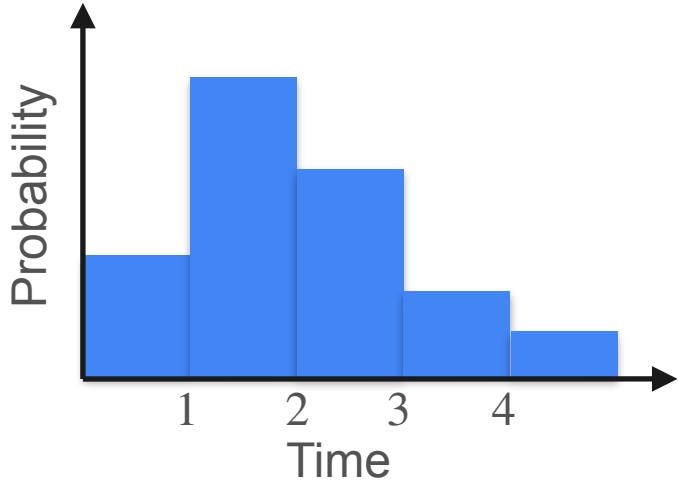
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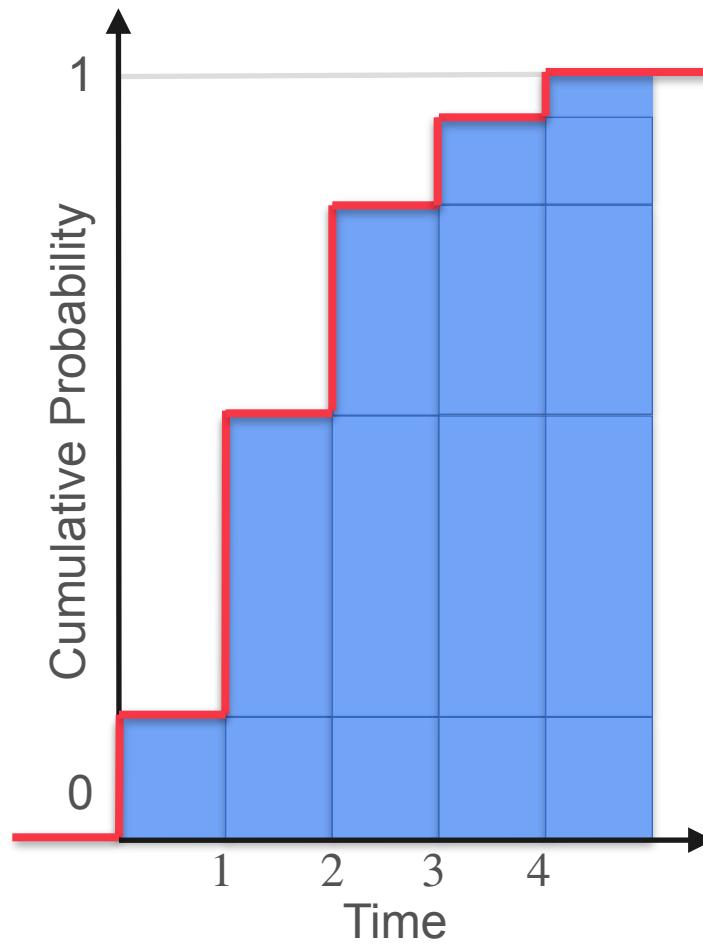
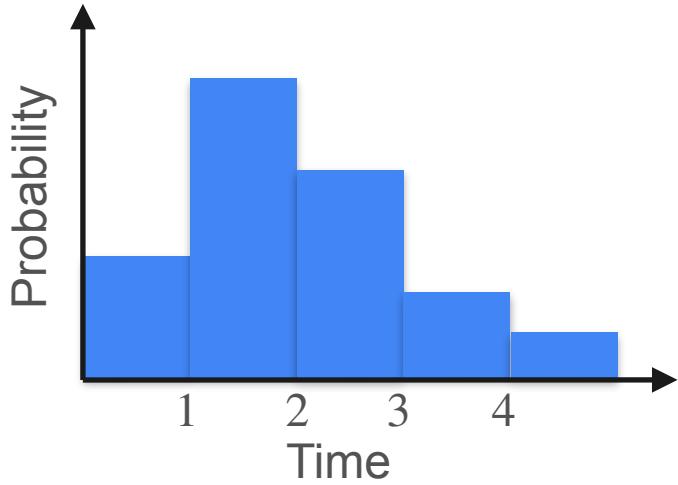
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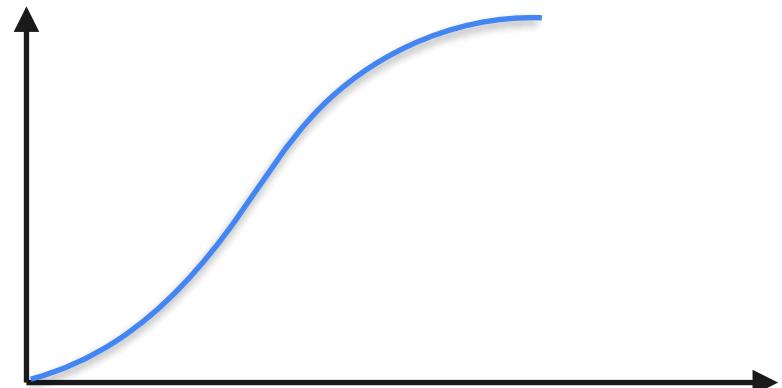
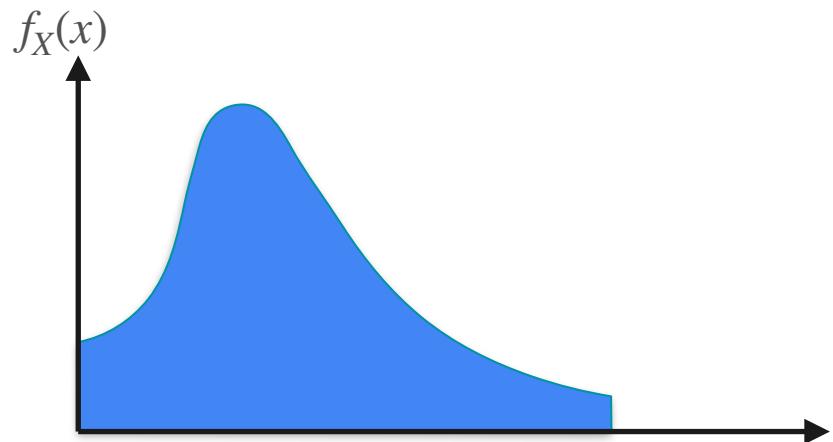
# Cumulative Distribution

CDF: Cumulative distribution function



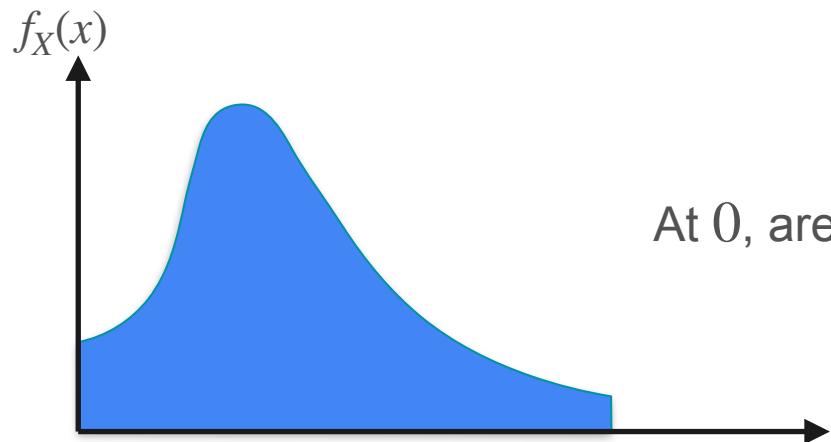
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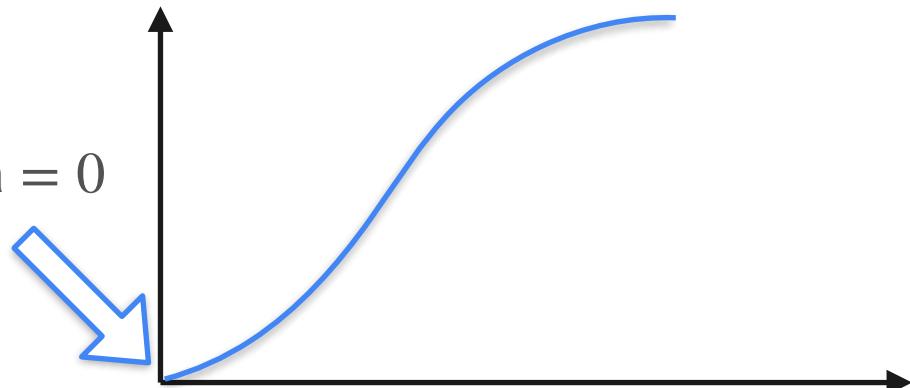


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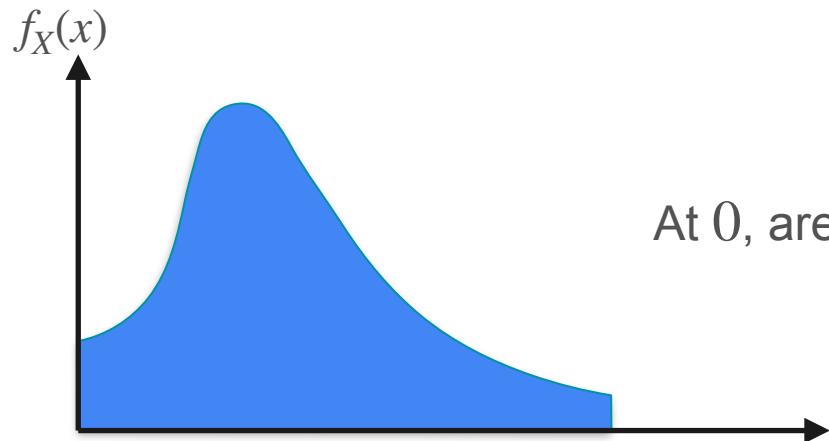


At 0, area = 0

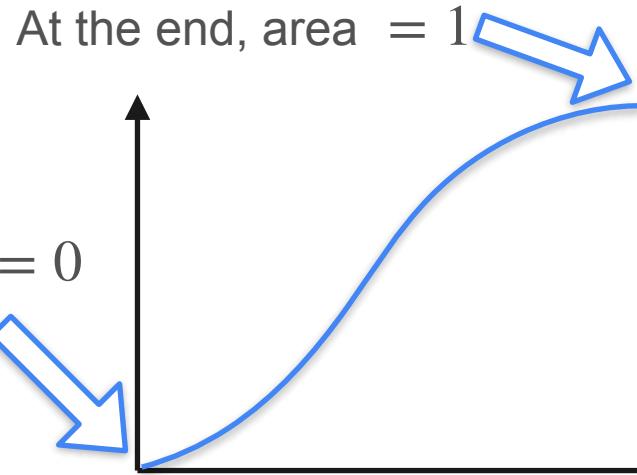


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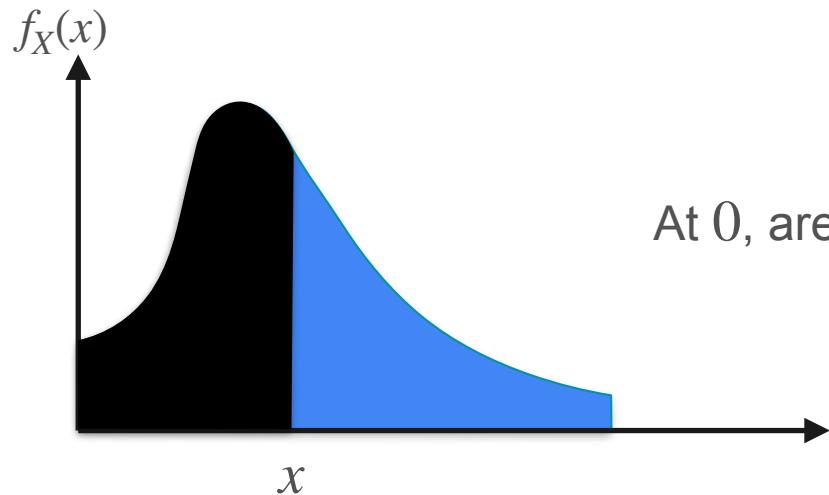


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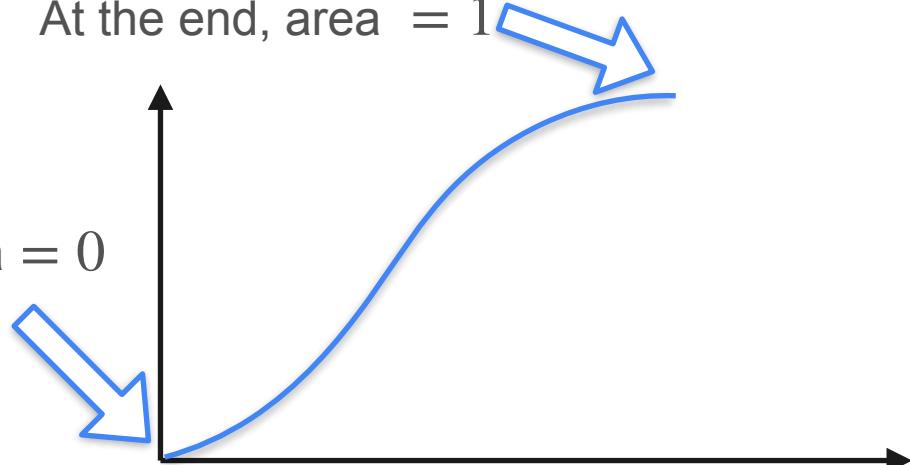
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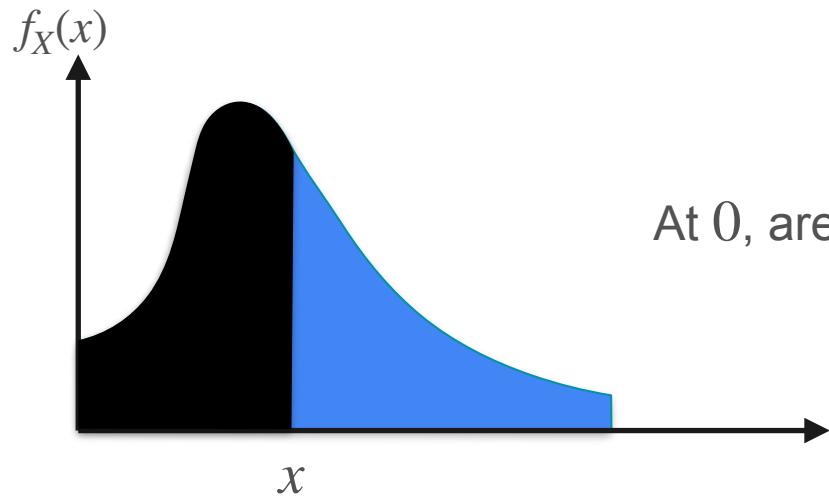
At 0, area = 0

At the end, area = 1



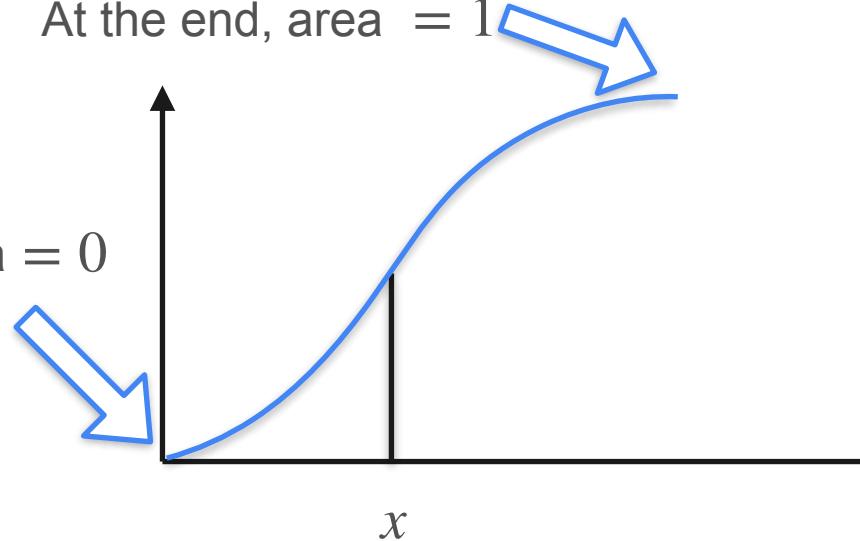
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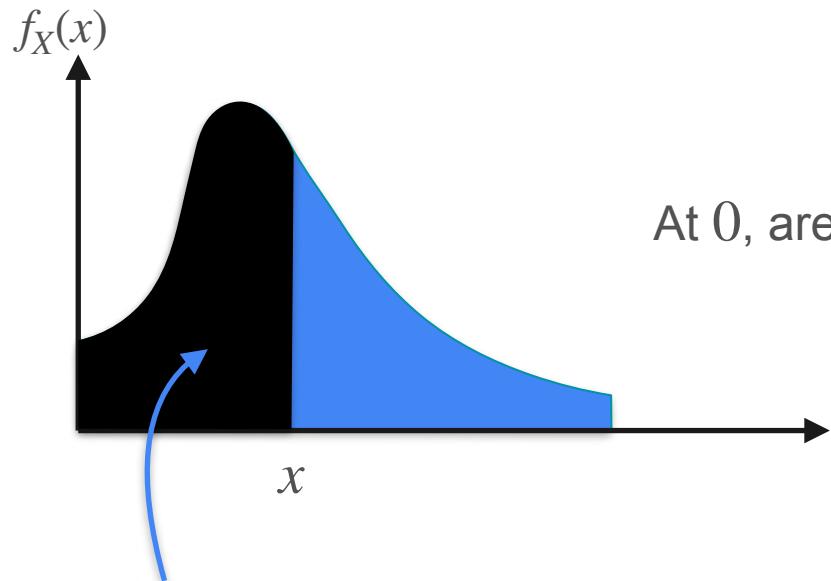
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At the end, area = 1



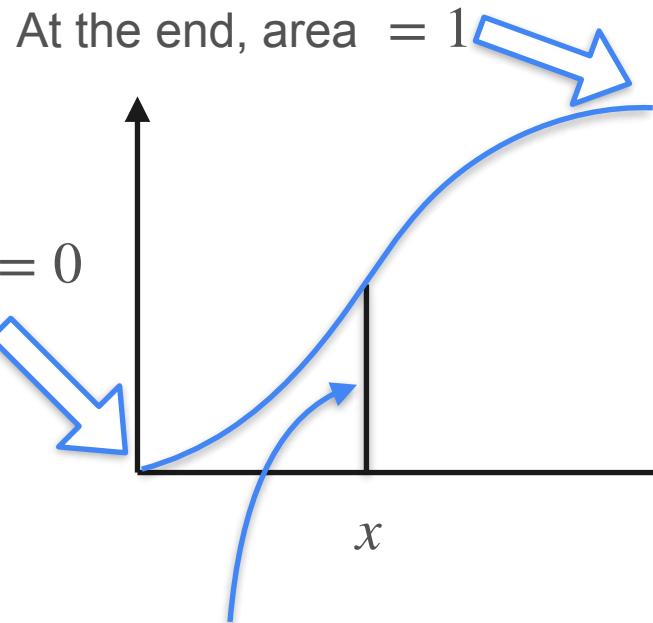
# Cumulative Distribution

CDF: Cumulative distribution function



At 0, area = 0

$P(\text{less than or equal to 2 minutes}) = 0.5$

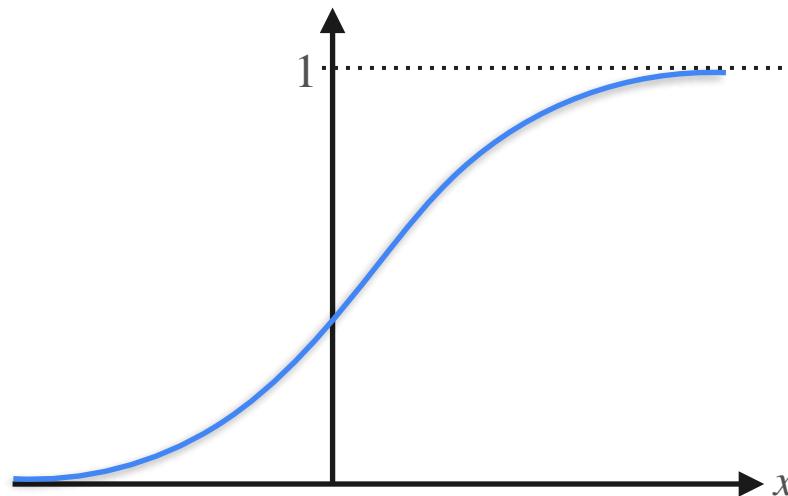
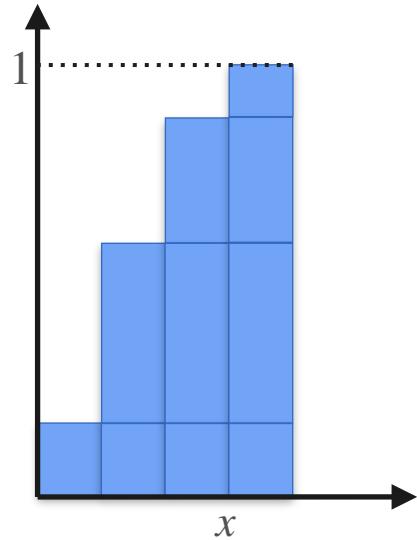


$P(\text{less than or equal to 2 minutes}) = 0.5$

# Cumulative Distribution Function: Formal Definition

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The CDF shows how much probability the variable has accumulated until a certain value

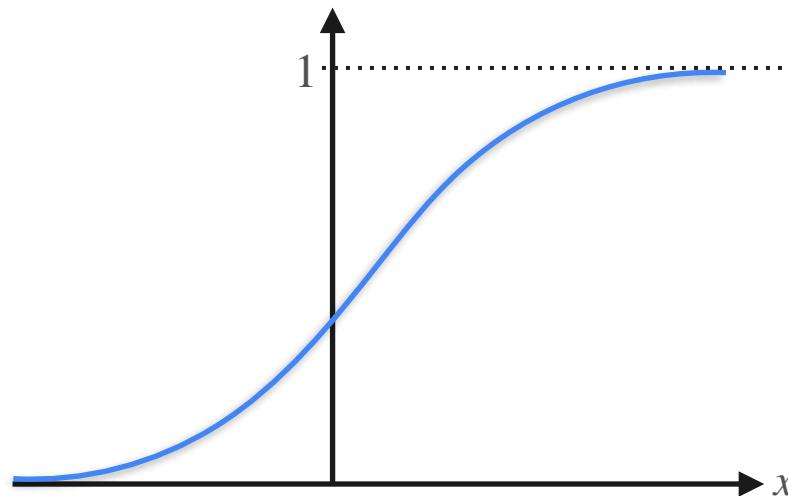
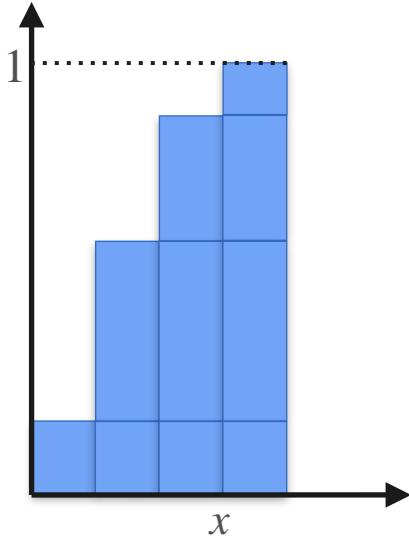


# Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$\text{CDF}(x) = \mathbf{P}(X \leq x)$$

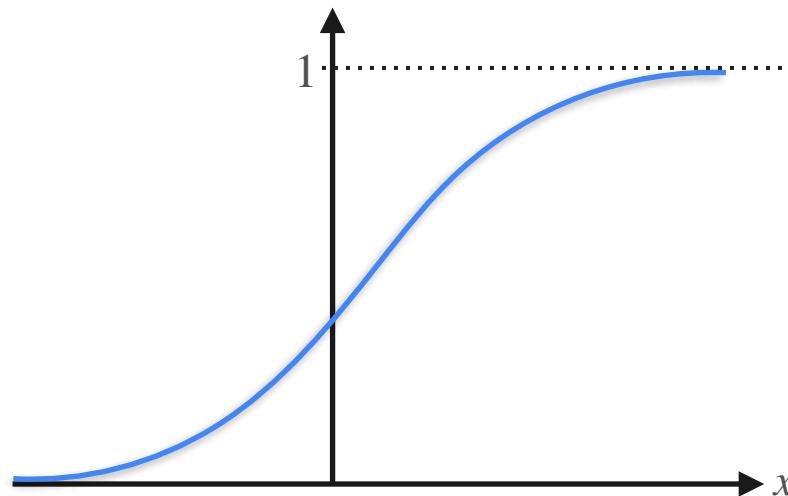
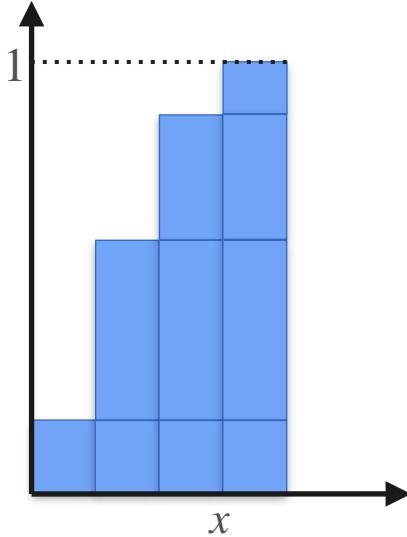


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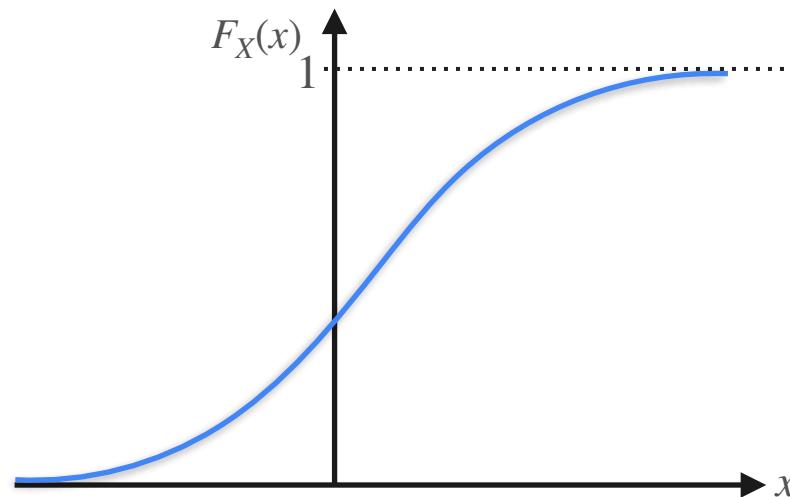
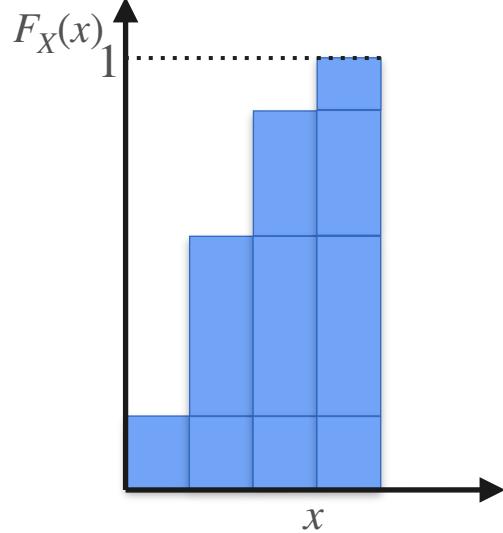


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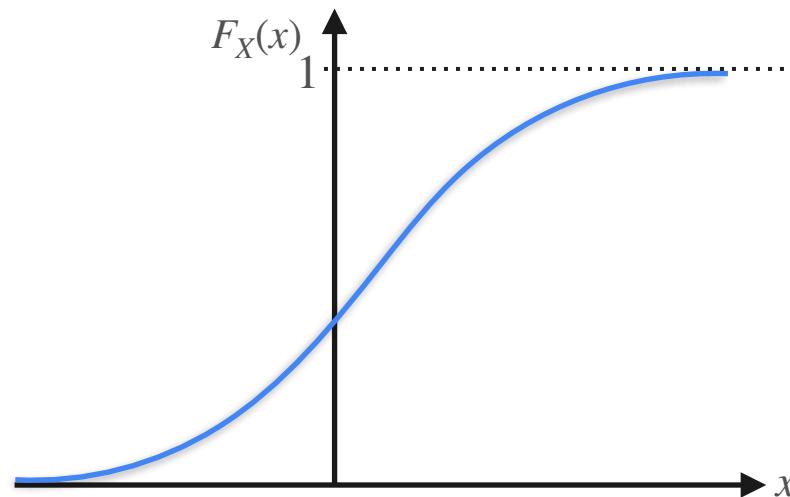
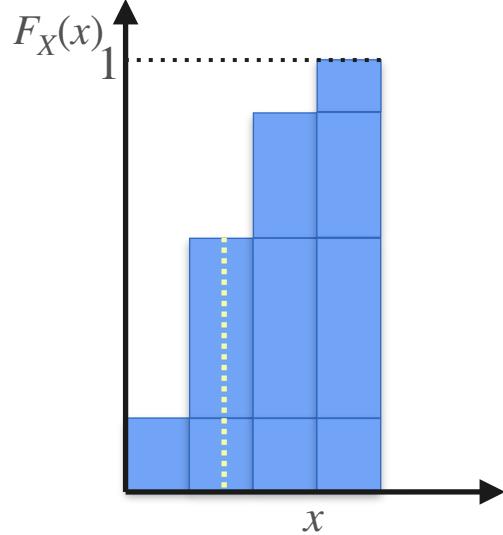


# Cumulative Distribution Function: Formal Definition

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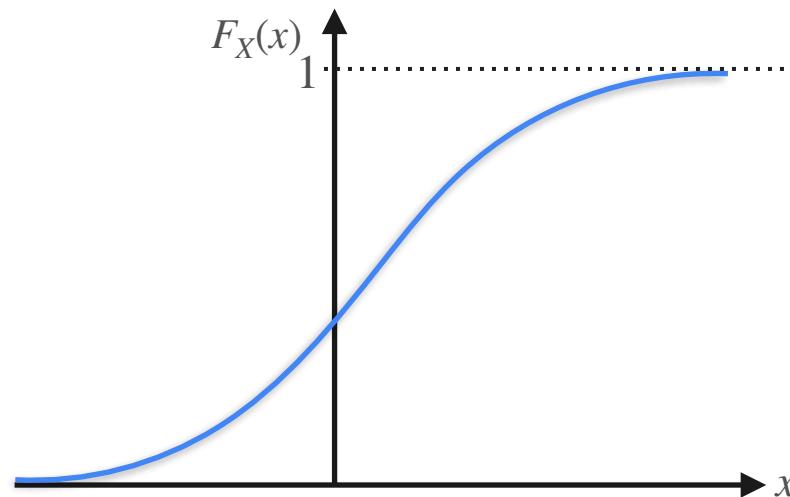
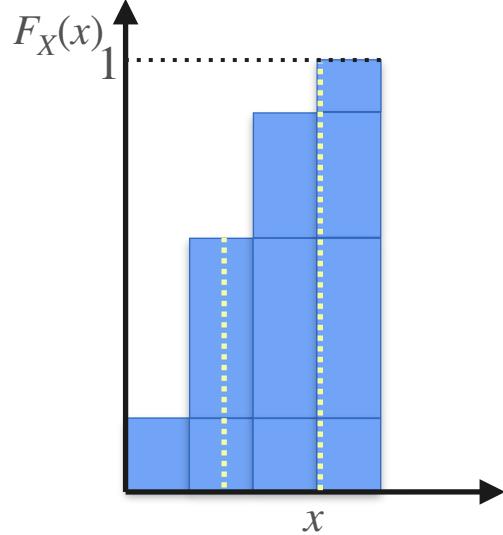


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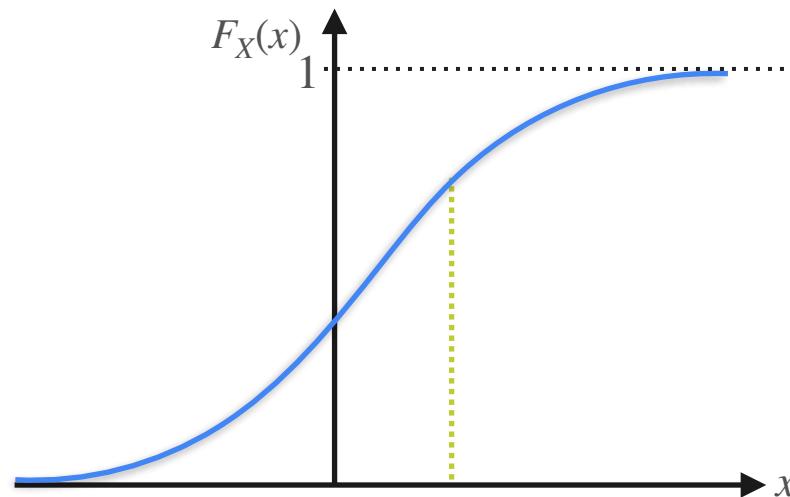
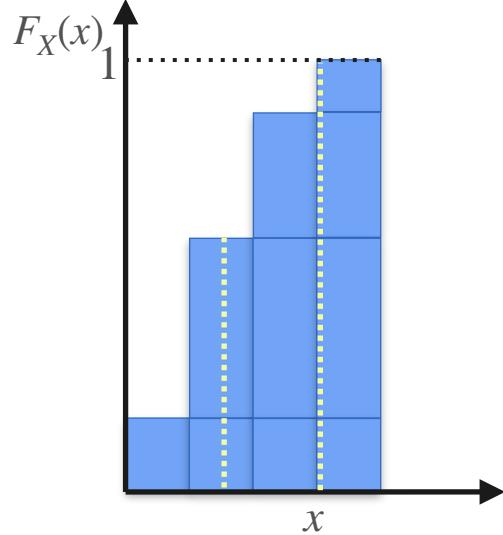


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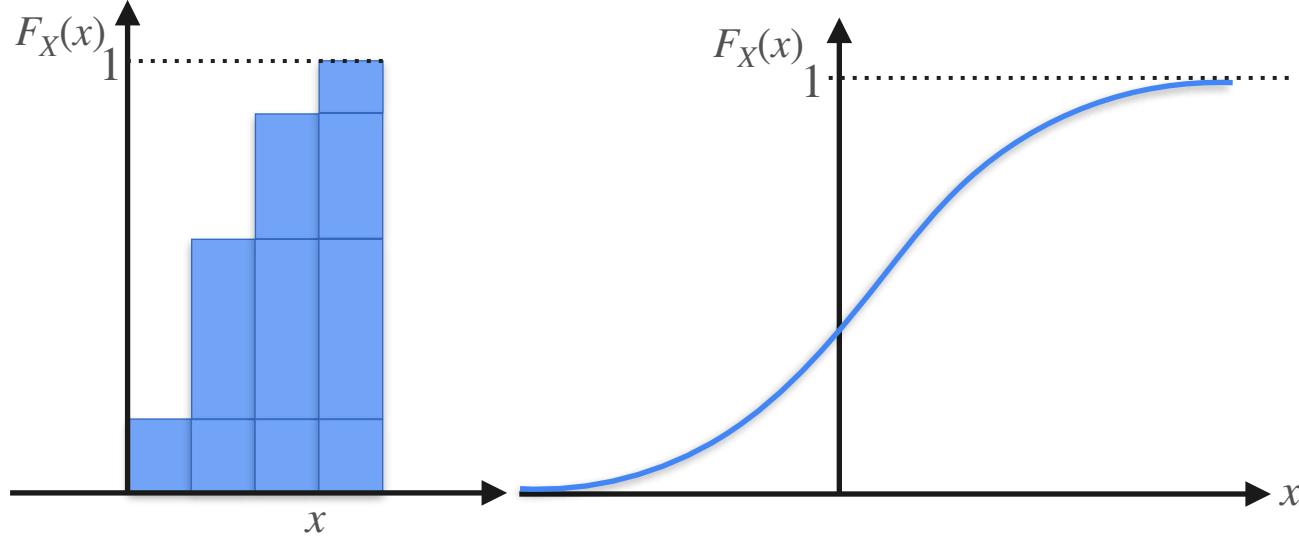


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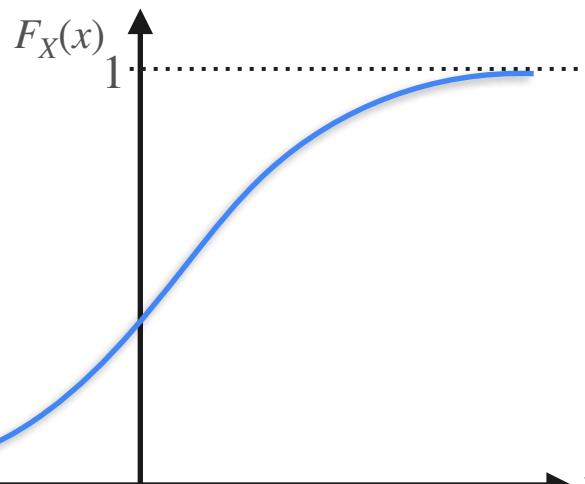
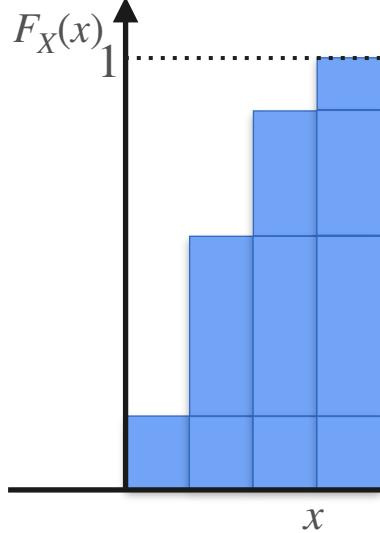


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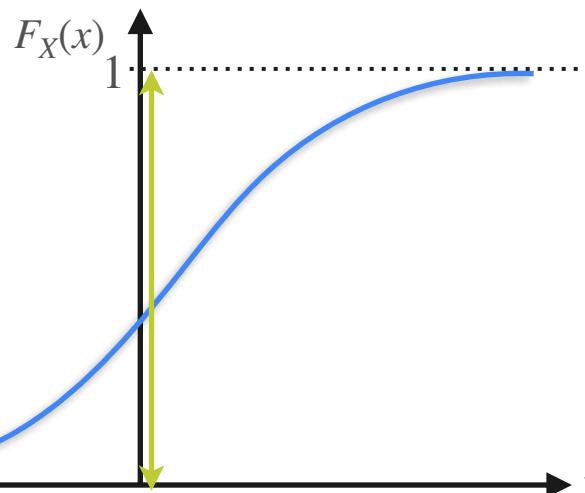
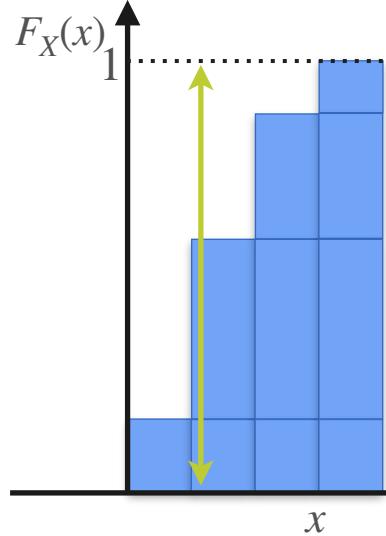
Properties

# Cumulative Distribution Function: Formal Definition

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## Properties

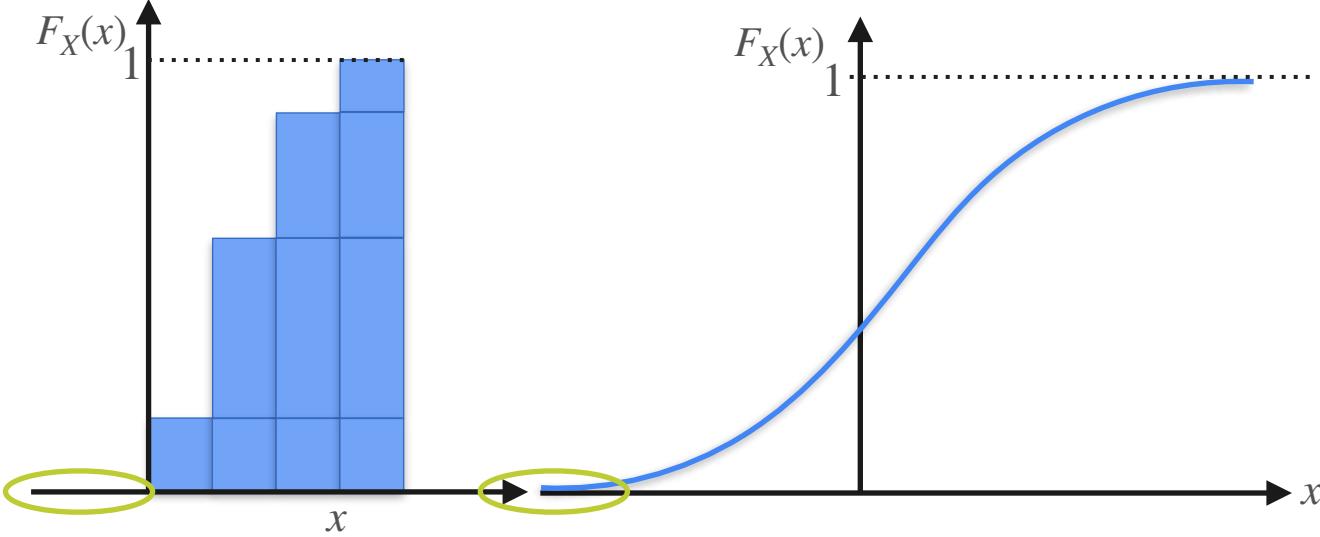
- $0 \leq F_X(x) \leq 1$

# Cumulative Distribution Function: Formal Definition

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## Properties

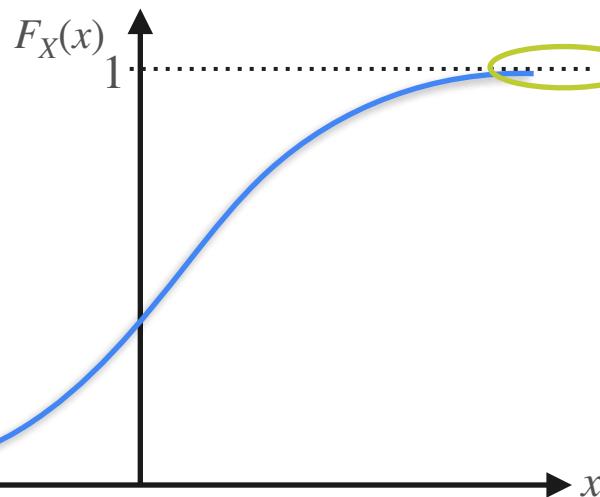
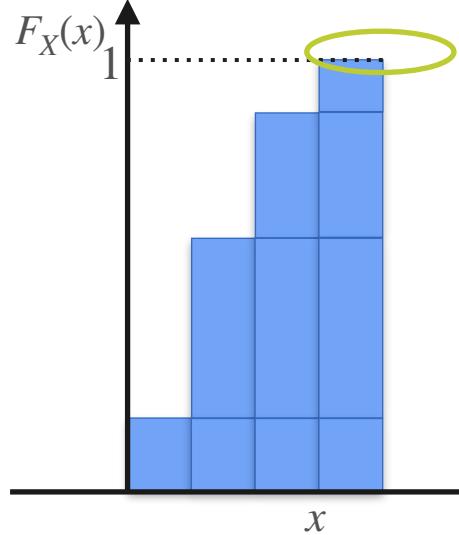
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- Left “endpoint” is 0

# Cumulative Distribution Function: Formal Definition

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## Properties

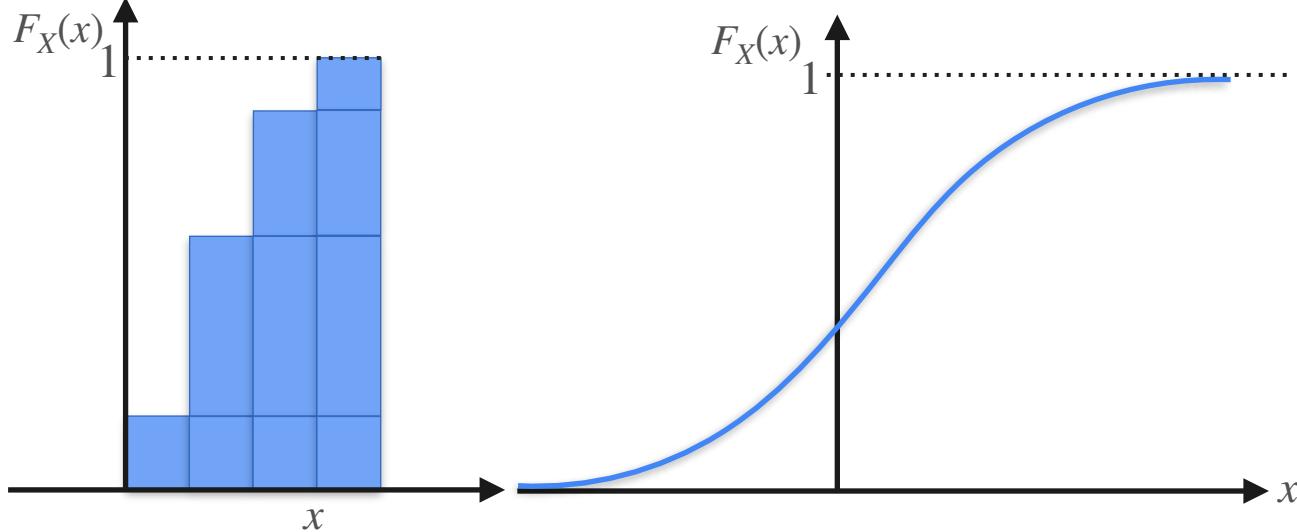
- $0 \leq F_X(x) \leq 1$
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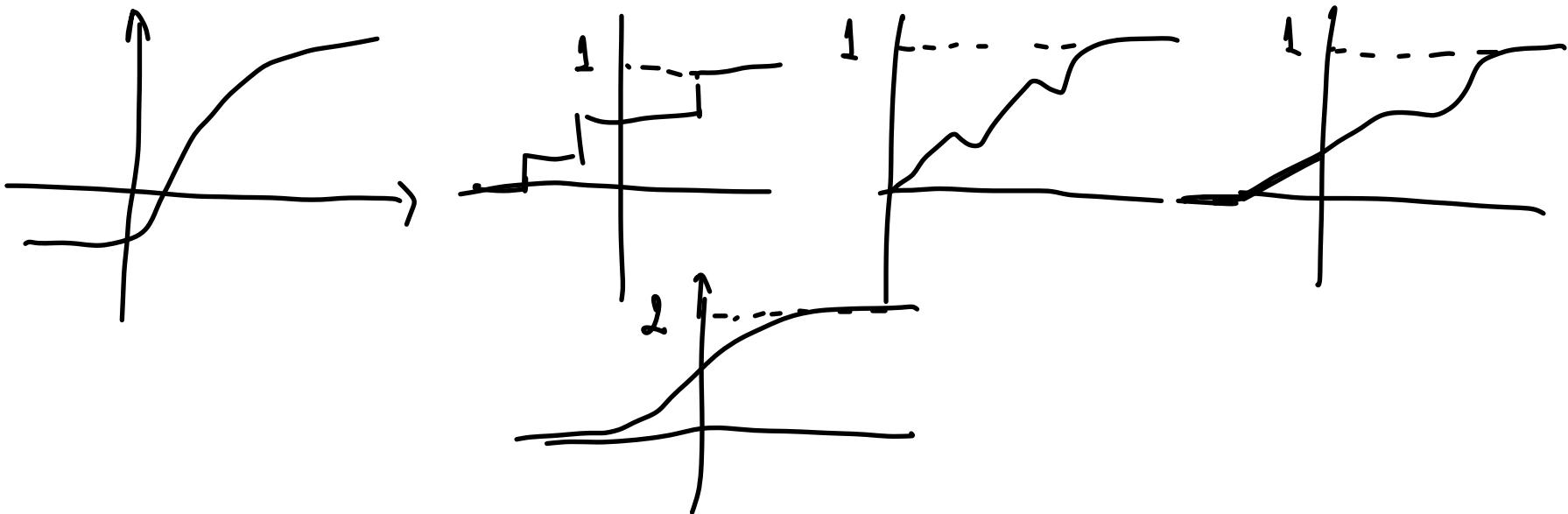


## Properties

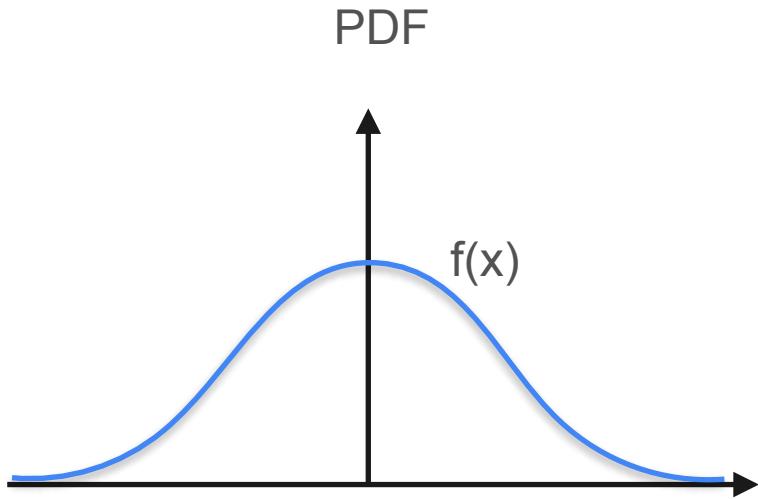
- $0 \leq F_X(x) \leq 1$
- Left “endpoint” is 0
- Right “endpoint” is 1
- Never decreases

# Quiz

- Which of the following functions could be a CDF?



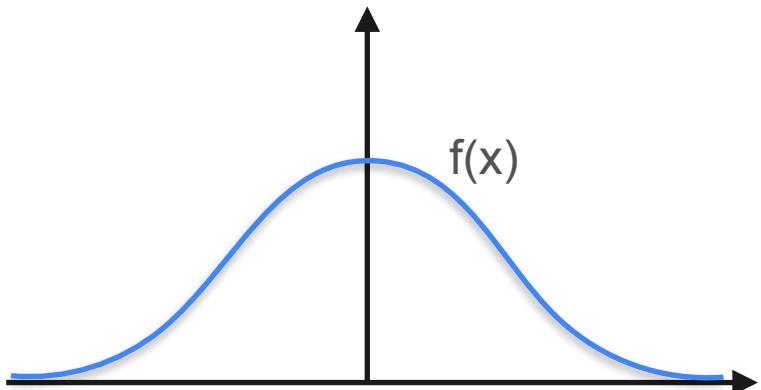
# PDF and CDF Summary



- area = 1
- Always positive

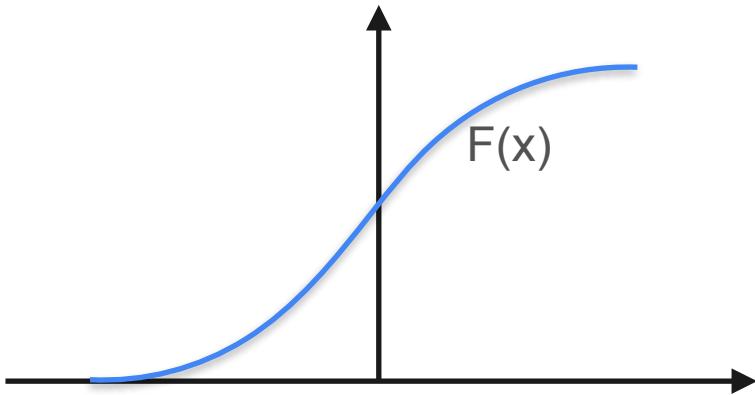
# PDF and CDF Summary

PDF



- area = 1
- Always positive

CDF



- left “endpoint” is 0
- right “endpoint” is 1
- (endpoints can be at infinity)
- Always positive and increasing



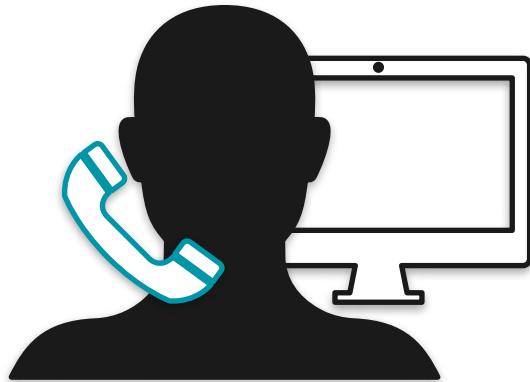
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# Probability Distributions

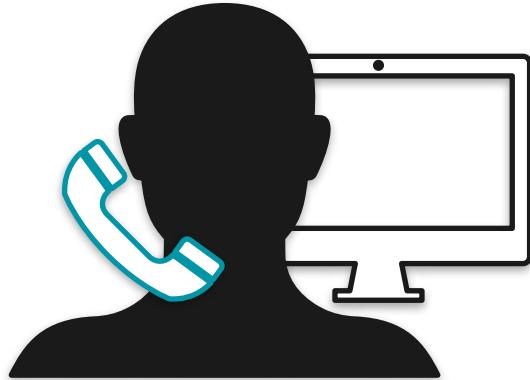
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## Uniform Distribution

# Uniform Distribution: Motivation

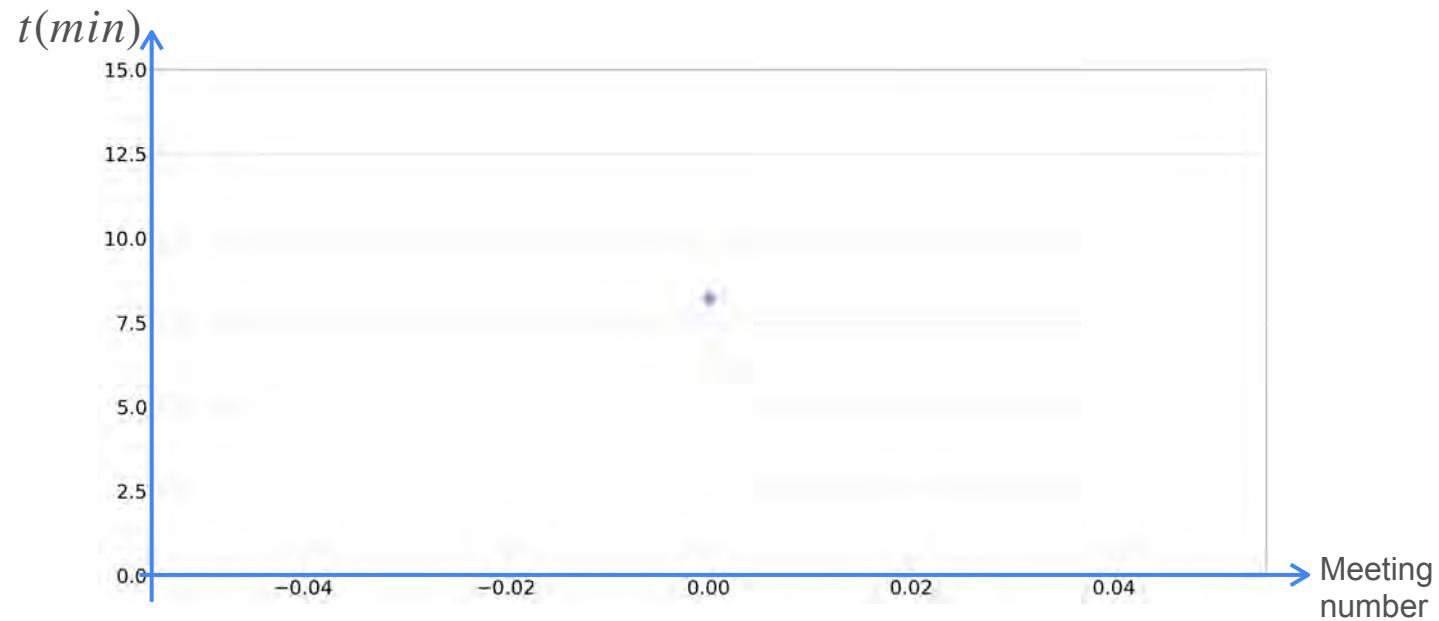


# Uniform Distribution: Motivation



You're calling a tech support line. They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

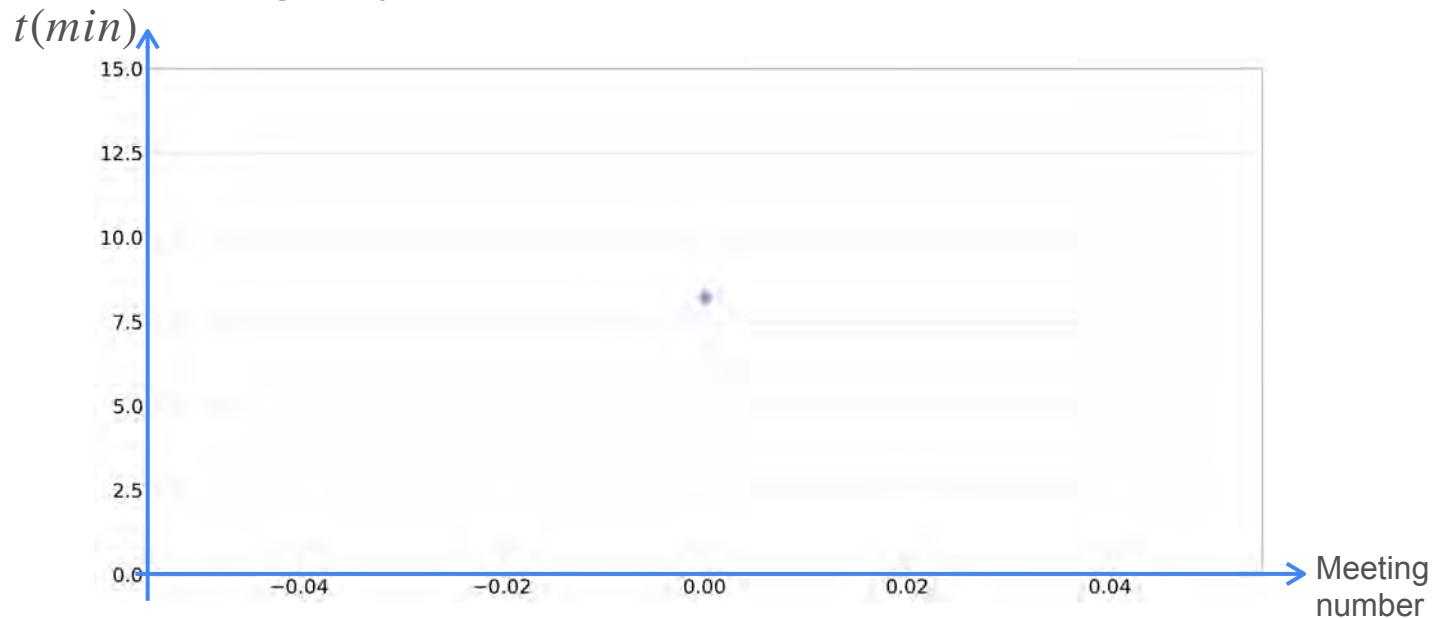
# Uniform Distribution: Motivation



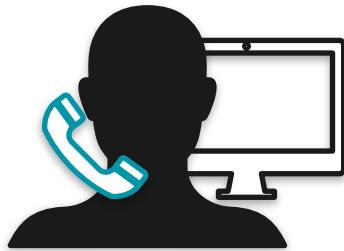
# Uniform Distribution: Motivation



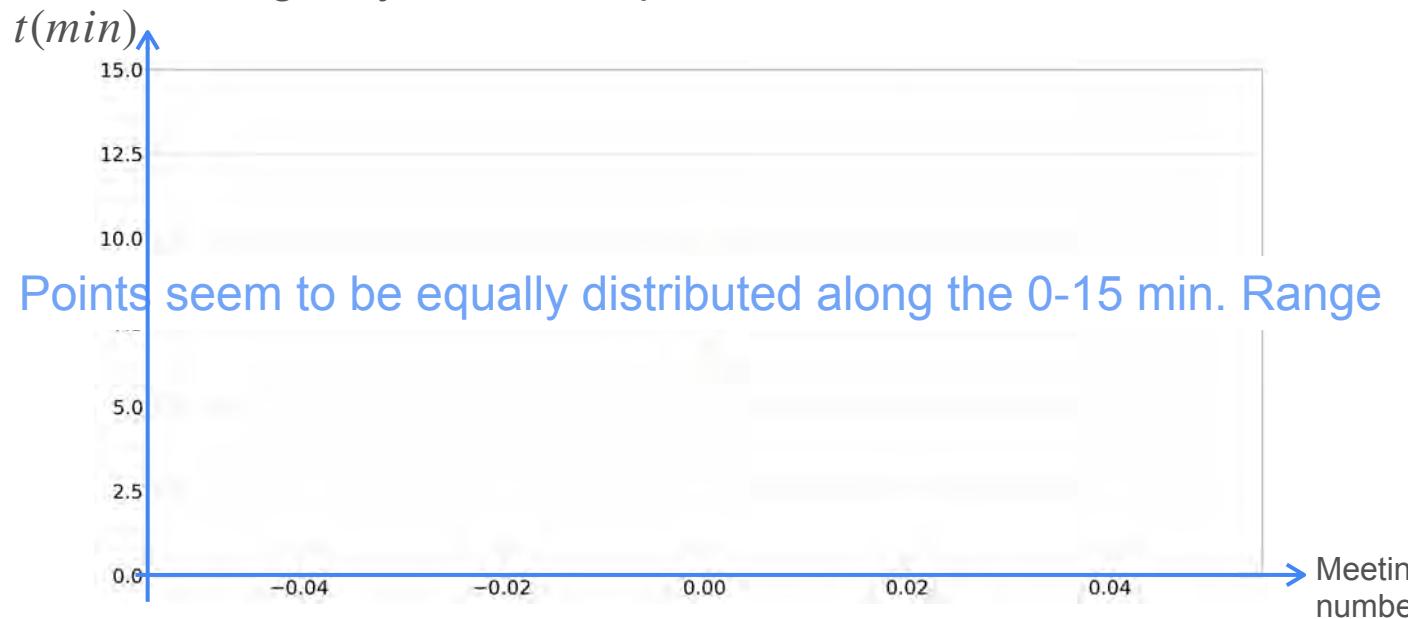
Last 200 times you called them, you took down notes of how long they took to respond



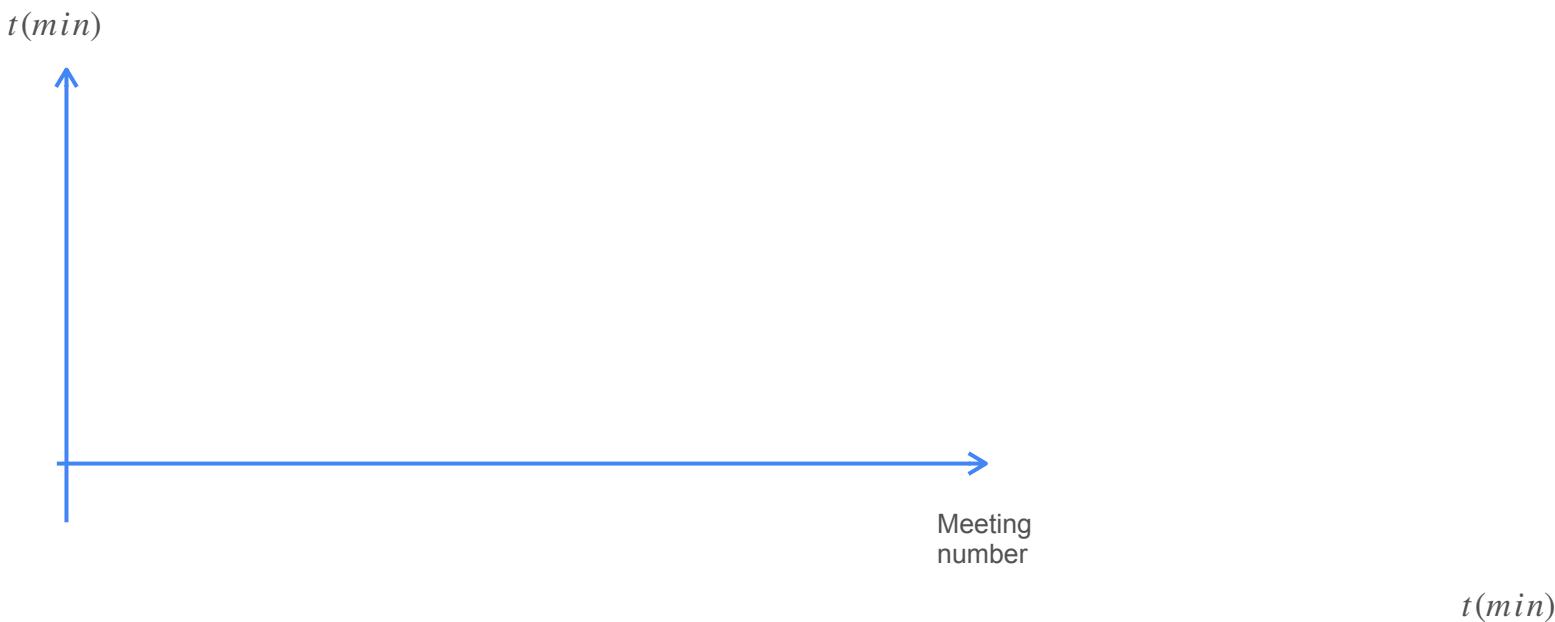
# Uniform Distribution: Motivation



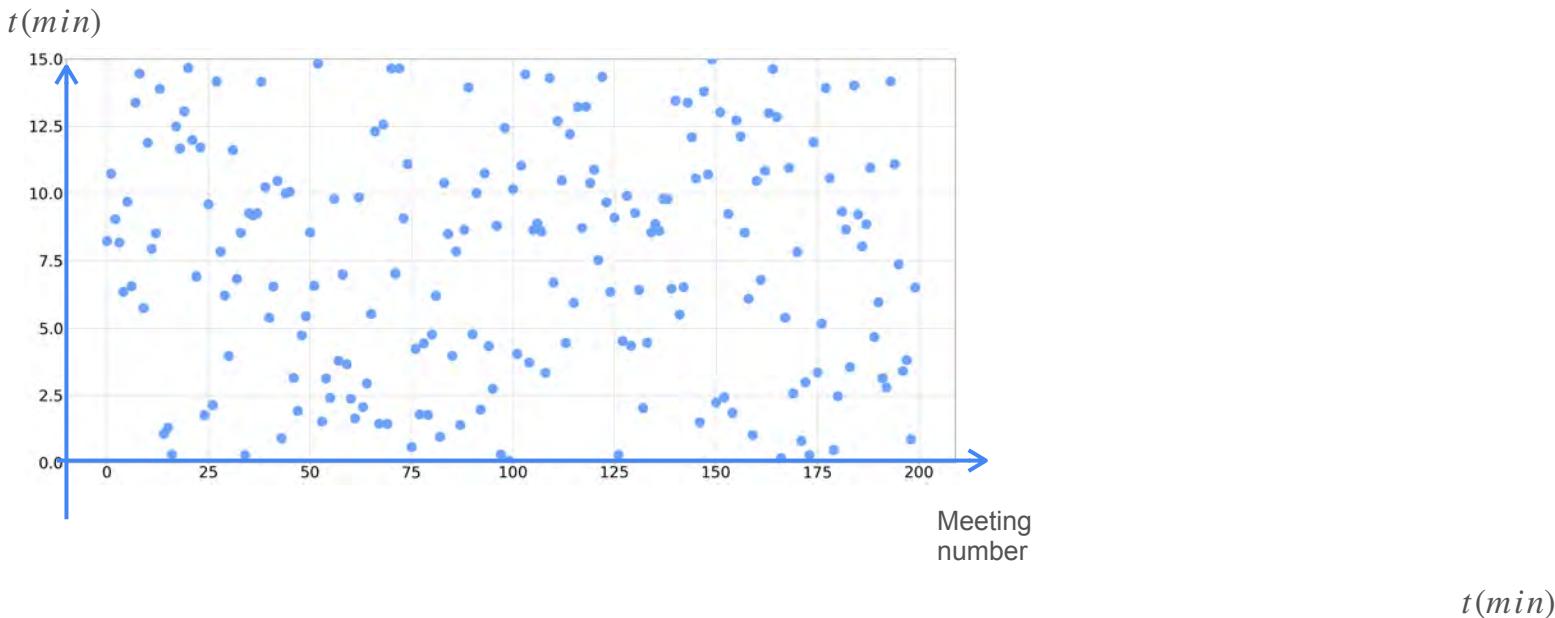
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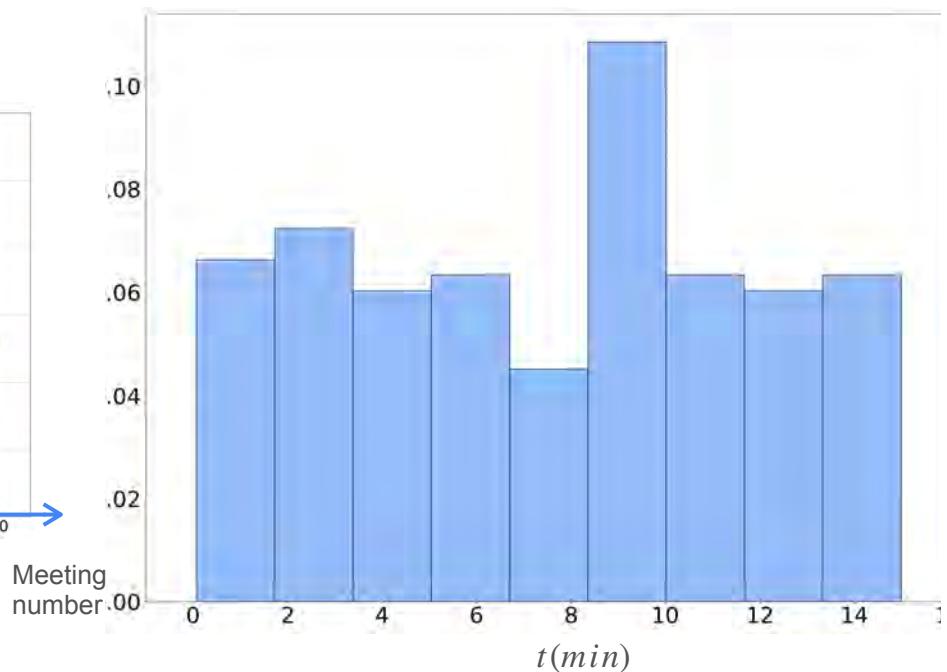
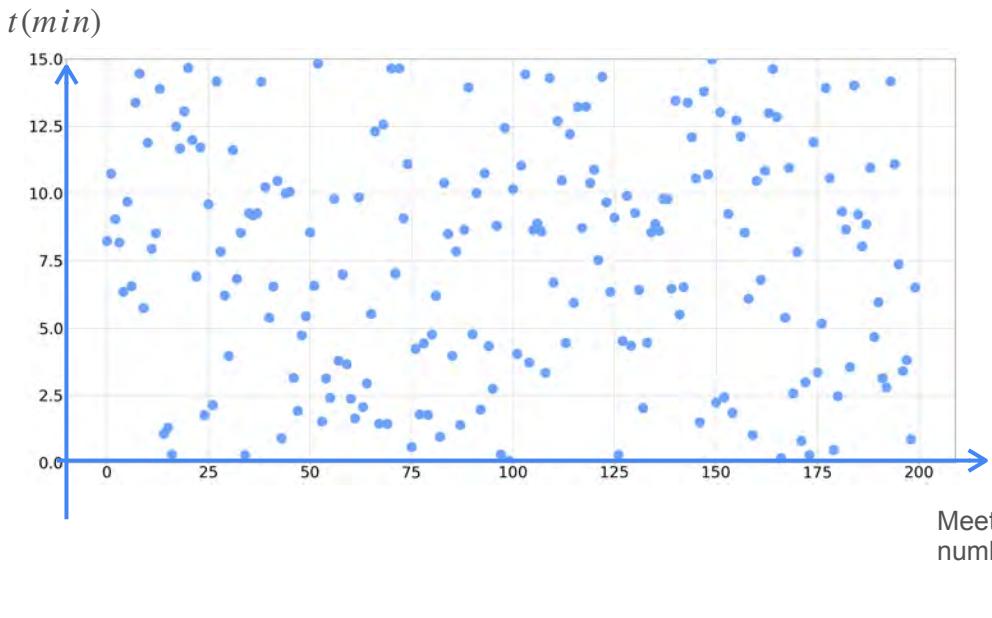
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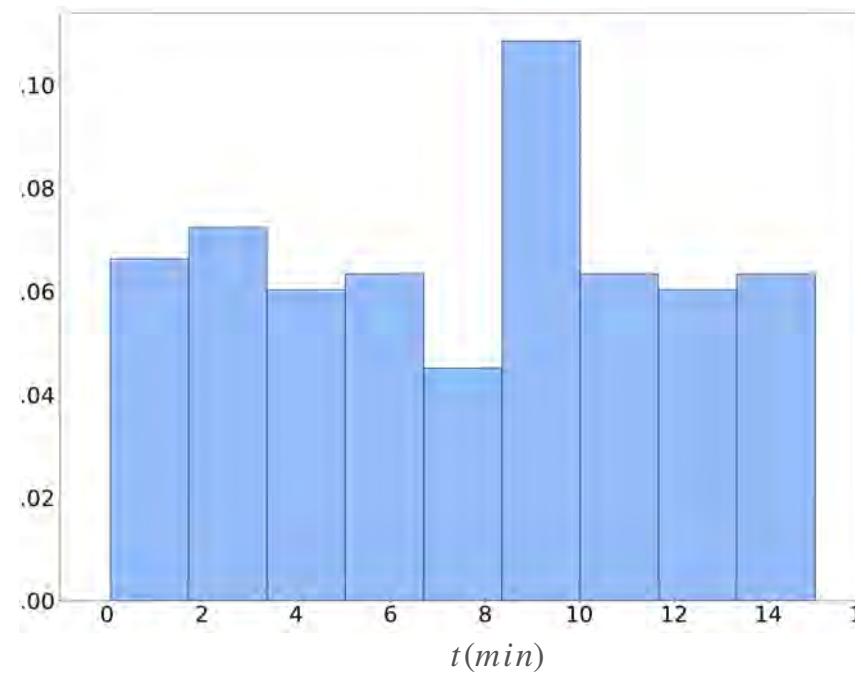
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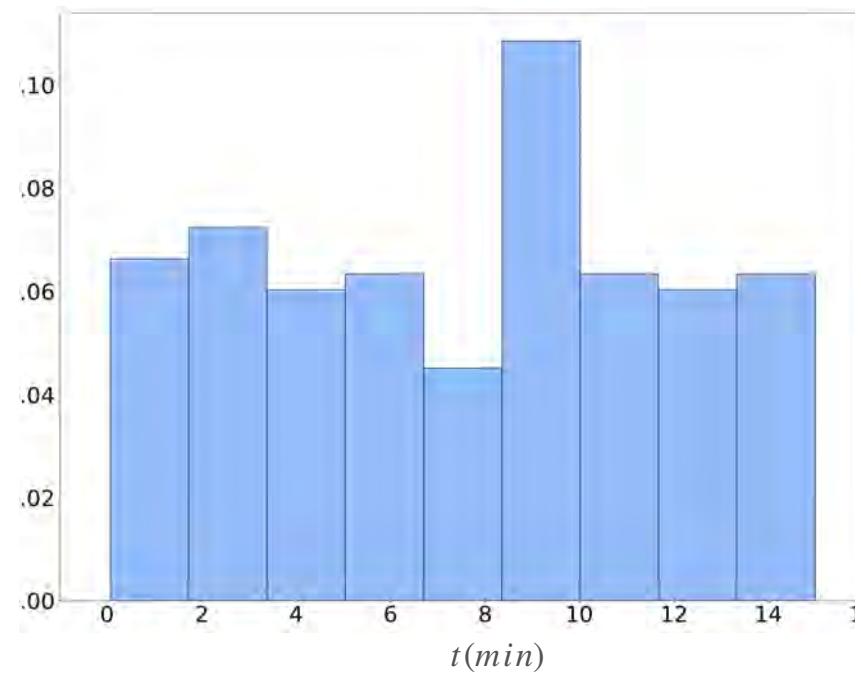


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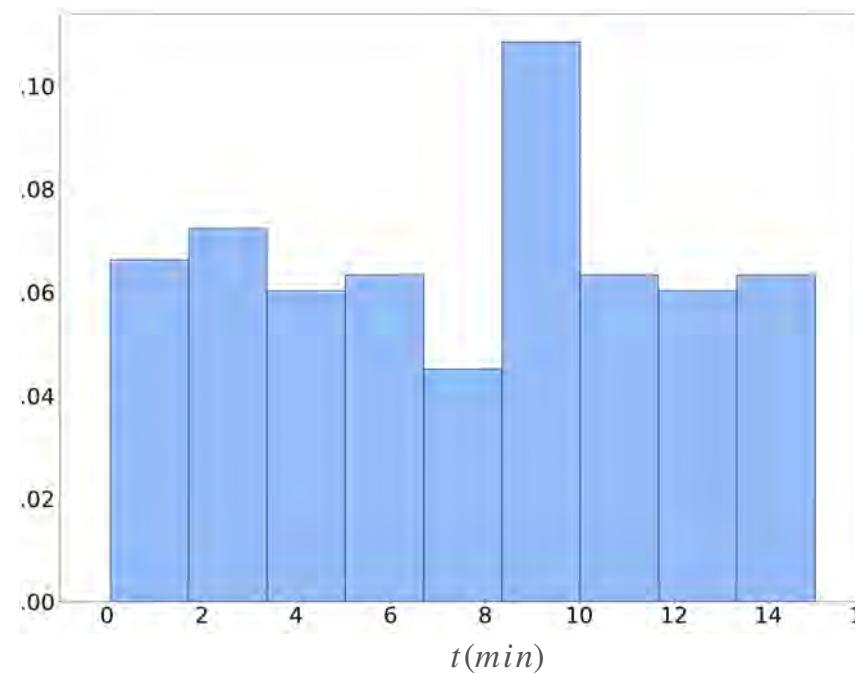
T: time (in minutes) you have to wait



# Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.



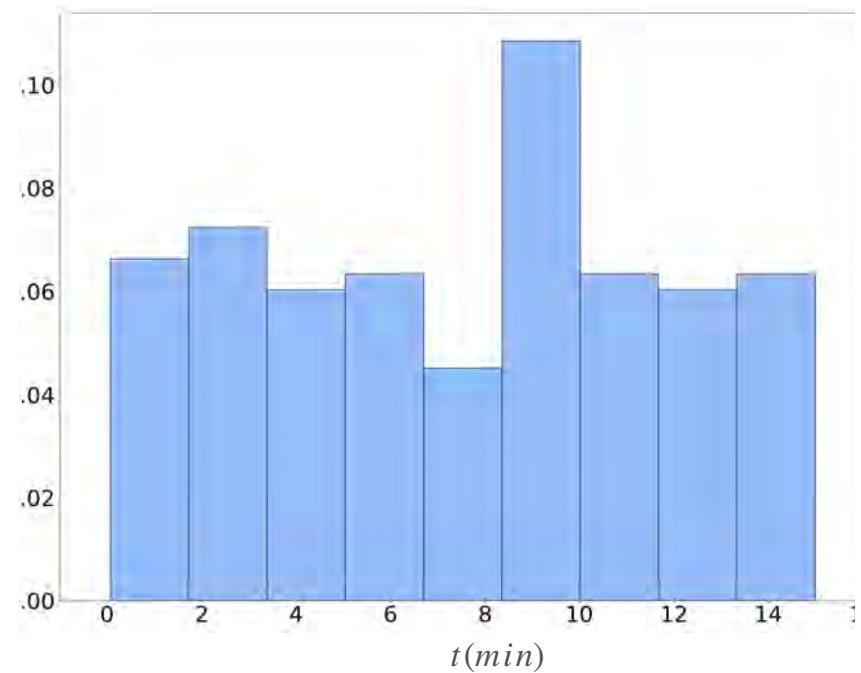
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The pdf must be constant for all values in the interval (0,15)



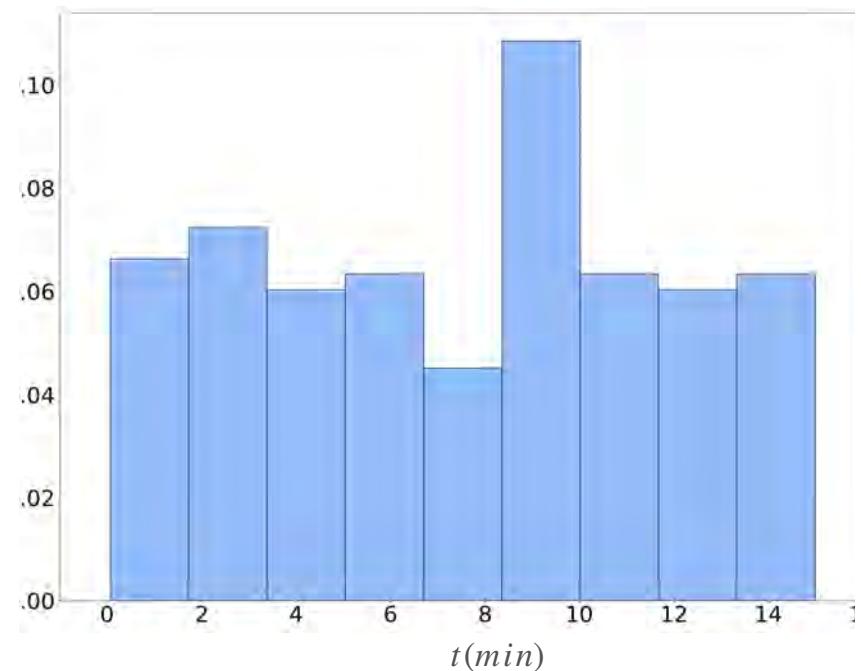
# Uniform Distribution: Motivation

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Which constant?



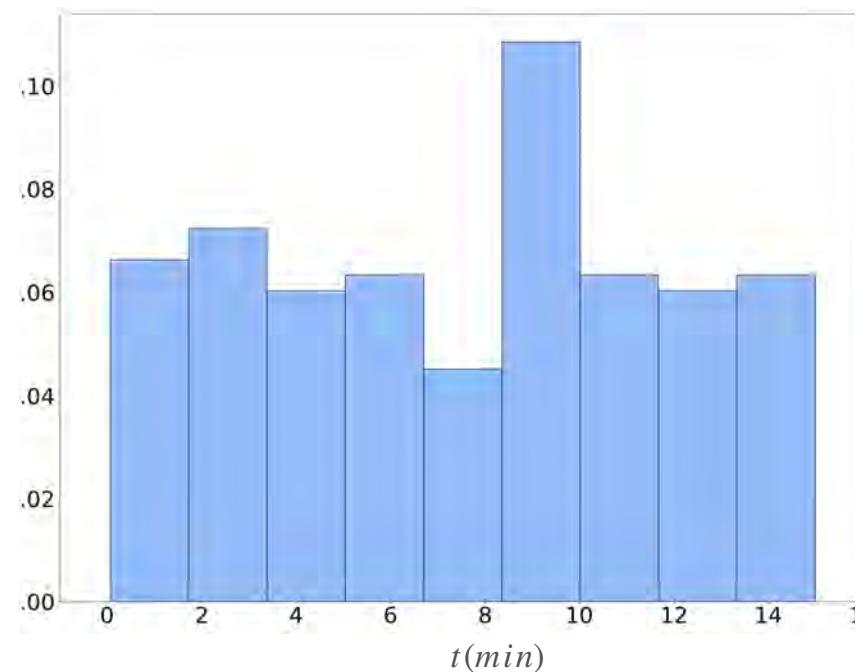
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T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval  $(0, 15)$

Which constant?  $\rightarrow 15 \times h = 1$



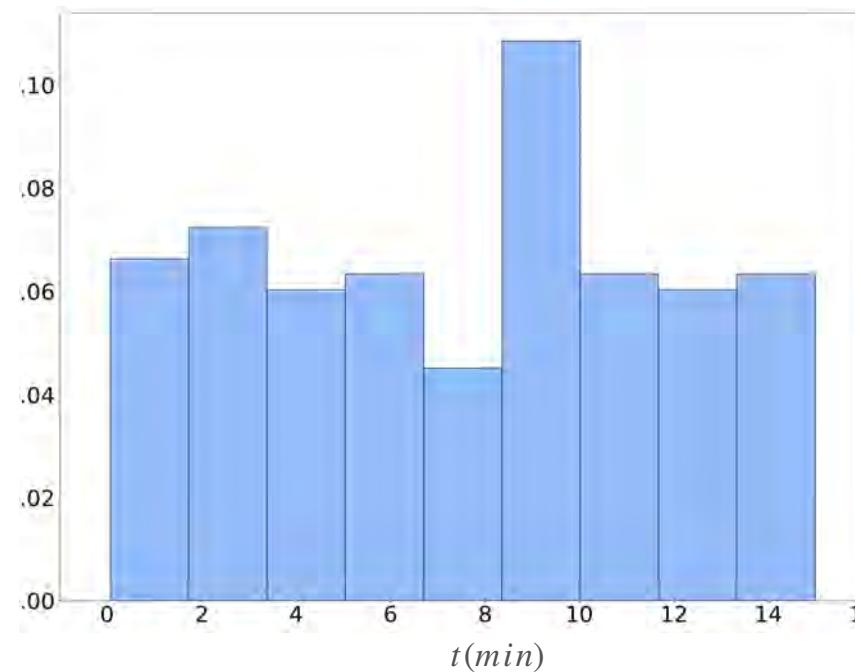
# Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant?  $\rightarrow 15 \times h = 1 \rightarrow h = \frac{1}{15} = 0.06$



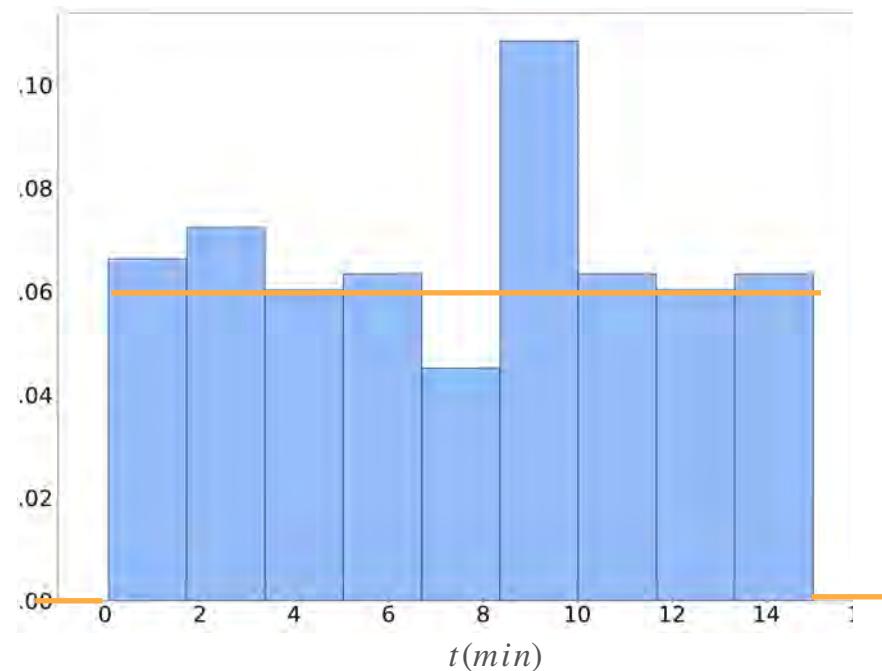
# Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant?  $\rightarrow 15 \times h = 1 \rightarrow h = \frac{1}{15} = 0.06$



# Uniform Distribution: Model

$x$

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A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

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Parameters:

$x$

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Parameters:

- $a$ : beginning of the interval

$x$

# Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

- $a$ : beginning of the interval
- $b$ : end of the interval

$x$

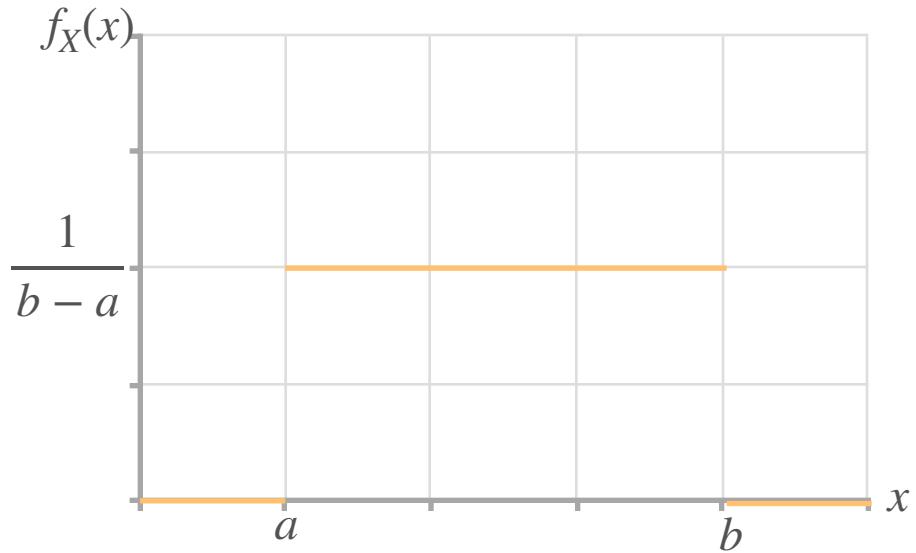
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A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

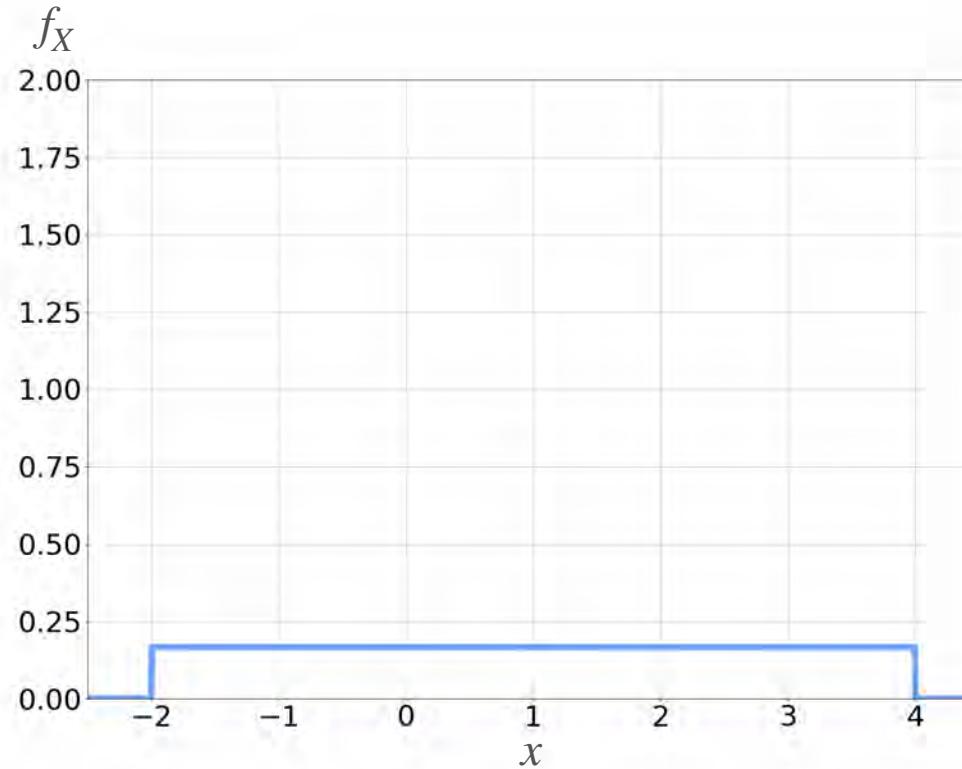
Parameters:

- $a$ : beginning of the interval
- $b$ : end of the interval

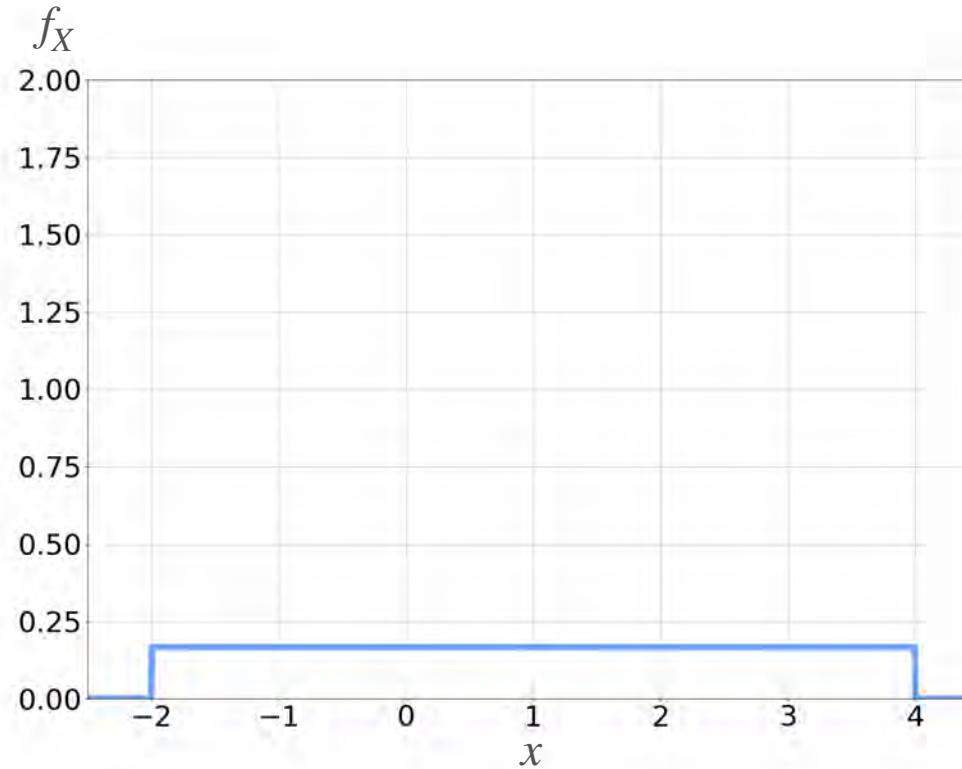
$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x \notin (a, b) \end{cases}$$



# Uniform Distribution: PDF



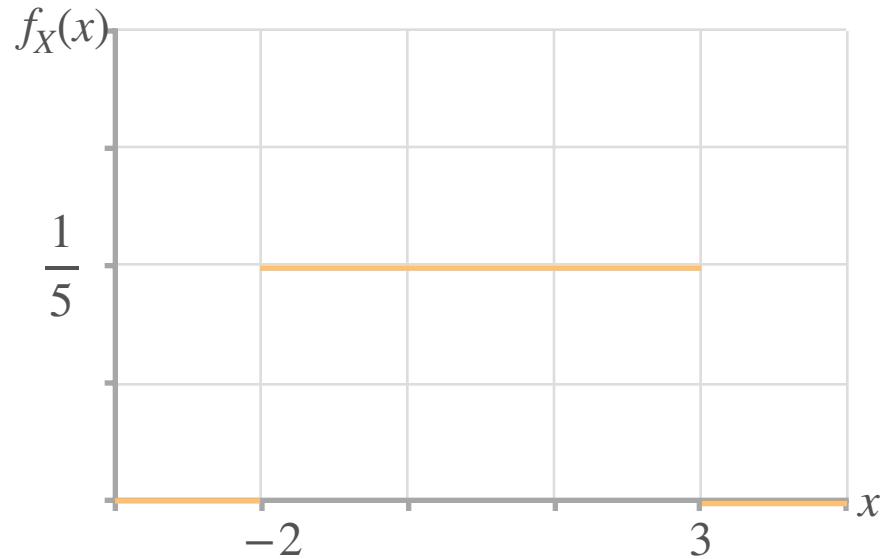
# Uniform Distribution: PDF



# Quiz

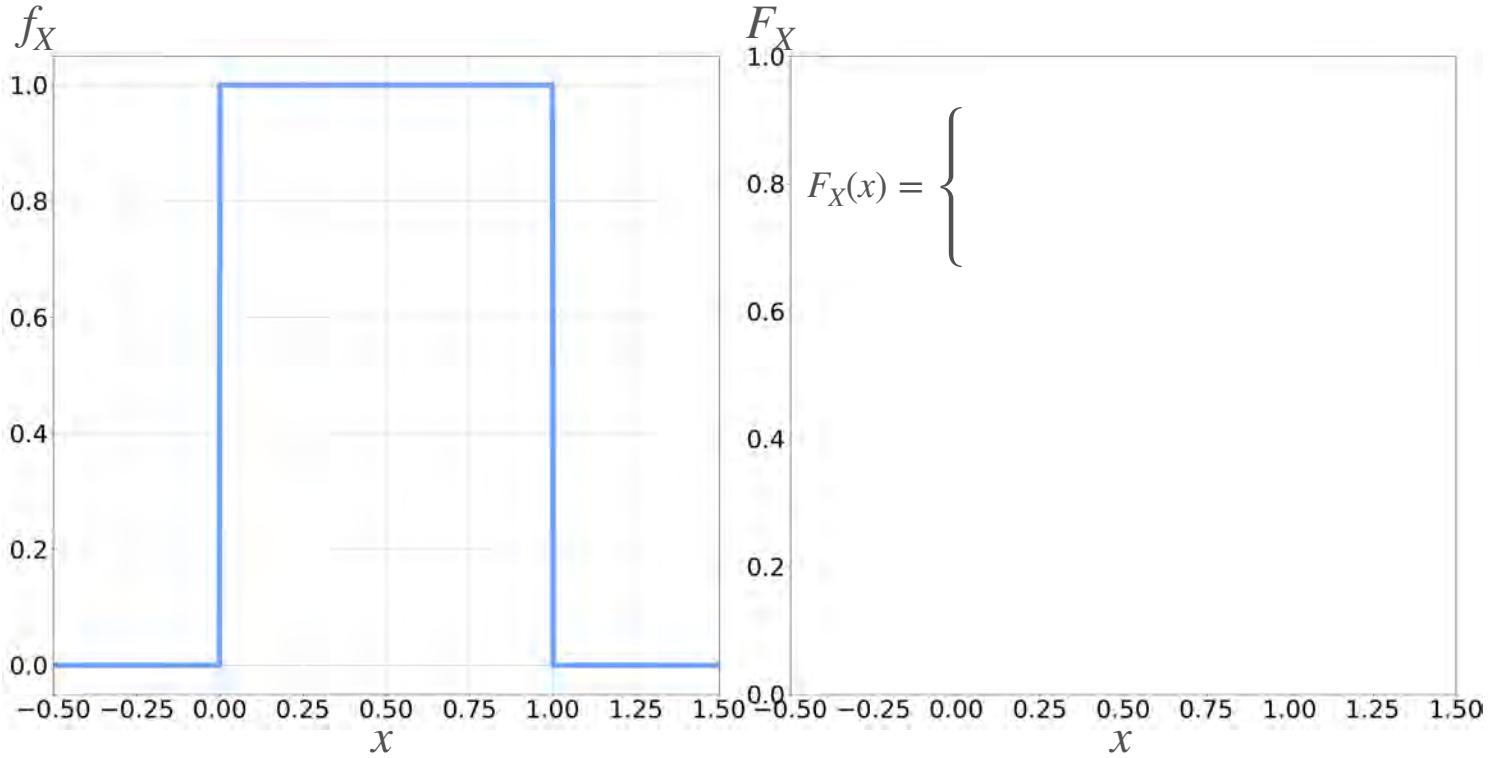
What is the probability of X being between -1 and 3?

What is the probability of X being between -1 and 4?

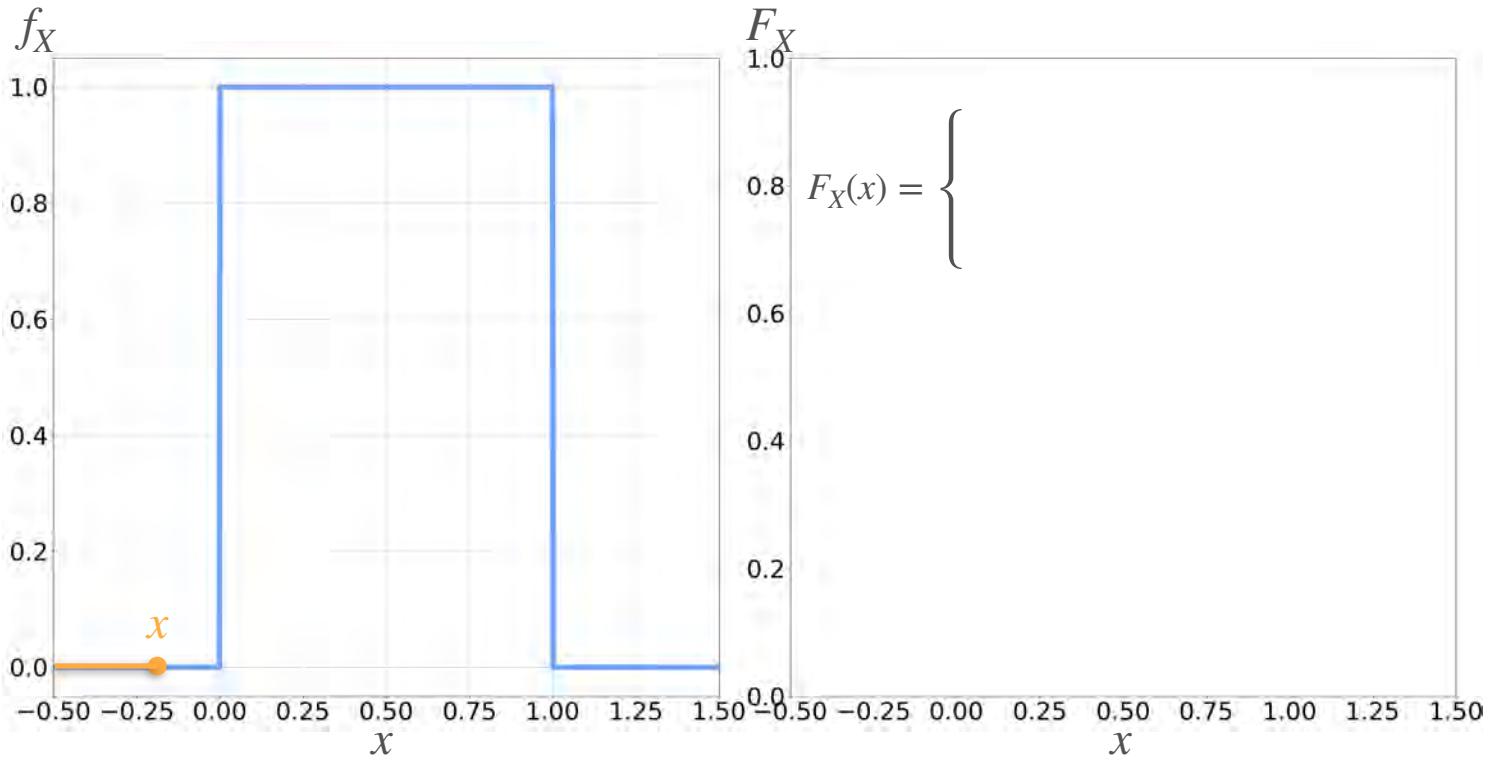


# Uniform Distribution: CDF

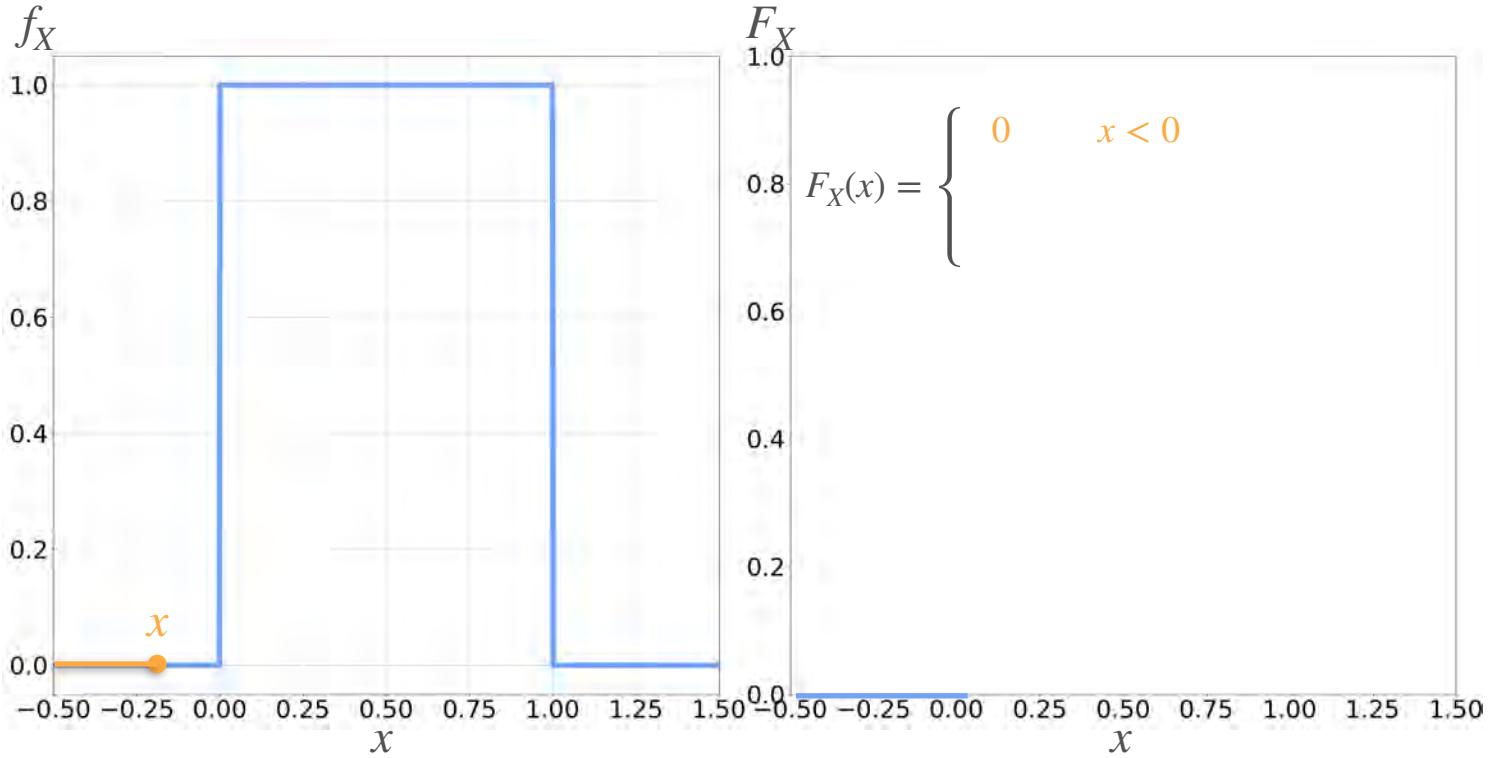
# Uniform Distribution: CDF



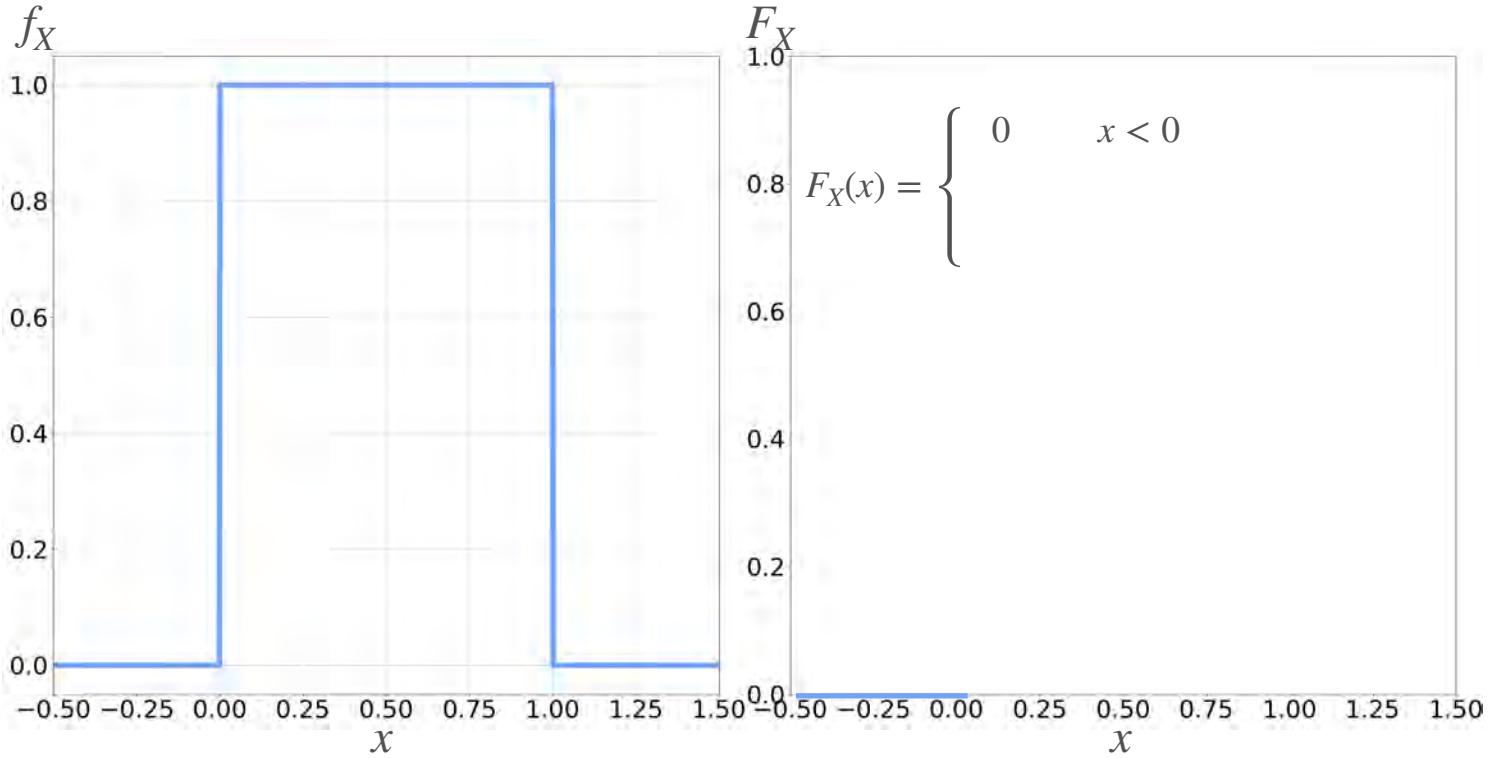
# Uniform Distribution: CDF



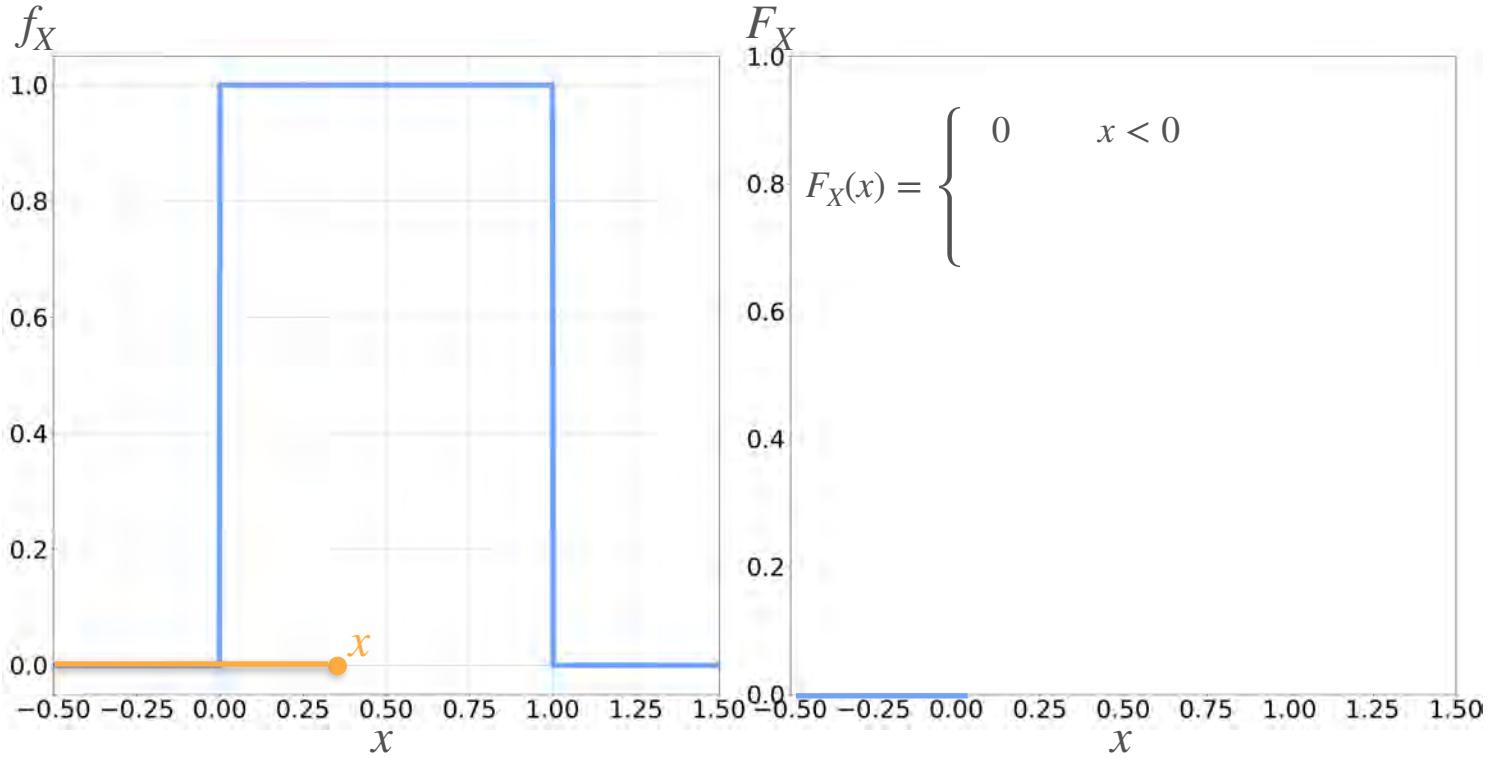
# Uniform Distribution: CDF



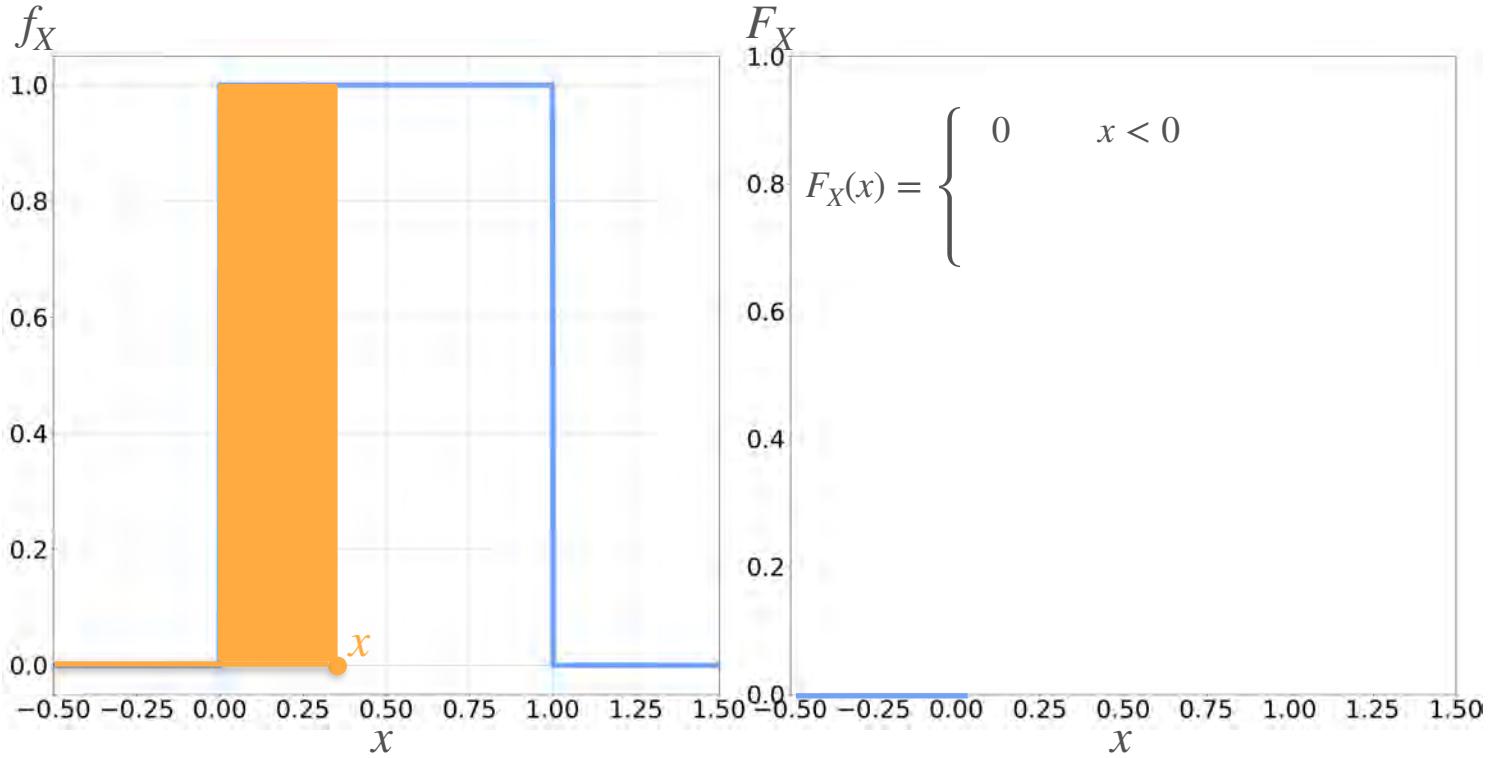
# Uniform Distribution: CDF



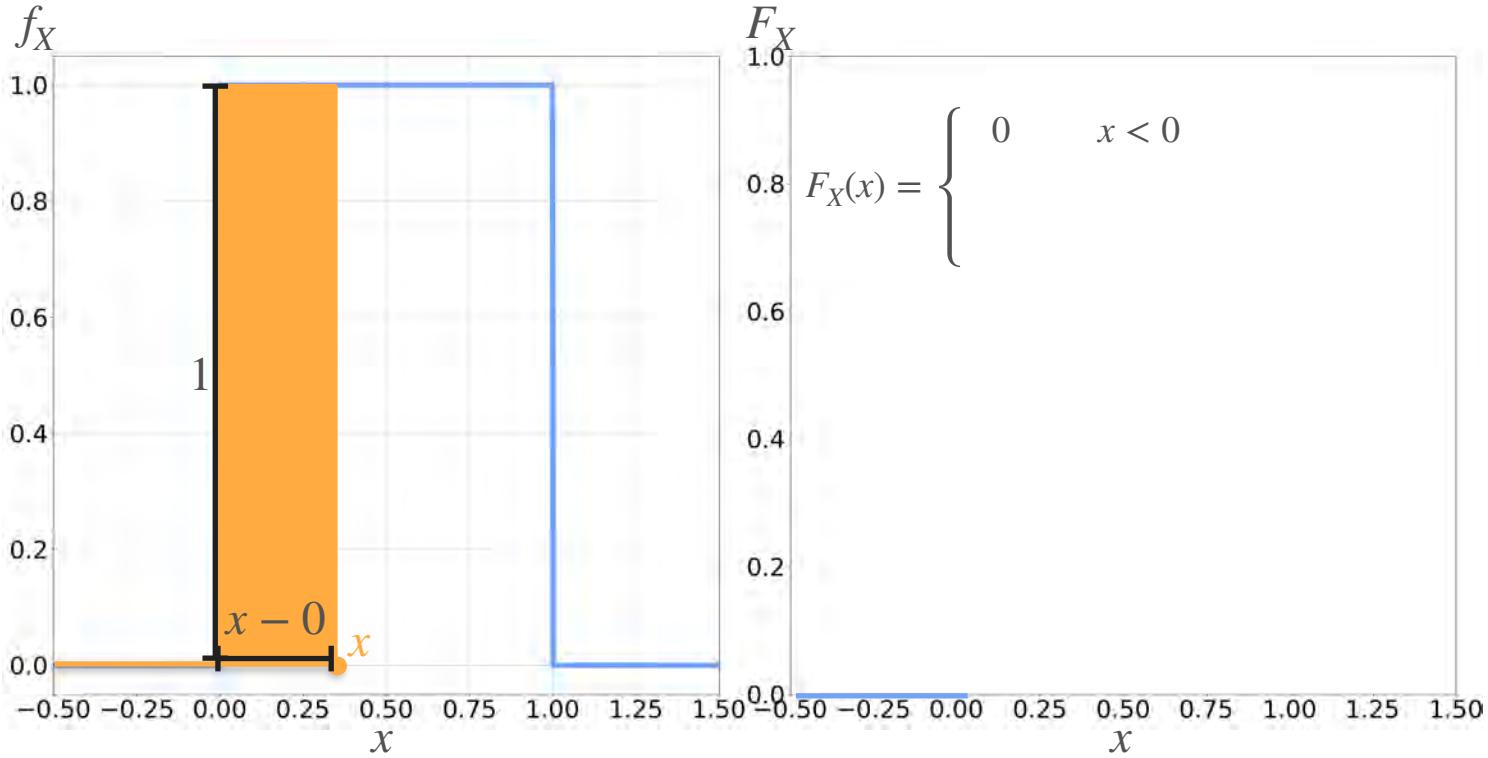
# Uniform Distribution: CDF



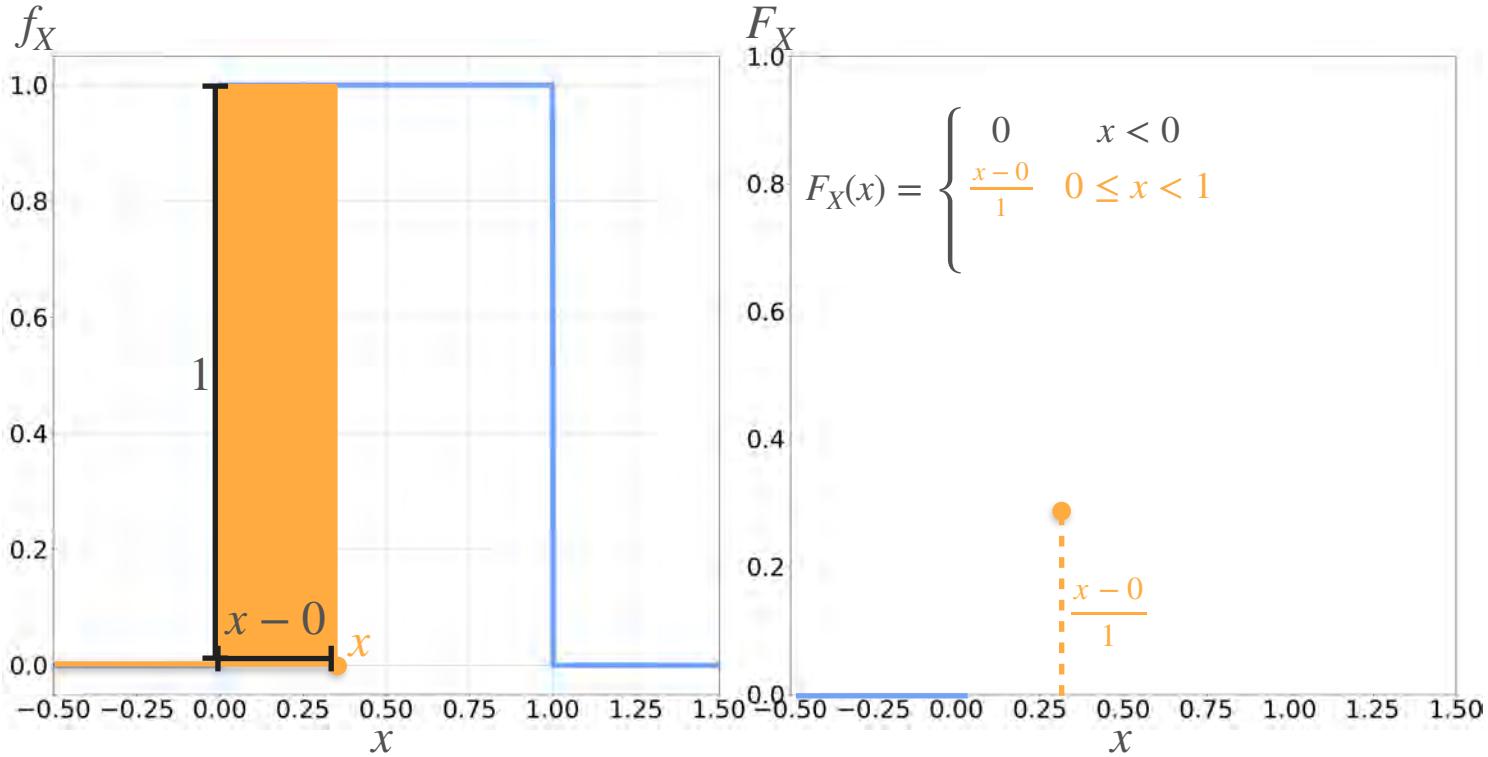
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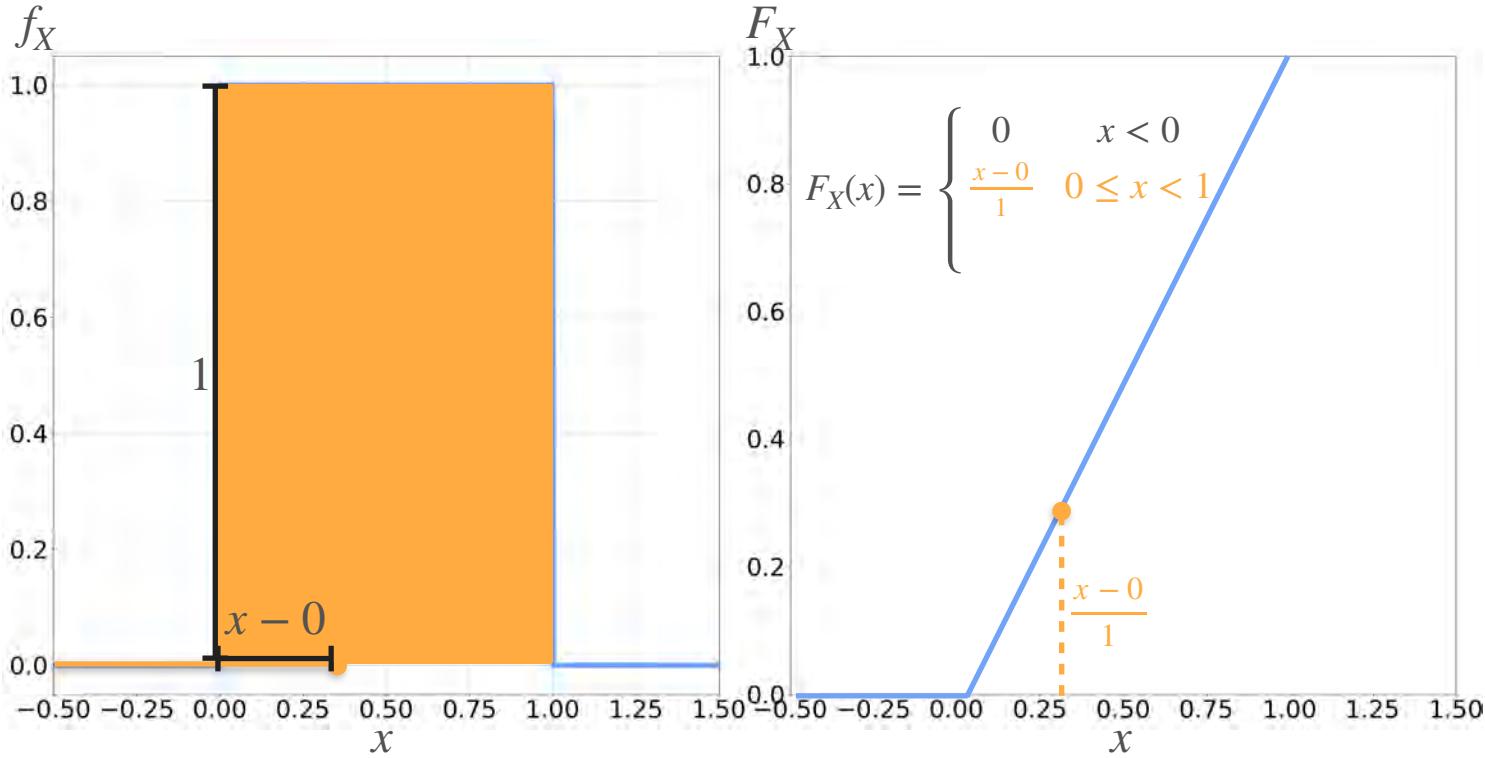
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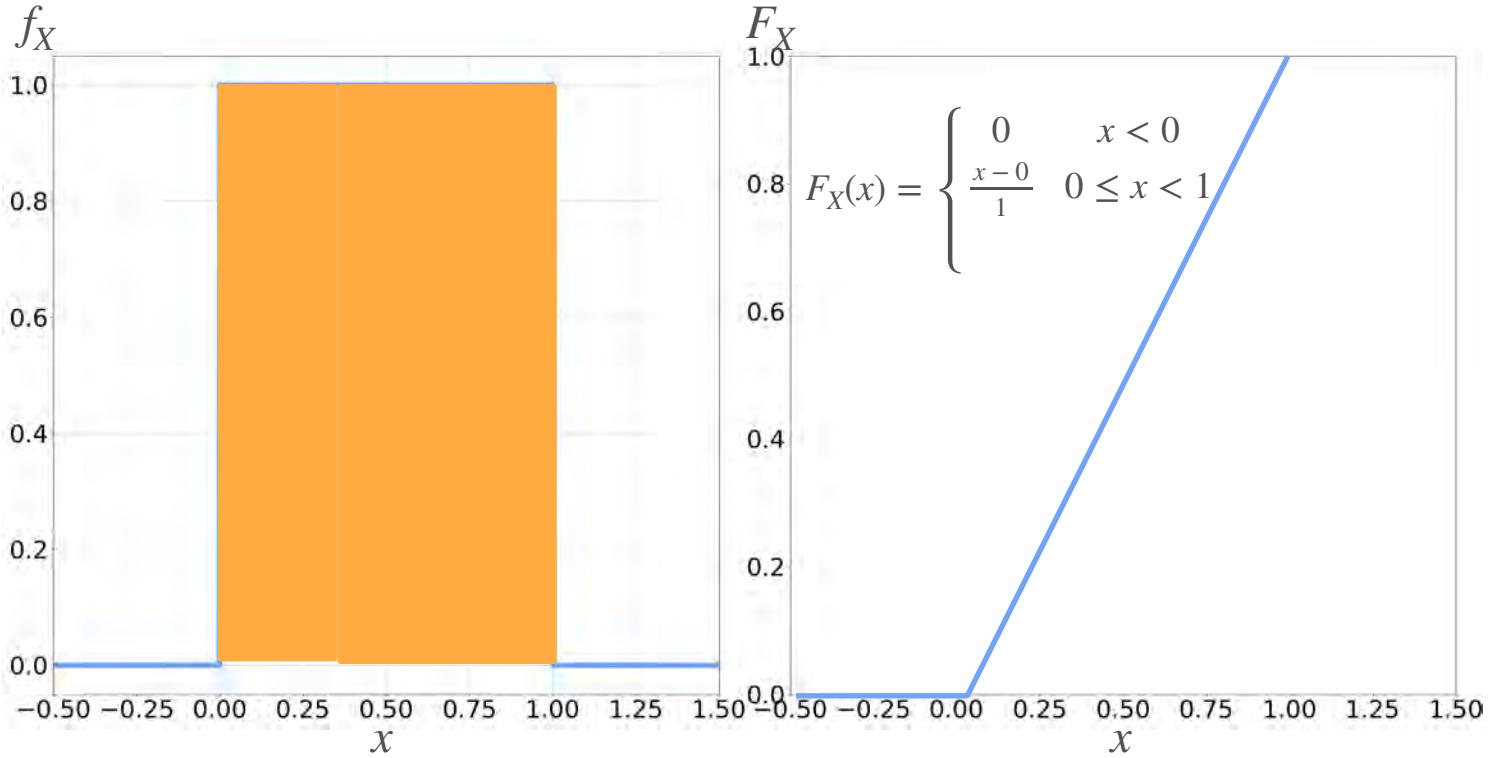
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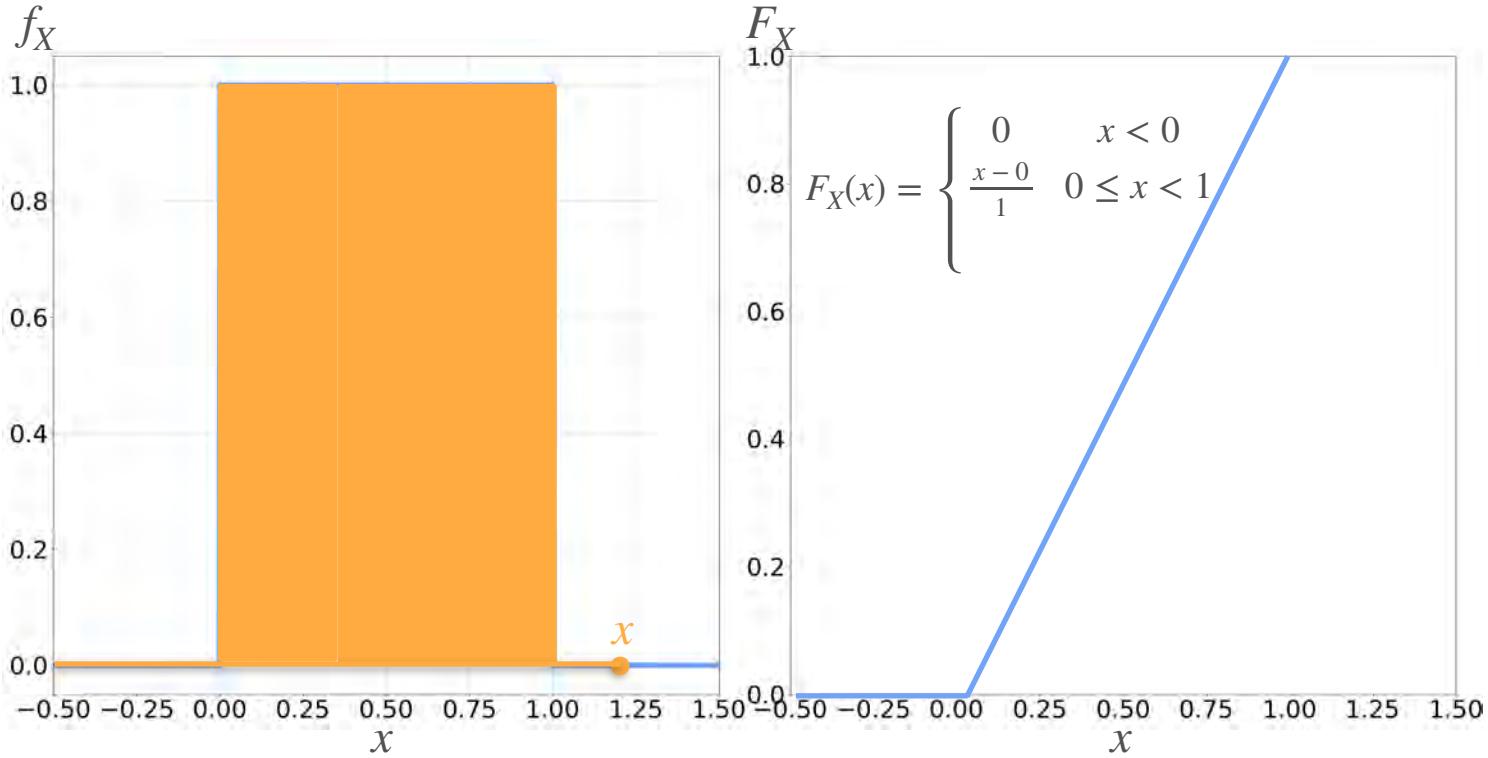
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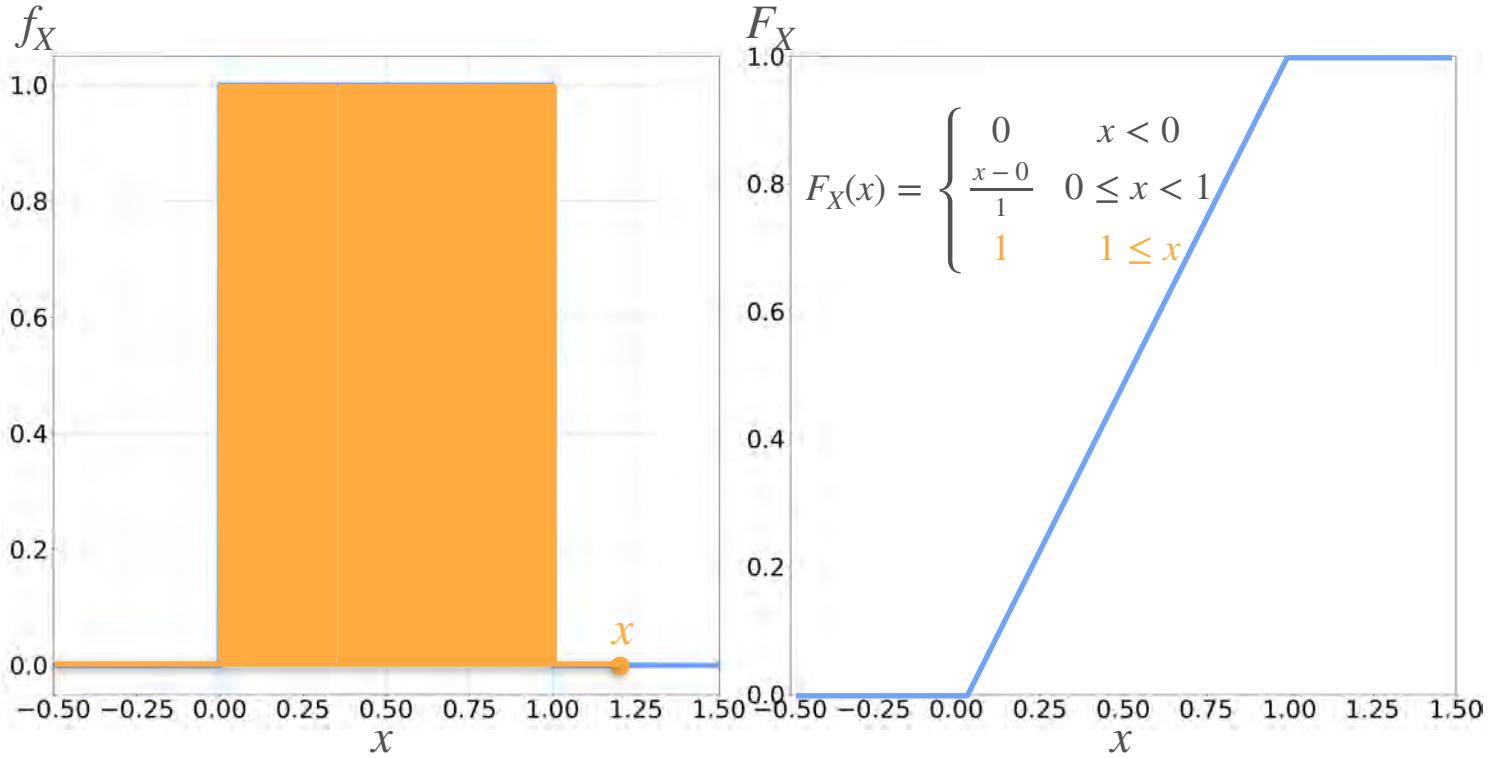
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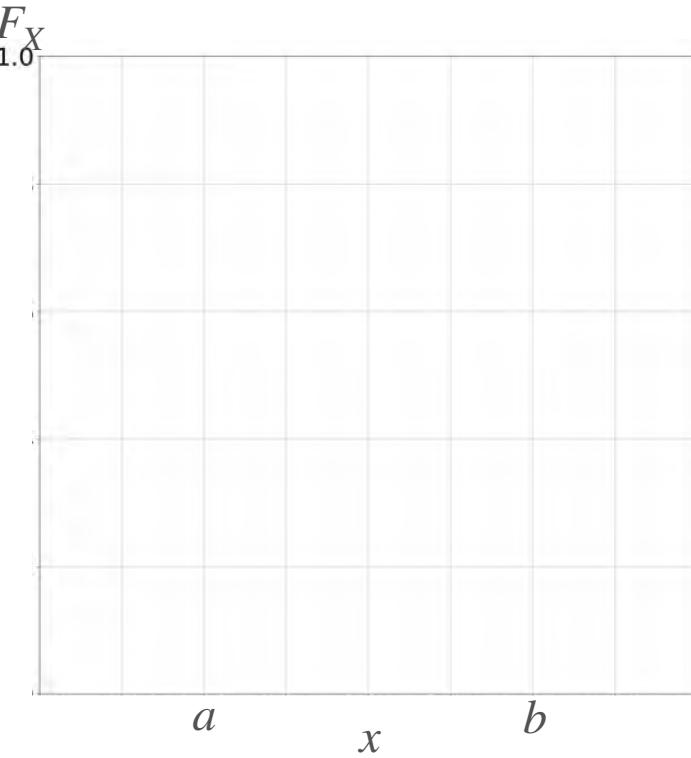
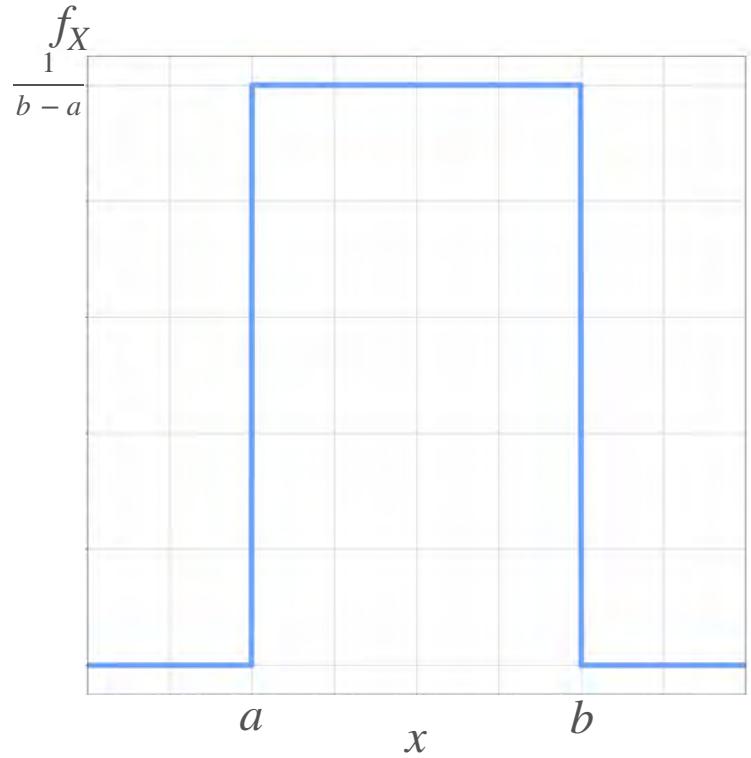
# Uniform Distribution: CDF



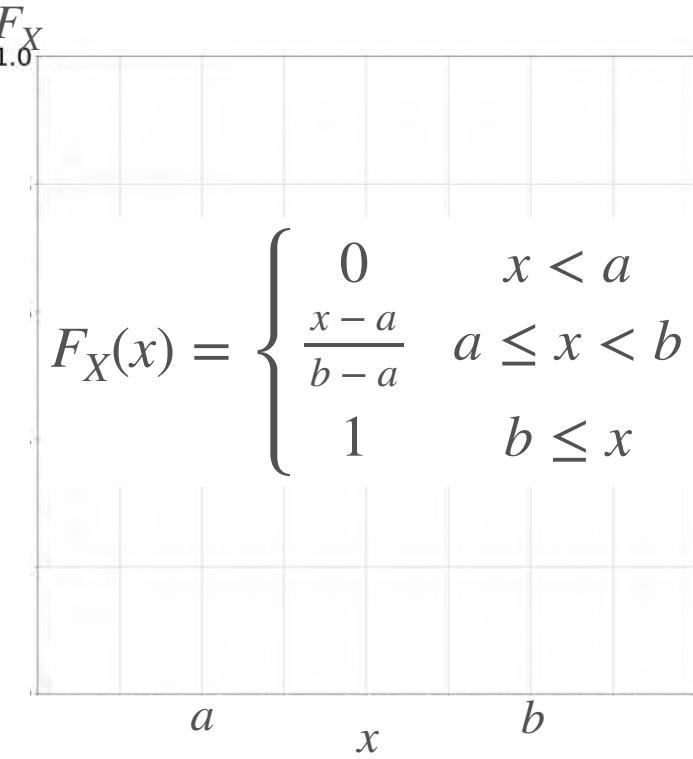
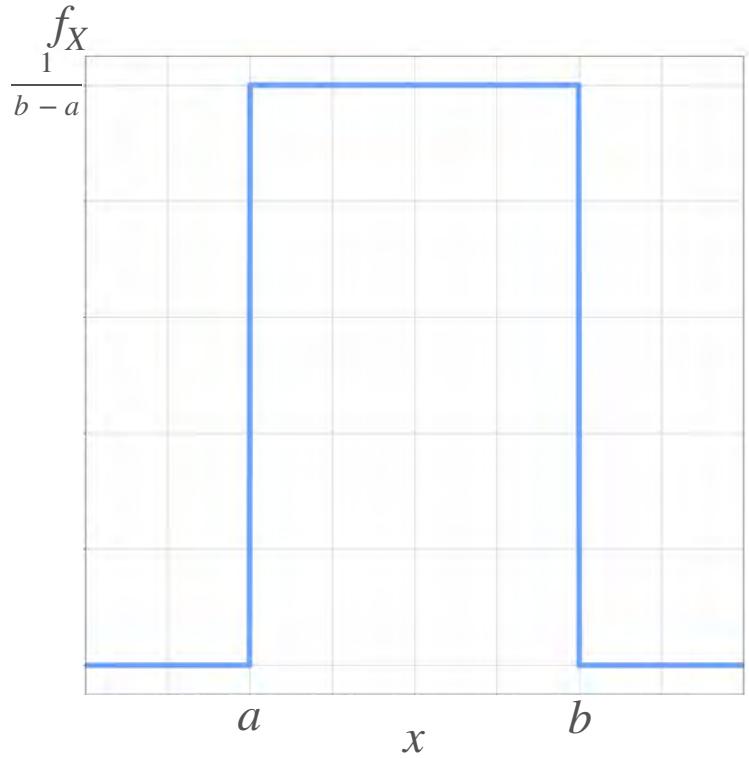
# Uniform Distribution: CDF



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# Uniform Distribution: CDF





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# Probability Distributions

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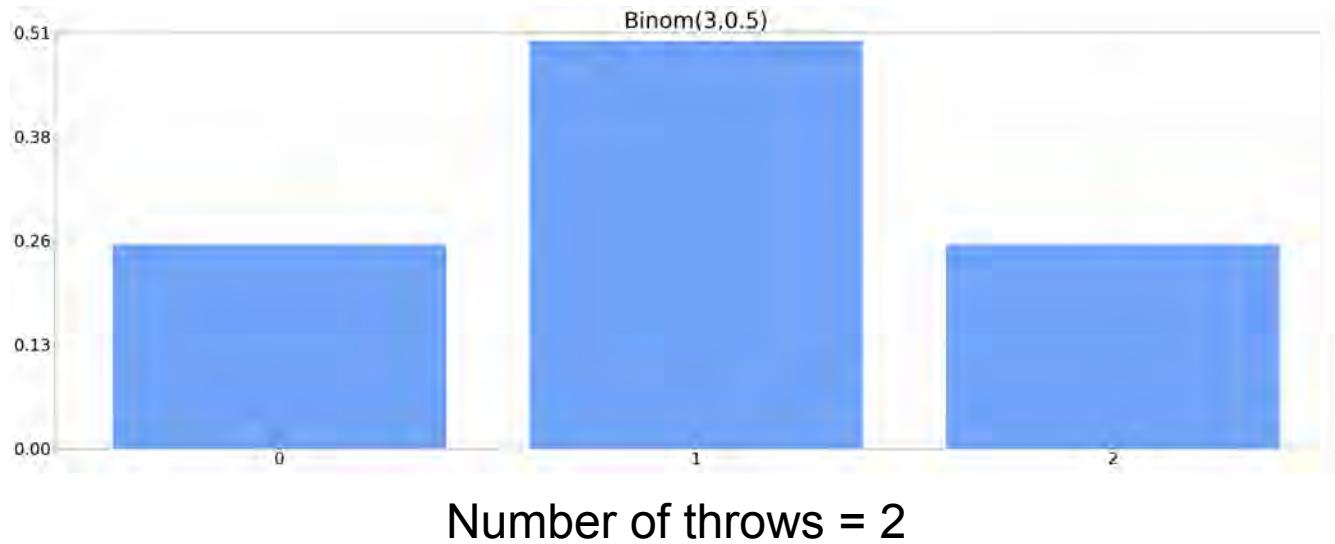
## Normal distribution

# Binomial Distribution With Very Large $n$

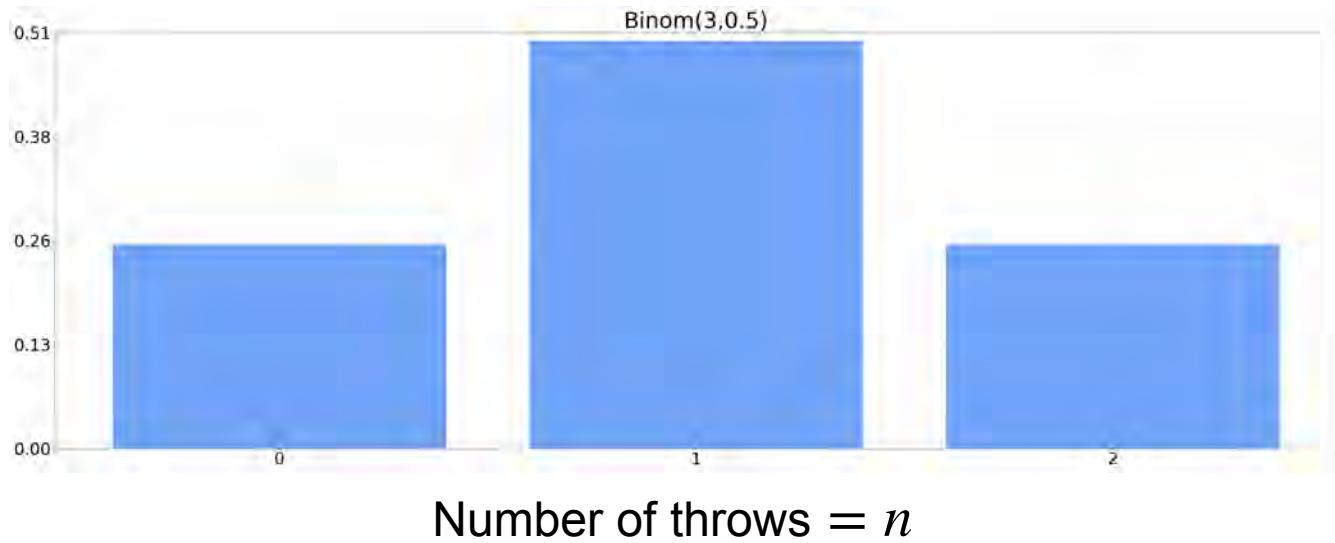
# Binomial Distribution With Very Large $n$



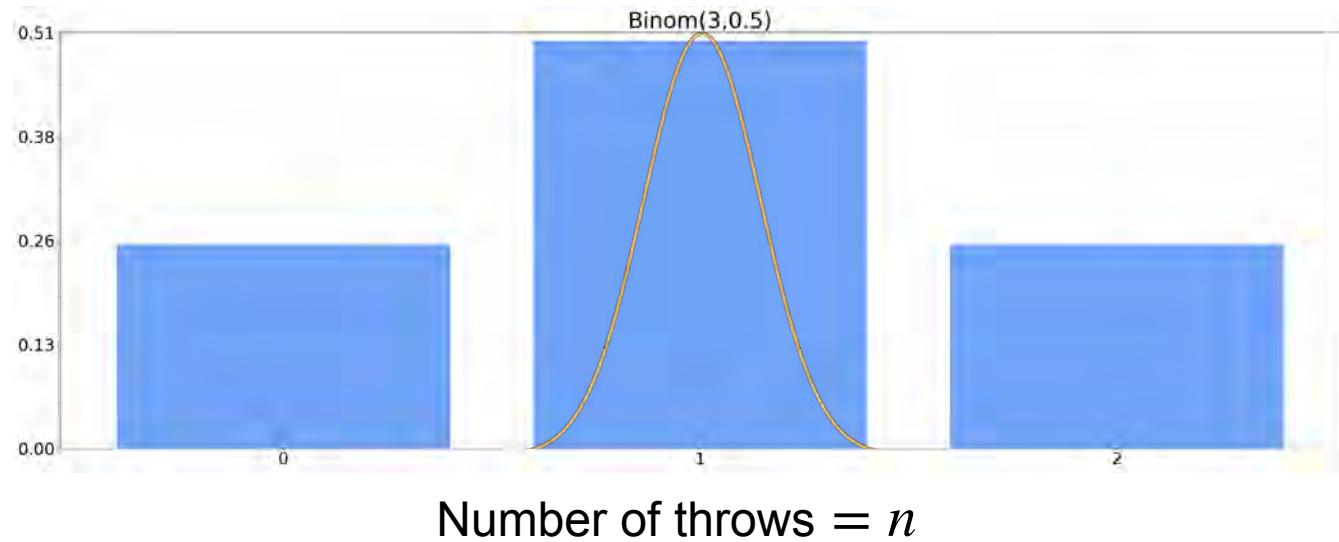
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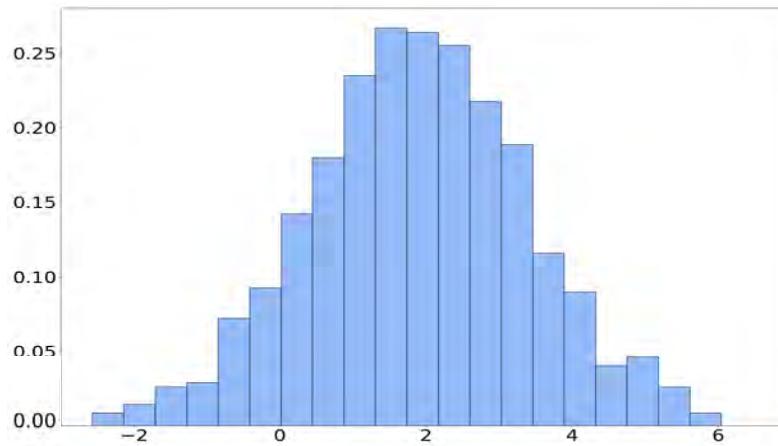
# Binomial Distribution With Very Large $n$



# Bell Shaped Data

# Bell Shaped Data

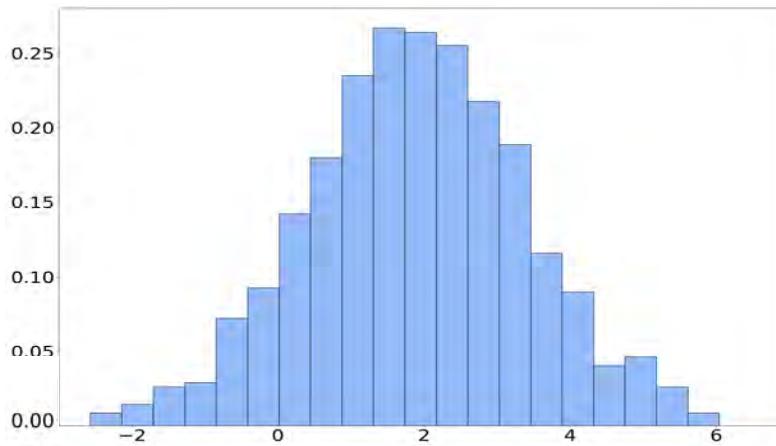
Data



# Bell Shaped Data

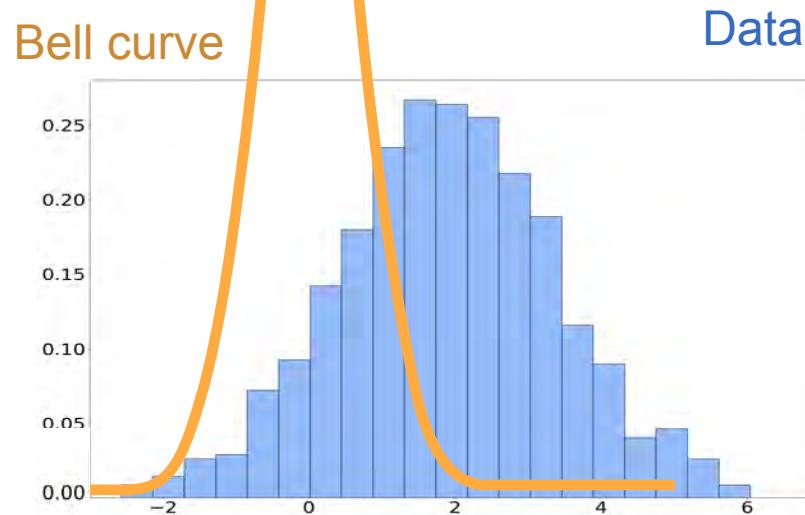
$$e^{-x^2}$$

Data



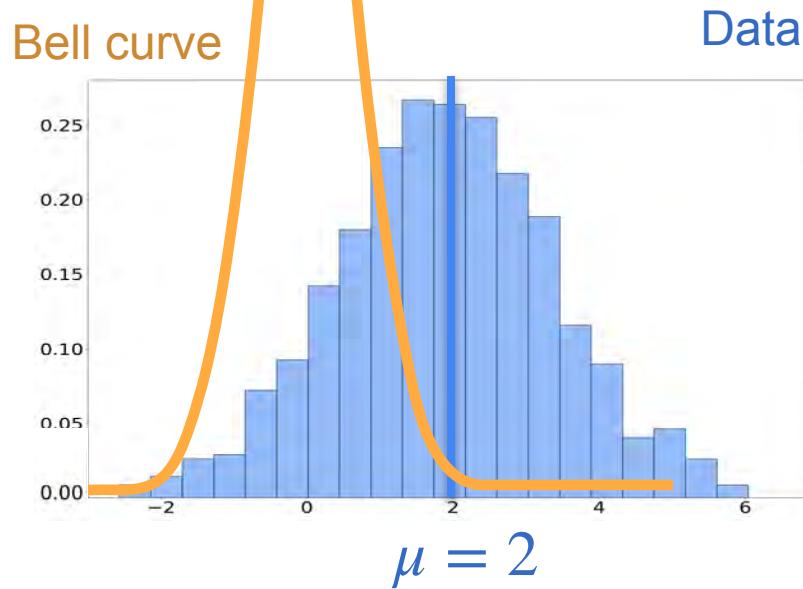
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$$e^{-x^2}$$



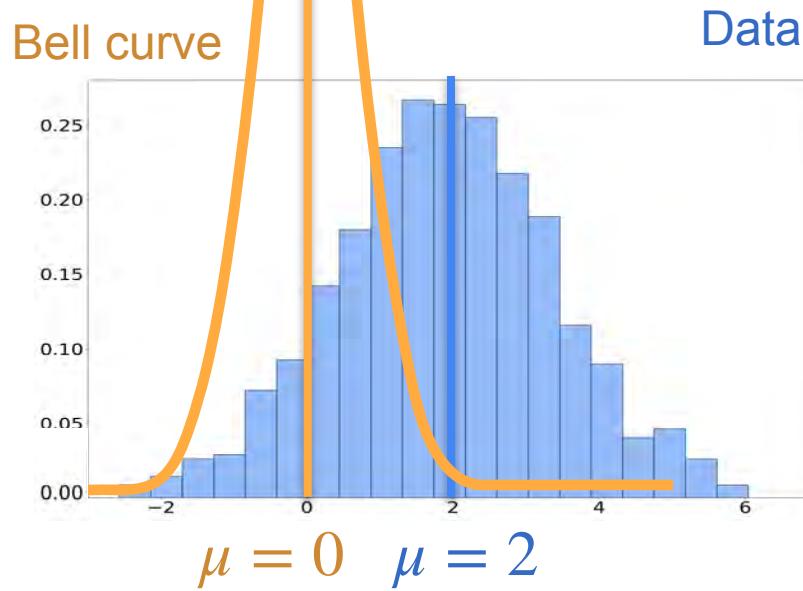
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$$e^{-x^2}$$



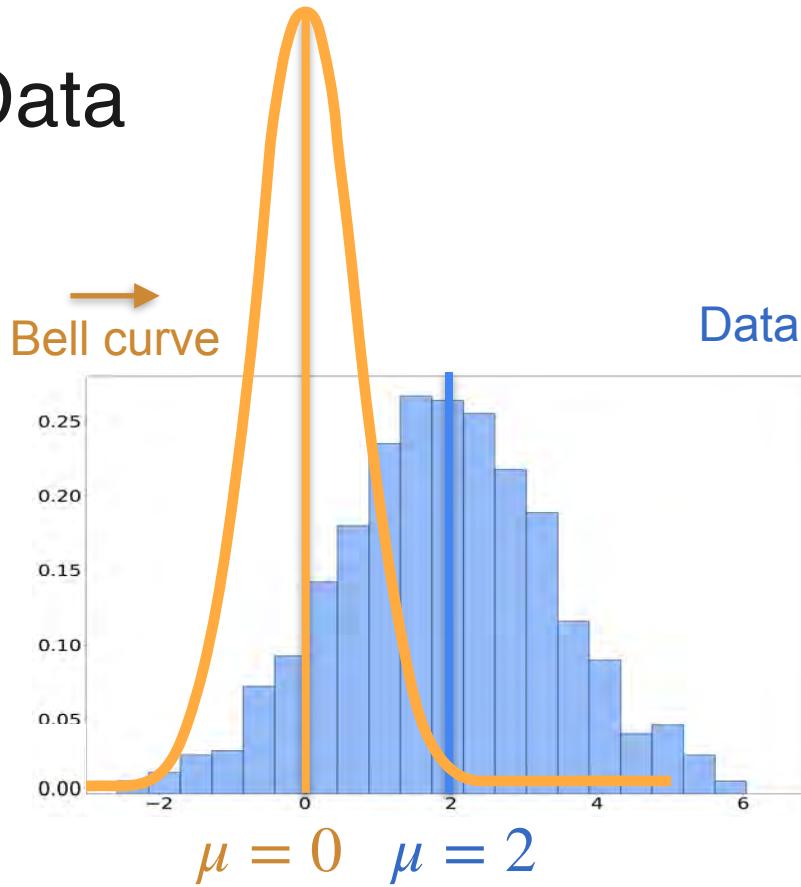
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$$e^{-x^2}$$



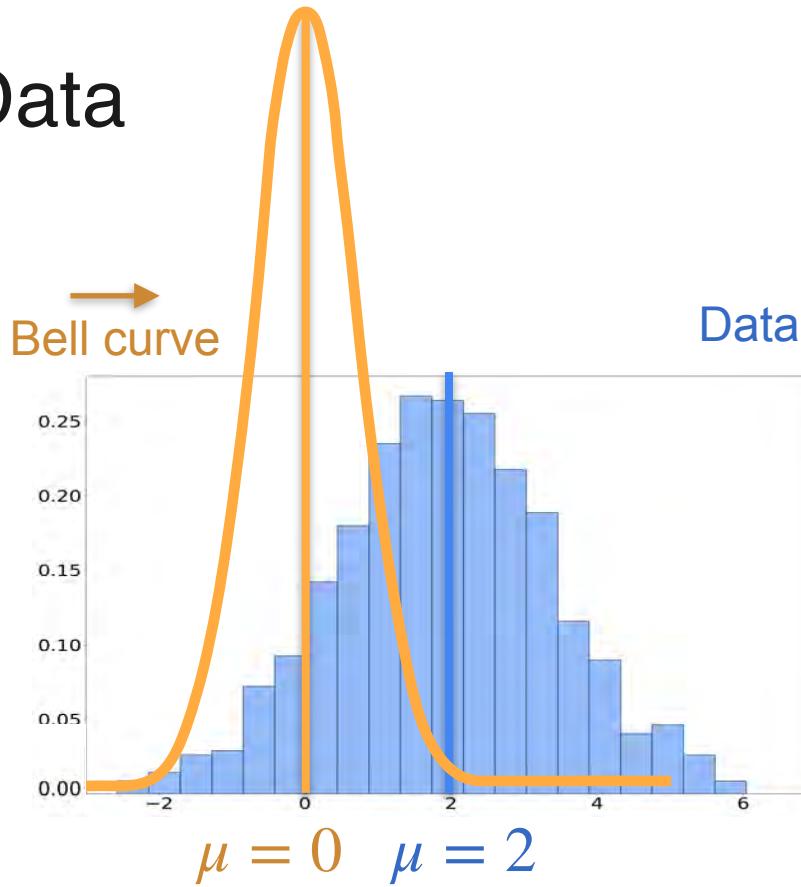
# Bell Shaped Data

$$e^{-x^2}$$



# Bell Shaped Data

$$e^{-(x-2)^2}$$

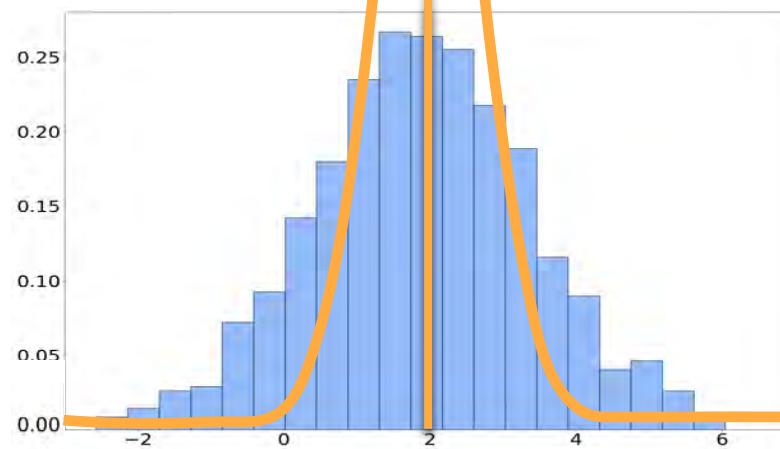


# Bell Shaped Data

$$e^{-(x-2)^2}$$

Bell curve

Data



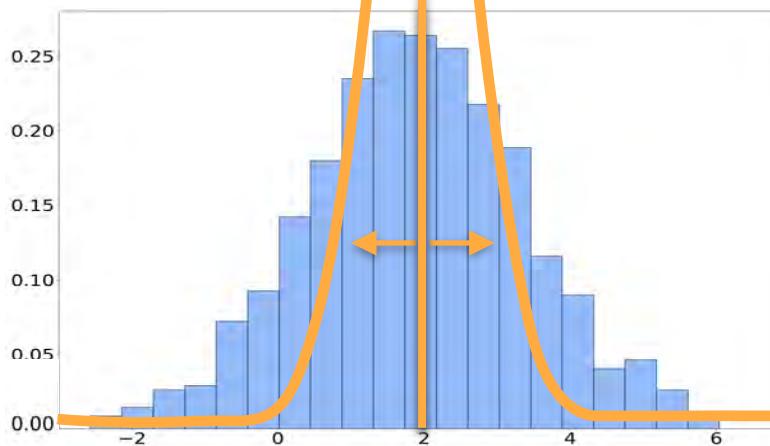
# Bell Shaped Data

$$e^{-(x-2)^2}$$

Bell curve

Data

$$\sigma = 1$$

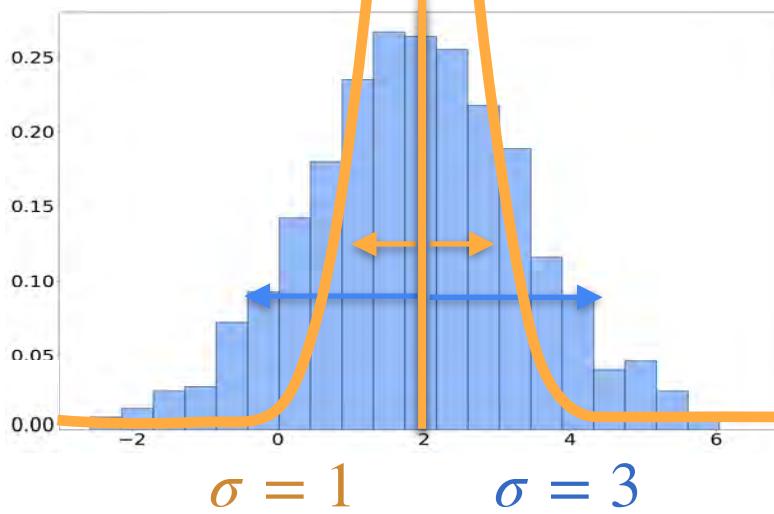


# Bell Shaped Data

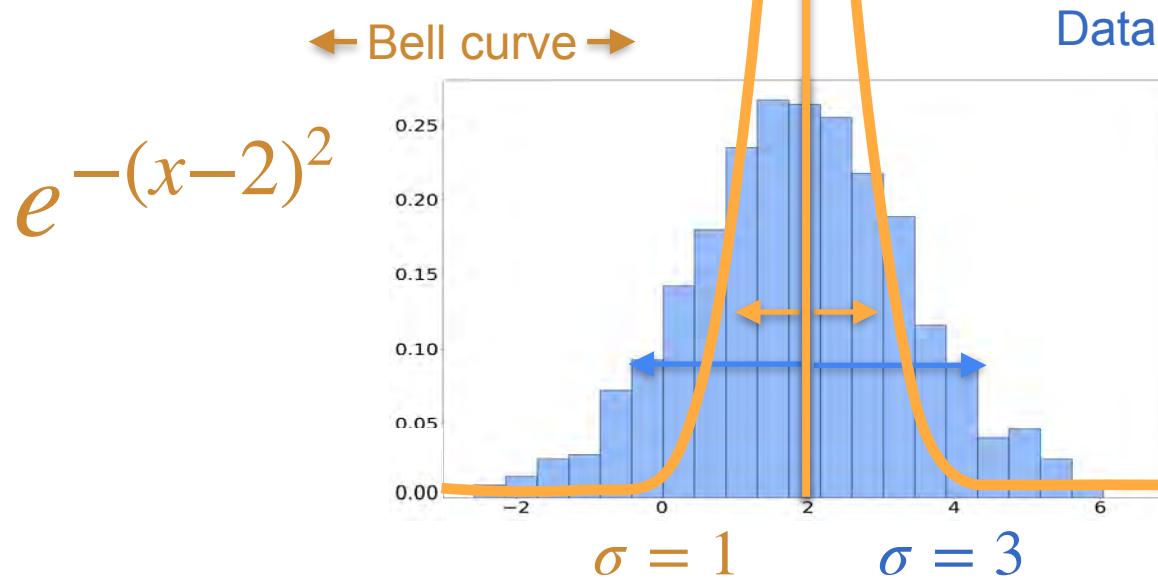
$$e^{-(x-2)^2}$$

Bell curve

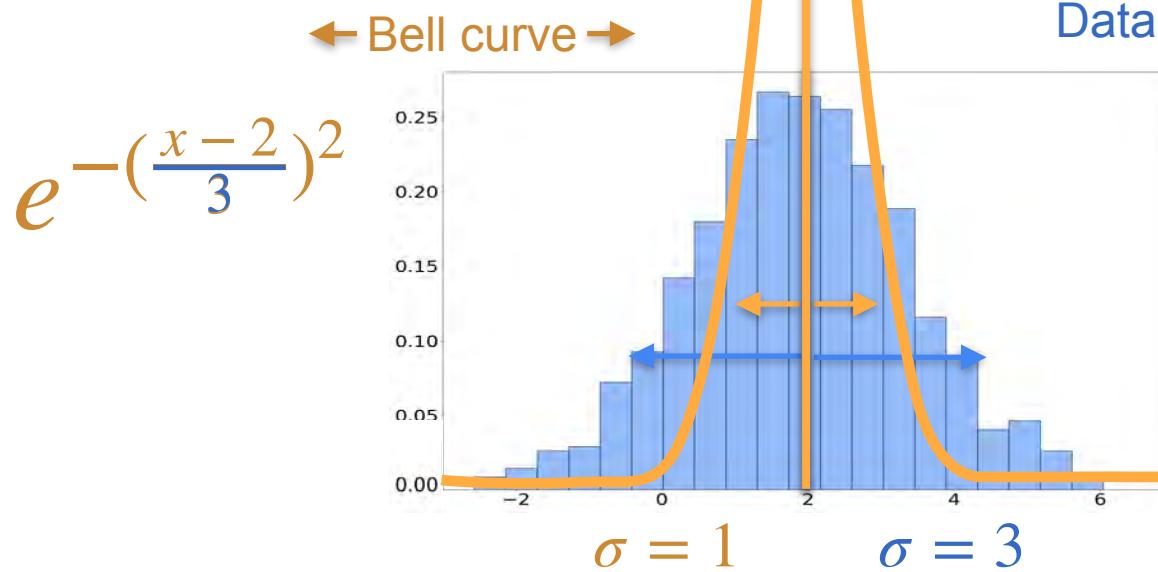
Data



# Bell Shaped Data



# Bell Shaped Data

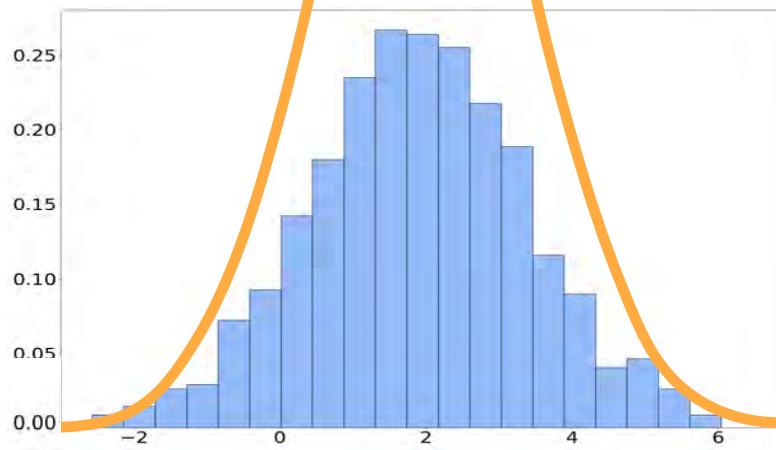


# Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data

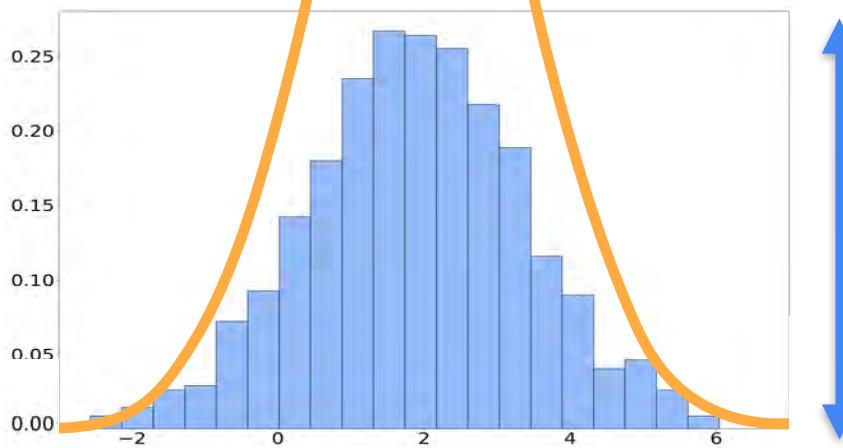


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$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data

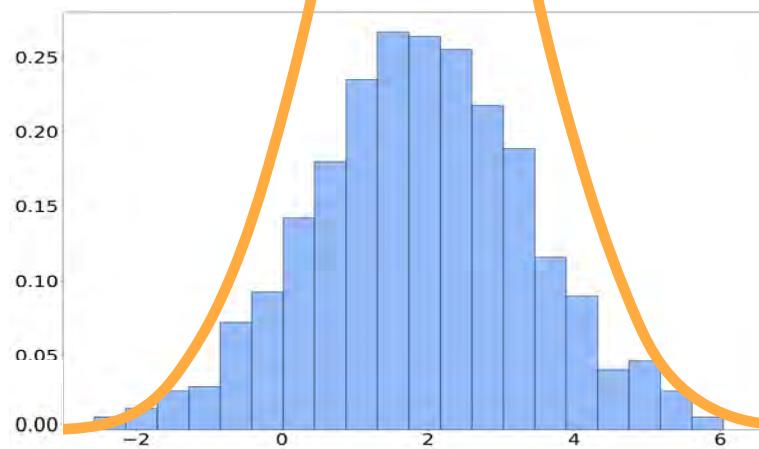


# Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data

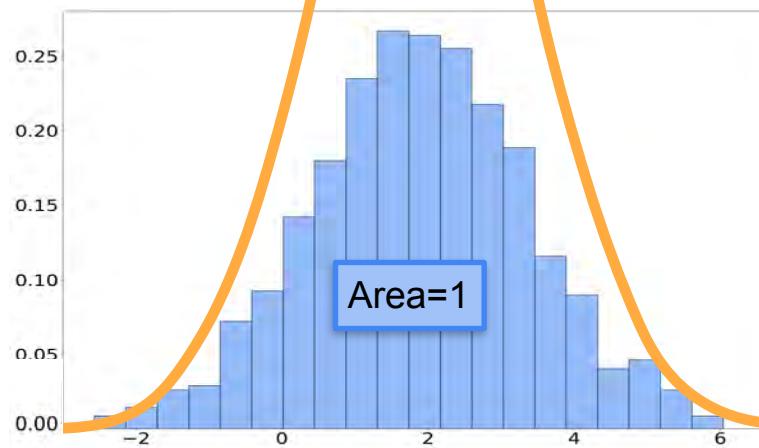


# Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Data



# Bell Shaped Data

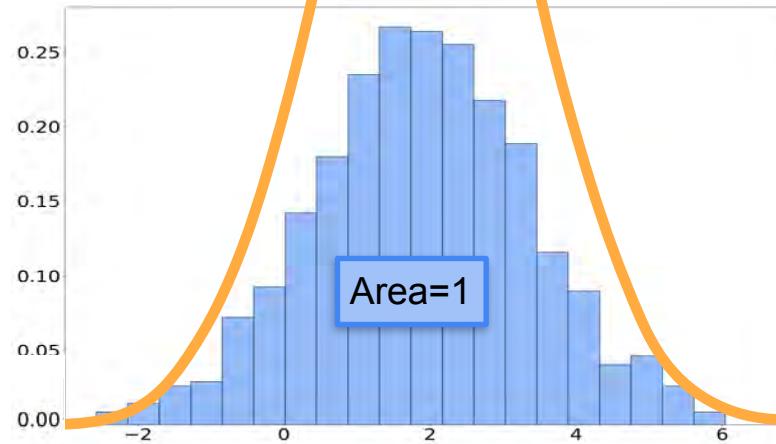
$$e^{-(\frac{x-2}{3})^2}$$

Bell curve

Area=??

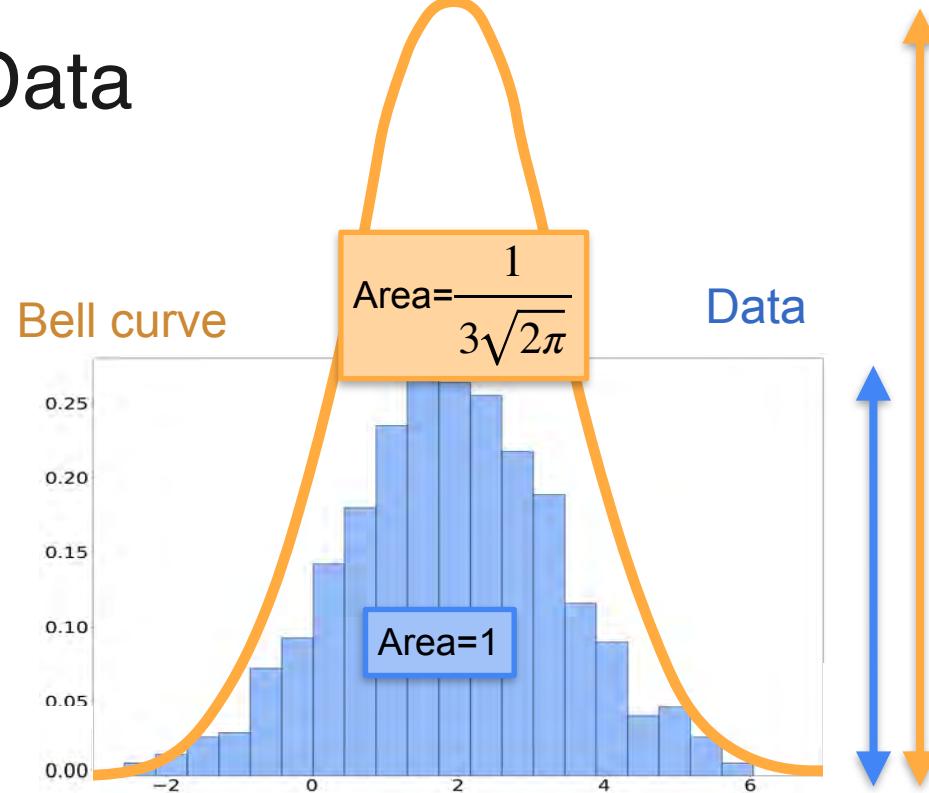
Data

Area=1



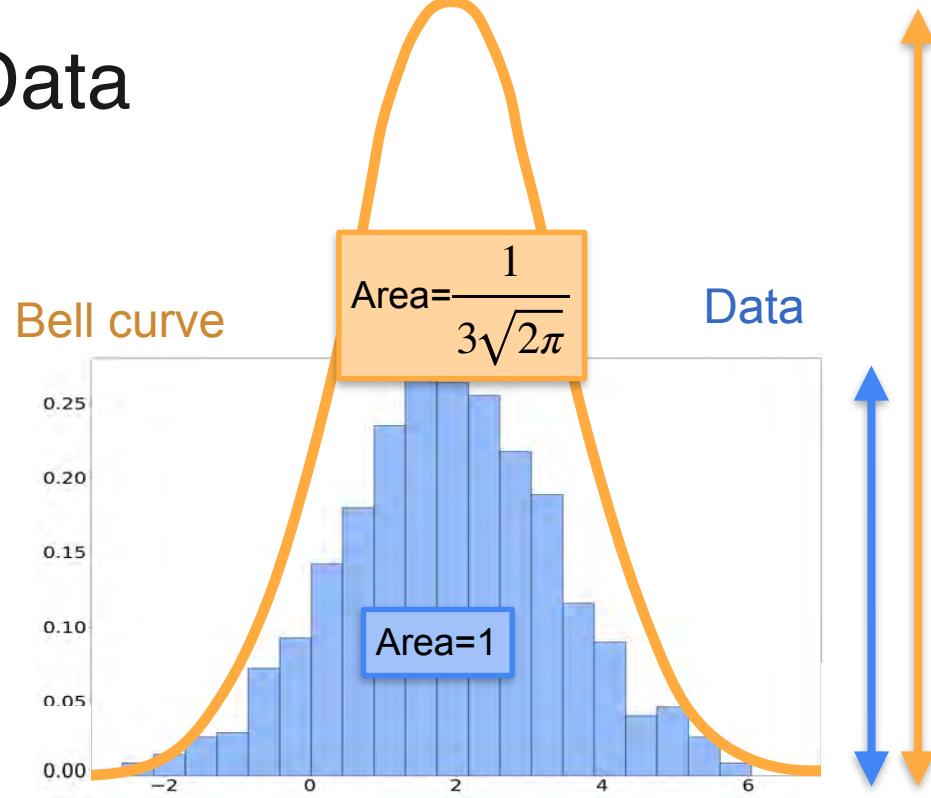
# Bell Shaped Data

$$e^{-(\frac{x-2}{3})^2}$$



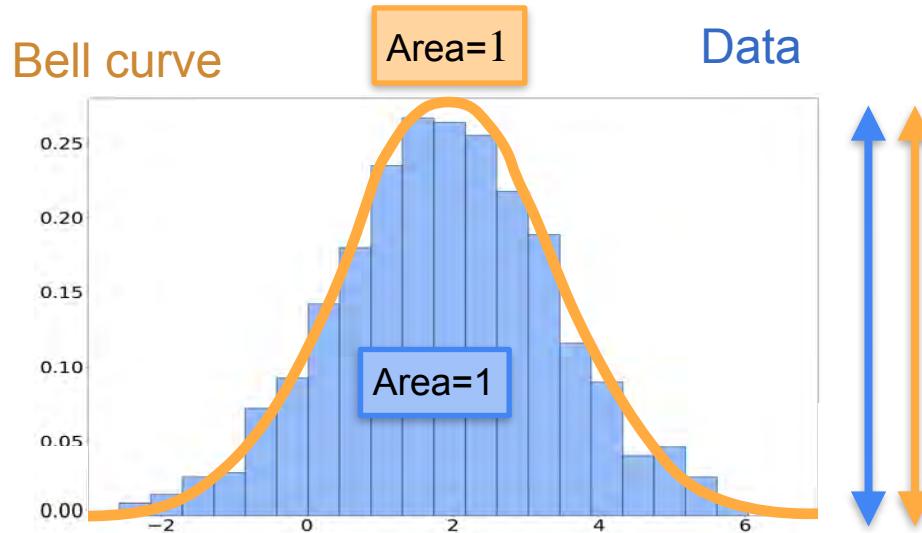
# Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}} e^{-(\frac{x-2}{3})^2}$$

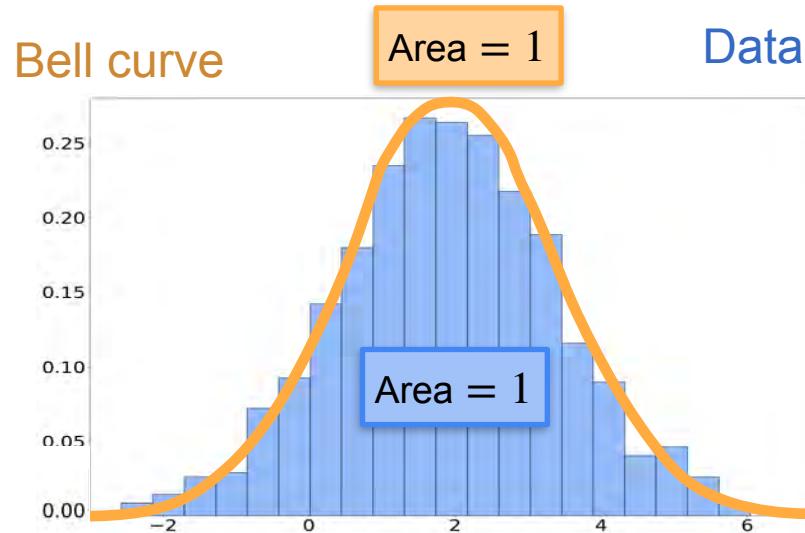


# Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}} e^{-(\frac{x-2}{3})^2}$$



# Bell Shaped Data



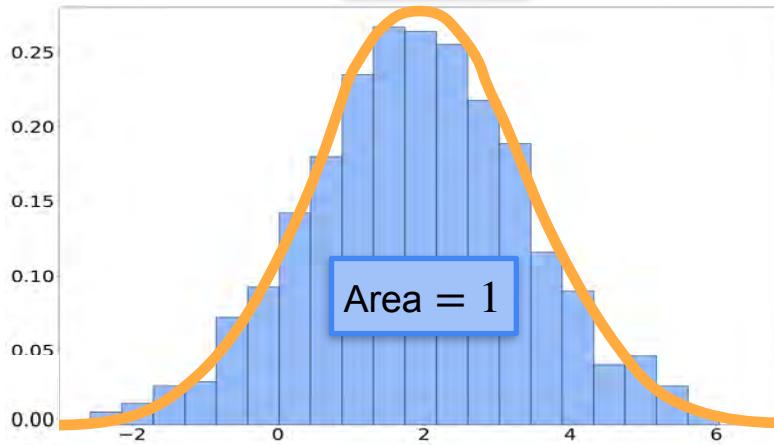
# Bell Shaped Data

Mean =  $\mu$

Bell curve

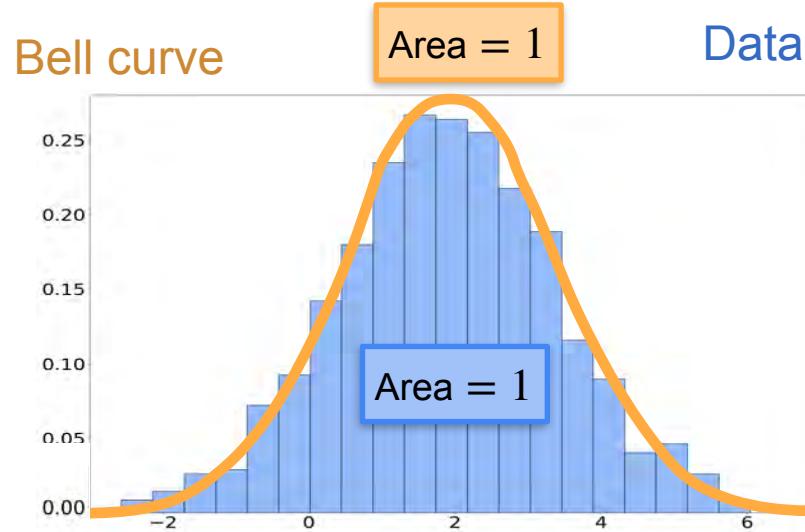
Area = 1

Data



# Bell Shaped Data

Bell curve  
Mean =  $\mu$   
Standard deviation =  $\sigma$

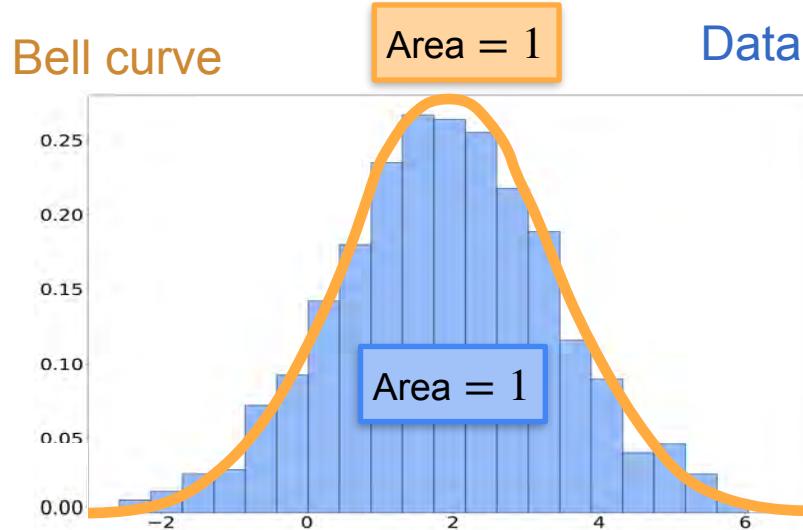


# Bell Shaped Data

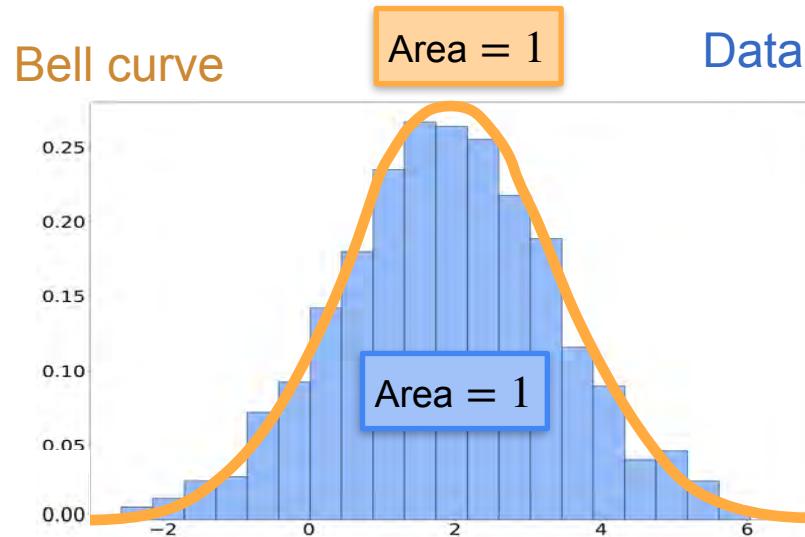
Mean =  $\mu$

Standard deviation =  $\sigma$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$



# Bell Shaped Data



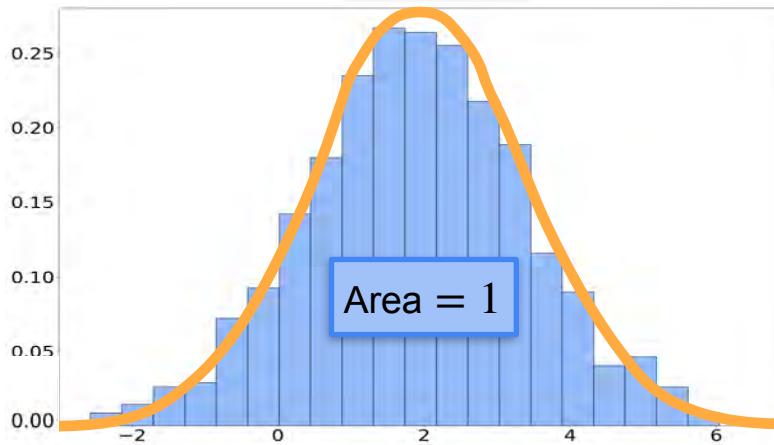
# Bell Shaped Data

Mean =  $\mu$

Bell curve

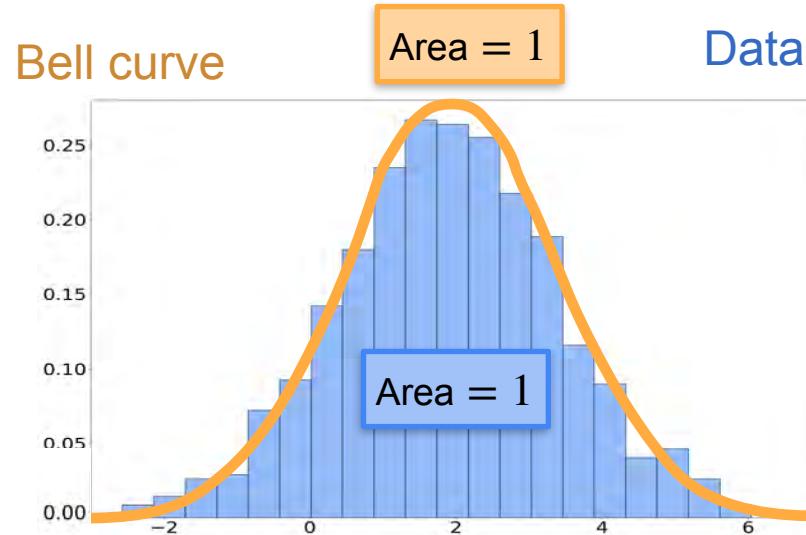
Area = 1

Data



# Bell Shaped Data

Bell curve  
Mean =  $\mu$   
Standard deviation =  $\sigma$

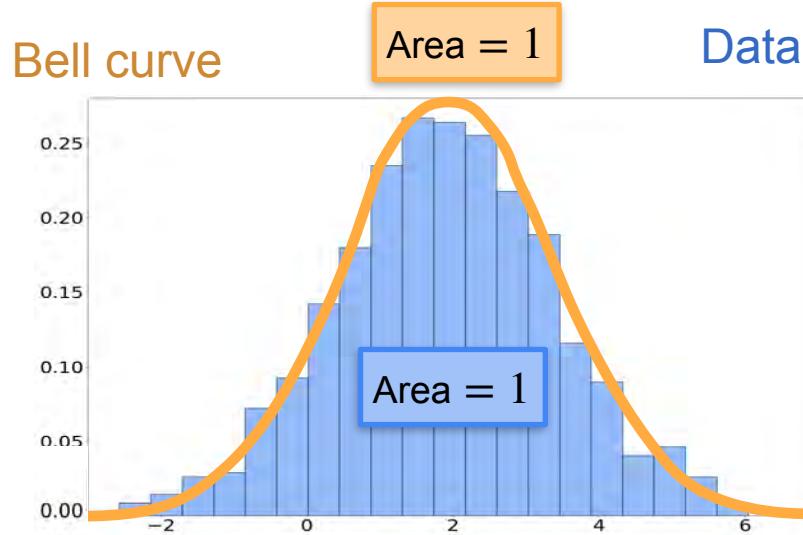


# Bell Shaped Data

Mean =  $\mu$

Standard deviation =  $\sigma$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$

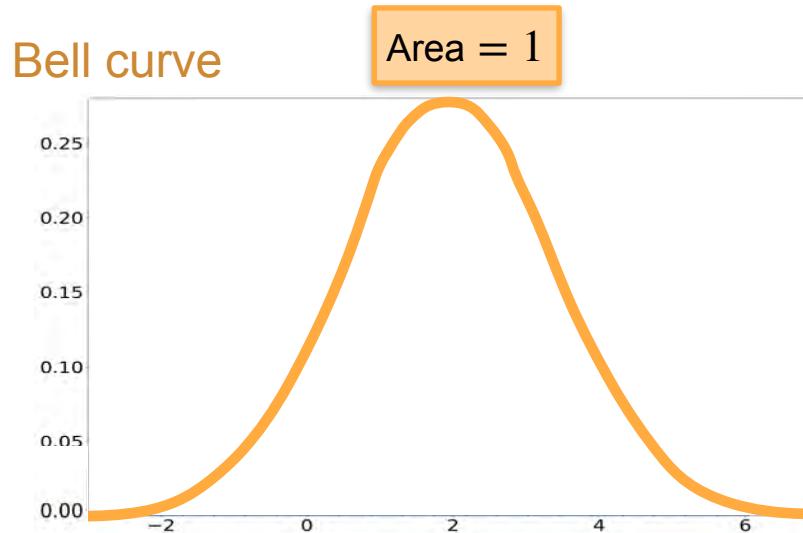


# Normal Distribution

Mean =  $\mu$

Standard deviation =  $\sigma$

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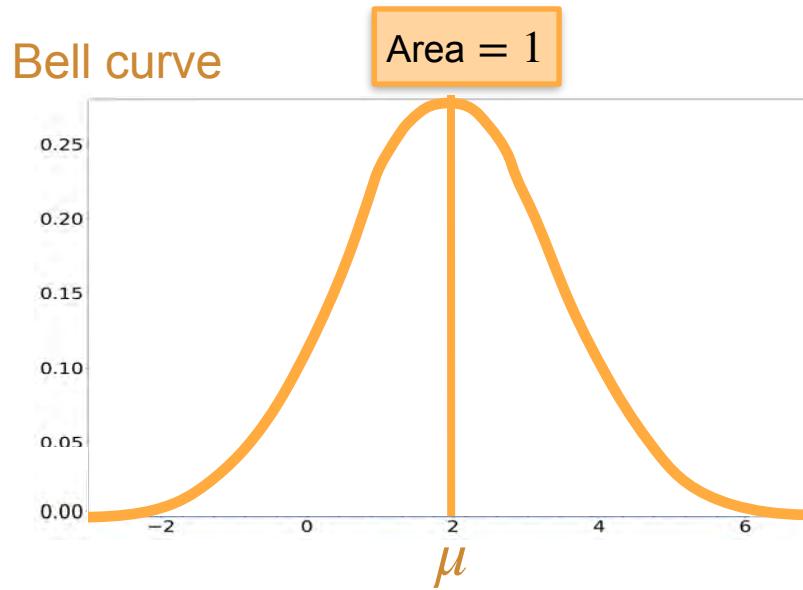


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Mean =  $\mu$

Standard deviation =  $\sigma$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$

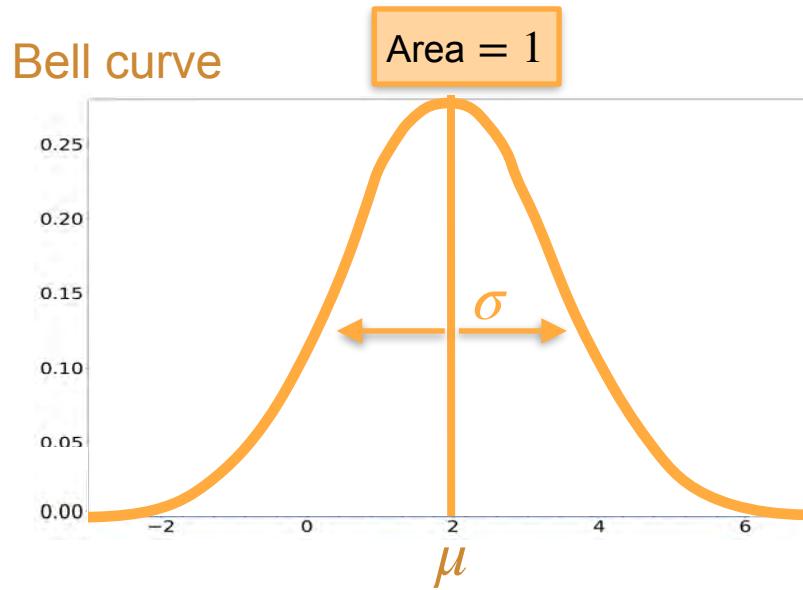


# Normal Distribution

Mean =  $\mu$

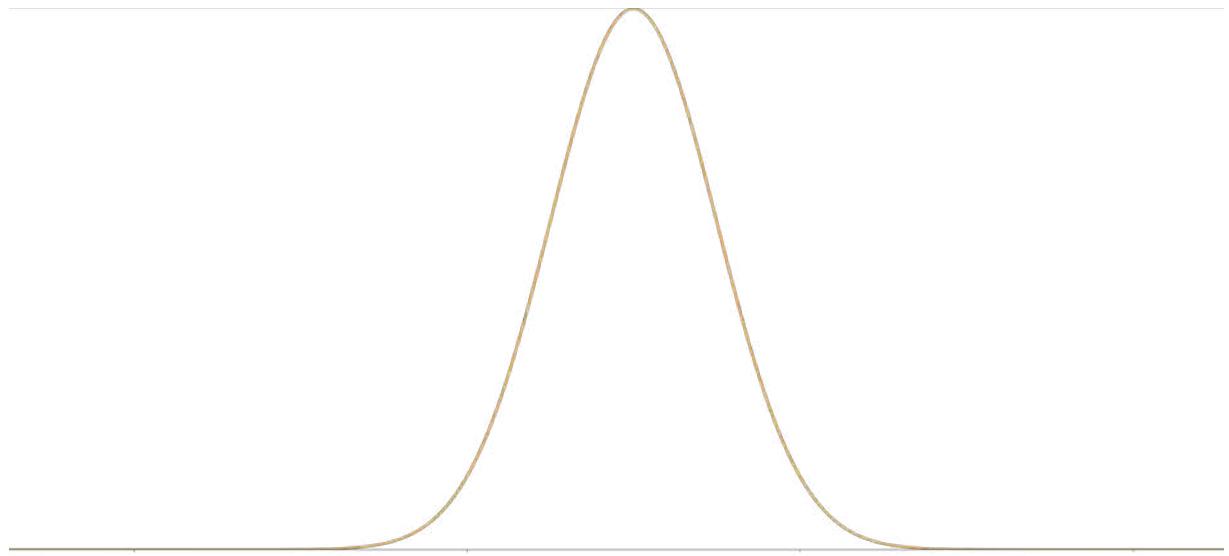
Standard deviation =  $\sigma$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-(\frac{x-\mu}{\sigma})^2}$$



# Normal Distribution

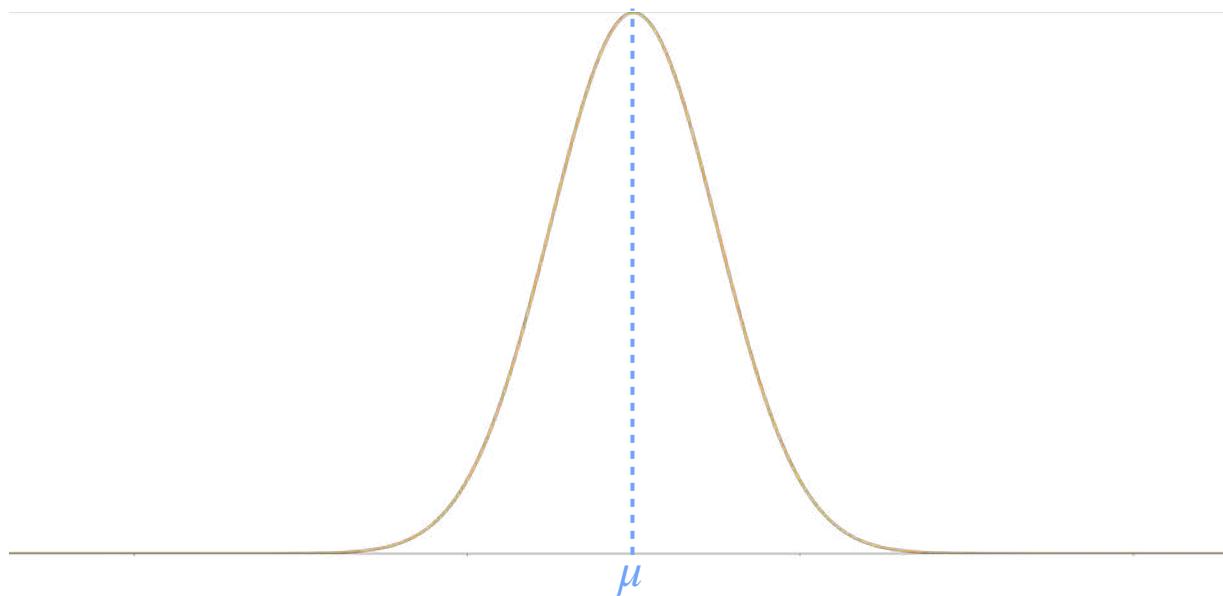
# Normal Distribution



# Normal Distribution

Parameters:

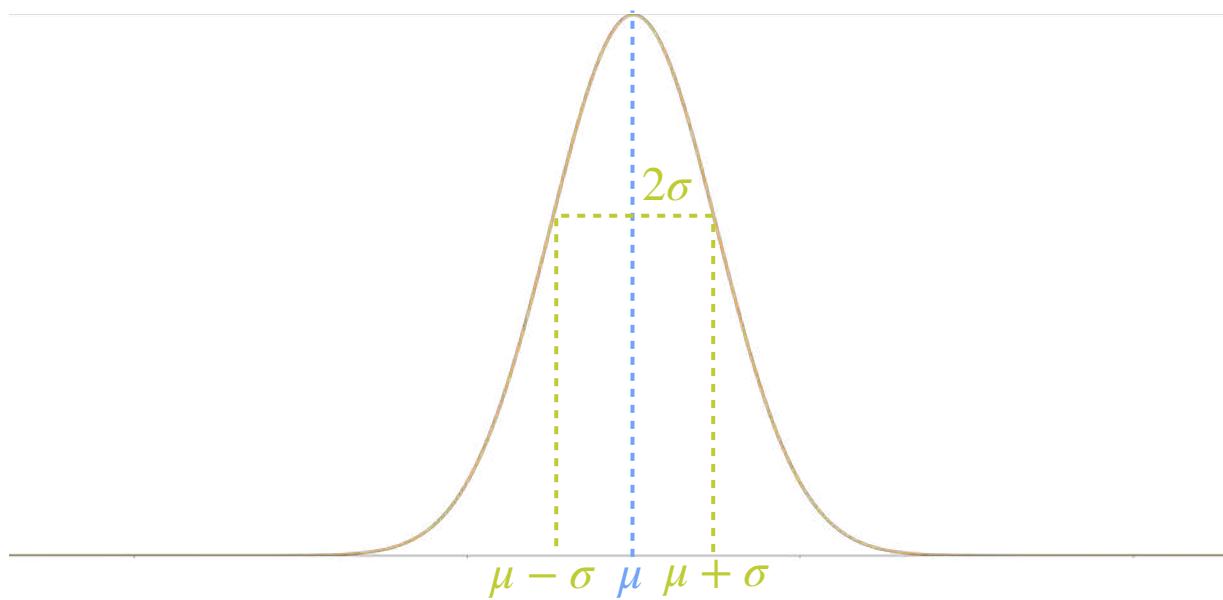
- $\mu$ : center of the bell



# Normal Distribution

Parameters:

- $\mu$ : center of the bell
- $\sigma$ : spread of the bell



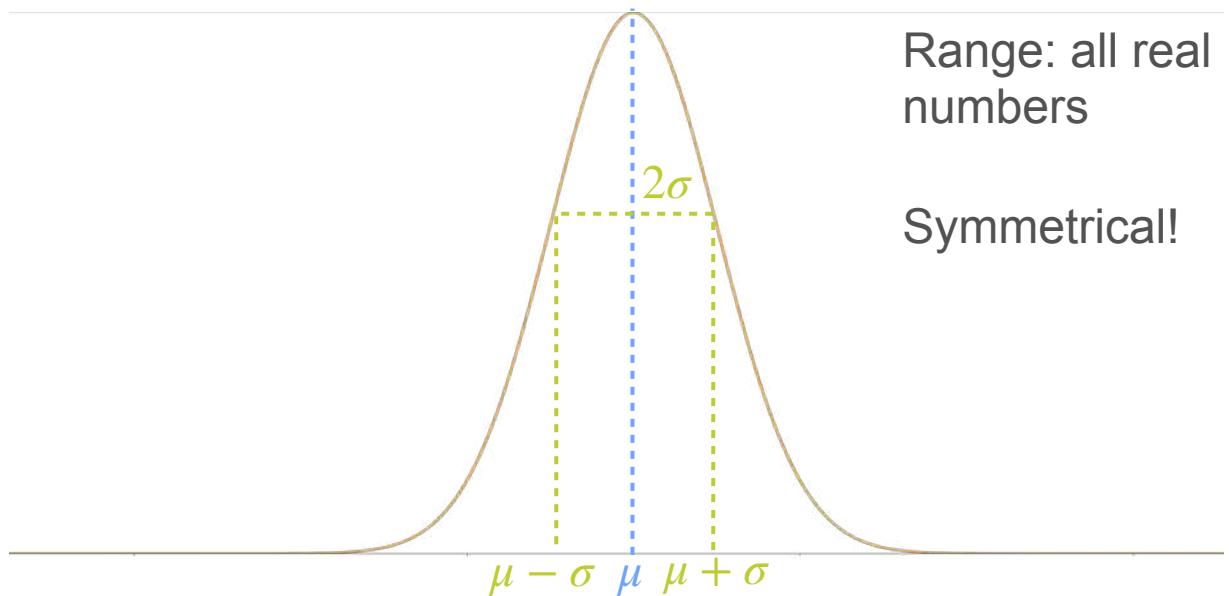
# Normal Distribution

Parameters:

- $\mu$ : center of the bell
- $\sigma$ : spread of the bell

Range: all real numbers

Symmetrical!



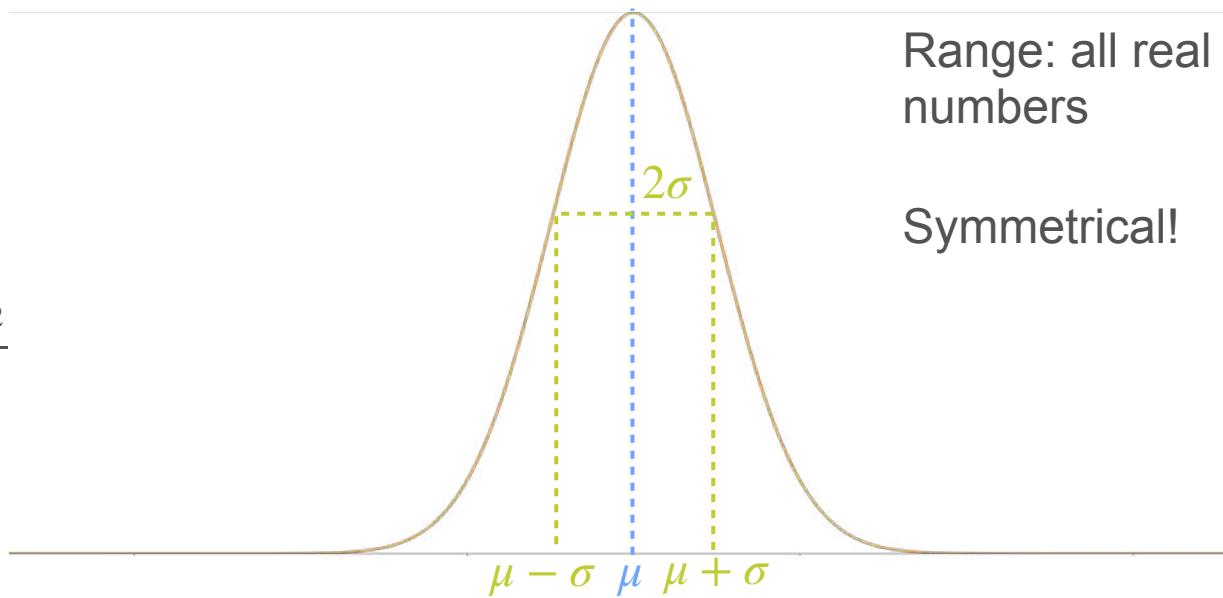
# Normal Distribution

Parameters:

- $\mu$ : center of the bell
- $\sigma$ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Scaling constant



Range: all real numbers

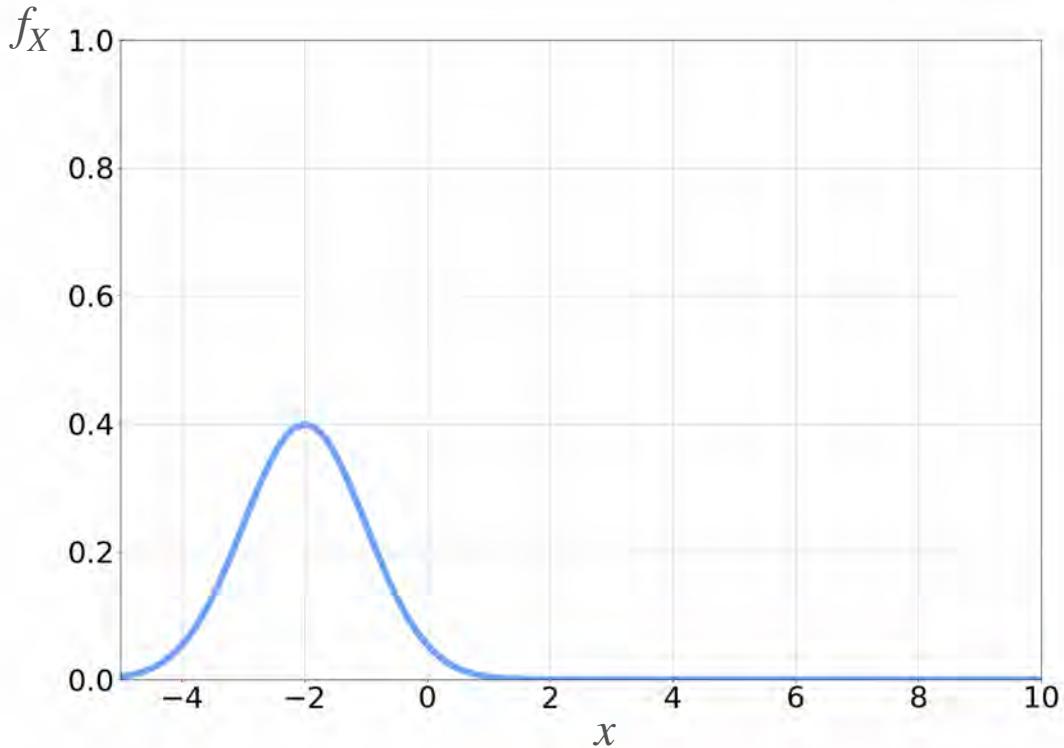
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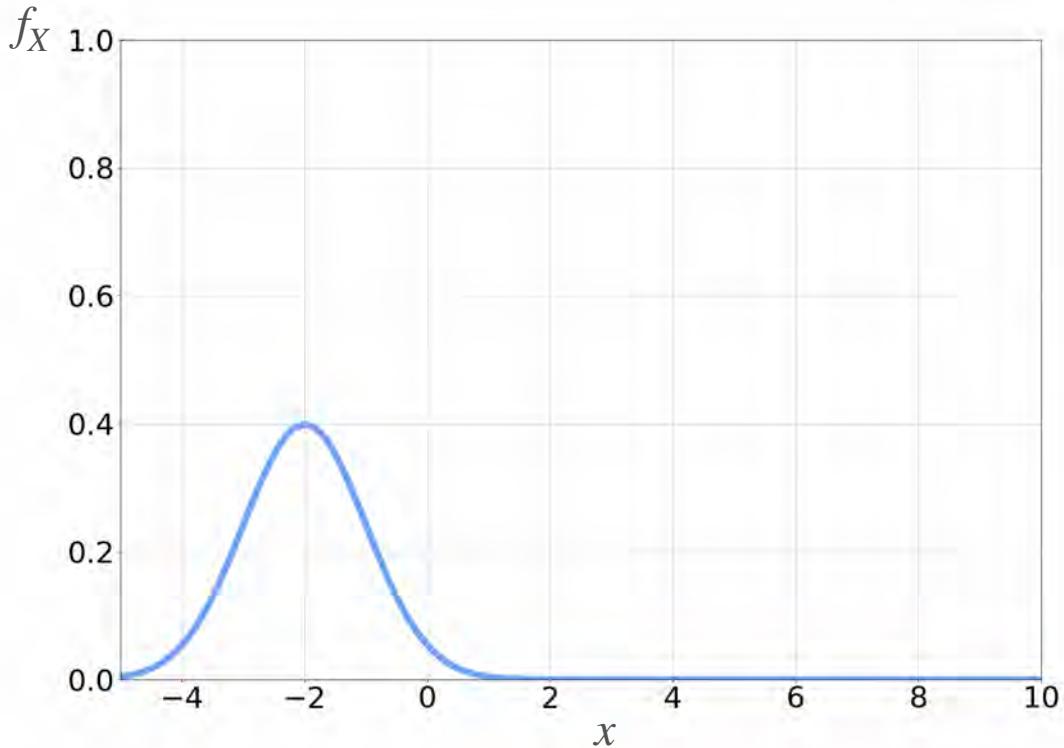


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# Normal Distribution - Notation

Parameters:

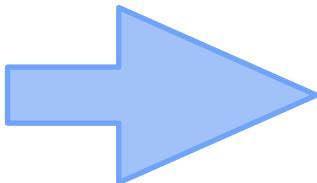
- $\mu$ : center of the bell
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$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

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Parameters:

- $\mu$ : center of the bell
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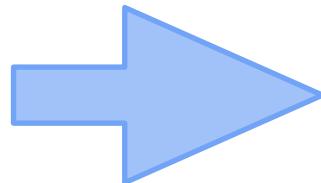
$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

# Normal Distribution - Notation

Parameters:

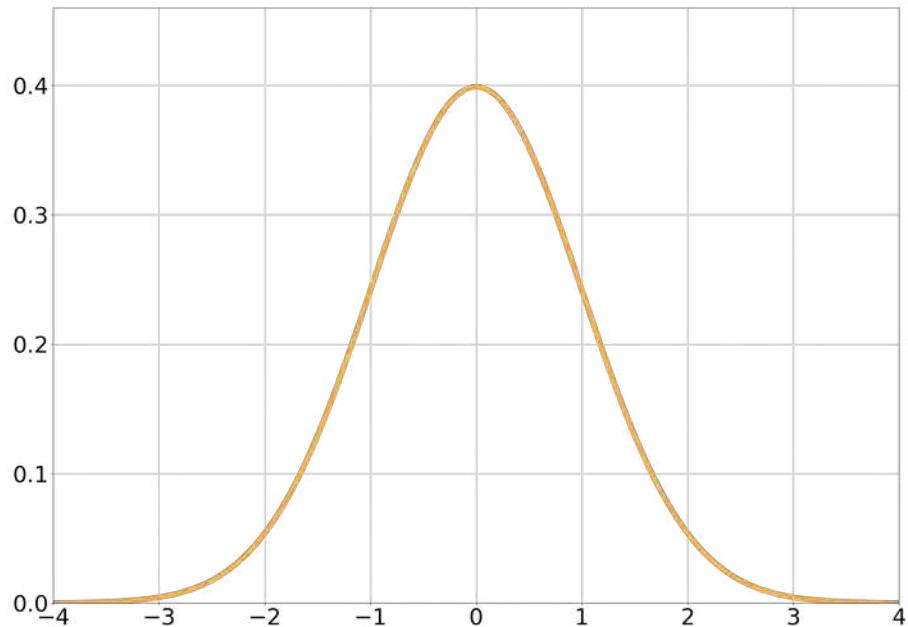
- $\mu$ : center of the bell
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$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

# Standard Normal Distribution

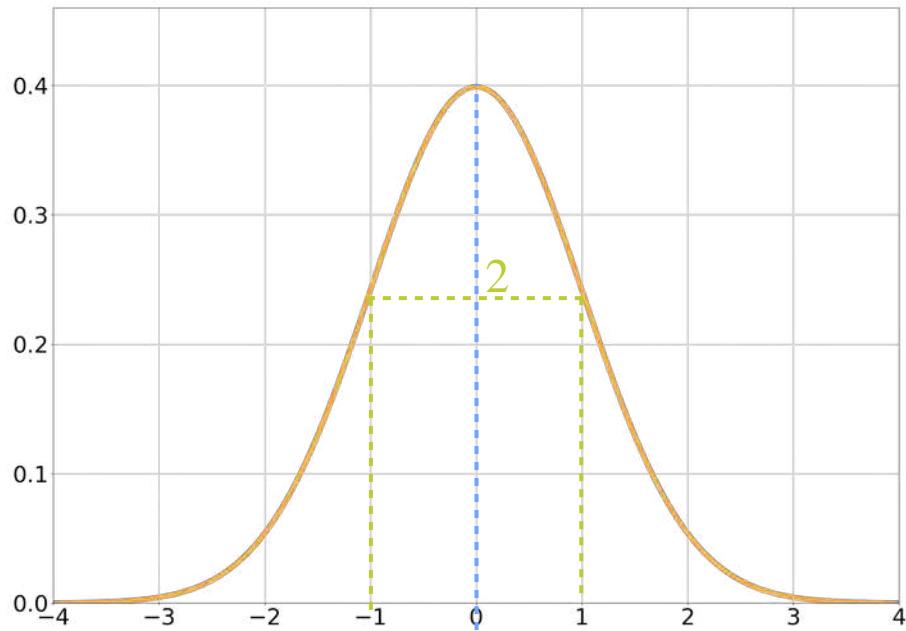


# Standard Normal Distribution

Parameters:

- $\mu$ : 0
- $\sigma$ : 1

$$X \sim \mathcal{N}(0, 1^2)$$



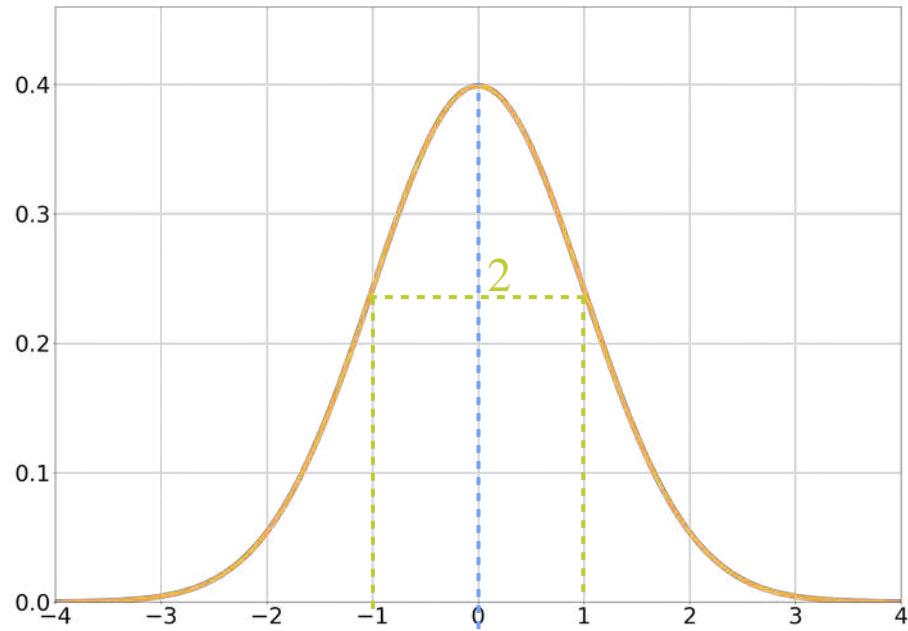
# Standard Normal Distribution

Parameters:

- $\mu$ : 0
- $\sigma$ : 1

$$X \sim \mathcal{N}(0, 1^2)$$

$$\begin{aligned}f_X(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-0)^2}{1^2}} \\&= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\end{aligned}$$



# Standardization

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There's a really easy way to convert any normal distribution to the standard one!

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$X$  distributes normally with

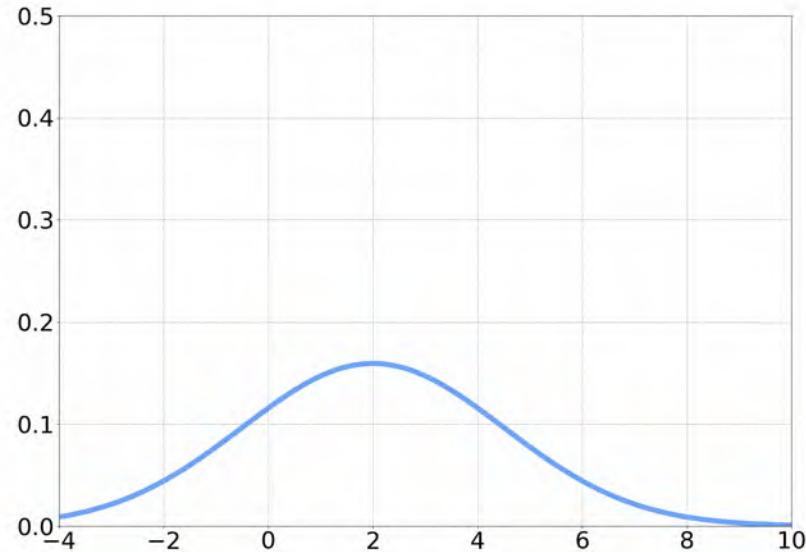
$$\mu = 2, \sigma = 2.5$$

# Standardization

There's a really easy way to convert any normal distribution to the standard one!

$X$  distributes normally with  
 $\mu = 2, \sigma = 2.5$

$$X - 2$$

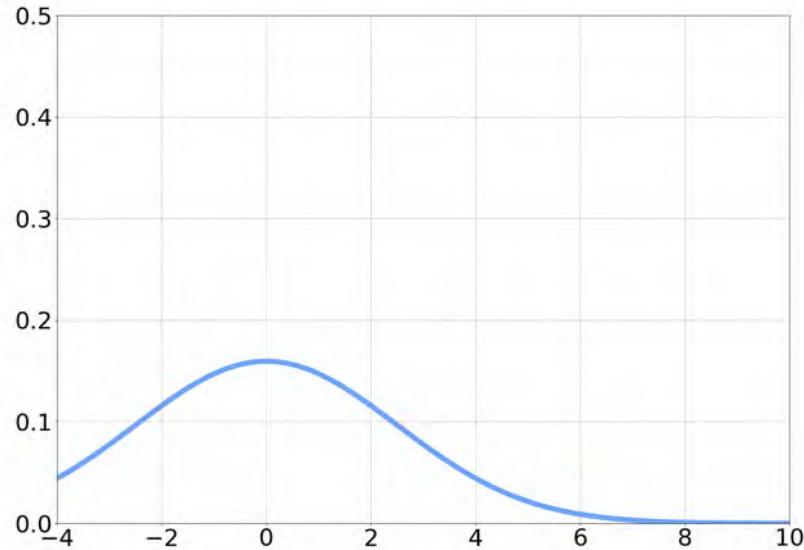


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$X$  distributes normally with  
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$$\frac{X - 2}{2.5}$$



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$X$  distributes normally with

$$\mu = 2, \sigma = 2.5$$

$$Z = \frac{X - 2}{2.5}$$

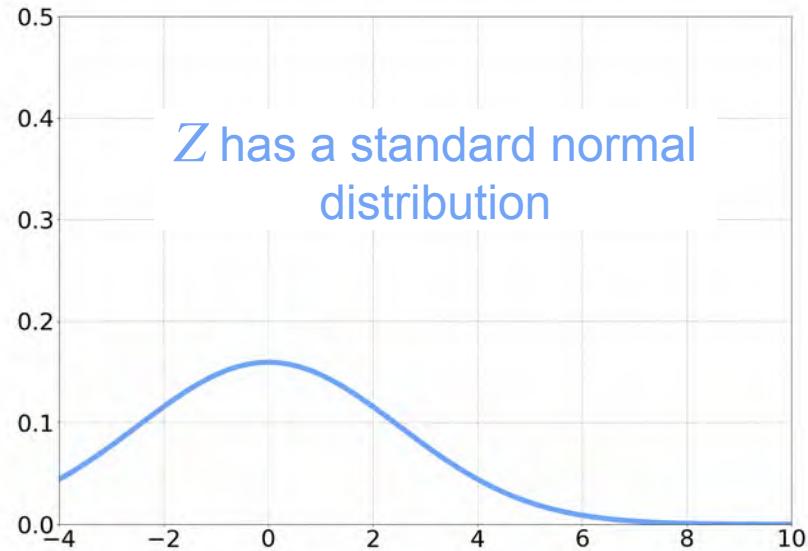


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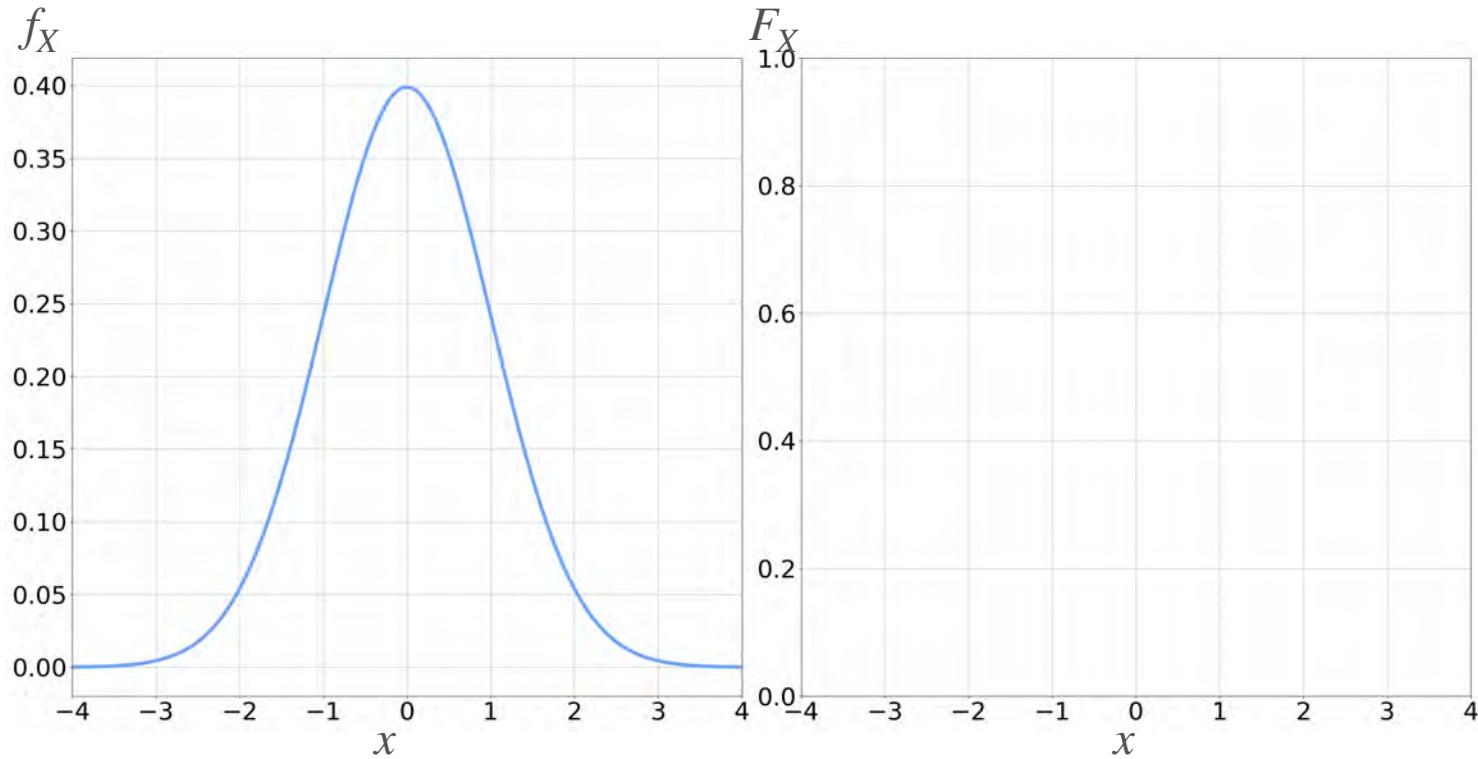
$X$  distributes normally with  
 $\mu = 2, \sigma = 2.5$

$$Z = \frac{X - \mu}{\sigma}$$

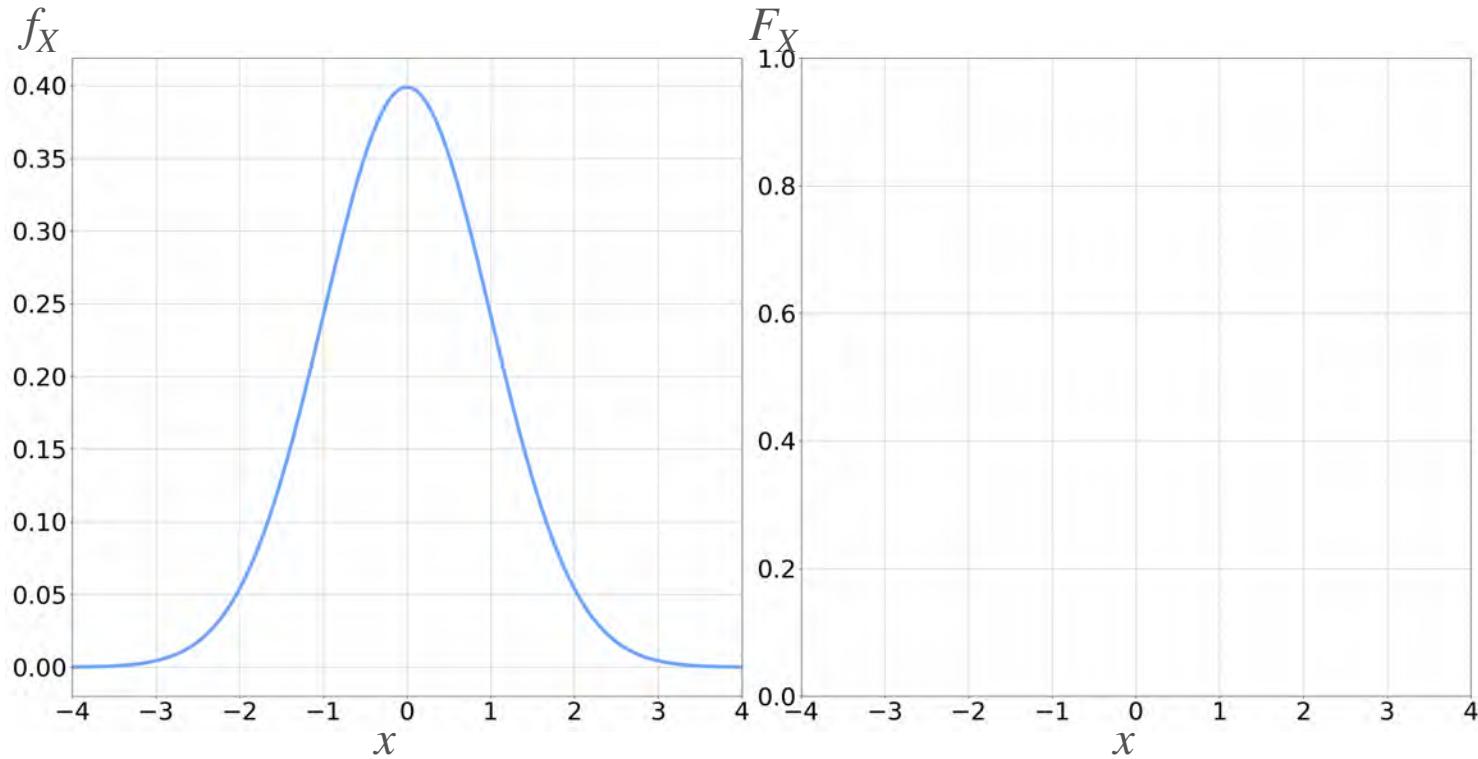
Standardization is crucial to compare variables of different magnitudes!



# Normal Distribution: CDF



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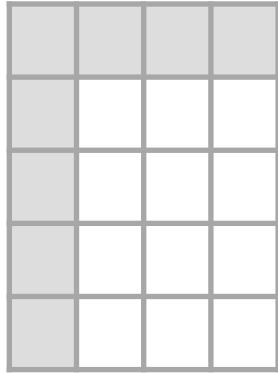
# Normal Distribution: Computing Probabilities

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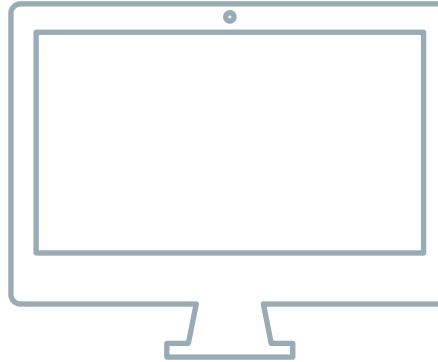
This math can't be done by hand

# Normal Distribution: Computing Probabilities

This math can't be done by hand



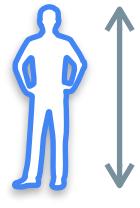
In the old days, people used tables of data



Now, you can use the help of some software to do the approximate area under the curve for you!

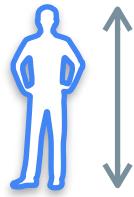
# Normal Distribution: Applications

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Height

# Normal Distribution: Applications

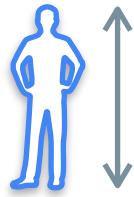


Height



Weight

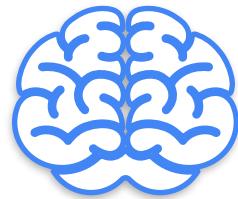
# Normal Distribution: Applications



Height

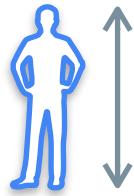


Weight



IQ

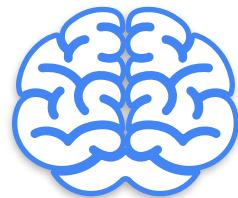
# Normal Distribution: Applications



Height



Weight

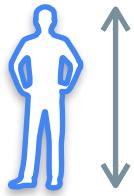


IQ



Noise in a  
communication channel

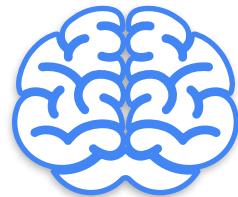
# Normal Distribution: Applications



Height



Weight



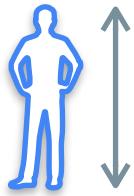
IQ



Noise in a  
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In general, characteristics that are the sum of many independent processes

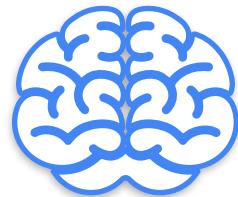
# Normal Distribution: Applications



Height



Weight



IQ



Noise in a  
communication channel

In general, characteristics that are the sum of many independent processes

Many models in ML are designed under the assumption that the variables follow a normal distribution



DeepLearning.AI

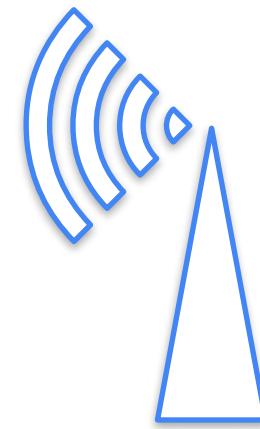
# Probability Distributions

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## Chi-squared distribution

# Chi-Square Distribution: Motivation

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Message sent: 10010

# Chi-Square Distribution: Motivation



Communication channel

Noise

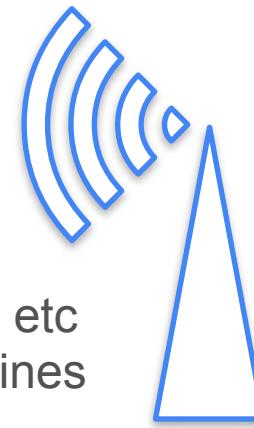
Interference from other devices

Obstructions like walls, trees, etc.

Atmospheric conditions: rain, humidity, etc

Electrical interference, i.e. from power lines

Others



Message sent: 10010

# Chi-Square Distribution: Motivation



Communication channel

Noise

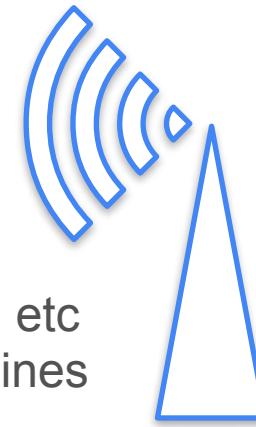
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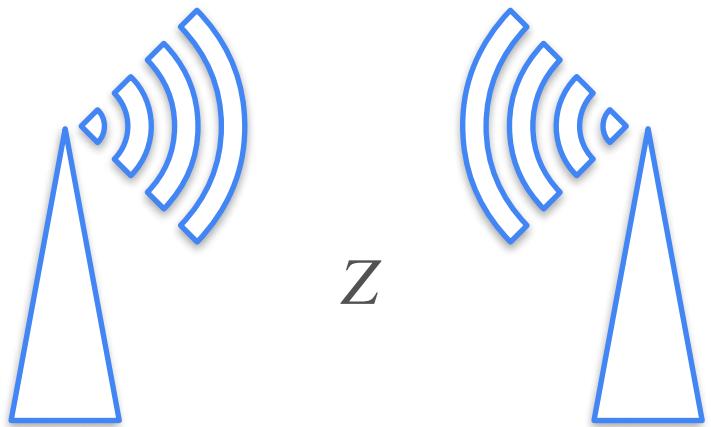
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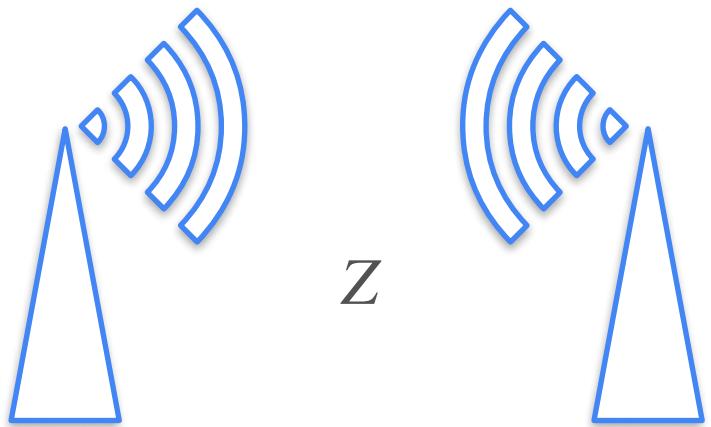
Message sent: 10010

Message received: 10010 +Z

# Chi-Square Distribution: Motivation

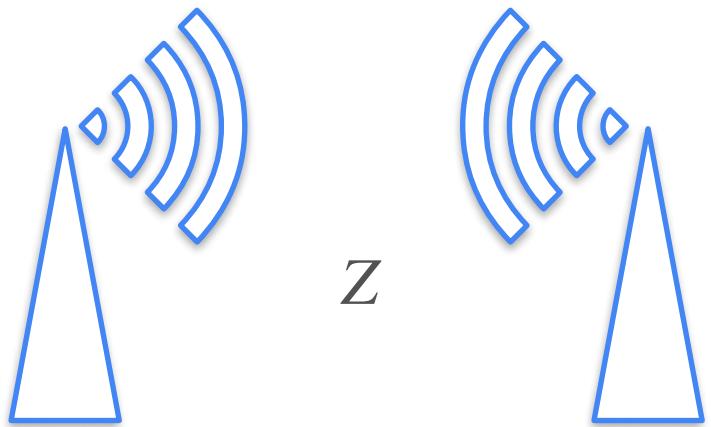


# Chi-Square Distribution: Motivation



The communication channel  
has noise with a standard  
normal distribution

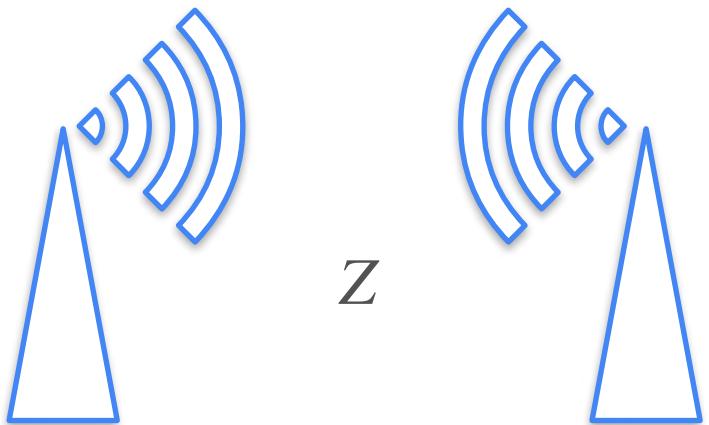
# Chi-Square Distribution: Motivation



What is the **power** of the noise in the channel?

The communication channel  
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# Chi-Square Distribution: Motivation

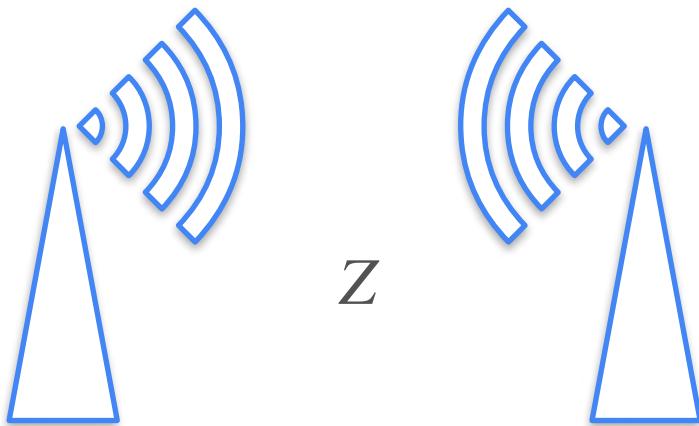


What is the **power** of the noise in the channel?

$$W = Z^2$$

The communication channel  
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# Chi-Square Distribution: Motivation



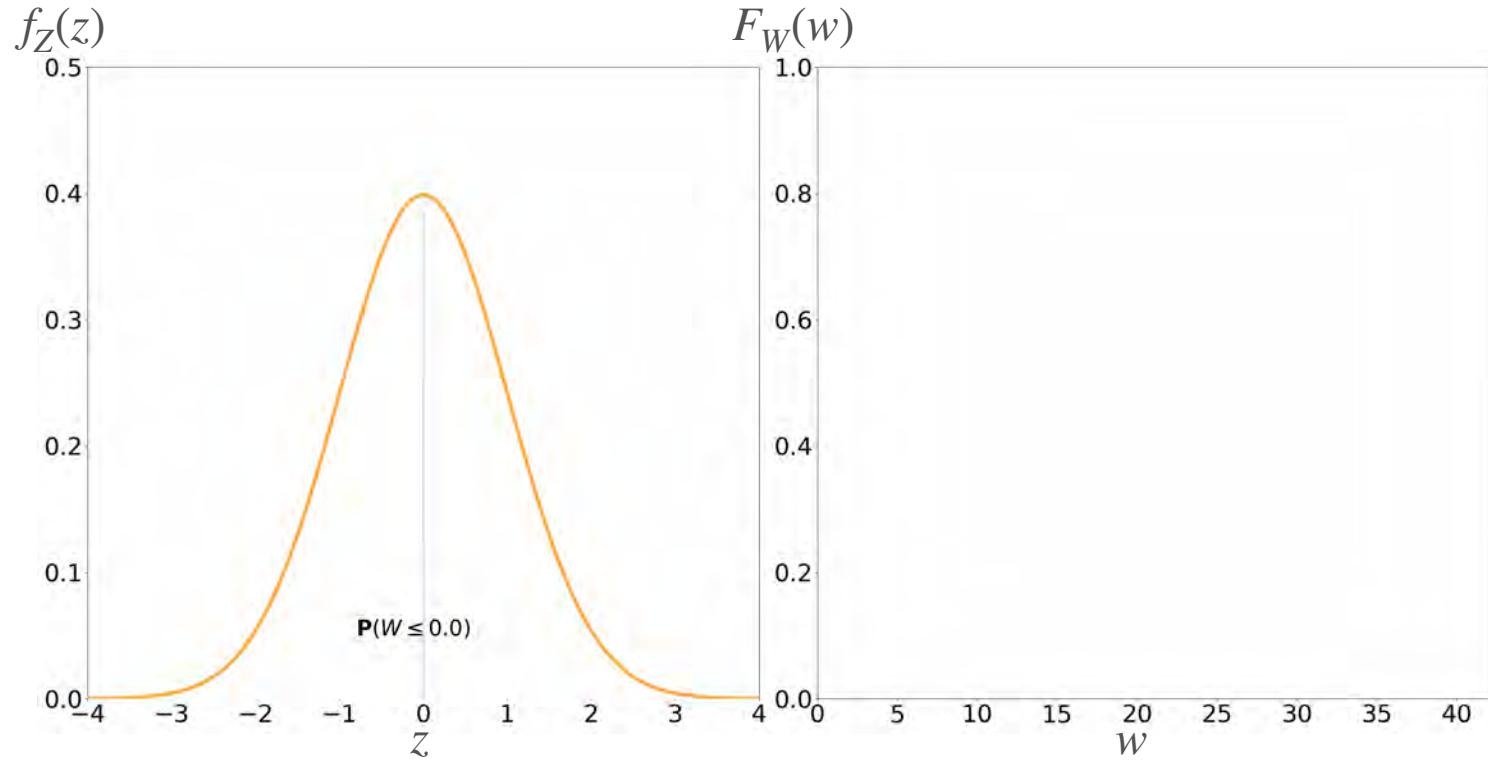
The communication channel has noise with a standard normal distribution

What is the **power** of the noise in the channel?

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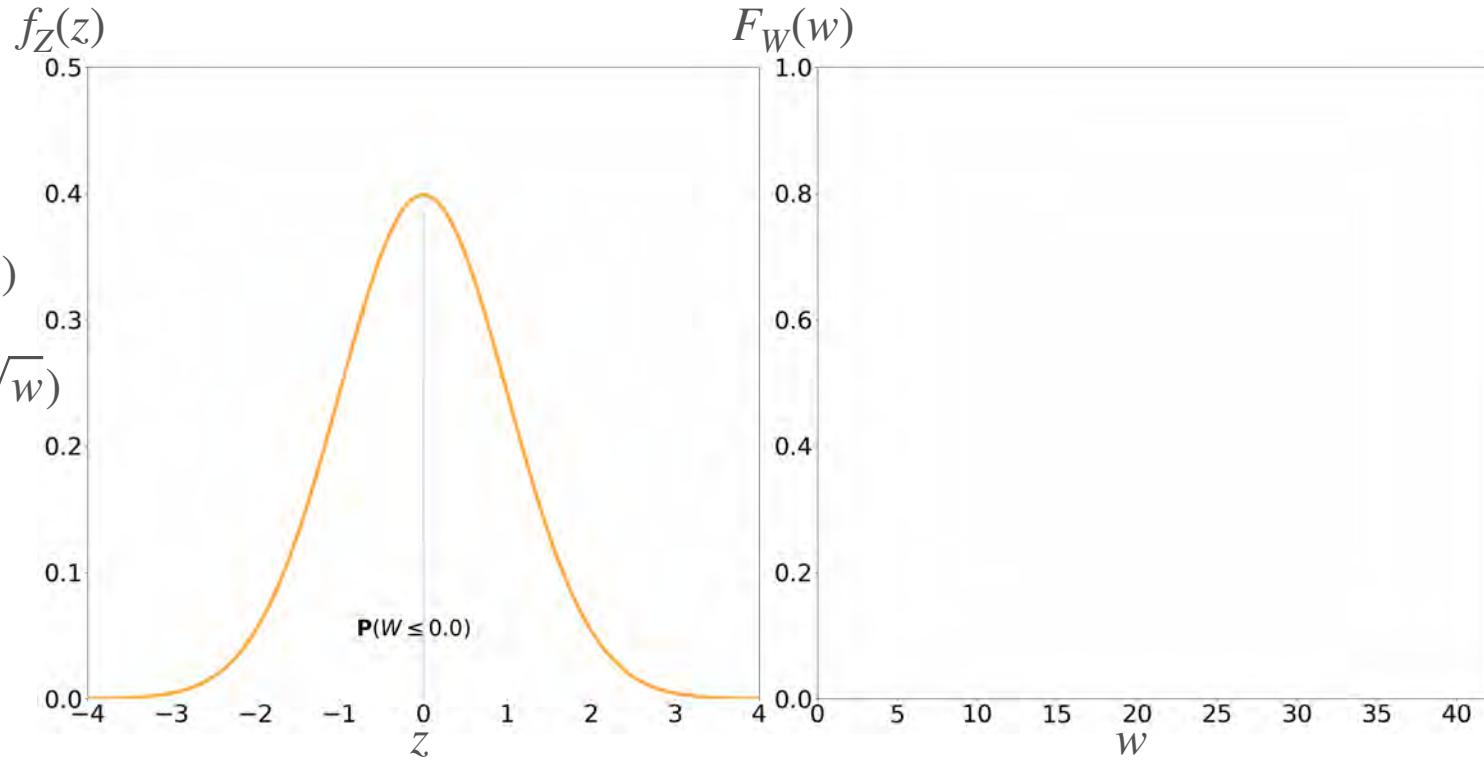
What is the distribution of  $W$ ?

# Chi Square Distribution



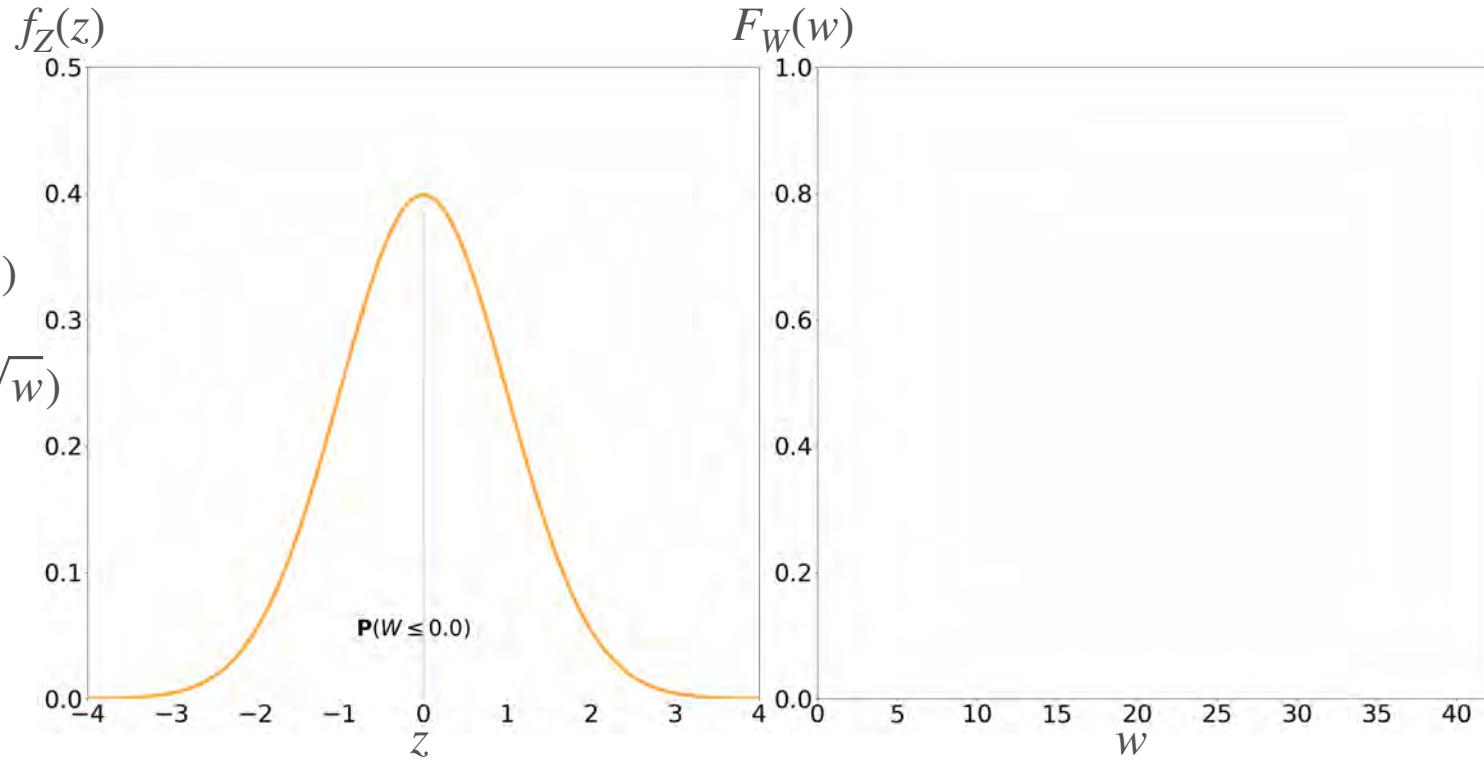
# Chi Square Distribution

$$\begin{aligned} F_W(w) &= \mathbf{P}(W \leq w) \\ &= \mathbf{P}(Z^2 \leq w) \\ &= \mathbf{P}(|Z| \leq \sqrt{w}) \\ &= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w}) \end{aligned}$$



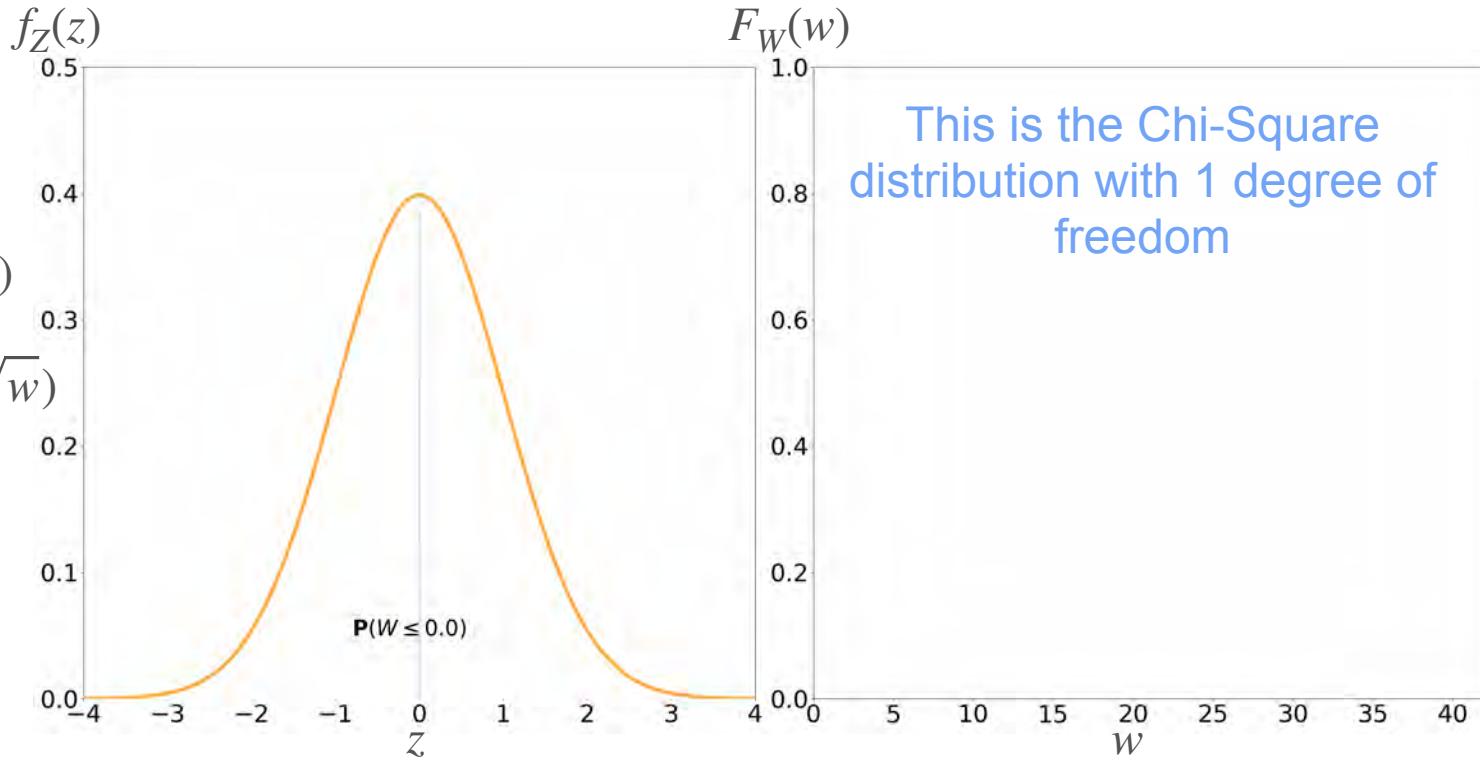
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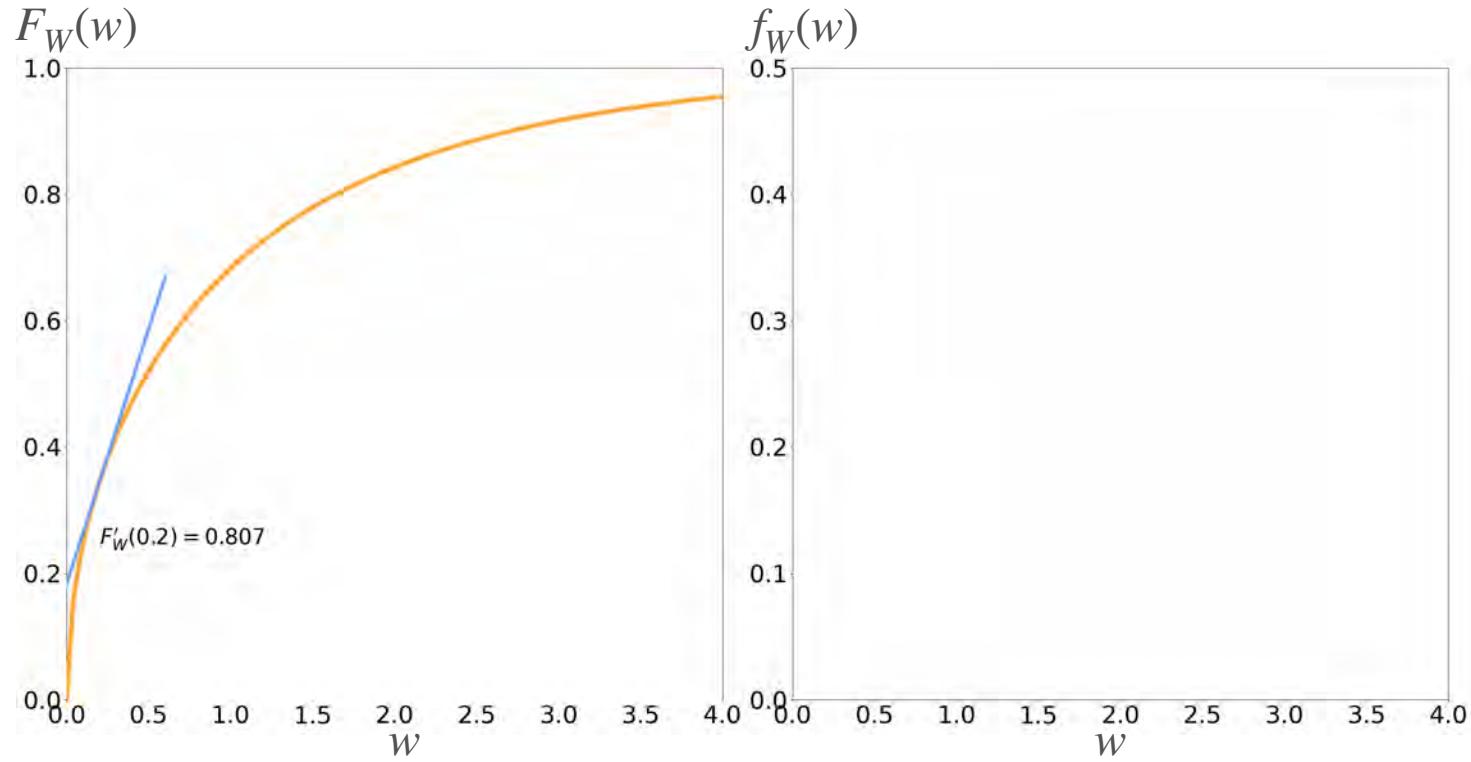


# Chi Square Distribution

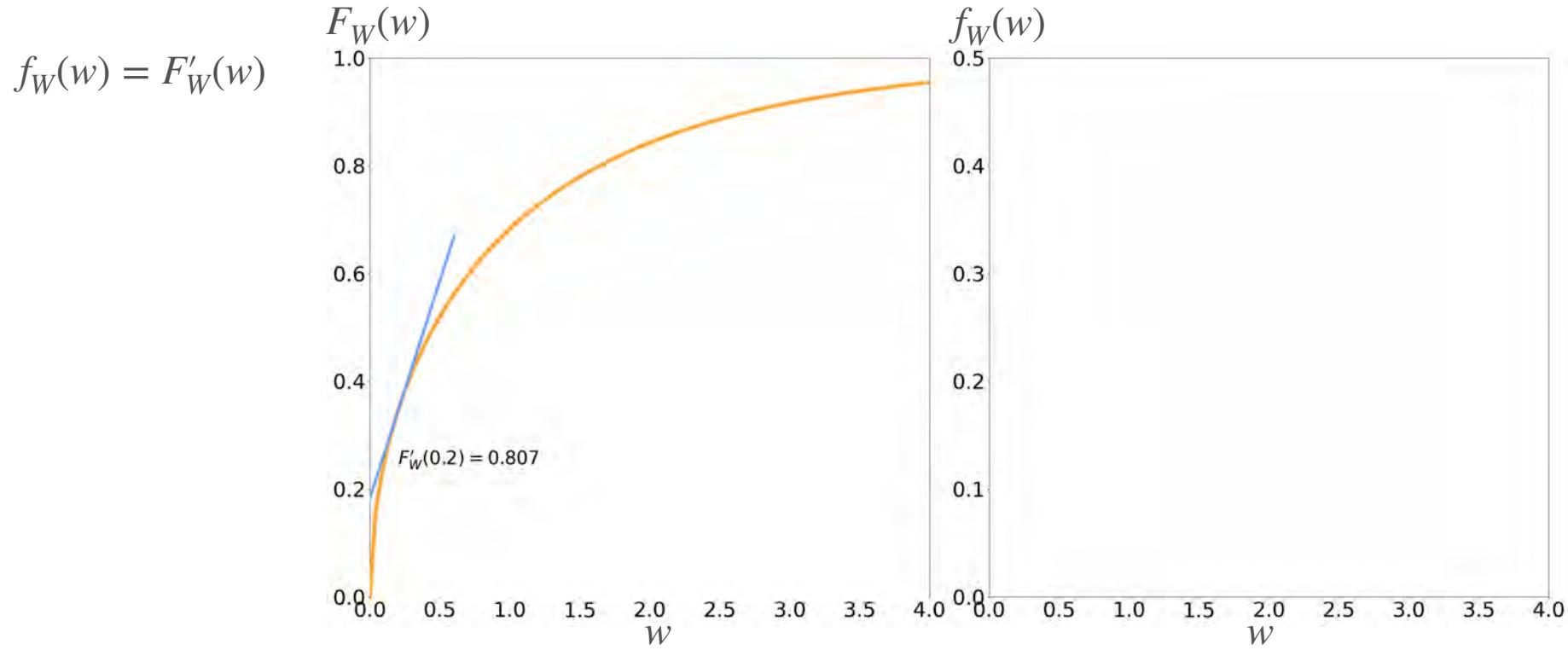
$$\begin{aligned}F_W(w) &= \mathbf{P}(W \leq w) \\&= \mathbf{P}(Z^2 \leq w) \\&= \mathbf{P}(|Z| \leq \sqrt{w}) \\&= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w})\end{aligned}$$



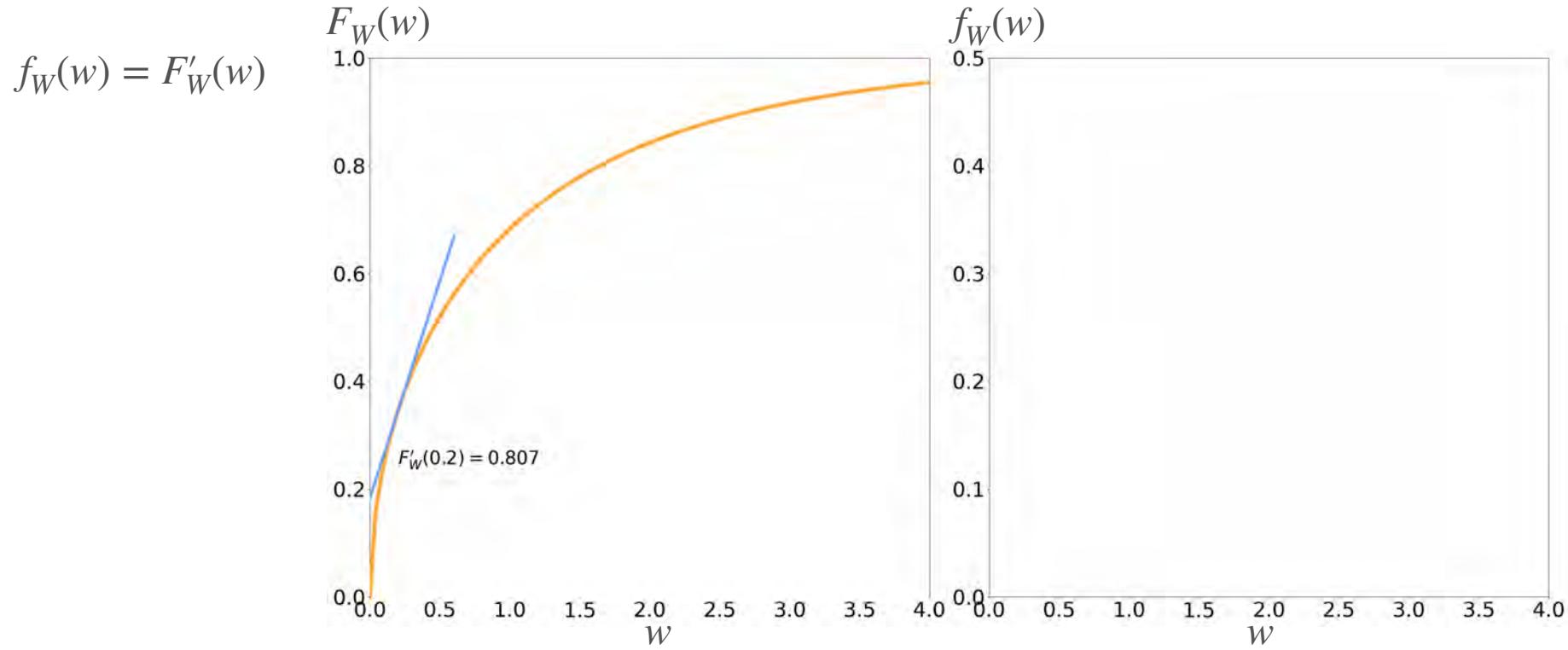
# Chi Square Distribution



# Chi Square Distribution



# Chi Square Distribution



# Chi-Square Distribution

# Chi-Square Distribution

Accumulated power over 2 transmissions?

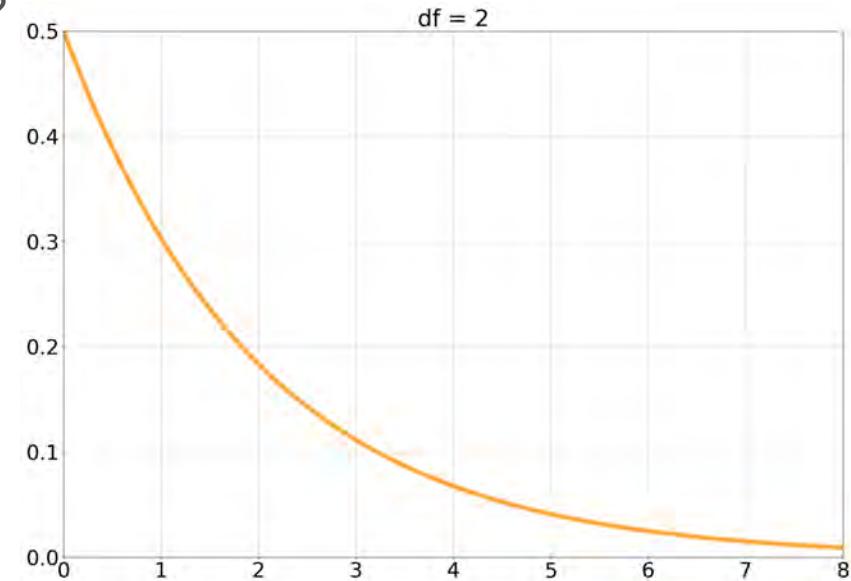
$$W_2 = Z_1^2 + Z_2^2$$

# Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df



# Chi-Square Distribution

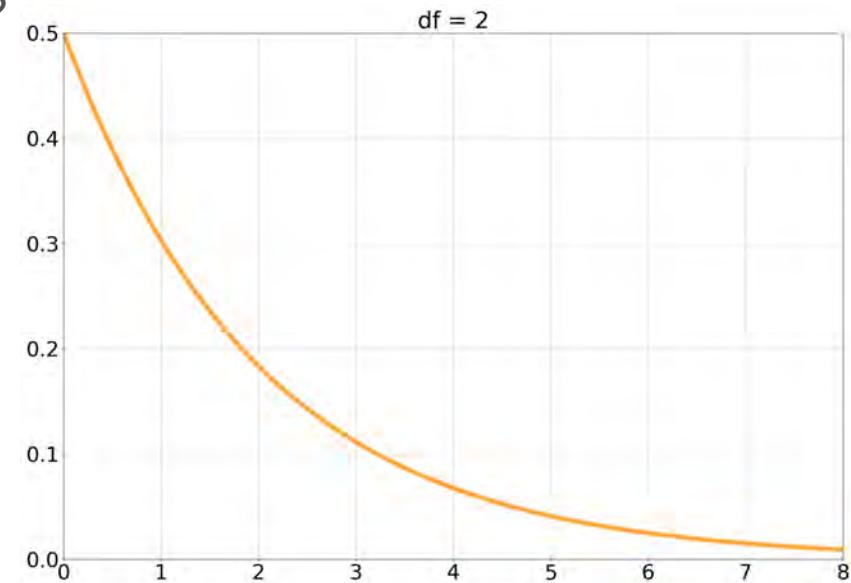
Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$



# Chi-Square Distribution

Accumulated power over 2 transmissions?

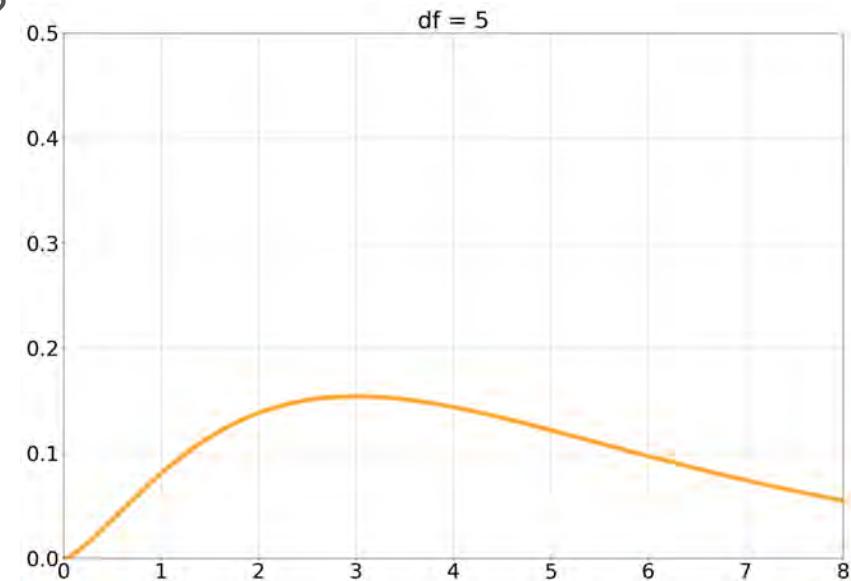
$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square  
with 5 df



# Chi-Square Distribution

Accumulated power over 2 transmissions?

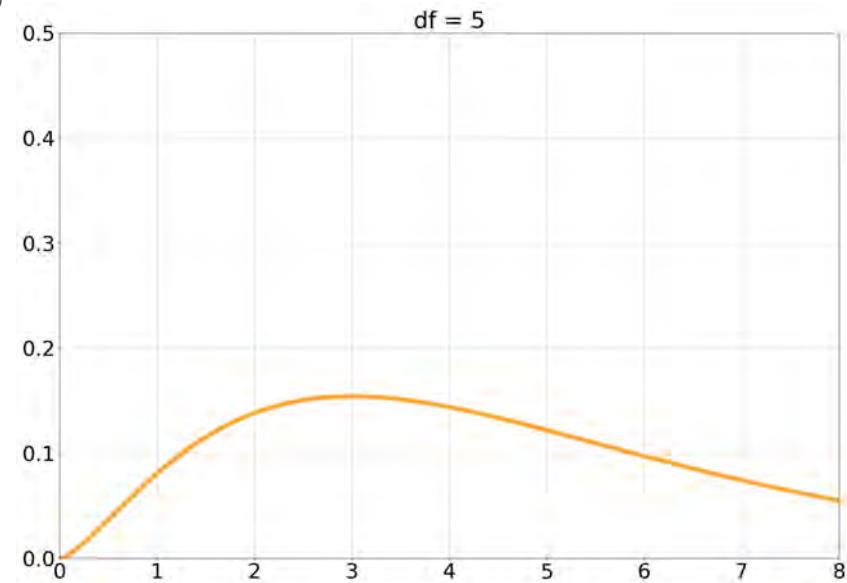
$$W_2 = Z_1^2 + Z_2^2 \quad \text{Chi-Square with 2 df}$$

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2 \quad \text{Chi-Square with 5 df}$$

Accumulated power over  $k$  transmissions?

$$W_k = \sum_{i=1}^k Z_i^2$$



# Chi-Square Distribution

Accumulated power over 2 transmissions?

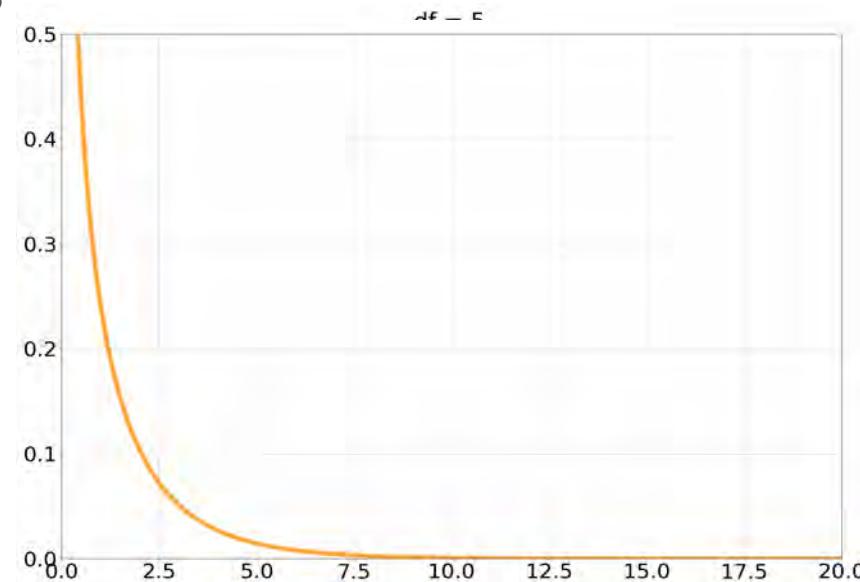
$$W_2 = Z_1^2 + Z_2^2 \quad \text{Chi-Square with 2 df}$$

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2 \quad \text{Chi-Square with 5 df}$$

Accumulated power over  $k$  transmissions?

$$W_k = \sum_{i=1}^k Z_i^2 \quad \text{Chi-Square with } k \text{ df}$$





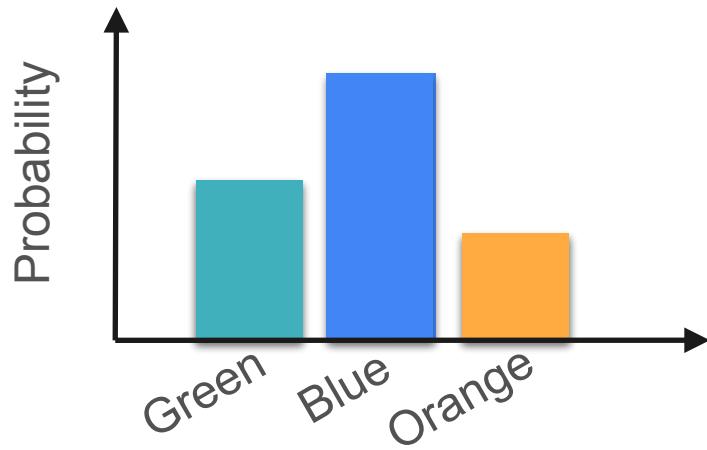
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# Probability Distributions

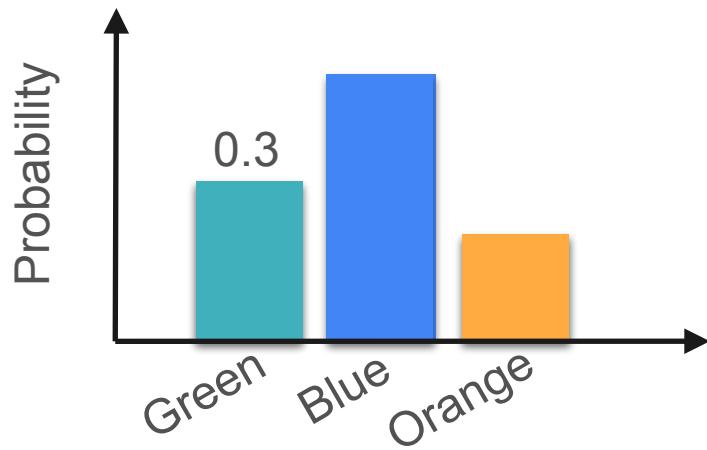
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## Sampling from a Distribution

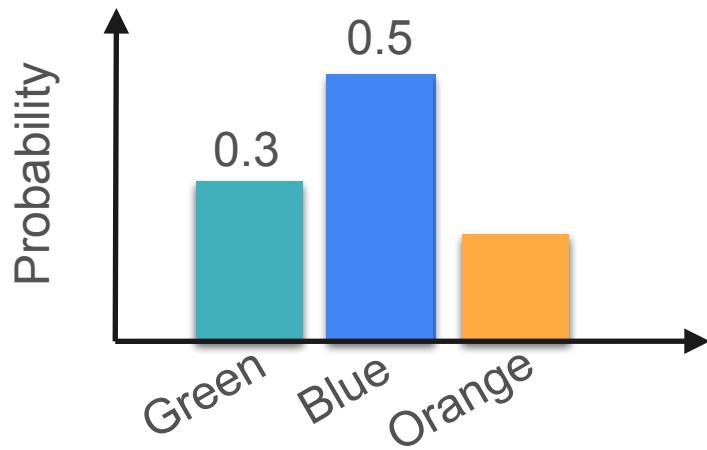
# Sampling From a Distribution



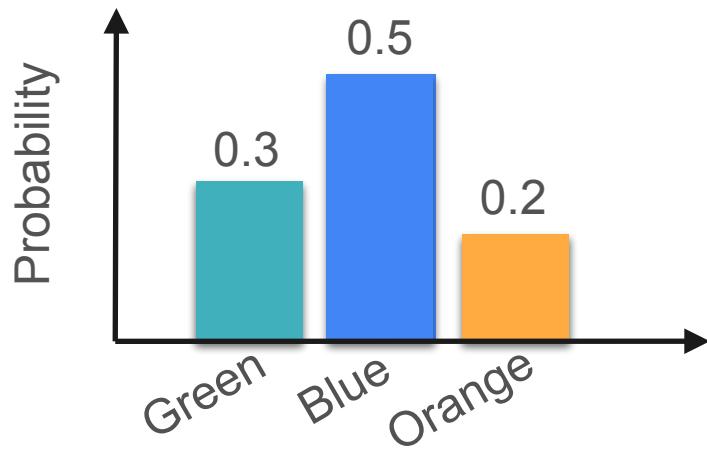
# Sampling From a Distribution



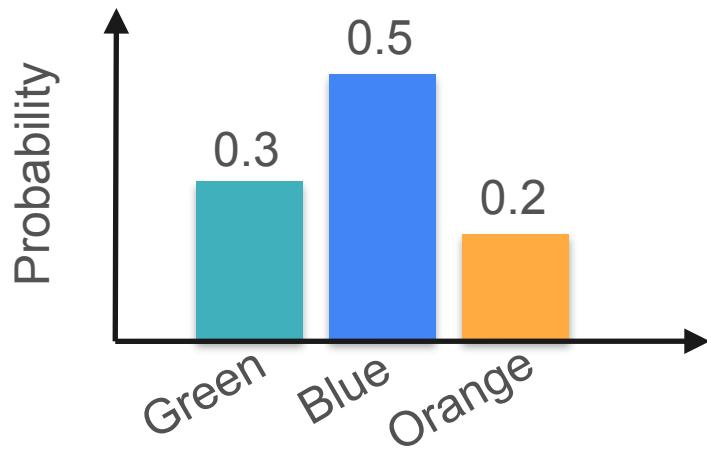
# Sampling From a Distribution



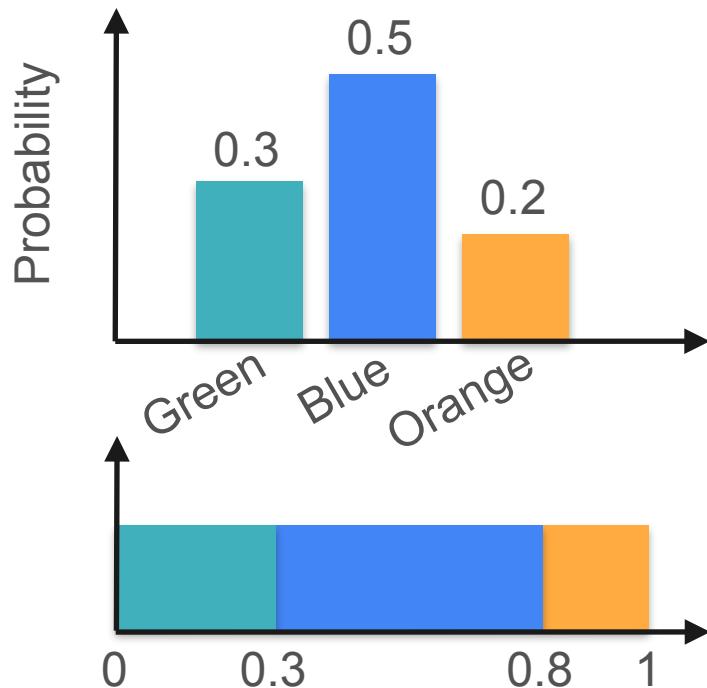
# Sampling From a Distribution



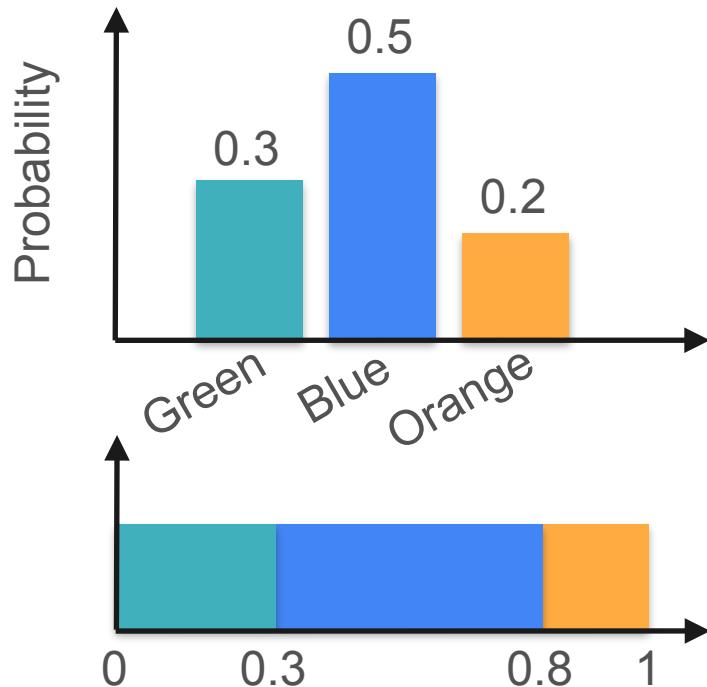
# Sampling From a Distribution



# Sampling From a Distribution

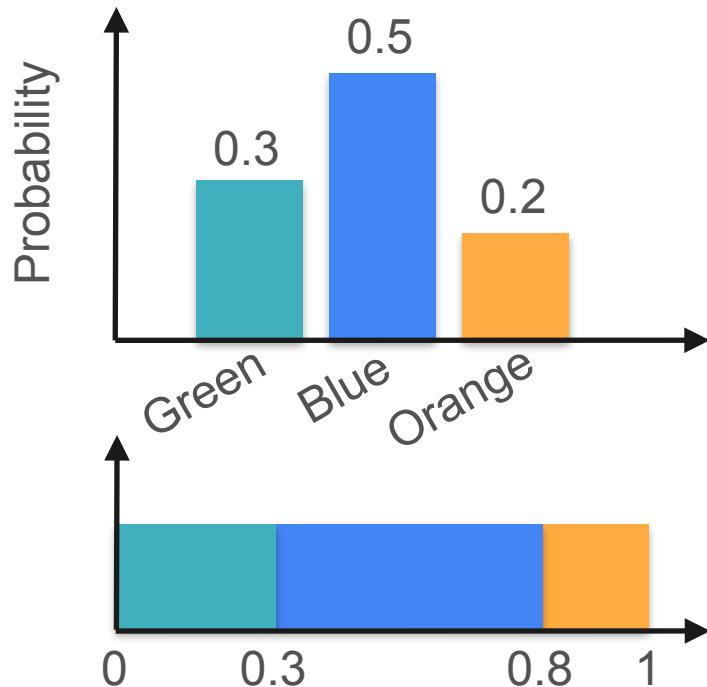


# Sampling From a Distribution



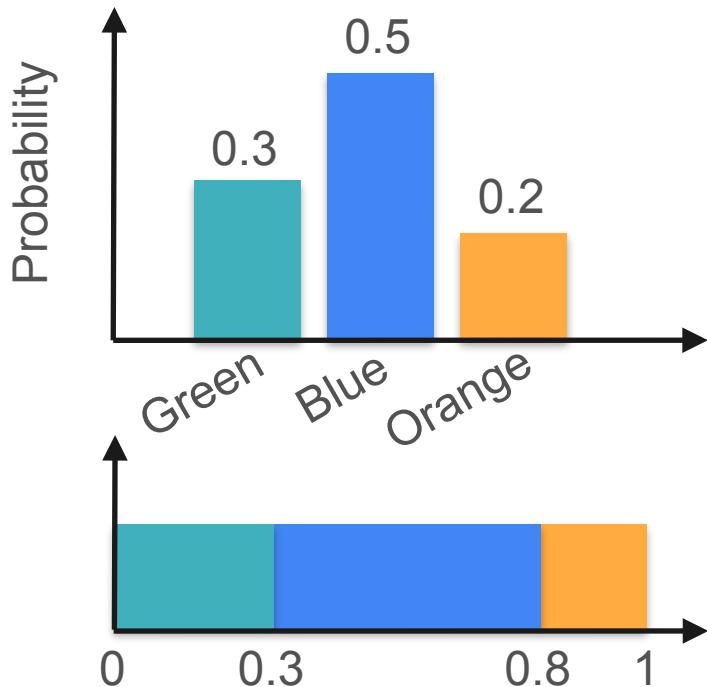
- **Step 1:** generate a random number between 0 and 1

# Sampling From a Distribution



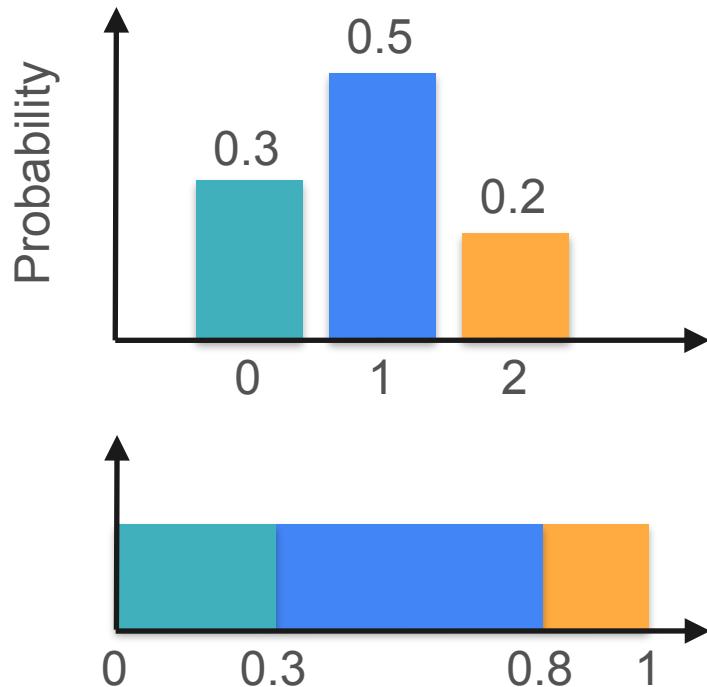
- **Step 1:** generate a random number between 0 and 1
- **Step 2:** find out which interval the number belongs to
  - [0, 0.3)
  - [0.3, 0.8)
  - [0.8, 1]

# Sampling From a Distribution



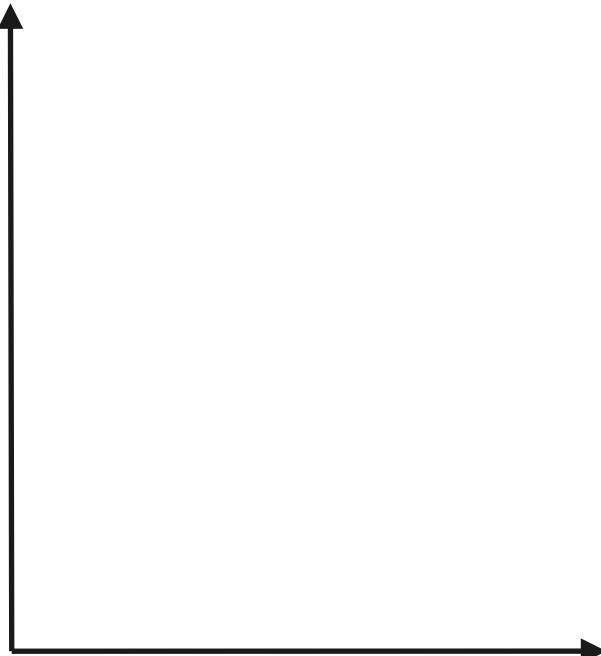
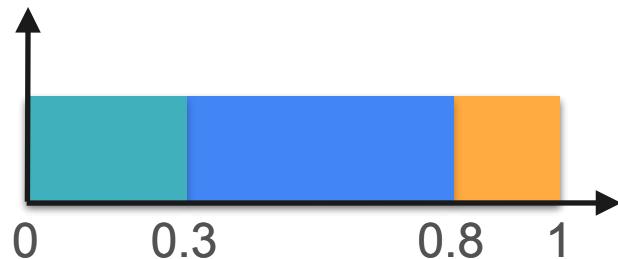
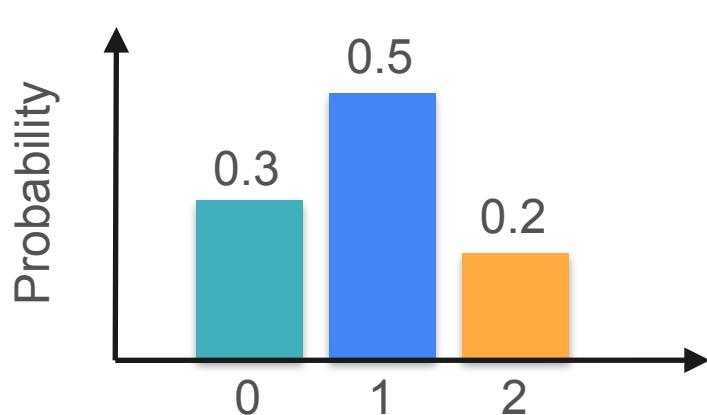
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  - [0.3, 0.8)
  - [0.8, 1]
- **Step 3:** Assign an outcome based on the interval

# Sampling From a Distribution

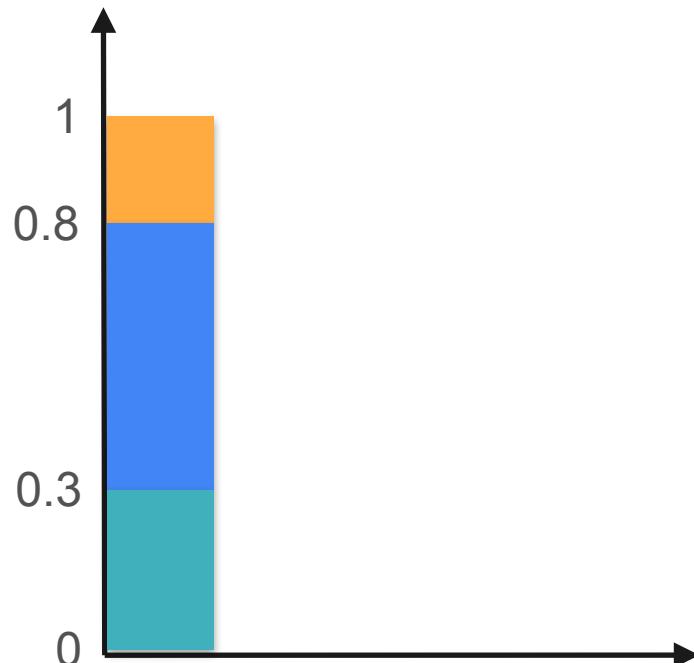
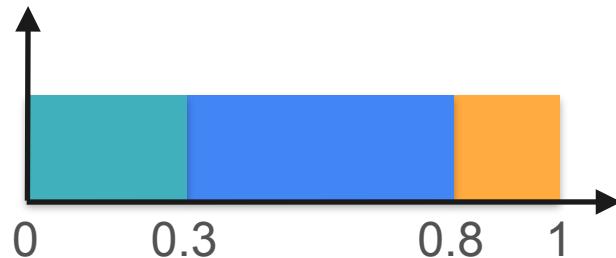
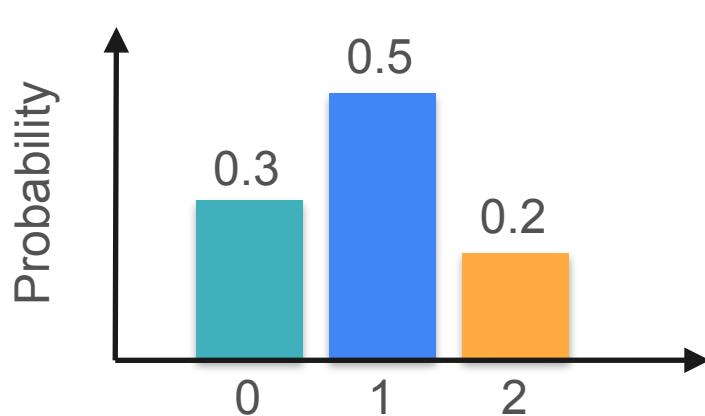


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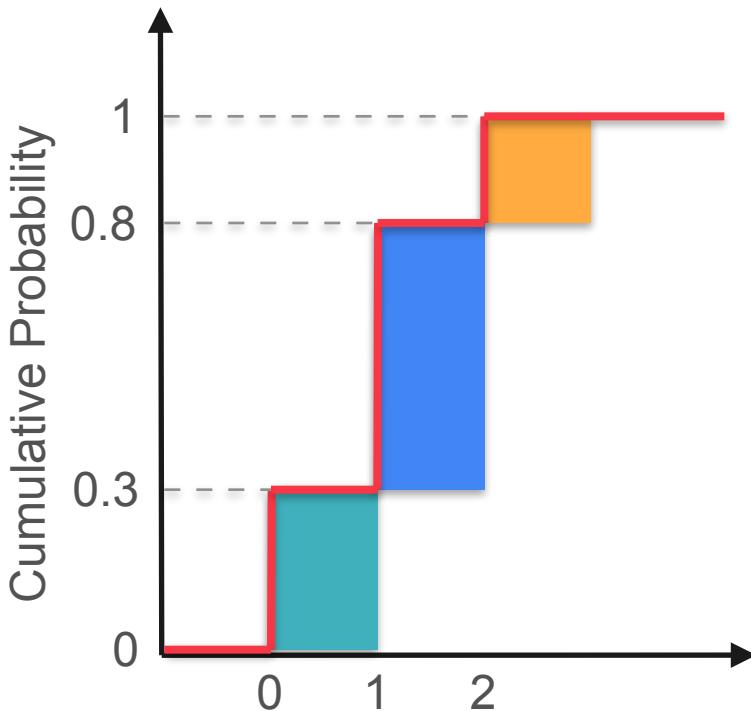
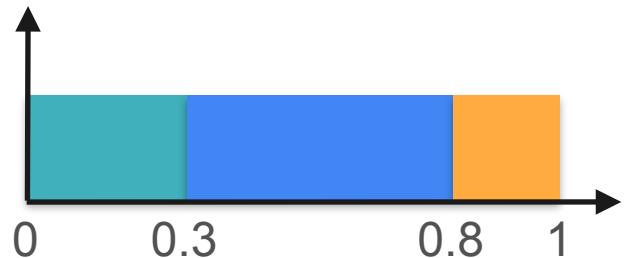
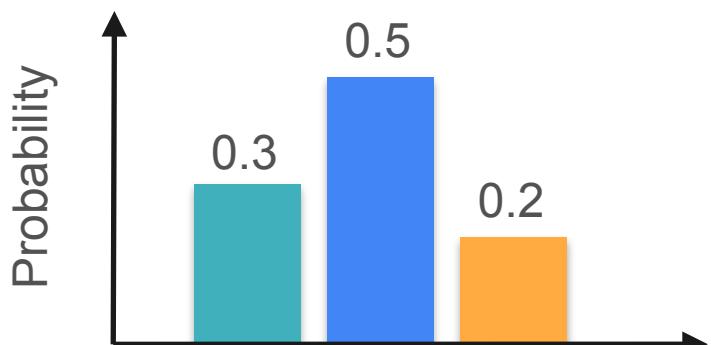
# Sampling From a Distribution



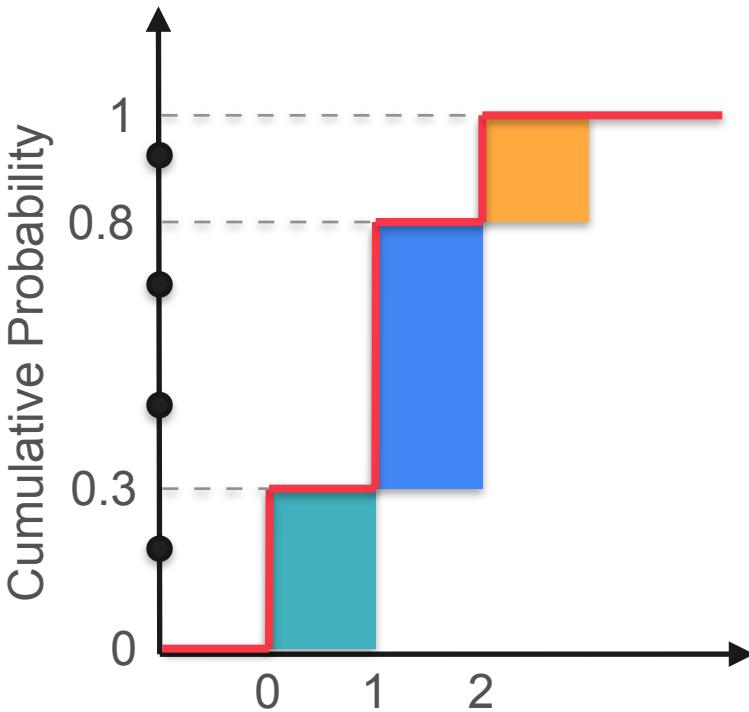
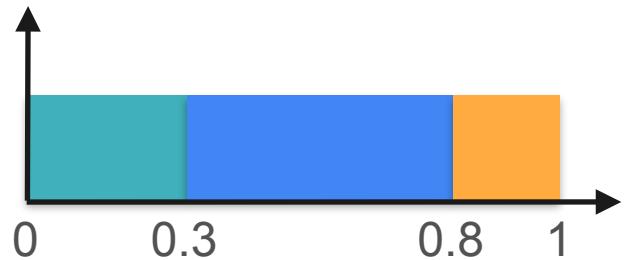
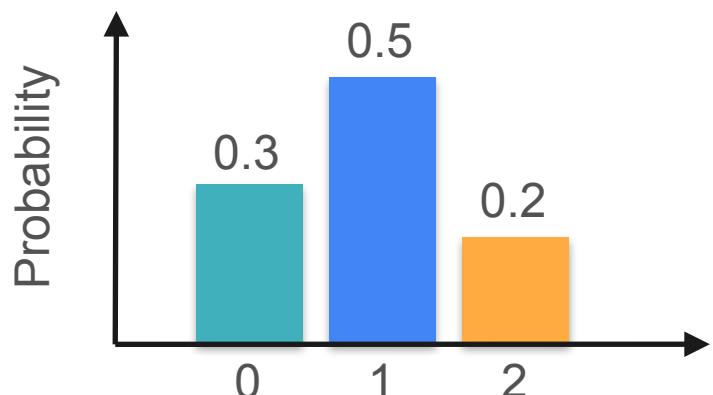
# Sampling From a Distribution



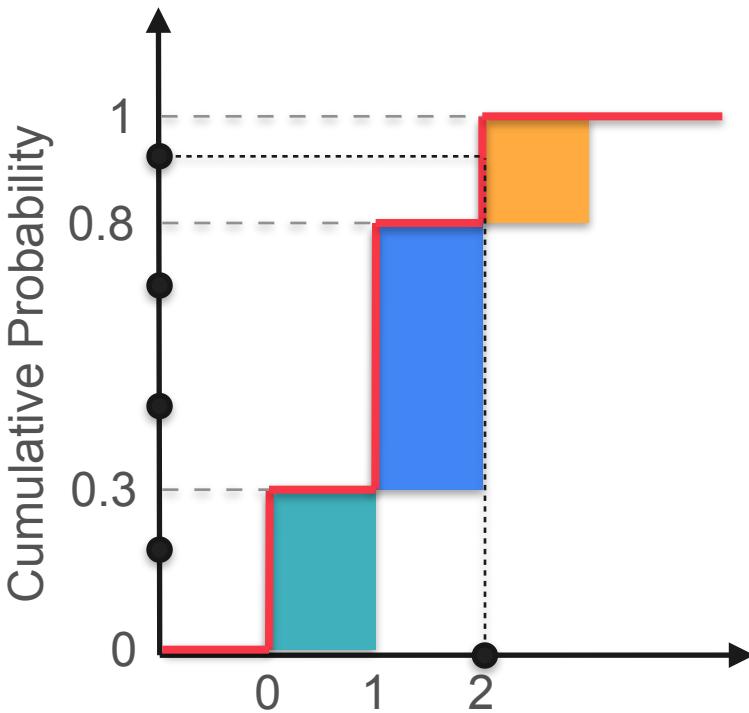
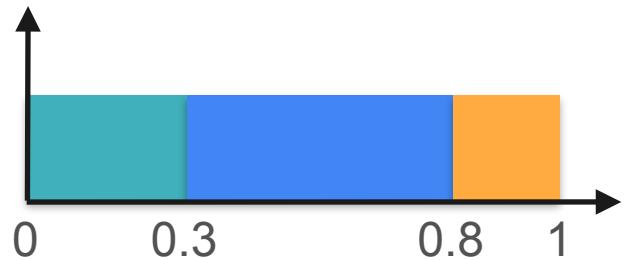
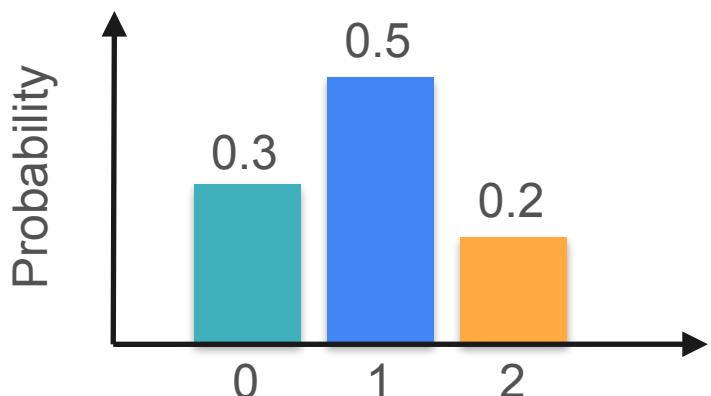
# Sampling From a Distribution



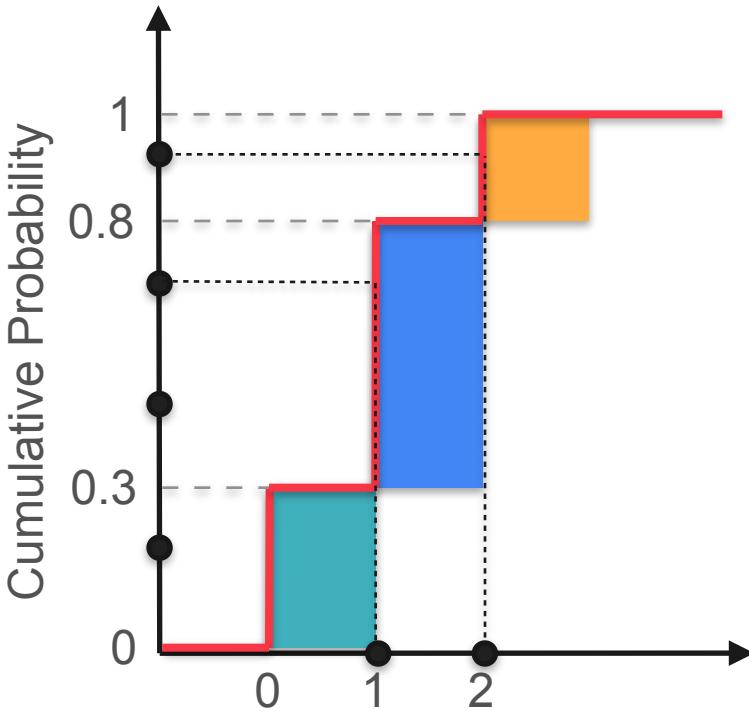
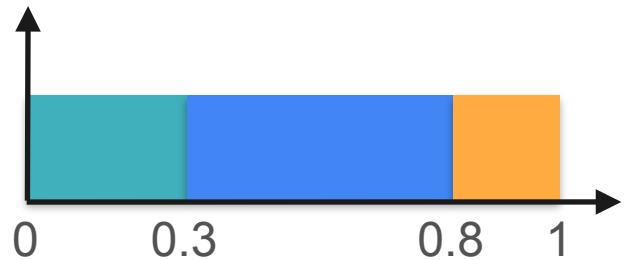
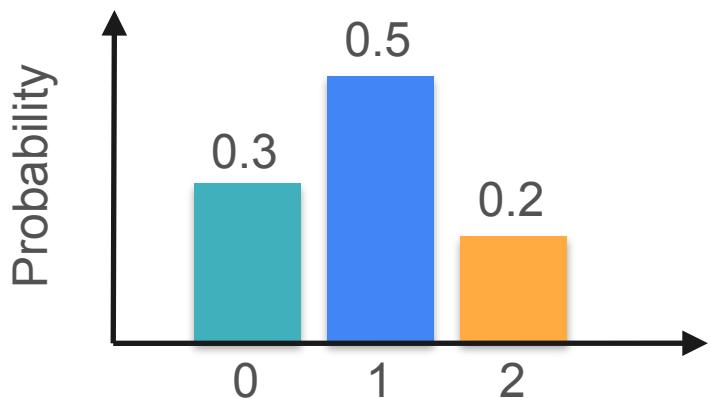
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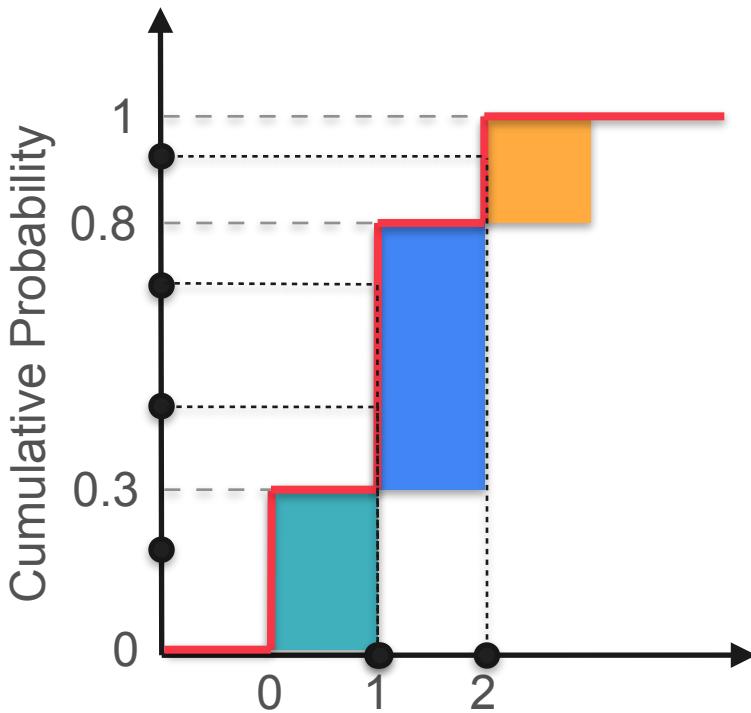
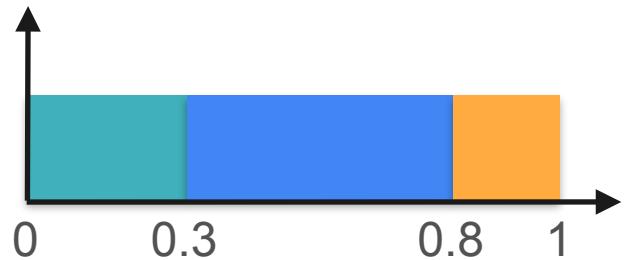
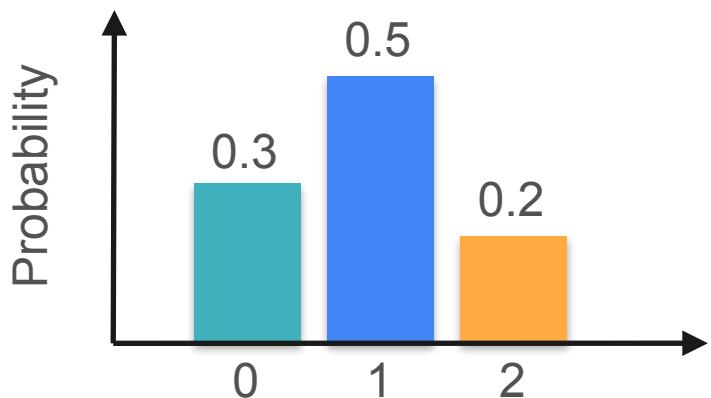
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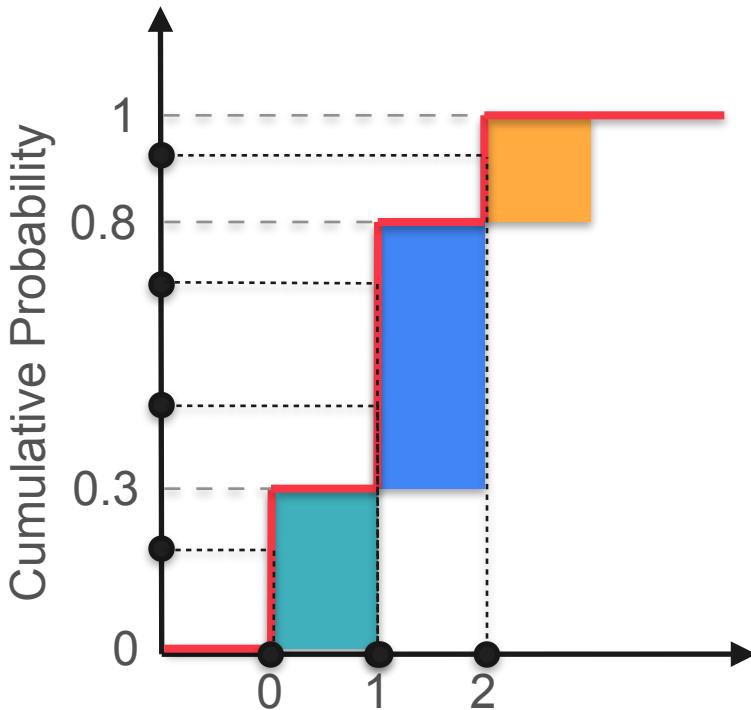
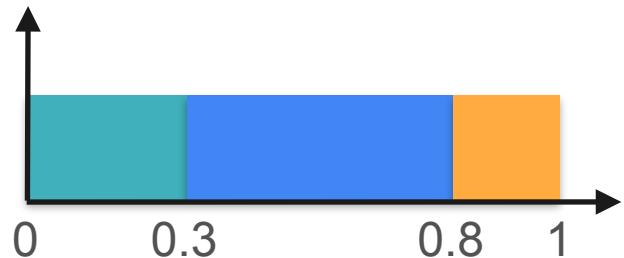
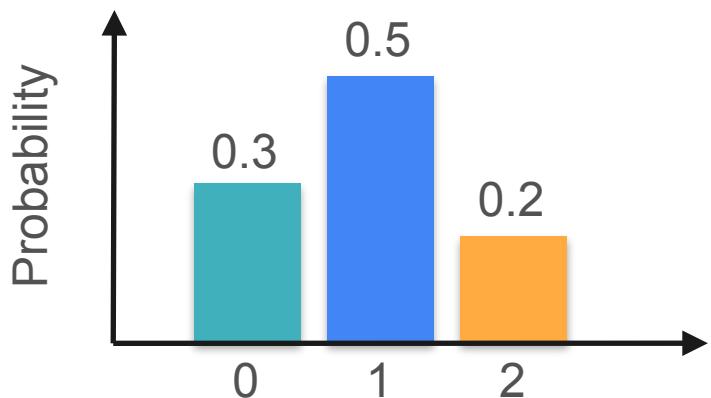
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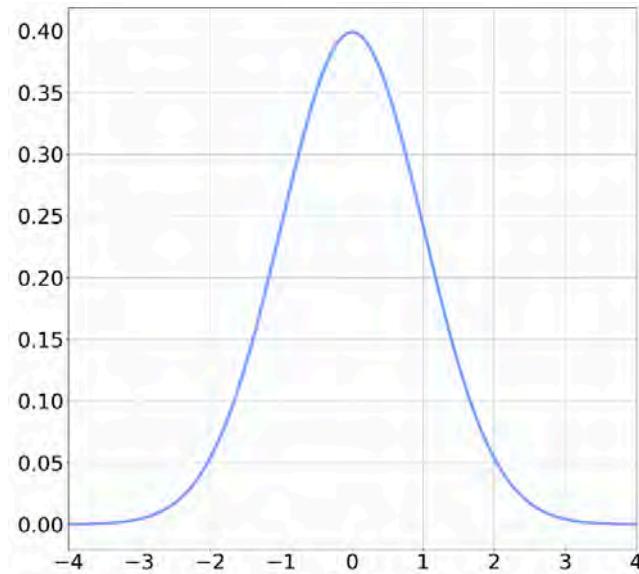
# Sampling From a Distribution



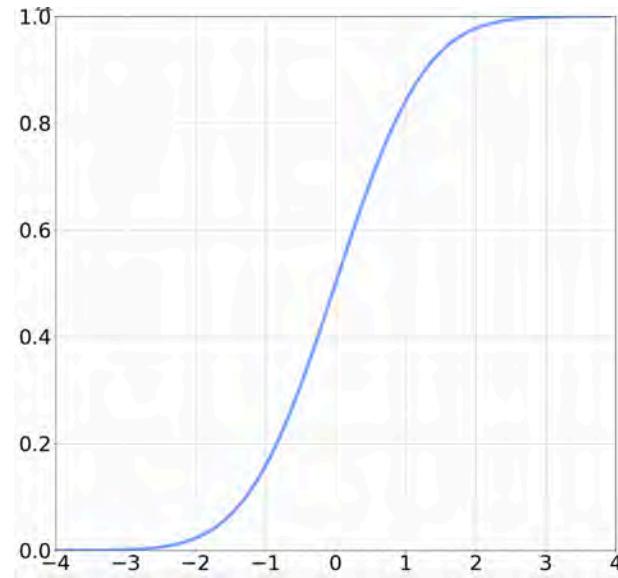
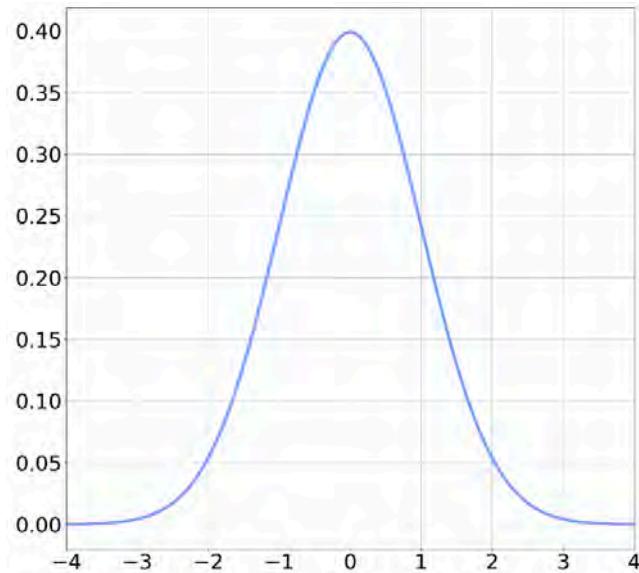
# Sampling From a Distribution



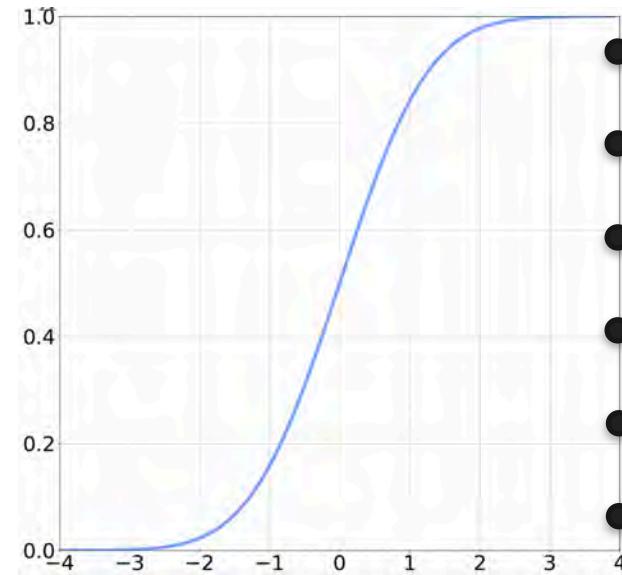
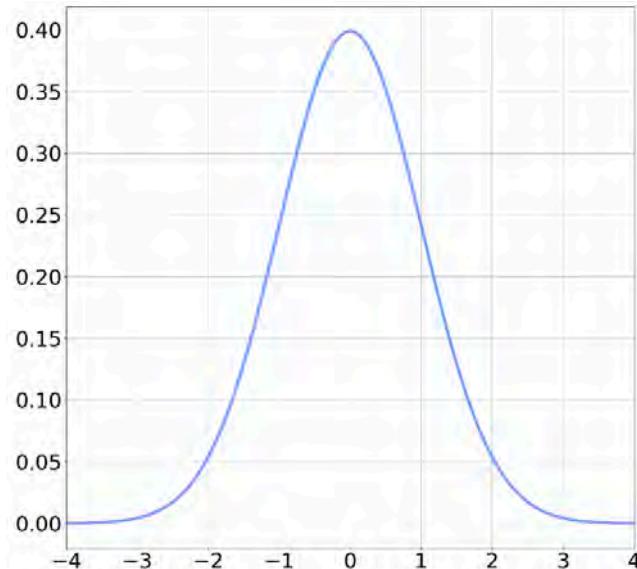
# Sampling From a Normal Distribution



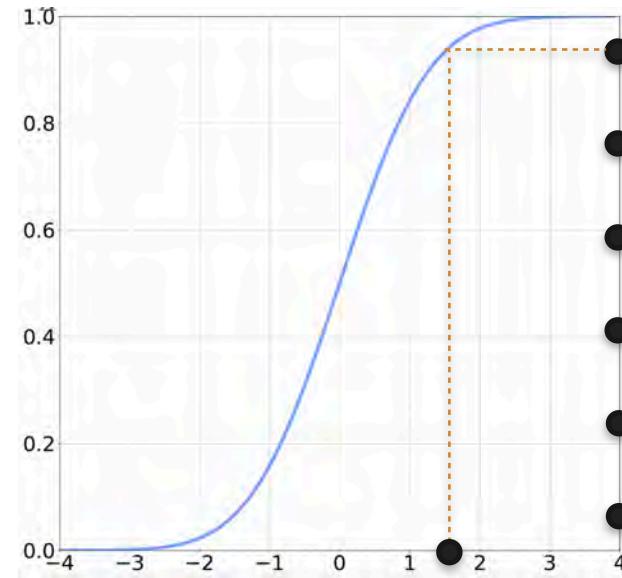
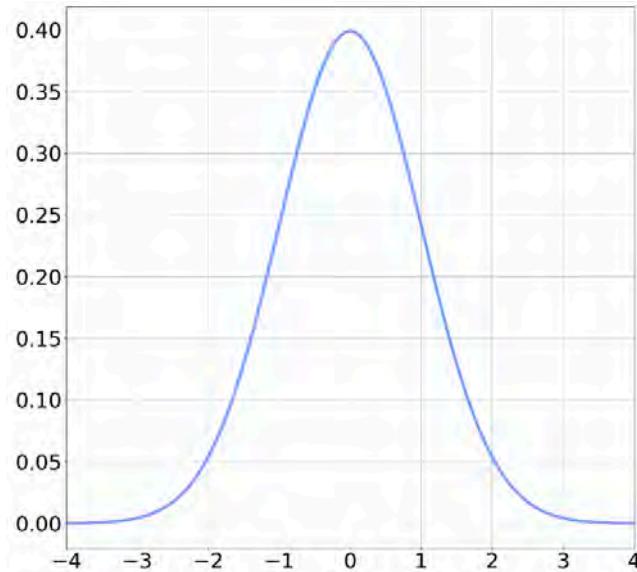
# Sampling From a Normal Distribution



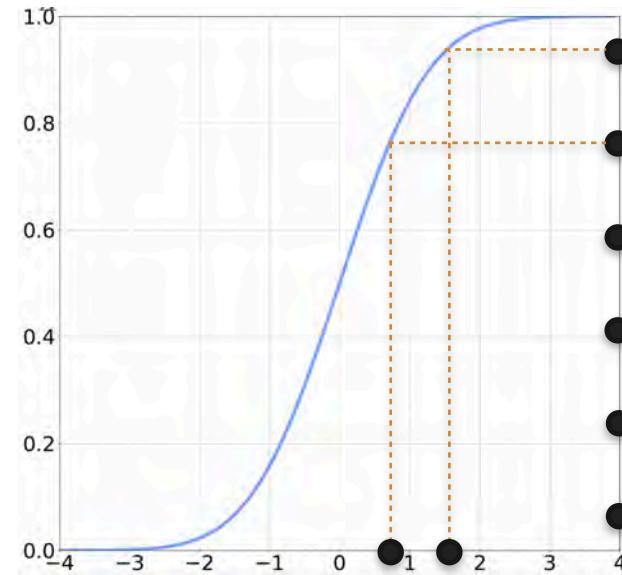
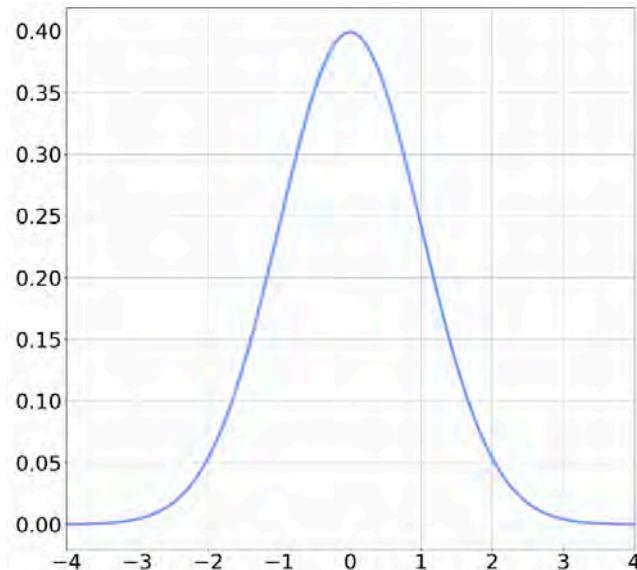
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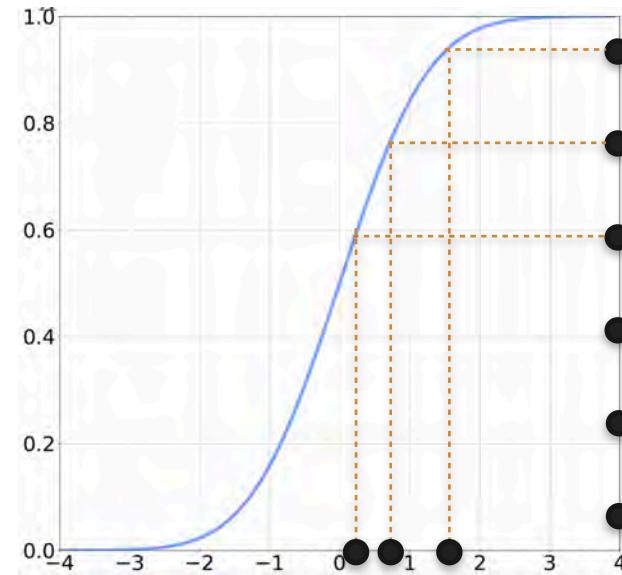
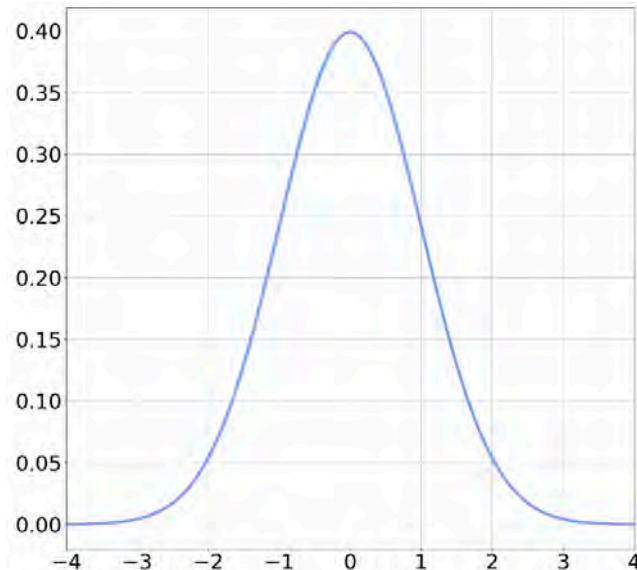
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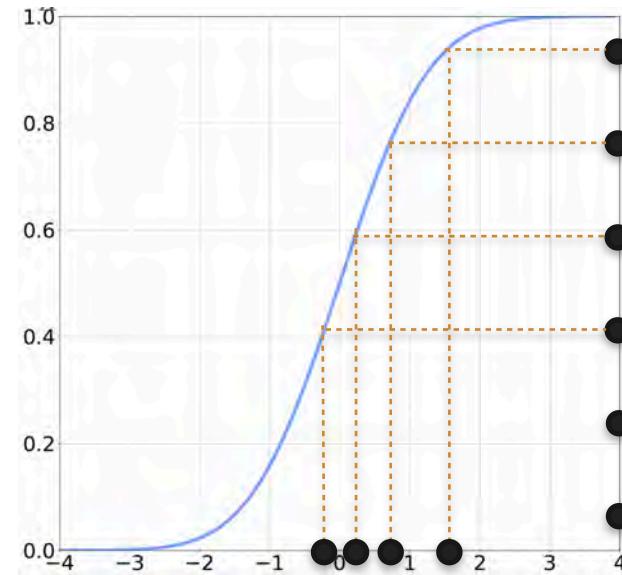
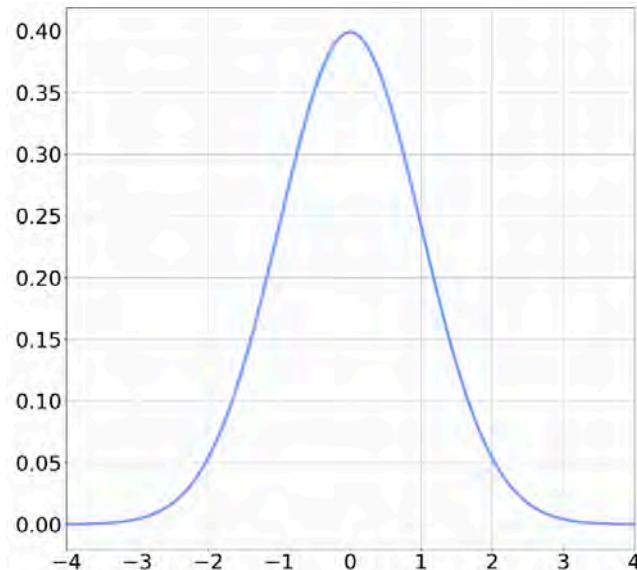
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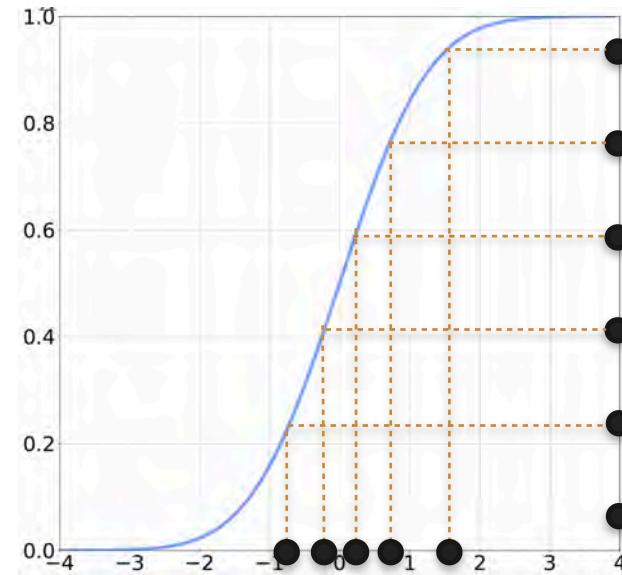
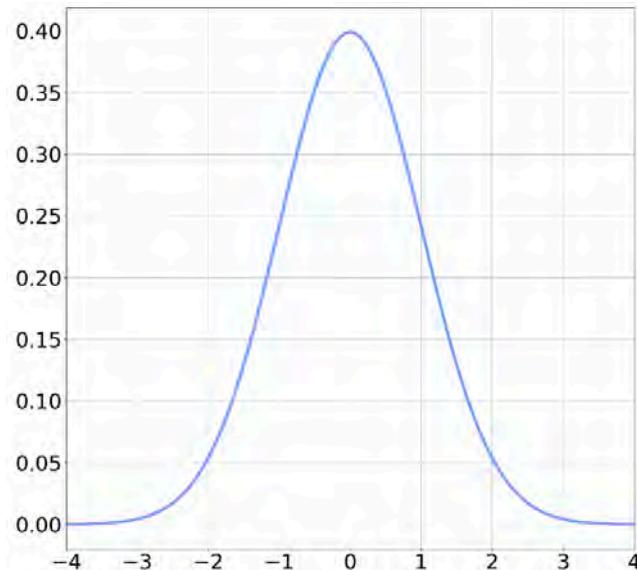
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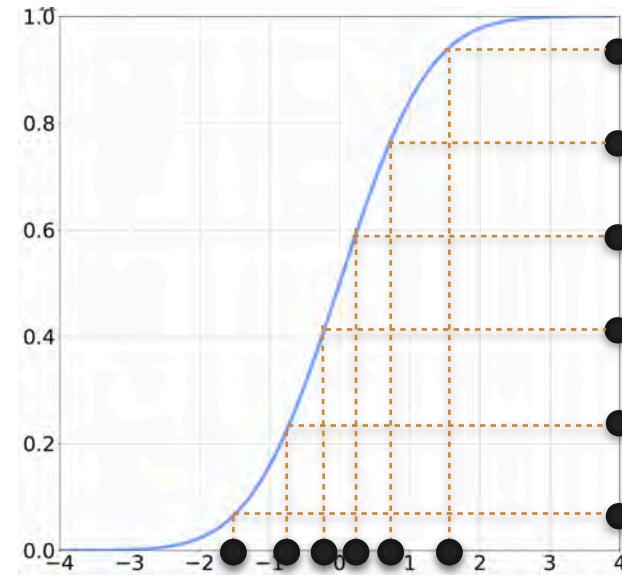
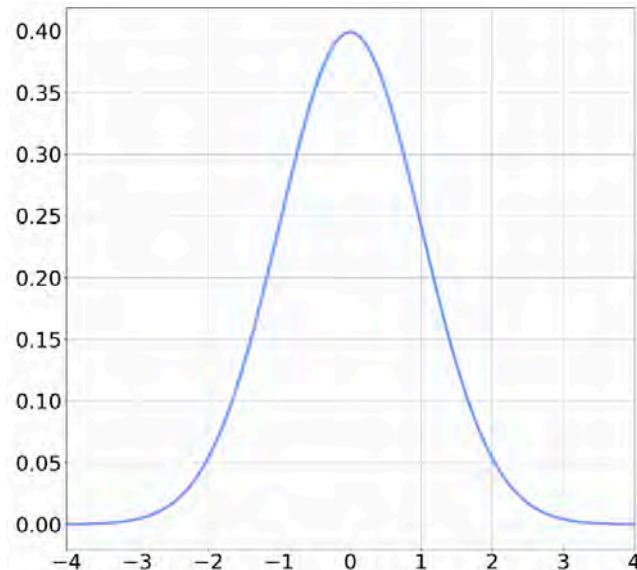
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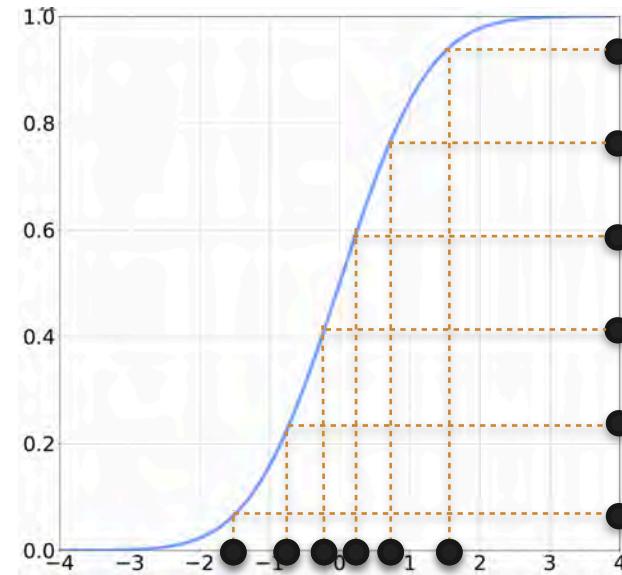
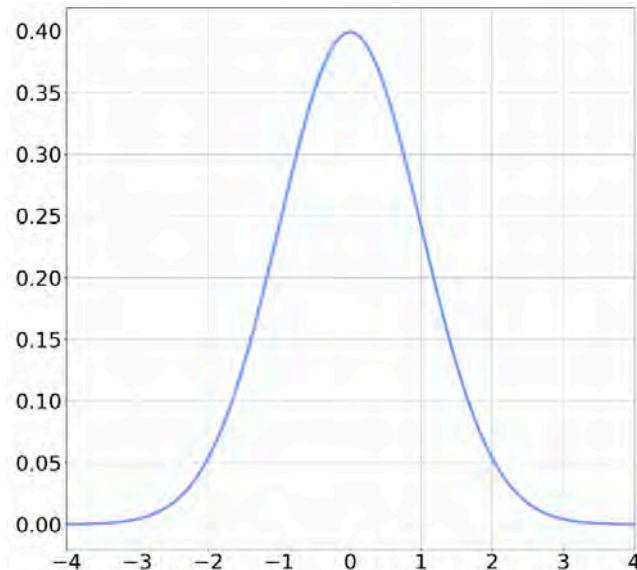
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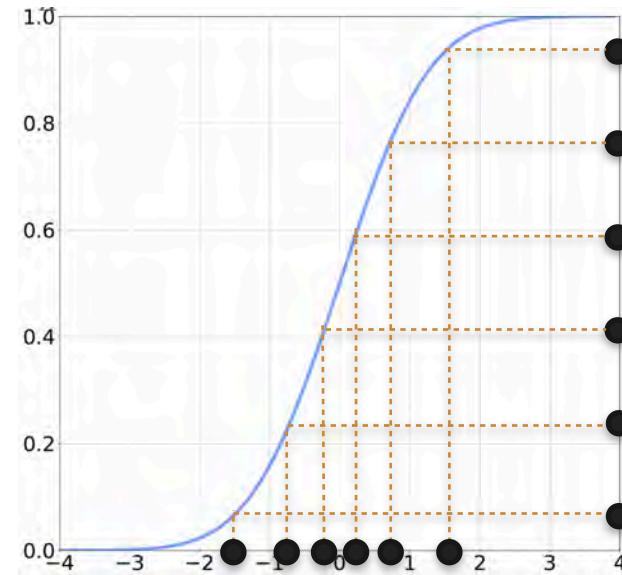
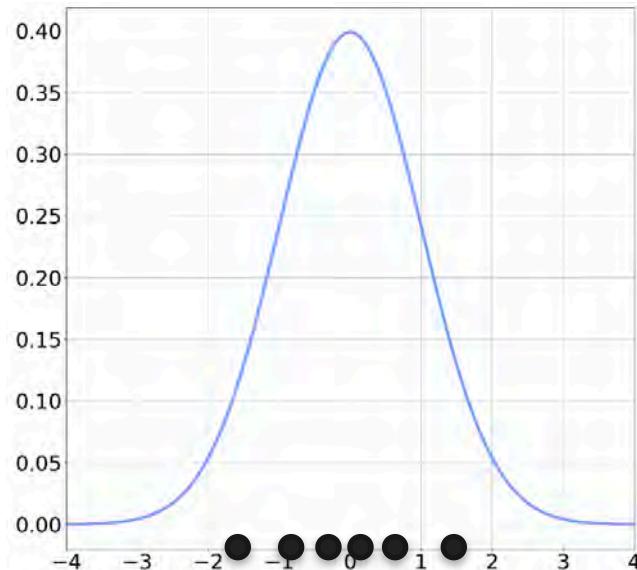
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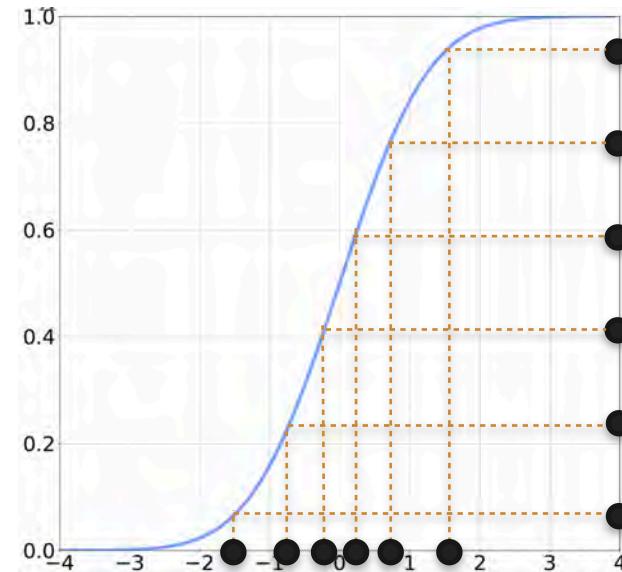
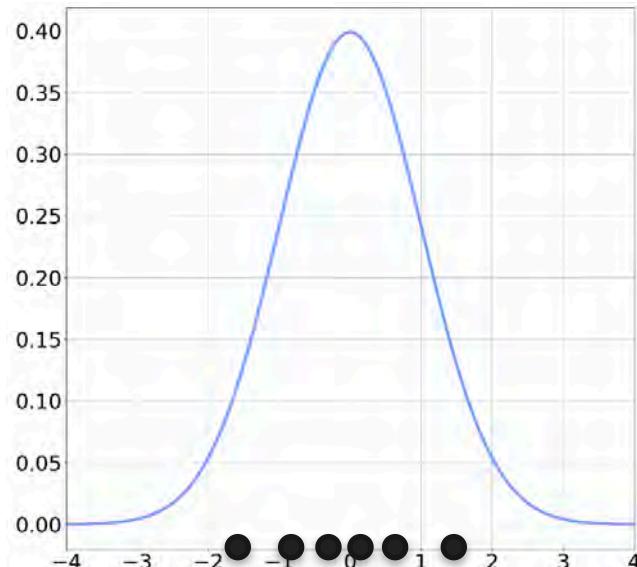
# Sampling From a Normal Distribution



# Sampling From a Normal Distribution



# Sampling From a Normal Distribution





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# Probability Distributions

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## Conclusion