DeepLearning.AI

Math for Machine Learning

Probability and Statistics

W1 Lesson 1

Introduction to Probability

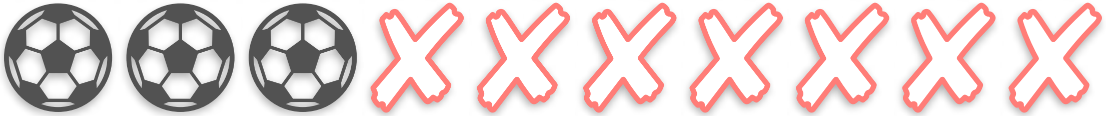
DeepLearning.AI

Introduction to probability

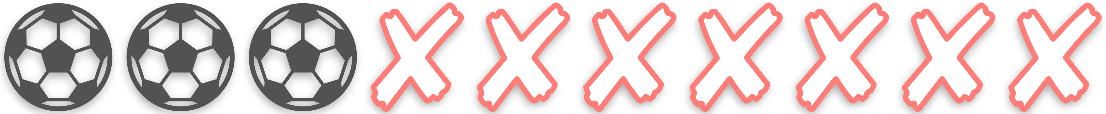
What is Probability?

Introduction to Probability

Introduction to Probability

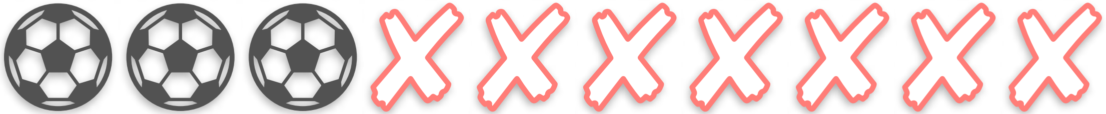


Introduction to Probability



Find the probability that a child picked at random plays soccer.

Introduction to Probability



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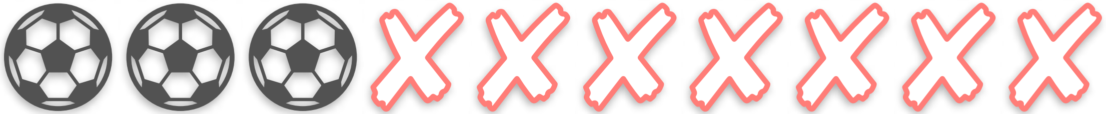

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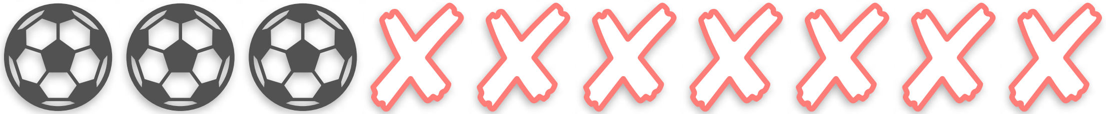

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$$\mathbf{P}(\text{soccer})$$

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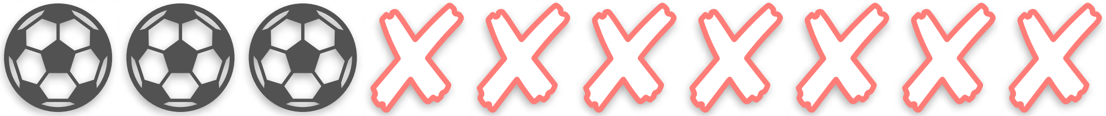

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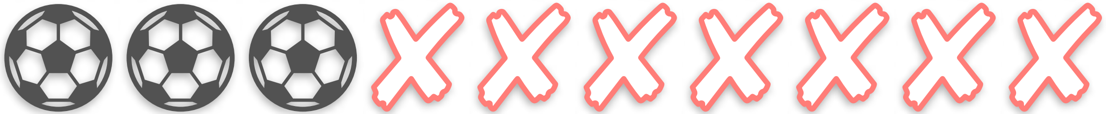

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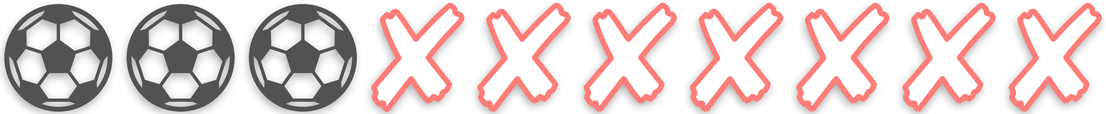

Find the probability that a child picked at random plays soccer.

$$\mathbf{P}(\text{soccer}) = \frac{\text{soccer}}{\text{total}}$$

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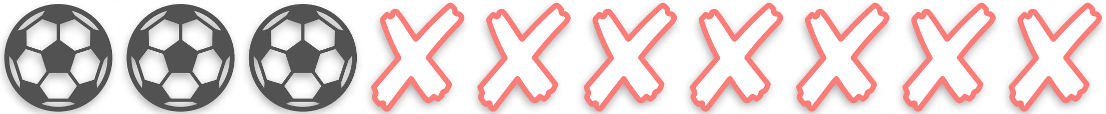

Find the probability that a child picked at random plays soccer.

$$\mathbf{P}(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \underline{\hspace{10cm}}$$

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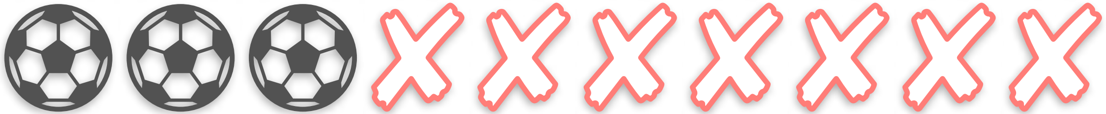

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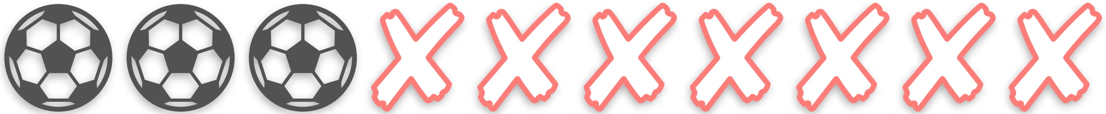

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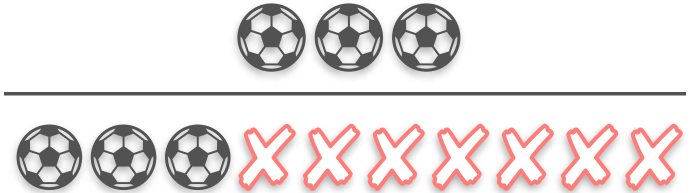

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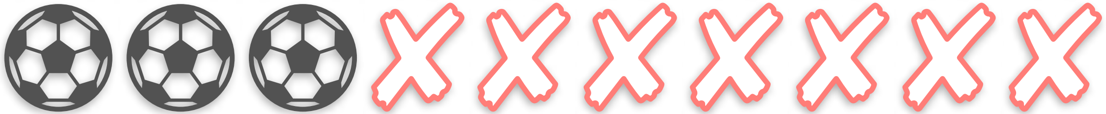

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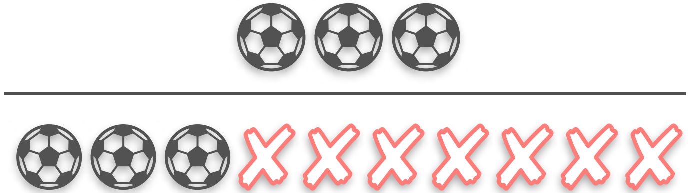

$$= \frac{3}{10}$$

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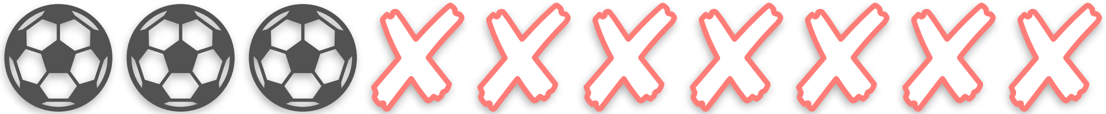

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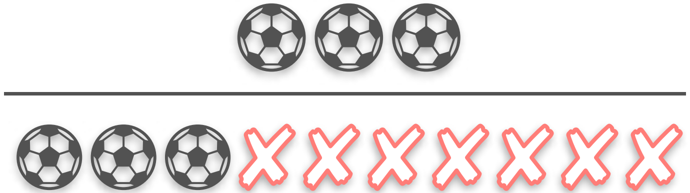

$$= \frac{3}{10} = 0.3$$

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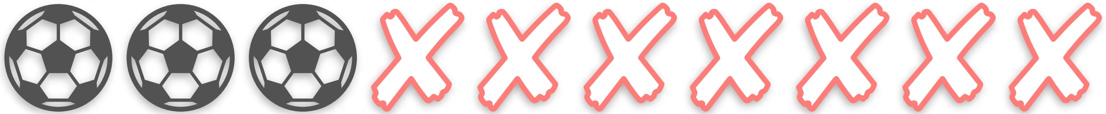

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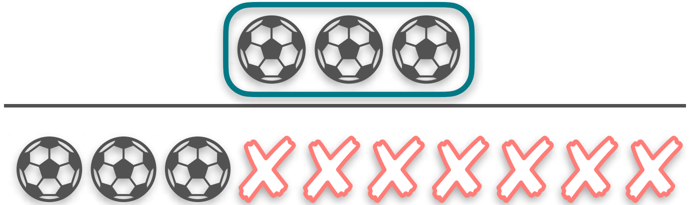

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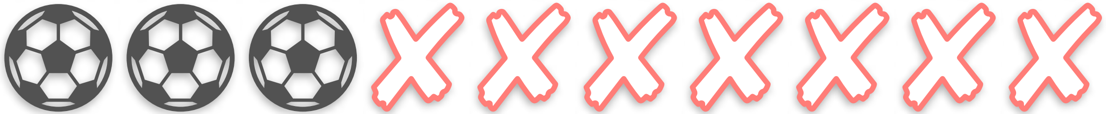

Find the probability that a child picked at random plays soccer.

Event

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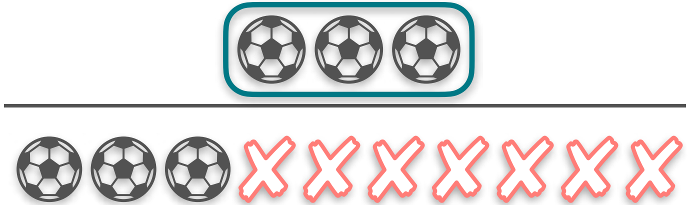

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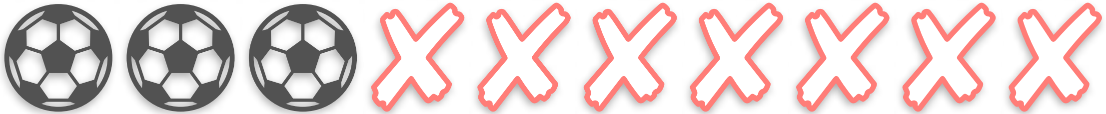

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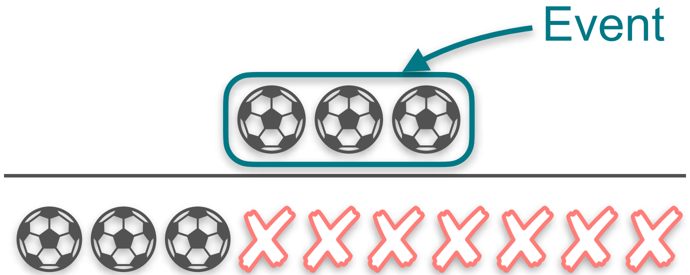

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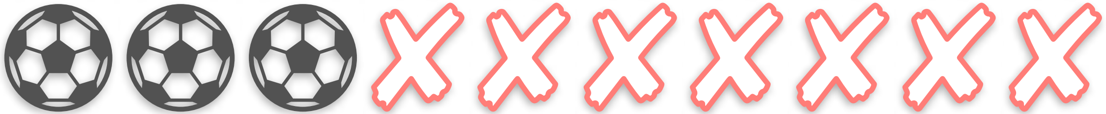

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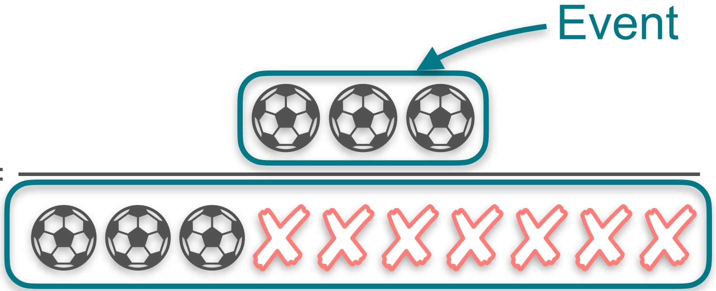

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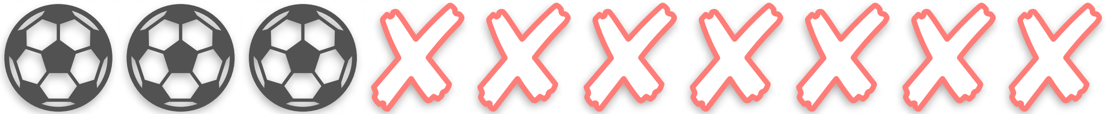

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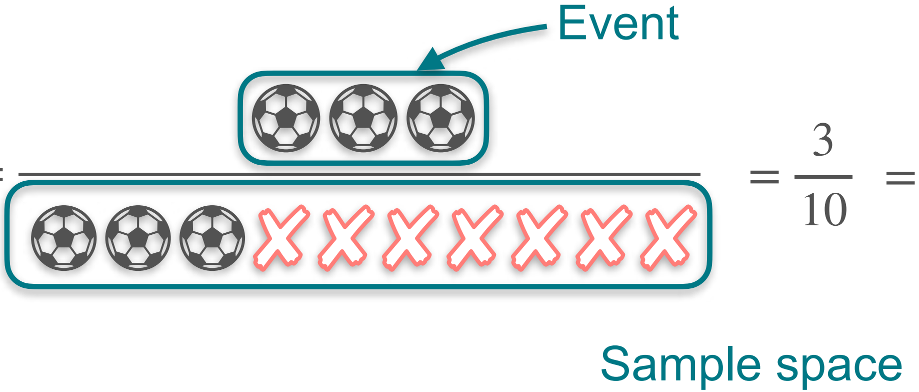

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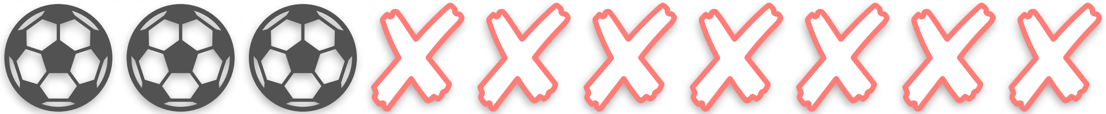

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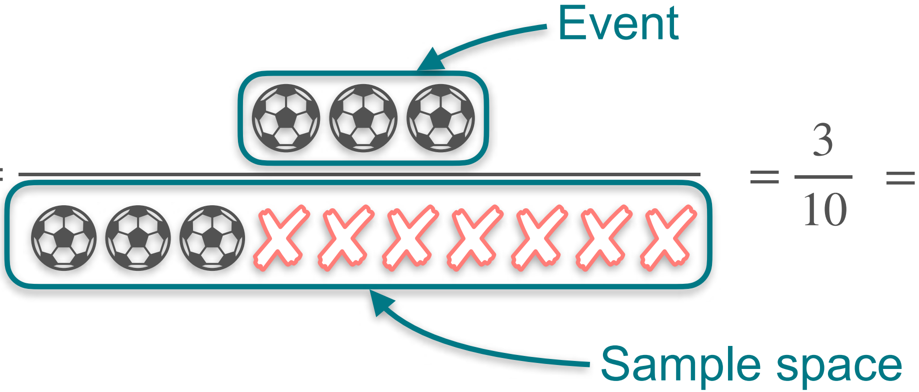

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Introduction to Probability: Venn Diagram

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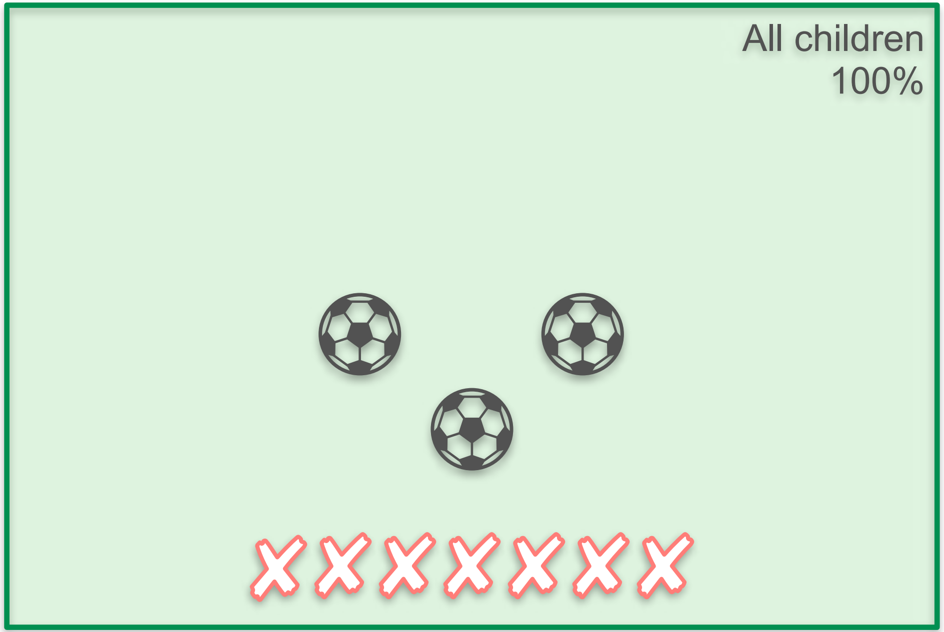

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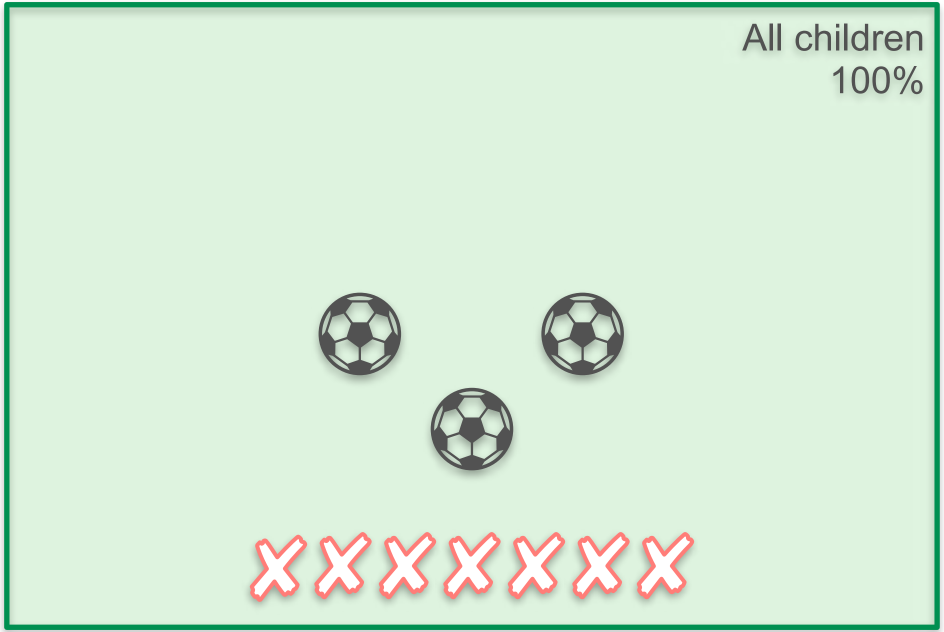
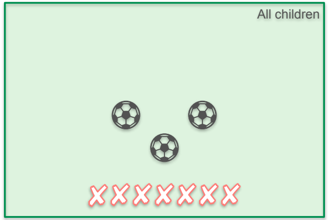
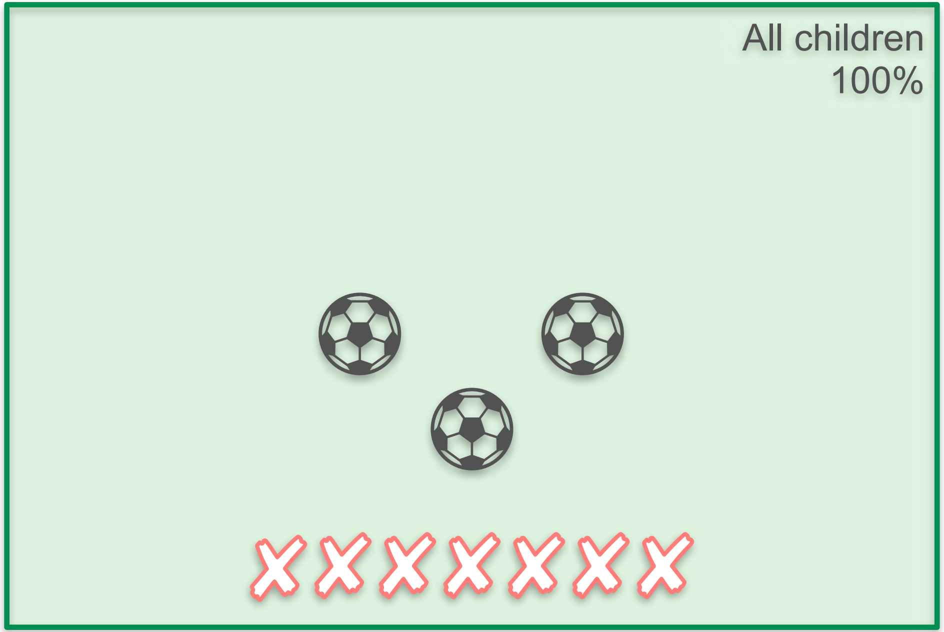
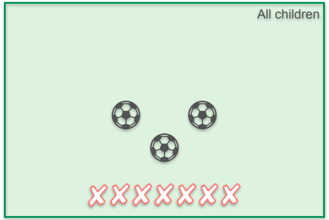




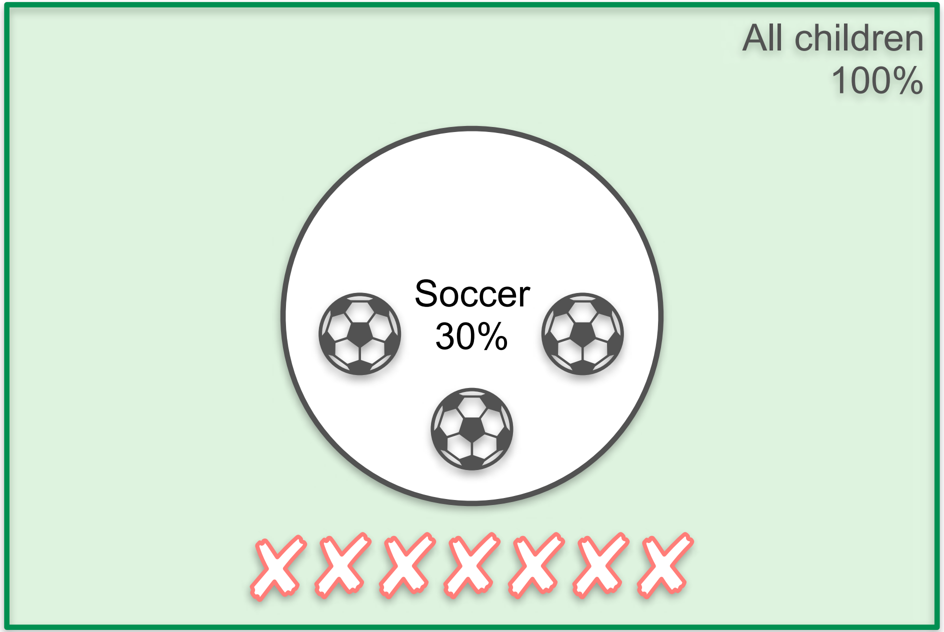
Introduction to Probability: Venn Diagram

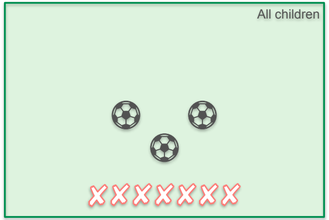




Sample Space

Introduction to Probability: Venn Diagram





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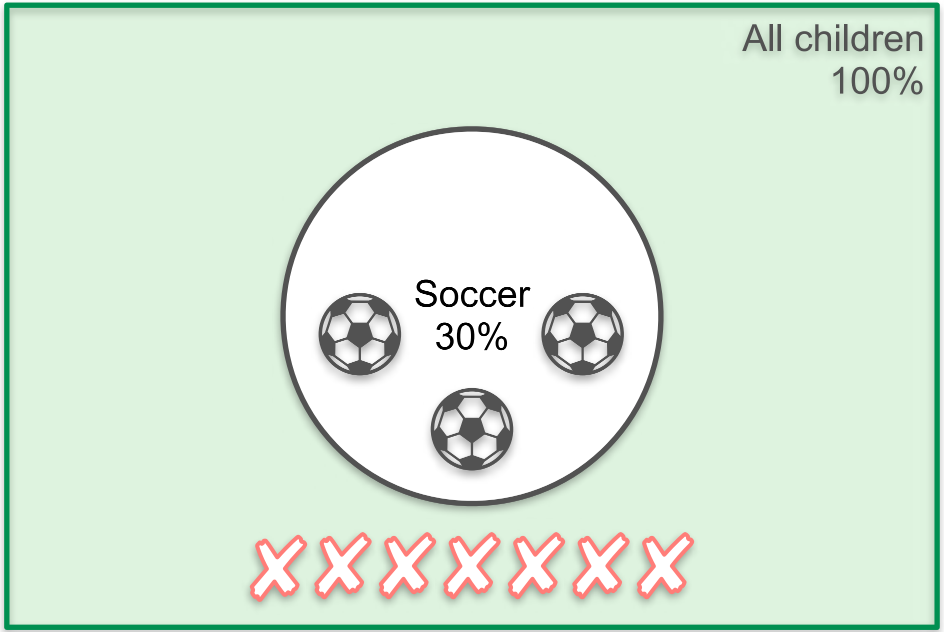
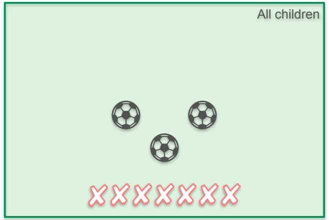
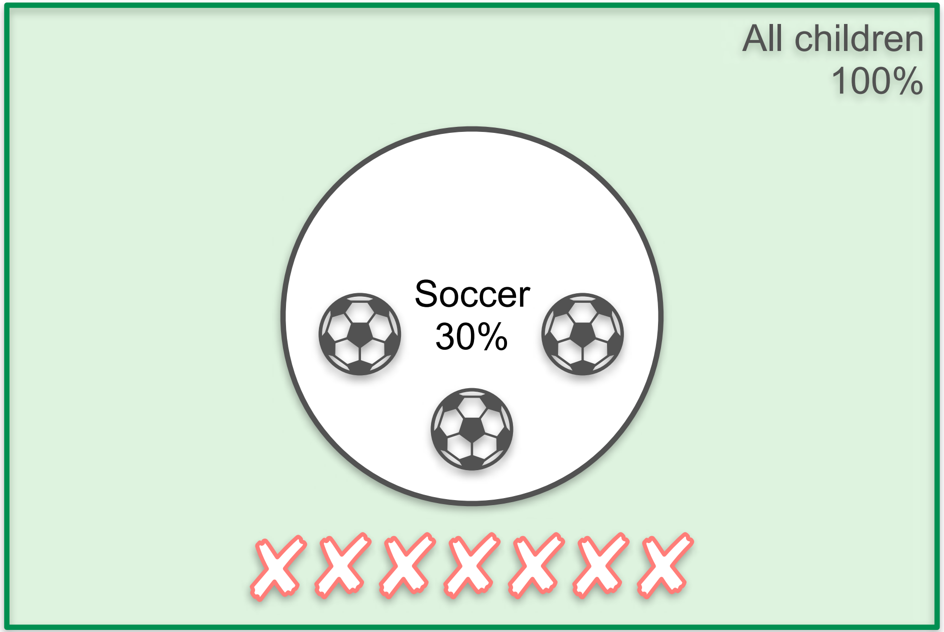
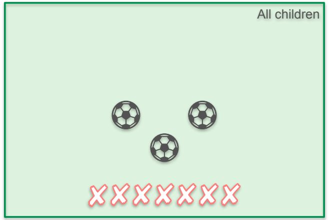




Sample Space

Introduction to Probability: Venn Diagram





Sample Space

Event

Introduction to Probability: Coin Example 1

Introduction to Probability: Coin Example 1

Introduction to Probability: Coin Example 1

Experiment

Introduction to Probability: Coin Example 1

Experiment

Probability of landing on heads

Introduction to Probability: Coin Example 1

Experiment

Probability of landing on heads

$$\mathbf{P}(\text{heads})$$

Introduction to Probability: Coin Example 1

Introduction to Probability: Coin Example 1

Introduction to Probability: Coin Example 1

50%

Introduction to Probability: Coin Example 1

50% 50%

Introduction to Probability: Coin Example 1

50% 50%

$$\mathbf{P}(\text{heads}) = \underline{\hspace{6cm}}$$

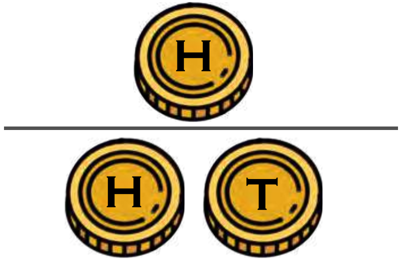
Introduction to Probability: Coin Example 1

50% 50%

$\mathbf{P}(\text{heads}) = \underline{\hspace{6cm}}$

Introduction to Probability: Coin Example 1

50% 50%

$\mathbf{P}(\text{heads}) =$ 

Introduction to Probability: Coin Example 1

50% 50%

$$\mathbf{P}(\text{heads}) = \frac{\text{H}}{\text{H} \quad \text{T}} = \frac{1}{2} = 0.5$$

Introduction to Probability: Coin Example 2

50% 50%

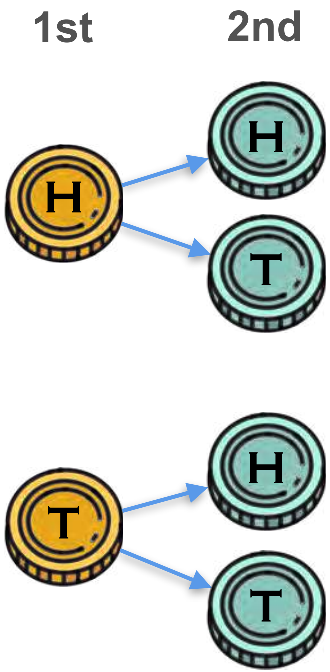
Introduction to Probability: Coin Example 2

50% 50%

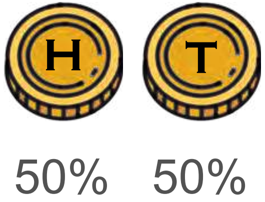
1st

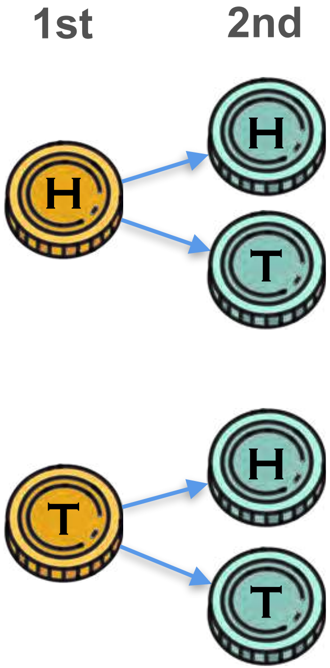
Introduction to Probability: Coin Example 2

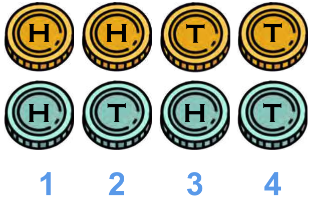
50% 50%



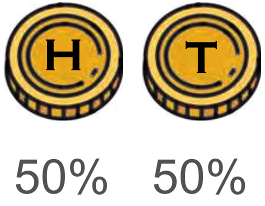
Introduction to Probability: Coin Example 2



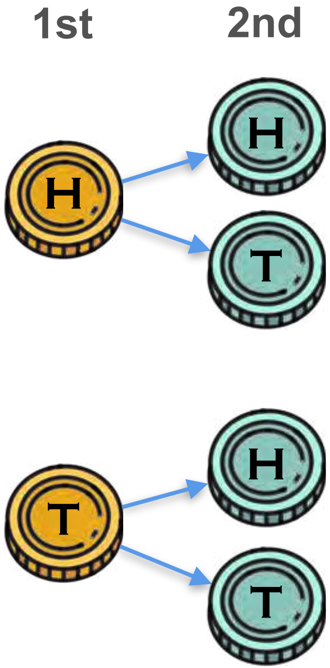




Introduction to Probability: Coin Example 2

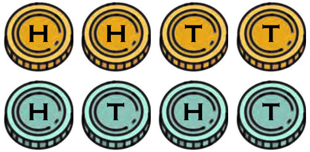


What is the probability of landing on heads twice?



Introduction to Probability: Coin Example 2

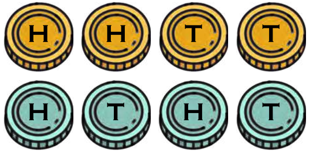
50%   50%



Introduction to Probability: Coin Example 2

50% 50%

$$\mathbf{P}(HH) = \underline{\hspace{6cm}}$$



Introduction to Probability: Coin Example 2

50%   50%

$$\mathbf{P}(HH) = \underline{\qquad\qquad\qquad\qquad}$$

Introduction to Probability: Coin Example 2

50% 50%

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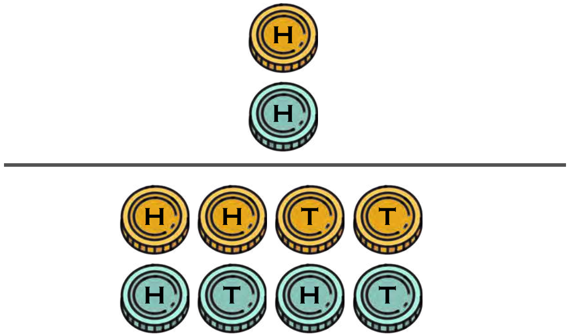
Introduction to Probability: Coin Example 2

50% 50%

$\mathbf{P}(HH) =$



Introduction to Probability: Coin Example 2

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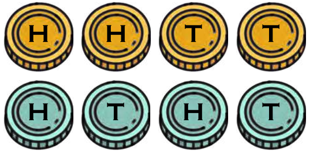
$\mathbf{P}(HH) =$



$= \frac{1}{4} = 0.25$

Introduction to Probability: Coin Example 3

50% 50%

Introduction to Probability: Coin Example 3

50% 50%

1st

Introduction to Probability: Coin Example 3

50% 50%

1st

Introduction to Probability: Coin Example 3

50% 50%

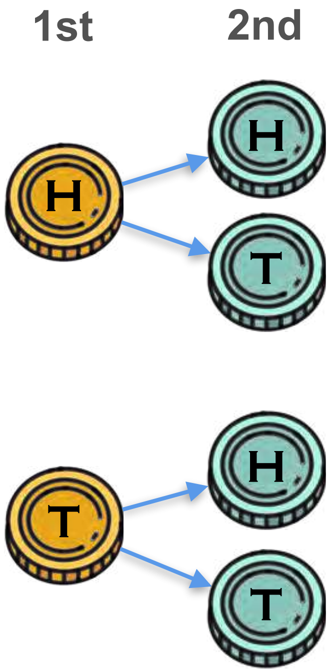
1st 2nd

Introduction to Probability: Coin Example 3

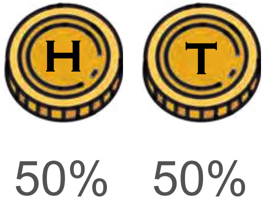
50% 50%

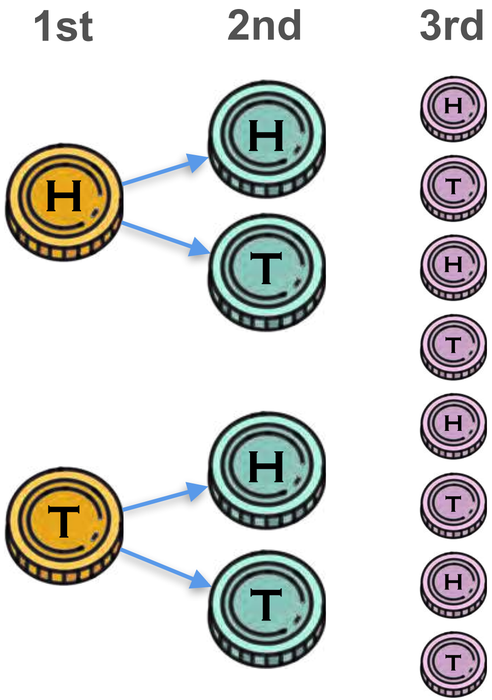
Introduction to Probability: Coin Example 3

50% 50%



Introduction to Probability: Coin Example 3





Introduction to Probability: Coin Example 3

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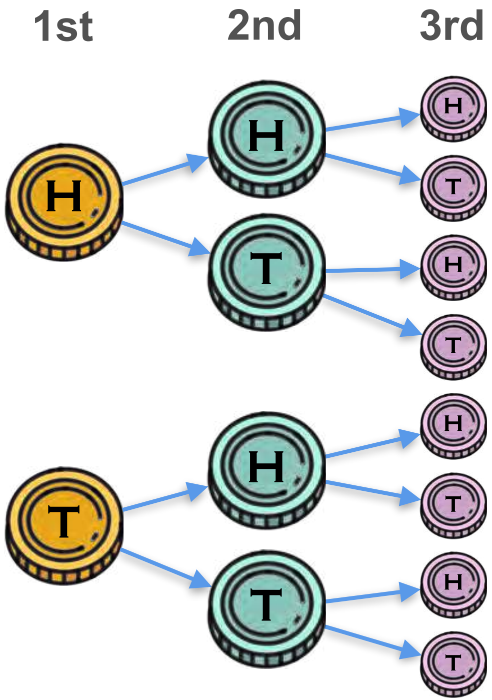

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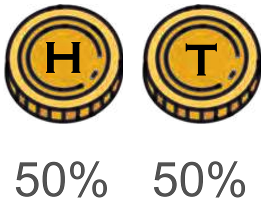

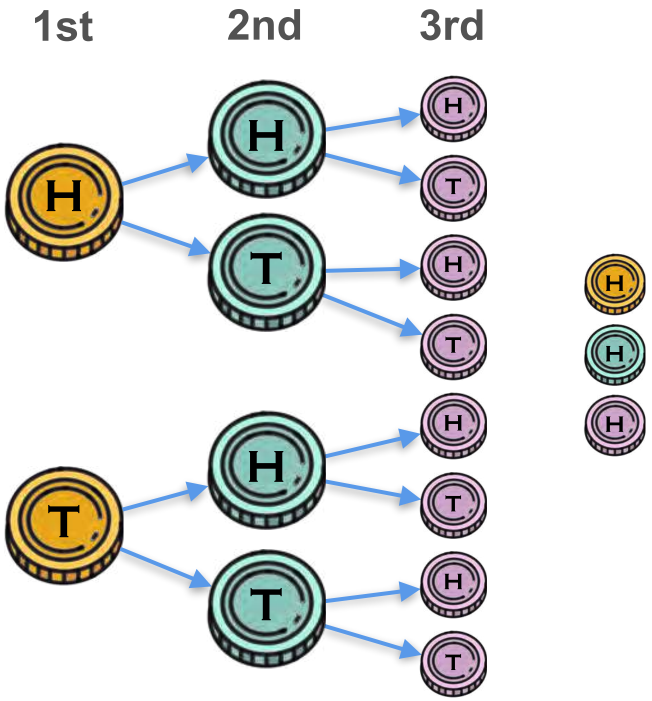

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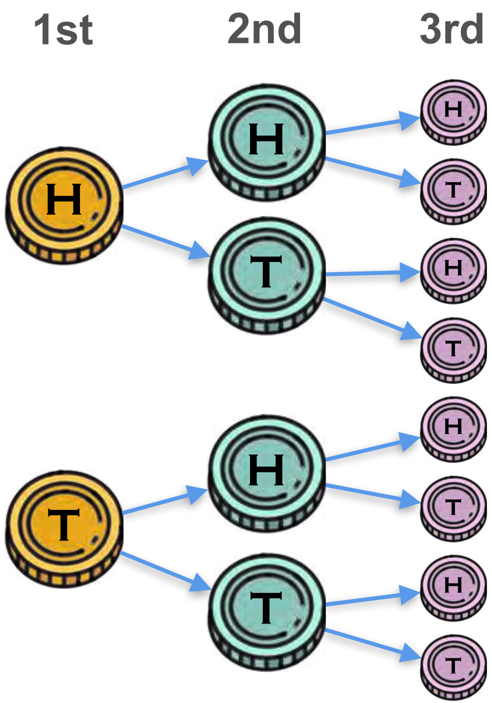

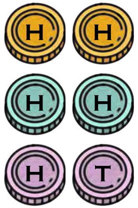

Introduction to Probability: Coin Example 3

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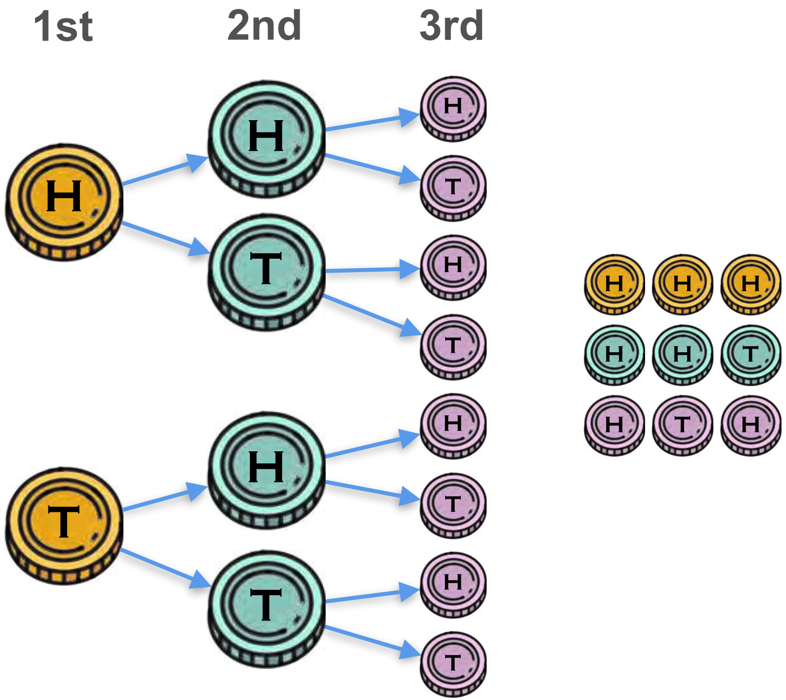

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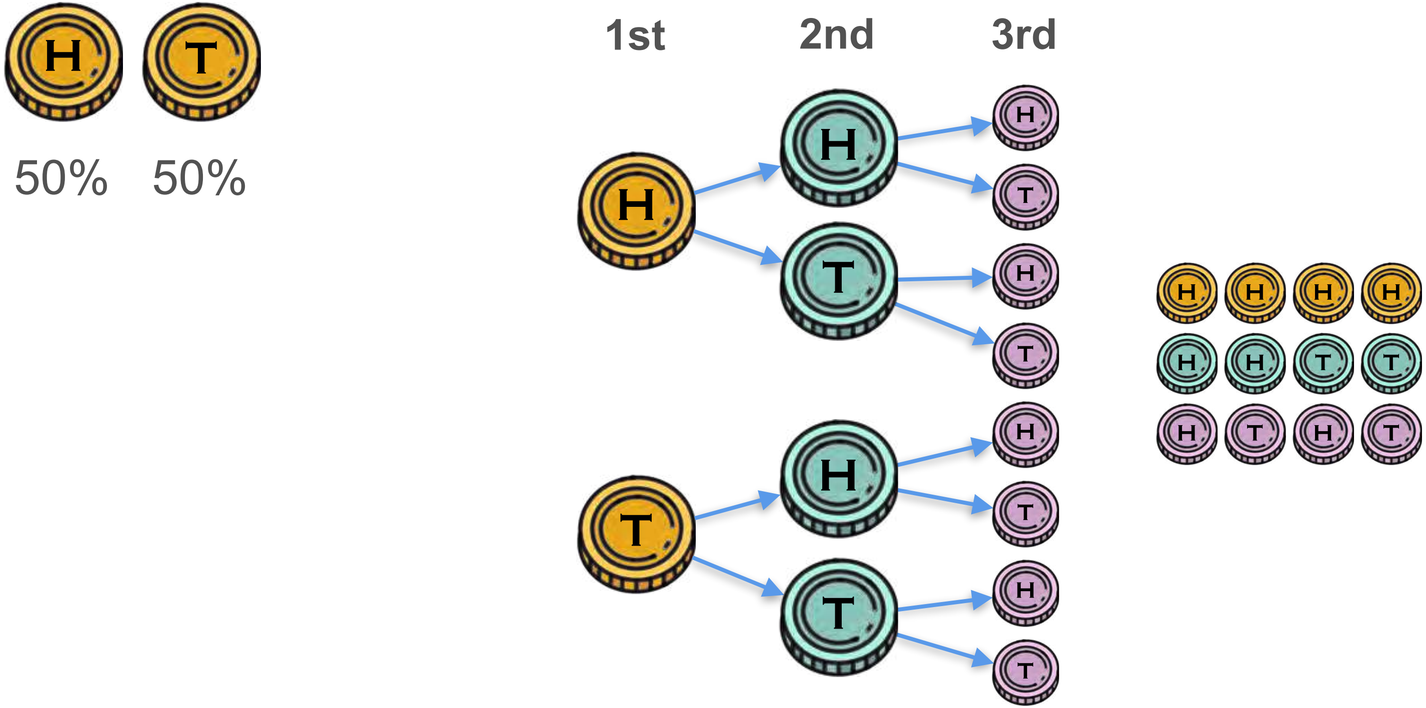
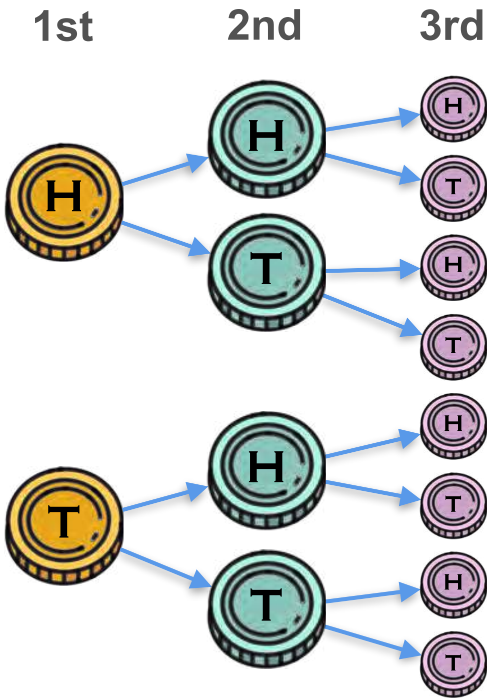
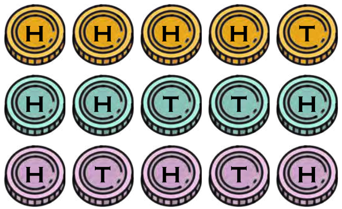


Introduction to Probability: Coin Example 3

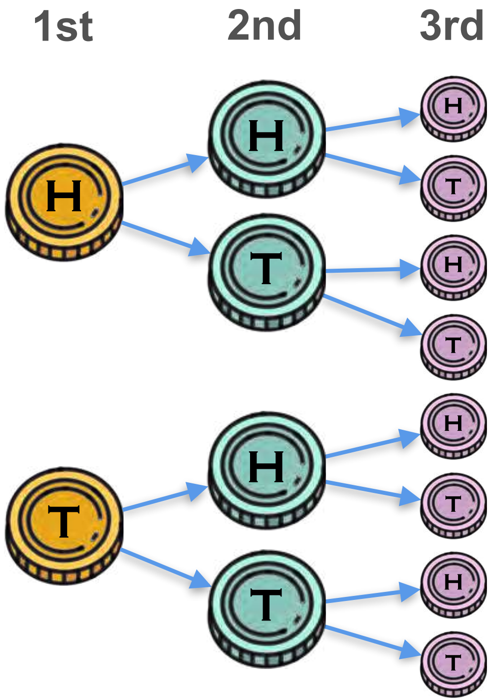
50% 50%

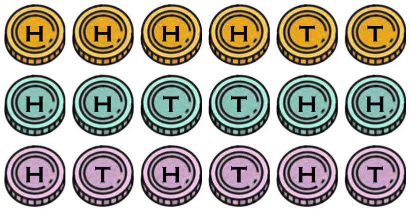




Introduction to Probability: Coin Example 3

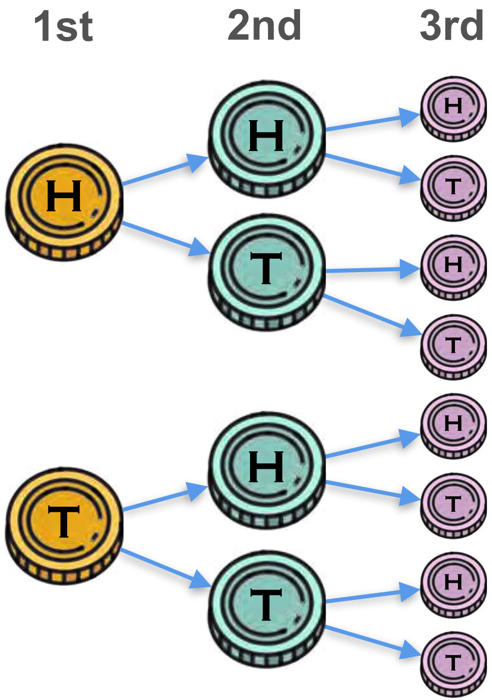
50% 50%

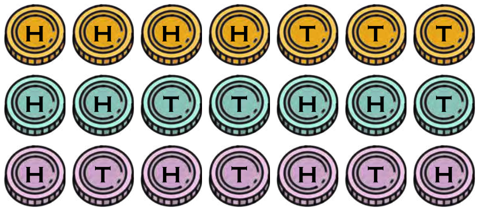




Introduction to Probability: Coin Example 3

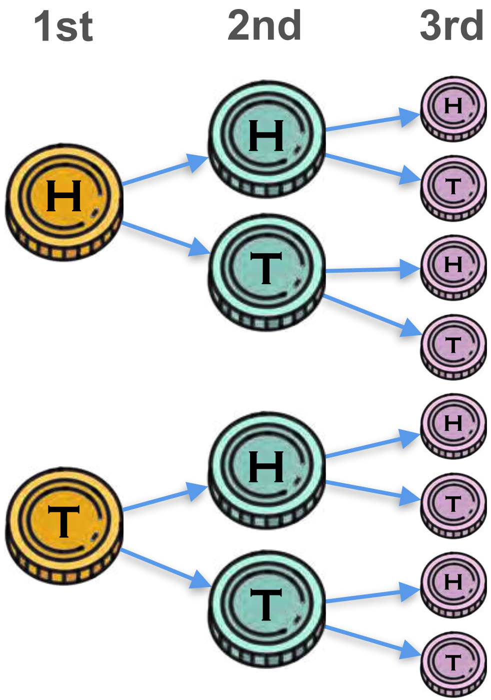
50% 50%

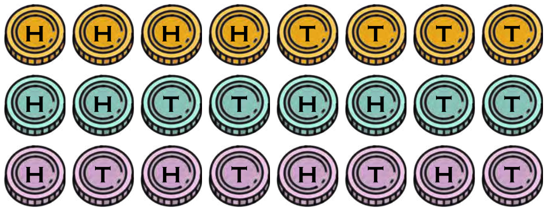




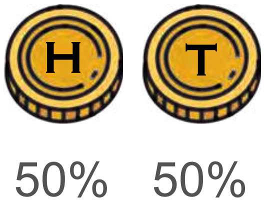
Introduction to Probability: Coin Example 3

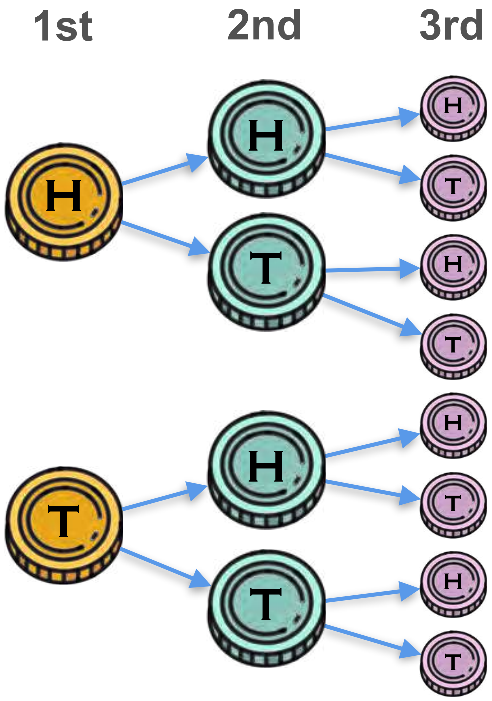
50% 50%

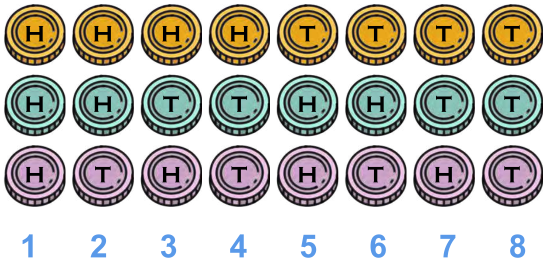




Introduction to Probability: Coin Example 3

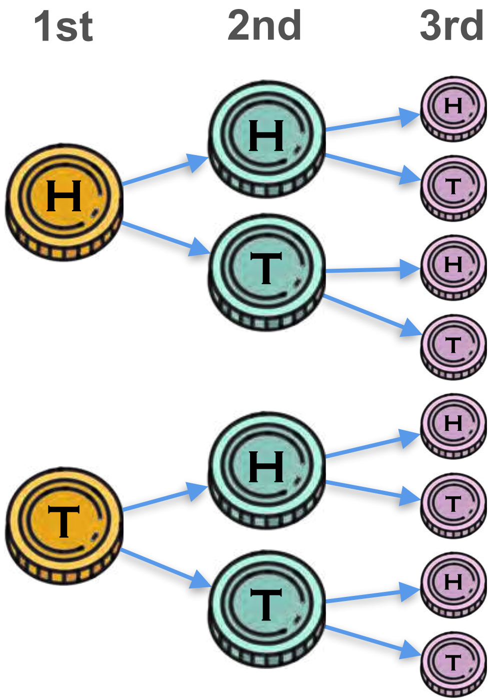


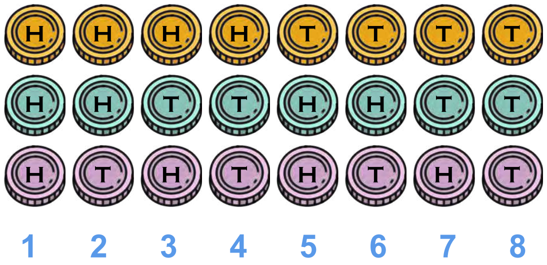




Introduction to Probability: Coin Example 3

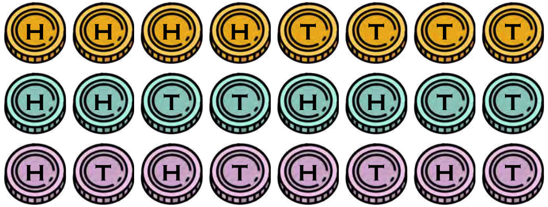
What is the probability of landing on heads 3 times?





Introduction to Probability: Coin Example 3

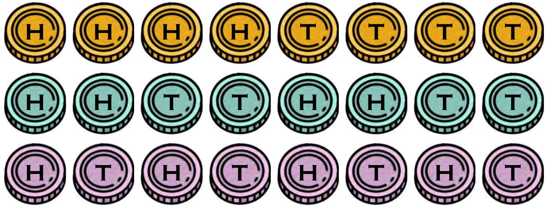
50% 50%



Introduction to Probability: Coin Example 3

50%   50%

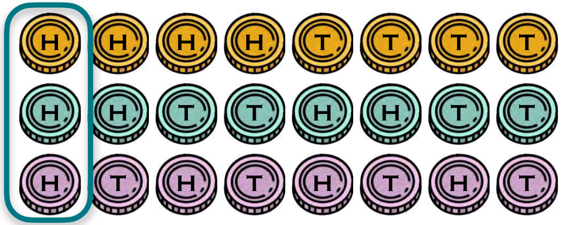
$$\mathbf{P}(HHH) = \frac{}{}$$



Introduction to Probability: Coin Example 3

50% 50%

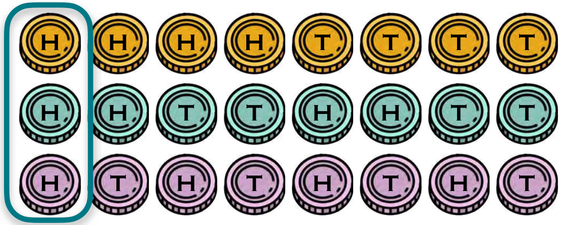
$$\mathbf{P}(HHH) = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$



Introduction to Probability: Coin Example 3

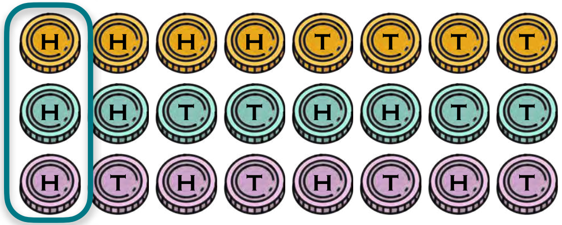
50% 50%

$$\mathbf{P}(HHH) = \underline{\qquad\qquad\qquad\qquad}$$



Introduction to Probability: Coin Example 3

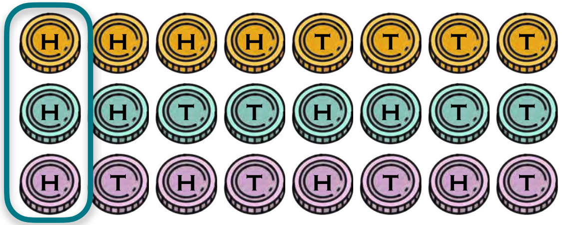
50%   50%



$$\mathbf{P}(HHH) = \frac{\text{[coins]}}{\text{[coins]}}$$

Introduction to Probability: Coin Example 3

50% 50%



$\mathbf{P}(HHH) =$ 

$= \frac{1}{8} = 0.125$

Introduction to Probability: Dice Example 1

Introduction to Probability: Dice Example 1

Introduction to Probability: Dice Example 1

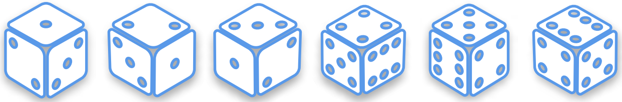
What is the probability of obtaining 6?

Introduction to Probability: Dice Example 1

What is the probability of obtaining 6?

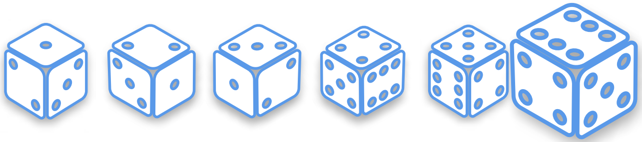
Introduction to Probability: Dice Example 1

What is the probability of obtaining 6?



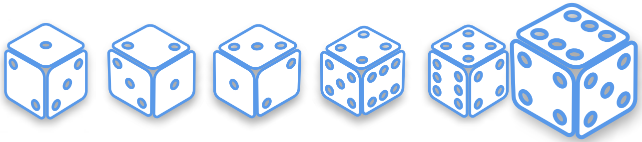
Introduction to Probability: Dice Example 1

What is the probability of obtaining 6?



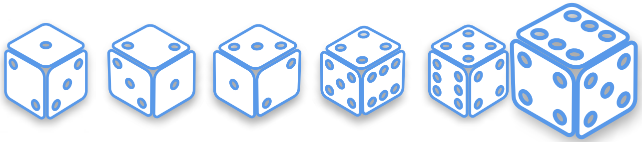
Introduction to Probability: Dice Example 1

What is the probability of obtaining 6?



$$\mathbf{P}(6) = \underline{\hspace{6cm}}$$

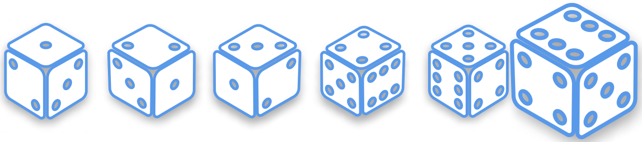
Introduction to Probability: Dice Example 1



What is the probability of obtaining 6?

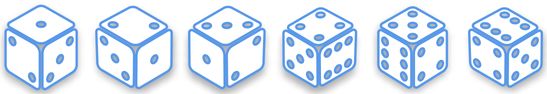
$$\mathbf{P}(6) = \frac{\quad}{\quad}$$

Introduction to Probability: Dice Example 1

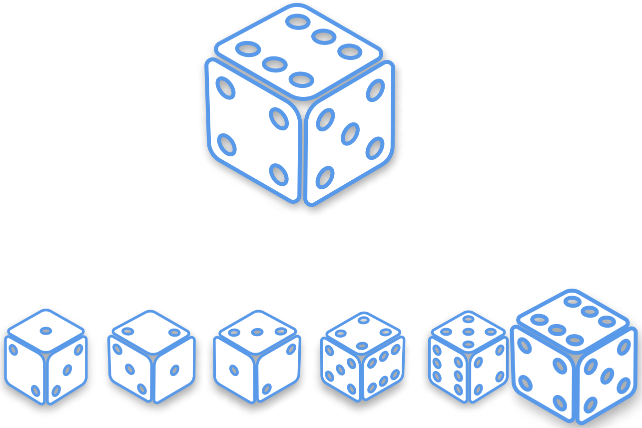


What is the probability of obtaining 6?

$\mathbf{P}(6) =$

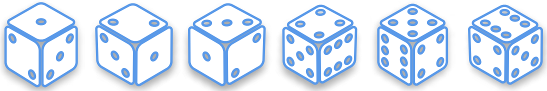


Introduction to Probability: Dice Example 1



What is the probability of obtaining 6?

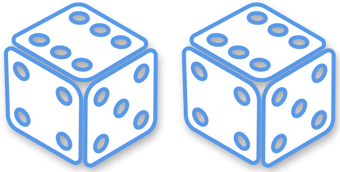
$\mathbf{P}(6) =$



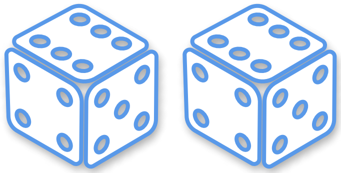
$= \frac{1}{6}$

Introduction to Probability: Dice Example 2

Introduction to Probability: Dice Example 2

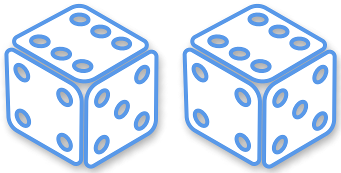


Introduction to Probability: Dice Example 2



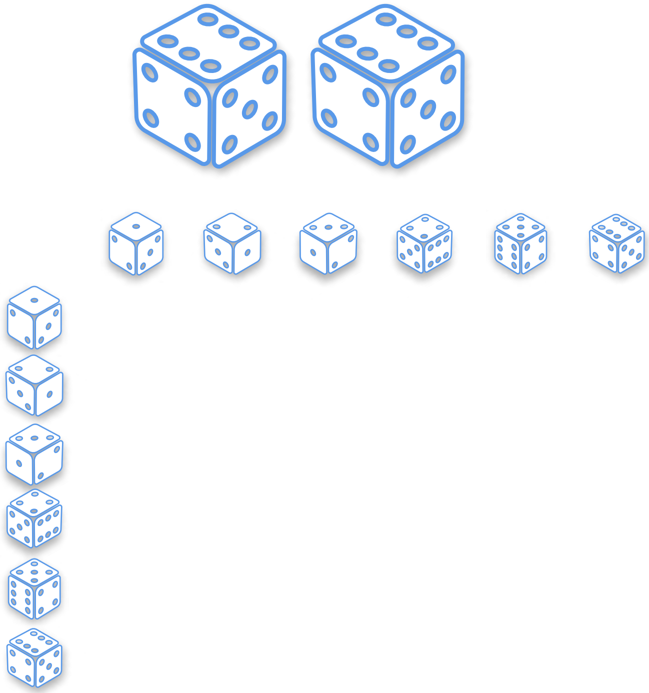
What is the probability of obtaining 6,6?

Introduction to Probability: Dice Example 2



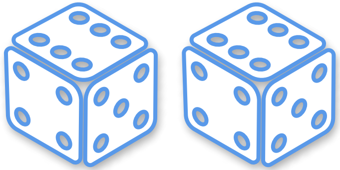
What is the probability of obtaining 6,6?

Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

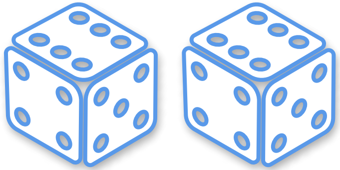
Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

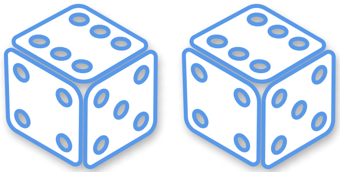
Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Introduction to Probability: Dice Example 2

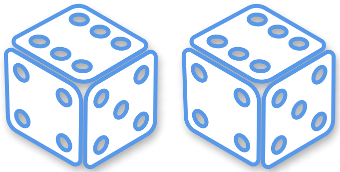


1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	$\boxed{6,6}$

What is the probability of obtaining 6,6?

$$\mathbf{P}(6,6) = \underline{\hspace{6cm}}$$

Introduction to Probability: Dice Example 2

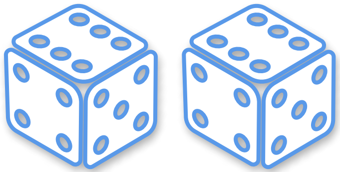


What is the probability of obtaining 6,6?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(6,6) = \frac{\boxed{6,6}}{}$$

Introduction to Probability: Dice Example 2



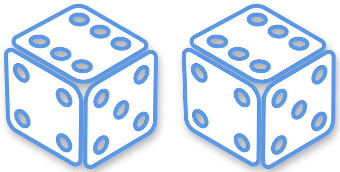
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	**6,6**

What is the probability of obtaining 6,6?

$$\mathbf{P}(6,6) = \frac{\boxed{6,6}}{\text{(all outcomes below)}}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Introduction to Probability: Dice Example 2



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

What is the probability of obtaining 6,6?

$$\mathbf{P}(6,6) = \frac{\boxed{6,6}}{\boxed{\begin{array}{cccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}} = \frac{1}{36}$$

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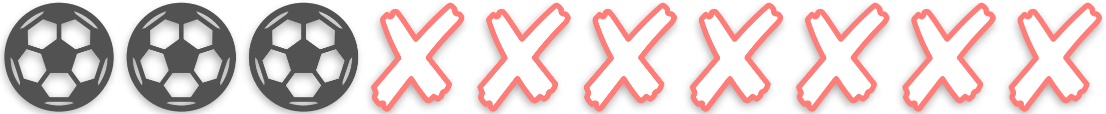
Introduction to probability

Complement of Probability

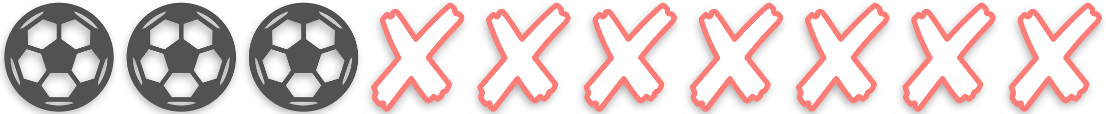
Complement of Probability

Complement of Probability



30%

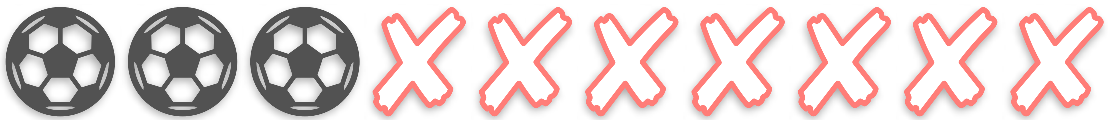
Complement of Probability



30%

What is the probability of a child NOT playing soccer?

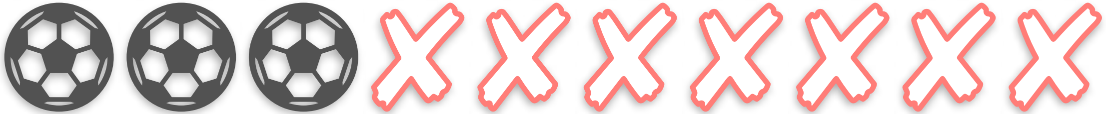
Complement of Probability



30%

What is the probability of a child NOT playing soccer?

Complement of Probability

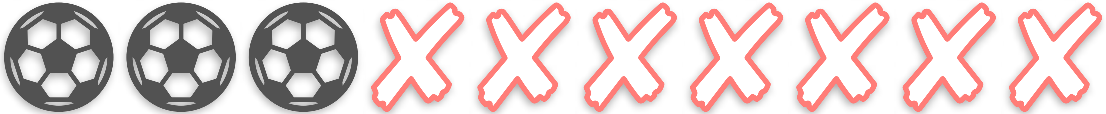


30%

What is the probability of a child NOT playing soccer?

$$\mathbf{P}(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}}$$

Complement of Probability

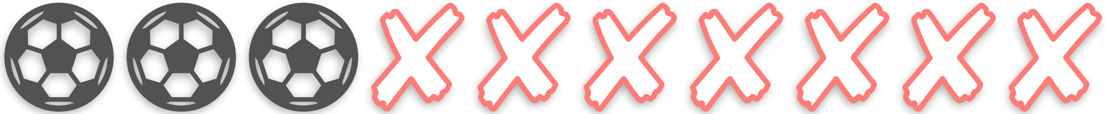


30%

What is the probability of a child NOT playing soccer?

$$\mathbf{P}(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \underline{\hspace{10cm}}$$

Complement of Probability

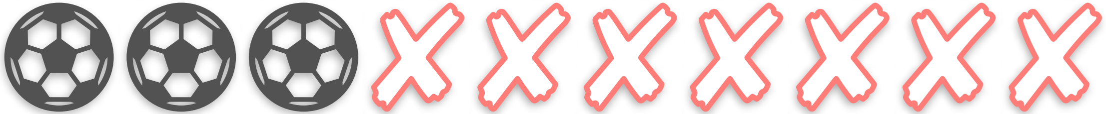


30%

What is the probability of a child NOT playing soccer?

$$\mathbf{P}(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} =$$

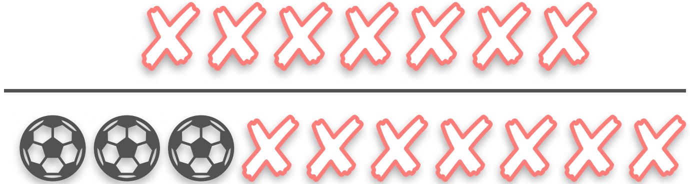
Complement of Probability



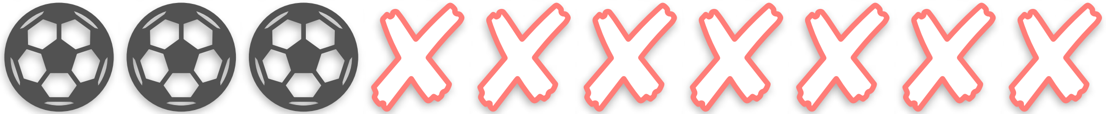
30%

What is the probability of a child NOT playing soccer?

$$\mathbf{P}(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} =$$



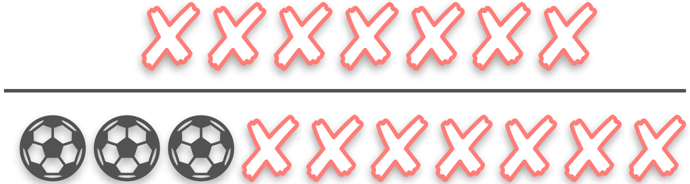
Complement of Probability



30%

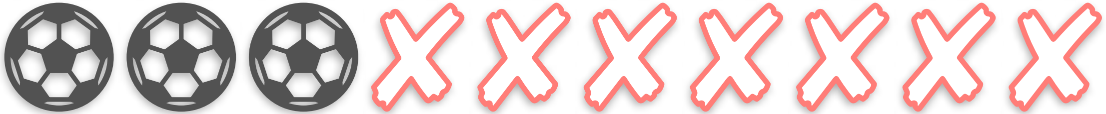
What is the probability of a child NOT playing soccer?

$$\mathbf{P}(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} =$$



$$= \frac{7}{10}$$

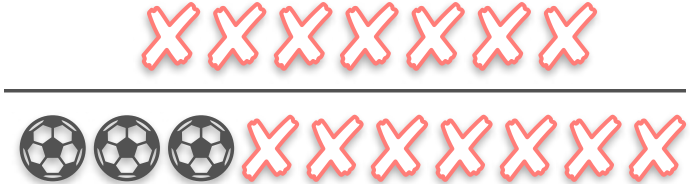
Complement of Probability



30%

What is the probability of a child NOT playing soccer?

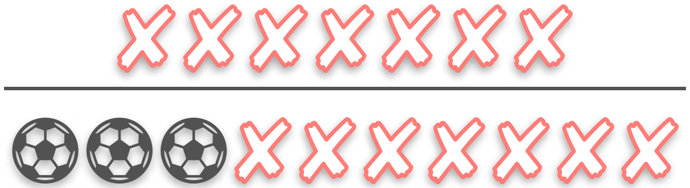
$$\mathbf{P}(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} =$$



$$= \frac{7}{10} = 0.7$$

Complement of Probability

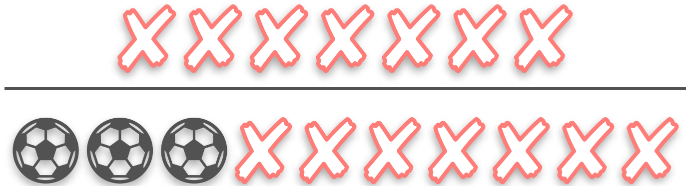
Complement of Probability

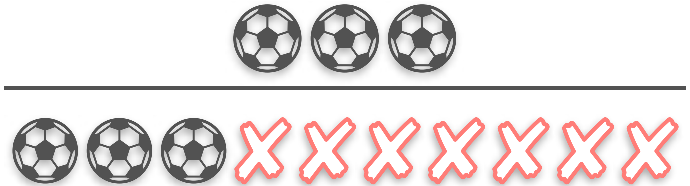


$\mathbf{P}$(not soccer)

0.7

Complement of Probability

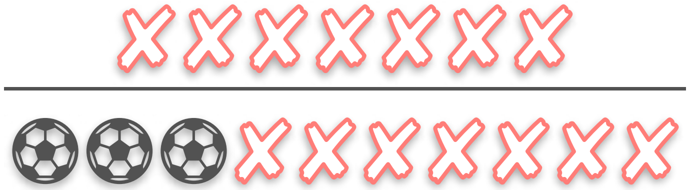


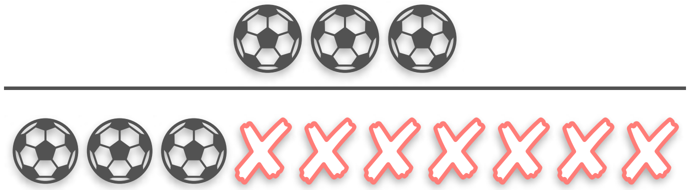


P(not soccer) **P**(soccer)

0.7 0.3

Complement of Probability

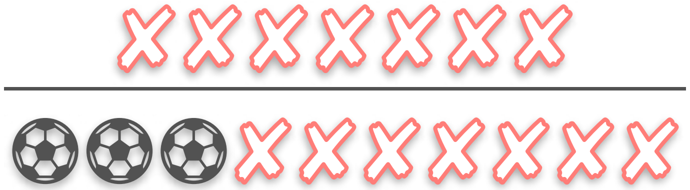


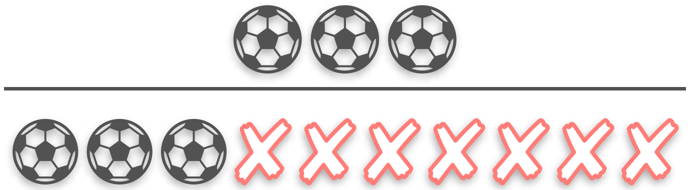


P(not soccer) P(soccer)

$$0.7 + 0.3 = 1$$

Complement of Probability

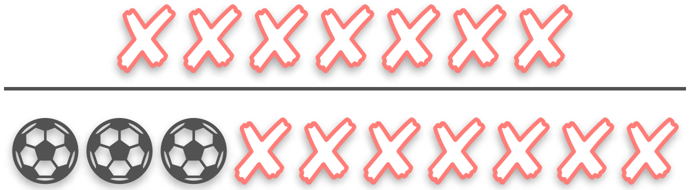


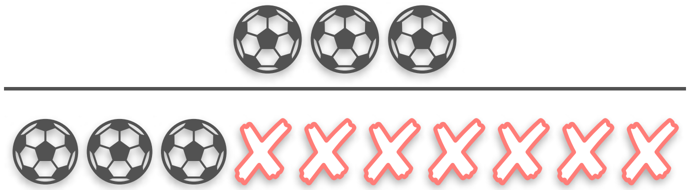


P(not soccer) **P**(soccer)

0.7 = 1 0.3

Complement of Probability

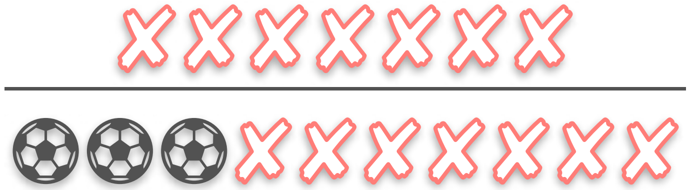


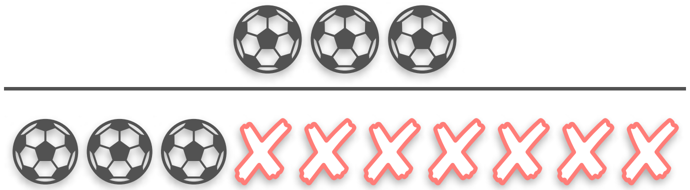


$\mathbf{P}(\text{not soccer})$ $\qquad\qquad$ $\mathbf{P}(\text{soccer})$

$$0.7 = 1 - 0.3$$

Complement of Probability





$$\mathbf{P}(\text{not soccer}) = 1 - \mathbf{P}(\text{soccer})$$

$$0.7 = 1 - 0.3$$

Complement of Probability

$$\boxed{\mathbf{P}(\text{not soccer}) = 1 - \mathbf{P}(\text{soccer})}$$

$$0.7 = 1 - 0.3$$

Complement of Probability

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$$0.7 = 1 - 0.3$$

Complement Rule

Complement of Probability

$$\mathbf{P}(\text{not soccer}) = 1 - \mathbf{P}(\text{soccer})$$

Complement Rule

Complement of Probability

$$\boxed{\mathbf{P}(\text{not soccer}) = 1 - \mathbf{P}(\text{soccer})}$$

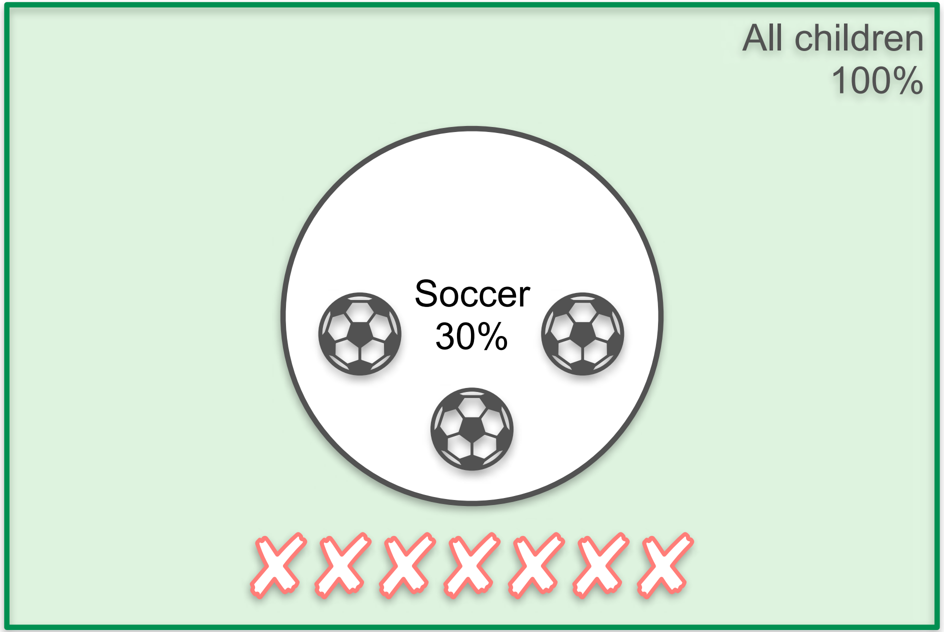
$$\mathbf{P}(A') = 1 - \mathbf{P}(A)$$

Complement Rule

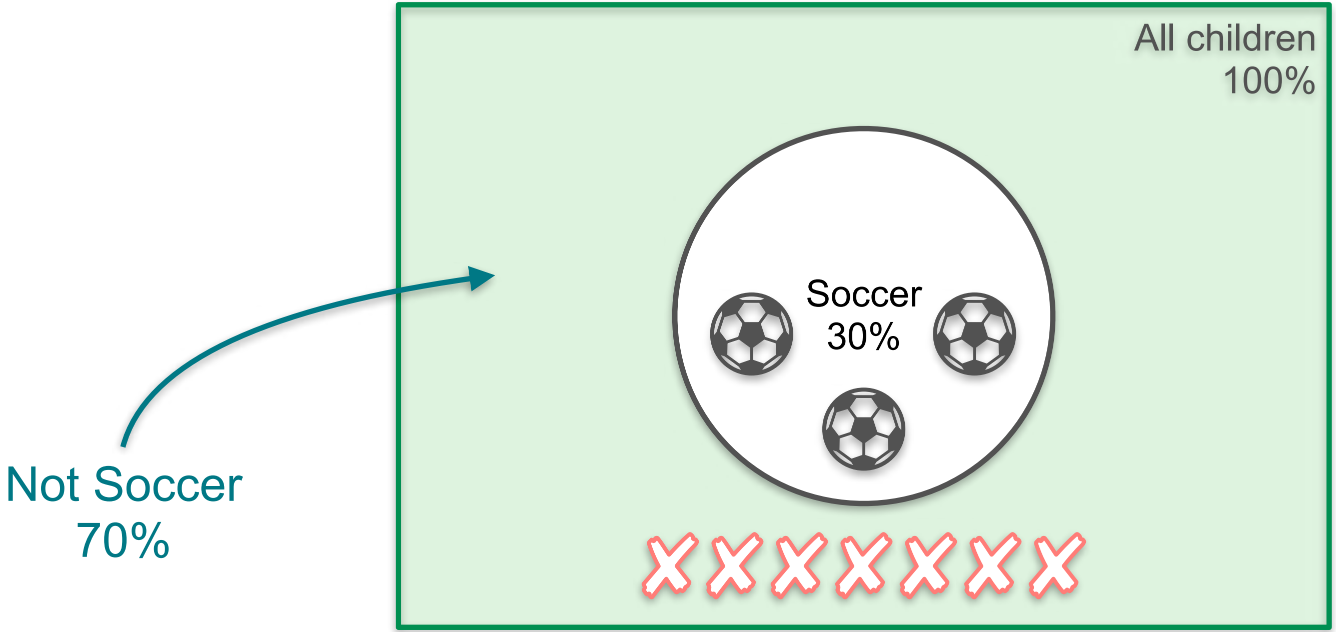
Complement of Probability: Venn Diagram

Complement of Probability: Venn Diagram

Complement of Probability: Venn Diagram



Complement of Probability: Venn Diagram



Complement of Probability: Coin Example 1

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$\mathbf{P}(\text{not } HHH)$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) \;=$$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1$$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

$$\mathbf{P}(HHH) =$$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

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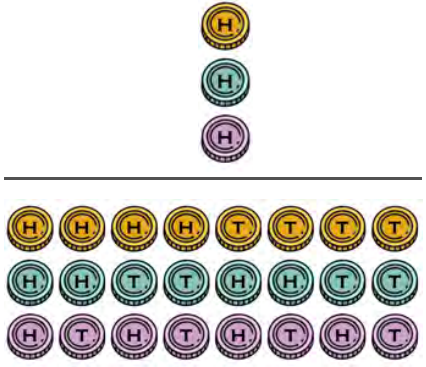
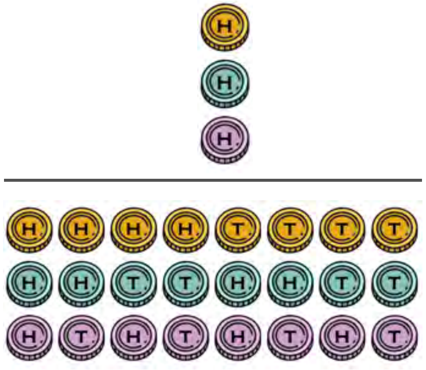


Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$



Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

$$\mathbf{P}(\text{not } HHH) = 1 -$$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

$$\mathbf{P}(\text{not } HHH) = 1 - \frac{1}{8}$$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

$$\mathbf{P}(\text{not } HHH) = 1 - \frac{1}{8}$$

Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

$$\mathbf{P}(\text{not } HHH) = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$

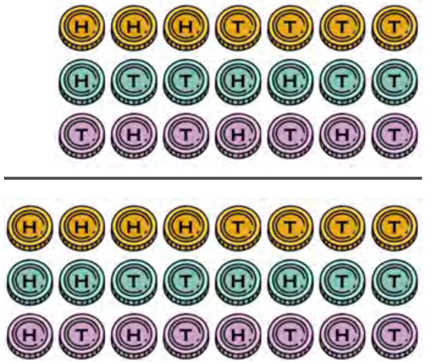
Complement of Probability: Coin Example 1

What is the probability of not landing on heads 3 times?

$$\mathbf{P}(\text{not } HHH) = 1 - \mathbf{P}(HHH)$$

$$\mathbf{P}(\text{not } HHH) = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$



Complement of Probability: Dice Example 1

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Complement of Probability: Dice Example 1

What is the probability of obtaining anything other than 6?

Complement of Probability: Dice Example 1

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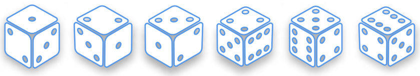
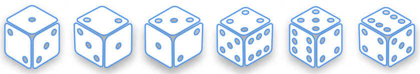


Complement of Probability: Dice Example 1

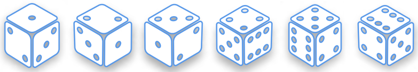
What is the probability of obtaining anything other than 6?



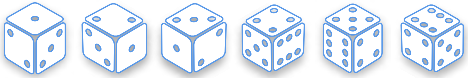
$\mathbf{P}(\text{not } 6) =$

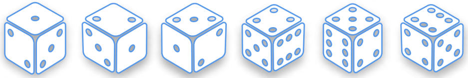
Complement of Probability: Dice Example 1

What is the probability of obtaining anything other than 6?



$\mathbf{P}(\text{not } 6) =$





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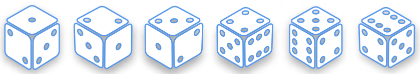

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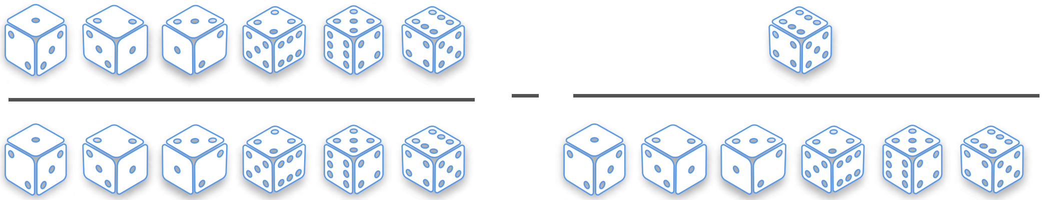
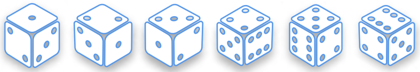


Complement of Probability: Dice Example 1

What is the probability of obtaining anything other than 6?



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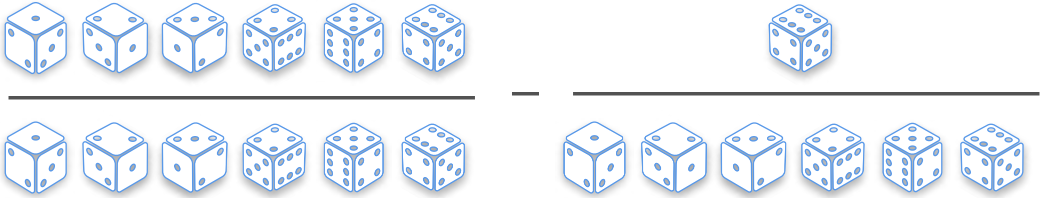

$=$

Complement of Probability: Dice Example 1

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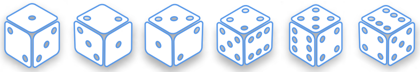
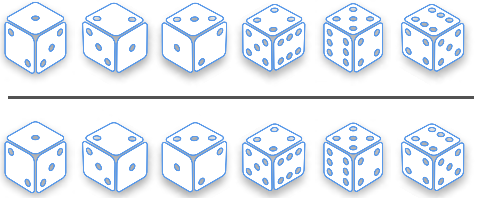
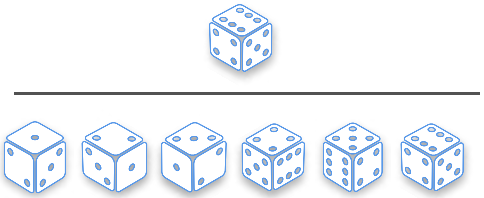


$\mathbf{P}(\text{not } 6) =$  $-$ 

$=$ ______

Complement of Probability: Dice Example 1

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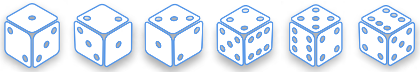

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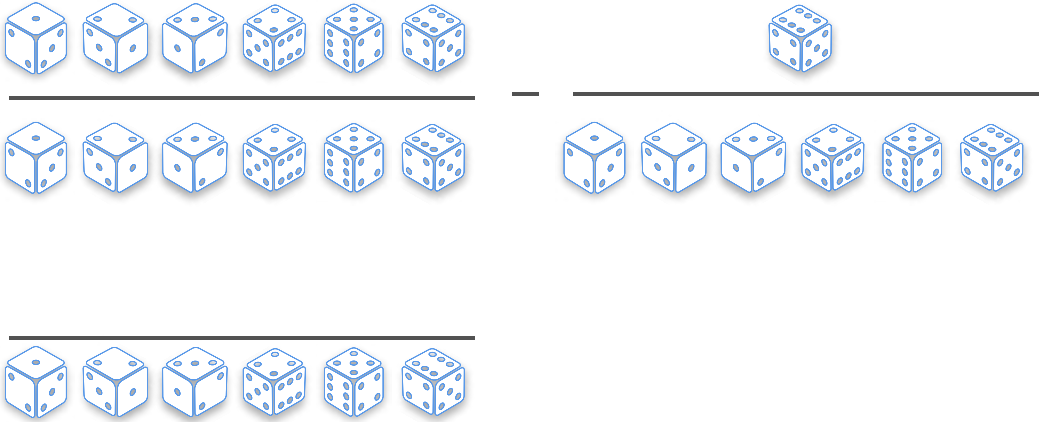

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Complement of Probability: Dice Example 1

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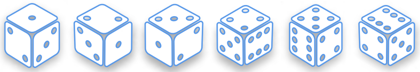

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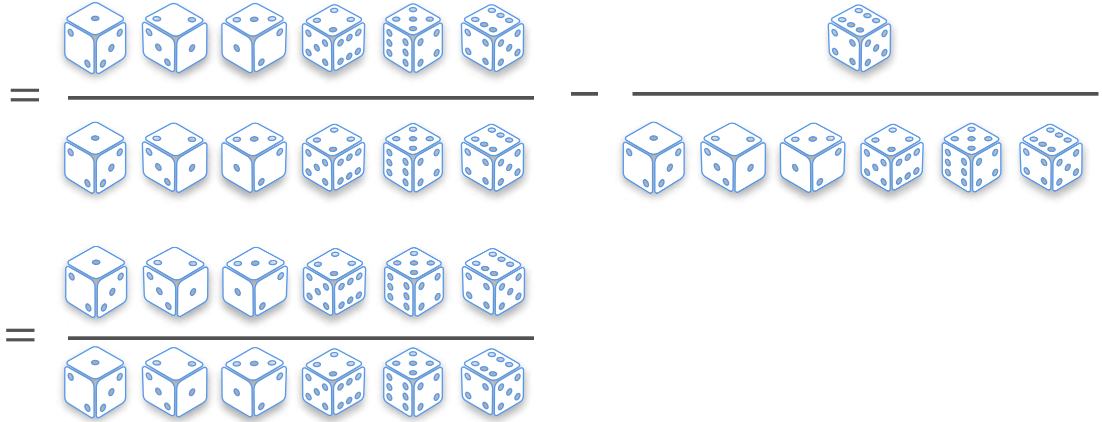

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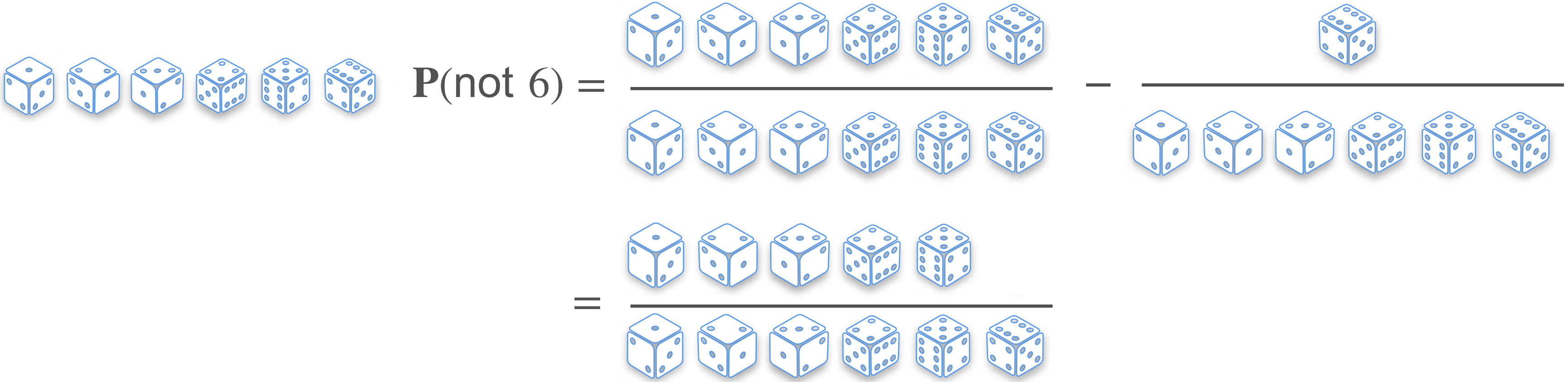

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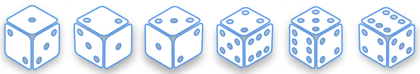

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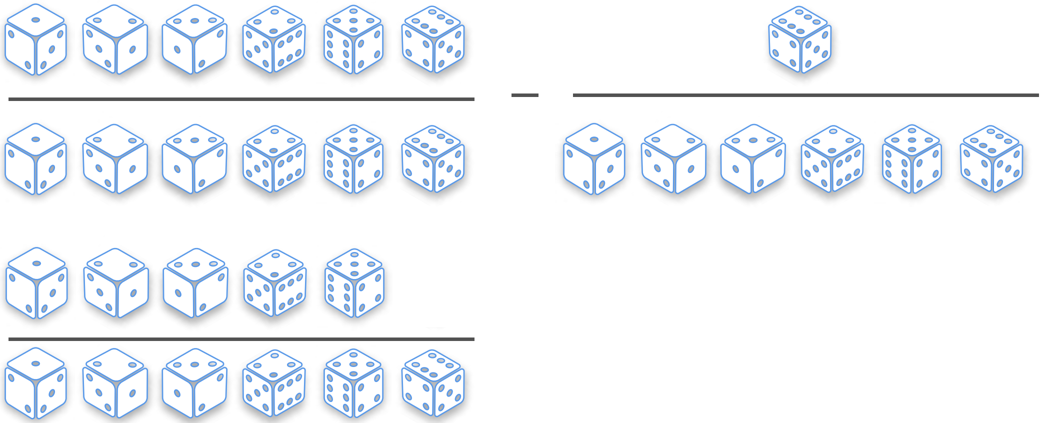

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$=$

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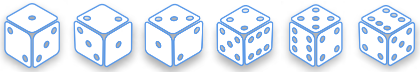
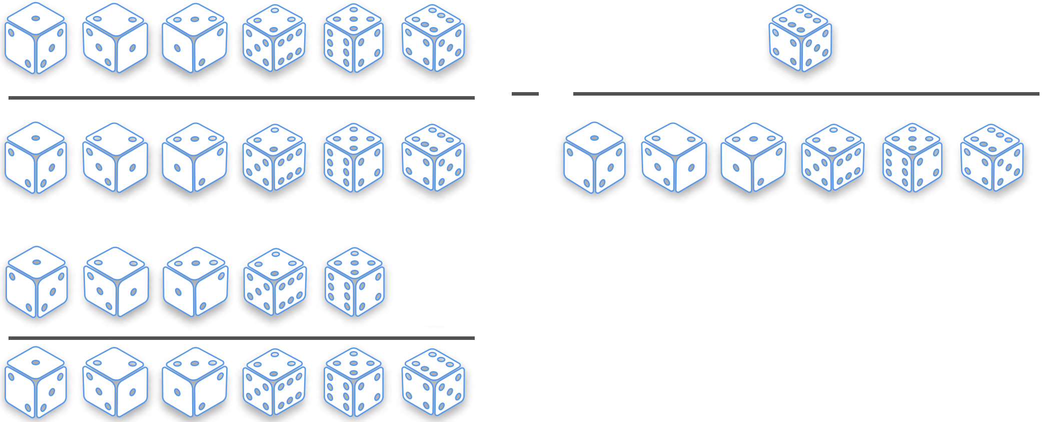


$P(\text{not } 6) =$



$= \frac{5}{6}$

DeepLearning.AI

Introduction to probability

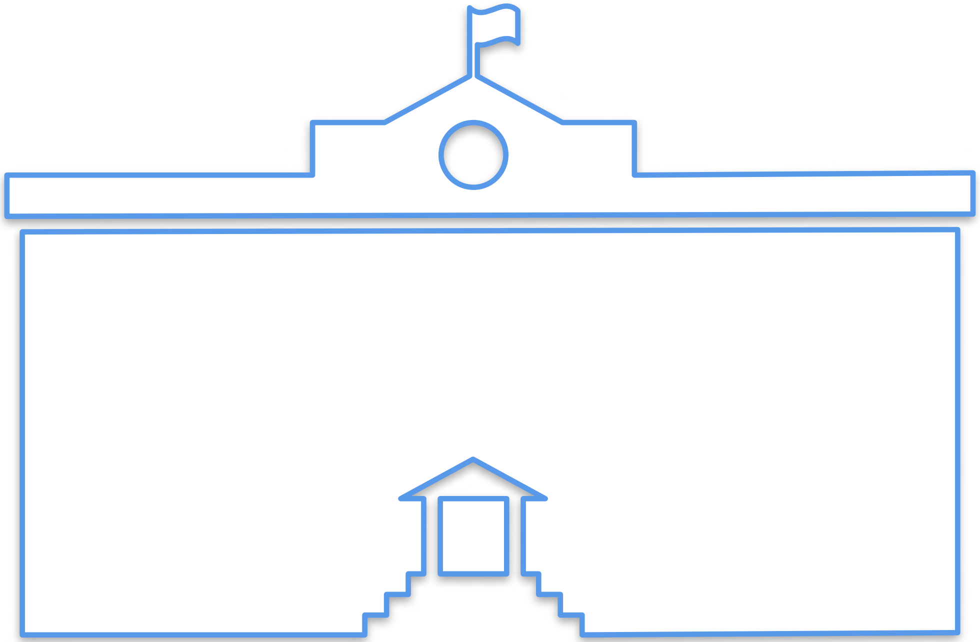
Sum of Probabilities

Sum of Probabilities: Quiz 1

Sum of Probabilities: Quiz 1

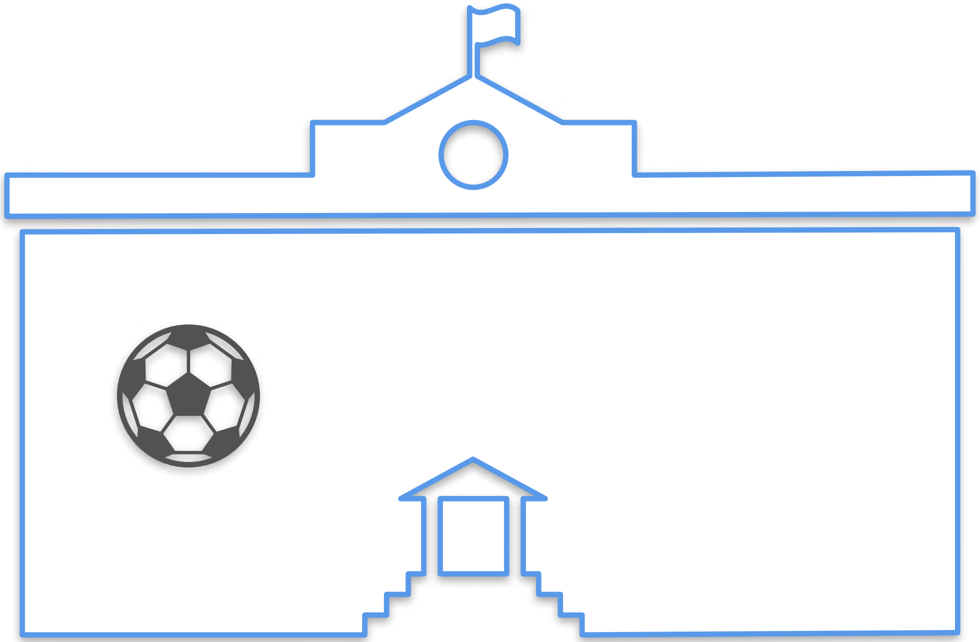
At a school, kids can only play one sport.

Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

Sum of Probabilities: Quiz 1



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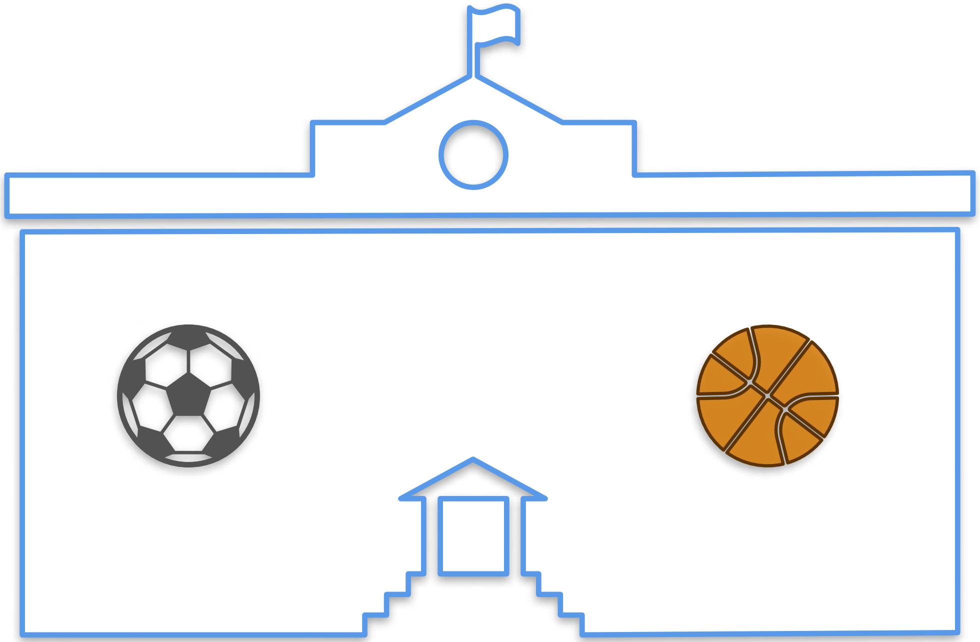

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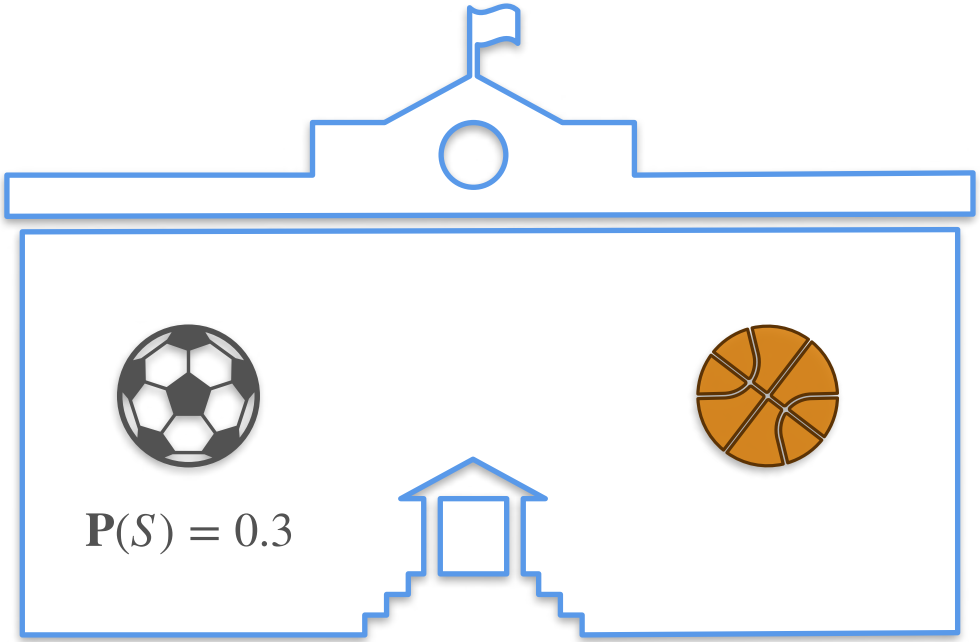

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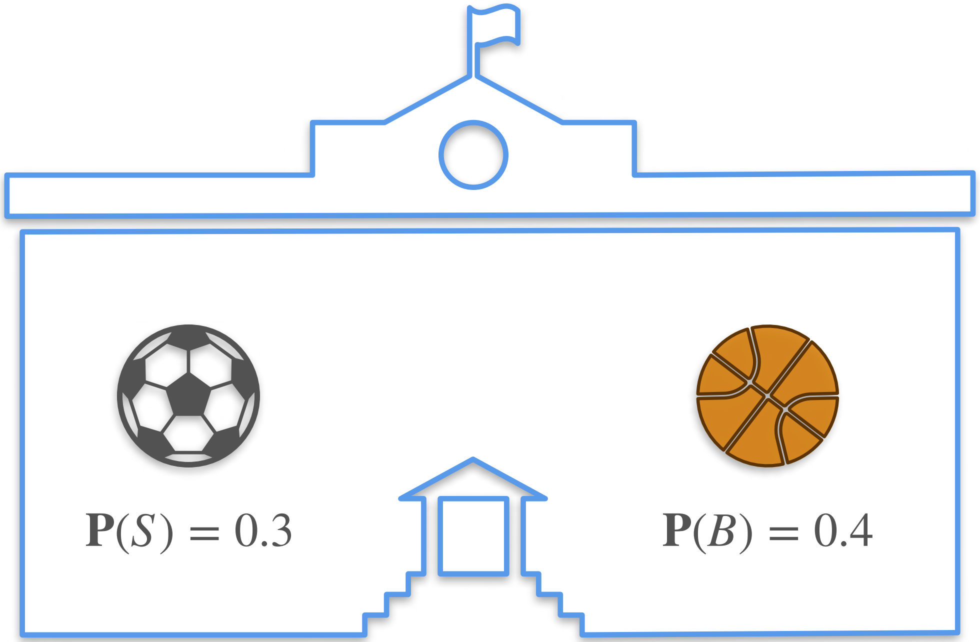

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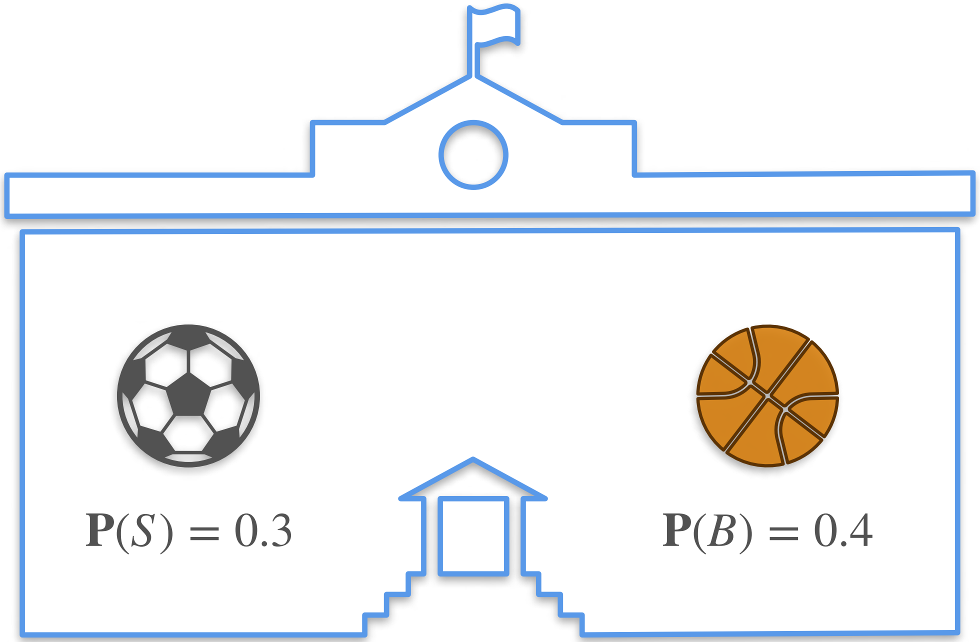
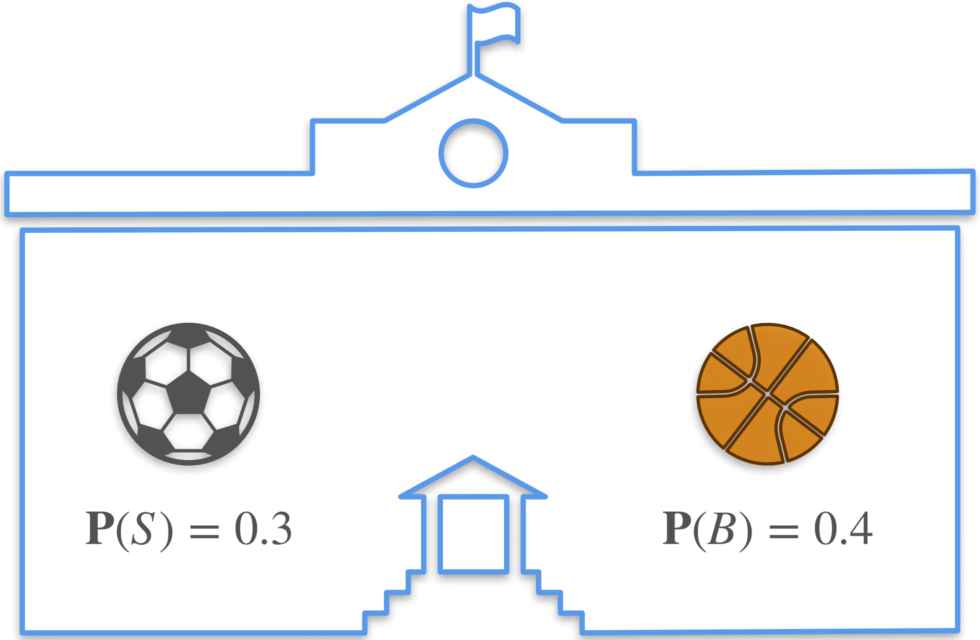
Sum of Probabilities: Quiz 1



At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

Sum of Probabilities: Quiz 1



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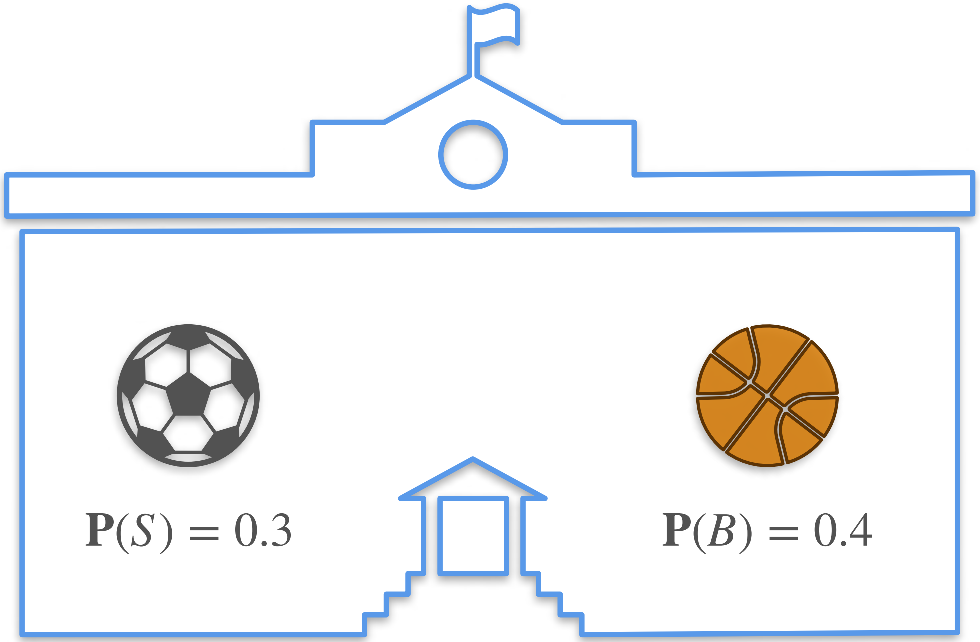

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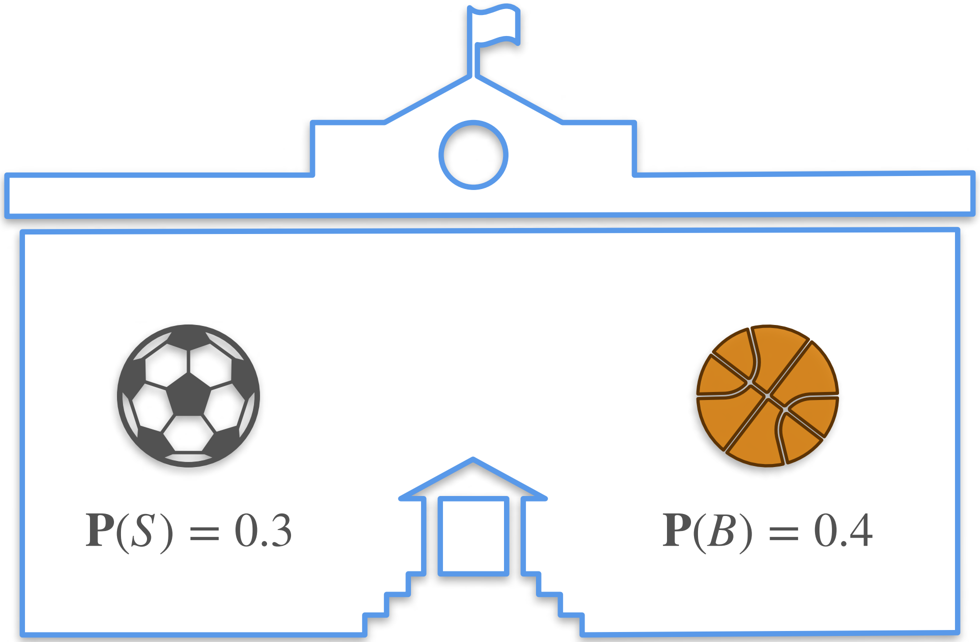


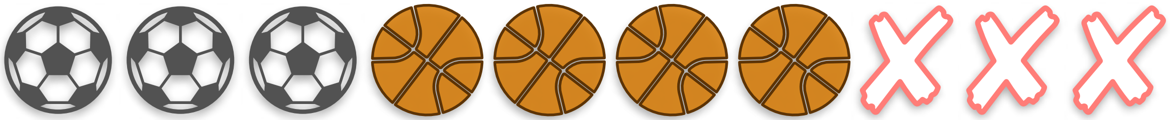
At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

Hint: What if there were only 10 kids?

Sum of Probabilities: Quiz 1 Solution

Sum of Probabilities: Quiz 1 Solution



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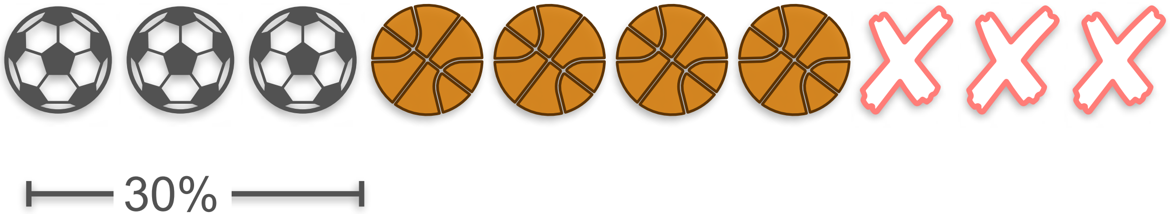

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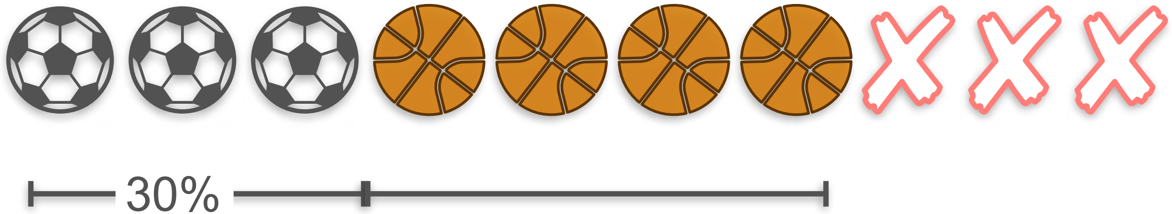

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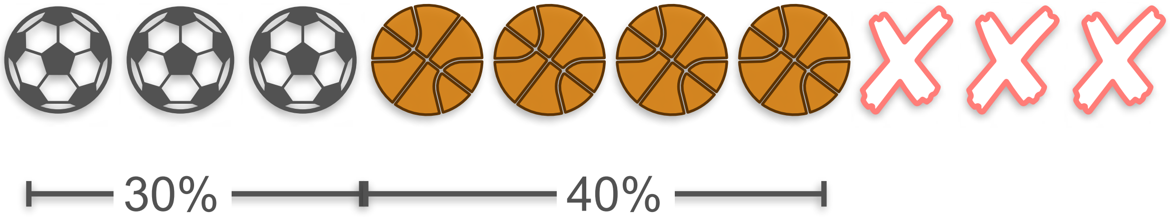
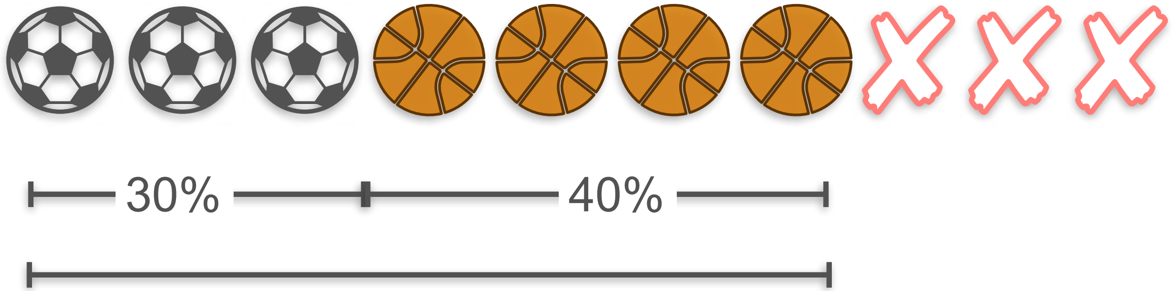
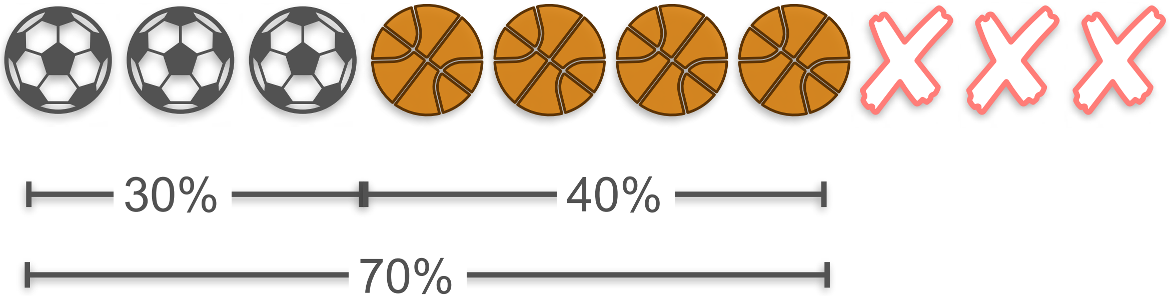


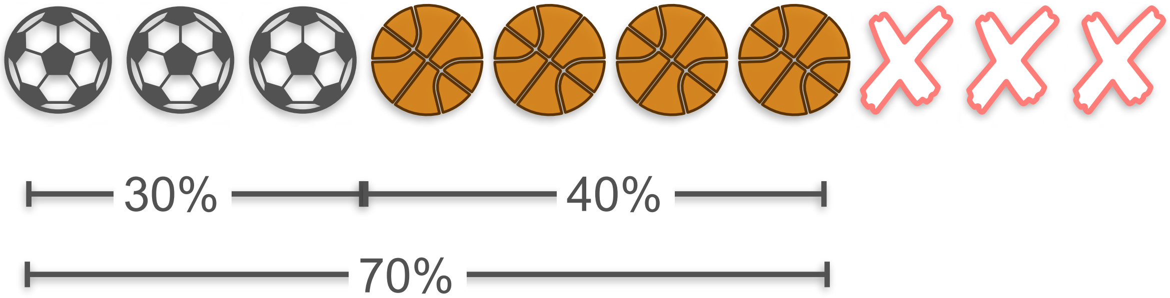
Sum of Probabilities: Quiz 1 Solution



Sum of Probabilities: Quiz 1 Solution

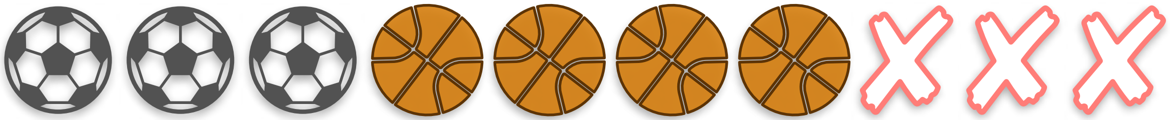


Sum of Probabilities: Quiz 1 Solution



$$\mathbf{P}(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}}$$

Sum of Probabilities: Quiz 1 Solution

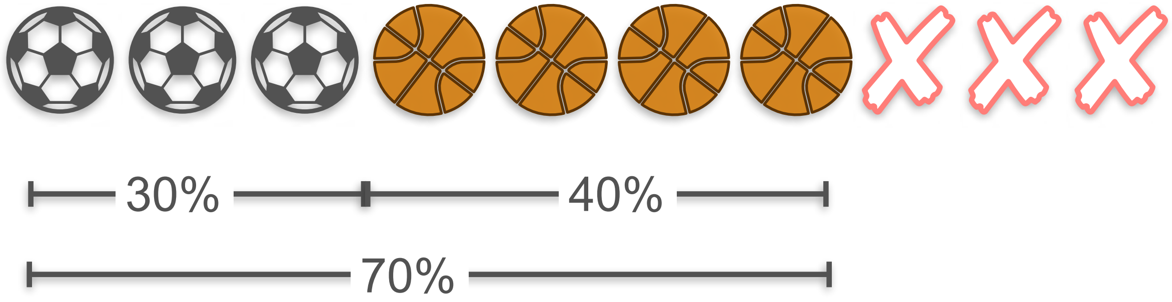


30% 40%

70%

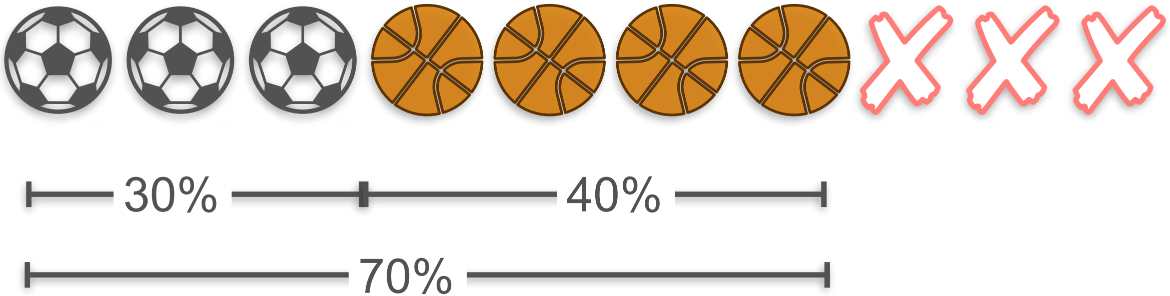
$$\mathbf{P}(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3+4}{10}$$

Sum of Probabilities: Quiz 1 Solution



$$\mathbf{P}(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3+4}{10} = 0.7$$

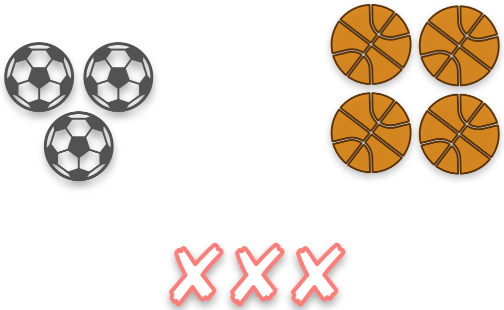
Sum of Probabilities: Quiz 1 Solution



$$\mathbf{P}(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3+4}{10} = 0.7$$

$$\mathbf{P}(\text{soccer or basketball}) = \mathbf{P}(\text{soccer}) + \mathbf{P}(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



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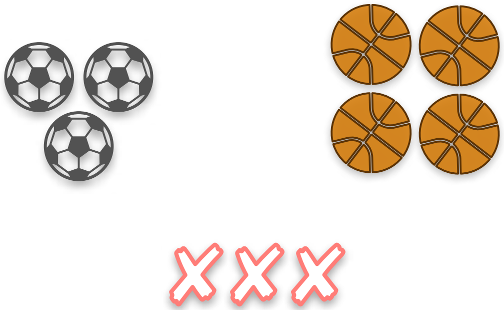
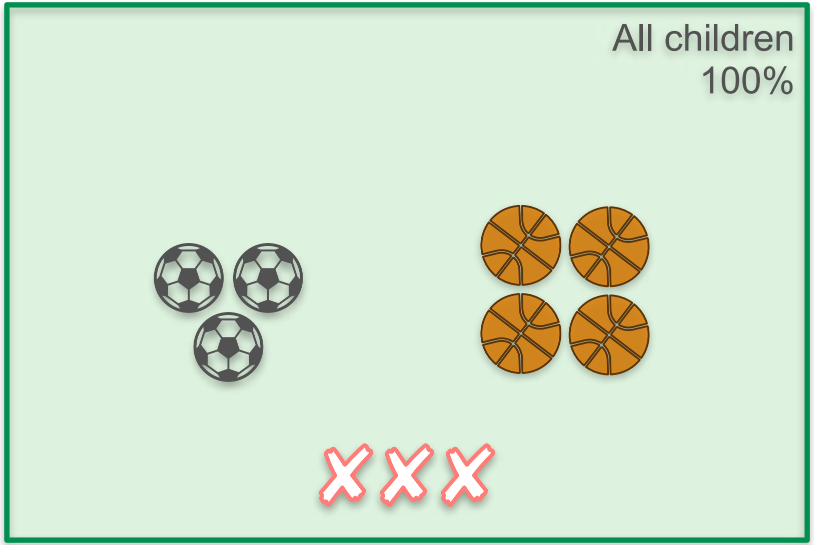


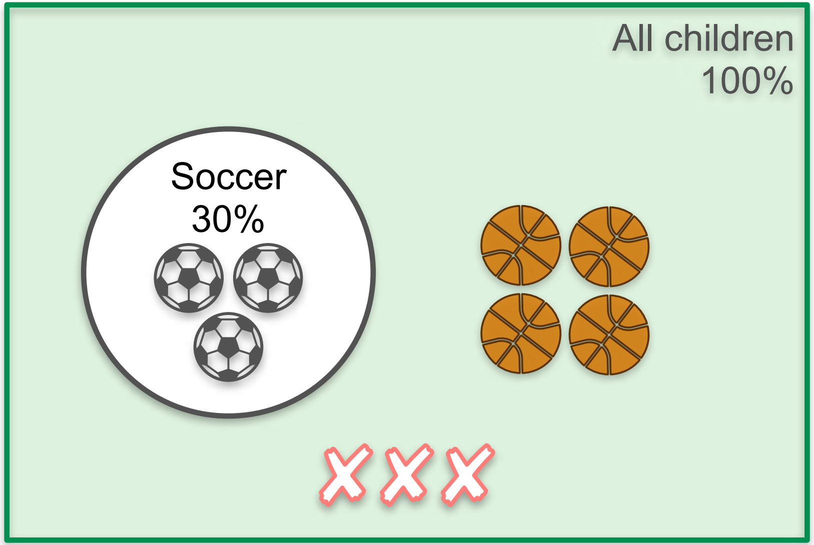
$$\mathbf{P}(\text{soccer} \cup \text{basketball}) = \mathbf{P}(\text{soccer}) + \mathbf{P}(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



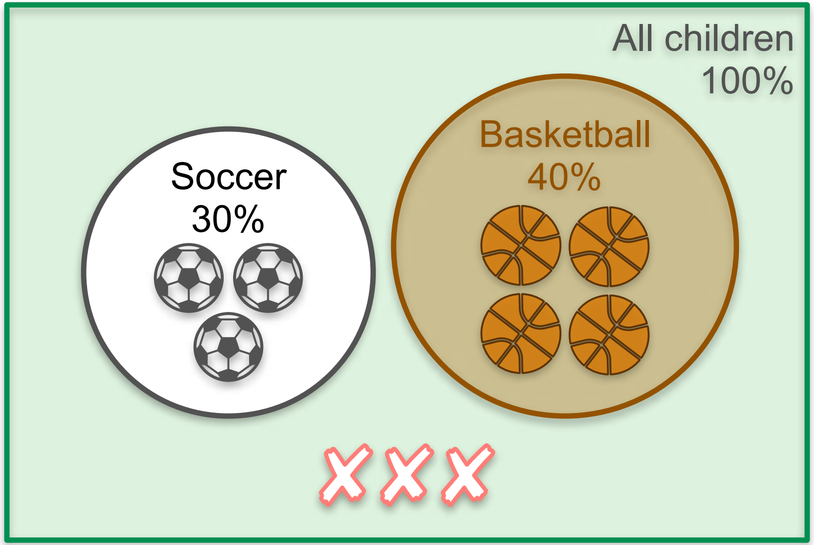
$$\mathbf{P}(\text{soccer} \cup \text{basketball}) = \mathbf{P}(\text{soccer}) + \mathbf{P}(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



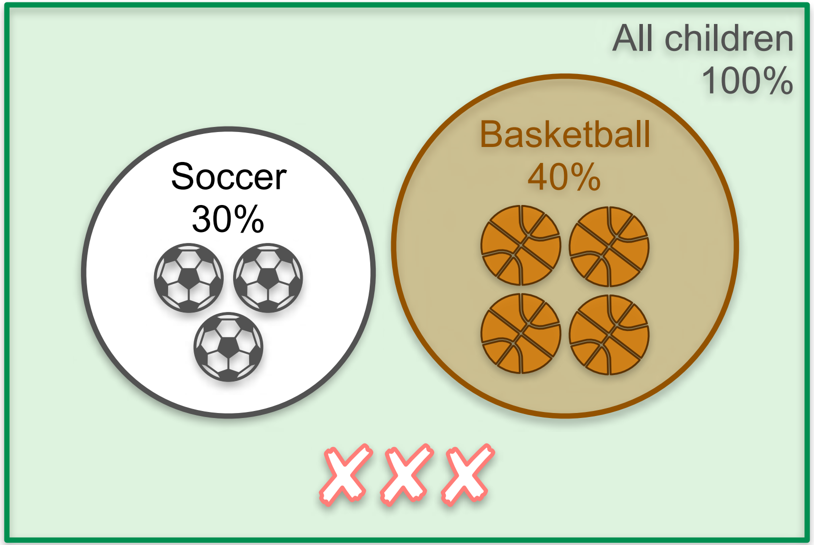
$$\mathbf{P}(\text{soccer} \cup \text{basketball}) = \mathbf{P}(\text{soccer}) + \mathbf{P}(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



$$\mathbf{P}(\text{soccer} \cup \text{basketball}) = \mathbf{P}(\text{soccer}) + \mathbf{P}(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



$$\mathbf{P}(\text{soccer} \cup \text{basketball}) = \mathbf{P}(\text{soccer}) + \mathbf{P}(\text{basketball})$$

$$\mathbf{P}(A \cup B) = \mathbf{P}(A) + \mathbf{P}(B)$$

Sum of Probabilities: Dice Example 1

Sum of Probabilities: Dice Example 1

Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?

Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?

Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?

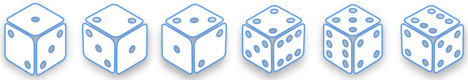
A

B

Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?

A



B

Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?

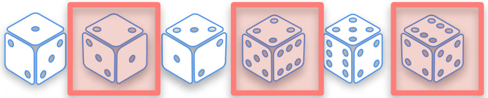
A

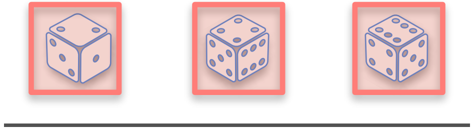
B

Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?

A





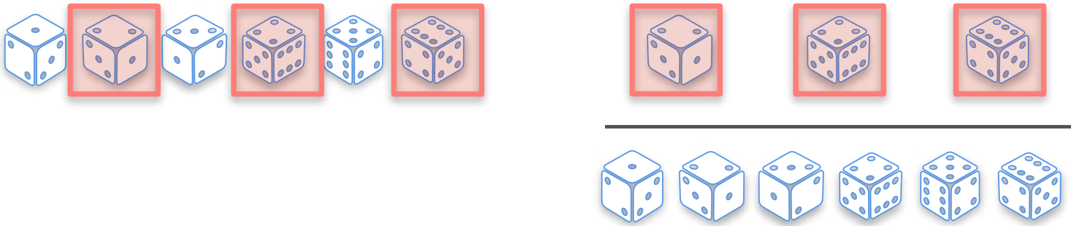
B

Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?

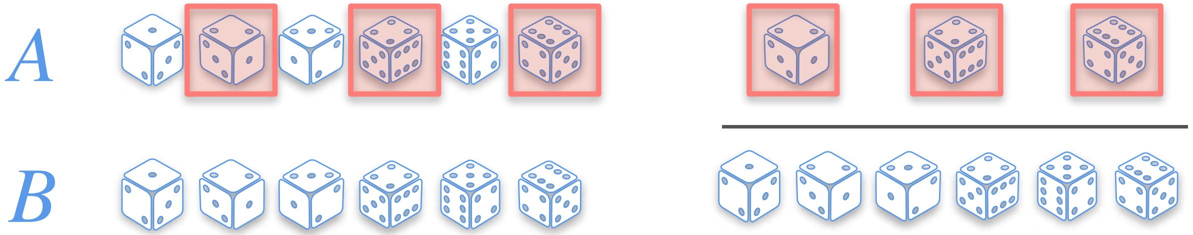
A

B



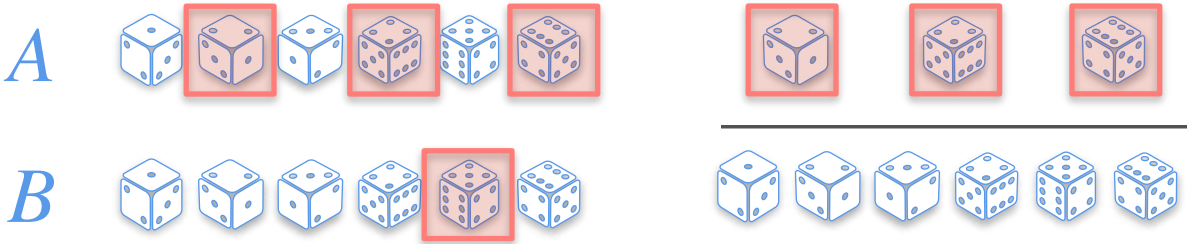
Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?



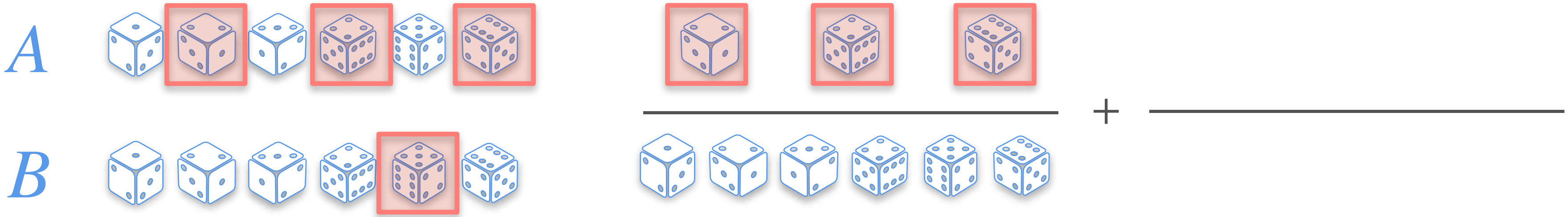
Sum of Probabilities: Dice Example 1

What is the probability of obtaining
an even number or a 5?



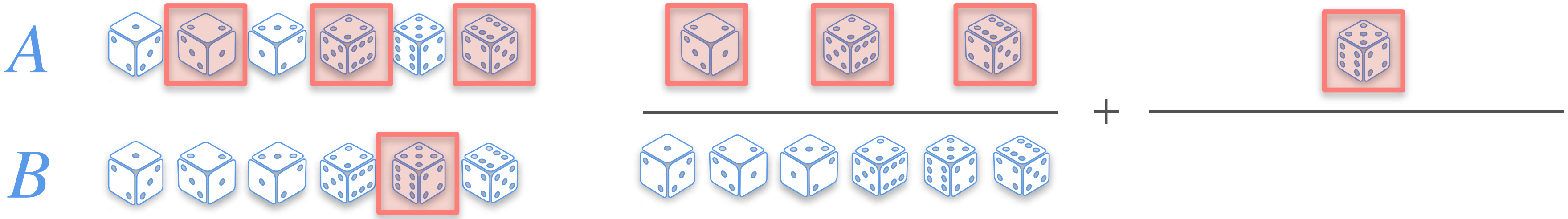
Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?



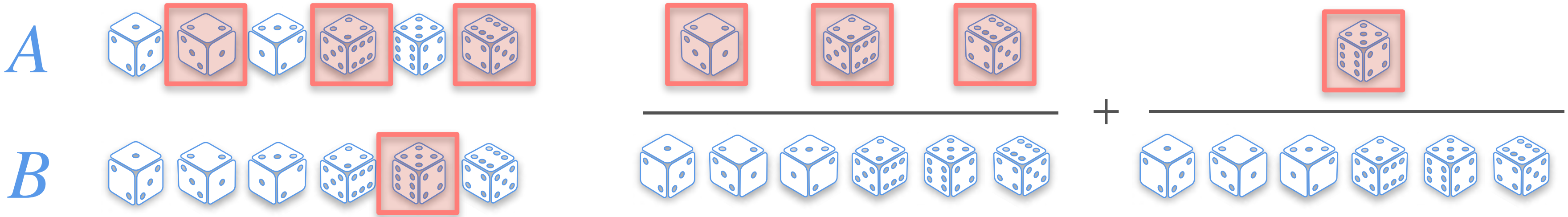
Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?



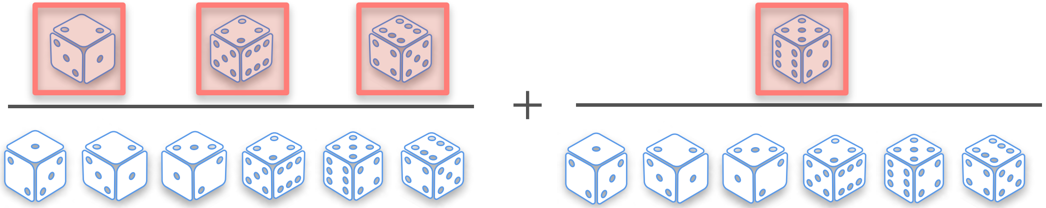
Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?



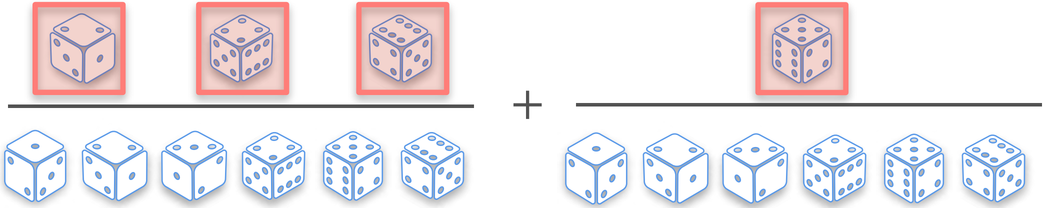
Sum of Probabilities: Dice Example 1

What is the probability of obtaining an even number or a 5?



Sum of Probabilities: Dice Example 1

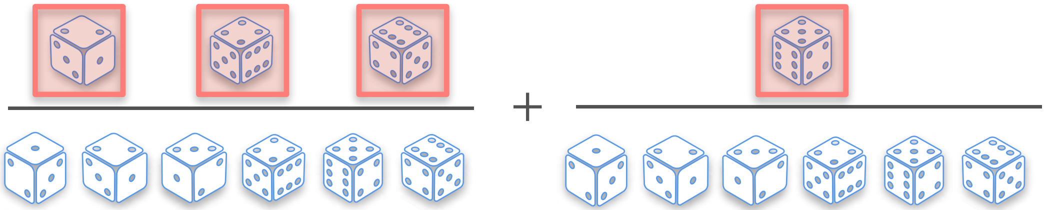
What is the probability of obtaining an even number or a 5?



$\mathbf{P}$(even number or 5) $=$

Sum of Probabilities: Dice Example 1

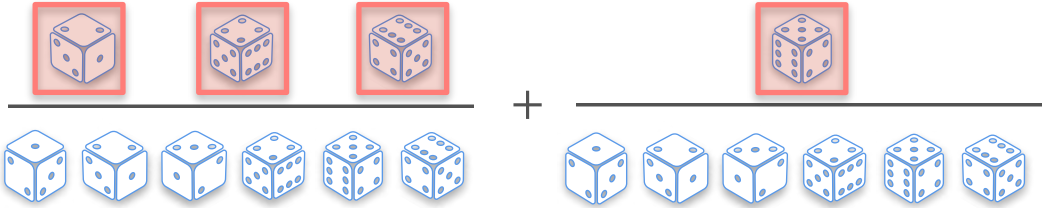
What is the probability of obtaining an even number or a 5?



$$\mathbf{P}(\text{even number or 5}) = \mathbf{P}(\text{even number})$$

Sum of Probabilities: Dice Example 1

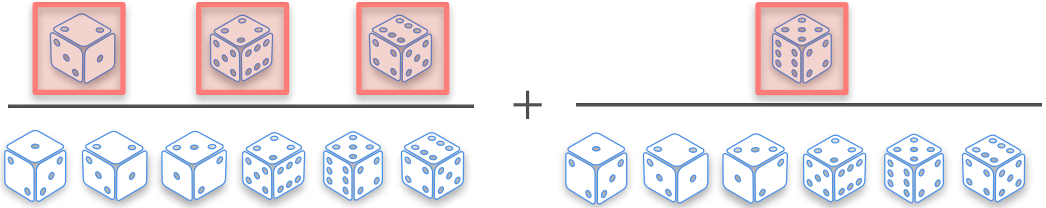
What is the probability of obtaining an even number or a 5?



$$\mathbf{P}(\text{even number or } 5) = \mathbf{P}(\text{even number}) + \mathbf{P}(5)$$

Sum of Probabilities: Dice Example 1

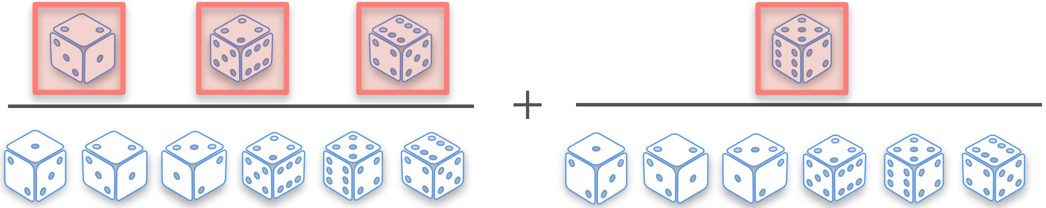
What is the probability of obtaining an even number or a 5?



$$\mathbf{P}(\text{even number or 5}) = \mathbf{P}(\text{even number}) + \mathbf{P}(5) = \frac{3}{6}$$

Sum of Probabilities: Dice Example 1

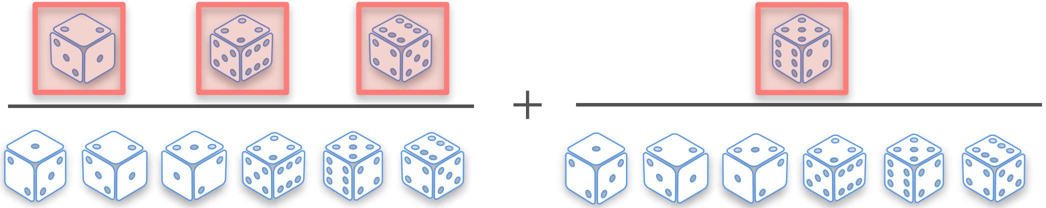
What is the probability of obtaining an even number or a 5?



$$\mathbf{P}(\text{even number or 5}) = \mathbf{P}(\text{even number}) + \mathbf{P}(5) = \frac{3}{6} + \frac{1}{6}$$

Sum of Probabilities: Dice Example 1

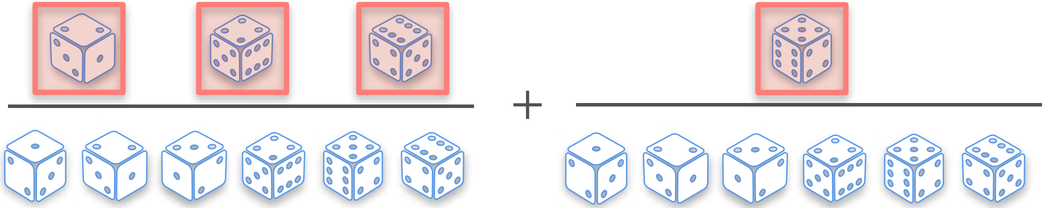
What is the probability of obtaining an even number or a 5?



$$\mathbf{P}(\text{even number or } 5) = \mathbf{P}(\text{even number}) + \mathbf{P}(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

Sum of Probabilities: Dice Example 1

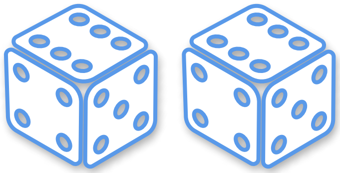
What is the probability of obtaining an even number or a 5?



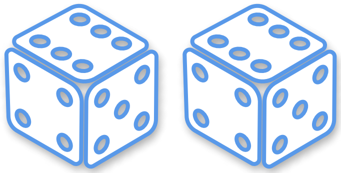
$$\mathbf{P}(\text{even number or 5}) = \mathbf{P}(\text{even number}) + \mathbf{P}(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Sum of Probabilities: Dice Example 2

Sum of Probabilities: Dice Example 2



Sum of Probabilities: Dice Example 2



What is the probability of obtaining a sum of 7 or a sum of 10?

Sum of Probabilities: Dice Example 2

Sum of Probabilities: Dice Example 2

A B

Sum of Probabilities: Dice Example 2

A

sum of 7

B

sum of 10

Sum of Probabilities: Dice Example 2

A

sum of 7

B

sum of 10

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

A

sum of 7

B

sum of 10

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

A

sum of 7

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

sum of 10

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

A or B

sum of 7 or sum of 10

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(sum of 7 or sum of 10)

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum of 7 or sum of 10})$

$= \underline{\qquad\qquad\qquad}$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(sum of 7 or sum of 10)

$$= \frac{\boxed{6,1}\ \boxed{5,2}\ \boxed{4,3}\ \boxed{3,4}\ \boxed{2,5}\ \boxed{1,6}}{}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

= (1,6 2,5 3,4 4,3 5,2 6,1) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

1,6
2,5
3,4
4,3
5,2
6,1

= ——————— + ———————

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

1,6
2,5
3,4
4,3
5,2
6,1

= ———————— + ————————

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

4,6
5,5
6,4

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

= (1,6 2,5 3,4 4,3 5,2 6,1) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

\+ (4,6 5,5 6,4) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

= (6,1 5,2 4,3 3,4 2,5 1,6) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

\+ (6,4 5,5 4,6) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{6}{36}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(sum of 7 or sum of 10)

$$= \frac{\text{6,1 } \ \text{5,2 } \ \text{4,3 } \ \text{3,4 } \ \text{2,5 } \ \text{1,6}}{\begin{array}{cccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}} + \frac{\text{6,4 } \ \text{5,5 } \ \text{4,6}}{\begin{array}{cccccc} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{array}}$$

$$= \frac{6}{36} + \frac{3}{36}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(sum of 7 or sum of 10)

$$= \frac{\{6,1;\ 5,2;\ 4,3;\ 3,4;\ 2,5;\ 1,6\}}{\text{36 outcomes}} + \frac{\{6,4;\ 5,5;\ 4,6\}}{\text{36 outcomes}}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{6}{36} + \frac{3}{36}$$

$$= \frac{9}{36}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum of 7 or sum of 10)

1,6 2,5 3,4 4,3 5,2 6,1

4,6 5,5 6,4

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

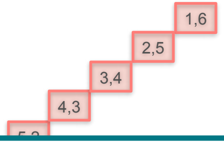
$$= \frac{6}{36} + \frac{3}{36}$$

$$= \frac{9}{36} = \frac{1}{4}$$

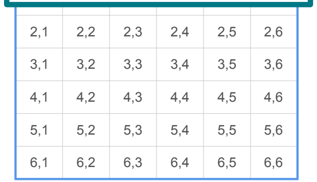
Sum of Probabilities: Dice Example 2

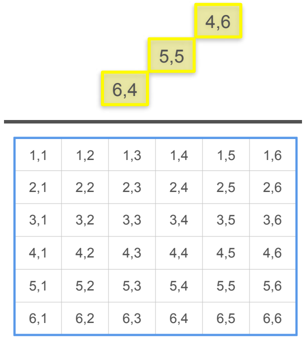
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(sum of 7 or sum of 10)



$= \boxed{\mathbf{P}(\text{sum of 7})} +$





$$= \frac{6}{36} + \frac{3}{36}$$

$$= \frac{9}{36} \qquad = \frac{1}{4}$$

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum of 7 or sum of 10})$

1,6 2,5 3,4 4,3

4,6 5,5

$$= \boxed{\mathbf{P}(\text{sum of 7})} + \boxed{\mathbf{P}(\text{sum of 10})}$$

2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

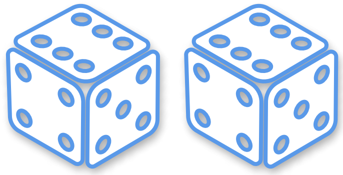
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{6}{36} + \frac{3}{36}$$

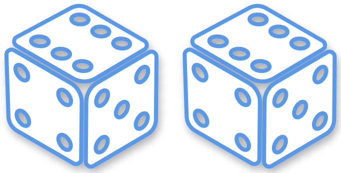
$$= \frac{9}{36} \quad = \frac{1}{4}$$

Sum of Probabilities: Dice Example 3

Sum of Probabilities: Dice Example 3



Sum of Probabilities: Dice Example 3



What is the probability of obtaining
a difference of 2 or a difference of 1?

Sum of Probabilities: Dice Example 3

Sum of Probabilities: Dice Example 3

A B

diff = 2 diff = 1

Sum of Probabilities: Dice Example 3

A

diff = 2

B

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A

diff = 2

B

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A

diff = 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A

diff = 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A

diff = 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A or B

diff = 2 or diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{diff} = 2 \text{ or diff} = 1)$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{diff} = 2 \text{ or diff} = 1)$

$= \underline{\hspace{4cm}}$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{diff} = 2 \text{ or diff} = 1)$

$$= \frac{1,3 \quad 2,4 \quad 3,5 \quad 4,6 \quad 3,1 \quad 4,2 \quad 5,3 \quad 6,4}{}$$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(diff = 2 or diff = 1)

=

1,3 2,4 3,1 3,5 4,2 4,6 5,3 6,4

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(diff = 2 or diff = 1)

1,3 2,4 3,5 4,6 3,1 4,2 5,3 6,4

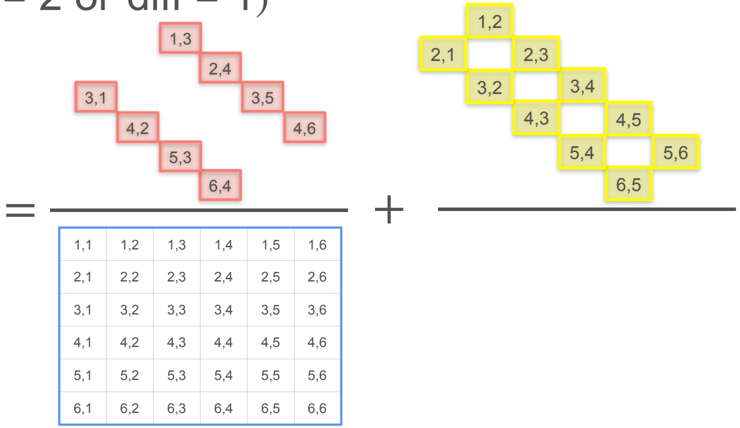
= ———— + ————

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

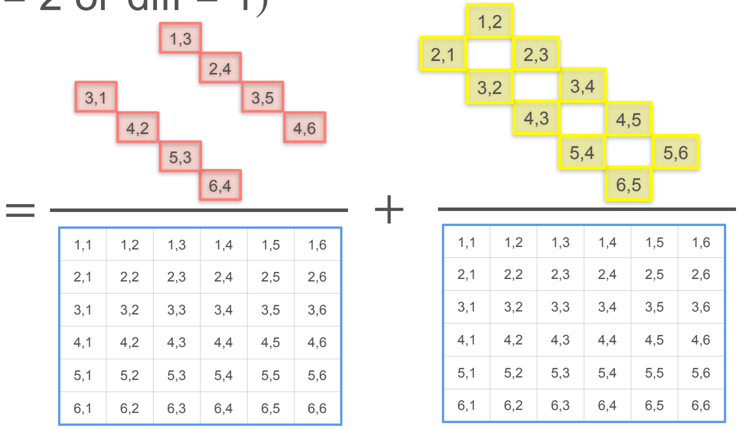
$\mathbf{P}$(diff = 2 or diff = 1)



Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(diff = 2 or diff = 1)



Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(diff = 2 or diff = 1)

$$= \frac{\{1,3;\ 2,4;\ 3,5;\ 4,6;\ 3,1;\ 4,2;\ 5,3;\ 6,4\}}{\text{36 outcomes}} + \frac{\{1,2;\ 2,1;\ 2,3;\ 3,2;\ 3,4;\ 4,3;\ 4,5;\ 5,4;\ 5,6;\ 6,5\}}{\text{36 outcomes}}$$

$$= \frac{8}{36}$$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{diff} = 2 \text{ or diff} = 1)$

= (1,3 2,4 3,5 4,6 3,1 4,2 5,3 6,4) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

+ (1,2 2,1 2,3 3,2 3,4 4,3 4,5 5,4 5,6 6,5) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{8}{36} + \frac{10}{36}$$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(diff = 2 or diff = 1)

= (1,3; 2,4; 3,5; 4,6; 3,1; 4,2; 5,3; 6,4) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

\+ (1,2; 2,1; 2,3; 3,2; 3,4; 4,3; 4,5; 5,4; 5,6; 6,5) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

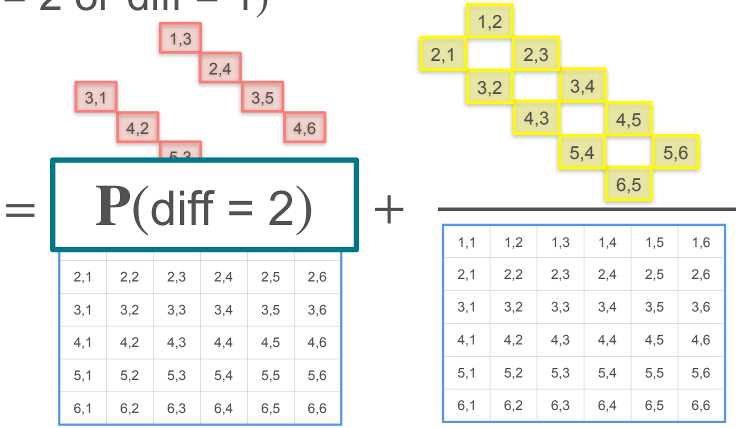
$$= \frac{8}{36} + \frac{10}{36}$$

$$= \frac{18}{36}$$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(diff = 2 or diff = 1)



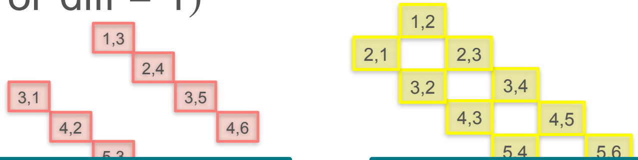
$$= \frac{8}{36} + \frac{10}{36}$$

$$= \frac{18}{36}$$

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(diff = 2 or diff = 1)



$= \boxed{\mathbf{P}(\text{diff} = 2)} + \boxed{\mathbf{P}(\text{diff} = 1)}$

2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{8}{36} + \frac{10}{36}$$

$$= \frac{18}{36}$$

DeepLearning.AI

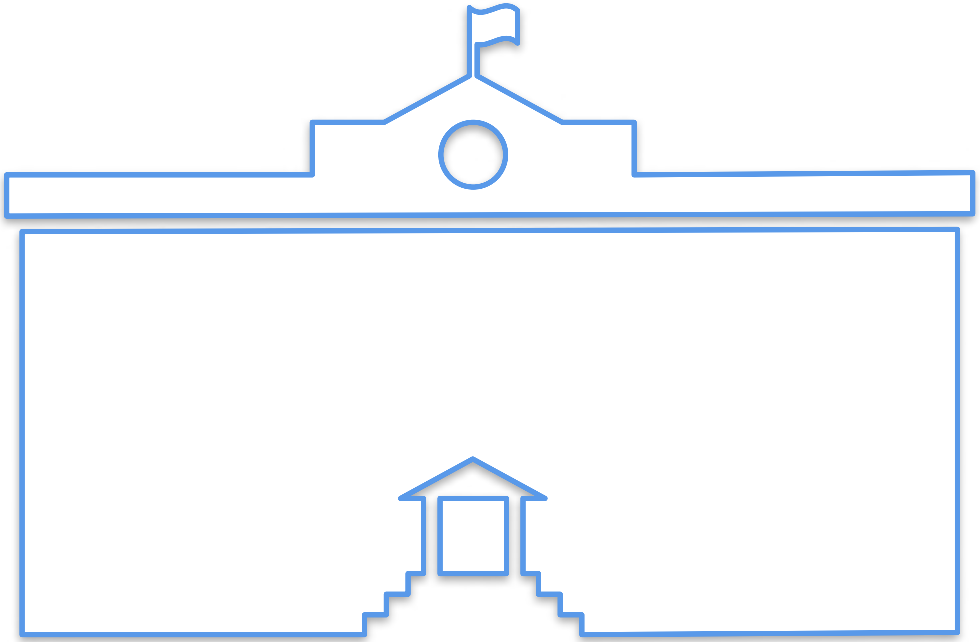
Introduction to probability

Sum of Probabilities (Joint Events)

Sum of Probabilities (Joint Events): Quiz 1

At a school, kids can play as many sports as they want.

Sum of Probabilities (Joint Events): Quiz 1



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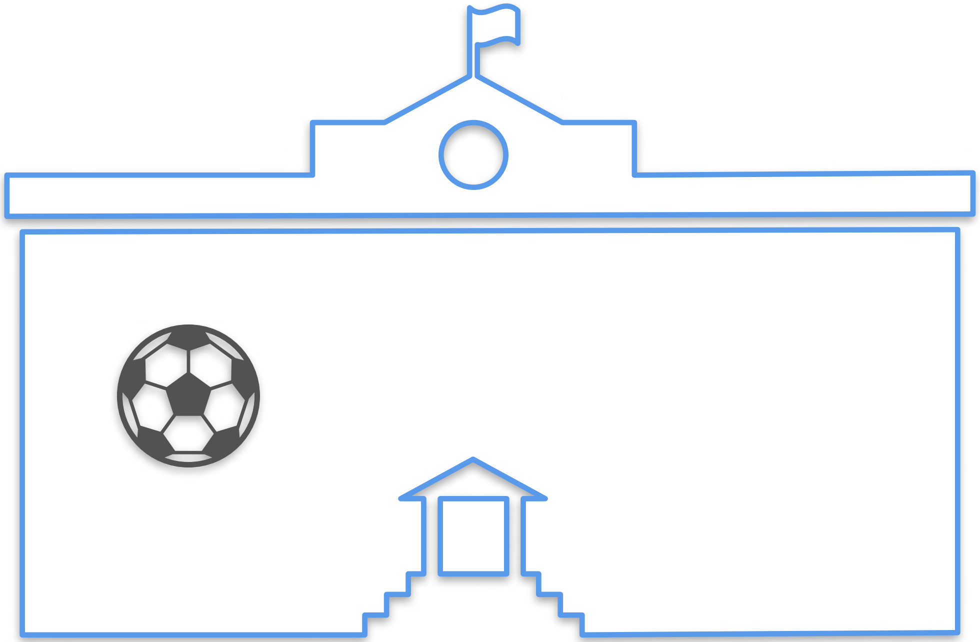

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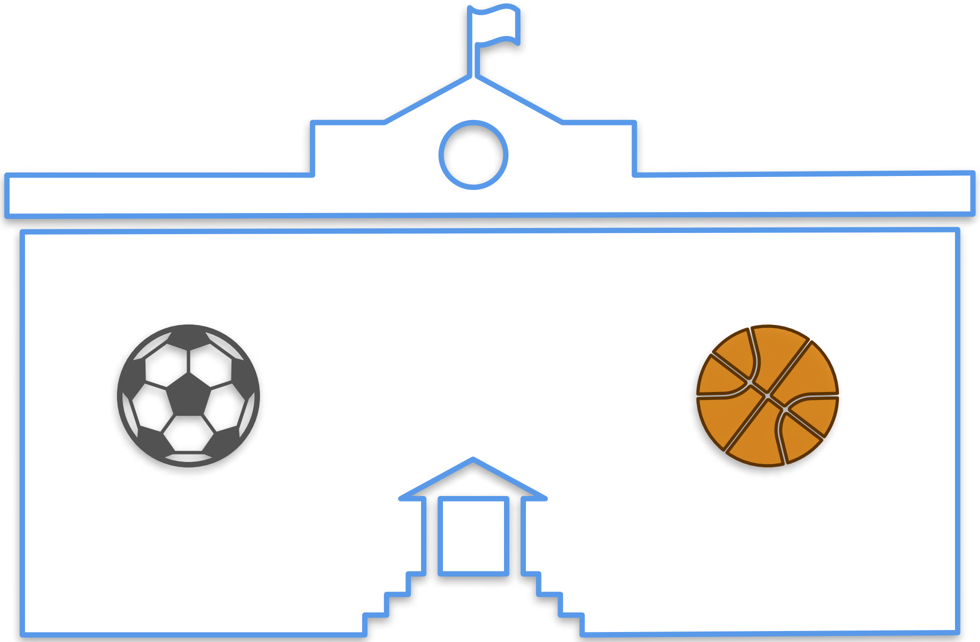

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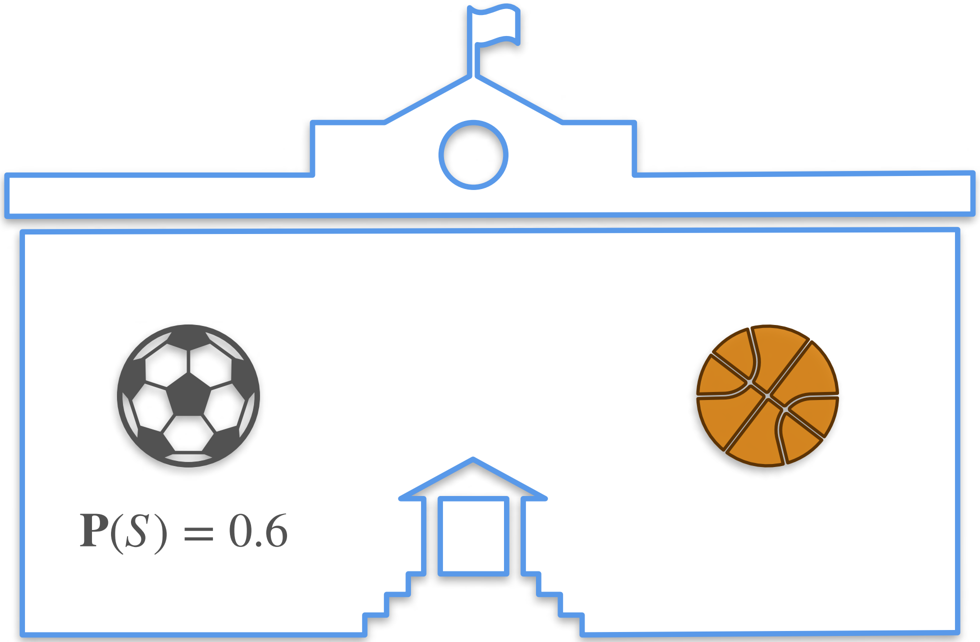

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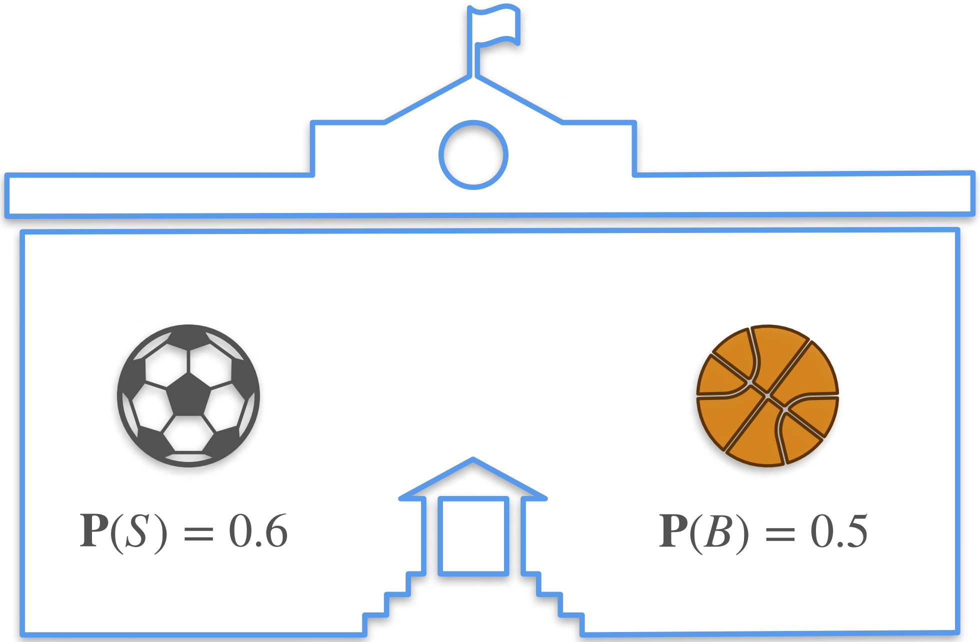

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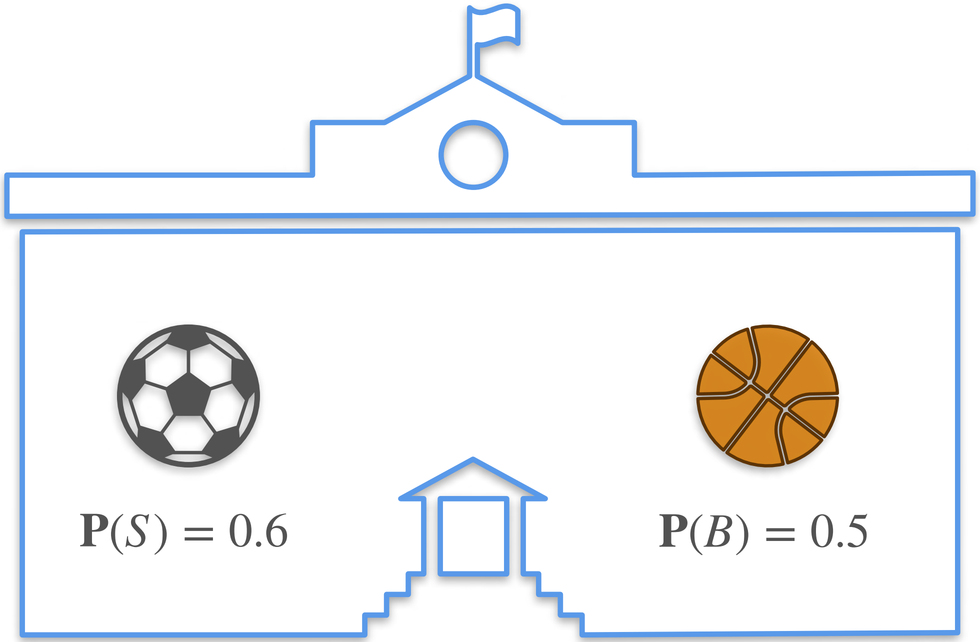

At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

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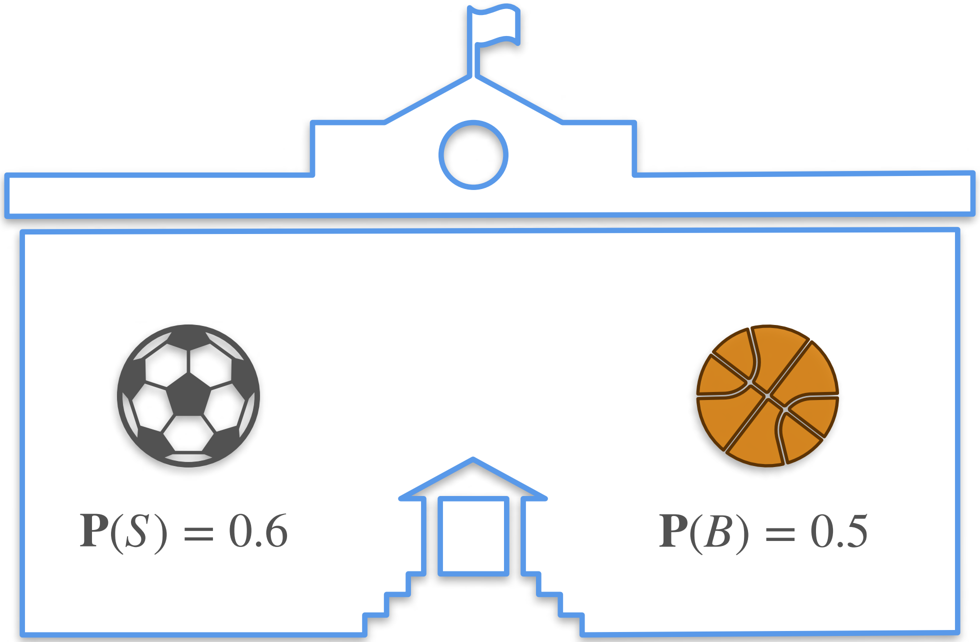

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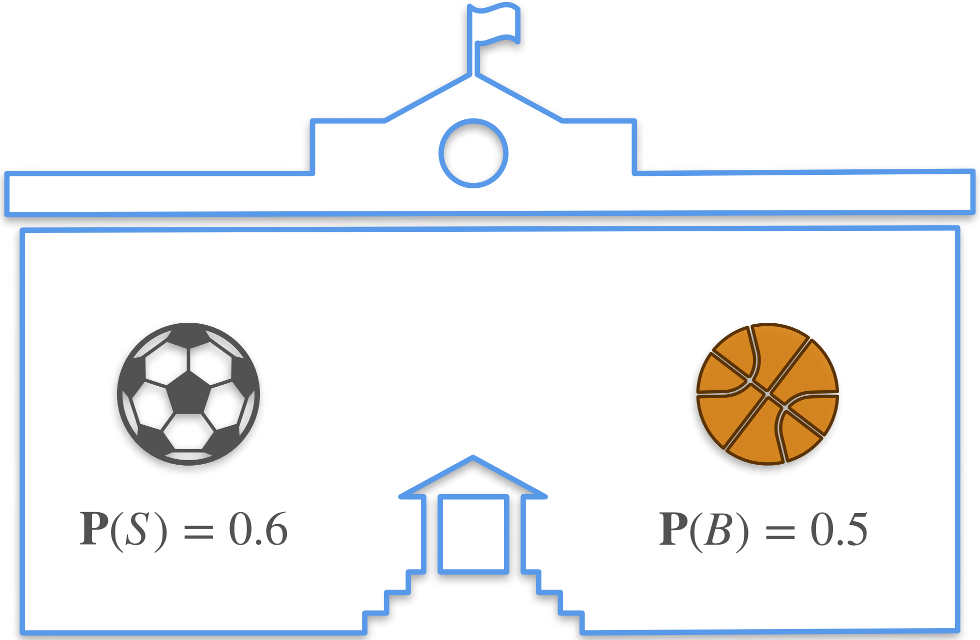

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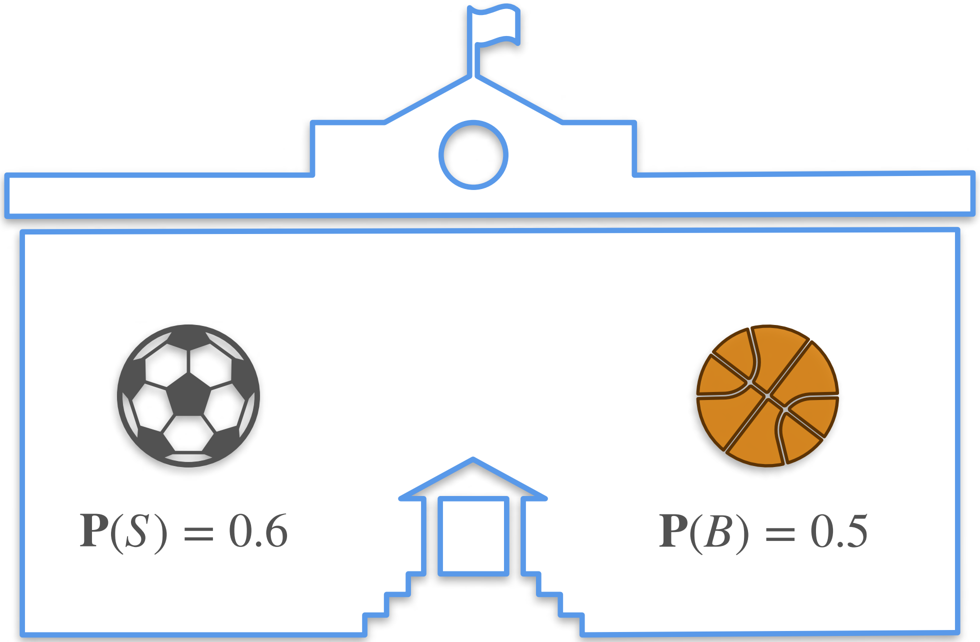

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What is the probability that a kid plays soccer or basketball?

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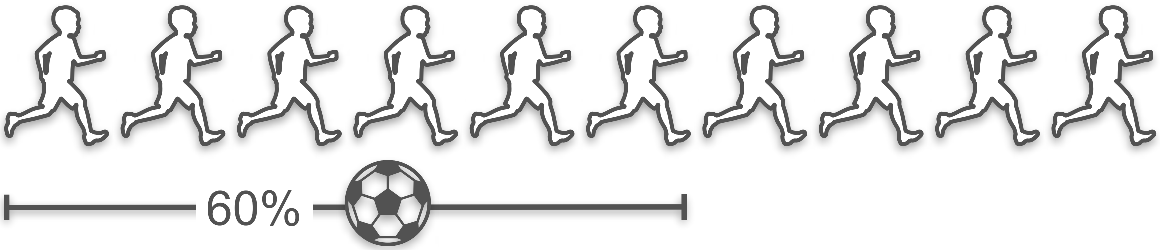

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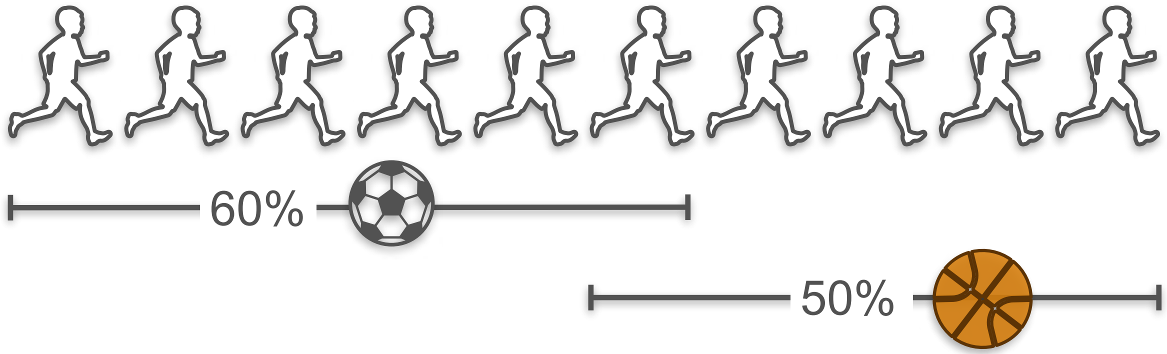

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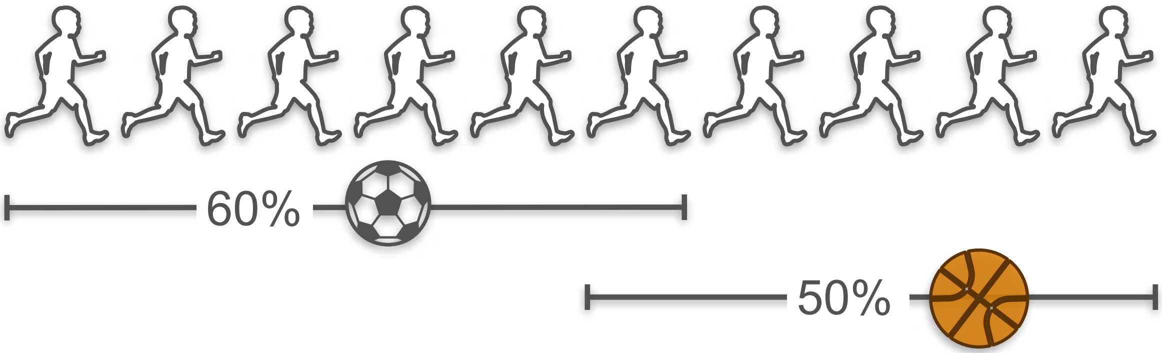

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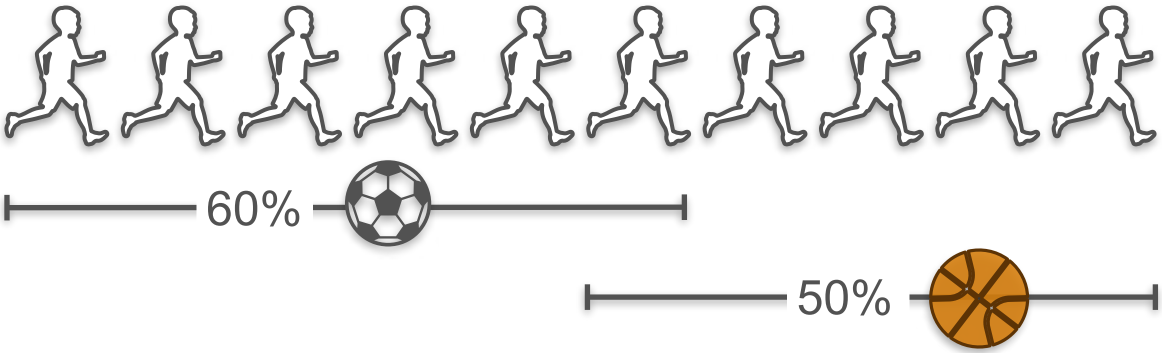

$\mathbf{P}(\text{soccer} \cup \text{basketball}) = ?$

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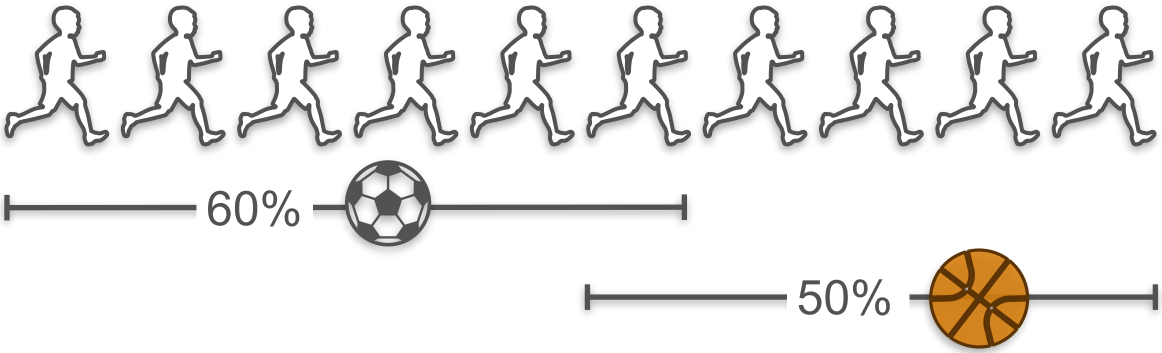

$$\mathbf{P}(\text{soccer} \cup \text{basketball}) = ?$$

We don't know how many children play multiple sports

Sum of Probabilities (Joint Events): Quiz 1 Solution

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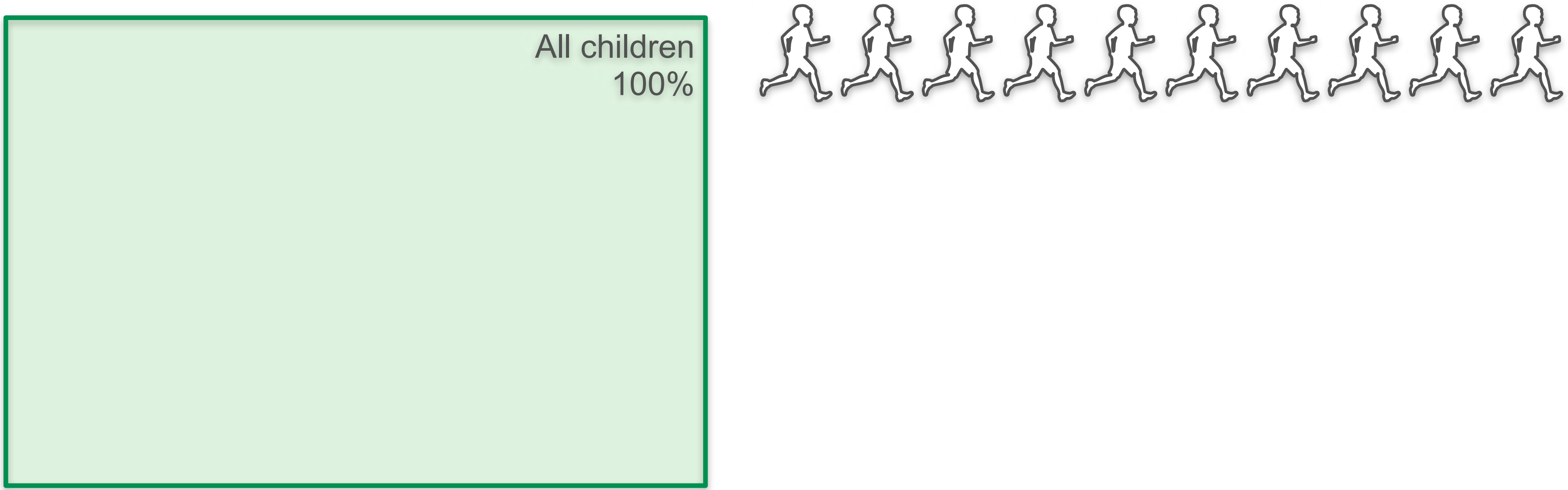

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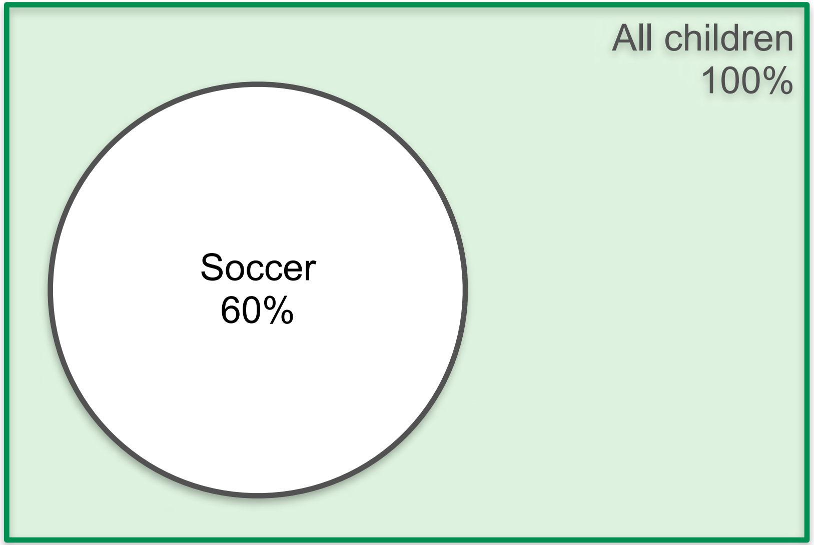

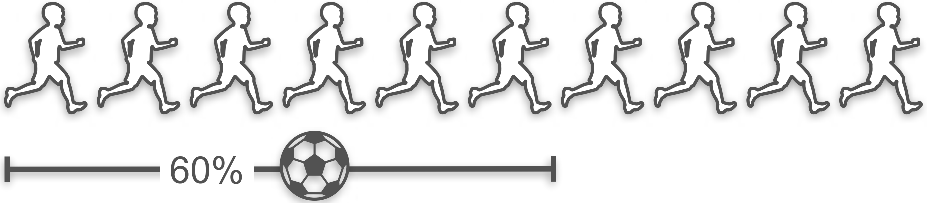

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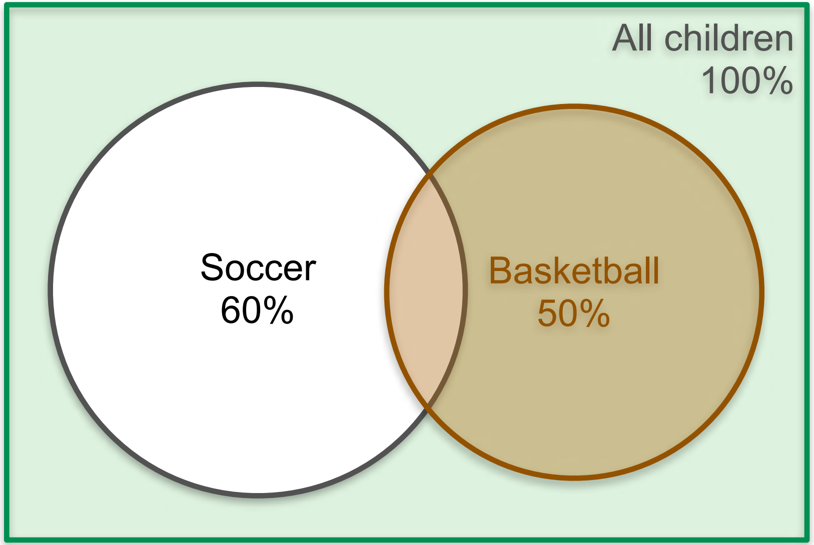

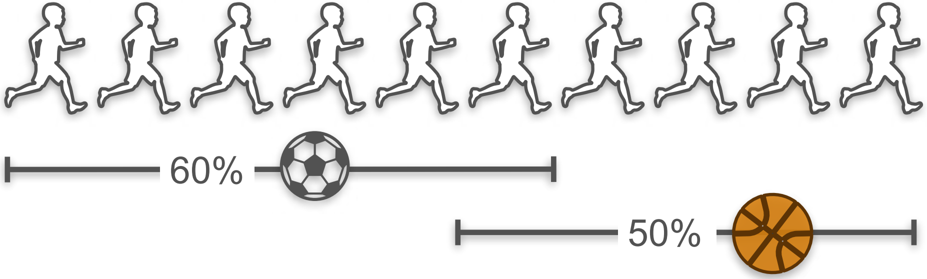

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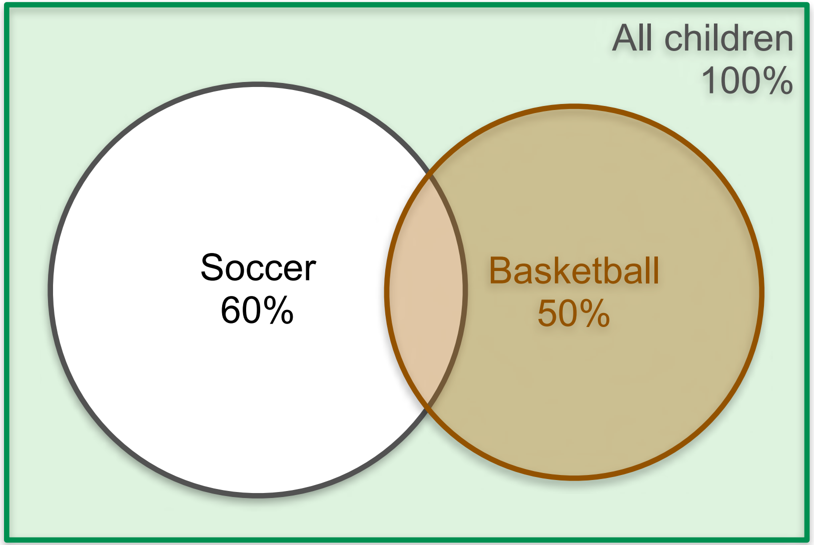
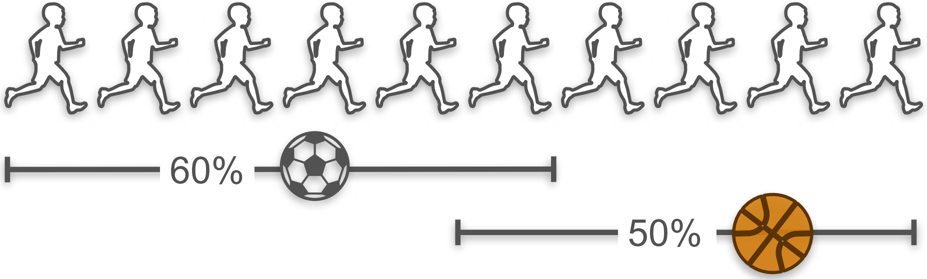
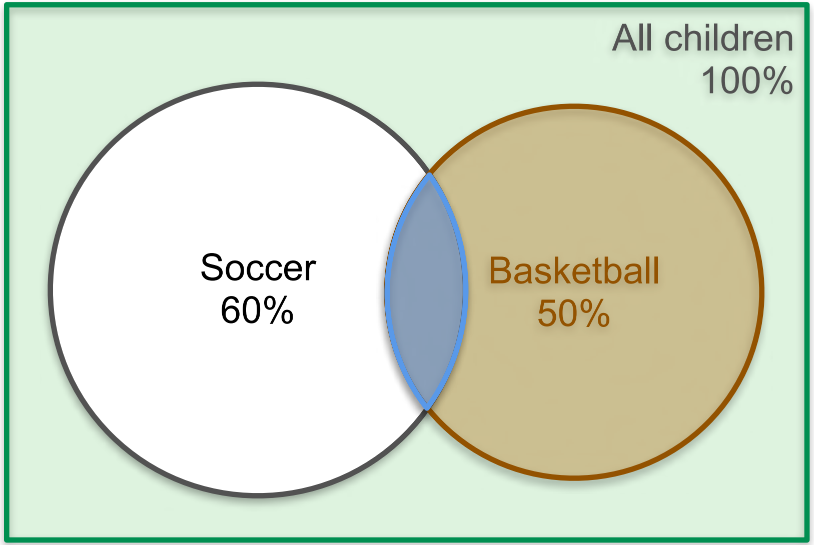
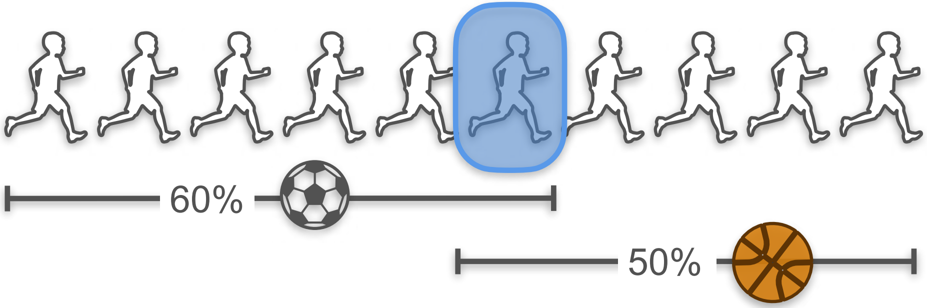




$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution





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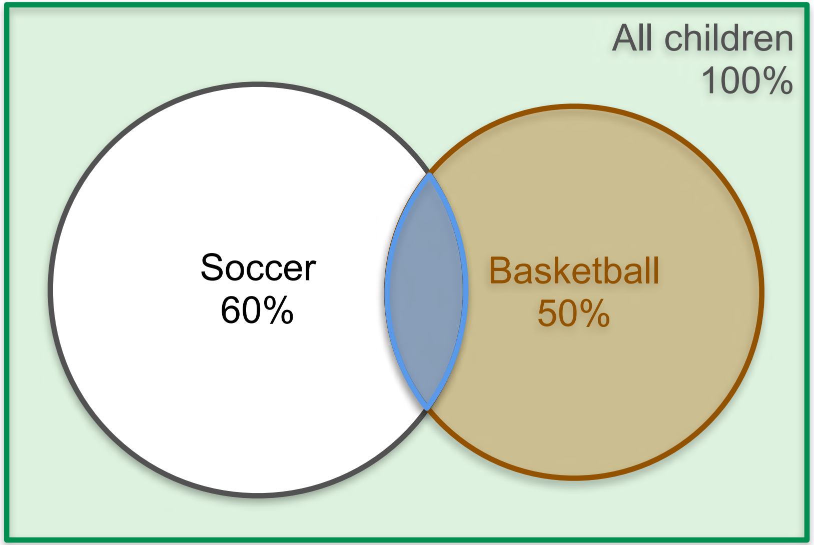
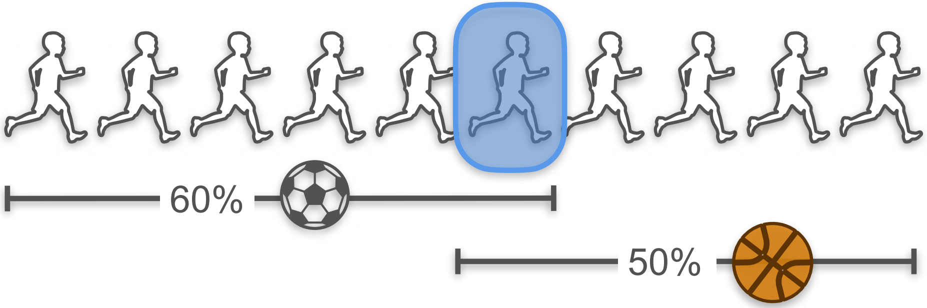


$\mathbf{P}(S \cap B)$



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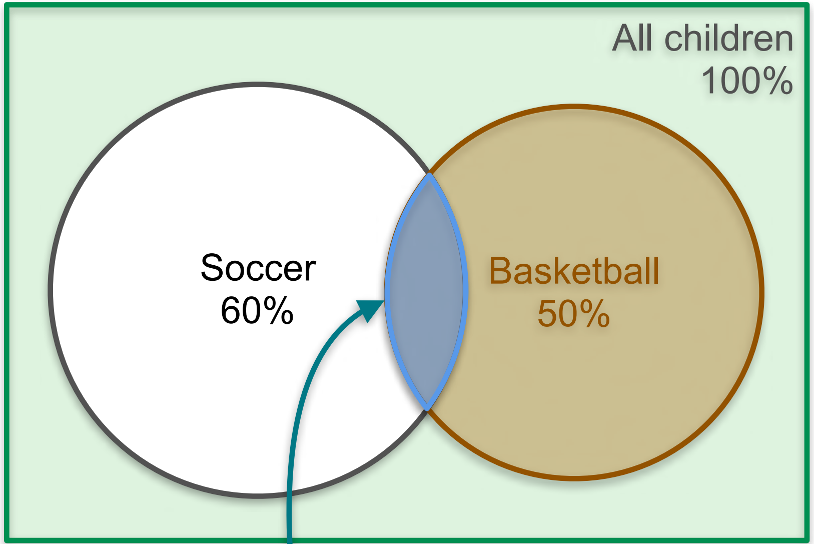

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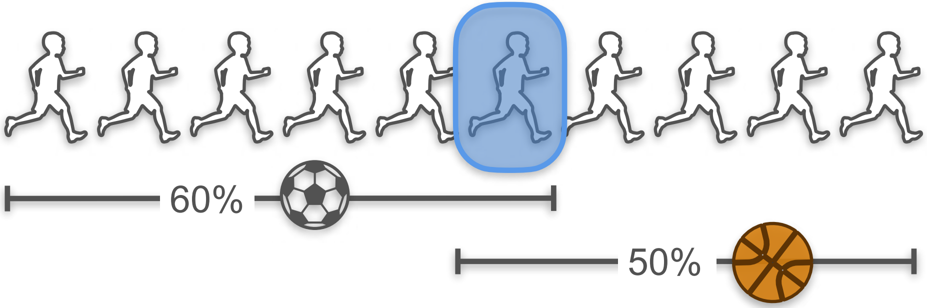

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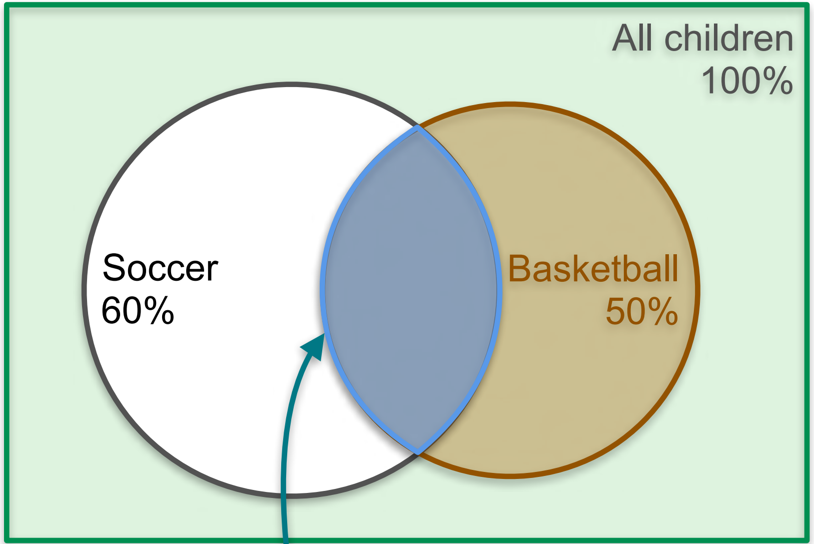

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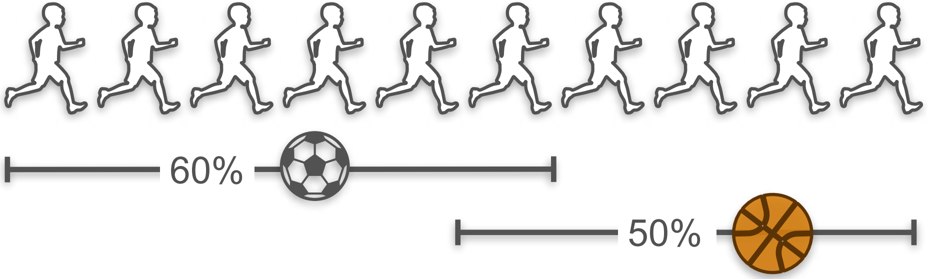

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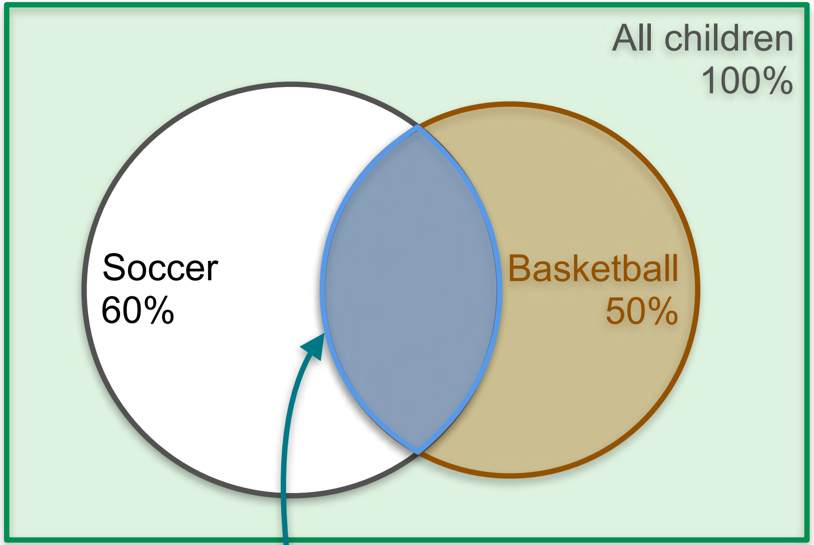

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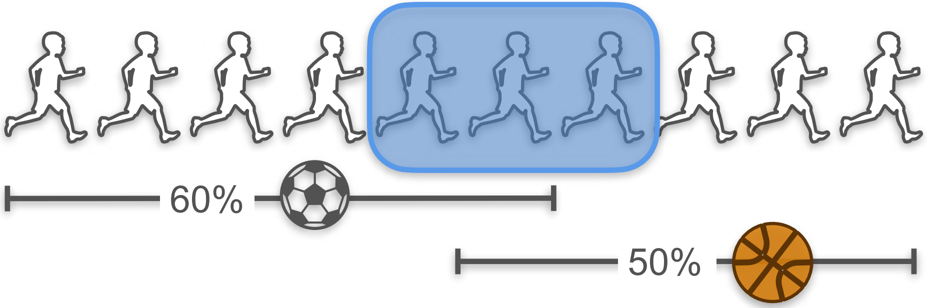

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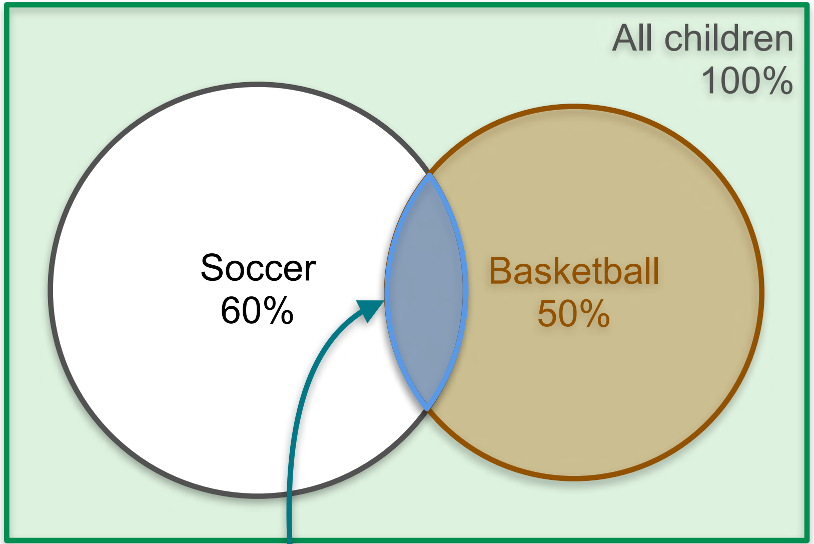

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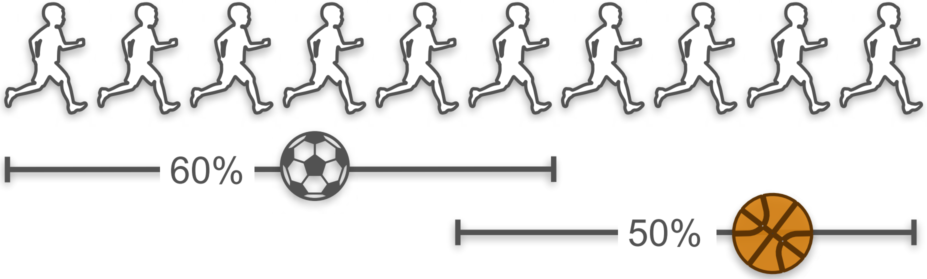

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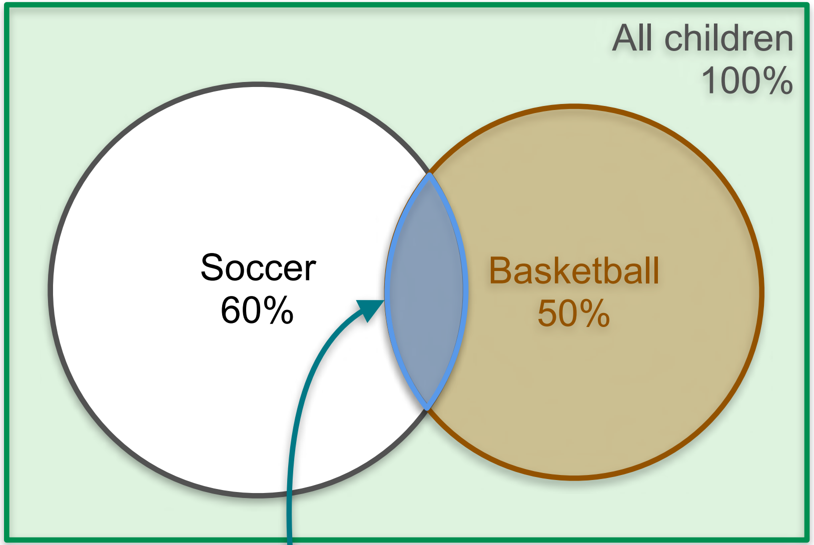

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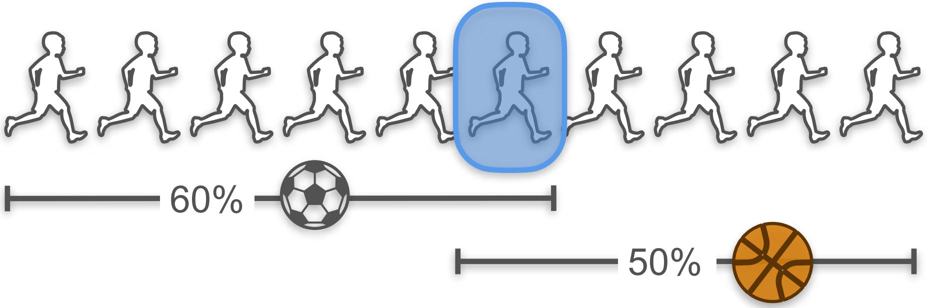

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Sum of Probabilities (Joint Events): Quiz 2

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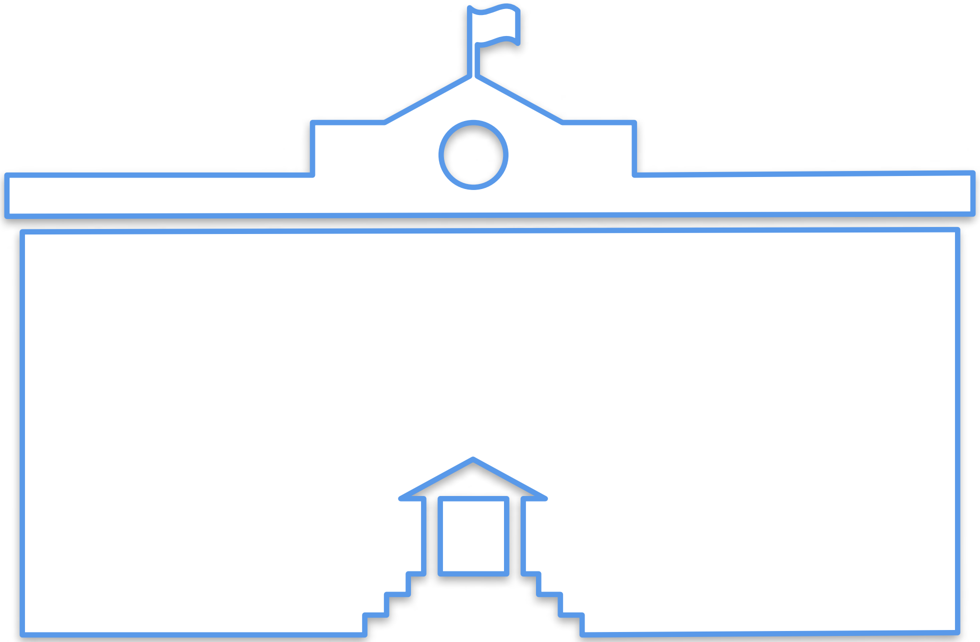

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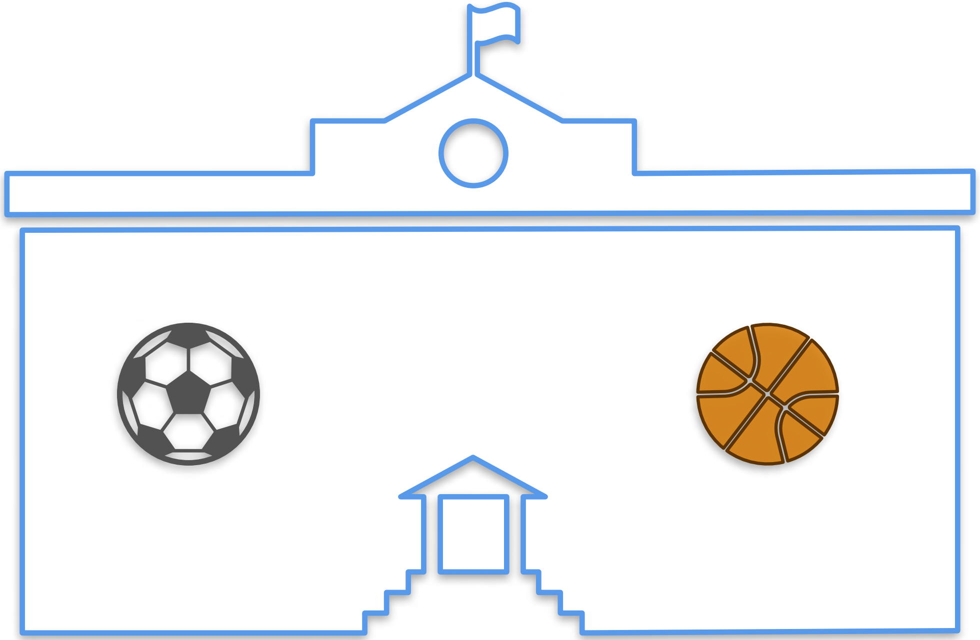

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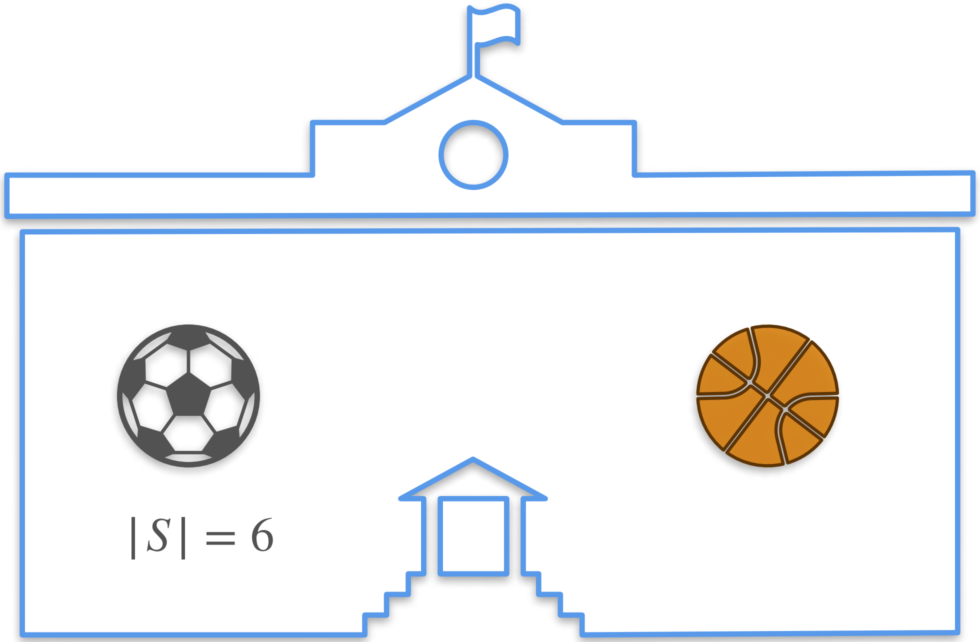

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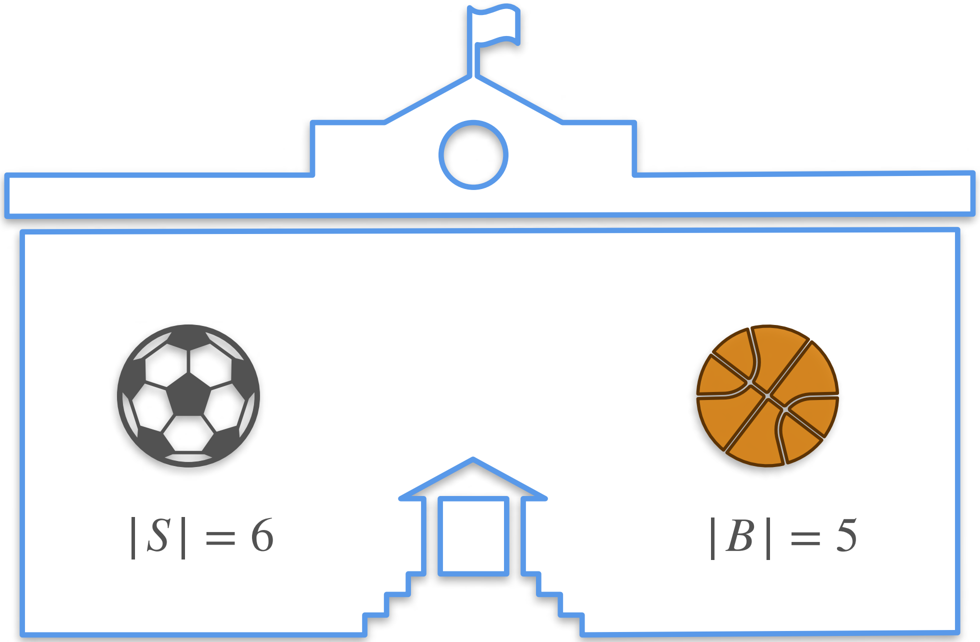

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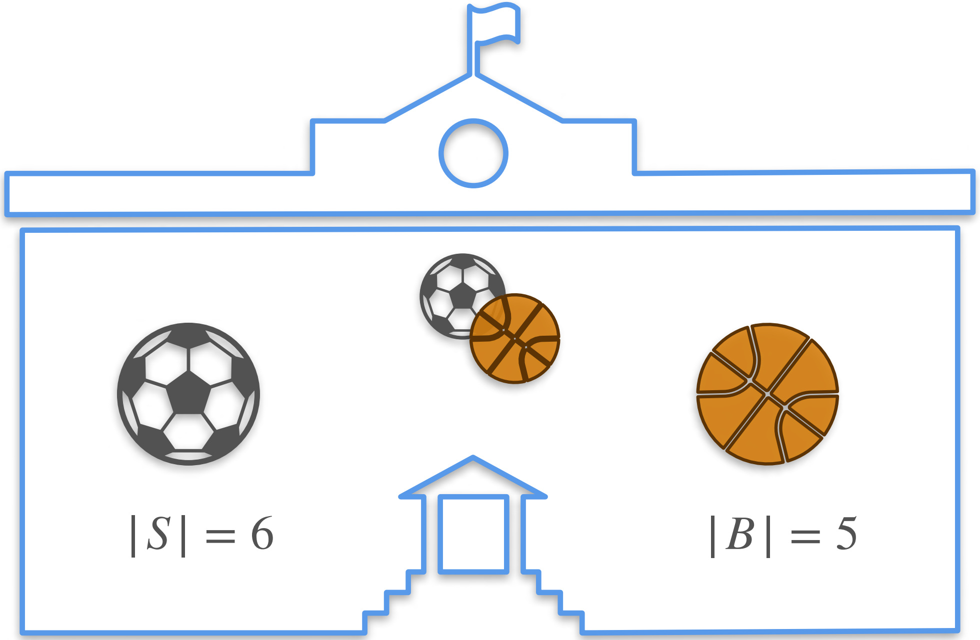

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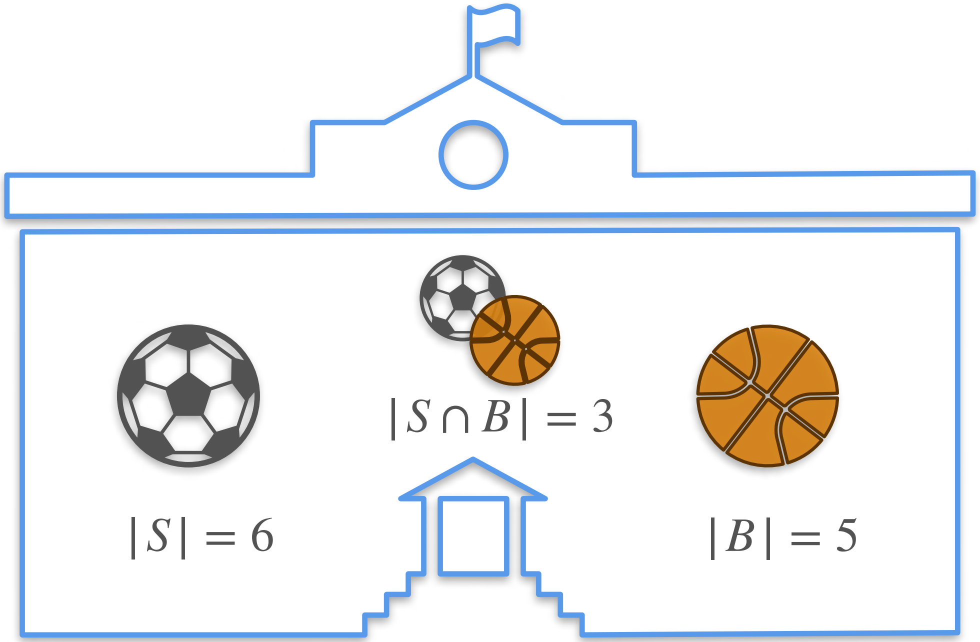

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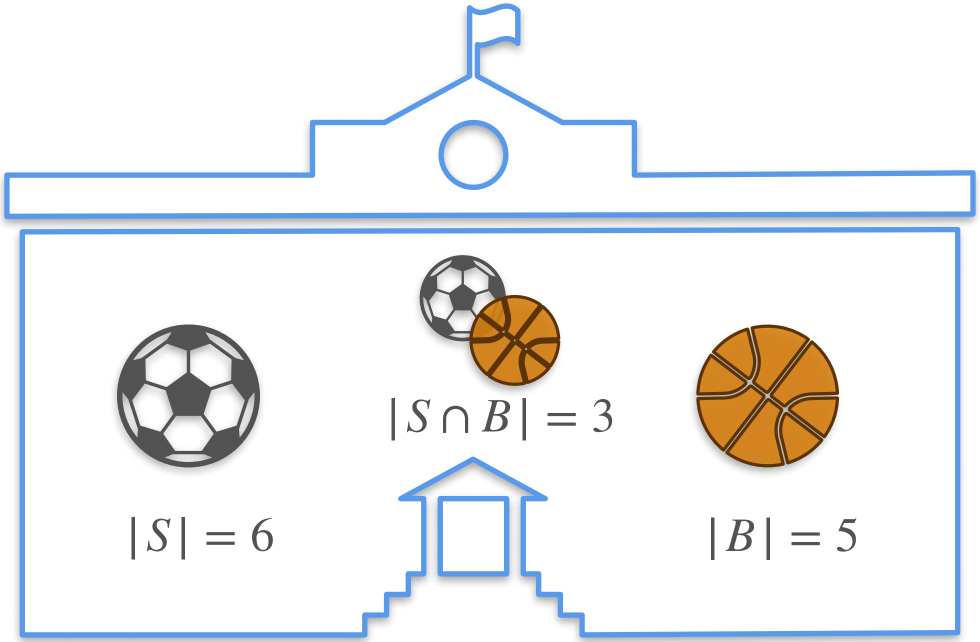

How many kids play soccer or basketball?

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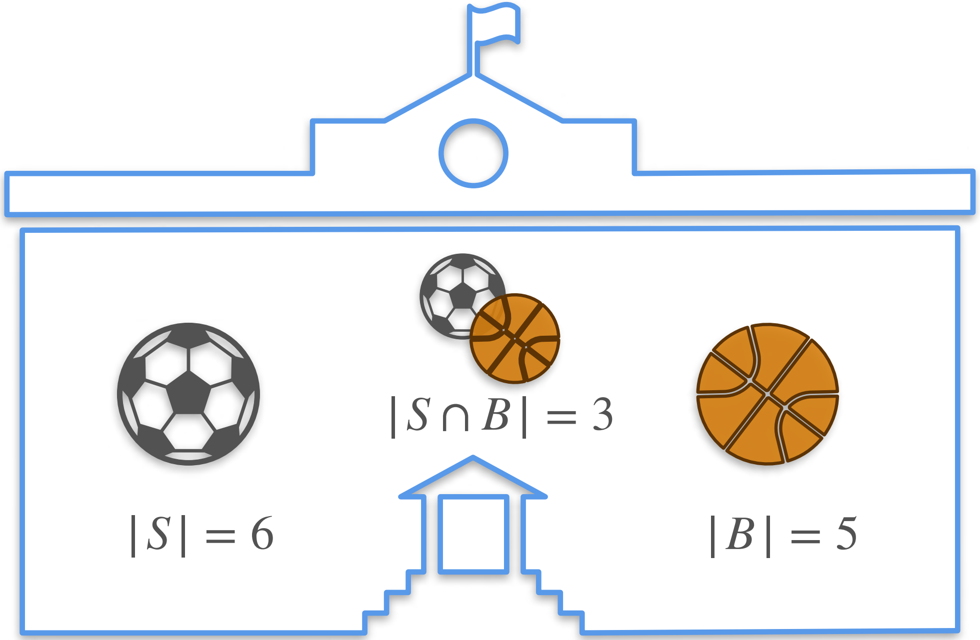

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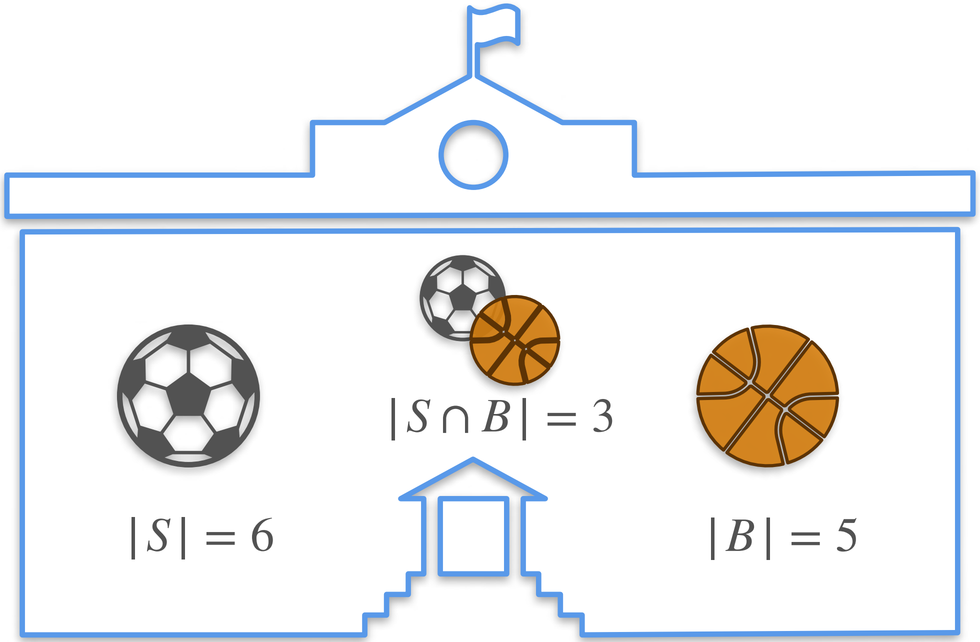

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Hint: What if there were only 10 kids?

Sum of Probabilities (Joint Events): Quiz 2 Solution

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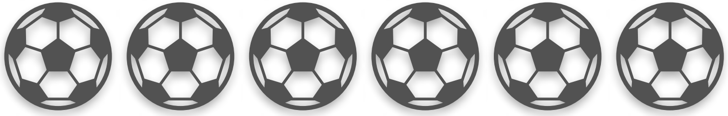

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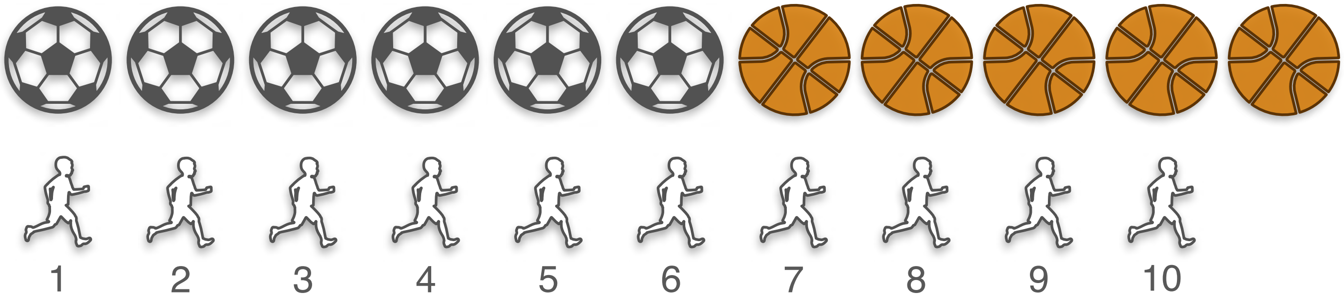

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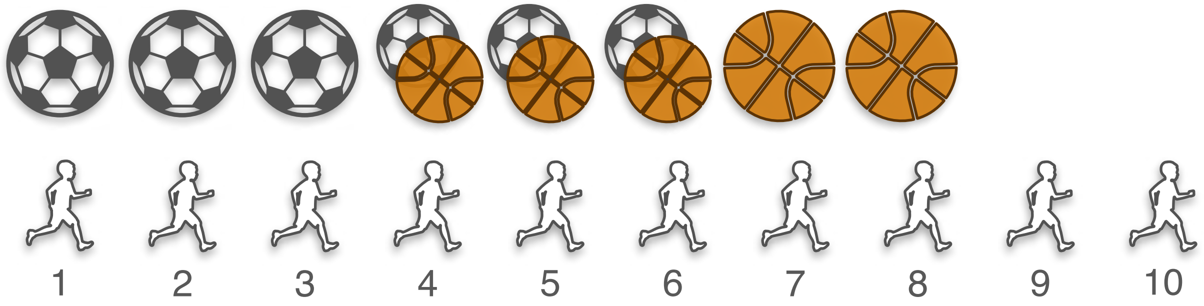

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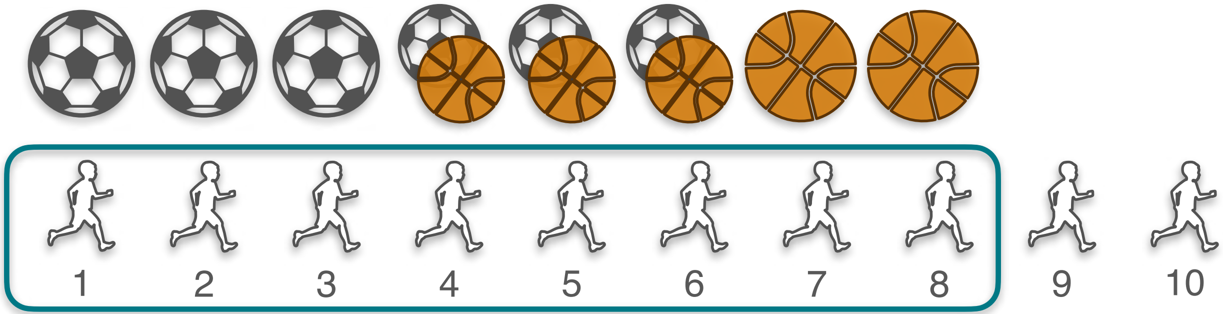

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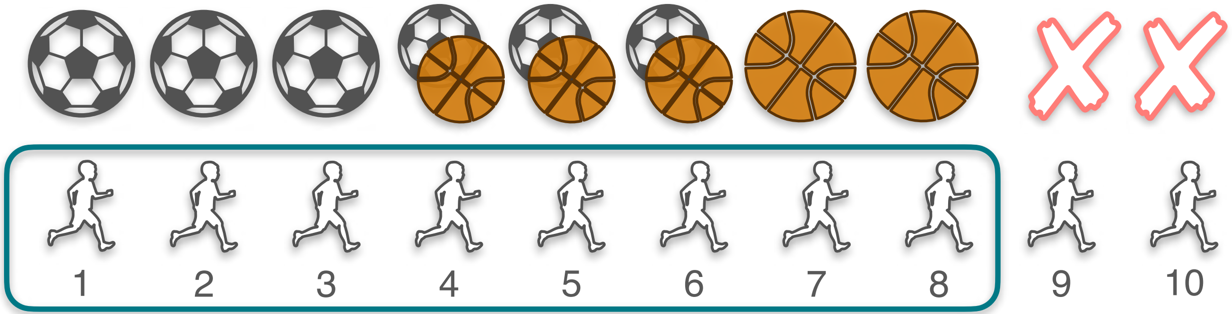
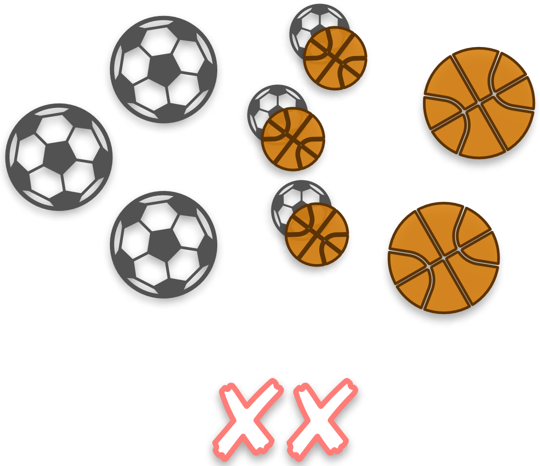


Sum of Probabilities (Joint Events): Venn Diagram



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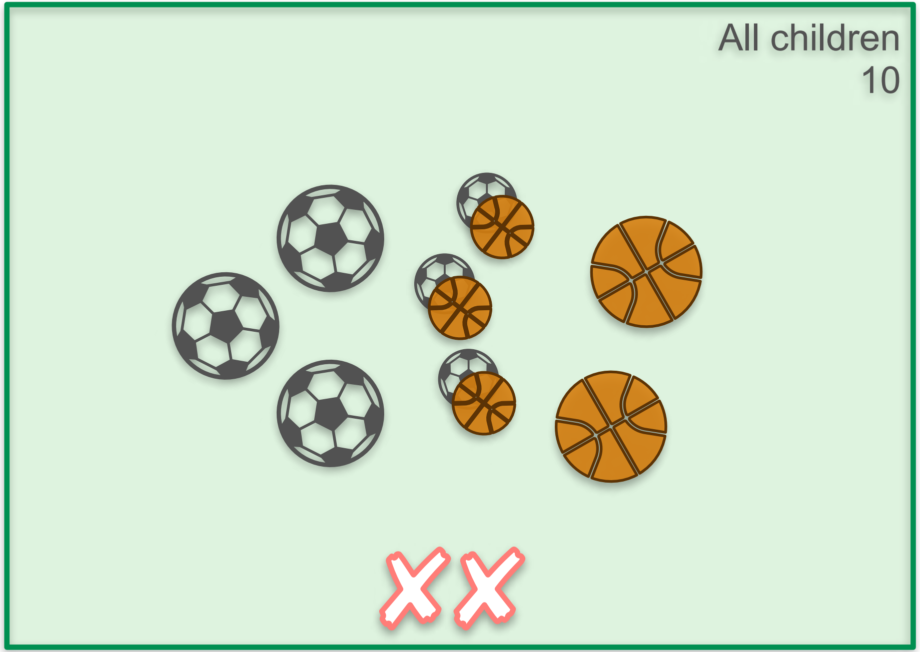

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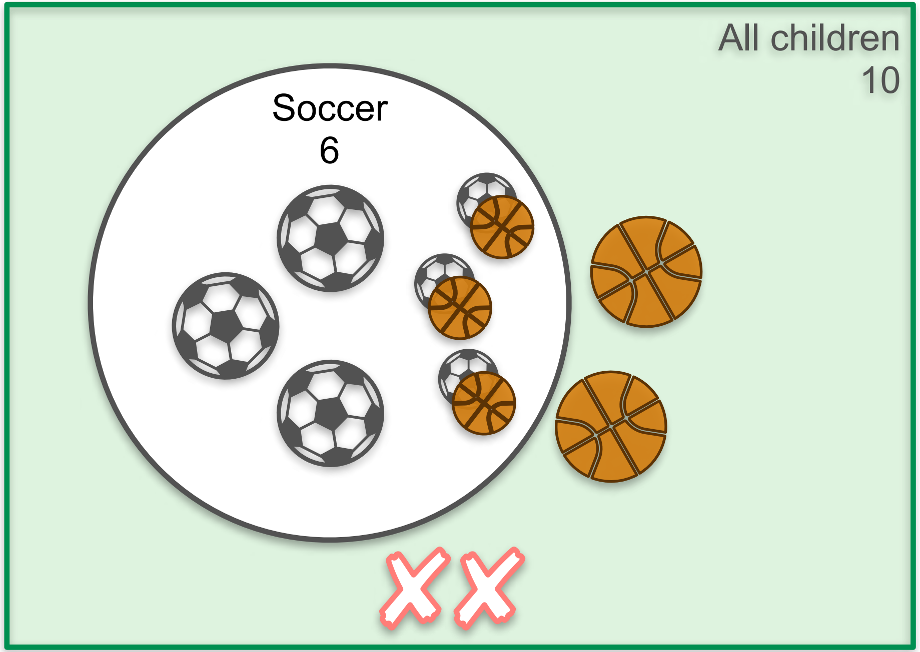

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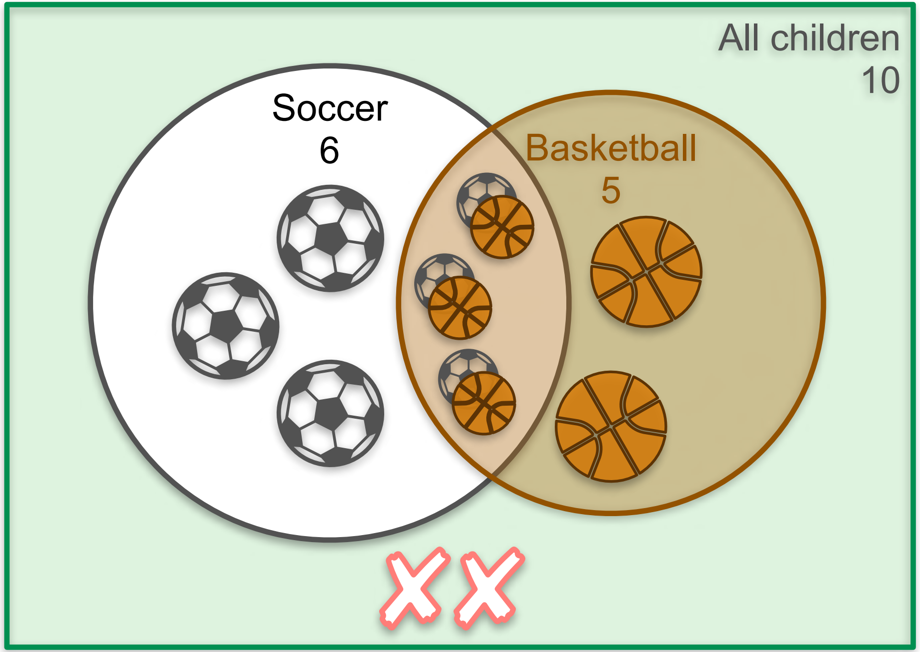

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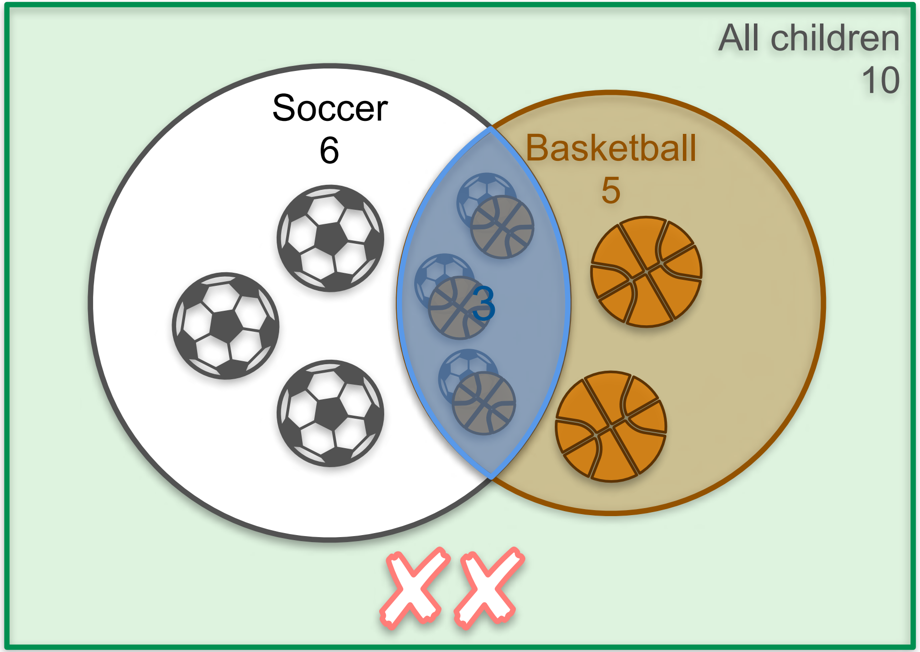

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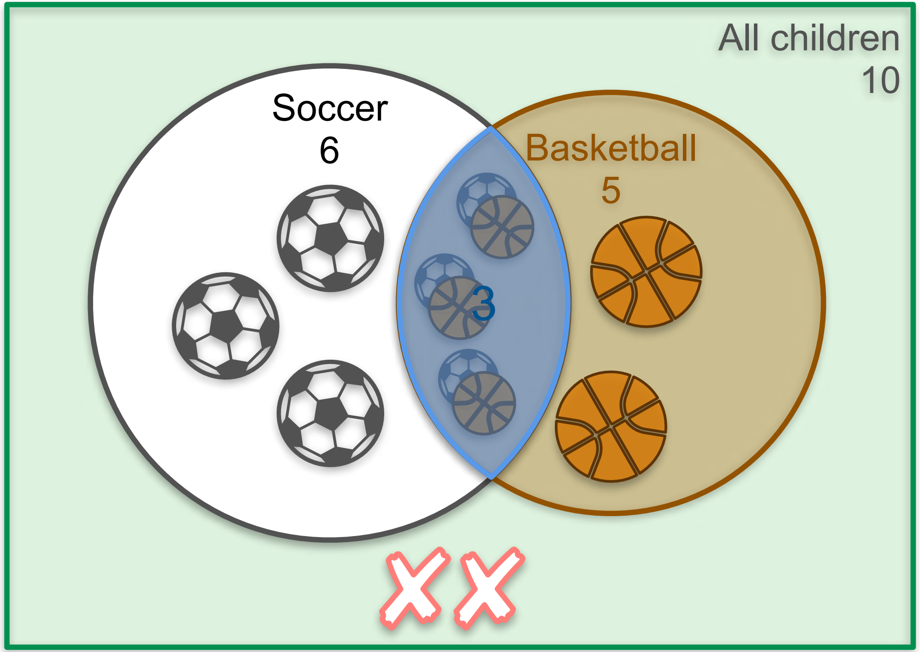
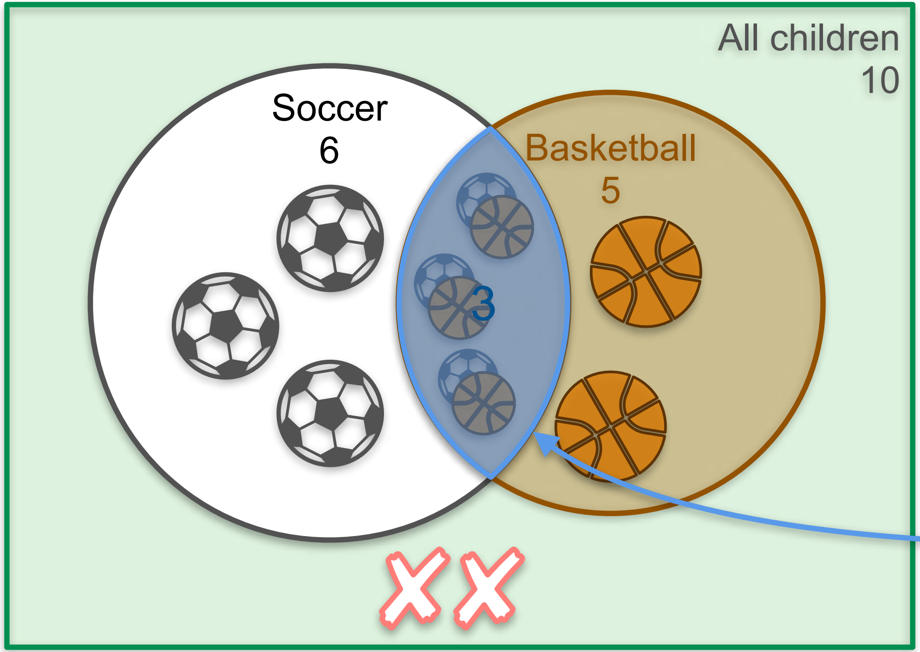


Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram



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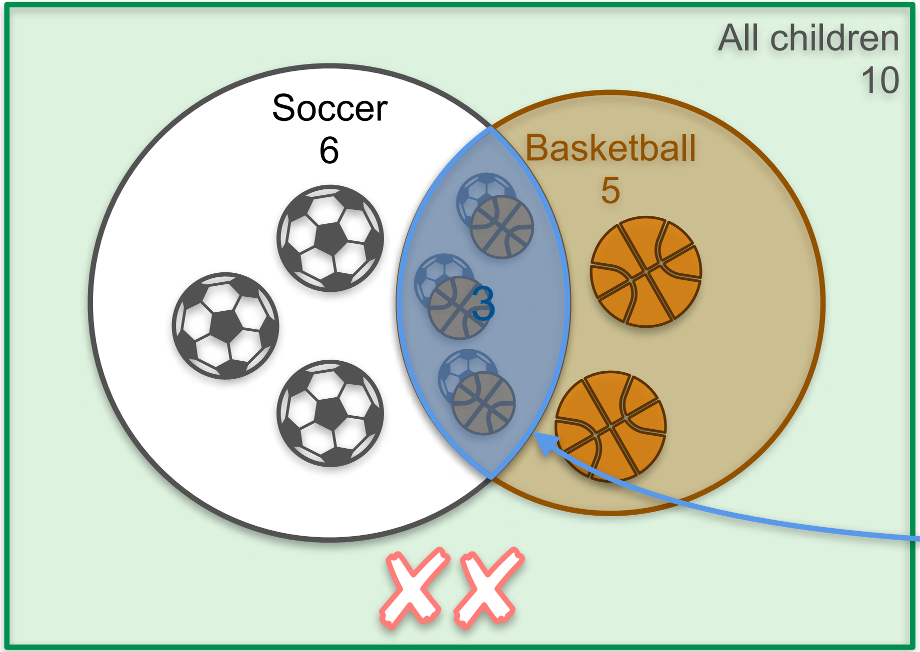

$$|S \cup B| \ =$$

Soccer and Basketball

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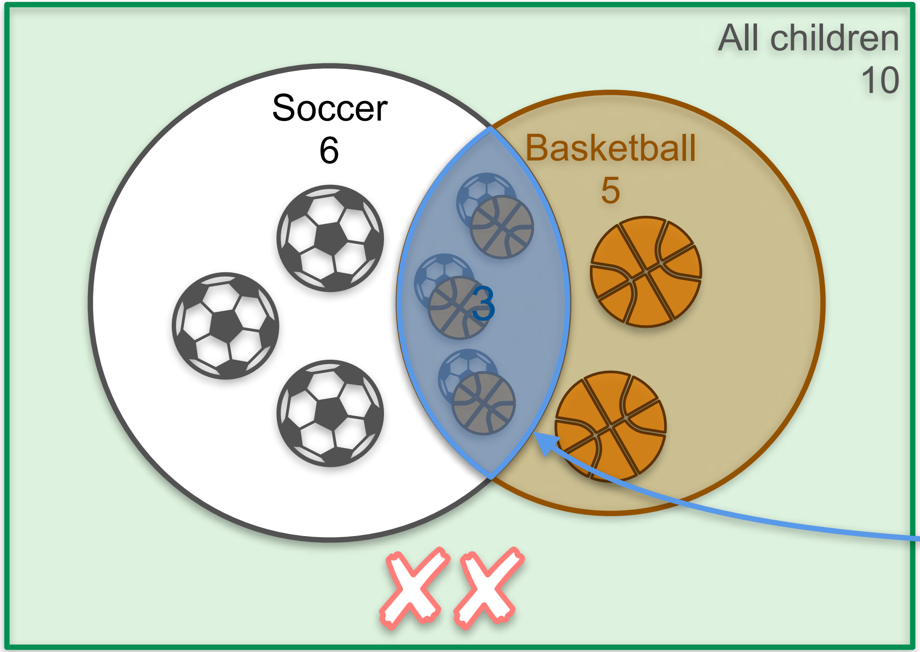

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Soccer and Basketball

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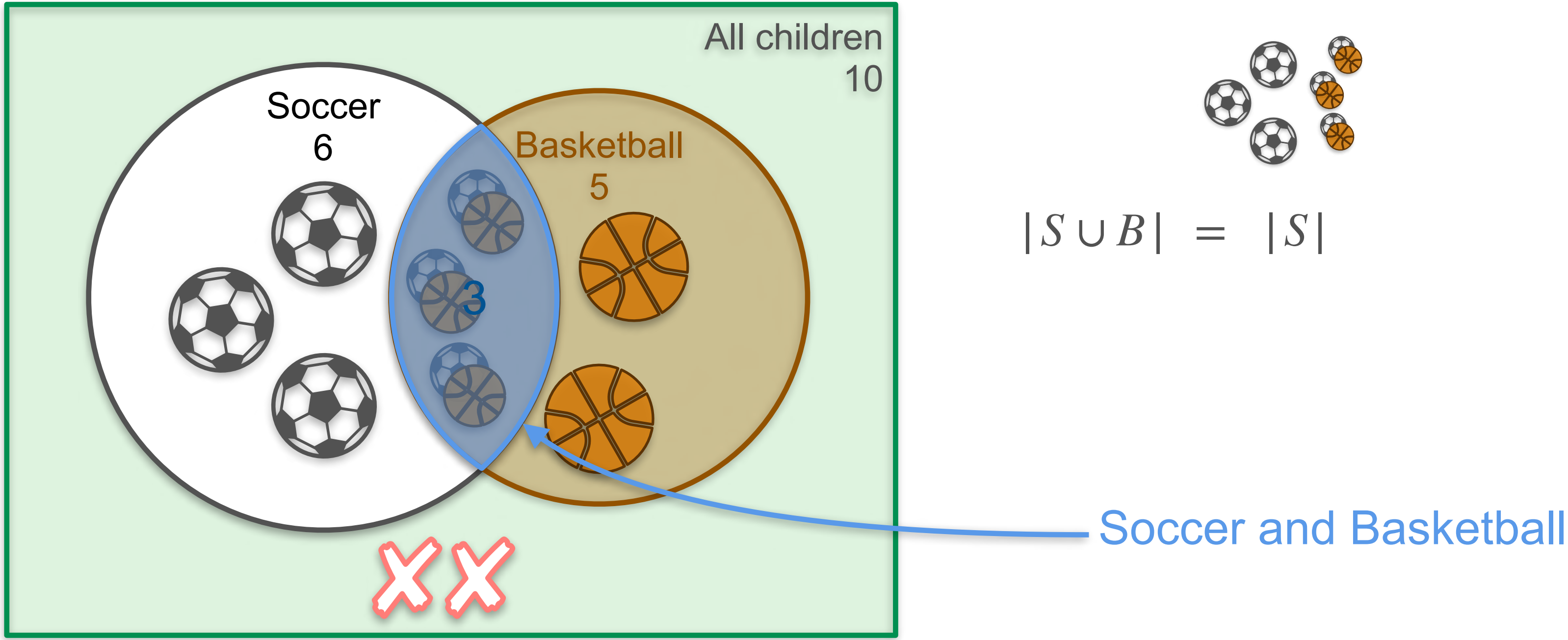
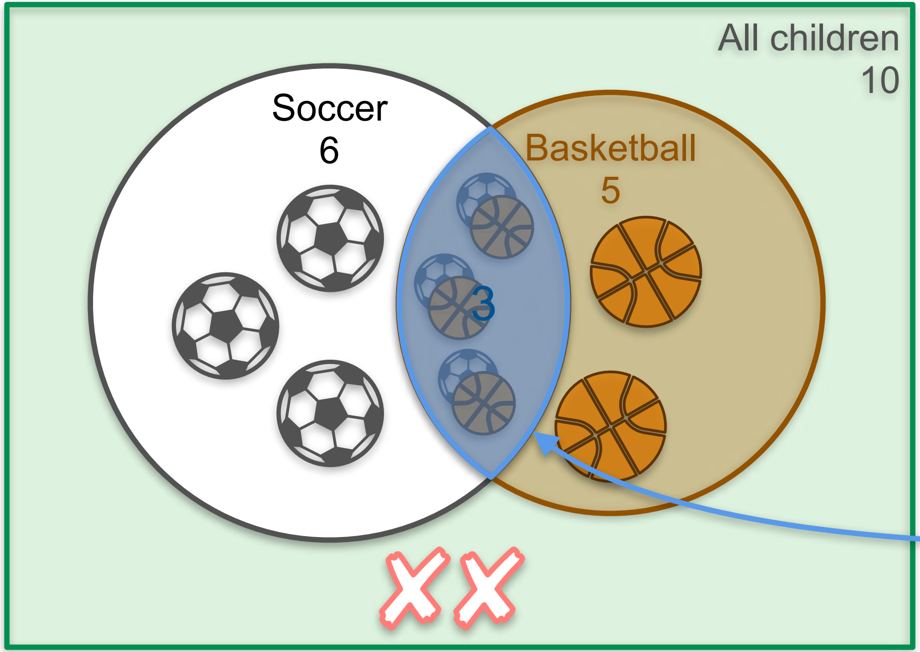
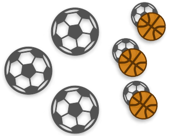


$$|S \cup B| = |S|$$

Sum of Probabilities (Joint Events): Venn Diagram

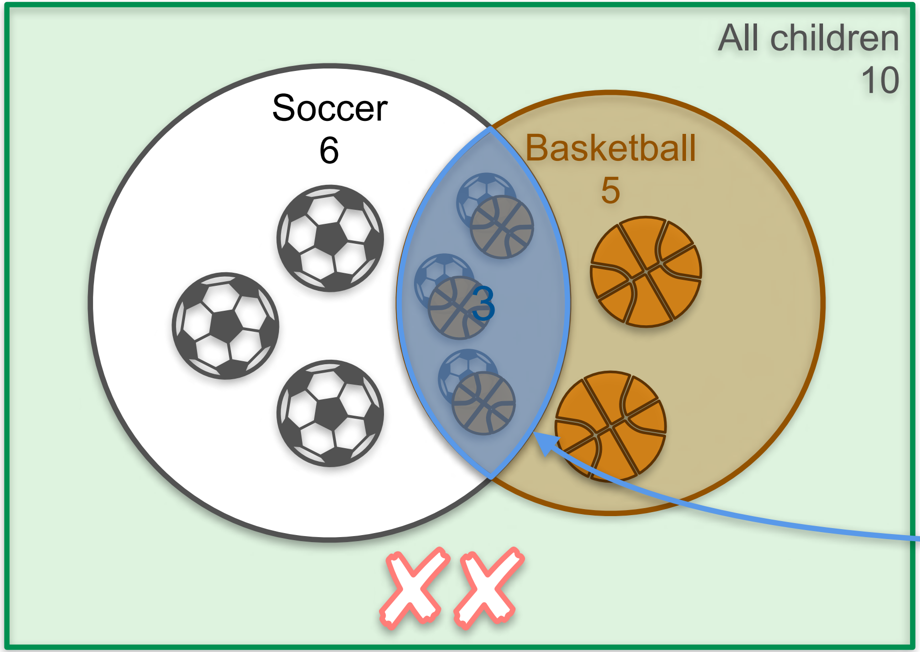


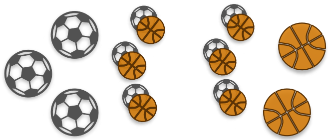


$$|S \cup B| = |S| + |B|$$

Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram

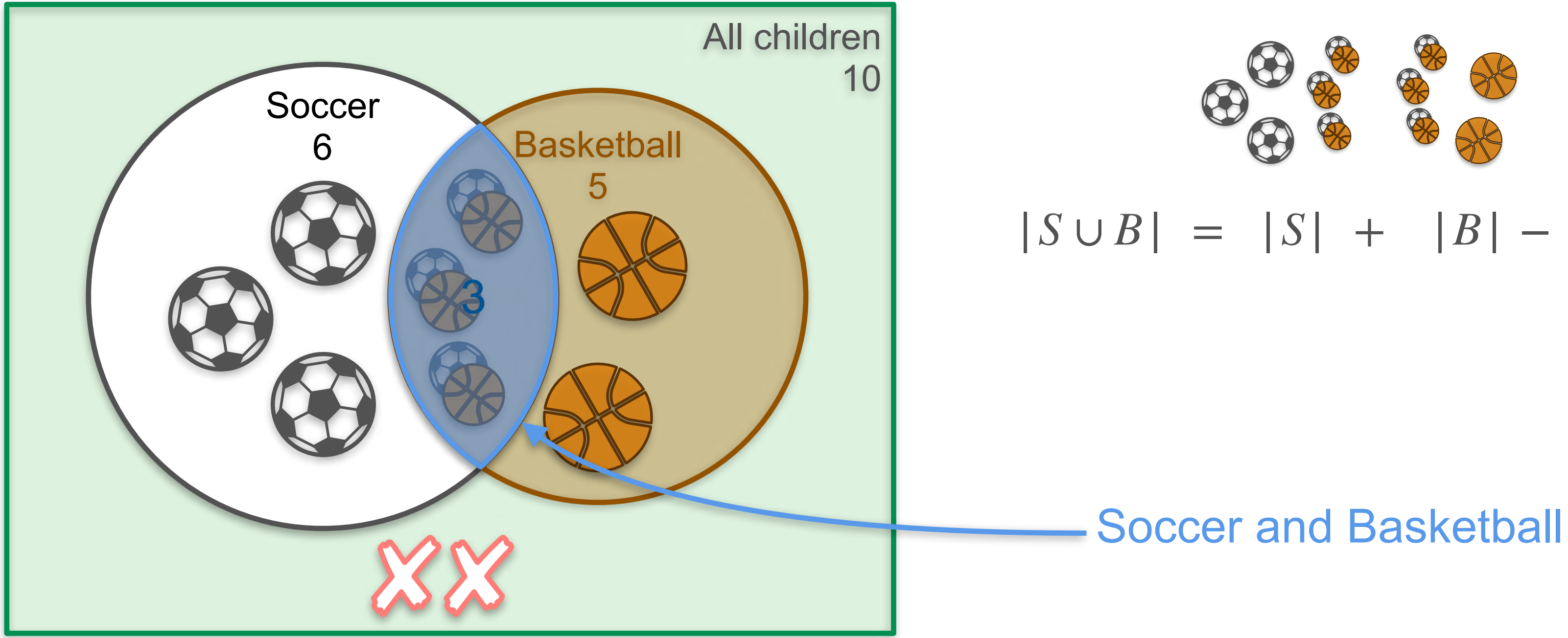




$$|S \cup B| \; = \; |S| \; + \; |B|$$

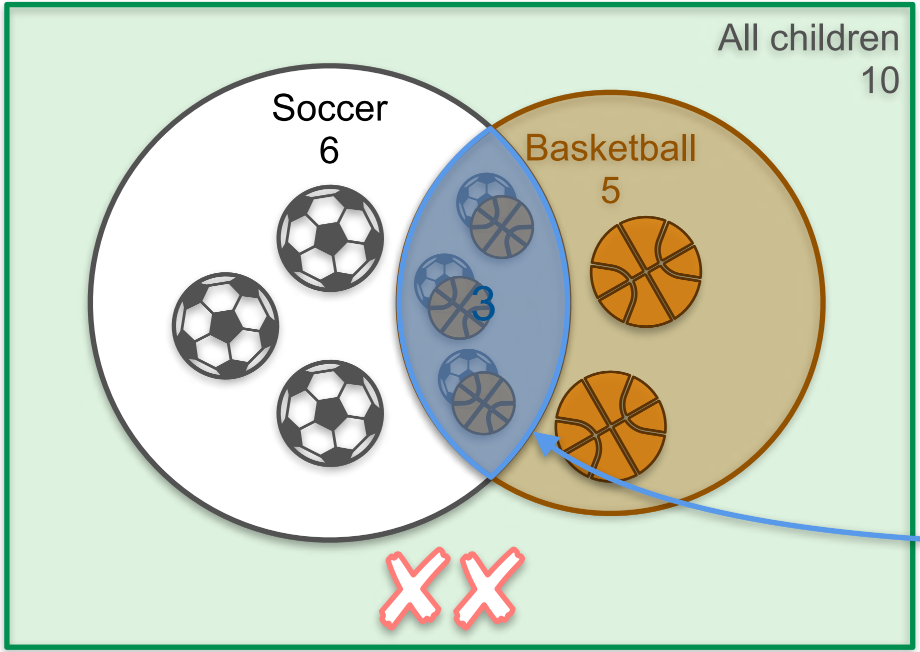
Soccer and Basketball

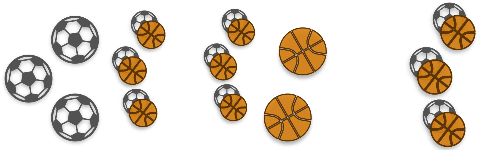
Sum of Probabilities (Joint Events): Venn Diagram



$$|S \cup B| = |S| + |B| - |S \cap B|$$

Sum of Probabilities (Joint Events): Venn Diagram

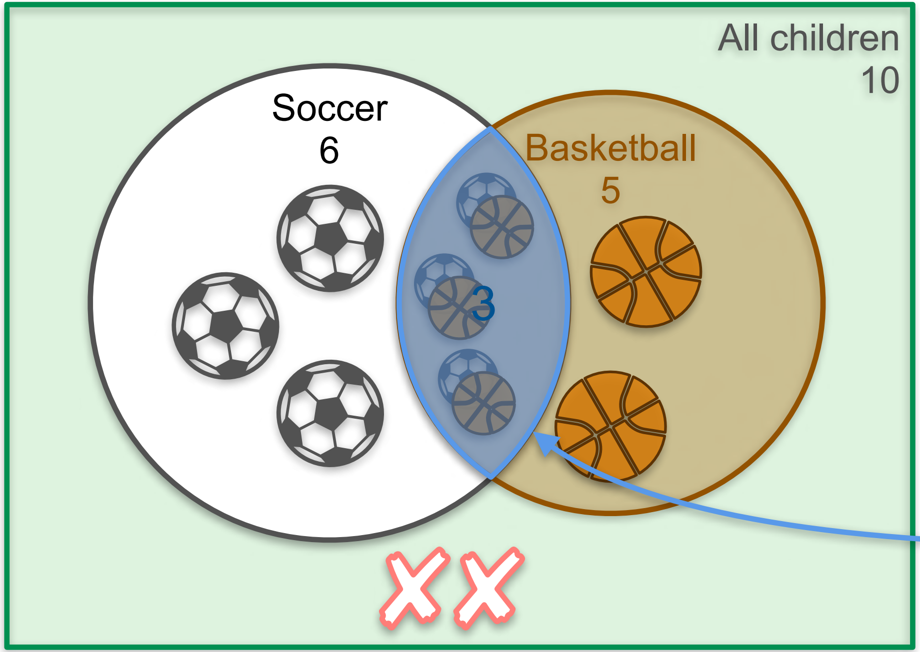


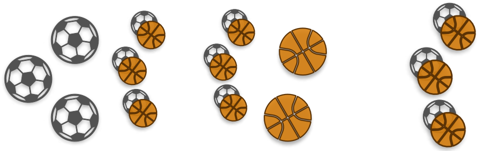


$$|S \cup B| = |S| + |B| - |S \cap B|$$

Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram



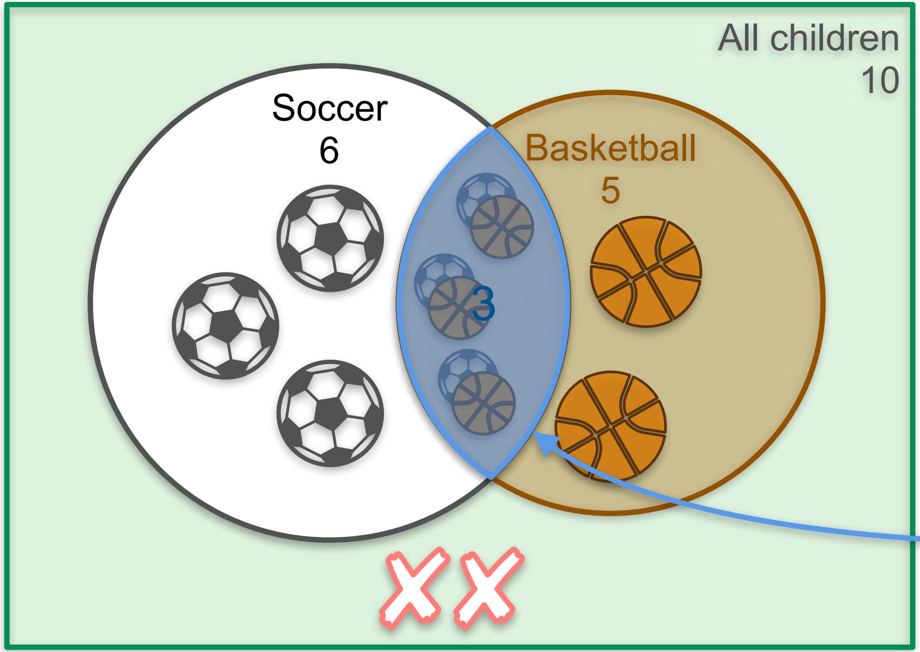


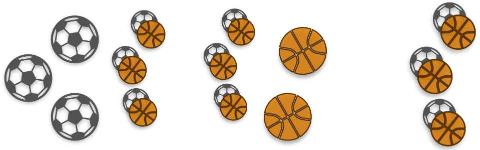
$$|S \cup B| = |S| + |B| - |S \cap B|$$

$$=$$

Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram



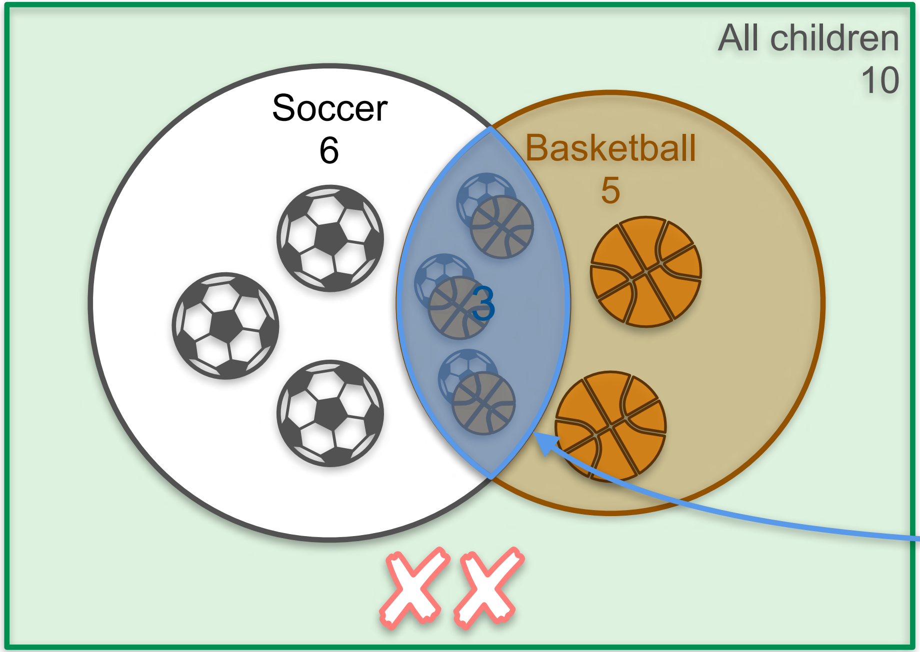


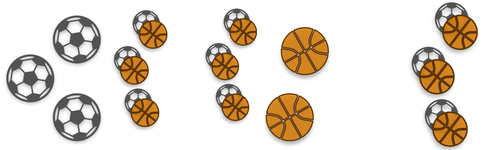
$$|S \cup B| = |S| + |B| - |S \cap B|$$

$$= 6 + 5 - 3$$

Soccer and Basketball

Sum of Probabilities (Joint Events): Venn Diagram



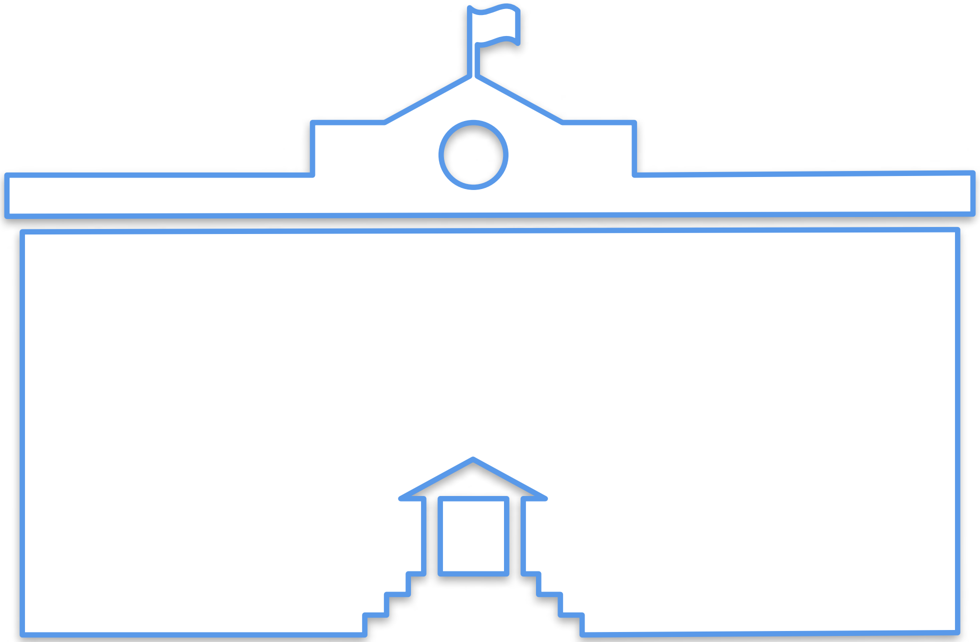


$$|S \cup B| = |S| + |B| - |S \cap B|$$
$$= 6 + 5 - 3$$
$$= 8$$

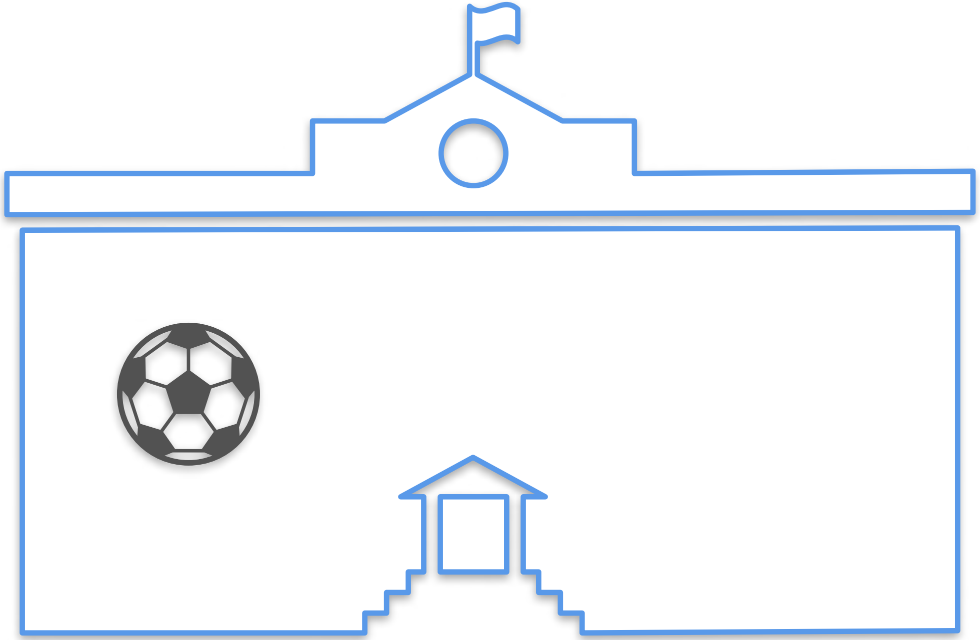
Soccer and Basketball

Sum of Probabilities (Joint Events): Quiz 3

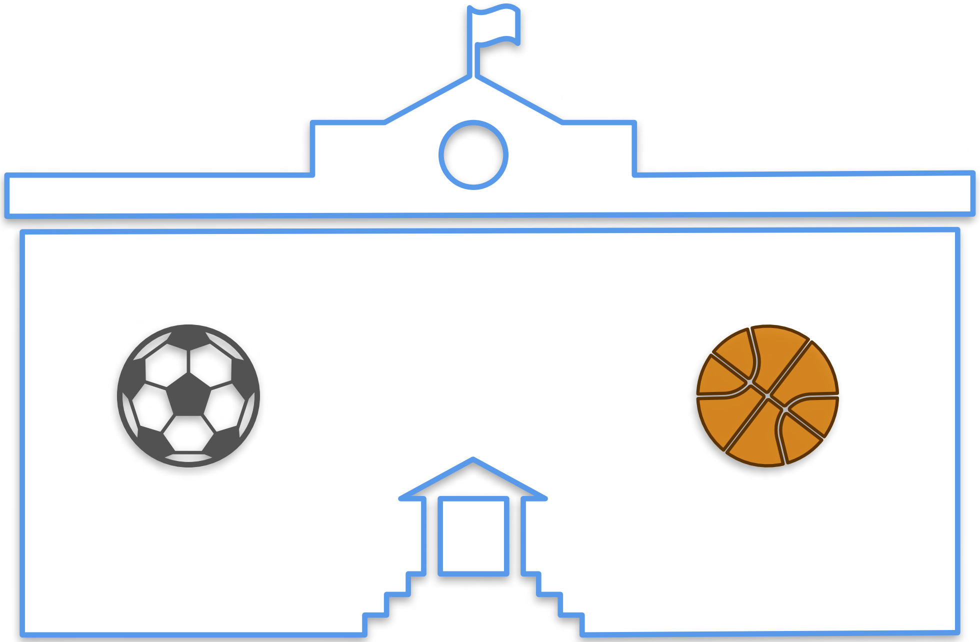
Sum of Probabilities (Joint Events): Quiz 3



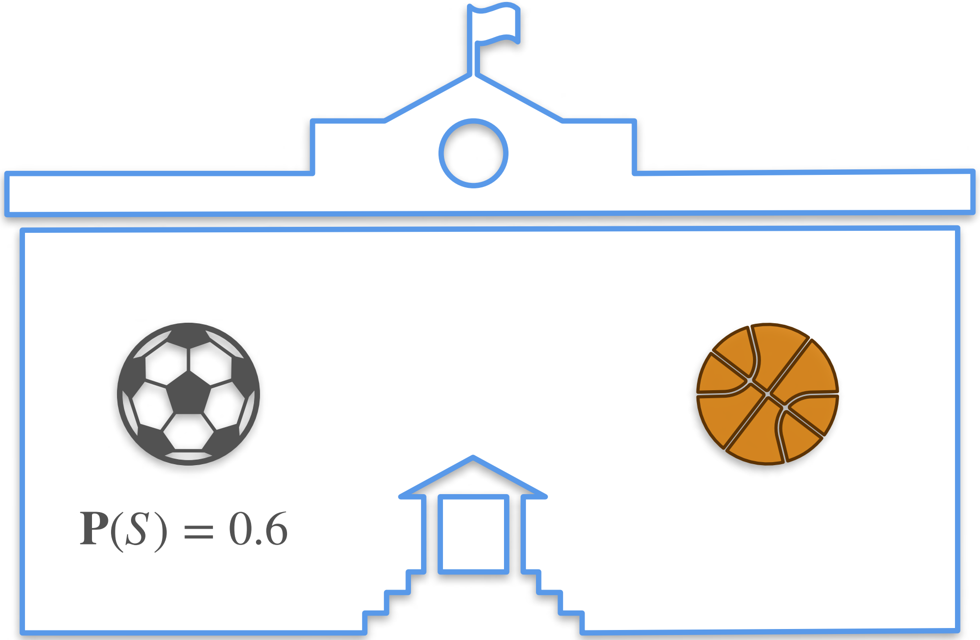
Sum of Probabilities (Joint Events): Quiz 3



Sum of Probabilities (Joint Events): Quiz 3

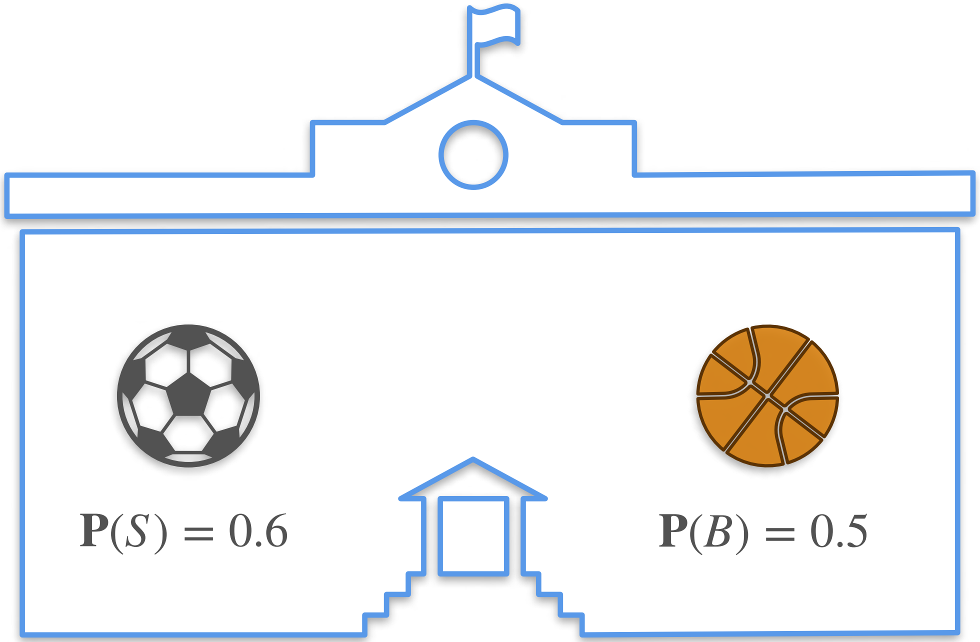


Sum of Probabilities (Joint Events): Quiz 3



$\mathbf{P}(S) = 0.6$

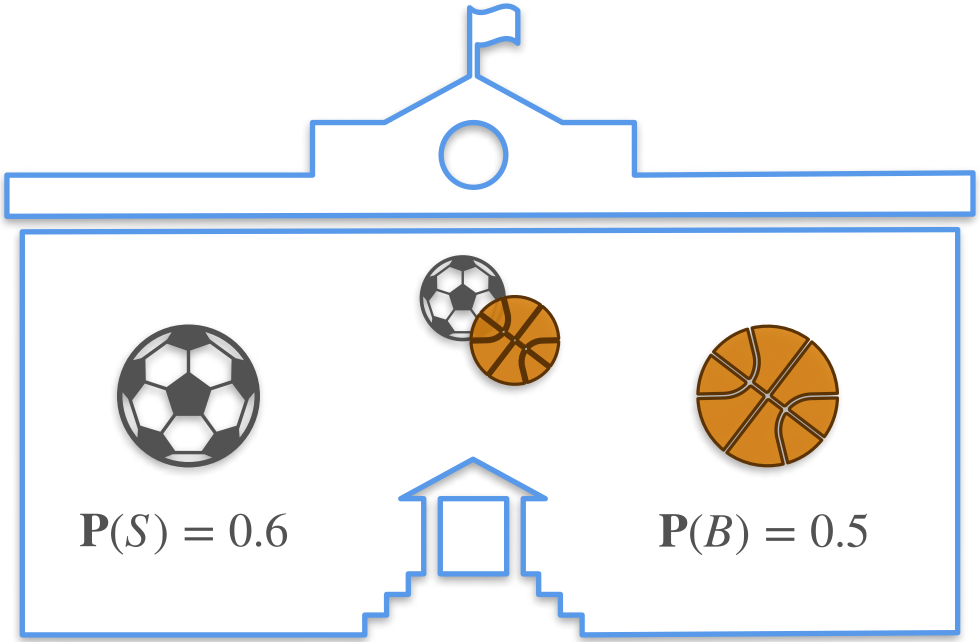
Sum of Probabilities (Joint Events): Quiz 3



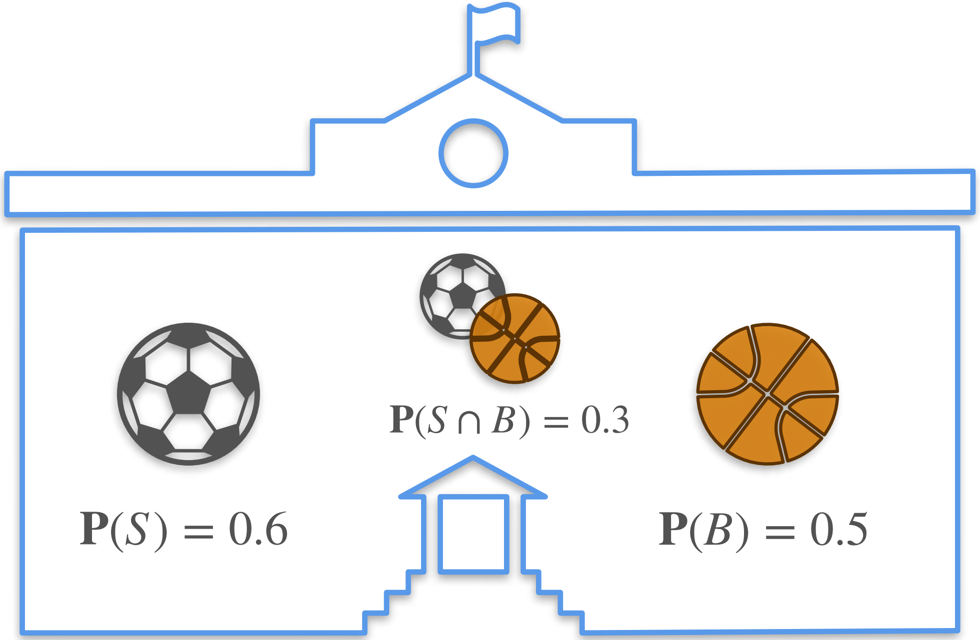
$\mathbf{P}(S) = 0.6$

$\mathbf{P}(B) = 0.5$

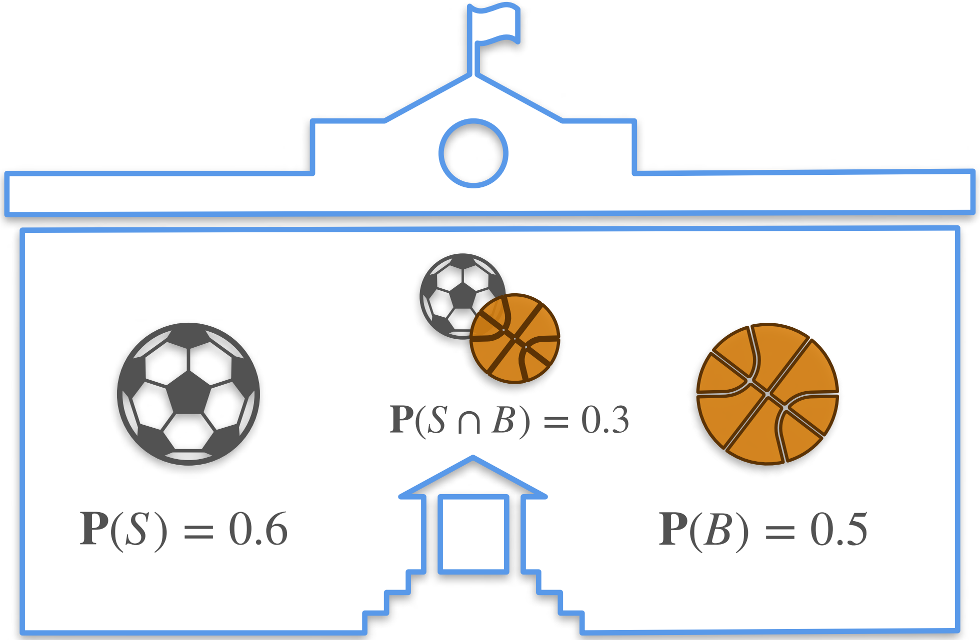
Sum of Probabilities (Joint Events): Quiz 3



Sum of Probabilities (Joint Events): Quiz 3



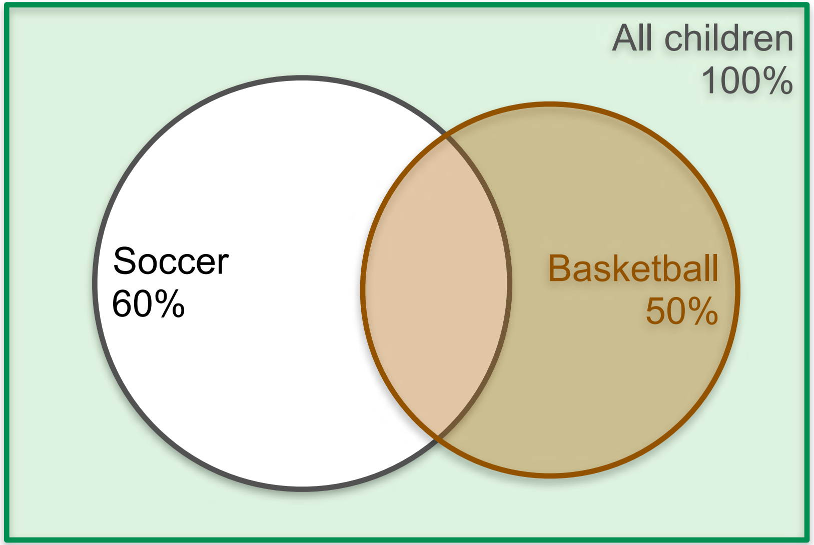
Sum of Probabilities (Joint Events): Quiz 3



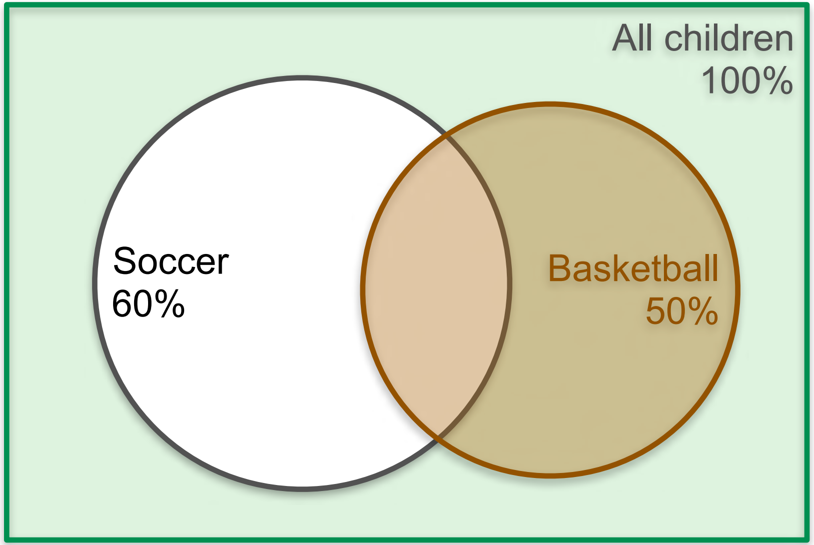
What is the probability that a child plays soccer or basketball?

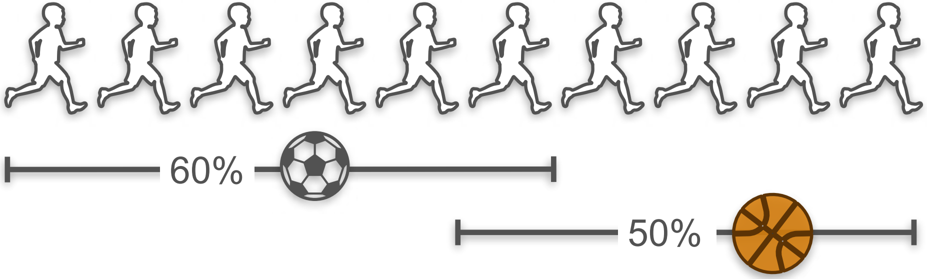
Sum of Probabilities (Joint Events): Quiz 3 Solution

Sum of Probabilities (Joint Events): Quiz 3 Solution

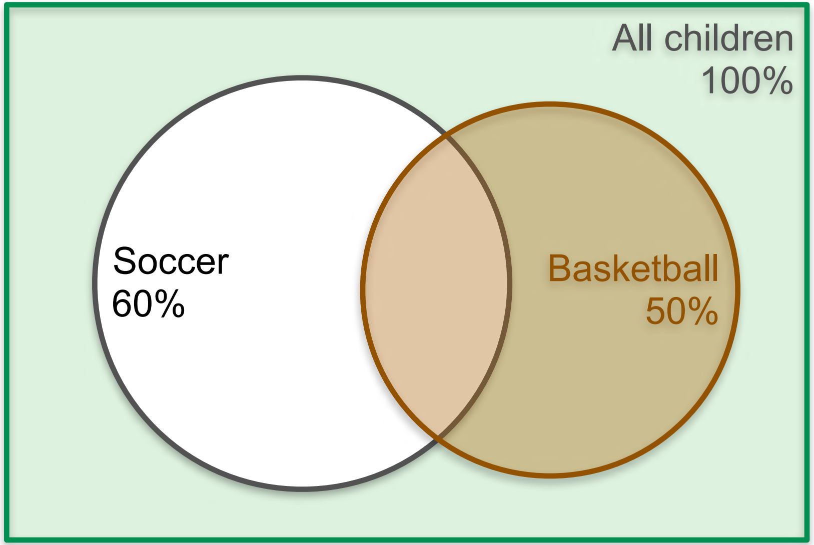


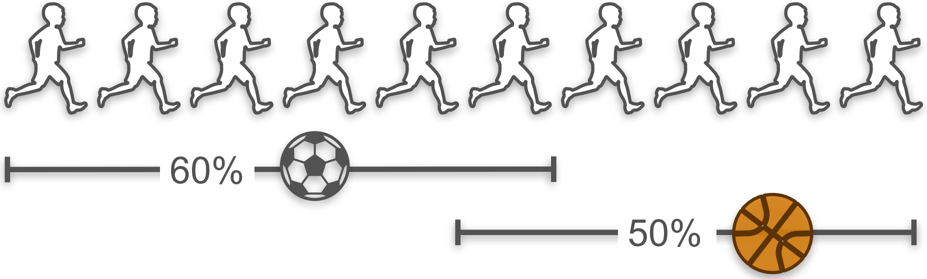
Sum of Probabilities (Joint Events): Quiz 3 Solution





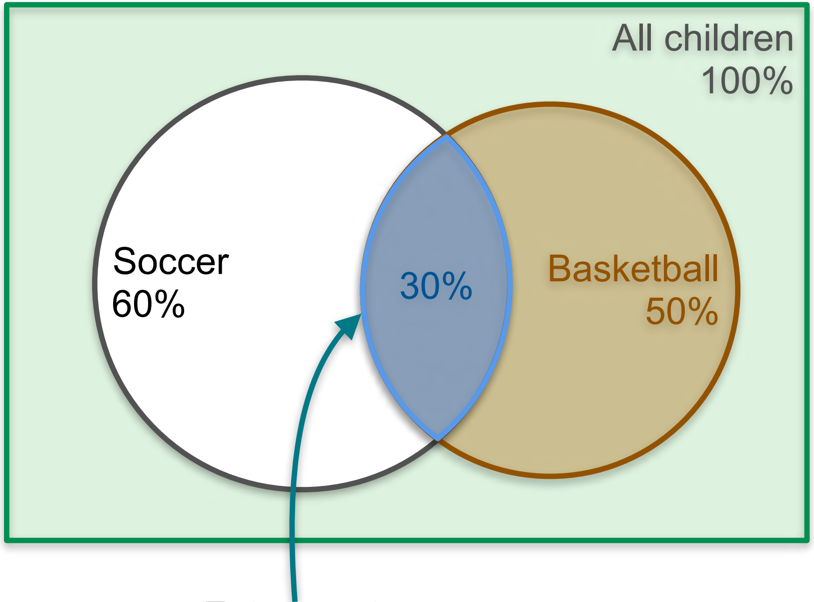
Sum of Probabilities (Joint Events): Quiz 3 Solution



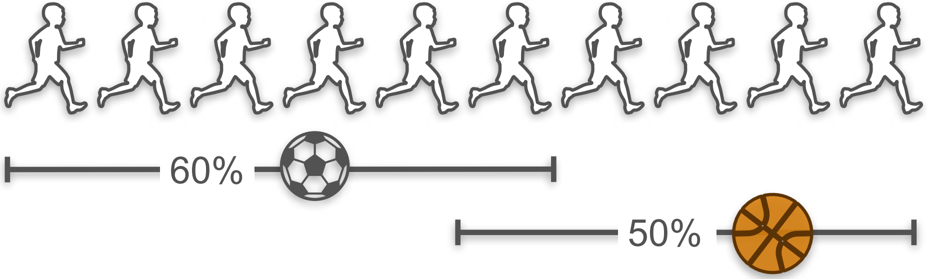


$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Sum of Probabilities (Joint Events): Quiz 3 Solution

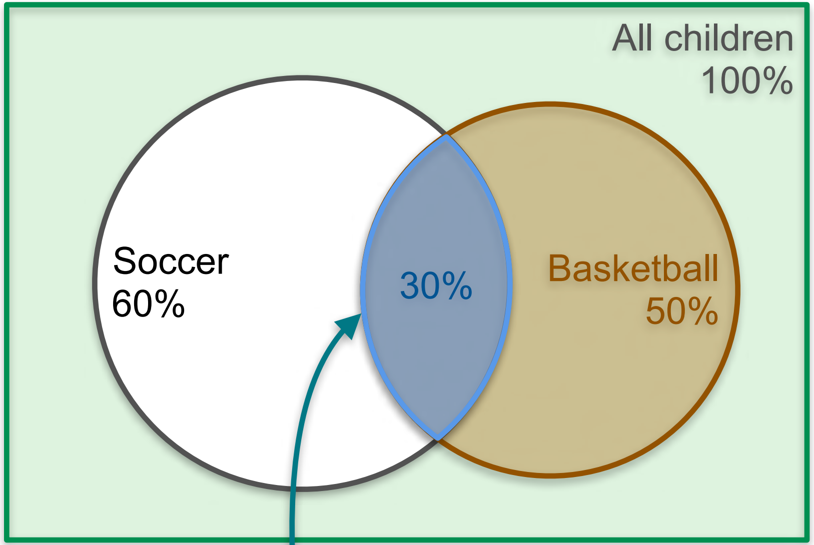


$\mathbf{P}(S \cap B)$

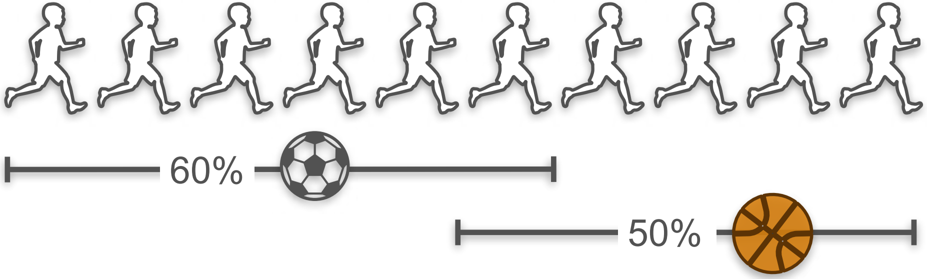


$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Sum of Probabilities (Joint Events): Quiz 3 Solution

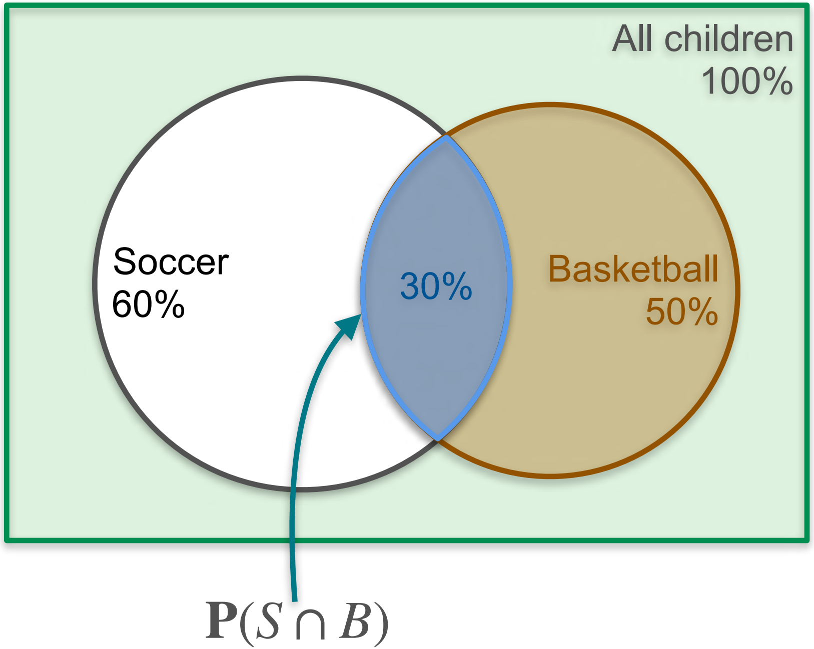


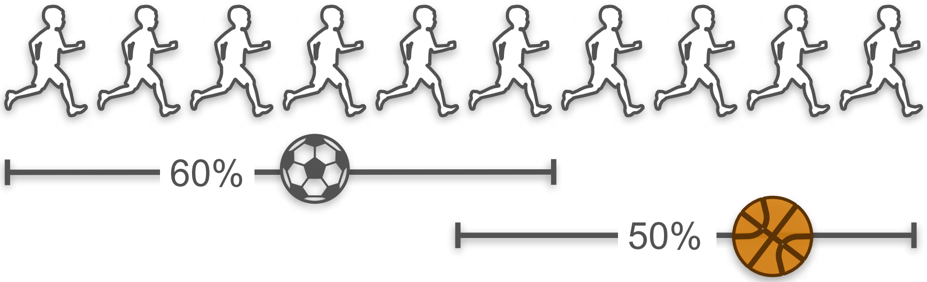
$\mathbf{P}(S \cap B)$



$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B)$$

Sum of Probabilities (Joint Events): Quiz 3 Solution





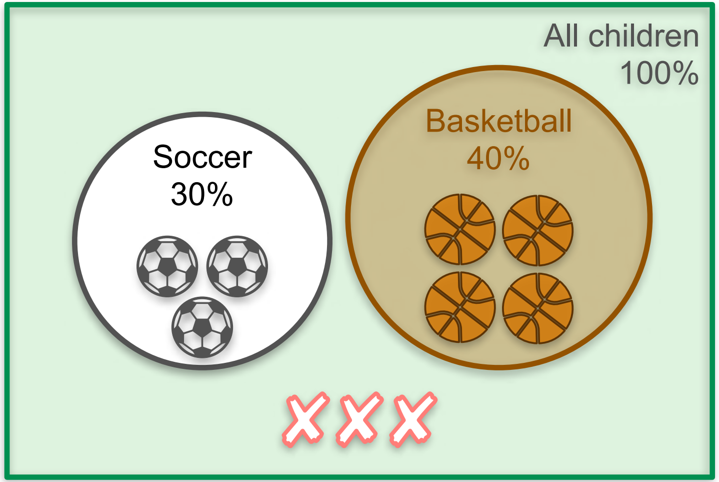
$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B)$$

$$= 0.6 + 0.5 - 0.3$$

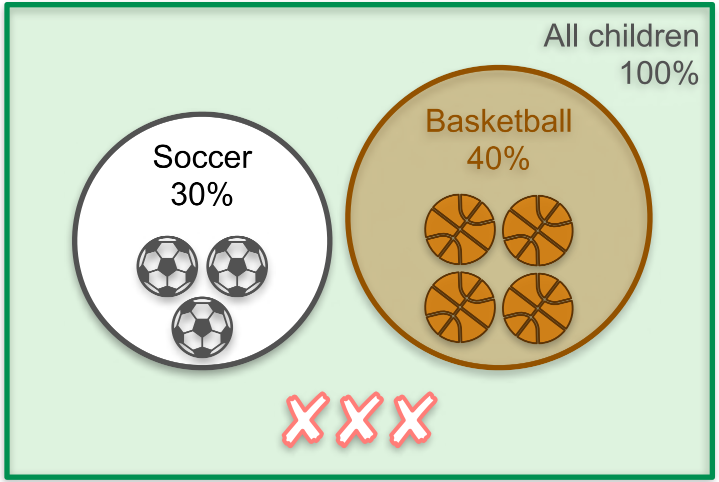
$$= 0.8$$

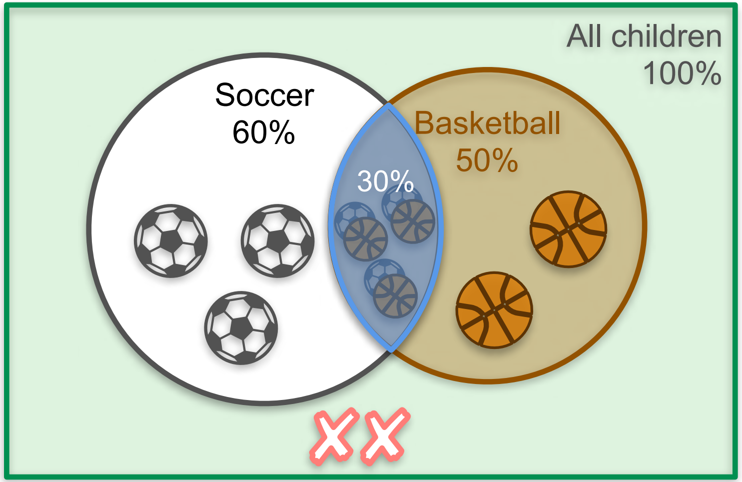
Disjoint Events Vs Joint Events

Disjoint Events Vs Joint Events

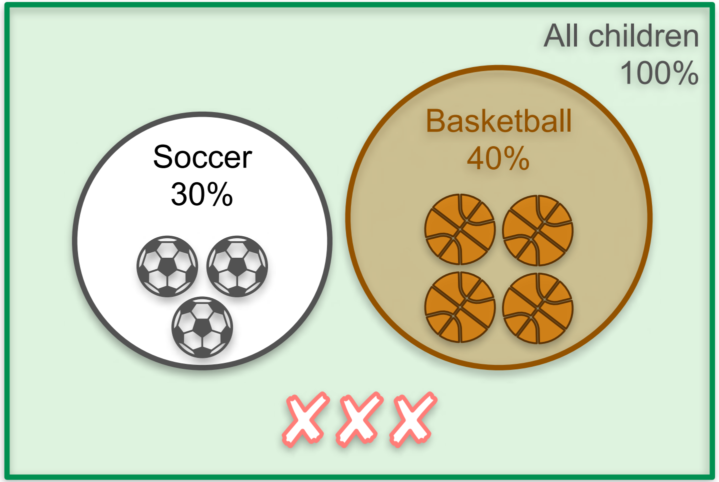


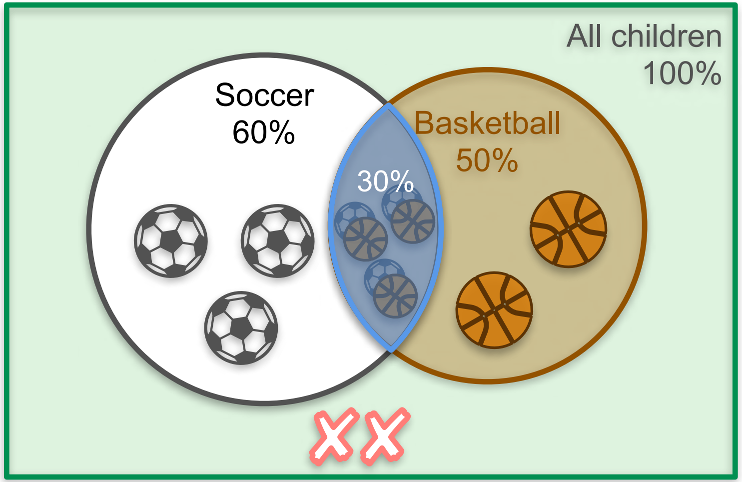
Disjoint Events Vs Joint Events



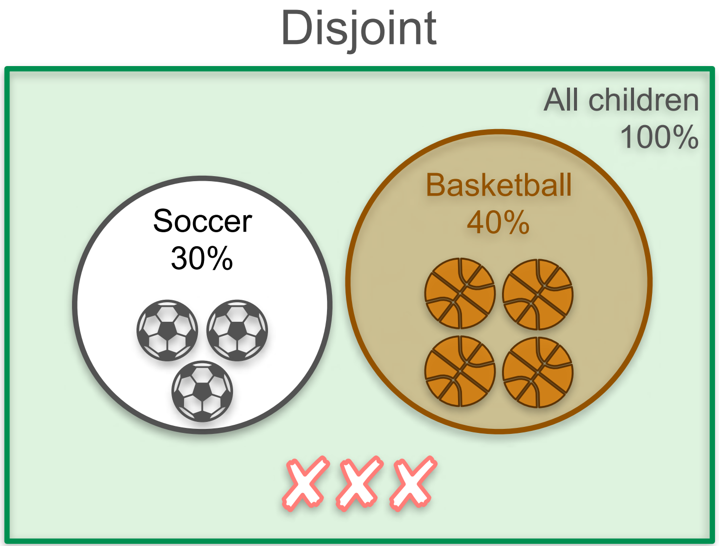


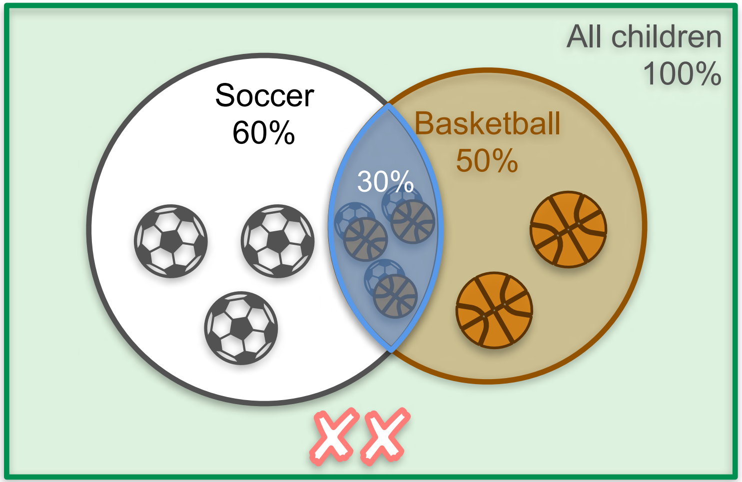
Disjoint Events Vs Joint Events





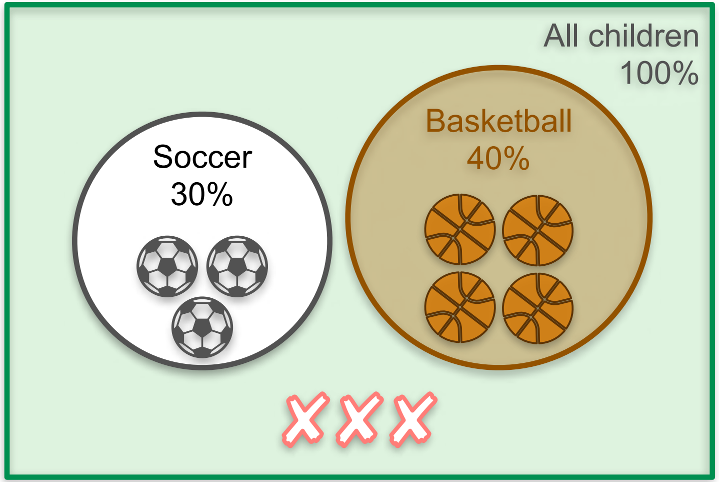
Disjoint Events Vs Joint Events

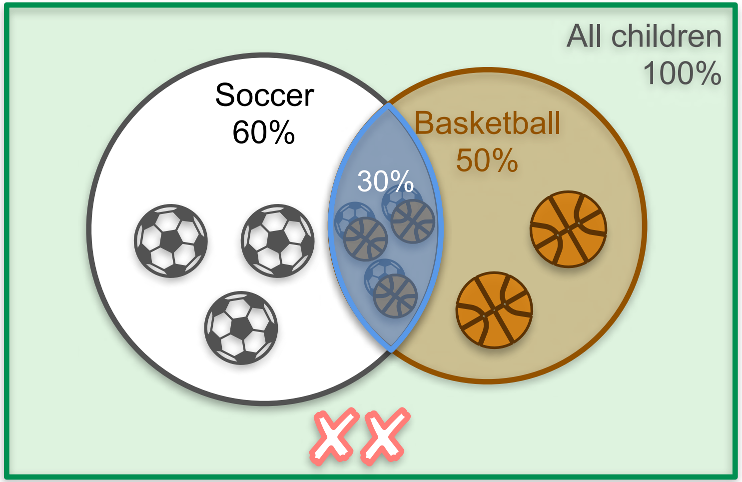




Disjoint Events Vs Joint Events

Disjoint

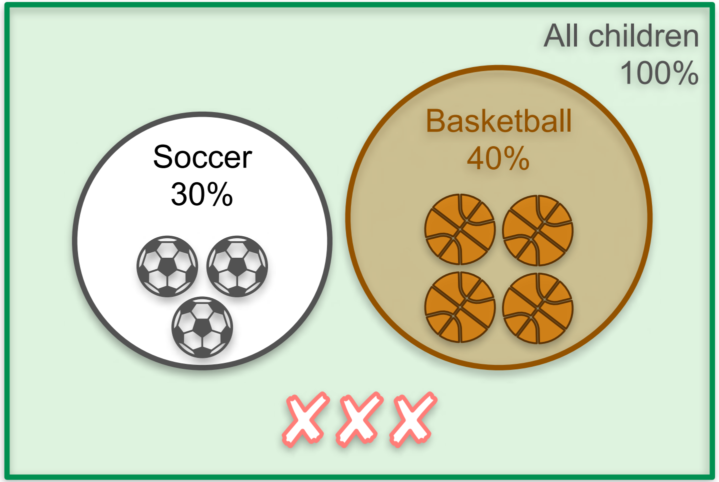


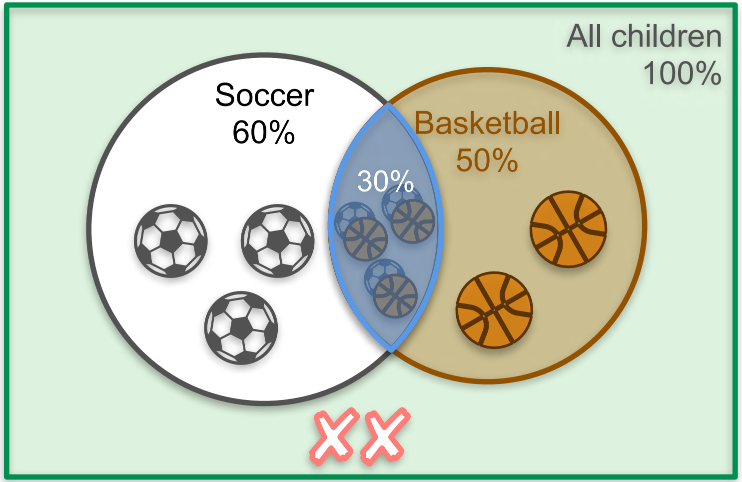


Mutually exclusive

Disjoint Events Vs Joint Events

Disjoint



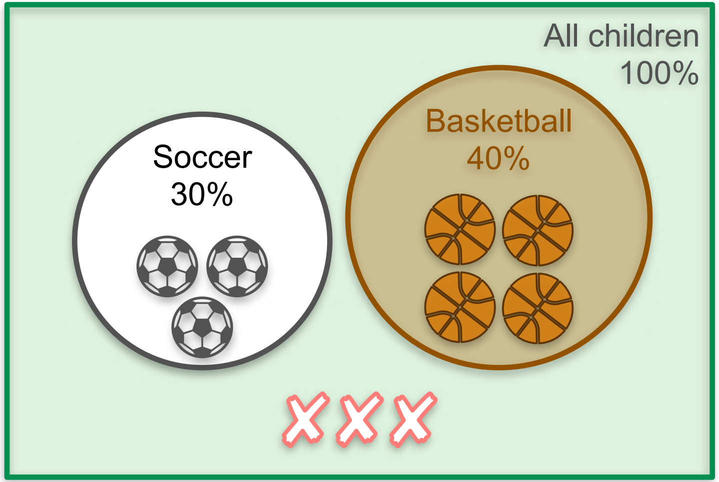


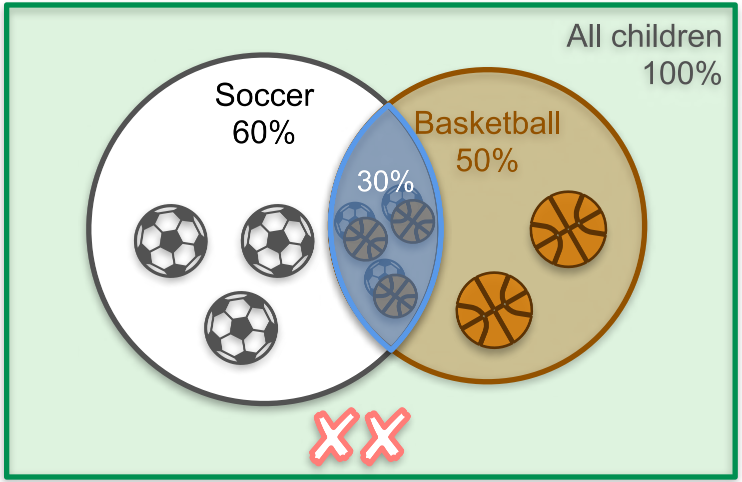
Mutually exclusive

$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Disjoint Events Vs Joint Events

Disjoint



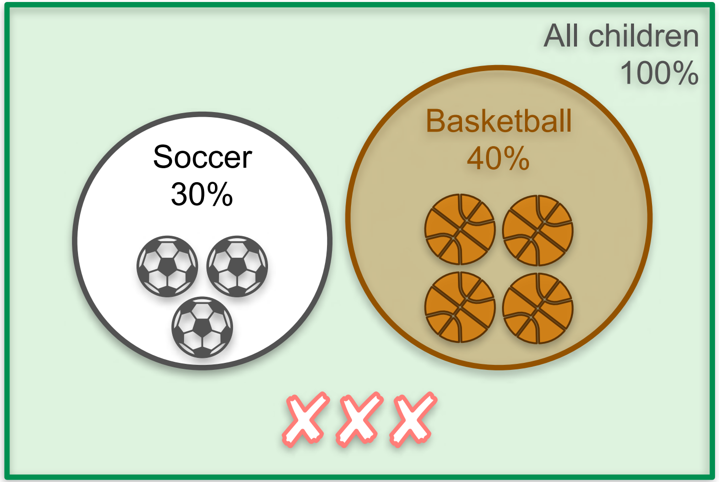


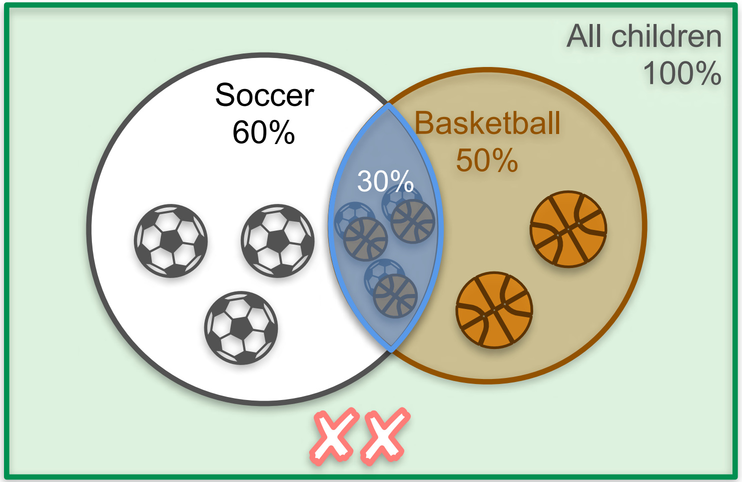
Mutually exclusive

$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Disjoint Events Vs Joint Events

Disjoint



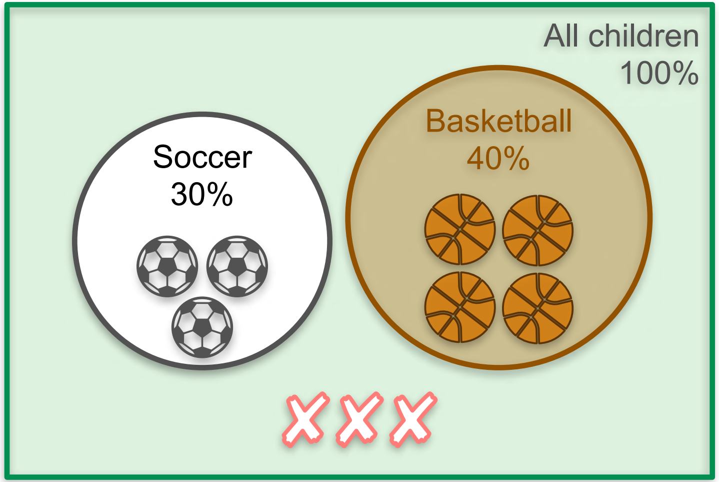


Mutually exclusive

$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Disjoint Events Vs Joint Events

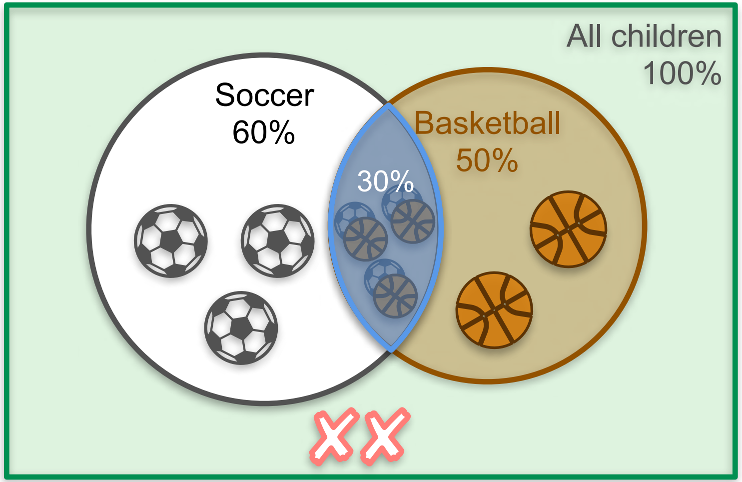
Disjoint



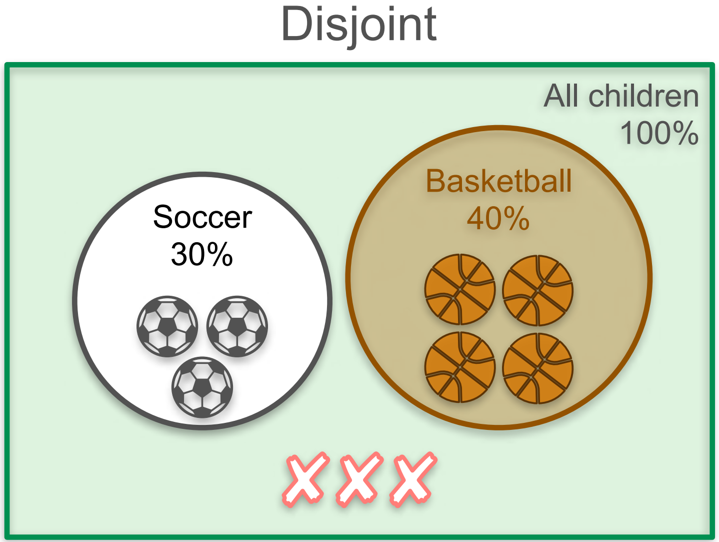
Mutually exclusive

$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Joint

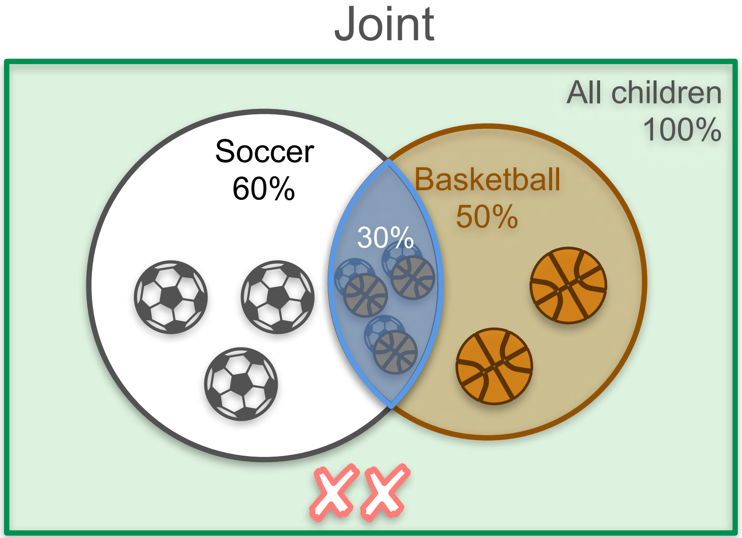


Disjoint Events Vs Joint Events



Mutually exclusive

$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

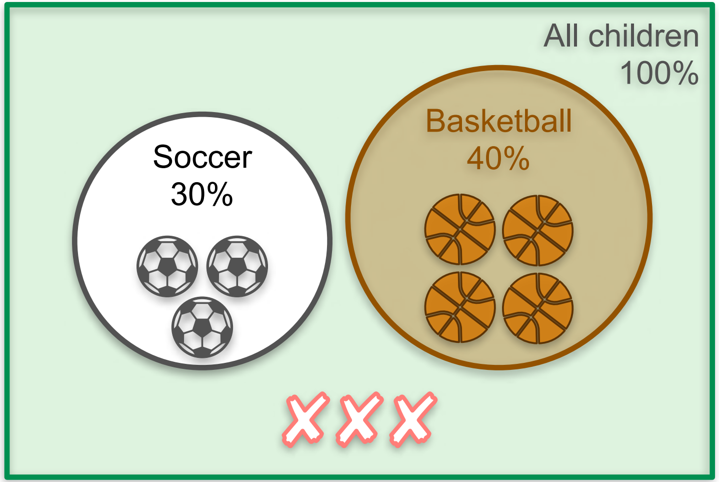


Non-mutually exclusive

DeepLearning.AI

Disjoint Events Vs Joint Events

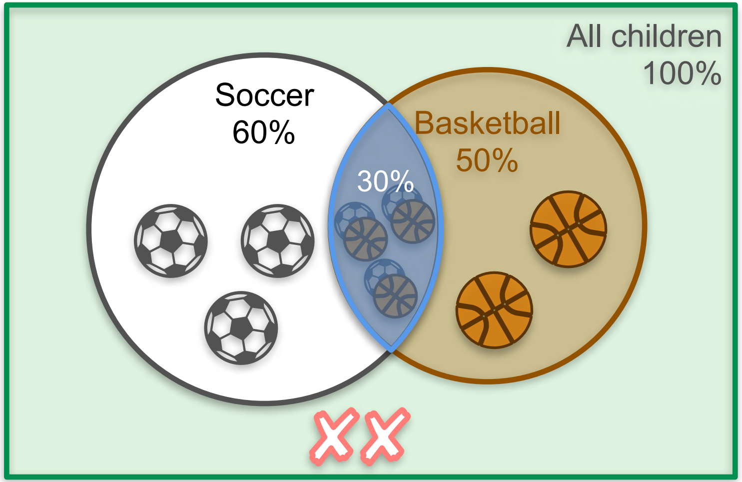
Disjoint



Mutually exclusive

$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B)$$

Joint

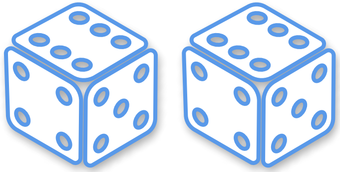


Non-mutually exclusive

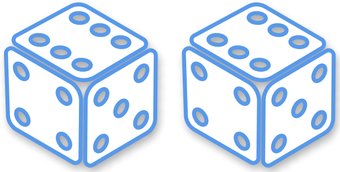
$$\mathbf{P}(S \cup B) = \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B)$$

Sum of Probabilities (Joint Events): Dice Example 1

Sum of Probabilities (Joint Events): Dice Example 1



Sum of Probabilities (Joint Events): Dice Example 1



What is the probability of obtaining a sum of 7 or a difference of 1?

Sum of Probabilities (Joint Events): Dice Example 1

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

sum = 7

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

A

sum = 7

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

A or B

sum = 7 or diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

A or B

sum = 7 or diff = 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

sum = 7 and diff = 1

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

$= \frac{\quad}{\quad}$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

1,6
2,5
3,4
4,3
5,2
6,1

$= \overline{}$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

1,6
2,5
3,4
4,3
5,2
6,1

=

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

1,6
2,5
3,4
4,3
5,2
6,1

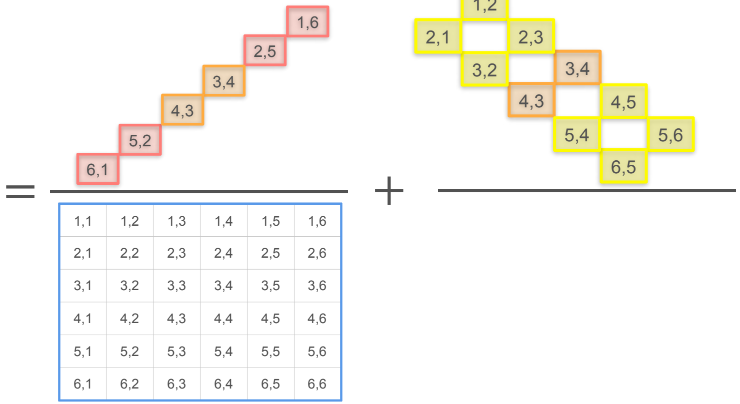
= ———— + ————

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

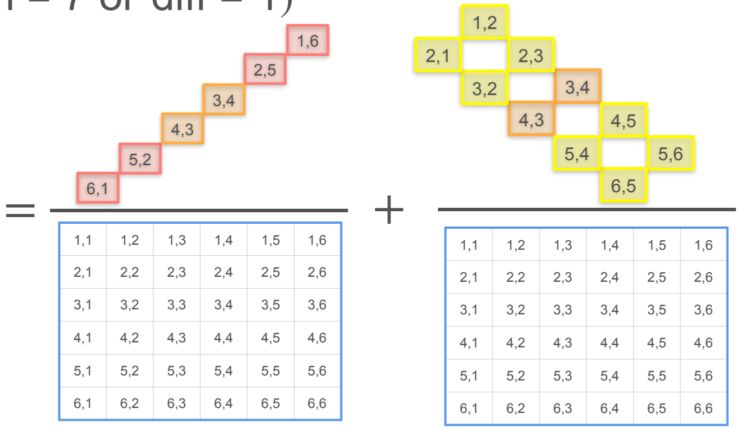
$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$



Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}$(sum = 7 or diff = 1)



Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

= (1,6 2,5 3,4 4,3 5,2 6,1) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

\+ (1,2 2,1 2,3 3,2 3,4 4,3 4,5 5,4 5,6 6,5) /

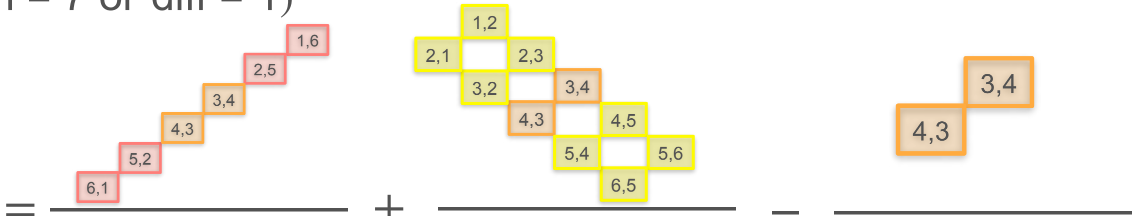
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

− ______

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

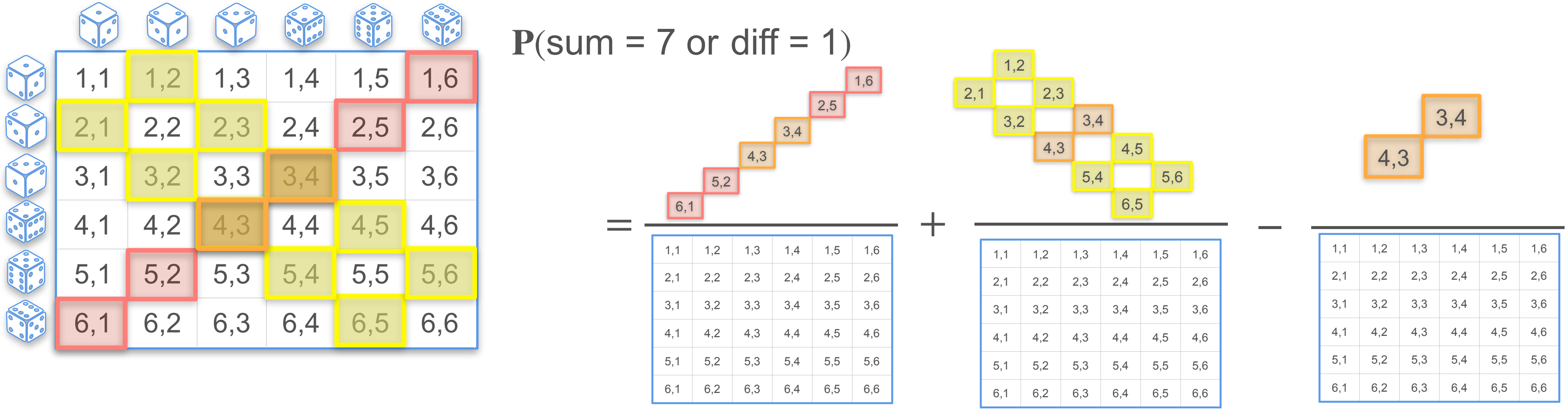
P(sum = 7 or diff = 1)



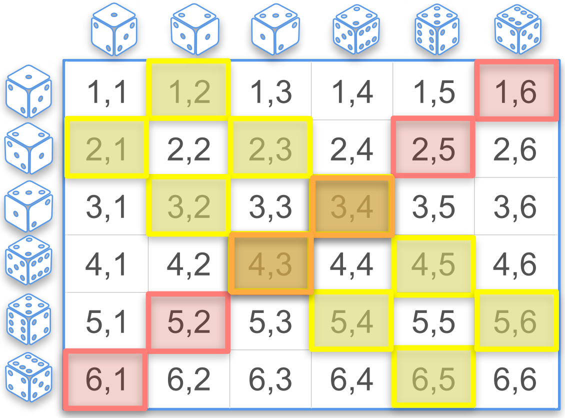
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

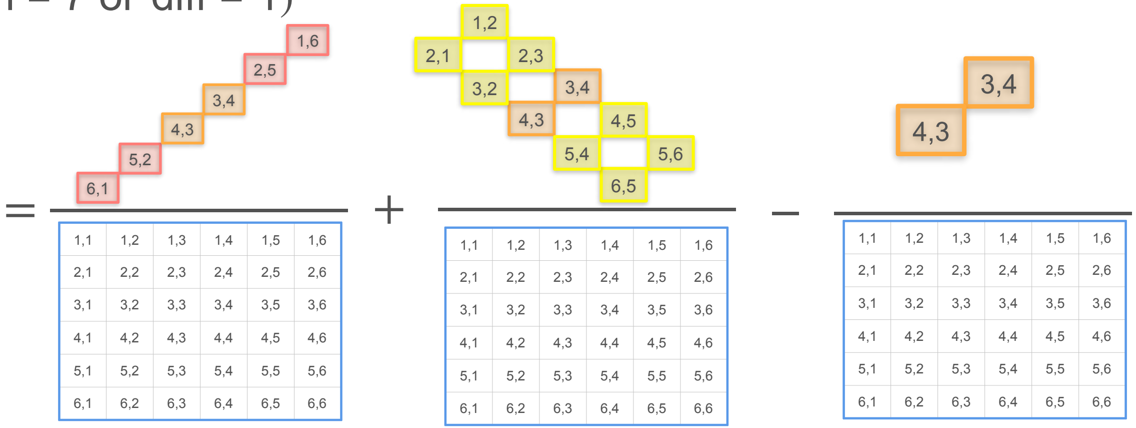


Sum of Probabilities (Joint Events): Dice Example 1



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

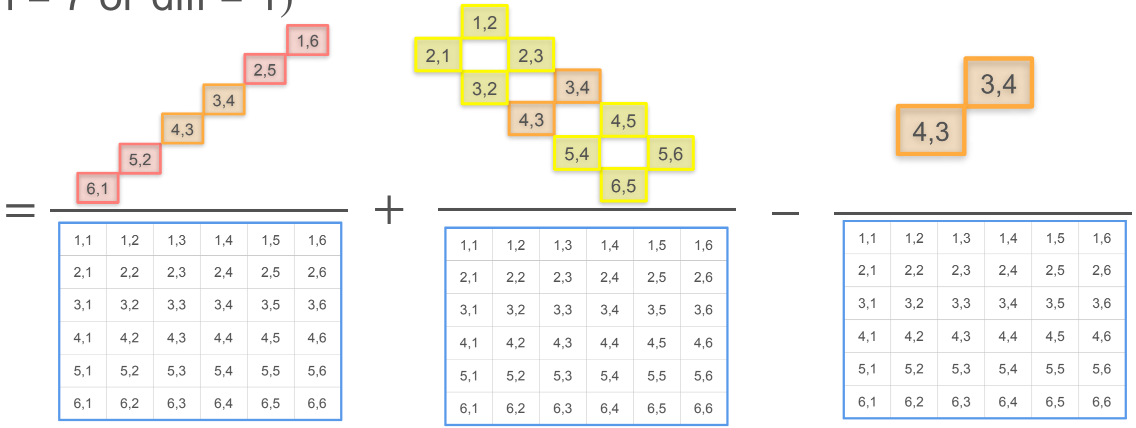


$= \dfrac{6}{36}$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

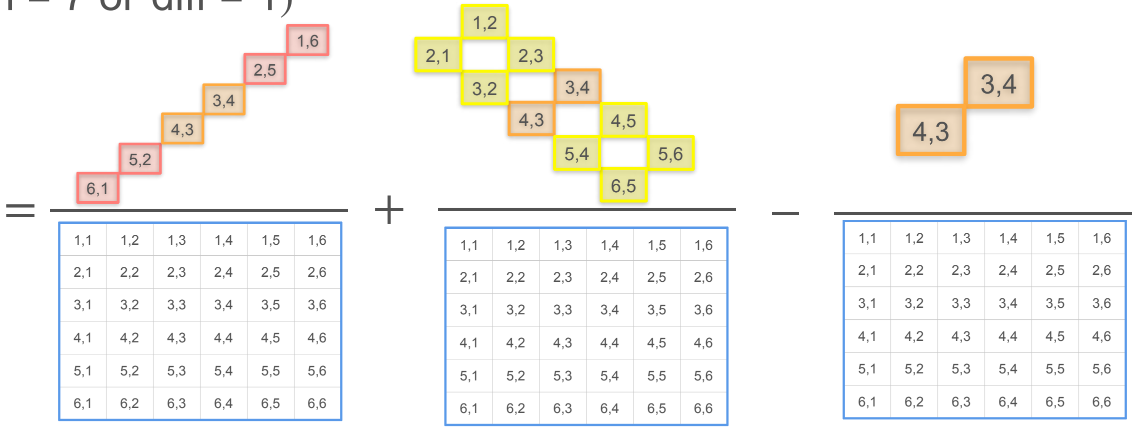


$$= \frac{6}{36} + \frac{10}{36}$$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$



$$= \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

P(sum = 7 or diff = 1)

= (6,1 5,2 4,3 3,4 2,5 1,6) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

\+ (1,2 2,1 2,3 3,2 3,4 4,3 4,5 5,4 5,6 6,5) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

− (3,4 4,3) /

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$$

$$= \frac{14}{36}$$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

$$= \boxed{\mathbf{P}(\text{sum} = 7)} + \frac{\{1,2;\ 2,1;\ 2,3;\ 3,2;\ 3,4;\ 4,3;\ 4,5;\ 5,4;\ 5,6;\ 6,5\}}{36 \text{ outcomes}} - \frac{\{3,4;\ 4,3\}}{36 \text{ outcomes}}$$

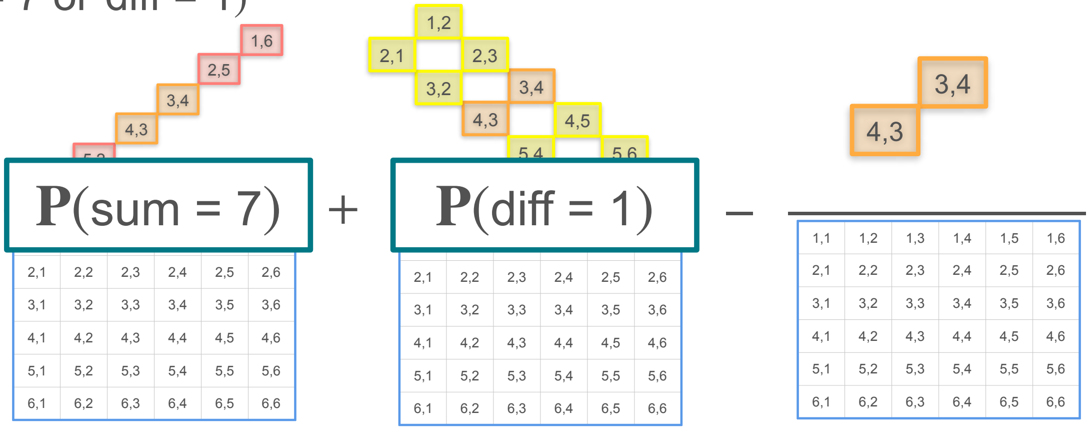
$$= \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$$

$$= \frac{14}{36}$$

Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$



$$= \boxed{\mathbf{P}(\text{sum} = 7)} + \boxed{\mathbf{P}(\text{diff} = 1)} -$$

$$= \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$$

$$= \frac{14}{36}$$

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Sum of Probabilities (Joint Events): Dice Example 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{sum} = 7 \text{ or diff} = 1)$

1,6 2,5 3,4 4,3

1,2 2,1 2,3 3,2 3,4 4,3 4,5 5,4 5,6

3,4 4,3

$= \boxed{\mathbf{P}(\text{sum} = 7)} + \boxed{\mathbf{P}(\text{diff} = 1)} - \boxed{\mathbf{P}(\text{sum} = 7 \cap \text{diff} = 1)}$

2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$$

$$= \frac{14}{36}$$

DeepLearning.AI

Introduction to probability

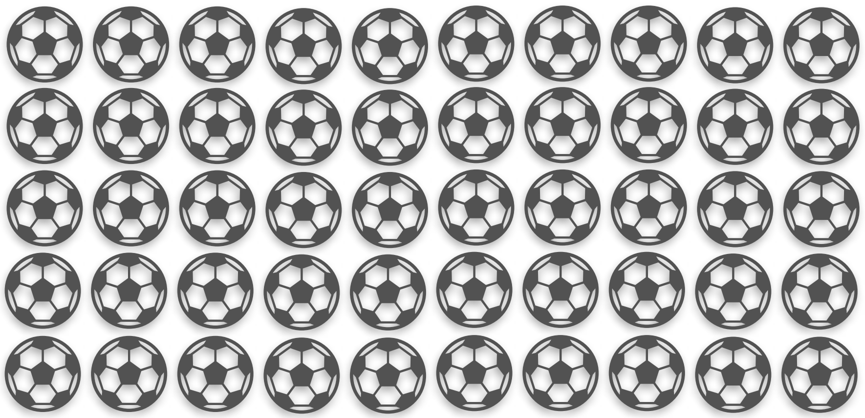
Independence

Independence: Quiz 1

Independence: Quiz 1

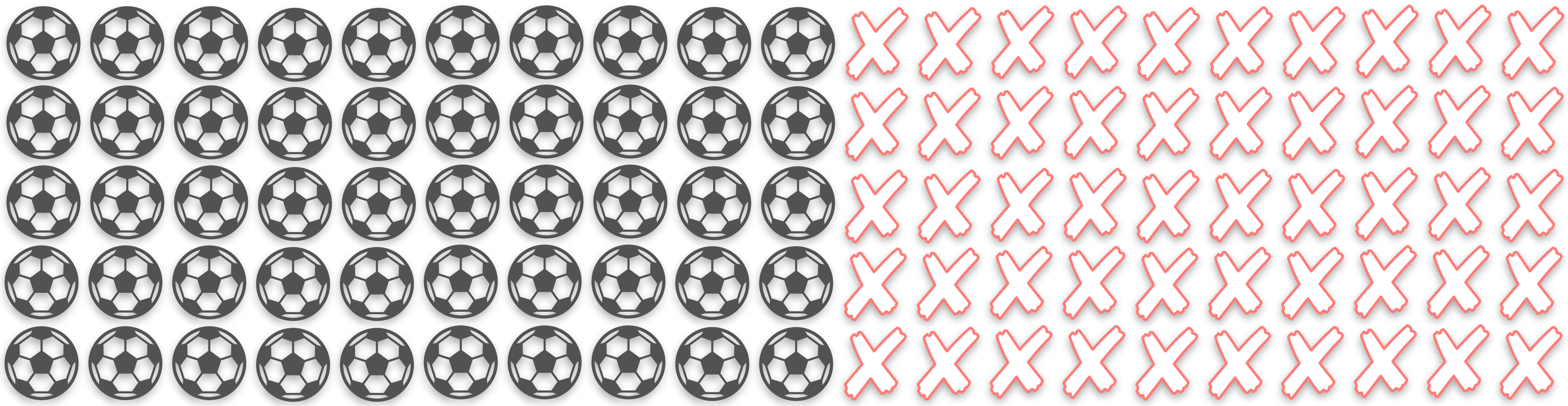
100 kids

Independence: Quiz 1



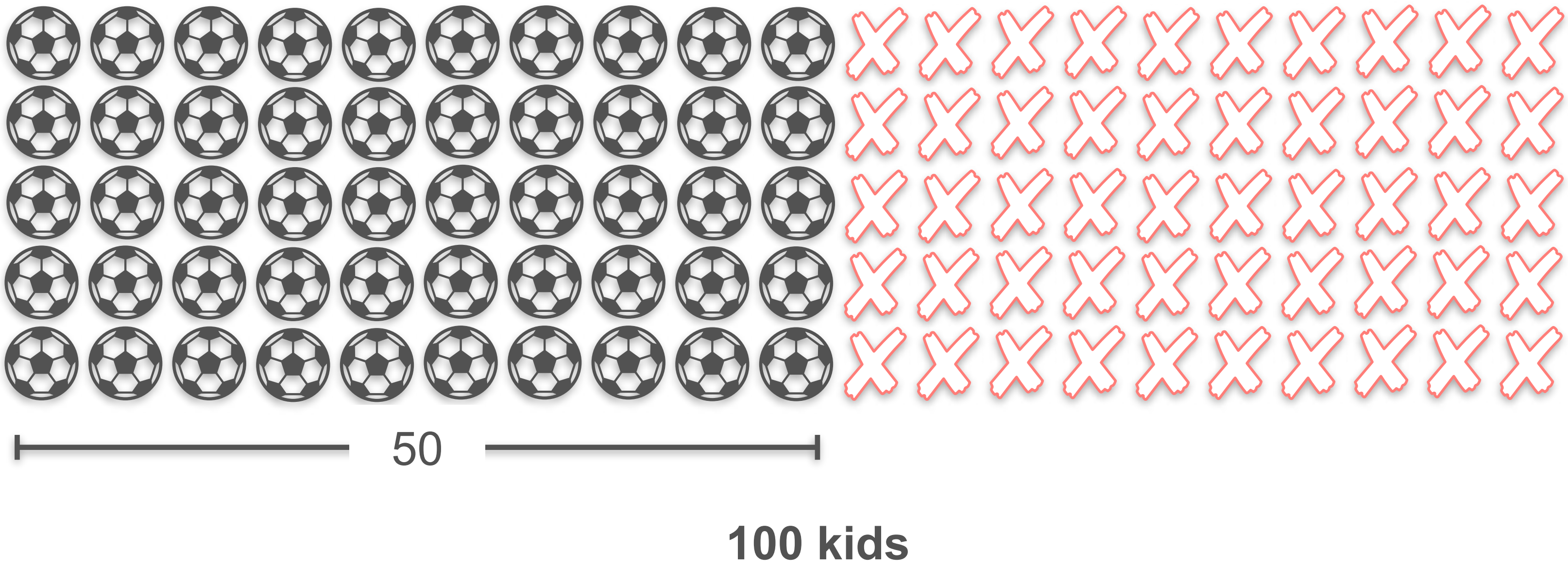
100 kids

Independence: Quiz 1

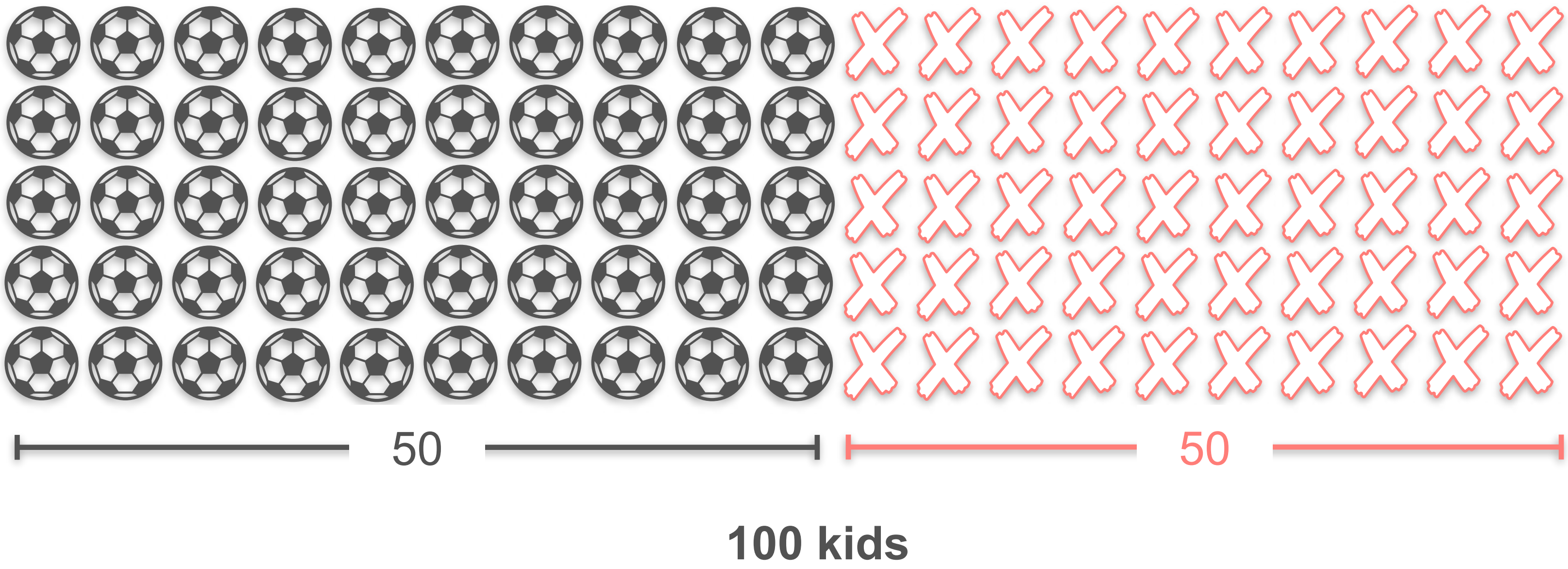


100 kids

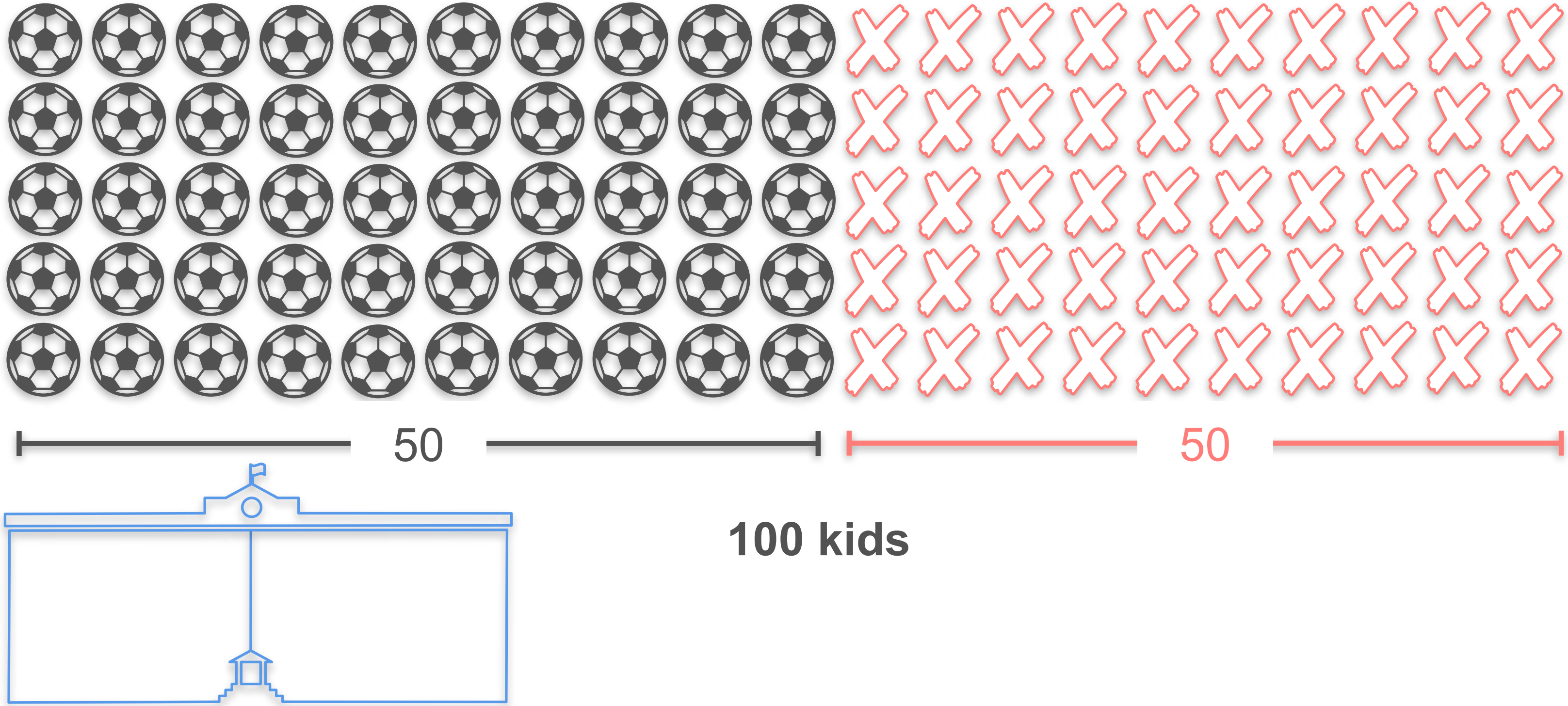
Independence: Quiz 1



Independence: Quiz 1

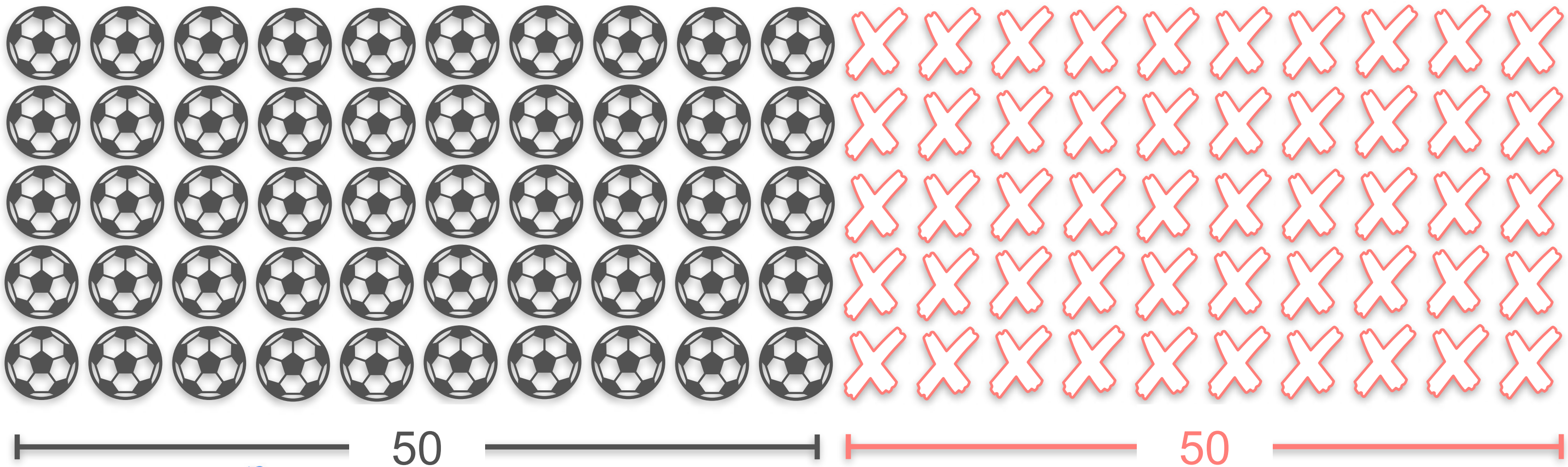


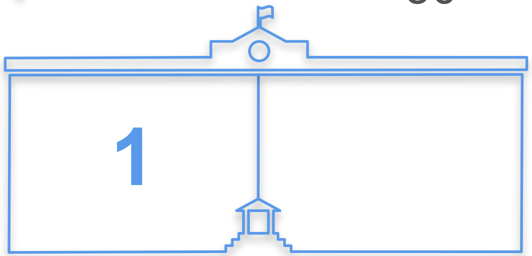
Independence: Quiz 1



100 kids

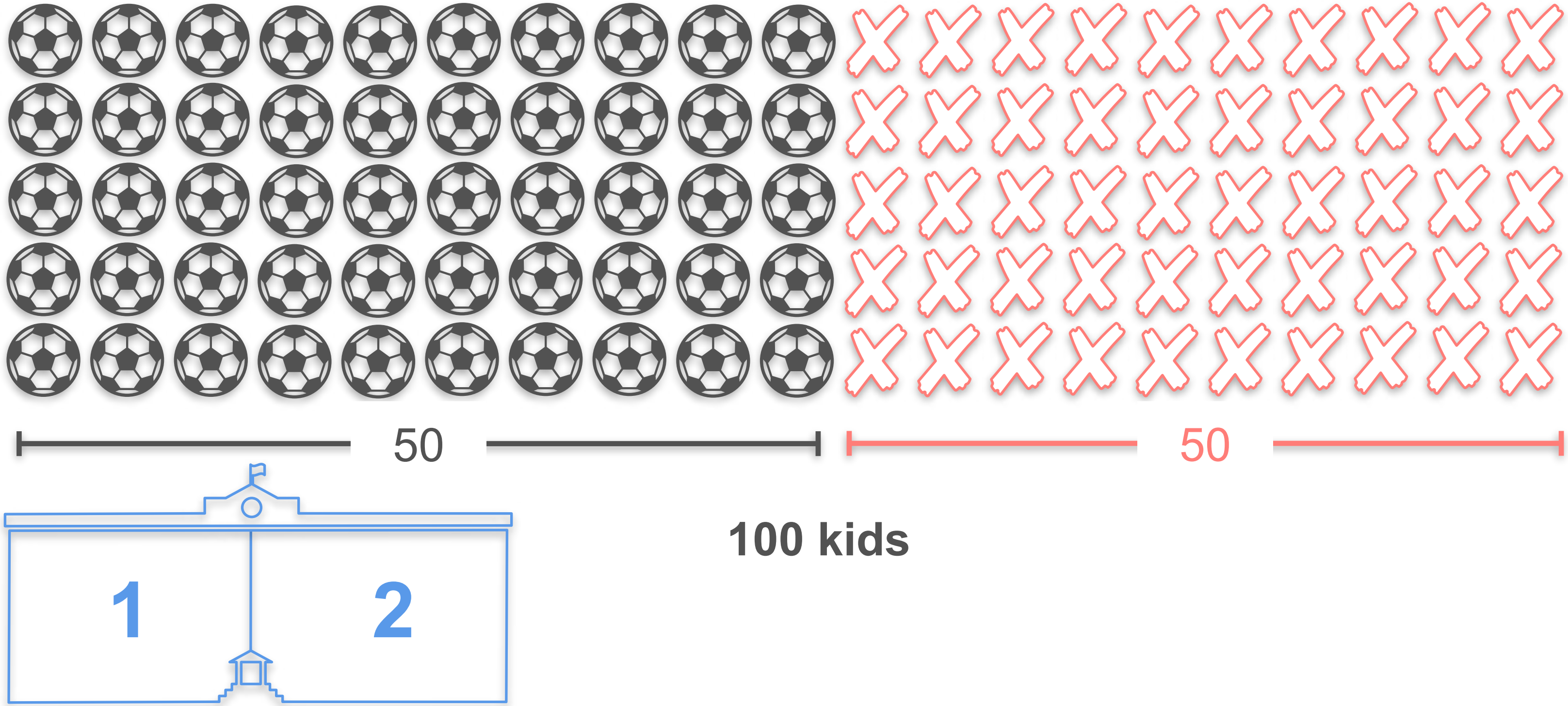
Independence: Quiz 1



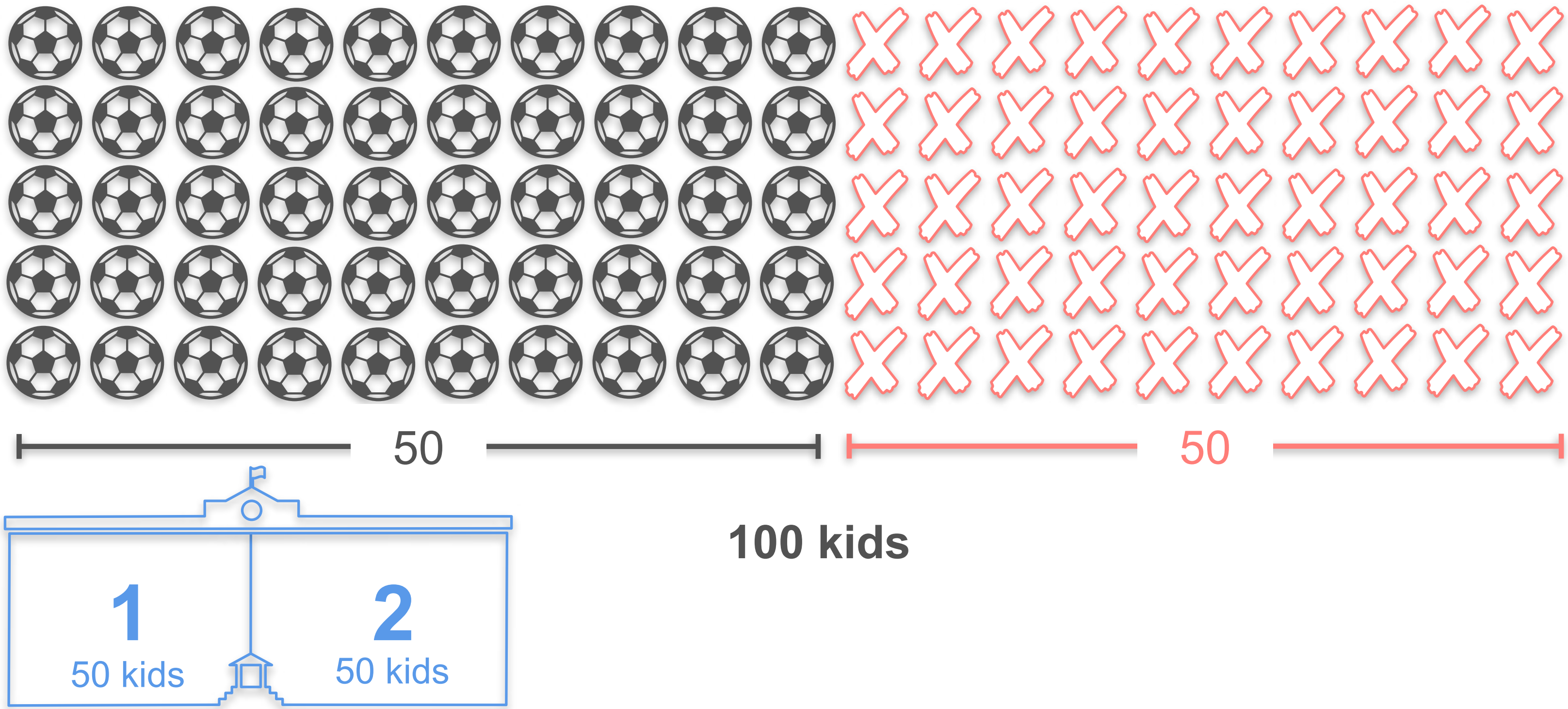


100 kids

Independence: Quiz 1

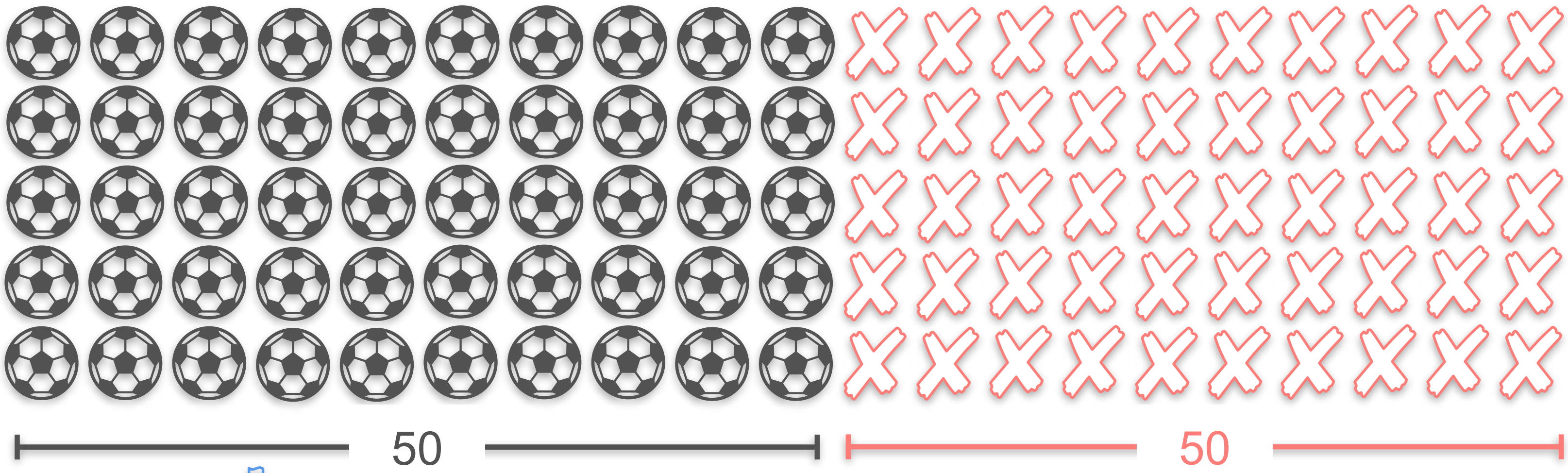


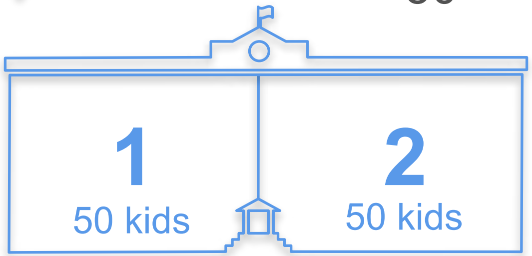
Independence: Quiz 1



100 kids

Independence: Quiz 1

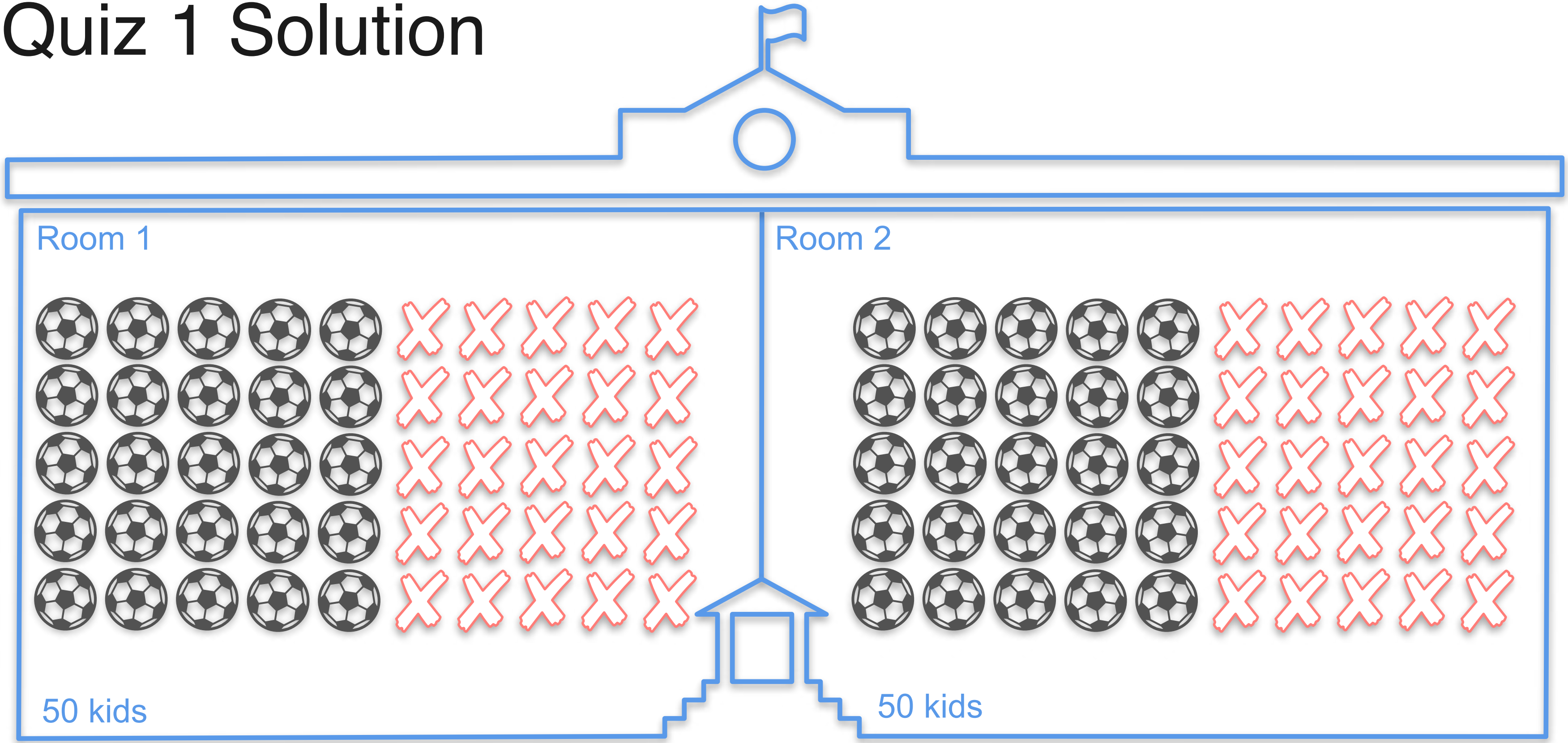




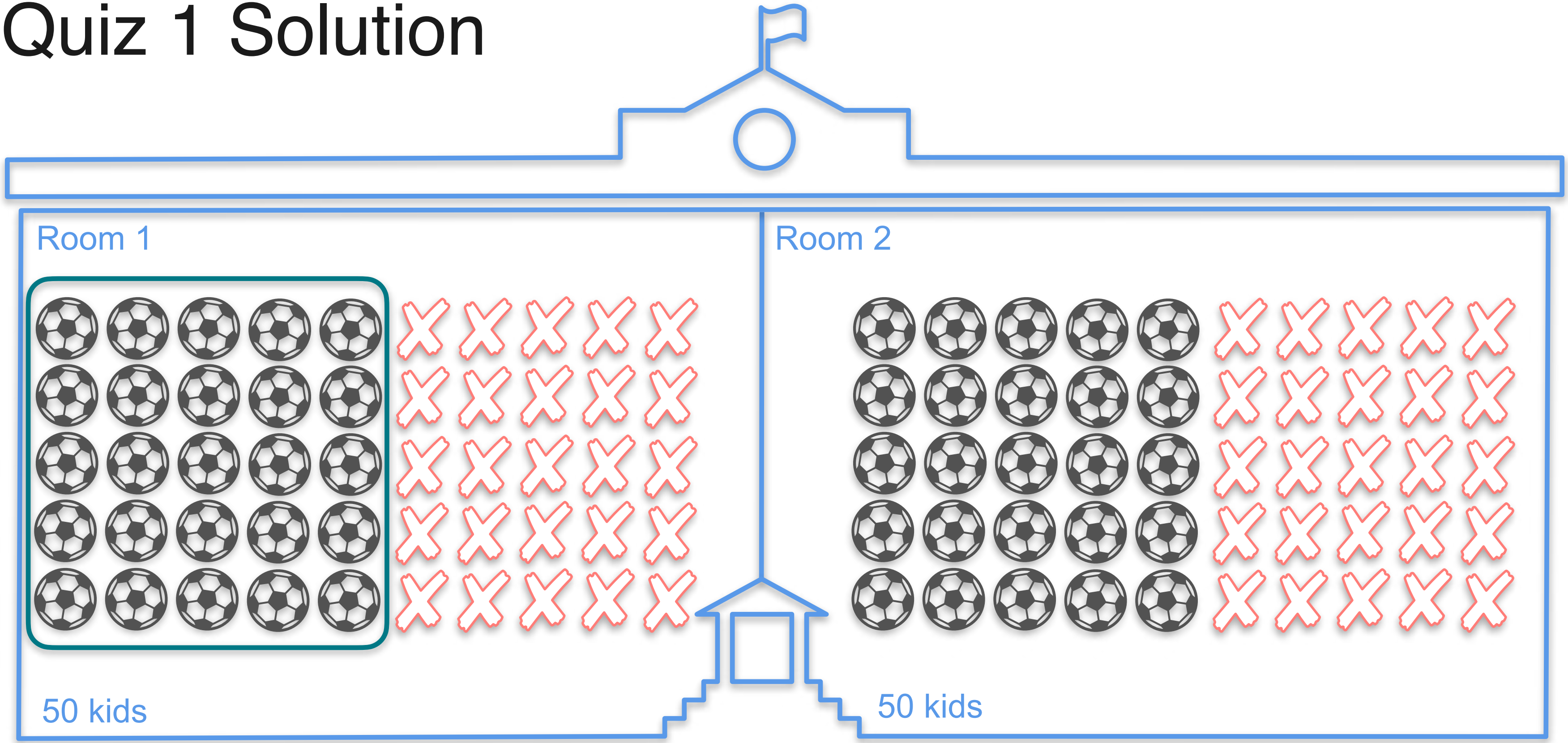
100 kids

How many kids in room 1 like soccer?

Quiz 1 Solution

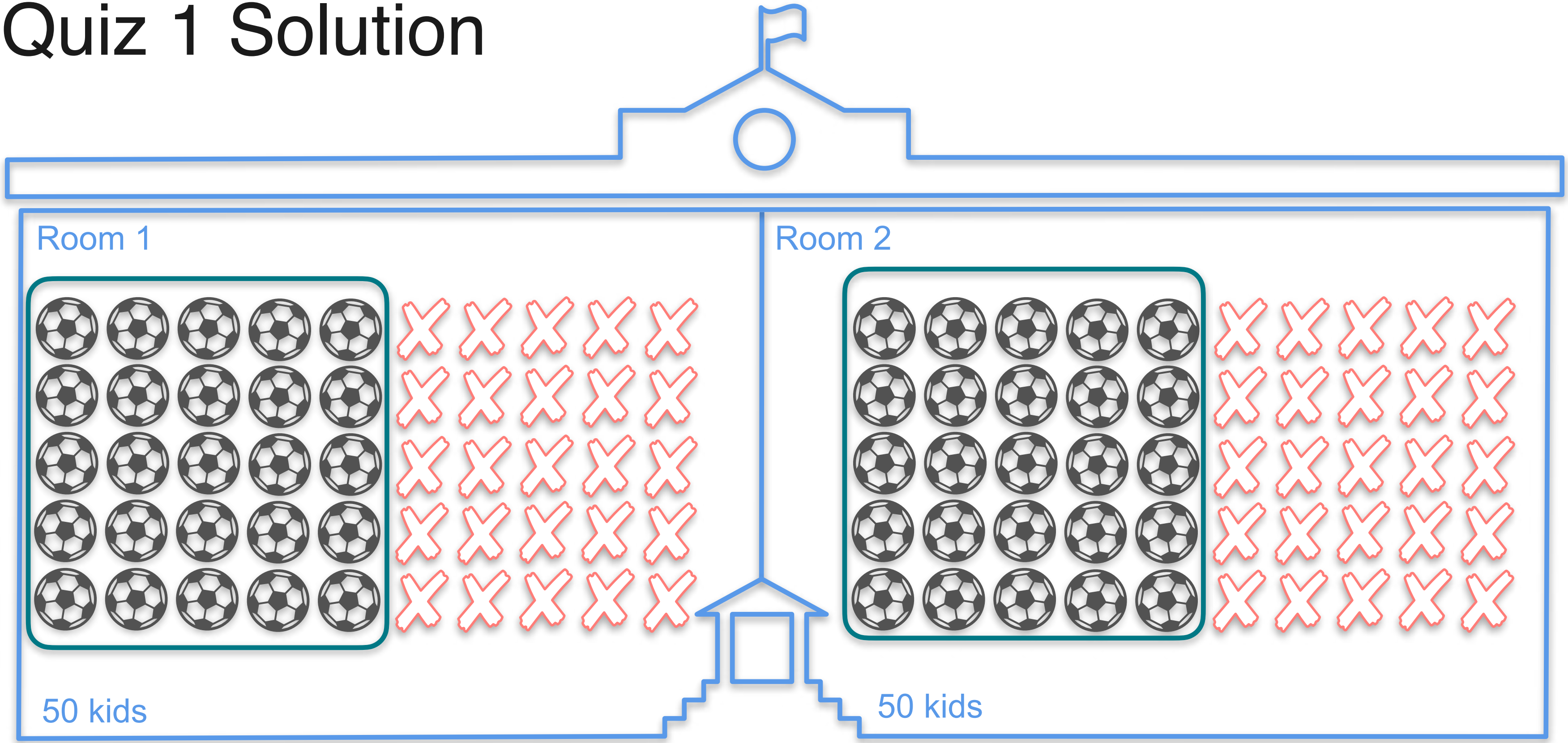


Quiz 1 Solution

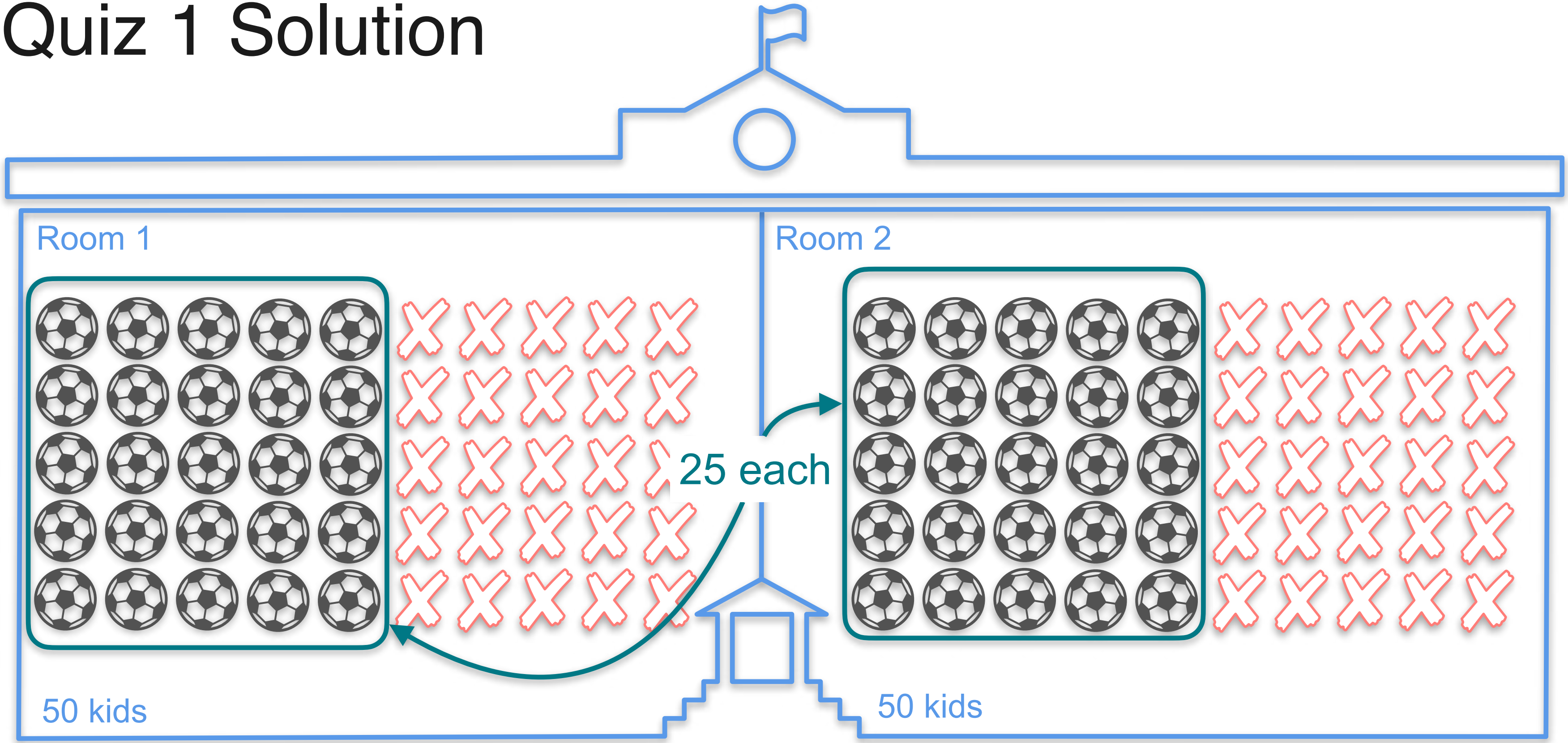


DeepLearning.AI

Quiz 1 Solution

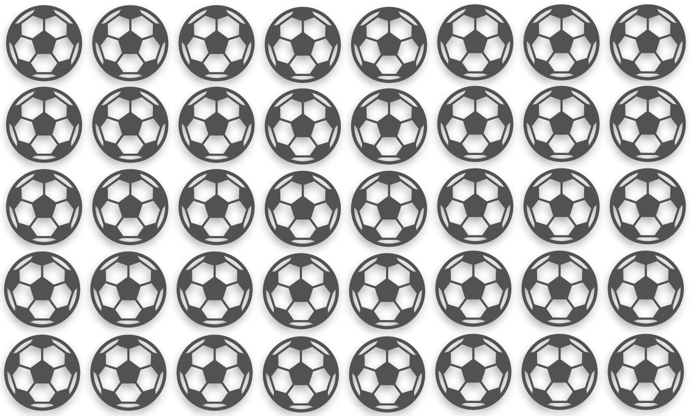


Quiz 1 Solution

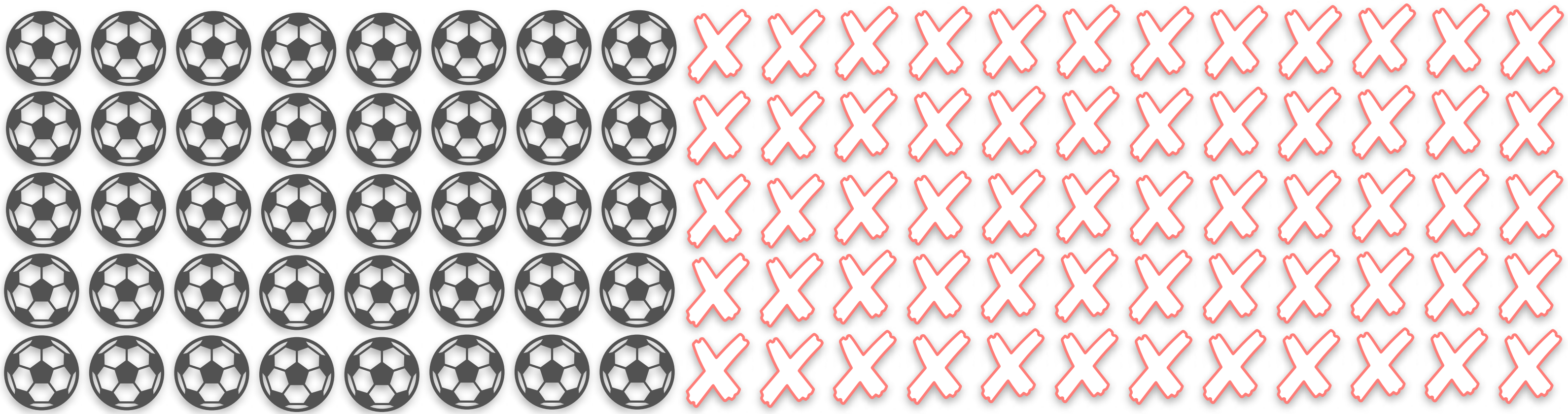


Independence: Quiz 2

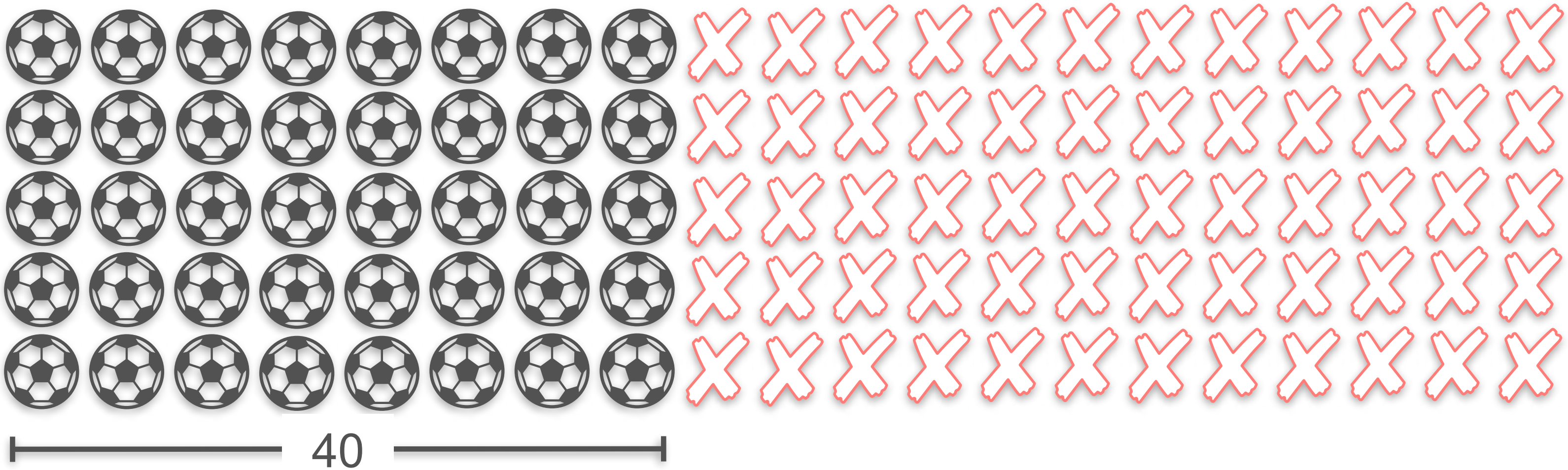
Independence: Quiz 2



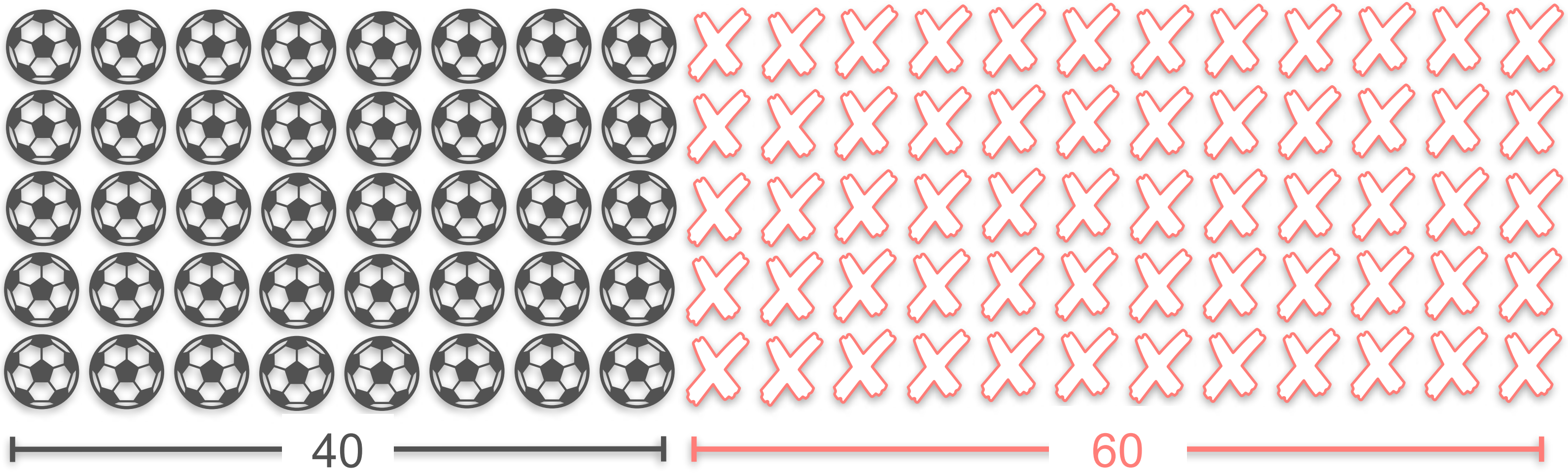
Independence: Quiz 2



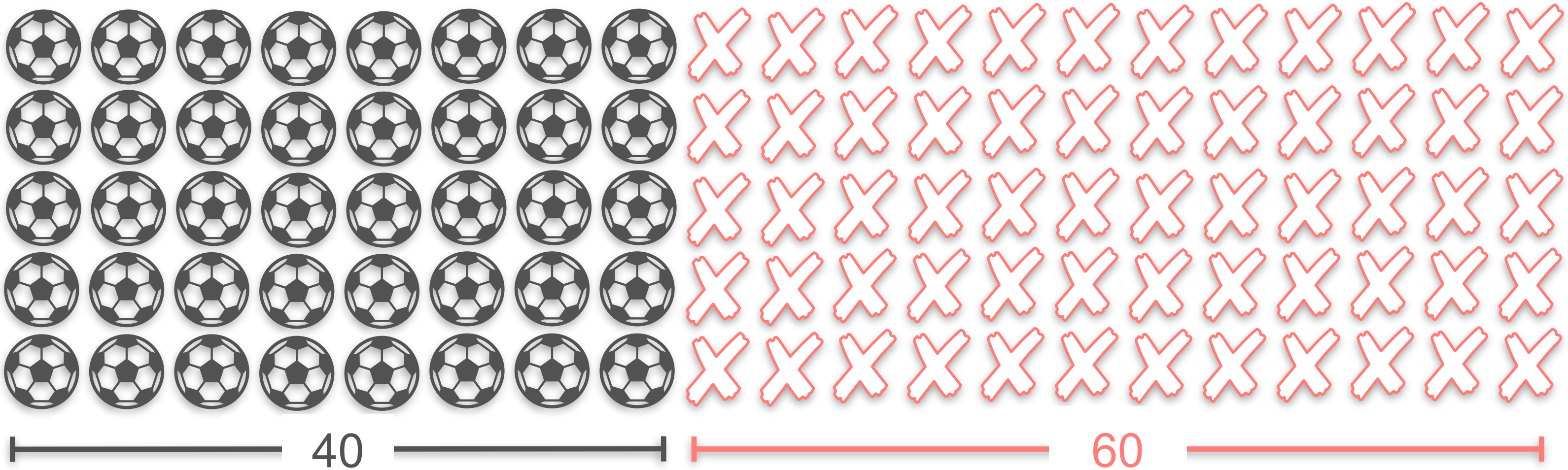
Independence: Quiz 2



Independence: Quiz 2

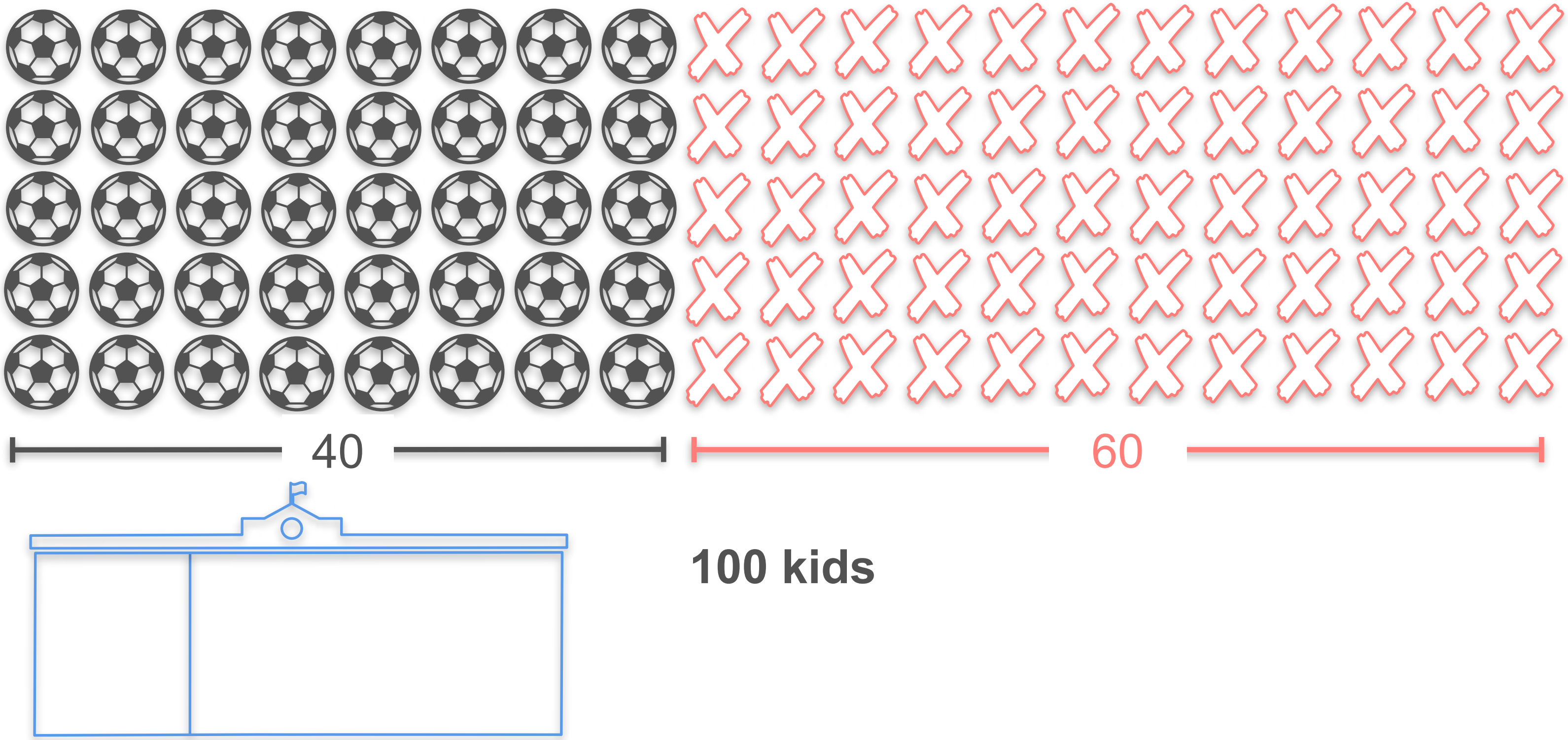


Independence: Quiz 2

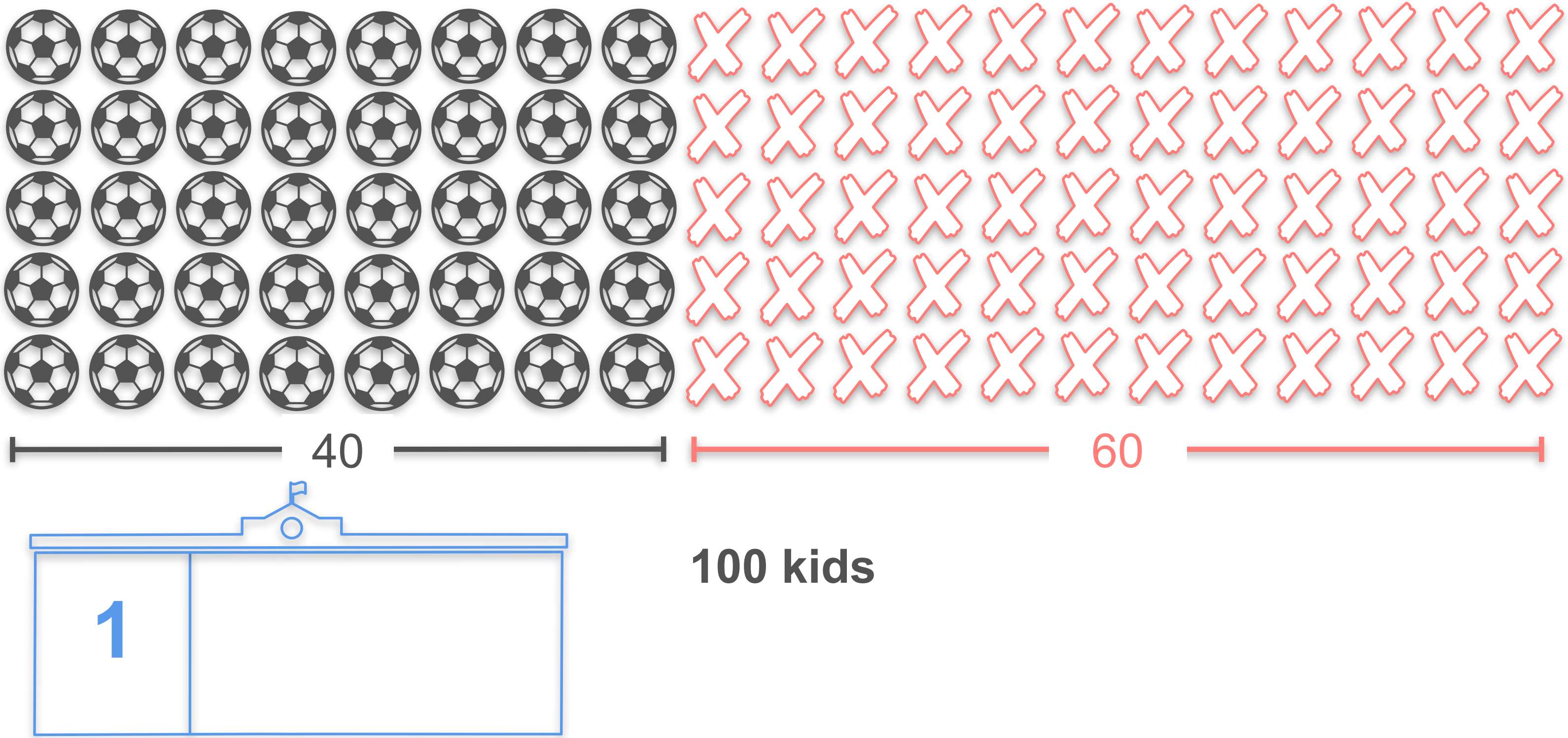


100 kids

Independence: Quiz 2

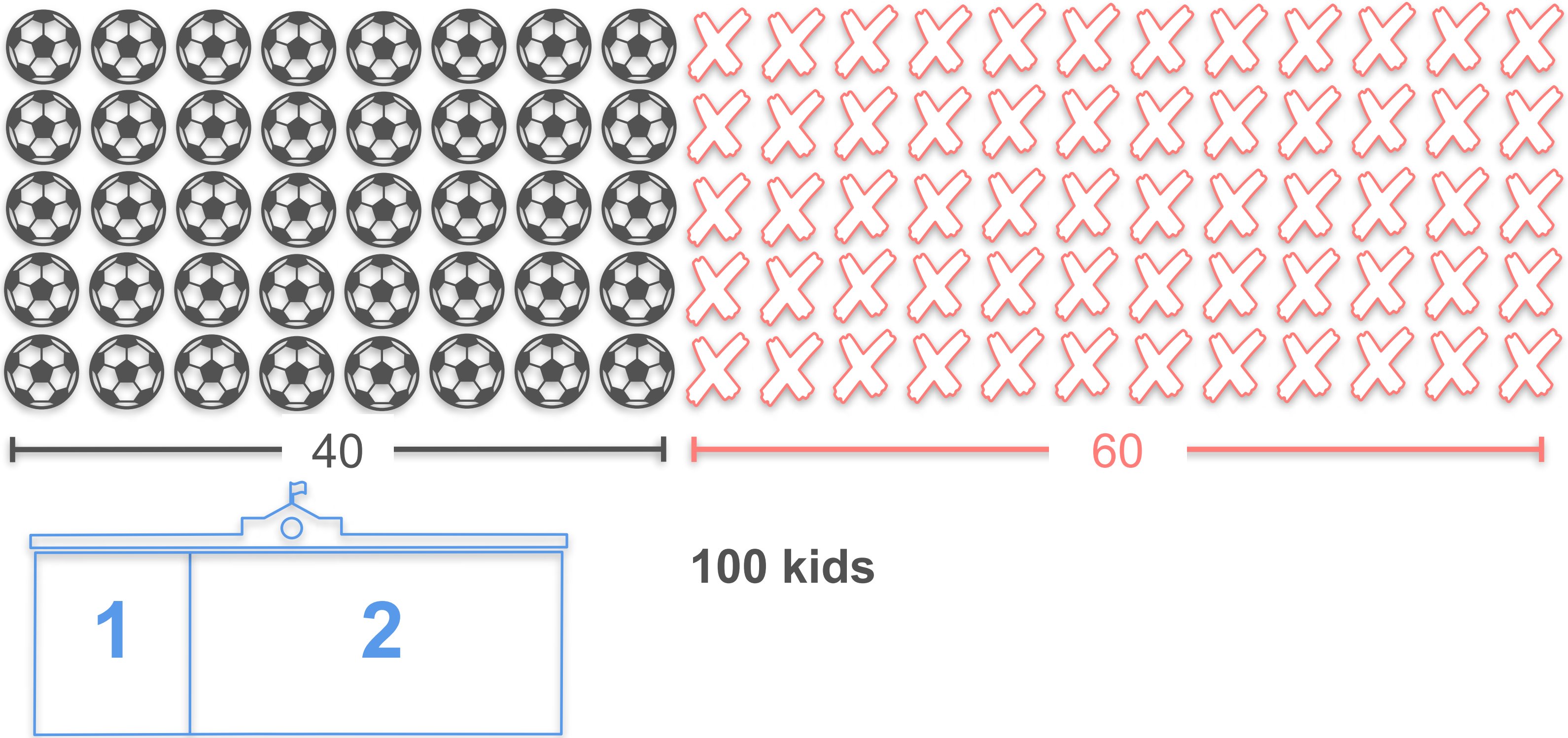


Independence: Quiz 2



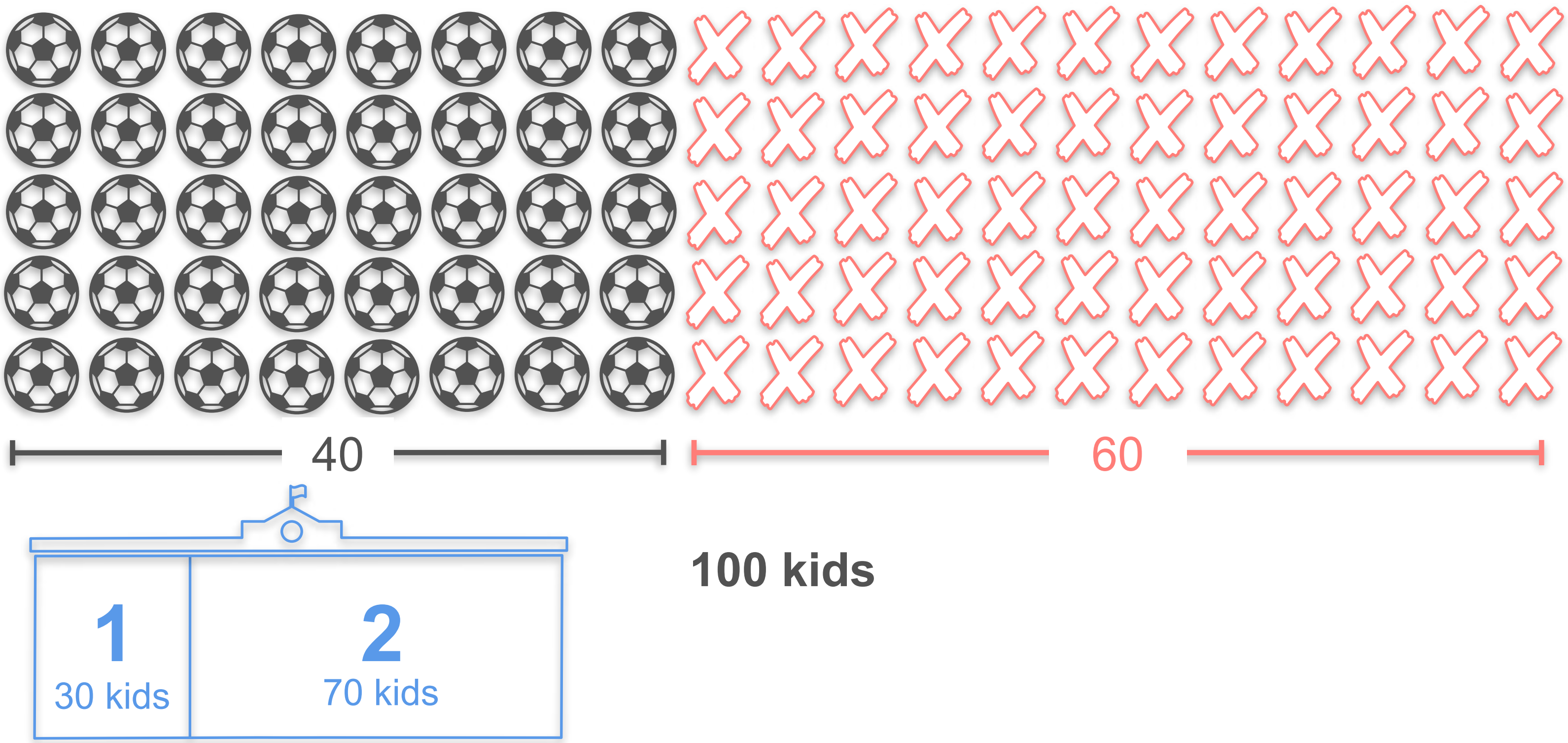
100 kids

Independence: Quiz 2

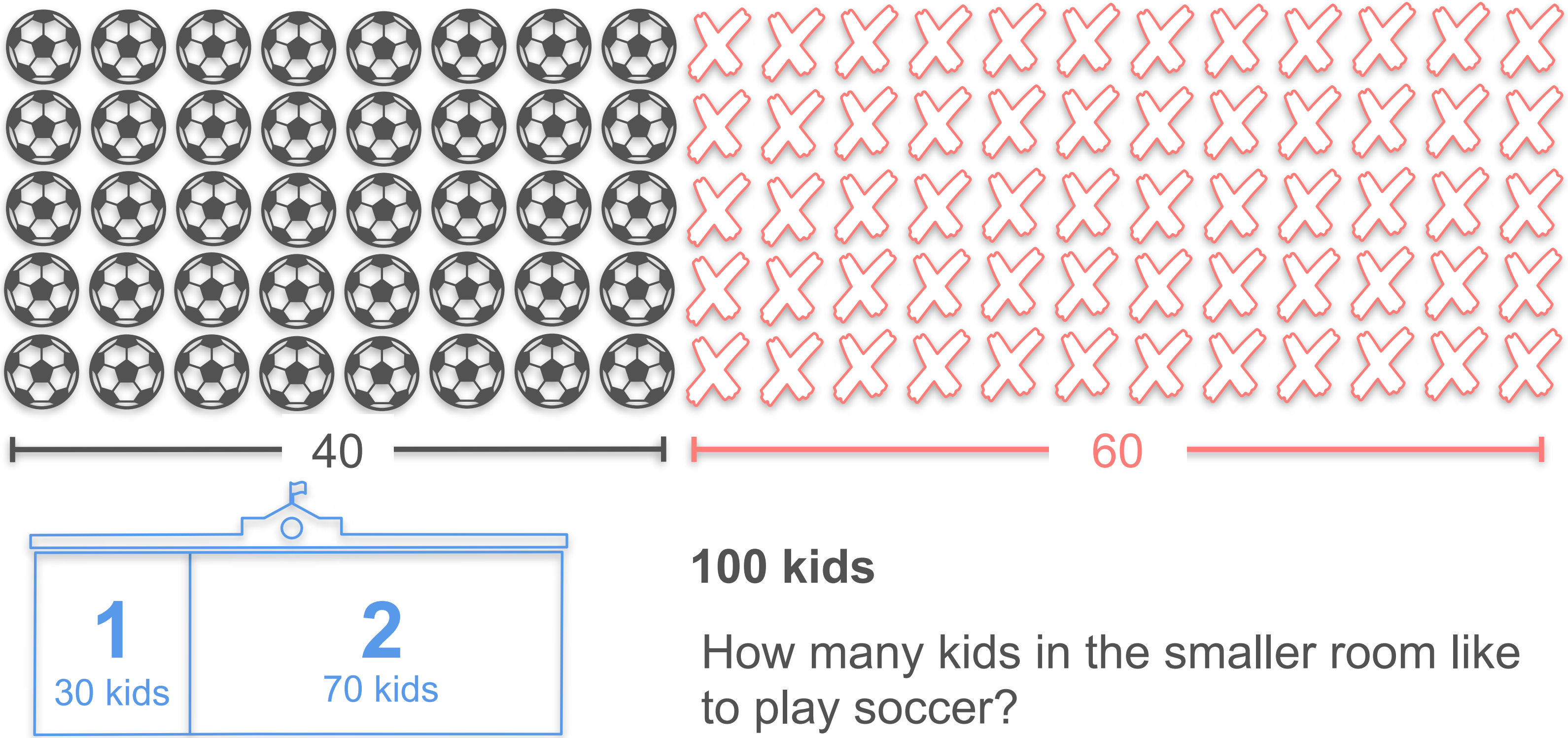


100 kids

Independence: Quiz 2



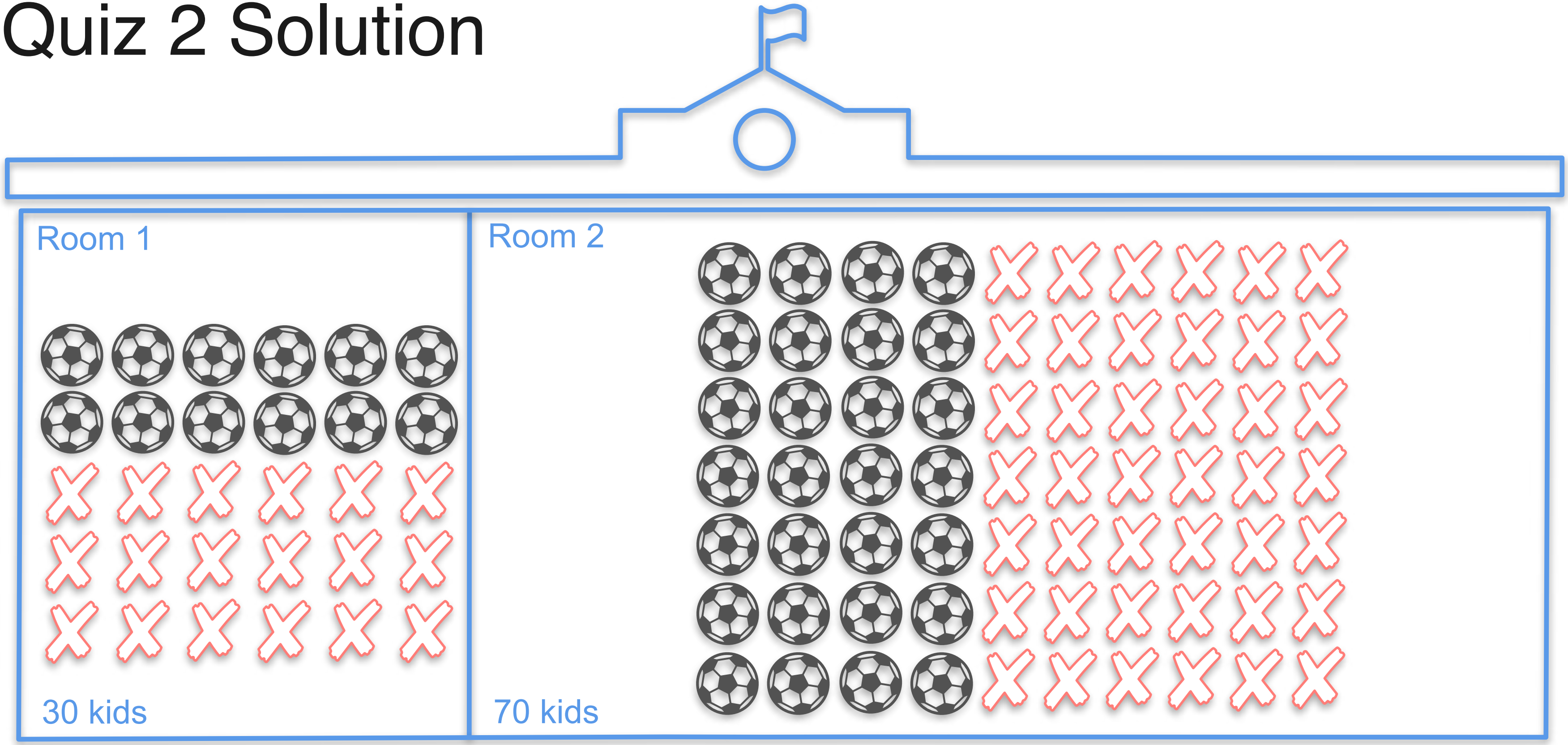
Independence: Quiz 2



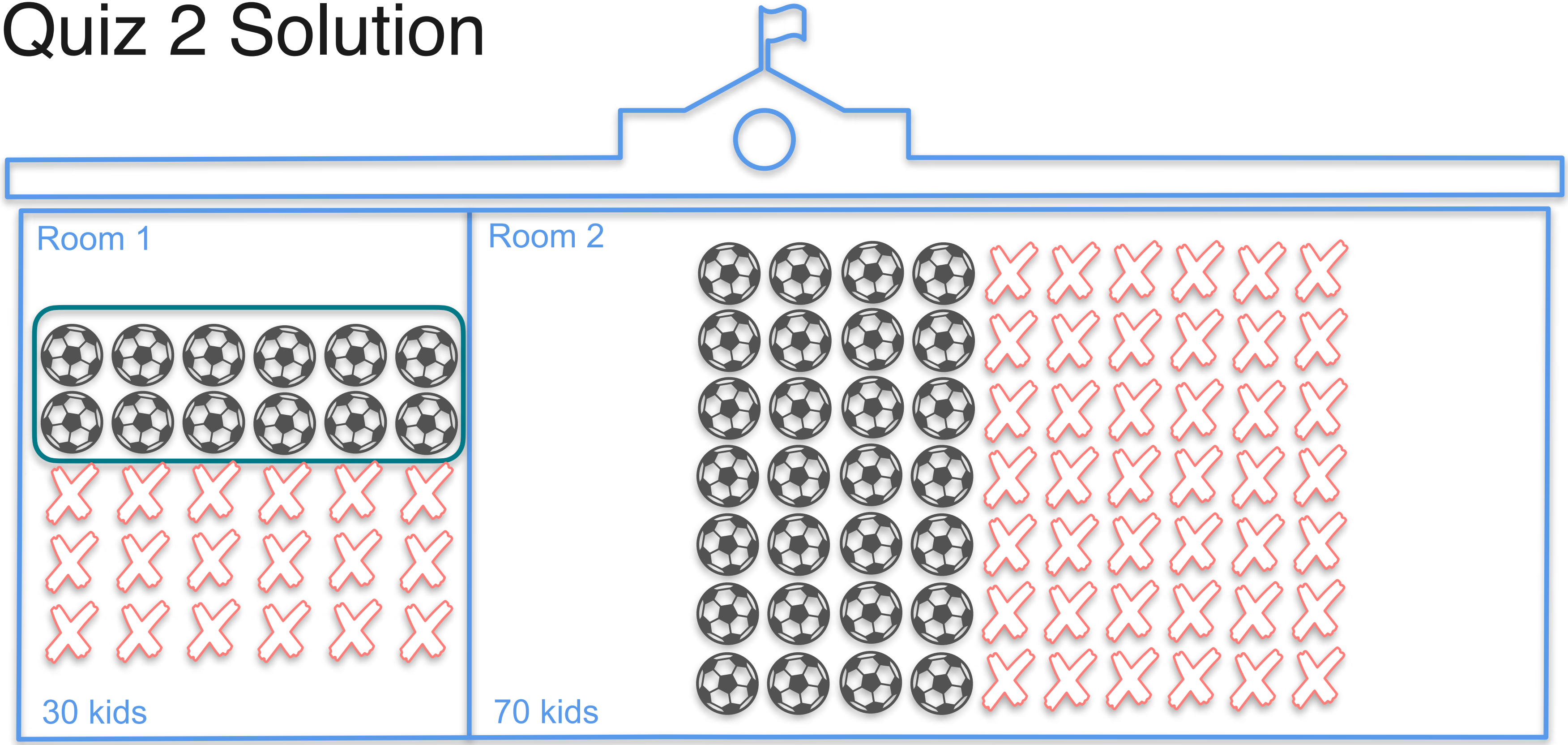
100 kids

How many kids in the smaller room like to play soccer?

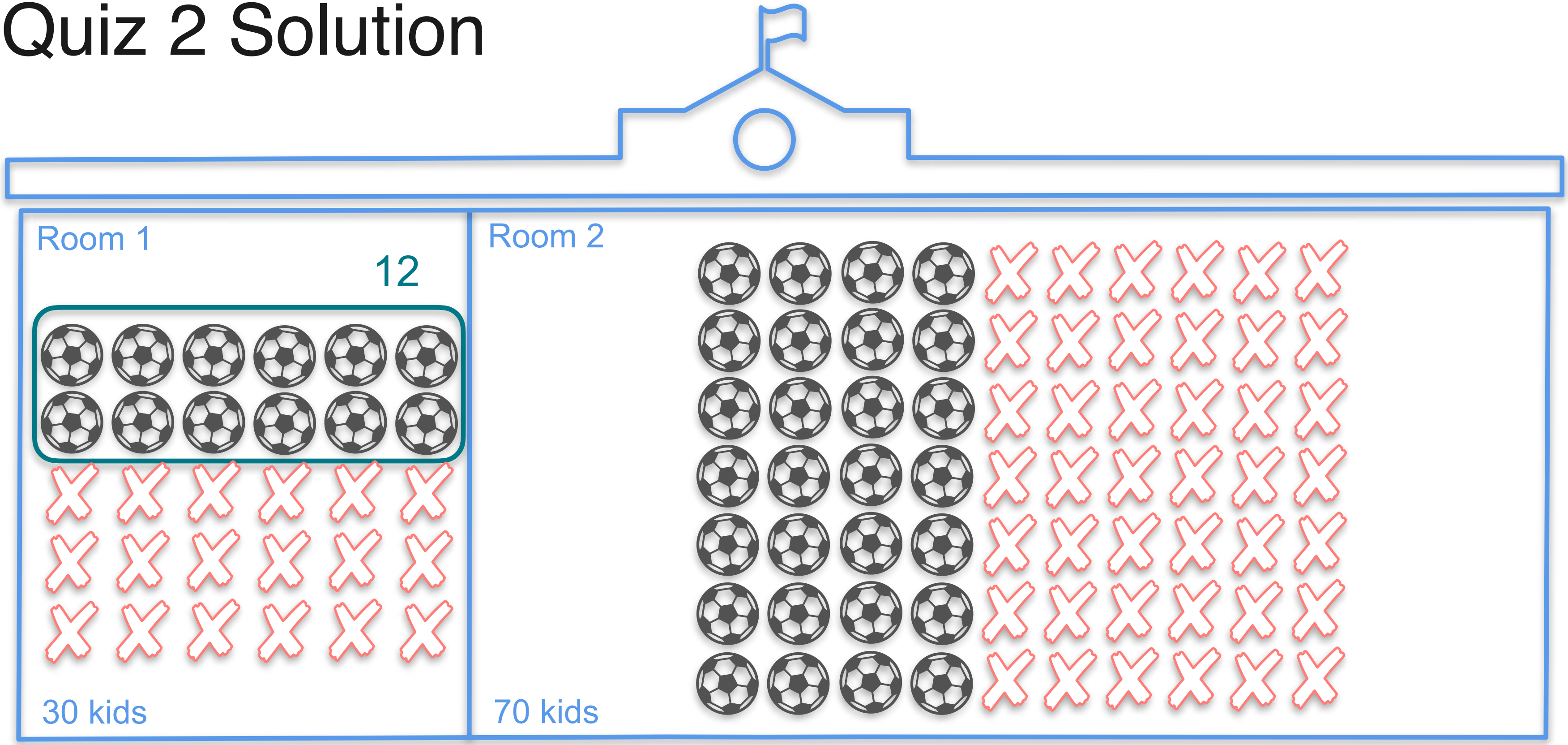
Quiz 2 Solution



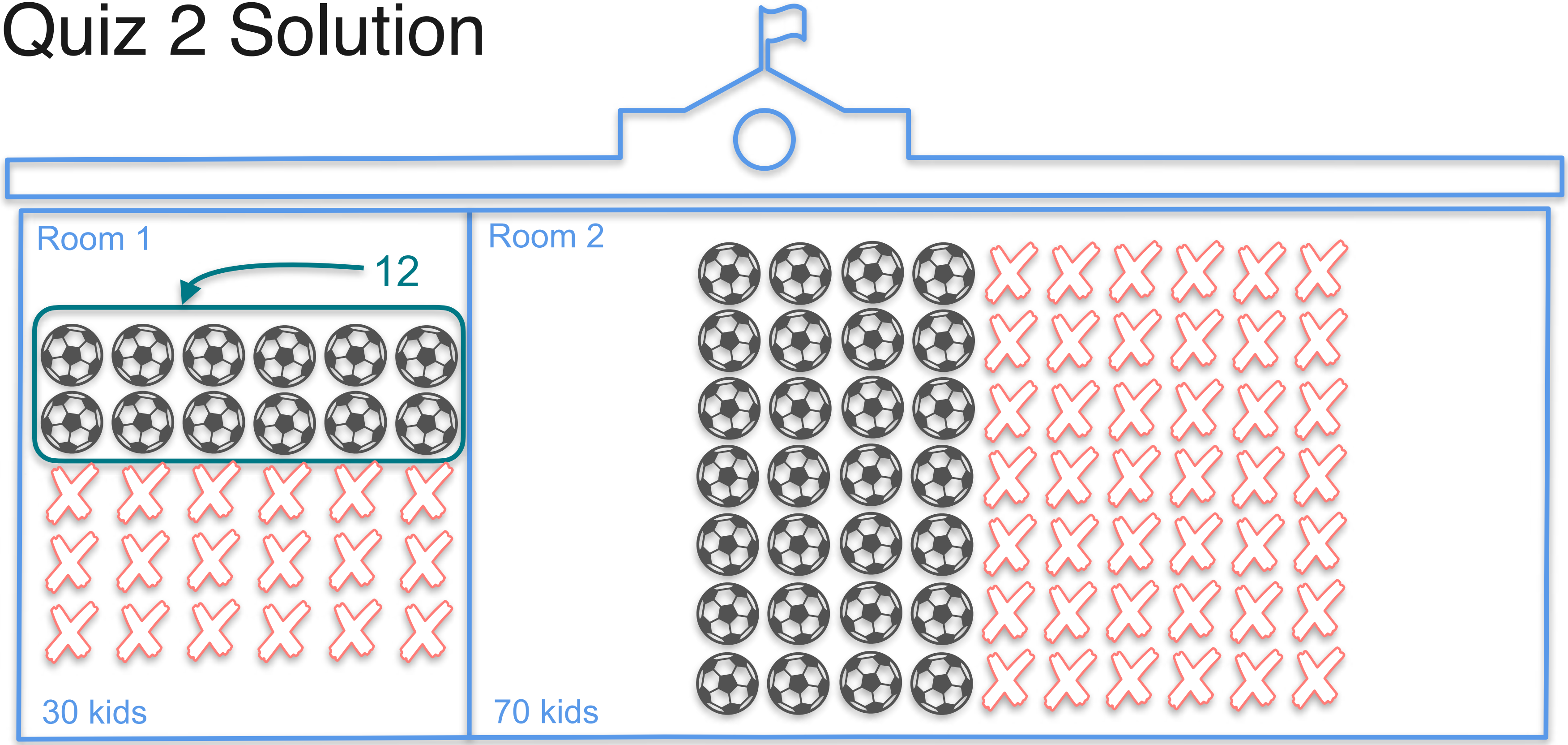
Quiz 2 Solution



Quiz 2 Solution

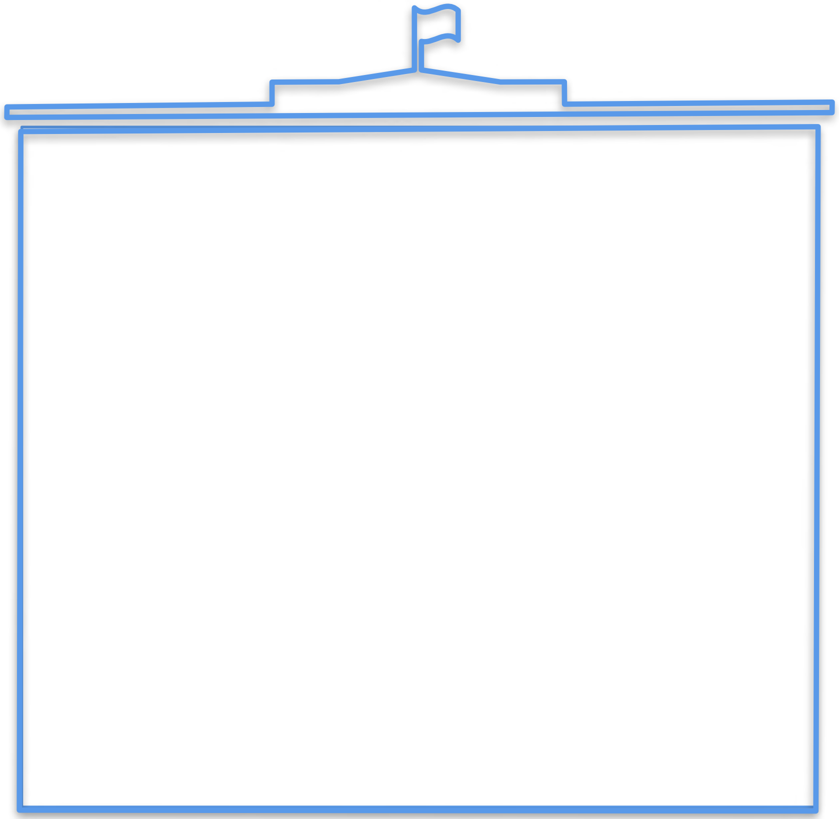


Quiz 2 Solution



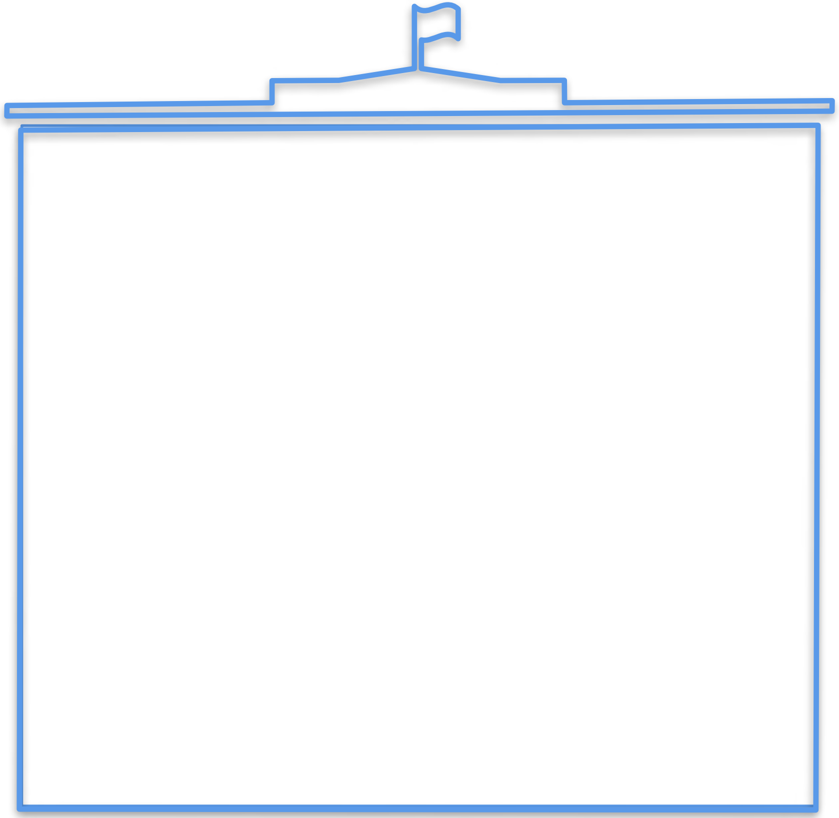
Independent Events

Independent Events



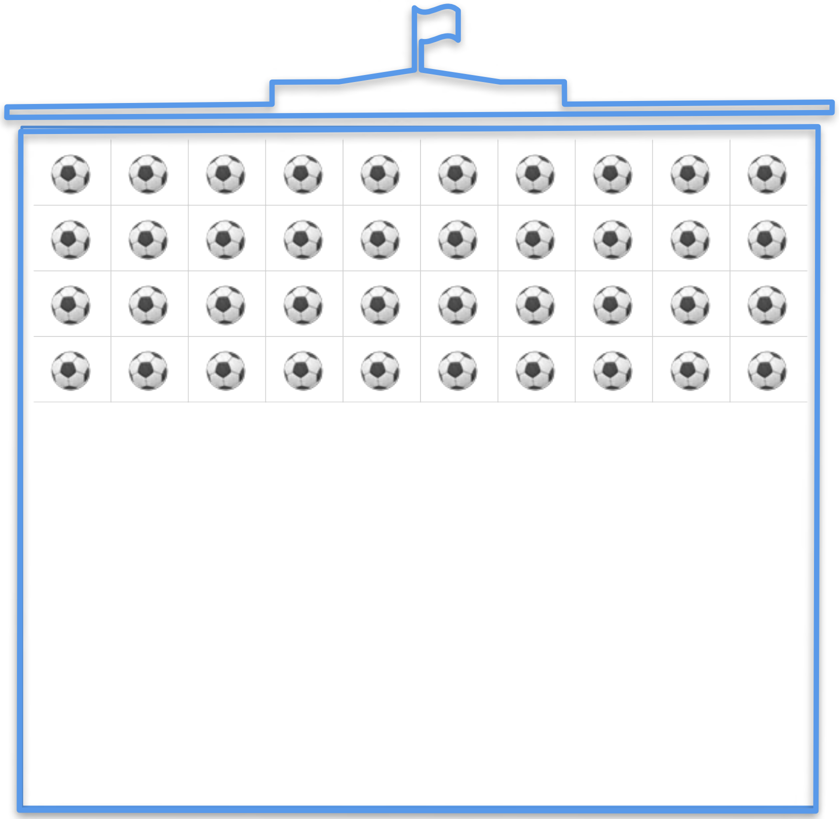
Independent Events

$\mathbf{P}(S) = 0.4$



Independent Events

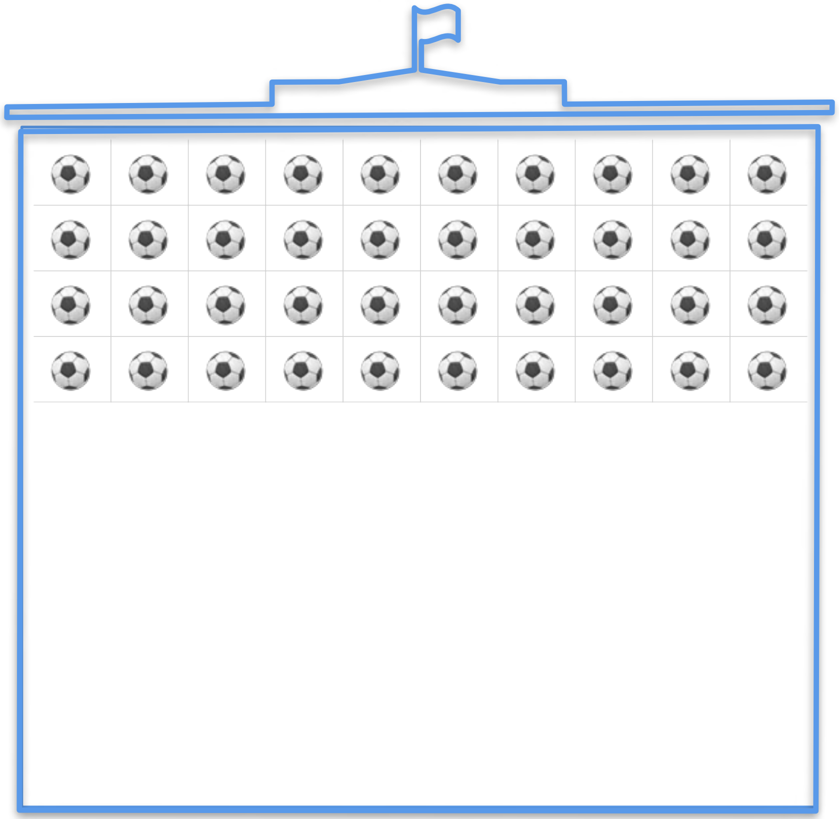
$\mathbf{P}(S) = 0.4$



Independent Events

$\mathbf{P}(S) = 0.4$

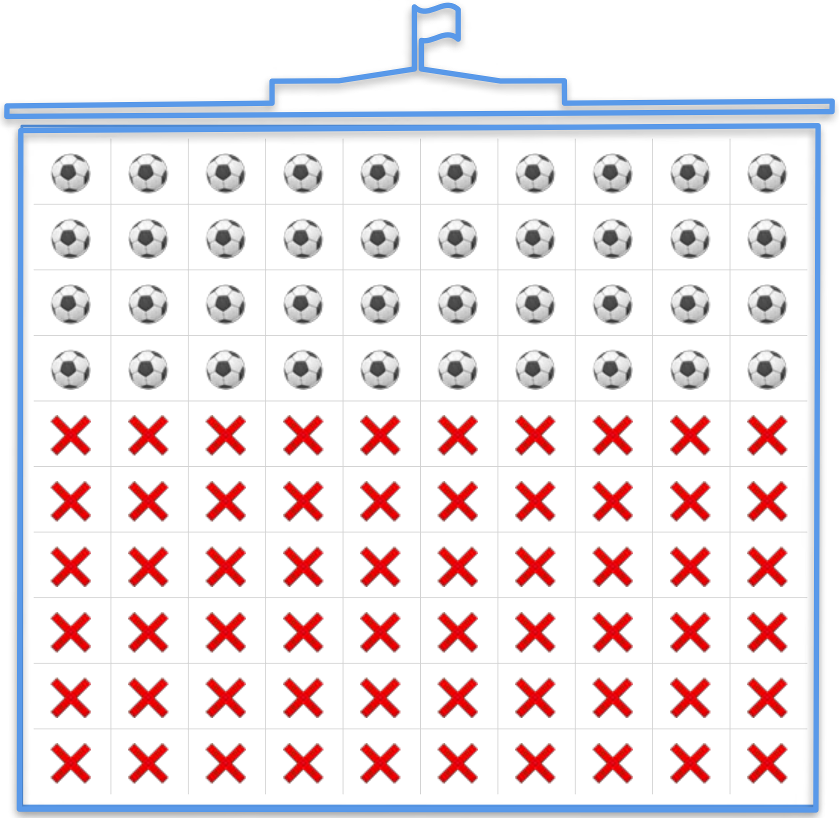
$\mathbf{P}(\text{not } S) = 0.6$



Independent Events

$\mathbf{P}(S) = 0.4$

$\mathbf{P}(\text{not } S) = 0.6$



Independent Events

$\mathbf{P}(S) = 0.4$

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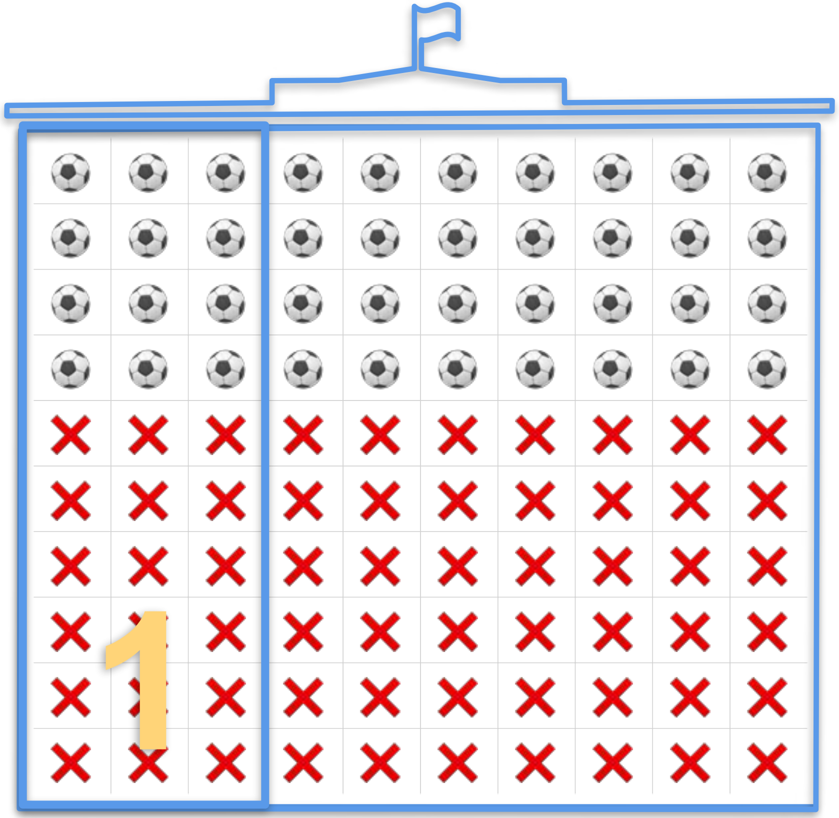
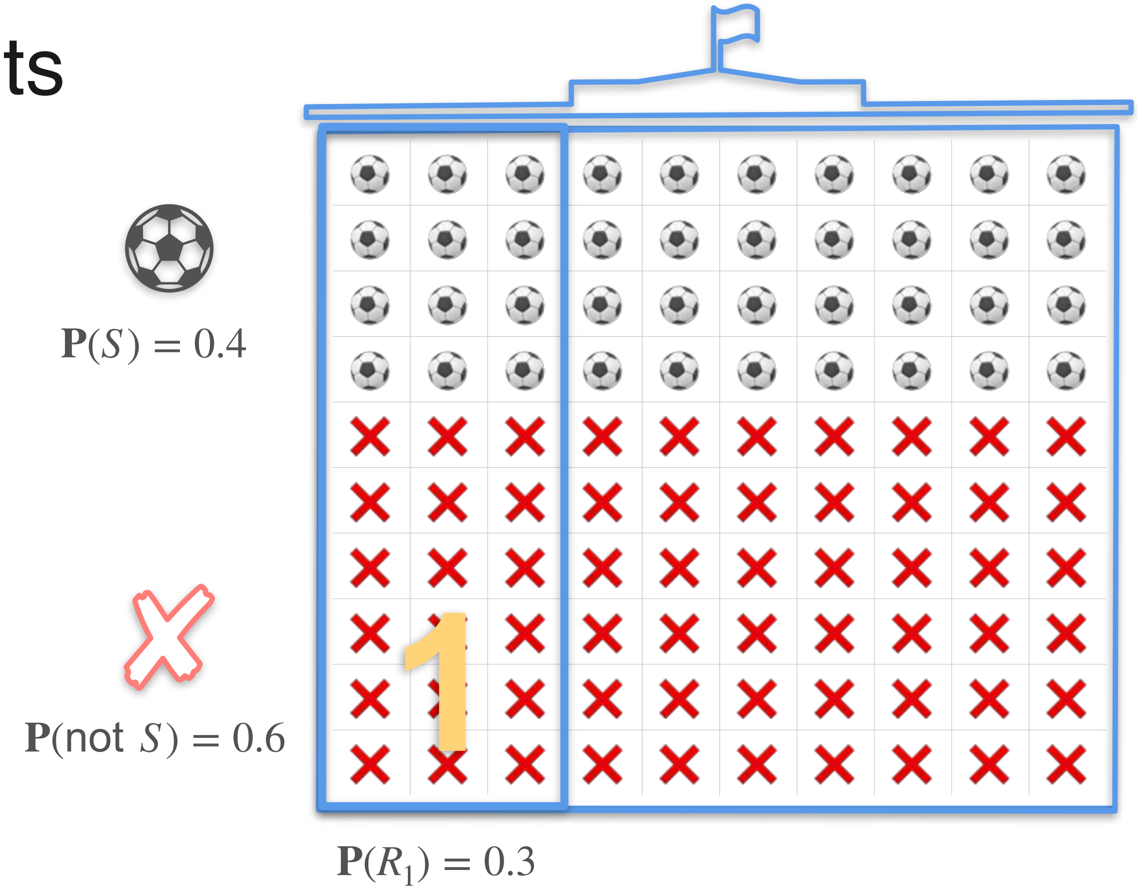
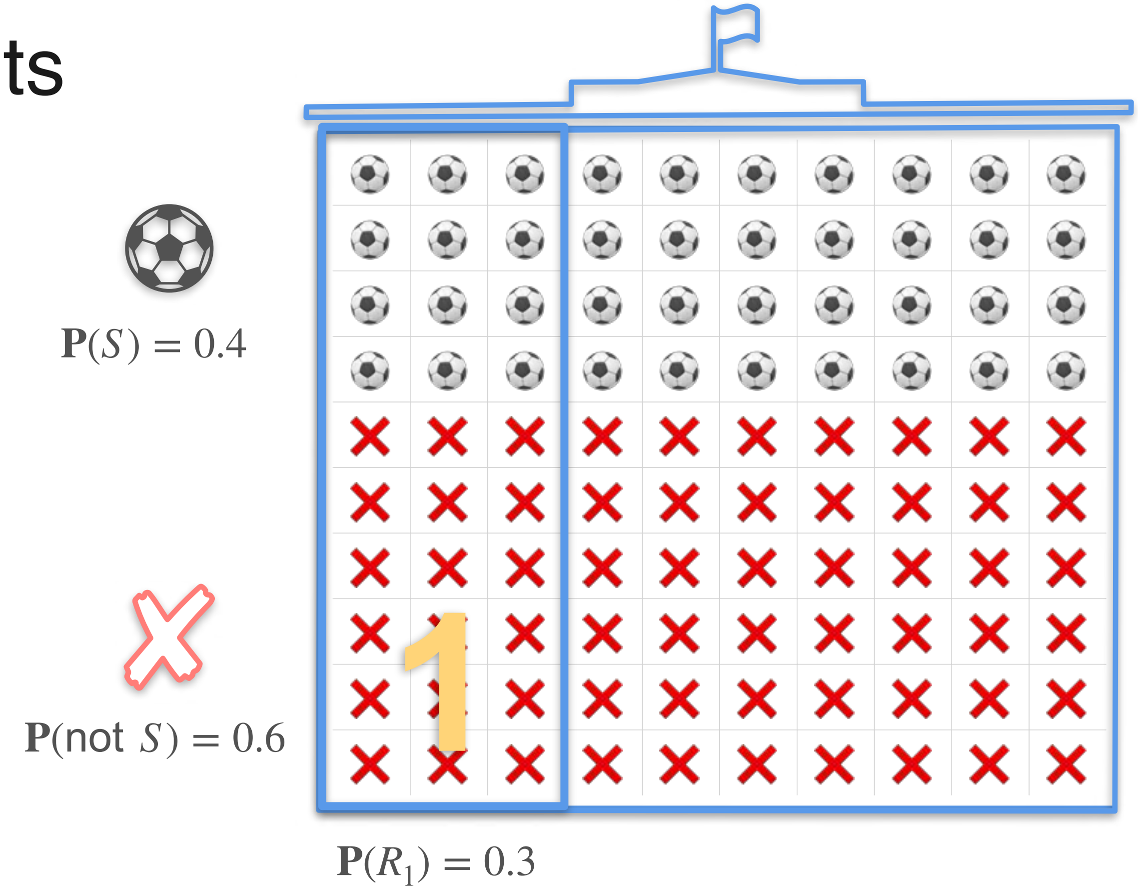


Independent Events



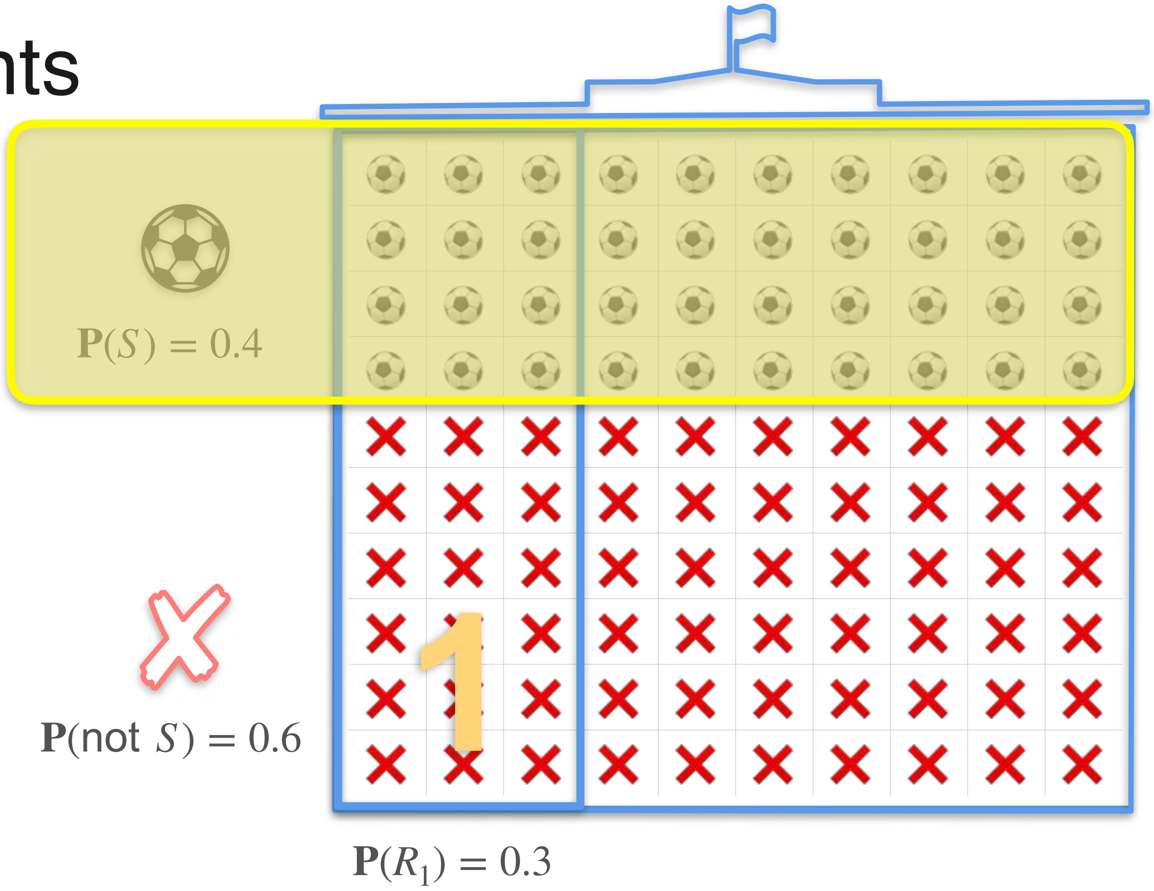
Independent Events

$\mathbf{P}$(Soccer and Room 1)



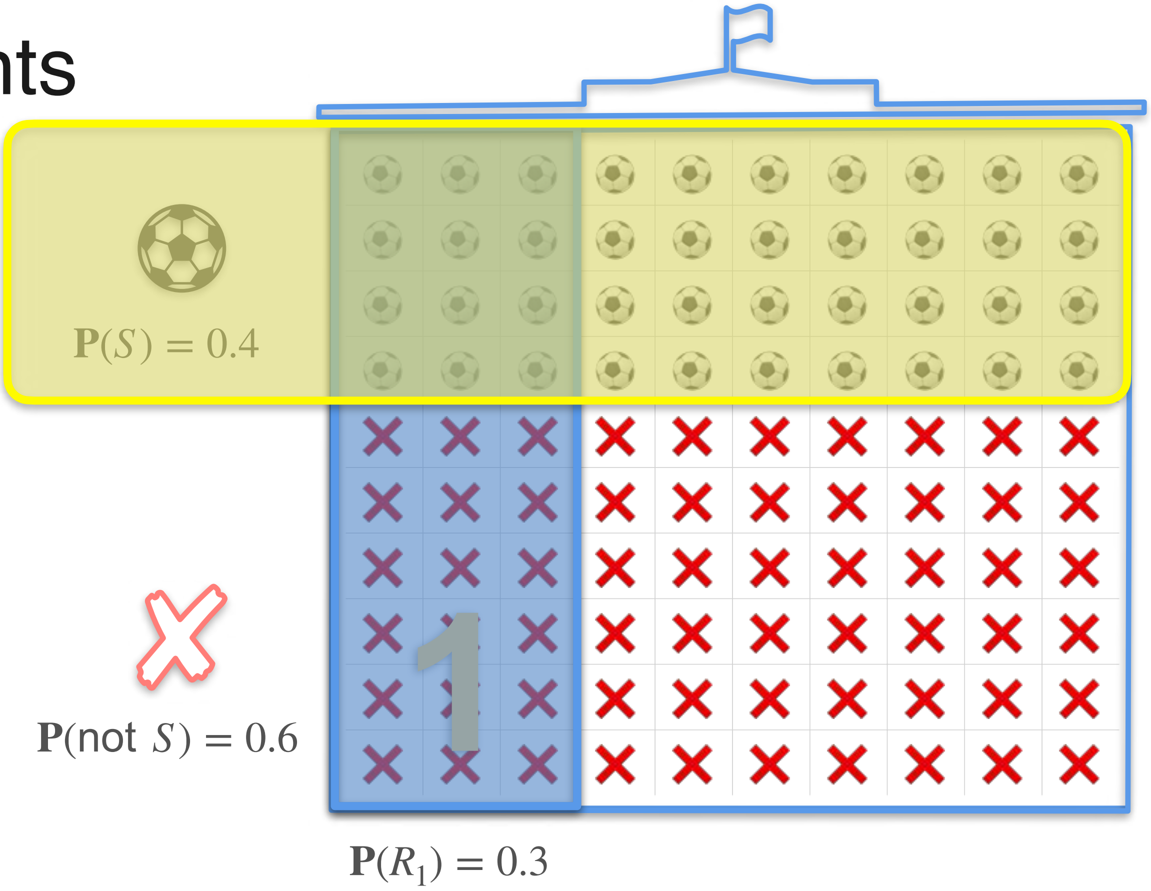
Independent Events

$\mathbf{P}$(Soccer and Room 1)



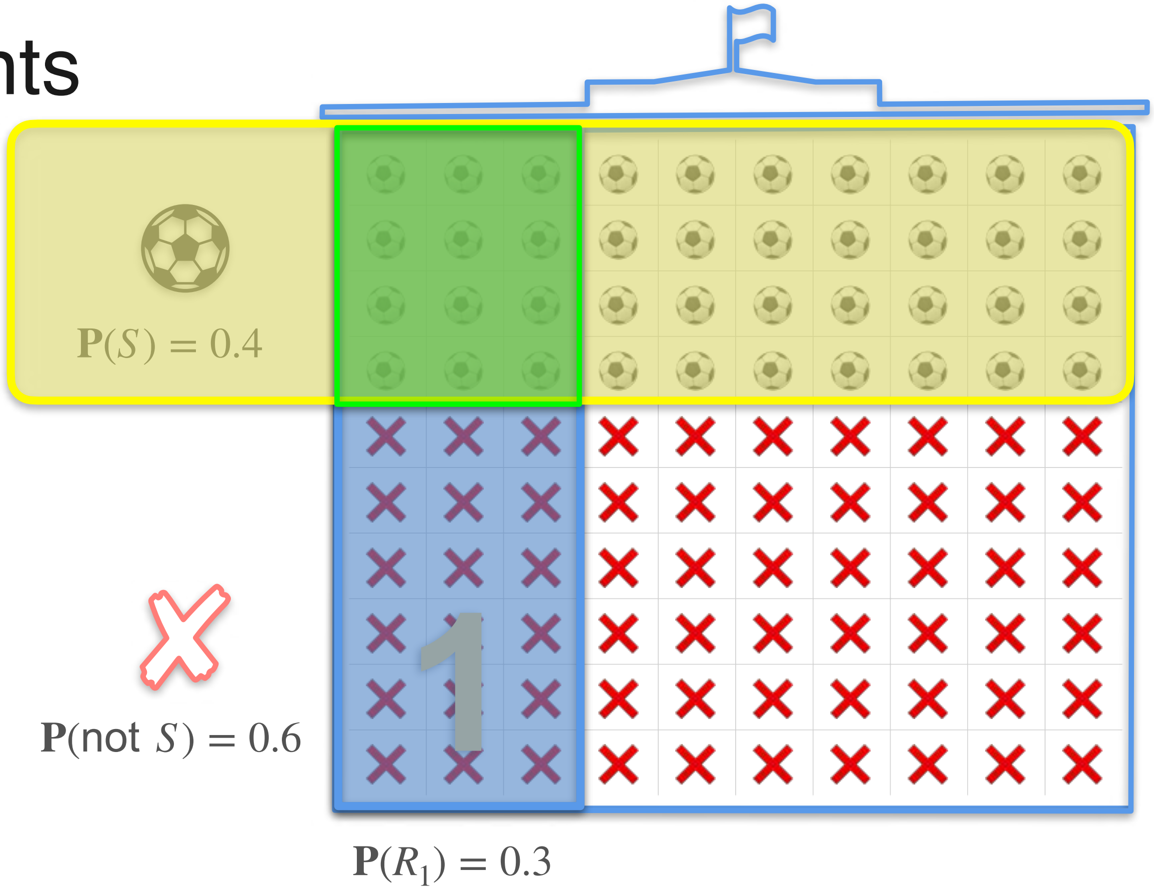
Independent Events

$\mathbf{P}$(Soccer and Room 1)



Independent Events

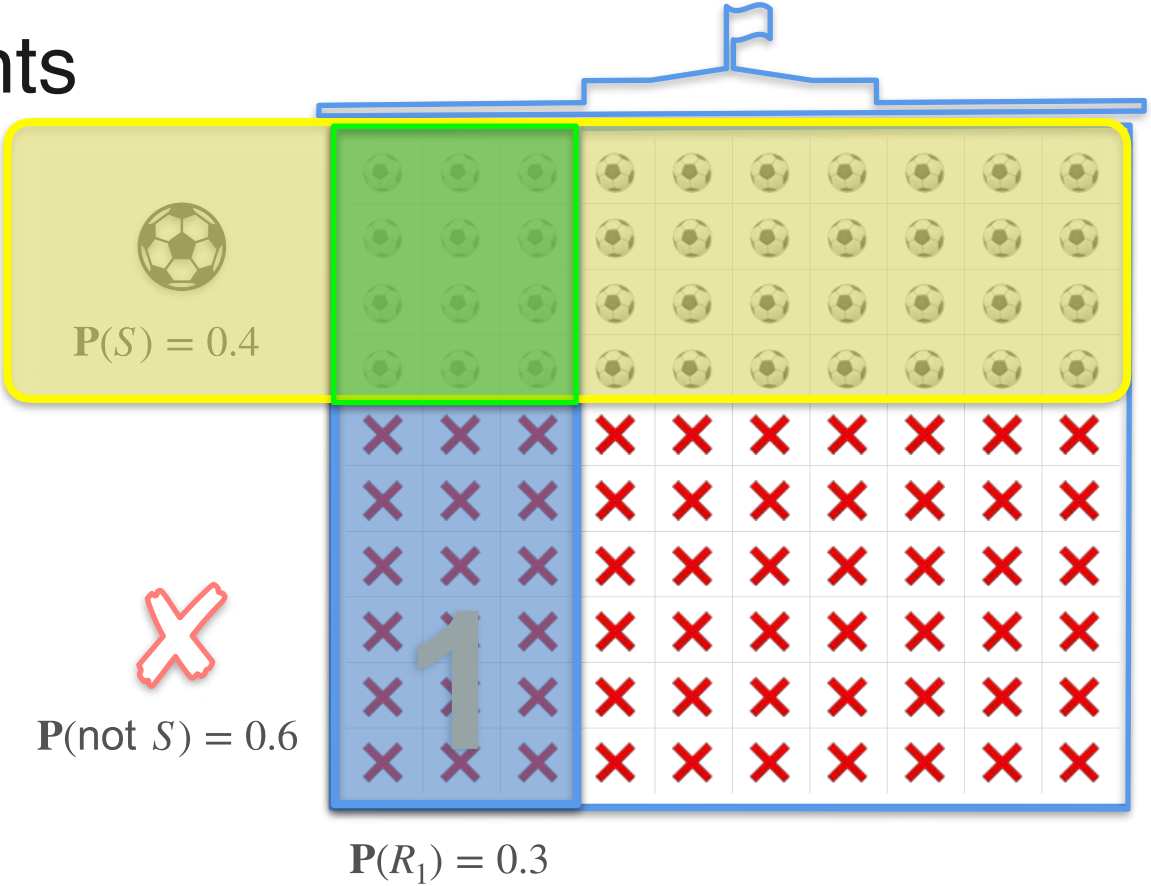
$\mathbf{P}$(Soccer and Room 1)



Independent Events

$\mathbf{P}$(Soccer and Room 1)

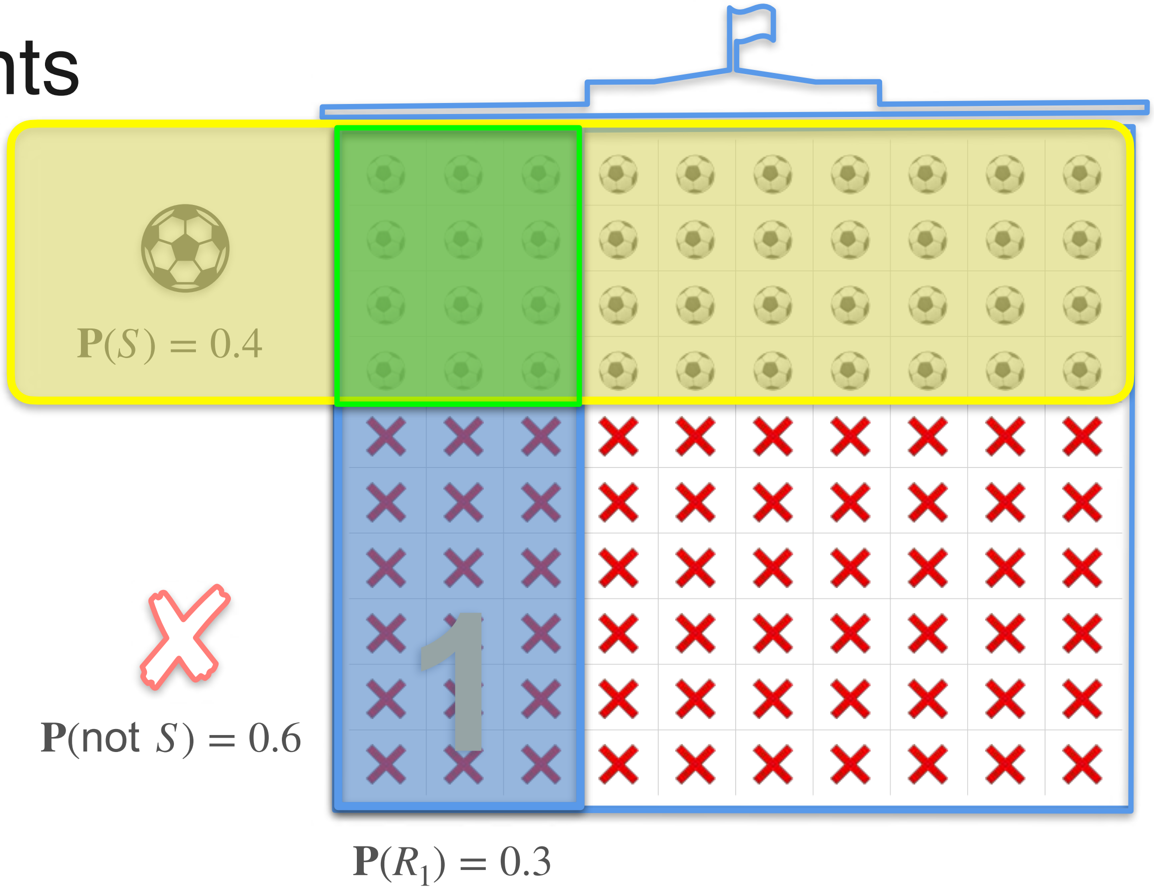
$\mathbf{P}(S \cap R_1) =$



Independent Events

P(Soccer and Room 1)

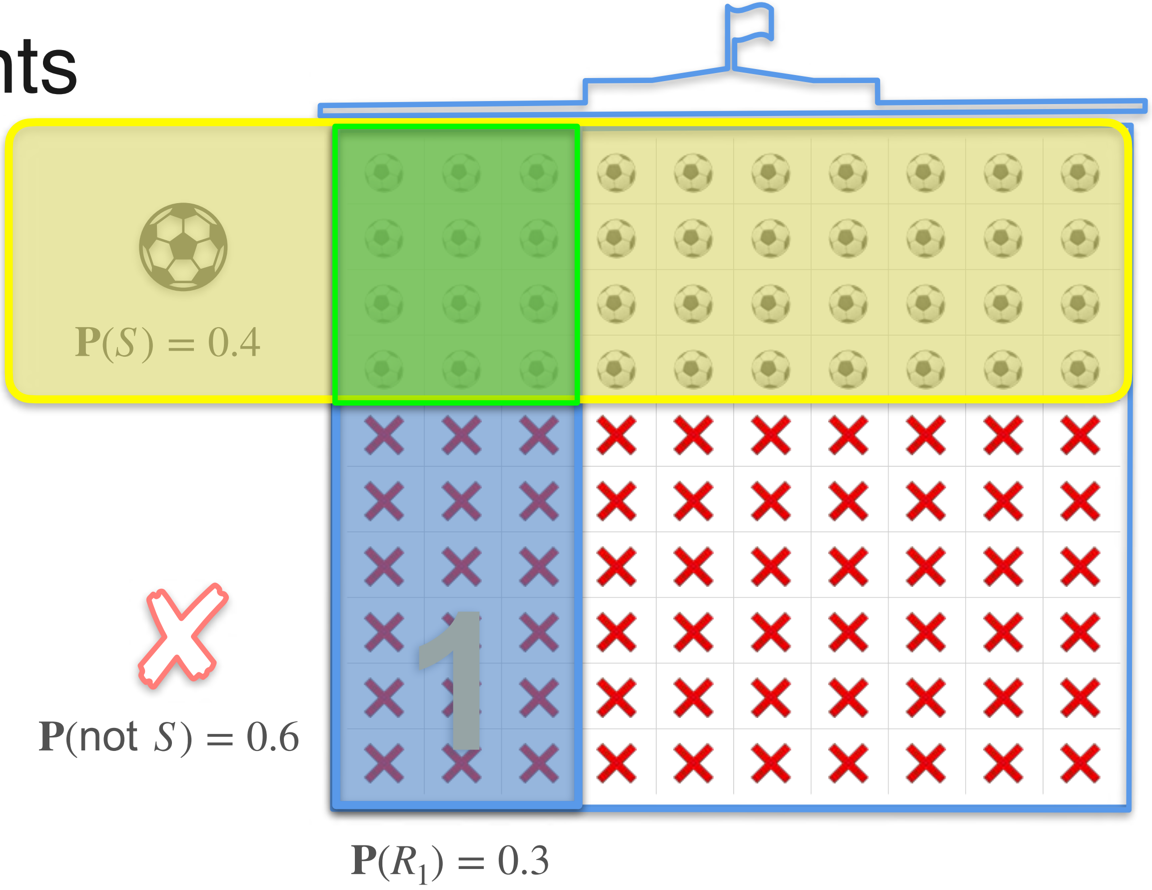
$\mathbf{P}(S \cap R_1) = \quad \mathbf{P}(S)$



Independent Events

$\mathbf{P}$(Soccer and Room 1)

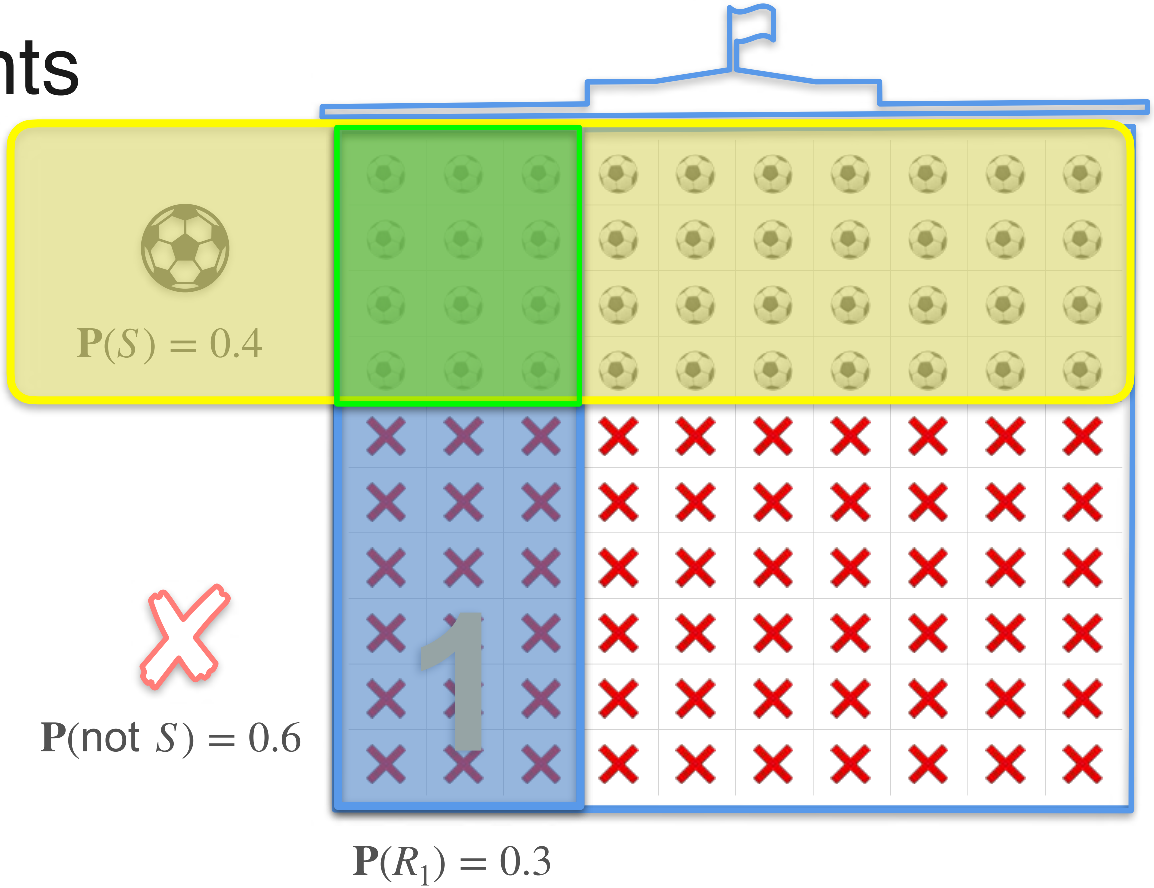
$\mathbf{P}(S \cap R_1) = \quad \mathbf{P}(S)$



Independent Events

P(Soccer and Room 1)

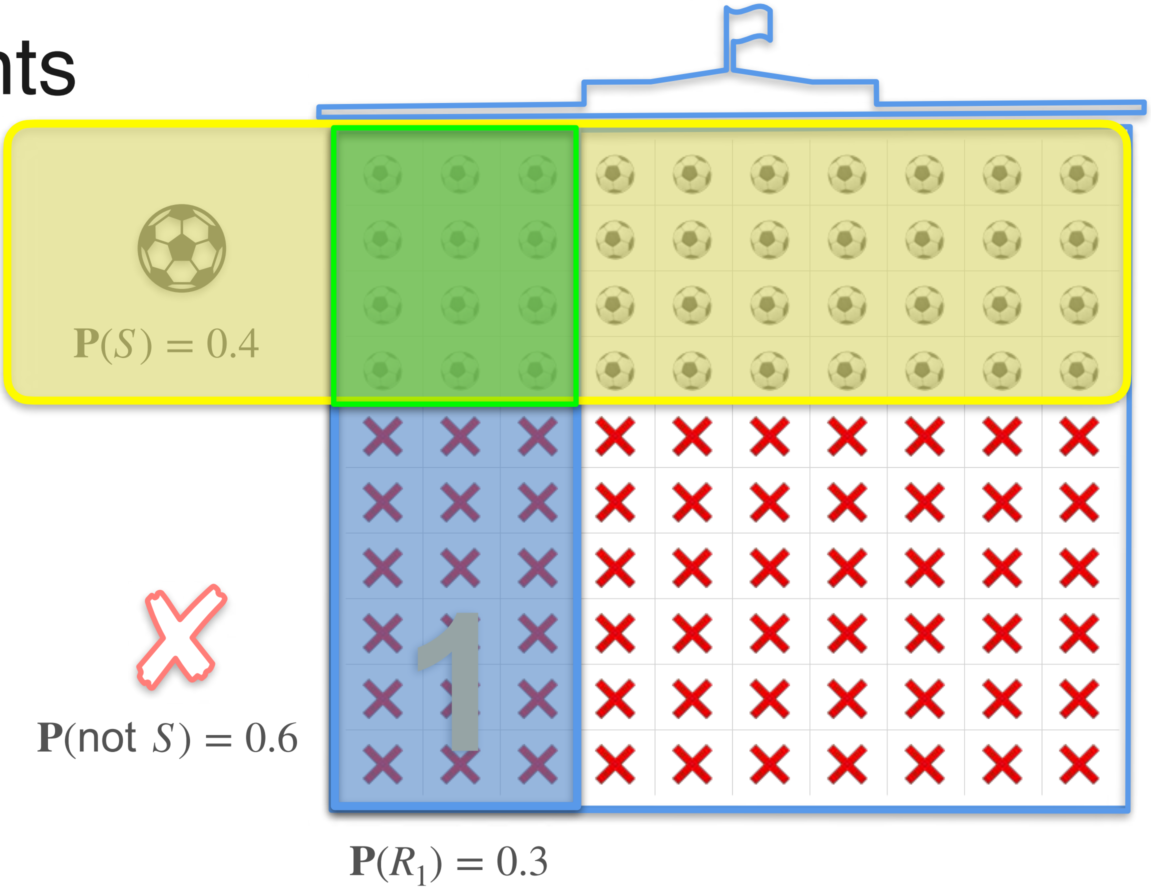
$\mathbf{P}(S \cap R_1) = \quad \mathbf{P}(S) \quad \bullet$



Independent Events

P(Soccer and Room 1)

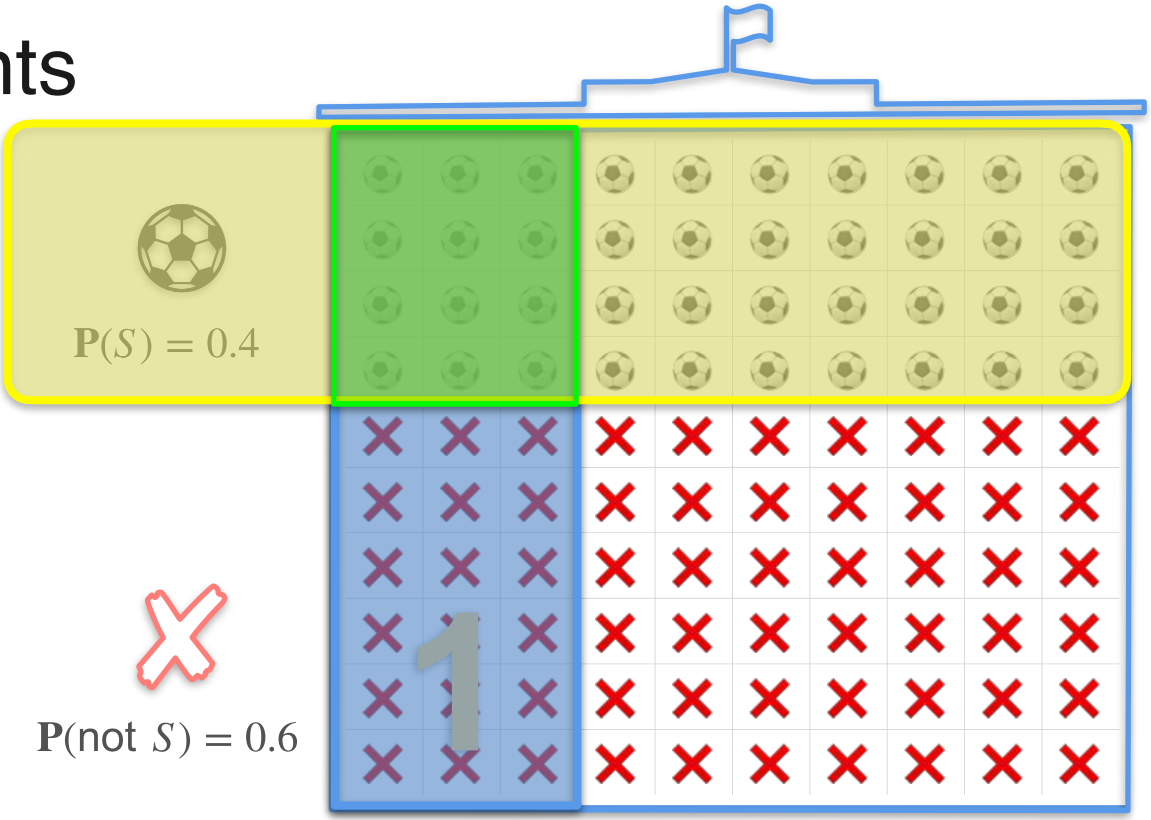
$\mathbf{P}(S \cap R_1) = \quad \mathbf{P}(S) \bullet \mathbf{P}(R_1)$



Independent Events

P(Soccer and Room 1)

$$\mathbf{P}(S \cap R_1) = \mathbf{P}(S) \bullet \mathbf{P}(R_1)$$



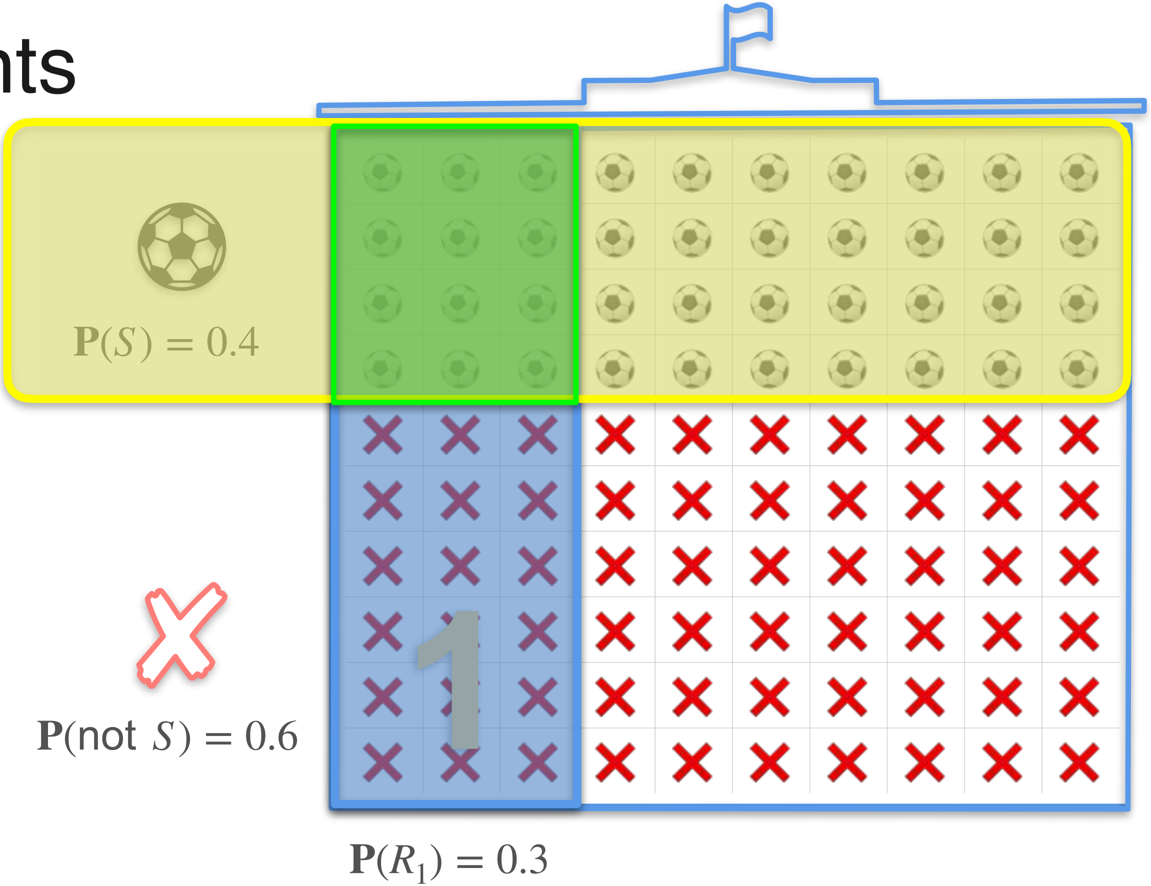
$\mathbf{P}(R_1) = 0.3$

Independent Events

$\mathbf{P}$(Soccer and Room 1)

$\mathbf{P}(S \cap R_1) = \mathbf{P}(S) \bullet \mathbf{P}(R_1)$

$= 0.4$

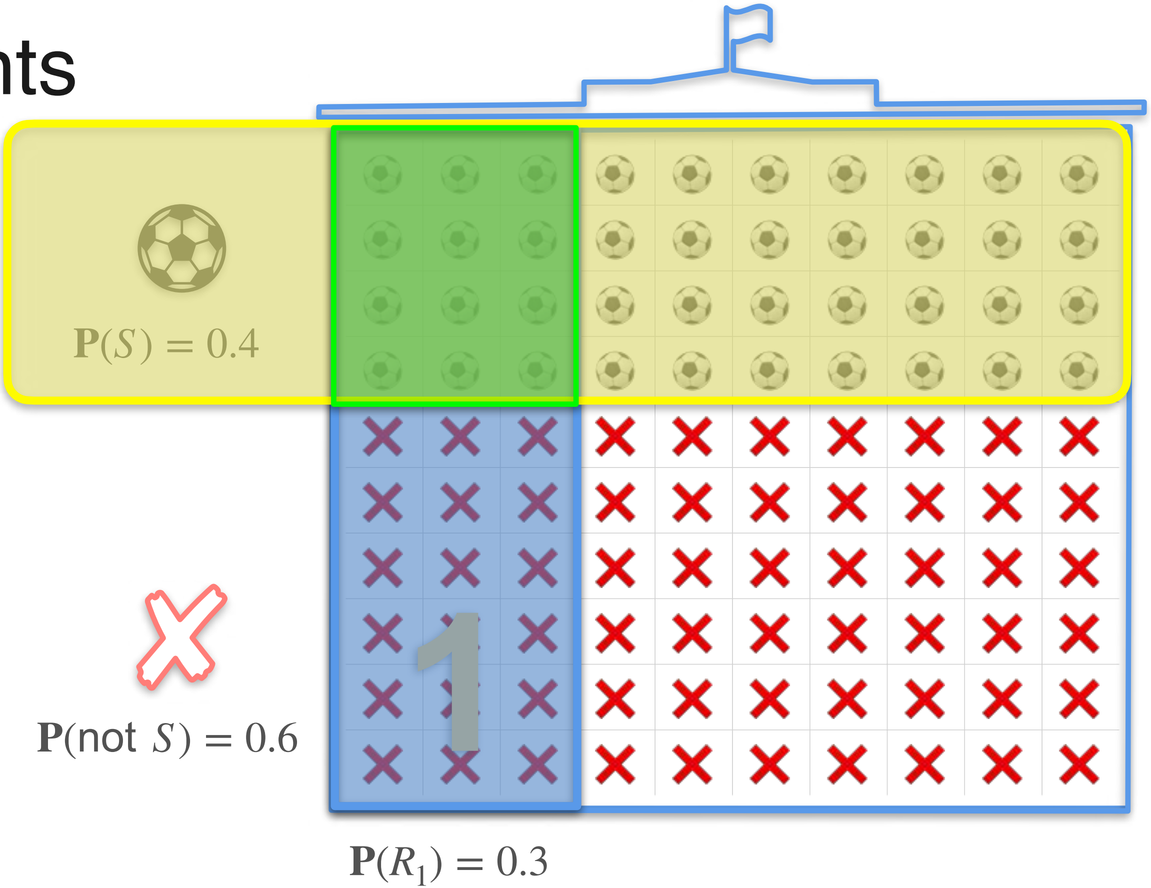


Independent Events

P(Soccer and Room 1)

$\mathbf{P}(S \cap R_1) = \mathbf{P}(S) \bullet \mathbf{P}(R_1)$

$= 0.4 \bullet$

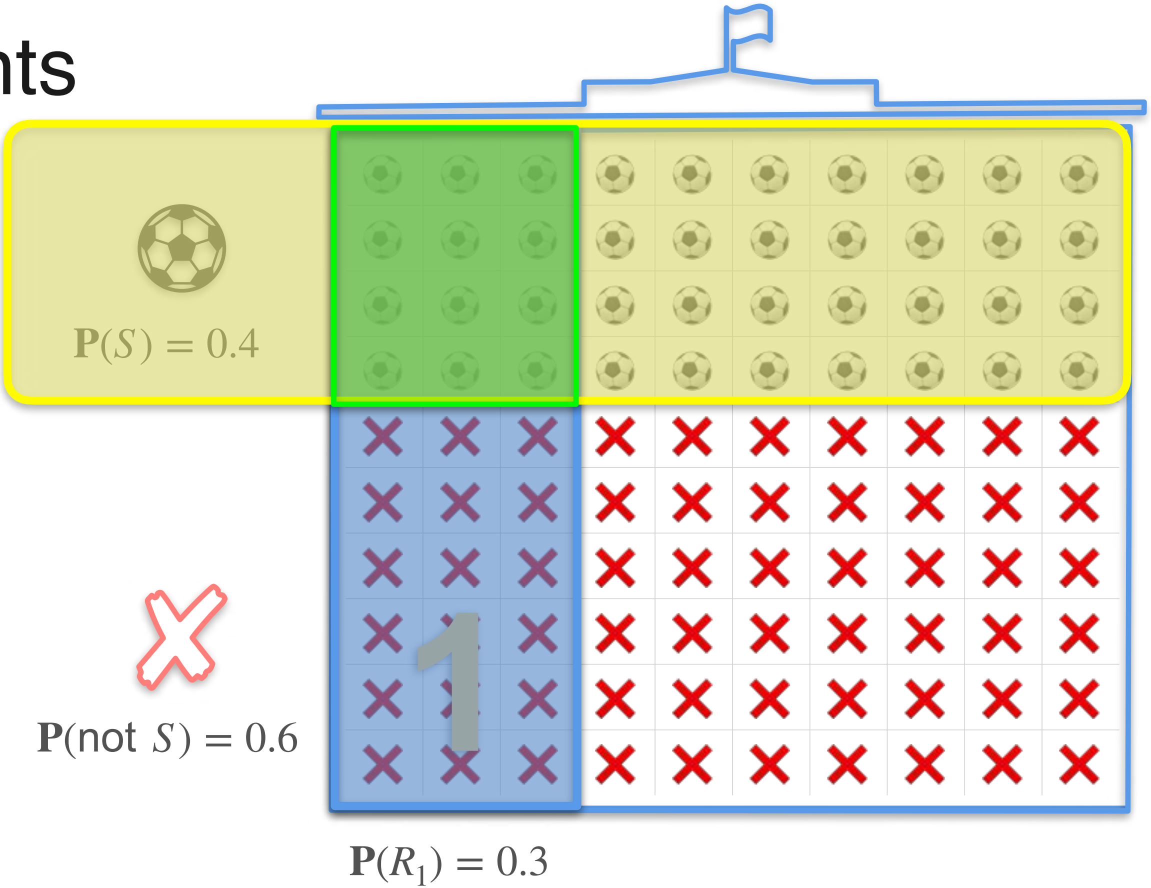


Independent Events

$\mathbf{P}(\text{Soccer and Room 1})$

$$\mathbf{P}(S \cap R_1) = \mathbf{P}(S) \bullet \mathbf{P}(R_1)$$

$$= 0.4 \bullet 0.3$$



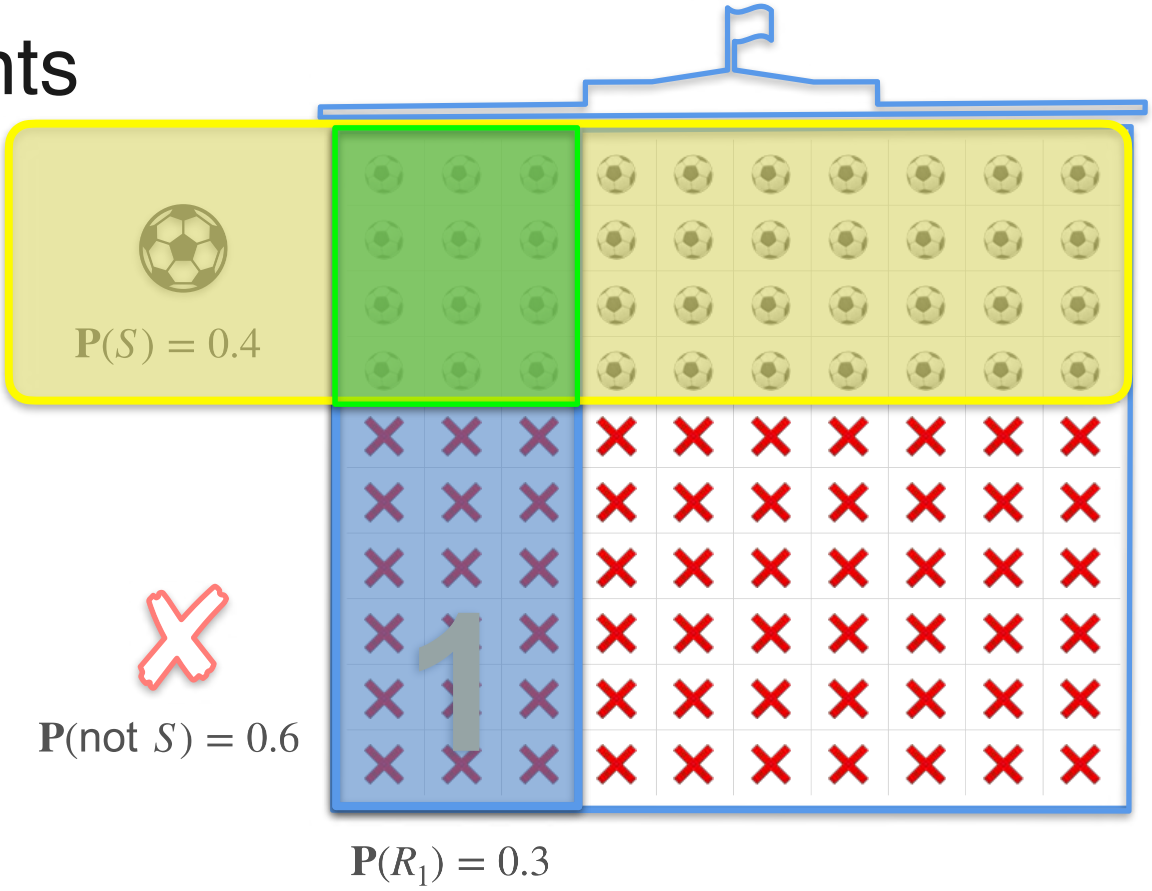
Independent Events

P(Soccer and Room 1)

$\mathbf{P}(S \cap R_1) = \mathbf{P}(S) \bullet \mathbf{P}(R_1)$

$= 0.4 \bullet 0.3$

$= 0.12$



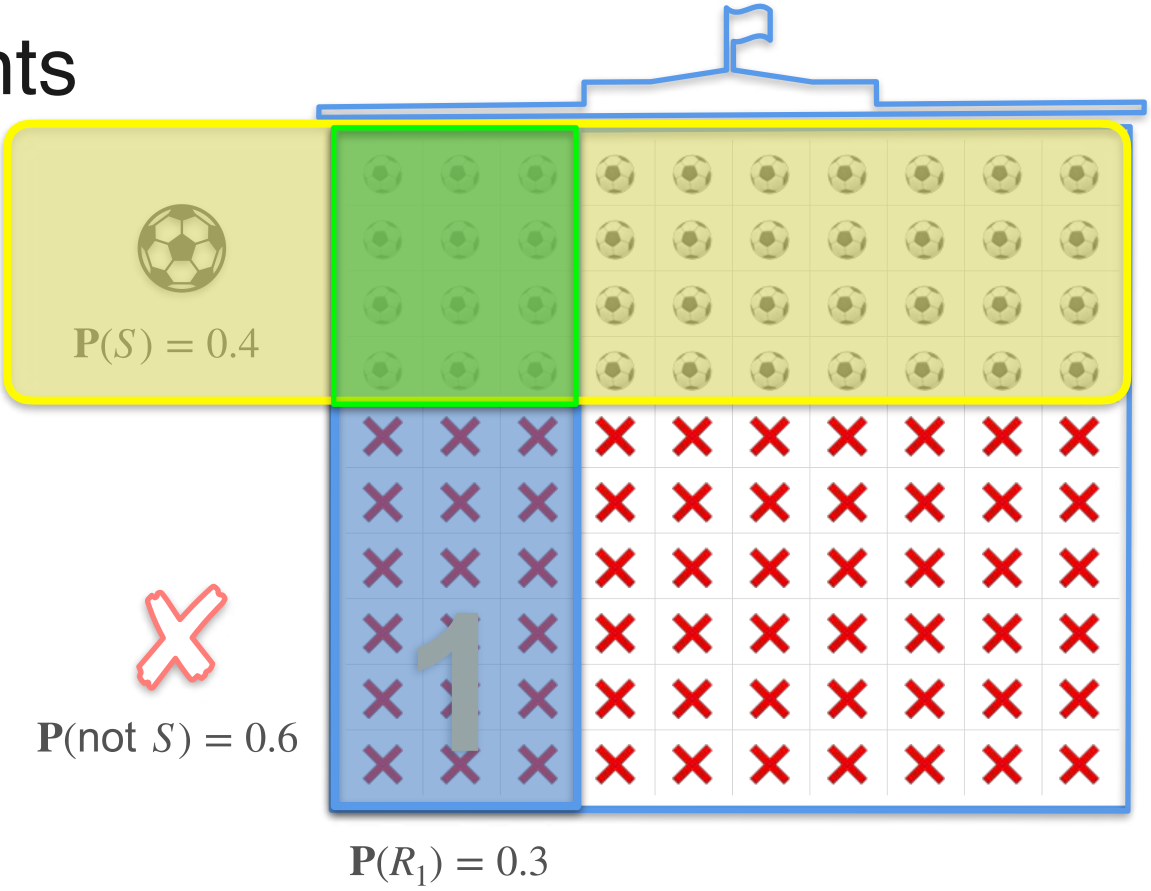
Independent Events

$\mathbf{P}$(Soccer and Room 1)

$$\mathbf{P}(S \cap R_1) = \mathbf{P}(S) \bullet \mathbf{P}(R_1)$$

$$= 0.4 \bullet 0.3$$

$$= 0.12$$



Product Rule (for Independent Events)

Product Rule (for Independent Events)

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Independent Events: Coin Example 1

50% 50%

Independent Events: Coin Example 1

50% 50%

Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?

Independent Events: Coin Example 1

50% 50%

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Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?

Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?

Independent Events: Coin Example 1

50% 50%

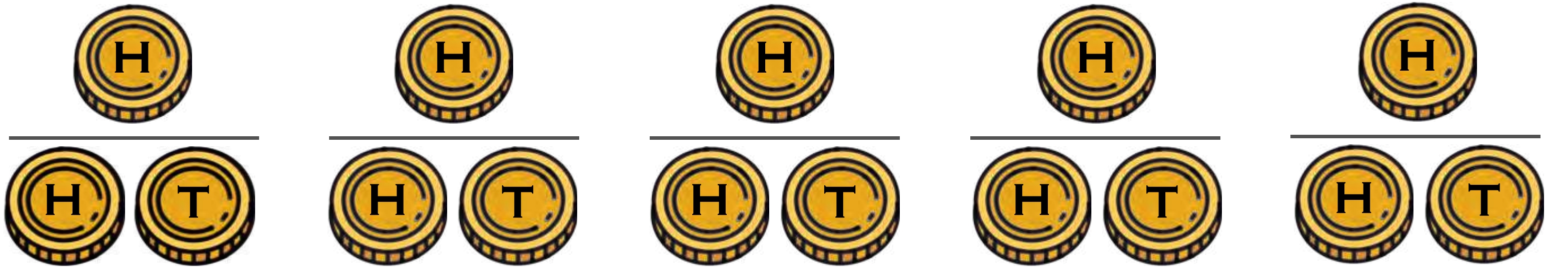
What is the probability of landing on heads five times?

$$\frac{1}{2}$$

Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?

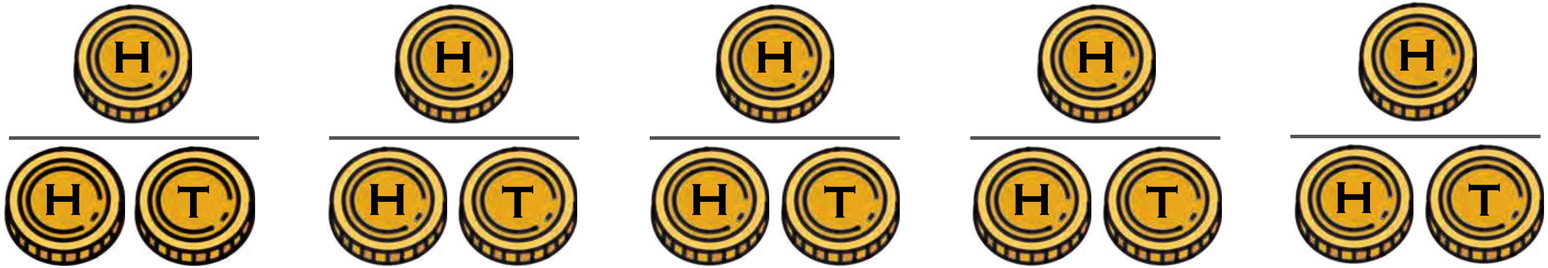


$$\frac{1}{2}$$

Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?



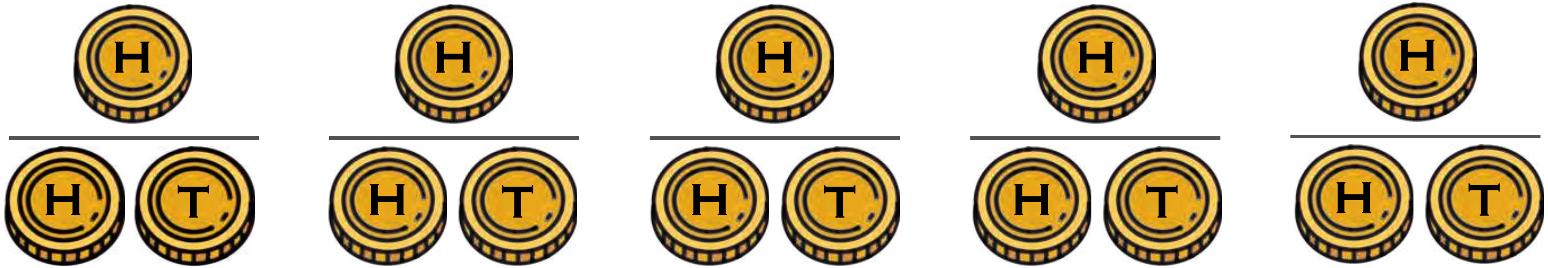
$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?

P(5 heads) =



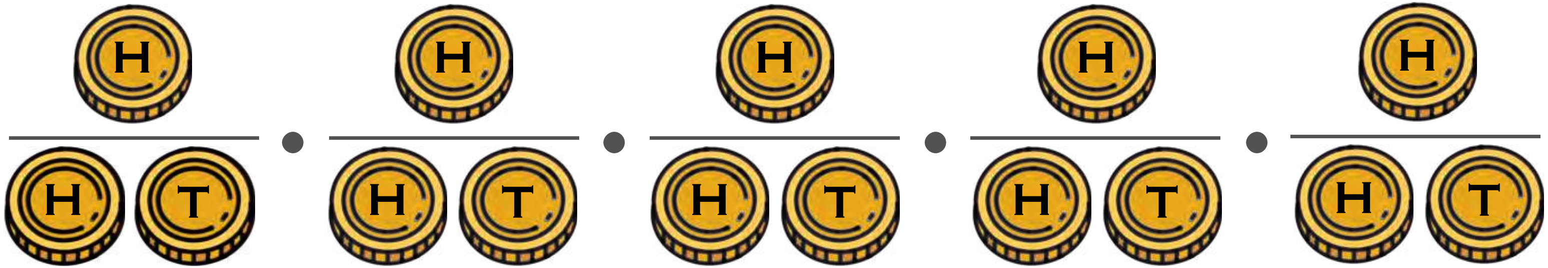
$$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}$$

Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?

$\mathbf{P}(5 \text{ heads}) =$



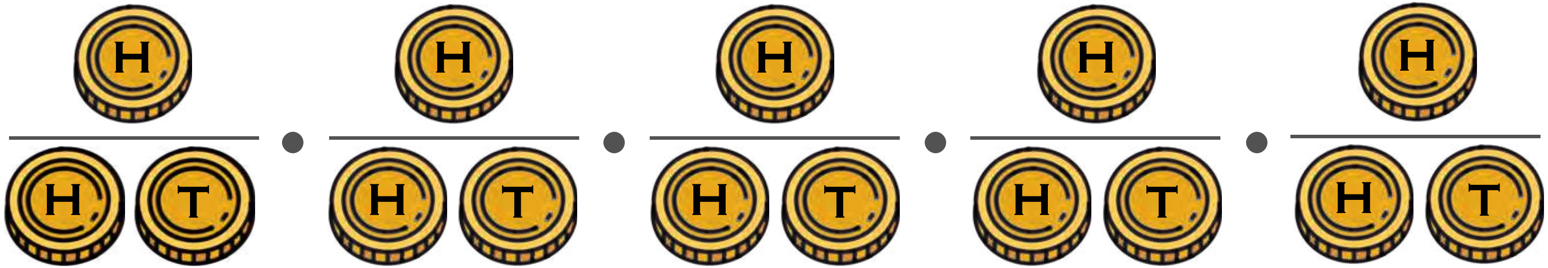
$$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}$$

Independent Events: Coin Example 1

50% 50%

What is the probability of landing on heads five times?

P(5 heads) =



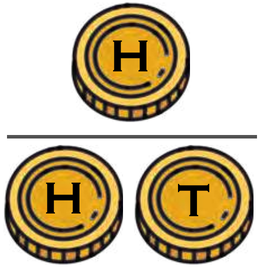
$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

Independent Events: Coin Example 2

50% 50%

What is the probability of landing on heads five times?

$\mathbf{P}(\text{5 heads}) =$

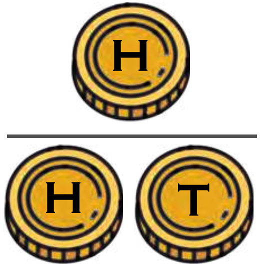


$$\frac{1}{2}$$

Independent Events: Coin Example 2

50%   50%

What is the probability of landing on heads five times?

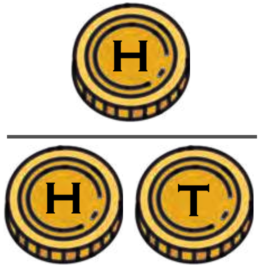
$\mathbf{P}(\text{5 heads}) = \Big($  $\Big)$

$$\frac{1}{2}$$

Independent Events: Coin Example 2

50% 50%

What is the probability of landing on heads five times?

$\mathbf{P}(\text{5 heads}) = \Bigg($  $\Bigg)$

$\frac{1}{2}$

Independent Events: Coin Example 2

50% 50%

What is the probability of landing on heads five times?

$$\mathbf{P}(\text{5 heads}) = \left(\frac{H}{H \quad T}\right)^{5}$$

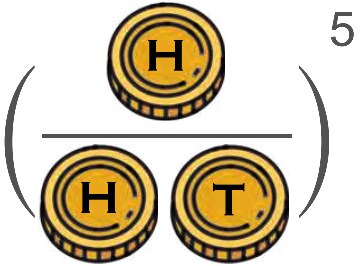
$$\frac{1}{2}$$

Independent Events: Coin Example 2

50% 50%

What is the probability of landing on heads five times?

$\mathbf{P}(5 \text{ heads}) =$



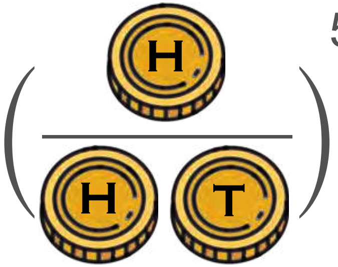
$$\left(\frac{1}{2}\right)^5$$

Independent Events: Coin Example 2

50% 50%

What is the probability of landing on heads five times?

$\mathbf{P}(5\text{ heads}) =$



5

$$\left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

Independent Events: Dice Example 1

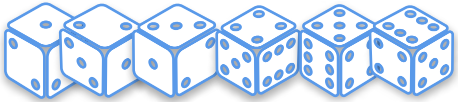
Independent Events: Dice Example 1

Independent Events: Dice Example 1

$$\mathbf{P}(6) = \frac{}{}$$

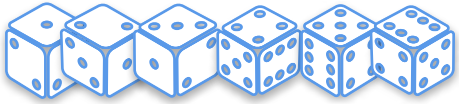
Independent Events: Dice Example 1

$\mathbf{P}(6) =$



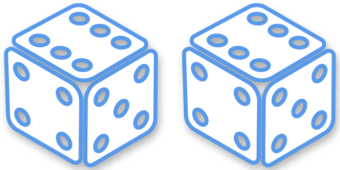
Independent Events: Dice Example 1

$$\mathbf{P}(6) = \frac{\qquad}{\qquad} = \frac{1}{6}$$

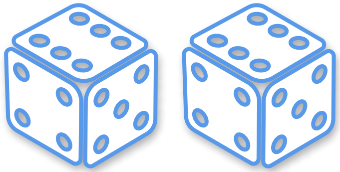


Independent Events: Dice Example 1

Independent Events: Dice Example 1

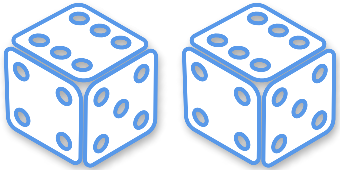


Independent Events: Dice Example 1



2 dice

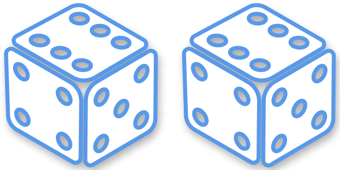
Independent Events: Dice Example 1



2 dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Independent Events: Dice Example 1

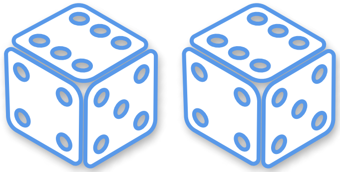


2 dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(6,6) = \underline{\hspace{6cm}}$$

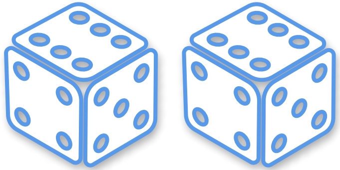
Independent Events: Dice Example 1



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	$\boxed{6,6}$

$$\mathbf{P}(6,6) = \underline{\hspace{6cm}}$$

Independent Events: Dice Example 1

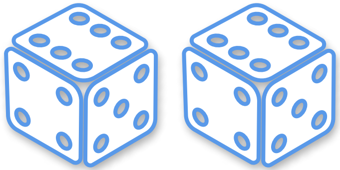


2 dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	$\boxed{6,6}$

$$\mathbf{P}(6,6) = \frac{\boxed{6,6}}{}$$

Independent Events: Dice Example 1



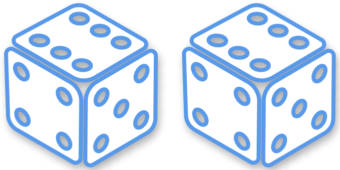
2 dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(6,6) =$ 6,6 divided by:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Independent Events: Dice Example 1



2 dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	$\boxed{6,6}$

$$\mathbf{P}(6,6) = \frac{\boxed{6,6}}{\begin{array}{|cccccc|} \hline 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \\ \hline \end{array}} = \frac{1}{36}$$

Independent Events: Dice Example 1

2 dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	$\boxed{6,6}$

Independent Events: Dice Example 1

2 dice

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	$\boxed{6,6}$

$$\mathbf{P}(6,6) =$$

Independent Events: Dice Example 1

2 dice

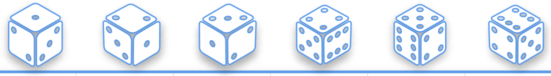
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

$\mathbf{P}(6,6) =$

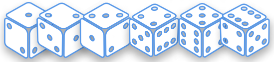
Independent Events: Dice Example 1

2 dice



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

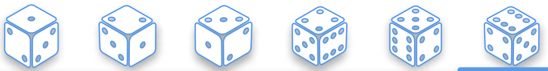
$\mathbf{P}(6,6) =$  / 

$\frac{1}{6}$

Independent Events: Dice Example 1

2 dice

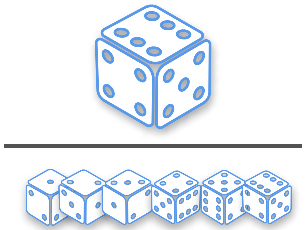
$\frac{1}{6}$



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

$\mathbf{P}(6,6) =$



$\frac{1}{6}$

Independent Events: Dice Example 1

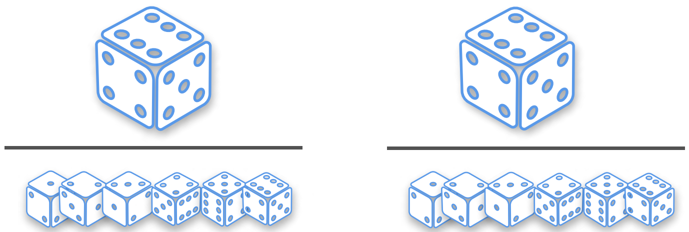
$\frac{1}{6}$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

2 dice

$\mathbf{P}(6,6) =$



$\frac{1}{6}$ $\frac{1}{6}$

Independent Events: Dice Example 1

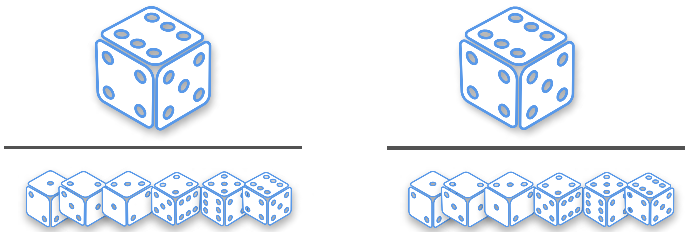
$\frac{1}{6}$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

2 dice

$\mathbf{P}(6,6) =$



$\frac{1}{6}$ $\frac{1}{6}$

Independent Events: Dice Example 1

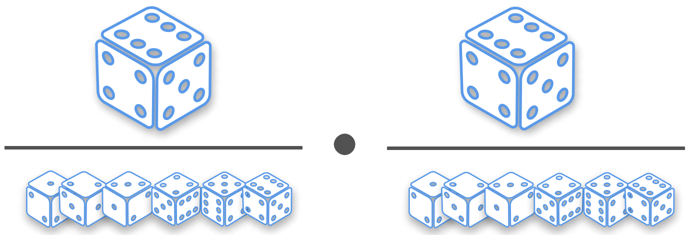
$\frac{1}{6}$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

2 dice

$P(6,6) =$



$$\frac{1}{6} \cdot \frac{1}{6}$$

Independent Events: Dice Example 1

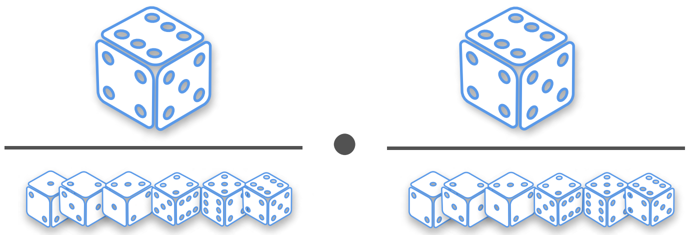
$\frac{1}{6}$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

2 dice

$\mathbf{P}(6,6) =$

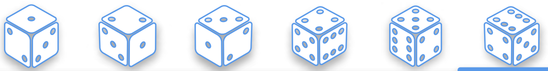


$$\frac{1}{6} \cdot \frac{1}{6}$$

$$= \left(\frac{1}{6}\right)$$

Independent Events: Dice Example 1

$$\frac{1}{6}$$

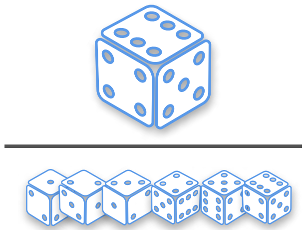


1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

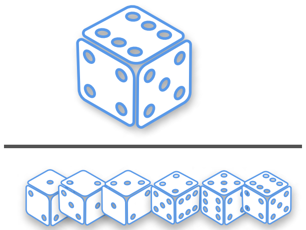
$$\frac{1}{6}$$

2 dice

$$\mathbf{P}(6,6) =$$



$$\bullet$$

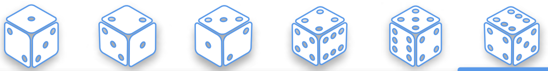


$$\frac{1}{6} \bullet \frac{1}{6}$$

$$= \left(\frac{1}{6}\right)$$

Independent Events: Dice Example 1

$\frac{1}{6}$

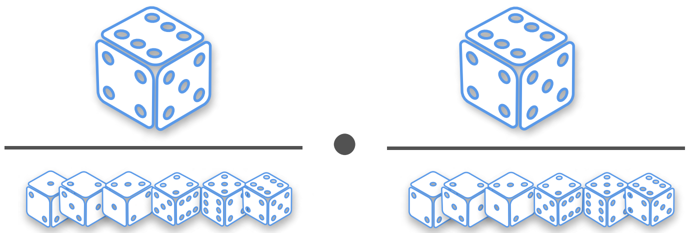


1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

2 dice

$\mathbf{P}(6,6) =$



$$\frac{1}{6} \cdot \frac{1}{6}$$

$$= \left(\frac{1}{6}\right)^2$$

Independent Events: Dice Example 1

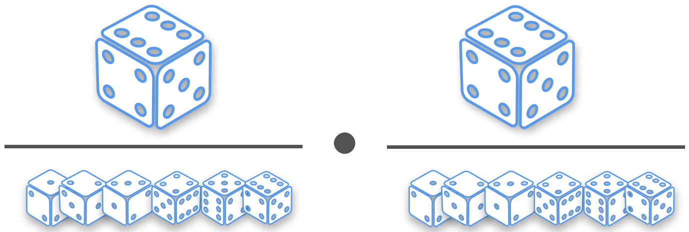
2 dice

$\frac{1}{6}$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\frac{1}{6}$

$\mathbf{P}(6,6) =$



$$\frac{1}{6} \bullet \frac{1}{6}$$

$$= \left(\frac{1}{6}\right)^2 \quad = \frac{1}{36}$$

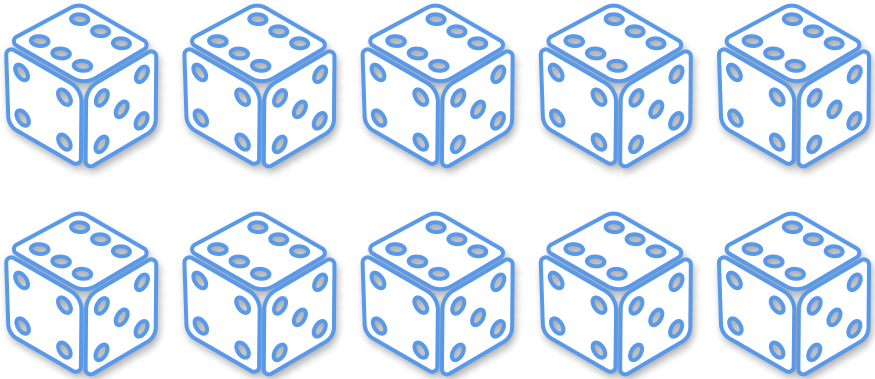
Independent Events: Dice Example 2

Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

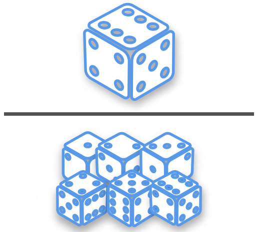
Independent Events: Dice Example 2

What is the probability of getting 10 sixes?



Independent Events: Dice Example 2

What is the probability of getting 10 sixes?



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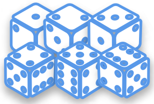
Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

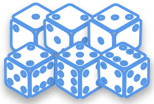
$\mathbf{P}(\text{10 sixes}) =$



Independent Events: Dice Example 2

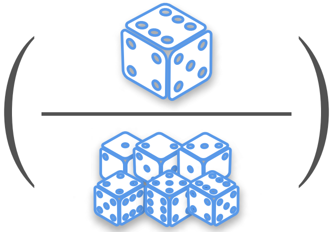
What is the probability of getting 10 sixes?

$$\mathbf{P}(\text{10 sixes}) = \left(\frac{\quad}{\quad} \right)$$



Independent Events: Dice Example 2

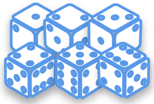
What is the probability of getting 10 sixes?

$\mathbf{P}(10 \text{ sixes}) =$  ^{10}

Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$\mathbf{P}(\text{10 sixes}) = \left(\frac{}{} \right)^{10}$$



$$= \left(\frac{1}{6}\right)^{10}$$

Introduction to probability

Birthday problem

6. The Birthday Problem

- Quiz: You have a party with 30 friends. What do you think is more likely, that there are two with the same birthday, or not?
  - Answer: Same birthday
- Calculate the probability that two people have the same birthday. Show that it's very close to 1.
- Question: How many people do you think there should be for the probability that 2 have the same birthday is 50?
  - Answer: 23
  - Show calculation

Quiz

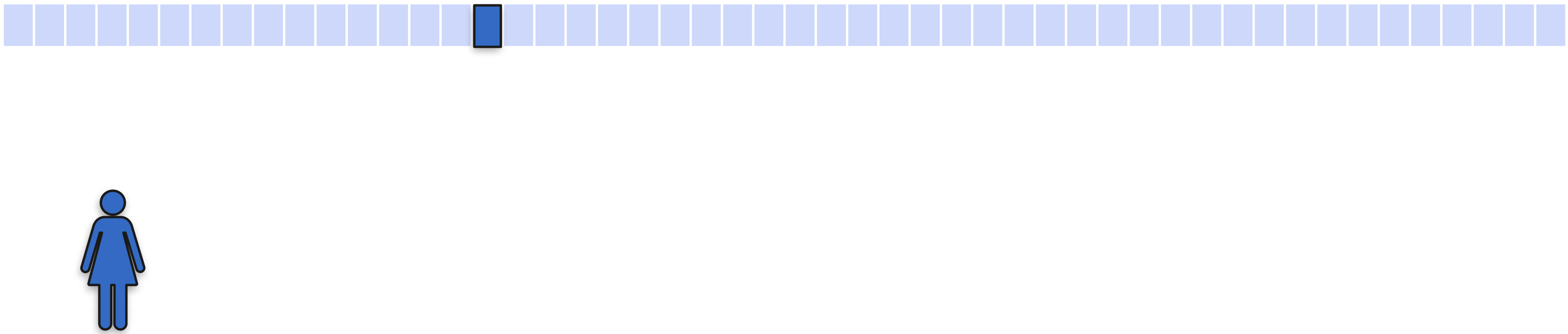
- You have 30 friends at a party. What do you think is more likely:
  - That there exist two people with the same birthday
  - That no two of them have the same birthday
- (Assume the year has 365 days, nobody has a birthday on Feb 29).

Quiz

- Answer: It’s more likely that 2 people have the same birthday.
- In fact, the probability of no two people having the same birthday is around 0.3.

Probability That Everyone Has a Different Birthday

Probability That Everyone Has a Different Birthday



$$\frac{365}{365}$$

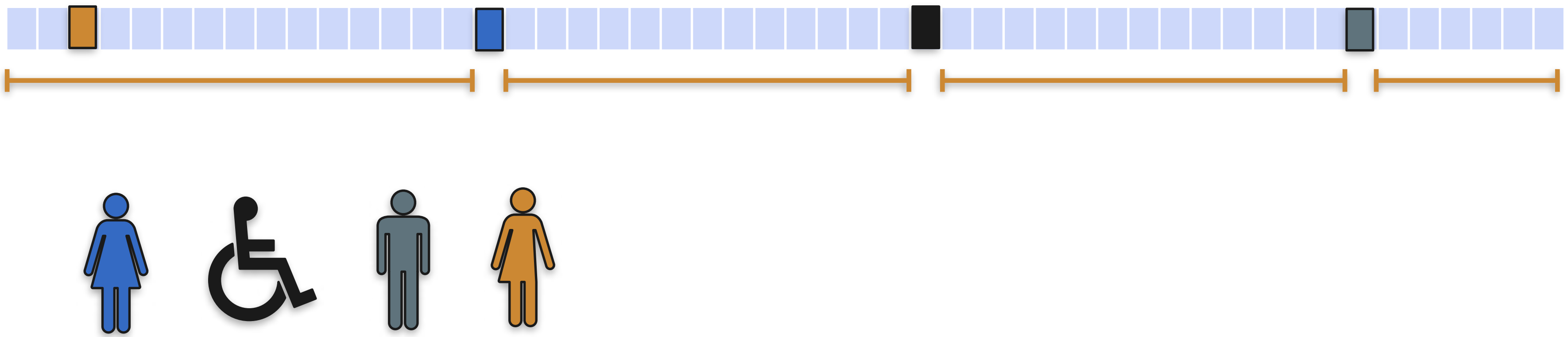
Probability That Everyone Has a Different Birthday

$$\frac{365}{365} \quad \frac{364}{365}$$

Probability That Everyone Has a Different Birthday

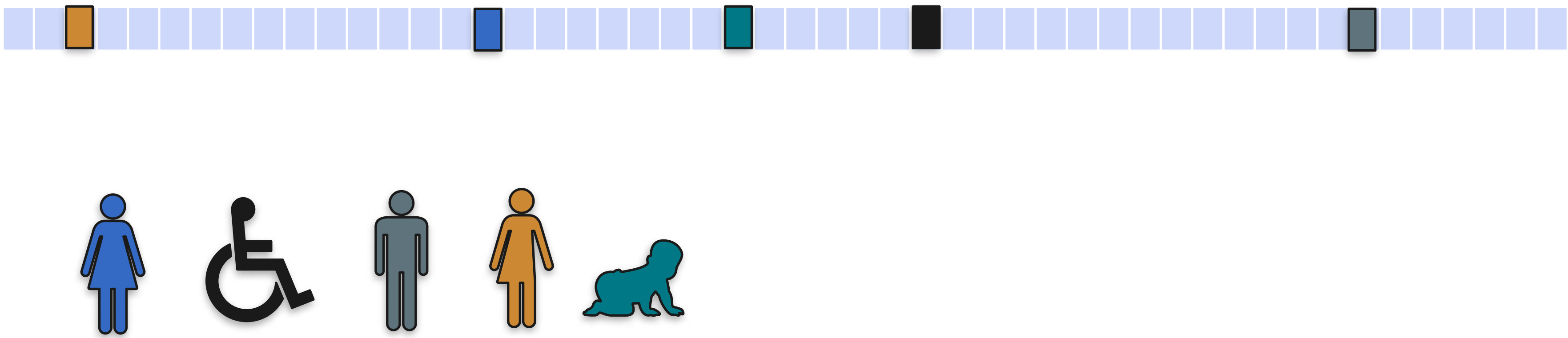
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365}$$

Probability That Everyone Has a Different Birthday



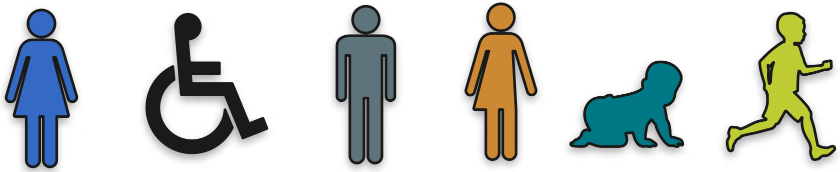
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365}$$

Probability That Everyone Has a Different Birthday



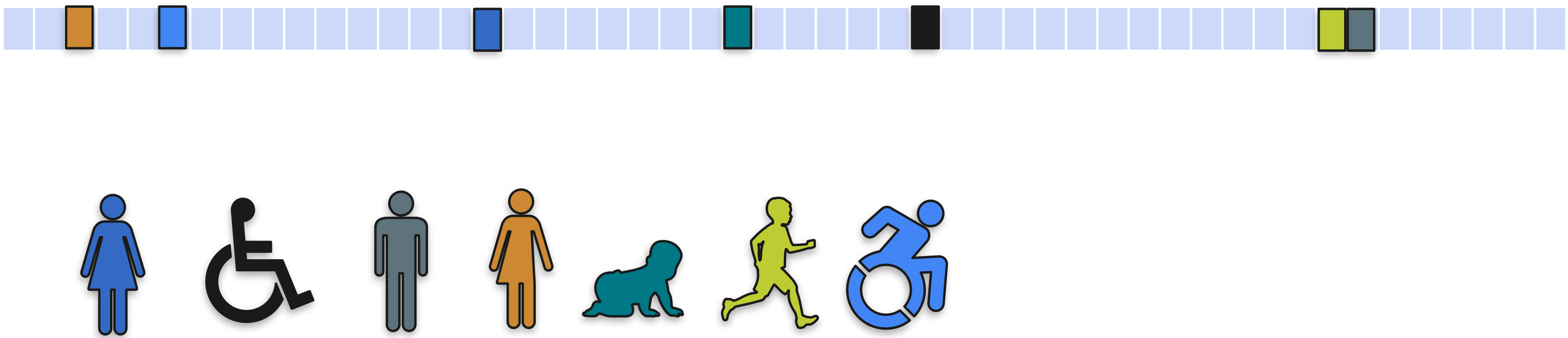
$\frac{365}{365}$ $\frac{364}{365}$ $\frac{363}{365}$ $\frac{362}{365}$ $\frac{361}{365}$

Probability That Everyone Has a Different Birthday



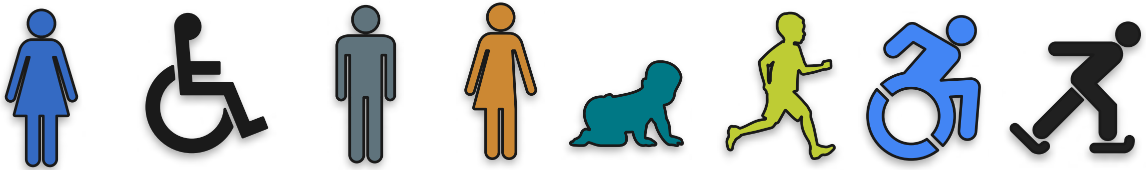
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365} \quad \frac{360}{365}$$

Probability That Everyone Has a Different Birthday



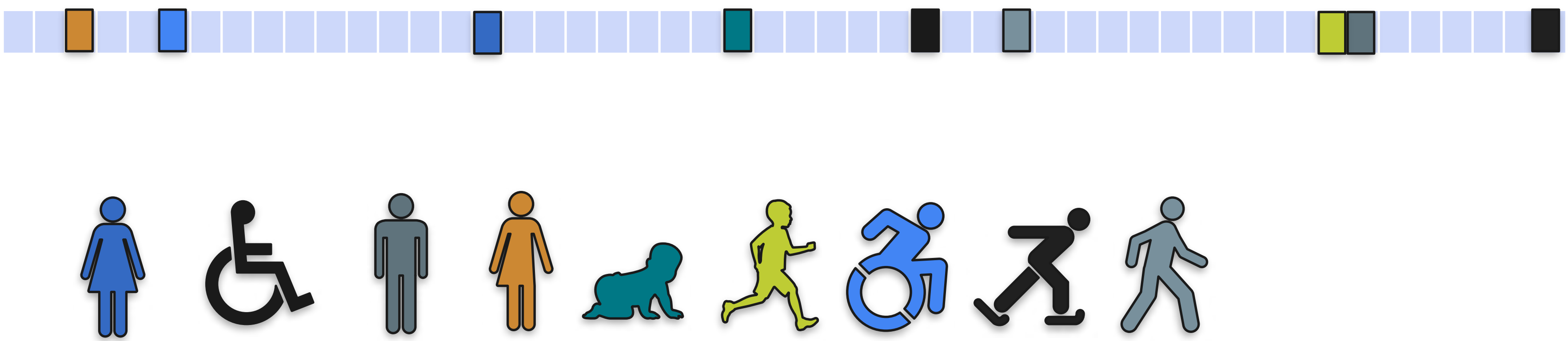
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365} \quad \frac{360}{365} \quad \frac{359}{365}$$

Probability That Everyone Has a Different Birthday



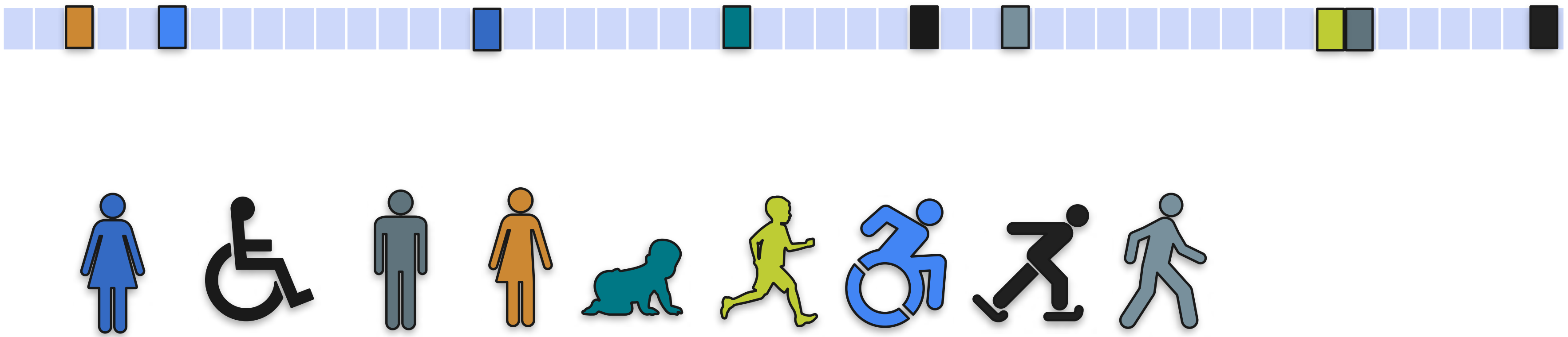
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365} \quad \frac{360}{365} \quad \frac{359}{365} \quad \frac{358}{365}$$

Probability That Everyone Has a Different Birthday



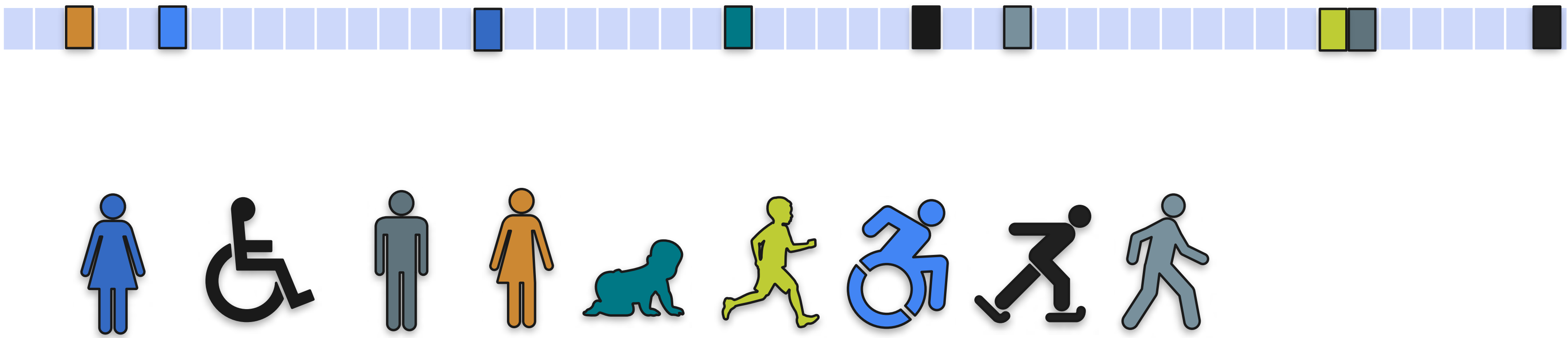
$$\frac{365}{365} \quad \frac{364}{365} \quad \frac{363}{365} \quad \frac{362}{365} \quad \frac{361}{365} \quad \frac{360}{365} \quad \frac{359}{365} \quad \frac{358}{365} \quad \frac{357}{365}$$

Probability That Everyone Has a Different Birthday



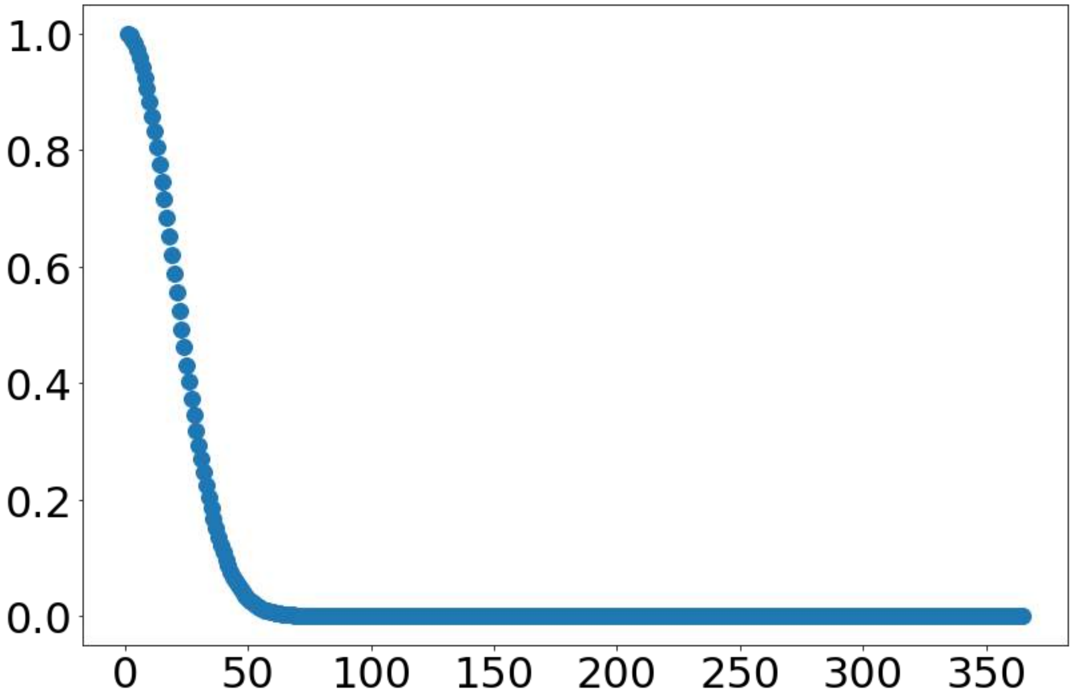
$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} =$$

Probability That Everyone Has a Different Birthday



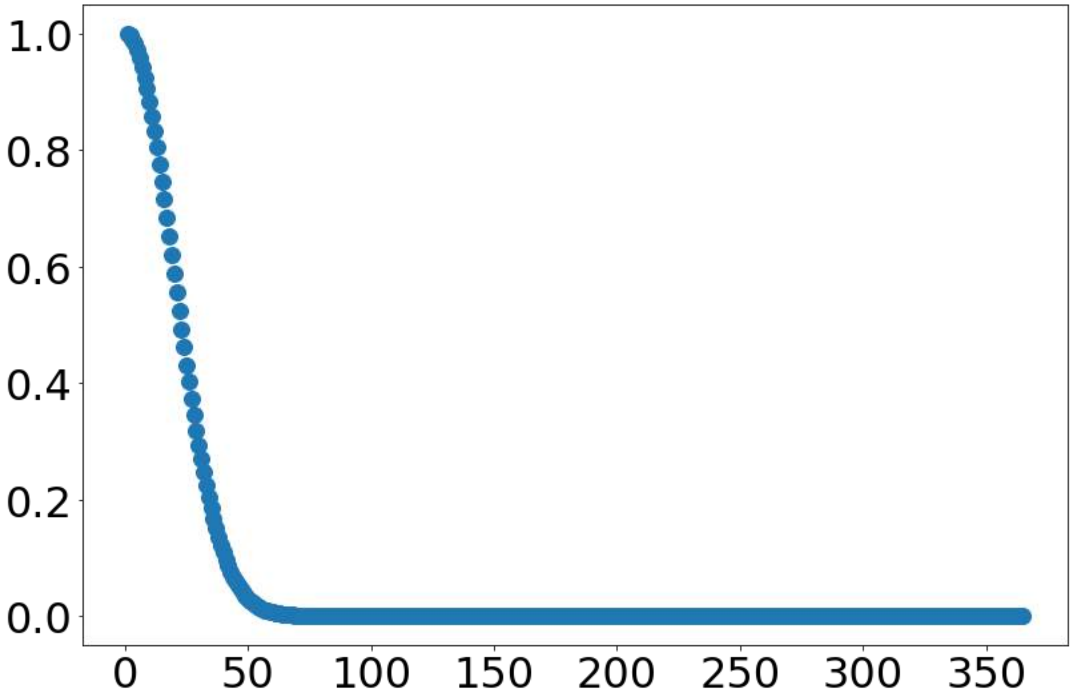
$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} = 0.905$$

Probability That no Two People Have the Same Birthday



Probability That no Two People Have the Same Birthday

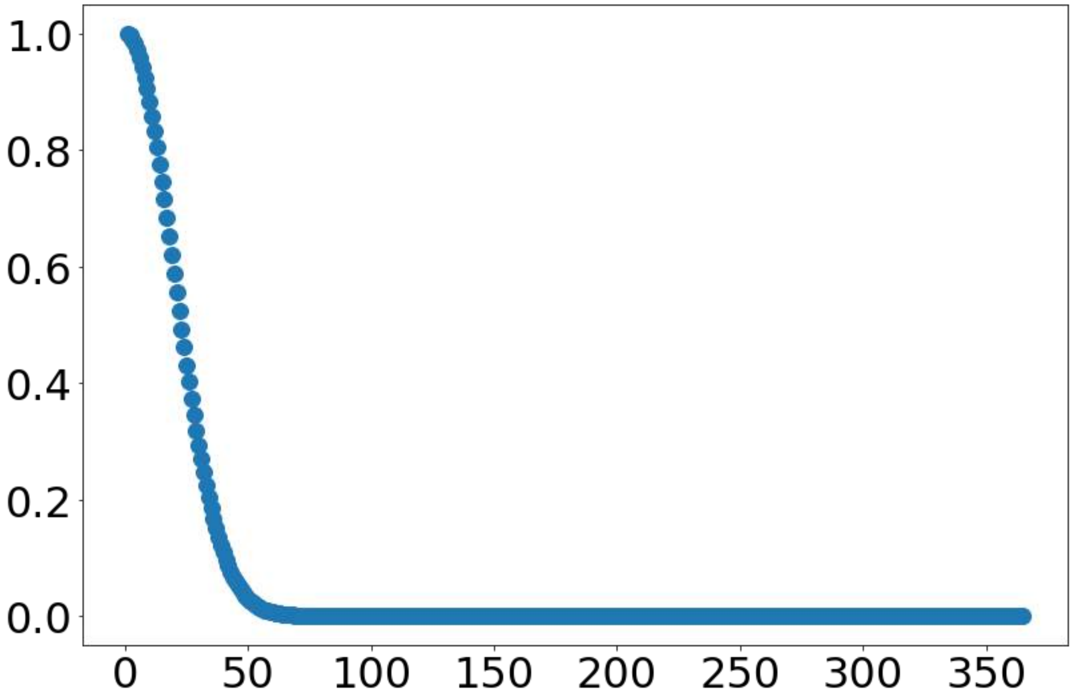
1 person: 1



Probability That no Two People Have the Same Birthday

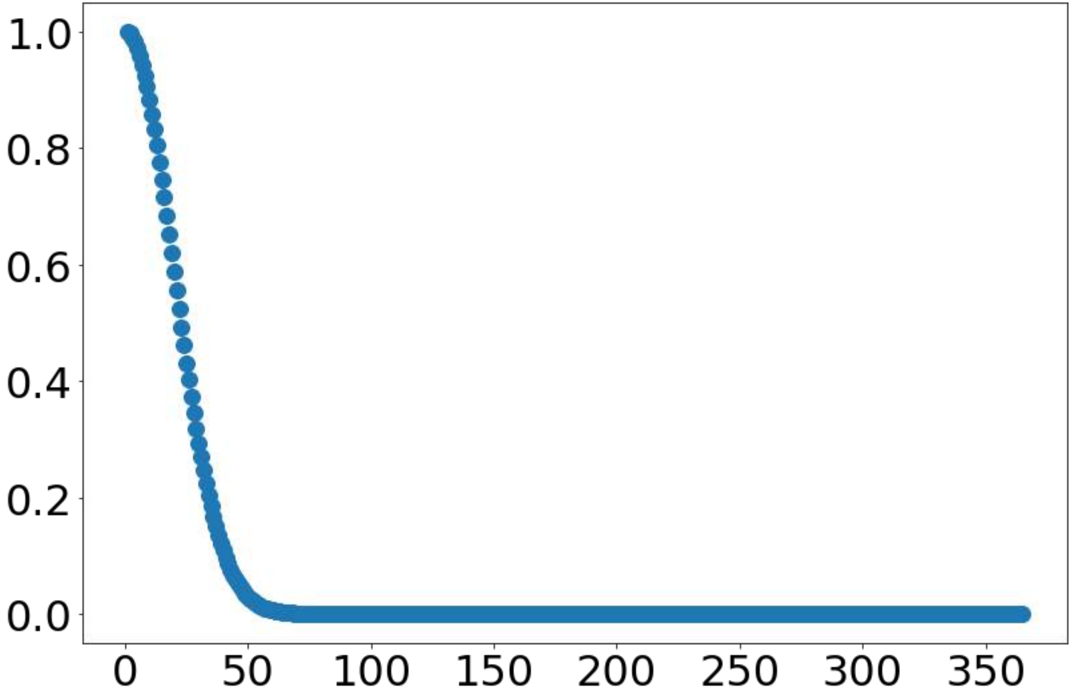
1 person: 1

2 people: 0.997



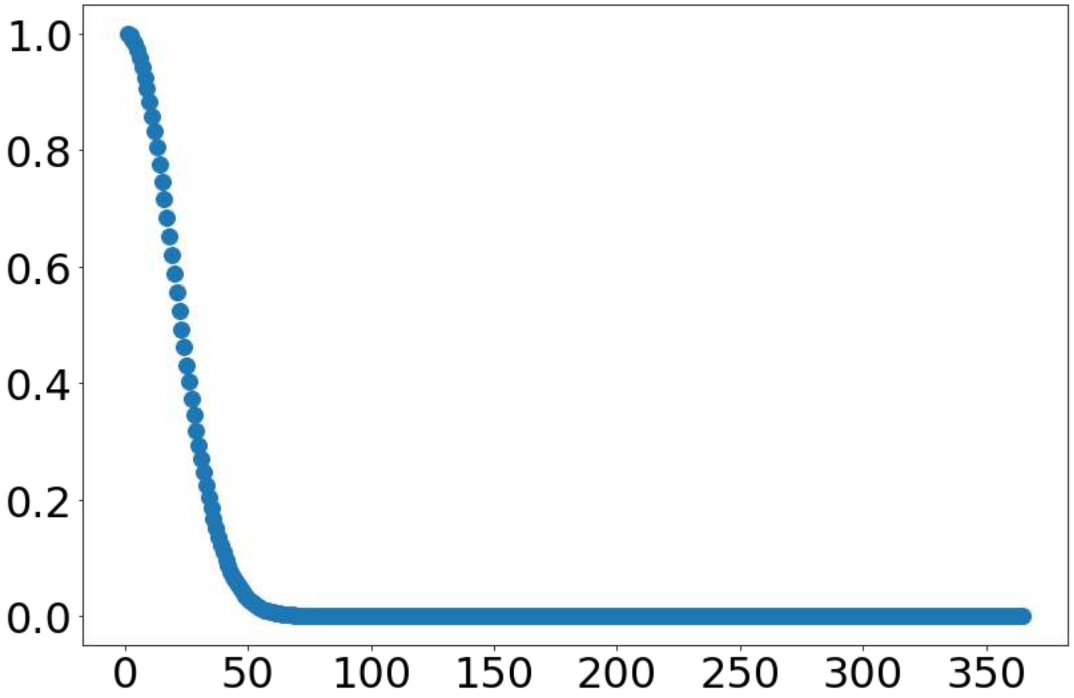
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992



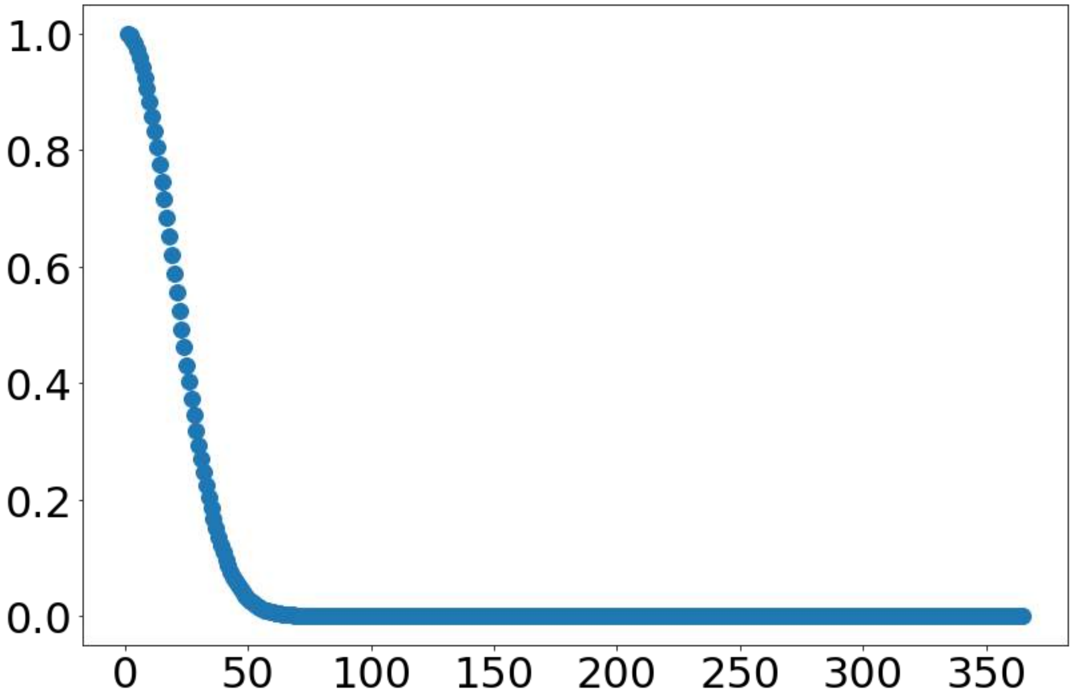
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984



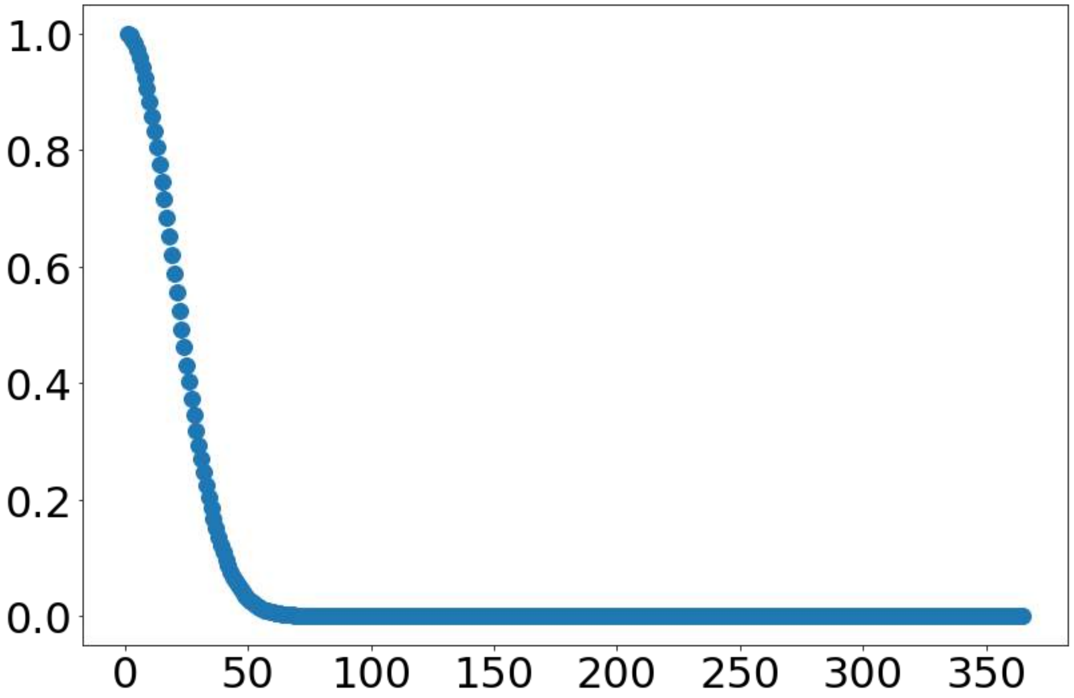
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973



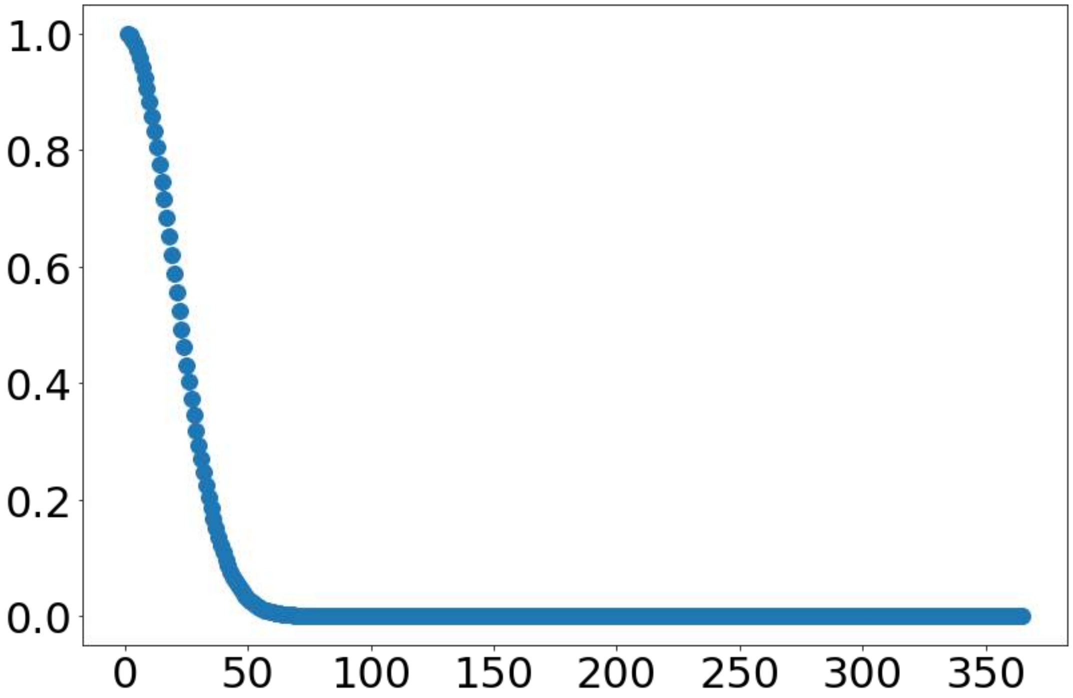
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883



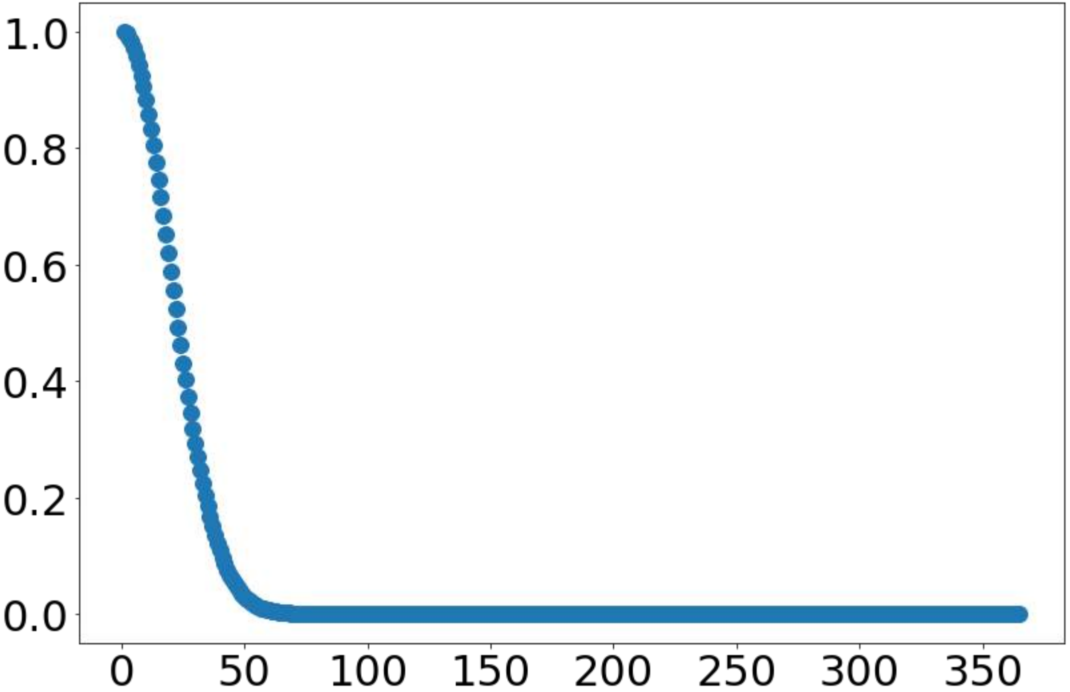
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589



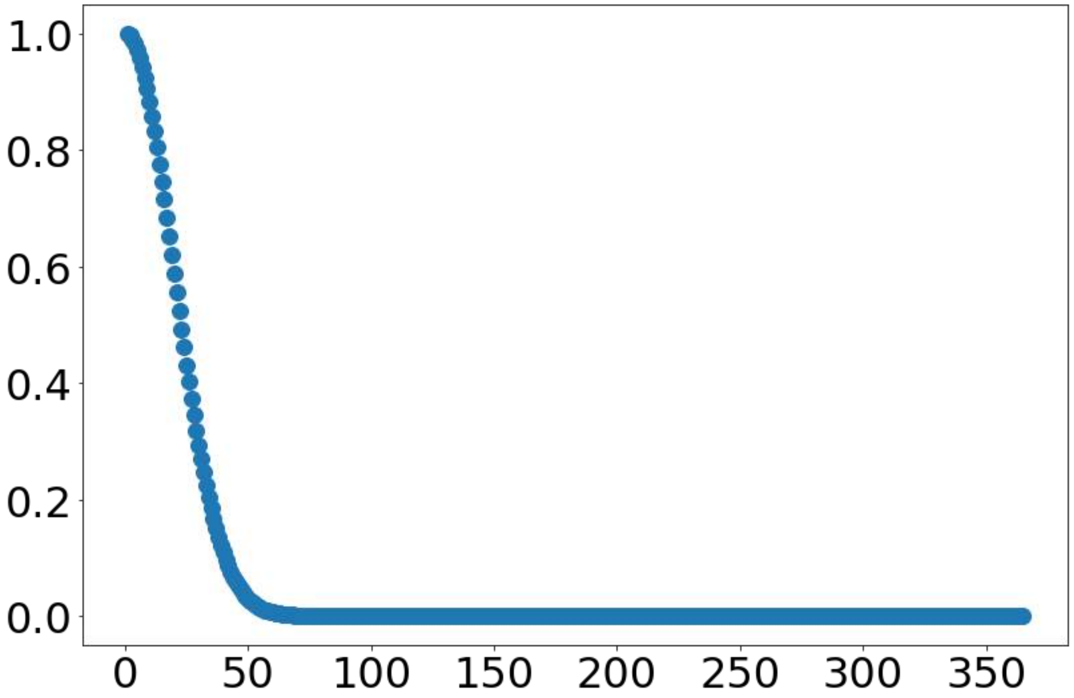
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493



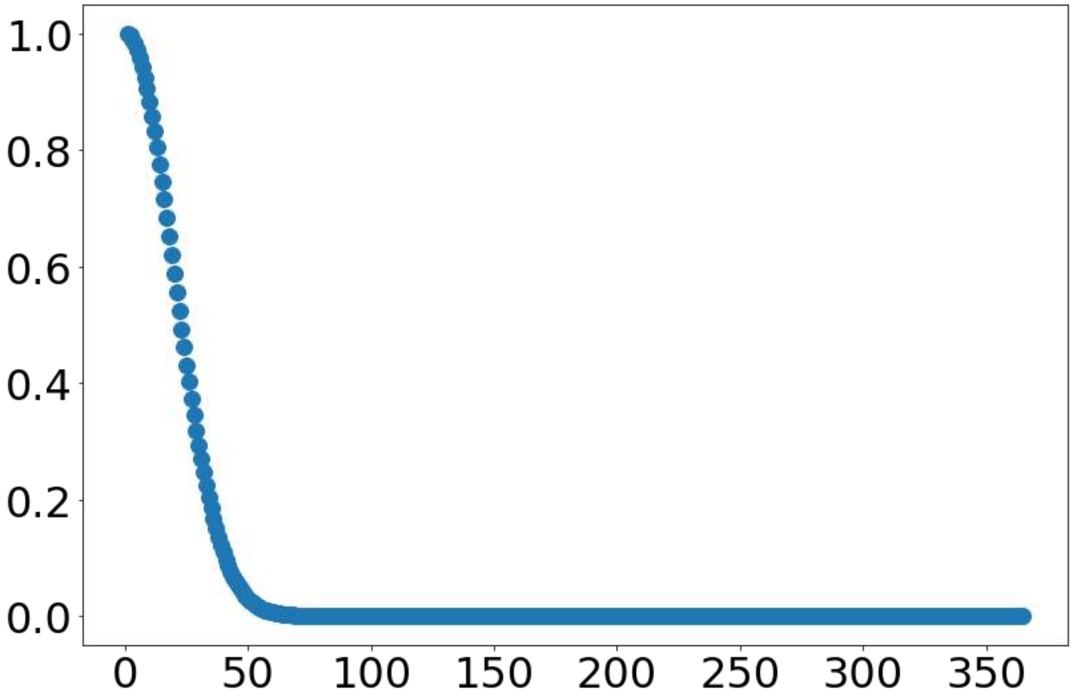
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294



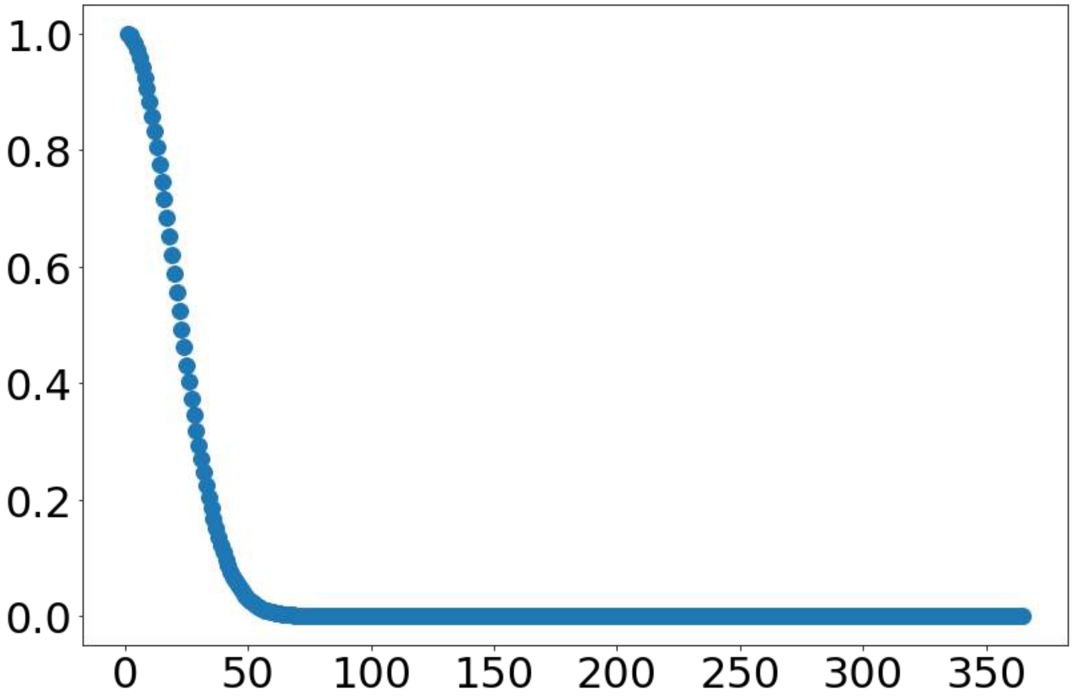
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030



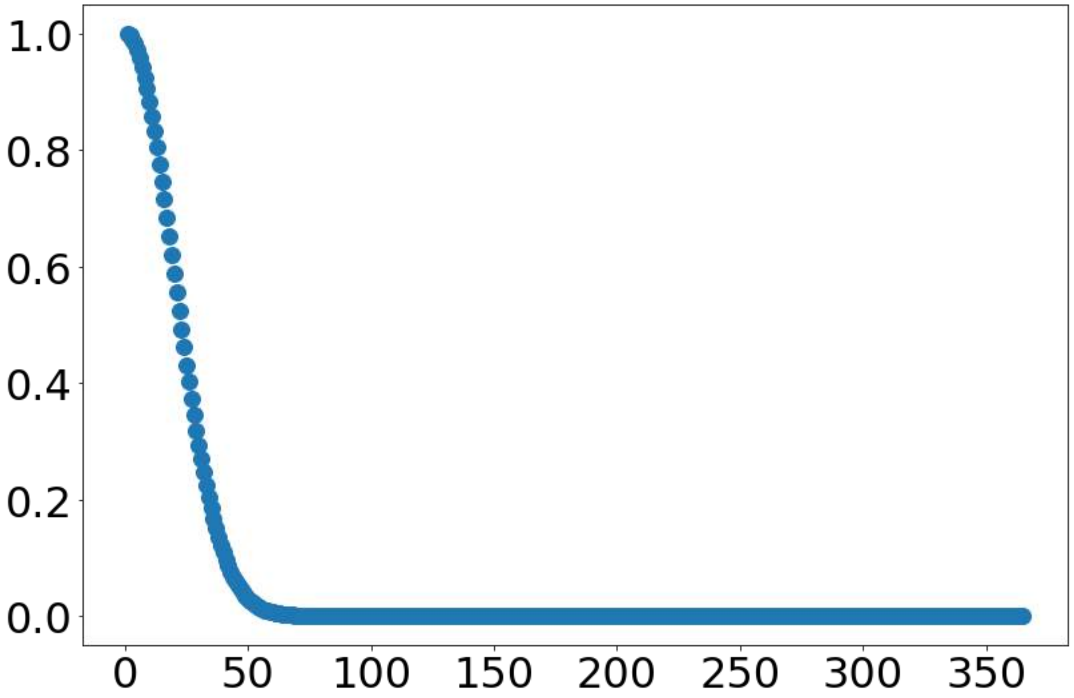
Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003



Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



Probability That no Two People Have the Same Birthday

1 person: 1
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5 people: 0.973
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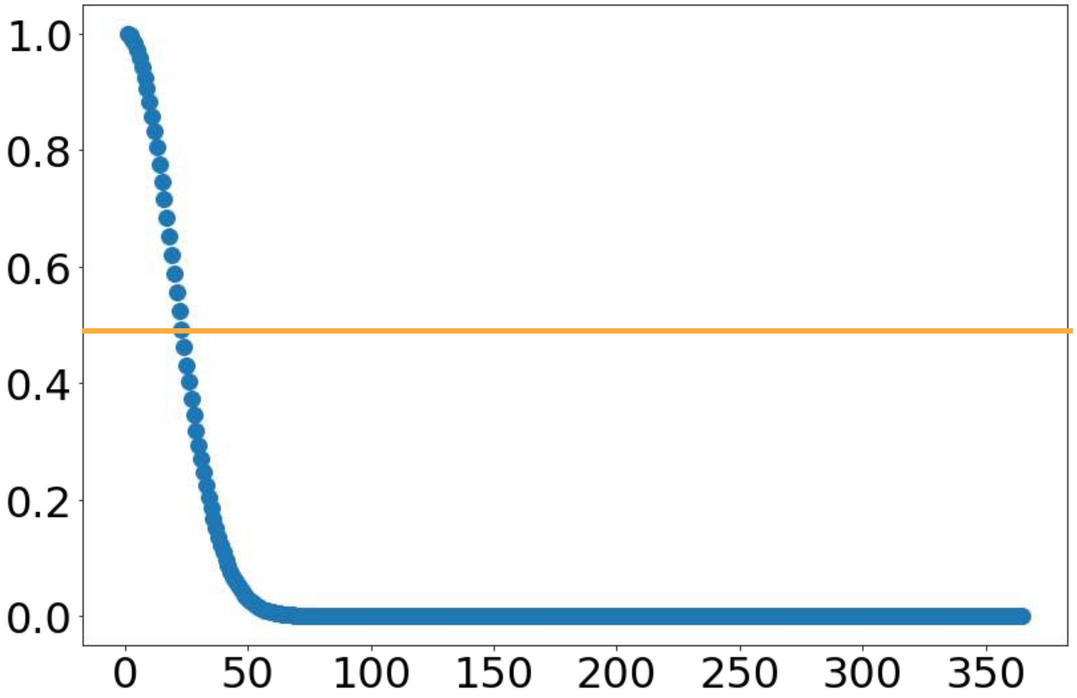
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0



Probability That no Two People Have the Same Birthday

1 person: 1
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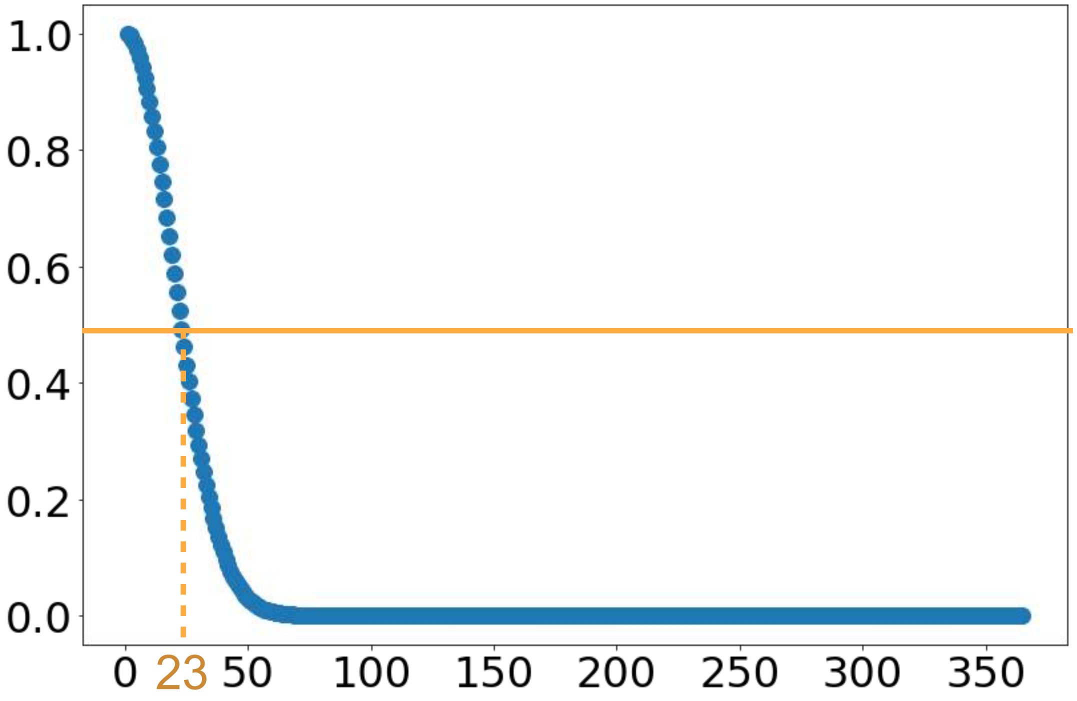
50 people: 0.030
100 people: 0.0000003
365 people: 0



Probability That no Two People Have the Same Birthday

1 person: 1
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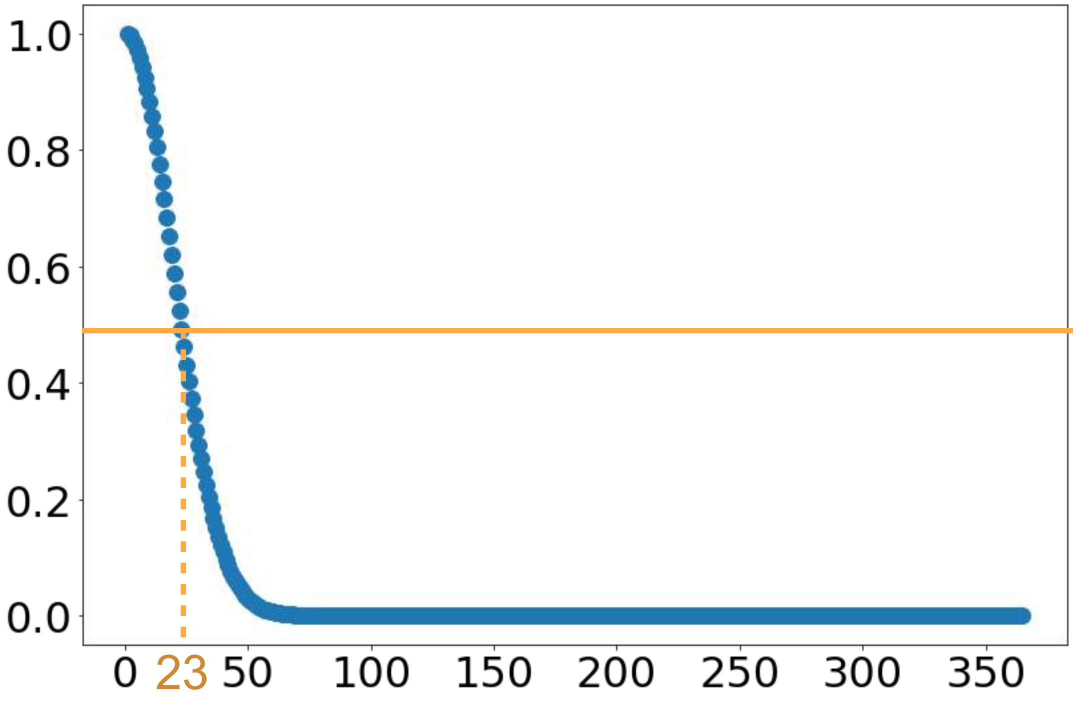
365 people: 0



DeepLearning.AI

Introduction to probability

Conditional probability

Conditional Probability: Coin Example 1

50%

50%

Conditional Probability: Coin Example 1

50%   50%

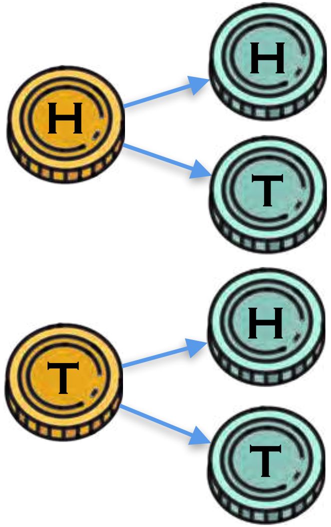
What is the probability of landing on heads twice?

Conditional Probability: Coin Example 1

50% 50%

What is the probability of landing on heads twice?

1st **2nd**

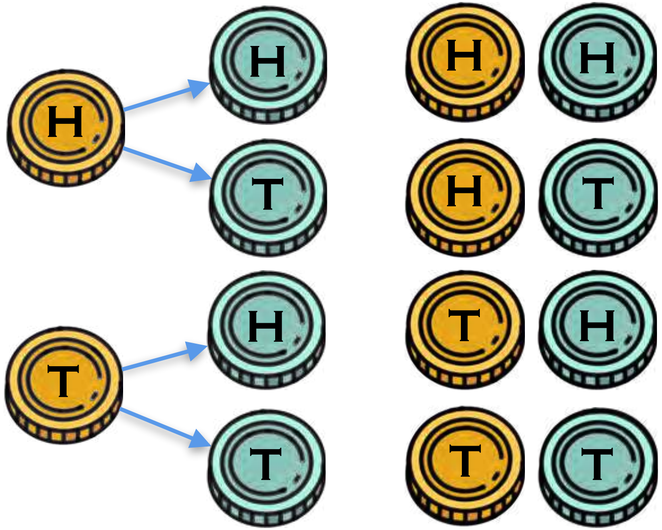


Conditional Probability: Coin Example 1

50% 50%

What is the probability of landing on heads twice?

1st **2nd**

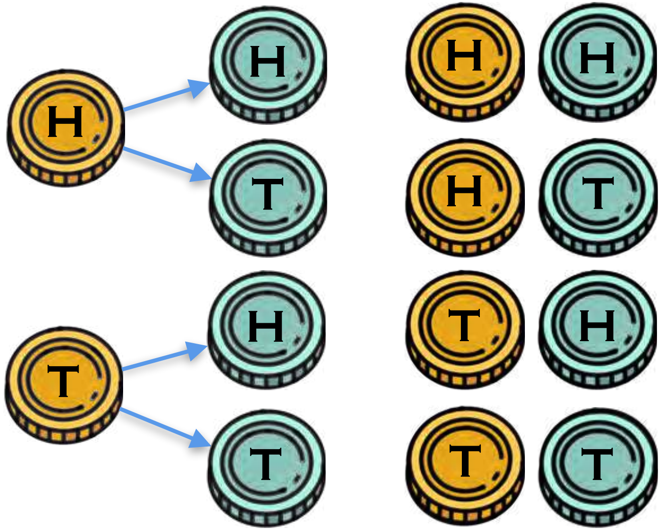


Conditional Probability: Coin Example 1

50% 50%

What is the probability of landing on heads twice?

1st **2nd**



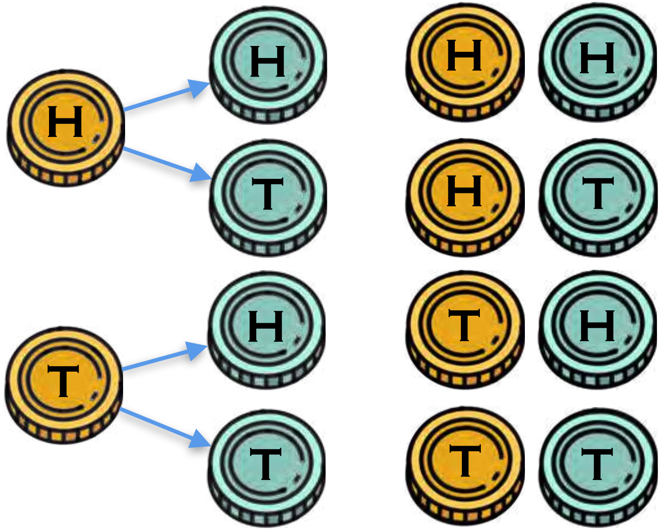
$$\mathbf{P}(HH) = \underline{\qquad\qquad\qquad}$$

Conditional Probability: Coin Example 1

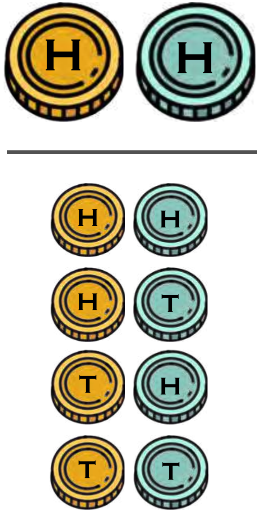
50% 50%

What is the probability of landing on heads twice?

1st **2nd**



$\mathbf{P}(HH) =$

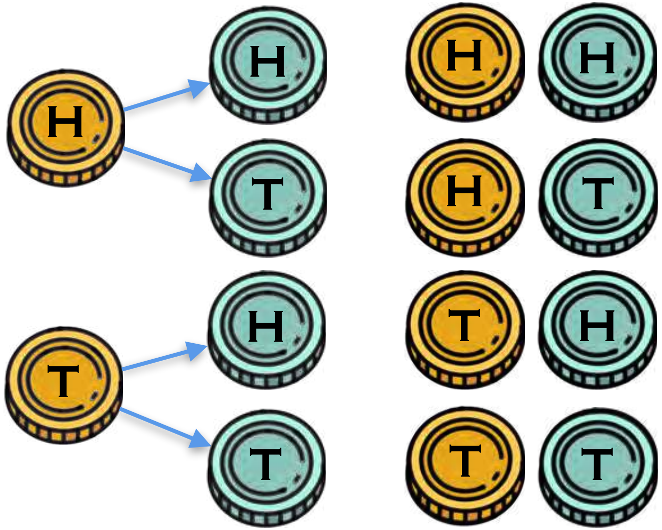


Conditional Probability: Coin Example 1

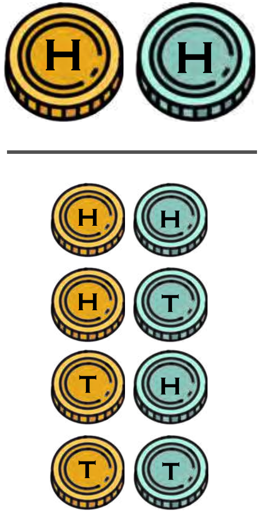
50% 50%

What is the probability of landing on heads twice?

1st **2nd**



$\mathbf{P}(HH) =$



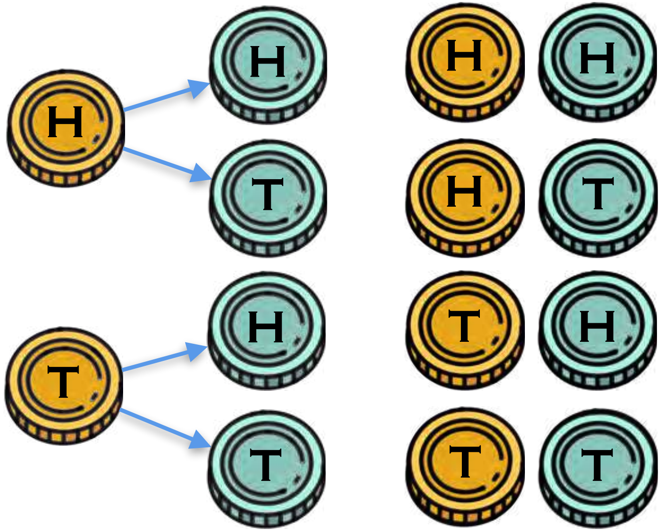
$= \frac{1}{4}$

Conditional Probability: Coin Example 1

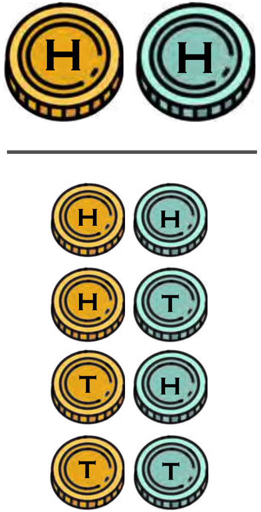
50% 50%

What is the probability of landing on heads twice?

1st 2nd **GIVEN that the first one is heads**



$\mathbf{P}(HH) =$



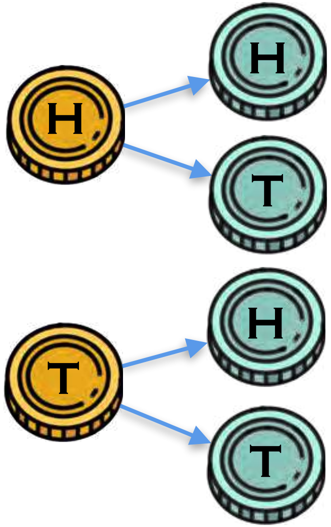
$= \frac{1}{4}$

Conditional Probability: Coin Example 1

50% 50%

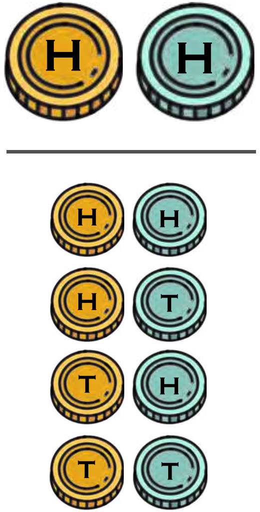
What is the probability of landing on heads twice?

1st **2nd**



GIVEN that the first one is heads

$\mathbf{P}(HH) =$



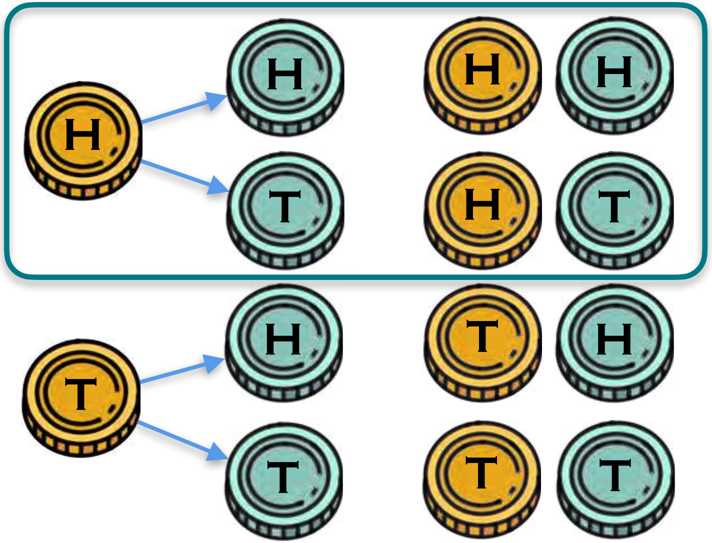
$= \frac{1}{4}$

Conditional Probability: Coin Example 1

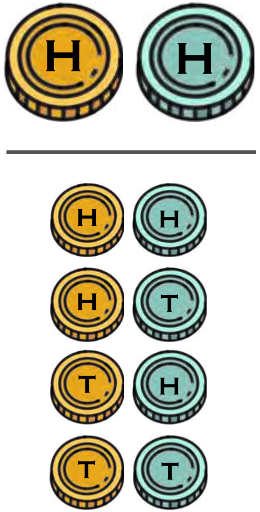
50% 50%

What is the probability of landing on heads twice?

1st **2nd** **GIVEN that the first one is heads**



$$\mathbf{P}(HH) = \frac{\text{HH}}{\text{HH, HT, TH, TT}} = \frac{1}{4}$$

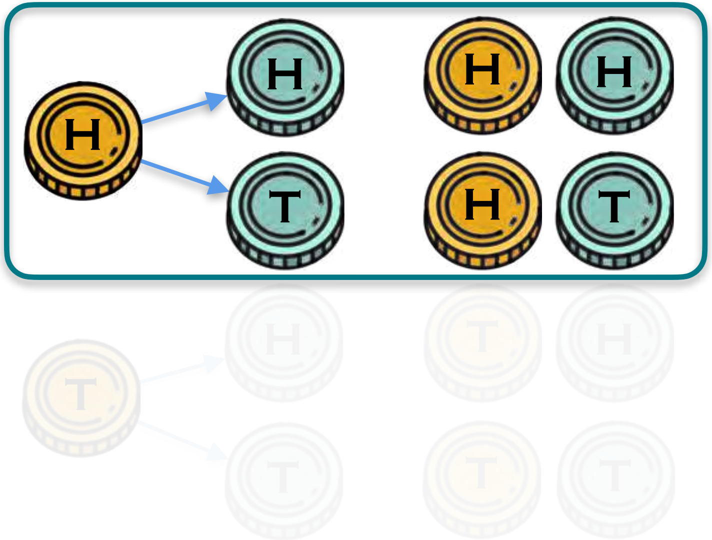


Conditional Probability: Coin Example 1

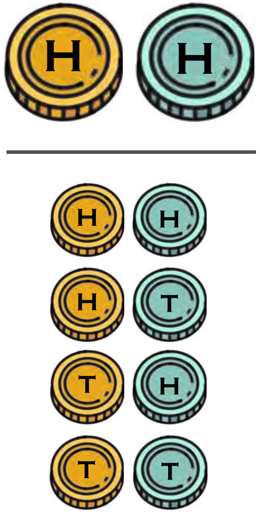
50% 50%

What is the probability of landing on heads twice?

1st 2nd **GIVEN that the first one is heads**



$$\mathbf{P}(HH) = \frac{\text{HH}}{\text{HH, HT, TH, TT}} = \frac{1}{4}$$

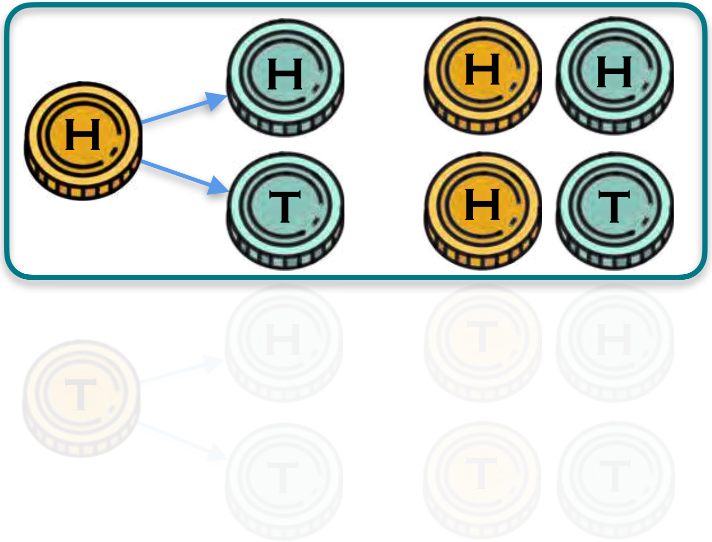


Conditional Probability: Coin Example 1

50% 50%

What is the probability of landing on heads twice?

1st **2nd** **GIVEN that the first one is heads**



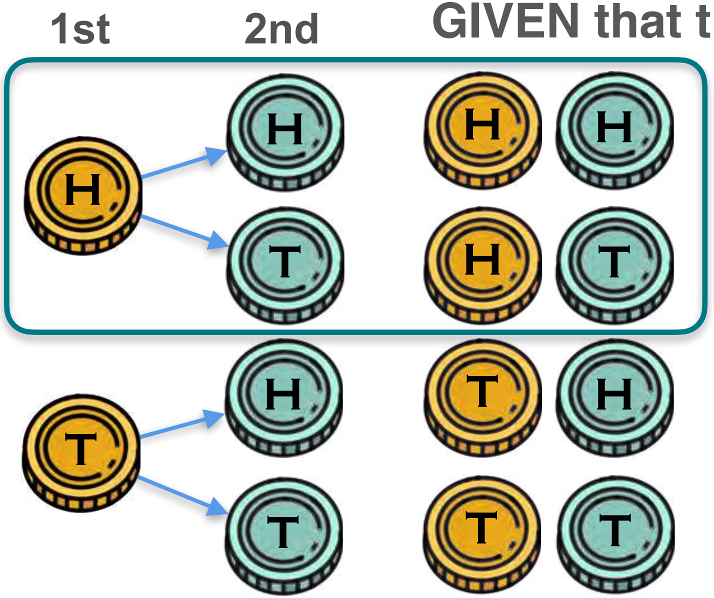
$$\mathbf{P}(HH) = \frac{HH}{HH \quad HT} = \frac{1}{4}$$

Conditional Probability: Coin Example 1

50% 50%

What is the probability of landing on heads twice?

GIVEN that the first one is heads



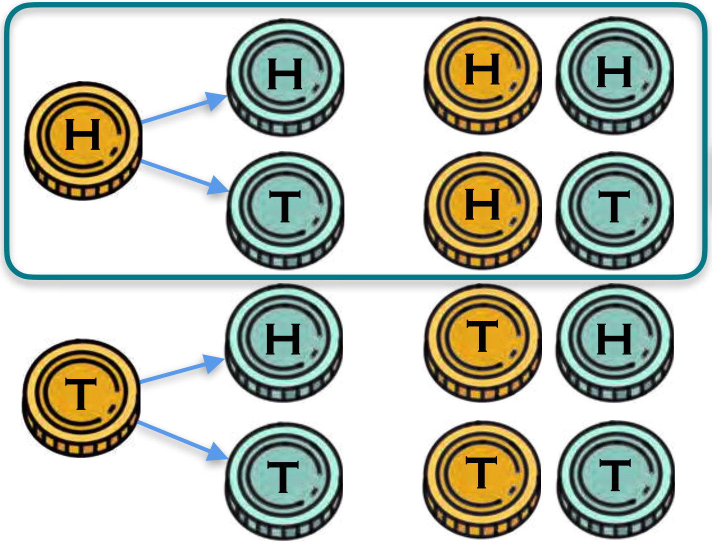
$$\mathbf{P}(HH \mid \text{1st is } H) = \frac{\text{HH}}{\text{HH, HT}} = \frac{1}{2}$$

Conditional Probability: Coin Example 1

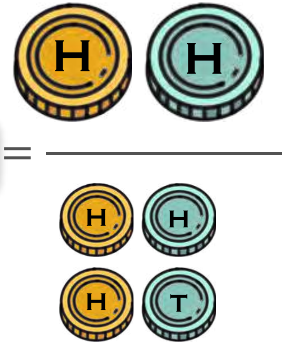
50% 50%

What is the probability of landing on heads twice?

1st 2nd **GIVEN that the first one is heads**



$\boxed{\mathbf{P}(HH \mid \text{1st is } H)} =$



$= \frac{1}{2}$

DeepLearning.AI

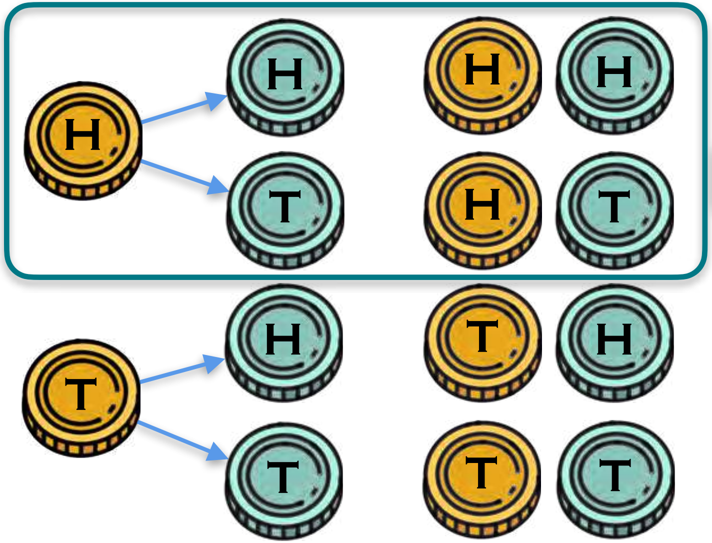
Conditional Probability: Coin Example 1

50% 50%

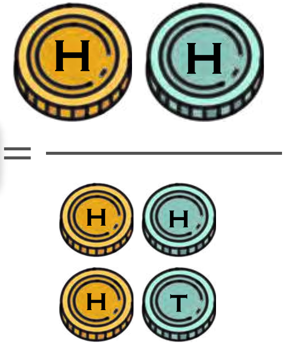
What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$\mathbf{P}(HH \mid \text{1st is } H) =$



$= \frac{1}{2}$

Conditional Probability: Coin Example 1

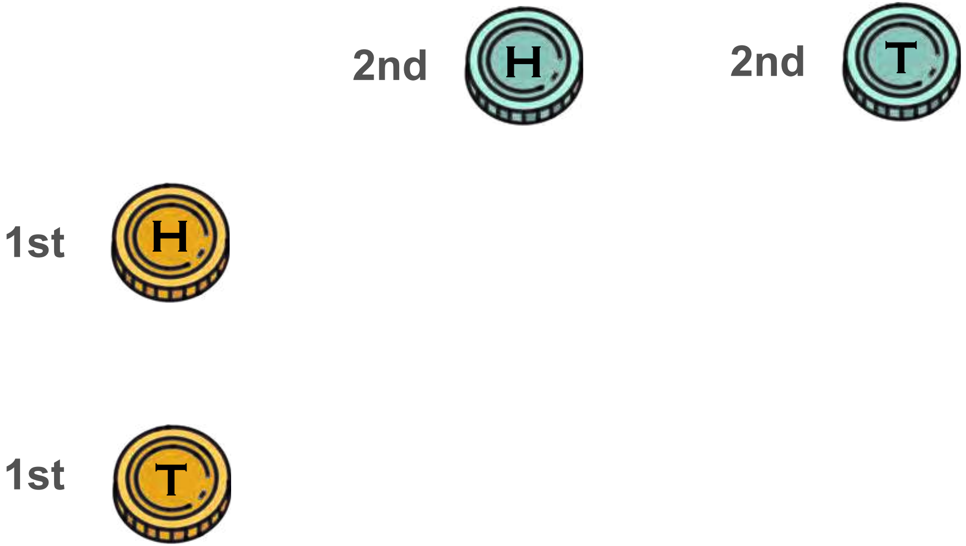
What is the probability of landing on heads twice?

GIVEN that the first one is heads

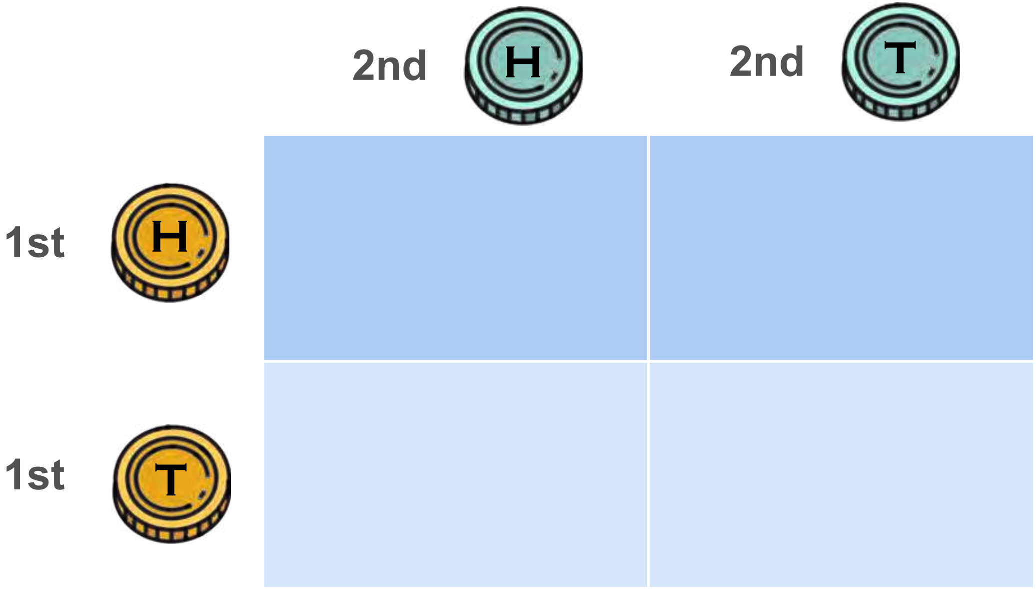
$$\mathbf{P}(HH \boxed{|\ \text{1st is } H)}$$

Conditional Probability: Coin Example 1

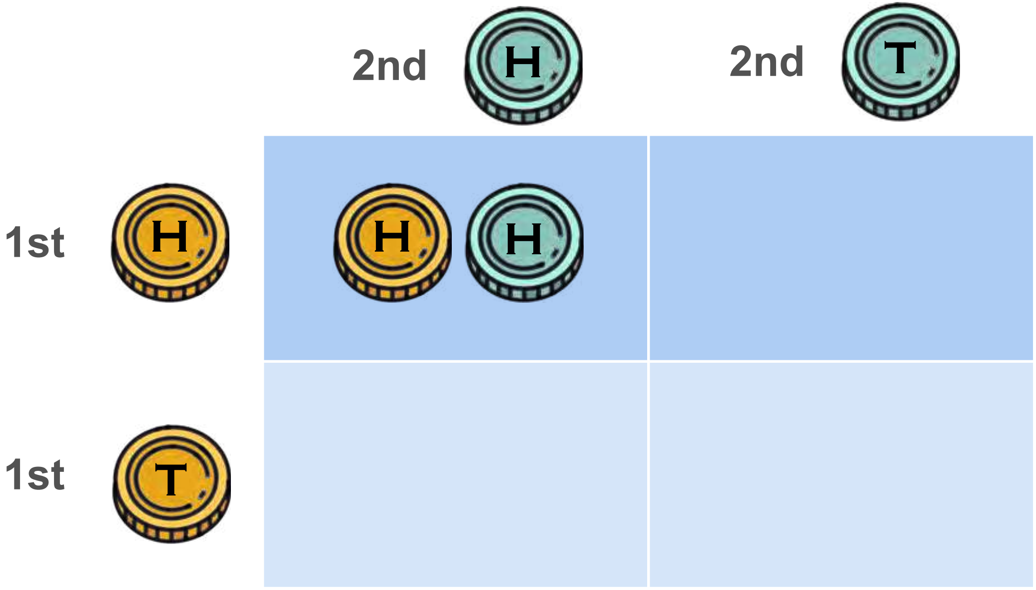
Conditional Probability: Coin Example 1



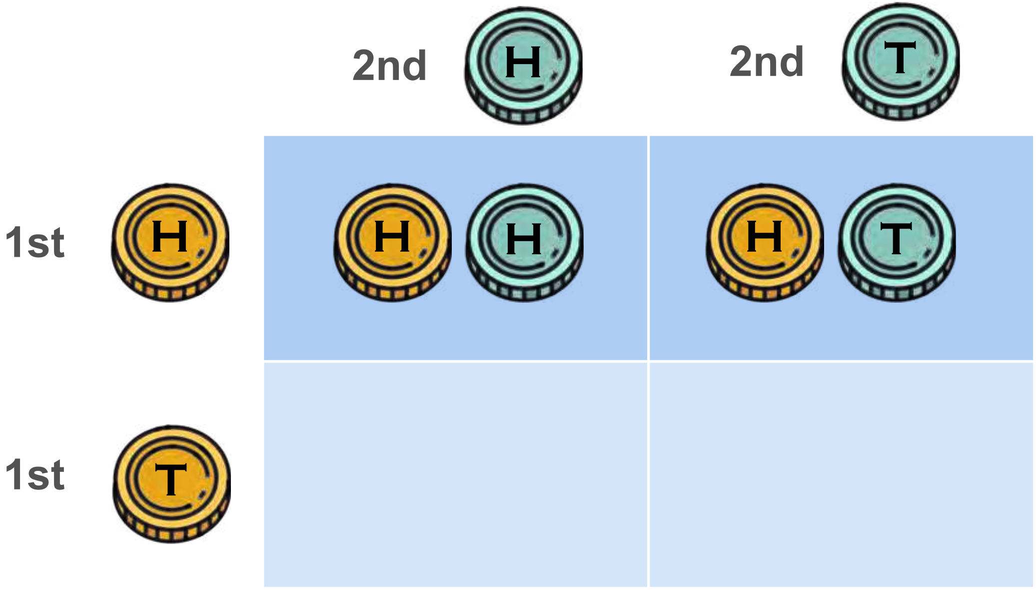
Conditional Probability: Coin Example 1



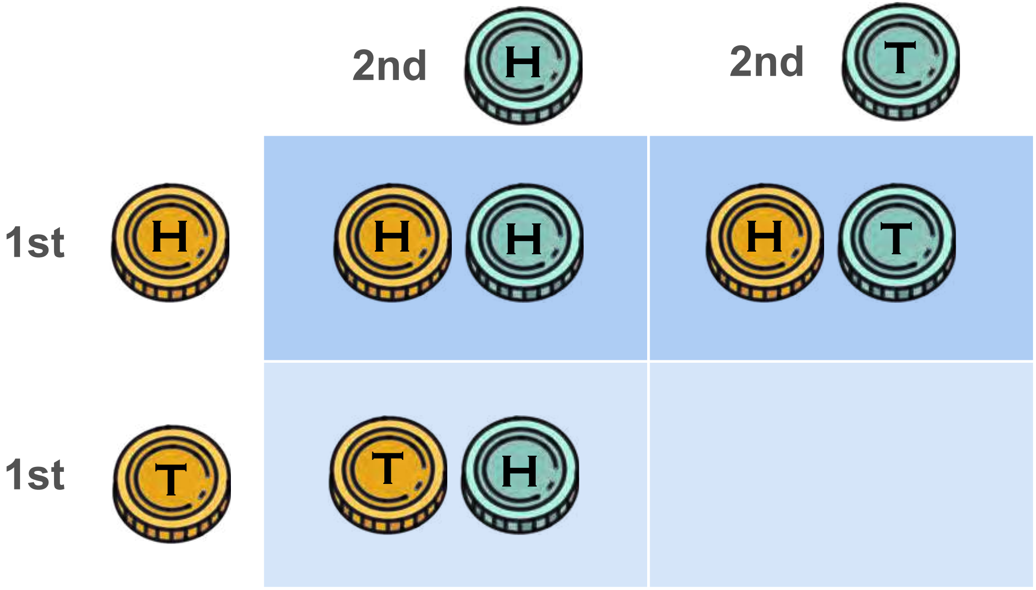
Conditional Probability: Coin Example 1



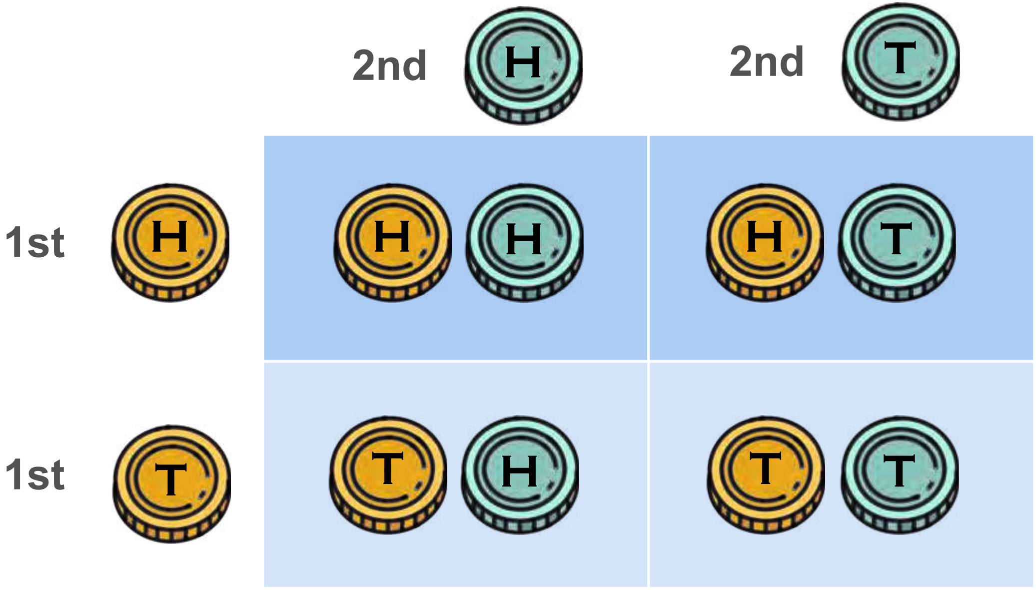
Conditional Probability: Coin Example 1



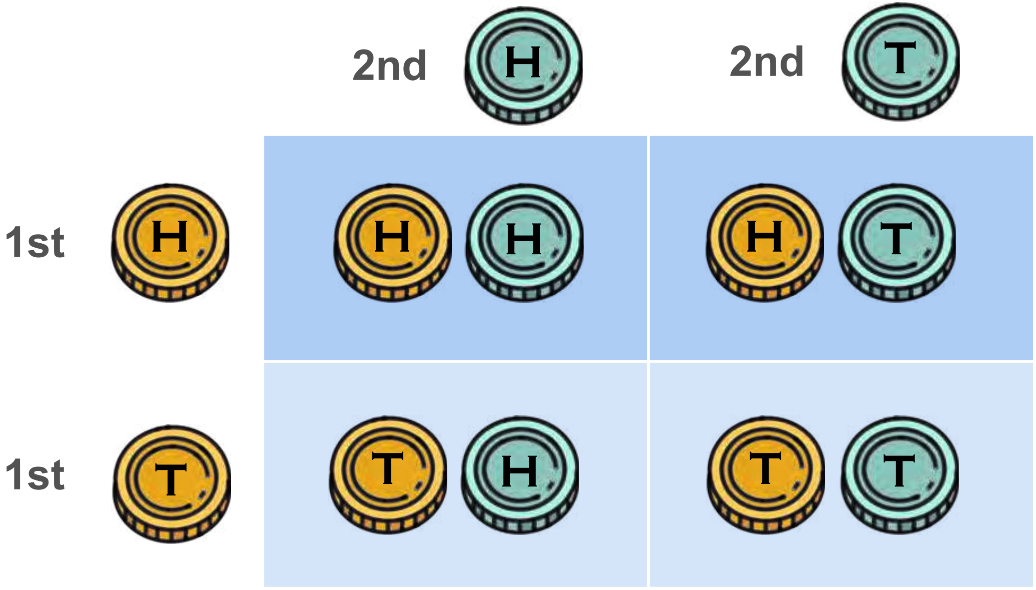
Conditional Probability: Coin Example 1



Conditional Probability: Coin Example 1

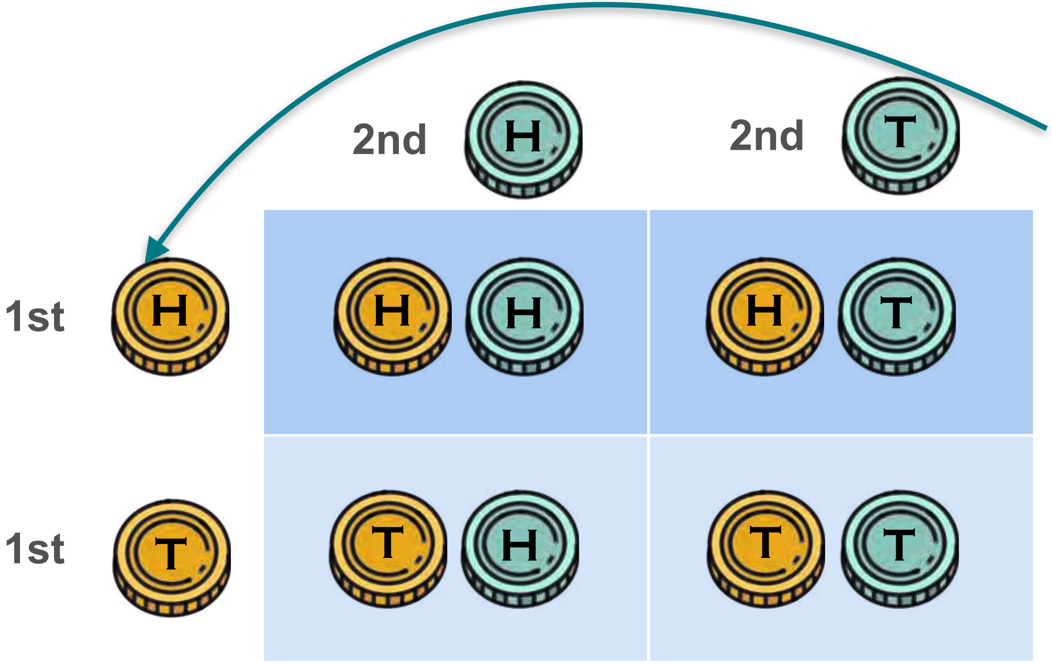


Conditional Probability: Coin Example 1



Probability of landing on heads twice

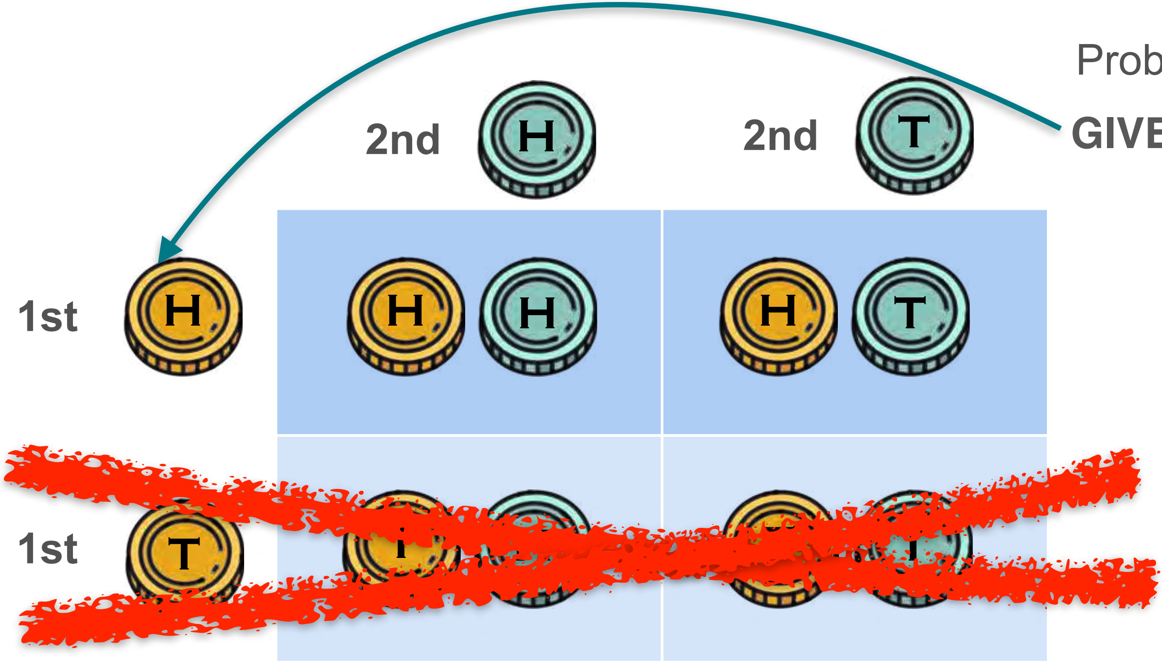
Conditional Probability: Coin Example 1



Probability of landing on heads twice

GIVEN that the first one is heads

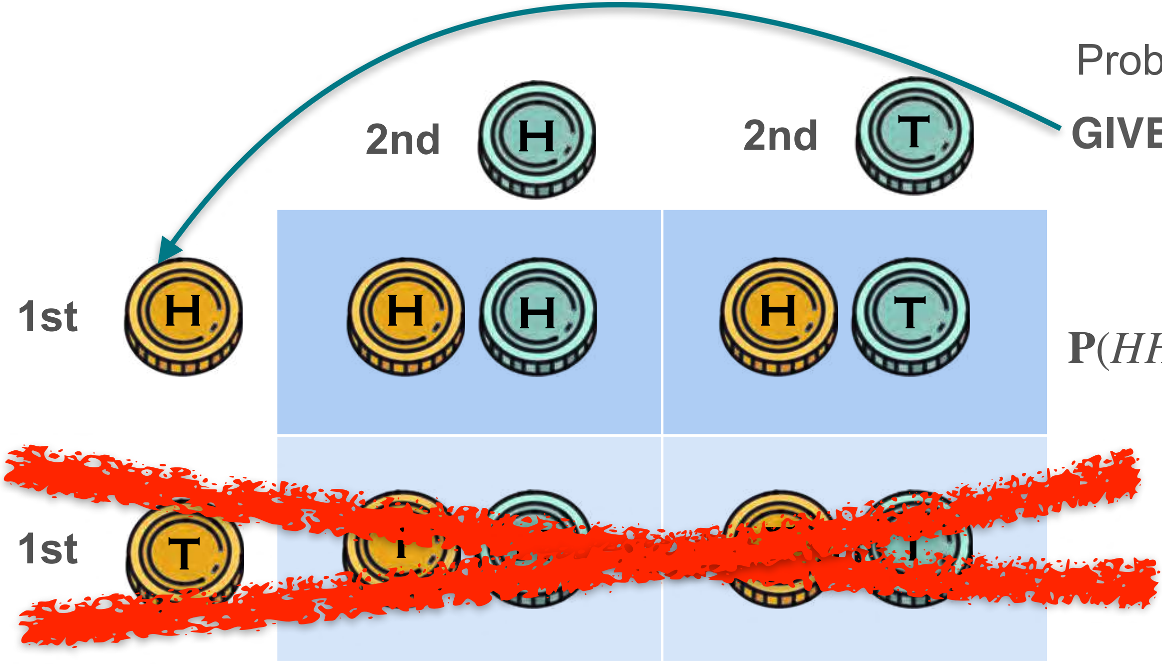
Conditional Probability: Coin Example 1



Probability of landing on heads twice

GIVEN that the first one is heads

Conditional Probability: Coin Example 1

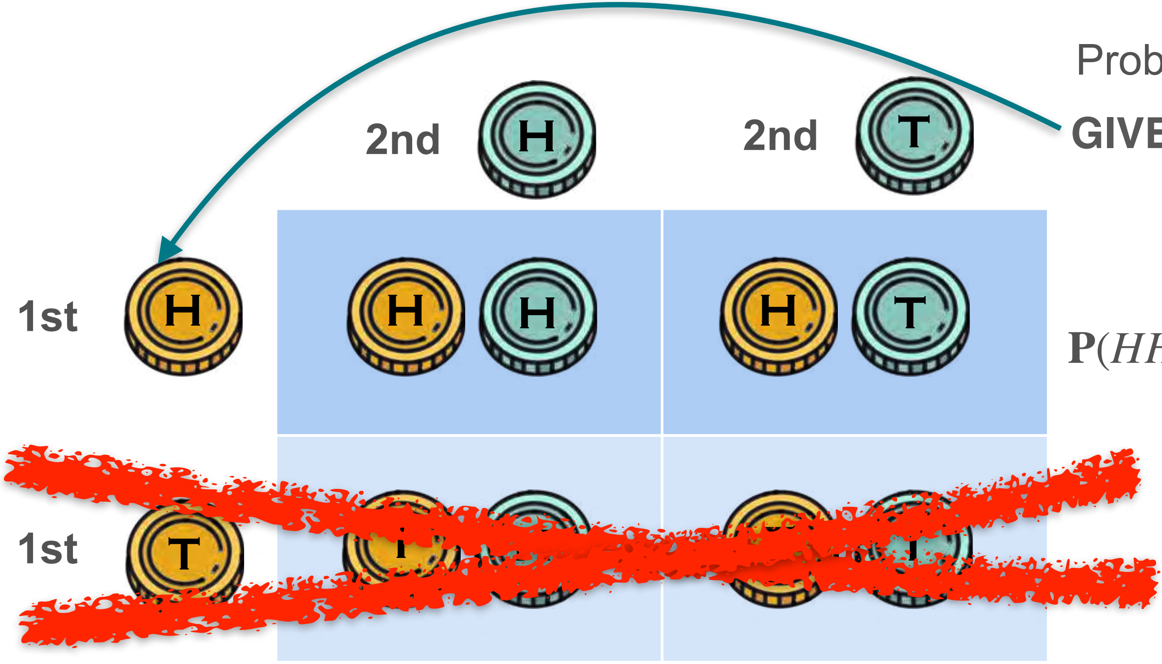


Probability of landing on heads twice

GIVEN that the first one is heads

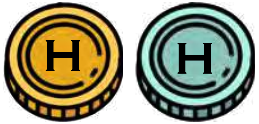
$$\mathbf{P}(HH \,|\, \text{1st is } H) = \underline{\qquad\qquad}$$

Conditional Probability: Coin Example 1

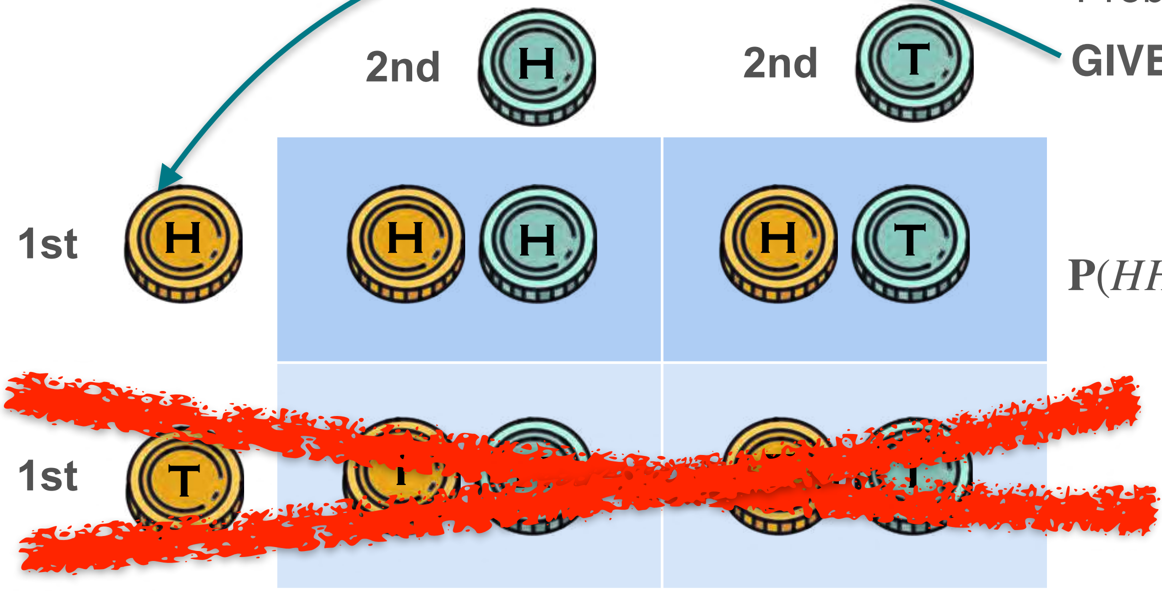


Probability of landing on heads twice

GIVEN that the first one is heads

$\mathbf{P}(HH \,|\, \text{1st is } H) =$  ―――

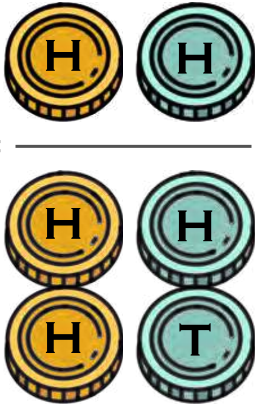
Conditional Probability: Coin Example 1



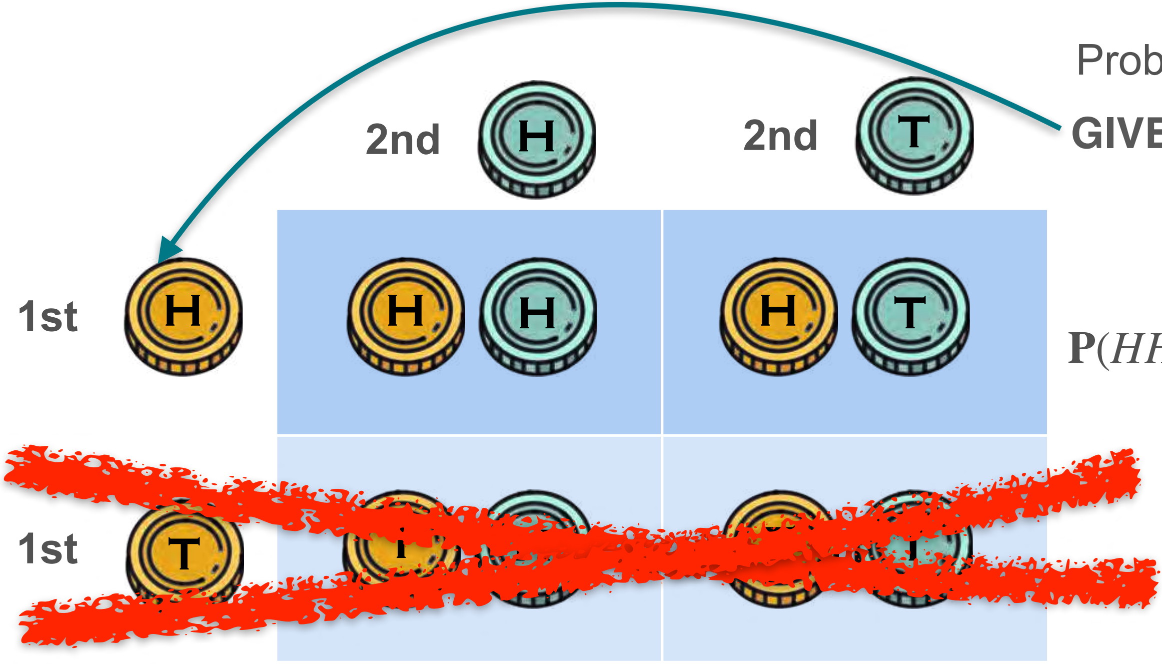
Probability of landing on heads twice

GIVEN that the first one is heads

$\mathbf{P}(HH \,|\, \text{1st is } H) =$



Conditional Probability: Coin Example 1



Probability of landing on heads twice

GIVEN that the first one is heads

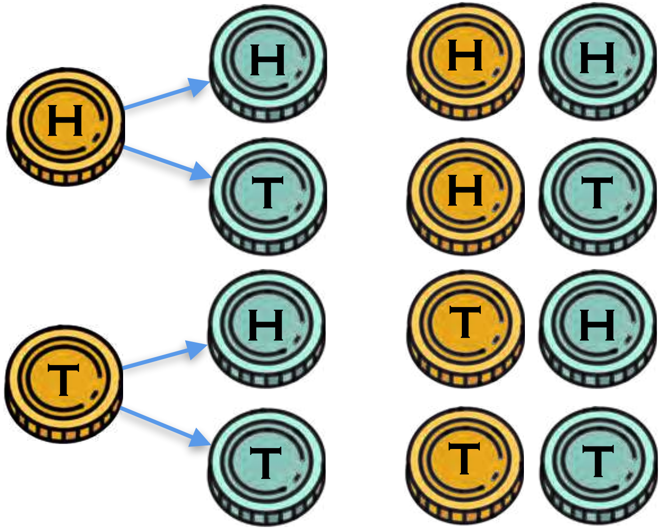
$$\mathbf{P}(HH \mid \text{1st is } H) = \frac{HH}{HH,\ HT} = \frac{1}{2}$$

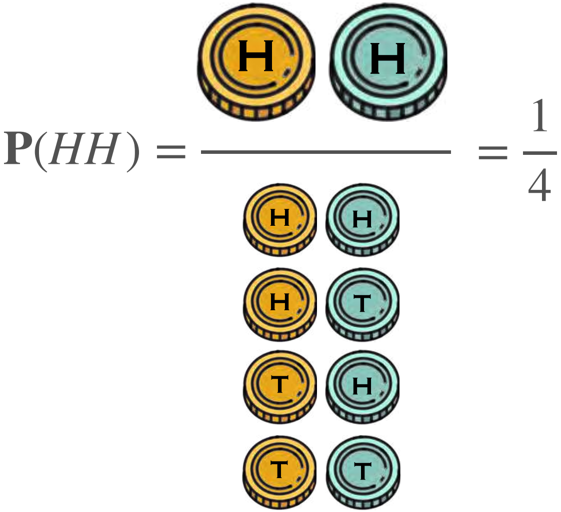
Conditional Probability: Coin Example 2

50% 50%

What is the probability of landing on heads twice?

1st **2nd**



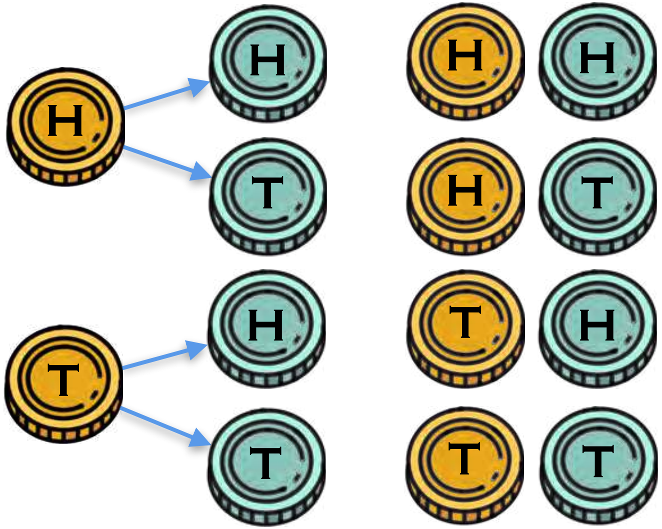


Conditional Probability: Coin Example 2

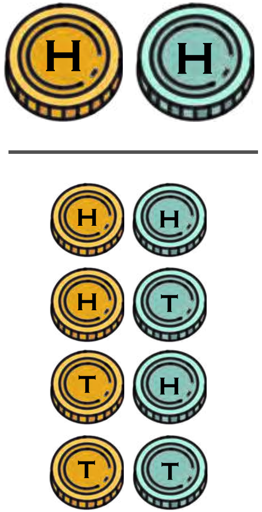
50% 50%

What is the probability of landing on heads twice?

1st **2nd** **GIVEN that the first one is tails**



$\mathbf{P}(HH) =$



$= \frac{1}{4}$

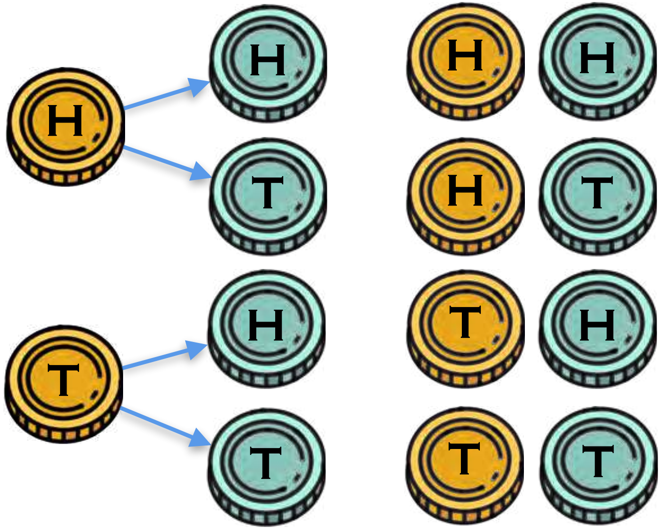
DeepLearning.AI

Conditional Probability: Coin Example 2

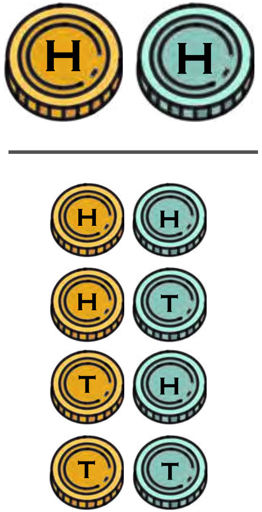
50% 50%

What is the probability of landing on heads twice?

1st **2nd** **GIVEN that the first one is tails**



$\mathbf{P}(HH) =$



$= \frac{1}{4}$

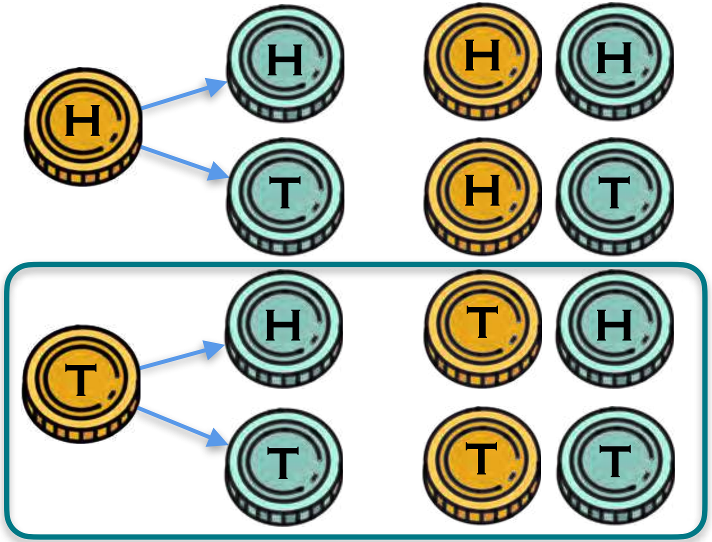
Conditional Probability: Coin Example 2

50% 50%

What is the probability of landing on heads twice?

GIVEN that the first one is tails

1st **2nd**



$\mathbf{P}(HH) =$

$= \frac{1}{4}$

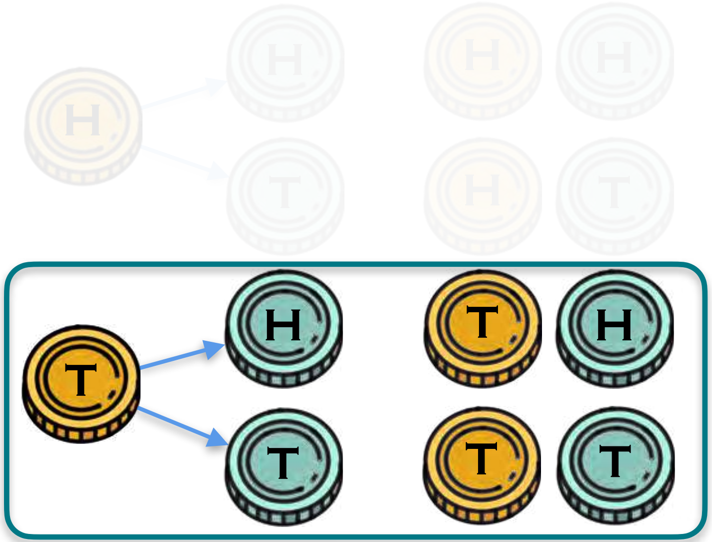
Conditional Probability: Coin Example 2

50% 50%

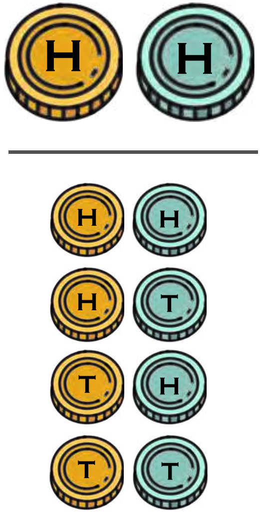
What is the probability of landing on heads twice?

GIVEN that the first one is tails

1st **2nd**



$\mathbf{P}(HH) =$



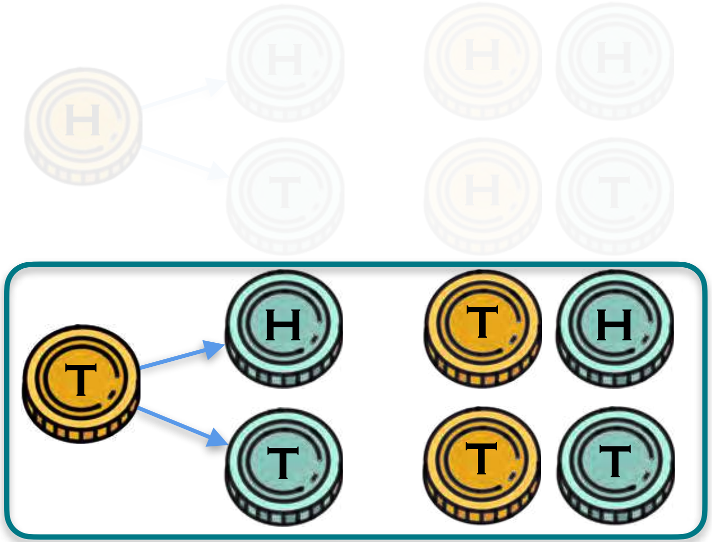
$= \frac{1}{4}$

Conditional Probability: Coin Example 2

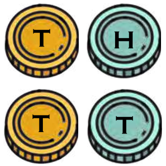
50% 50%

What is the probability of landing on heads twice?

1st **2nd** **GIVEN that the first one is tails**



$\mathbf{P}(HH) =$  $= \frac{1}{4}$

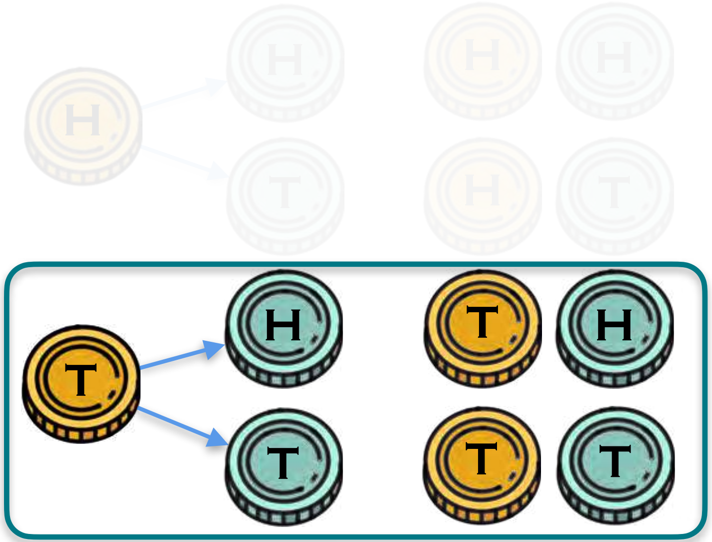


Conditional Probability: Coin Example 2

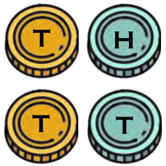
50% 50%

What is the probability of landing on heads twice?

1st **2nd** **GIVEN that the first one is tails**



$$\mathbf{P}(HH) = \underline{\hspace{4cm}} = \frac{1}{4}$$

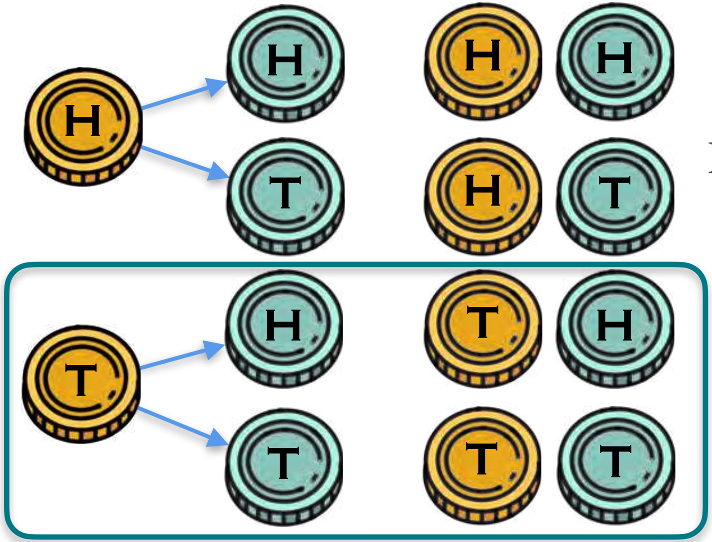


Conditional Probability: Coin Example 2

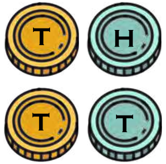
50% 50%

What is the probability of landing on heads twice?

1st **2nd** **GIVEN that the first one is tails**



$$\mathbf{P}(HH \mid \text{1st is } T) = \frac{0}{} = 0$$



Conditional Probability: Coin Example 2

Conditional Probability: Coin Example 2

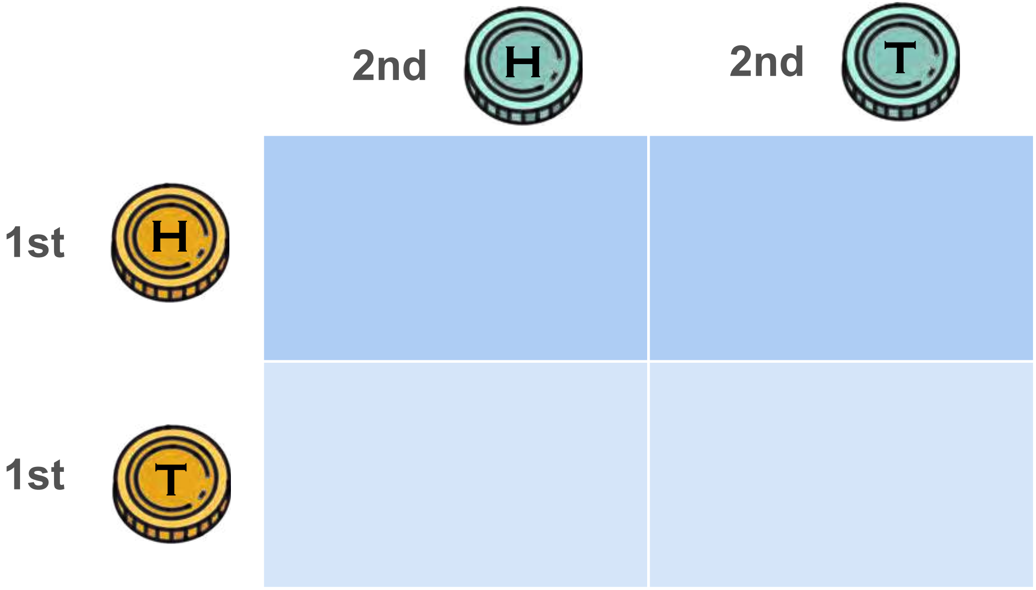
2nd 

2nd 

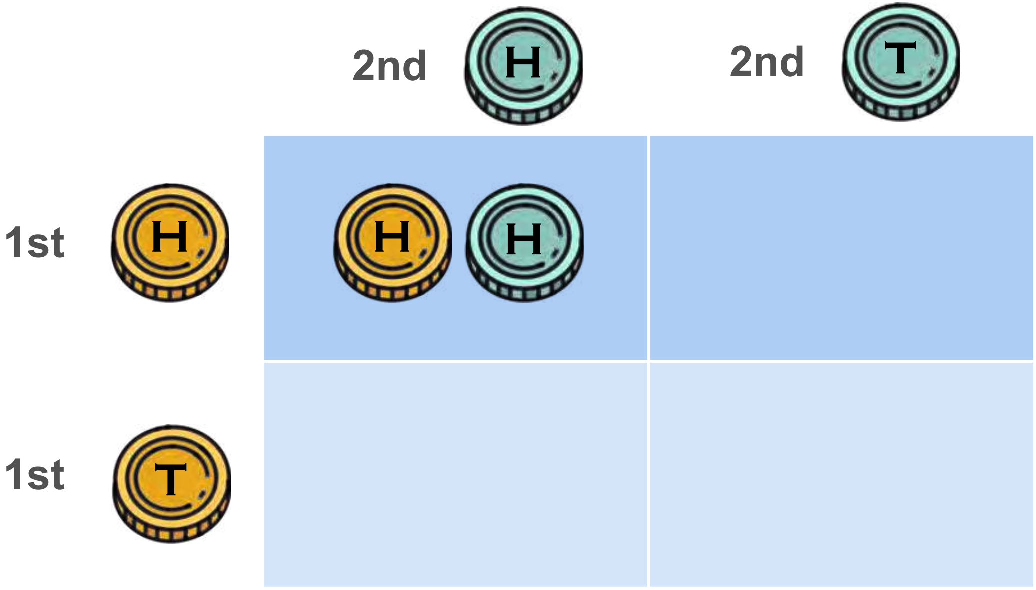
1st 

1st 

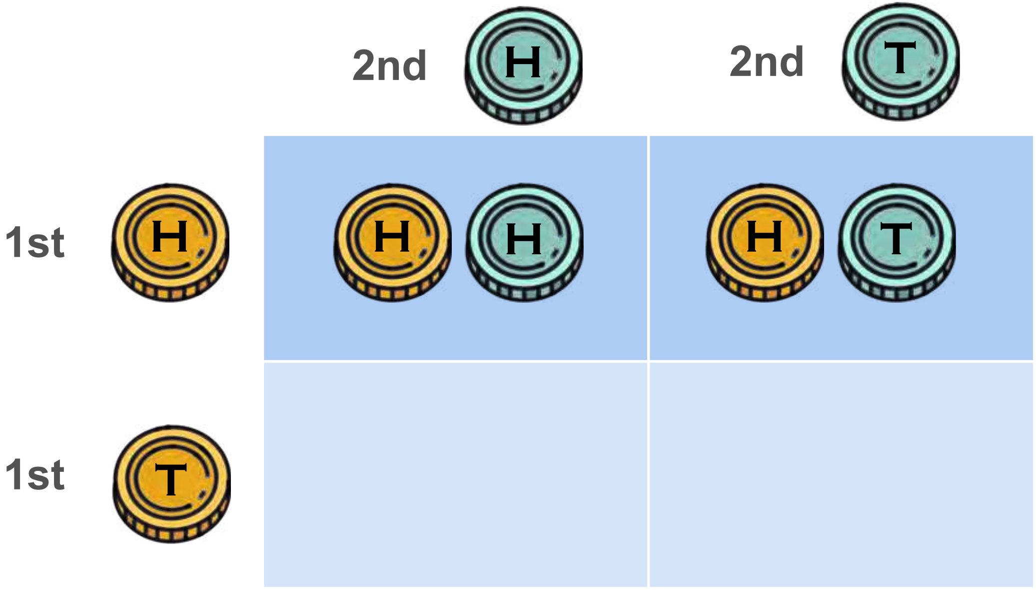
Conditional Probability: Coin Example 2



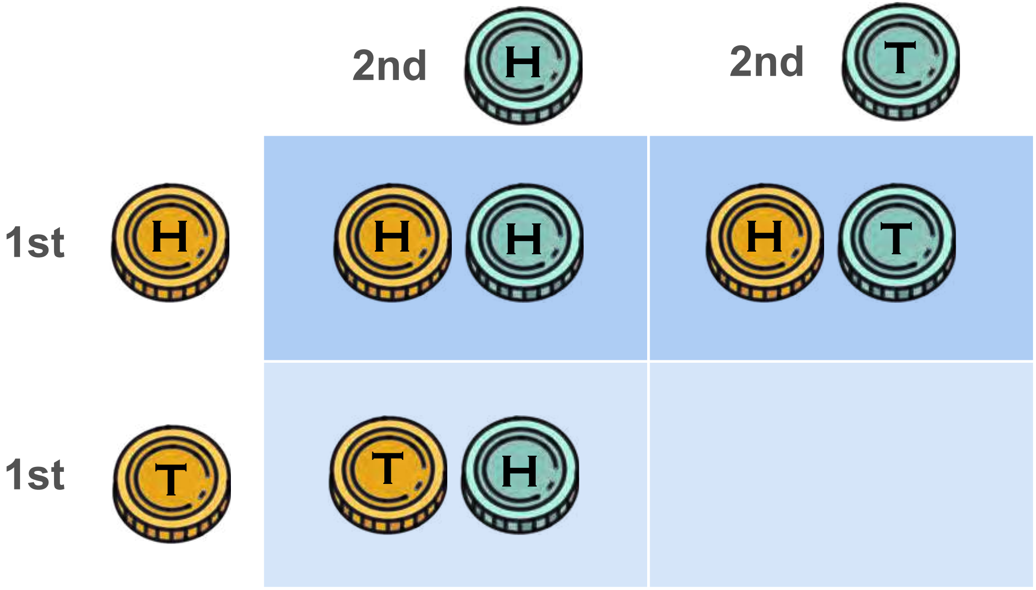
Conditional Probability: Coin Example 2



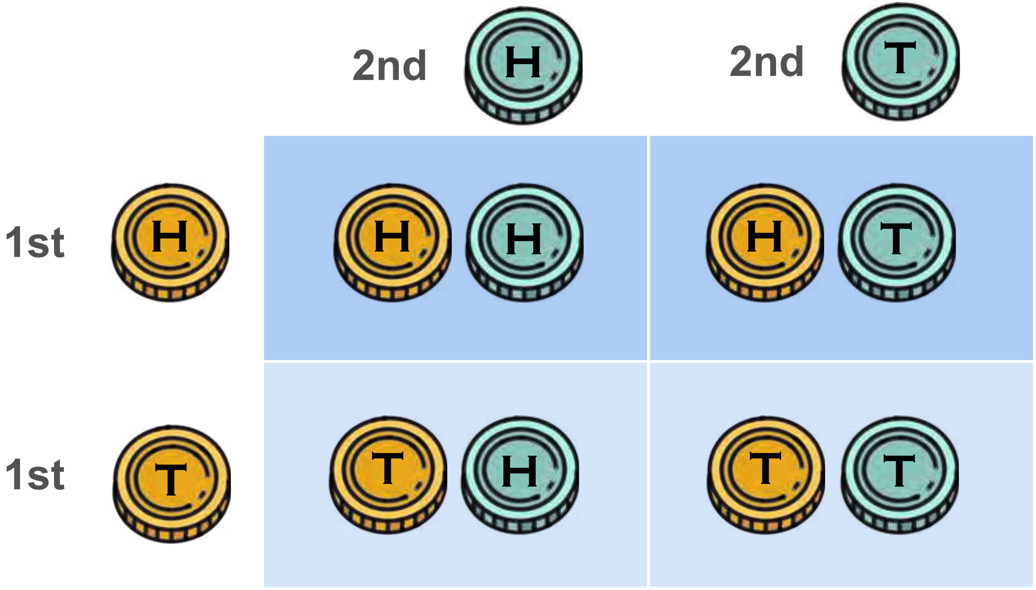
Conditional Probability: Coin Example 2



Conditional Probability: Coin Example 2

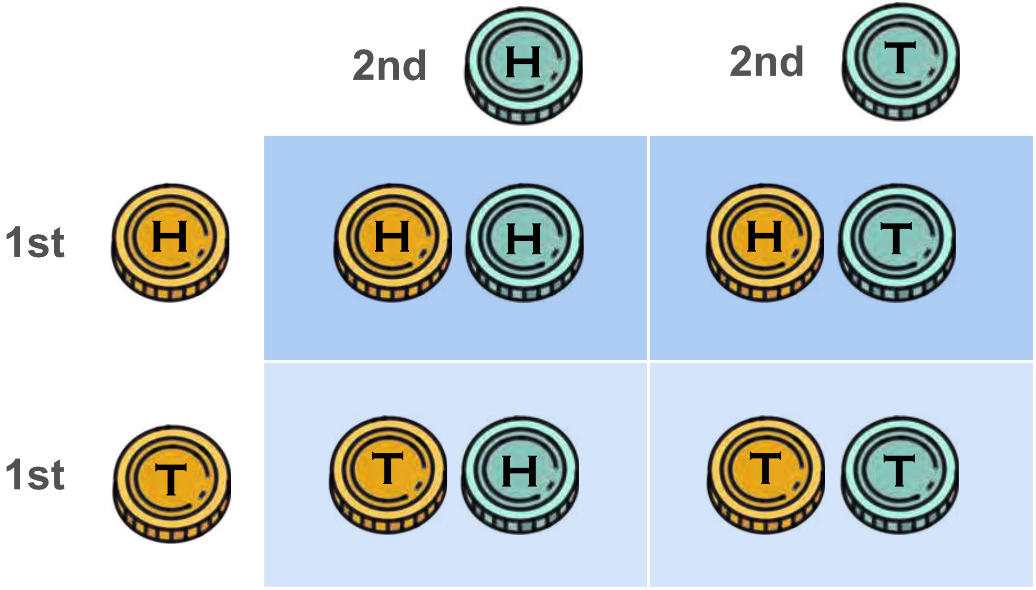


Conditional Probability: Coin Example 2



Conditional Probability: Coin Example 2

Probability of landing on heads twice

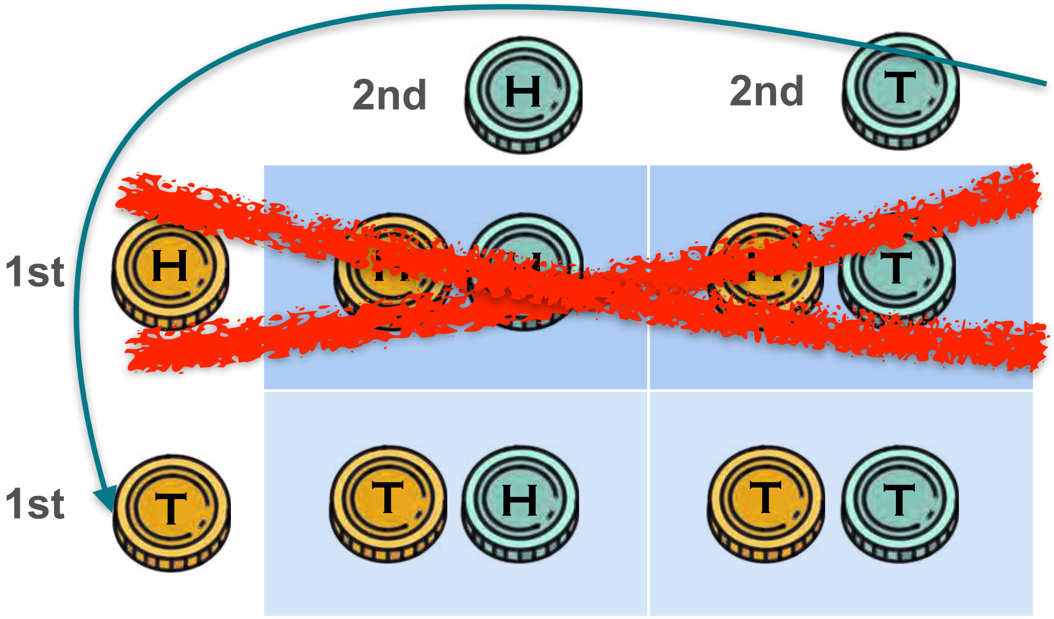


Conditional Probability: Coin Example 2

Probability of landing on heads twice

GIVEN that the first one is tails

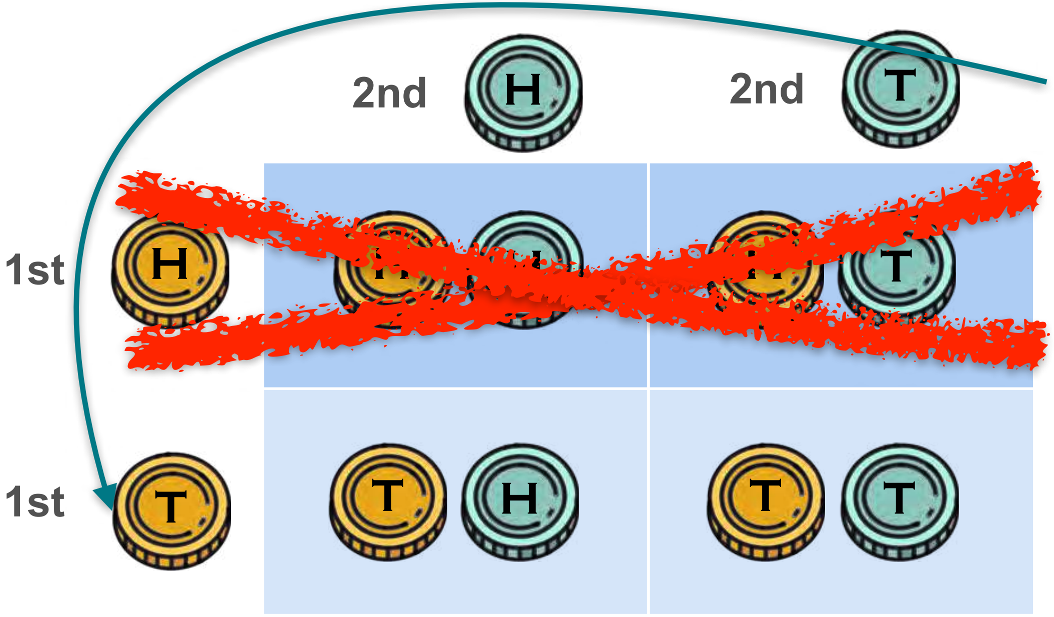
Conditional Probability: Coin Example 2



Probability of landing on heads twice

GIVEN that the first one is tails

Conditional Probability: Coin Example 2

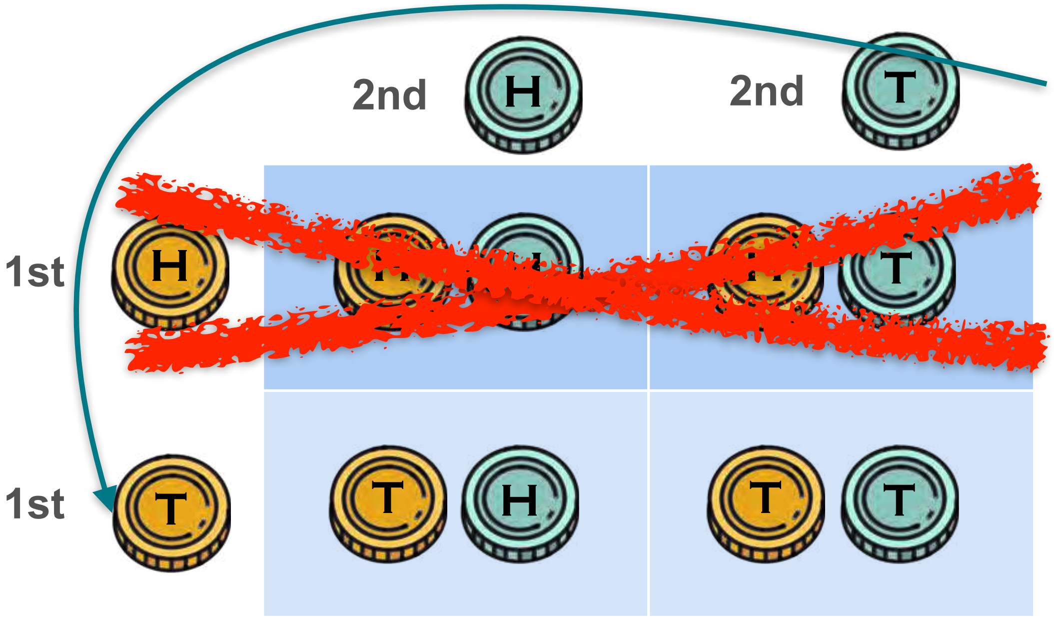


Probability of landing on heads twice

GIVEN that the first one is tails

$\mathbf{P}(HH \mid \text{1st is } T) = \underline{\hspace{3cm}}$

Conditional Probability: Coin Example 2

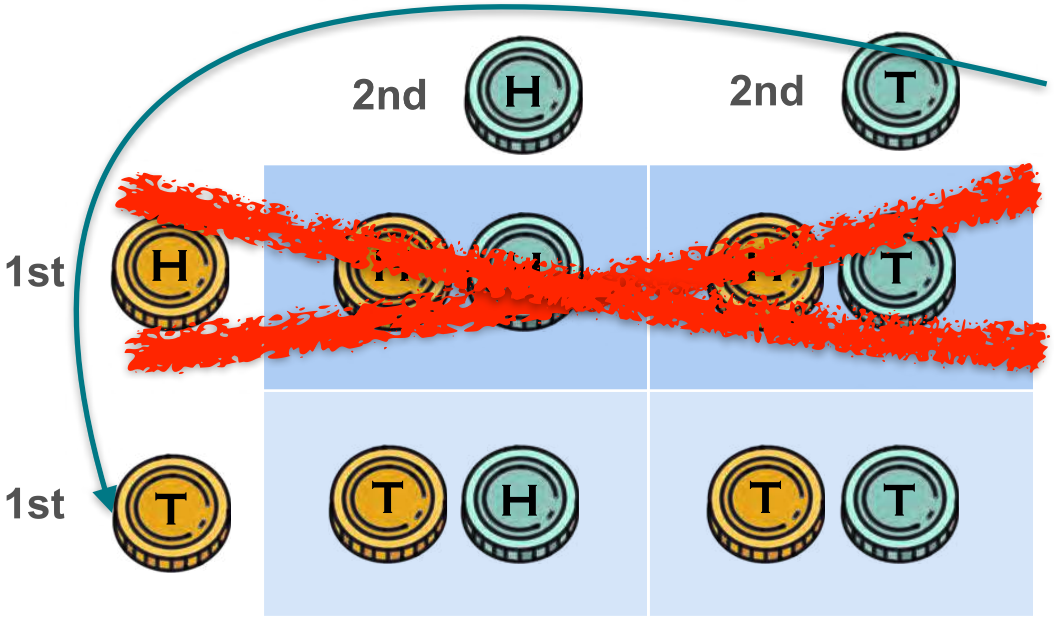


Probability of landing on heads twice

GIVEN that the first one is tails

$\mathbf{P}(HH \mid \text{1st is } T) = \underline{\qquad\qquad}$

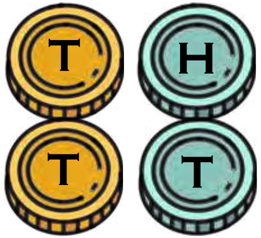
Conditional Probability: Coin Example 2



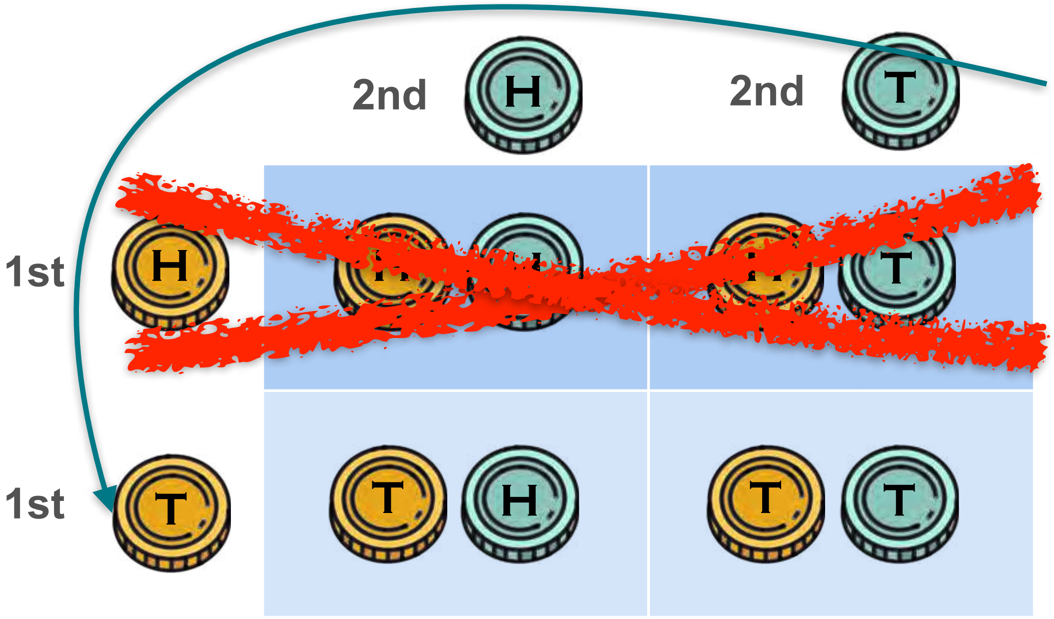
Probability of landing on heads twice

GIVEN that the first one is tails

$$\mathbf{P}(HH \mid \text{1st is } T) = \frac{\quad}{\quad}$$



Conditional Probability: Coin Example 2

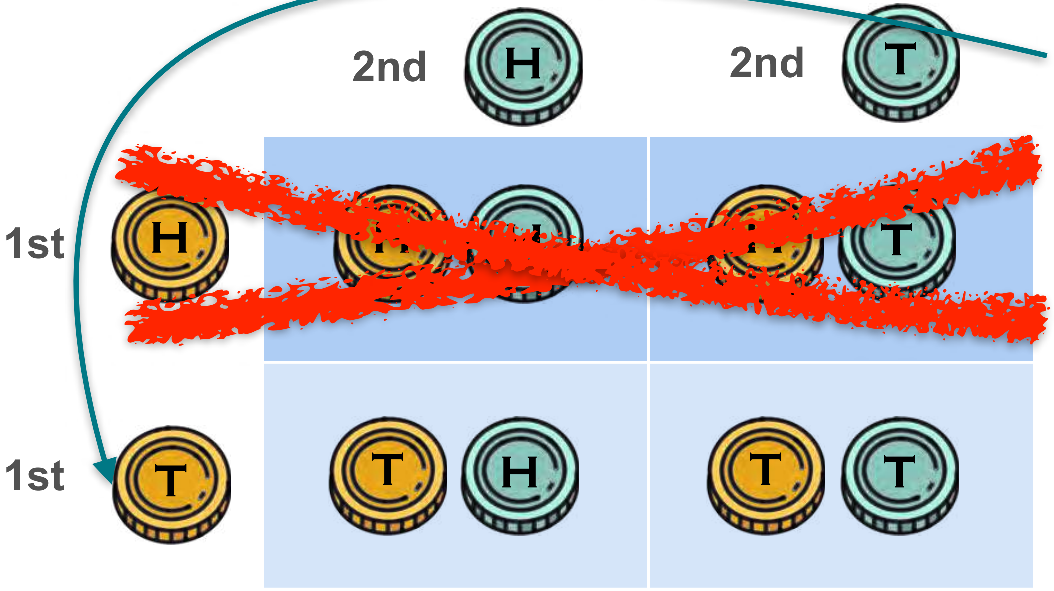


Probability of landing on heads twice

GIVEN that the first one is tails

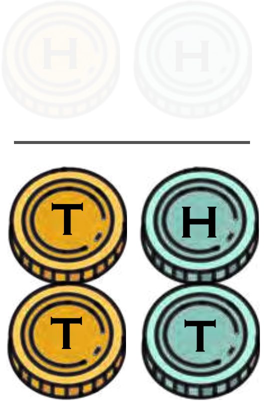
$$\mathbf{P}(HH \mid \text{1st is } T) = \frac{}{\text{TH, TT}}$$

Conditional Probability: Coin Example 2

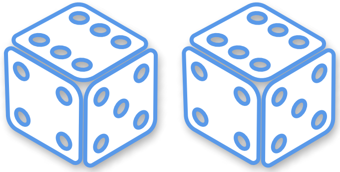


Probability of landing on heads twice

GIVEN that the first one is tails

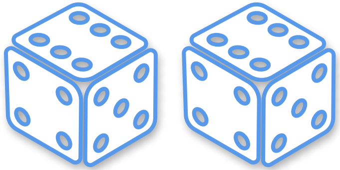
$\mathbf{P}(HH \mid \text{1st is } T) =$  $= 0$

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

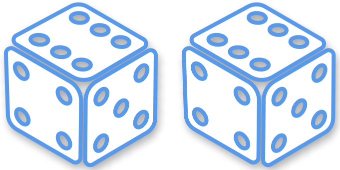
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

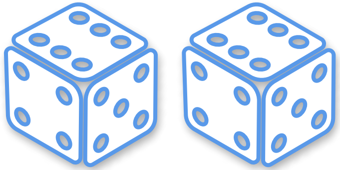
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	$\boxed{4,6}$
5,1	5,2	5,3	5,4	$\boxed{5,5}$	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

Conditional Probability: Dice Example 1

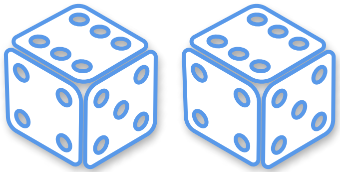


1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

What is the probability that the sum is 10?

$\mathbf{P}(\text{sum} = 10) = \_\_\_\_\_\_\_\_\_\_$

Conditional Probability: Dice Example 1



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	$\boxed{4,6}$
5,1	5,2	5,3	5,4	$\boxed{5,5}$	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

What is the probability that the sum is 10?

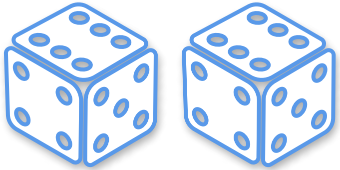
4,6

5,5

6,4

$$\mathbf{P}(\text{sum} = 10) = \underline{\hspace{6cm}}$$

Conditional Probability: Dice Example 1



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

What is the probability that the sum is 10?

$$\mathbf{P}(\text{sum} = 10) = \frac{\{6,4\},\ \{5,5\},\ \{4,6\}}{\text{all 36 outcomes}}$$

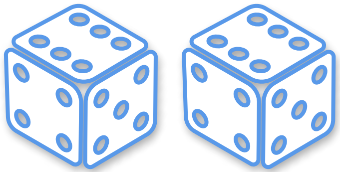
Numerator:

4,6
5,5
6,4

Denominator:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	$\boxed{4,6}$
5,1	5,2	5,3	5,4	$\boxed{5,5}$	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

What is the probability that the sum is 10?

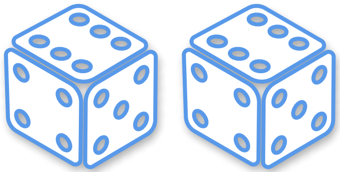
$$\mathbf{P}(\text{sum} = 10) = \frac{\{6,4;\ 5,5;\ 4,6\}}{\text{all 36 outcomes}} = \frac{3}{36}$$

Numerator: 6,4 · 5,5 · 4,6

Denominator:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1



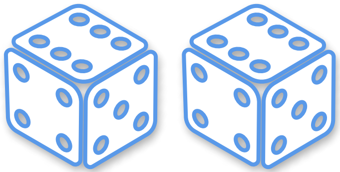
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	$\boxed{4,6}$
5,1	5,2	5,3	5,4	$\boxed{5,5}$	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

What is the probability that the sum is 10?

$$\mathbf{P}(\text{sum} = 10) = \frac{\boxed{6,4}\ \boxed{5,5}\ \boxed{4,6}}{\text{all 36 outcomes}} = \frac{1}{12}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

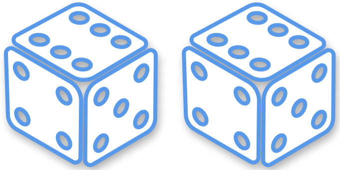
Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 1

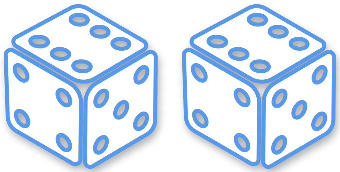


What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	**4,6**
5,1	5,2	5,3	5,4	**5,5**	5,6
6,1	6,2	6,3	**6,4**	6,5	6,6

Conditional Probability: Dice Example 1



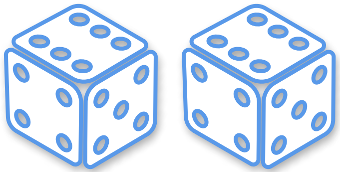
What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	**4,6**
5,1	5,2	5,3	5,4	**5,5**	5,6
6,1	6,2	6,3	**6,4**	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\qquad\qquad\qquad\qquad}$$

Conditional Probability: Dice Example 1



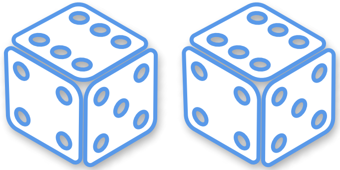
What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

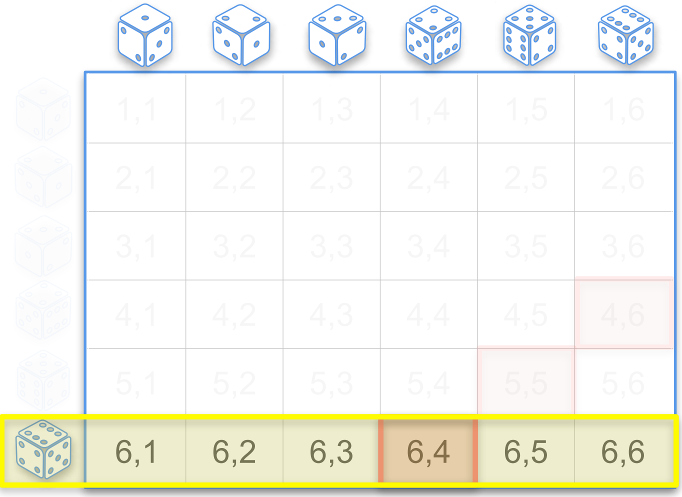
$$\mathbf{P}(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{6cm}}$$

Conditional Probability: Dice Example 1



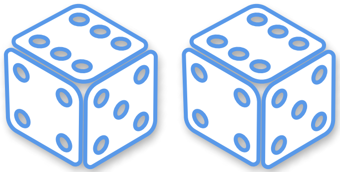
What is the probability that the sum is 10?

GIVEN that the first one is 6



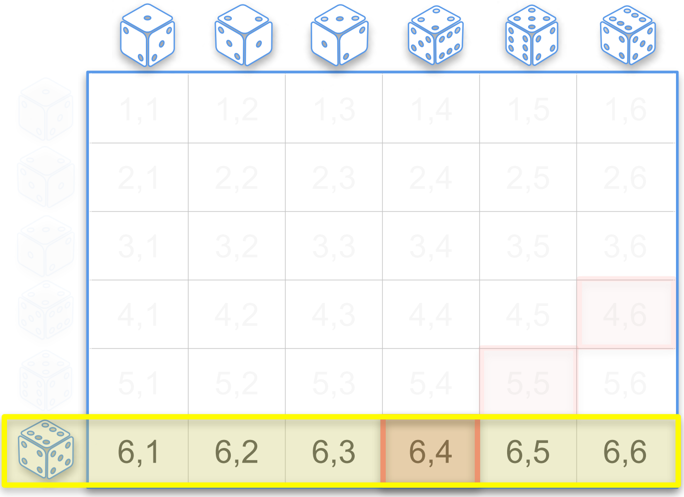
$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\qquad\qquad\qquad}$$

Conditional Probability: Dice Example 1



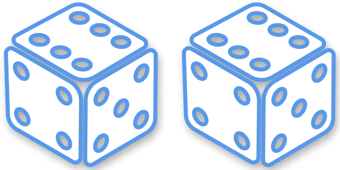
What is the probability that the sum is 10?

GIVEN that the first one is 6



$$P(\text{sum} = 10 \mid \text{1st is } 6) = \frac{\quad}{\boxed{6,1 \quad 6,2 \quad 6,3 \quad 6,4 \quad 6,5 \quad 6,6}}$$

Conditional Probability: Dice Example 1



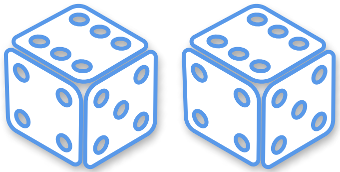
What is the probability that the sum is 10?

GIVEN that the first one is 6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

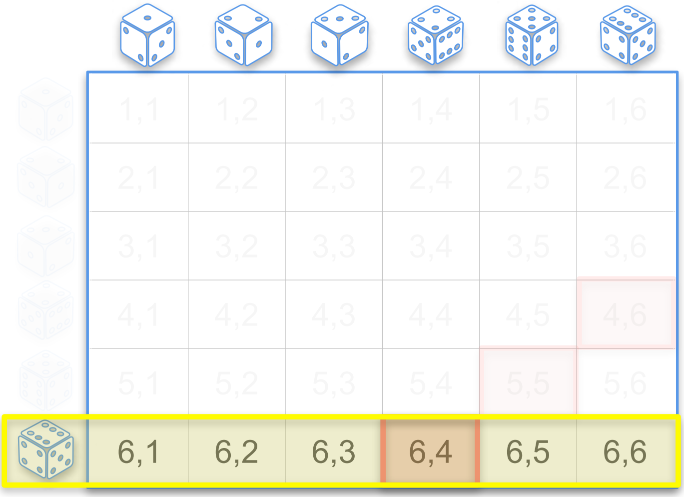
$$P(\text{sum} = 10 \mid \text{1st is 6}) = \frac{\boxed{6,4}}{\boxed{6,1 \quad 6,2 \quad 6,3 \quad 6,4 \quad 6,5 \quad 6,6}}$$

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

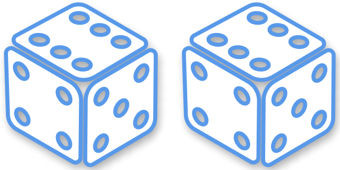
GIVEN that the first one is 6



$$P(\text{sum} = 10 \mid \text{1st is 6}) = \frac{\boxed{6,4}}{\boxed{6,1 \quad 6,2 \quad 6,3 \quad 6,4 \quad 6,5 \quad 6,6}}$$

$$= \frac{1}{6}$$

Conditional Probability: Dice Example 2



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	$\boxed{4,6}$
5,1	5,2	5,3	5,4	$\boxed{5,5}$	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

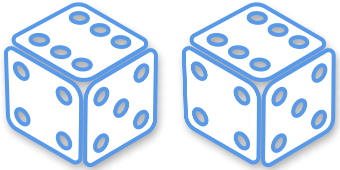
What is the probability that the sum is 10?

4,6
5,5
6,4

$$\mathbf{P}(\text{sum} = 10) = \frac{\boxed{6,4}\ \boxed{5,5}\ \boxed{4,6}}{\text{(table below)}} = \frac{3}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

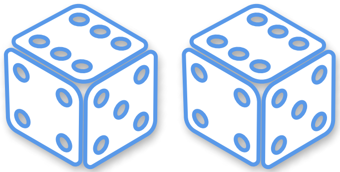
Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 2

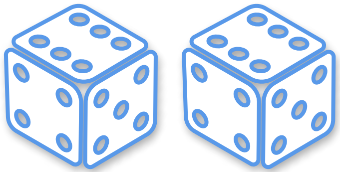


What is the probability that the sum is 10?

GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 2



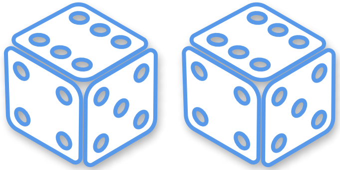
What is the probability that the sum is 10?

GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is 1}) = \_\_\_\_\_\_\_\_\_\_$$

Conditional Probability: Dice Example 2



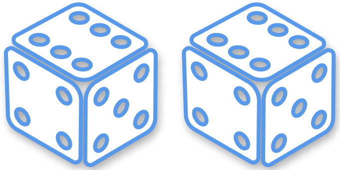
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

What is the probability that the sum is 10?

GIVEN that the first one is 1

$\mathbf{P}(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\qquad\qquad\qquad}$

Conditional Probability: Dice Example 2



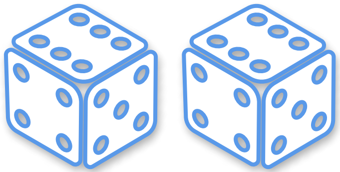
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

What is the probability that the sum is 10?

GIVEN that the first one is 1

$$\mathbf{P}(\text{sum} = 10 \mid \text{1st is 1}) = \underline{\qquad\qquad\qquad}$$

Conditional Probability: Dice Example 2



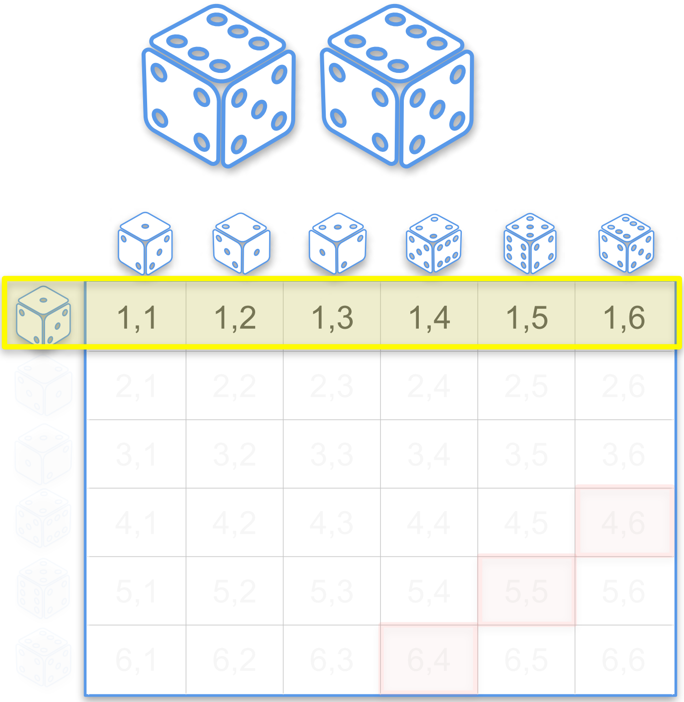
What is the probability that the sum is 10?

GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is 1}) = \frac{}{\boxed{1,1 \quad 1,2 \quad 1,3 \quad 1,4 \quad 1,5 \quad 1,6}}$$

Conditional Probability: Dice Example 2

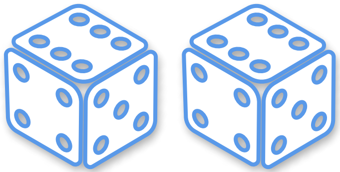


What is the probability that the sum is 10?

GIVEN that the first one is 1

$$\mathbf{P}(\text{sum} = 10 \mid \text{1st is 1}) = \frac{\boxed{1,1 \quad 1,2 \quad 1,3 \quad 1,4 \quad 1,5 \quad 1,6}}{\boxed{1,1 \quad 1,2 \quad 1,3 \quad 1,4 \quad 1,5 \quad 1,6}}$$

Conditional Probability: Dice Example 2



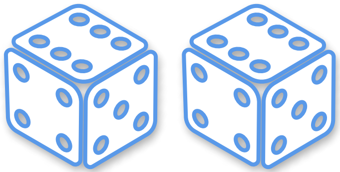
What is the probability that the sum is 10?

GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is 1}) = \frac{0}{\boxed{1,1 \quad 1,2 \quad 1,3 \quad 1,4 \quad 1,5 \quad 1,6}}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10 \mid \text{1st is 1}) = \frac{0}{\boxed{1,1 \quad 1,2 \quad 1,3 \quad 1,4 \quad 1,5 \quad 1,6}}$$

$$= 0$$

Product Rule (for Independent Events)

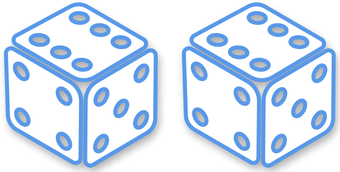
$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Product Rule (for Independent Events)

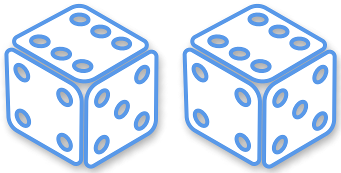
When A and B independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Conditional Probability: Dice Example 3

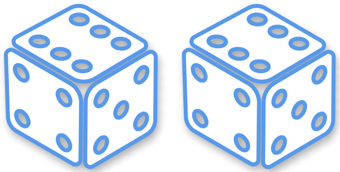


Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

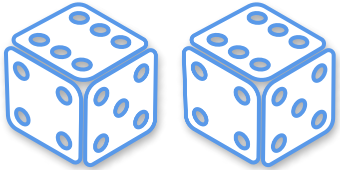
Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

Conditional Probability: Dice Example 3

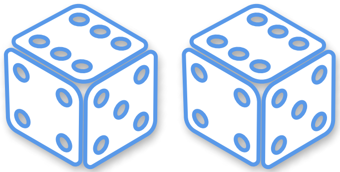


What is the probability that
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{1st is 6} \cap \text{sum = 10}) = \underline{\qquad\qquad\qquad}$$

Conditional Probability: Dice Example 3

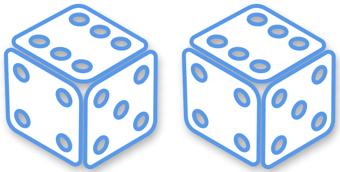


What is the probability that
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

$$\mathbf{P}(\text{1st is } 6 \cap \text{sum} = 10) = \frac{\boxed{6,4}}{\qquad}$$

Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{1st is 6} \cap \text{sum = 10}) = \frac{\boxed{6,4}}{\text{(all 36 outcomes)}}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$= \frac{1}{36}$$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6})$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6})$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6})$

6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is } 6 \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is } 6)$

6,1 6,2 6,3 6,4 6,5 6,6

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6})$

6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6})$

6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{6}{36}$$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is } 6 \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is } 6)$

6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum = 10}) =$

$\mathbf{P}(\text{1st is 6})$ $\mathbf{P}(\text{sum = 10} \mid \text{1st 6})$

6,1	6,2	6,3	6,4	6,5	6,6

divided by

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6})$

$\mathbf{P}(\text{sum} = 10 \mid \text{1st 6})$

6,1	6,2	6,3	6,4	6,5	6,6

divided by

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

6,4

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum = 10}) =$

$\mathbf{P}(\text{1st is 6})$

6,1	6,2	6,3	6,4	6,5	6,6

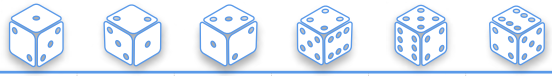
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

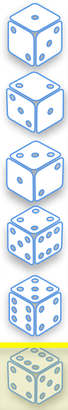
$$\frac{1}{6}$$

$\mathbf{P}(\text{sum = 10} \mid \text{1st 6})$

6,4

Conditional Probability: Dice Example 3





	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6})$ $\mathbf{P}(\text{sum} = 10 \mid \text{1st 6})$

6,1	6,2	6,3	6,4	6,5	6,6

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

6,4

6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{1}{6}$$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) =$

$\mathbf{P}(\text{1st is 6}) \quad \bullet \quad \mathbf{P}(\text{sum} = 10 \mid \text{1st 6})$

$$\frac{\{6,1 \;\; 6,2 \;\; 6,3 \;\; 6,4 \;\; 6,5 \;\; 6,6\}}{\text{all 36 outcomes } 1,1 \ldots 6,6} \quad \bullet \quad \frac{\{6,4\}}{\{6,1 \;\; 6,2 \;\; 6,3 \;\; 6,4 \;\; 6,5 \;\; 6,6\}}$$

$$\frac{1}{6} \quad \bullet \quad \frac{1}{6} = \frac{1}{36}$$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

=

6,1	6,2	6,3	6,4	6,5	6,6

over

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

•

6,4

over

6,1	6,2	6,3	6,4	6,5	6,6

$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) = \mathbf{P}(\text{1st is 6}) \bullet \mathbf{P}(\text{sum} = 10 \mid \text{1st 6})$

Conditional Probability: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

=

6,1	6,2	6,3	6,4	6,5	6,6

over

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

•

6,4

over

6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{1st is 6} \cap \text{sum} = 10) = \mathbf{P}(\text{1st is 6}) \bullet \mathbf{P}(\text{sum} = 10 \mid \text{1st 6})$$

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B \mid A)$$

The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B \mid A)$$

The General Product Rule

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B \mid A)$$

When independent

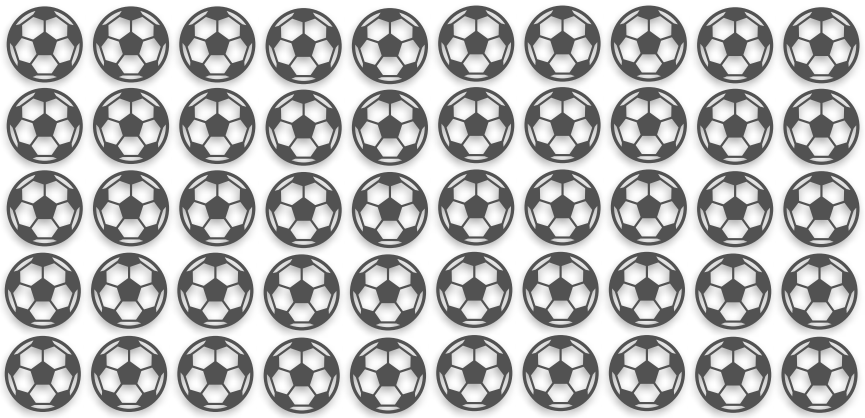
$$\mathbf{P}(B \mid A) = \mathbf{P}(B)$$

Quiz 1

Quiz 1

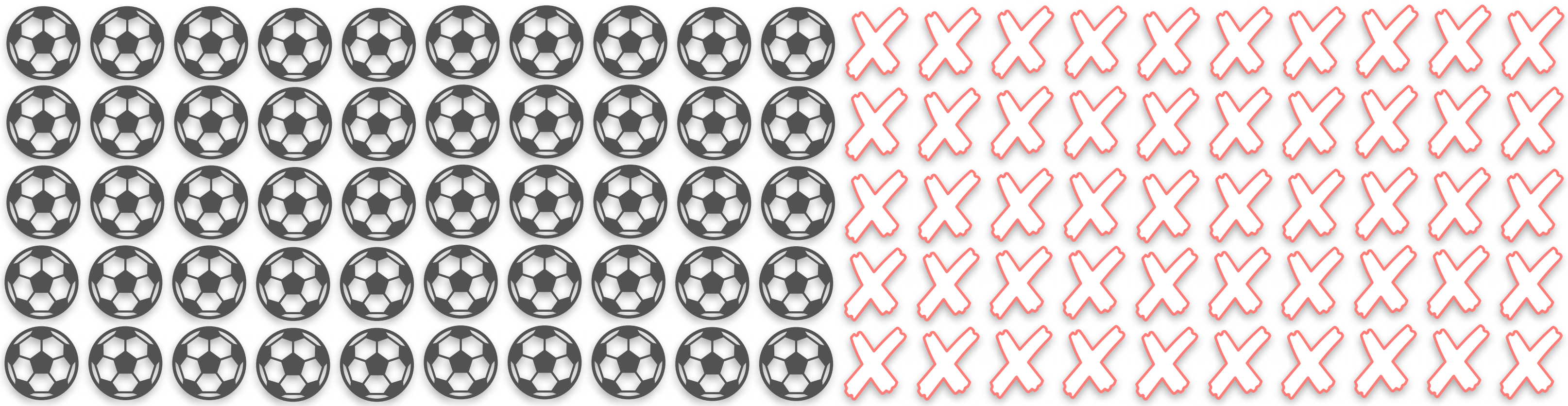
100 kids

Quiz 1



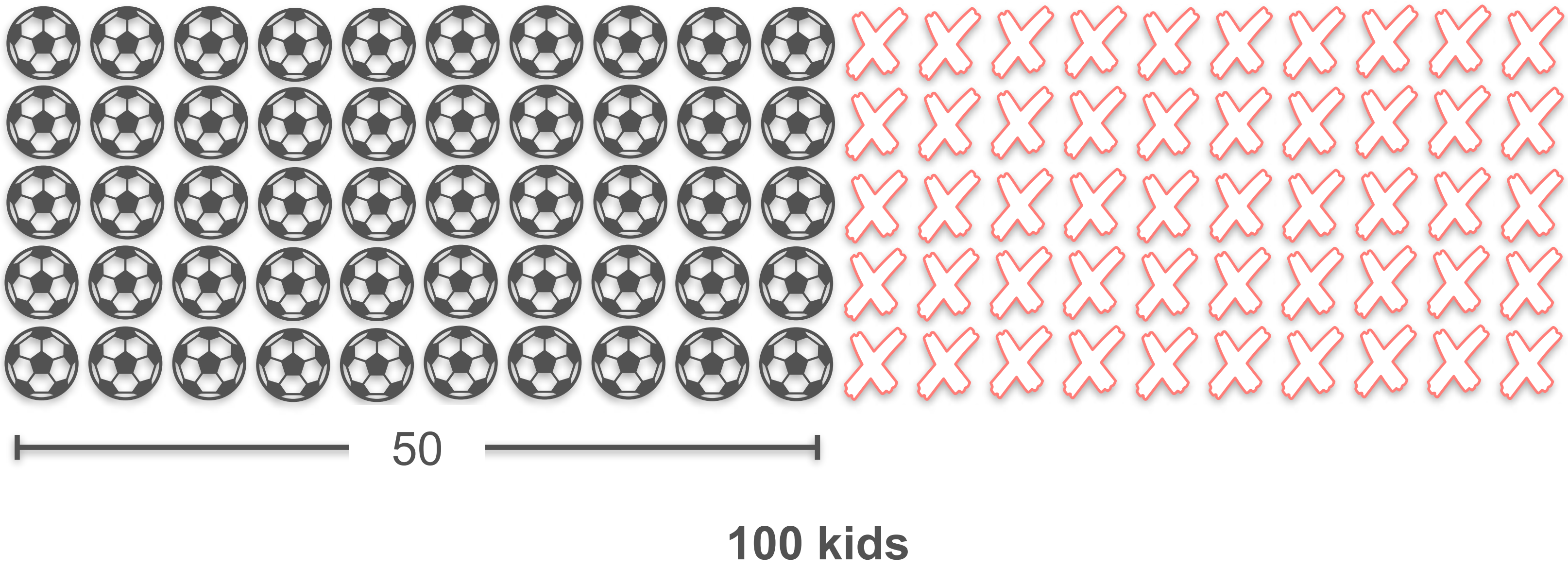
100 kids

Quiz 1

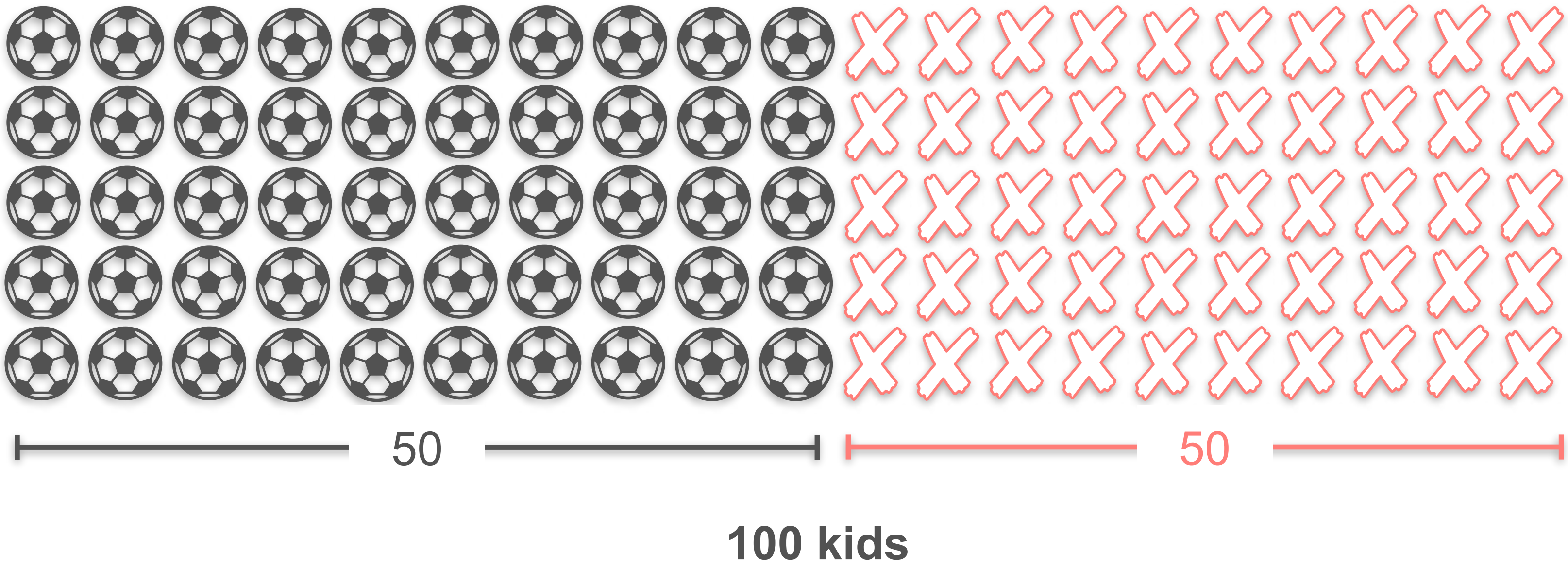


100 kids

Quiz 1

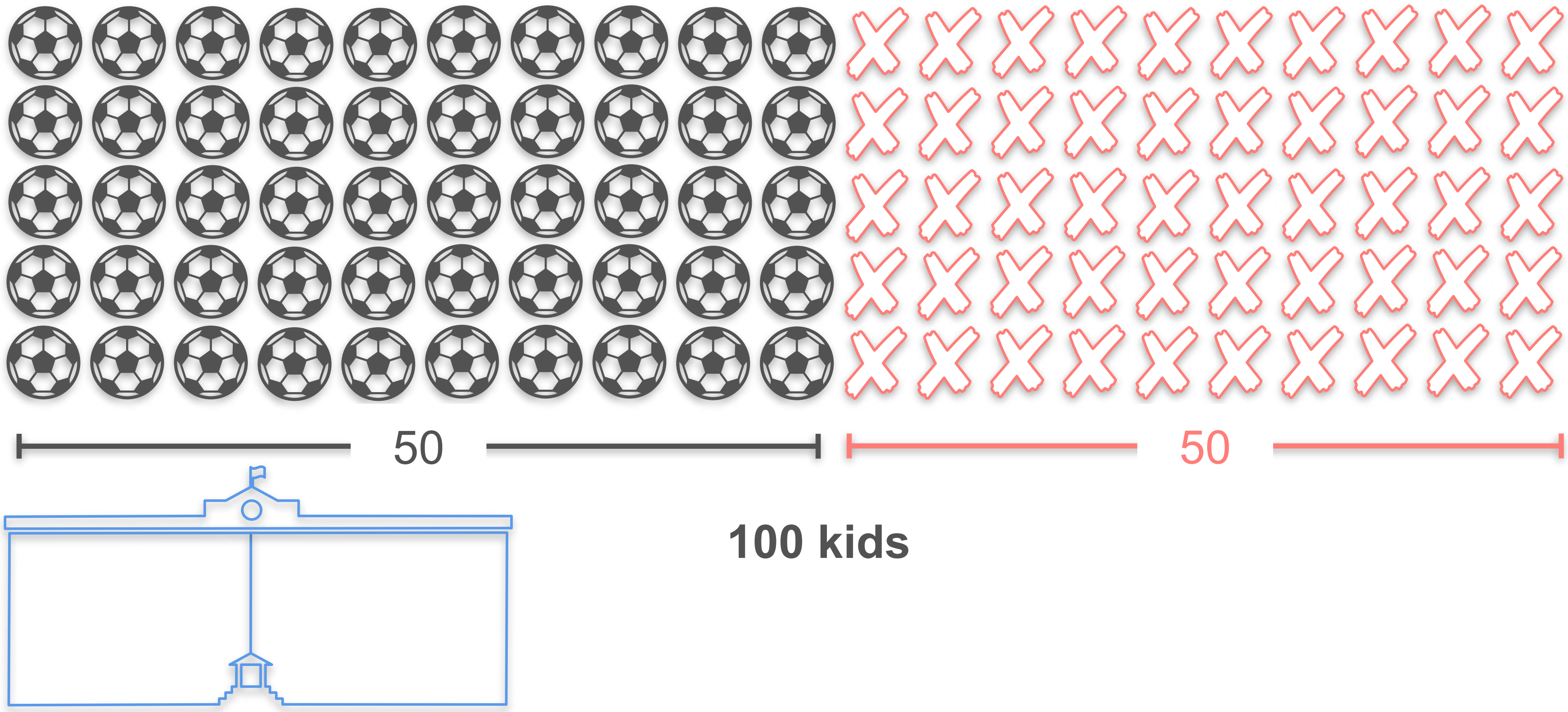


Quiz 1

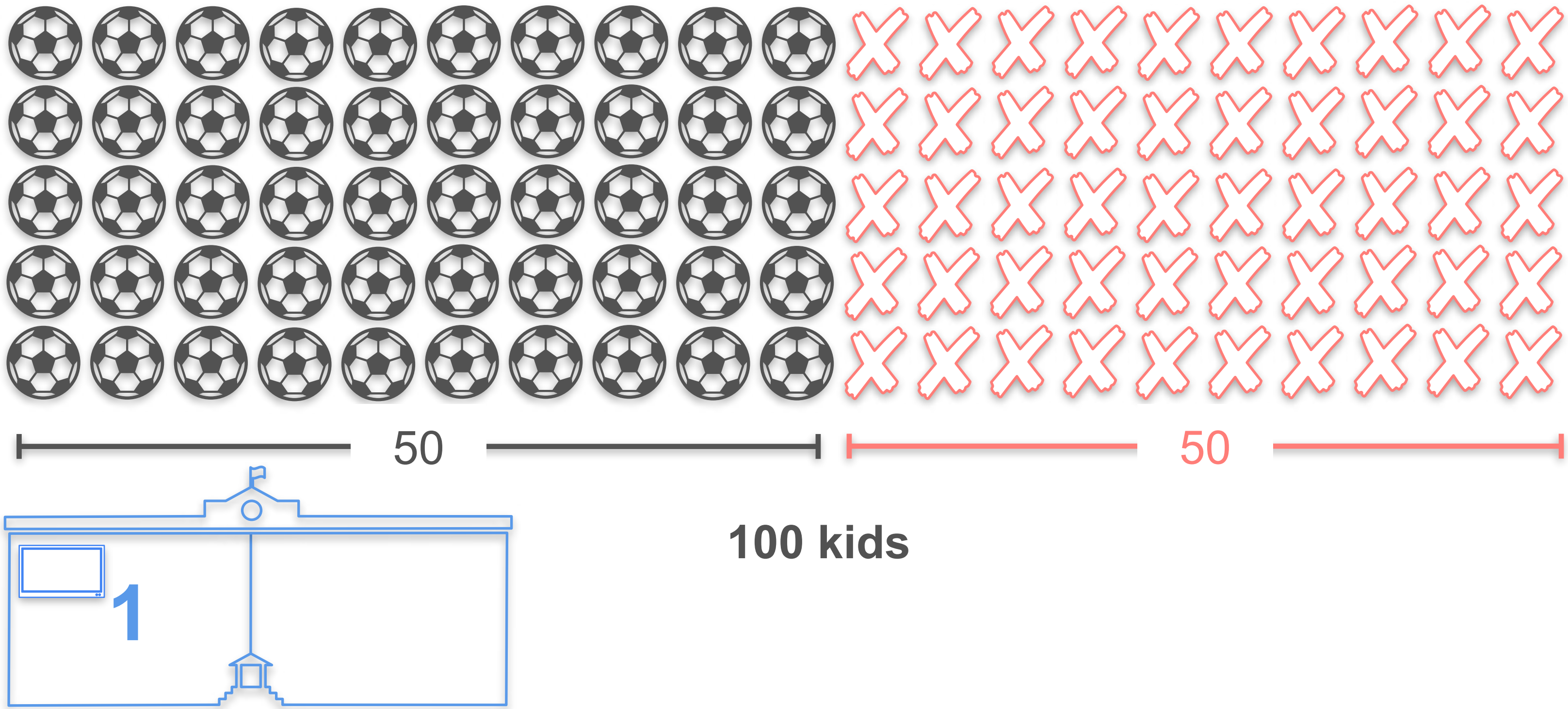


DeepLearning.AI

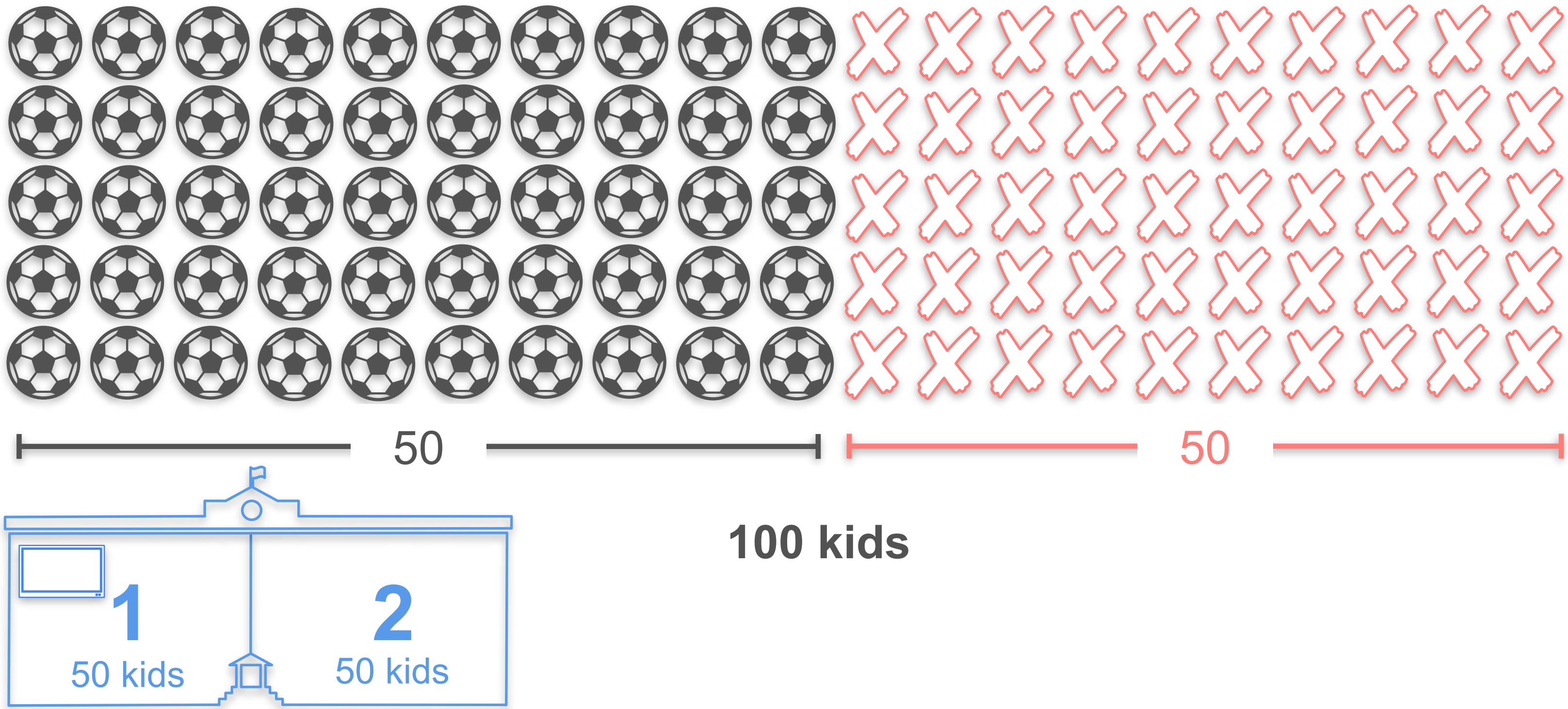
Quiz 1



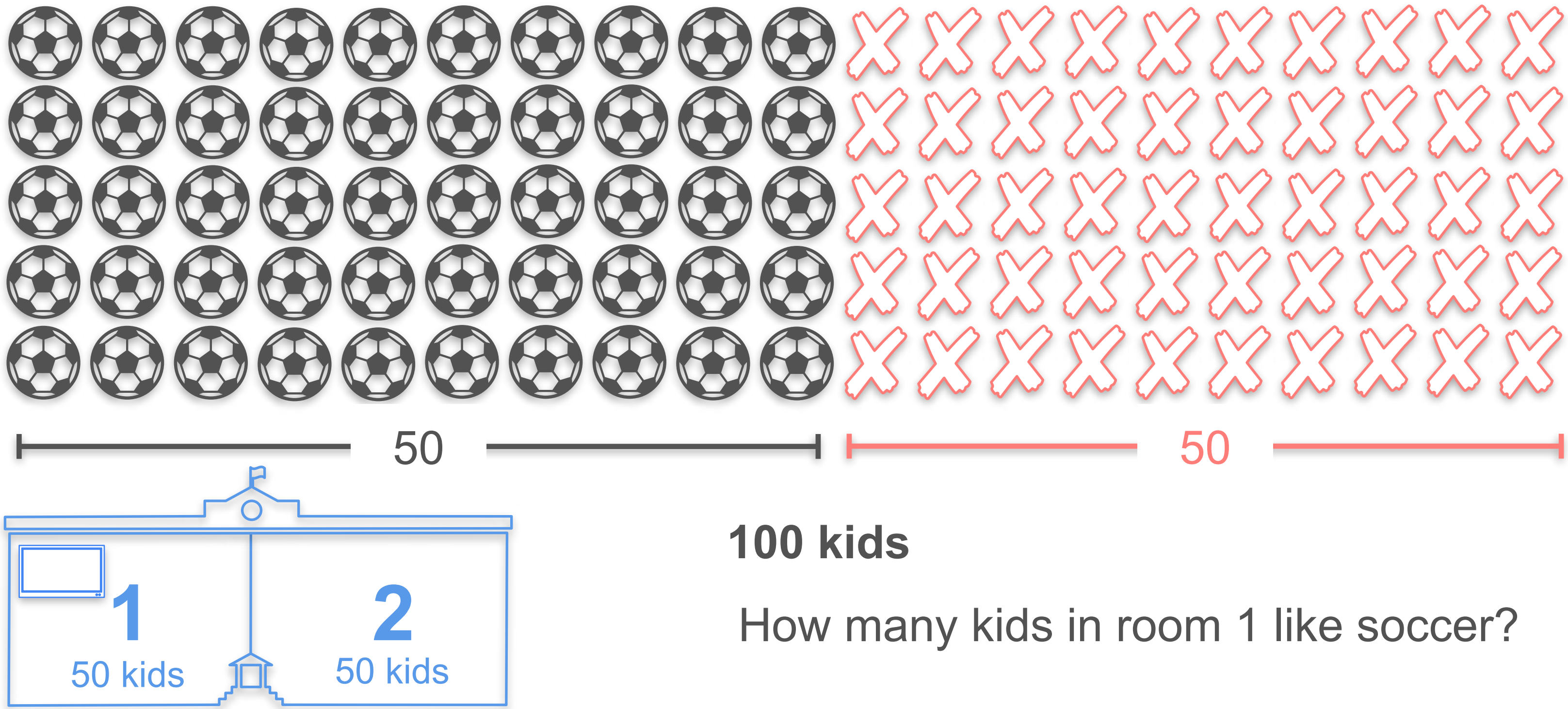
Quiz 1



Quiz 1



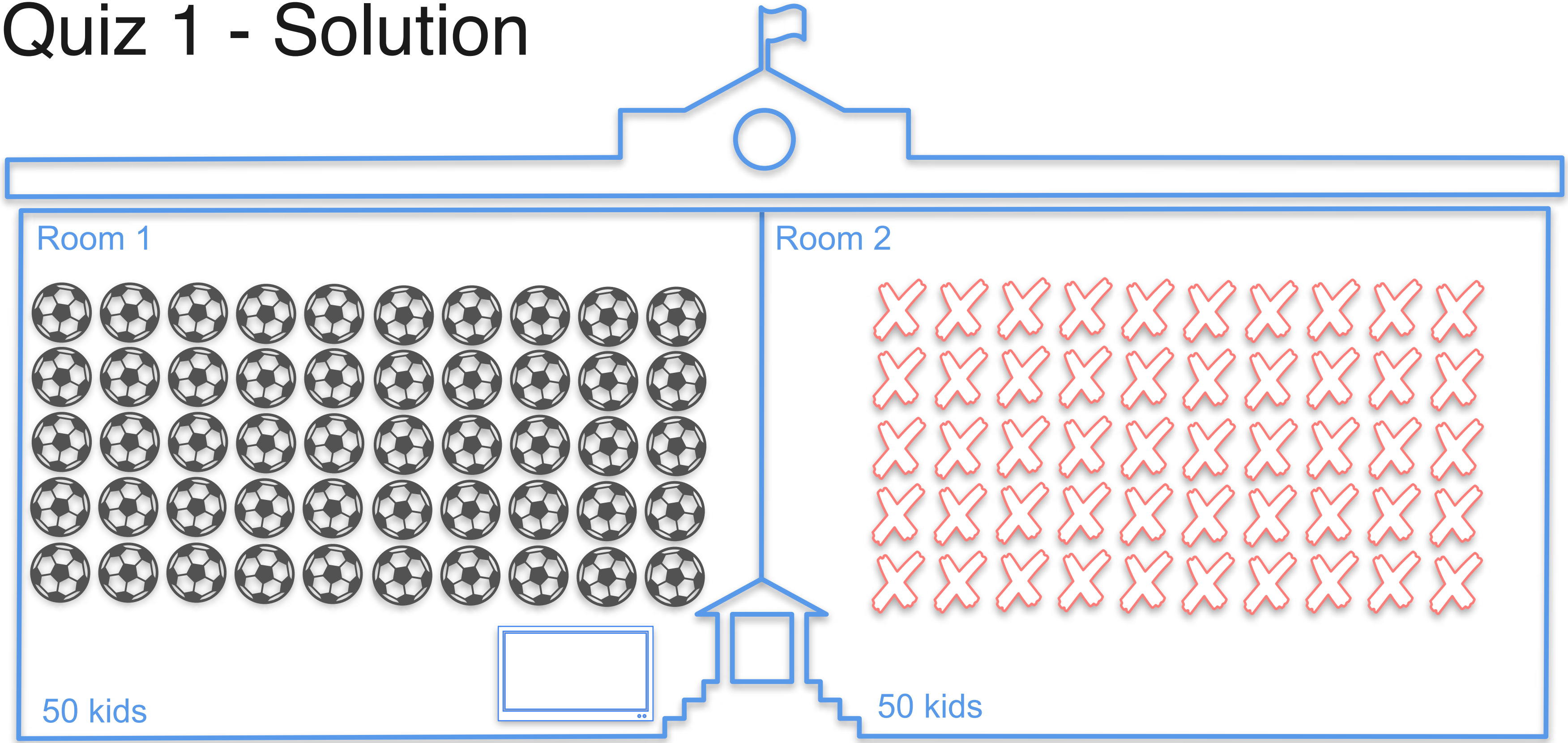
Quiz 1



100 kids

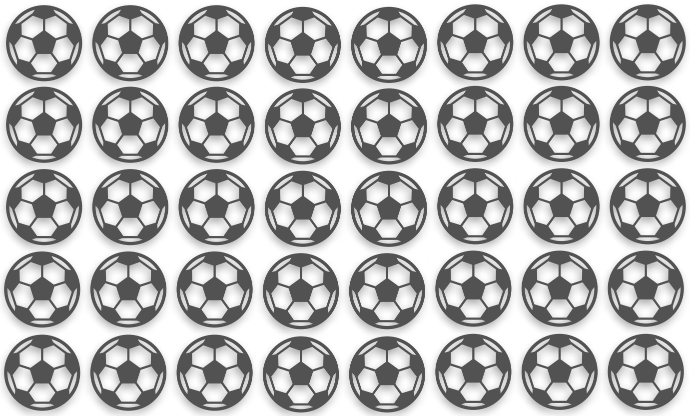
How many kids in room 1 like soccer?

Quiz 1 - Solution

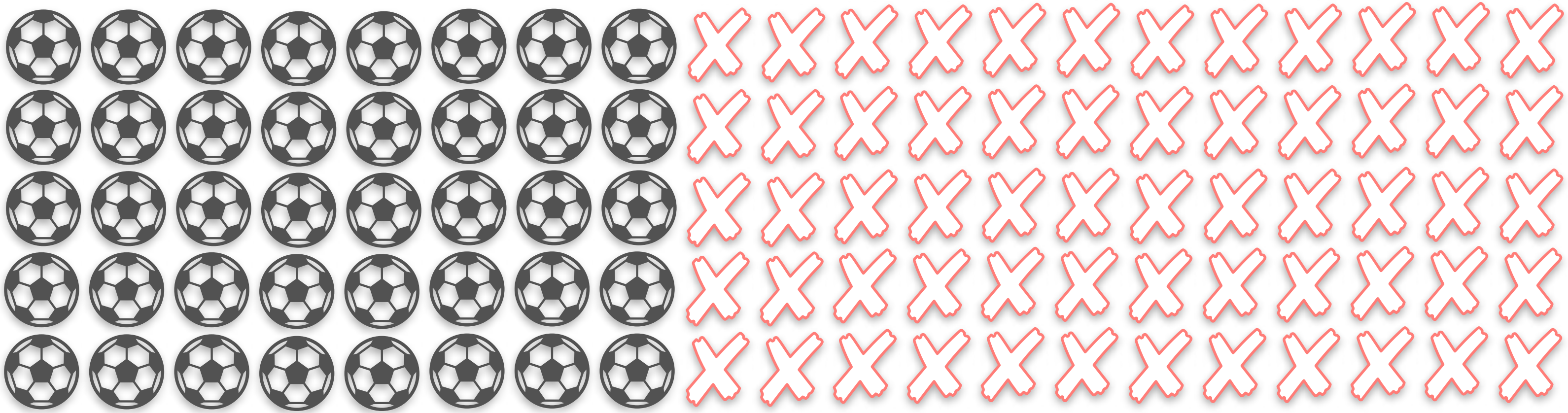


Quiz 2

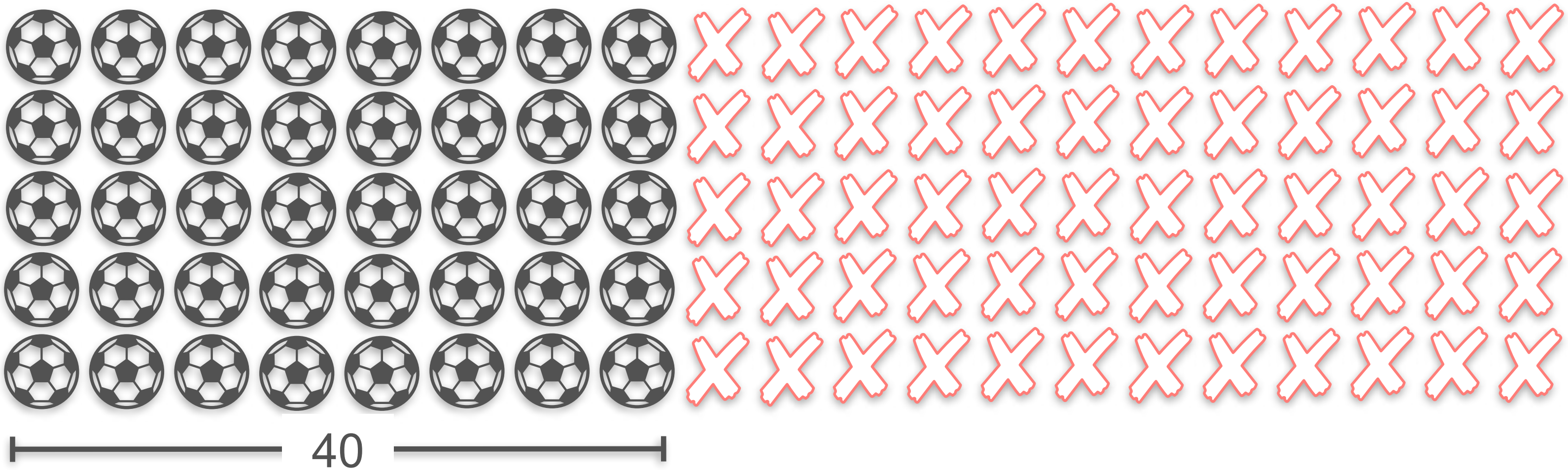
Quiz 2



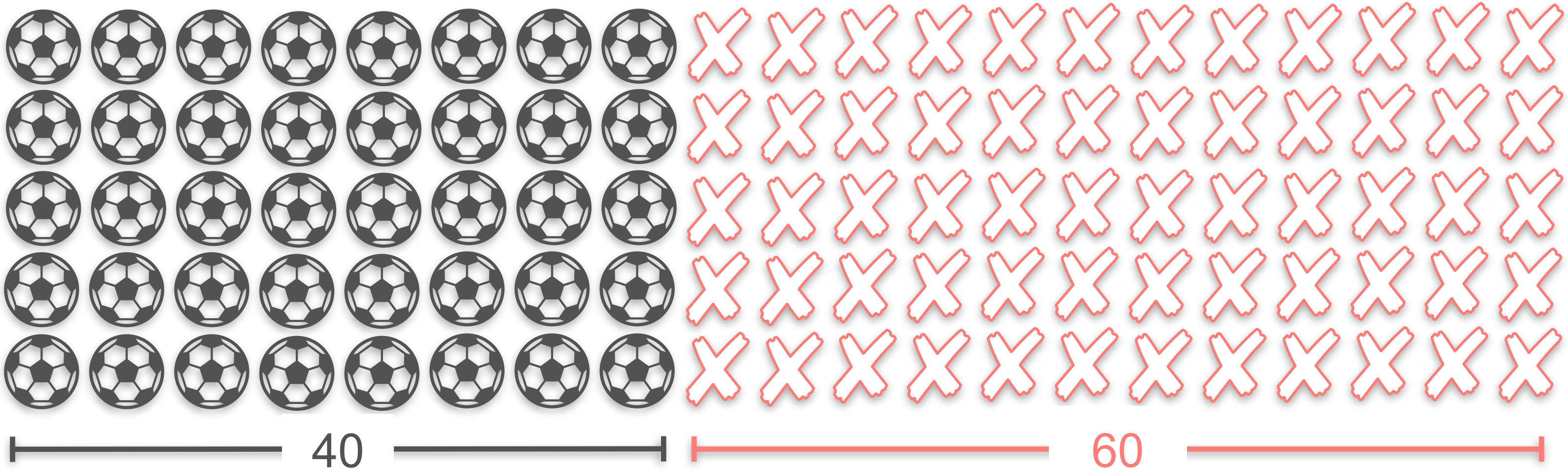
Quiz 2



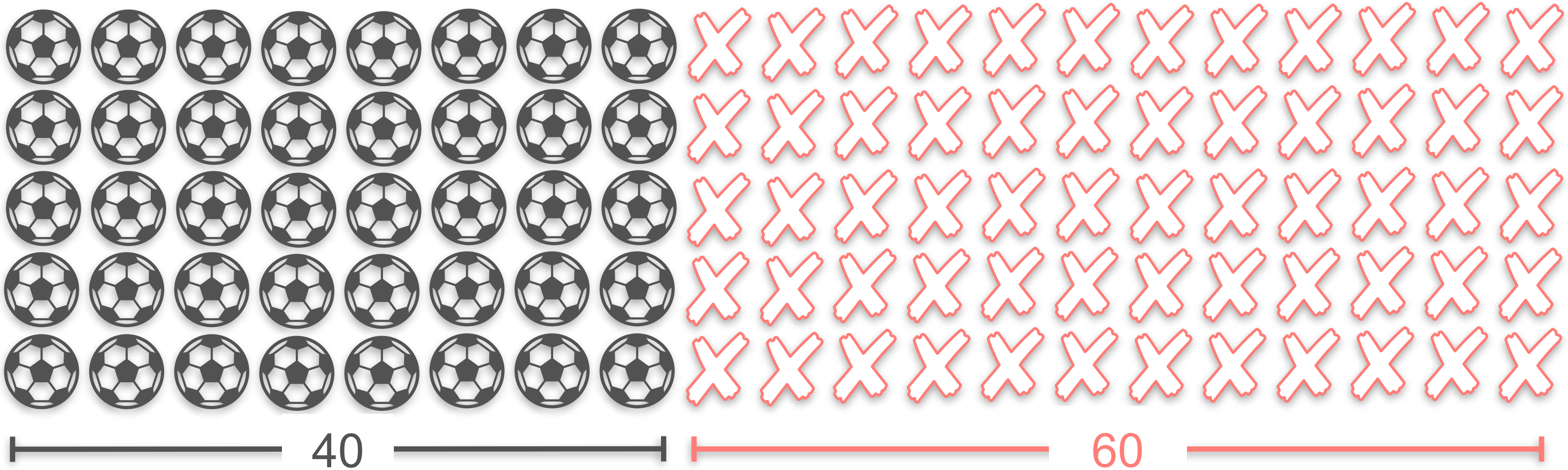
Quiz 2



Quiz 2

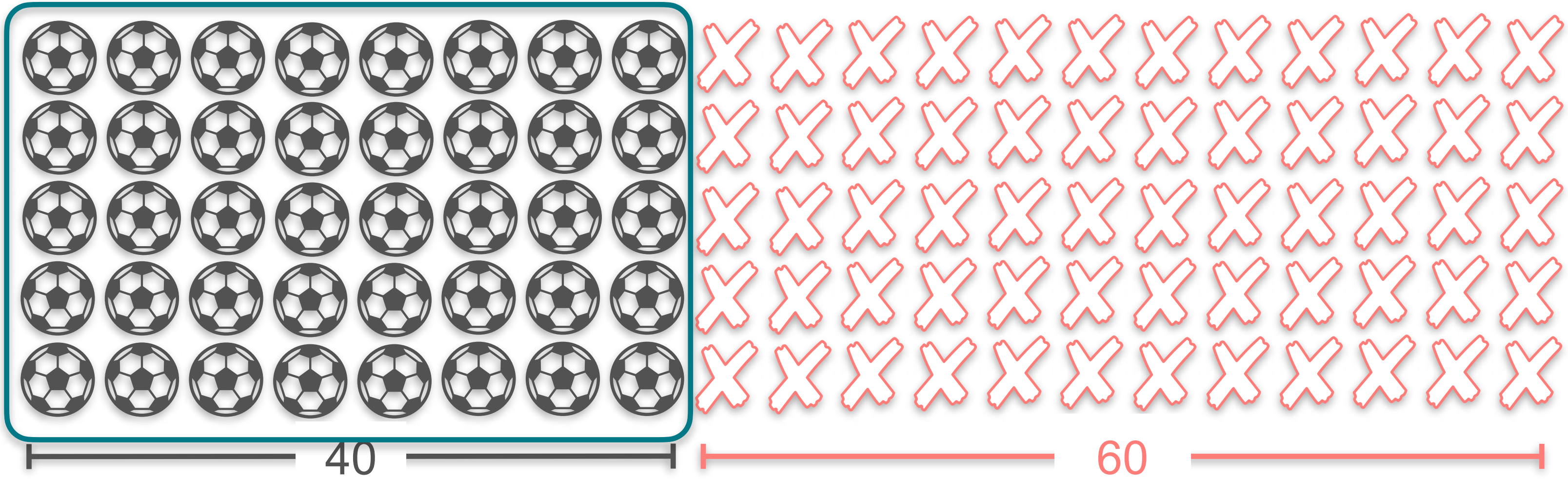


Quiz 2

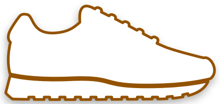


100 kids

Quiz 2

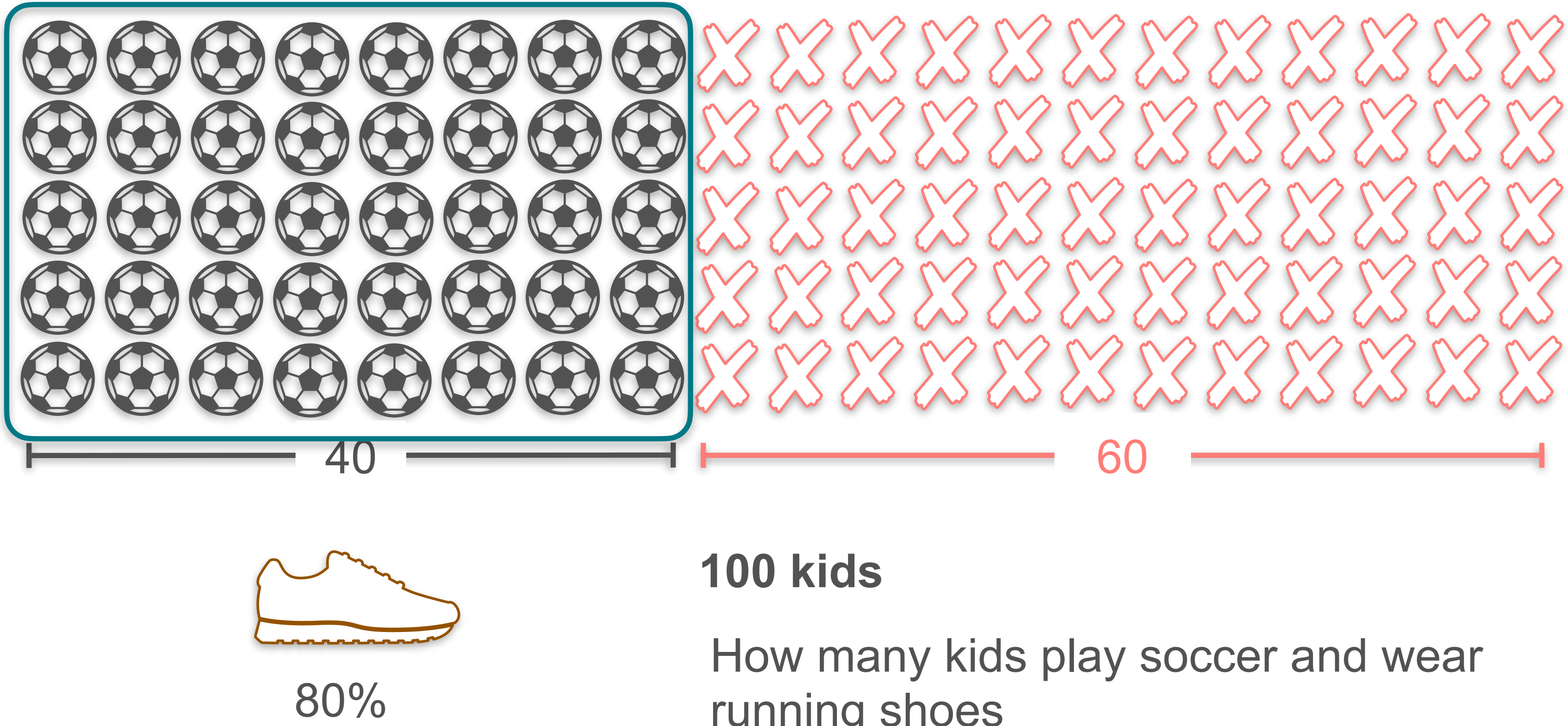


100 kids



80%

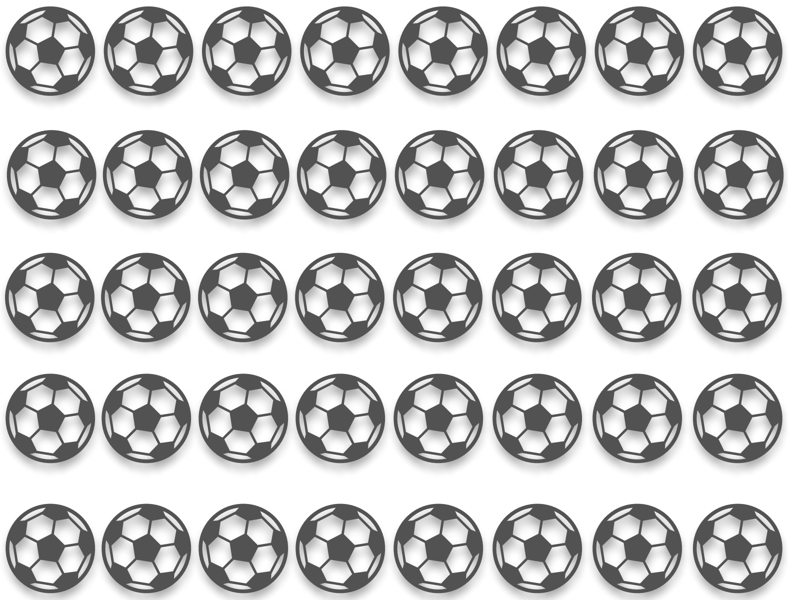
Quiz 2



100 kids

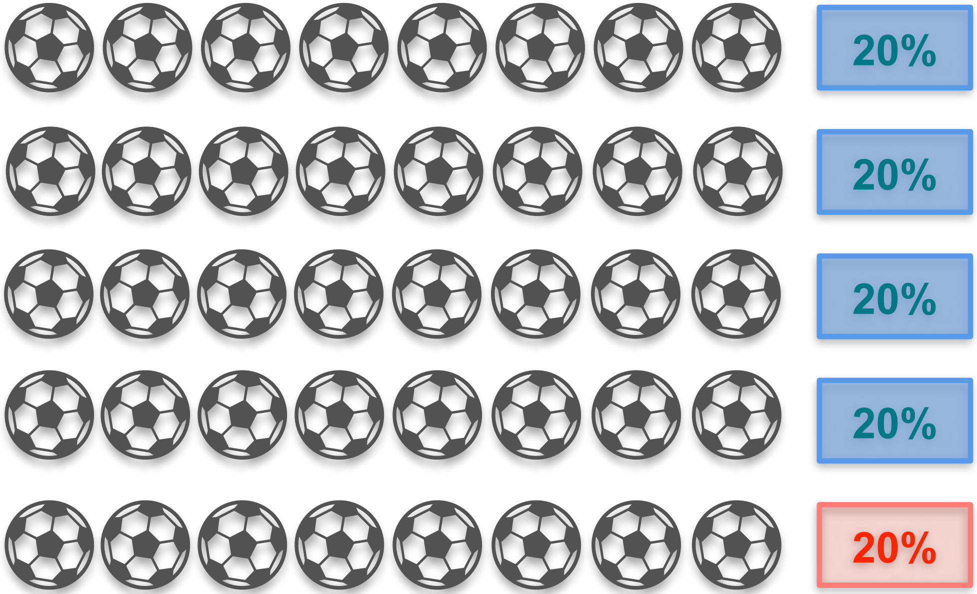
How many kids play soccer and wear running shoes

Quiz 2 - Solution

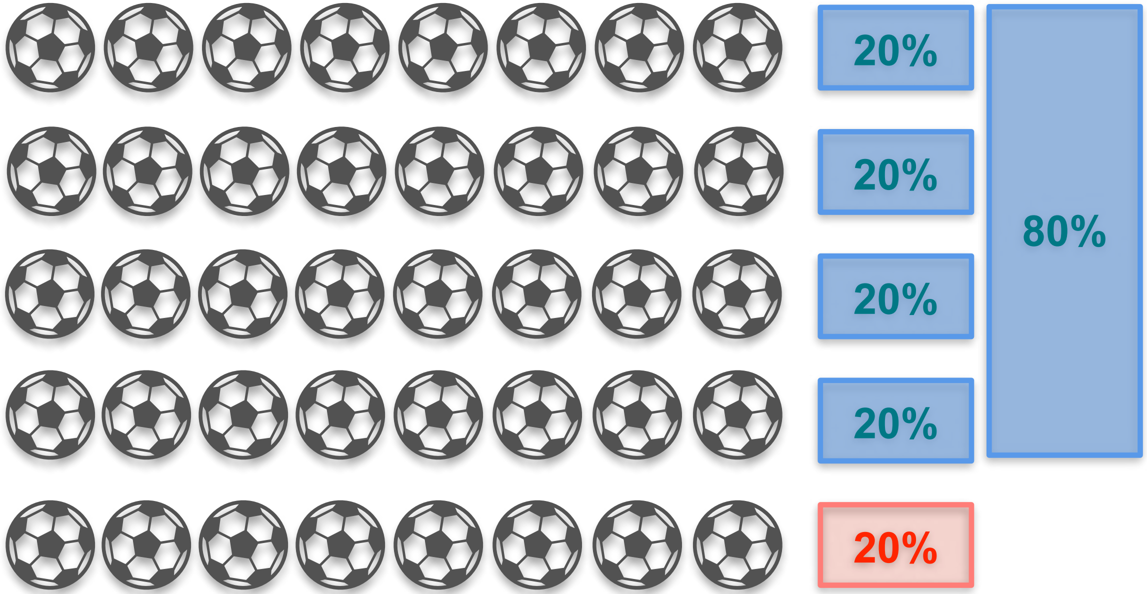


DeepLearning.AI

Quiz 2 - Solution



Quiz 2 - Solution



Quiz 2 - Solution

Conditional Probability

Conditional Probability

Conditional Probability

$P(S) = 0.4$

Conditional Probability

$P(S) = 0.4$

Conditional Probability

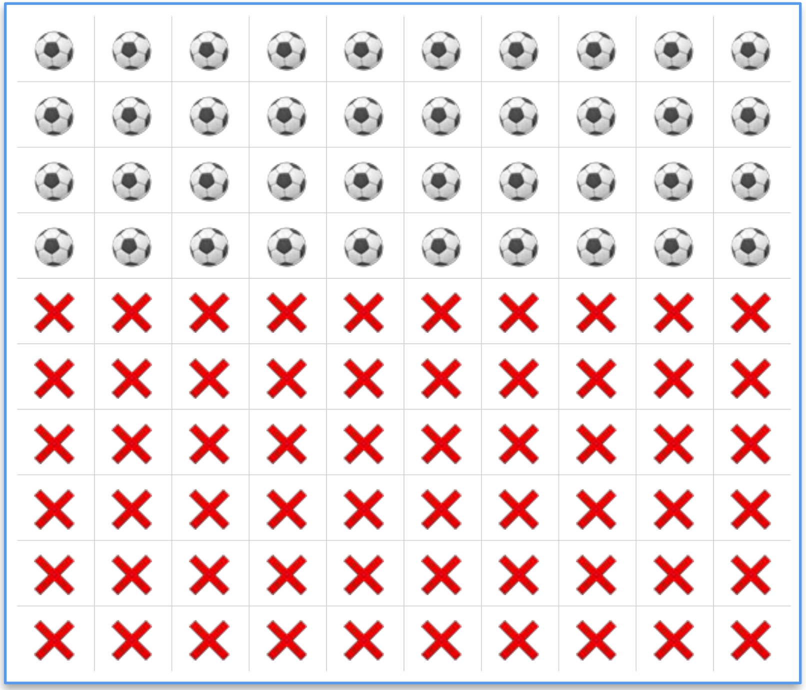
$P(S) = 0.4$

$P(\text{not } S) = 0.6$

Conditional Probability

$P(S) = 0.4$

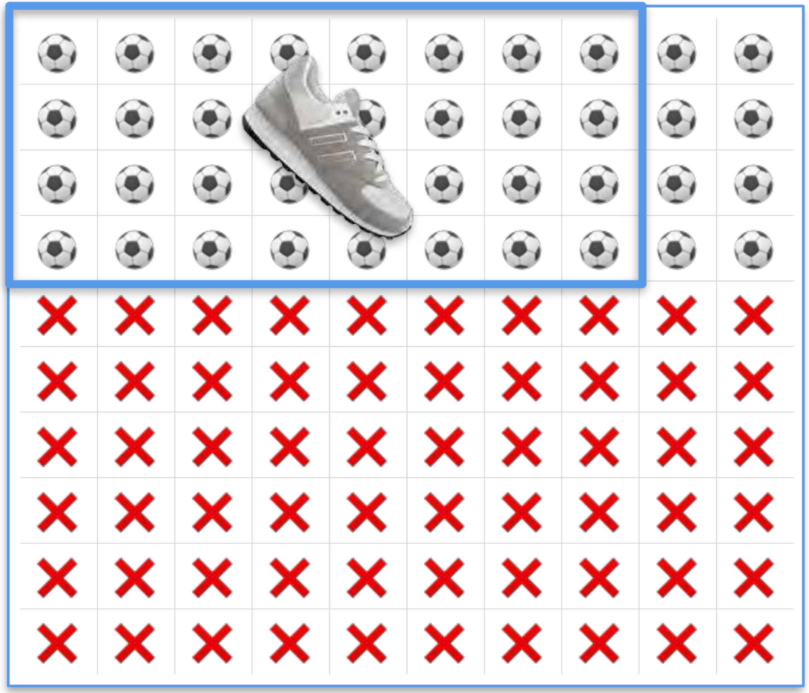
$P(\text{not } S) = 0.6$



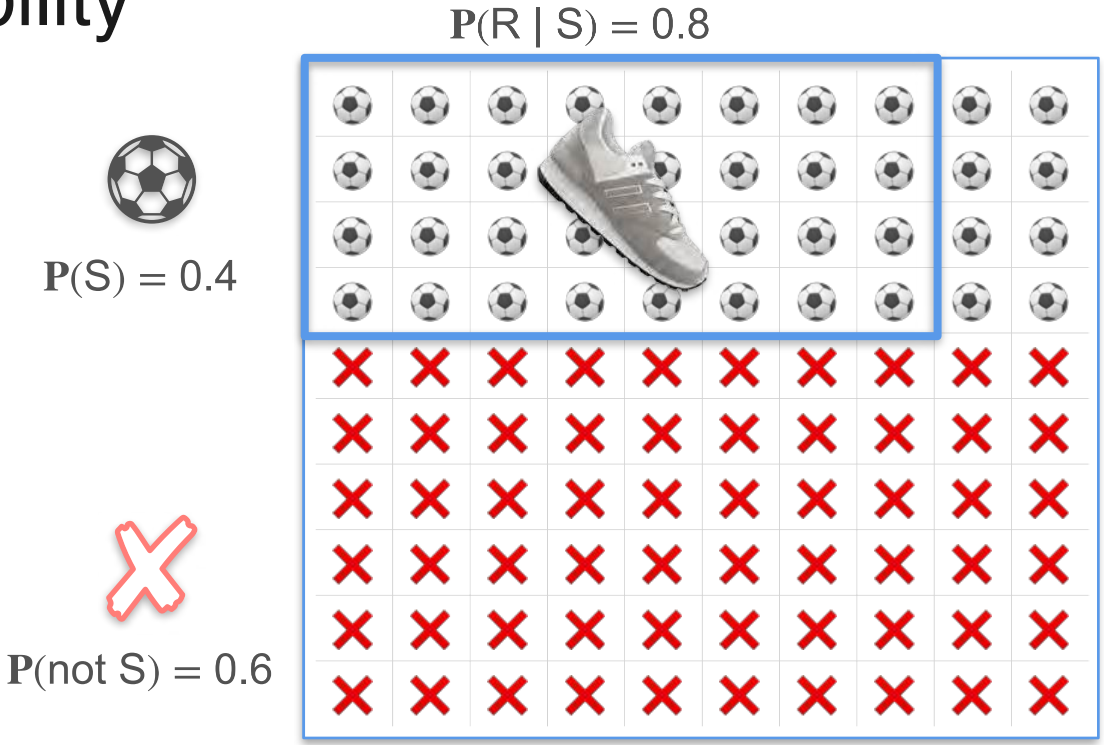
Conditional Probability

$P(S) = 0.4$

$P(\text{not } S) = 0.6$



Conditional Probability



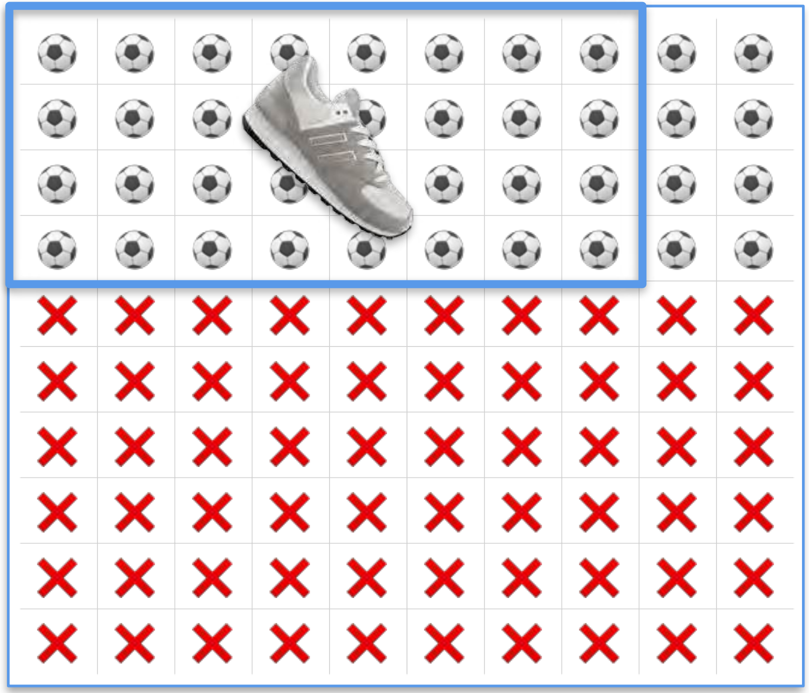
Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

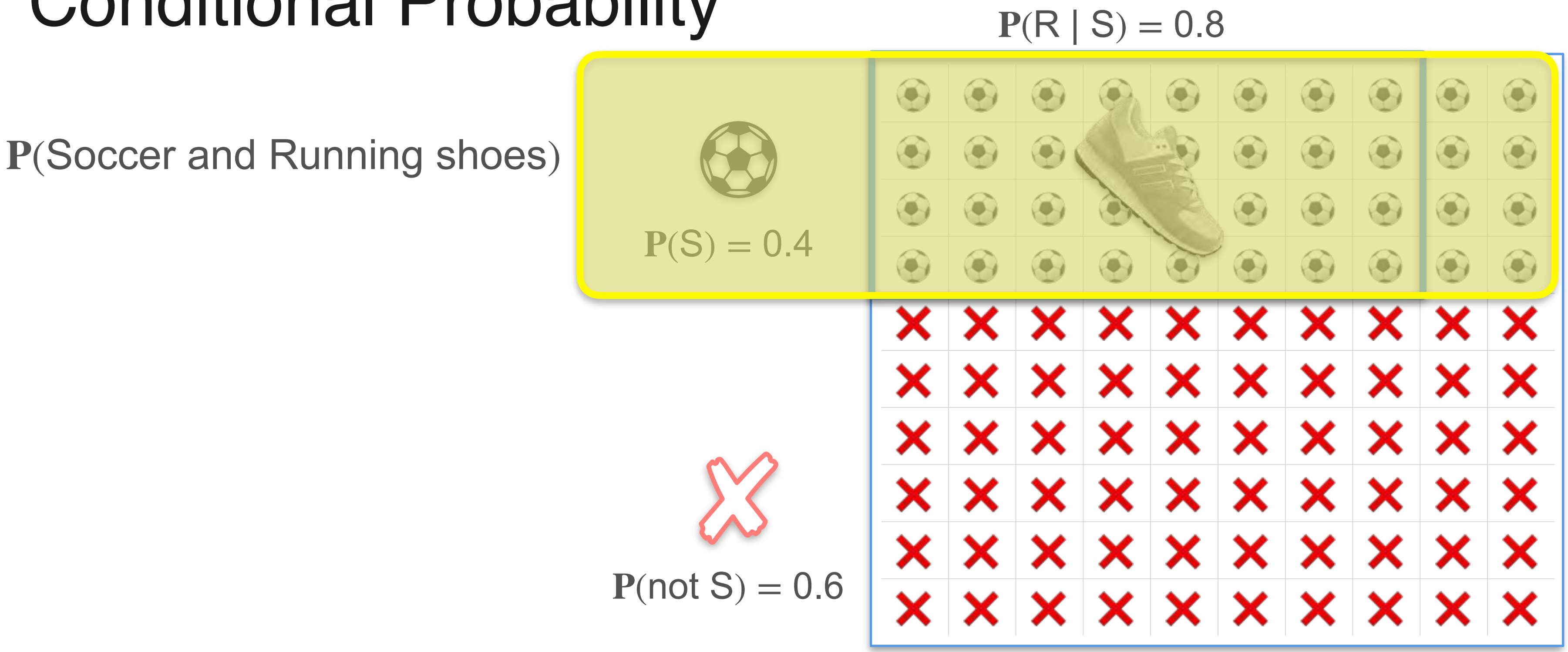
$\mathbf{P}(\text{S}) = 0.4$

$\mathbf{P}(\text{not S}) = 0.6$

$\mathbf{P}(\text{R} \mid \text{S}) = 0.8$

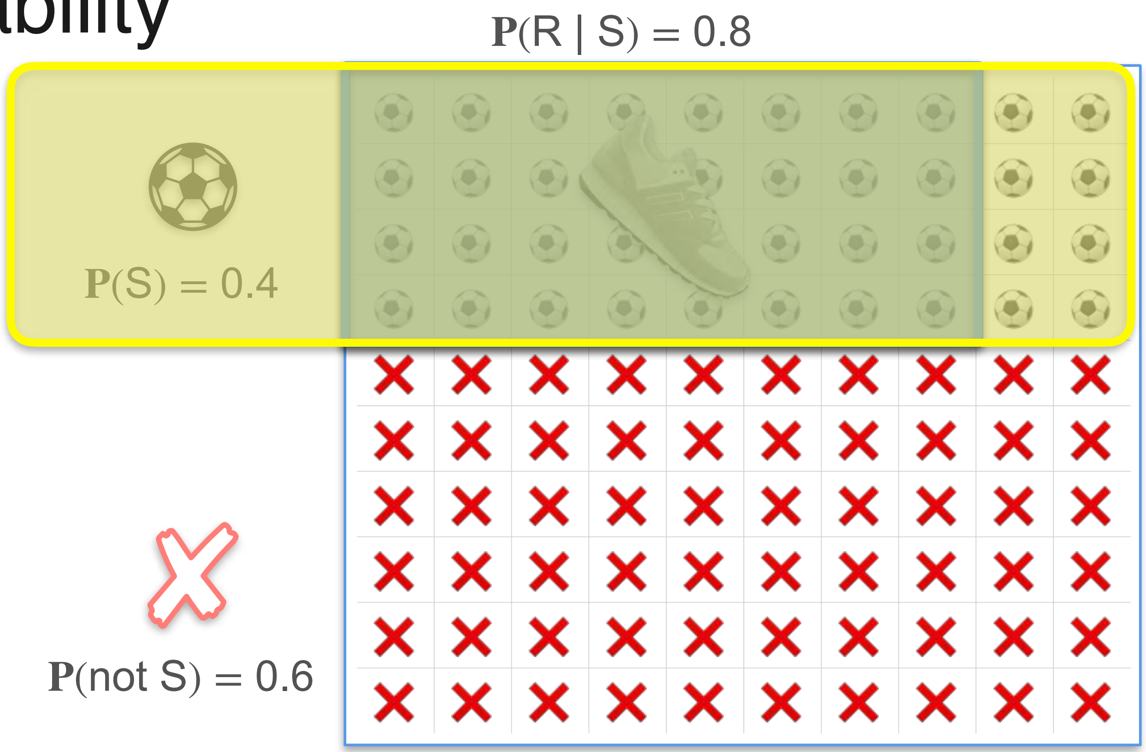


Conditional Probability



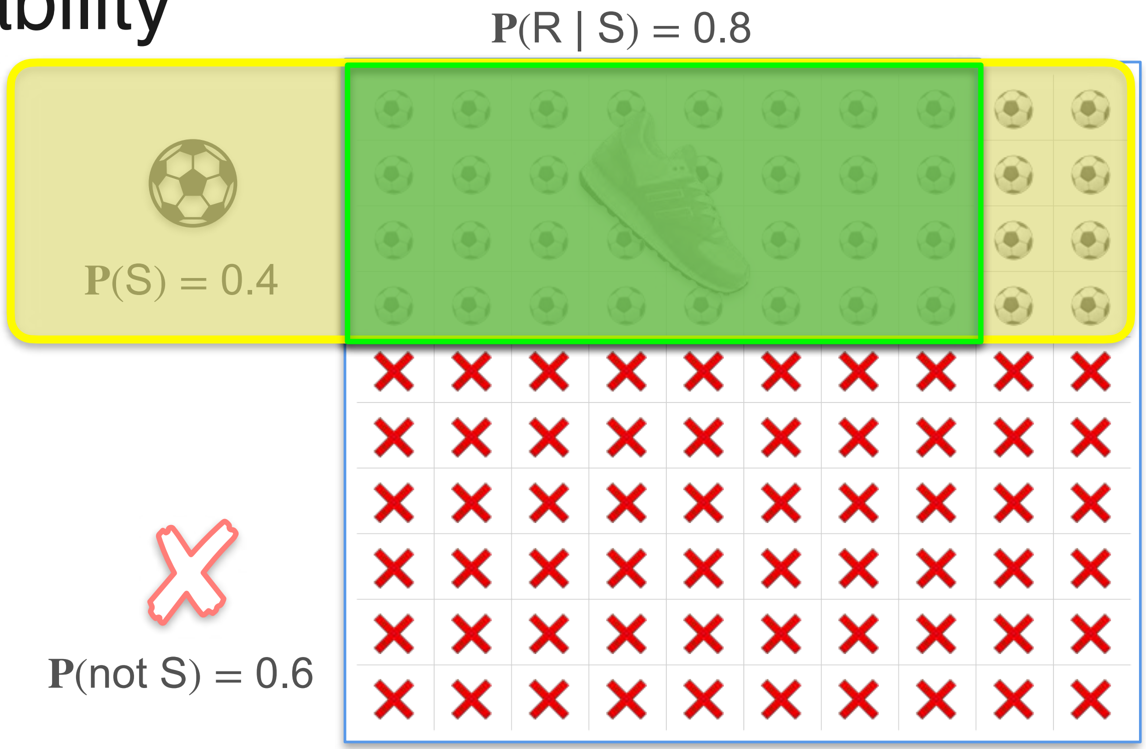
Conditional Probability

$\mathbf{P}(\text{Soccer and Running shoes})$



Conditional Probability

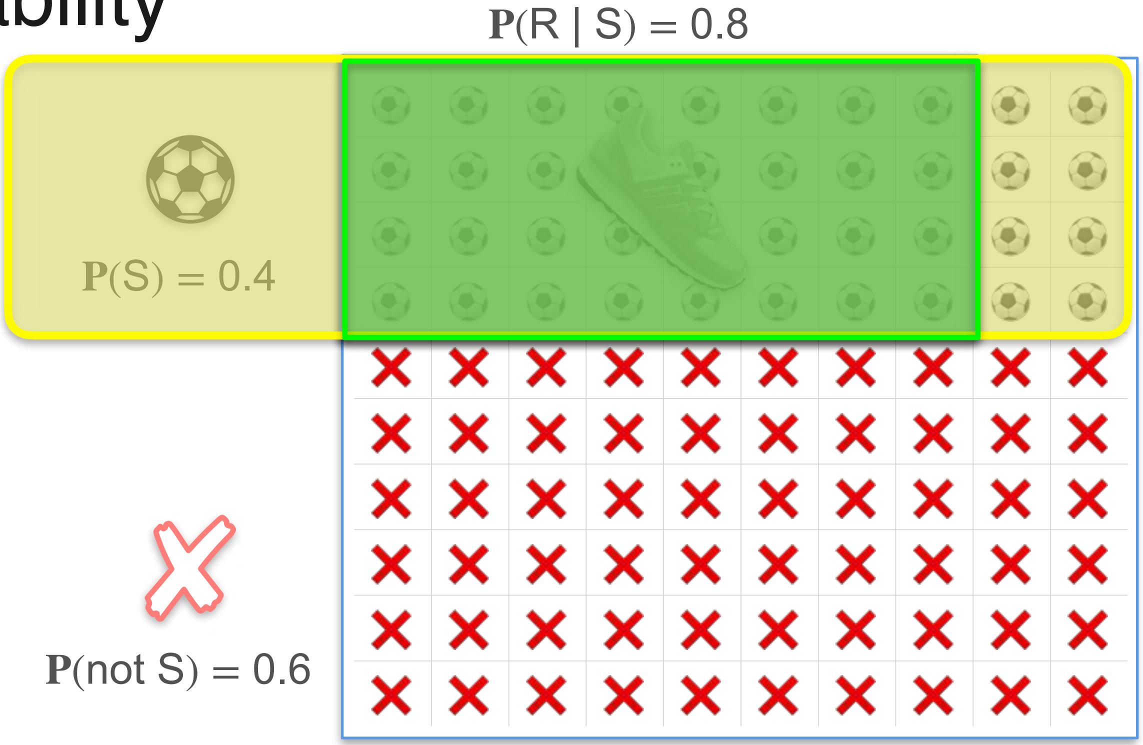
$\mathbf{P}(\text{Soccer and Running shoes})$



Conditional Probability

$\mathbf{P}(\text{Soccer and Running shoes})$

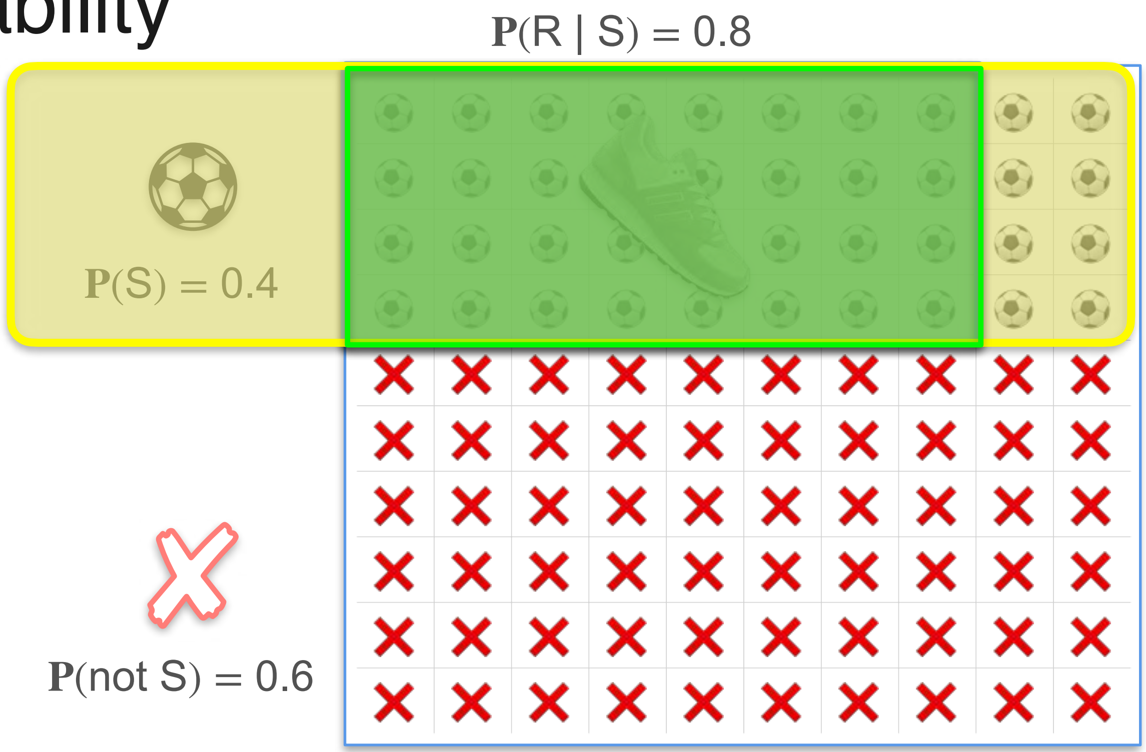
$\mathbf{P}(S \cap R) =$



Conditional Probability

P(Soccer and Running shoes)

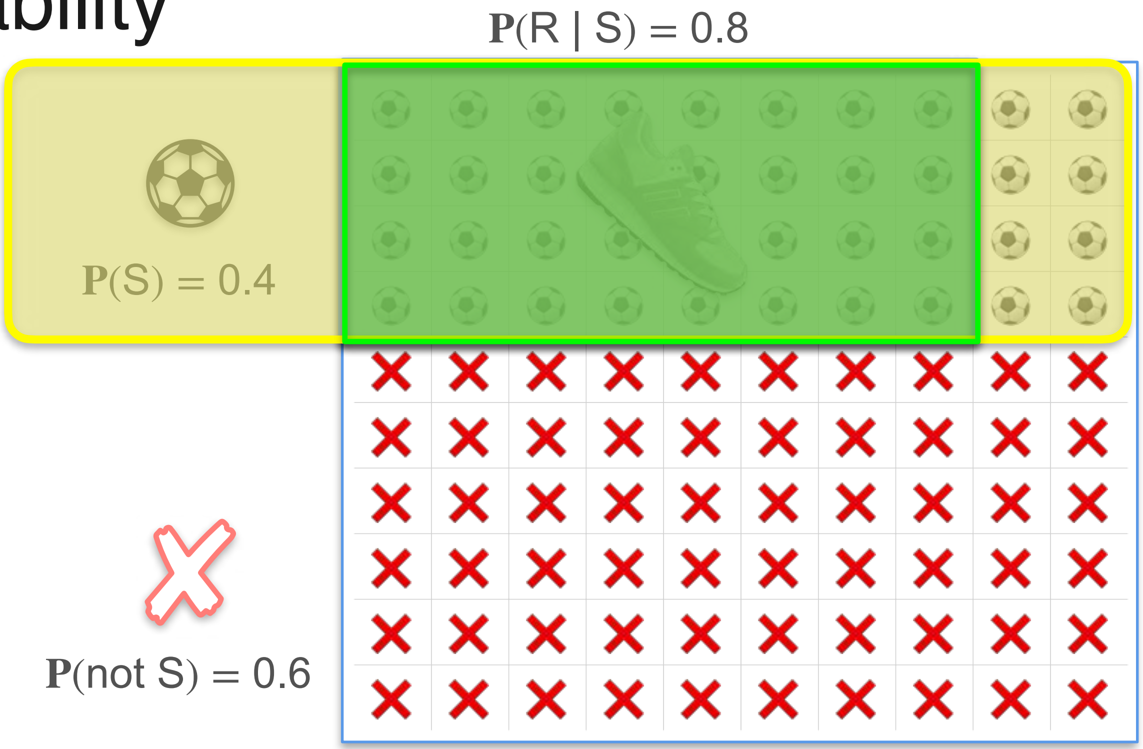
$P(S \cap R) = \quad P(S)$



Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

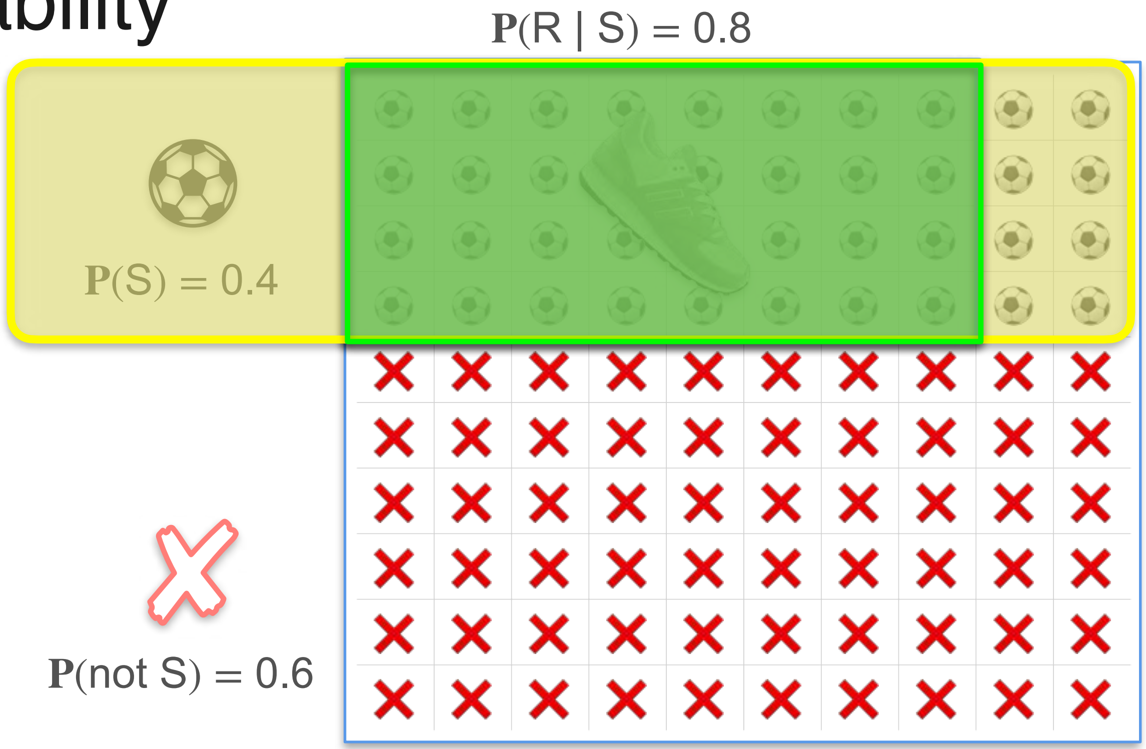
$\mathbf{P}(S \cap R) = \quad \mathbf{P}(S)$



Conditional Probability

P(Soccer and Running shoes)

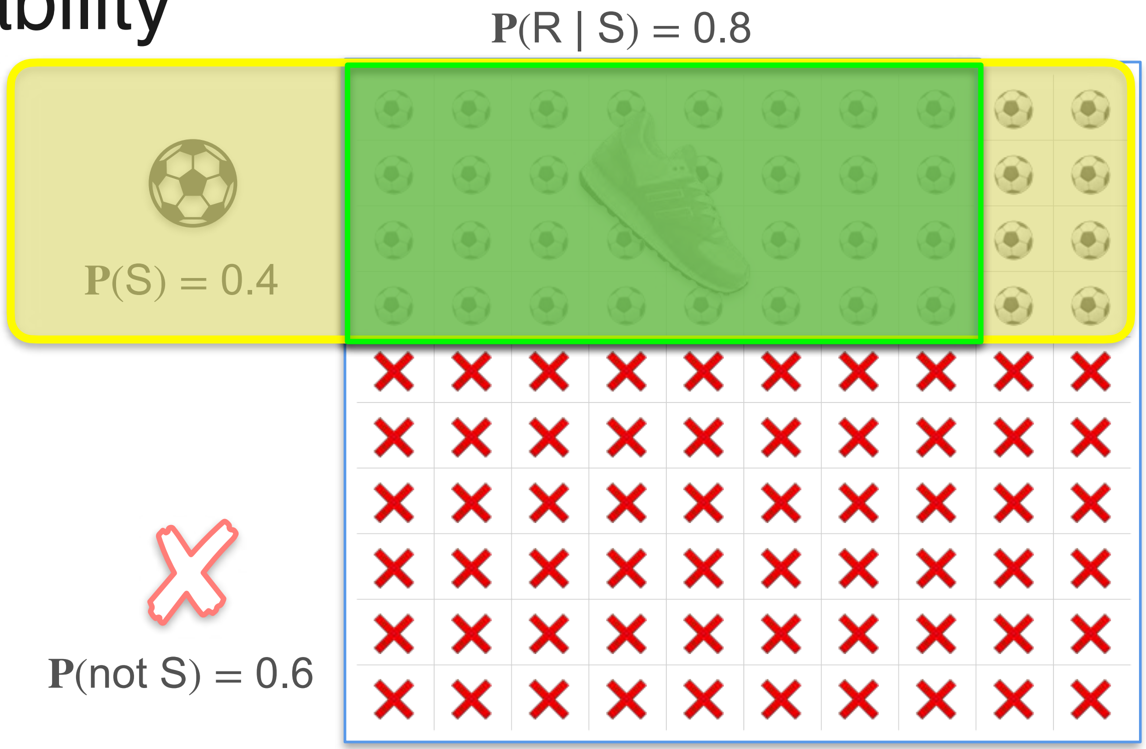
$P(S \cap R) = P(S) \cdot$



Conditional Probability

P(Soccer and Running shoes)

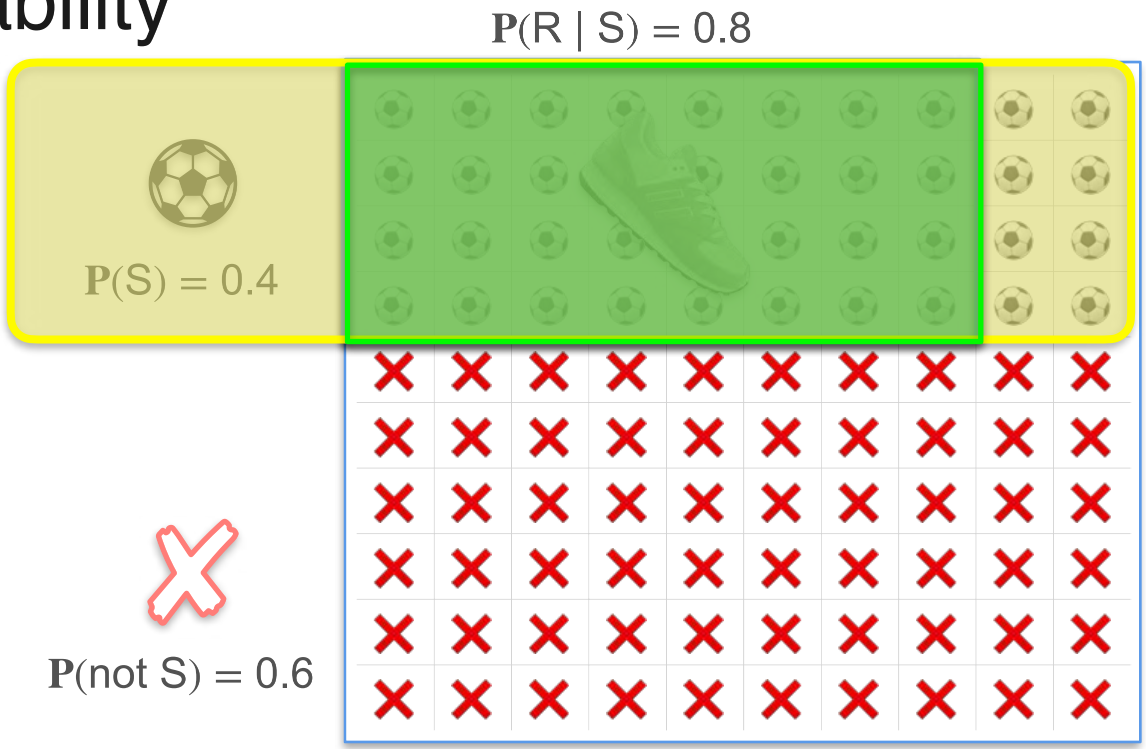
$$P(S \cap R) = P(S) \bullet P(R \mid S)$$



Conditional Probability

$P(\text{Soccer and Running shoes})$

$P(S \cap R) = P(S) \bullet P(R \mid S)$

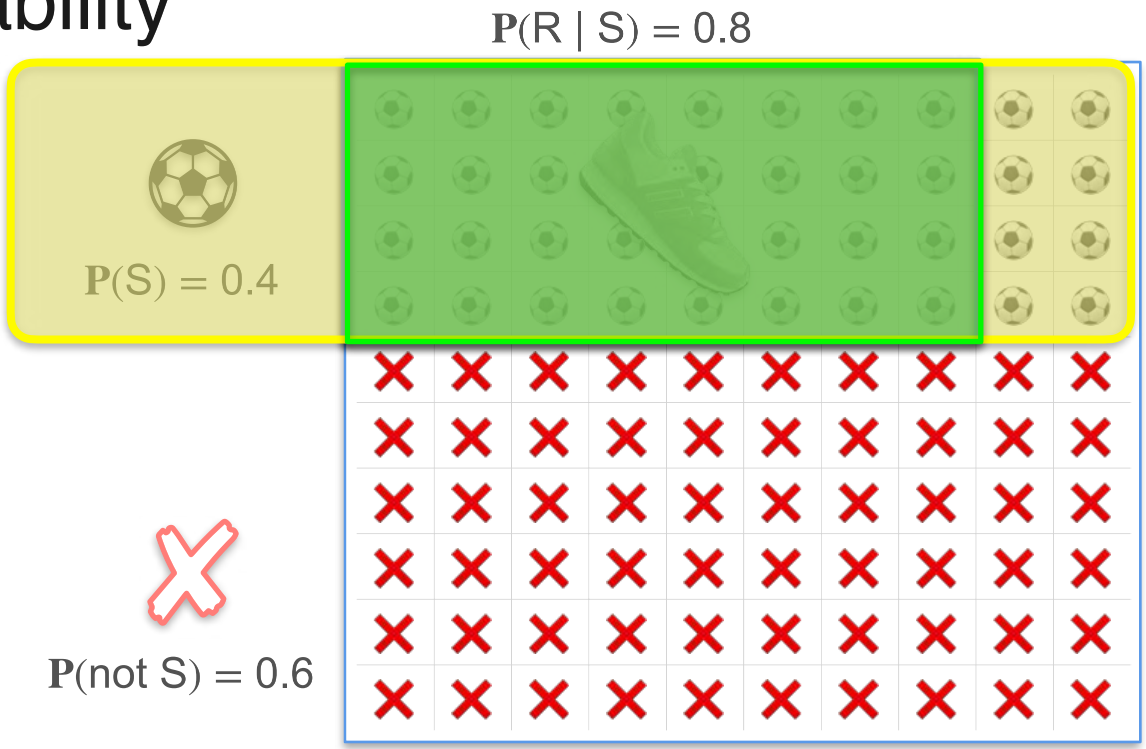


Conditional Probability

P(Soccer and Running shoes)

$$P(S \cap R) = P(S) \bullet P(R \mid S)$$

$$= 0.4$$

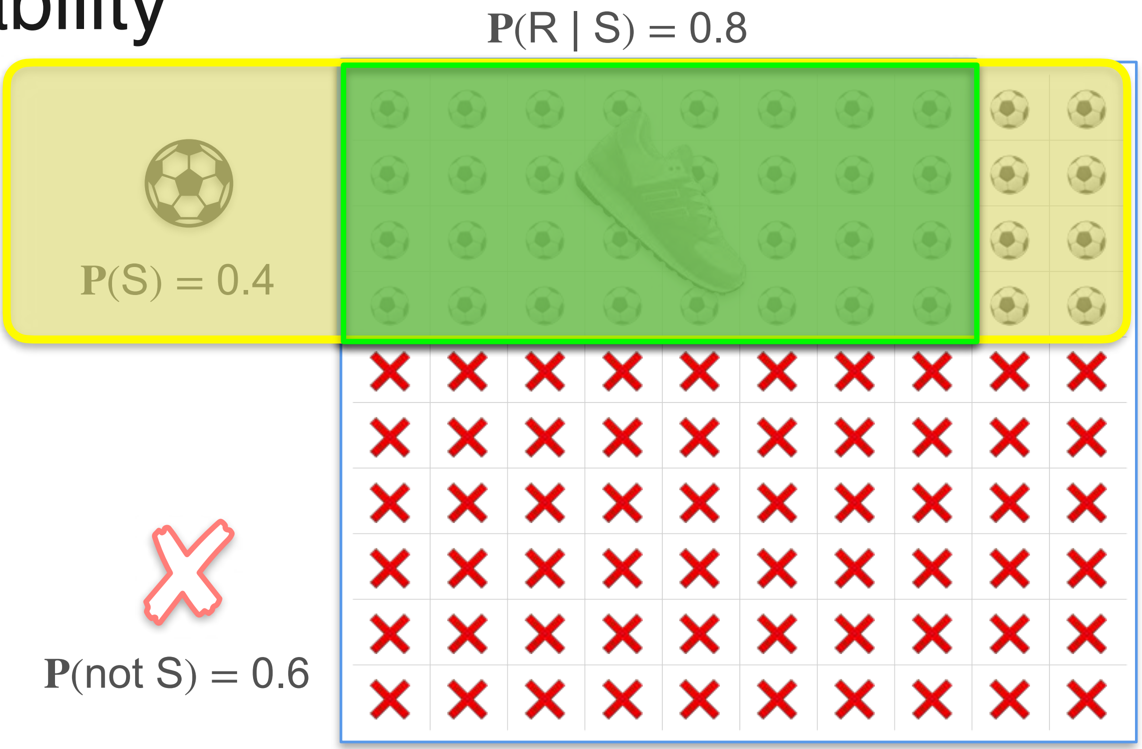


Conditional Probability

P(Soccer and Running shoes)

$P(S \cap R) = P(S) \bullet P(R|S)$

$= 0.4 \quad \bullet$

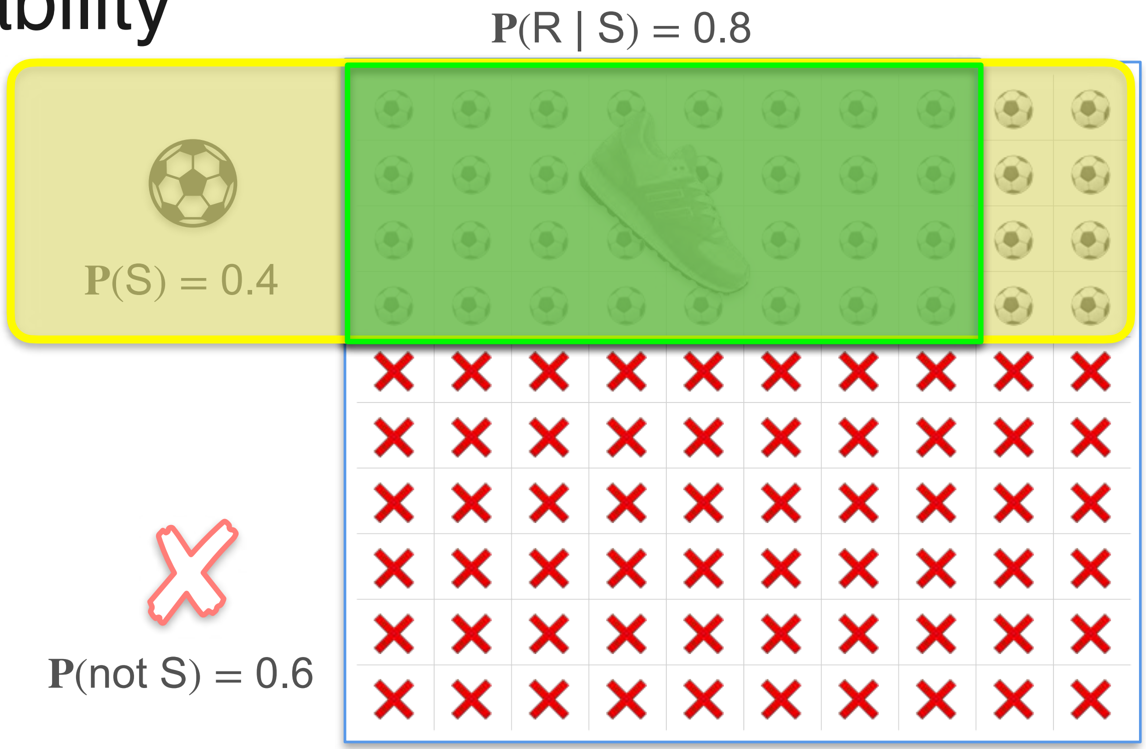


Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$$\mathbf{P}(S \cap R) = \mathbf{P}(S) \bullet \mathbf{P}(R \mid S)$$

$$= 0.4 \bullet 0.8$$



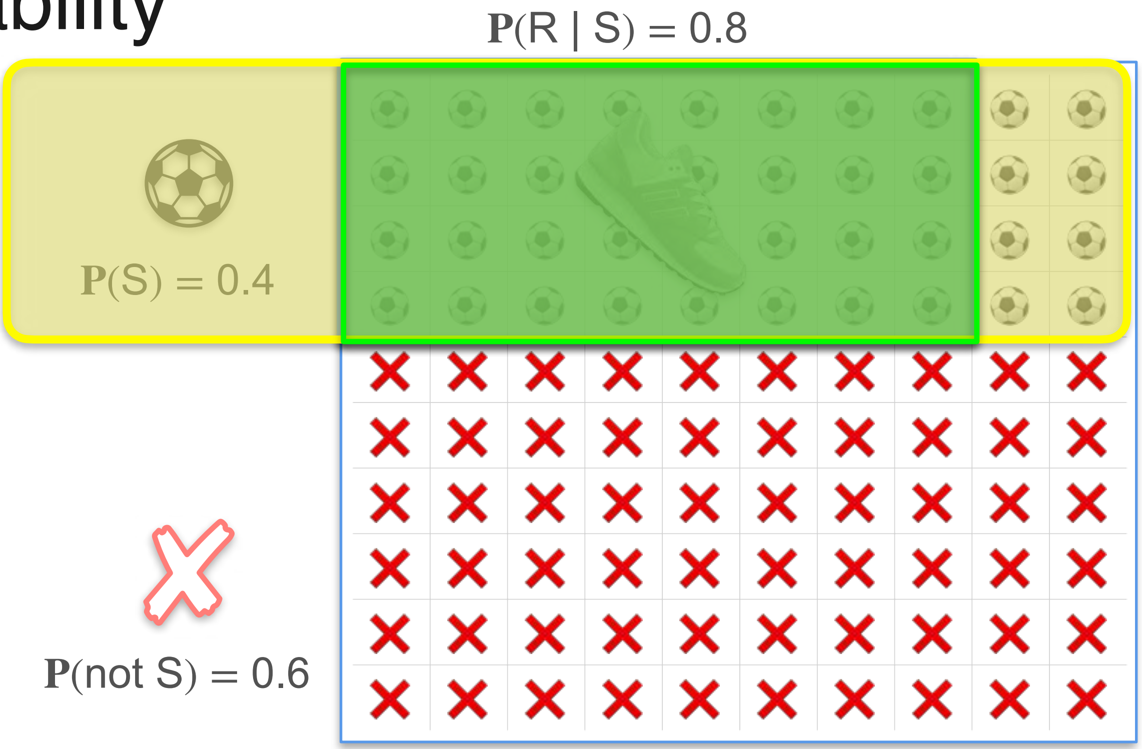
Conditional Probability

P(Soccer and Running shoes)

$$P(S \cap R) = P(S) \bullet P(R|S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



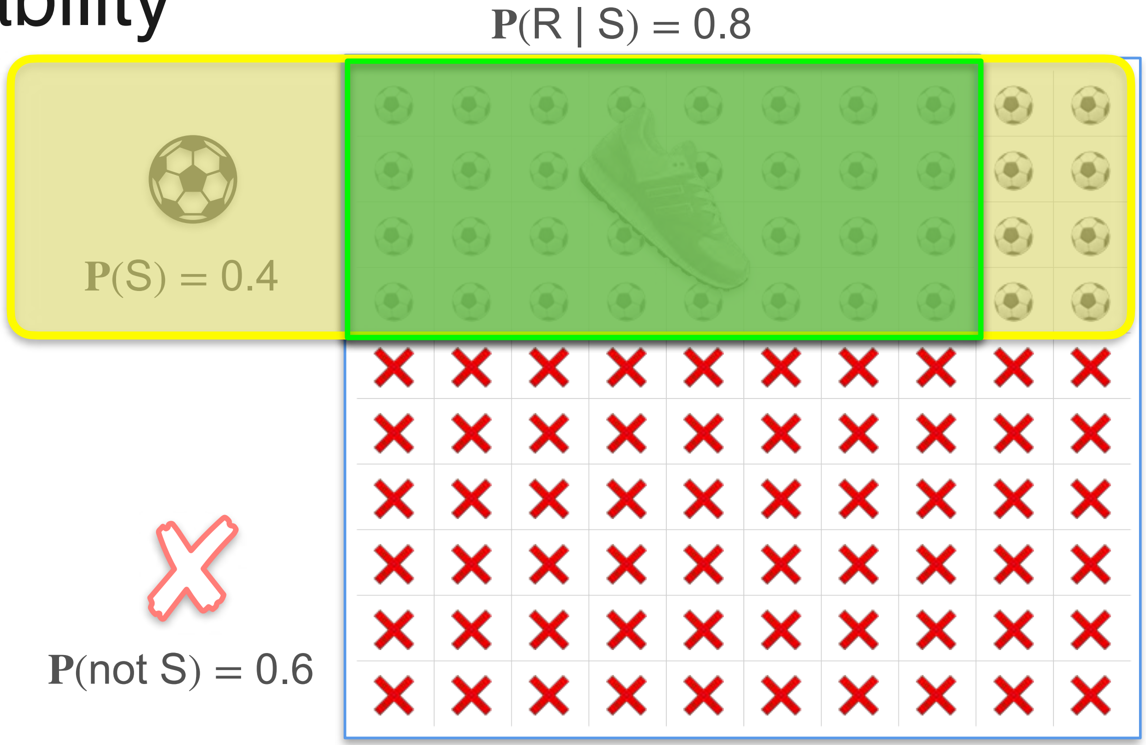
Conditional Probability

P(Soccer and Running shoes)

$$P(S \cap R) = P(S) \bullet P(R \mid S)$$

$$= 0.4 \bullet 0.8$$

$$= 0.32$$



Conditional Probability

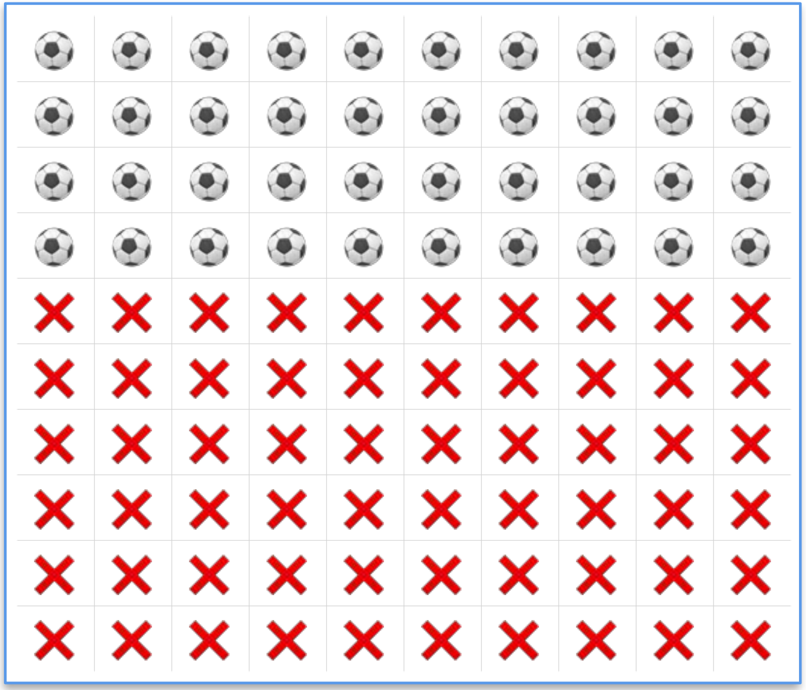
$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



Conditional Probability

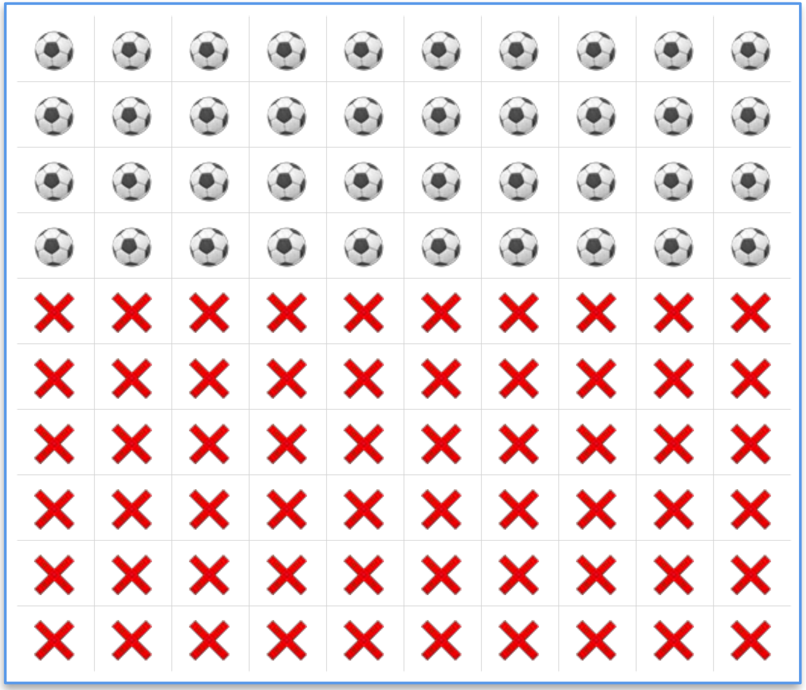
$\mathbf{P}(R \mid S) = 0.8$

P(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}(\text{not } S) = 0.6$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

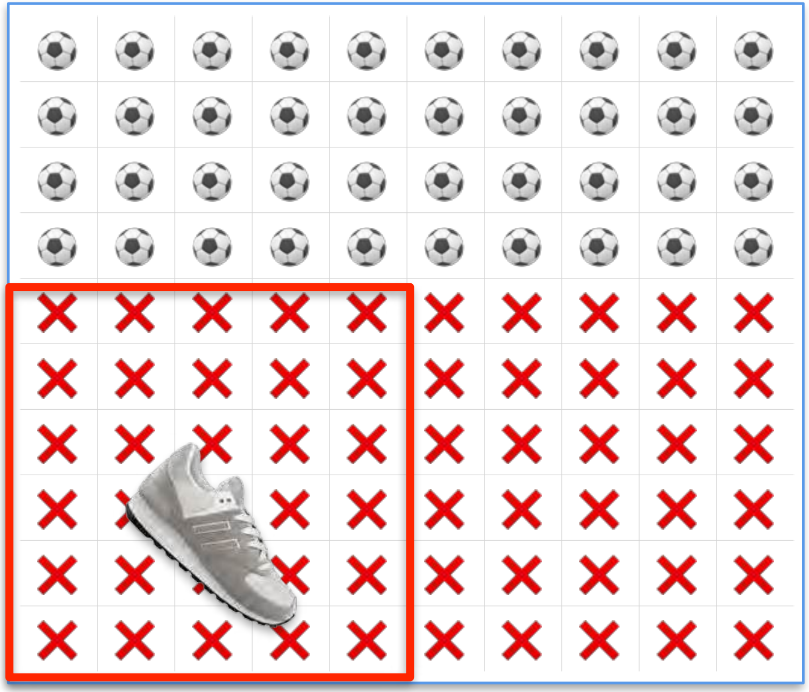
P(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

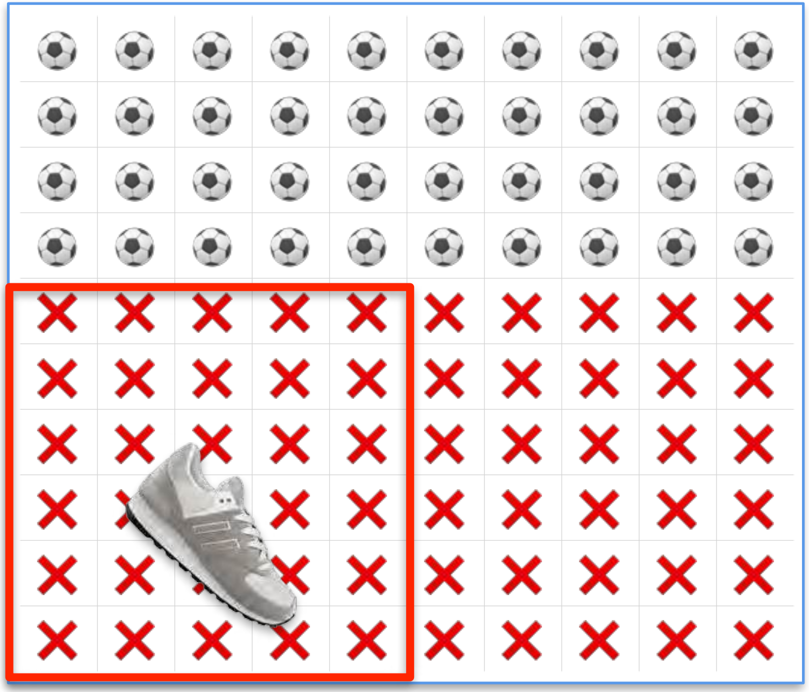
$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}$(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

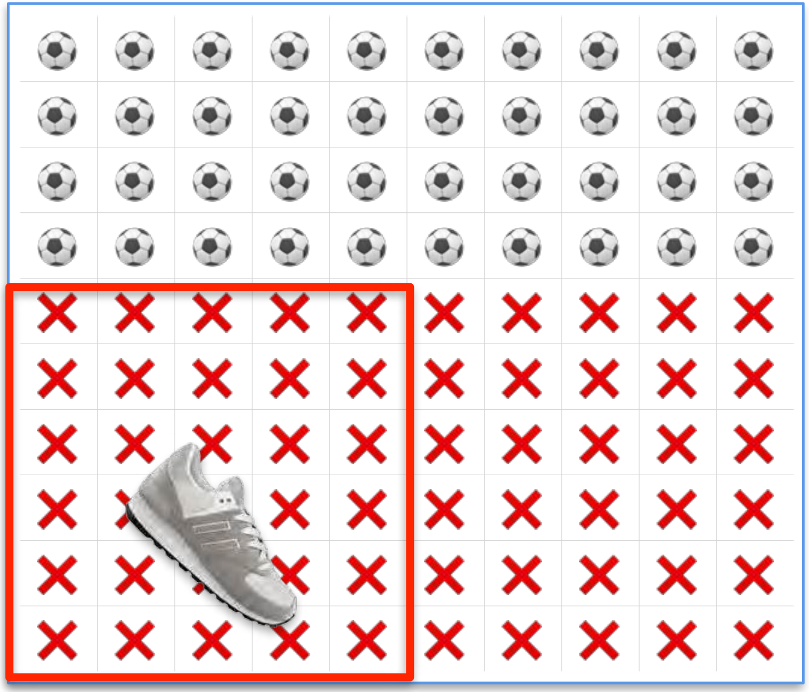
Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}$(not Soccer and Running shoes)

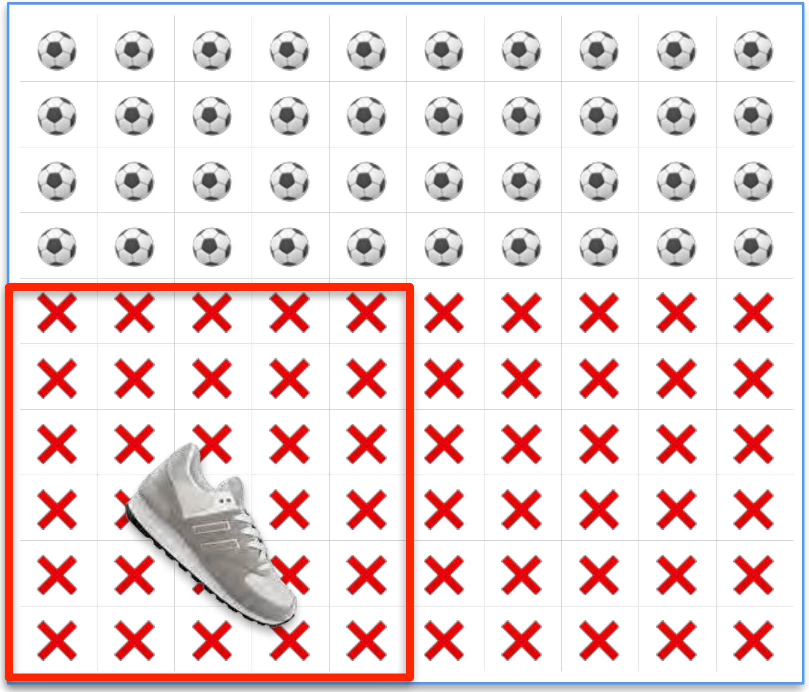
$\mathbf{P}(\text{not } S \cap R) =$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}$(not Soccer and Running shoes)

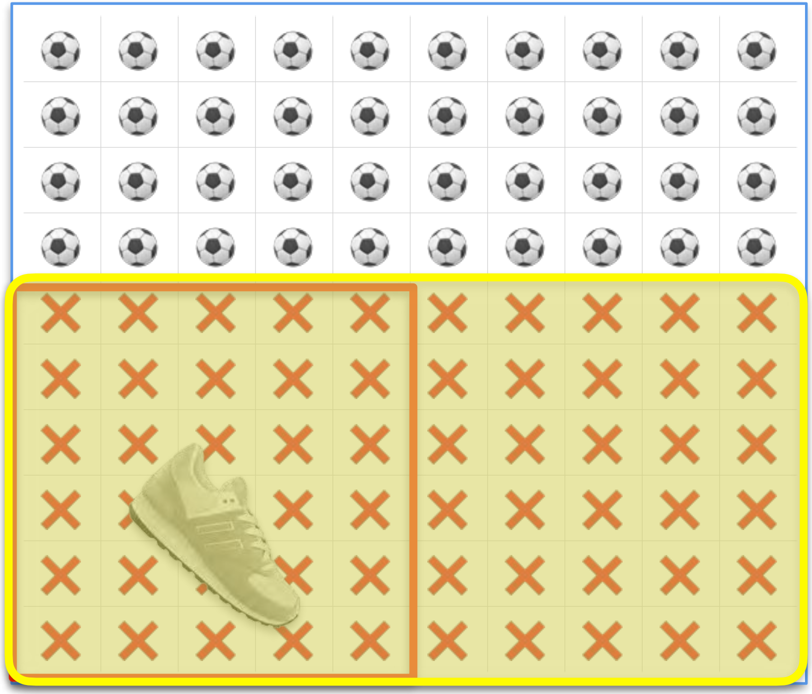
$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S)$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}$(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S \cap R) = \quad \mathbf{P}(\text{not } S)$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

P(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

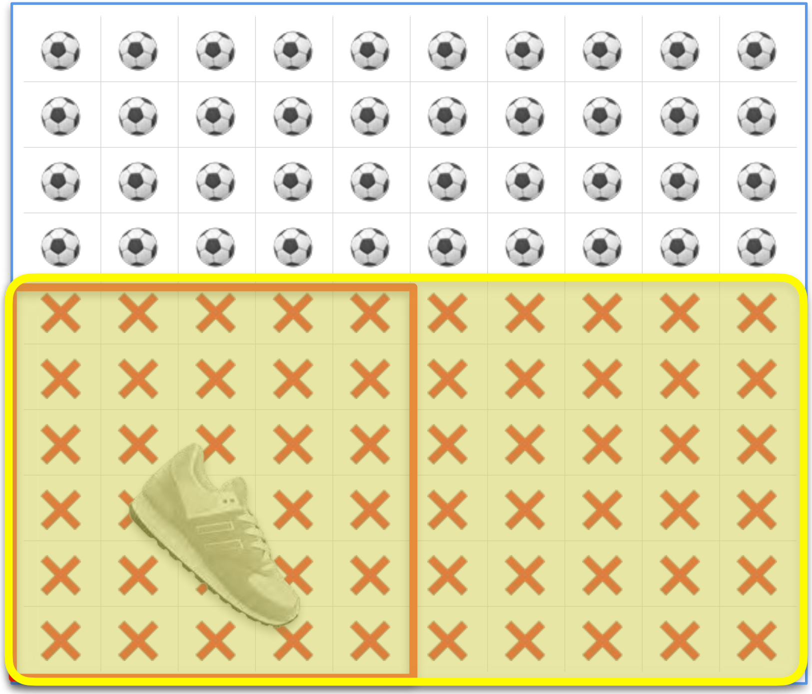
$\mathbf{P}(S) = 0.4$

P(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

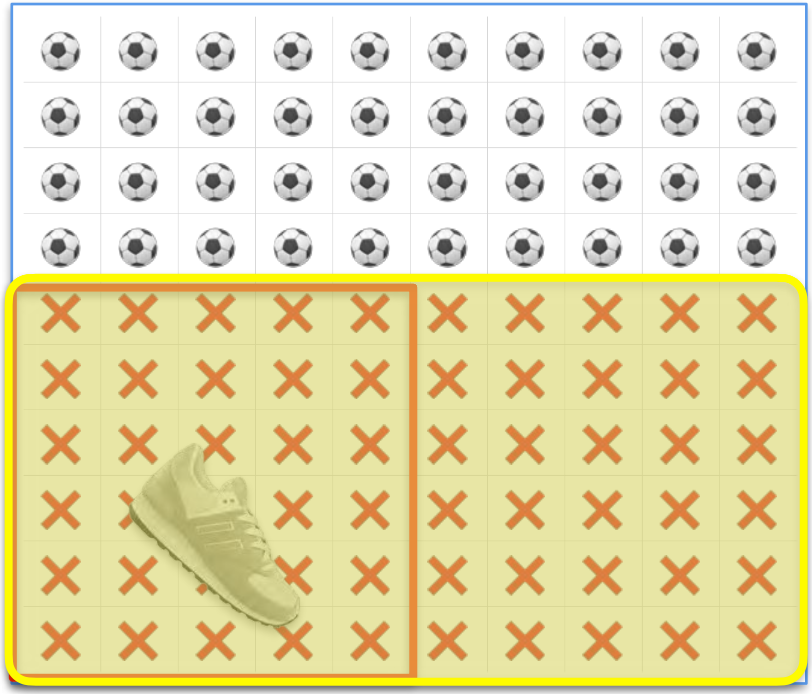
$\mathbf{P}(S) = 0.4$

$\mathbf{P}$(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet \mathbf{P}(R \mid \text{not } S)$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

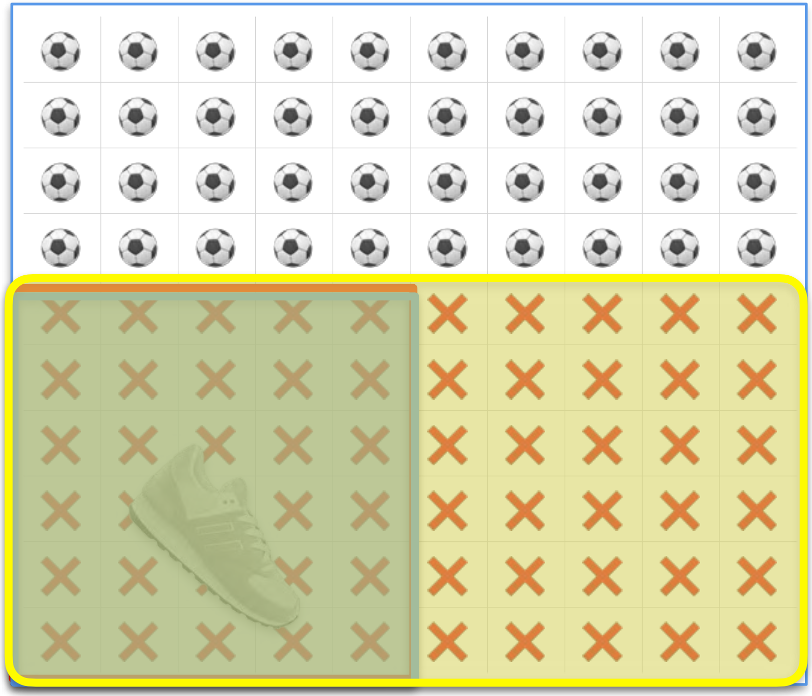
$\mathbf{P}(S) = 0.4$

$\mathbf{P}$(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet \mathbf{P}(R \mid \text{not } S)$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

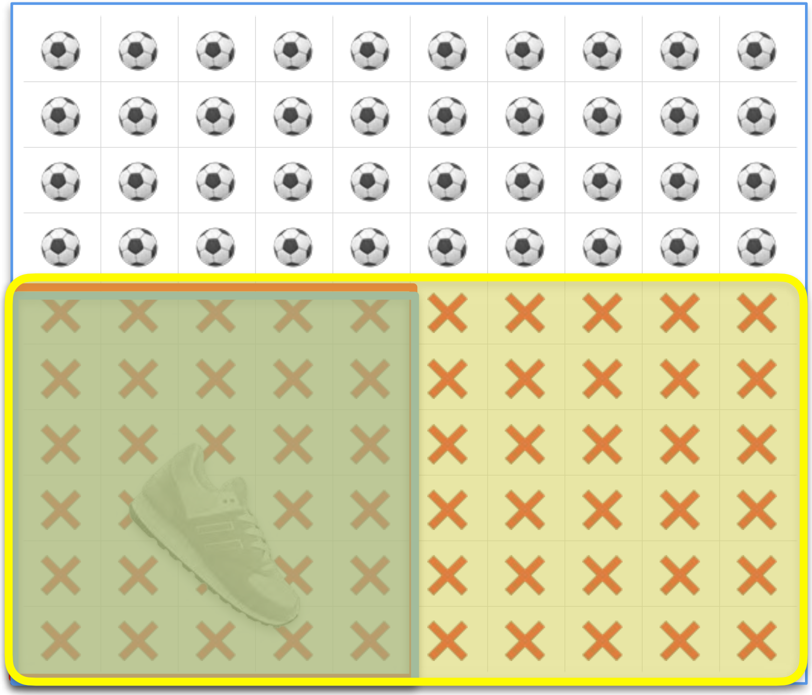
$\mathbf{P}$(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet \mathbf{P}(R \,|\, \text{not } S)$

$= 0.6$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \,|\, S) = 0.8$



$\mathbf{P}(R \,|\, \text{not } S) = 0.5$

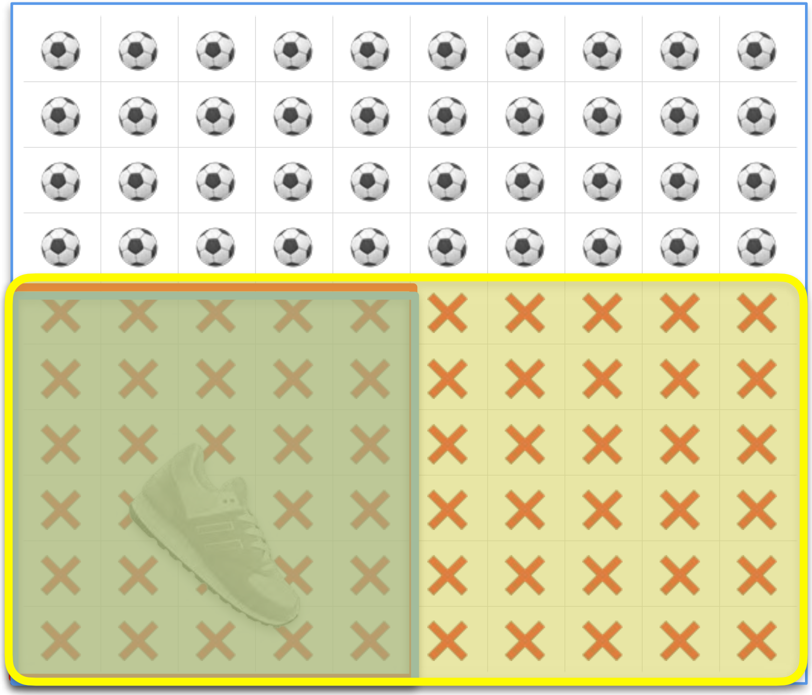
Conditional Probability

$\mathbf{P}(R \mid S) = 0.8$

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$



$\mathbf{P}$(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet \mathbf{P}(R \mid \text{not } S)$

$= 0.6 \bullet$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

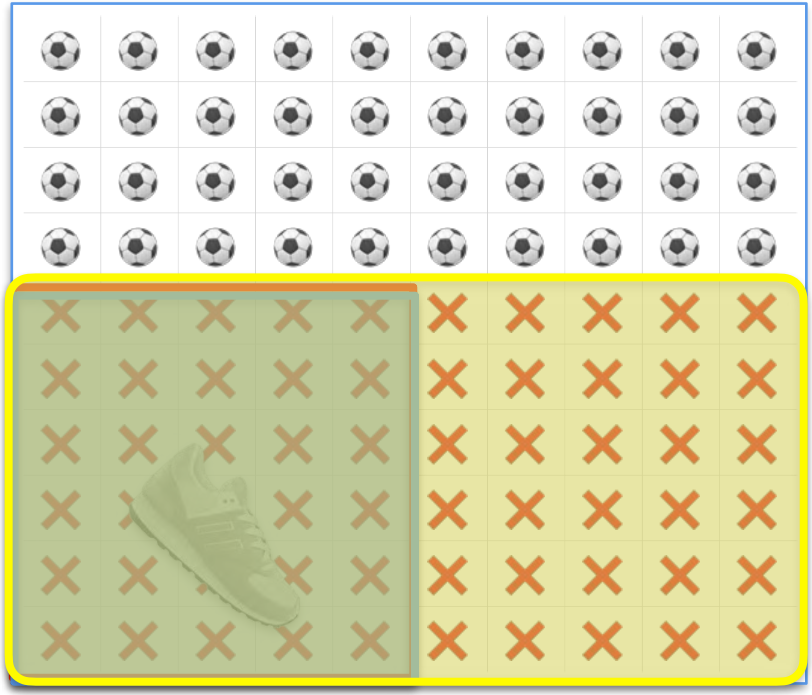
$\mathbf{P}$(not Soccer and Running shoes)

$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet \mathbf{P}(R \mid \text{not } S)$

$= 0.6 \bullet 0.5$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}$(not Soccer and Running shoes)

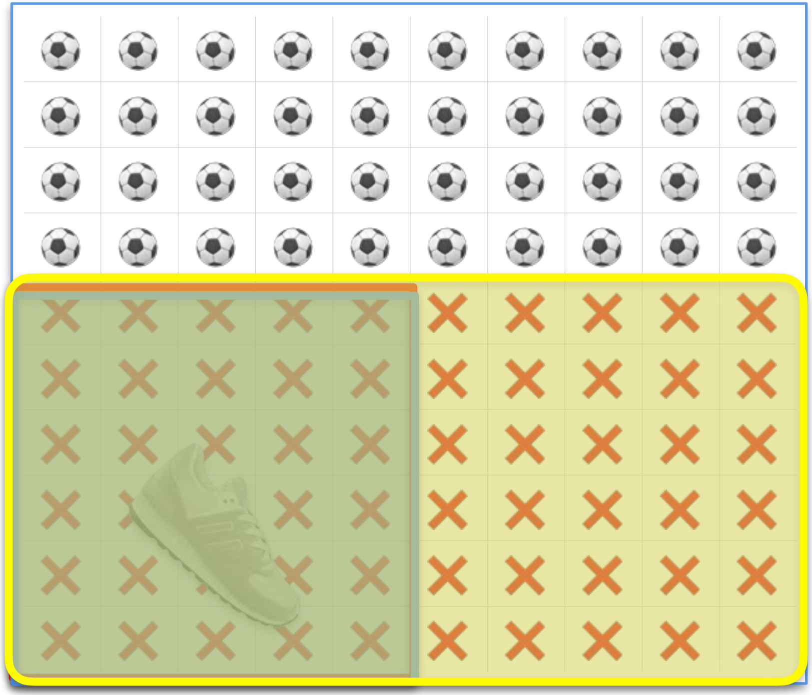
$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet \mathbf{P}(R \mid \text{not } S)$

$= 0.6 \bullet 0.5$

$= 0.3$

$\mathbf{P}(\text{not } S) = 0.6$

$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

DeepLearning.AI

Conditional Probability

$\mathbf{P}$(Soccer and Running shoes)

$\mathbf{P}(S \cap R) = 0.32$

$\mathbf{P}(S) = 0.4$

$\mathbf{P}$(not Soccer and Running shoes)

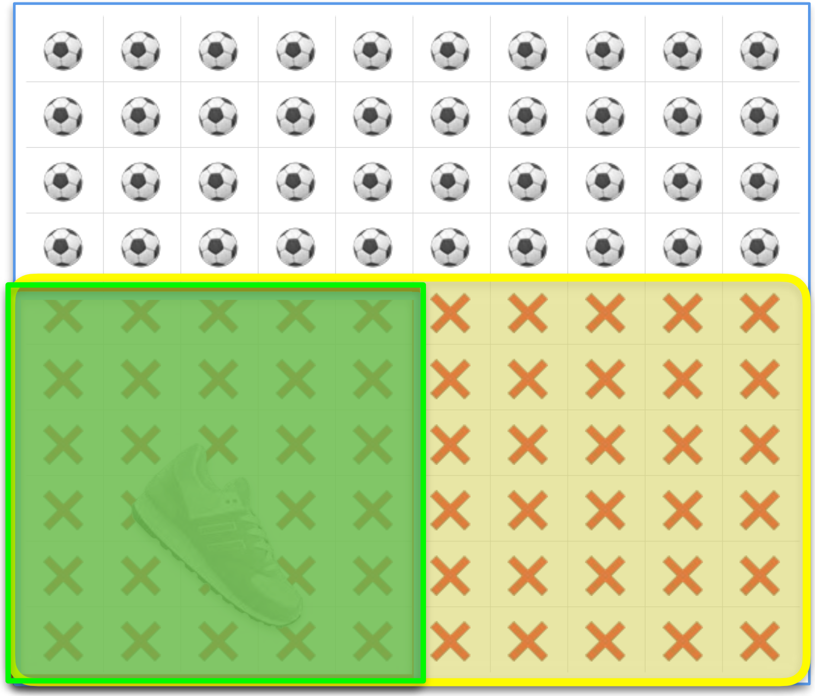
$\mathbf{P}(\text{not } S \cap R) = \mathbf{P}(\text{not } S) \bullet \mathbf{P}(R \mid \text{not } S)$

$= 0.6 \bullet 0.5$

$= 0.3$

$\mathbf{P}(\text{not } S) = 0.6$

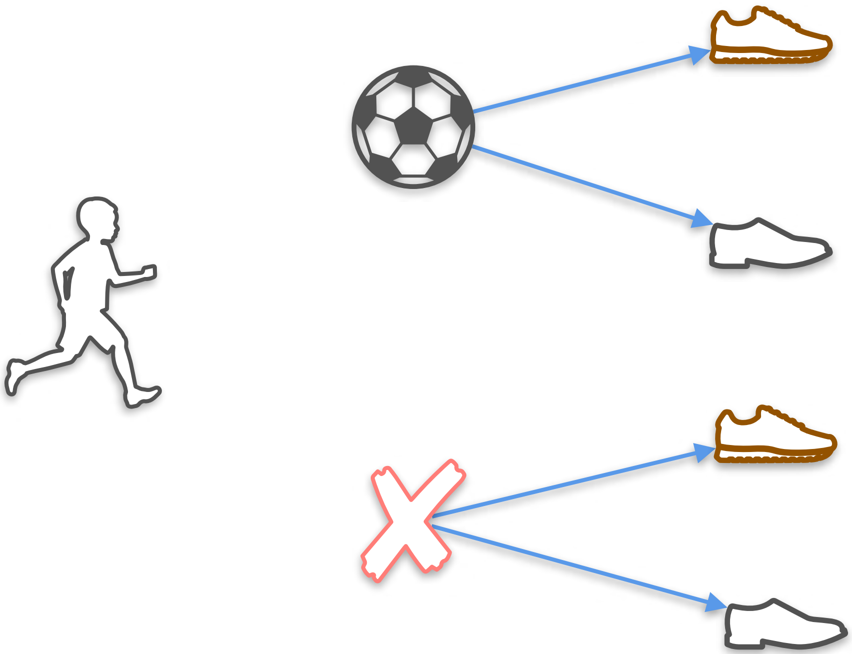
$\mathbf{P}(R \mid S) = 0.8$



$\mathbf{P}(R \mid \text{not } S) = 0.5$

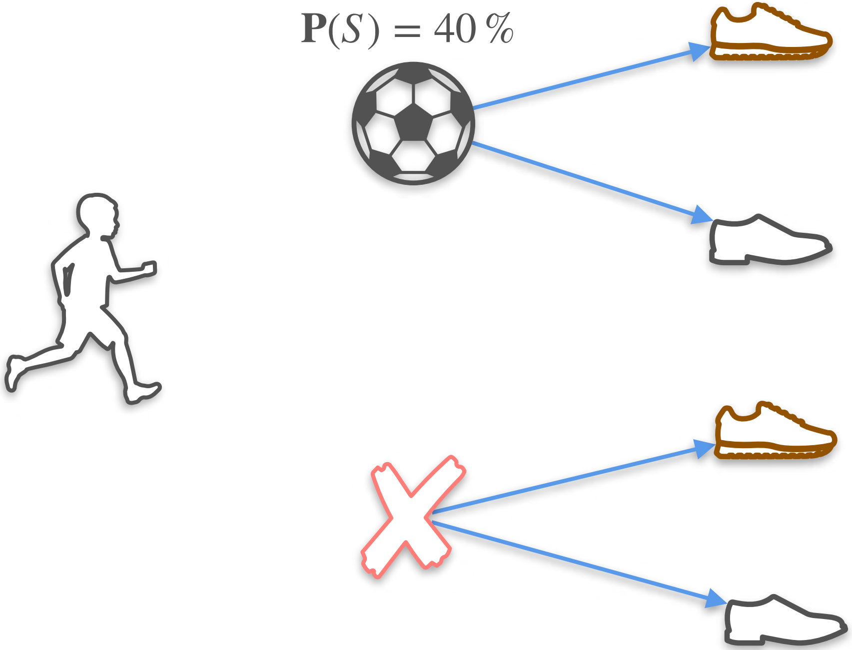
Conditional Probability

Conditional Probability

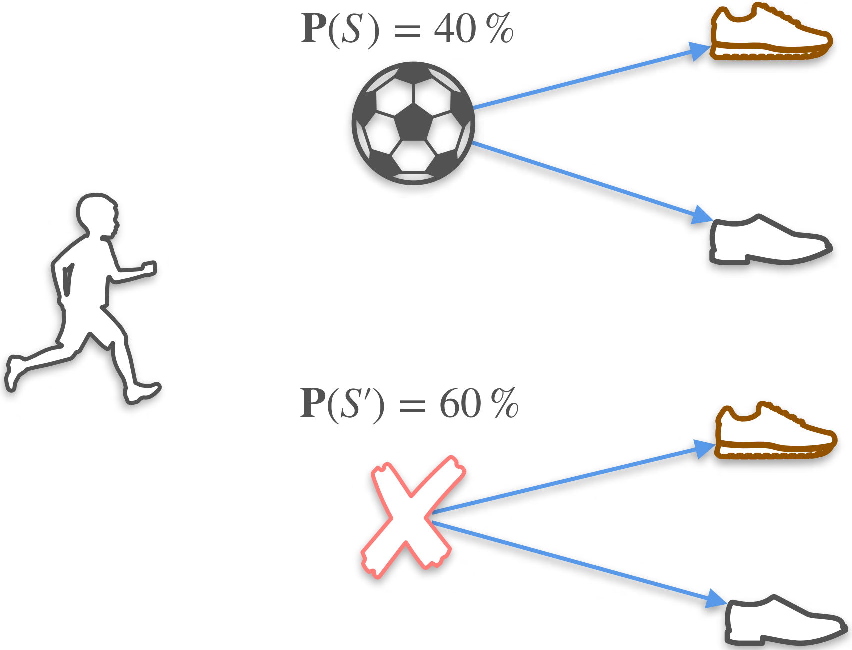


Conditional Probability

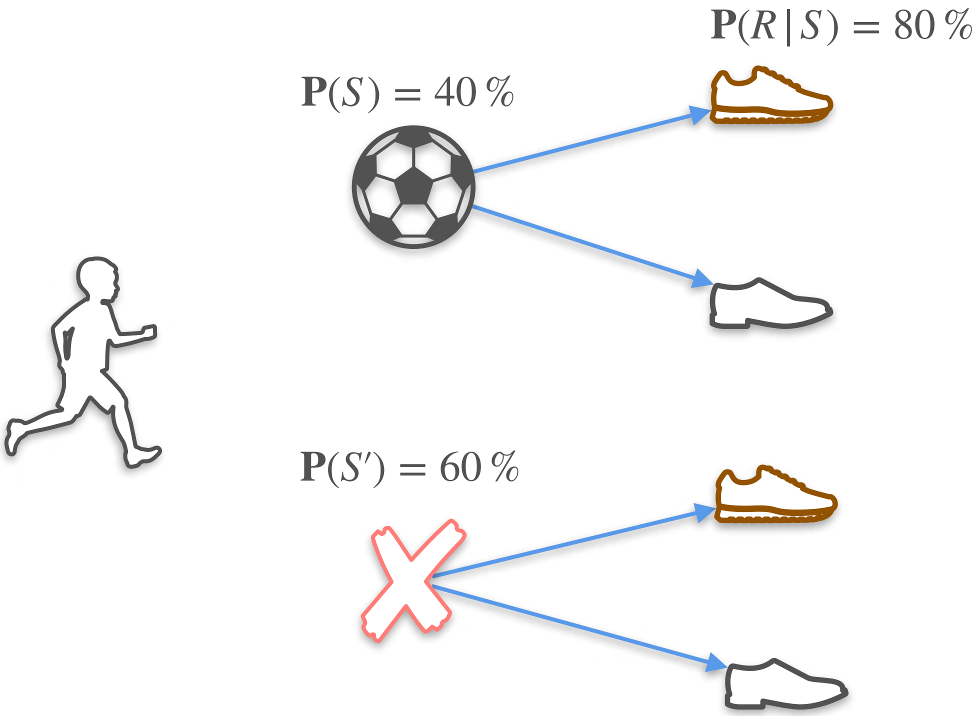
$\mathbf{P}(S) = 40\,\%$



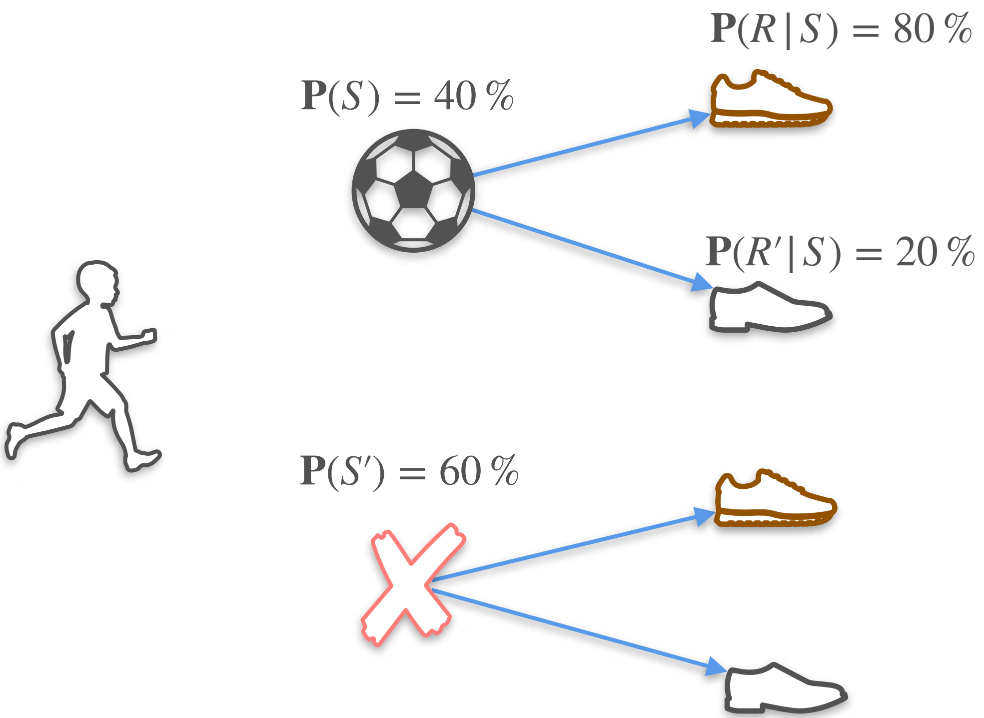
Conditional Probability



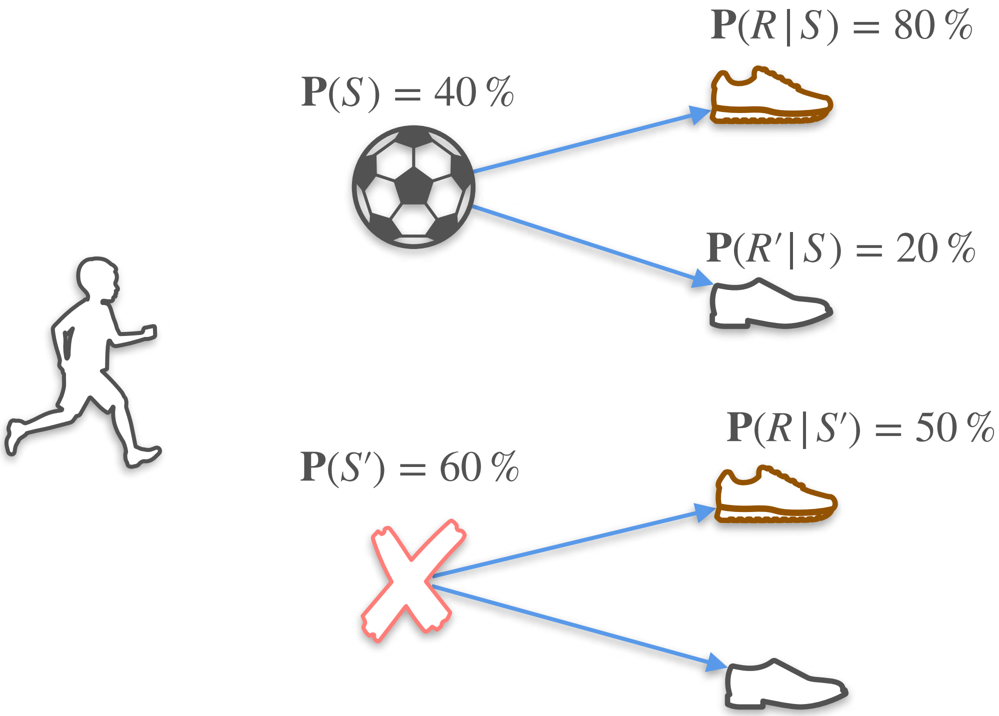
Conditional Probability



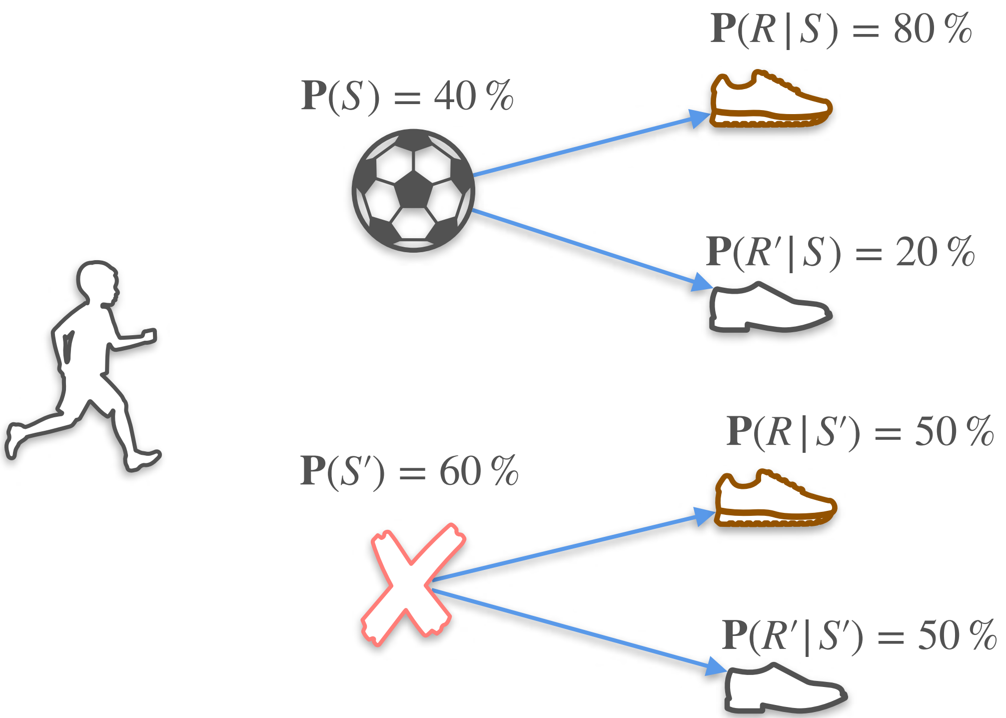
Conditional Probability



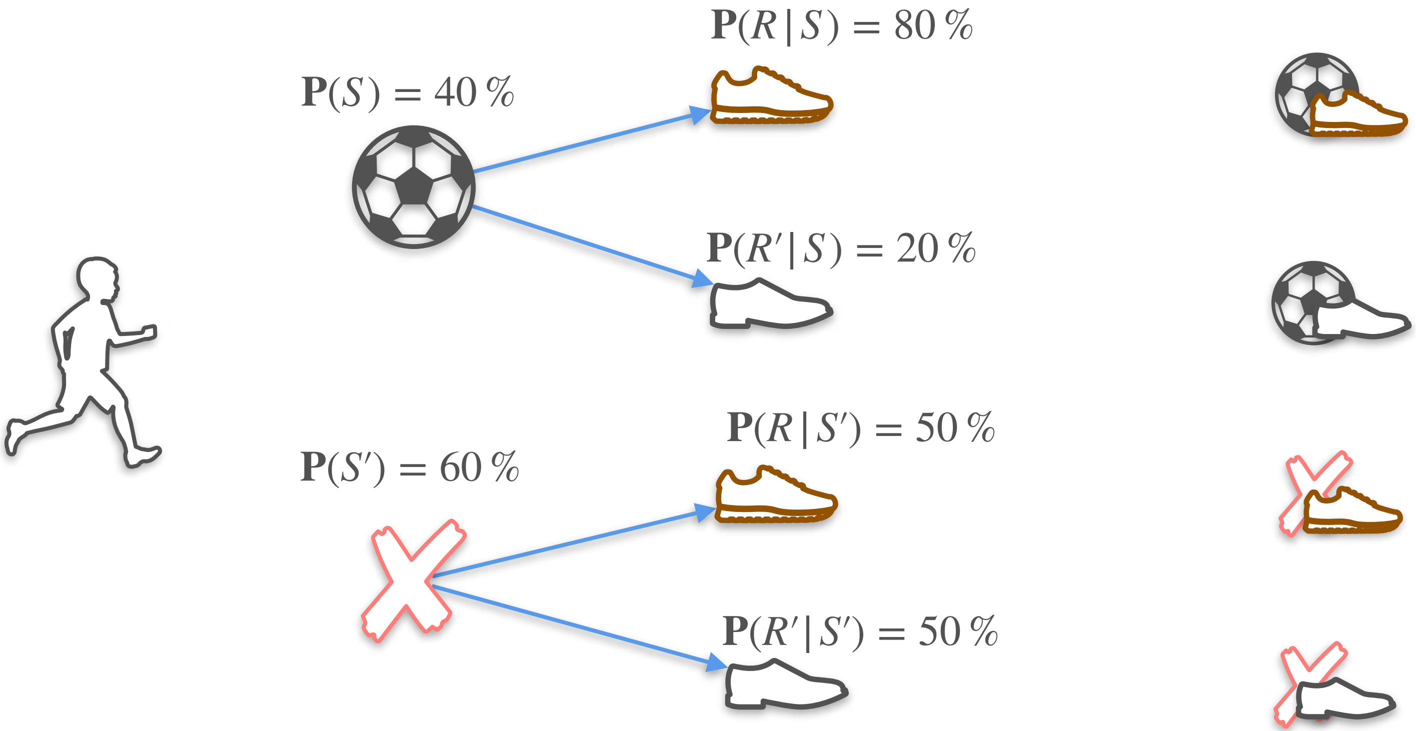
Conditional Probability



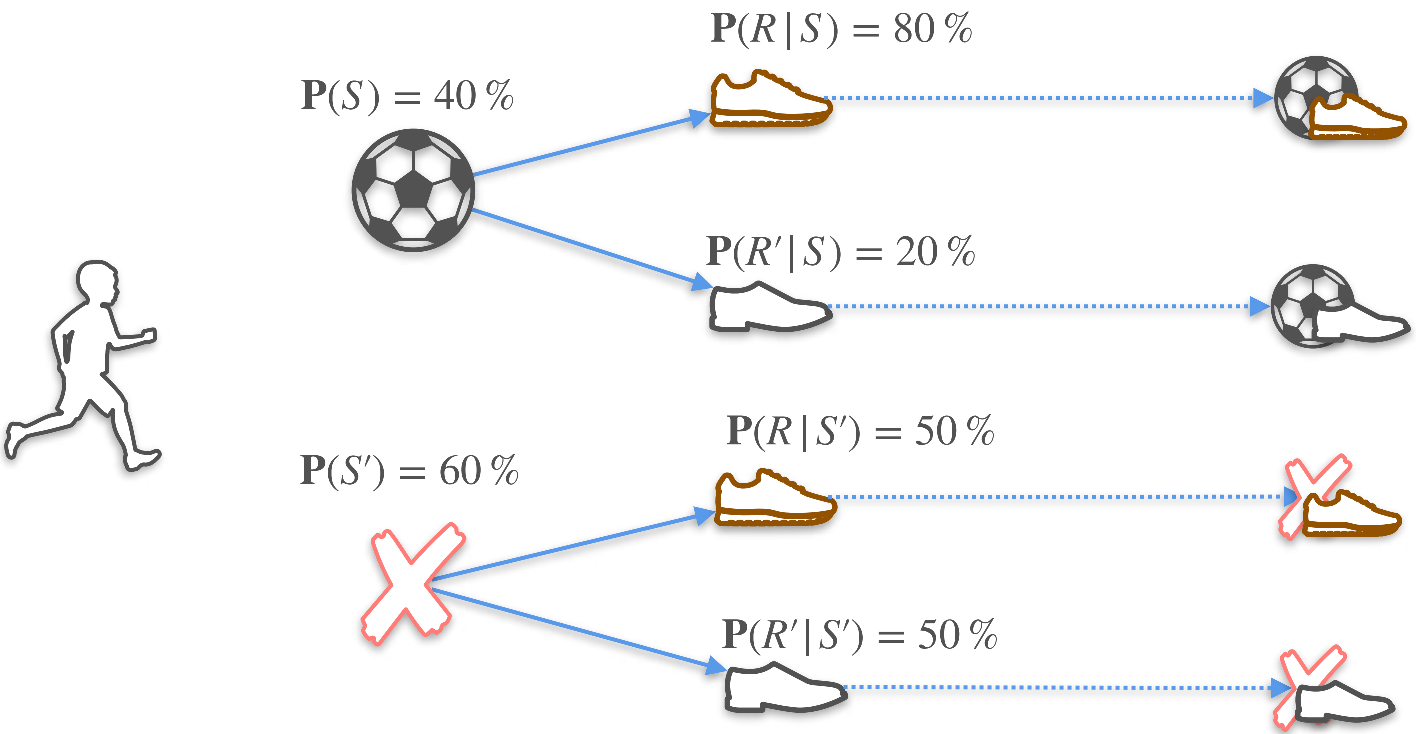
Conditional Probability



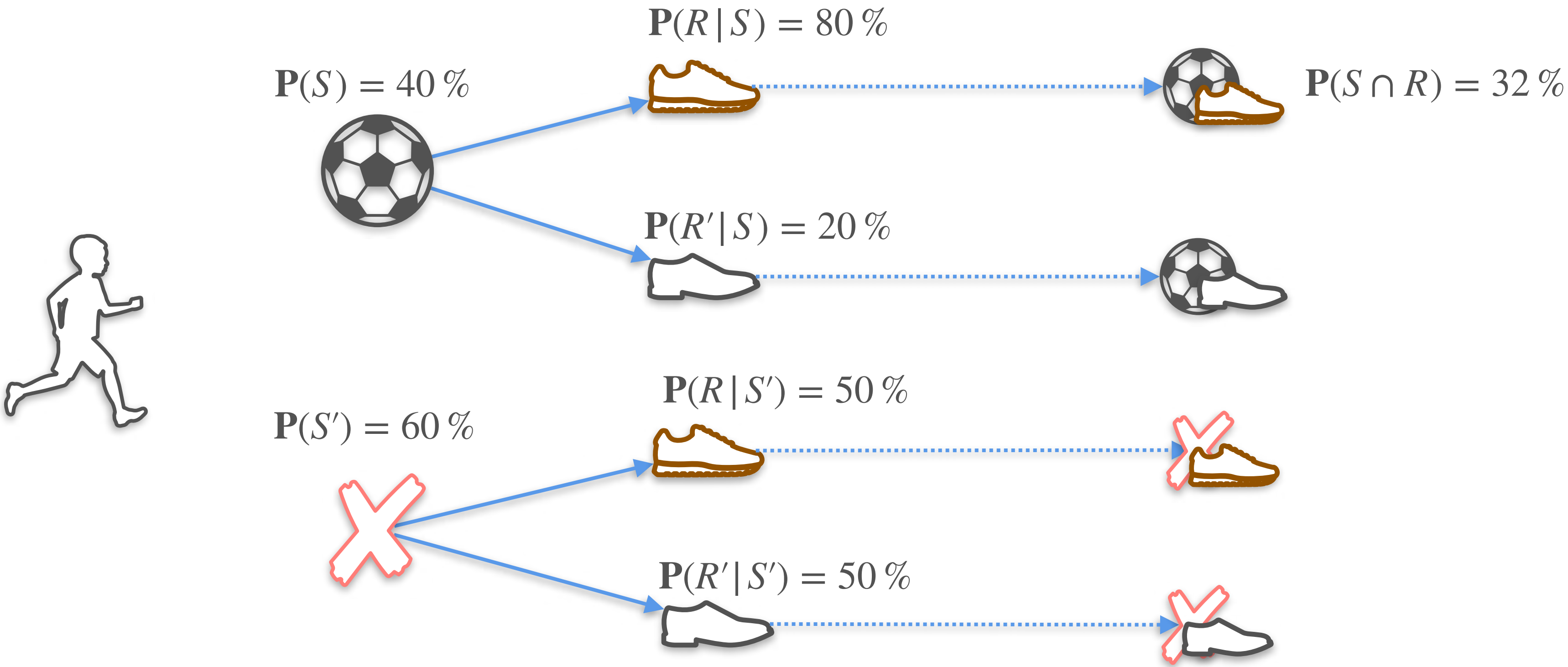
Conditional Probability



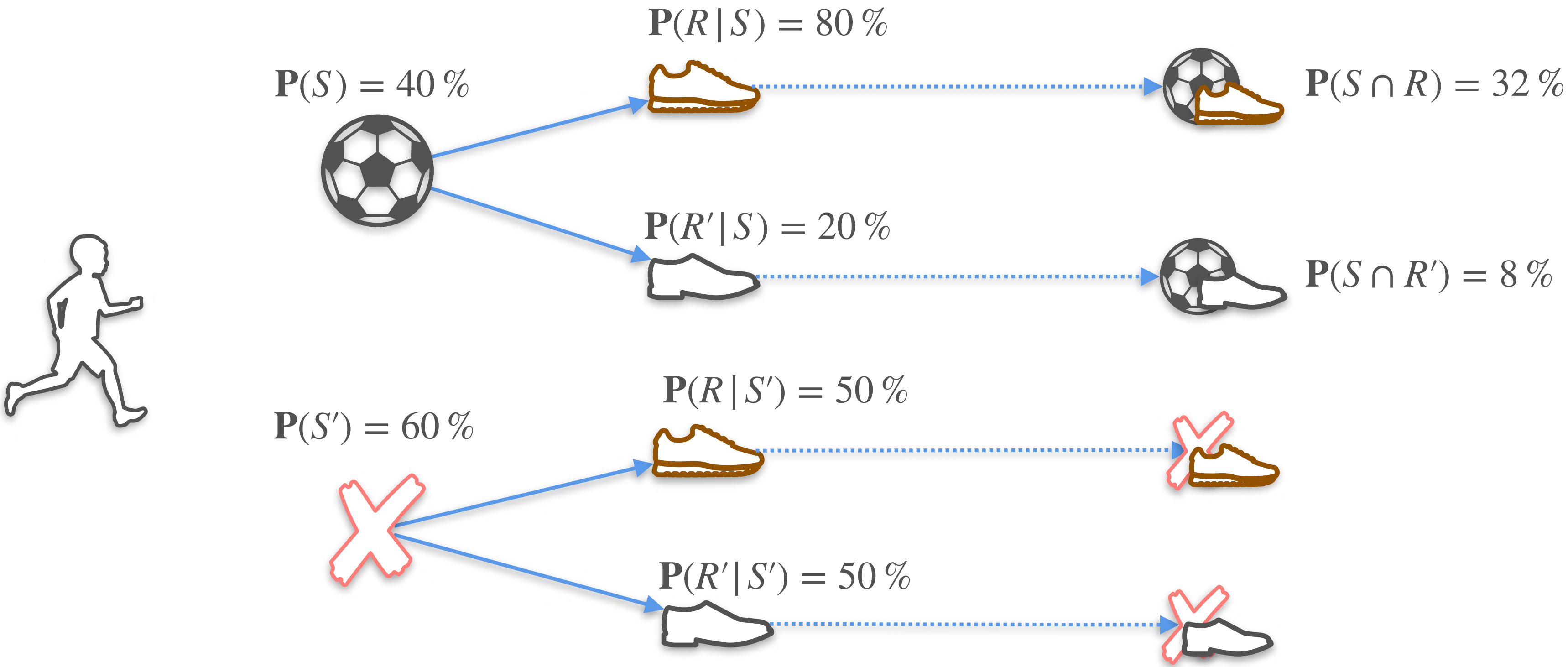
Conditional Probability



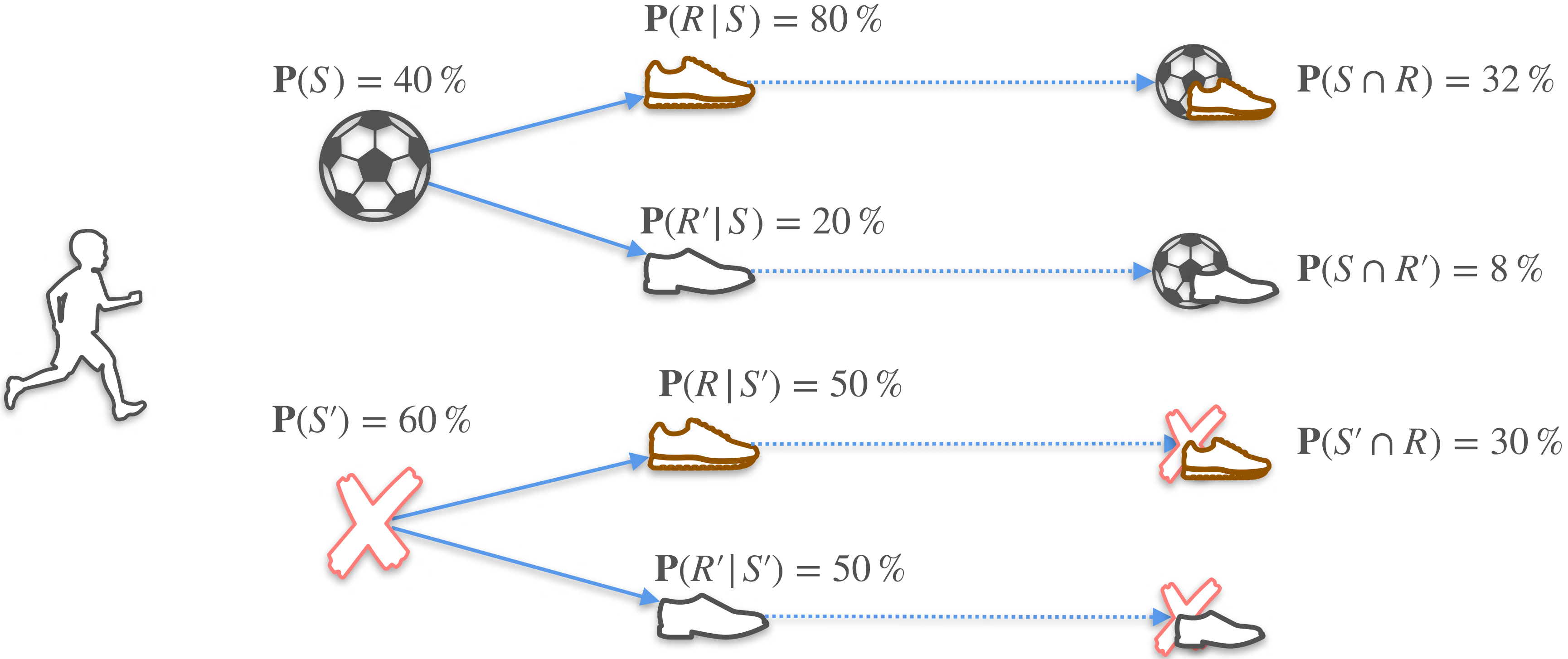
Conditional Probability



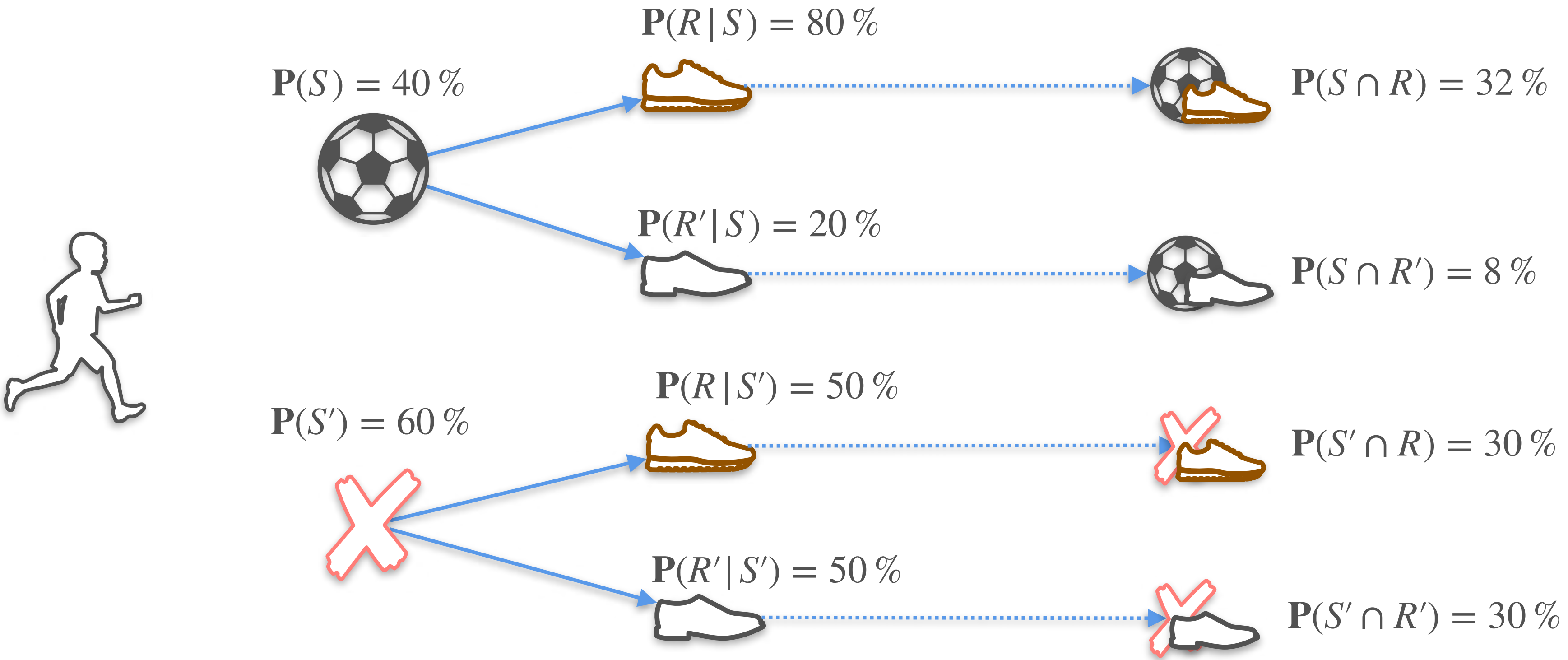
Conditional Probability



Conditional Probability

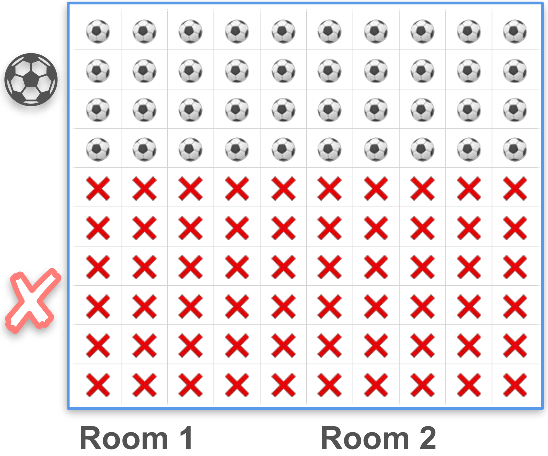


Conditional Probability

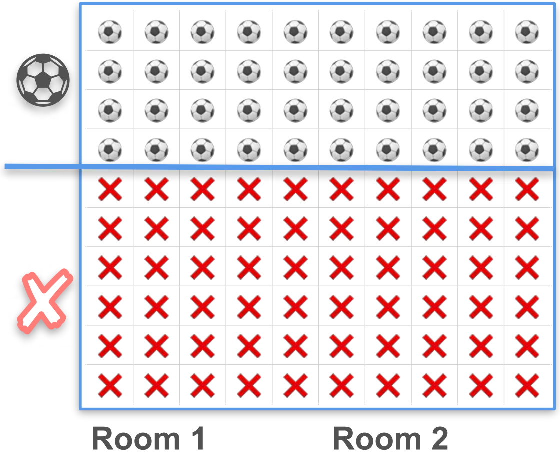


Independent vs Dependent Events

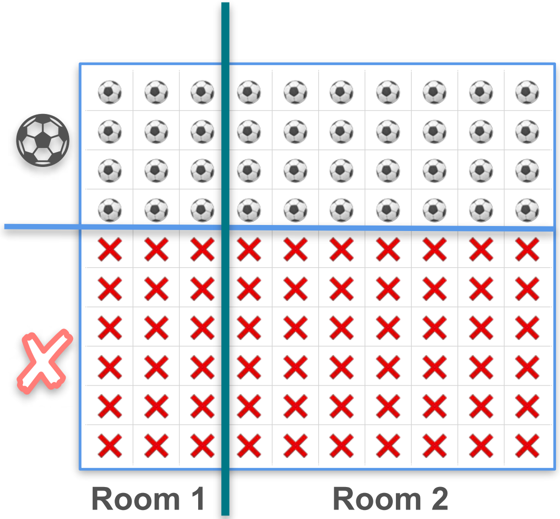
Independent vs Dependent Events



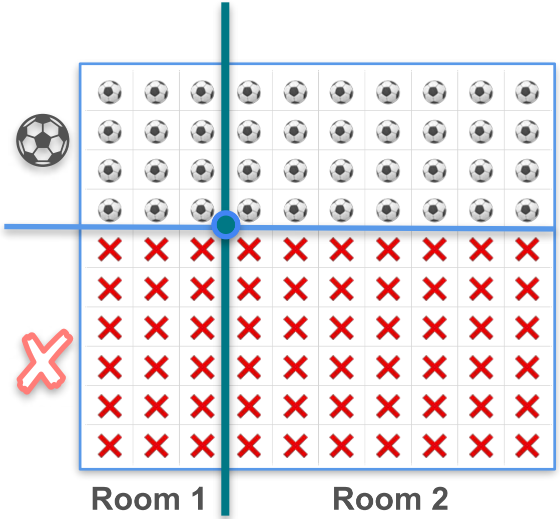
Independent vs Dependent Events



Independent vs Dependent Events

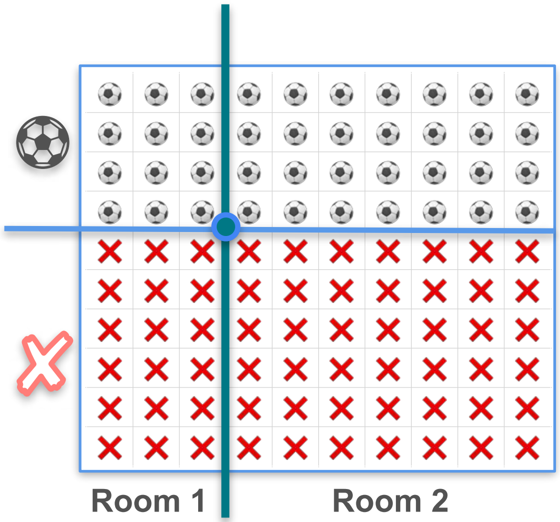


Independent vs Dependent Events



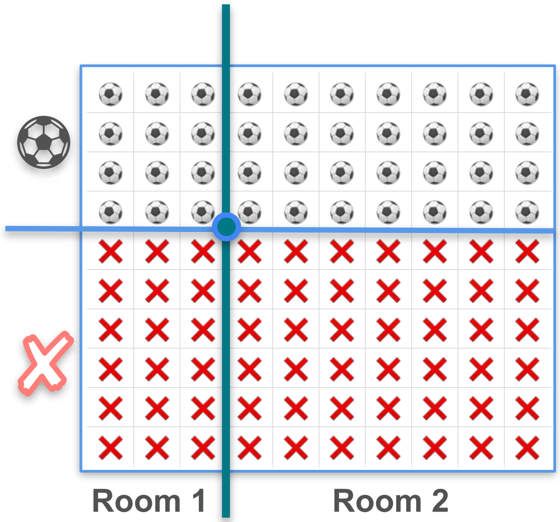
Independent vs Dependent Events

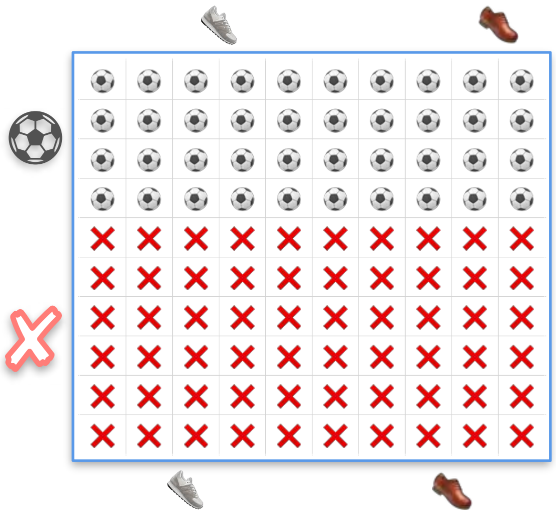
Independent



Independent vs Dependent Events

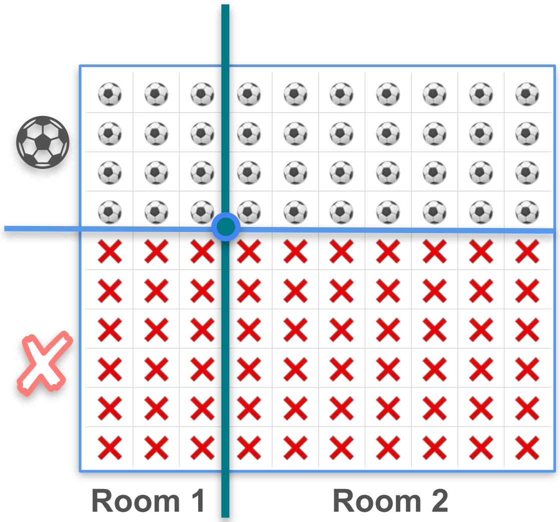
Independent

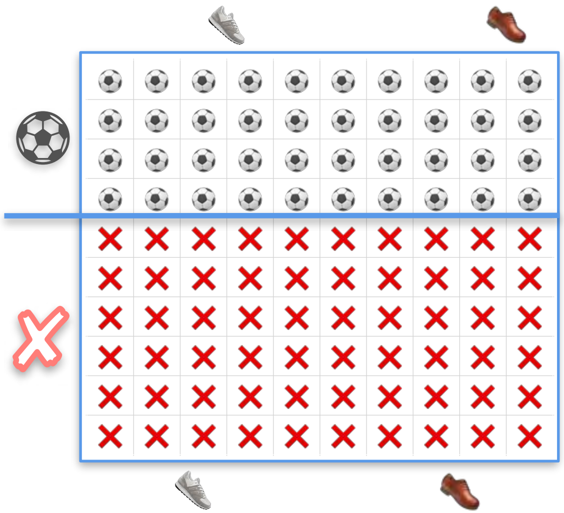




Independent vs Dependent Events

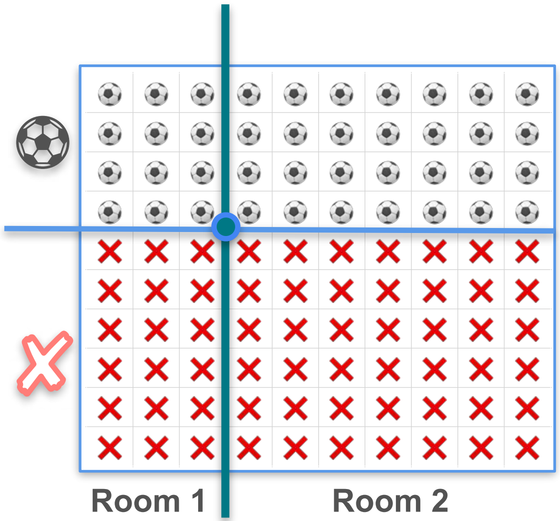
Independent

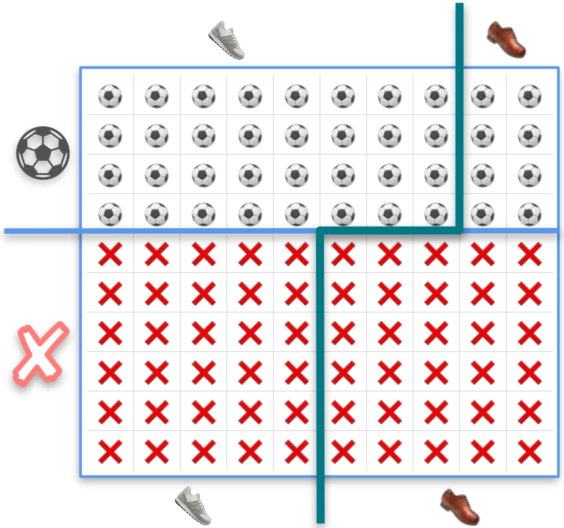




Independent vs Dependent Events

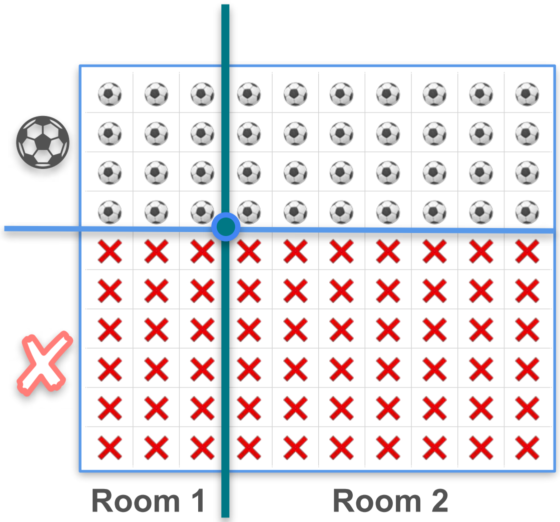
Independent



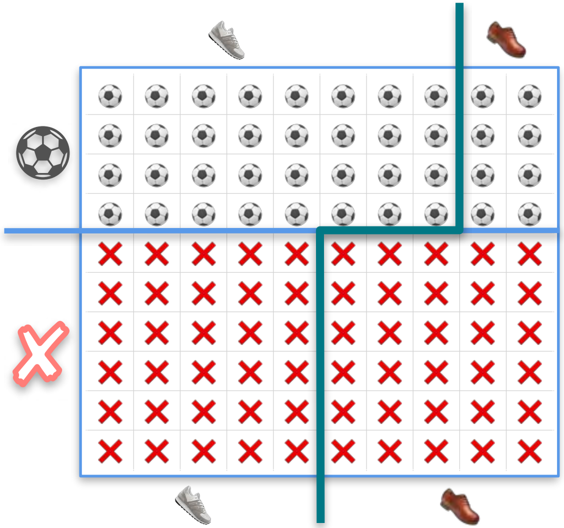


Independent vs Dependent Events

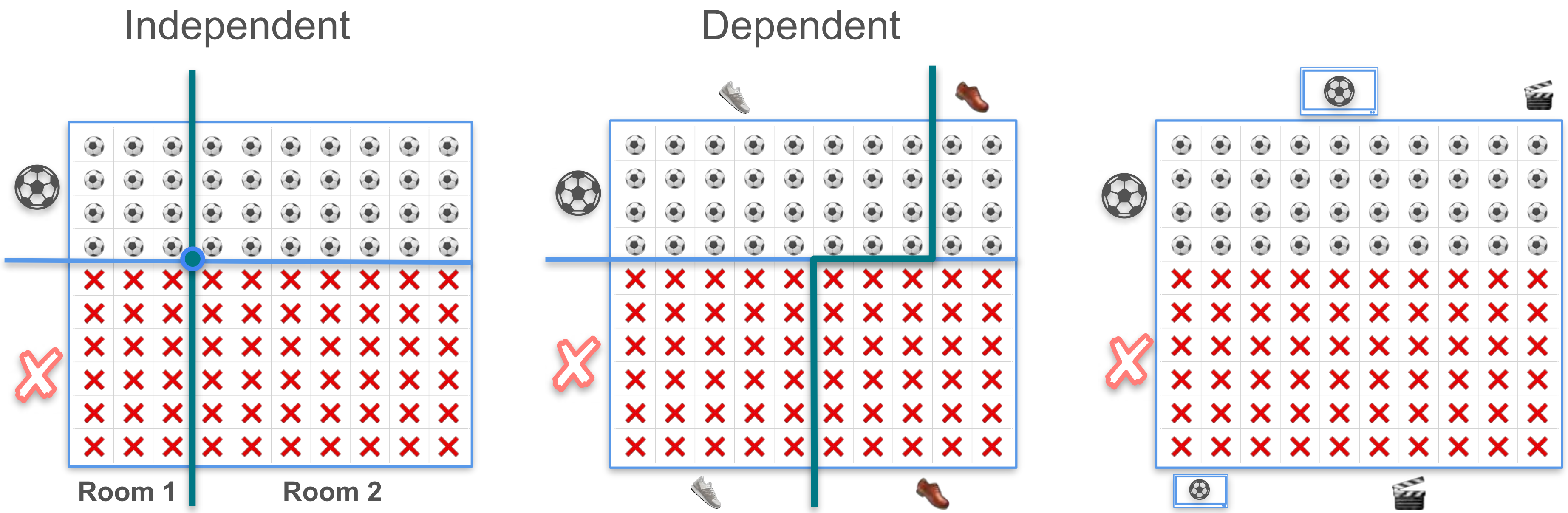
Independent



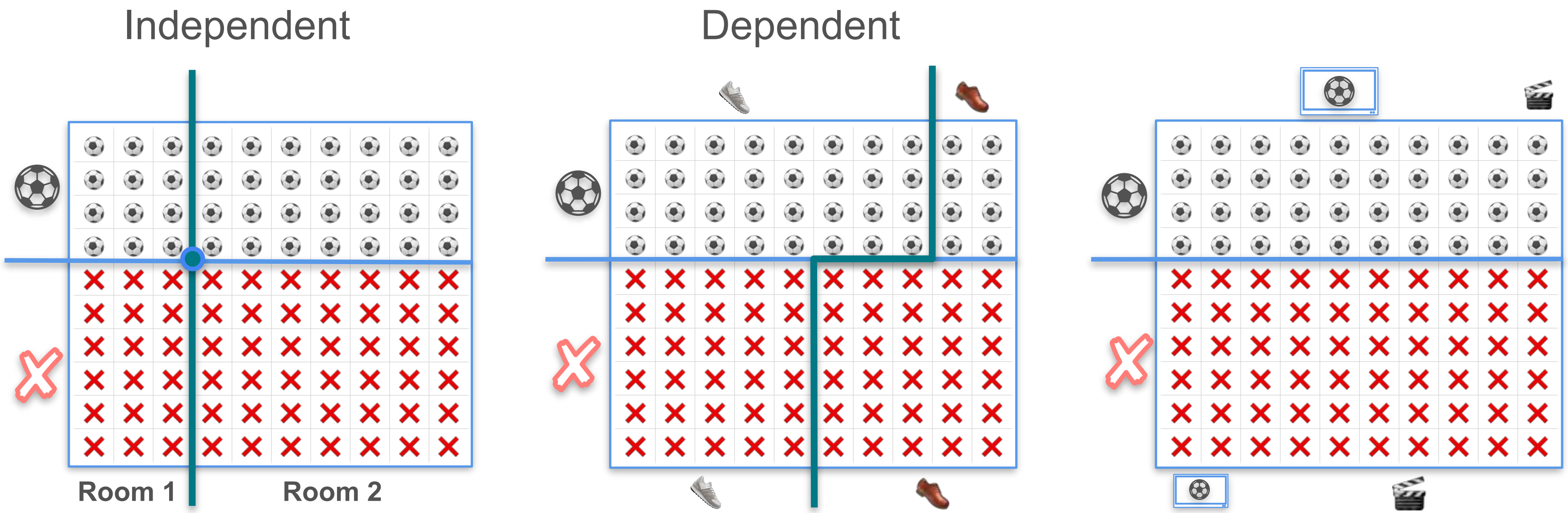
Dependent



Independent vs Dependent Events



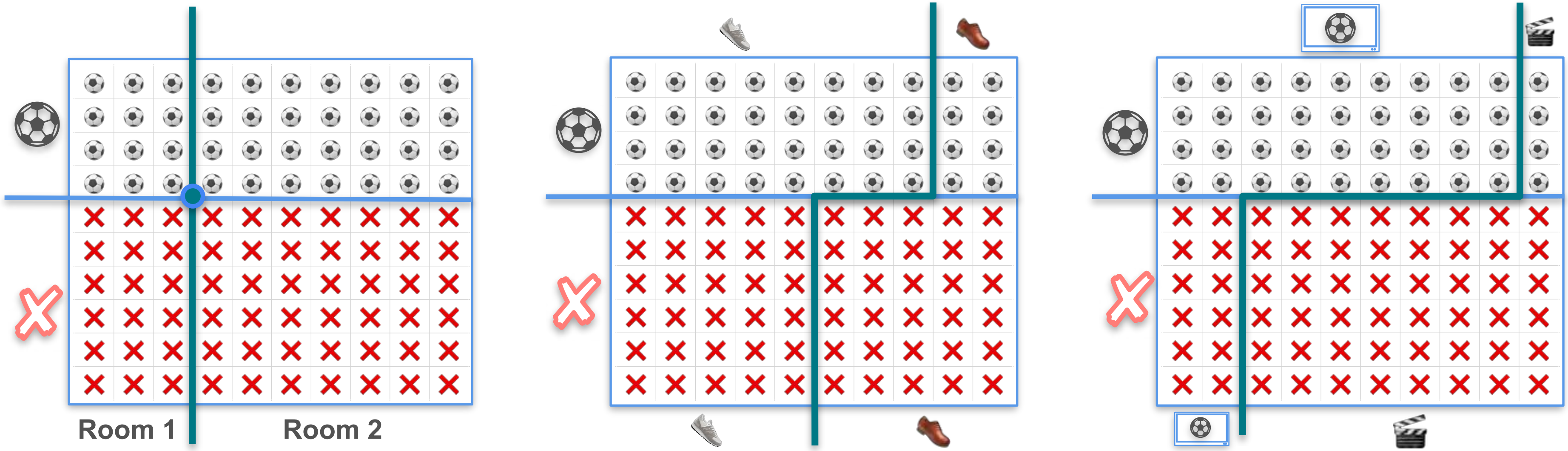
Independent vs Dependent Events



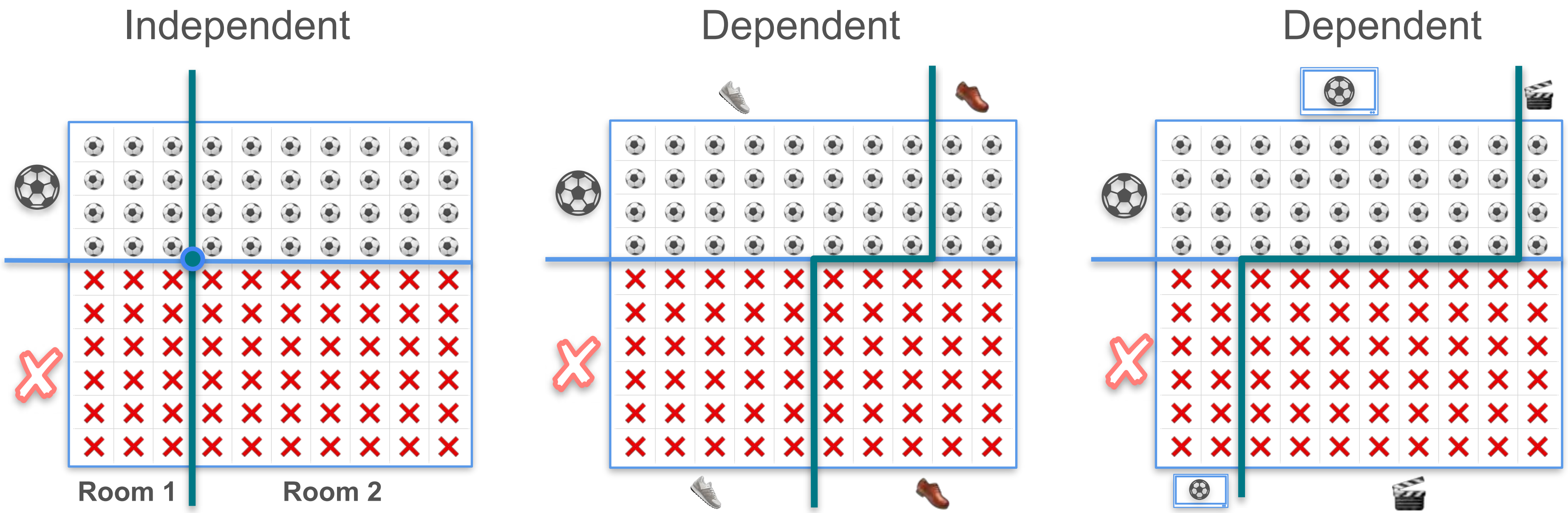
Independent vs Dependent Events

Independent

Dependent



Independent vs Dependent Events



Conditional Probability

Conditional Probability

Independent

Conditional Probability

Independent

$\mathbf{P}(R \mid S) = 50\,\%$

$\mathbf{P}(S) = 40\,\%$

Room 1

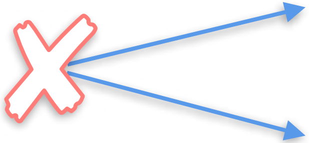
$\mathbf{P}(R' \mid S) = 50\,\%$

Room 2

$\mathbf{P}(R \mid S') = 50\,\%$

$\mathbf{P}(S') = 60\,\%$

Room 1



$\mathbf{P}(R' \mid S) = 50\,\%$

Room 2

Conditional Probability

Independent

$\mathbf{P}(S) = 40\,\%$

$\mathbf{P}(R \mid S) = 50\,\%$

Room 1

$\mathbf{P}(R' \mid S) = 50\,\%$

Room 2

$\mathbf{P}(S') = 60\,\%$

$\mathbf{P}(R \mid S') = 50\,\%$

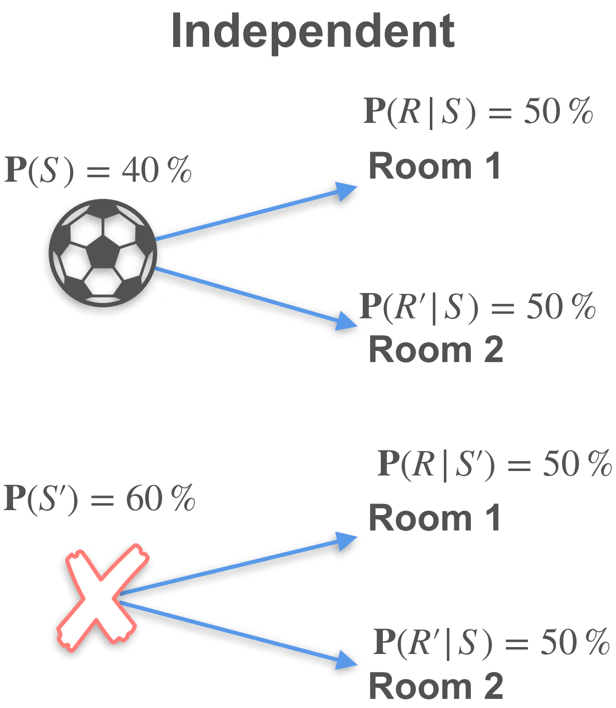
Room 1

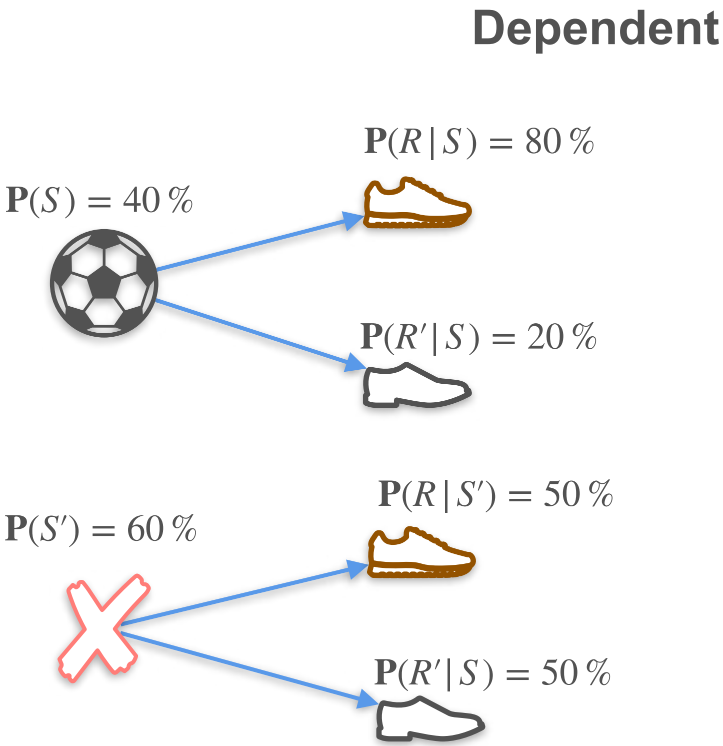
$\mathbf{P}(R' \mid S) = 50\,\%$

Room 2

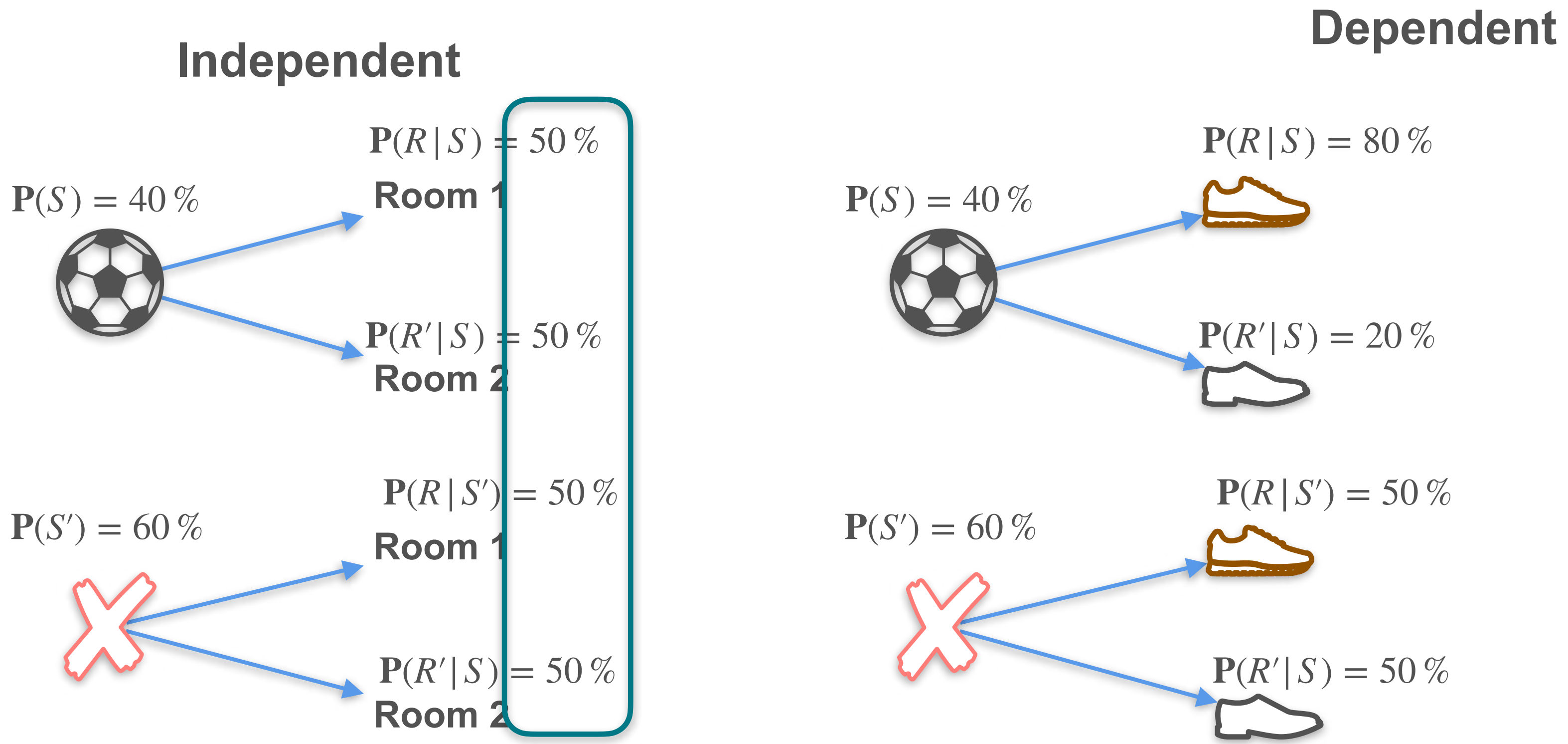
Dependent

Conditional Probability

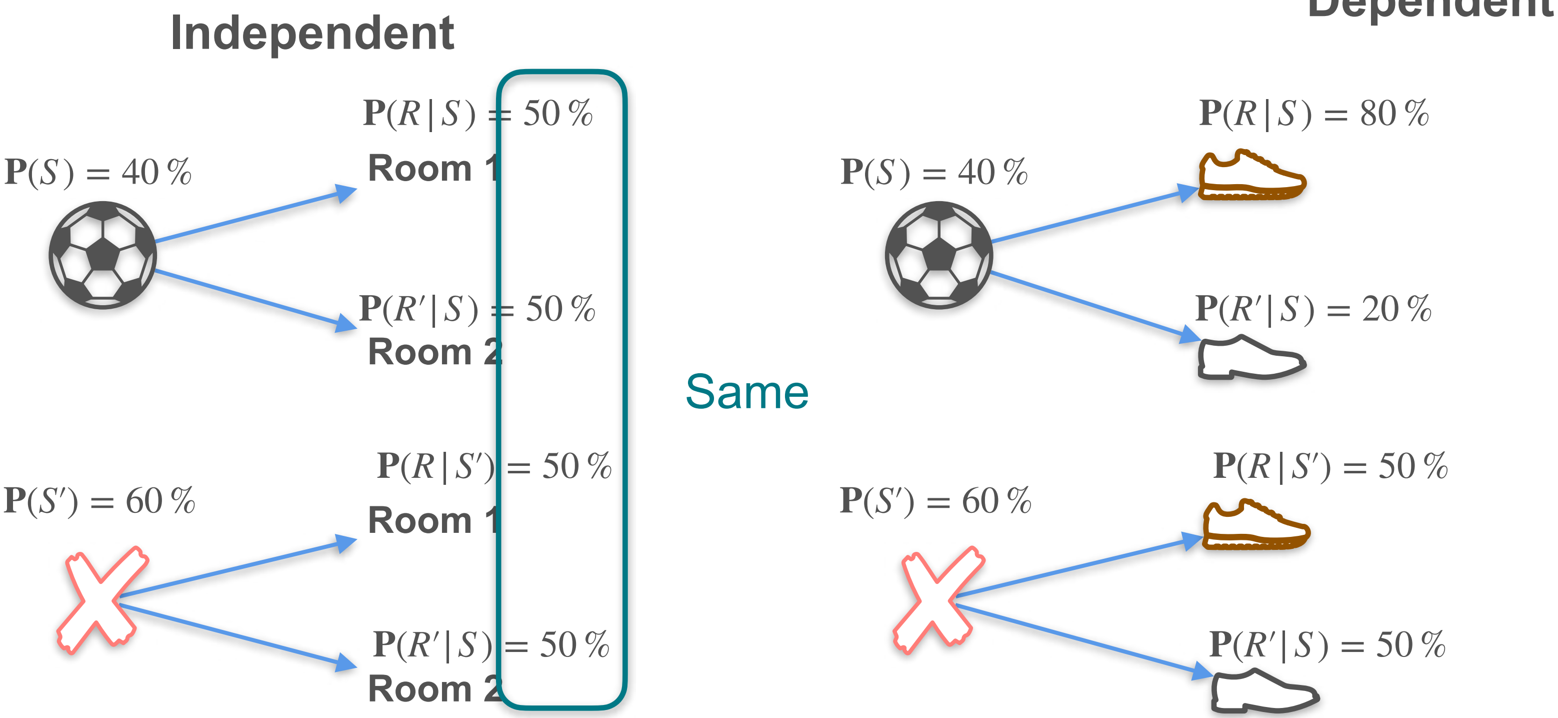




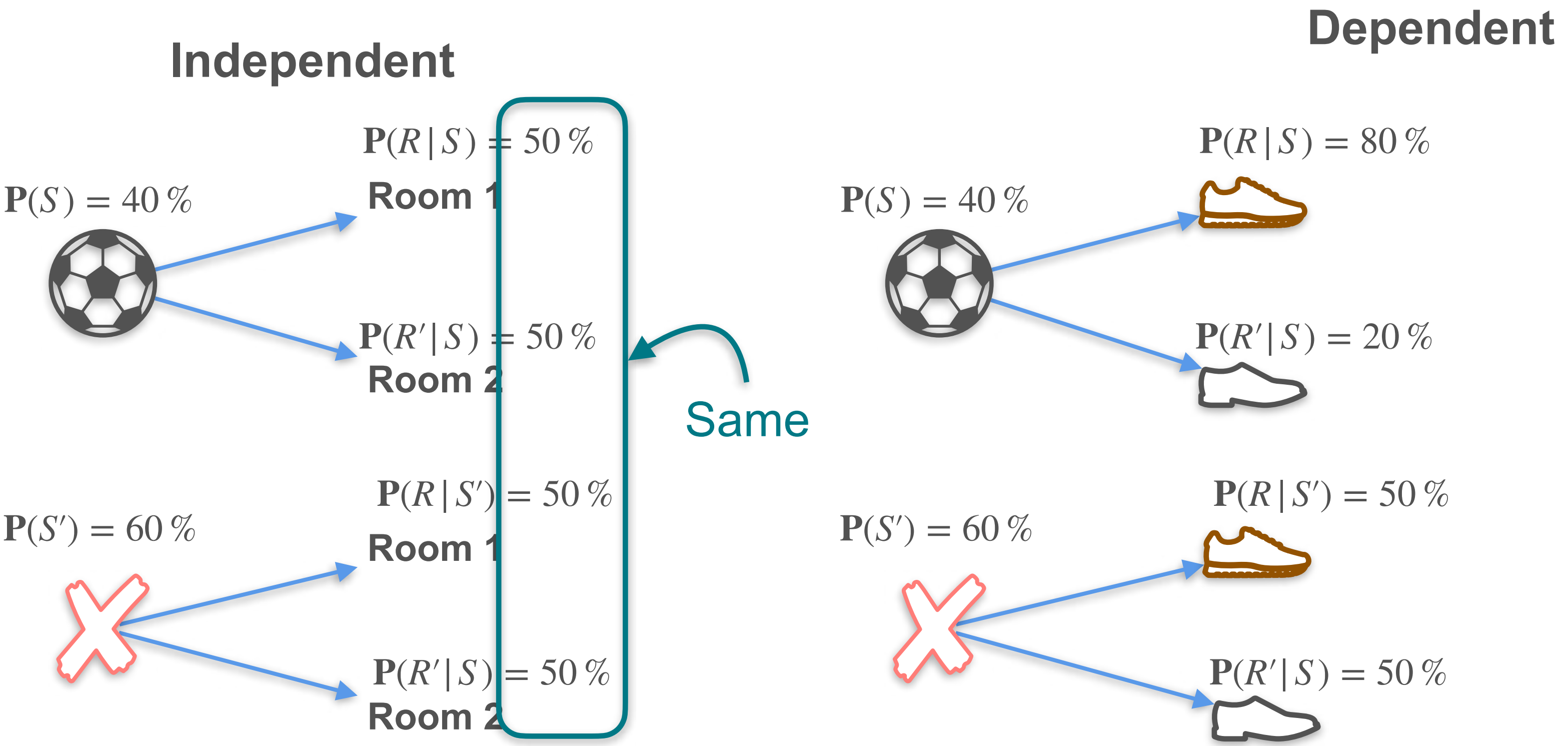
Conditional Probability



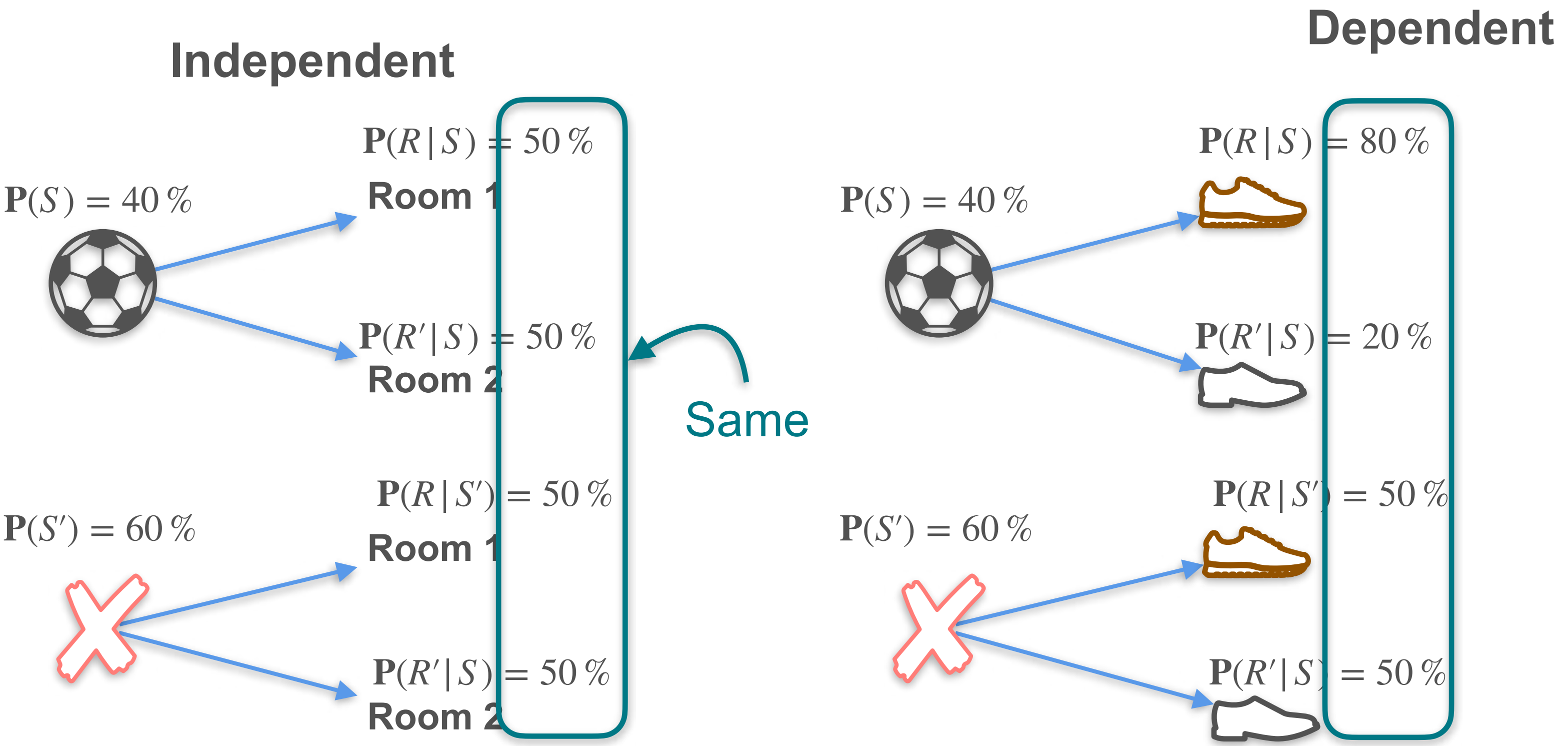
Conditional Probability



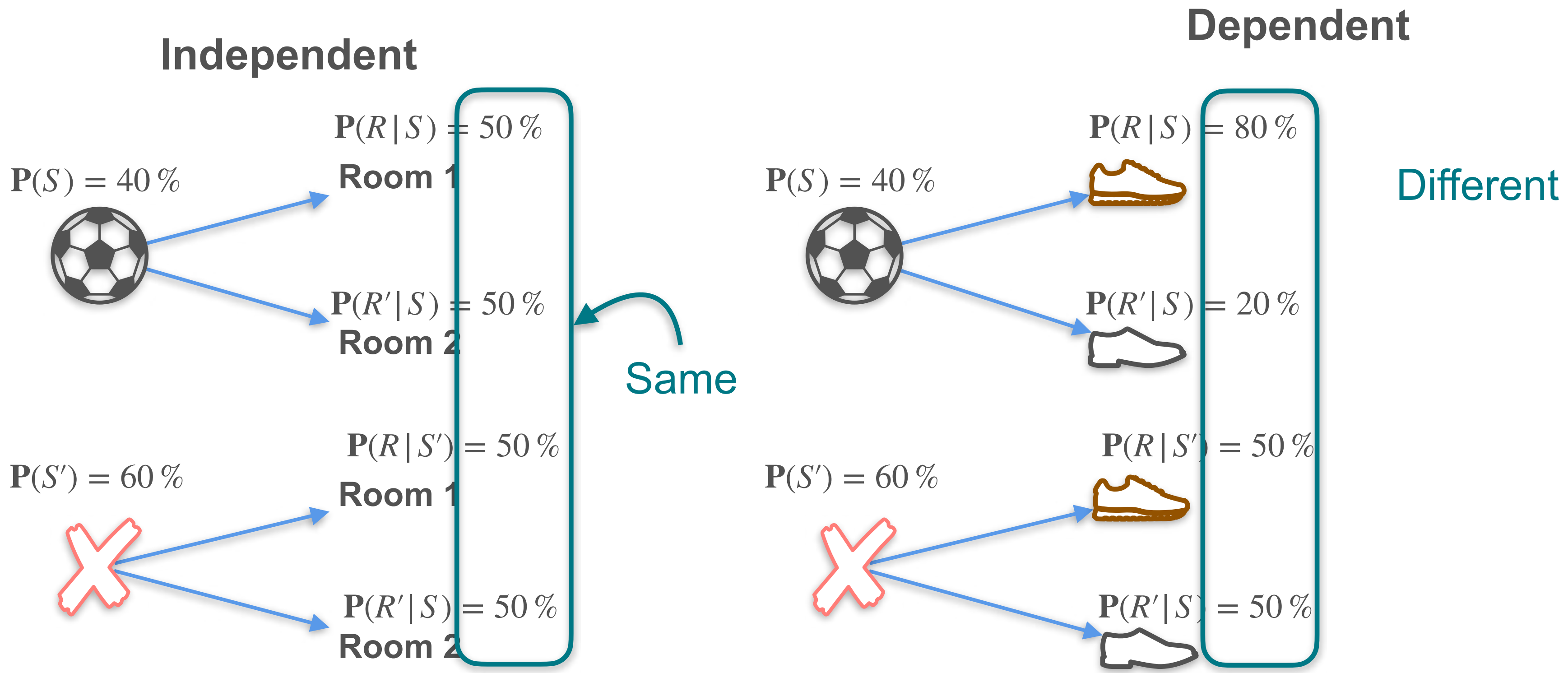
Conditional Probability



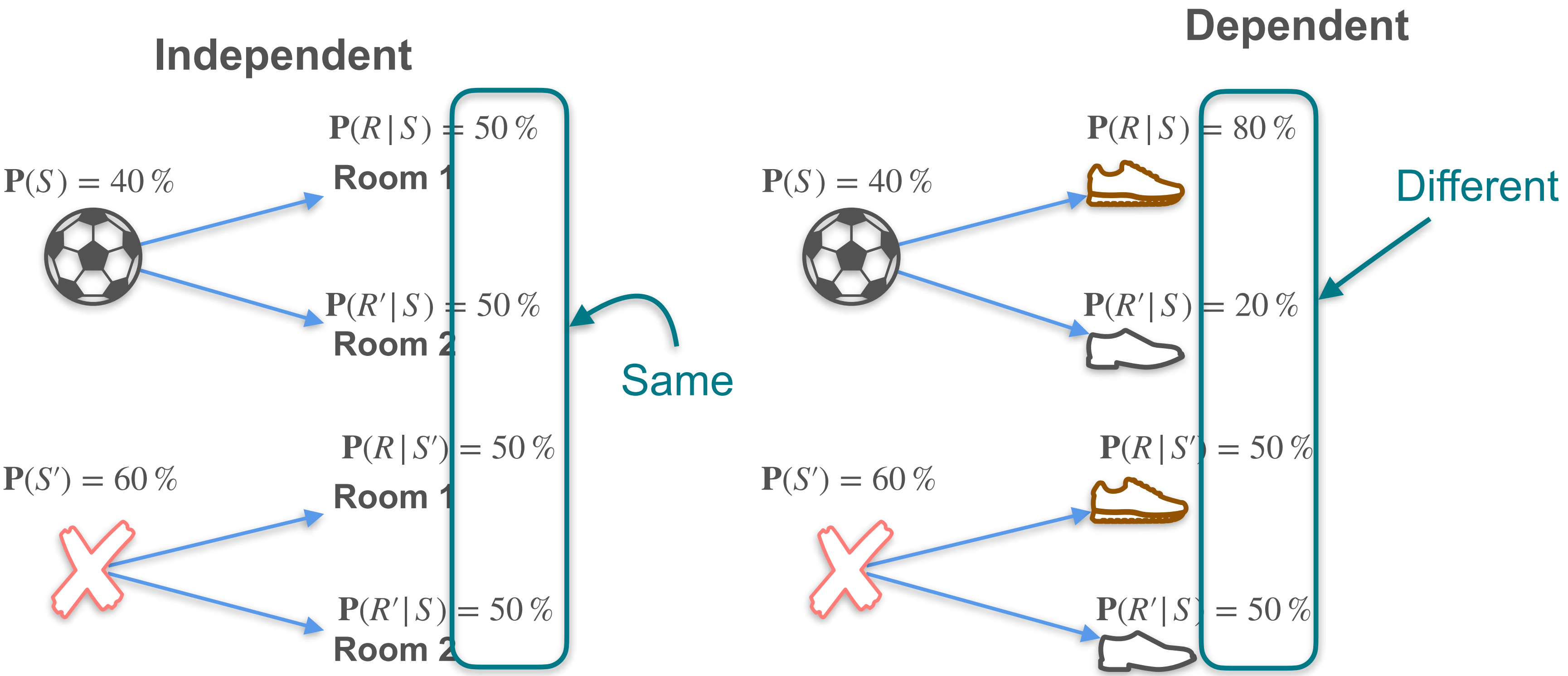
Conditional Probability



Conditional Probability



Conditional Probability



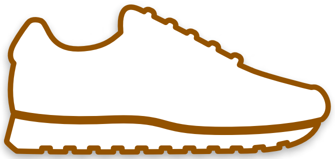
DeepLearning.AI

Introduction to probability

Bayes theorem

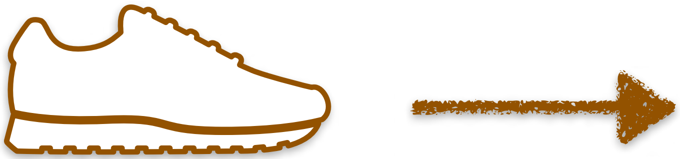
Bayes Theorem - Motivation

Bayes Theorem - Motivation



60%

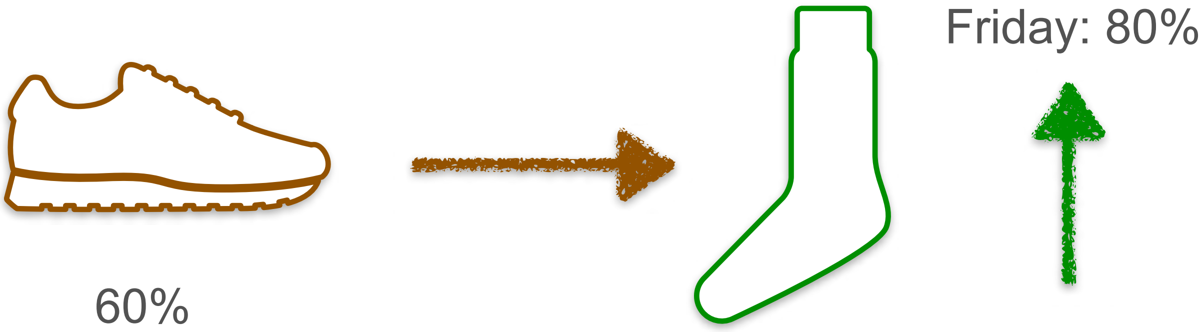
Bayes Theorem - Motivation



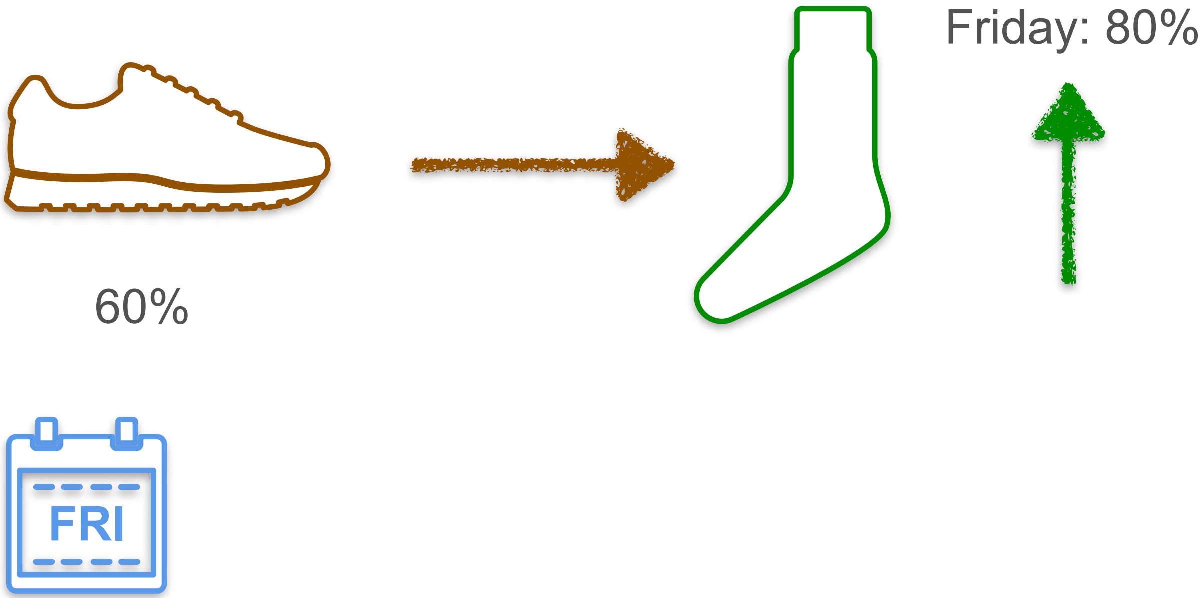
60%

Bayes Theorem - Motivation

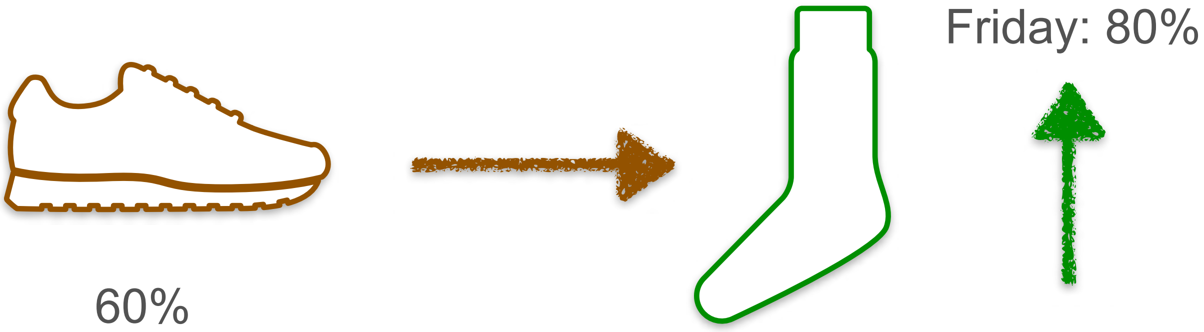
Bayes Theorem - Motivation



Bayes Theorem - Motivation



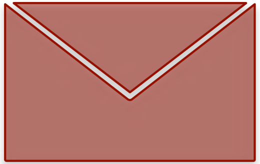
Bayes Theorem - Motivation



What is the probability that a customer will purchase a pair of socks given that they purchased shoes?

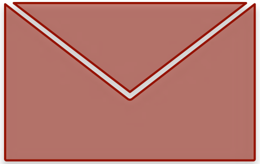
Bayes Theorem: Motivation

Bayes Theorem: Motivation



spam

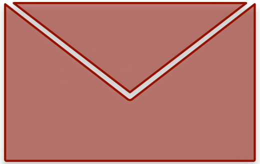
Bayes Theorem: Motivation



spam

contains
“lottery”

Bayes Theorem: Motivation

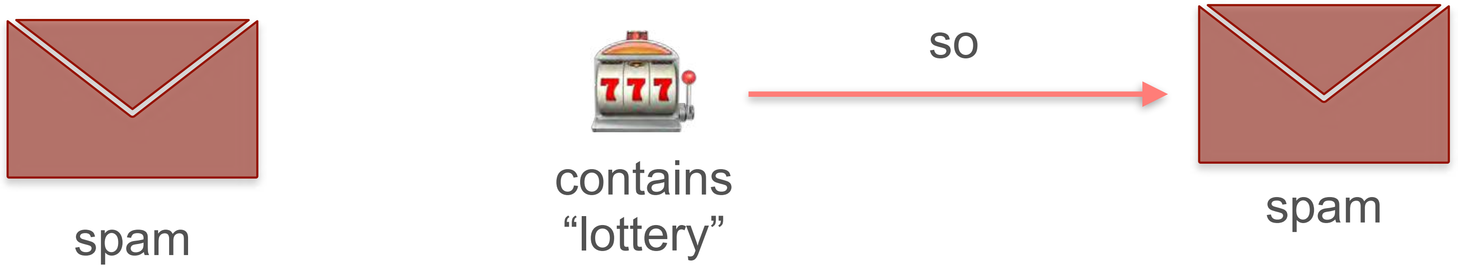


spam

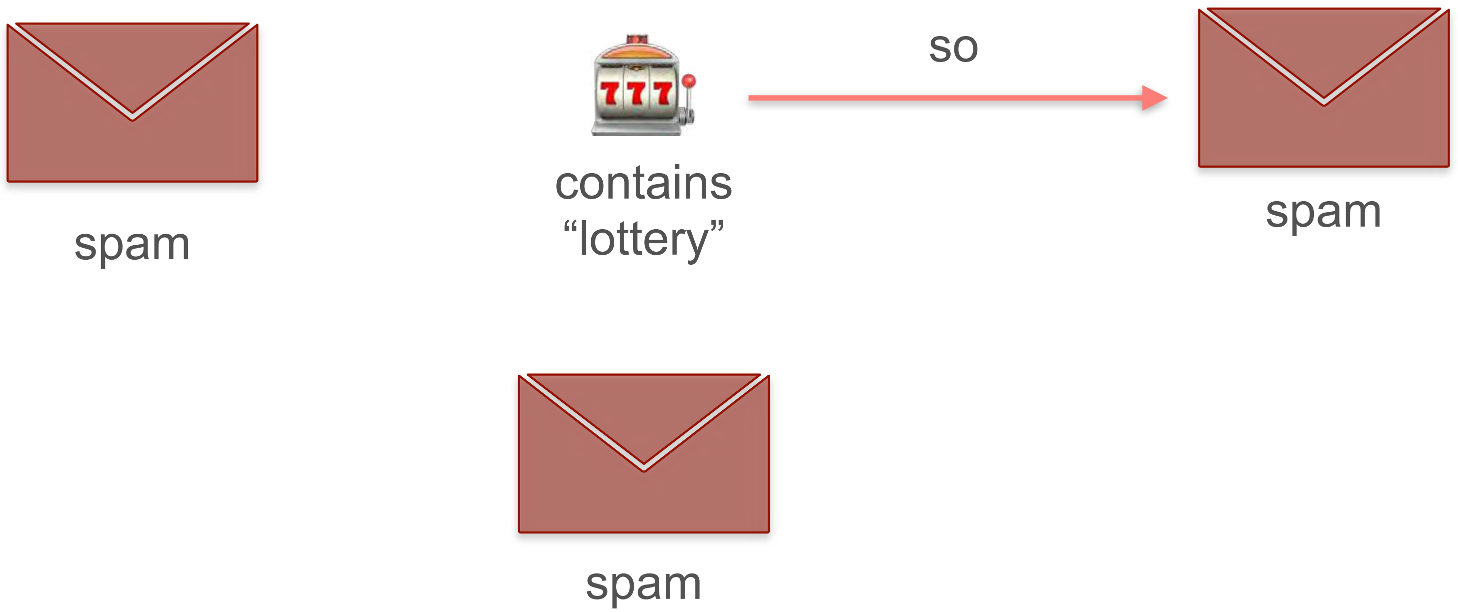
contains
“lottery”

so

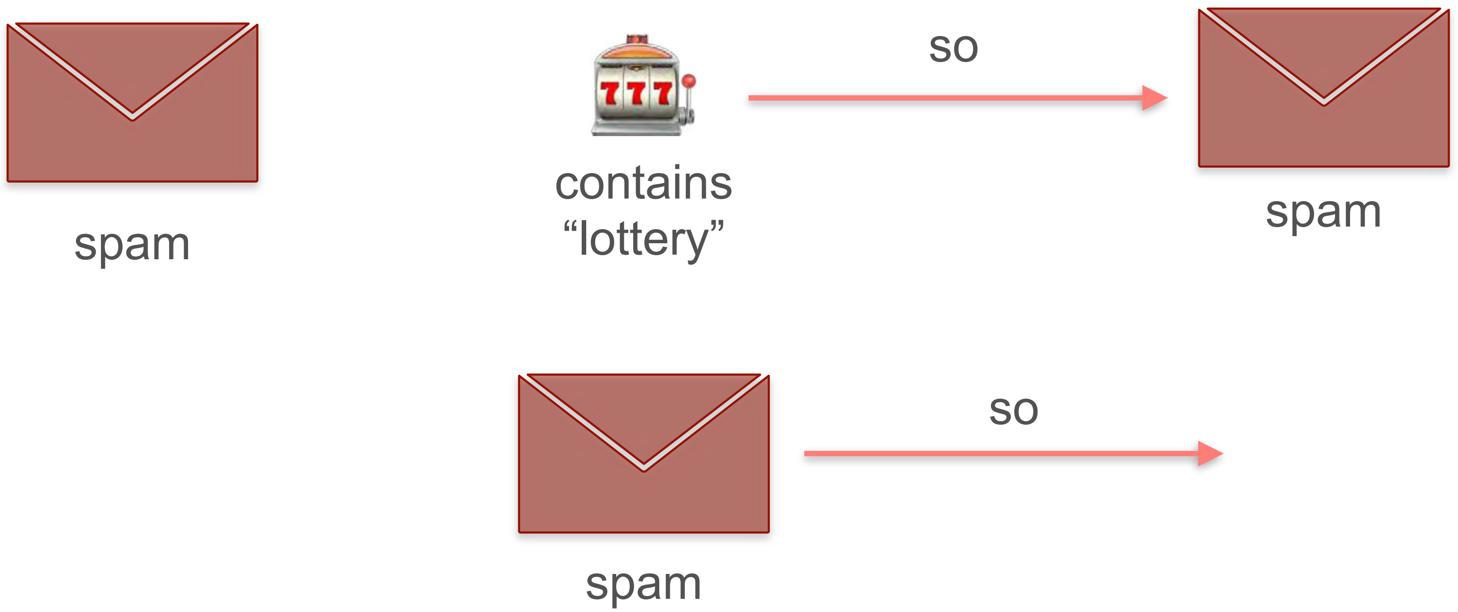
Bayes Theorem: Motivation



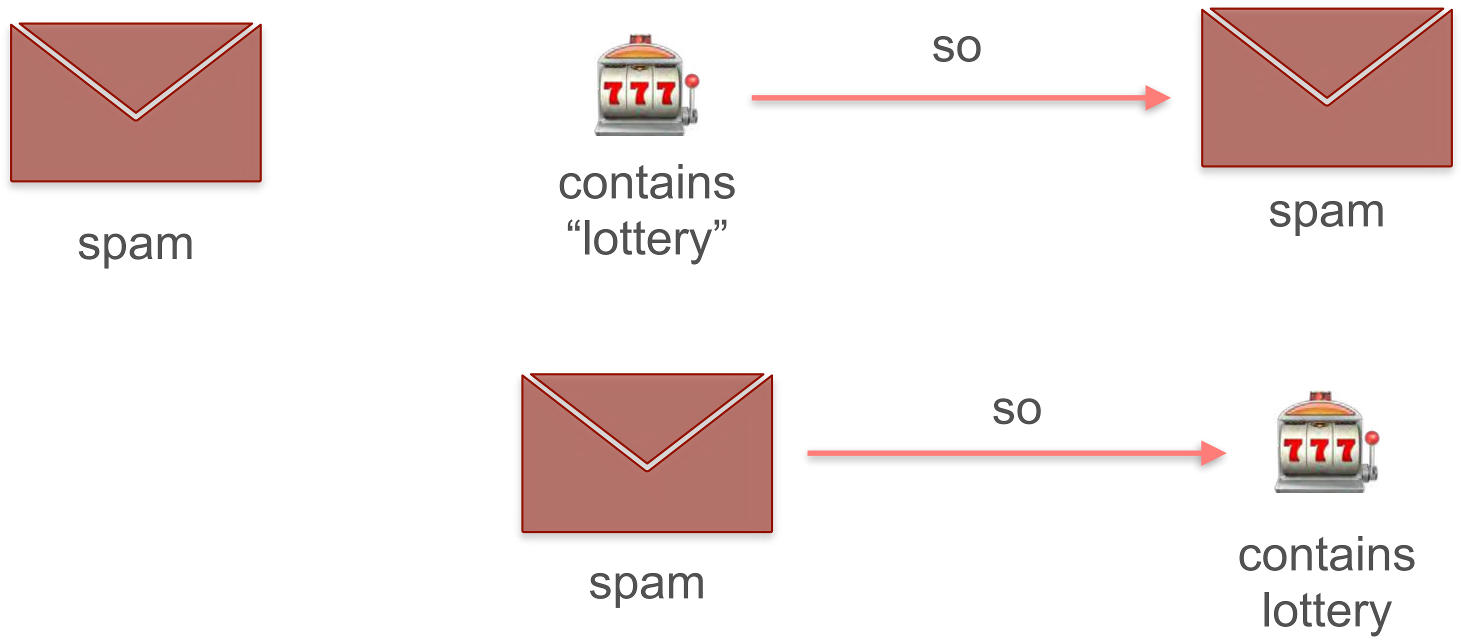
Bayes Theorem: Motivation



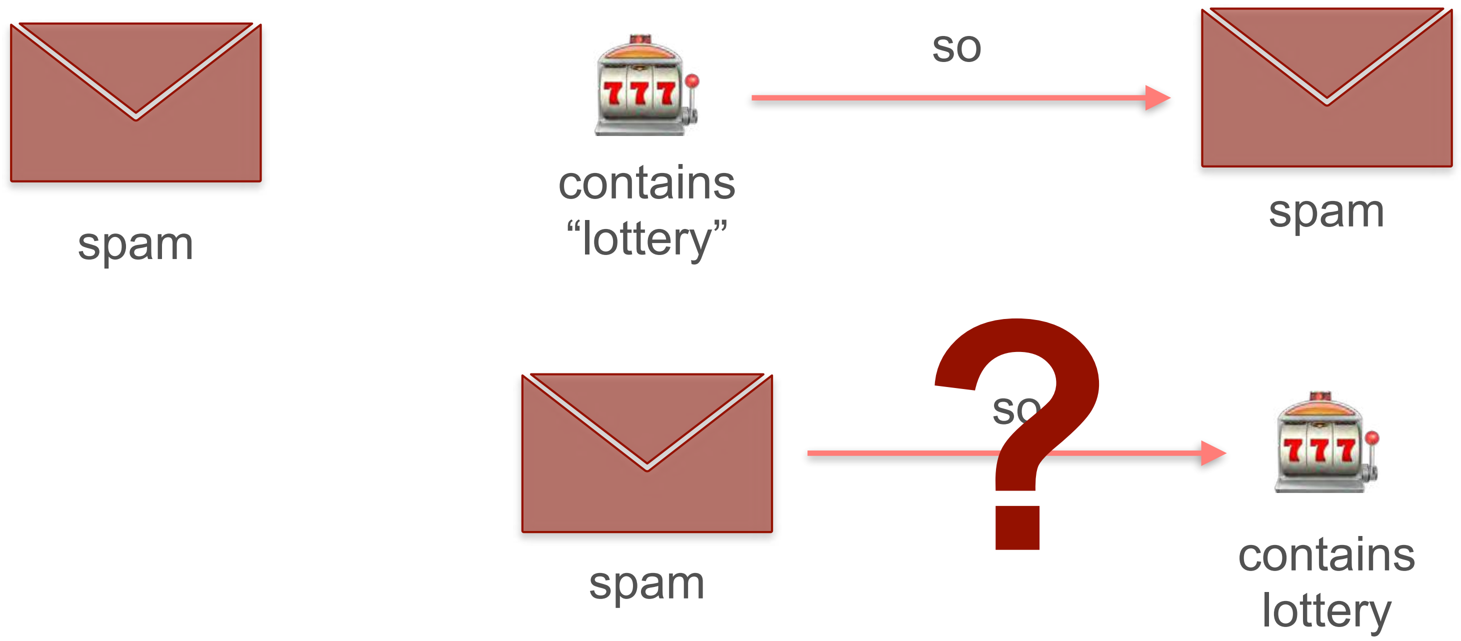
Bayes Theorem: Motivation



Bayes Theorem: Motivation

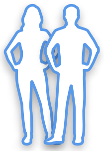


Bayes Theorem: Motivation

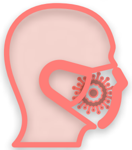


Bayes Theorem: Intuition

Bayes Theorem: Intuition

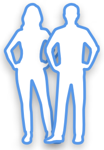


1,000,000 people

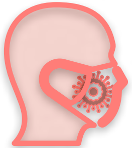


1 / 10,000 people

Bayes Theorem: Intuition



1,000,000 people

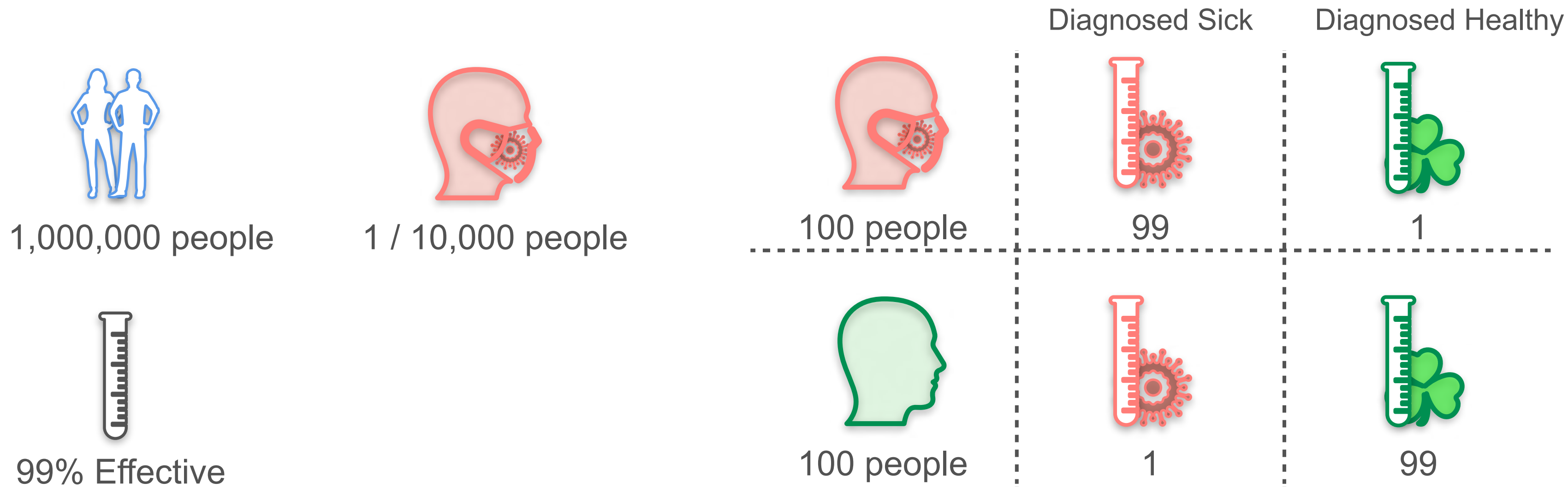


1 / 10,000 people

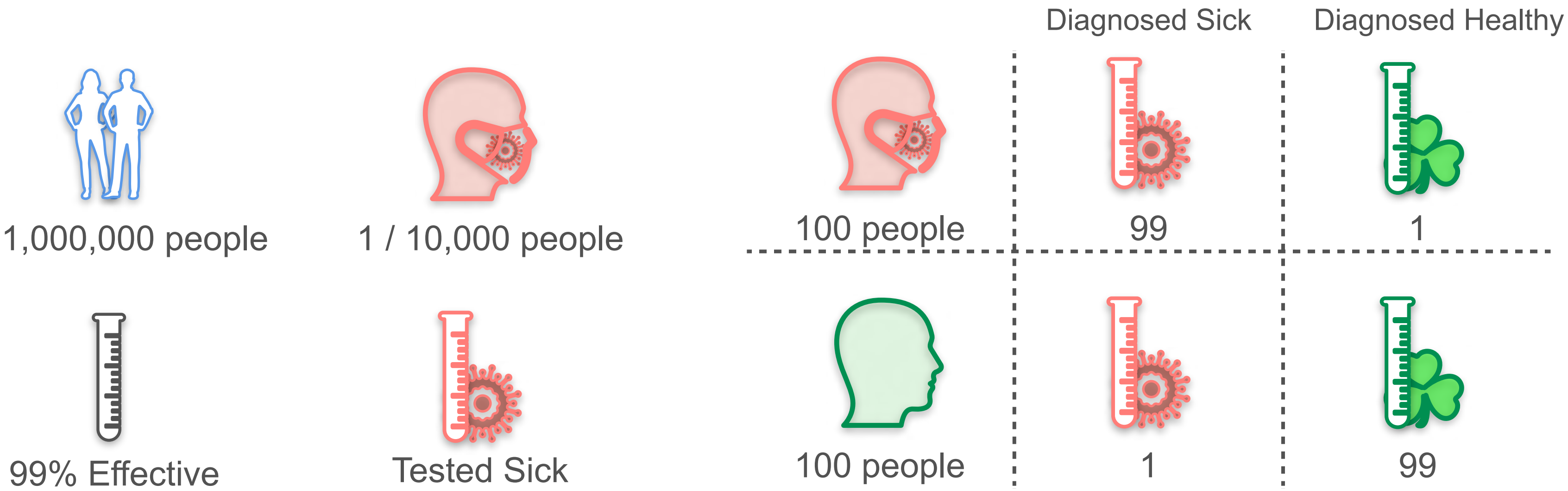
99% Effective

Bayes Theorem: Intuition

Bayes Theorem: Intuition



Bayes Theorem: Intuition



Bayes Theorem: Intuition

1,000,000 people

1 / 10,000 people

99% Effective

Tested Sick

	Diagnosed Sick	Diagnosed Healthy
100 people	99	1
100 people	1	99

What's the probability that **you are sick**

Bayes Theorem: Intuition

1,000,000 people

1 / 10,000 people

99% Effective

Tested Sick

	Diagnosed Sick	Diagnosed Healthy
100 people	99	1
100 people	1	99

What's the probability that **you are sick GIVEN that you tested sick**?

Bayes Theorem: Intuition

1,000,000 people

1 / 10,000 people

99% Effective

Tested Sick

	Diagnosed Sick	Diagnosed Healthy
100 people	99	1
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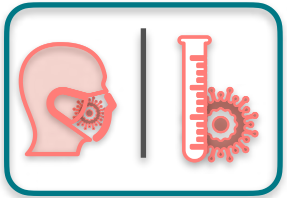
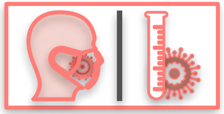
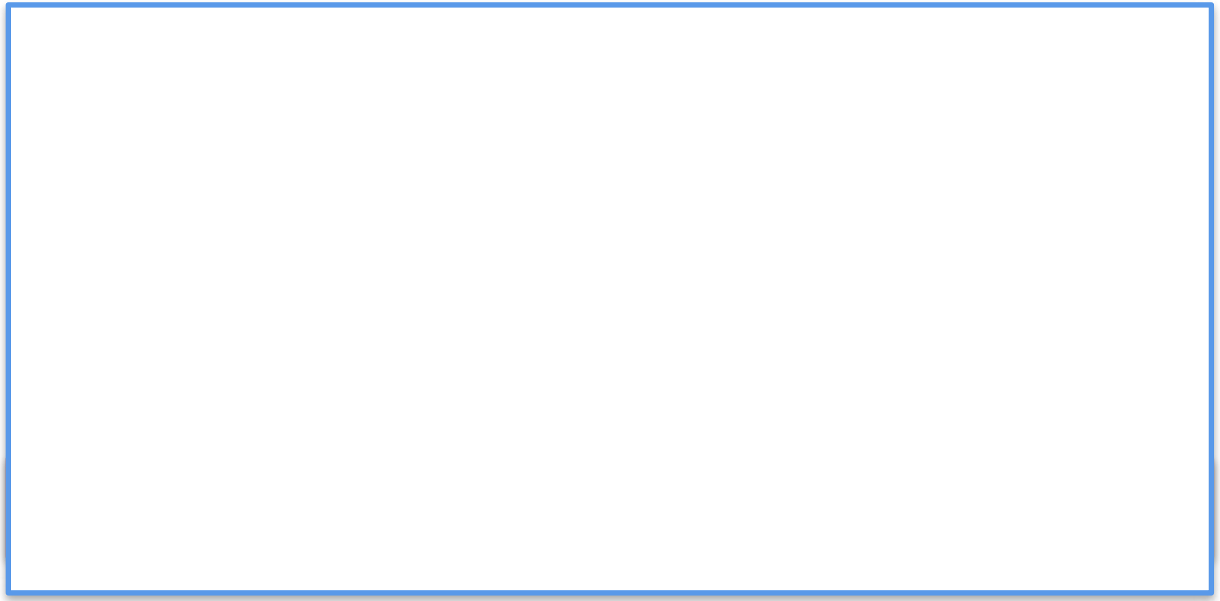


What's the probability that **you are sick GIVEN that you tested sick**?

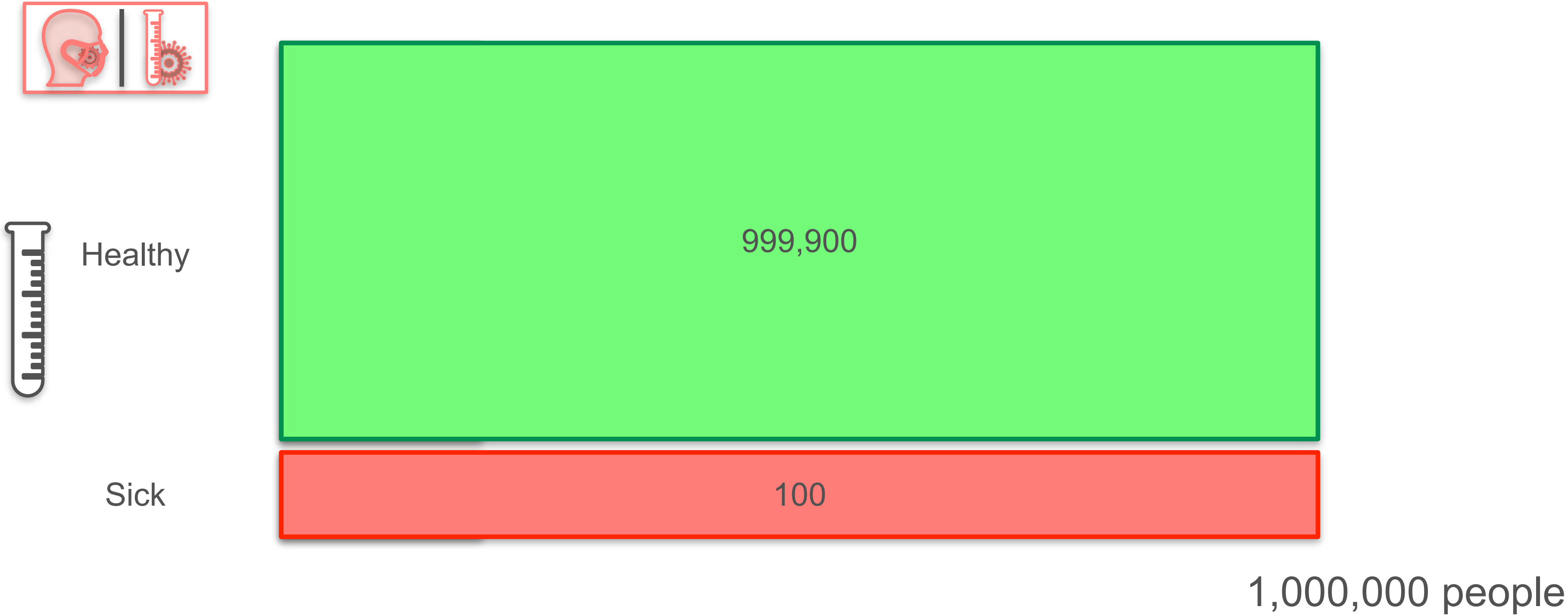
Bayes Theorem: Intuition



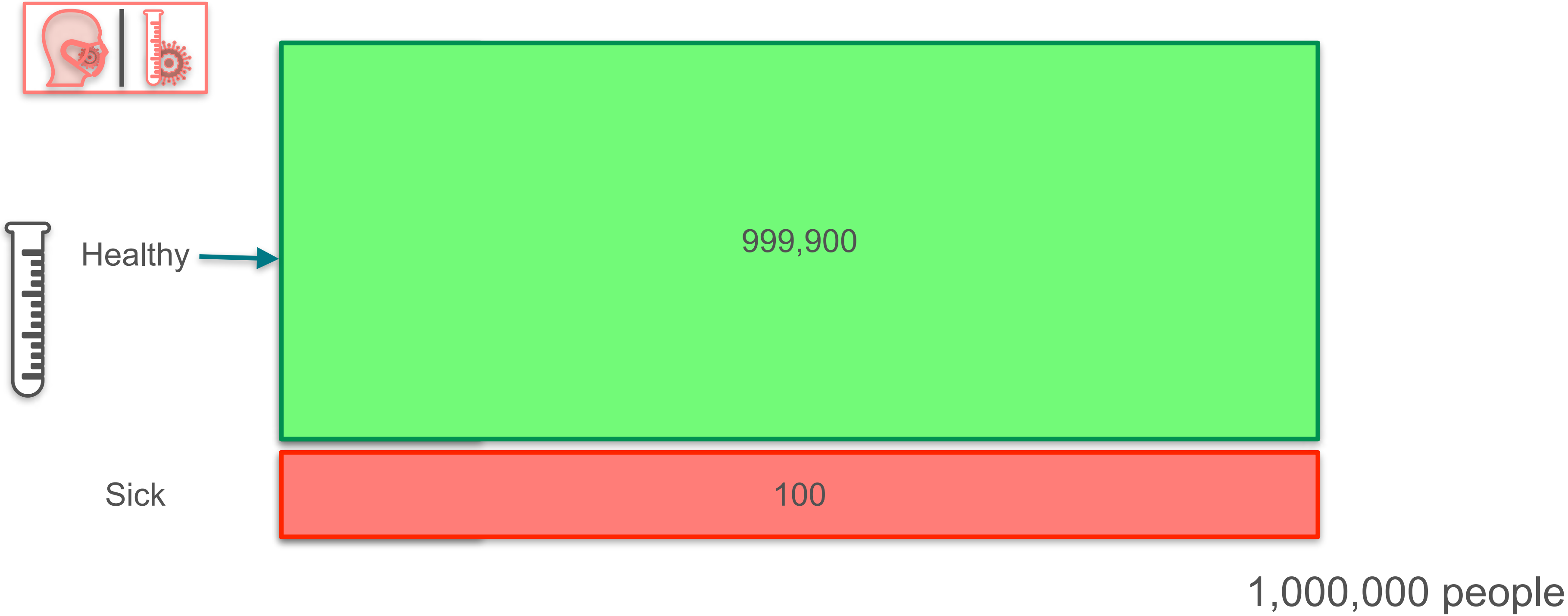


1,000,000 people

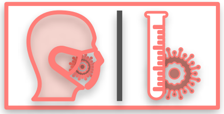
Bayes Theorem: Intuition

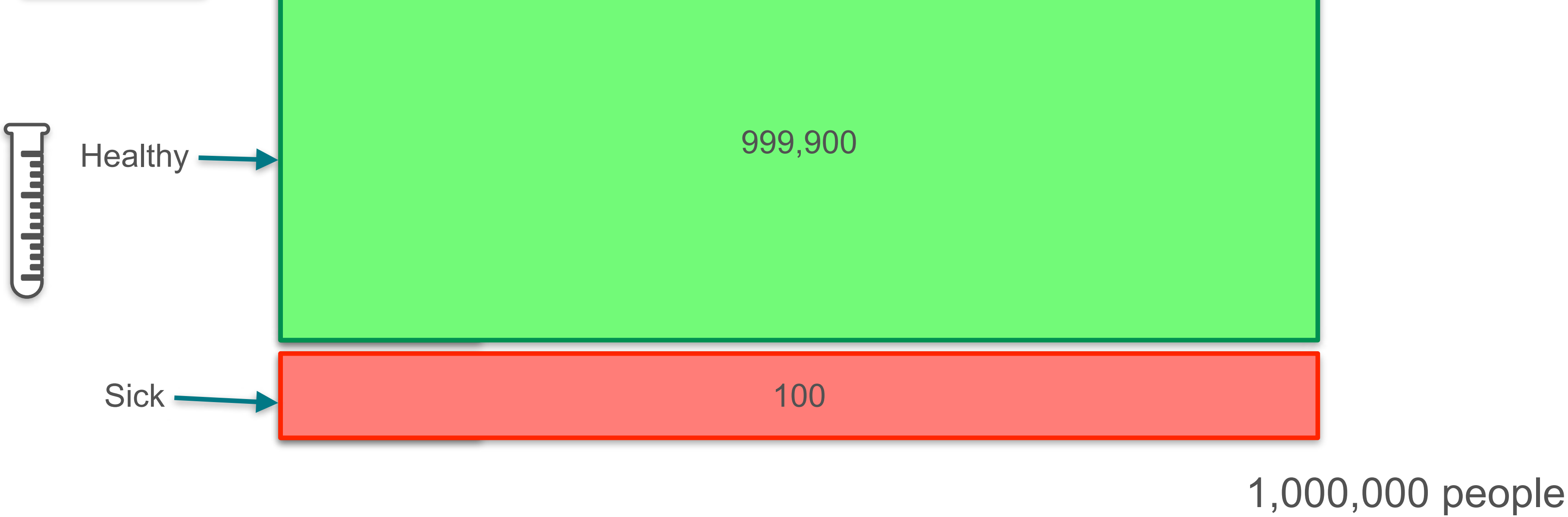


Bayes Theorem: Intuition

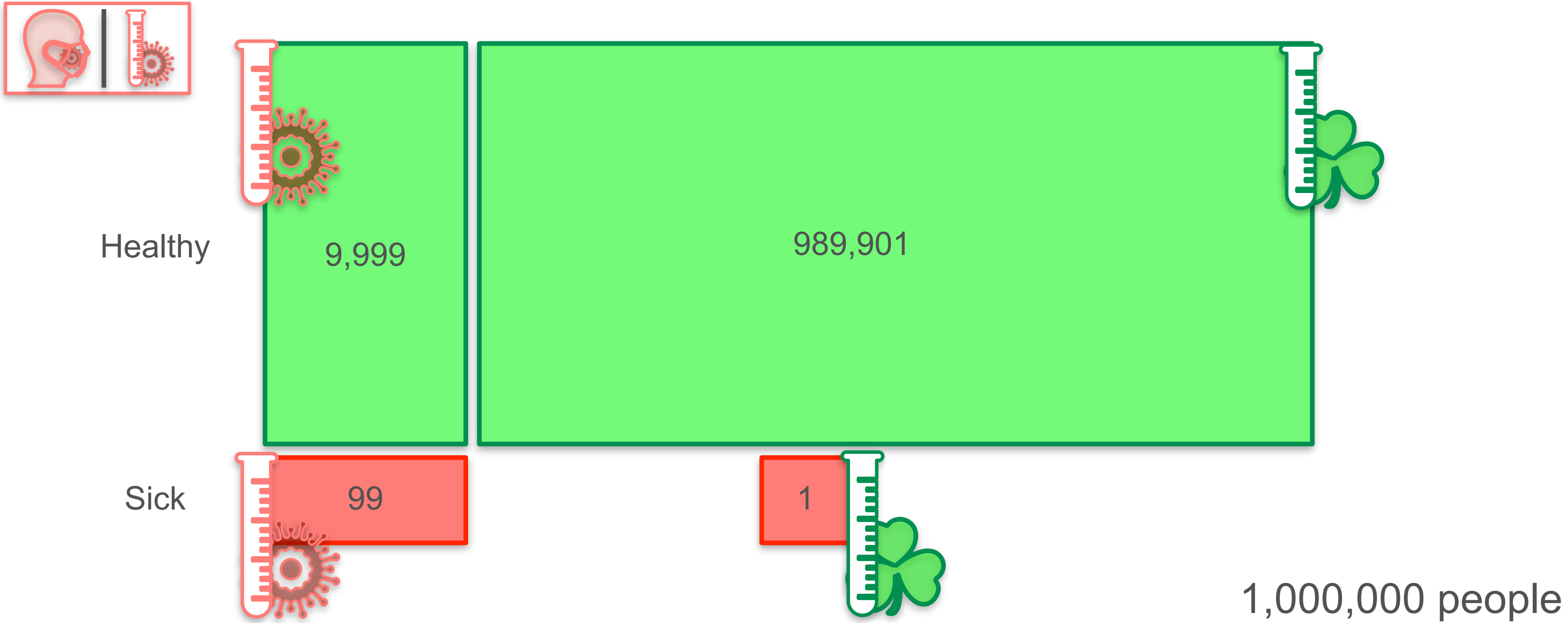


Bayes Theorem: Intuition

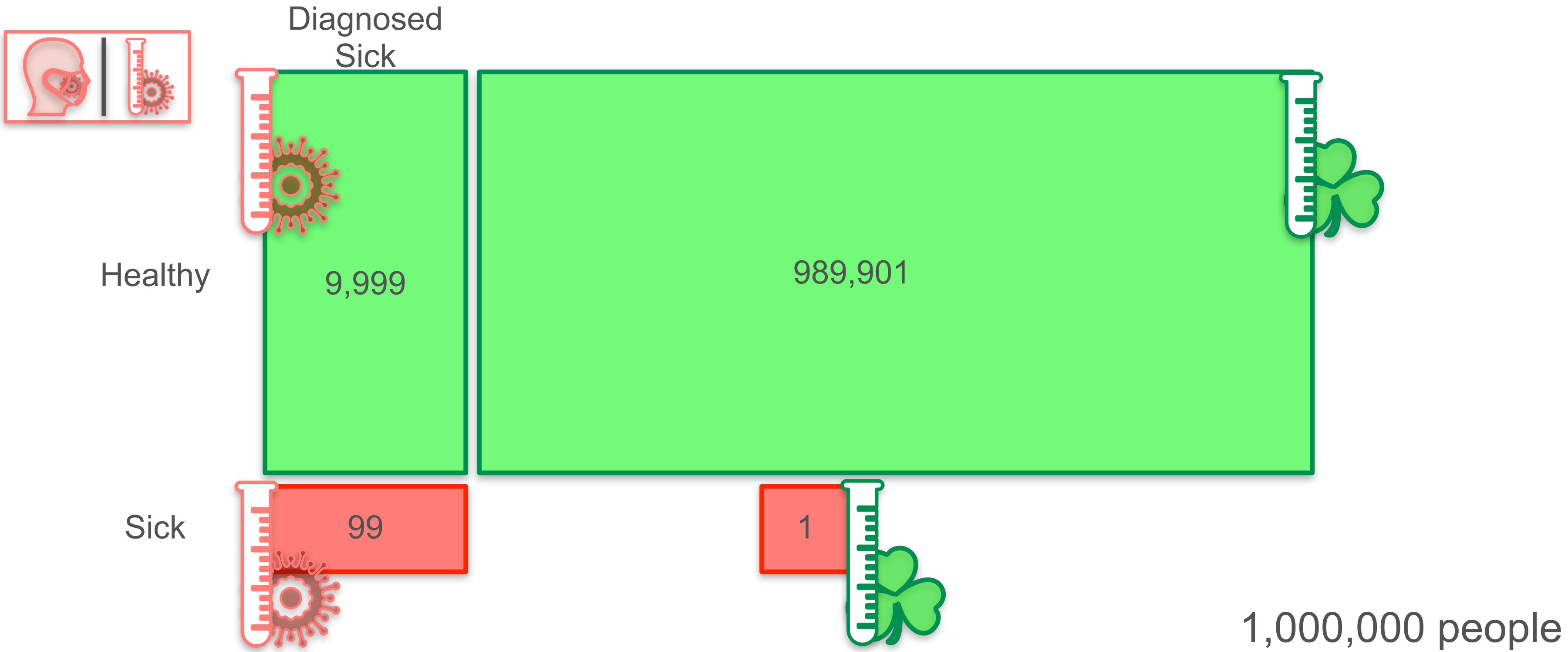




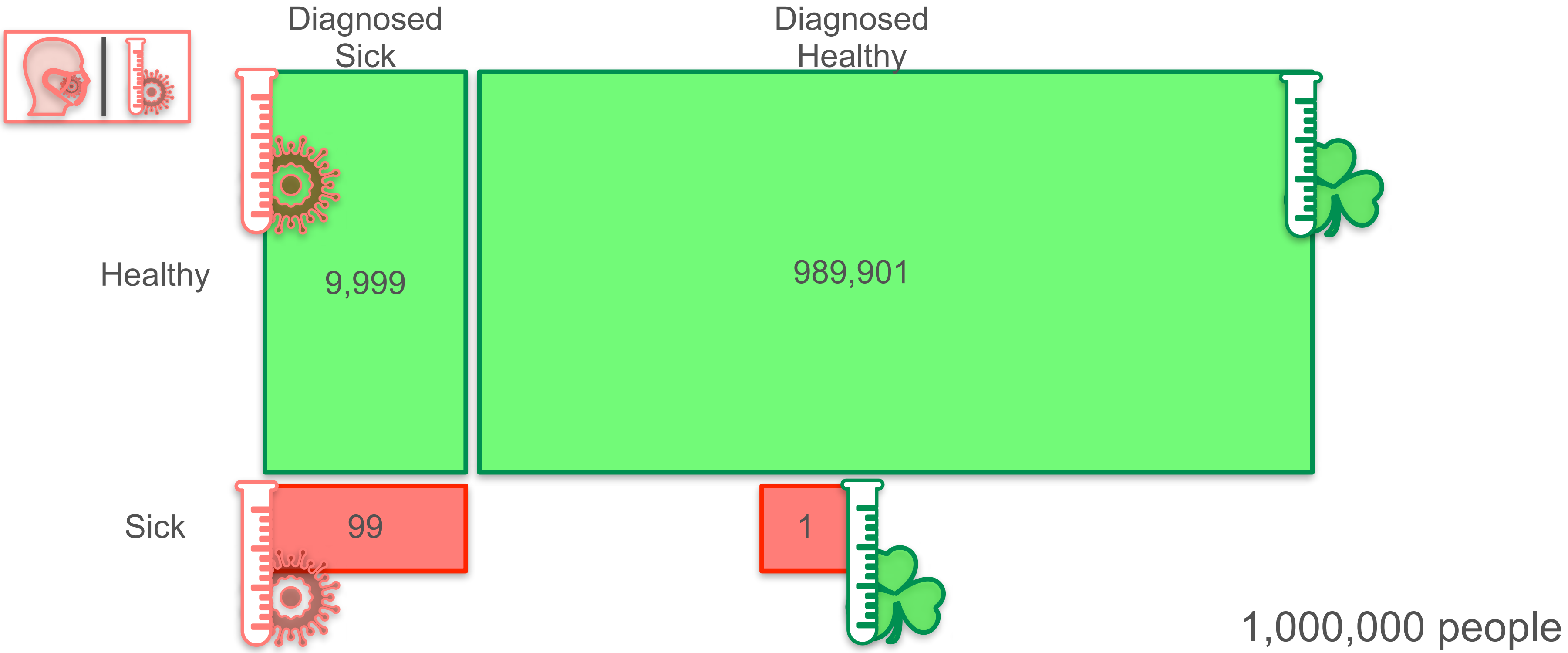
Bayes Theorem: Intuition



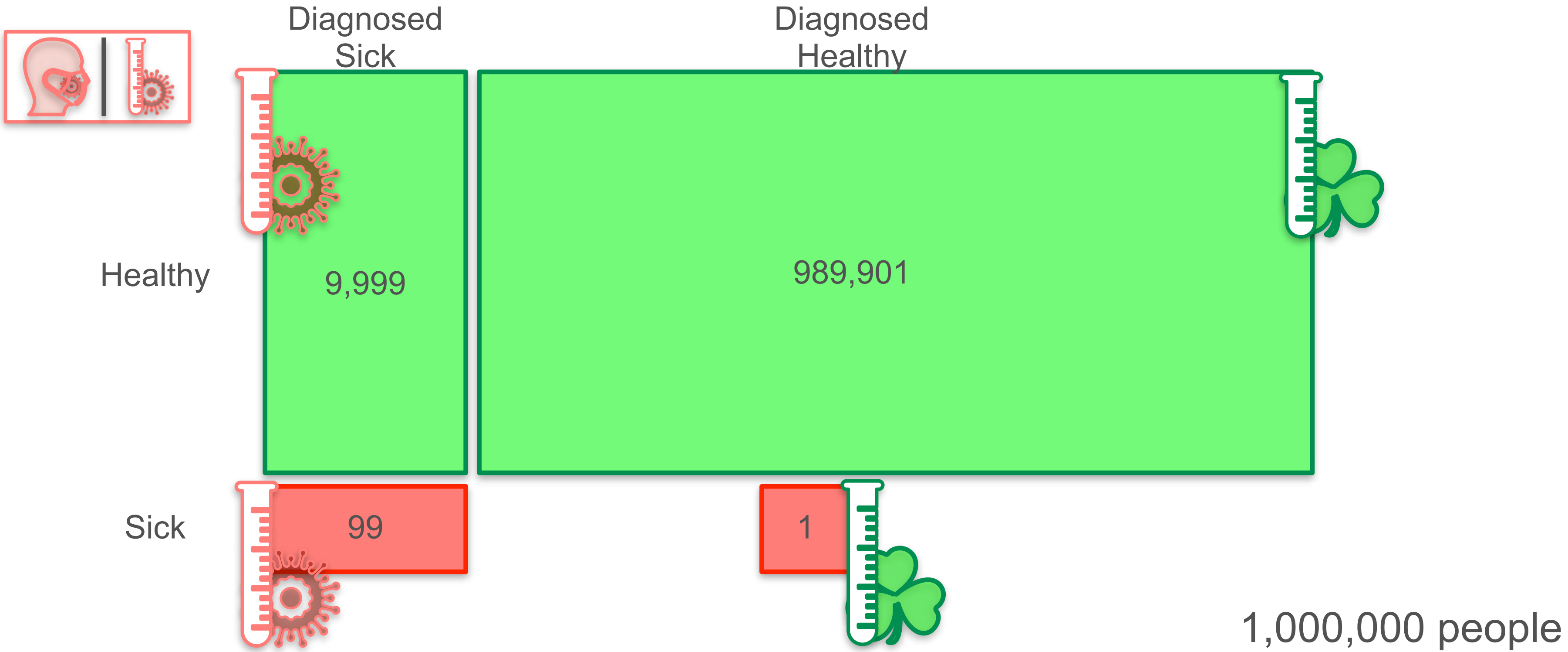
Bayes Theorem: Intuition



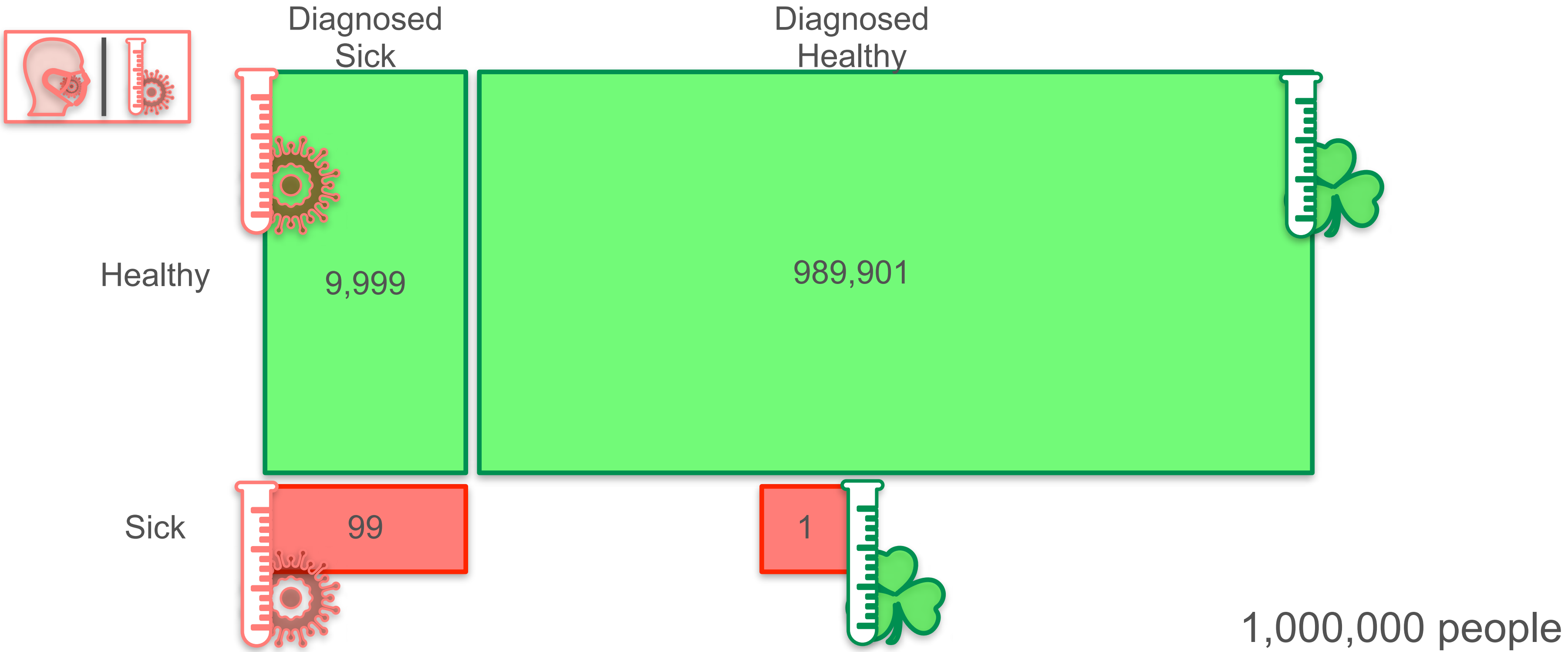
Bayes Theorem: Intuition



Bayes Theorem: Intuition



Bayes Theorem: Intuition



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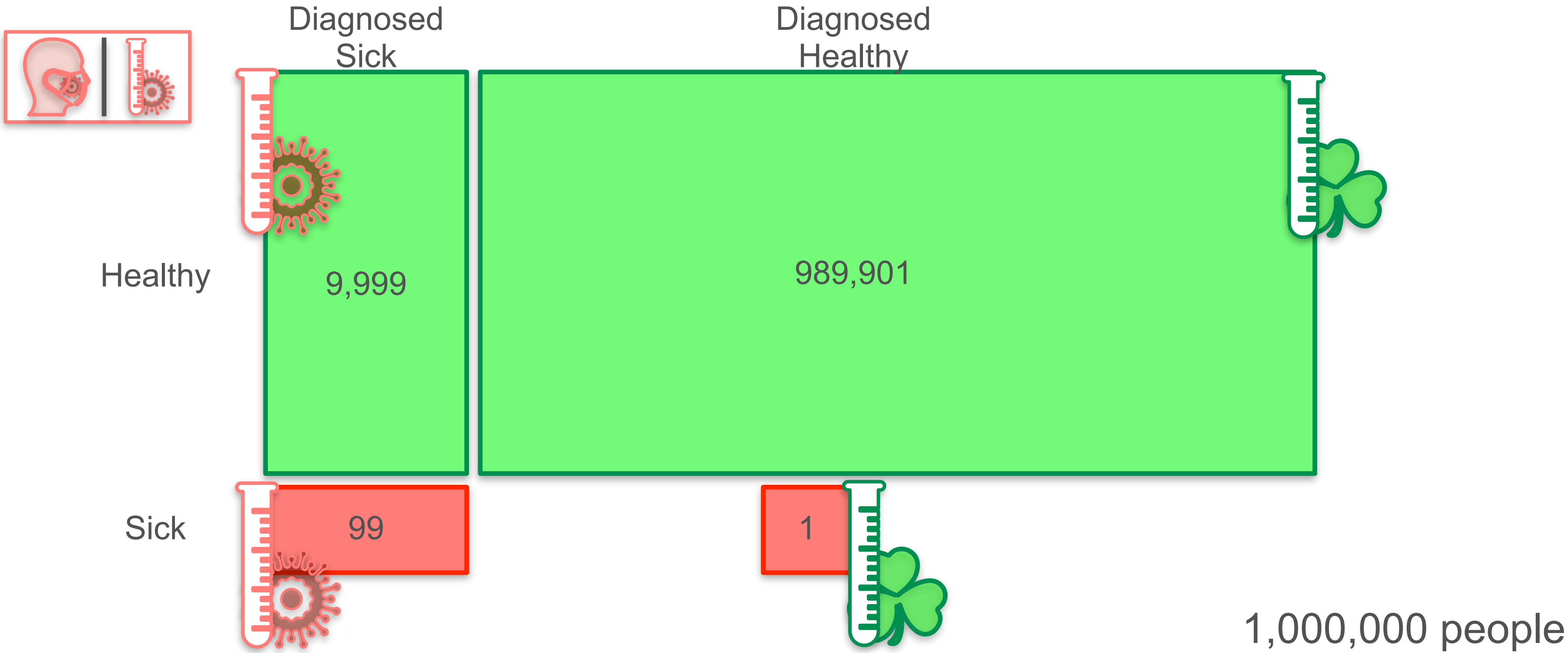
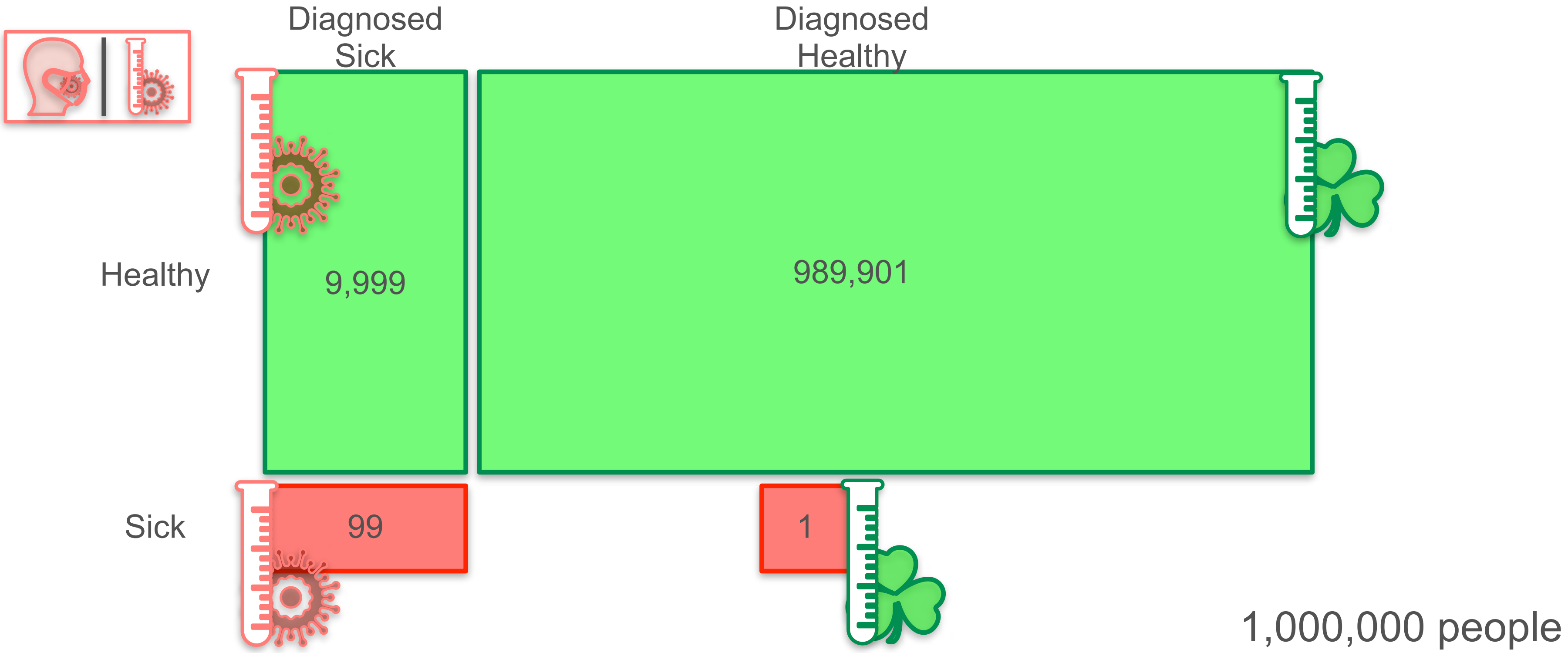
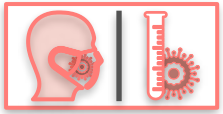
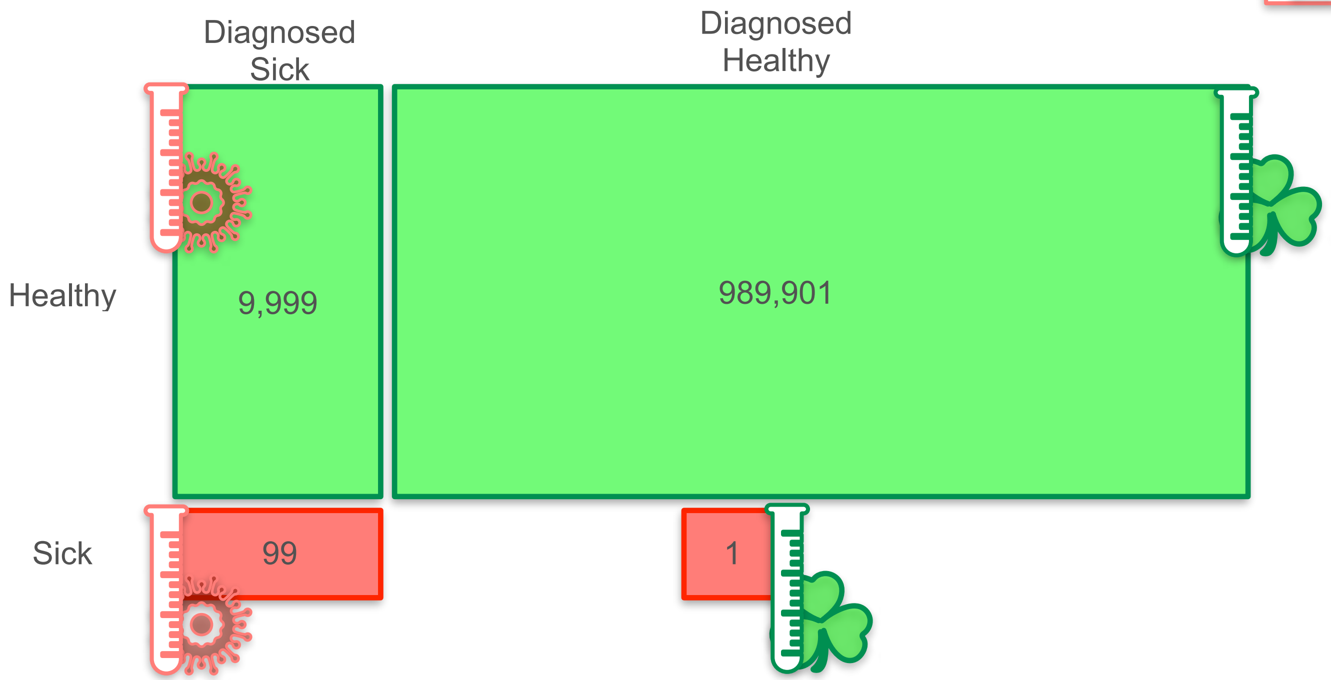


Bayes Theorem: Intuition



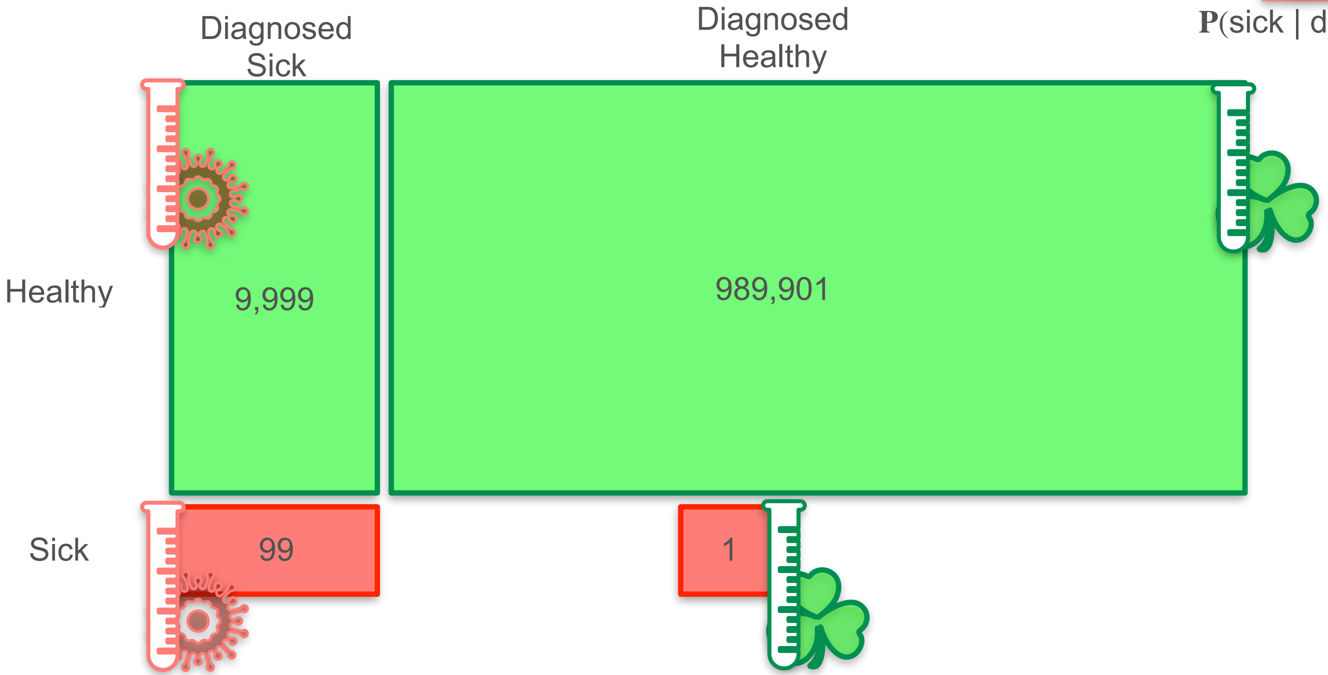
Bayes Theorem: Intuition





1,000,000 people

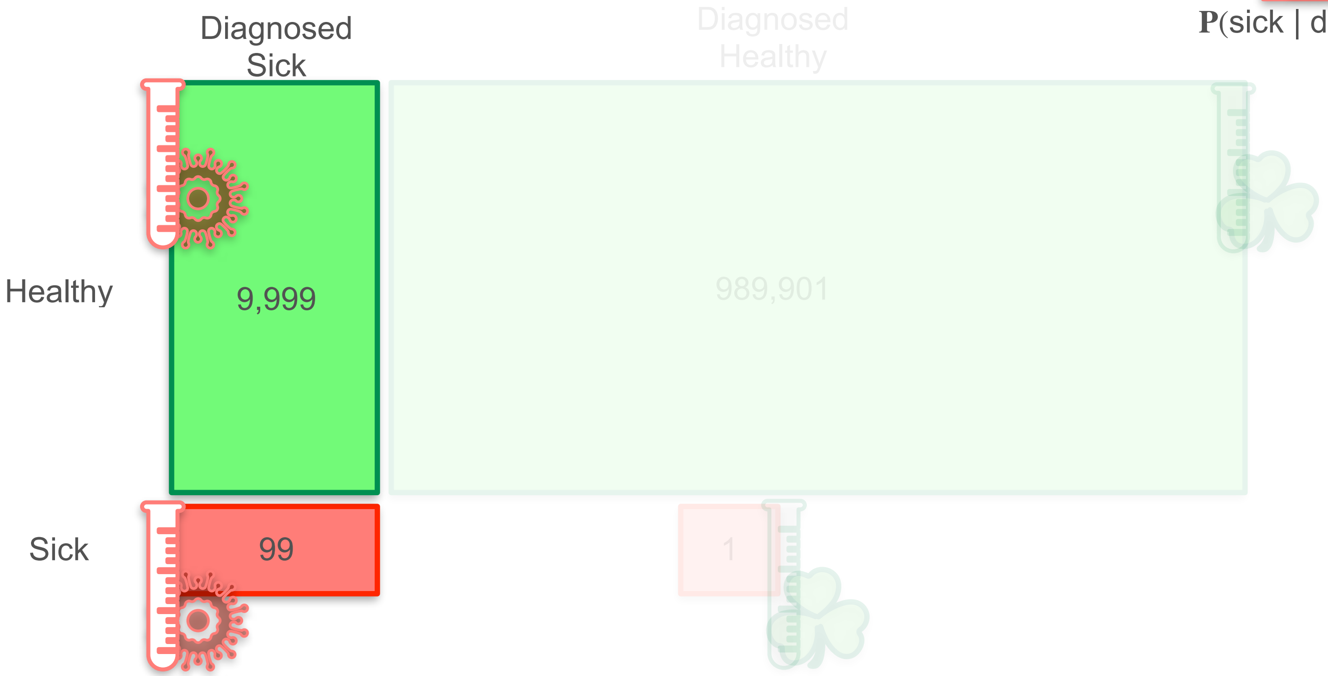
Bayes Theorem: Intuition

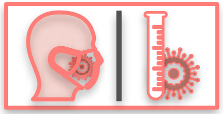


$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$ ______________

1,000,000 people

Bayes Theorem: Intuition

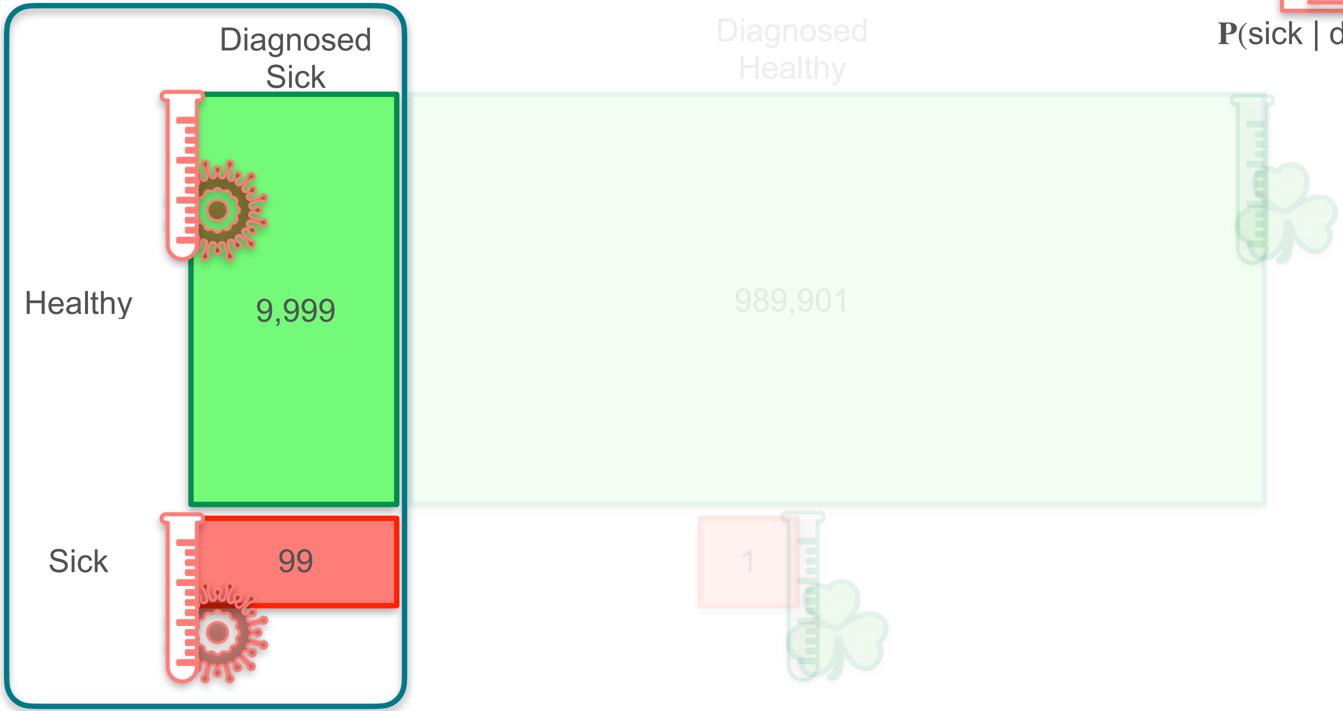


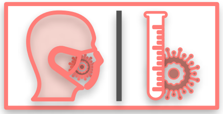


P(sick | diagnosed sick) = ____________

1,000,000 people

Bayes Theorem: Intuition

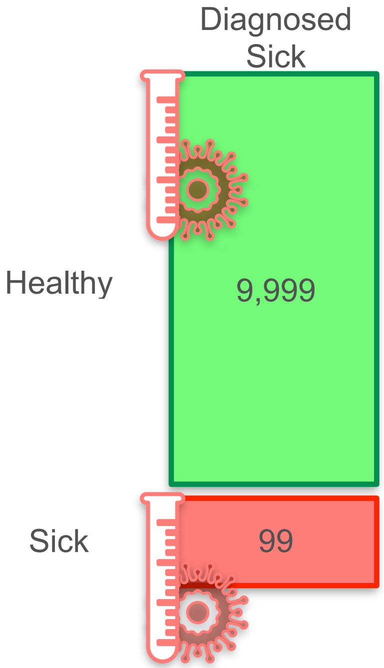


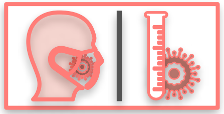


$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \overline{}$

1,000,000 people

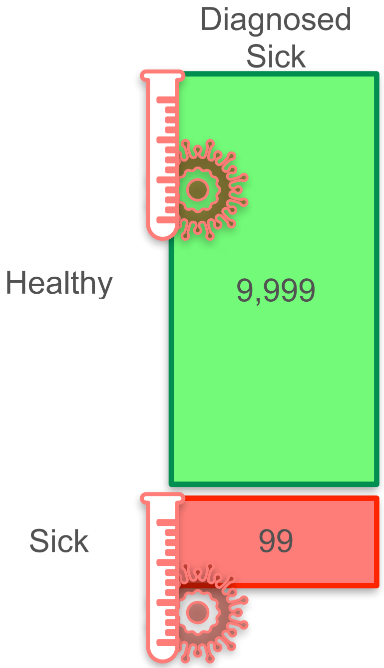
Bayes Theorem: Intuition

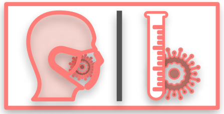




$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \overline{\hspace{6em}}$$

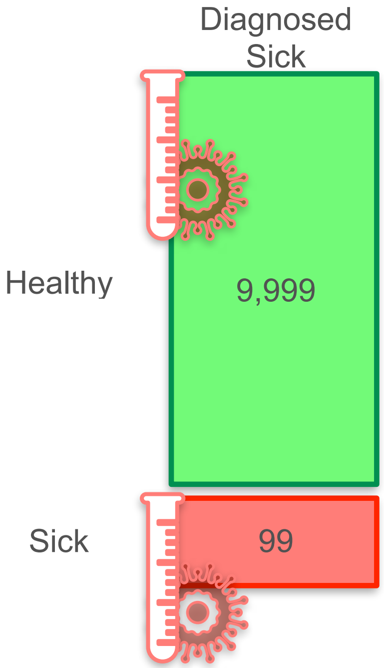
Bayes Theorem: Intuition

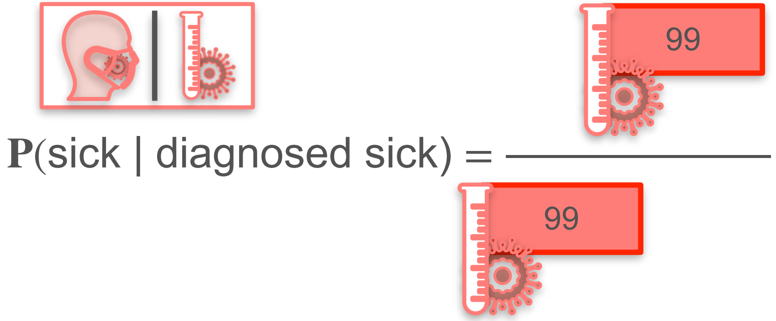




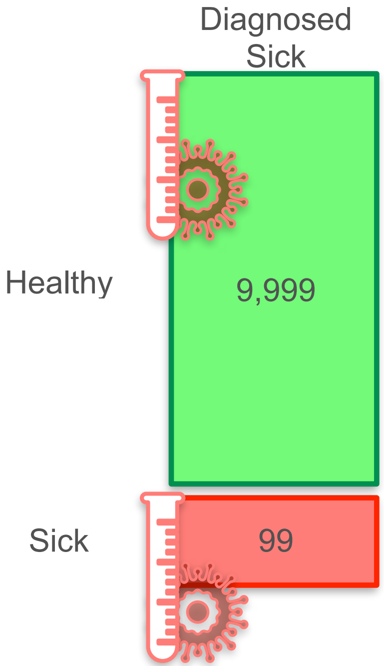
$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{}$$

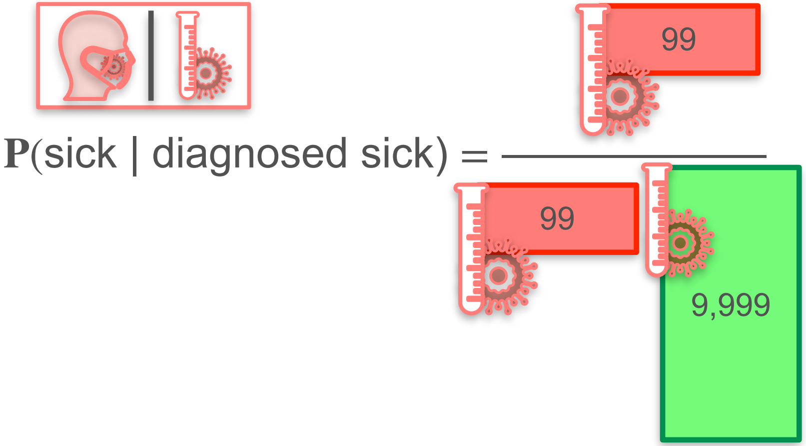
Bayes Theorem: Intuition



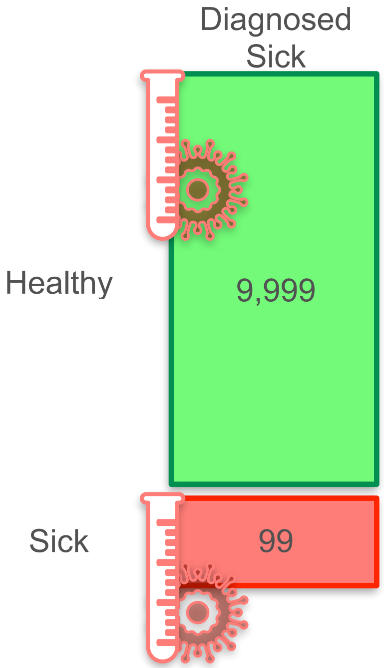


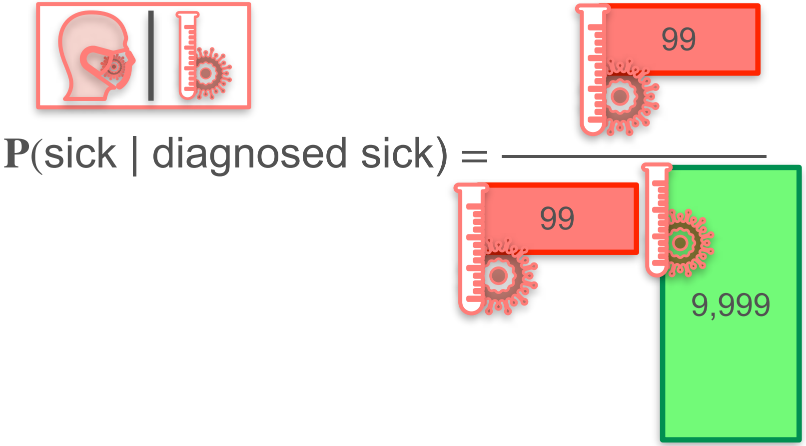
Bayes Theorem: Intuition





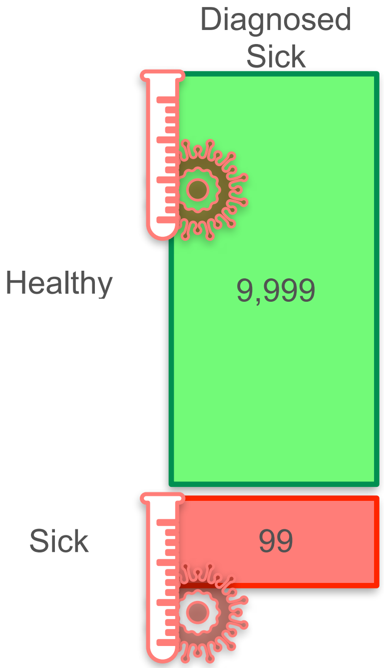
Bayes Theorem: Intuition

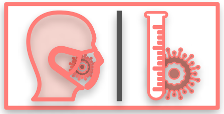




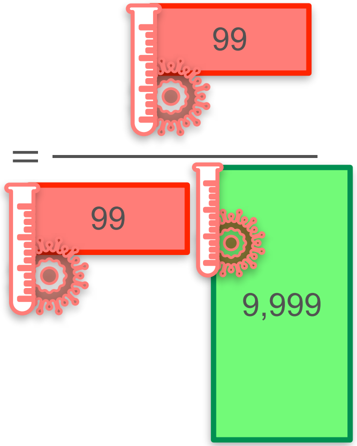
$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

Bayes Theorem: Intuition





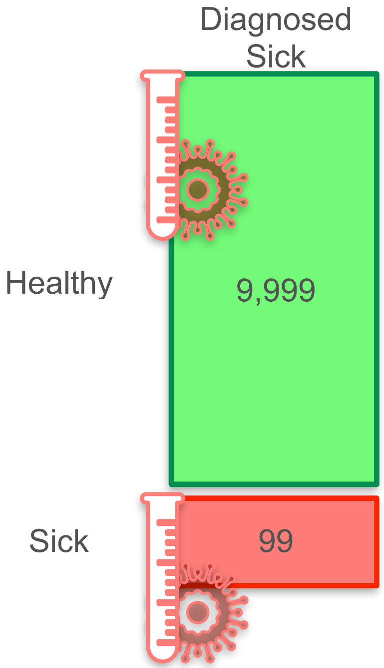
$\mathbf{P}$(sick | diagnosed sick) =

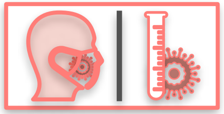


$$= \frac{99}{99 + 9999}$$

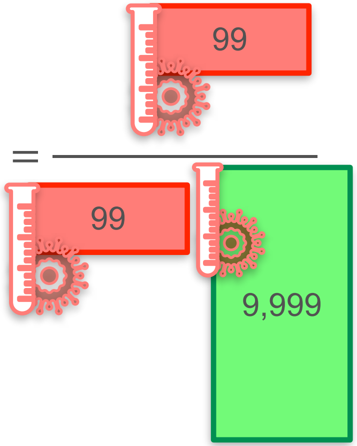
$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

Bayes Theorem: Intuition





$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$

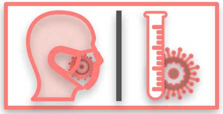


$$= \frac{99}{99 + 9999}$$

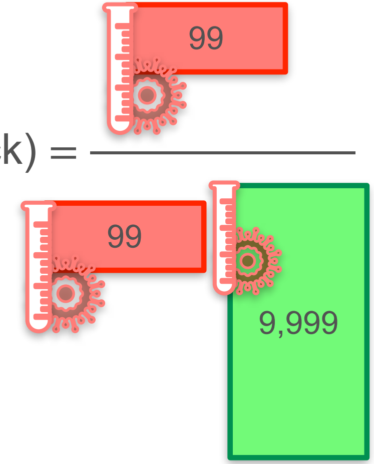
$$= \frac{99}{10098} = 0.0098$$

$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

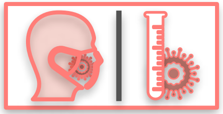
Bayes Theorem: Intuition



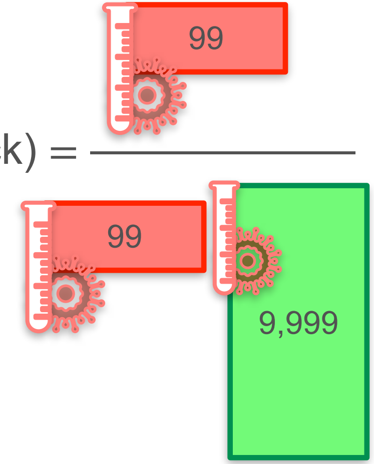
P(sick | diagnosed sick) =



Bayes Theorem: Intuition

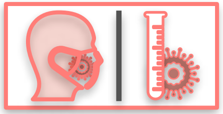


$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$

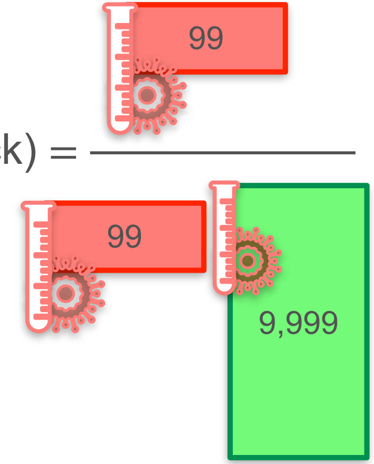


$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

Bayes Theorem: Intuition



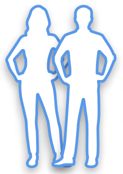
$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$



$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick + healthy and diagnosed sick}}$$

$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

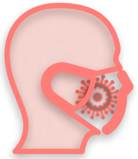
Bayes Theorem: Intuition

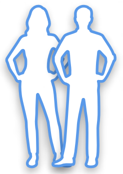


1,000,000
people

Bayes Theorem: Intuition

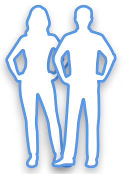
sick = 100





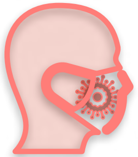
1,000,000
people

Bayes Theorem: Intuition



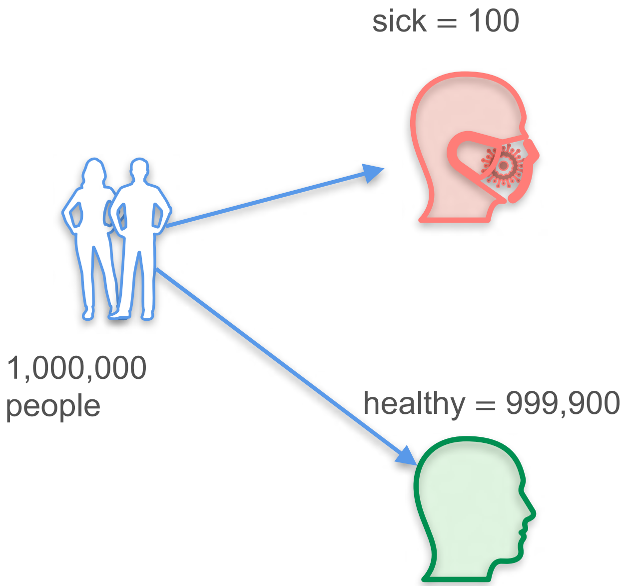
1,000,000
people

sick = 100

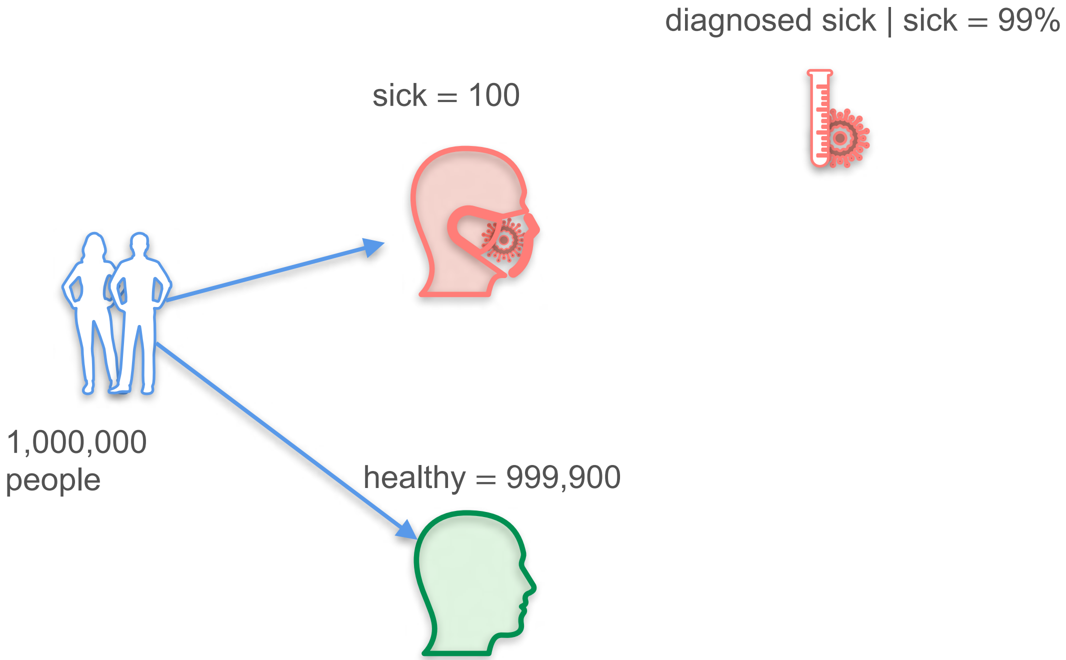


healthy = 999,900

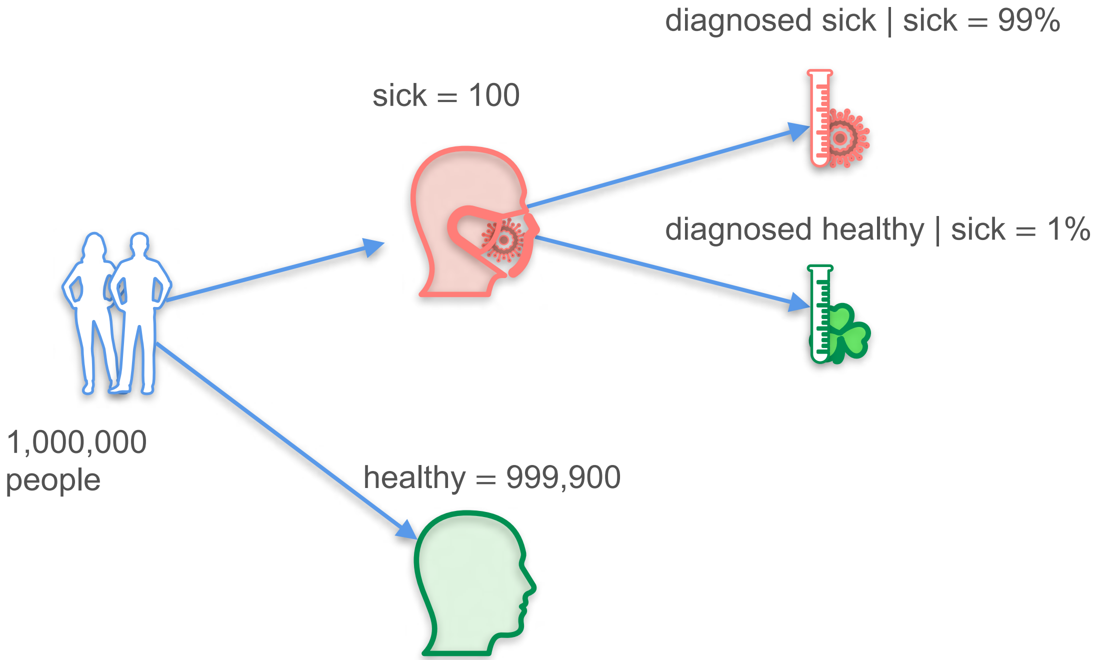
Bayes Theorem: Intuition



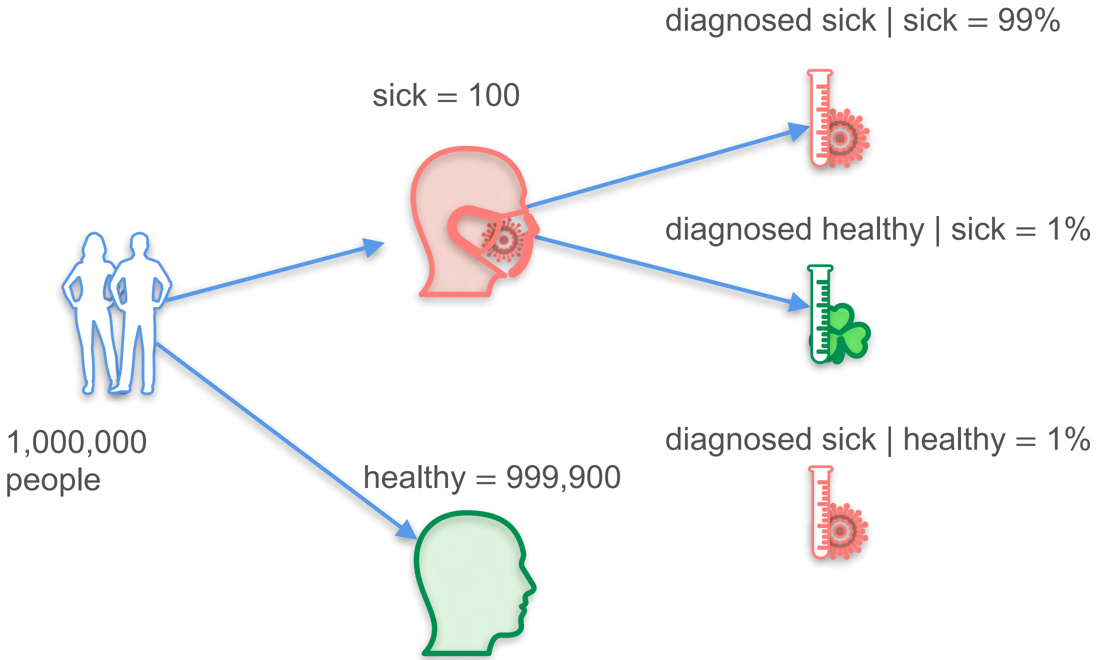
Bayes Theorem: Intuition



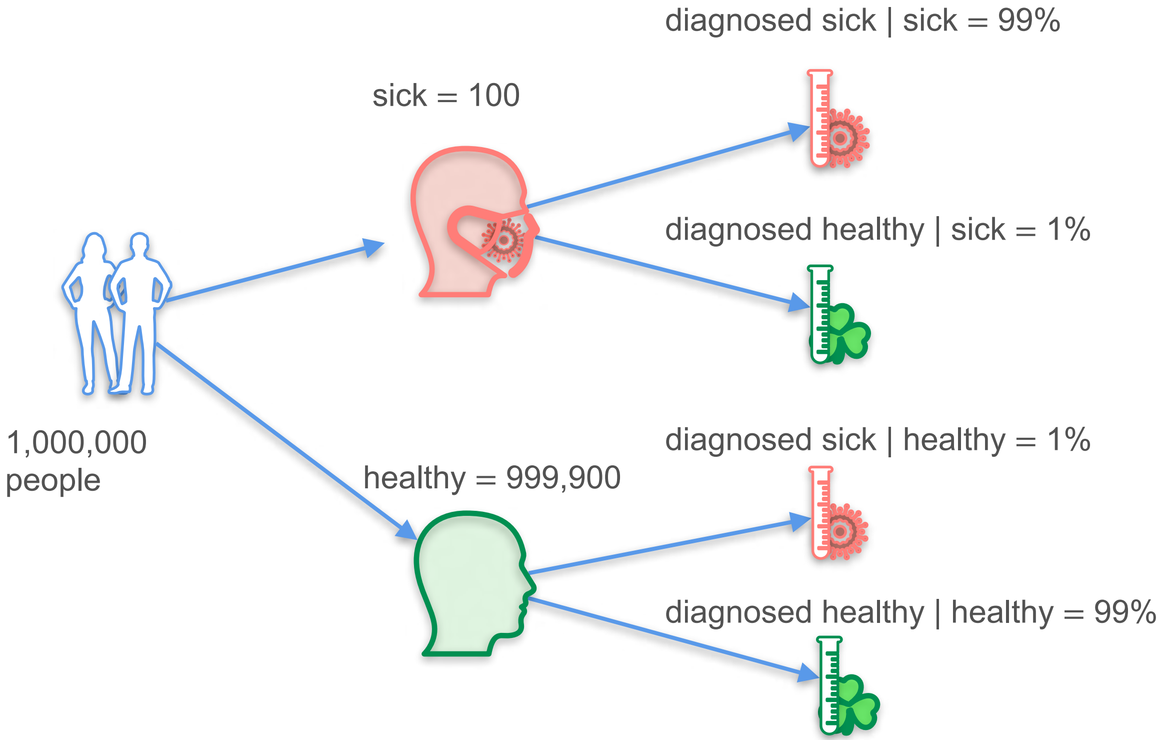
Bayes Theorem: Intuition



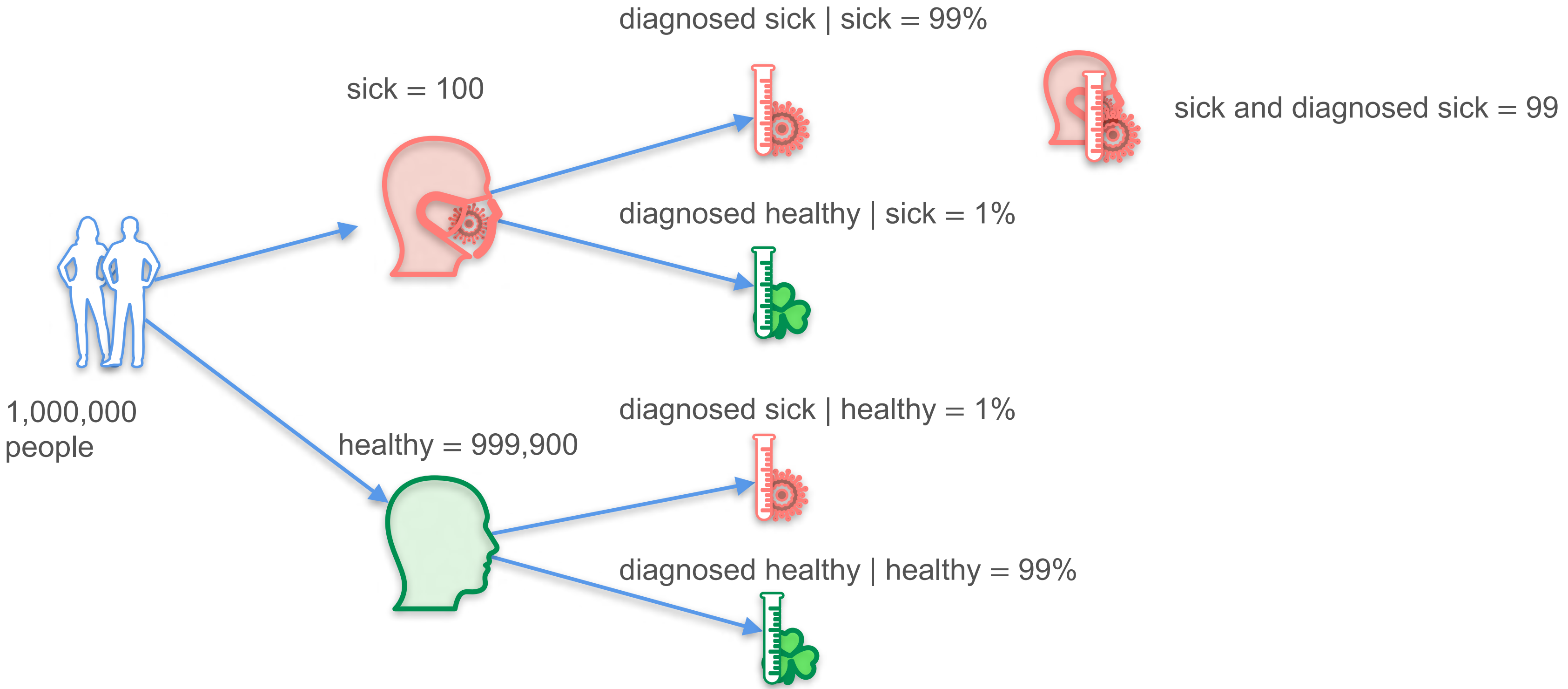
Bayes Theorem: Intuition



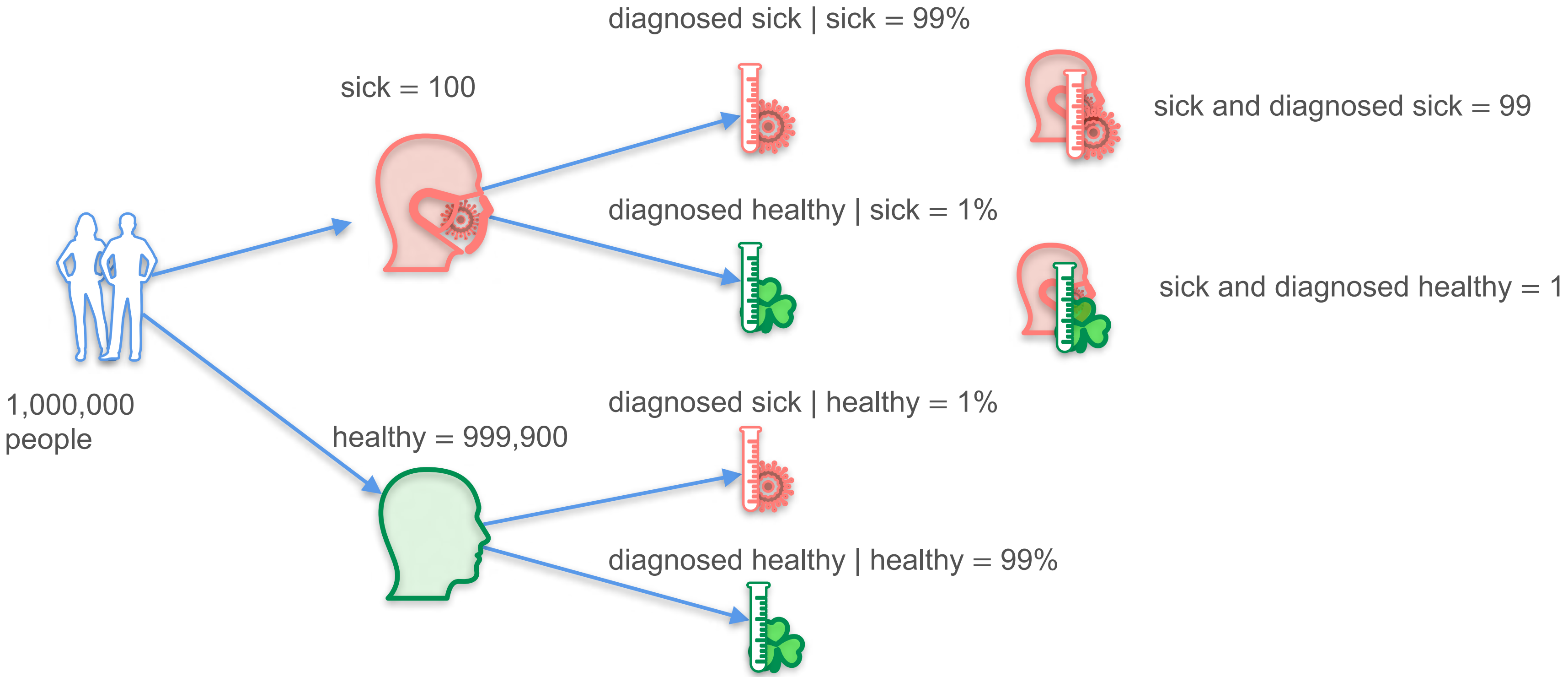
Bayes Theorem: Intuition



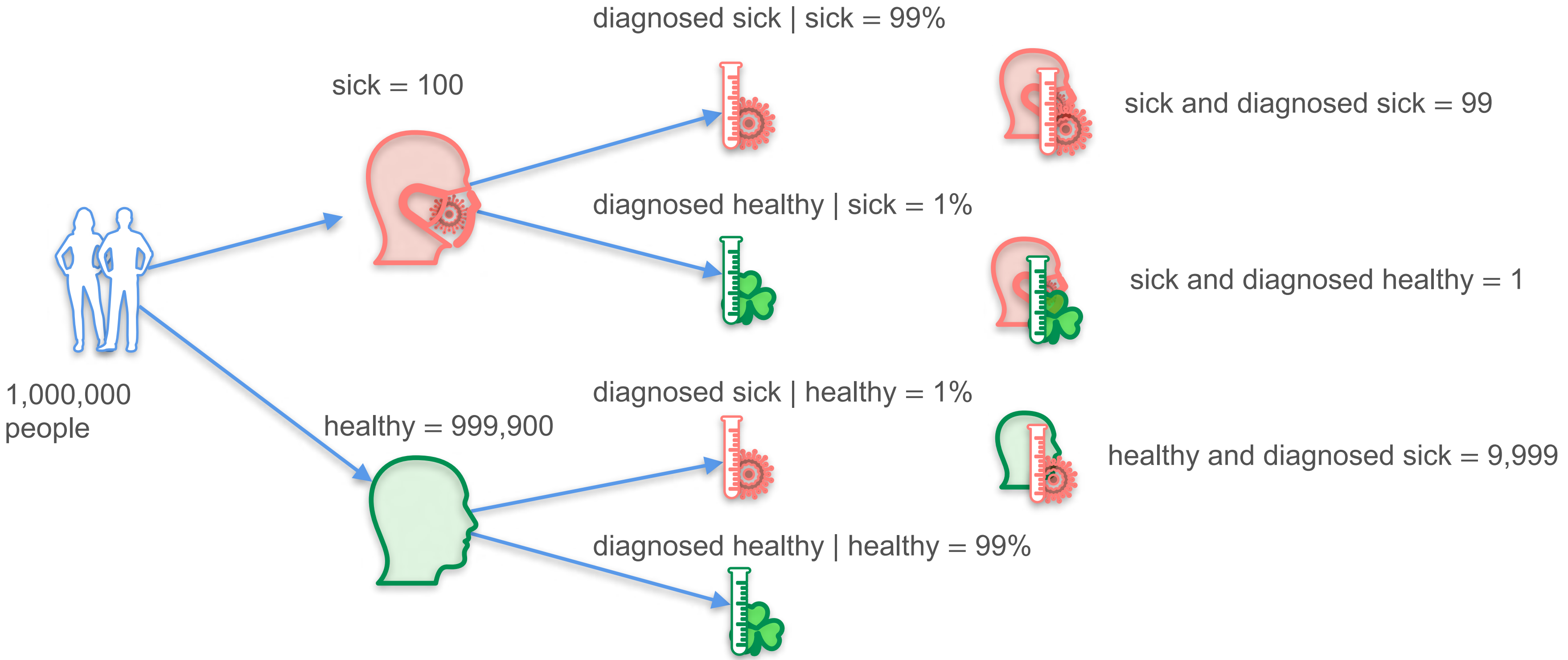
Bayes Theorem: Intuition



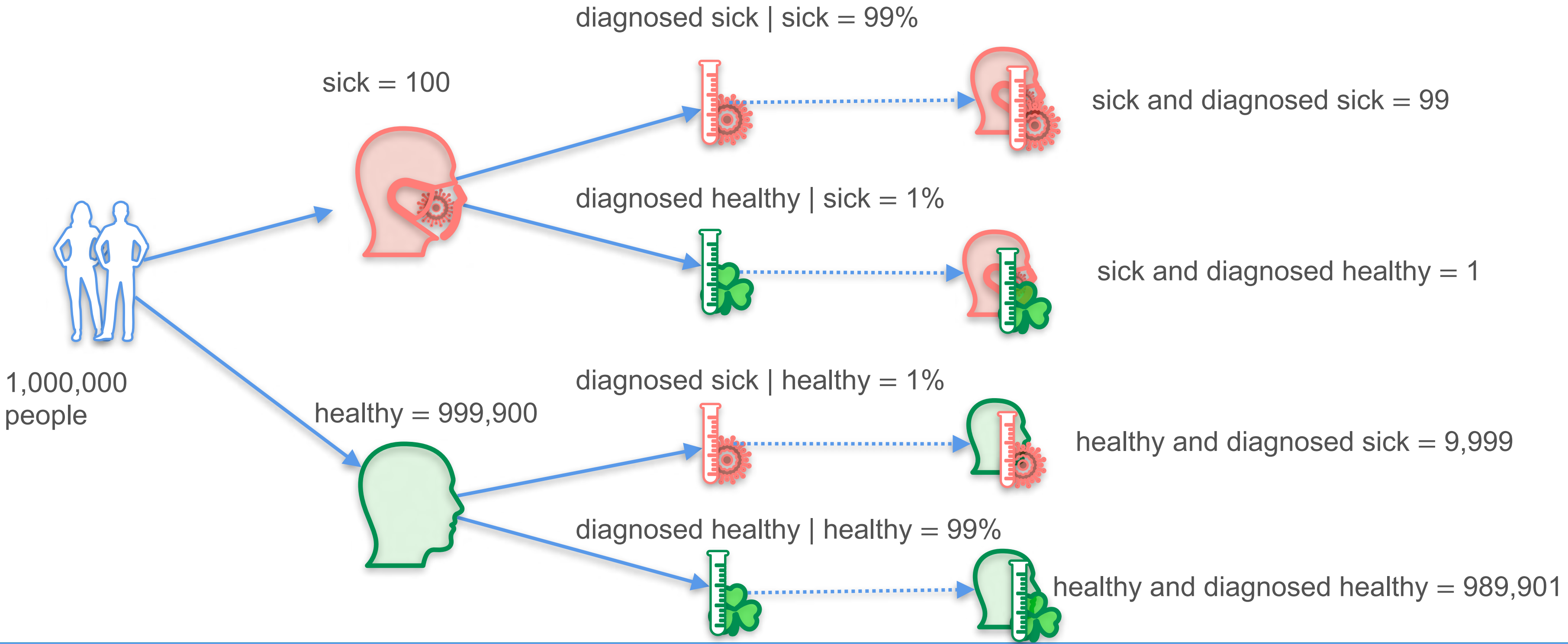
Bayes Theorem: Intuition



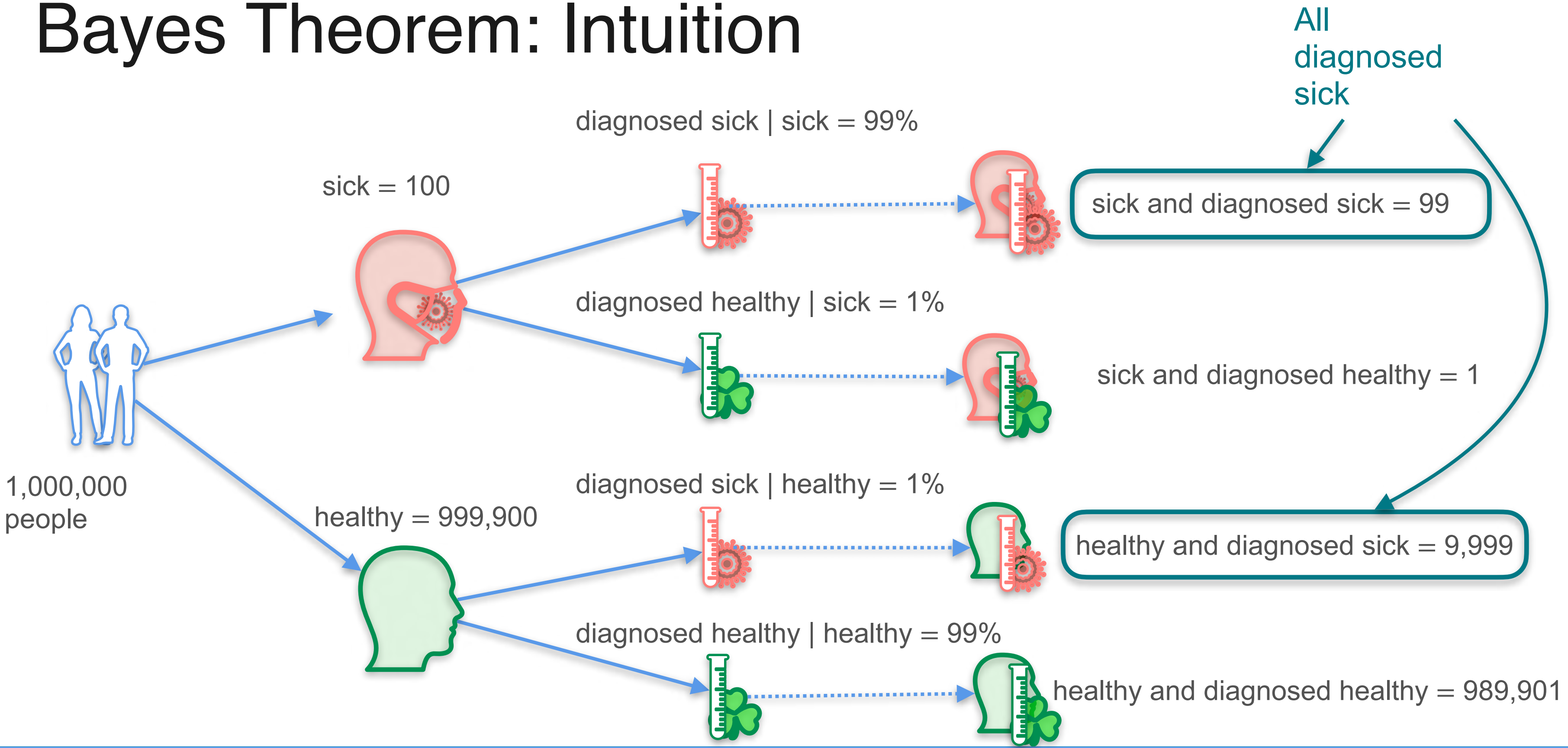
Bayes Theorem: Intuition



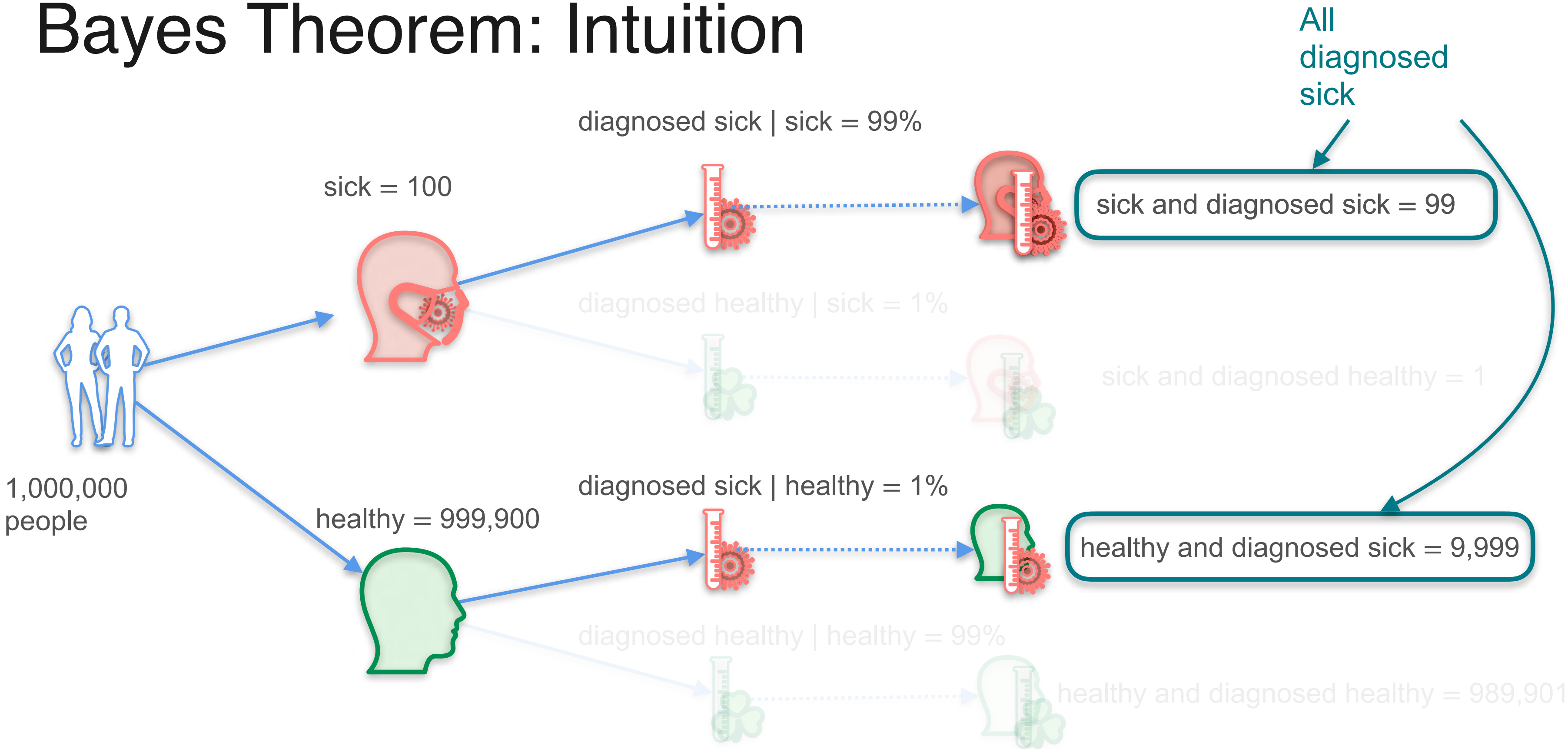
Bayes Theorem: Intuition



Bayes Theorem: Intuition

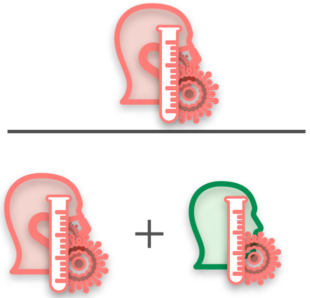


Bayes Theorem: Intuition



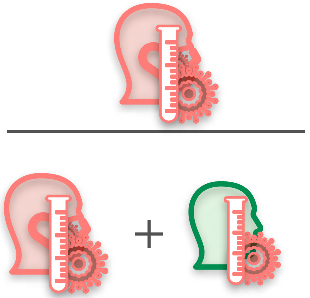
Bayes Theorem: Intuition

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$



$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9{,}999 + \text{sick and diagnosed sick} = 99}$$

Bayes Theorem: Intuition

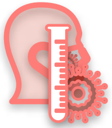
$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$ 

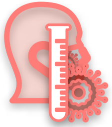
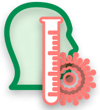
$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9{,}999 + \text{sick and diagnosed sick} = 99}$$

$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098}$$

Bayes Theorem: Intuition

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) =$



 + 

$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9{,}999 + \text{sick and diagnosed sick} = 99}$$

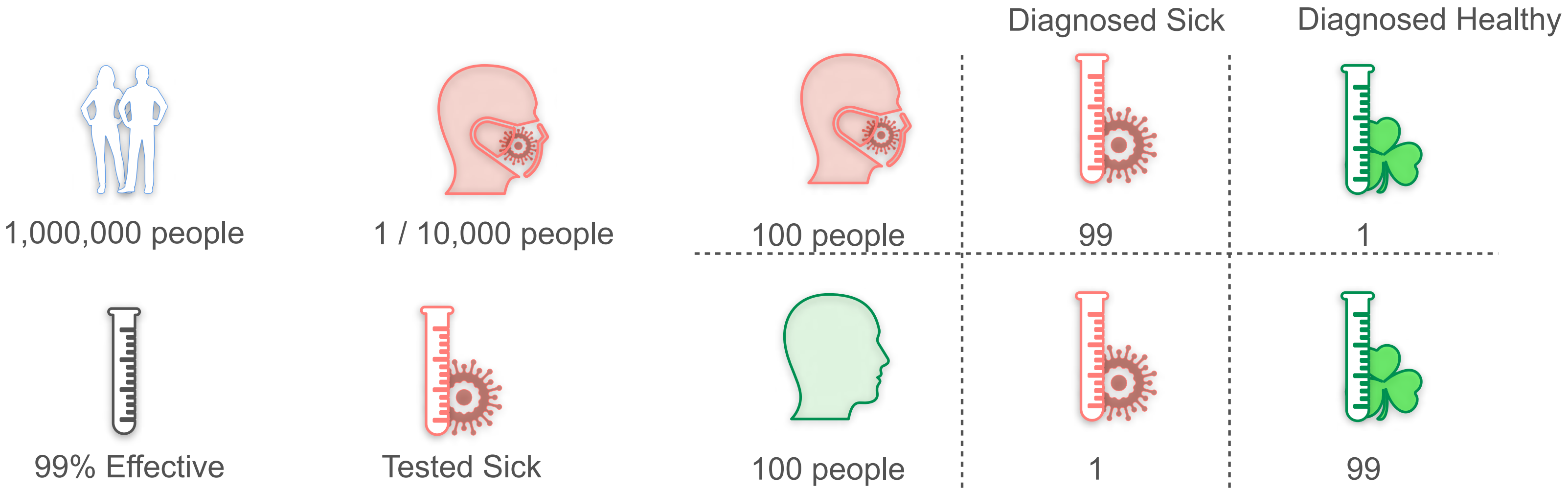
$$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098} = 0.0098$$

Bayes Theorem: Formula

Bayes Theorem: Formula

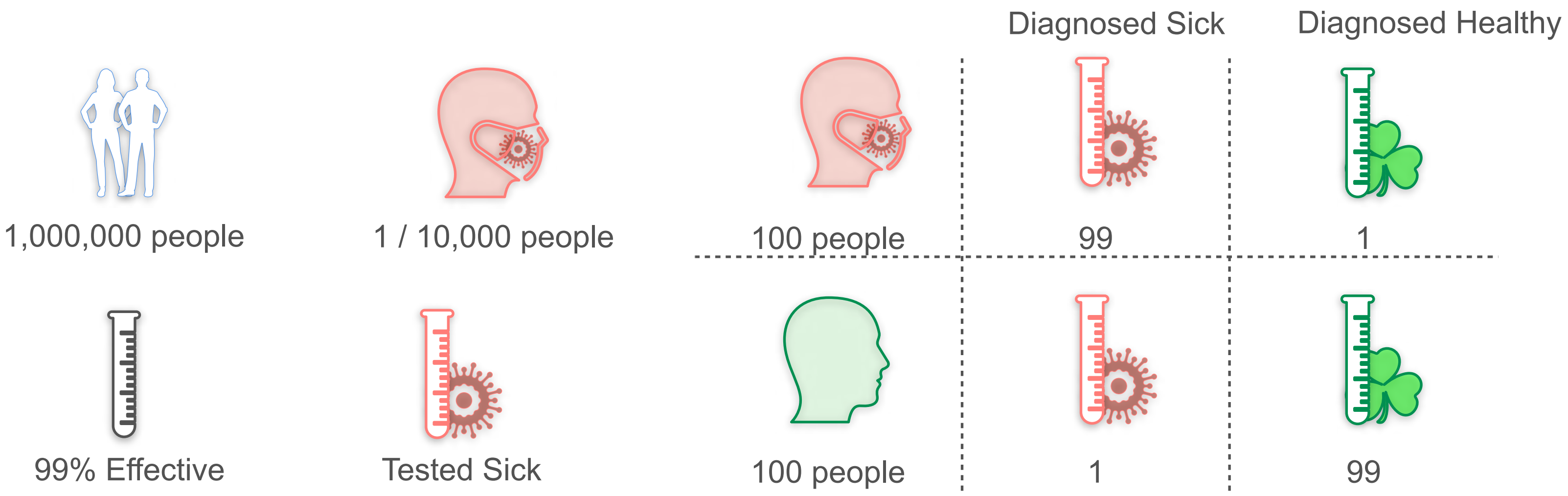
The probability that **you are sick GIVEN that you tested sick**

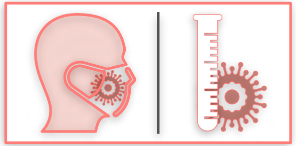
Bayes Theorem: Formula



The probability that **you are sick GIVEN that you tested sick**

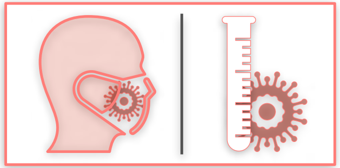
Bayes Theorem: Formula





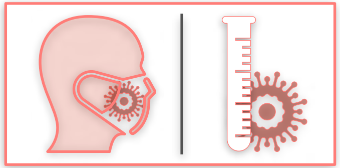
The probability that **you are sick GIVEN that you tested sick**

Bayes Theorem: Formula



The probability that **you are sick GIVEN that you tested sick**

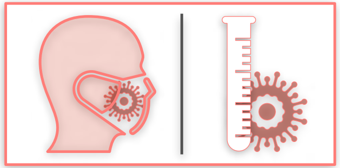
Bayes Theorem: Formula



$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

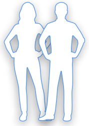
The probability that **you are sick GIVEN that you tested sick**

Bayes Theorem: Formula



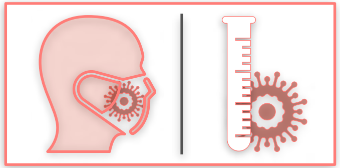
$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick GIVEN that you tested sick**



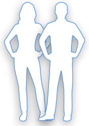
1,000,000

Bayes Theorem: Formula

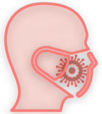


$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

The probability that **you are sick GIVEN that you tested sick**

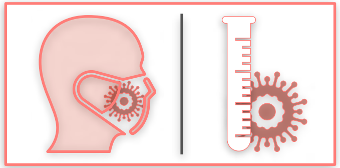


1,000,000



1 / 10,000

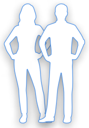
Bayes Theorem: Formula



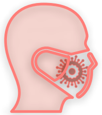
$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

The probability that **you are sick GIVEN that you tested sick**

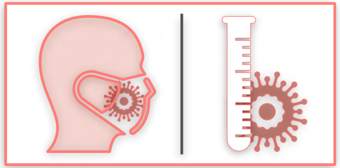


1,000,000



1 / 10,000

Bayes Theorem: Formula

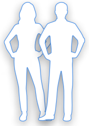


$\mathrm{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathrm{P}(\text{sick}) = 0.01\%$

$\mathrm{P}(\text{not sick}) = 99.99\%$

The probability that **you are sick GIVEN that you tested sick**



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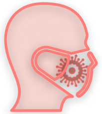
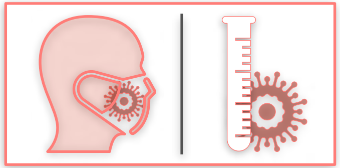


1 / 10,000

Bayes Theorem: Formula

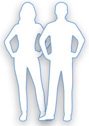


$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

The probability that **you are sick GIVEN that you tested sick**



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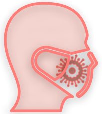
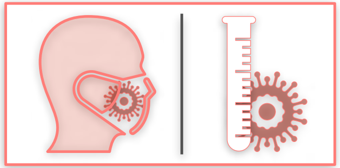


1 / 10,000

99% Effective

Bayes Theorem: Formula



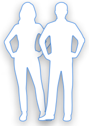
$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

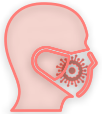
$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

The probability that **you are sick GIVEN that you tested sick**



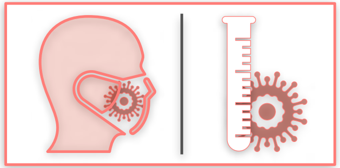
1,000,000



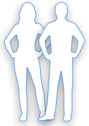
1 / 10,000

99% Effective

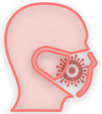
Bayes Theorem: Formula



The probability that **you are sick GIVEN that you tested sick**



1,000,000



1 / 10,000

99% Effective

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

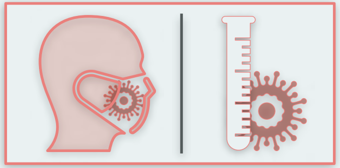
$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

Bayes Theorem: Formula



$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

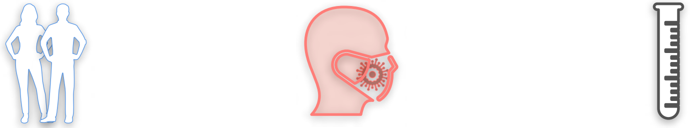
$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

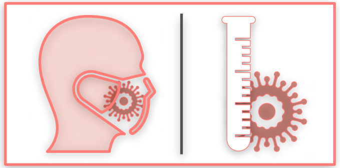
$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

The probability that **you are sick GIVEN that you tested sick**



1,000,000 | 1 / 10,000 | 99% Effective

Bayes Theorem: Formula



$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

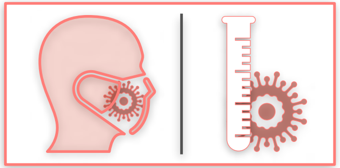
$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

DeepLearning.AI

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

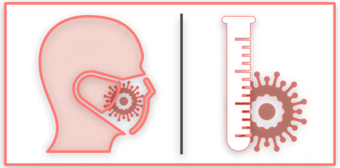
$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(A \mid B) = ?$

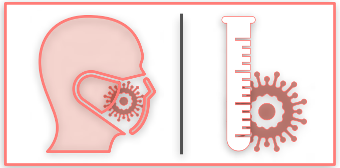
$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(A \mid B) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

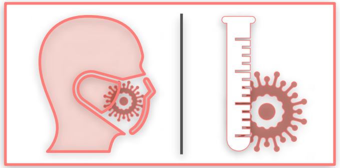
$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

From Conditional Probability

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(A \mid B) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

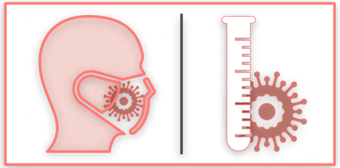
$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

From Conditional Probability

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

Bayes Theorem: Formula



$\mathbf{P}(\text{sick} \mid \text{diagnosed sick}) = ?$

$\mathbf{P}(\text{sick}) = 0.01\%$

$\mathbf{P}(\text{not sick}) = 99.99\%$

$\mathbf{P}(\text{diagnosed sick} \mid \text{sick}) = 99\%$

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A: sick

B: diagnosed sick

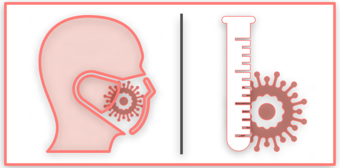
$\mathbf{P}(A \mid B) = ?$

From Conditional Probability

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A \mid B) = \underline{\hspace{8cm}}$$

Bayes Theorem: Formula



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$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(\text{sick and diagnosed sick})}{}$$

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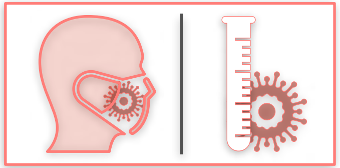

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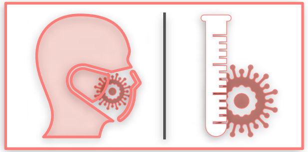
Bayes Theorem: Formula



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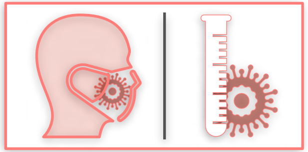

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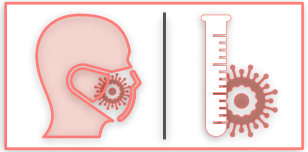

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$\mathbf{P}(\text{sick and diagnosed sick}) = ?$

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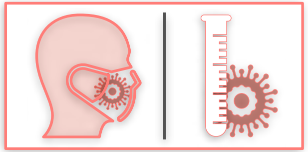

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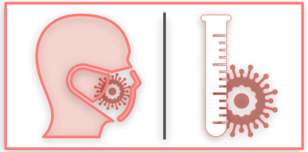

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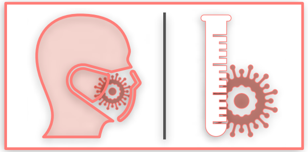
$\mathbf{P}(\text{diagnosed sick}) = ?$

BAYES THEOREM FORMULA CAN HELP

Bayes Theorem: Formula



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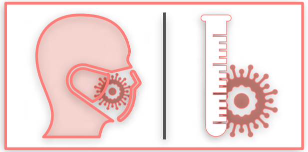

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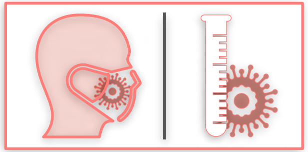
From Conditional Probability

Bayes Theorem: Formula



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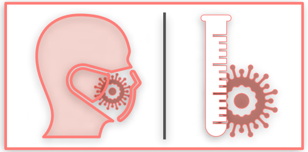
From Conditional Probability

$$\mathbf{P}(\mathrm{A} \cap \mathrm{B}) = \mathbf{P}(\mathrm{A}) \cdot \mathbf{P}(\mathrm{B} \mid \mathrm{A})$$

Bayes Theorem: Formula



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From Conditional Probability

$$\mathbf{P}(\mathrm{A} \cap \mathrm{B}) = \mathbf{P}(\mathrm{A}) \cdot \mathbf{P}(\mathrm{B} \mid \mathrm{A})$$

$$\mathbf{P}(\text{sick and diagnosed sick}) =$$

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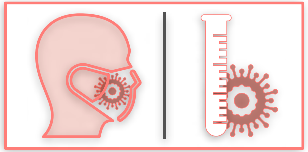

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$$\mathbf{P}(\text{sick and diagnosed sick}) = \quad \mathbf{P}(\text{sick})$$

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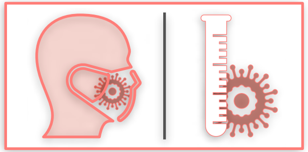

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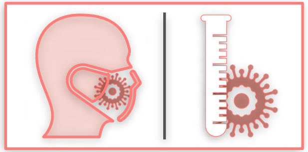
Bayes Theorem: Formula



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$$\mathbf{P}(\mathrm{A} \cap \mathrm{B}) = \mathbf{P}(\mathrm{A}) \cdot \mathbf{P}(\mathrm{B} \mid \mathrm{A})$$

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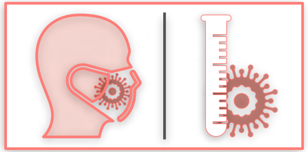

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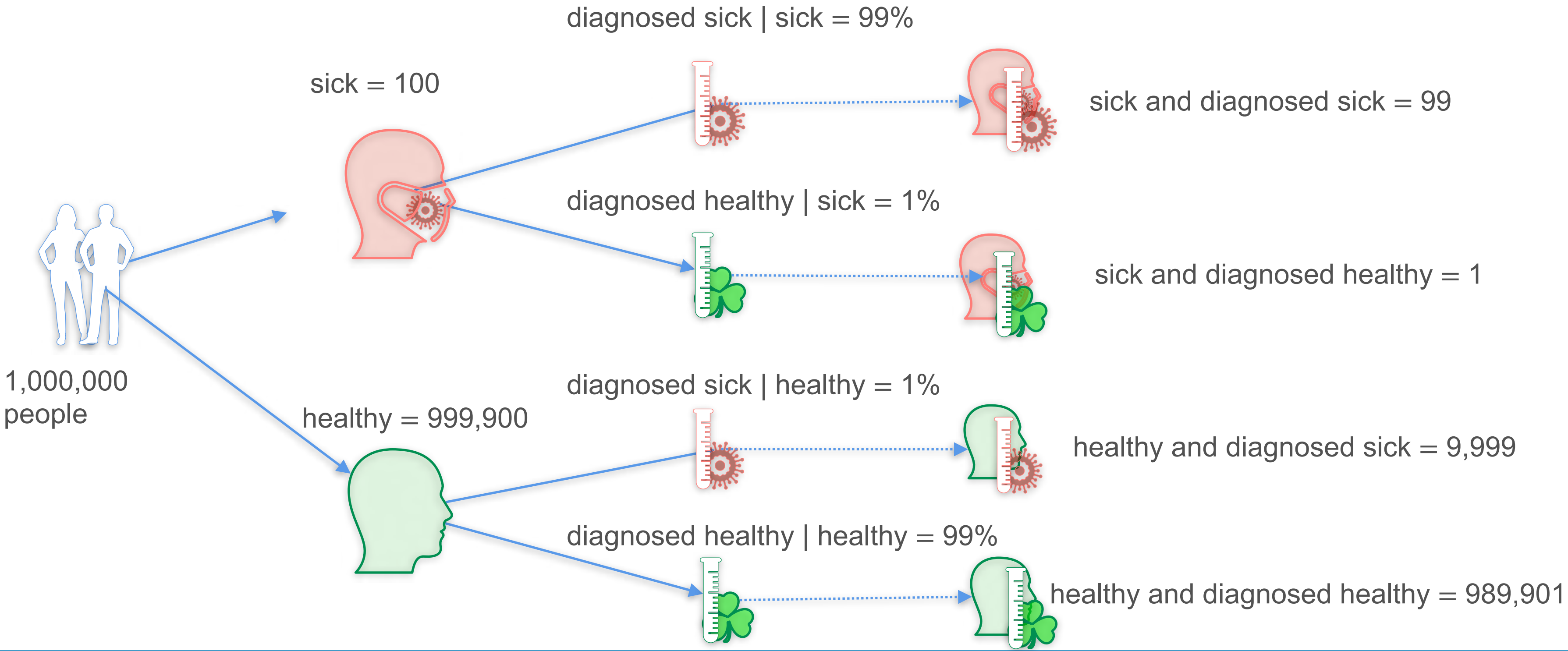

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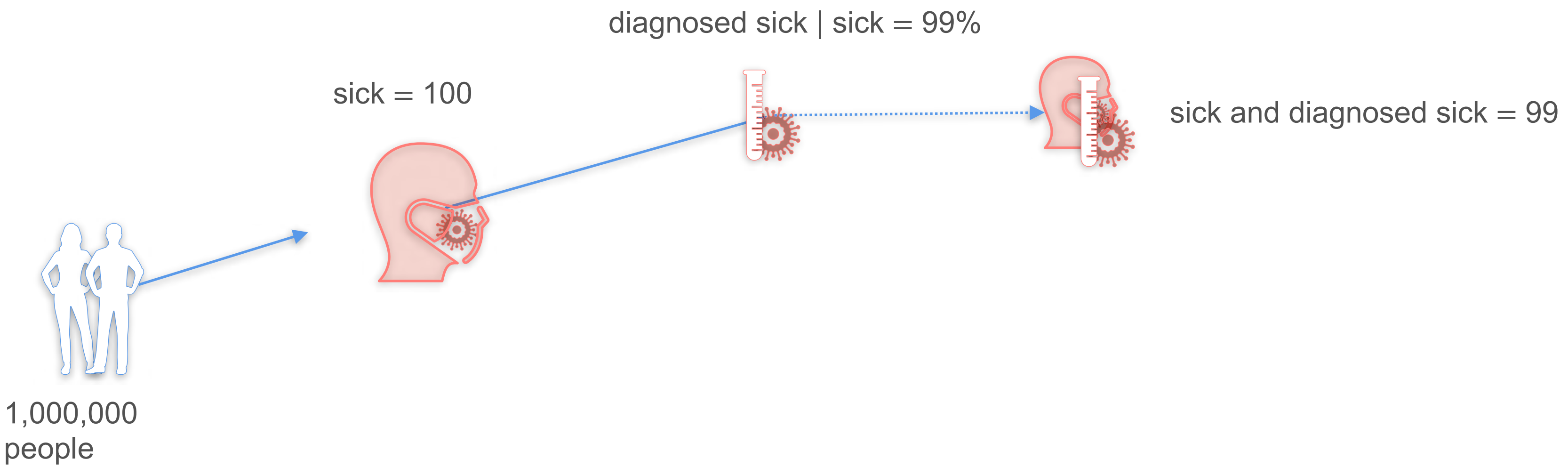

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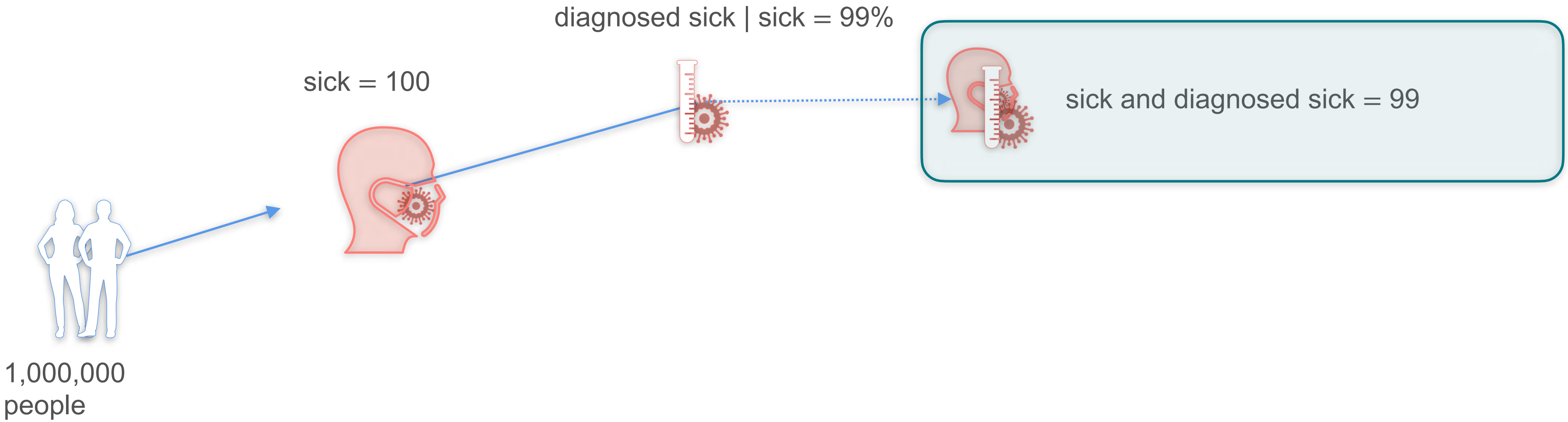

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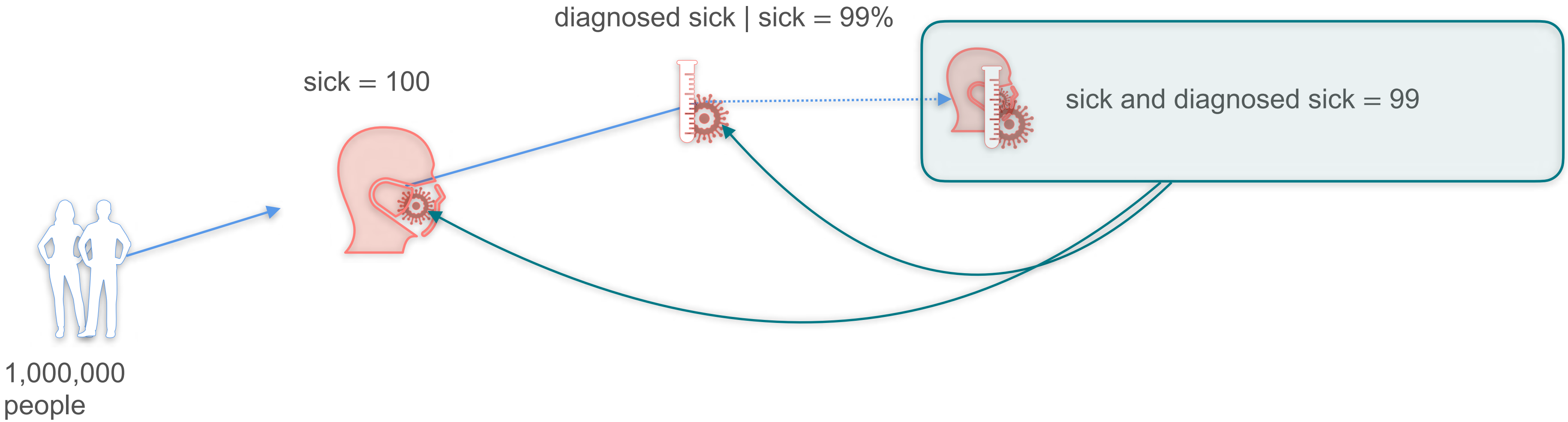

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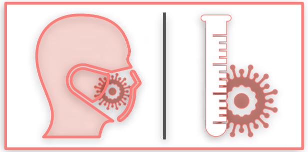

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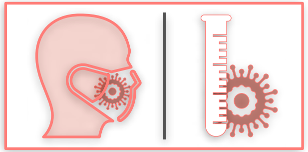

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Bayes Theorem: Formula

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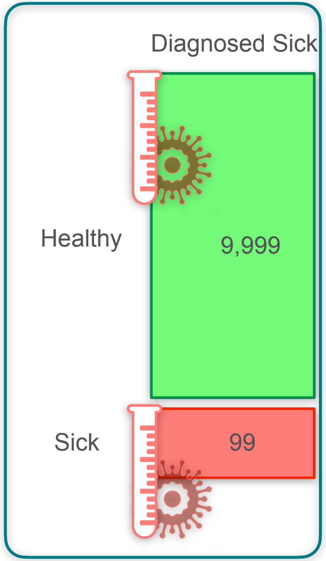
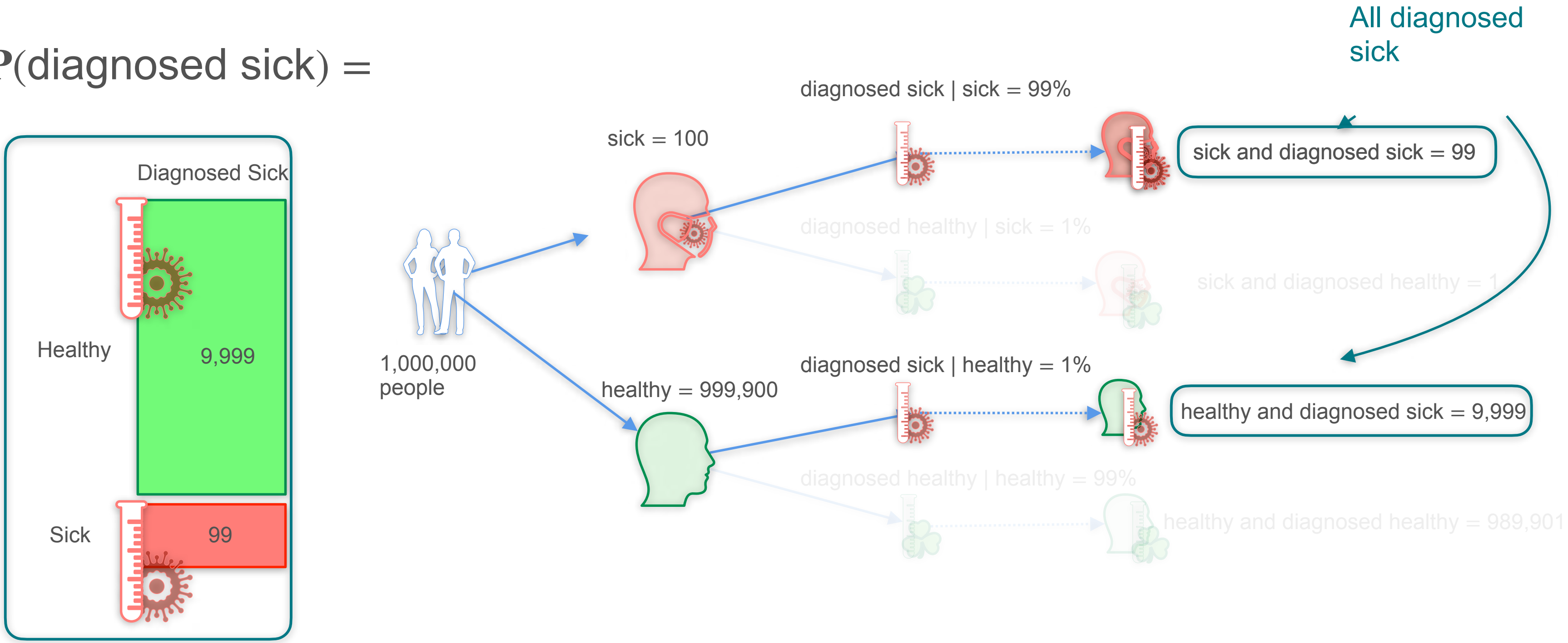


Bayes Theorem: Formula

$\mathbf{P}(\text{diagnosed sick}) =$



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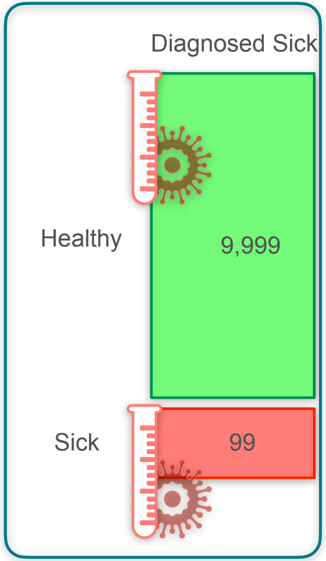
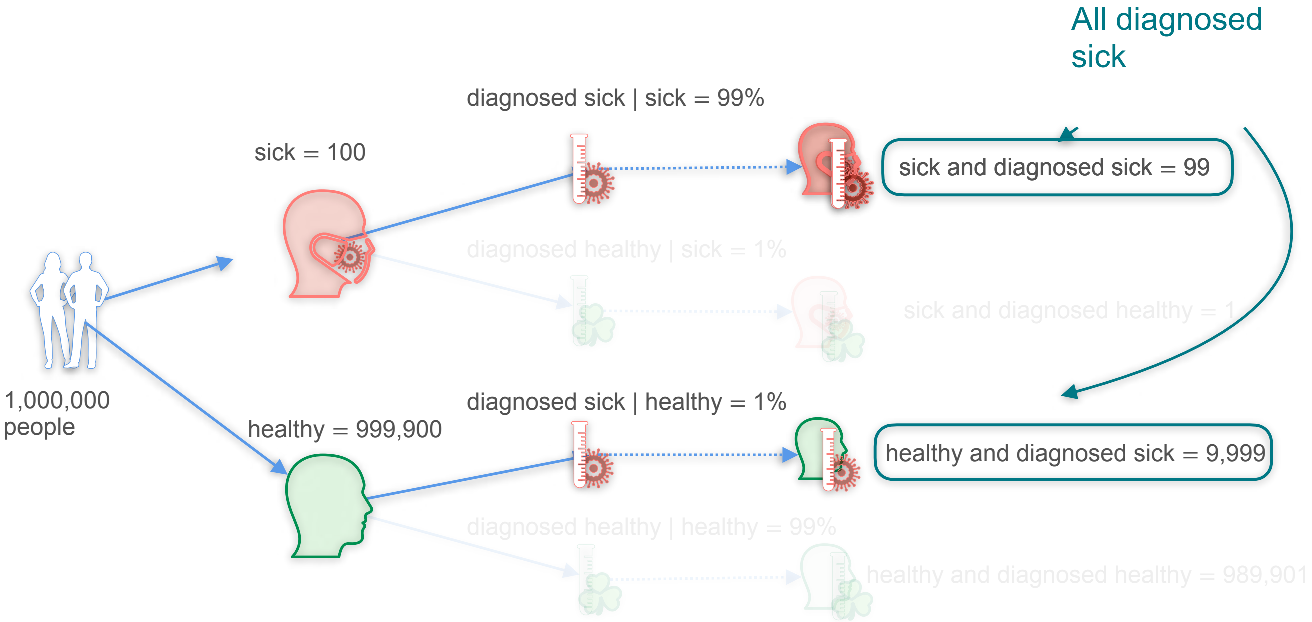
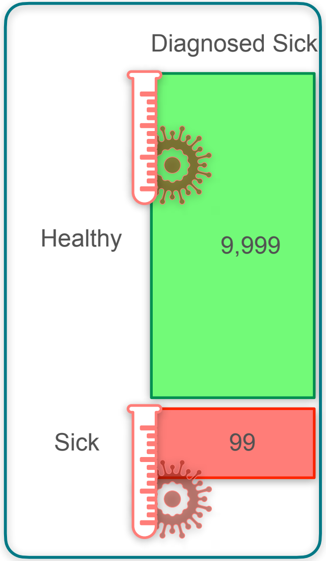


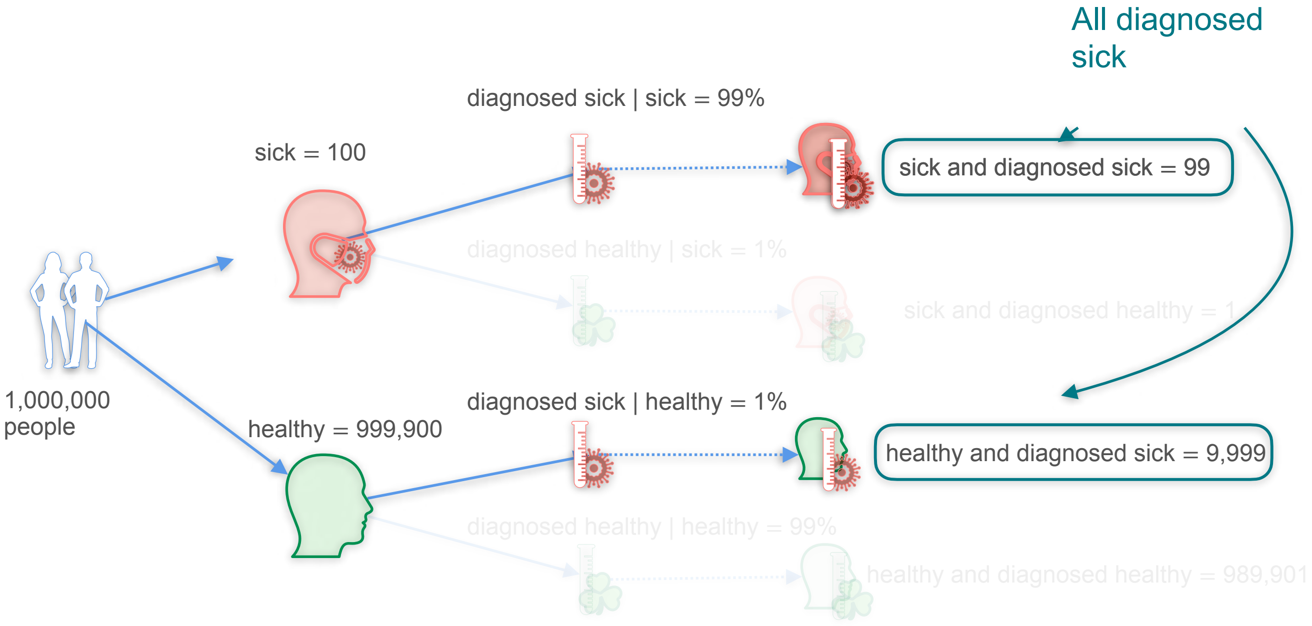


$\mathbf{P}(\text{sick} \cap \text{diagnosed sick})$

Bayes Theorem: Formula

$\mathbf{P}(\text{diagnosed sick}) =$

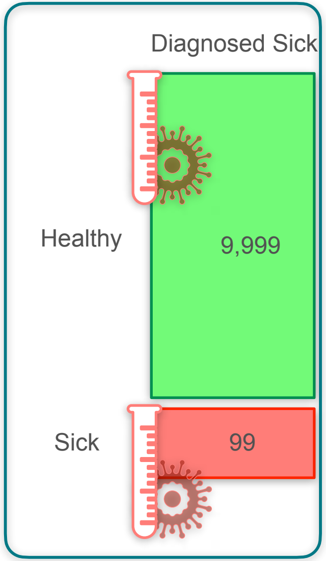


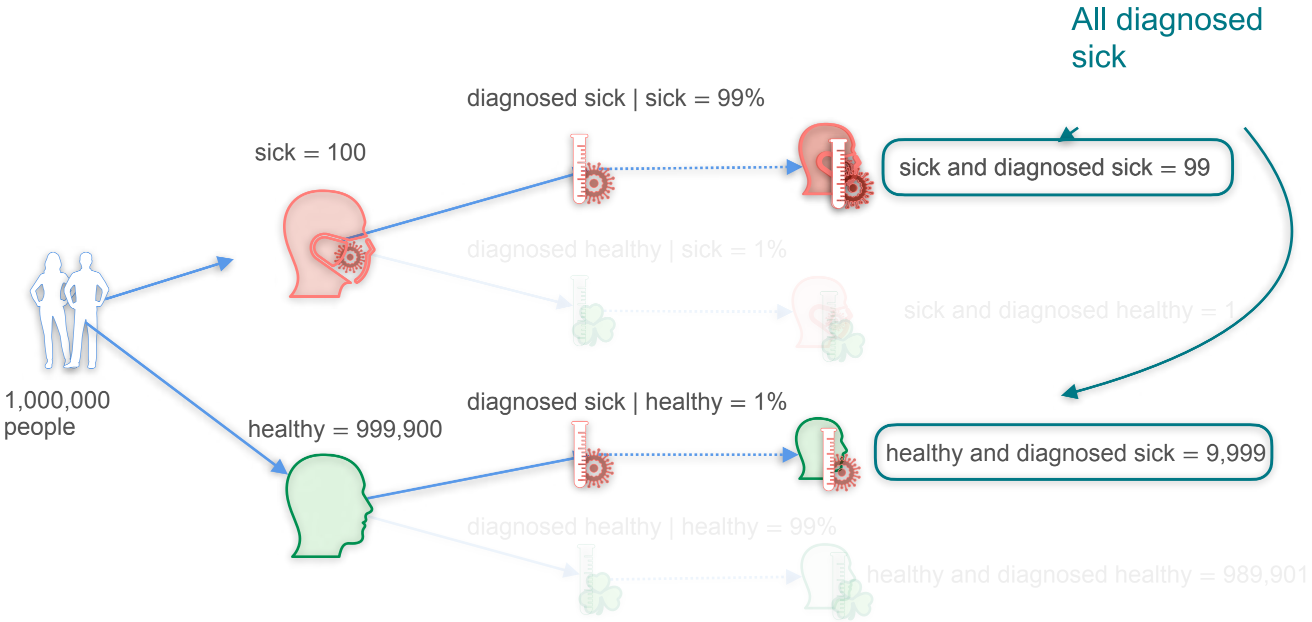


$\mathbf{P}(\text{sick} \cap \text{diagnosed sick}) +$

Bayes Theorem: Formula

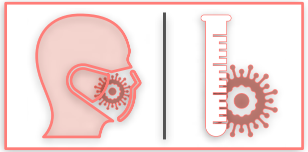
$\mathbf{P}(\text{diagnosed sick}) =$





$\mathbf{P}(\text{sick} \cap \text{diagnosed sick}) + \mathbf{P}(\text{not sick} \cap \text{diagnosed sick})$

Bayes Theorem: Formula



A: sick

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$$\mathbf{P}(\text{diagnosed sick}) = \mathbf{P}(\text{sick} \cap \text{diagnosed sick}) + \mathbf{P}(\text{not sick} \cap \text{diagnosed sick})$$

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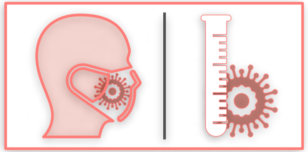

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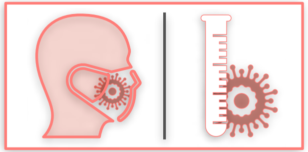
Bayes Theorem: Formula



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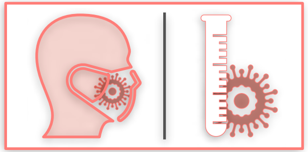
Bayes Theorem: Formula



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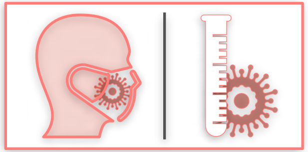

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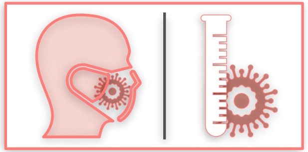

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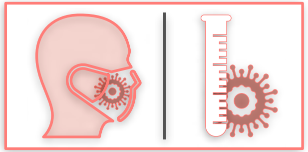

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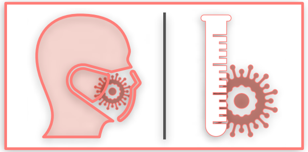

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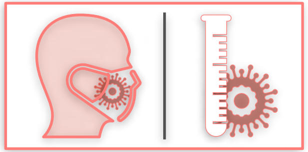

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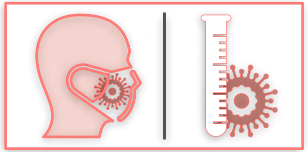

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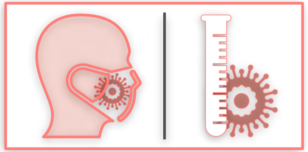

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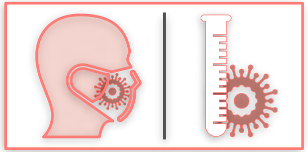

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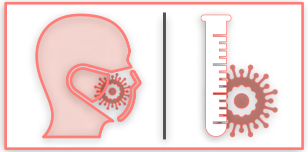

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BAYES THEOREM FORMULA

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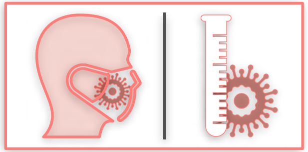

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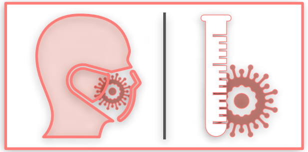

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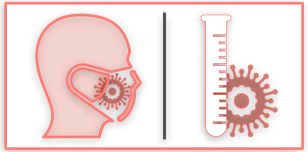

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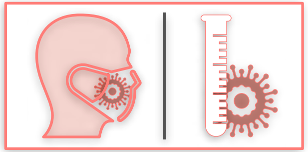

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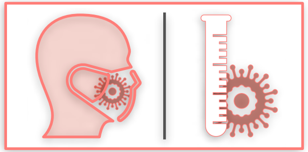

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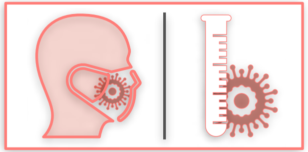

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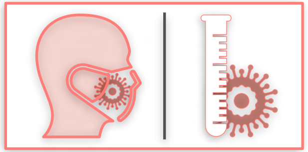

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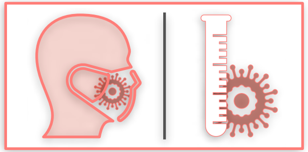

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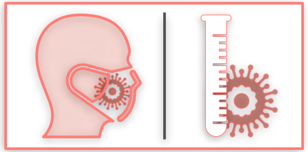

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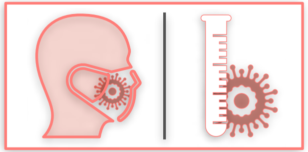

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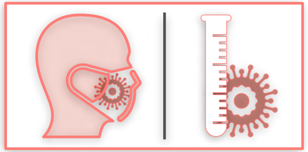

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BAYES THEOREM
FORMULA

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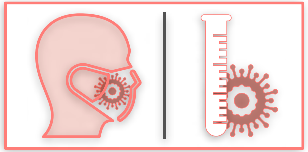

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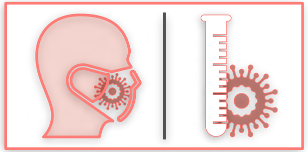

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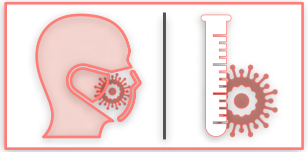

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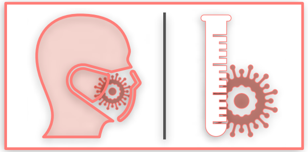

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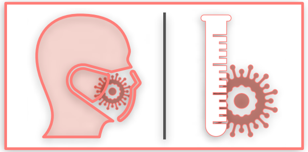
$\mathbf{P}(B \mid A) = 99\%$

$\mathbf{P}(B \mid A') = 1\%$

BAYES THEOREM FORMULA

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \bullet \mathbf{P}(B \mid A)}{\mathbf{P}(\text{sick}) \bullet \mathbf{P}(\text{diagnosed sick} \mid \text{sick}) + \mathbf{P}(\text{not sick}) \bullet \mathbf{P}(\text{diagnosed sick} \mid \text{not sick})}$$

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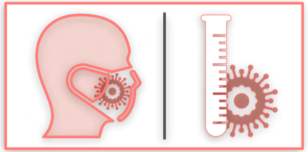

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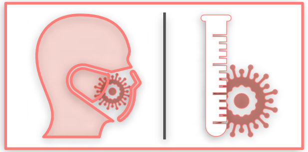

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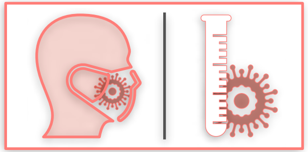

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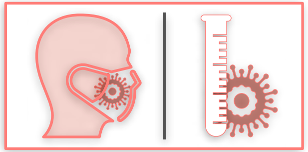

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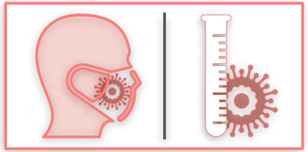

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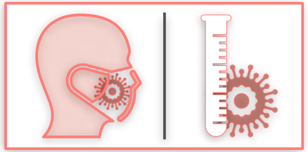

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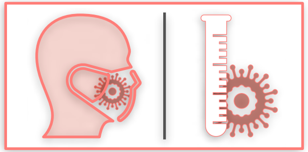

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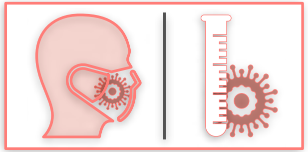
Bayes Theorem: Formula



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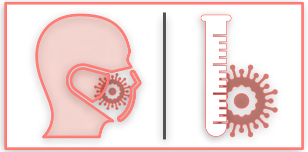
$\mathbf{P}(B \mid A)$ = 99%

$\mathbf{P}(B \mid A')$ = 1%

$$\mathbf{P}(A \mid B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

BAYES THEOREM FORMULA

Bayes Theorem: Formula



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$\mathbf{P}(B \mid A)$ = 99%

$\mathbf{P}(B \mid A')$ = 1%

$$\mathbf{P}(A \mid B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

$$\mathbf{P}(A \mid B) = 0.0098$$

BAYES THEOREM FORMULA

Bayes Theorem: Spam Example

Bayes Theorem: Spam Example

Bayes Theorem: Spam Example

Bayes Theorem: Spam Example

20 spam

DeepLearning.AI

Bayes Theorem: Spam Example

20 spam 80 not spam (ham)

DeepLearning.AI

Bayes Theorem: Spam Example

20 spam

80 not spam (ham)

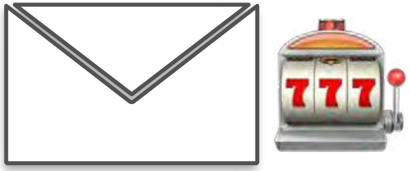
“lottery”

Bayes Theorem: Spam Example

20 spam

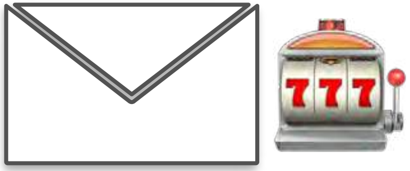
80 not spam (ham)

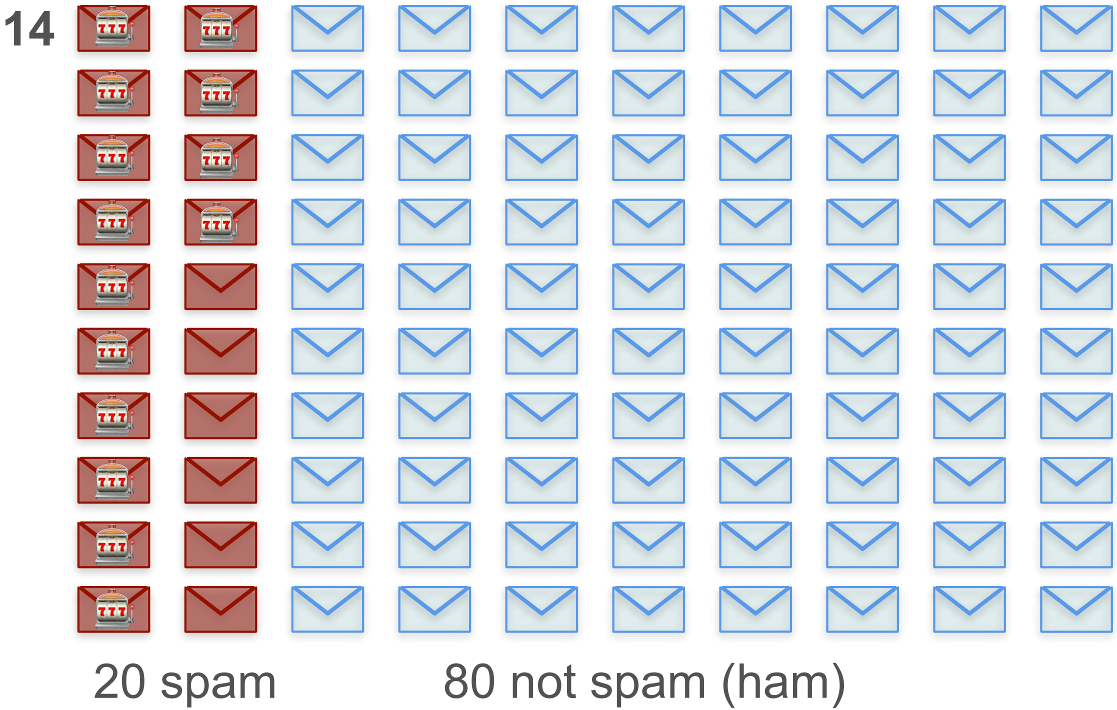
Bayes Theorem: Spam Example



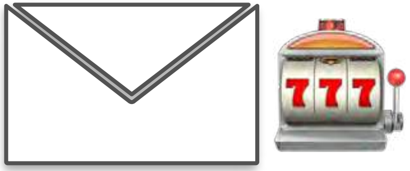
20 spam 80 not spam (ham)

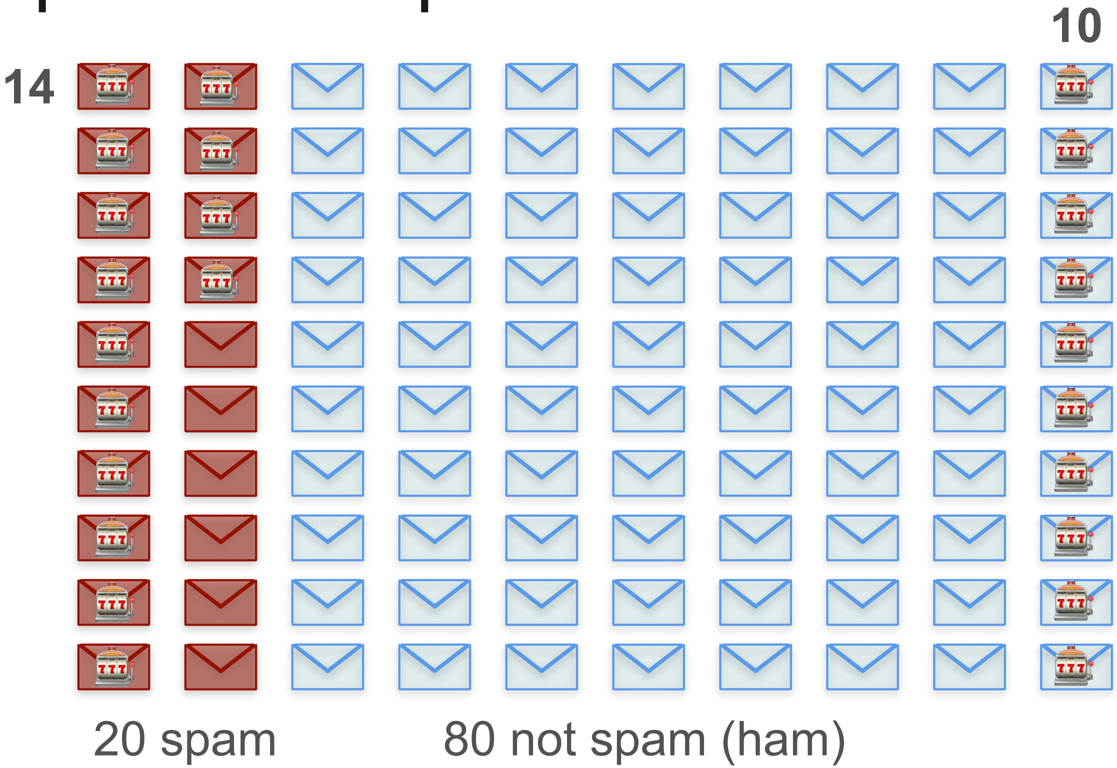
Bayes Theorem: Spam Example



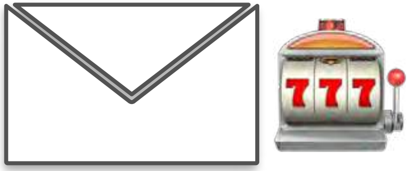


Bayes Theorem: Spam Example

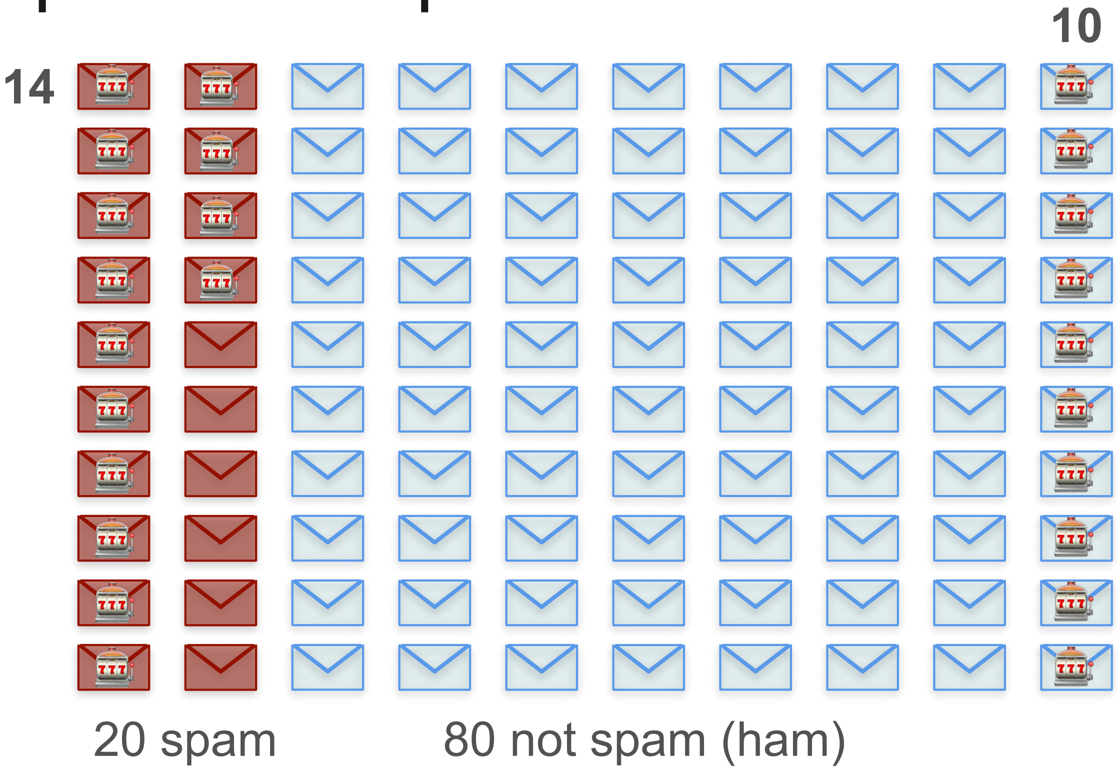




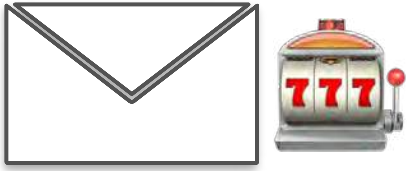
Bayes Theorem: Spam Example



What is the probability that an email containing lottery is a spam?

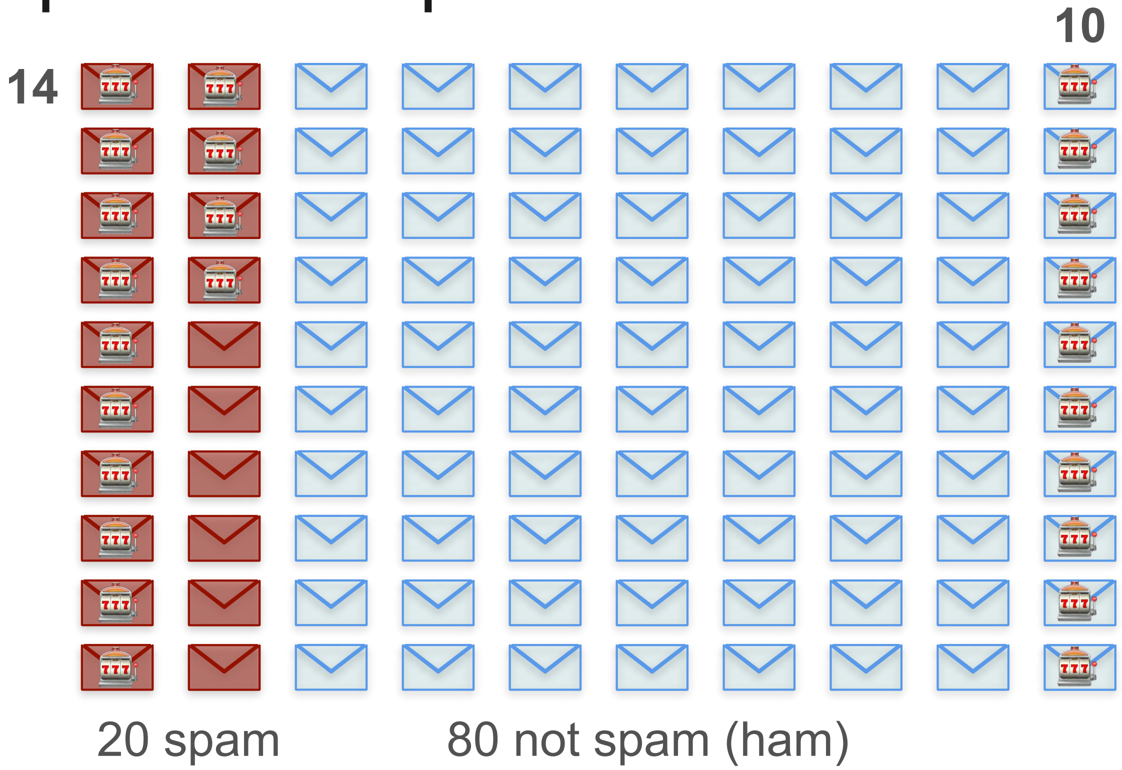


Bayes Theorem: Spam Example

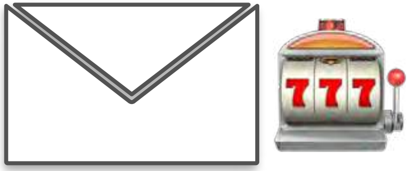


What is the probability that an email containing lottery is a spam?

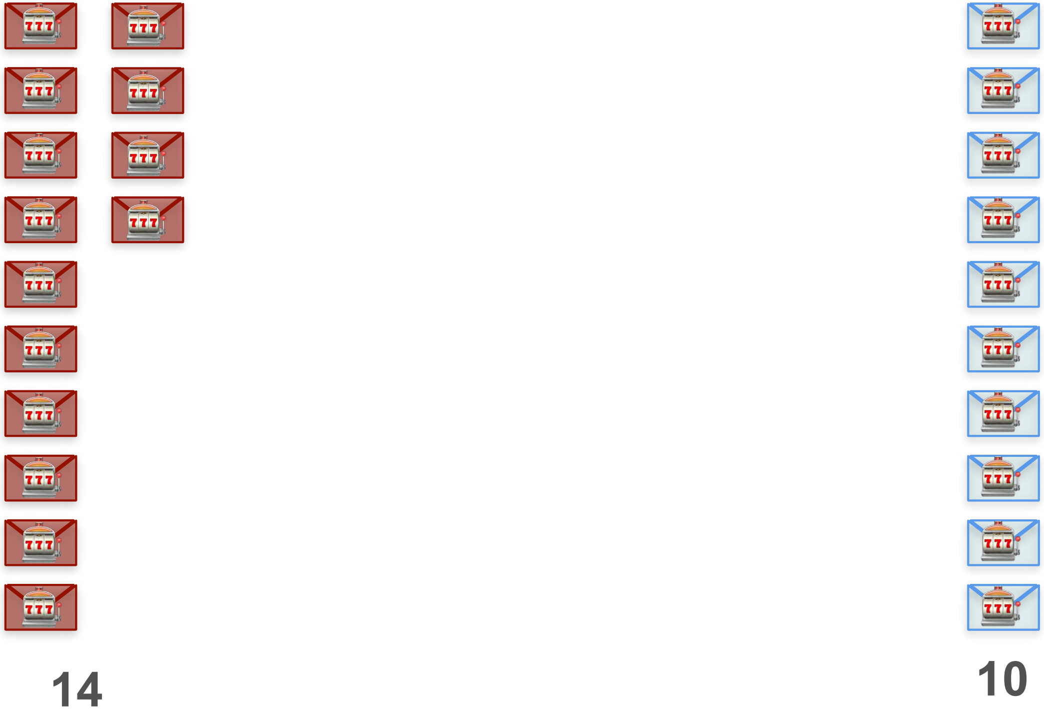
$\mathbf{P}(\text{spam} \mid \text{lottery})$



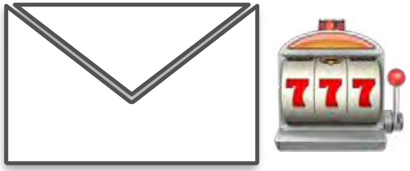
Bayes Theorem: Spam Example (Intuition Solution)



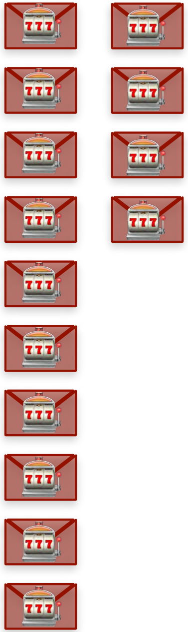
$\mathbf{P}(\text{spam} \mid \text{lottery})$



Bayes Theorem: Spam Example (Intuition Solution)



$\mathbf{P}$(spam | lottery)



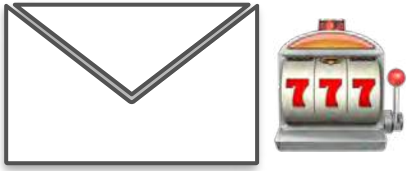
14

24 emails

containing lottery

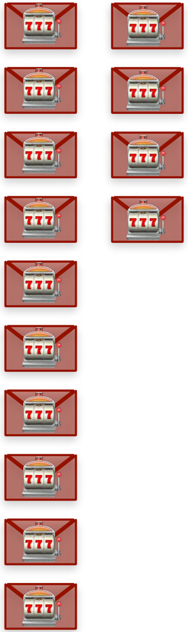
10

Bayes Theorem: Spam Example (Intuition Solution)



$\mathbf{P}(\text{spam} \mid \text{lottery})$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$



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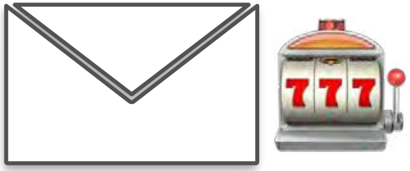
Bayes Theorem: Spam Example (Intuition Solution)



P(spam | lottery)

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$$

14

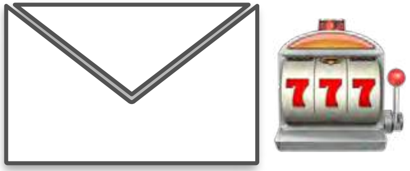
24 emails

containing lottery

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

10

Bayes Theorem: Spam Example (Intuition Solution)



$\mathbf{P}(\text{spam} \mid \text{lottery})$

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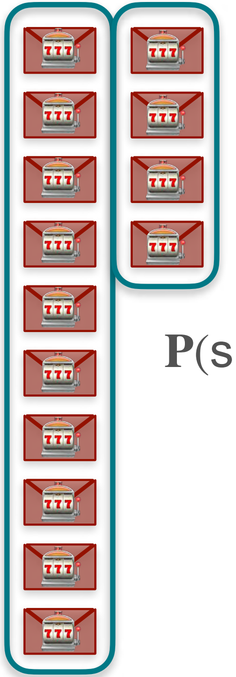

14

24 emails

containing lottery

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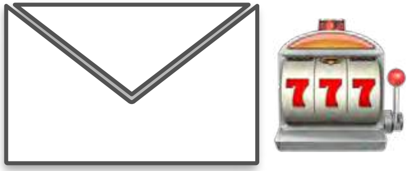
10

Bayes Theorem: Spam Example (Intuition Solution)



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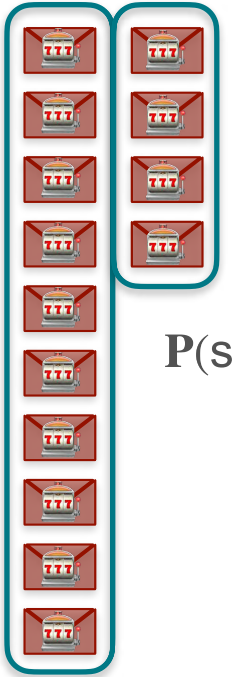


14

24 emails

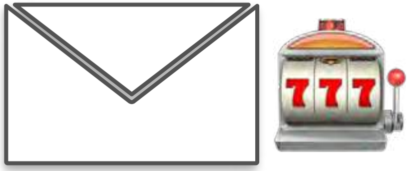
containing lottery

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

10

Bayes Theorem: Spam Example (Intuition Solution)



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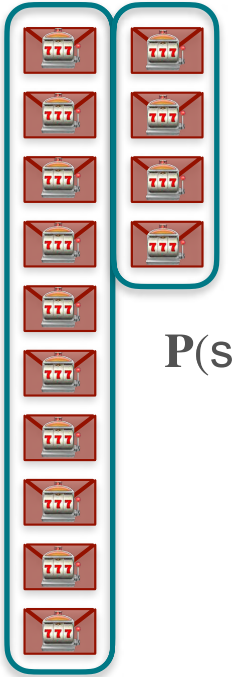


14

24 emails

containing lottery

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

$$= \frac{7}{12} = 0.583$$

10

Bayes Theorem: Spam Example (Formula Solution)

Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam} \mid \text{lottery})$$

Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam} \mid \text{lottery})$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A)}{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A) + \mathbf{P}(A') \cdot \mathbf{P}(B \mid A')}$$

Bayes Theorem: Spam Example (Formula Solution)

$\mathbf{P}(\text{spam} \mid \text{lottery})$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A)}{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A) + \mathbf{P}(A') \cdot \mathbf{P}(B \mid A')}$$

A: Email is spam

Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam} \mid \text{lottery})$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A)}{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A) + \mathbf{P}(A') \cdot \mathbf{P}(B \mid A')}$$

A: Email is spam

B: Email contains lottery

Bayes Theorem: Spam Example (Formula Solution)

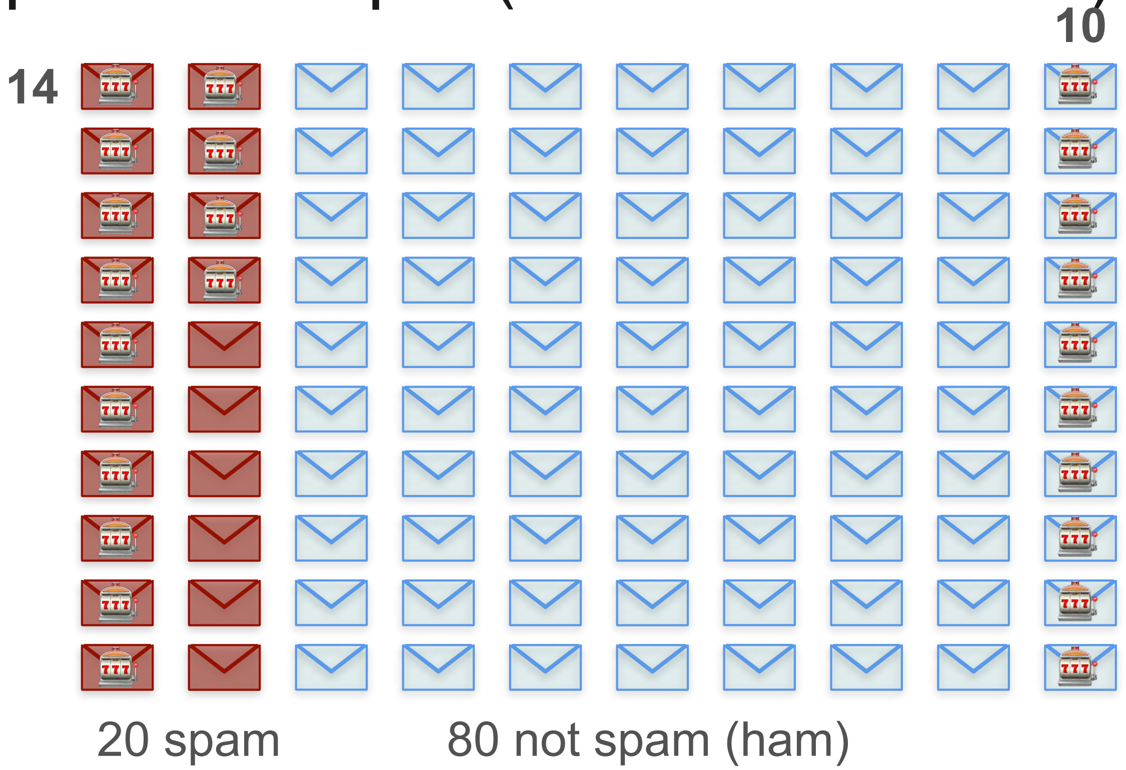
$$\mathbf{P}(\text{spam} \mid \text{lottery})$$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A)}{\mathbf{P}(A) \cdot \mathbf{P}(B \mid A) + \mathbf{P}(A') \cdot \mathbf{P}(B \mid A')}$$

A: Email is spam $\quad$ B: Email contains lottery

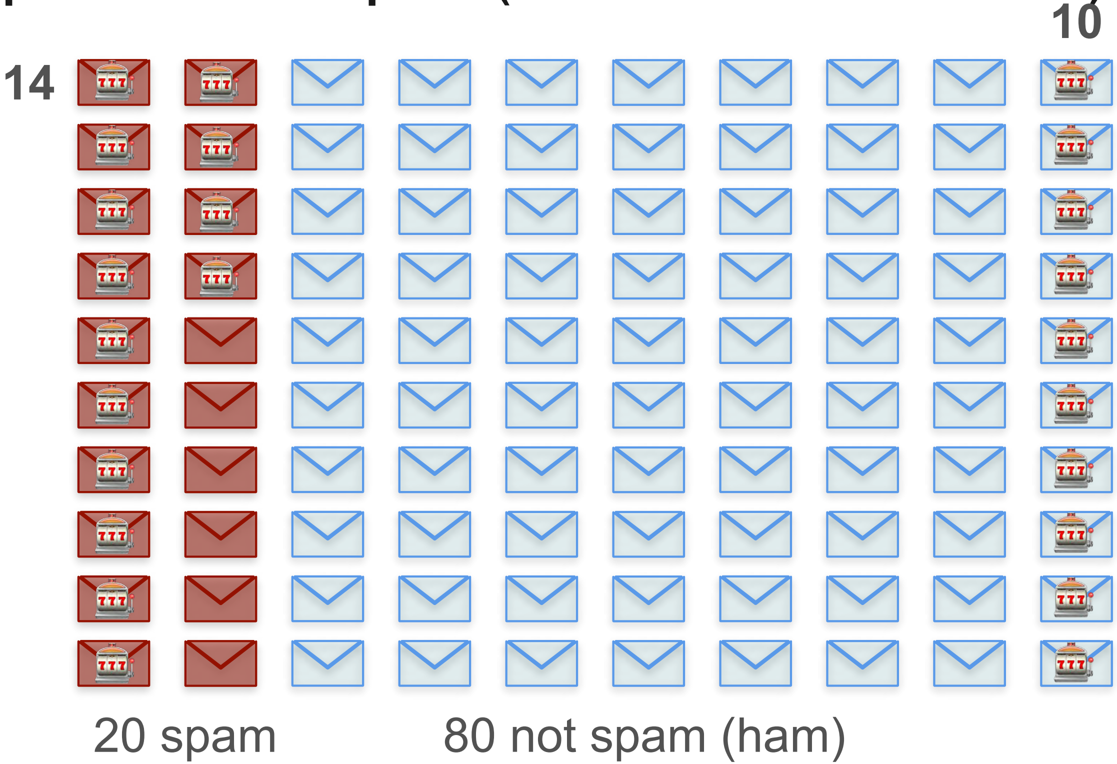
$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

Bayes Theorem: Spam Example (Formula Solution)



Bayes Theorem: Spam Example (Formula Solution)

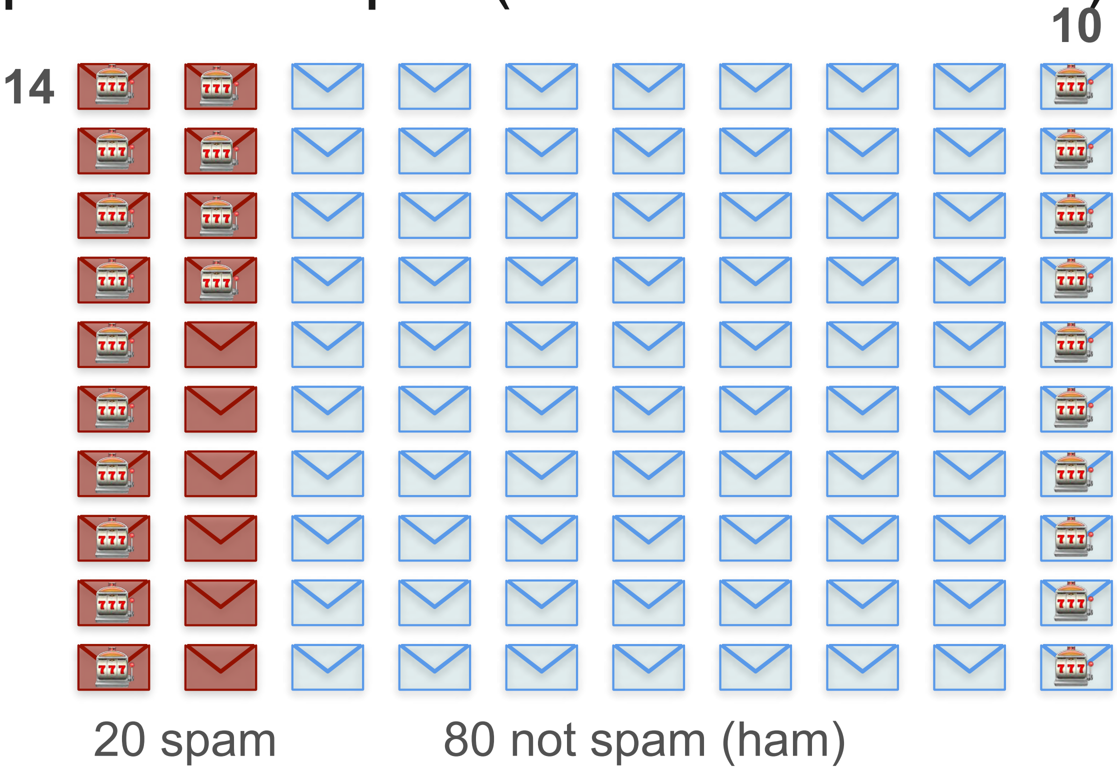
$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$



Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{not spam}) = \frac{80}{100} = 0.8$$

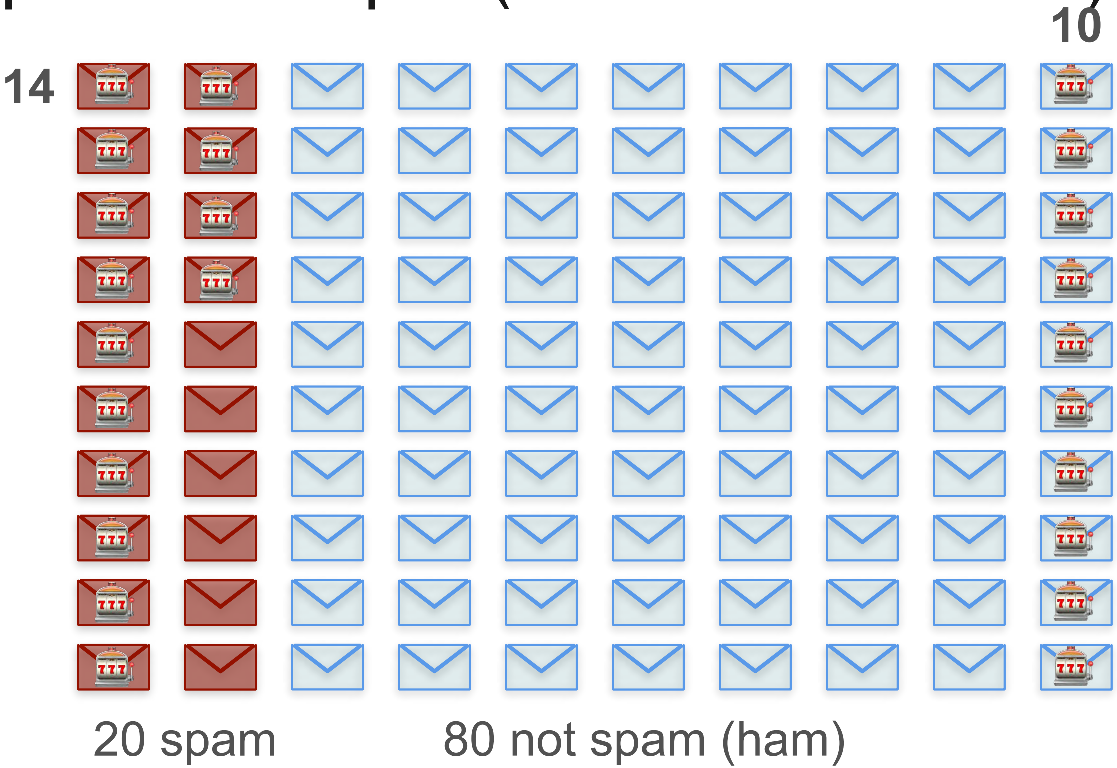


Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{not spam}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



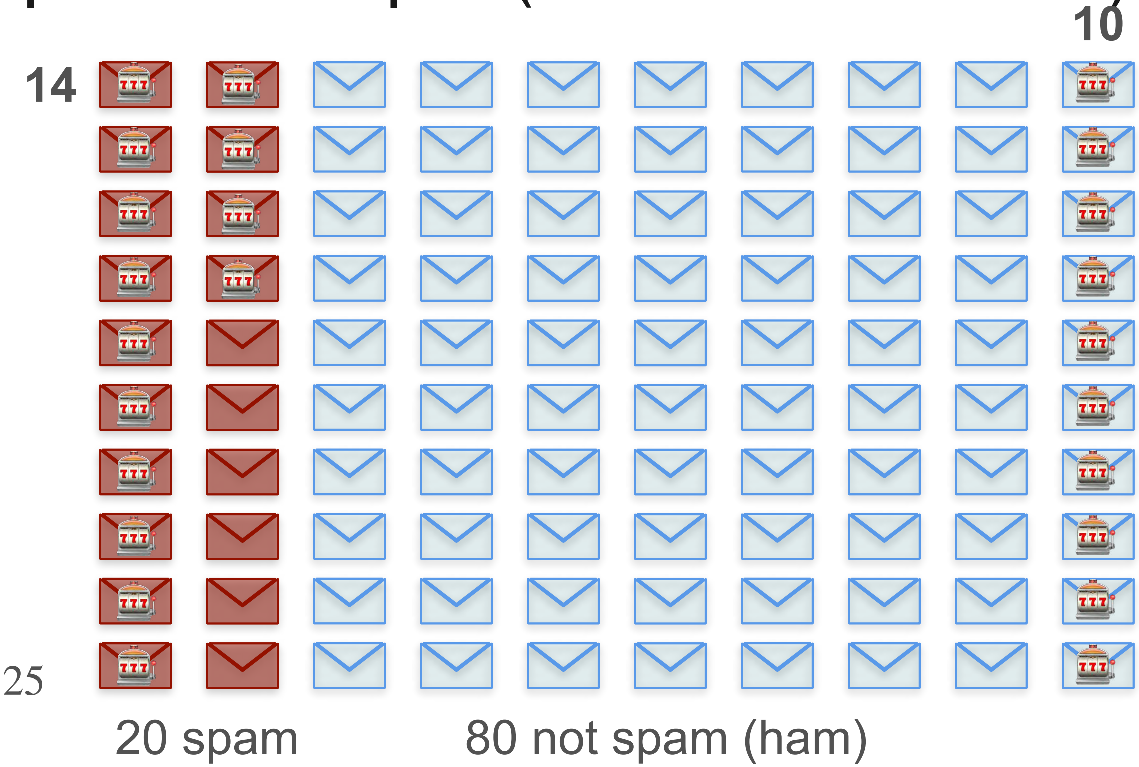
Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{not spam}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$

$$\mathbf{P}(\text{lottery} \mid \text{not spam}) = \frac{10}{80} = 0.125$$



Bayes Theorem: Spam Example (Formula Solution)

Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam}) = 0.2$$

Bayes Theorem: Spam Example (Formula Solution)

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

Bayes Theorem: Spam Example (Formula Solution)

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{not spam}) = 0.8$

Bayes Theorem: Spam Example (Formula Solution)

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{not spam}) = 0.125$

Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam}) = 0.2 \qquad \mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$$

$$\mathbf{P}(\text{not spam}) = 0.8 \qquad \mathbf{P}(\text{lottery} \mid \text{not spam}) = 0.125$$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

Bayes Theorem: Spam Example (Formula Solution)

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{not spam}) = 0.125$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)}$$

Bayes Theorem: Spam Example (Formula Solution)

$$\mathbf{P}(\text{spam}) = 0.2 \qquad \mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$$

$$\mathbf{P}(\text{not spam}) = 0.8 \qquad \mathbf{P}(\text{lottery} \mid \text{not spam}) = 0.125$$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) + \mathbf{P}(\text{not spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{not spam})}$$

$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)} = 0.583$$

Bayes Theorem

Bayes Theorem

PRIOR

Bayes Theorem

PRIOR

EVENT

Bayes Theorem

PRIOR

EVENT

POSTERIOR

Bayes Theorem

PRIOR

EVENT

POSTERIOR

$$\mathbf{P}(A)$$

Bayes Theorem

PRIOR

EVENT

POSTERIOR

$\mathbf{P}(A)$

E

Bayes Theorem

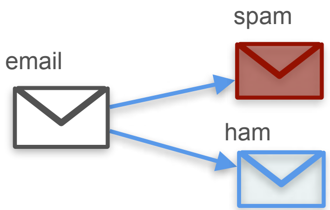
PRIOR	EVENT	POSTERIOR
$\mathbf{P}(A)$	E	$\mathbf{P}(A \mid E)$

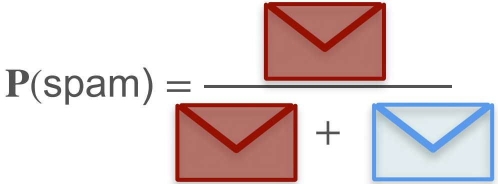
Prior and Posterior

PRIOR **EVENT** **POSTERIOR**

Prior and Posterior

PRIOR **EVENT** **POSTERIOR**



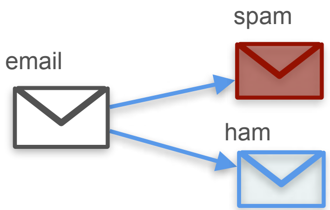


Prior and Posterior

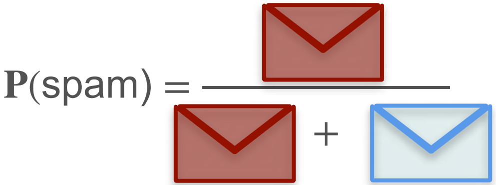
PRIOR

EVENT

POSTERIOR

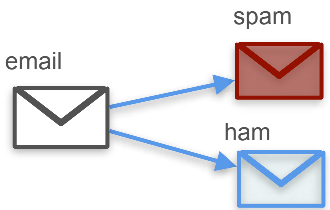


Email contains lottery

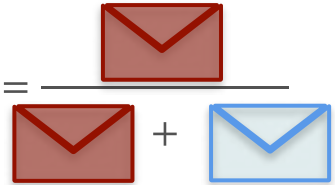


Prior and Posterior

PRIOR



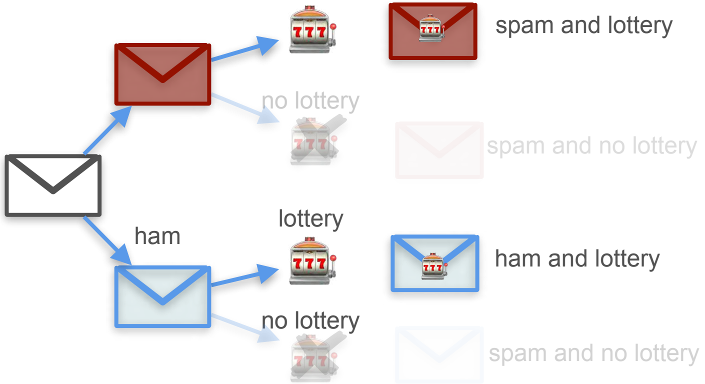
$P(\text{spam}) =$



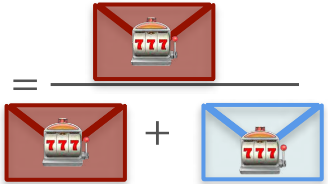
EVENT

Email contains lottery

POSTERIOR



$P(\text{spam} \mid \text{lottery}) =$



Prior and Posterior

PRIOR **EVENT** **POSTERIOR**

Prior and Posterior

PRIOR **EVENT** **POSTERIOR**

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Prior and Posterior

PRIOR | **EVENT** | **POSTERIOR**

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	$\boxed{4,6}$
5,1	5,2	5,3	5,4	$\boxed{5,5}$	5,6
6,1	6,2	6,3	$\boxed{6,4}$	6,5	6,6

Prior and Posterior

PRIOR **EVENT** **POSTERIOR**

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10) = \frac{3}{36}$$

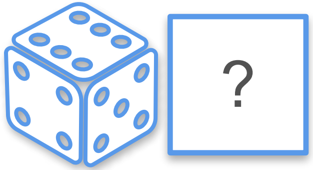
Prior and Posterior

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10) = \frac{3}{36}$$

EVENT



1st dice is 6

POSTERIOR

Prior and Posterior

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10) = \frac{3}{36}$$

EVENT

?

1st dice is 6

POSTERIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

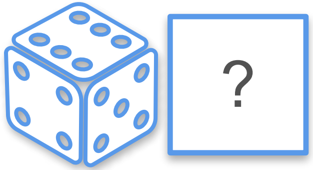
Prior and Posterior

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10) = \frac{3}{36}$$

EVENT



1st dice is 6

POSTERIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Prior and Posterior

PRIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$\mathbf{P}(\text{sum} = 10) = \frac{3}{36}$$

EVENT

?

1st dice is 6

POSTERIOR

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

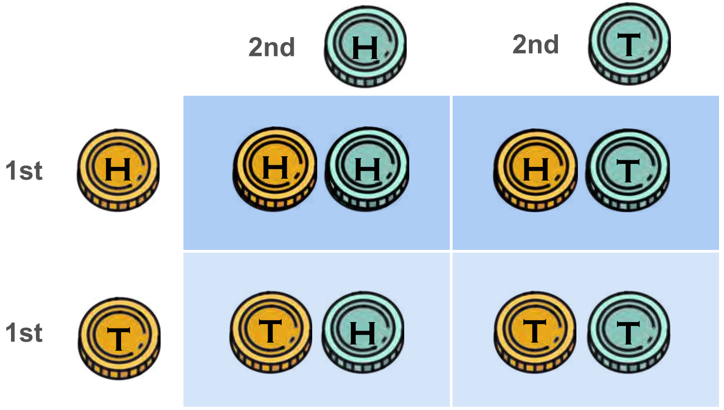
$$\mathbf{P}(\text{sum} = 10 \mid \text{1st is } 6) = \frac{1}{6}$$

Prior and Posterior

PRIOR **EVENT** **POSTERIOR**

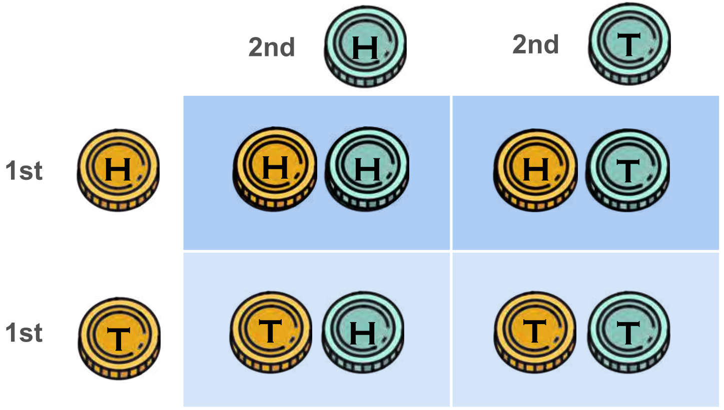
Prior and Posterior

PRIOR **EVENT** **POSTERIOR**



Prior and Posterior

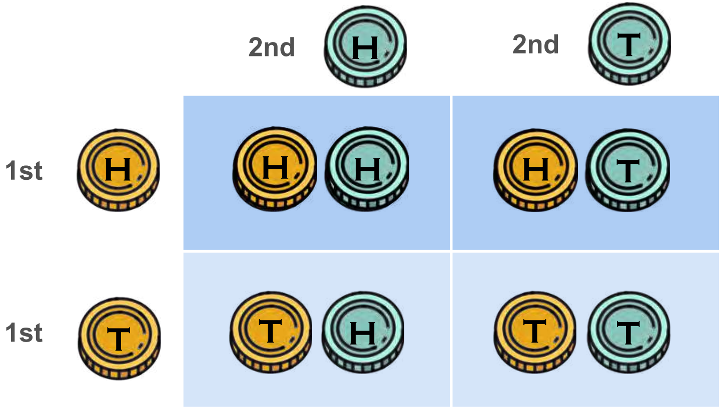
PRIOR **EVENT** **POSTERIOR**



$$\mathbf{P}(HH) = \frac{1}{4}$$

Prior and Posterior

PRIOR | **EVENT** | **POSTERIOR**

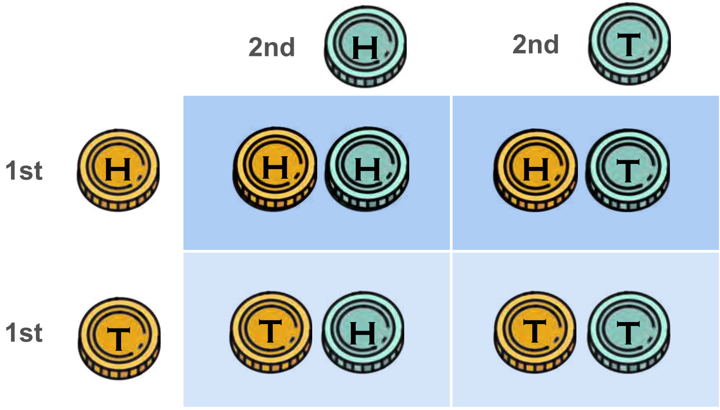


$$\mathbf{P}(HH) = \frac{1}{4}$$

1st coin is H

Prior and Posterior

PRIOR



$$\mathbf{P}(HH) = \frac{1}{4}$$

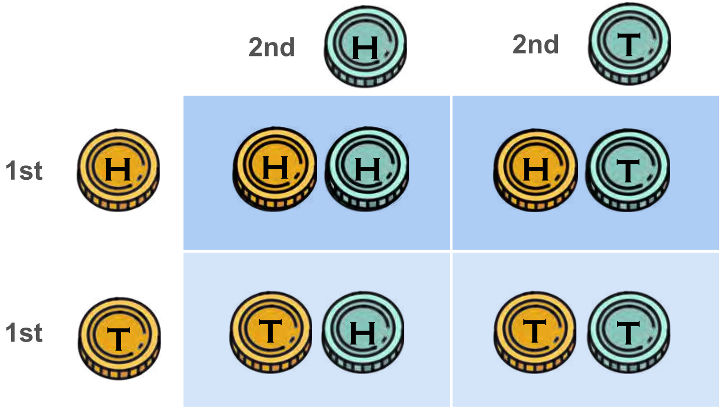
EVENT

1st coin is H

POSTERIOR

Prior and Posterior

PRIOR



$$\mathbf{P}(HH) = \frac{1}{4}$$

EVENT

1st coin is H

POSTERIOR

$$\mathbf{P}(HH \,|\, \text{1st is } H) = \frac{1}{2}$$

Video 8e: the Naive Bayes Model

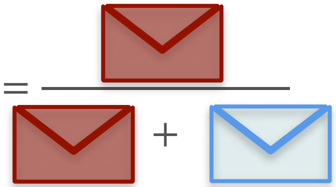
What About 2 Events?

PRIOR	EVENT	POSTERIOR

What About 2 Events?

PRIOR **EVENT** **POSTERIOR**

P(spam) =

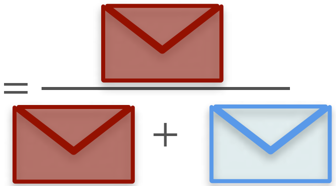


What About 2 Events?

PRIOR

EVENT

POSTERIOR

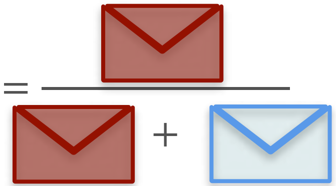
$P(\text{spam}) =$ 

Email contains ‘lottery’

What About 2 Events?

PRIOR

P(spam) =



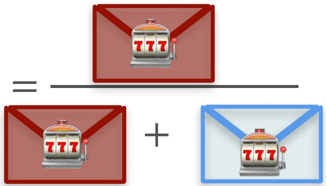
+

EVENT

Email contains ‘lottery’

POSTERIOR

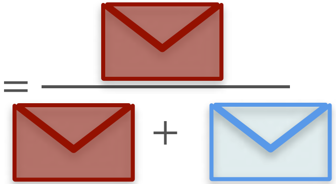
P(spam | lottery) =



+

What About 2 Events?

PRIOR

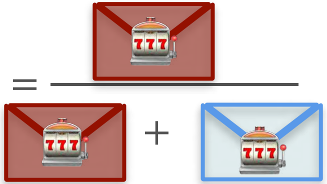
P(spam) = 

EVENT

Email contains ‘lottery’

Email contains ‘winning'

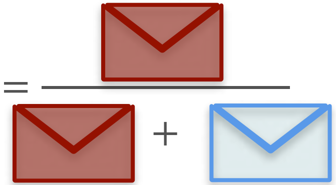
POSTERIOR

P(spam | lottery) = 

What About 2 Events?

PRIOR

P(spam) =



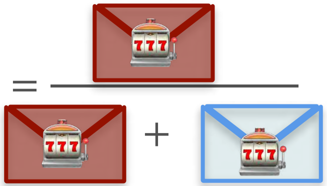
EVENT

Email contains ‘lottery’

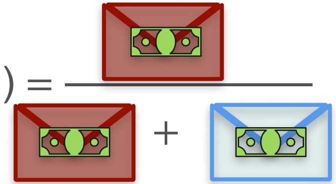
Email contains ‘winning'

POSTERIOR

P(spam | lottery) =

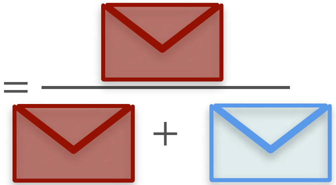


P(spam | winning) =



What About 2 Events?

PRIOR

P(spam) = 

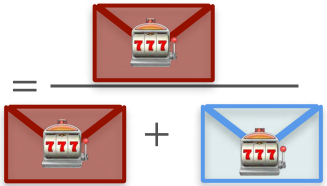
EVENT

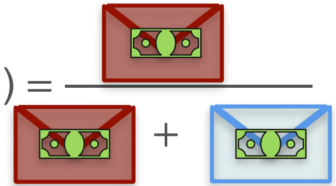
Email contains ‘lottery’

Email contains ‘winning'

Email contains ‘lottery’ and ‘winning’

POSTERIOR

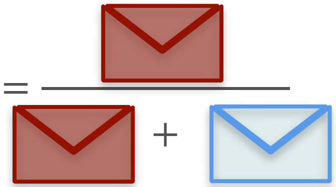
P(spam | lottery) = 

P(spam | winning) = 

What About 2 Events?

PRIOR

$\mathbf{P}(\text{spam}) =$



EVENT

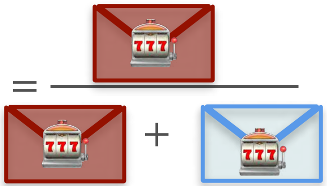
Email contains 'lottery'

Email contains 'winning'

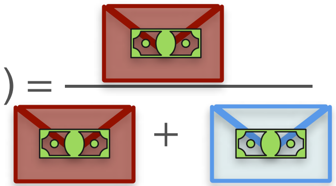
Email contains 'lottery' and 'winning'

POSTERIOR

$\mathbf{P}(\text{spam} \mid \text{lottery}) =$



$\mathbf{P}(\text{spam} \mid \text{winning}) =$



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What About 2 Events?

EVENT

POSTERIOR

🎰 💵

Email contains ‘lottery’ and ‘winning’

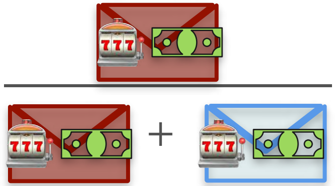
What About 2 Events?

EVENT

Email contains 'lottery' and 'winning'

POSTERIOR

P(spam | lottery & winning) =

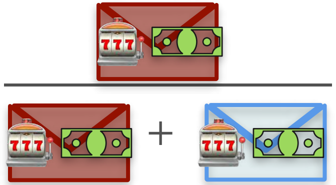


What About 2 Events?

EVENT

Email contains ‘lottery’ and ‘winning’

POSTERIOR

$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) =$ 

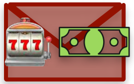
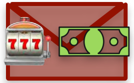
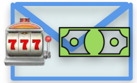
$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

What About 2 Events?

EVENT

Email contains ‘lottery’ and ‘winning’

POSTERIOR

$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) =$  / ( + )

?

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

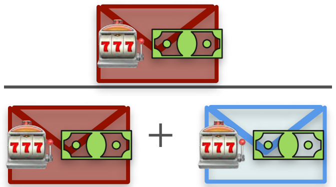
What About 2 Events?

EVENT

Email contains ‘lottery’ and ‘winning’

POSTERIOR

$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) =$



$$\frac{\text{\# Spam emails with 'lottery' and 'winning'}}{\text{\# Emails with 'lottery' and 'winning'}}$$

?

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

What About More Than 2 Events?

EVENT | **POSTERIOR**

What About More Than 2 Events?

EVENT | **POSTERIOR**

Email contains $w_1, w_2, \ldots, w_{100}$

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains $w_1, w_2, \ldots, w_{100}$

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_{100}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{ham})}$$

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains $w_1, w_2, \ldots, w_{100}$

?

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_{100}) = \frac{\mathbf{P}(\text{spam})\ \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{ham})}$$

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains $w_1, w_2, \ldots, w_{100}$

$$\frac{\text{\# Spam emails with } w_1, \ldots, w_{100}}{\text{\# Emails with } w_1, \ldots, w_{100}}$$

?

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_{100}) = \frac{\mathbf{P}(\text{spam})\ \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{ham})}$$

What About More Than 2 Events?

EVENT

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Email contains $w_1, w_2, \ldots, w_{100}$

$$\frac{\cancel{\text{\# Spam emails with } w_1, \ldots, w_{100}}\ 0}{\text{\# Emails with } w_1, \ldots, w_{100}}$$

?

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_{100}) = \frac{\mathbf{P}(\text{spam})\ \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1, \ldots, w_{100} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

🎰 💵

The appearances of ‘lottery’ and ‘winning’ are independent

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of ‘lottery’ and ‘winning’ are independent

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \overbrace{\mathbf{P}(\text{lottery \& winning} \mid \text{spam})}^{\mathbf{P}(A \cap B)}}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of ‘lottery’ and ‘winning’ are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery \& winning} \mid \text{ham})}$$

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The appearances of ‘lottery’ and ‘winning’ are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

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Is There a Quicker Way To Estimate the Probability?

Naive assumption

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$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

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Is There a Quicker Way To Estimate the Probability?

Naive assumption

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_n) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_n \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_n \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1, \ldots, w_n \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of the words $w_1, w_2, \ldots, w_n$ are independent

$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_n) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_n \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1, \ldots, w_n \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1, \ldots, w_n \mid \text{ham})}$$

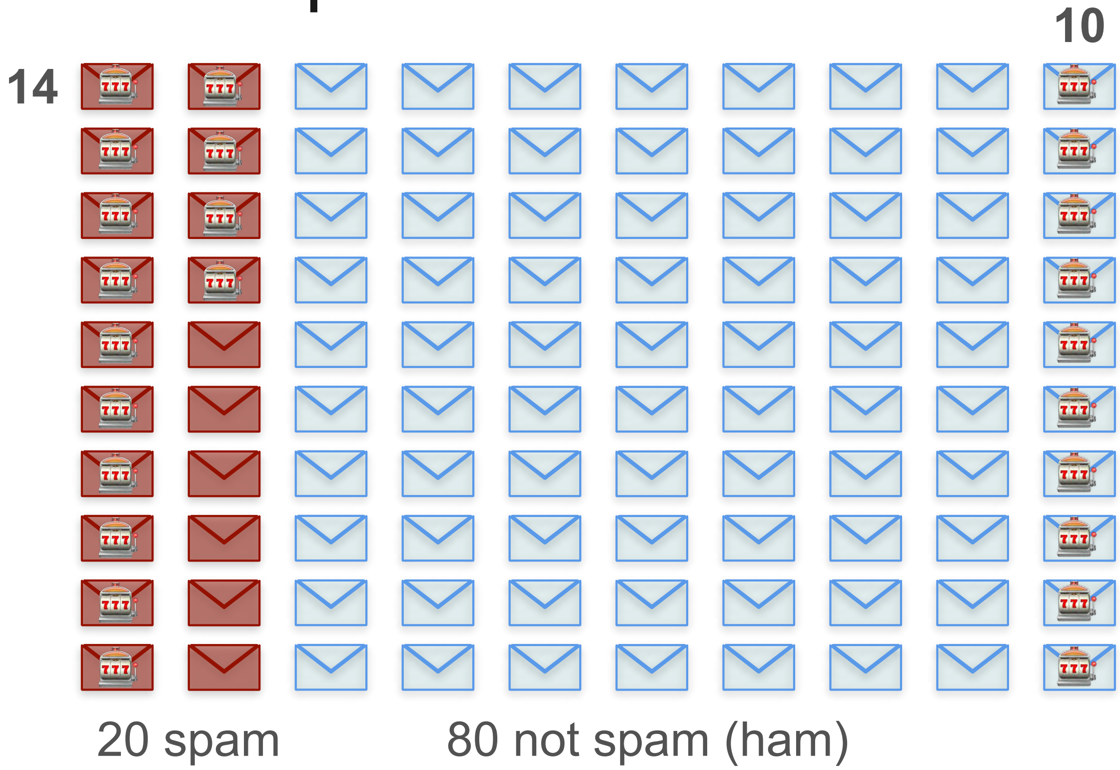
Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of the words $w_1, w_2, \ldots, w_n$ are independent

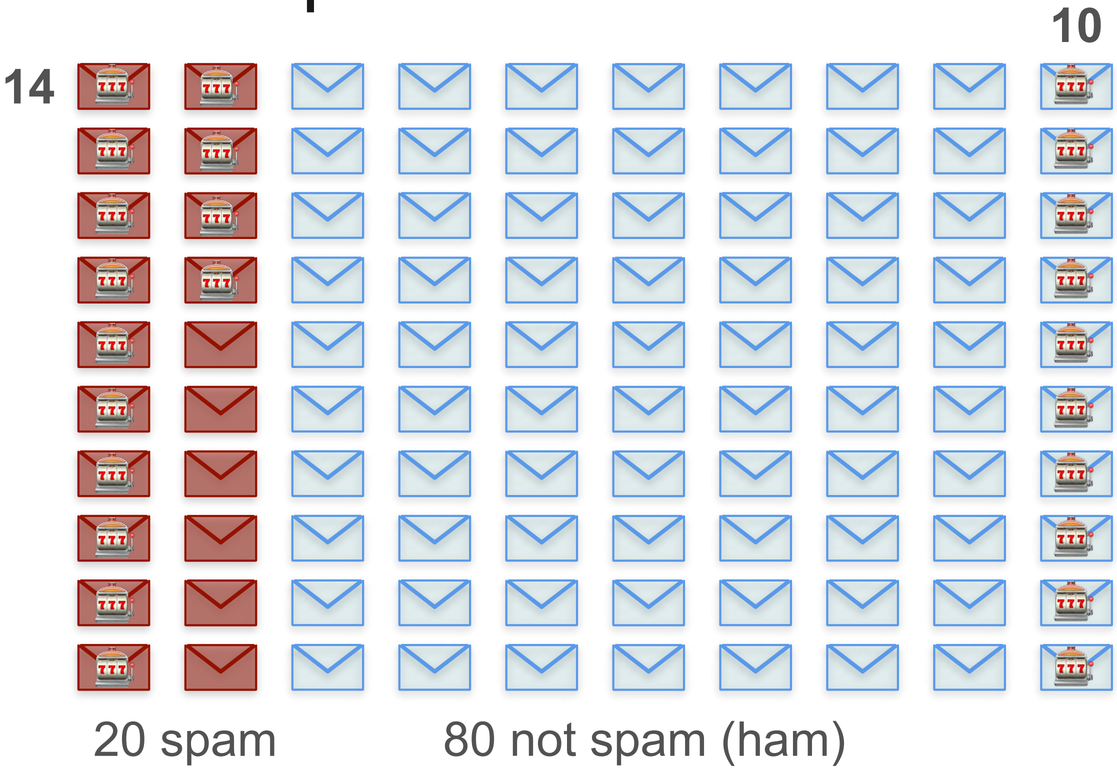
$$\mathbf{P}(\text{spam} \mid w_1, \ldots, w_n) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1 \mid \text{spam}) \cdots \mathbf{P}(w_n \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(w_1 \mid \text{spam}) \cdots \mathbf{P}(w_n \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(w_1 \mid \text{ham}) \cdots \mathbf{P}(w_n \mid \text{ham})}$$

Naive Bayes: Spam Example



Naive Bayes: Spam Example

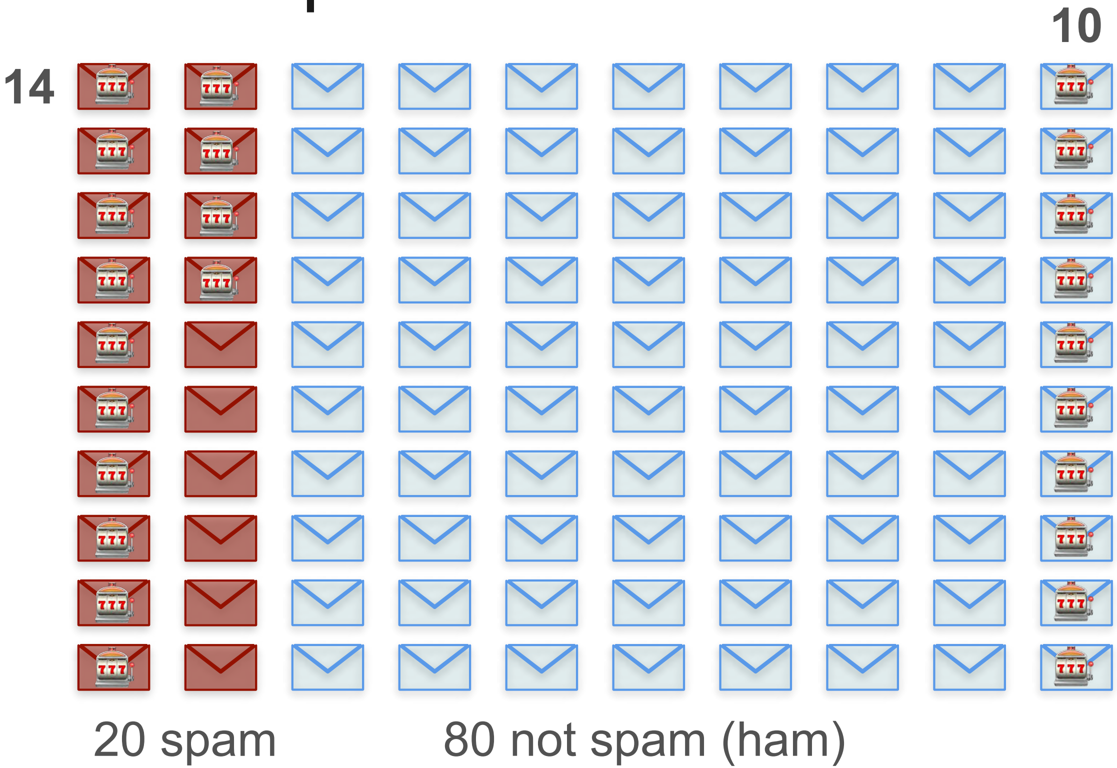
$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$



Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$

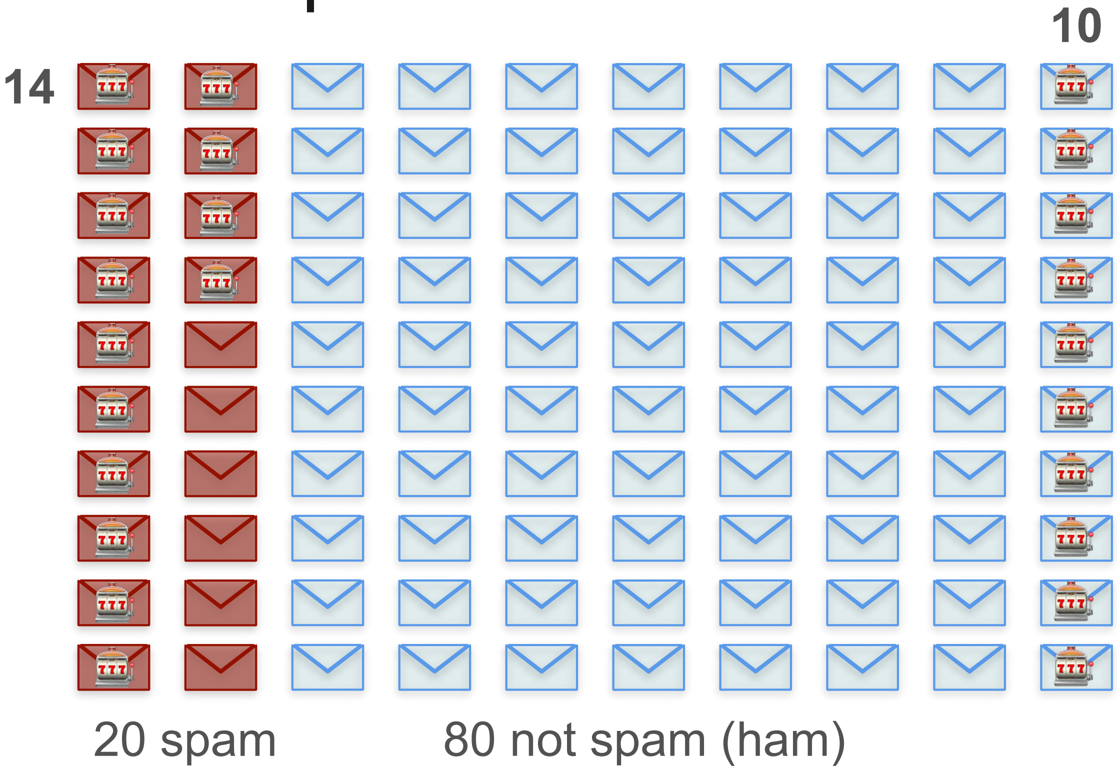


Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$



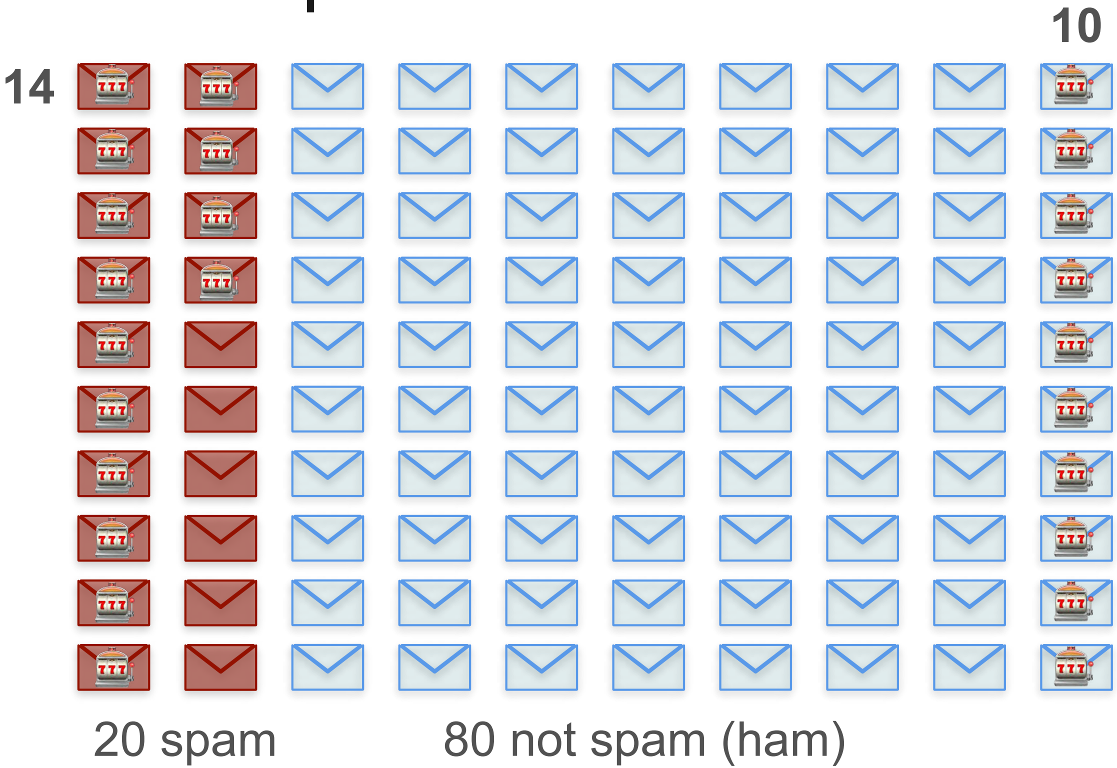
Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$

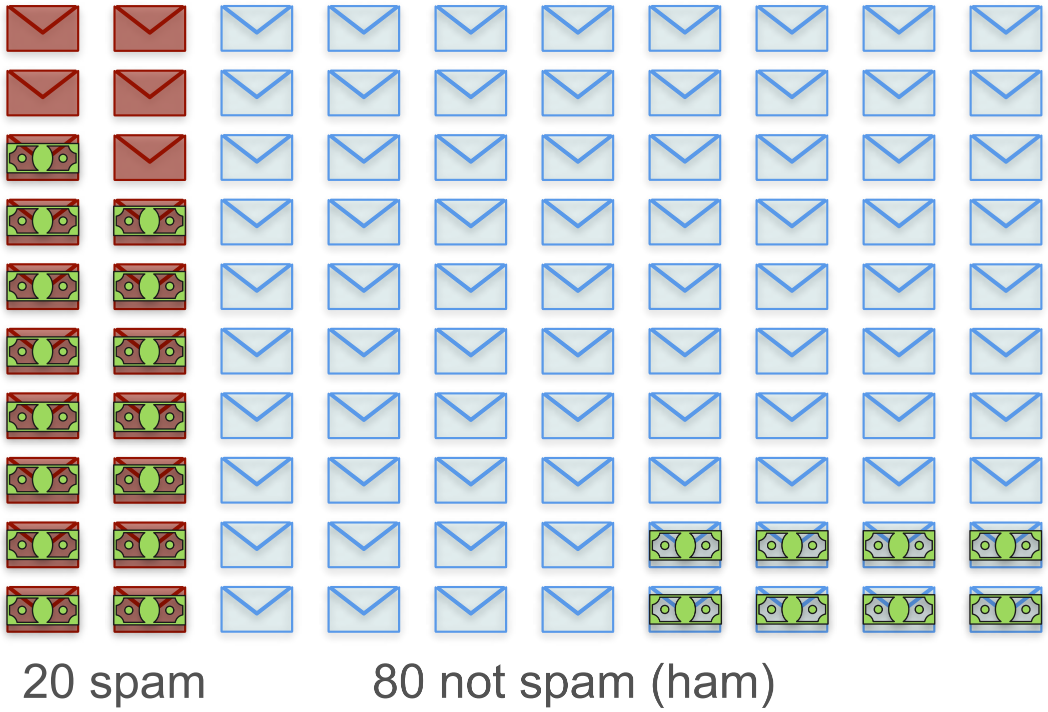
$$\mathbf{P}(\text{lottery} \mid \text{ham}) = \frac{10}{80} = 0.125$$



Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$



Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

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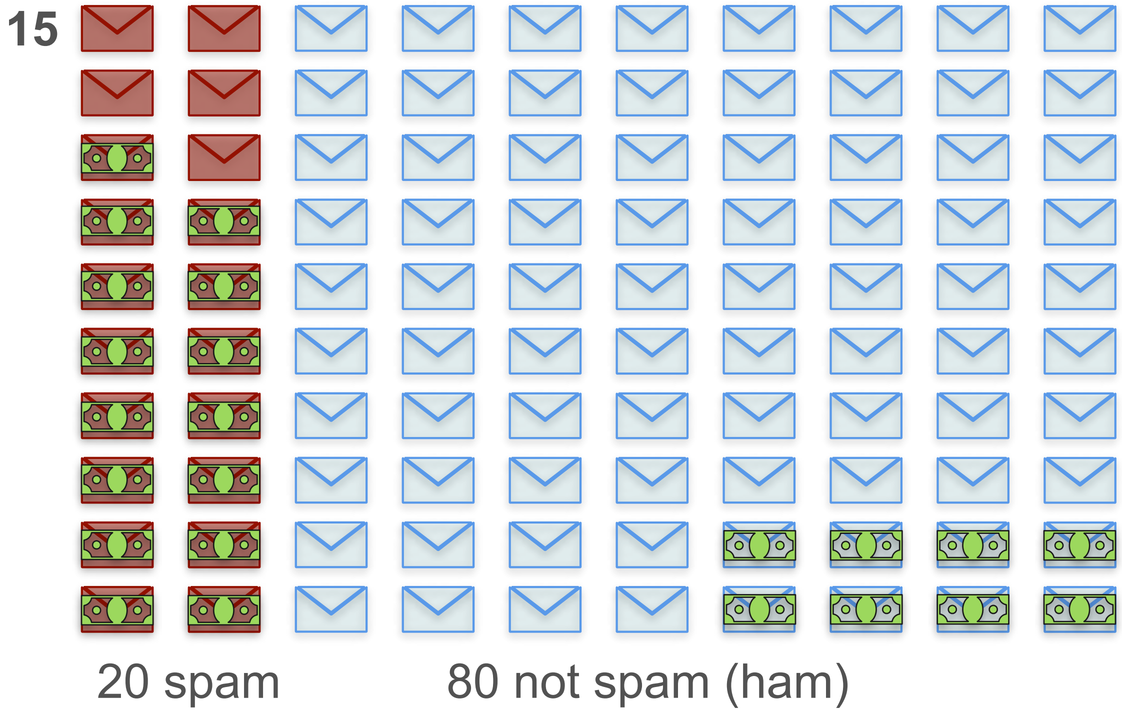
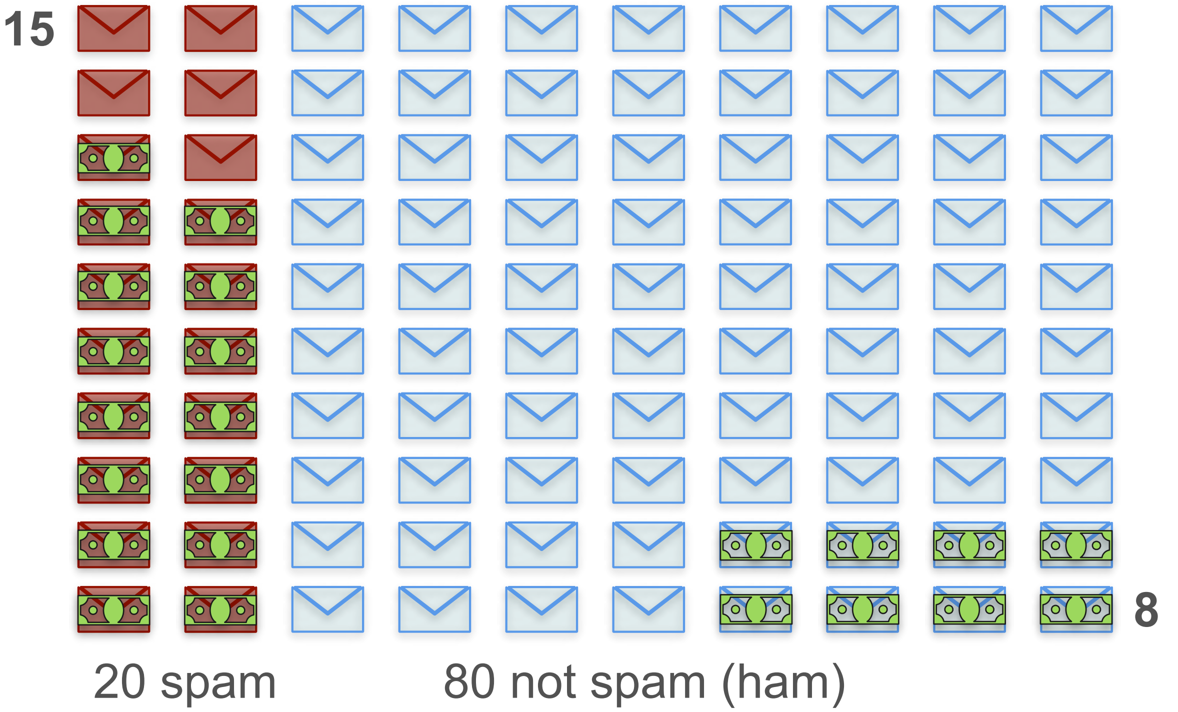


Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$

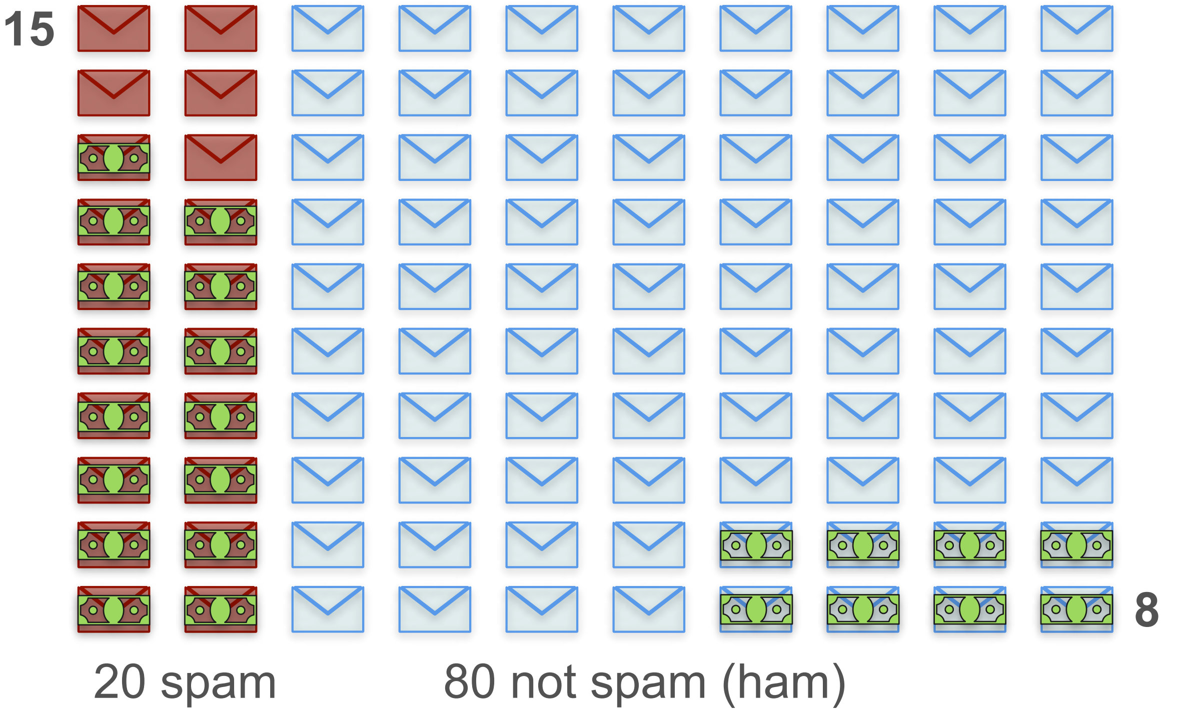


Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$



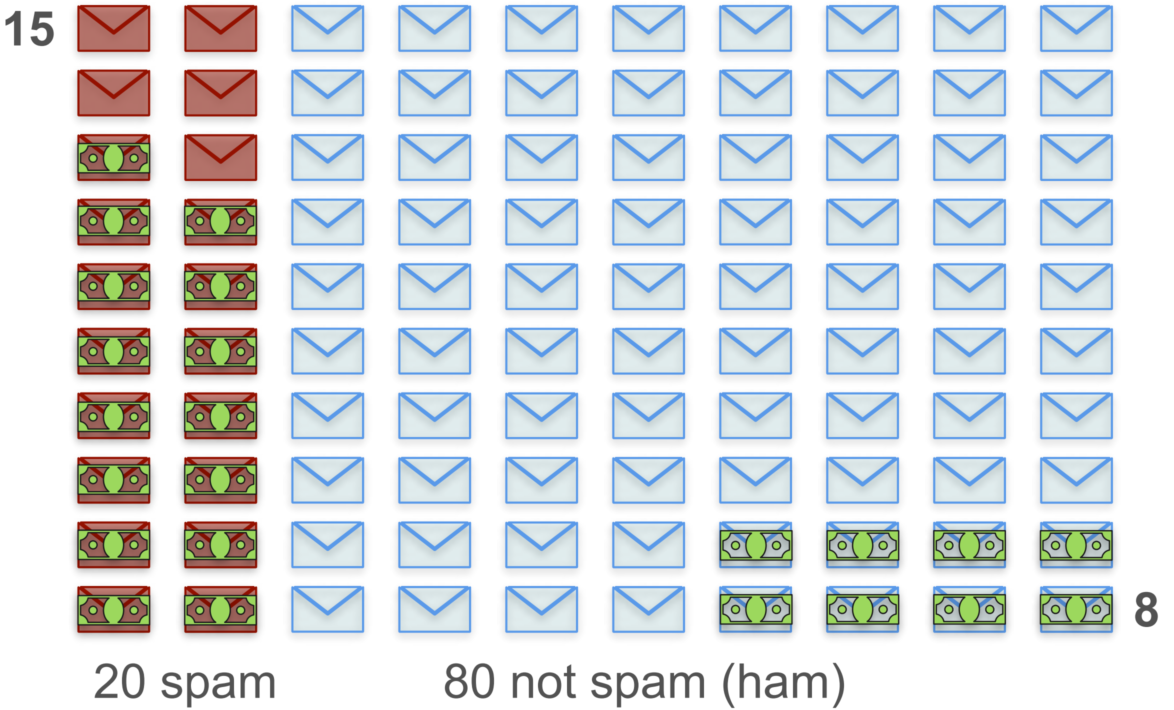
Naive Bayes: Spam Example

$$\mathbf{P}(\text{spam}) = \frac{20}{100} = 0.2$$

$$\mathbf{P}(\text{ham}) = \frac{80}{100} = 0.8$$

$$\mathbf{P}(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$

$$\mathbf{P}(\text{winning} \mid \text{ham}) = \frac{8}{80} = 0.1$$



Naive Bayes: Spam Example

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{lottery} \mid \text{ham}) = 0.125$

$\mathbf{P}(\text{winning} \mid \text{spam}) = 0.75$

$\mathbf{P}(\text{winning} \mid \text{ham}) = 0.1$

Naive Bayes: Spam Example

$\mathbf{P}(\text{spam}) = 0.2$ $\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$ $\mathbf{P}(\text{winning} \mid \text{spam}) = 0.75$

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$$\mathbf{P}(\text{spam} \mid \text{lottery}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery} \mid \text{ham}) \cdot \mathbf{P}(\text{winning} \mid \text{ham})}$$

Naive Bayes: Spam Example

$\mathbf{P}(\text{spam}) = 0.2$ $\qquad$ $\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$ $\qquad$ $\mathbf{P}(\text{winning} \mid \text{spam}) = 0.75$

$\mathbf{P}(\text{not spam}) = 0.8$ $\qquad$ $\mathbf{P}(\text{lottery} \mid \text{ham}) = 0.125$ $\qquad$ $\mathbf{P}(\text{winning} \mid \text{ham}) = 0.1$

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$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)}$$

Naive Bayes: Spam Example

$\mathbf{P}(\text{spam}) = 0.2$

$\mathbf{P}(\text{not spam}) = 0.8$

$\mathbf{P}(\text{lottery} \mid \text{spam}) = 0.7$

$\mathbf{P}(\text{lottery} \mid \text{ham}) = 0.125$

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$$\mathbf{P}(\text{spam} \mid \text{lottery \& winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)} = 0.913$$

DeepLearning.AI

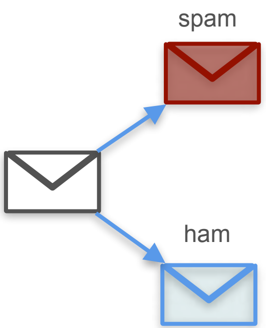
Introduction to probability

Probability in Machine Learning

Bayes Theorem

Bayes Theorem

PRIOR



Bayes Theorem

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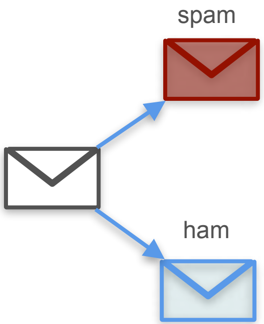

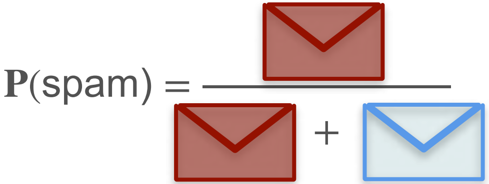

Bayes Theorem

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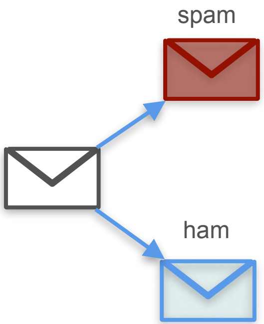

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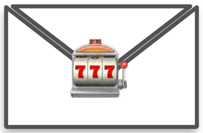

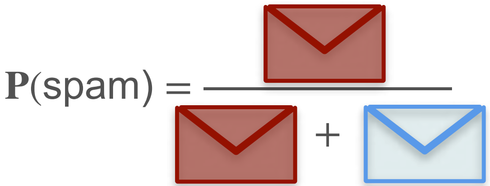

Bayes Theorem

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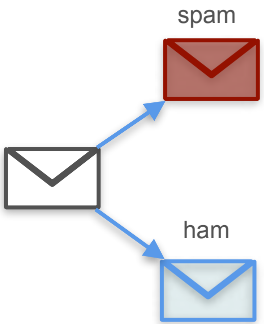

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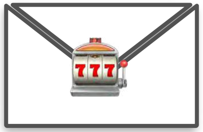

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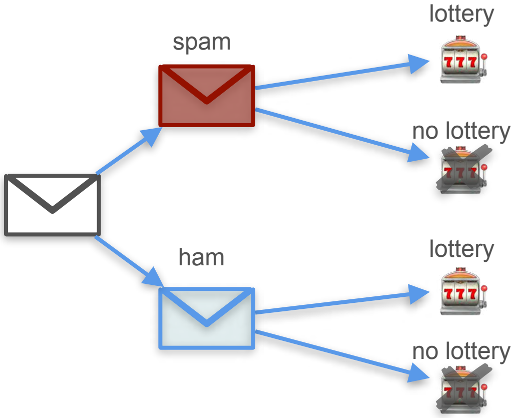

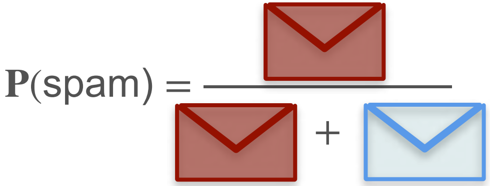

Bayes Theorem

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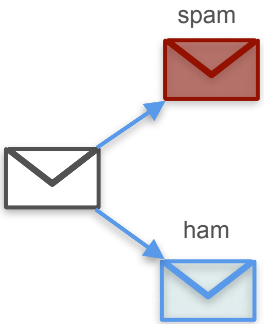

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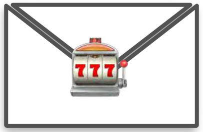

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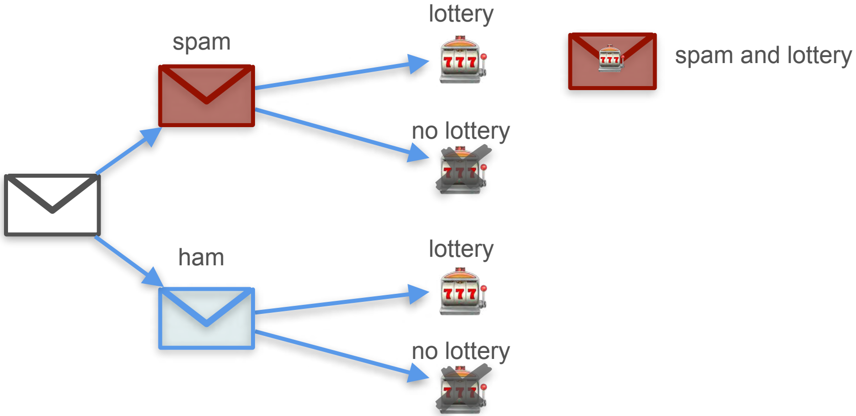
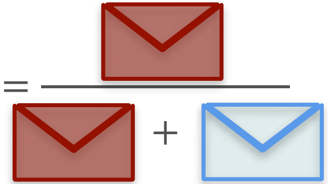


P(spam) =



Bayes Theorem

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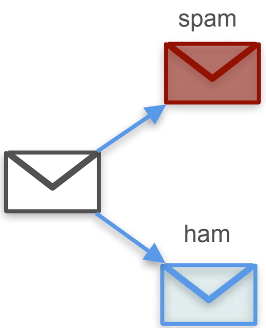

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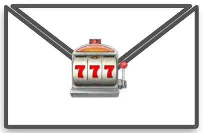

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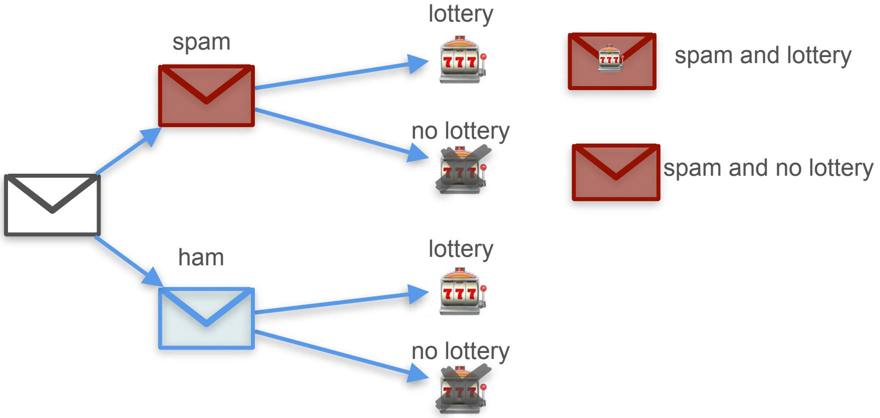

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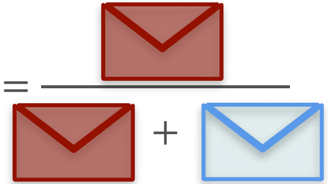

Bayes Theorem

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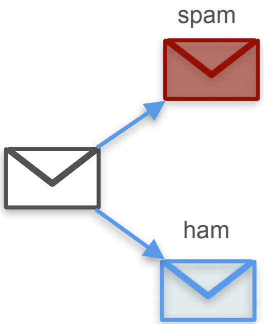

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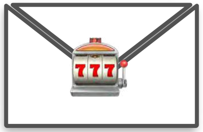

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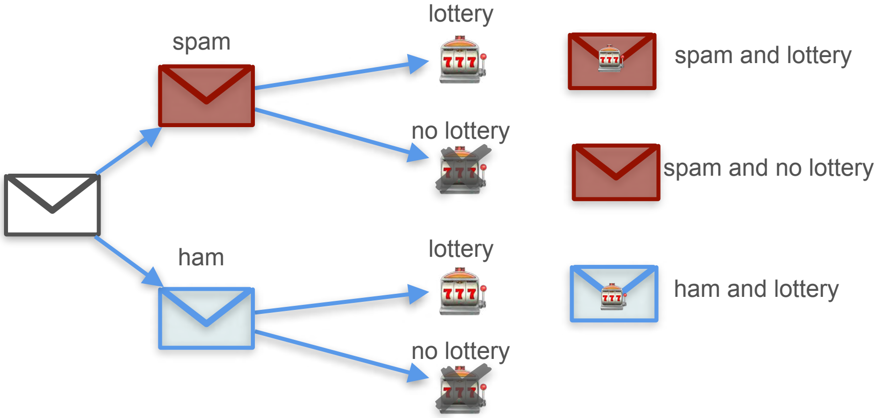
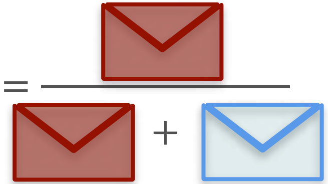


$P(\text{spam}) =$



Bayes Theorem

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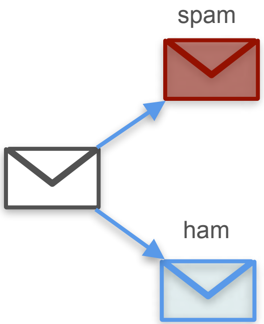

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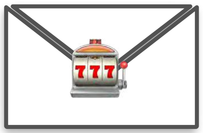

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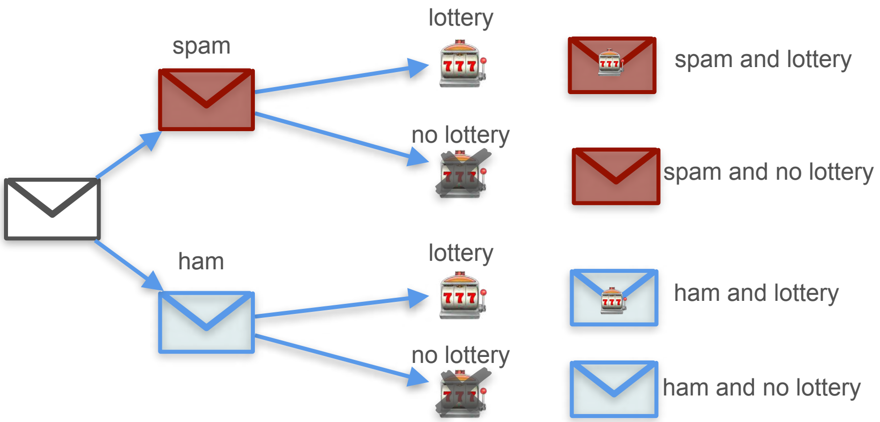

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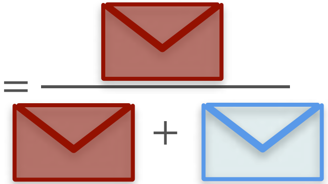

Bayes Theorem

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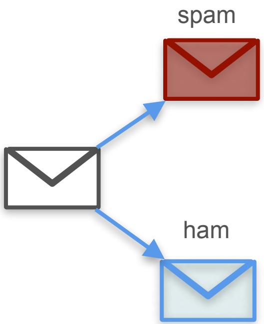
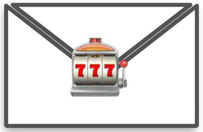
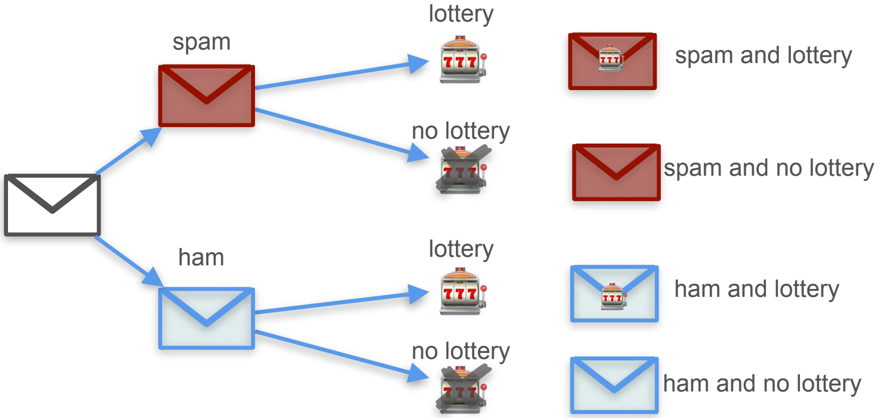


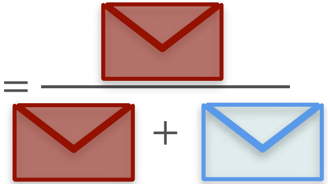
EVENT



POSTERIOR



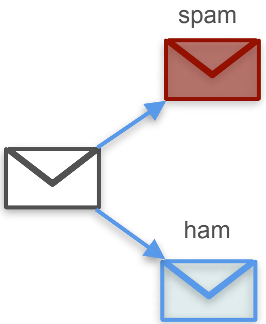
$\mathrm{P}(\text{spam}) =$



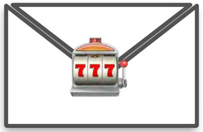
$\mathrm{P}(\text{spam} \mid \text{lottery}) =$

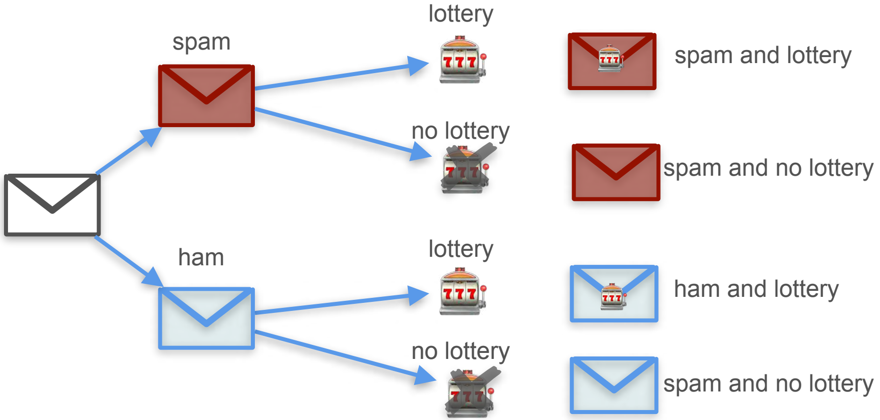
Bayes Theorem

PRIOR

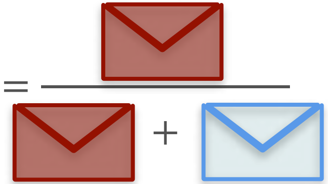


EVENT





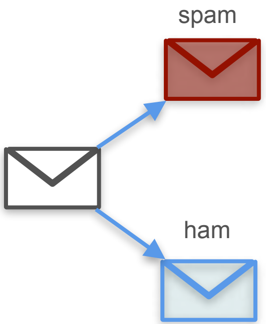
$\mathbf{P}(\text{spam}) =$



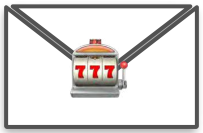
$\mathbf{P}(\text{spam} \mid \text{lottery}) =$

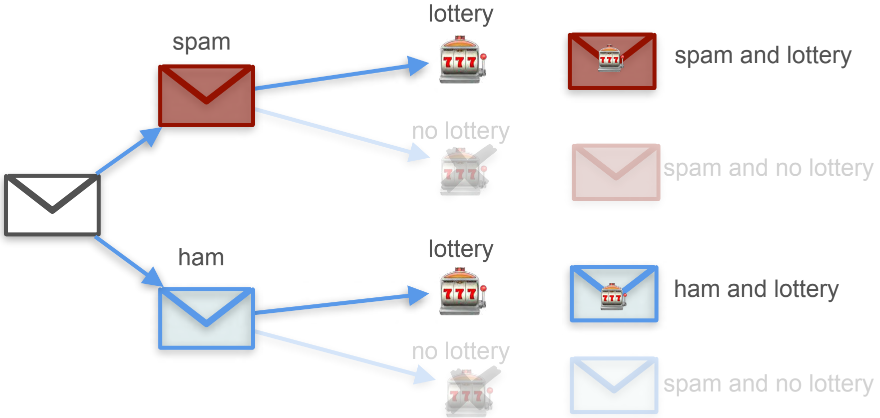
Bayes Theorem

PRIOR

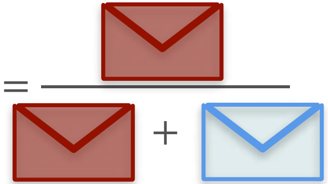


EVENT





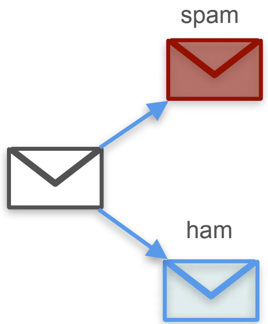
$\mathbf{P}(\text{spam}) =$



$\mathbf{P}(\text{spam} \mid \text{lottery}) =$

Bayes Theorem

PRIOR



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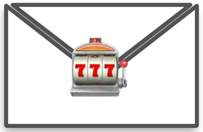
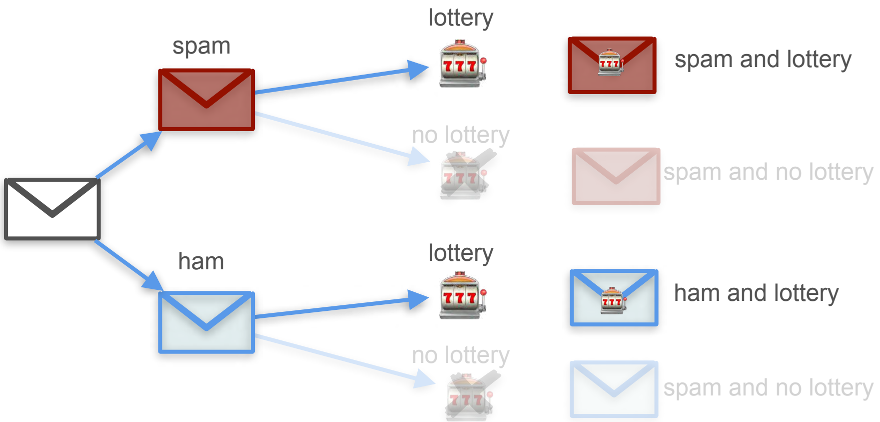
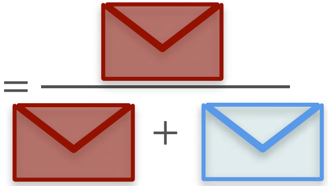


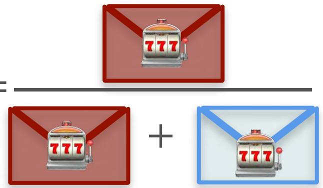
POSTERIOR



P(spam) =



P(spam | lottery) =



Example Problem

Example Problem

Image recognition

Example Problem

Image recognition

- What is the probability that there is a cat in the image



Example Problem

Image recognition

- What is the probability that there is a cat in the image
- $\mathbf{P}(\text{cat} \mid \text{image}) = \mathbf{P}(\text{cat} \mid \text{pixel}_1, \text{pixel}_2, \ldots, \text{pixel}_n)$



Example Problem

Example Problem

Classification

Example Problem

Classification

Patient 1	
Age	29
Gender	Female
Height	169 cm
Weight	62 kg
Smoker	No
...	...
Heart rate	63
Blood pressure	120 90

- Is this patient healthy?

Example Problem

Classification

Patient 1	
Age	29
Gender	Female
Height	169 cm
Weight	62 kg
Smoker	No
...	...
Heart rate	63
Blood pressure	120 90

- Is this patient healthy?
- Calculate $\mathbf{P}(\text{healthy} \mid \text{symptoms and history})$

Example Problem

Example Problem

Sentiment analysis

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

Matsuo Bashō

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

Matsuo Bashō

- Is this a happy sentence?

Example Problem

Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

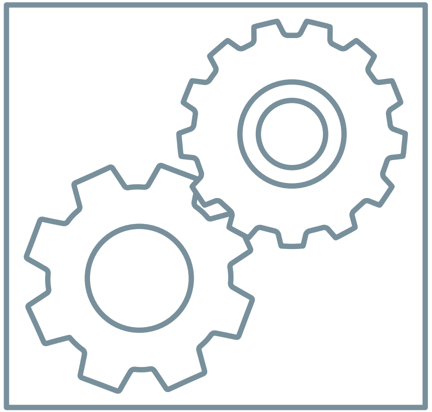
Matsuo Bashō

- Is this a happy sentence?
- Calculate P(happy I words in the sentence)

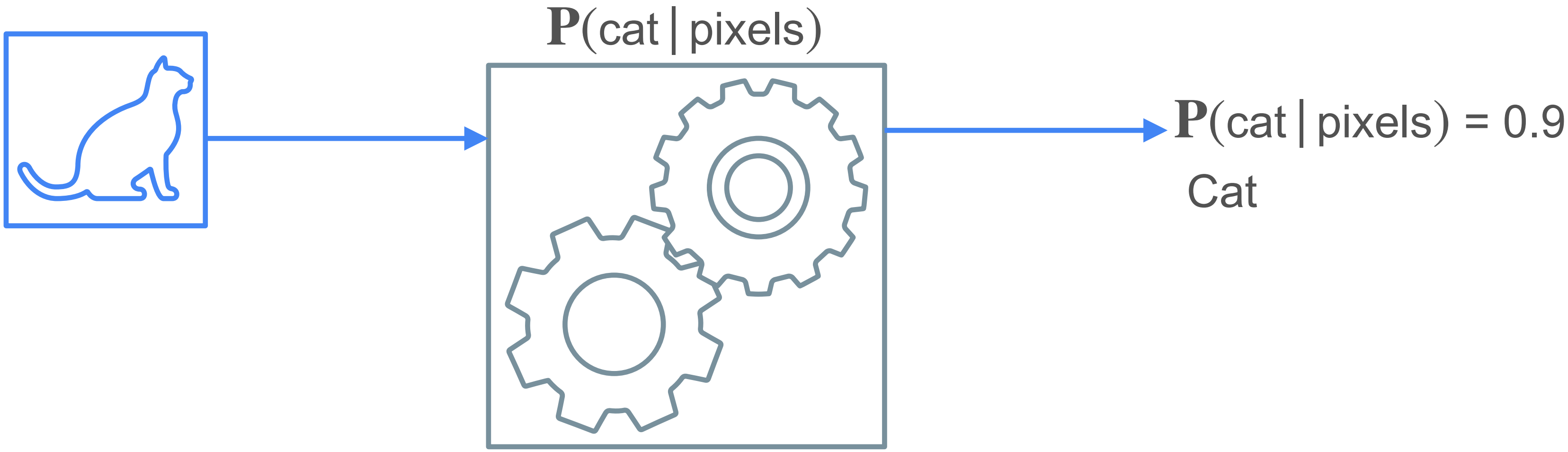
Example Solution

Example Solution

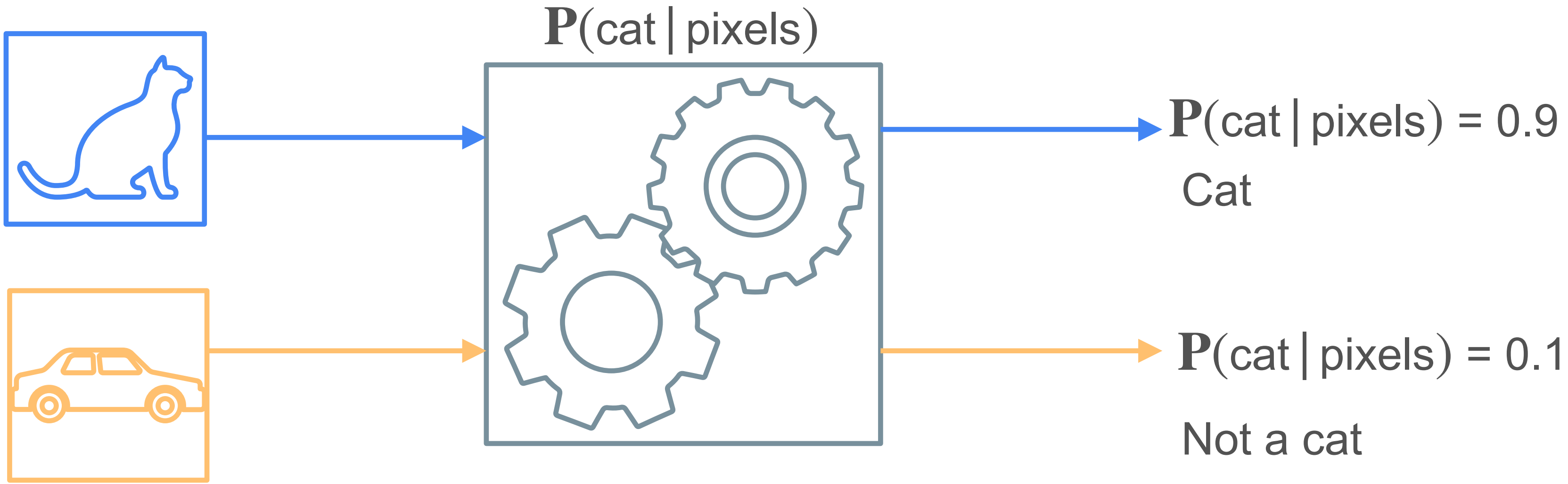
$\mathbf{P}(\text{cat} \mid \text{pixels})$



Example Solution



Example Solution



Example Problem: Generative Models

Example Problem: Generative Models

Face generation

Example Problem: Generative Models

Face generation

Image generated by a StyleGAN

Example Problem: Generative Models

Face generation

- Generate a group of pixels such that the resulting image looks like a human face.
- Goal: generate images such that $\mathbf{P}(\text{face} \mid \text{pixels})$ is high.

Image generated by a StyleGAN

W1 Lesson 2
Probability Distributions

DeepLearning.AI

DeepLearning.AI

Probability Distributions

Random Variables

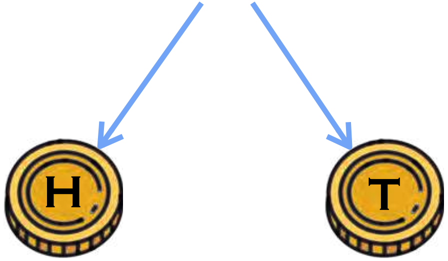
From Events to Random Variables

From Events to Random Variables

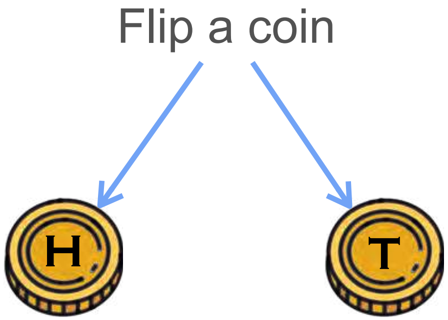
Flip a coin

From Events to Random Variables

Flip a coin



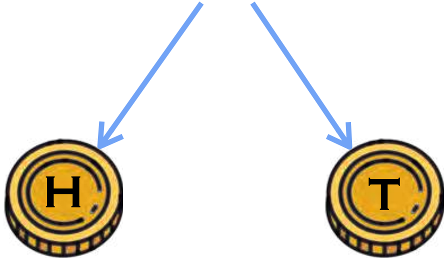
From Events to Random Variables



$$\mathbf{P}(H) = 0.5$$

From Events to Random Variables

Flip a coin

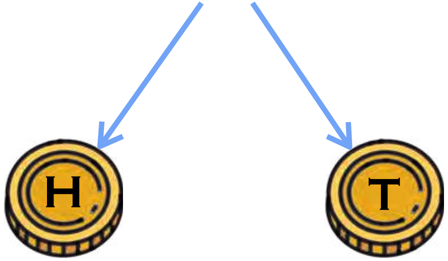


$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

From Events to Random Variables

$X =$ Number of heads

Flip a coin



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From Events to Random Variables

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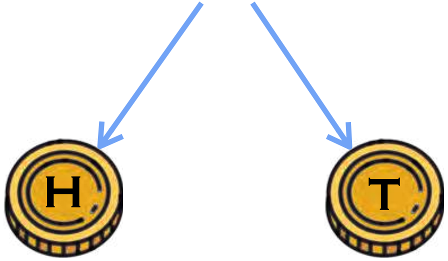


$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads

0 heads

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads

0 heads

$X = 1$

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads $X = 1$

0 heads $X = 0$

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads

$X = 1$

0 heads

$X = 0$

$\mathbf{P}(X = 1) = 0.5$

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\quad$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads $\quad$ 0 heads

$X = 1$ $\quad$ $X = 0$

$\mathbf{P}(X = 1) = 0.5$ $\quad$ $\mathbf{P}(X = 0) = 0.5$

From Events to Random Variables

Flip a coin

$\mathbf{P}(H) = 0.5$ $\mathbf{P}(T) = 0.5$

$X =$ Number of heads

Flip a coin

1 heads

$X = 1$

$\mathbf{P}(X = 1) = 0.5$

0 heads

$X = 0$

$\mathbf{P}(X = 0) = 0.5$

X is a random variable

From Events to Random Variables

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 10$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 10$

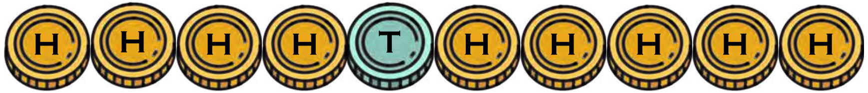
$X = 9$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 10$

$X = 9$



$X = 9$

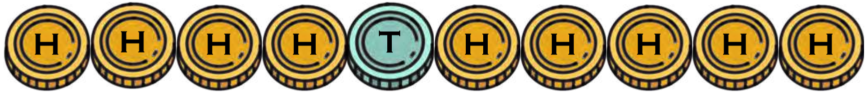
From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$

$X = 9$



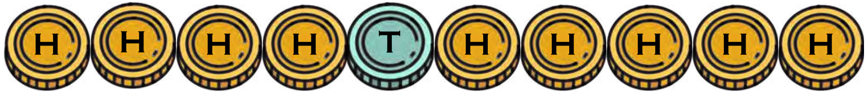
$X = 9$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses $\qquad \mathbf{P}(H) = 0.5$

$X = 10 \qquad 0.5^{10}$

$X = 9$



$X = 9$

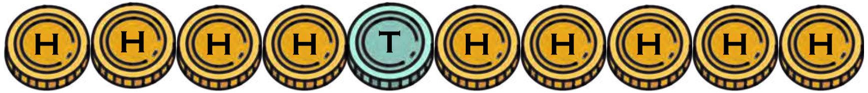
From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$X = 9$ $\quad 0.5^{9}0.5$



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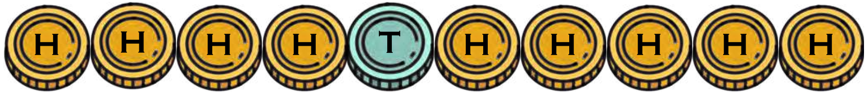


$X = 9$ $\quad 0.5^{9}0.5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$X = 9$ $\quad 0.5^{9}0.5$

$X = 9$ $\quad 0.5^{9}0.5$

$\mathbf{P}(X = 0)$?

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

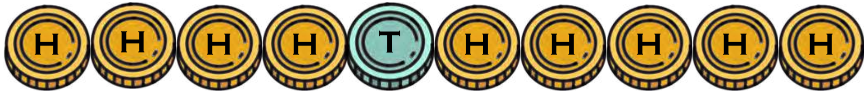
$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$\mathbf{P}(X = 0)?$

$\mathbf{P}(X = 1)?$

$X = 9$ $\quad 0.5^{9}0.5$



$X = 9$ $\quad 0.5^{9}0.5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$X = 9$ $\quad 0.5^{9}0.5$

$X = 9$ $\quad 0.5^{9}0.5$

$\mathbf{P}(X = 0)$?

$\mathbf{P}(X = 1)$?

$\dots$

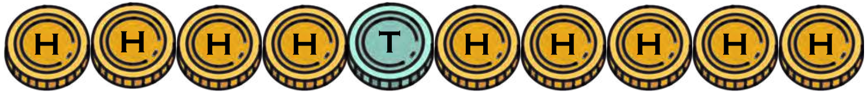
From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$\mathbf{P}(H) = 0.5$

$X = 10$ $\quad 0.5^{10}$

$X = 9$ $\quad 0.5^9 0.5$



$X = 9$ $\quad 0.5^9 0.5$

$\mathbf{P}(X = 0)$?

$\mathbf{P}(X = 1)$?

$\dots$

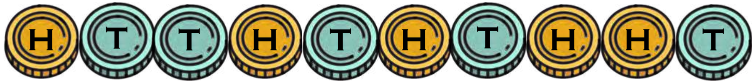
$\mathbf{P}(X = 10)$?

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses



$X = 5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

Possible outcomes:

$X = 5$

$X = 6$

$X = 5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	

$\mathbf{P}(H) = 0.5$

From Events to Random Variables

$X =$ Number of heads in 10 coin tosses

$X = 5$

$X = 6$

$X = 5$

Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	

$\mathbf{P}(H) = 0.5$

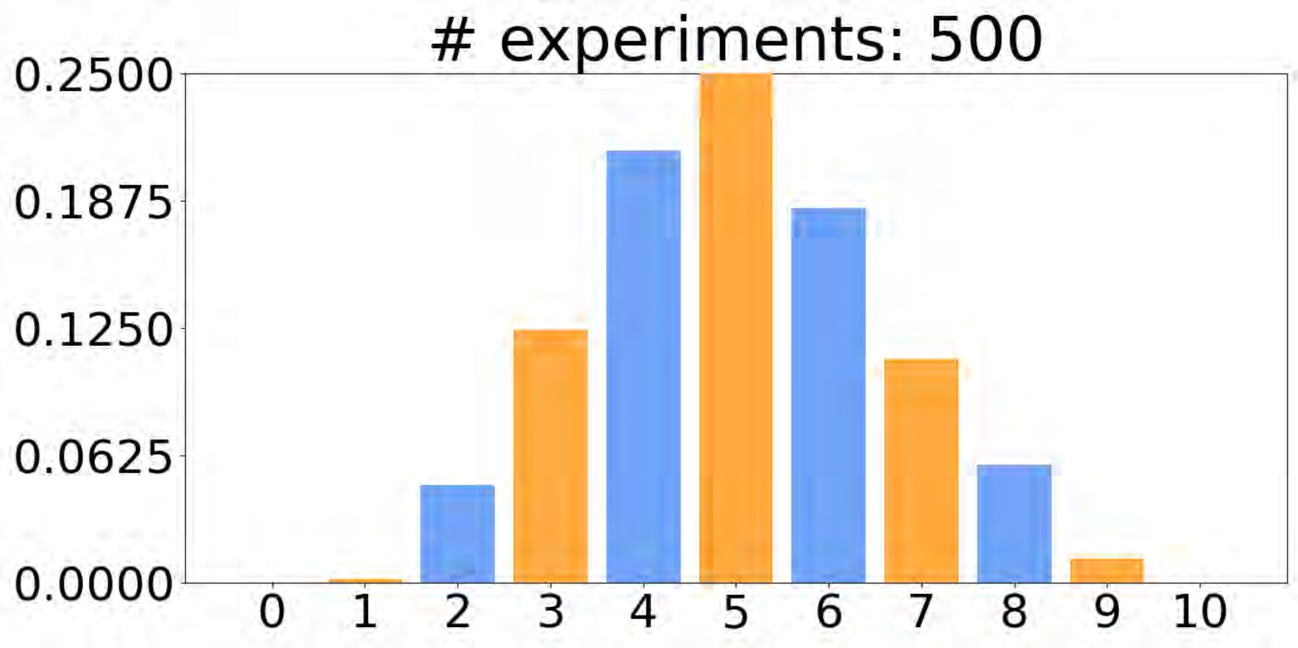
Repeat 500 times

Flipping a Fair Coin 500 Times

$$\mathbf{P}(H) = 0.5$$

Repeat 500 times

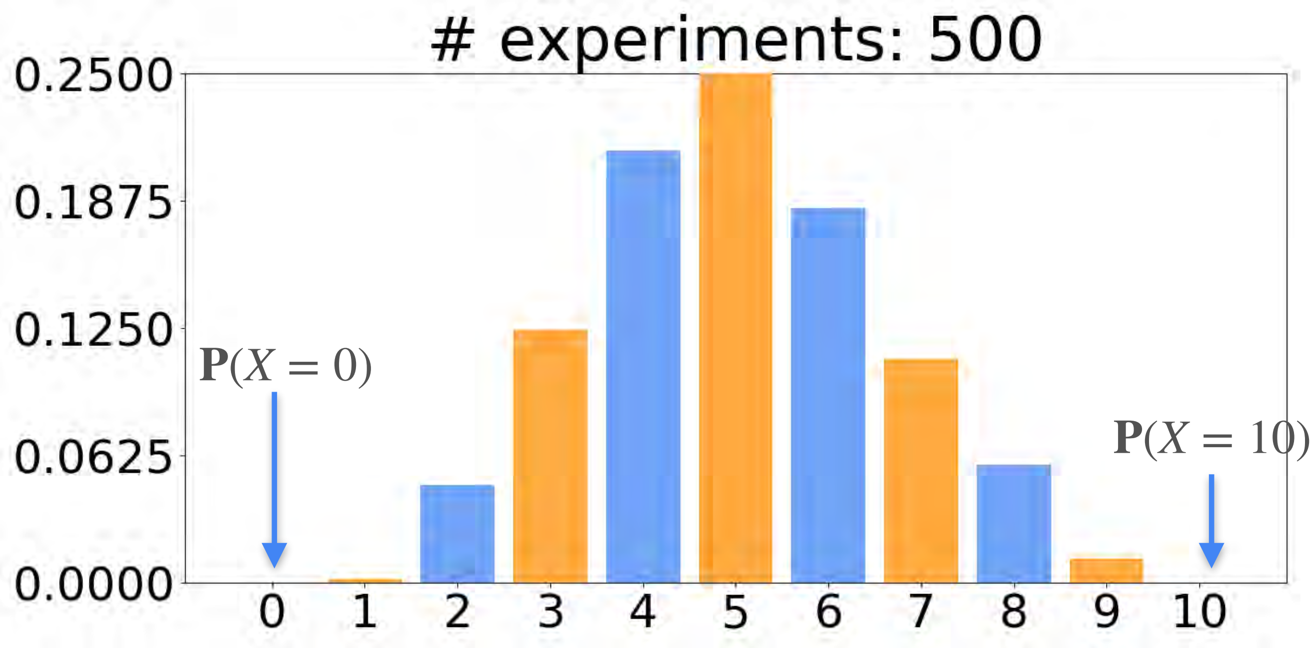
Flipping a Fair Coin 500 Times



$\mathbf{P}(H) = 0.5$

Repeat 500 times

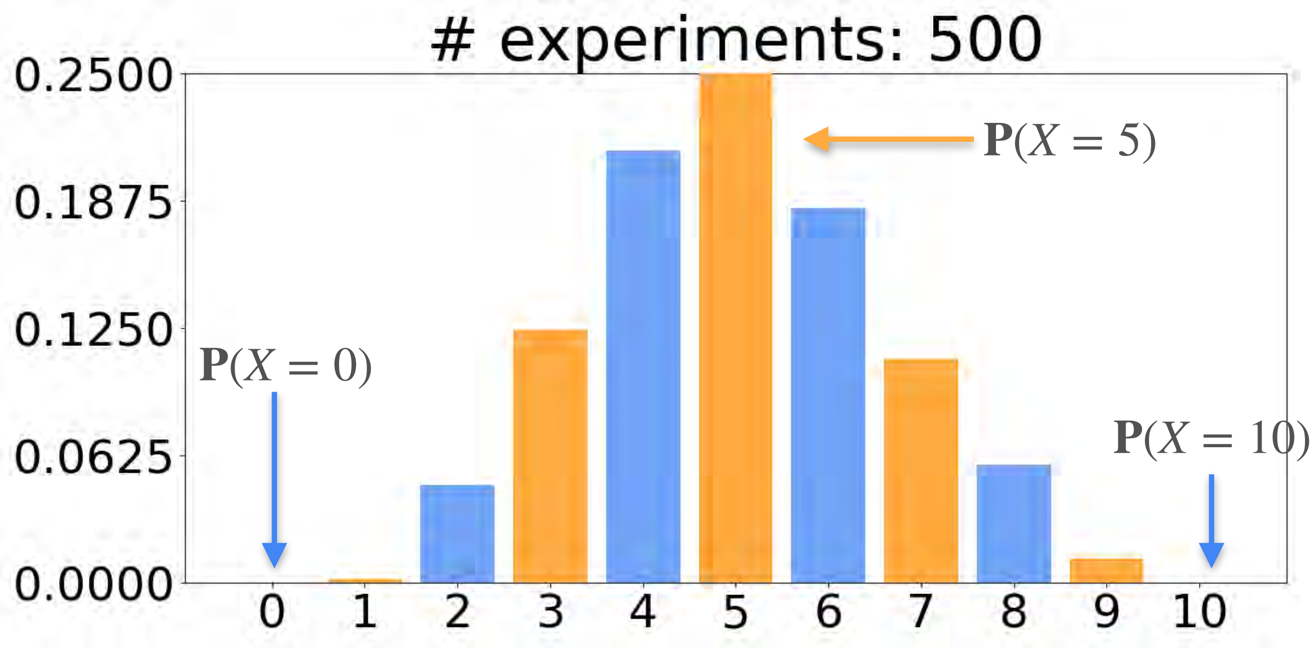
Flipping a Fair Coin 500 Times



$\mathbf{P}(H) = 0.5$

Repeat 500 times

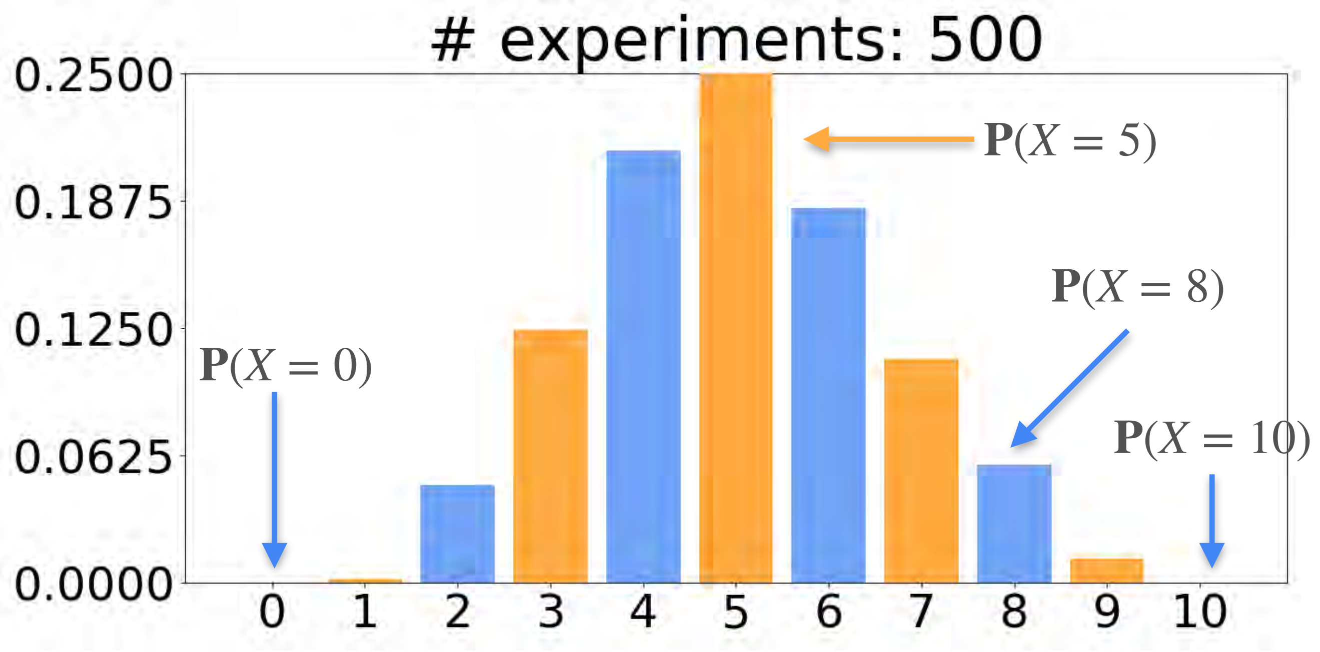
Flipping a Fair Coin 500 Times



$\mathbf{P}(H) = 0.5$

Repeat 500 times

Flipping a Fair Coin 500 Times



$\mathbf{P}(H) = 0.5$

Repeat 500 times

Why Random Variables?

- Random variables allow you to model the whole experiment at once

Why Random Variables?

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$X =$ Number of heads

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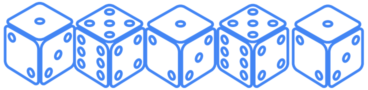
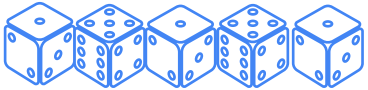


$X =$ Number of 1's

Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



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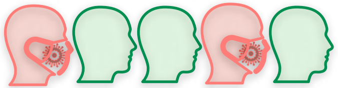
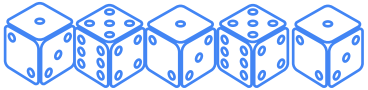


$X =$ Number of sick patients

Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



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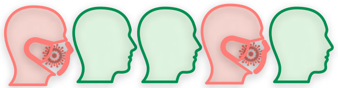

$X =$ Number of sick patients

?

Why Random Variables?

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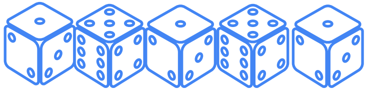
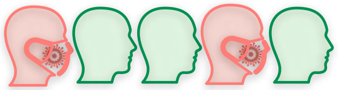
$X =$ Number of heads



$X =$ Number of 1's



$X =$ Number of sick patients

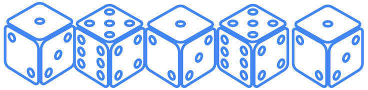
?

$\mathbf{P}(X = 1) = 0.5$

Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



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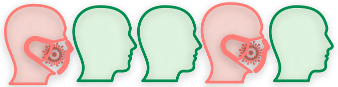


$X =$ Number of sick patients

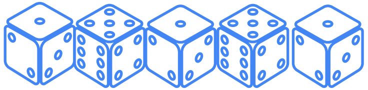
?

$$\mathbf{P}(X = 1) = 0.5$$
$$\mathbf{P}(X = -7) = 0.2$$

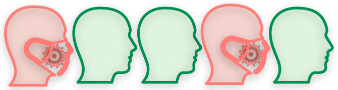
Why Random Variables?

- Random variables allow you to model the whole experiment at once

$X =$ Number of heads



$X =$ Number of 1's



$X =$ Number of sick patients

?

$$\mathbf{P}(X = 1) = 0.5$$
$$\mathbf{P}(X = -7) = 0.2$$
$$\mathbf{P}(X = 3.14159) = 0.3$$

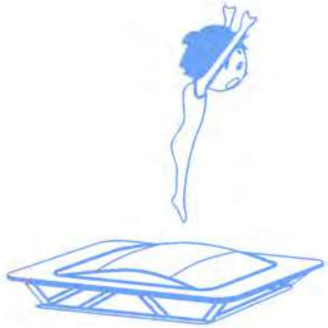
Other Random Variables

Other Random Variables

Wait time until the next bus arrives

Other Random Variables

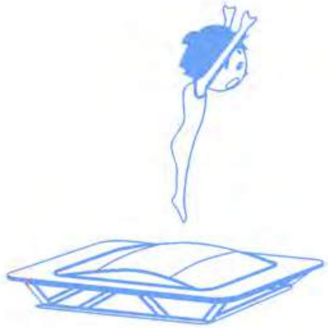
Wait time until the next bus arrives



Height of an gymnast's jump

Other Random Variables

Wait time until the next bus arrives



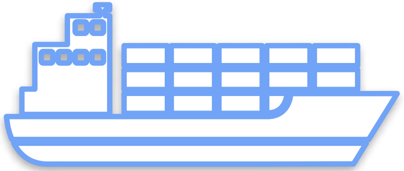
Height of an gymnast's jump

Number of defective products in a shipment

Other Random Variables

Wait time until the next bus arrives

Height of an gymnast’s jump



Number of defective products in a shipment

mm. of rain in November

Discrete and Continuous Random Variables

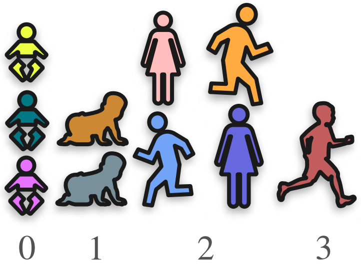
Discrete and Continuous Random Variables

Discrete random variables

Continuous random variables

Discrete and Continuous Random Variables

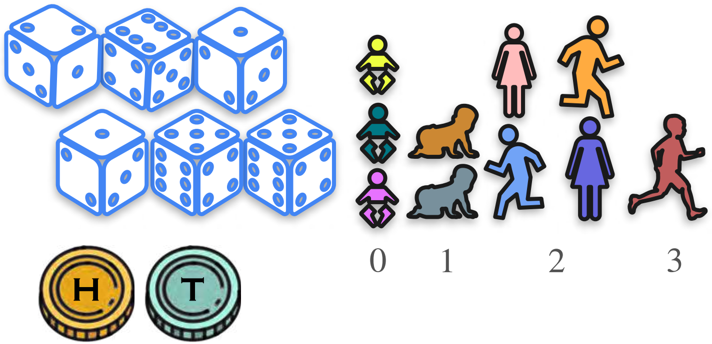
Discrete random variables



Continuous random variables

Discrete and Continuous Random Variables

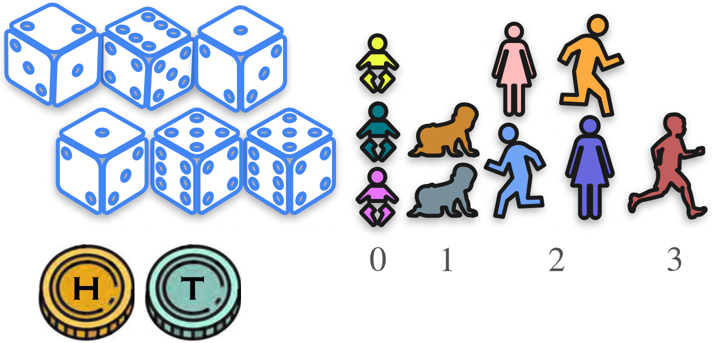
Discrete random variables



Continuous random variables

Discrete and Continuous Random Variables

Discrete random variables

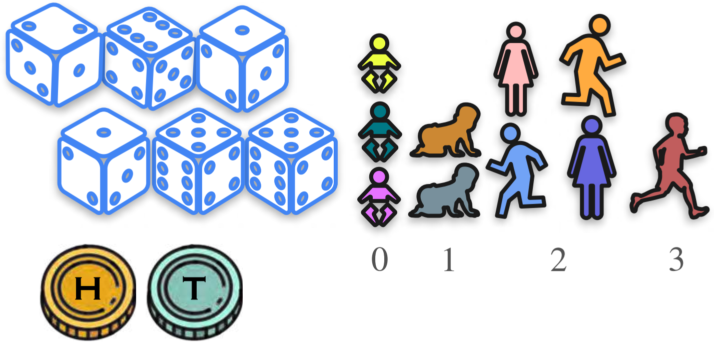


Finite number of values

Continuous random variables

Discrete and Continuous Random Variables

Discrete random variables



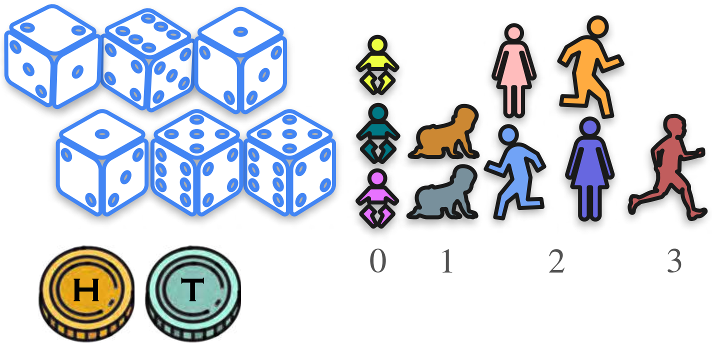
Finite number of values

Continuous random variables

Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

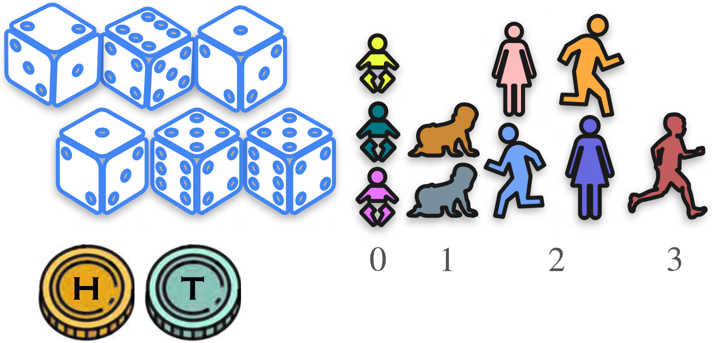
(Could be infinite too)

Continuous random variables

Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

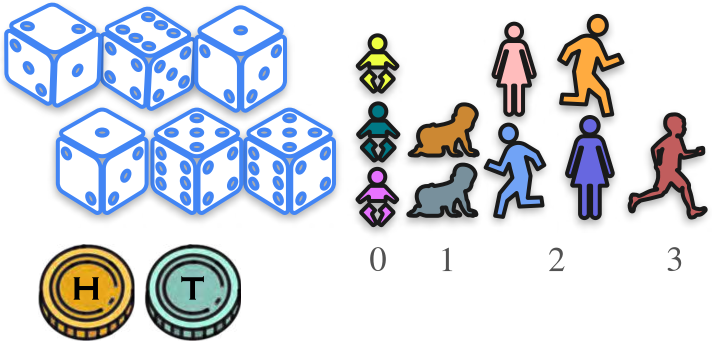
Can take only a **countable** number of values

Continuous random variables

Infinite number of values

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

Can take only a **countable** number of values

Continuous random variables

Infinite number of values

Takes values on an interval

Random Variable Vs. Deterministic Variable

Random Variable Vs. Deterministic Variable

Deterministic

Random

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2,\ f(x) = x^2$$

Random

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2,\ f(x) = x^2$$

Random

X = number of defective items in a shipment

Random Variable Vs. Deterministic Variable

Deterministic	**Random**
$x = 2,\ f(x) = x^2$	X = number of defective items in a shipment
Fixed outcome	

Random Variable Vs. Deterministic Variable

Deterministic	**Random**
$x = 2,\ f(x) = x^2$	X = number of defective items in a shipment
Fixed outcome	Uncertain outcome

Probability Distributions

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Probability Distributions (Discrete)

Discrete Distributions: Flip Three Coins

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses

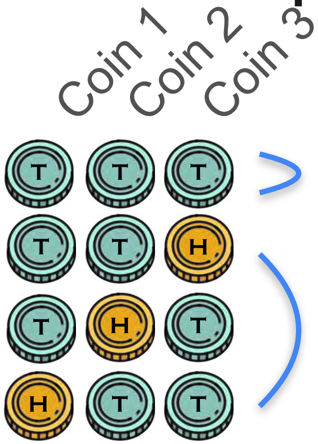
Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses

All three tails $(X = 0)$

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses

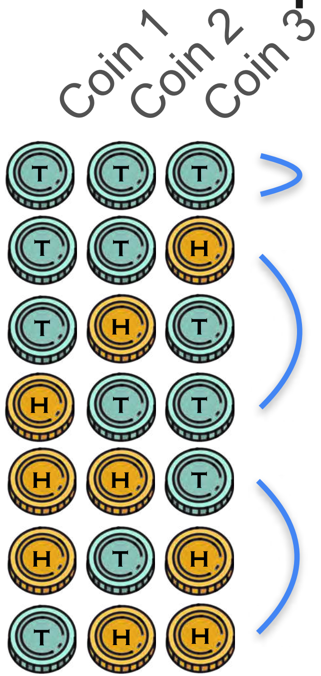


All three tails $(X = 0)$

One heads $(X = 1)$

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses



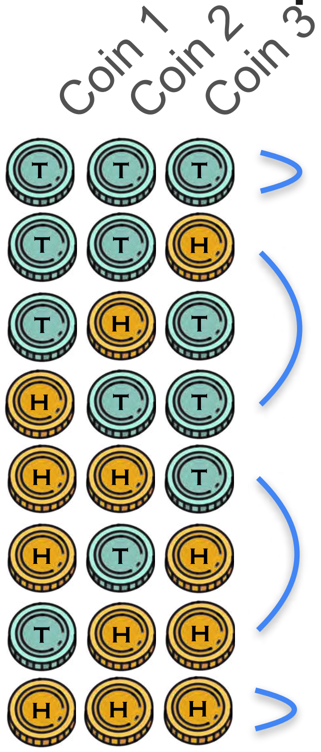
All three tails $(X = 0)$

One heads $(X = 1)$

Two heads $(X = 2)$

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses



All three tails $(X = 0)$

One heads $(X = 1)$

Two heads $(X = 2)$

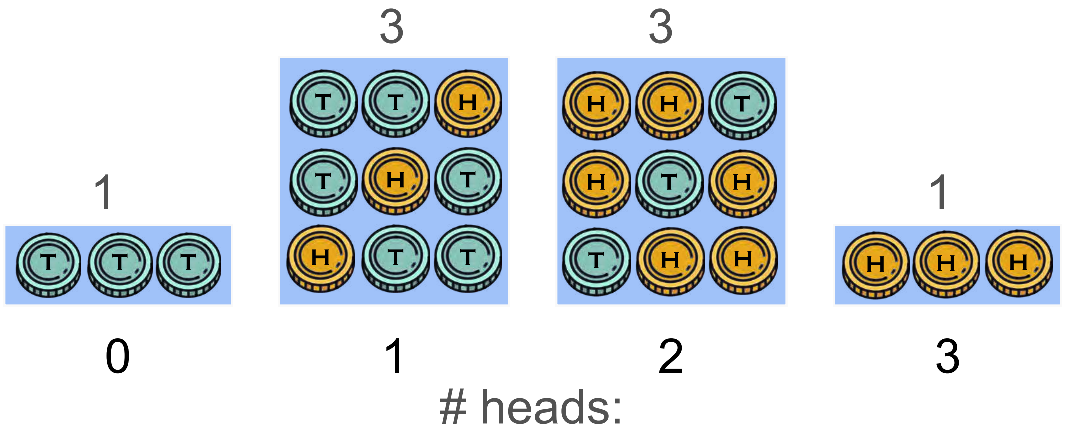
All three heads $(X = 3)$

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses

Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses



Discrete Distributions: Flip Three Coins

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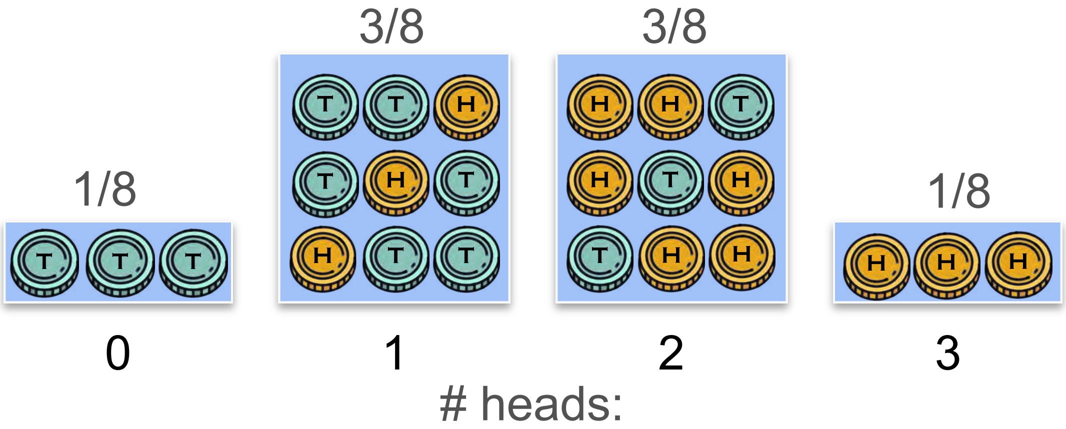
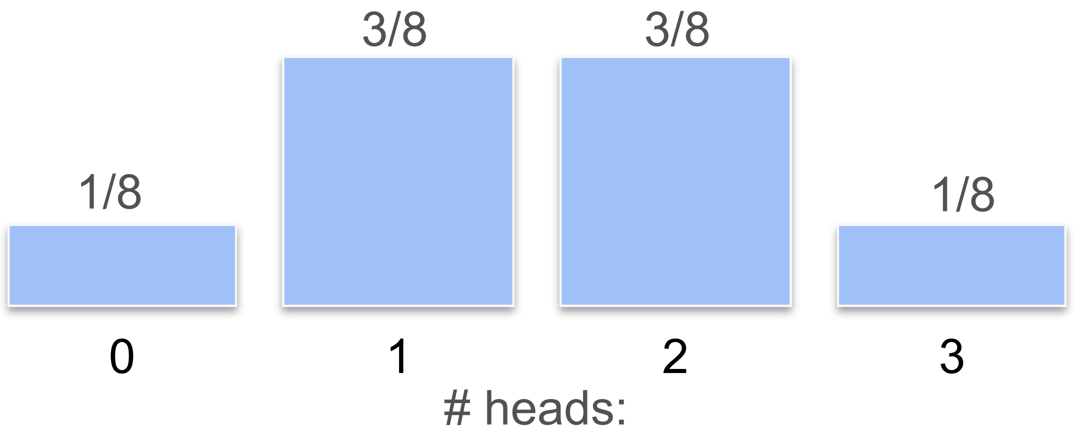


Discrete Distributions: Flip Three Coins

X_1: number of heads in **3** coin tosses



Discrete Distributions: Flip Four Coins

Discrete Distributions: Flip Four Coins

X_2: number of heads in **4** coin tosses

Discrete Distributions: Flip Four Coins

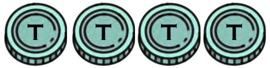
X_2: number of heads in **4** coin tosses

0	1	2	3	4

heads:

Discrete Distributions: Flip Four Coins

X_2: number of heads in **4** coin tosses

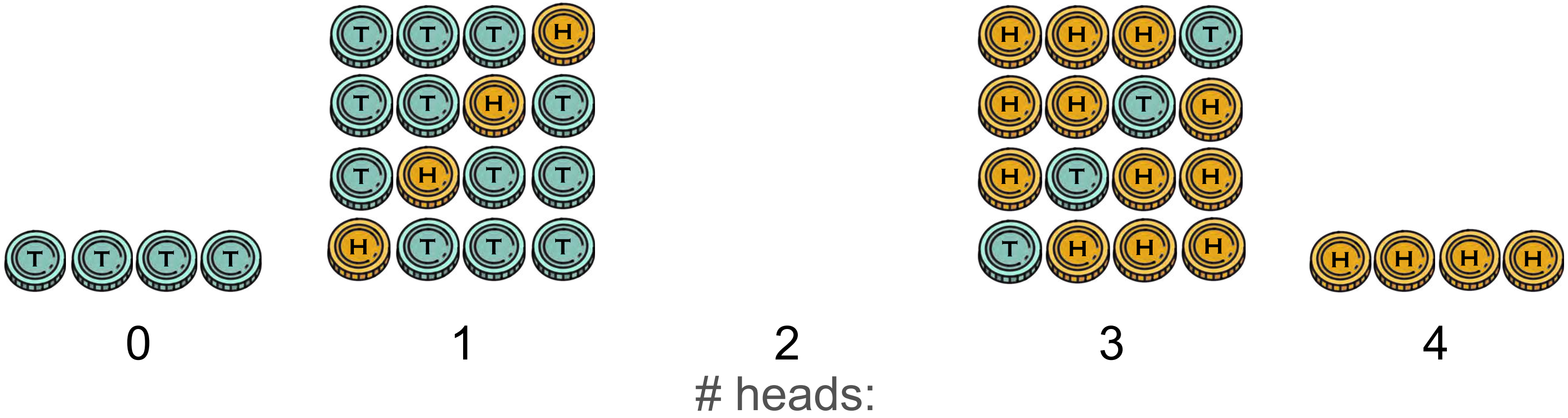


0 1 2 3 4

heads:

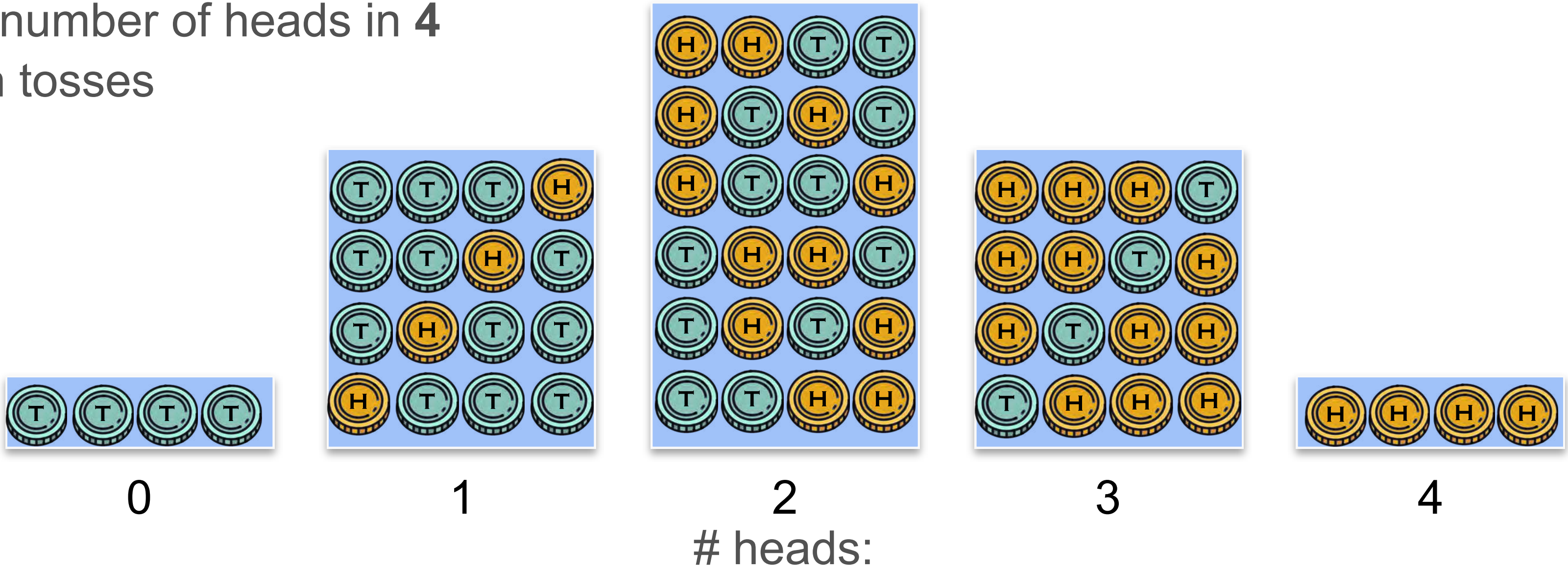
Discrete Distributions: Flip Four Coins

X_2: number of heads in **4** coin tosses



Discrete Distributions: Flip Four Coins

X_2: number of heads in **4** coin tosses



Discrete Distributions: Flip Four Coins

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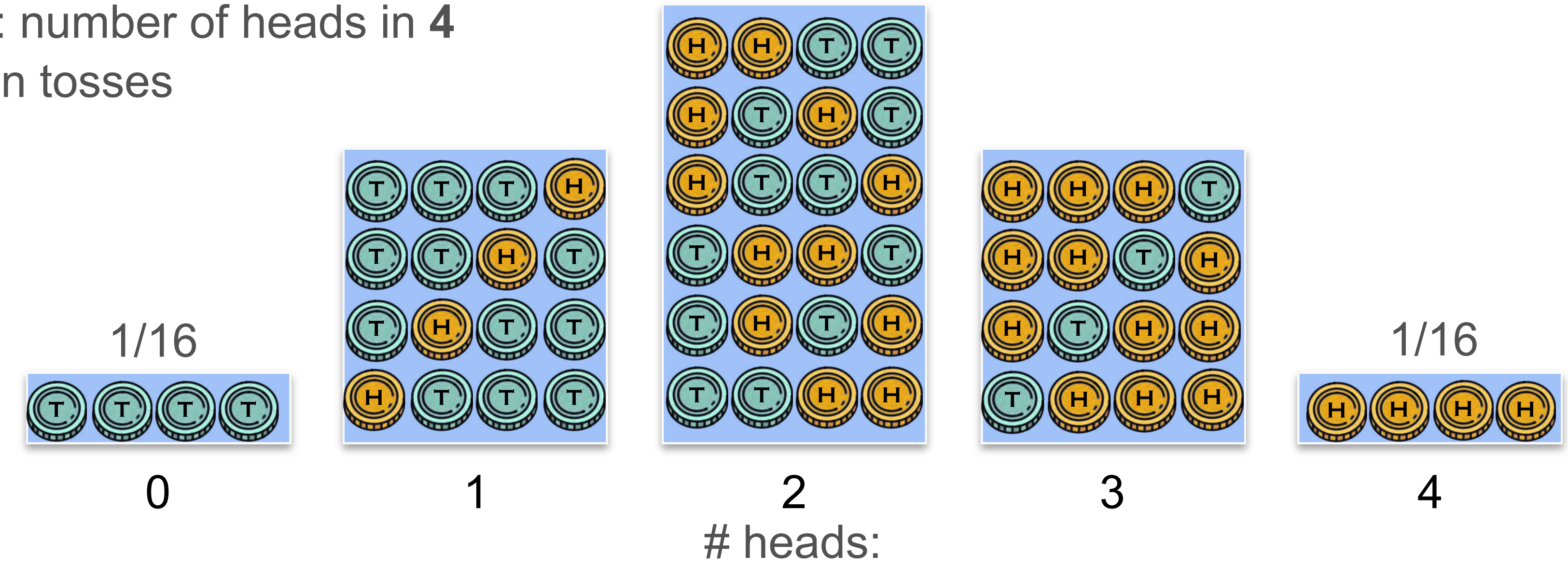

Discrete Distributions: Flip Four Coins

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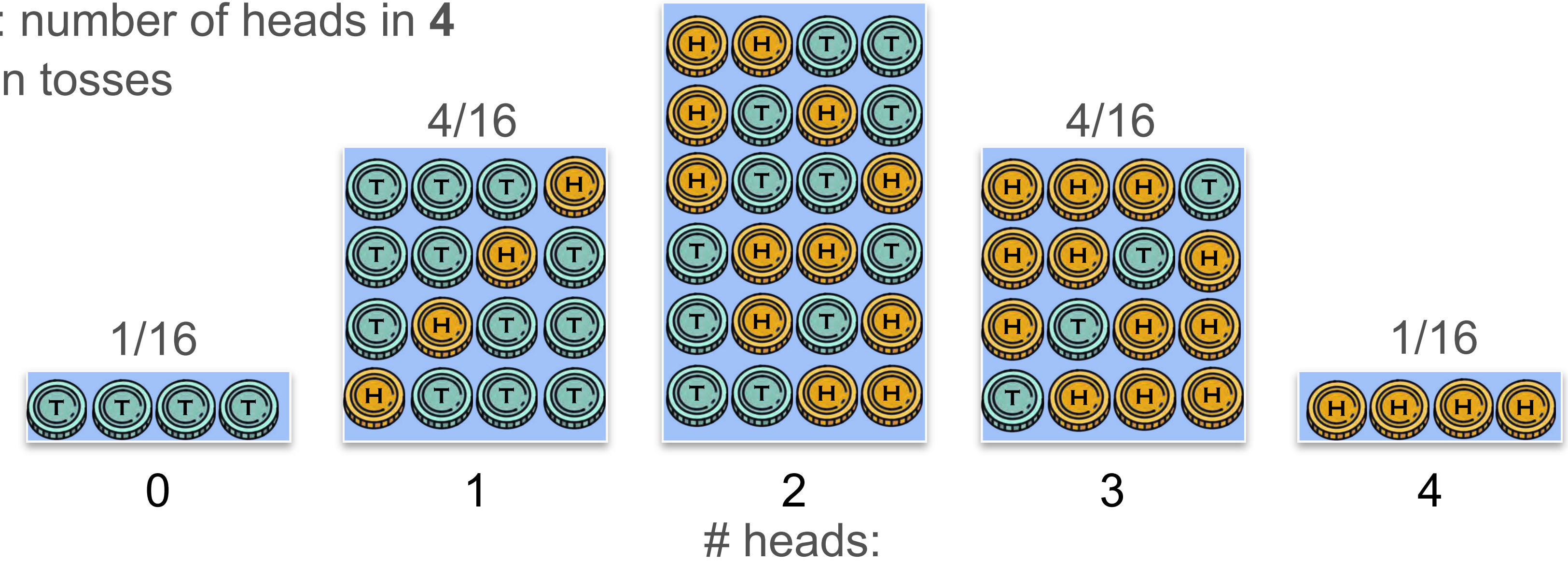
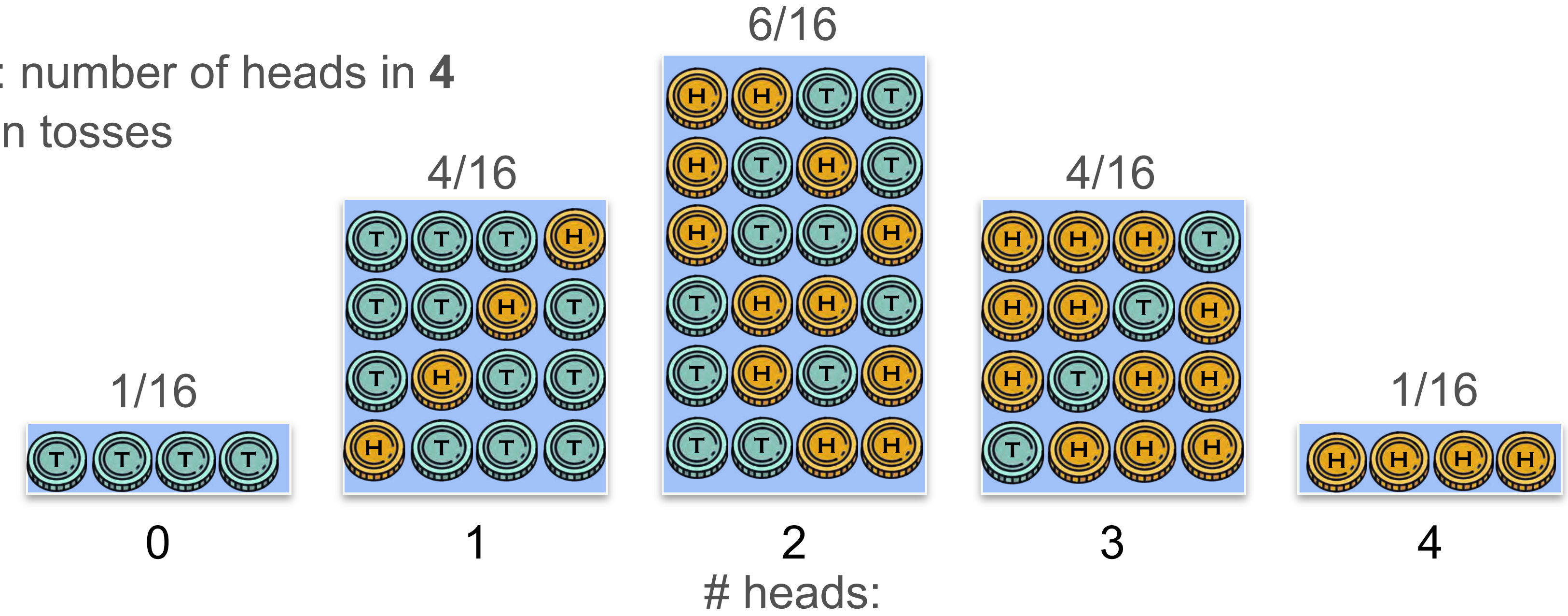
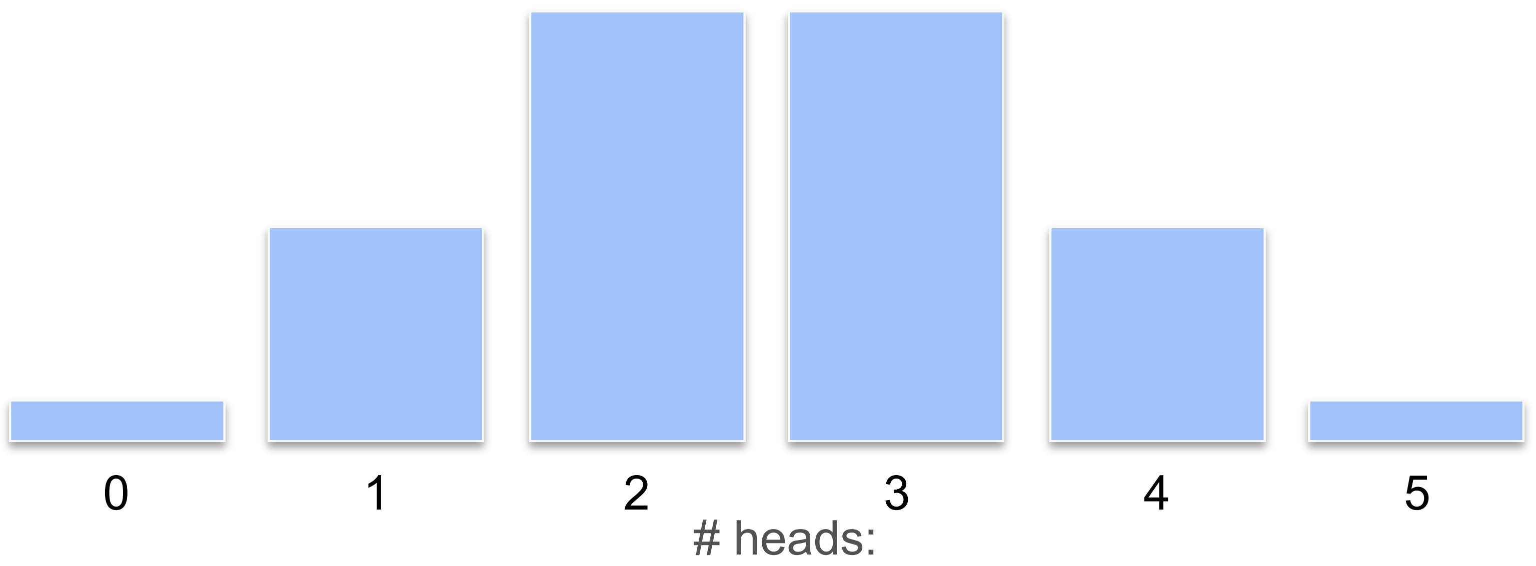


Discrete Distributions: Flip Four Coins

X_2: number of heads in **4** coin tosses

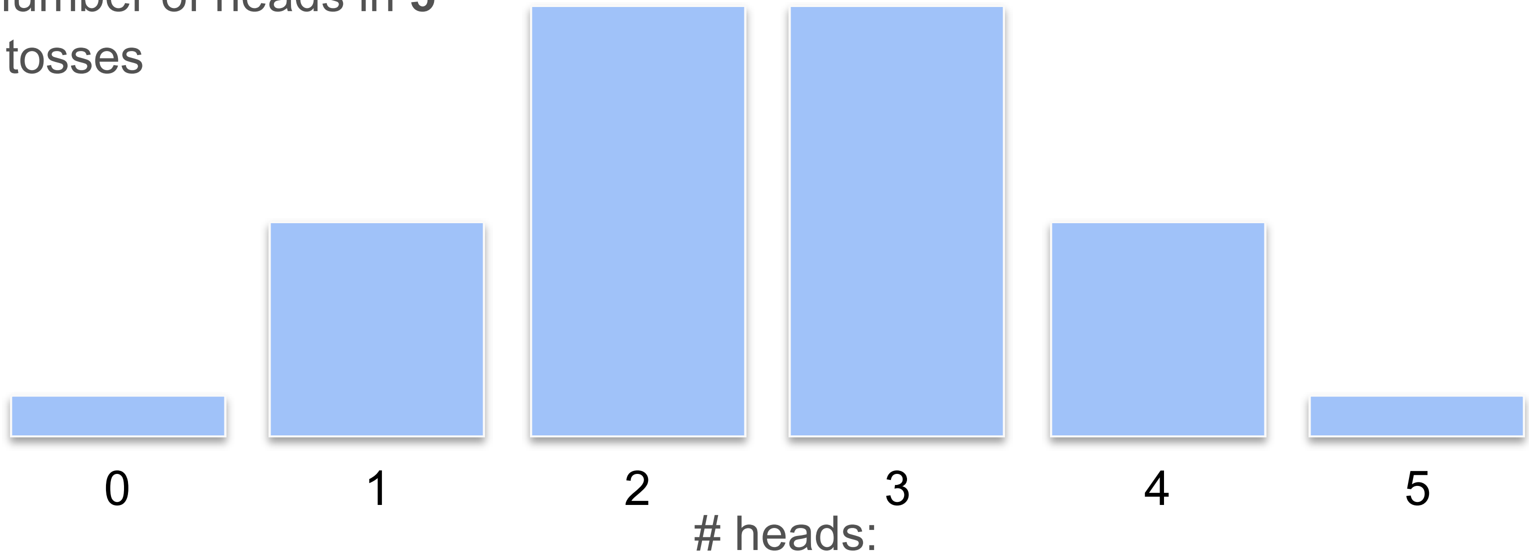


Discrete Distributions: Flip Five Coins



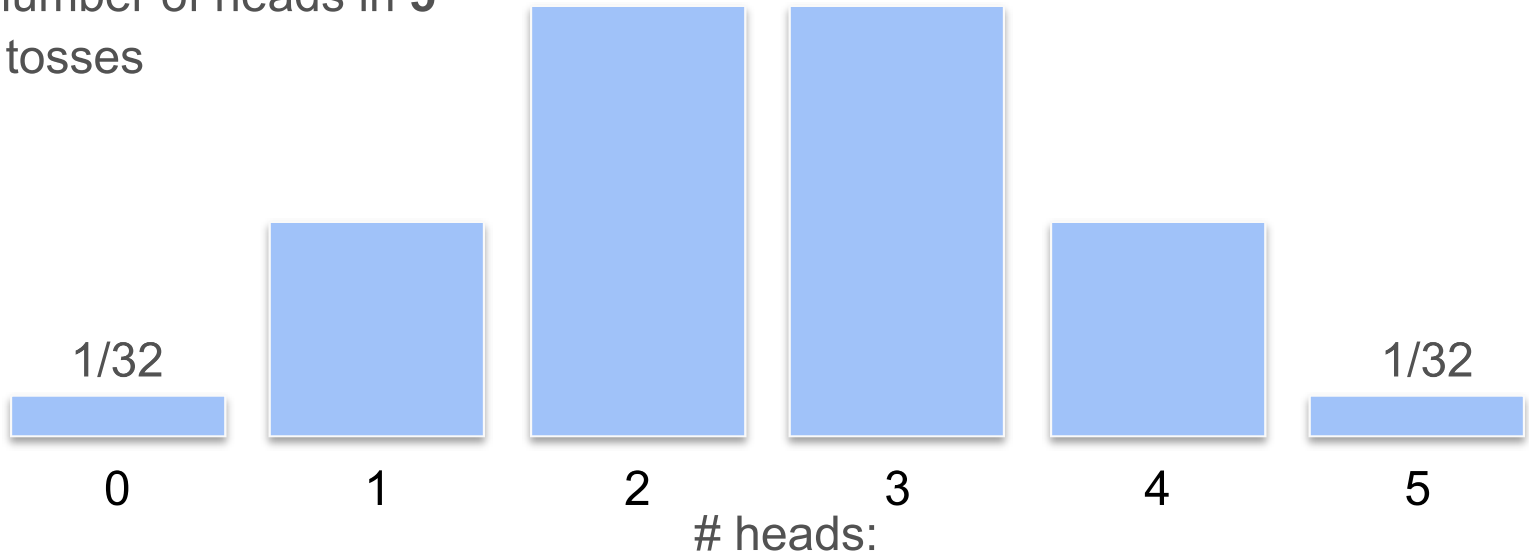
Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



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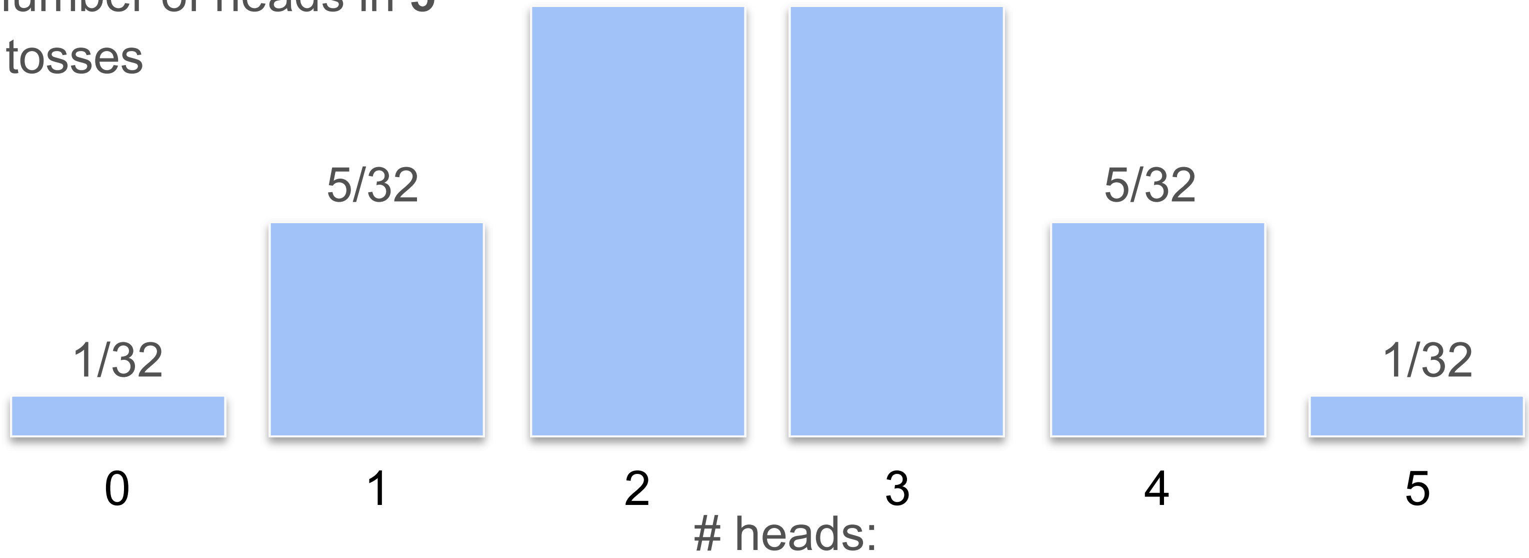
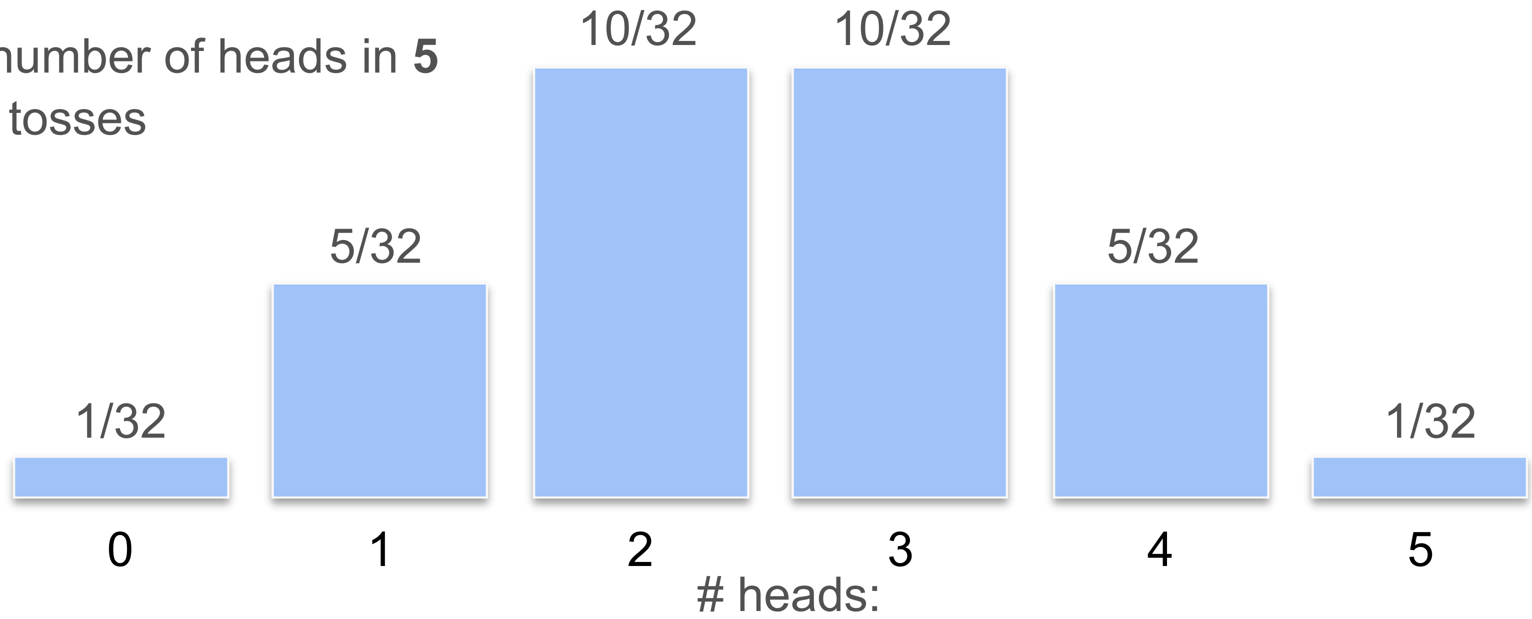
Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



Discrete Distributions: Flip Five Coins

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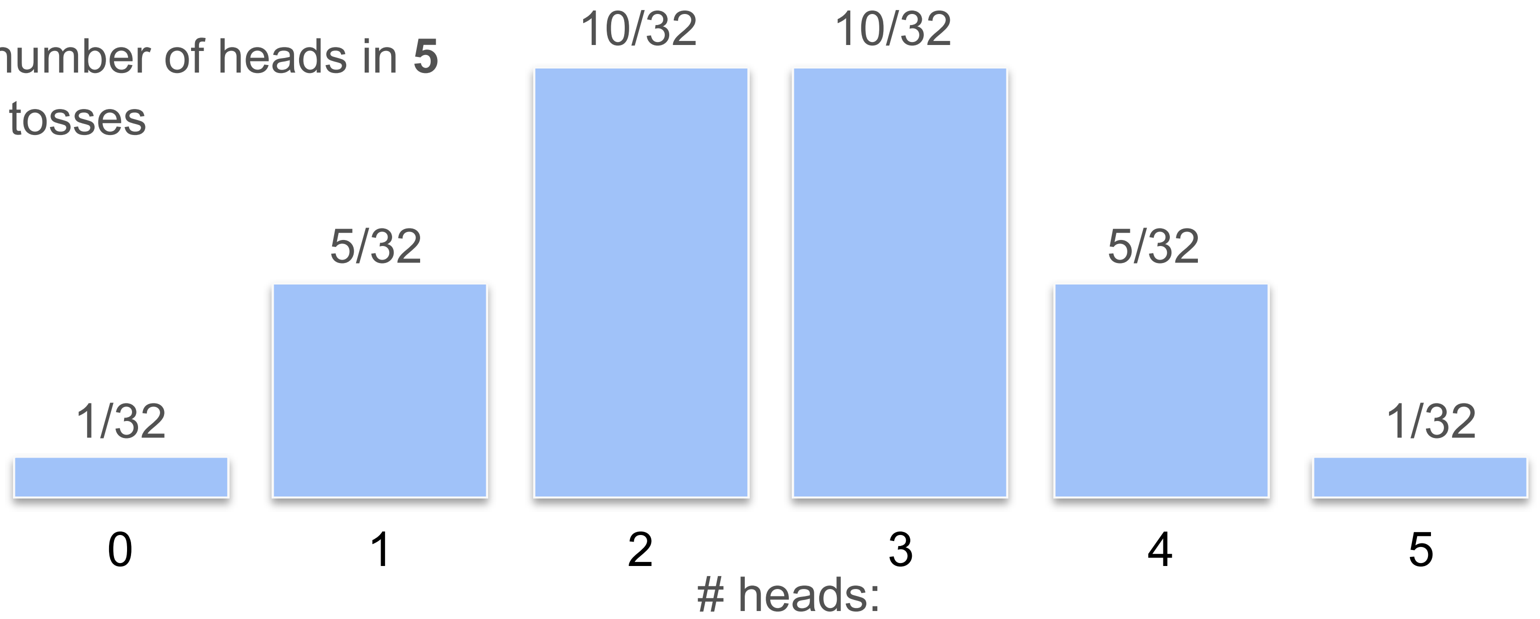

Discrete Distributions: Flip Five Coins

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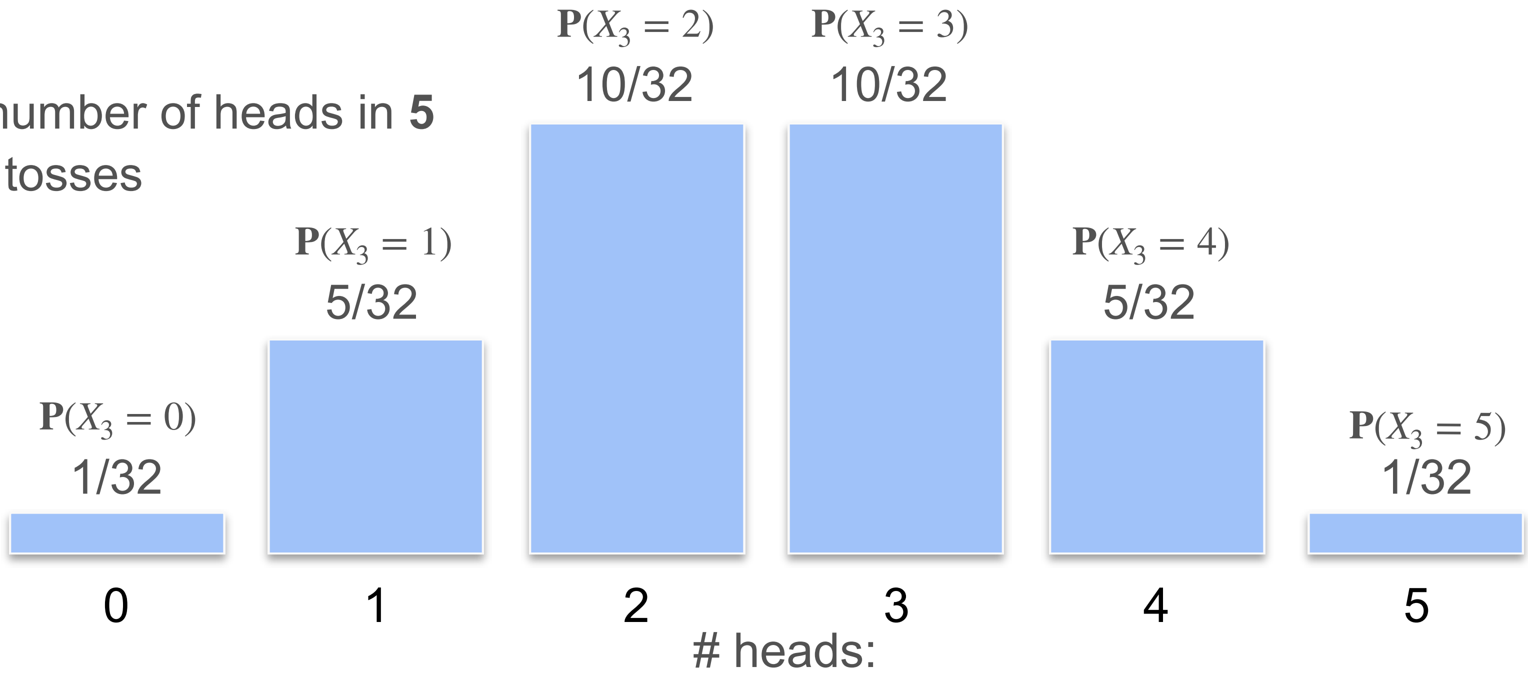

Discrete Distributions: Flip Five Coins

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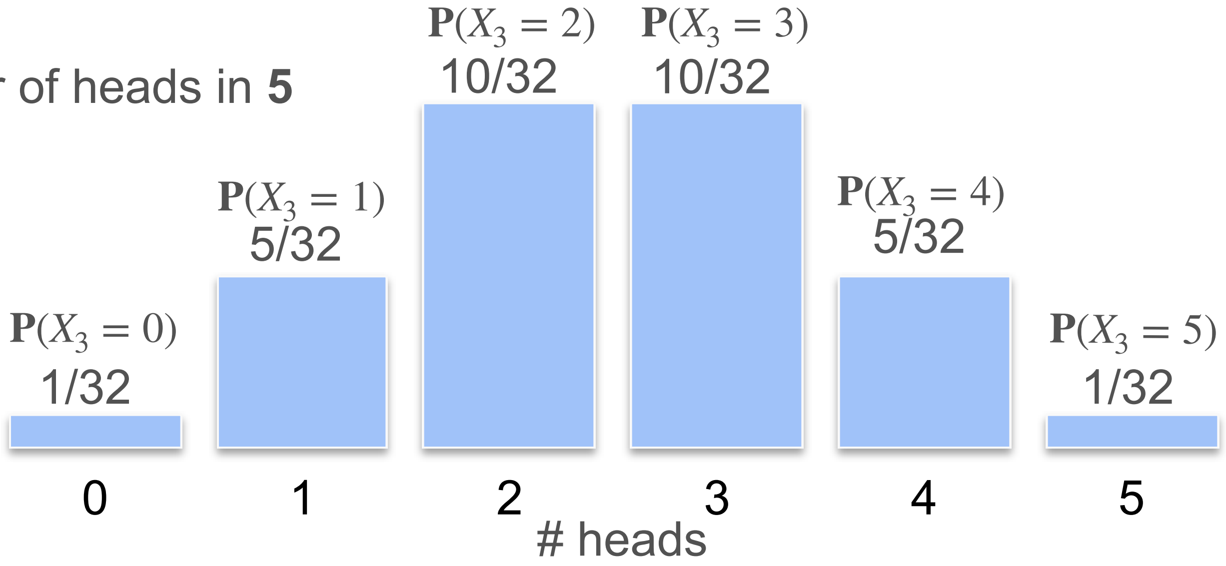

Discrete Distributions: Flip Five Coins

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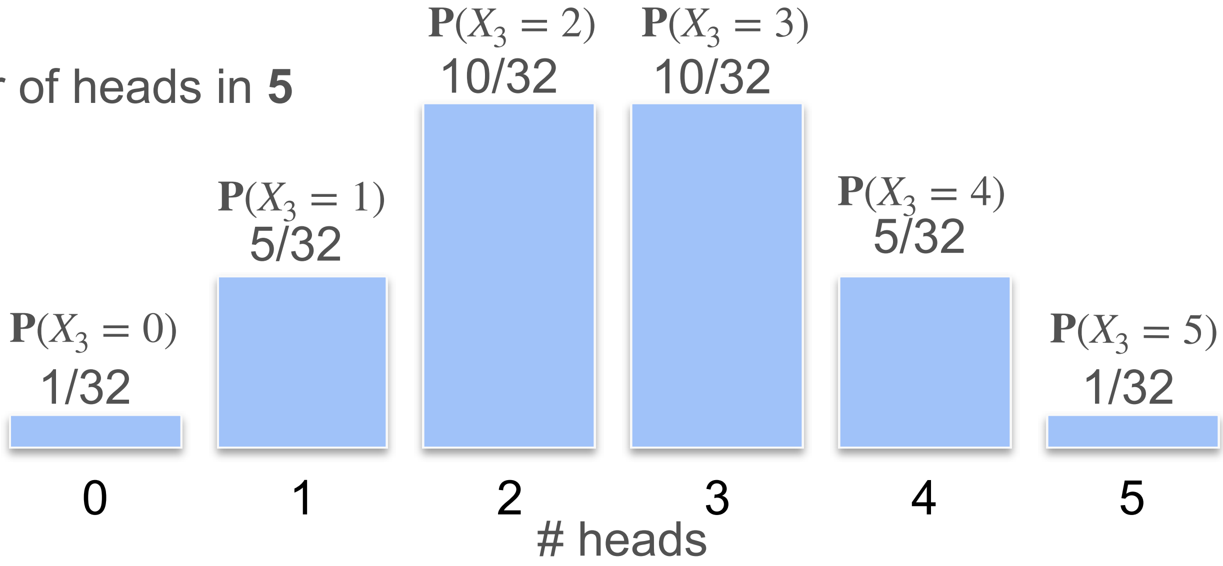
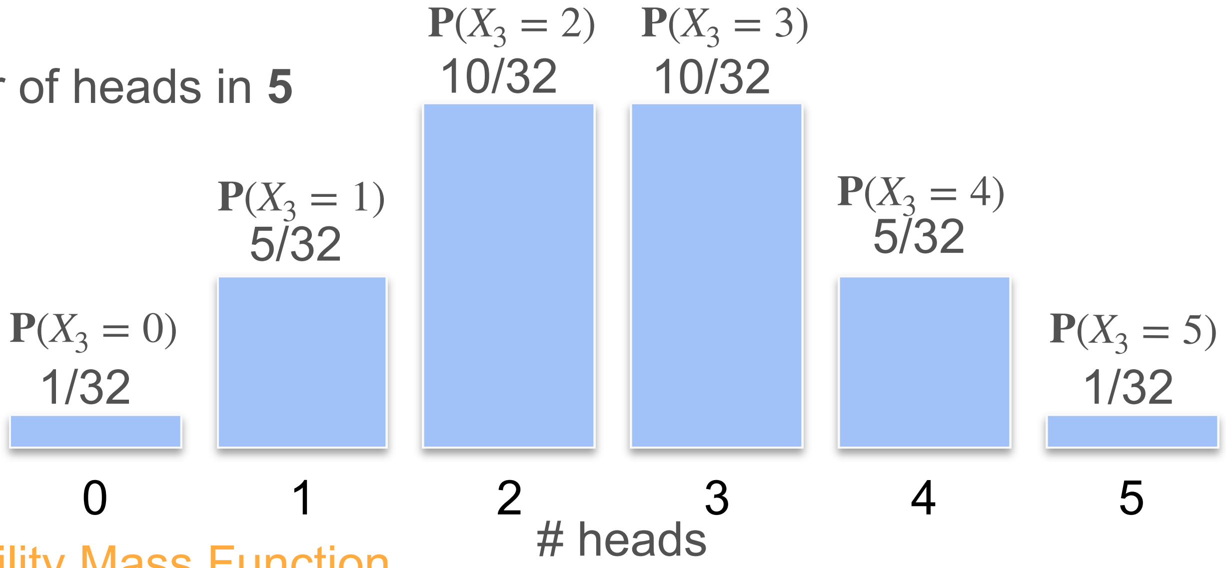


$$\mathbf{P}(X_3 = x), \quad x = 0,1,2,3,4,5$$

Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses

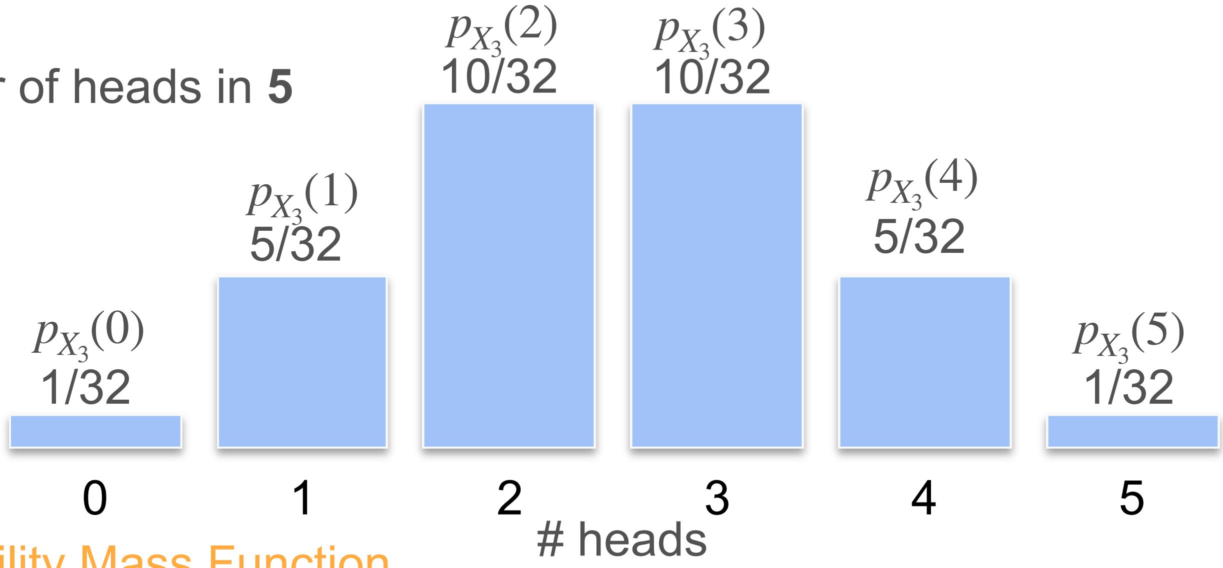


Probability Mass Function (PMF)

$$p_X(x) = \mathbf{P}(X_3 = x), \quad x = 0,1,2,3,4,5$$

Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



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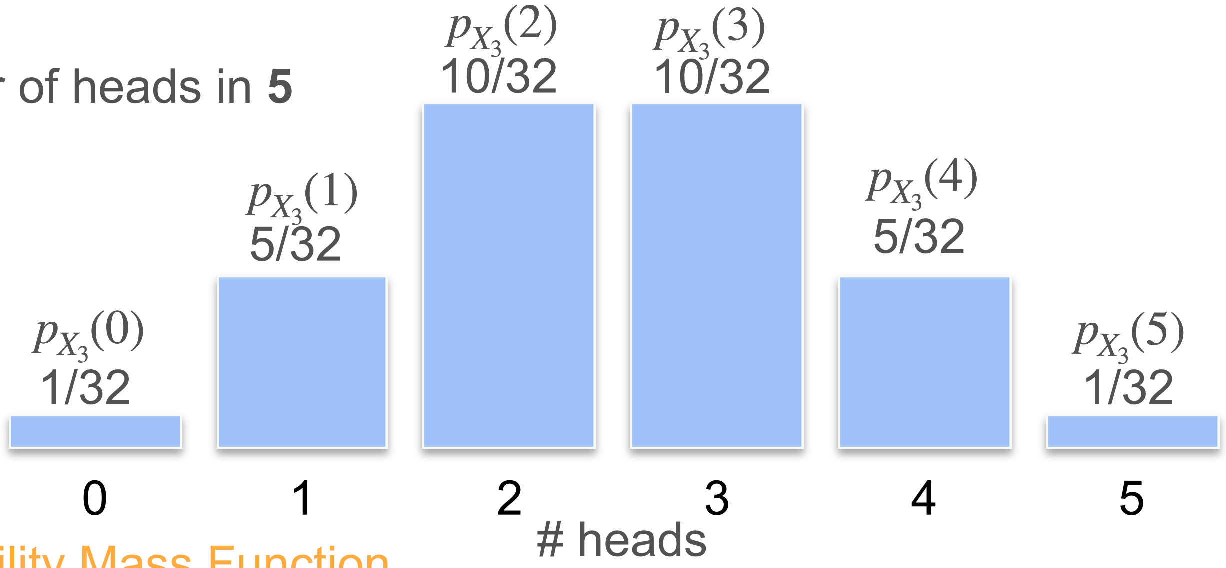
Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



Probability Mass Function (PMF)

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Discrete Distributions: Flip Five Coins

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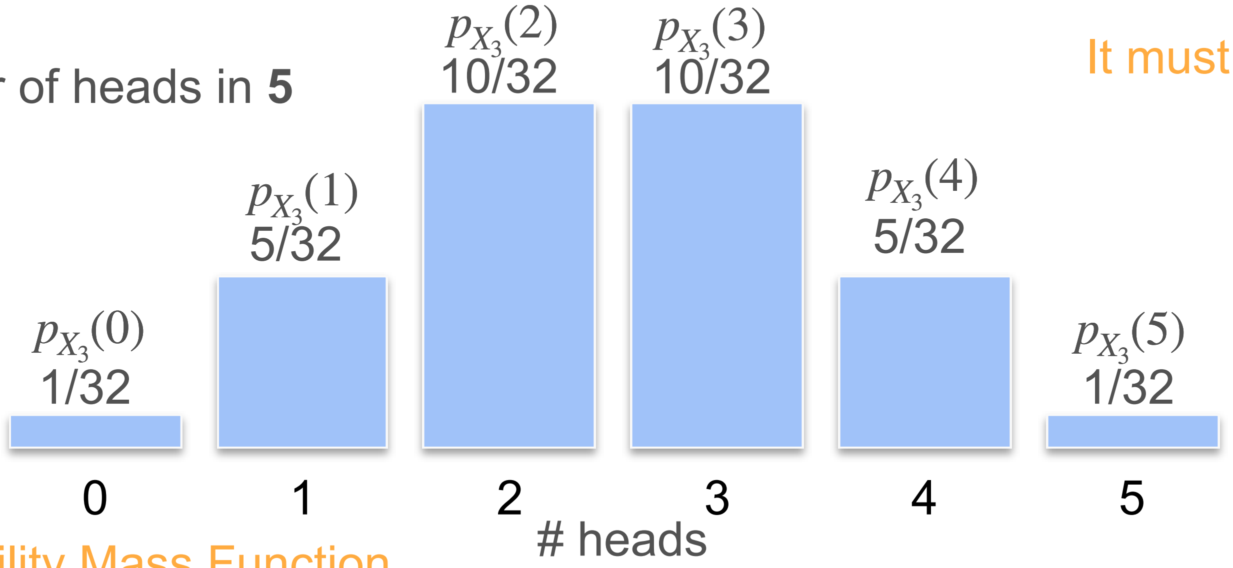


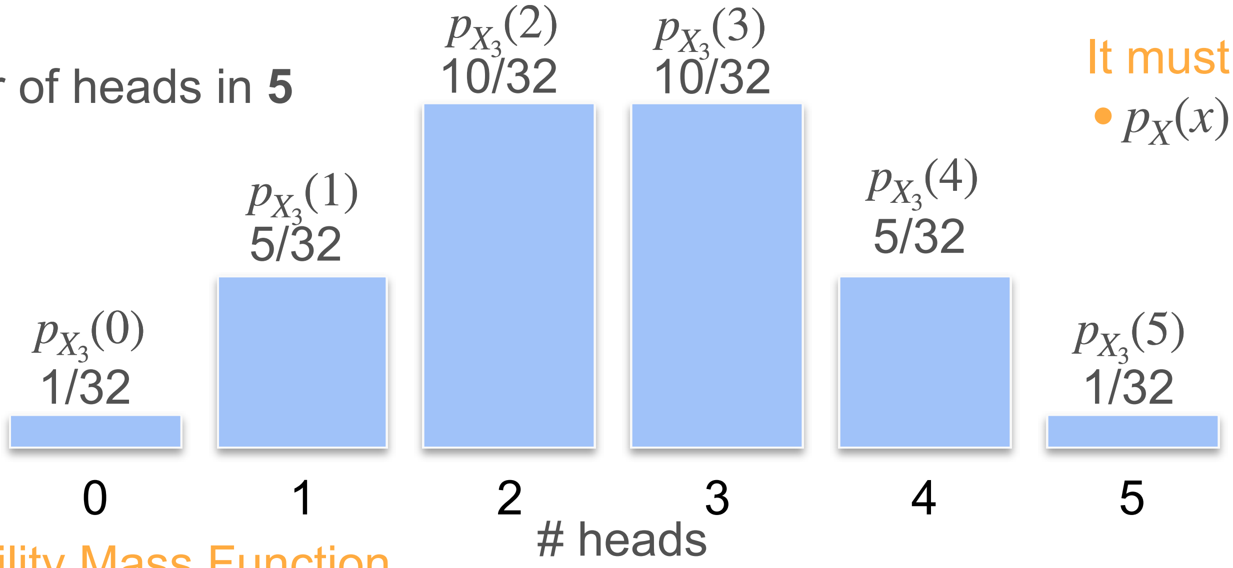
It must verify:

Probability Mass Function (PMF)

$$p_X(x) = \mathbf{P}(X_4 = x), \quad x = 0,1,2,3,4,5$$

Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



It must verify:

- $p_X(x) \geq 0$

Probability Mass Function (PMF)

$$p_X(x) = \mathbf{P}(X_4 = x), \quad x = 0,1,2,3,4,5$$

Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses

It must verify:

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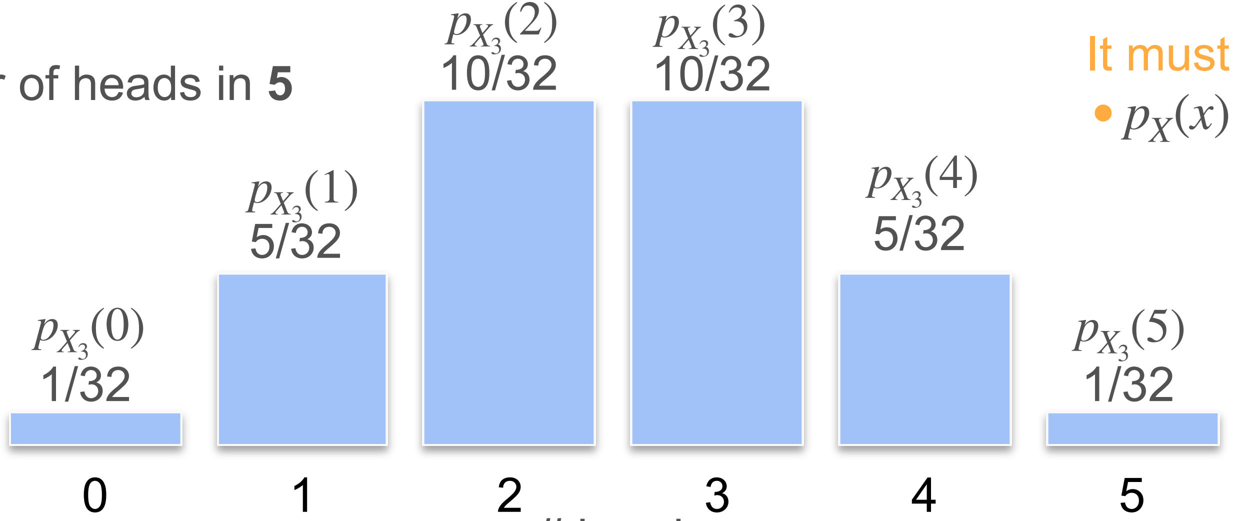


\# heads

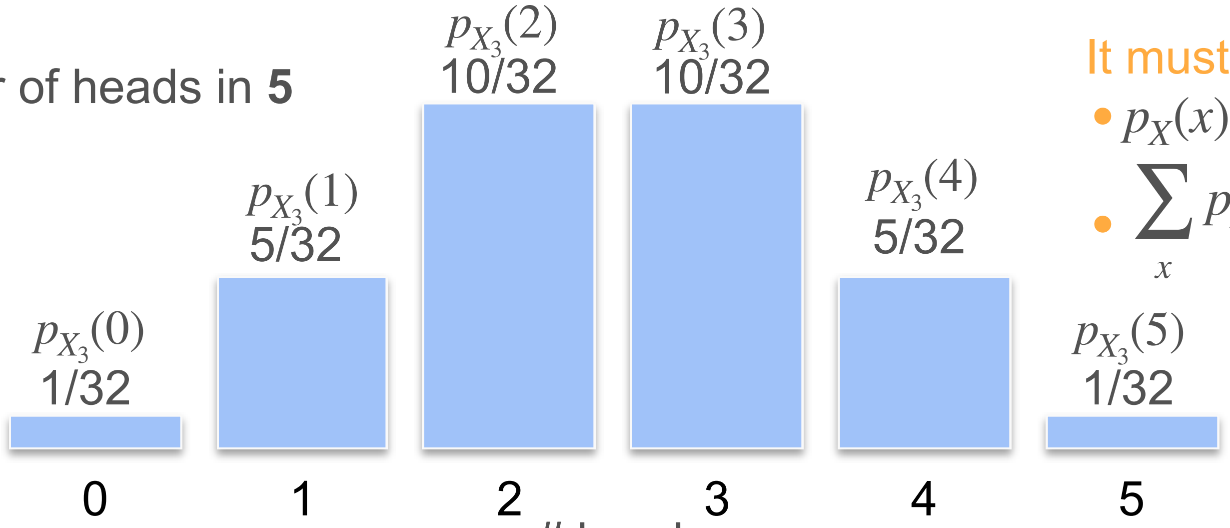
Probability Mass Function (PMF)

$$p_X(x) = \mathbf{P}(X_4 = x), \quad x = 0,1,2,3,4,5$$

$$p_{X_3}(0) + p_{X_3}(1) + p_{X_3}(2) + p_{X_3}(3) + p_{X_3}(4) + p_{X_3}(5) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = 1$$

Discrete Distributions: Flip Five Coins

X_3: number of heads in **5** coin tosses



It must verify:

- $p_X(x) \geq 0$
- $\sum_x p_X(x) = 1$

Probability Mass Function (PMF)

$$p_X(x) = \mathbf{P}(X_4 = x), \quad x = 0,1,2,3,4,5$$

$$p_{X_3}(0) + p_{X_3}(1) + p_{X_3}(2) + p_{X_3}(3) + p_{X_3}(4) + p_{X_3}(5) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} + \frac{10}{32} + \frac{5}{32} + \frac{1}{32} = 1$$

Binomial Distribution

Binomial Distribution

X_1, X_2, X_3, X_4 are very similar

They all represent **number of heads in n experiments**

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The way the probability distributes along the possible outcomes seems to have a similar pattern

Binomial Distribution

X_1, X_2, X_3, X_4 are very similar

They all represent **number of heads in n experiments**

The way the probability distributes along the possible outcomes seems to have a similar pattern

Could there be a **single model** to represent all this variables?

↓

Binomial distribution

DeepLearning.AI

Probability Distributions

Binomial Distribution

Binomial Distribution: Example

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

0.5 0.5 0.5 0.5 0.5

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

Binomial Distribution: Example

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$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

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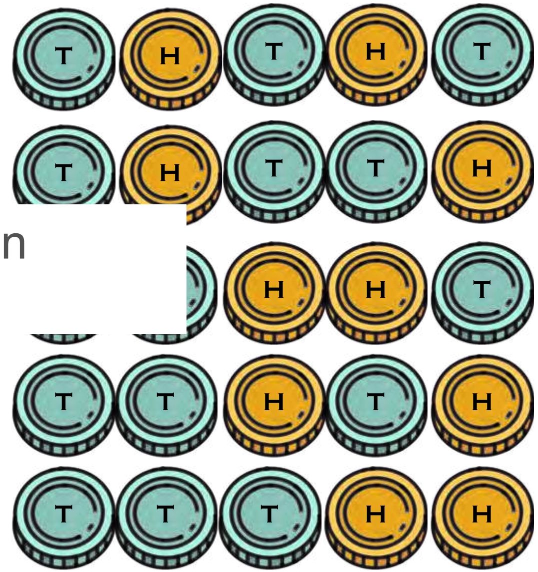
Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

Binomial Distribution: Example

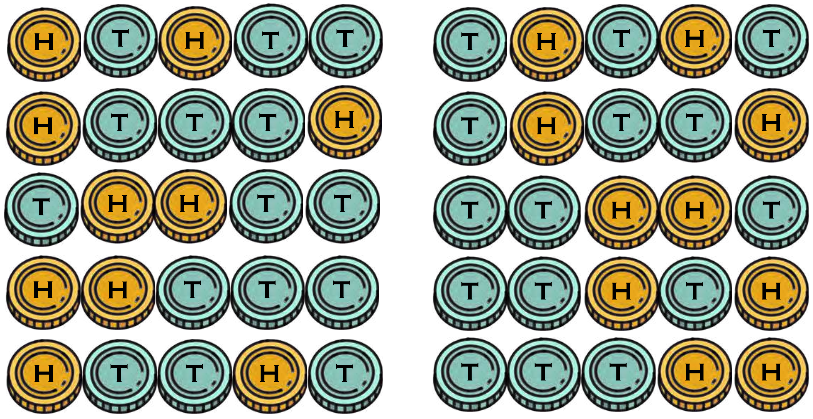
What is the probability that if I flip 5 coins, 2 of them land in heads?

10 ways to have 2 heads in 5 coin tosses



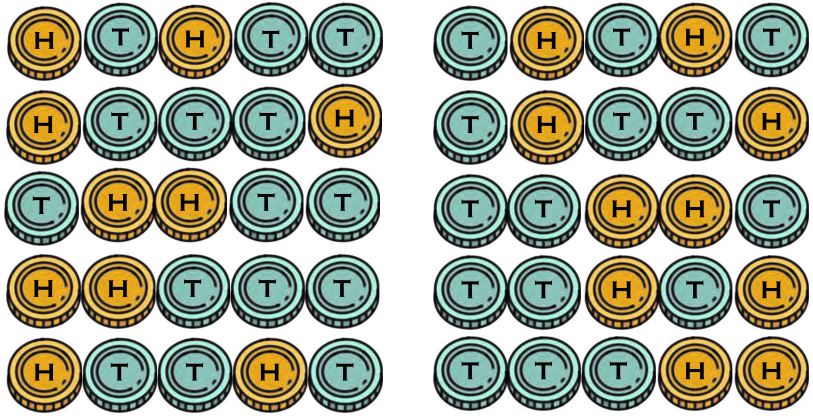
Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



Binomial Distribution: Example

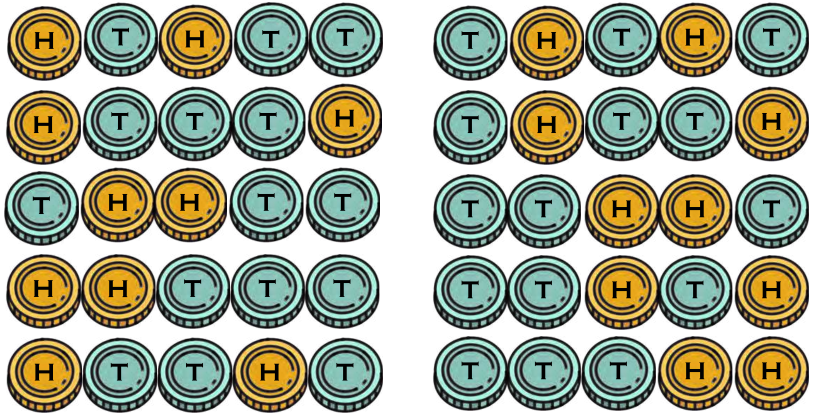
What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \rule{2cm}{0.4pt}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?

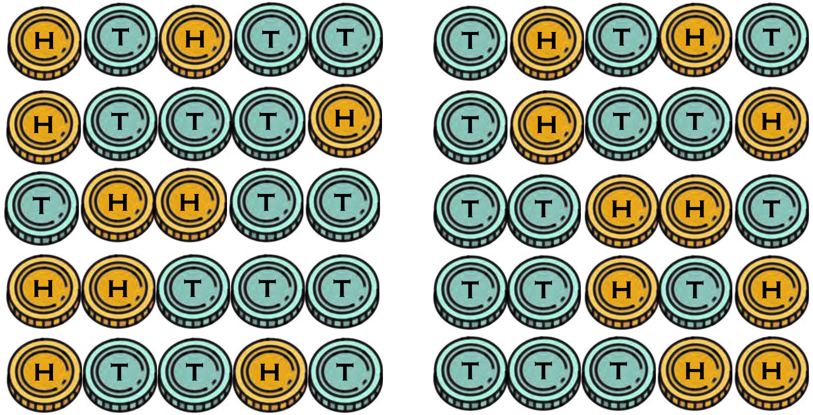


$$10 = \frac{5!}{\quad\quad}$$

Number of ways you can order 5 coins

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



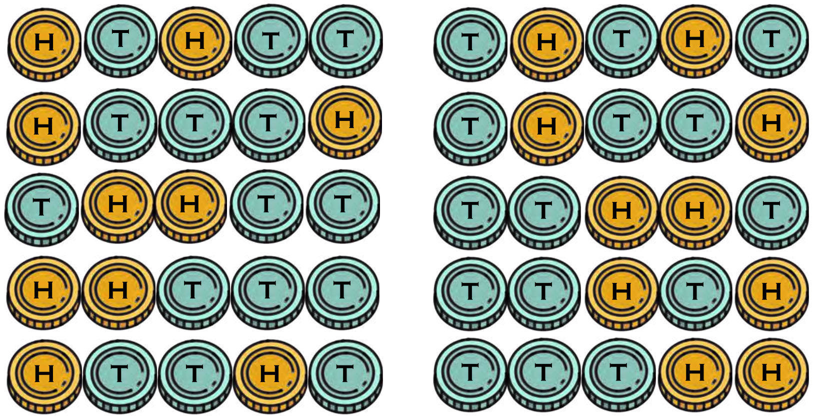
$$10 = \frac{5!}{2!}$$

Number of ways you can order 5 coins

Number of H

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



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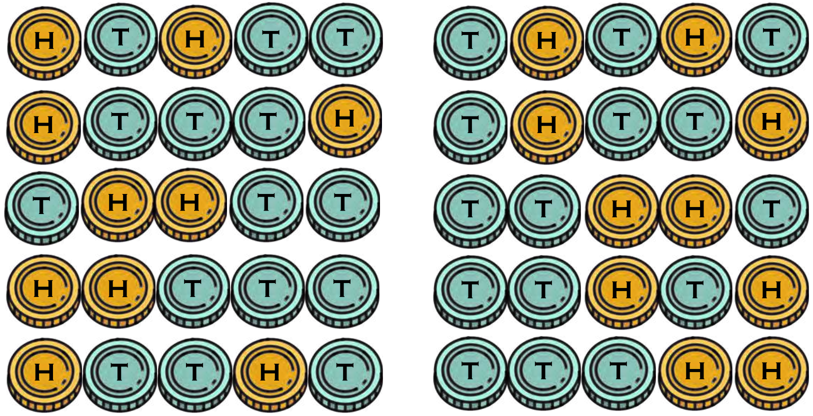
Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



Number of ways you can order 5 coins

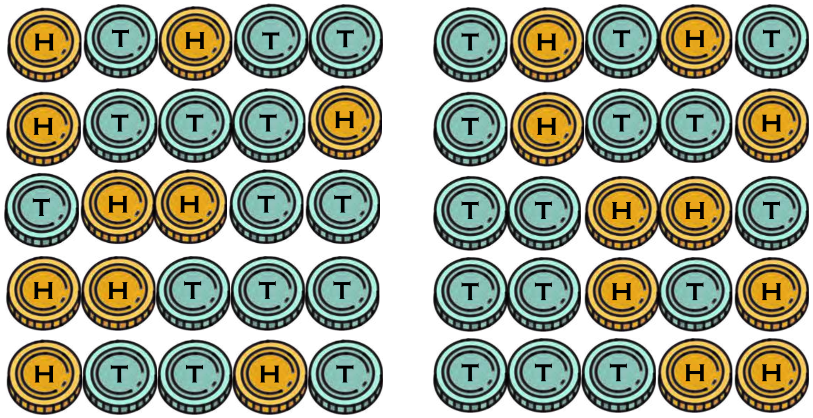
$$10 = \frac{5!}{2!(5-2)!}$$

Number of H

Number of T

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



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Binomial Distribution: Example

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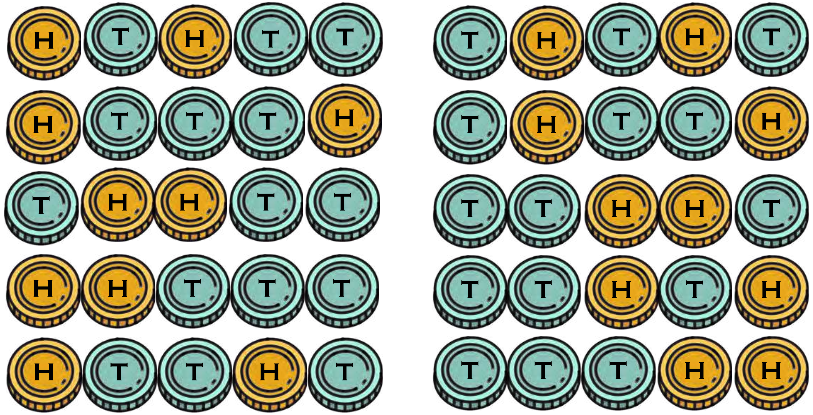

$$10 = \frac{5!}{2!(5-2)!} = \binom{5}{2}$$

Binomial coefficient

Number of ways you can get 2 heads in 5 coin tosses

Binomial Distribution: Binomial Coefficient

Binomial Distribution: Binomial Coefficient

In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

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Property**:**

$$\binom{n}{k} = \binom{n}{n-k}$$

Binomial Distribution: Binomial Coefficient

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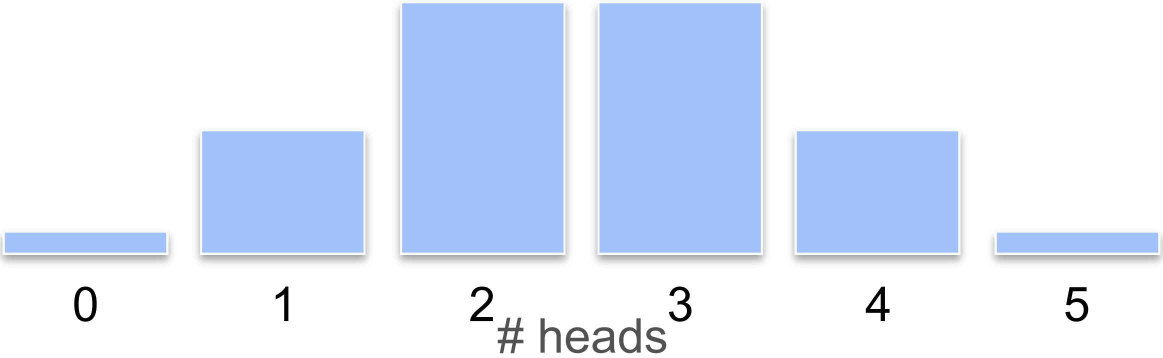
Property**:**

$$\binom{n}{k} = \binom{n}{n-k}$$

The PMF with a fair coin is symmetrical



Binomial Distribution: Binomial Coefficient

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Binomial Distribution: Binomial Coefficient

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Your coin has $\mathbf{P}(H) = p$

Binomial Distribution: Binomial Coefficient

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Event: $X = x$: x heads in 5 tosses

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\boxed{p^x}$$

Probability of seeing x heads

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

Probability of seeing $5 - x$ tails

$$\boxed{p^x}\boxed{(1-p)^{5-x}}$$

Probability of seeing x heads

Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

Probability of seeing $5 - x$ tails

All the possible orders

$$\boxed{\binom{5}{x}}\boxed{p^x}\boxed{(1-p)^{5-x}}$$

Probability of seeing x heads

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1-p)^{5-x}$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

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X **follows a binomial distribution**

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

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X **follows a binomial distribution**

$$X \sim Binomial(\boxed{5},\boxed{p})$$

Number of flips

$\mathbf{P}(H)$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

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Number of flips

$\mathbf{P}(H)$

Binomial Distribution

Binomial Distribution

$$\boxed{\begin{array}{c} n = 5 \\ p = 0.5 \end{array}}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

Binomial Distribution

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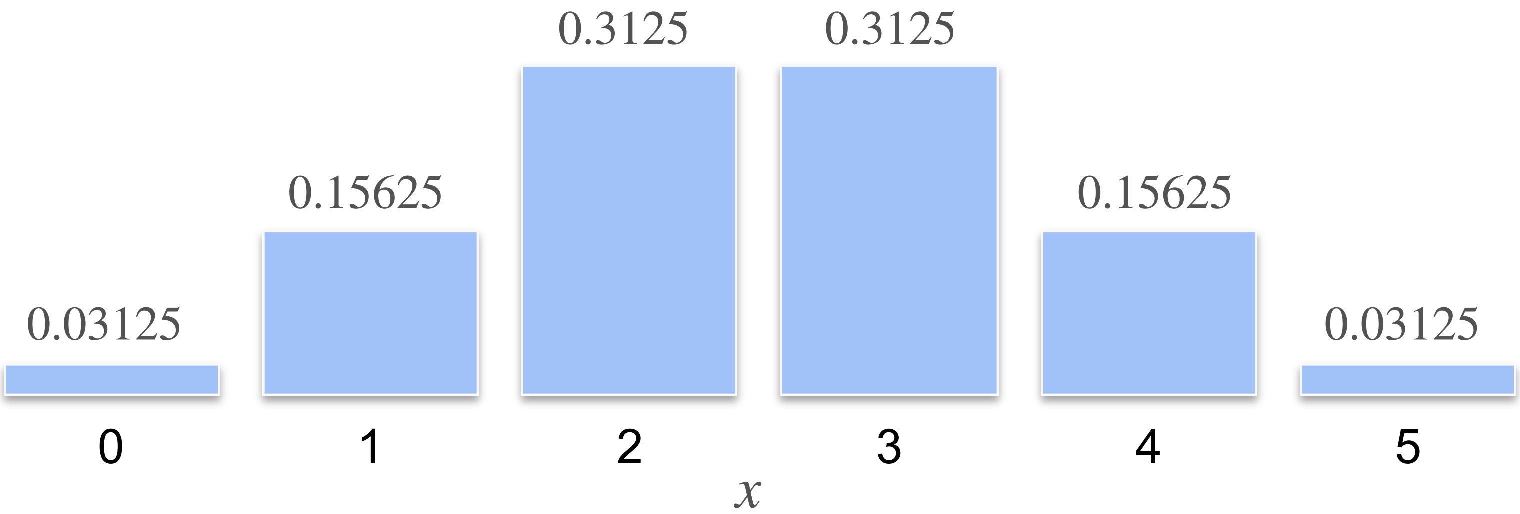
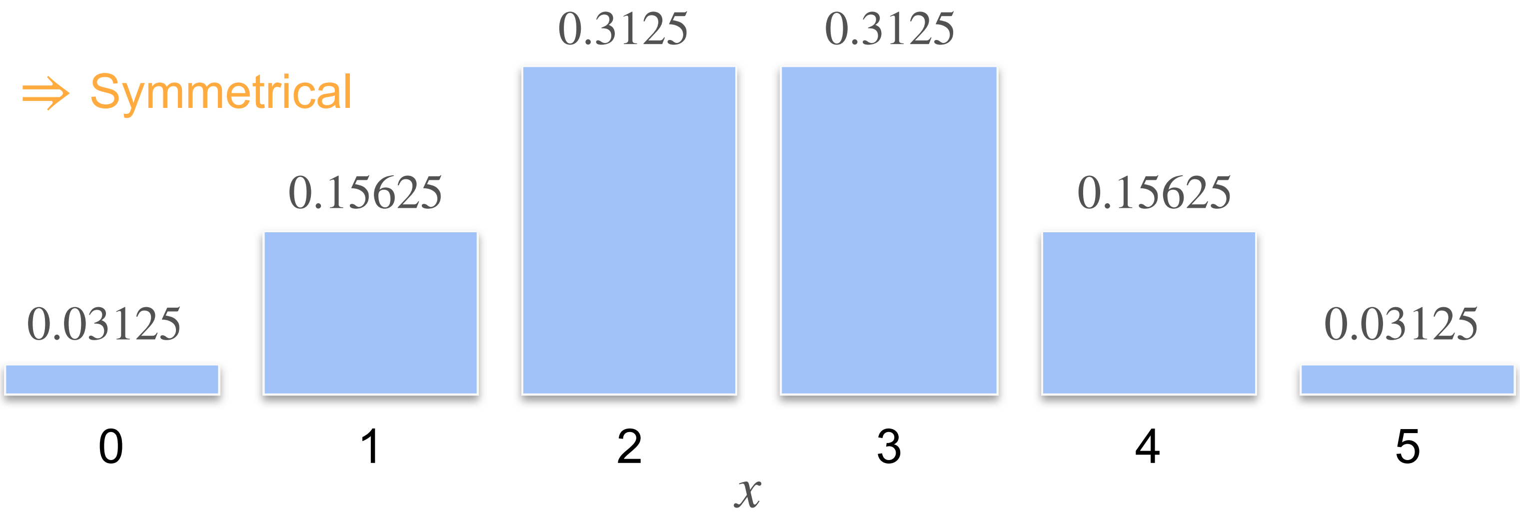


Binomial Distribution

$$n = 5$$
$$p = 0.5$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

$p = 0.5 \Rightarrow$ Symmetrical



Binomial Distribution

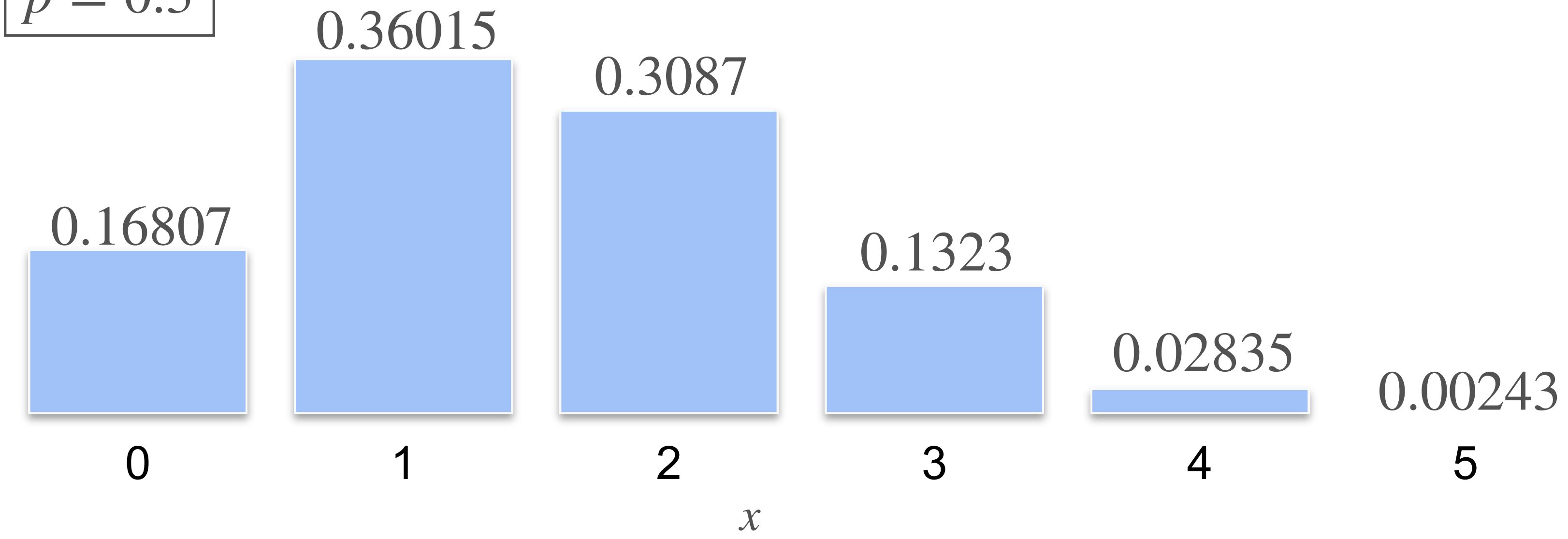
$$\boxed{\begin{array}{c} n = 5 \\ p = 0.3 \end{array}}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$

Binomial Distribution

$n = 5$
$p = 0.3$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$



Binomial Distribution

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

$$X \sim Binomial(5,p)$$

Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,1,2,\ldots,n$$

$$X \sim Binomial(n, p)$$

Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,1,2,\ldots,n$$

$$X \sim Binomial(n, p)$$

n and p are called the **parameters** of the binomial distribution

Binomial Coefficient

Binomial Coefficient

Pick 1st number

n options

Binomial Coefficient

Pick 1st number   Pick 2nd number

n options $n-1$ options

Binomial Coefficient

Pick 1st number Pick 2nd number Pick 3rd number

n options $n-1$ options $n-2$ options

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number

n options $\quad n-1$ options $\quad n-2$ options $\quad \ldots$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options $\quad n-1$ options $\quad n-2$ options $\quad \ldots \quad n-(k-1)$options

Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — Pick k-th number

n options — $n-1$ options — $n-2$ options — $\dots$ — $n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\dots,k$

$2,1,3,4,\dots,k$

$5,1,3,k,\dots,2$

$n,1,3,4,\dots,2$

$\dots$

Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — … — Pick k-th number

n options — $n-1$ options — $n-2$ options — $\dots$ — $n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\dots,k$

$2,1,3,4,\dots,k$

$5,1,3,k,\dots,2$

$n,1,3,4,\dots,2$

$\dots$

$$n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → $n-1$ options → $n-2$ options → … → $n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

Some of them repeat

$$n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))$$

Binomial Coefficient

Binomial Coefficient

Pick 1st number

n options

Binomial Coefficient

Pick 1st number | Pick 2nd number

n options → n-1 options

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number
n options	n-1 options	n-2 options

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number	
n options	n-1 options	n-2 options	...

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options | n-1 options | n-2 options | ... | n-(k-1) options

Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options | n-1 options | n-2 options | ... | n-(k-1) options

Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

$$n * (n-1) * (n-2) * \dots * (n-(k-1))$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

5, 1, 3, k, ... 2

n, 1, 3, 4, ... 2

...

$$n * (n-1) * (n-2) * ... * (n-(k-1))$$

Some of them repeat

Binomial Coefficient

Binomial Coefficient

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st Pick 2nd

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd
1,2,3,4			
1,2,4,3			
1,3,2,4			
1,3,4,2			
…			
4,3,2,1			

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4				
1,2,4,3				
1,3,2,4				
1,3,4,2				
...				
4,3,2,1				

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options			

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options		

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options	2 options	

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

…

4,3,2,1

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options	2 options	1 option

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

Binomial Coefficient

Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
4 options	3 options	2 options	1 option

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

$$4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

Binomial Coefficient

Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
4 options	3 options	2 options	1 option

1,2,3,4

1,2,4,3

1,3,2,4

1,3,4,2

...

4,3,2,1

$$4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

For five numbers:

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$$

Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
	4 options	3 options	2 options	1 option

$1,2,3,4$

$1,2,4,3$

$1,3,2,4$

$1,3,4,2$

$\dots$

$4,3,2,1$

$$4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

For five numbers:

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$$

General solution: $k!$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options $\quad n-1$ options $\quad n-2$ options $\quad \dots \quad n-(k-1)$options

Ordered sets of length k

$1,2,3,4,\dots,k$

$2,1,3,4,\dots,k$

$5,1,3,k,\dots,2 \qquad n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))$

$n,1,3,4,\dots,2$

$\dots$

Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — Pick k-th number

n options — $n-1$ options — $n-2$ options — $\dots$ — $n-(k-1)$options

Ordered sets of length k → Unordered sets of length k

$1,2,3,4,\dots,k$

$2,1,3,4,\dots,k$

$5,1,3,k,\dots,2$

$n,1,3,4,\dots,2$

$\dots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options $\quad$ $n-1$ options $\quad$ $n-2$ options $\quad \ldots \quad$ $n-(k-1)$options

Ordered sets of length k $\longrightarrow$ Unordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!} = \_\_\_\_\_\_$$

Binomial Coefficient

Pick 1st number　Pick 2nd number　Pick 3rd number　Pick k-th number

n options　$n - 1$ options　$n - 2$ options　$\ldots$　$n - (k - 1)$options

Ordered sets of length k ⟶ Unordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$\frac{n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-(k-1))}{k!} = \frac{n!}{(n-k)!}$$

Binomial Coefficient

Pick 1st number — Pick 2nd number — Pick 3rd number — Pick k-th number

n options — $n-1$ options — $n-2$ options $\ldots$ $n-(k-1)$options

Ordered sets of length k → Unordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!}=\frac{n!}{(n-k)!\,k!}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → $n-1$ options → $n-2$ options → $\ldots$ → $n-(k-1)$options

Ordered sets of length k → Unordered sets of length k

$1,2,3,4,\ldots,k$

$2,1,3,4,\ldots,k$

$5,1,3,k,\ldots,2$

$n,1,3,4,\ldots,2$

$\ldots$

$$\frac{n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot(n-(k-1))}{k!}=\frac{n!}{(n-k)!\,k!}=\binom{n}{k}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → n-1 options → n-2 options → ... → n-(k-1) options

Ordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

n * (n-1) * (n-2) * ... * (n-(k-1))

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → n-1 options → n-2 options → ... → n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \dots * (n-(k-1))}{k!} = \frac{\quad}{\quad}$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!} = \frac{n!}{(n-k)!}$$

Binomial Coefficient

Pick 1st number | Pick 2nd number | Pick 3rd number | Pick k-th number

n options → n-1 options → n-2 options → ... → n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!} = \frac{n!}{(n-k)! * k!}$$

Binomial Coefficient

Pick 1st number	Pick 2nd number	Pick 3rd number		Pick k-th number
n options	n-1 options	n-2 options	...	n-(k-1) options

Ordered sets of length k → Unordered sets of length k

1, 2, 3, 4, ... k

2, 1, 3, 4, ... k

4, 1, 3, k, ... 2

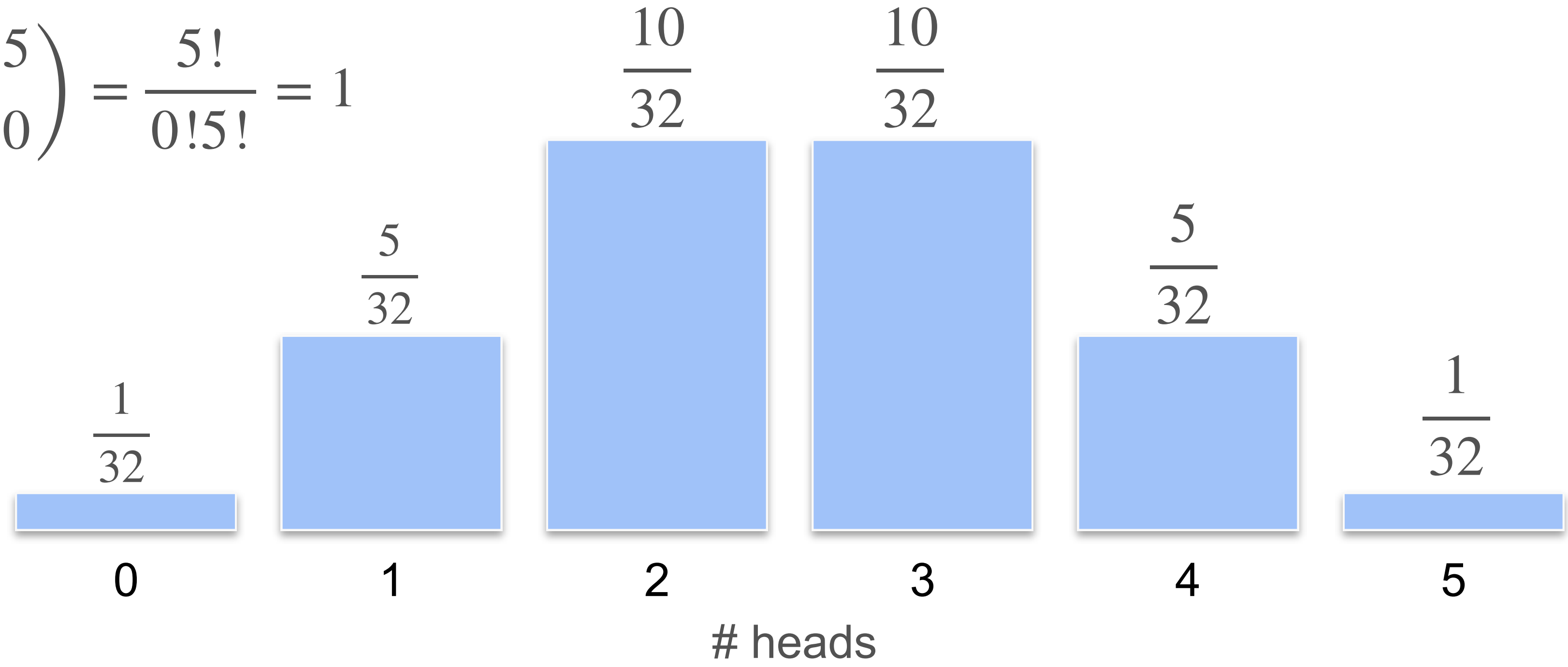
n, 1, k, 4, ... 2

...

$$\frac{n * (n-1) * (n-2) * \ldots * (n-(k-1))}{k!} = \frac{n!}{(n-k)! * k!} = \binom{n}{k}$$

Binomial Distribution: Fair Coins

$$\binom{5}{0} = \frac{5!}{0!5!} = 1$$



Binomial Distribution: Fair Coins

$50\,\%$

$50\,\%$

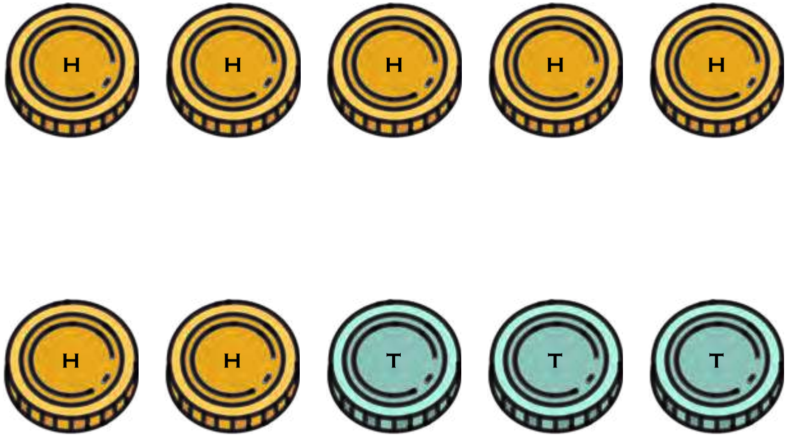
$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

Binomial Distribution: Biased Coins

30 %

70 %



Binomial Distribution: Biased Coins

30 %

70 %

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$

Binomial Distribution: Biased Coins

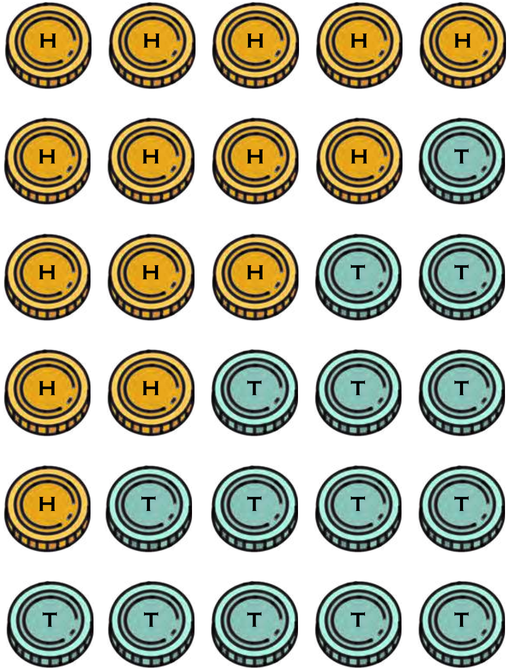
30 %

70 %

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.01323$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7$$

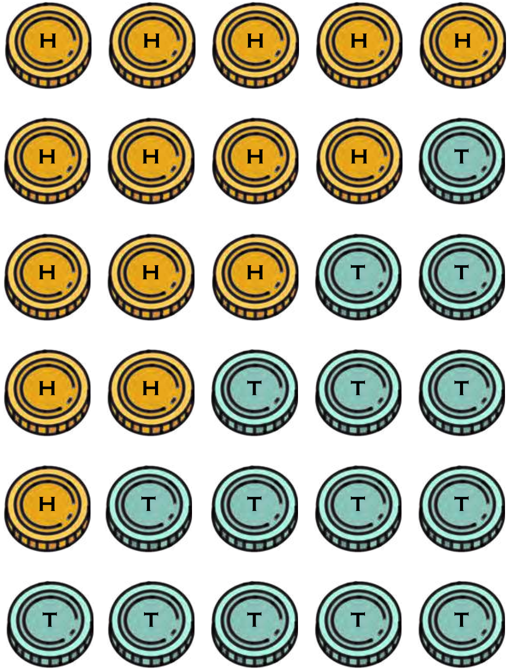
$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7$$

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

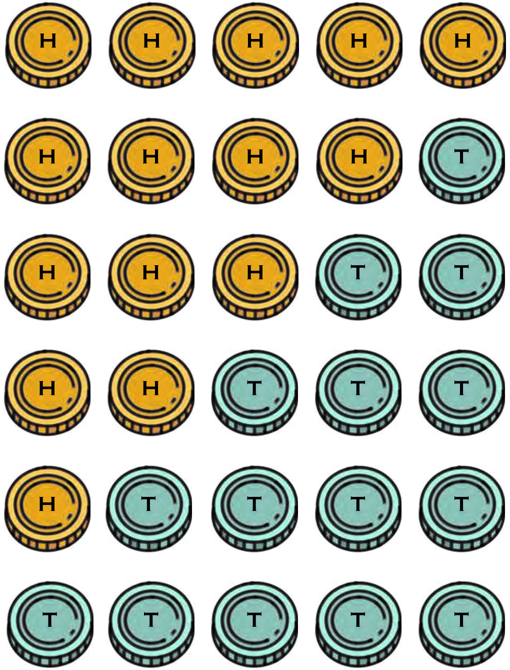
$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

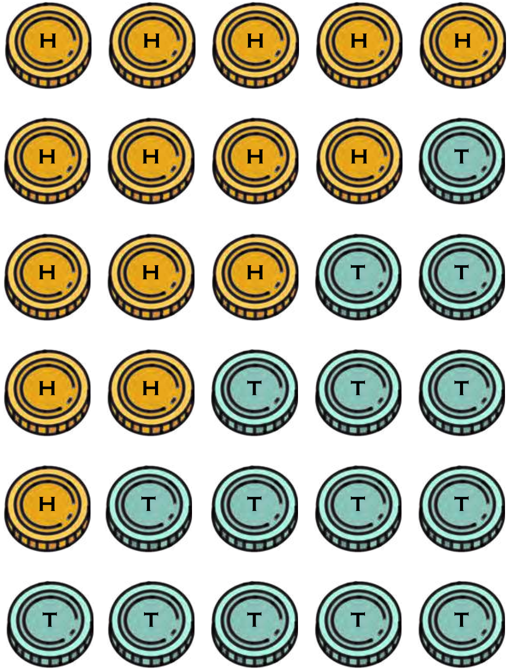
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

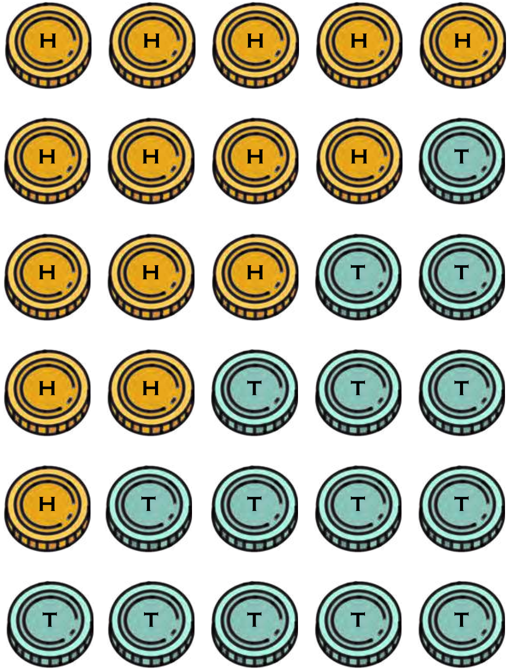
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 =$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 =$$

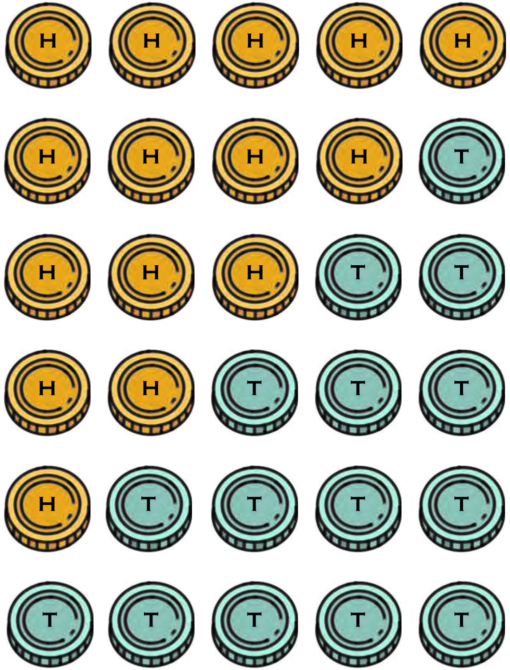
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 =$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 = 0.3^4 \cdot 0.7^1$$

$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 = 0.3^3 \cdot 0.7^2$$

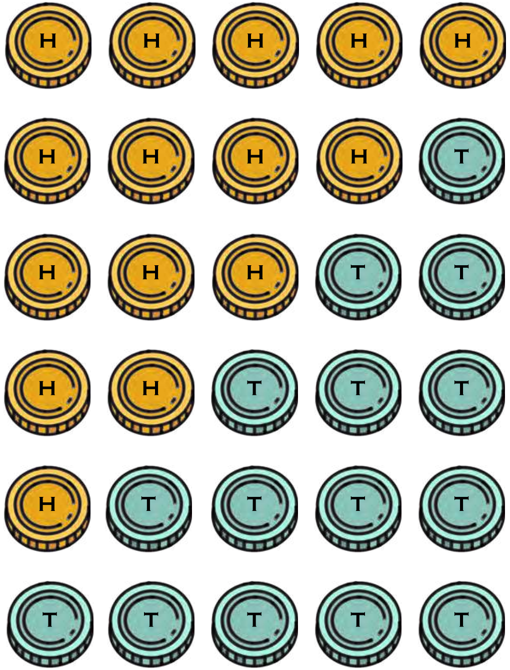
$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^2 \cdot 0.7^3$$

$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^1 \cdot 0.7^4$$

$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$$0.3^5 \cdot 0.7^0$$

$$0.3^4 \cdot 0.7^1$$

$$0.3^3 \cdot 0.7^2$$

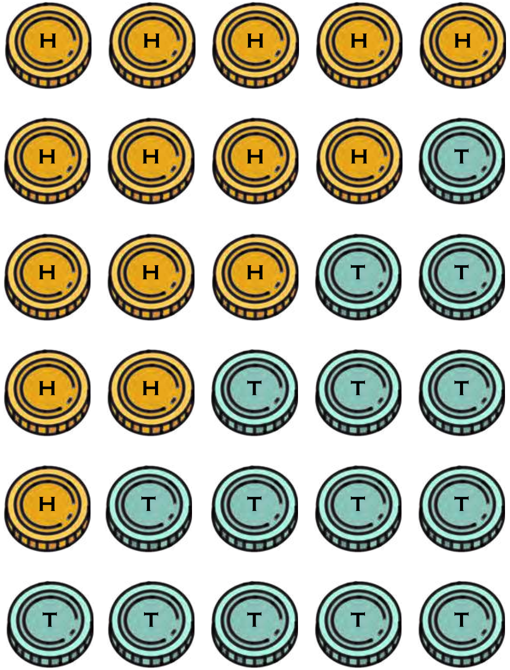
$$0.3^2 \cdot 0.7^3$$

$$0.3^1 \cdot 0.7^4$$

$$0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins



$0.3^5 \cdot 0.7^0$

$0.3^4 \cdot 0.7^1$

$0.3^3 \cdot 0.7^2$

$0.3^2 \cdot 0.7^3$

$0.3^1 \cdot 0.7^4$

$0.3^0 \cdot 0.7^5$

$$= 0.3^k \cdot 0.7^{n-k} \Rightarrow \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Account for all possible orders of heads and tails

Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins

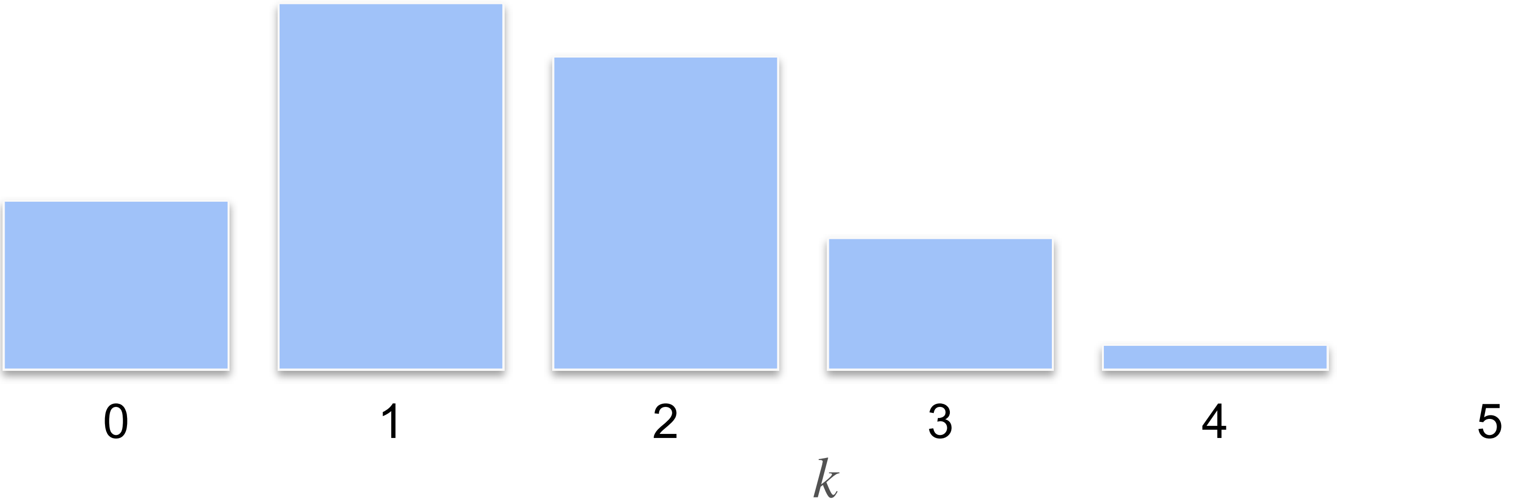
n = 5

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

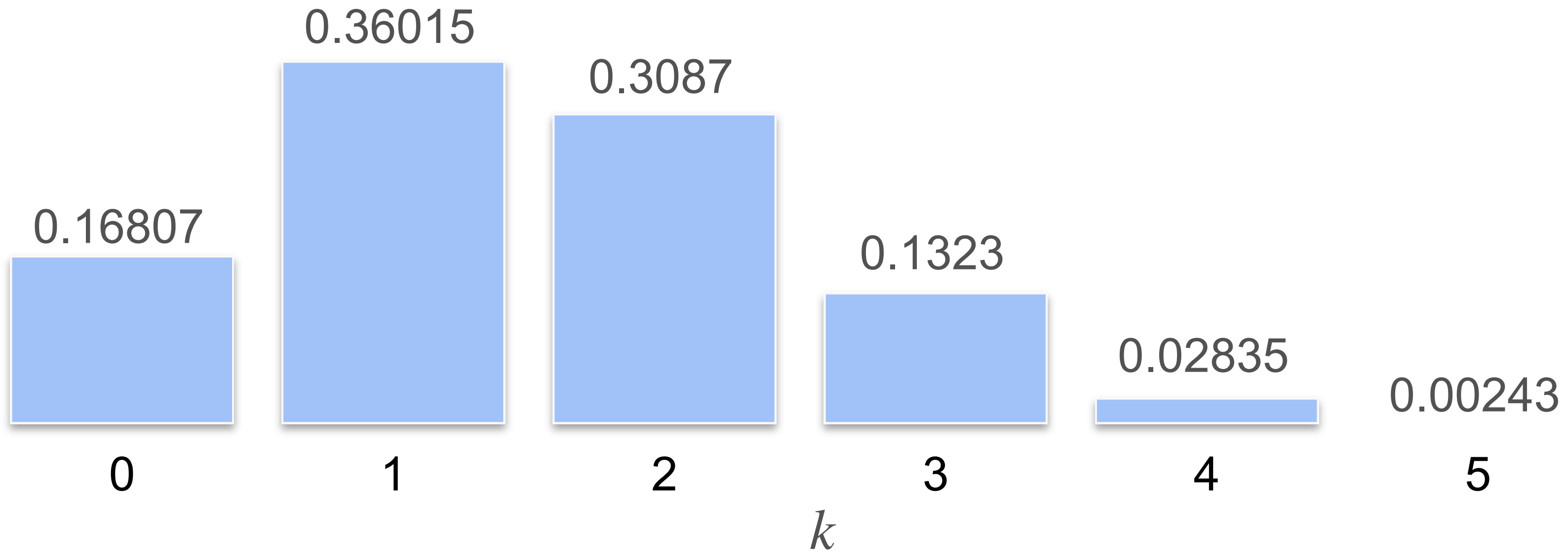
n = 5
p = 0.3



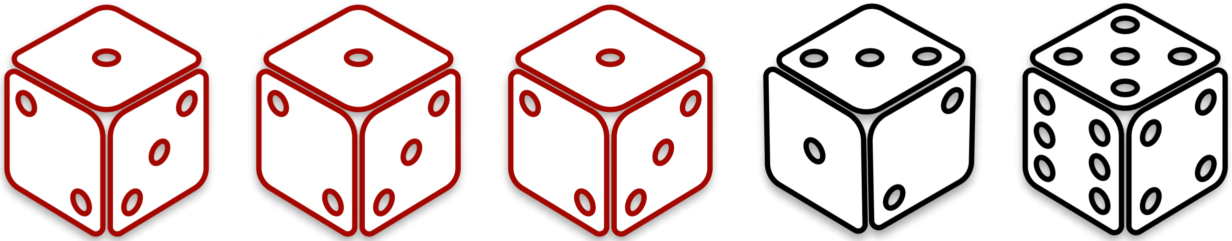
Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

n = 5
p = 0.3

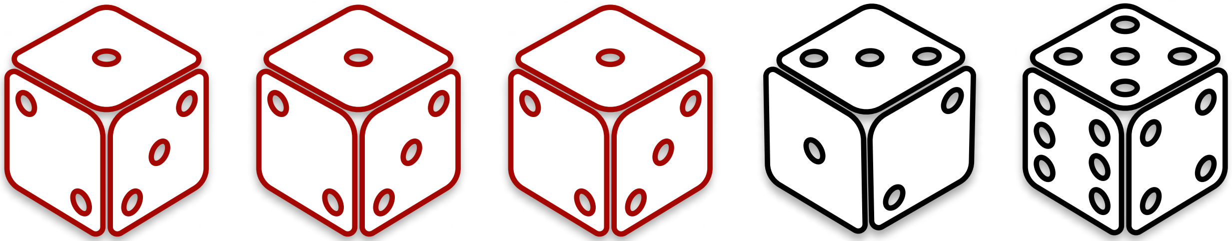


Binomial Distribution: Quiz

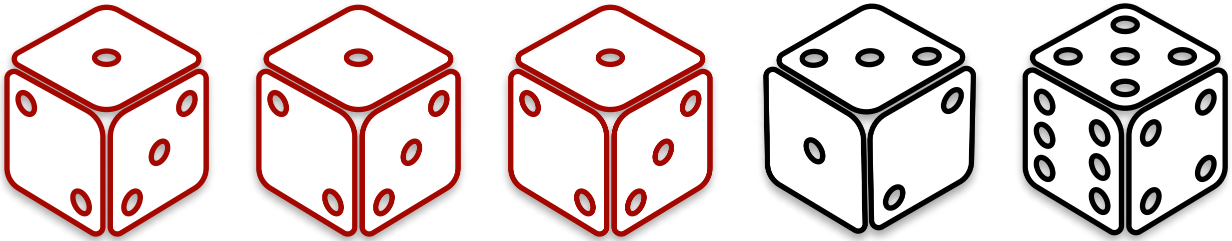


Binomial Distribution: Quiz

What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?

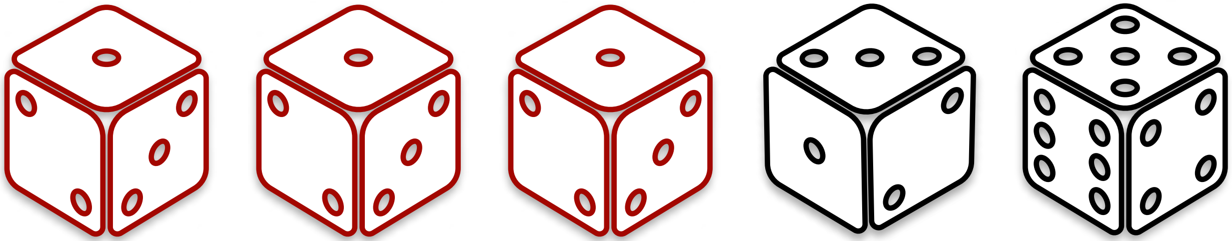


Binomial Distribution: Quiz

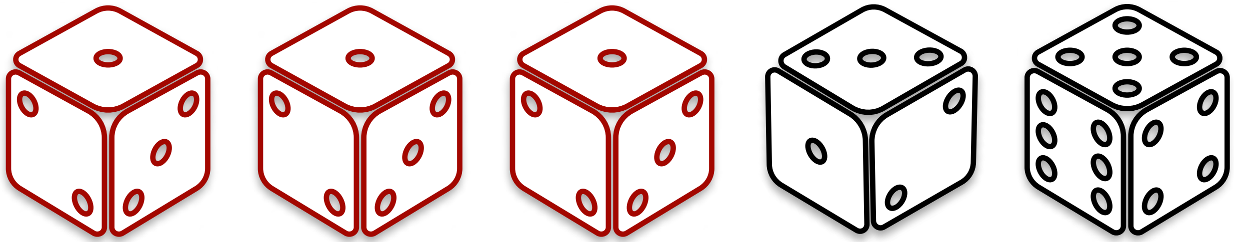


Binomial Distribution: Quiz

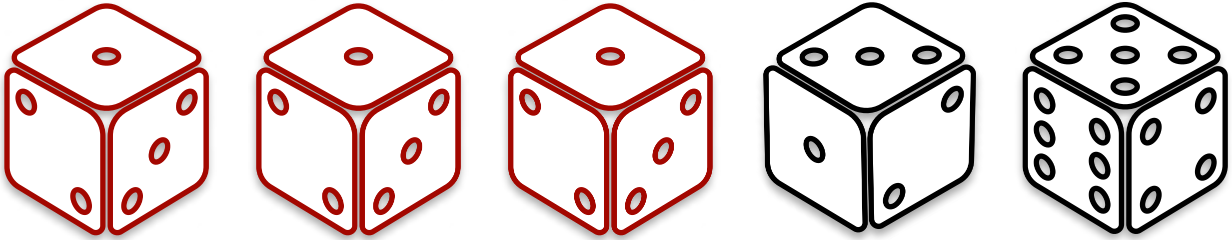
What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?



Binomial Distribution: Dice Is a Biased Coin!

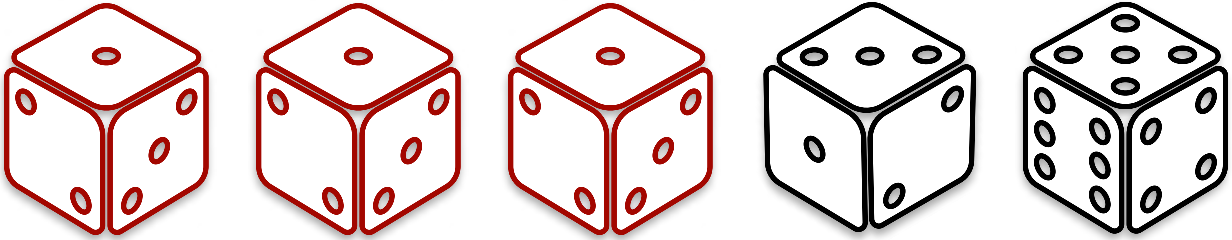


Binomial Distribution: Dice Is a Biased Coin!



one	one	one	not one	not one
$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{5}{6}$	$p = \frac{5}{6}$

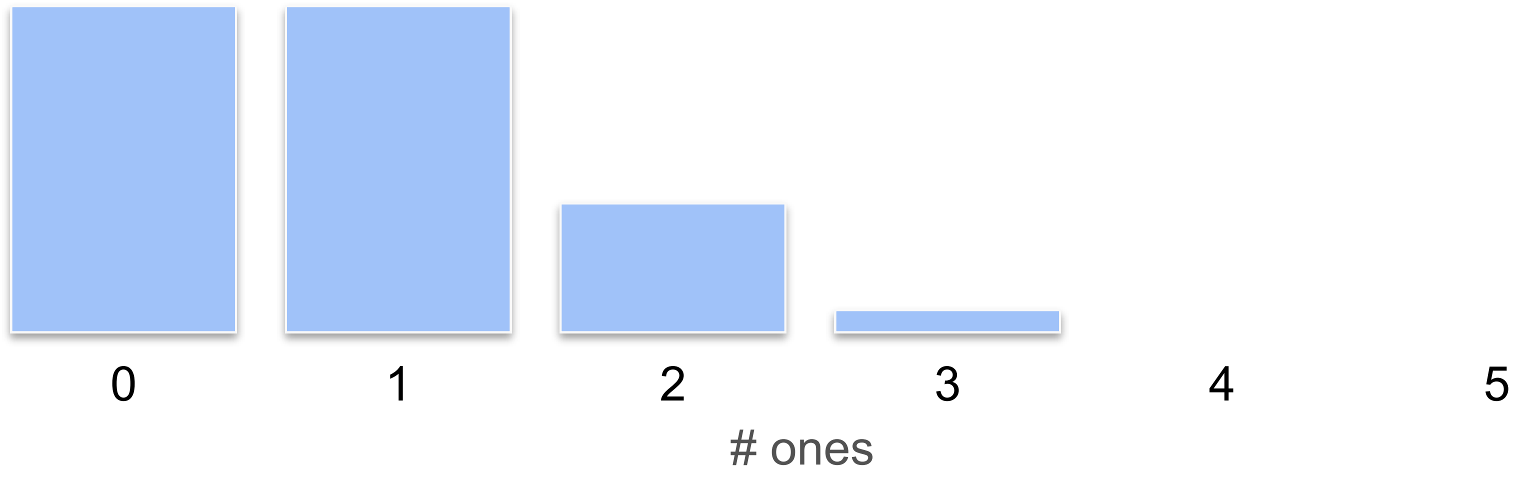
Binomial Distribution: Dice Is a Biased Coin!



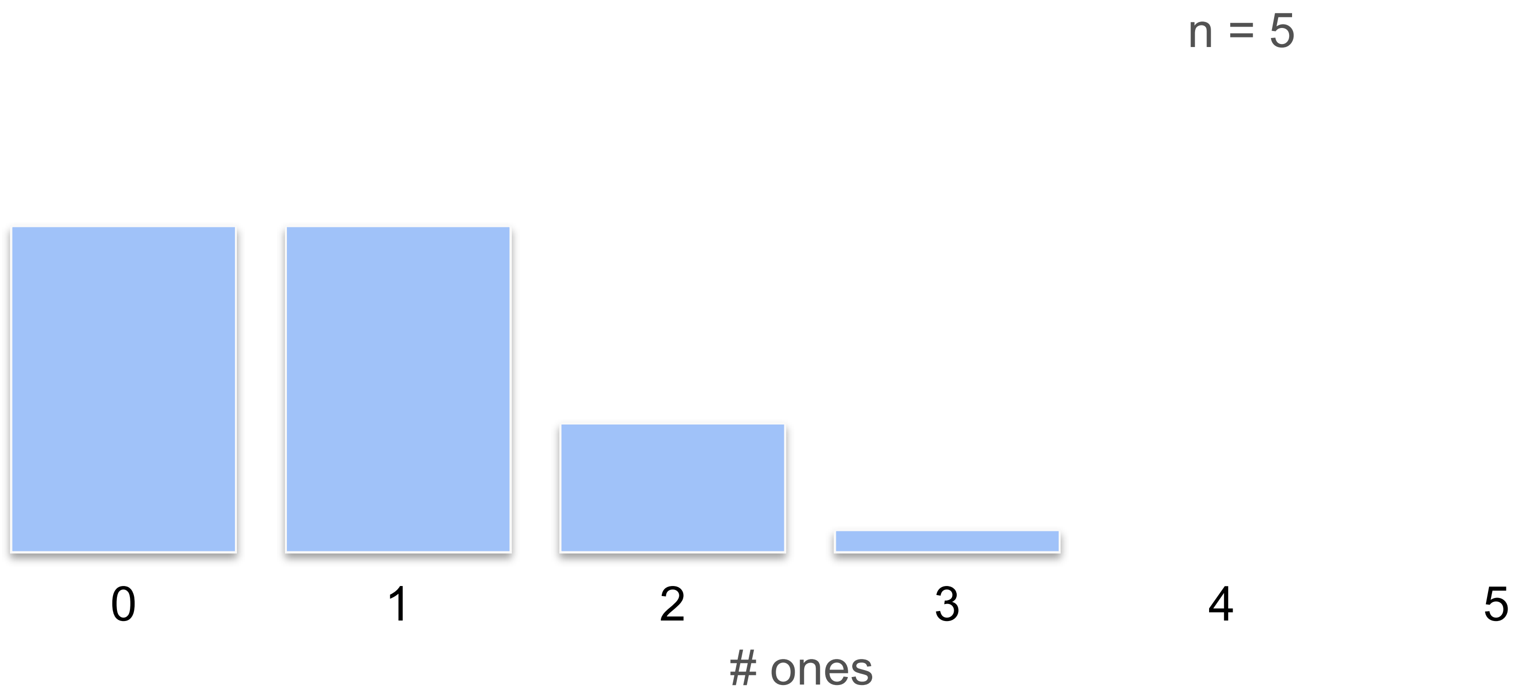
one	one	one	not one	not one
$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{5}{6}$	$p = \frac{5}{6}$

heads	heads	heads	not heads	not heads
$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{1}{6}$	$p = \frac{5}{6}$	$p = \frac{5}{6}$

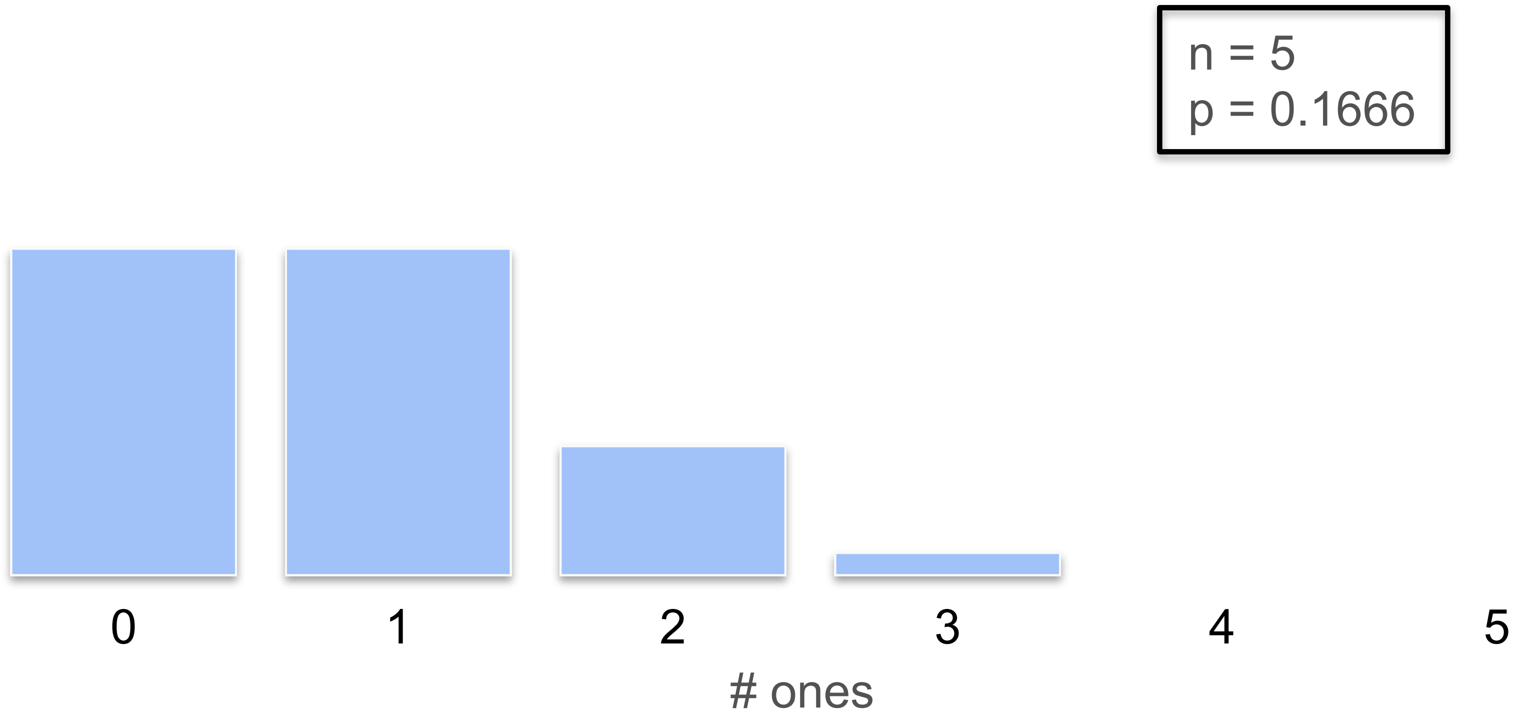
Binomial Distribution: Dice Is a Biased Coin!



Binomial Distribution: Dice Is a Biased Coin!

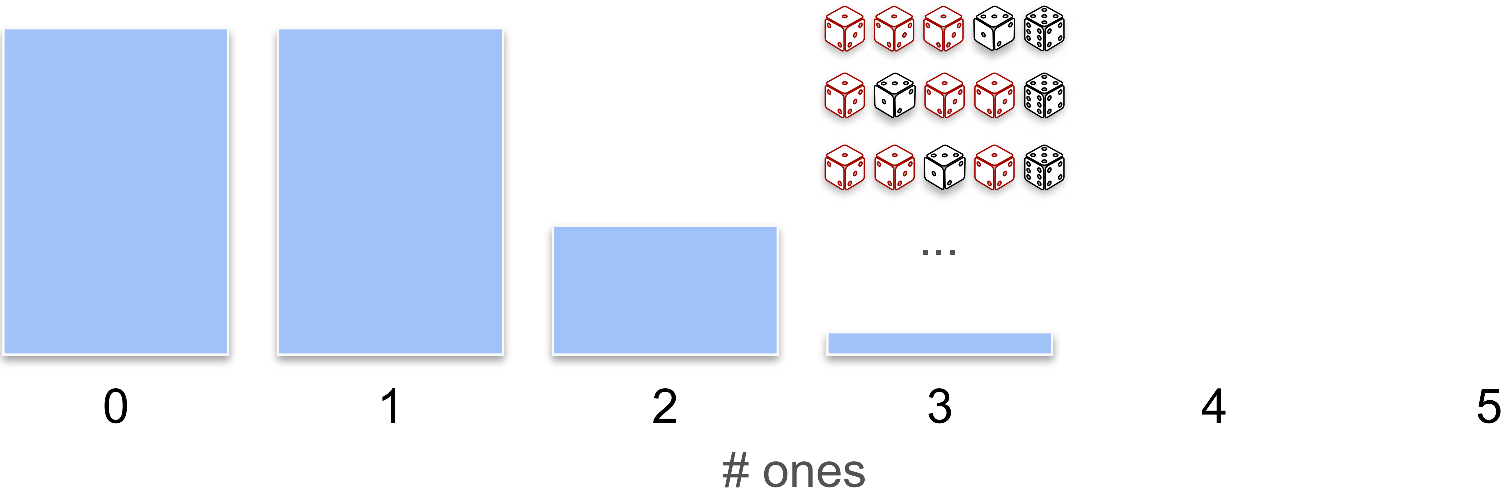


Binomial Distribution: Dice Is a Biased Coin!

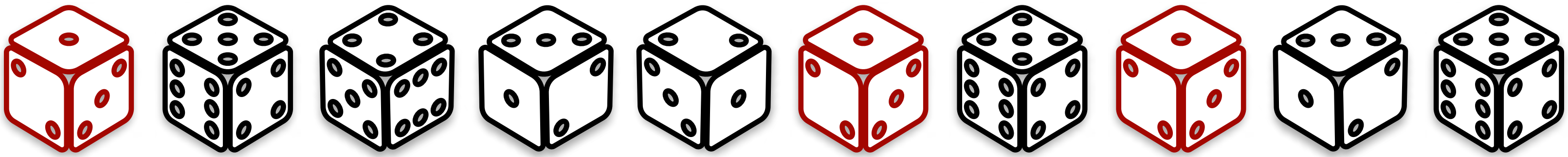


Binomial Distribution: Dice Is a Biased Coin!

n = 5
p = 0.1666

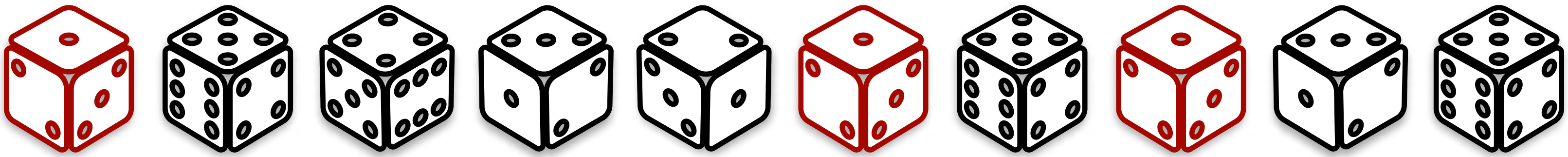


Binomial Distribution: Quiz

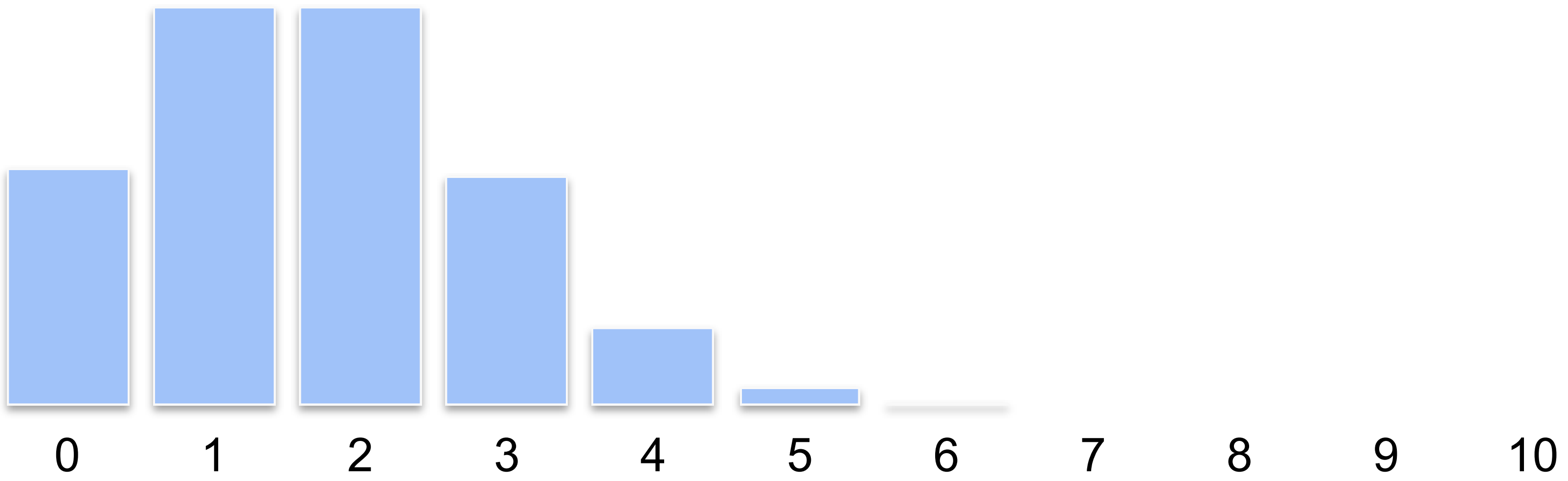


Binomial Distribution: Quiz

- Quiz: What are the parameters for the following binomial distribution:
  - I roll 10 dice
  - I want to record the number of times I obtain the number 1

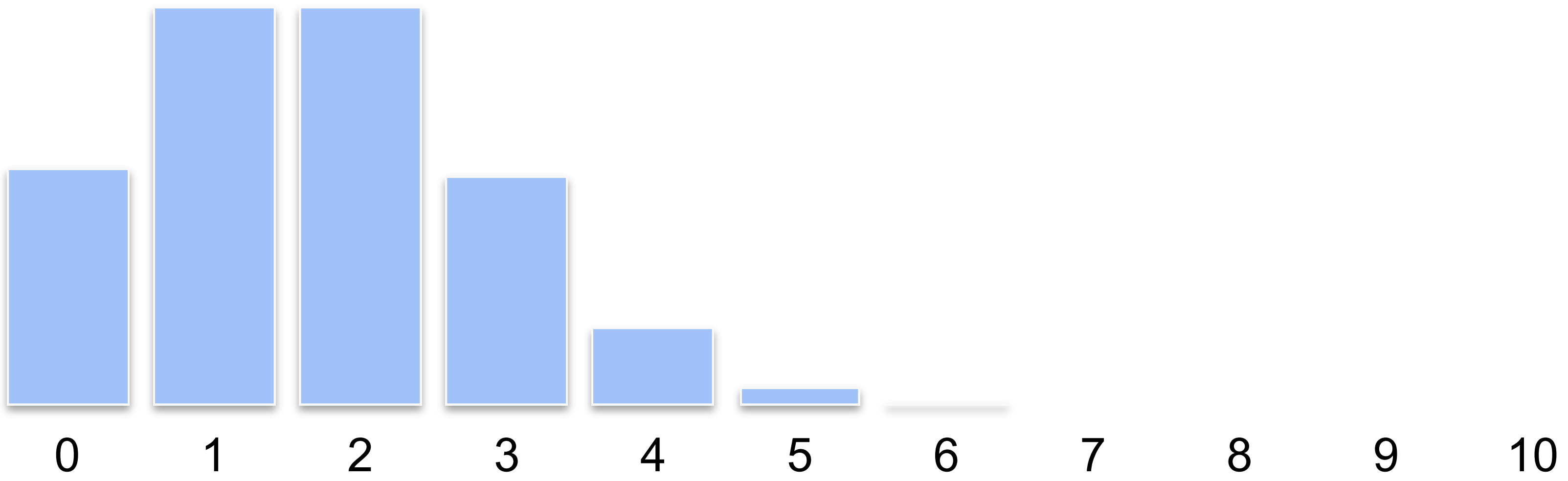


Binomial Distribution: Quiz



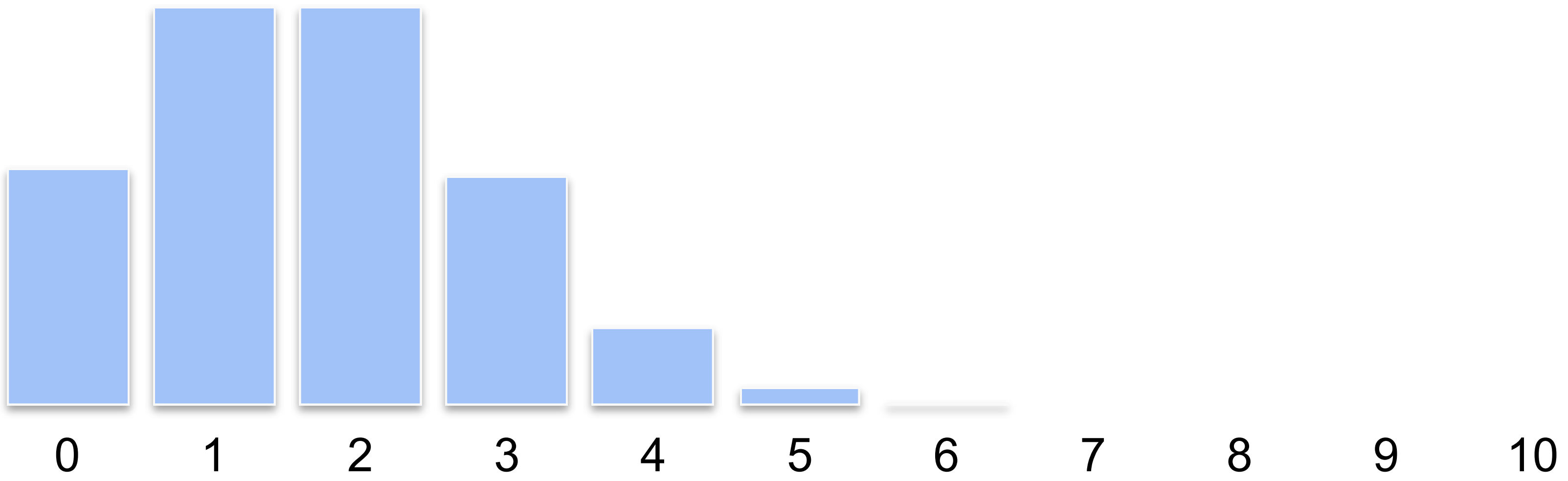
Binomial Distribution: Quiz

$$n = 10$$



Binomial Distribution: Quiz

$$n = 10$$
$$p = 0.1666$$



Bernoulli Distribution

Bernoulli Distribution

$X =$ Number of heads

Bernoulli Distribution

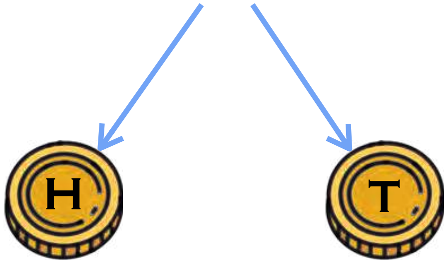
$X =$ Number of heads

Flip a coin

Bernoulli Distribution

$X =$ Number of heads

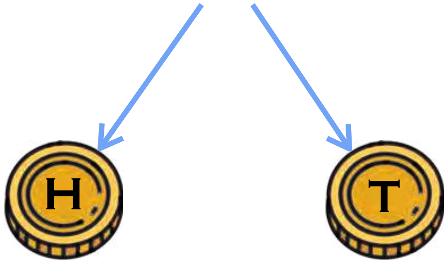
Flip a coin



Bernoulli Distribution

$X =$ Number of heads

Flip a coin

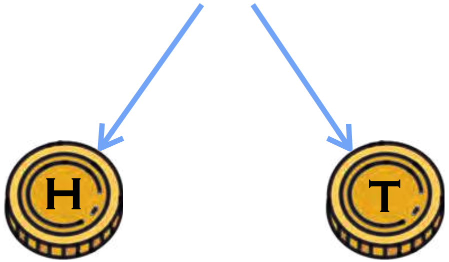


$\mathrm{P}(X = 1) = 0.5$

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

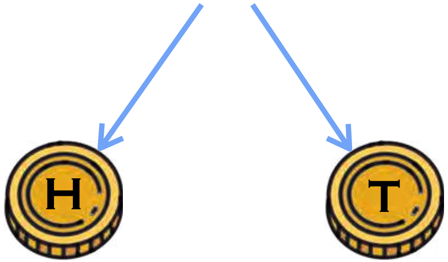


$\mathbf{P}(X = 1) = 0.5$ $\mathbf{P}(X = 0) = 0.5$

Bernoulli Distribution

X = Number of heads

Flip a coin



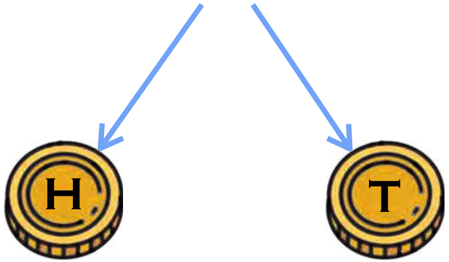
$P(X = 1) = 0.5$ $P(X = 0) = 0.5$

Success

Bernoulli Distribution

$X =$ Number of heads

Flip a coin



$\mathrm{P}(X = 1) = 0.5$ $\mathrm{P}(X = 0) = 0.5$

Success Failure

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

$\mathrm{P}(X = 1) = 0.5$

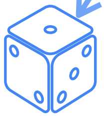
Success

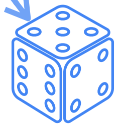
$\mathrm{P}(X = 0) = 0.5$

Failure

$X =$ Number of 1's

Throw a dice





$\mathrm{P}(X = 1) = \frac{1}{6}$

Success

$\mathrm{P}(X = 0) = \frac{5}{6}$

Failure

Bernoulli Distribution

$X =$ Number of heads

Flip a coin

$\mathrm{P}(X = 1) = 0.5$

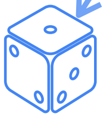
Success

$\mathrm{P}(X = 0) = 0.5$

Failure

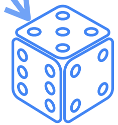
$X =$ Number of 1's

Throw a dice



$\mathrm{P}(X = 1) = \frac{1}{6}$

Success

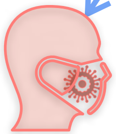


$\mathrm{P}(X = 0) = \frac{5}{6}$

Failure

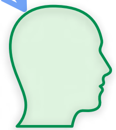
$X =$ Number of sick patients

Flip a coin



$\mathrm{P}(X = 1) = p$

Success



$\mathrm{P}(X = 0) = 1 - p$

Failure

Bernoulli Distribution

$X =$ Number of heads

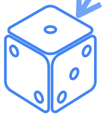
Flip a coin

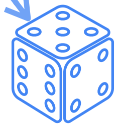
$\mathrm{P}(X = 1) = 0.5$ Success

$\mathrm{P}(X = 0) = 0.5$ Failure

$X =$ Number of 1's

Throw a dice



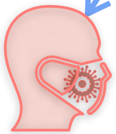


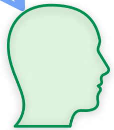
$\mathrm{P}(X = 1) = \frac{1}{6}$ Success

$\mathrm{P}(X = 0) = \frac{5}{6}$ Failure

$X =$ Number of sick patients

Flip a coin





$\mathrm{P}(X = 1) = p$ Success

$\mathrm{P}(X = 0) = 1 - p$ Failure

$$X \sim Bernoulli(p)$$

Bernoulli Distribution

X = Number of heads

Flip a coin

$\mathrm{P}(X = 1) = 0.5$

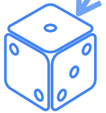
Success

$\mathrm{P}(X = 0) = 0.5$

Failure

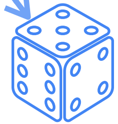
X = Number of 1's

Throw a dice



$\mathrm{P}(X = 1) = \frac{1}{6}$

Success

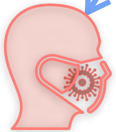


$\mathrm{P}(X = 0) = \frac{5}{6}$

Failure

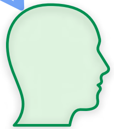
X = Number of sick patients

Flip a coin



$\mathrm{P}(X = 1) = p$

Success



$\mathrm{P}(X = 0) = 1 - p$

Failure

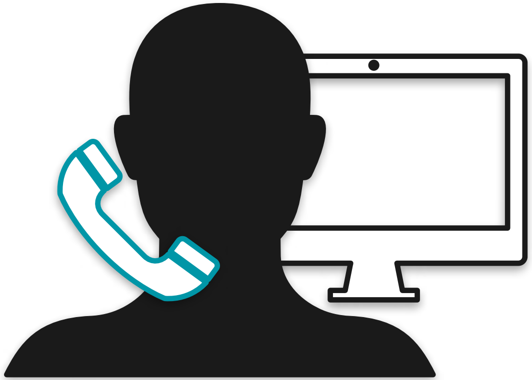
$X \sim Bernoulli(p)$ ← p is the parameter of the Bernoulli distribution

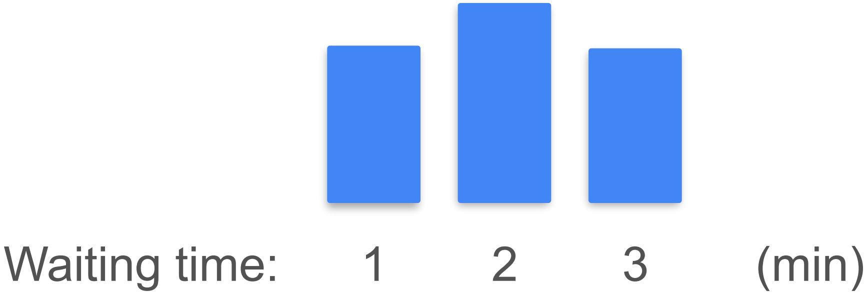
DeepLearning.AI

Probability Distributions

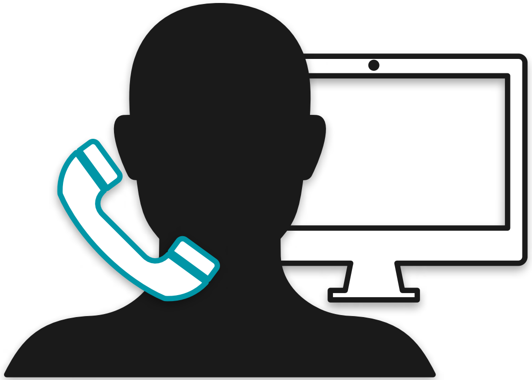
Probability Distributions (Continuous)

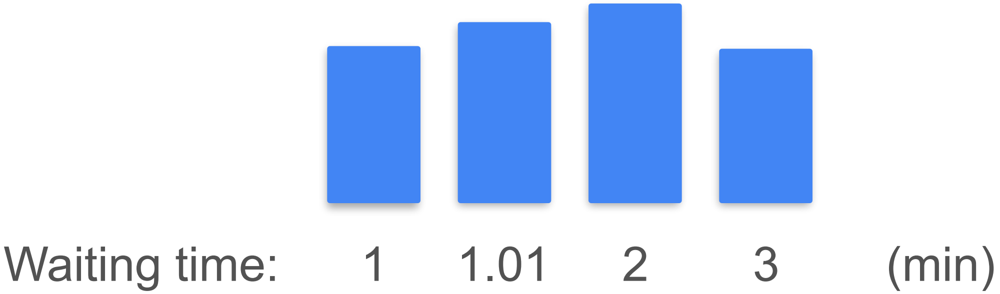
From Discrete to Continuous Distributions



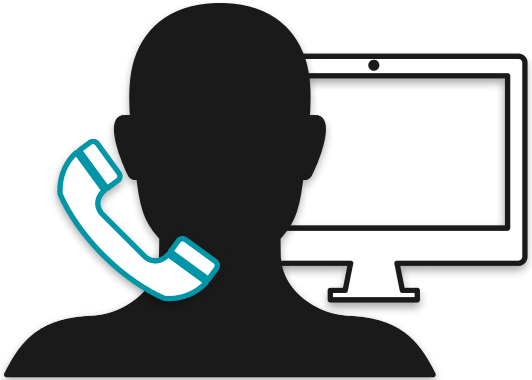


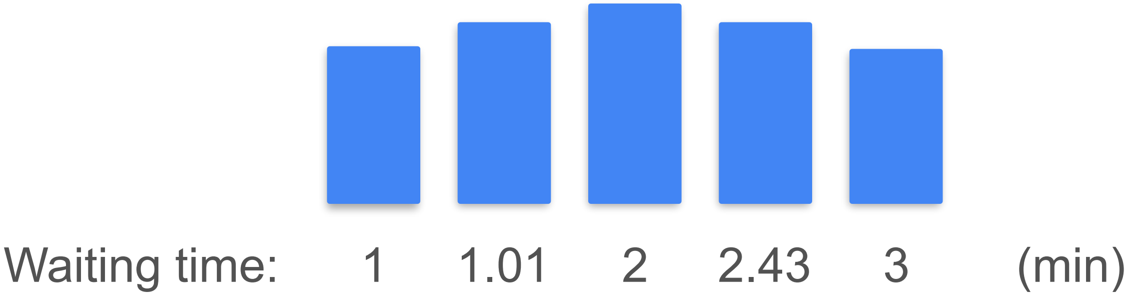
From Discrete to Continuous Distributions



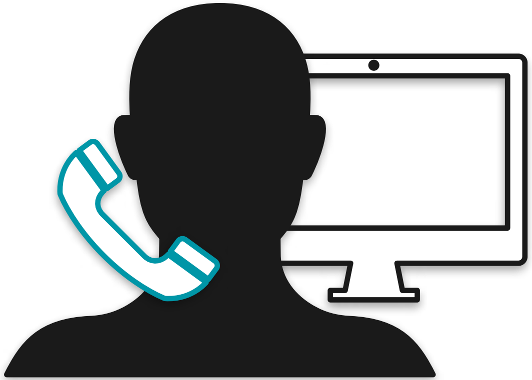


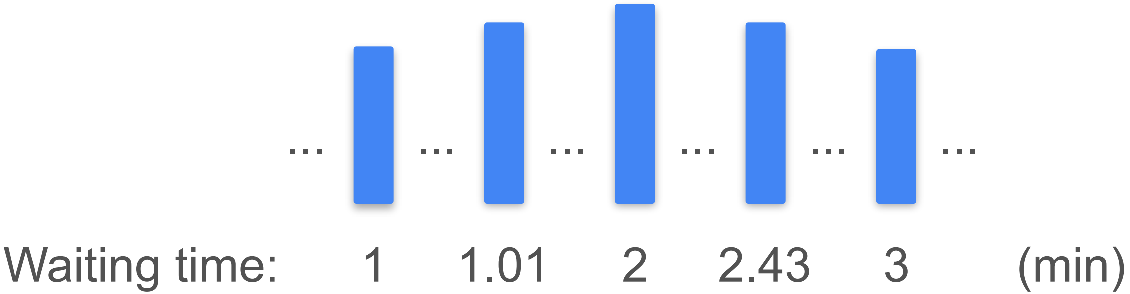
From Discrete to Continuous Distributions



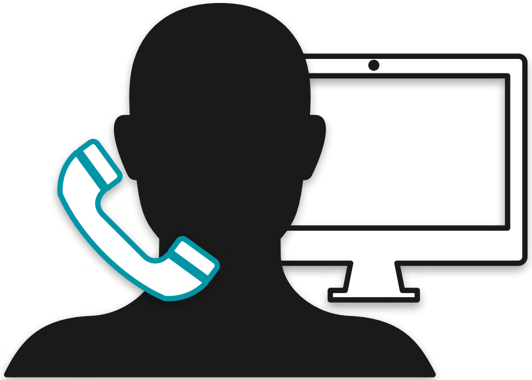


From Discrete to Continuous Distributions

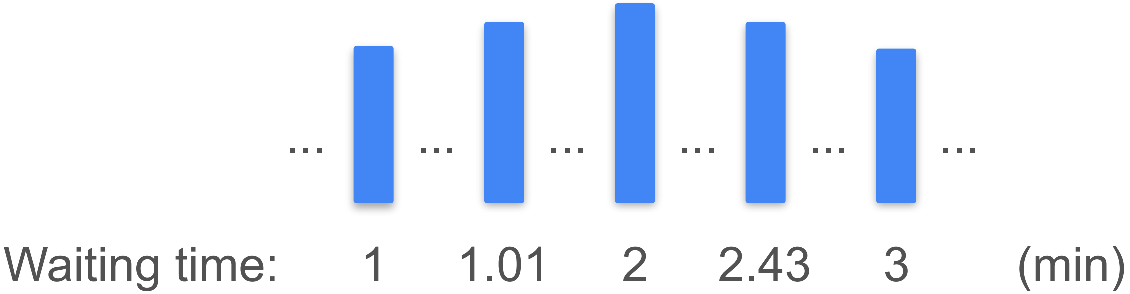




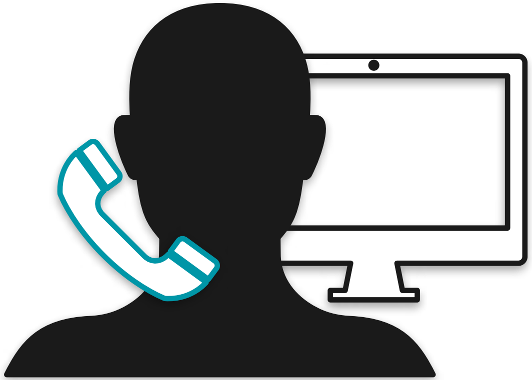
From Discrete to Continuous Distributions



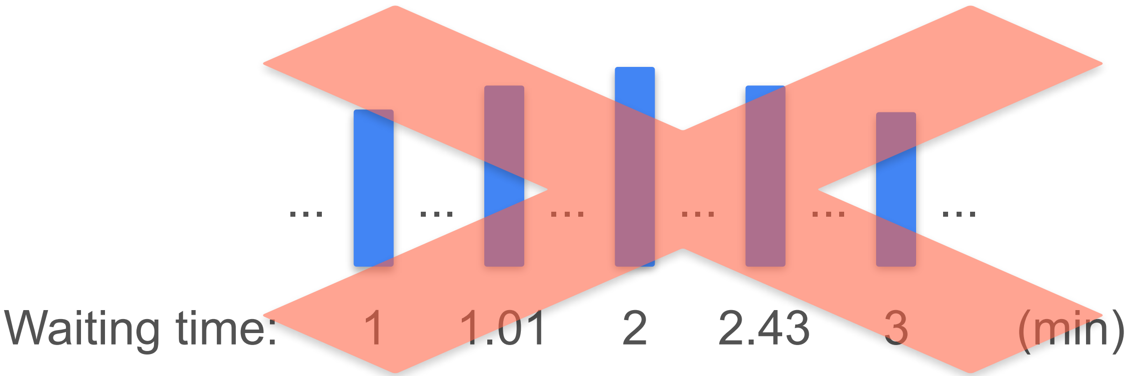
This is a continuous distribution!



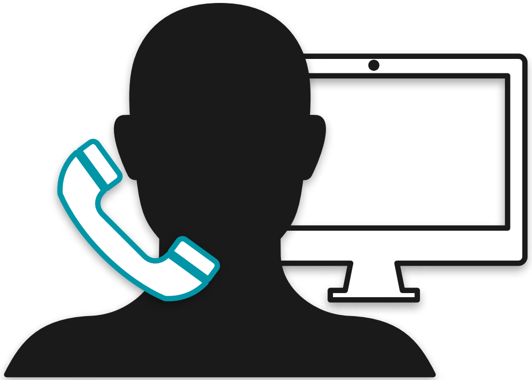
From Discrete to Continuous Distributions



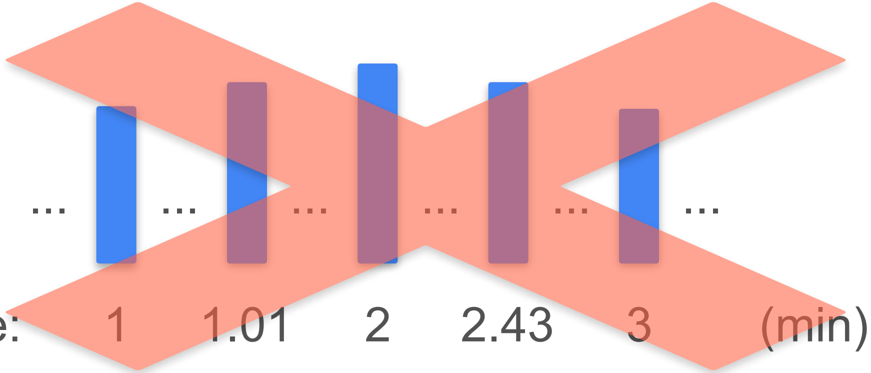
This is a continuous distribution!



From Discrete to Continuous Distributions



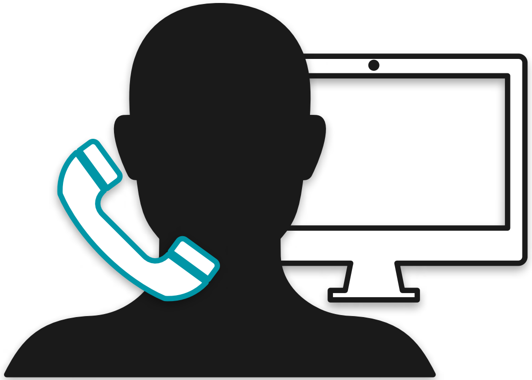
This is a continuous distribution!



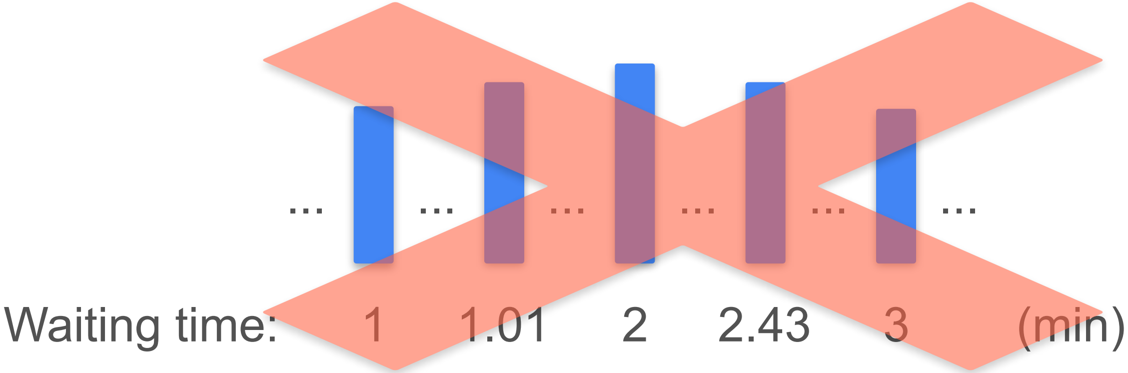
Waiting time:

What is the probability that you will wait EXACTLY one minute for the call?

From Discrete to Continuous Distributions



This is a continuous distribution!



What is the probability that you will wait EXACTLY one minute for the call?

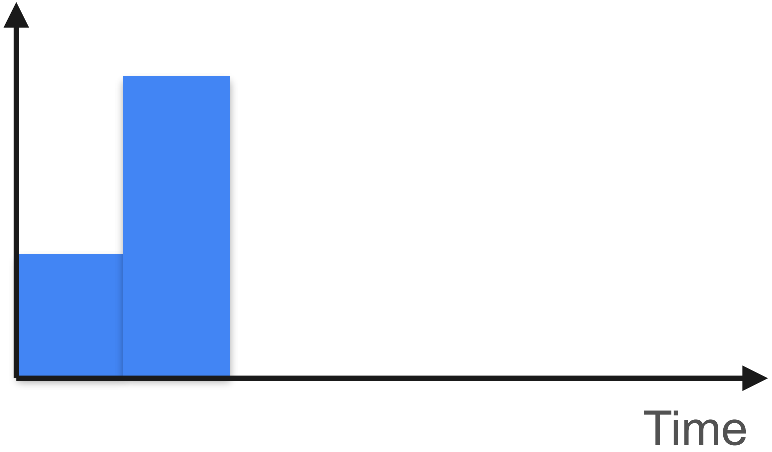
Answer: ZERO

From Discrete to Continuous Distributions

From Discrete to Continuous Distributions

$\mathbf{P}$(between 0 and 1 mins)

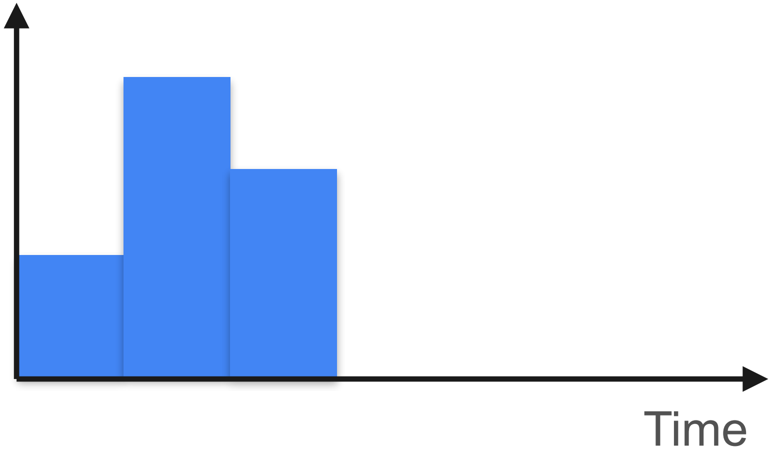
From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 1 mins)

$\mathbf{P}$(between 1 and 2 mins)

From Discrete to Continuous Distributions

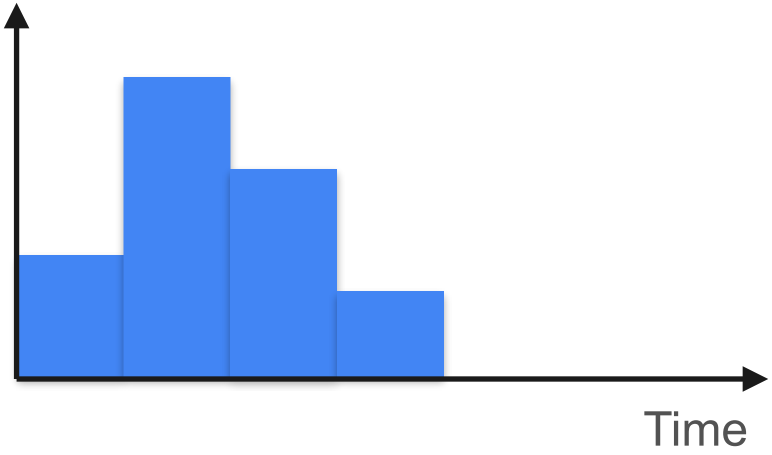


$\mathbf{P}$(between 0 and 1 mins)

$\mathbf{P}$(between 1 and 2 mins)

$\mathbf{P}$(between 2 and 3 mins)

From Discrete to Continuous Distributions



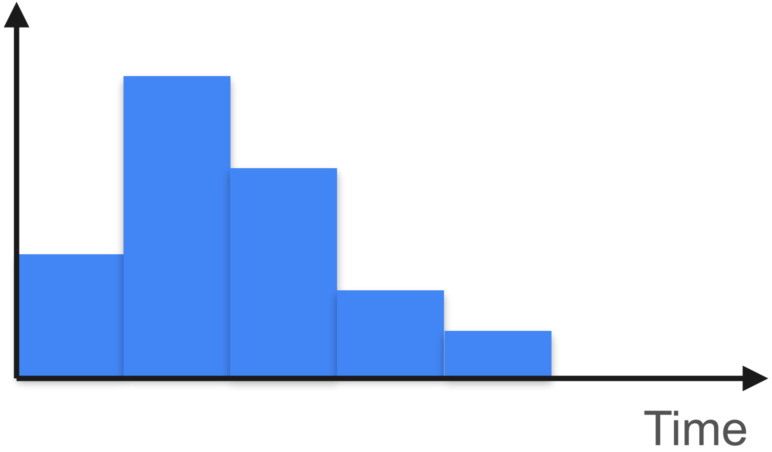
$\mathbf{P}$(between 0 and 1 mins)

$\mathbf{P}$(between 1 and 2 mins)

$\mathbf{P}$(between 2 and 3 mins)

$\mathbf{P}$(between 3 and 4 mins)

From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 1 mins)

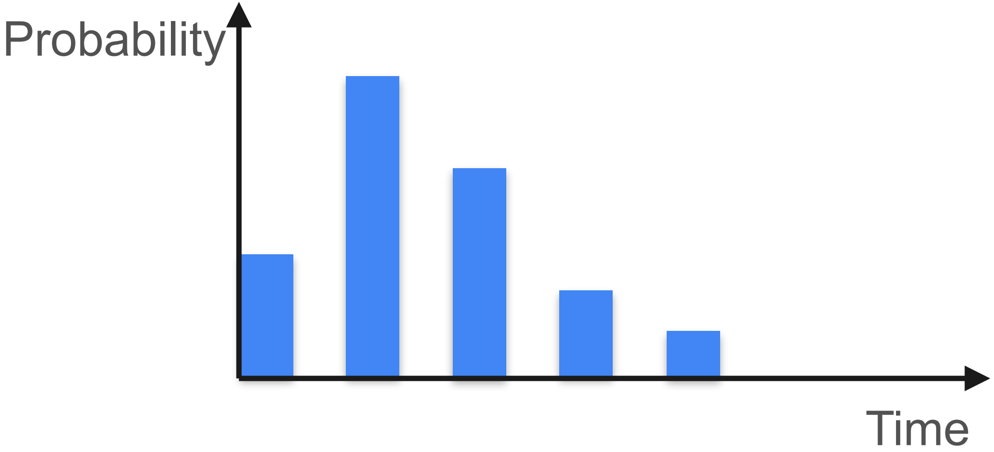
$\mathbf{P}$(between 1 and 2 mins)

$\mathbf{P}$(between 2 and 3 mins)

$\mathbf{P}$(between 3 and 4 mins)

$\mathbf{P}$(between 4 and 5 mins)

From Discrete to Continuous Distributions



P(between 0 and 1 mins)

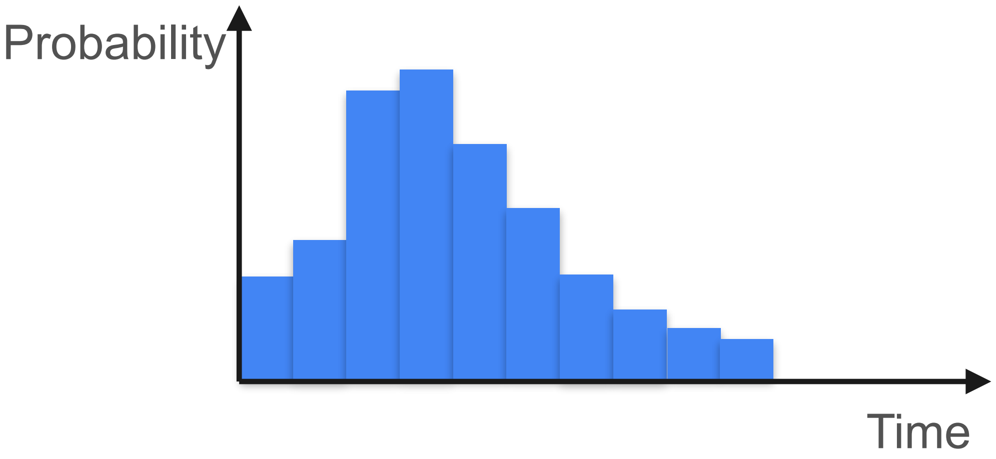
P(between 1 and 2 mins)

P(between 2 and 3 mins)

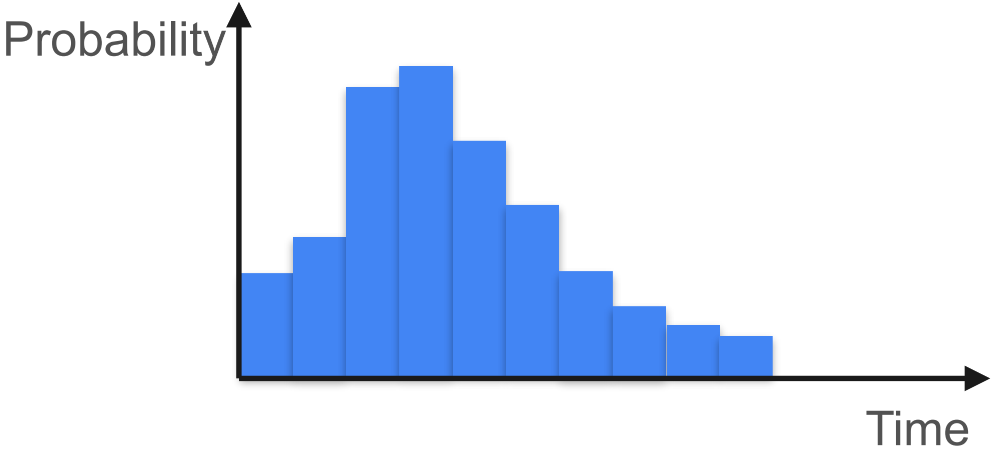
P(between 3 and 4 mins)

P(between 4 and 5 mins)

From Discrete to Continuous Distributions

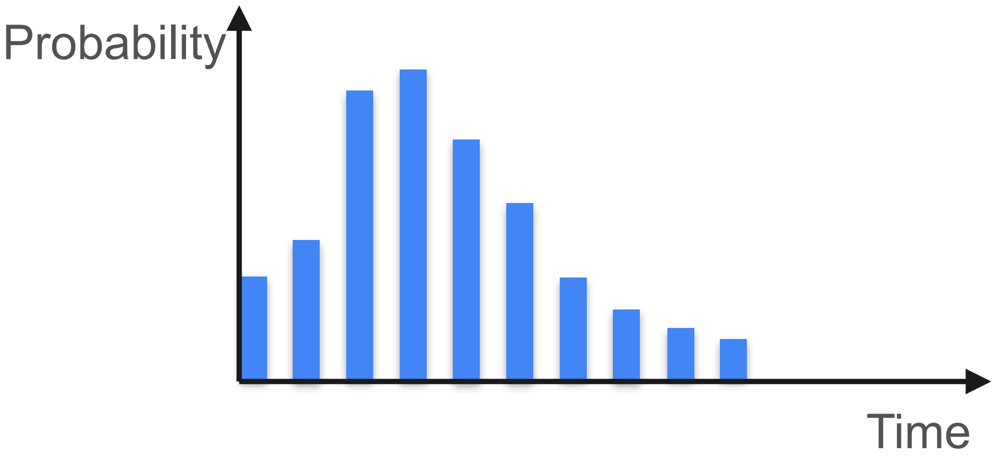


From Discrete to Continuous Distributions



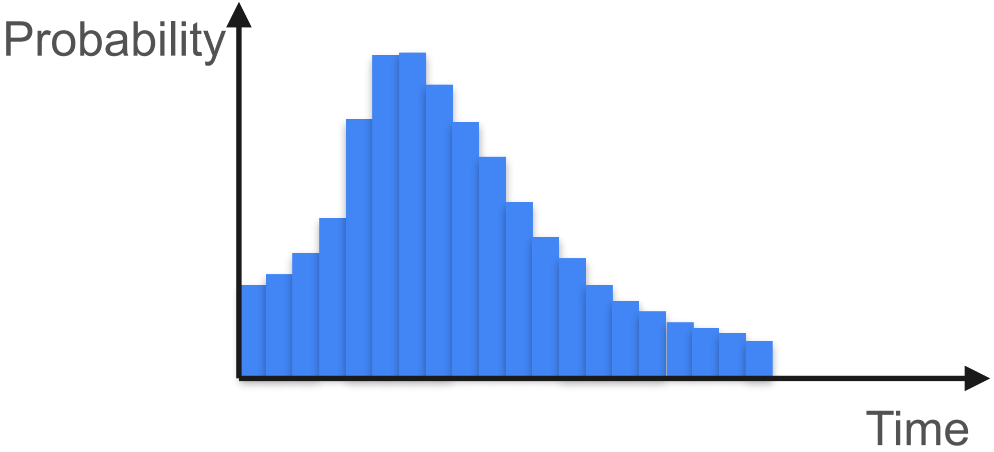
$\mathbf{P}$(between 0 and 0.5 mins)
$\mathbf{P}$(between 0.5 and 1 mins)
$\mathbf{P}$(between 1 and 1.5 mins)
⋮
$\mathbf{P}$(between 3.5 and 4 mins)
$\mathbf{P}$(between 4 and 4.5 mins)
$\mathbf{P}$(between 4.5 and 5 mins)

From Discrete to Continuous Distributions

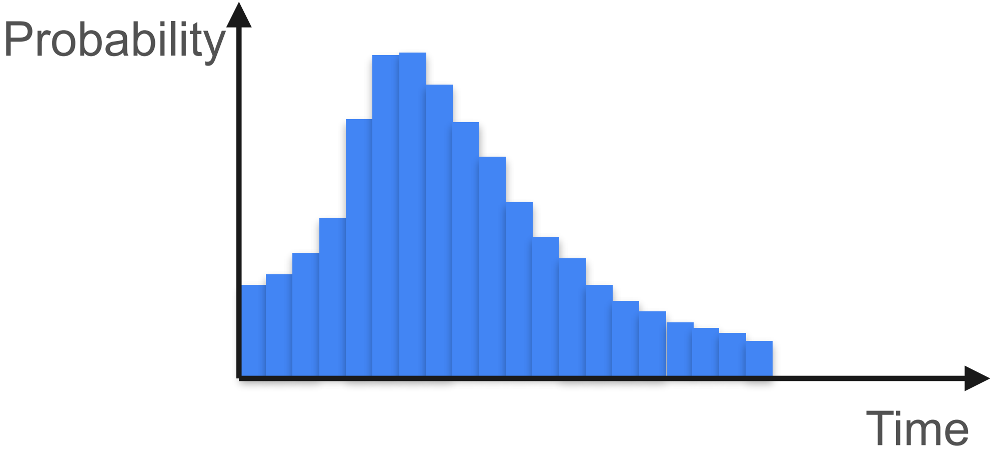


P(between 0 and 0.5 mins)
P(between 0.5 and 1 mins)
P(between 1 and 1.5 mins)
⋮
P(between 3.5 and 4 mins)
P(between 4 and 4.5 mins)
P(between 4.5 and 5 mins)

From Discrete to Continuous Distributions



From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 0.25 mins)

$\mathbf{P}$(between 0.25 and 0.5 mins)

$\mathbf{P}$(between 0.5 and 0.75 mins)

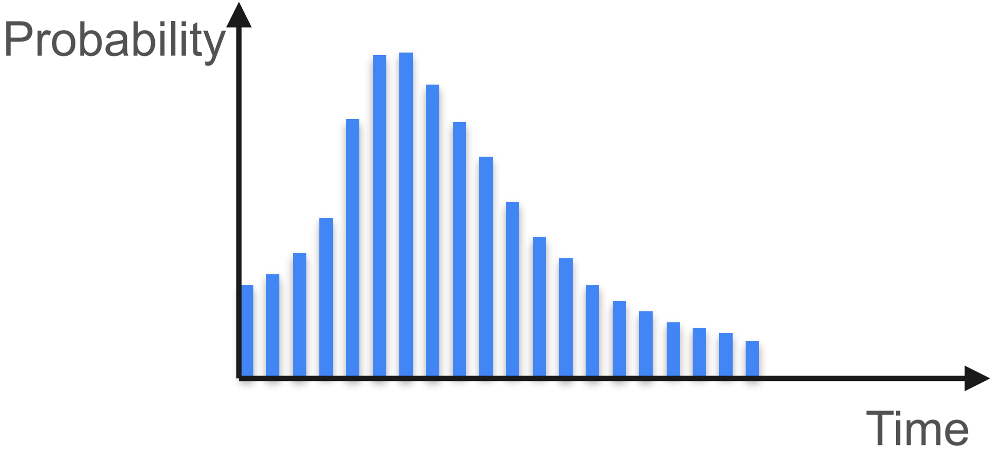
⋮

$\mathbf{P}$(between 4.25 and 4.5 mins)

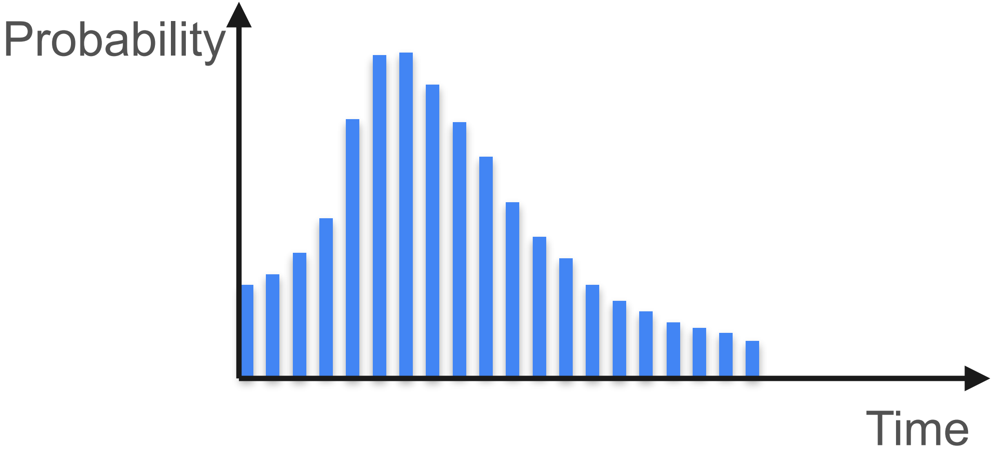
$\mathbf{P}$(between 4.5 and 4.75 mins)

$\mathbf{P}$(between 4.75 and 5 mins)

From Discrete to Continuous Distributions

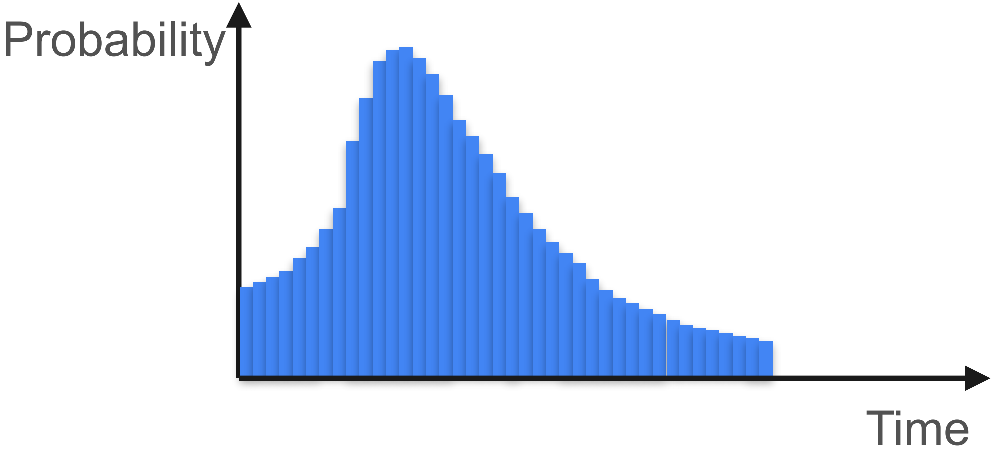


From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 0.25 mins)
$\mathbf{P}$(between 0.25 and 0.5 mins)
$\mathbf{P}$(between 0.5 and 0.75 mins)
⋮
$\mathbf{P}$(between 4.25 and 4.5 mins)
$\mathbf{P}$(between 4.5 and 4.75 mins)
$\mathbf{P}$(between 4.75 and 5 mins)

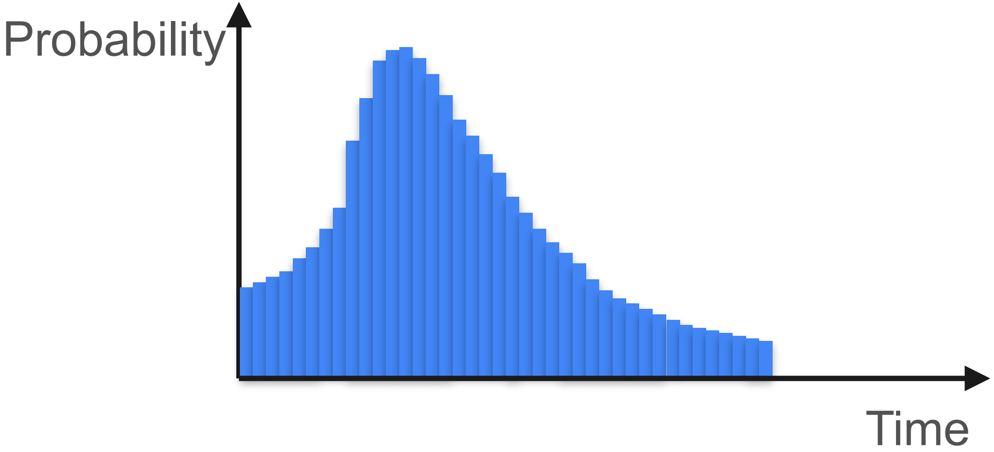
From Discrete to Continuous Distributions



$\mathbf{P}$(between 0 and 0.125 mins)

$\mathbf{P}$(between 4.875 and 5 mins)

From Discrete to Continuous Distributions

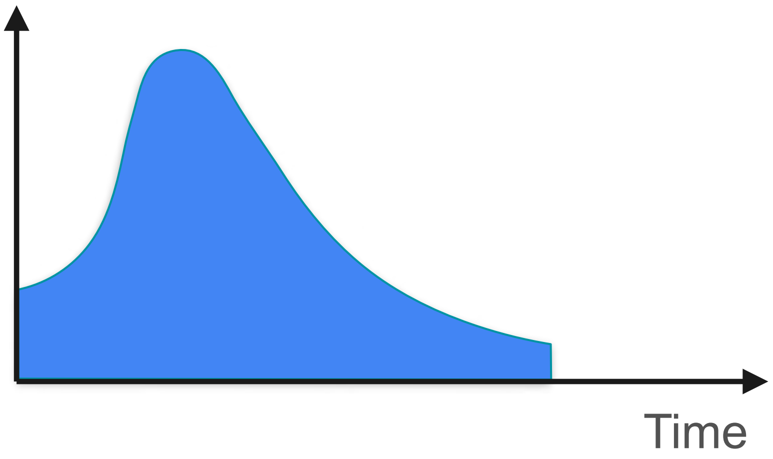


$\mathbf{P}$(between 0 and 0.125 mins)

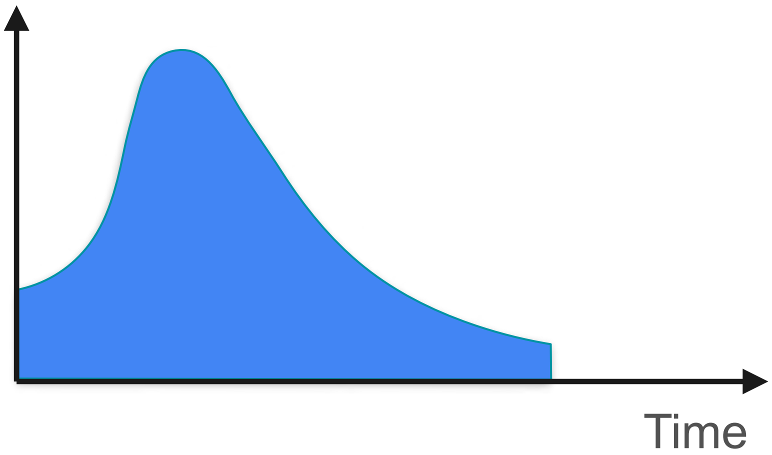
⋮

$\mathbf{P}$(between 4.875 and 5 mins)

From Discrete to Continuous Distributions

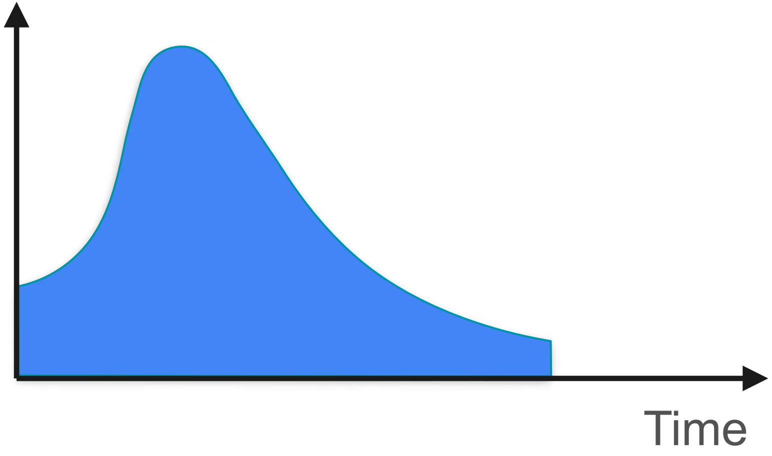


From Discrete to Continuous Distributions



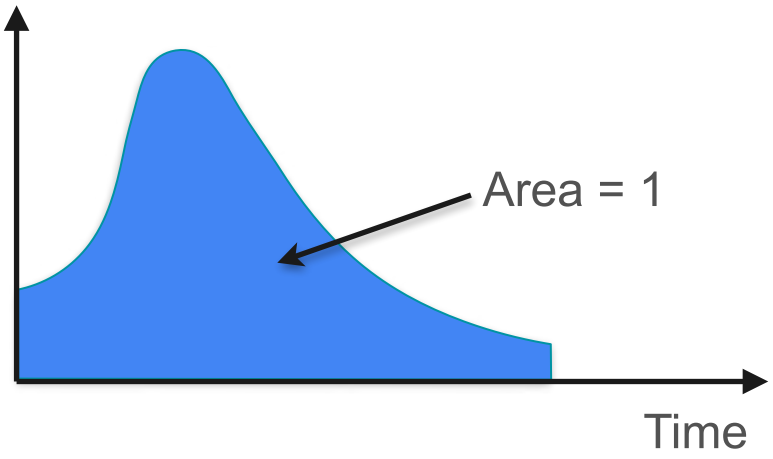
- Discrete:
  - Sum of heights equals 1

From Discrete to Continuous Distributions



- Discrete:
  - Sum of heights equals 1
- Continuous:
  - Area under the curve equals 1

From Discrete to Continuous Distributions



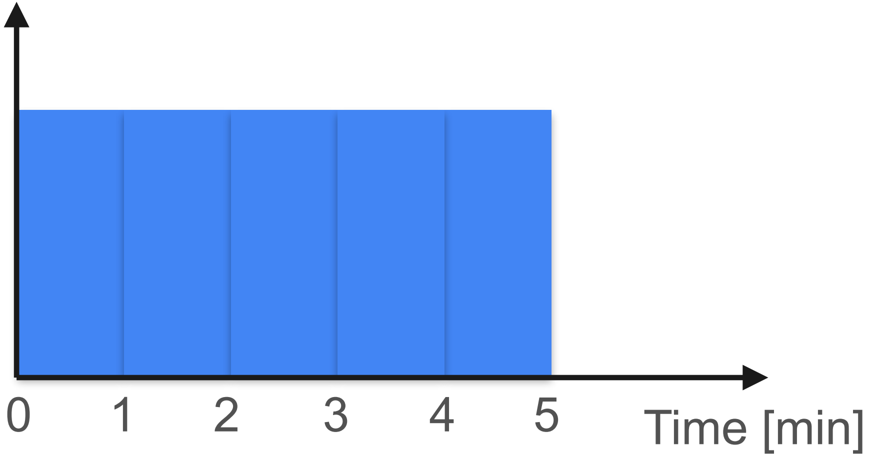
- Discrete:
  - Sum of heights equals 1
- Continuous:
  - Area under the curve equals 1

DeepLearning.AI

Probability Distributions

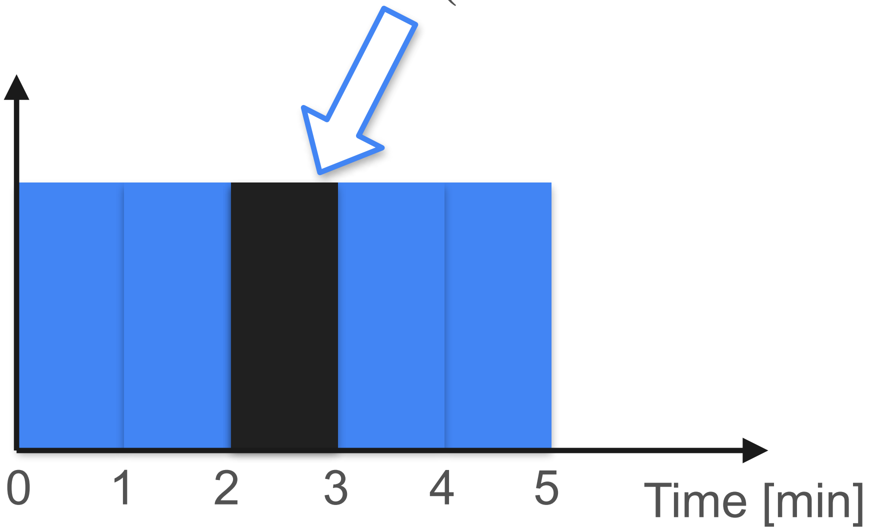
Probability density function

Probability Density Function



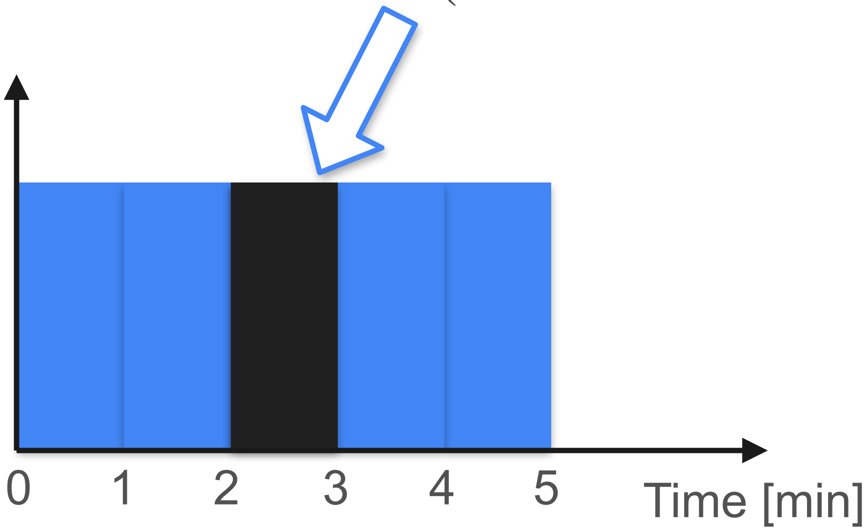
Probability Density Function

$\mathbf{P}(\text{between 2 and 3 min}) = ?$



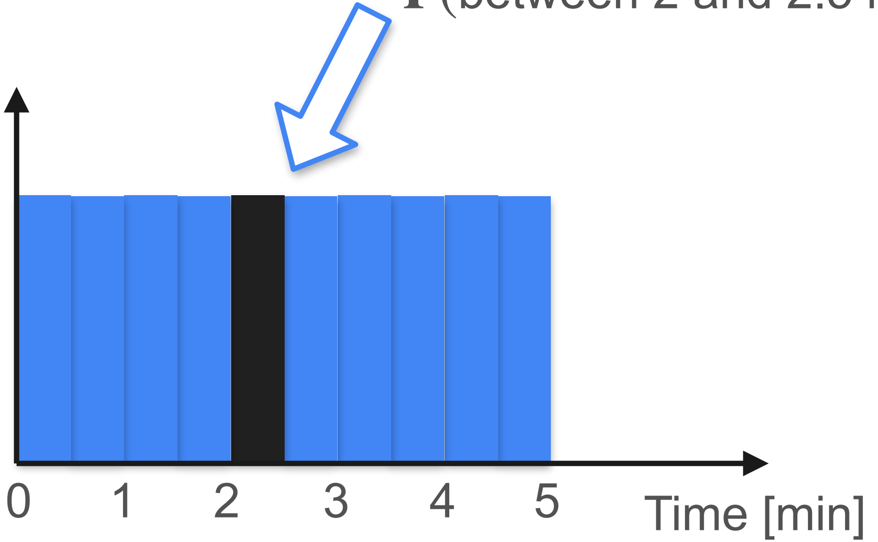
Probability Density Function

$\mathbf{P}(\text{between 2 and 3 min}) = 1/5$



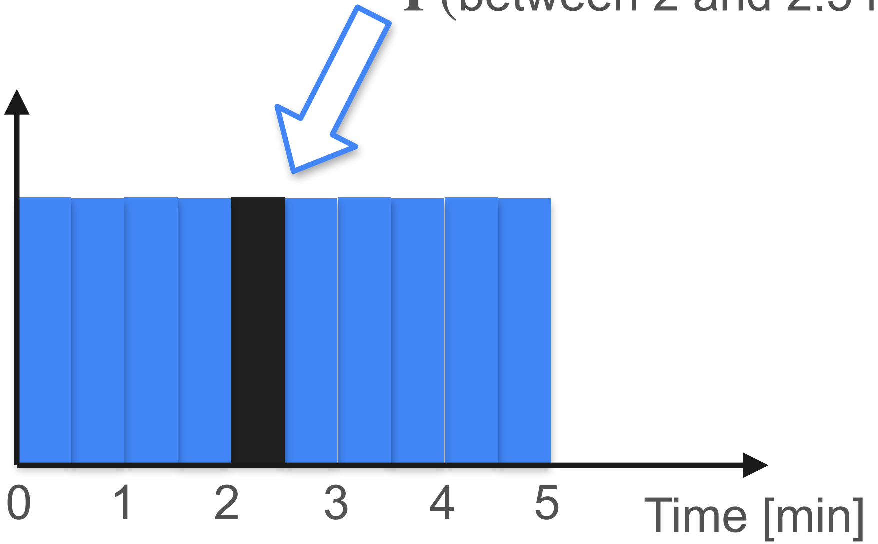
Probability Density Function

$\mathbf{P}(\text{between 2 and 2.5 min}) = ?$

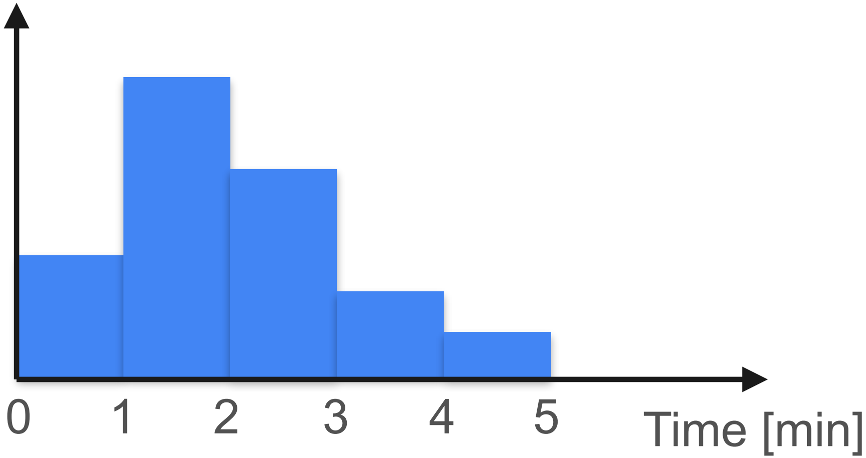


Probability Density Function

$\mathbf{P}(\text{between 2 and 2.5 min}) = 0.1$

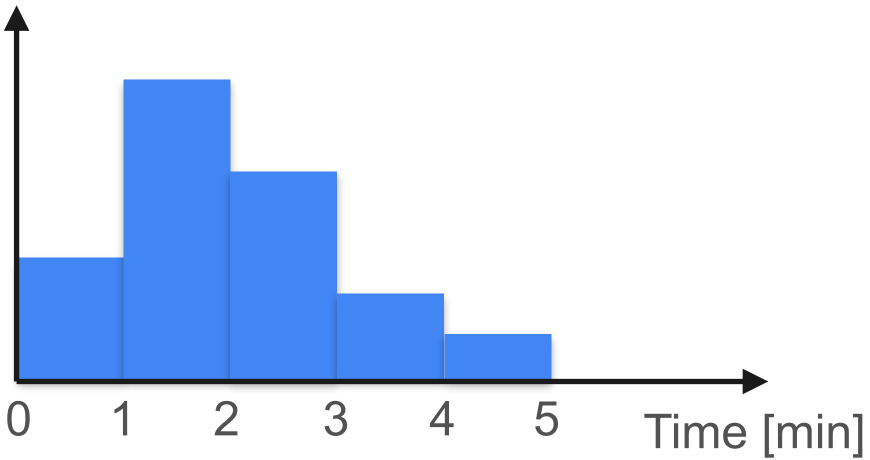


Probability Density Function



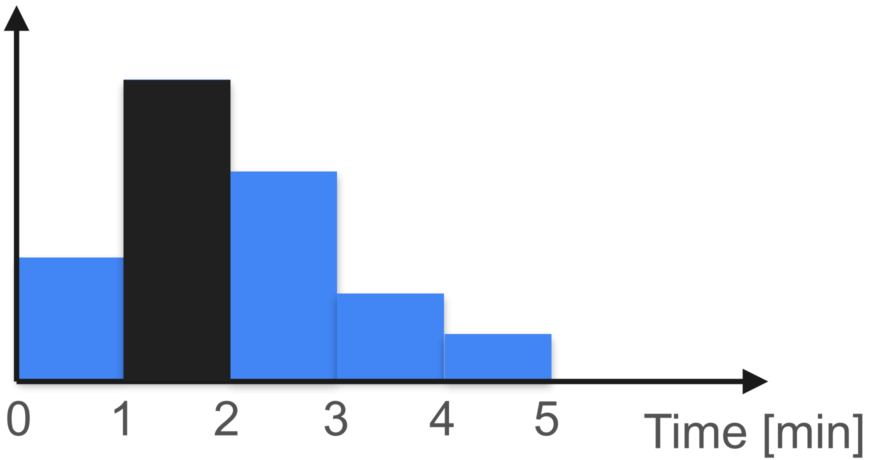
Probability Density Function

$\mathbf{P}$(between 1 and 2 min)

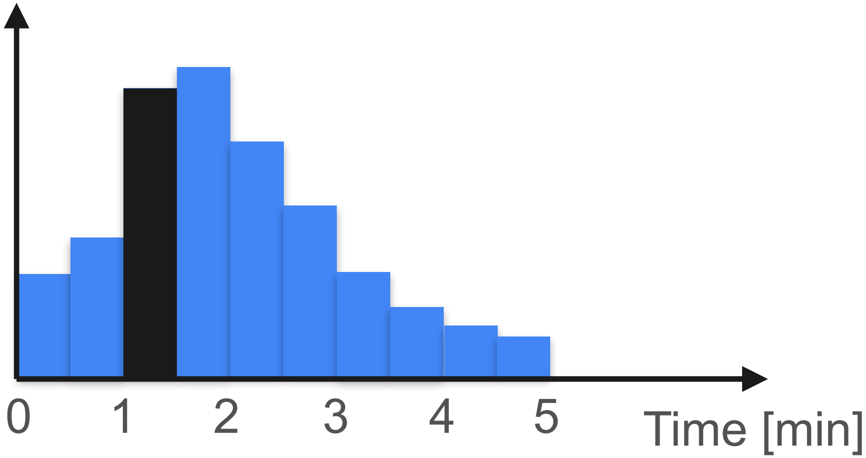


Probability Density Function

$\mathbf{P}$(between 1 and 2 min)

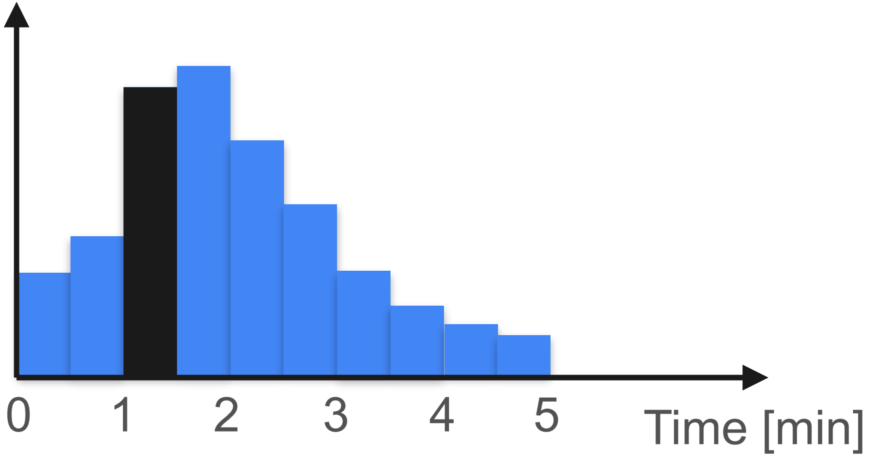


Probability Density Function

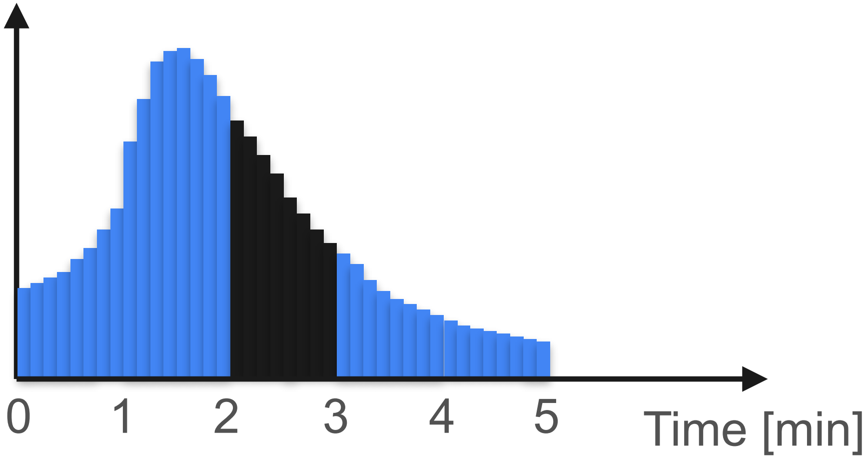


Probability Density Function

$\mathbf{P}$(between 1 and 1:30)

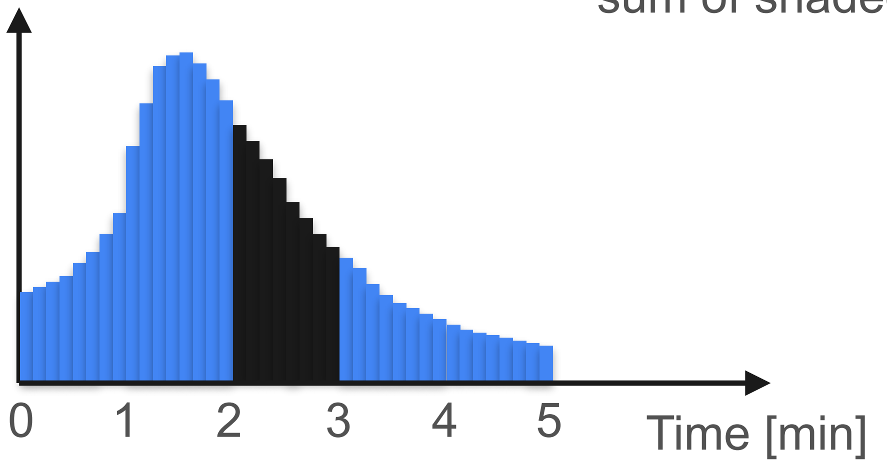


Probability Density Function

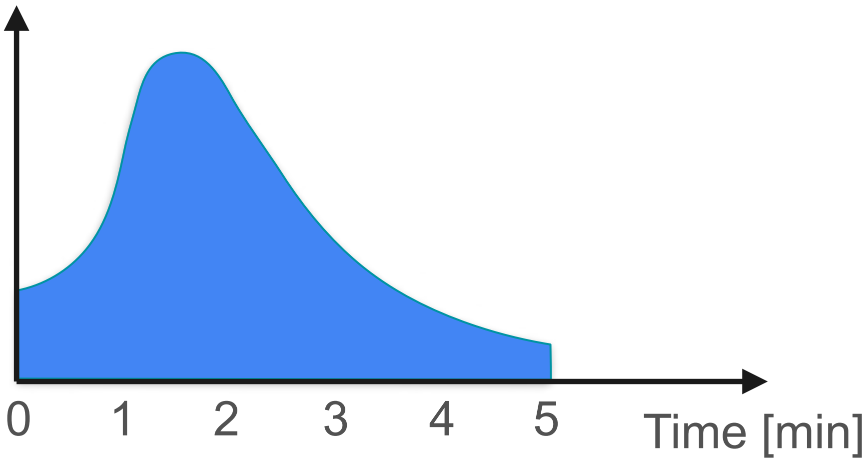


Probability Density Function

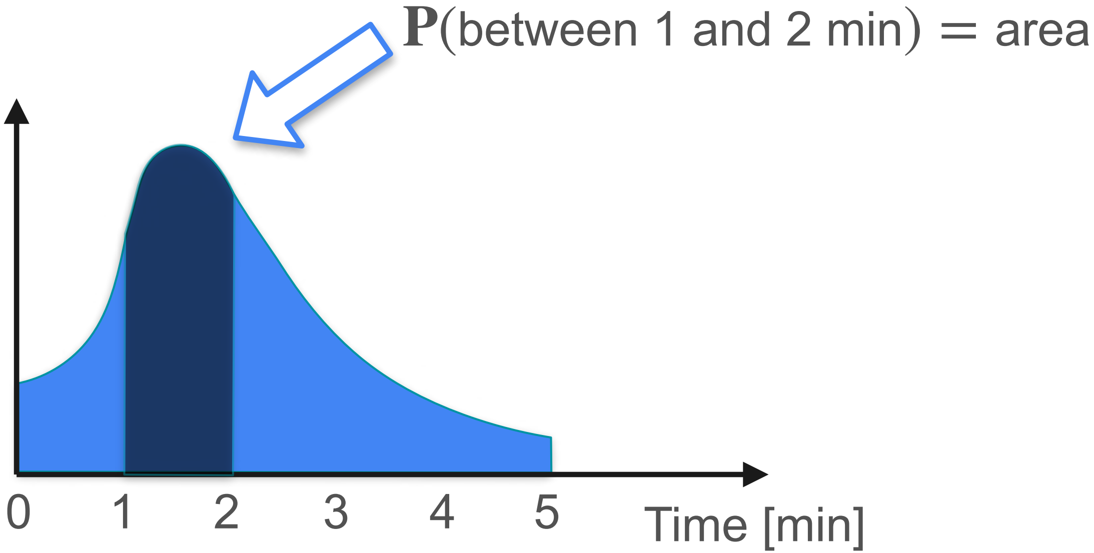
$\mathbf{P}(\text{between 2 and 3 min}) =$
sum of shaded areas



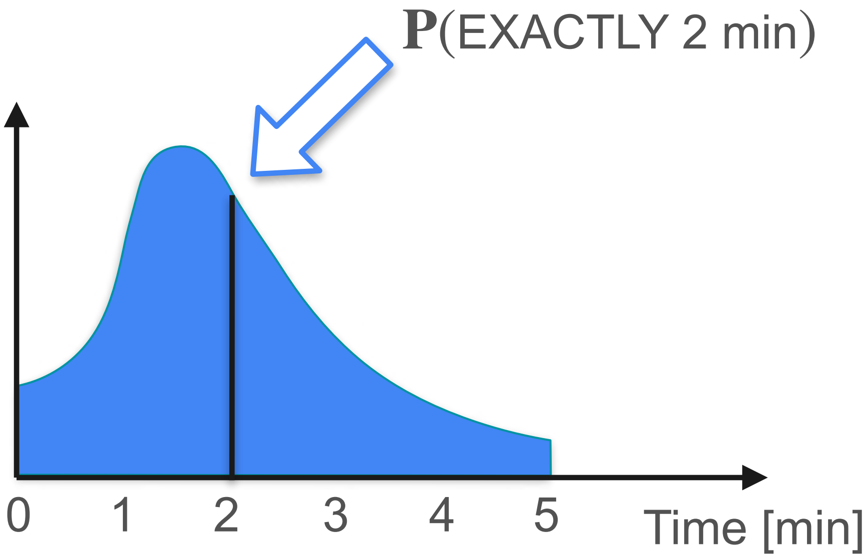
Probability Density Function



Probability Density Function

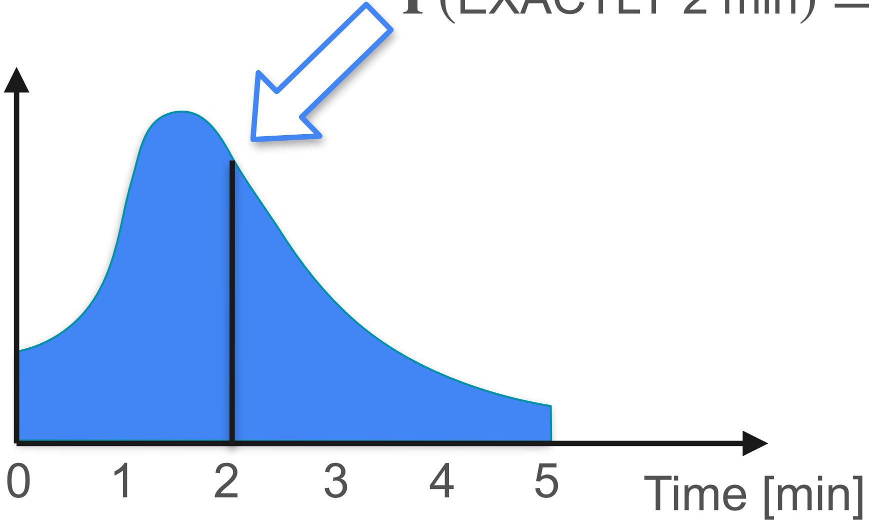


Probability Density Function

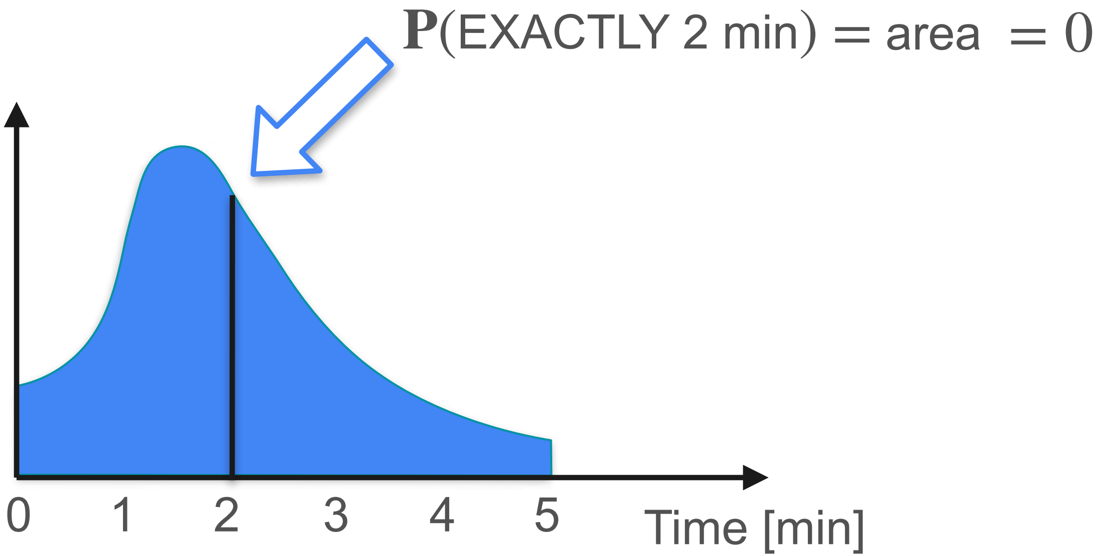


Probability Density Function

$\mathbf{P}(\text{EXACTLY 2 min}) = \text{area}$

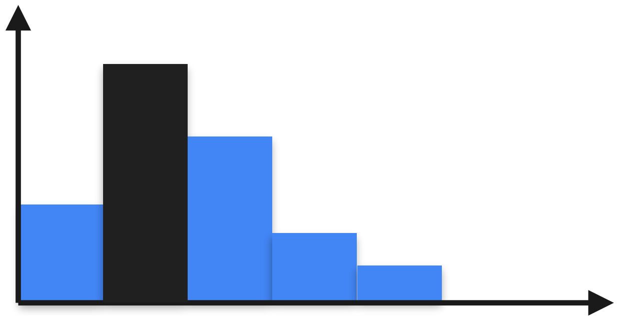


Probability Density Function

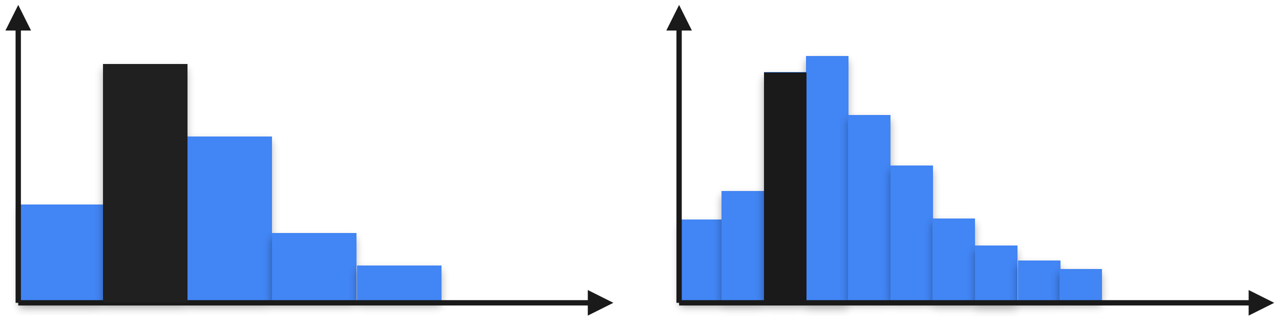


Probability Density Function

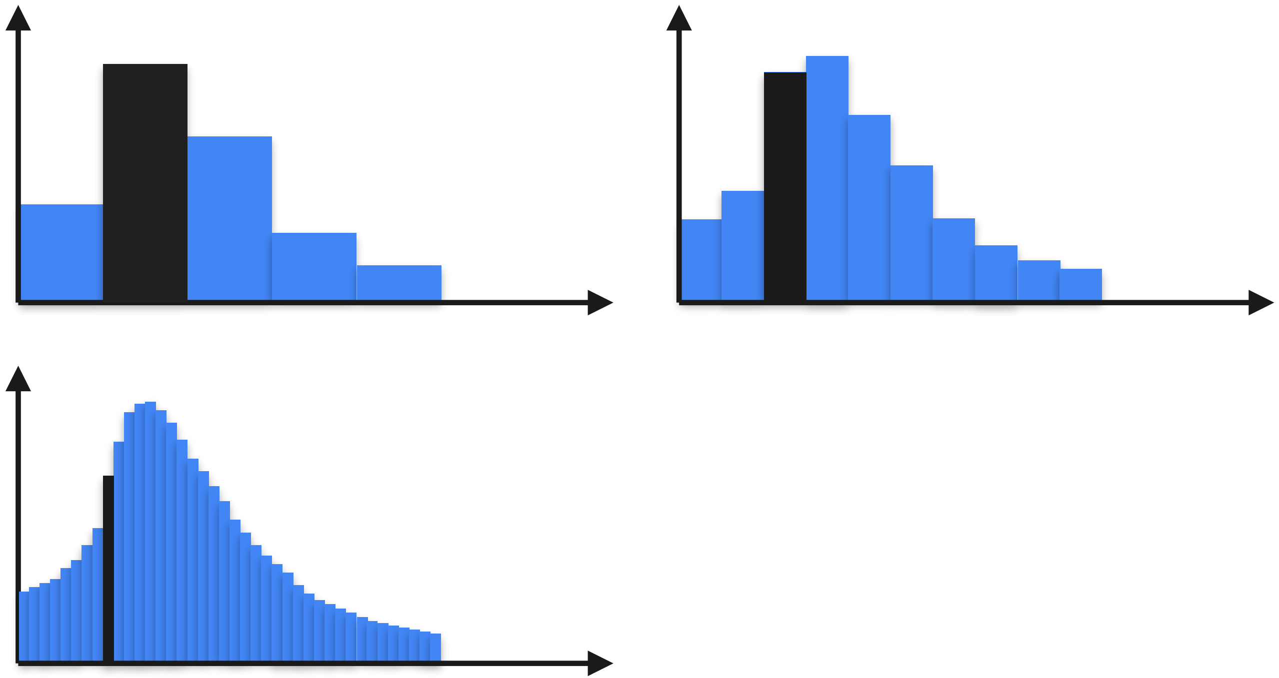
Probability Density Function



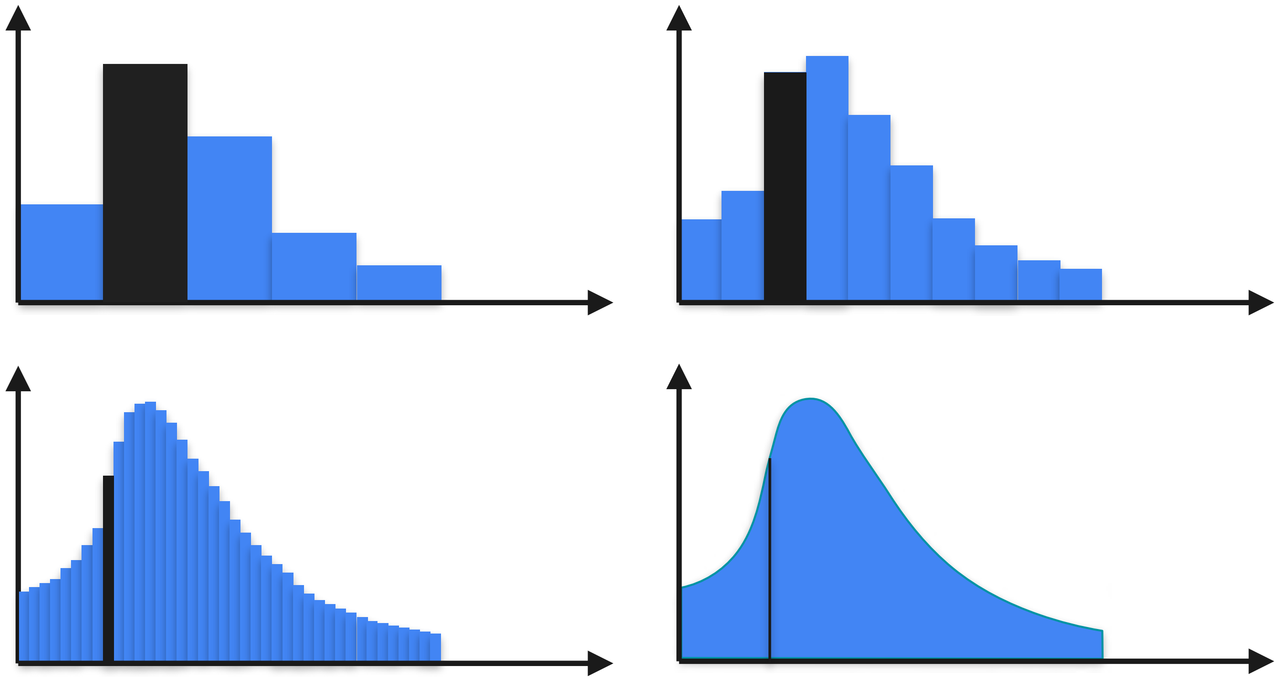
Probability Density Function



Probability Density Function



Probability Density Function



Probability Density Function: Formal Definition

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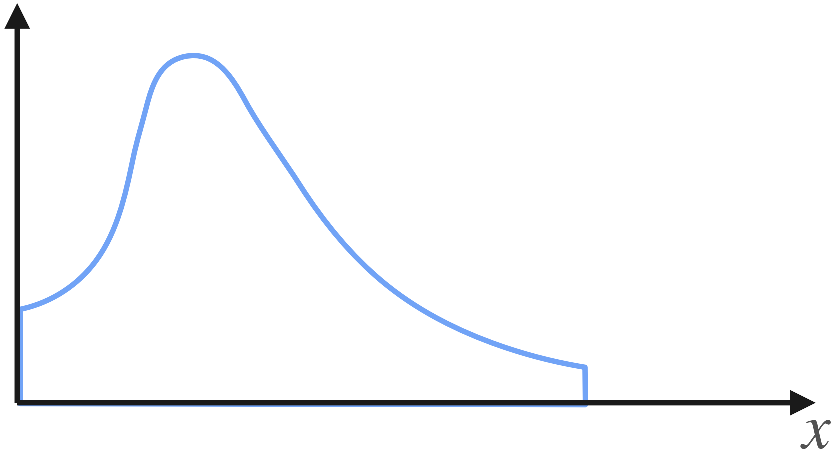
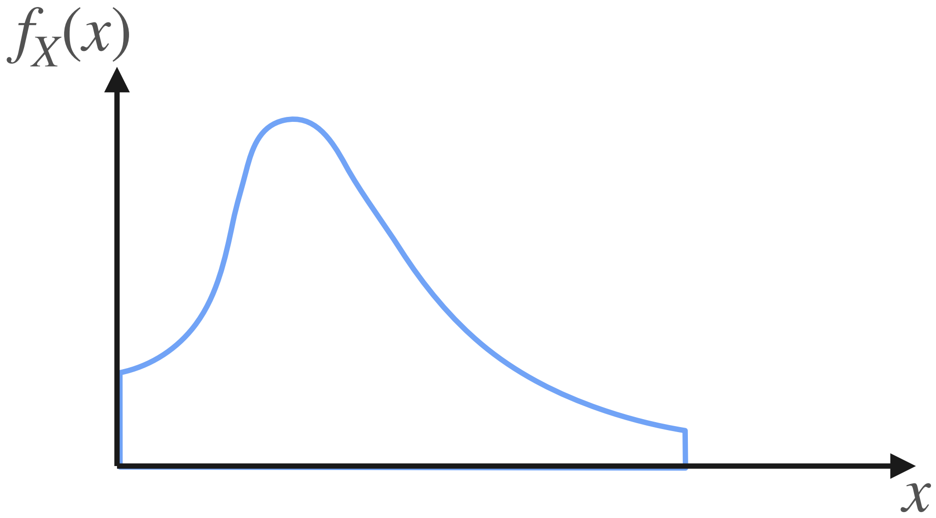


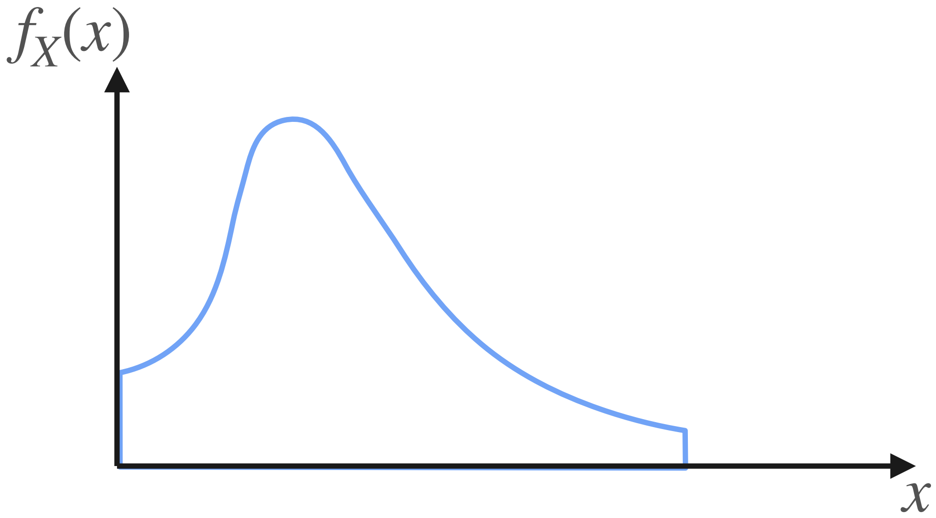
Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Probability Density Function: Formal Definition

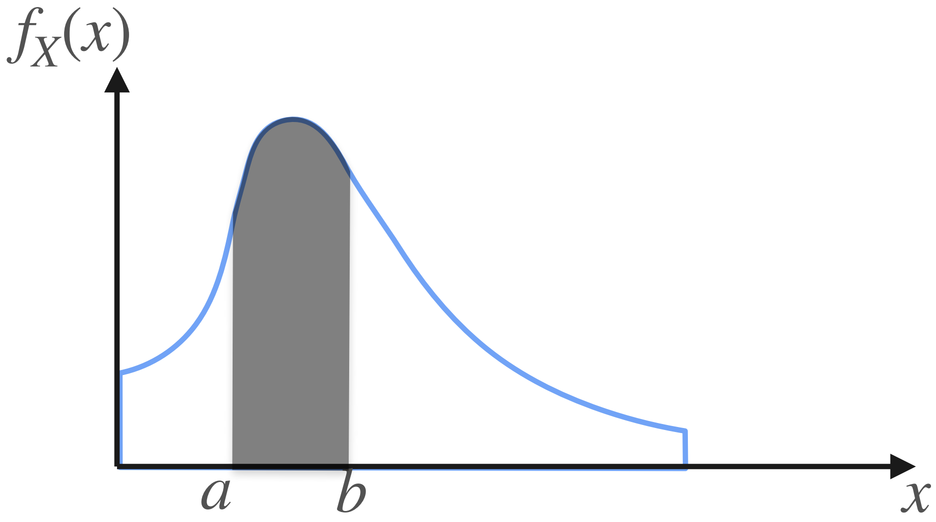


Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

Probability Density Function: Formal Definition



Probability Density Function (PDF)

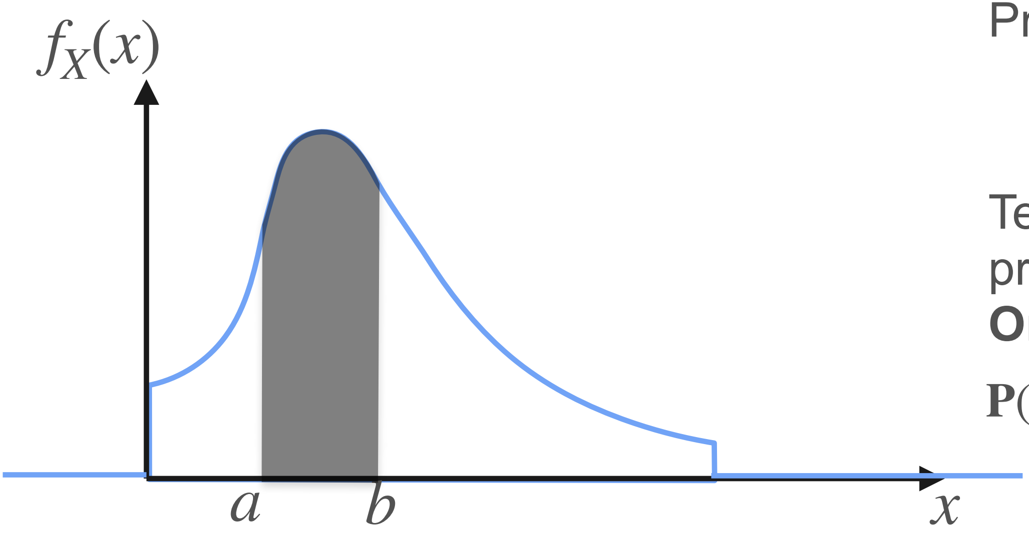
$$f_X(x)$$

Tells you the rate you accumulate probability around each point.

Only defined for continuous variables!

$\mathbf{P}(a < X < b)$ = area under $f_X(x)$

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$f_X(x)$

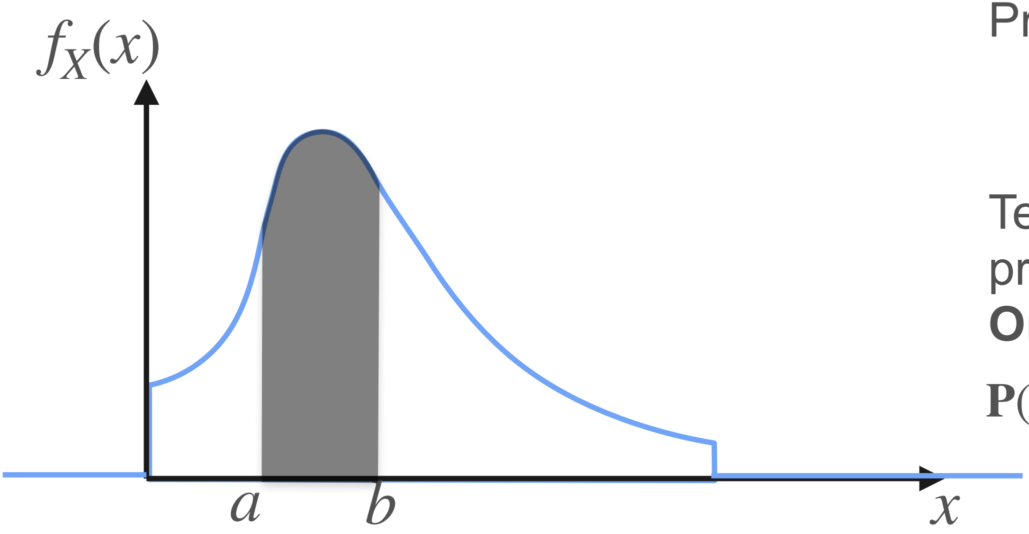
Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

$\mathbf{P}(a < X < b) =$ area under $f_X(x)$

$f_X(x)$ needs to satisfy:

- It is defined for all numbers

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$f_X(x)$

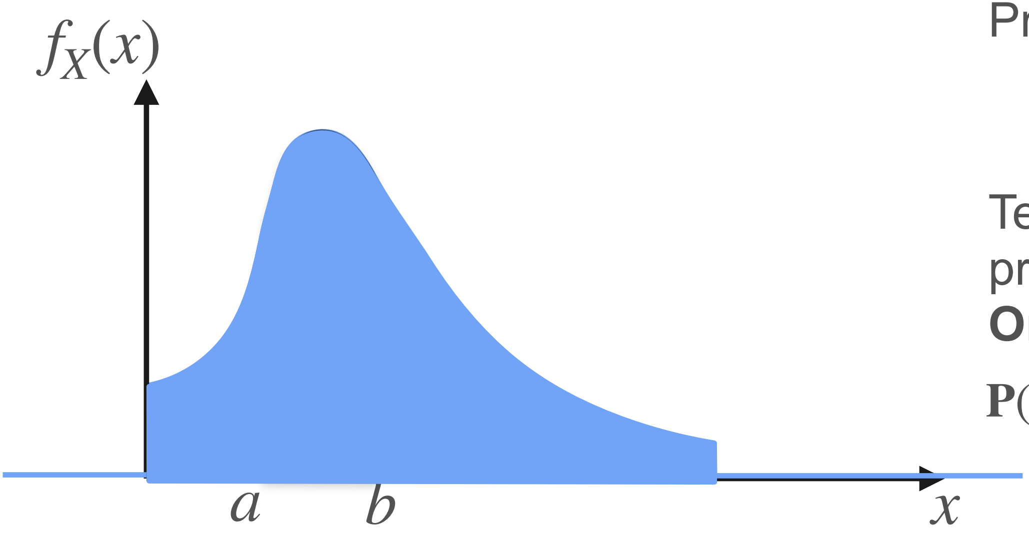
Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

$\mathbf{P}(a < X < b) =$ area under $f_X(x)$

$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$

Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.
Only defined for continuous variables!

$\mathbf{P}(a < X < b) =$ area under $f_X(x)$

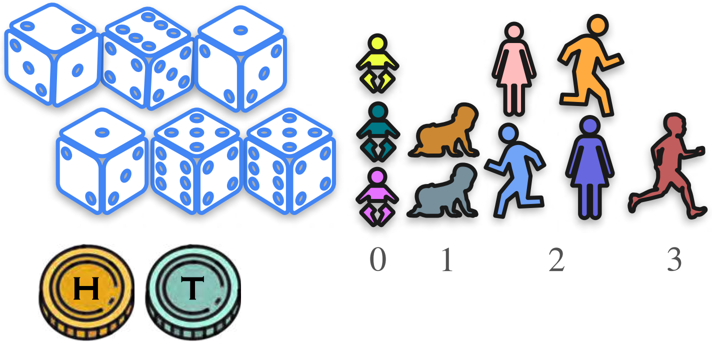
$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$
- Area under $f_X(x) = 1$

Discrete and Continuous Random Variables

Discrete and Continuous Random Variables

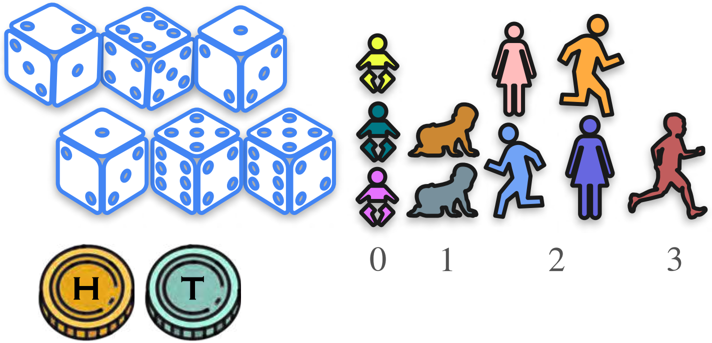
Discrete random variables



Can take only a **finite** or at most countable number of values

Discrete and Continuous Random Variables

Discrete random variables



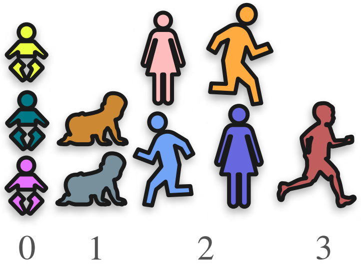
Can take only a **finite** or at most countable number of values

Continuous random variables

Takes values on an interval (infinite possibilities!)

Discrete and Continuous Random Variables

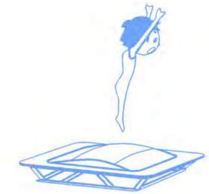
Discrete random variables



Can take only a **finite** or at most countable number of values

PMF: $p_X(x) = \mathbf{P}(X = x)$

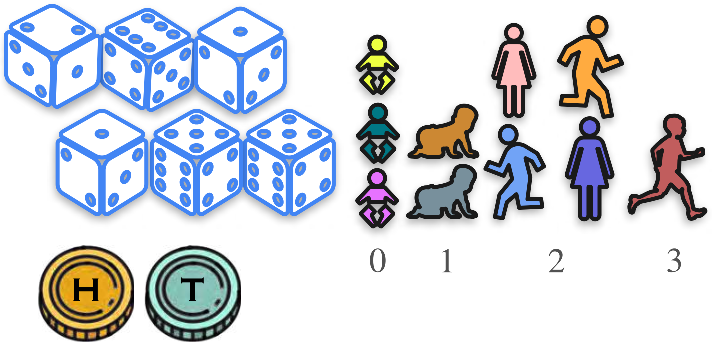
Continuous random variables



Takes values on an interval (infinite possibilities!)

Discrete and Continuous Random Variables

Discrete random variables



Can take only a **finite** or at most countable number of values

PMF: $p_X(x) = \mathbf{P}(X = x)$

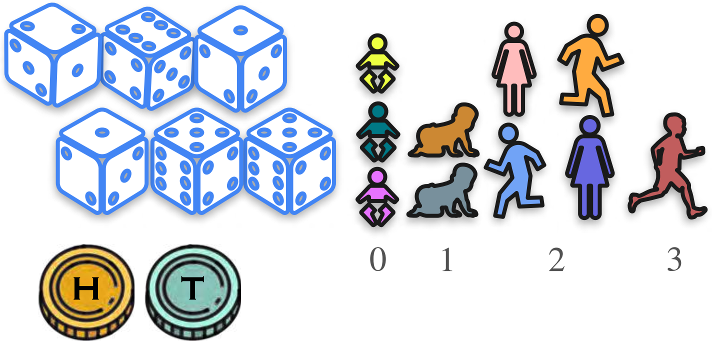
Continuous random variables

Takes values on an interval (infinite possibilities!)

PDF: $f_X(x)$

Discrete and Continuous Random Variables

Discrete random variables



Can take only a **finite** or at most countable number of values

PMF: $p_X(x) = \mathbf{P}(X = x)$

Continuous random variables

Takes values on an interval (infinite possibilities!)

PDF: $f_X(x)$

$\mathbf{P}(X = x) = 0 \quad \forall x$

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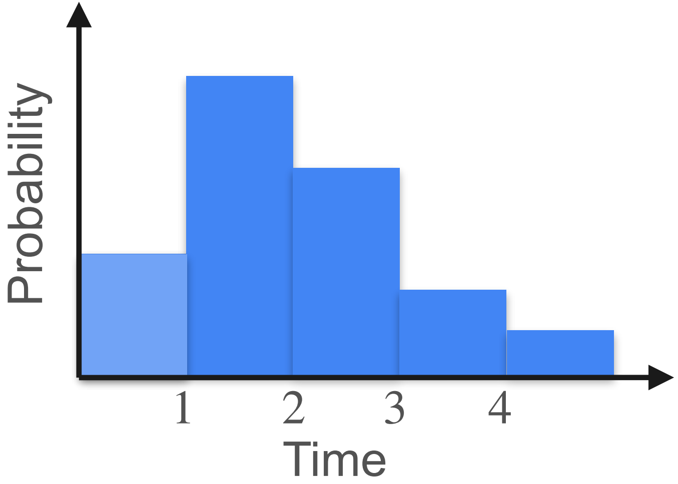
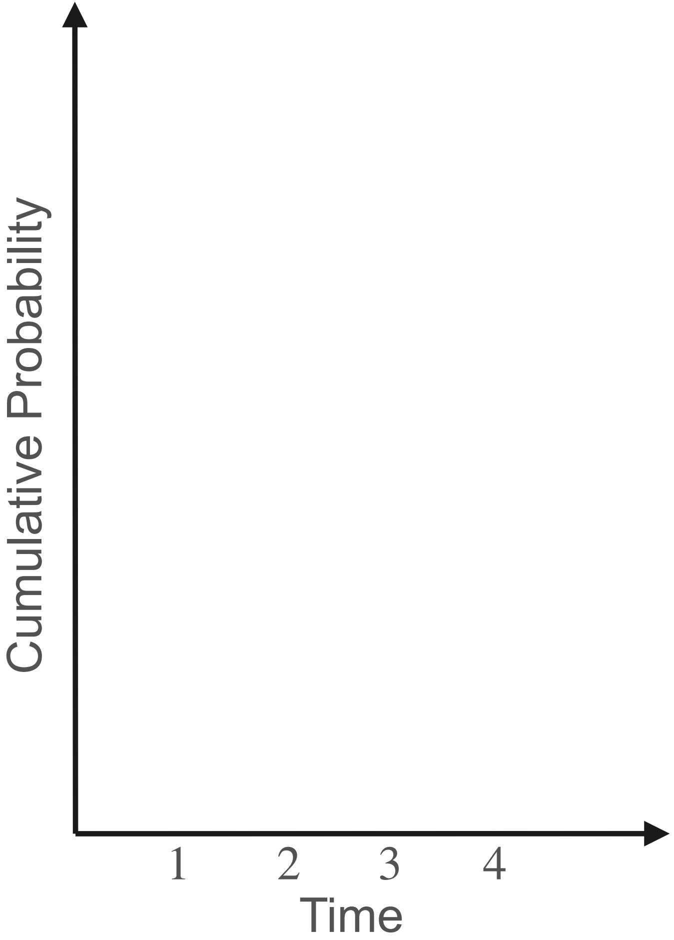
Probability Distributions

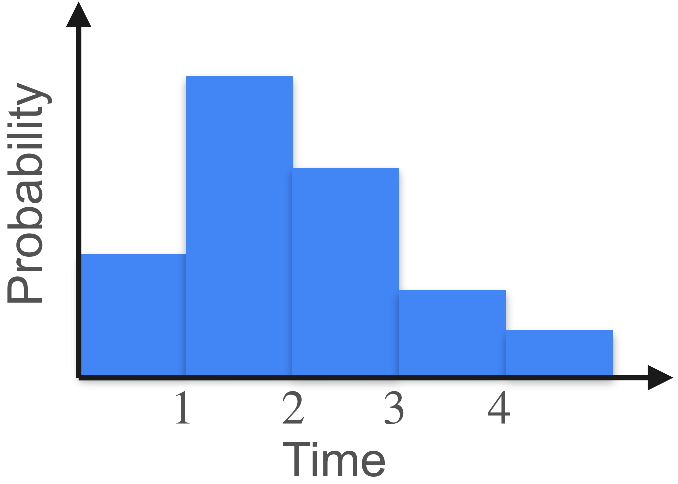
Cumulative Distribution Function

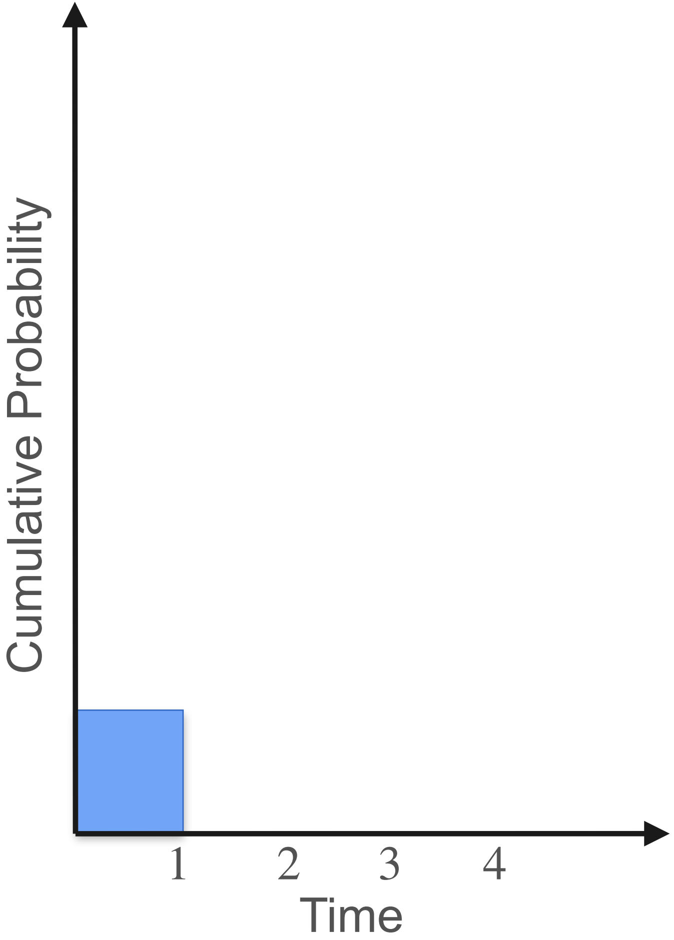
Cumulative Distribution





Cumulative Distribution





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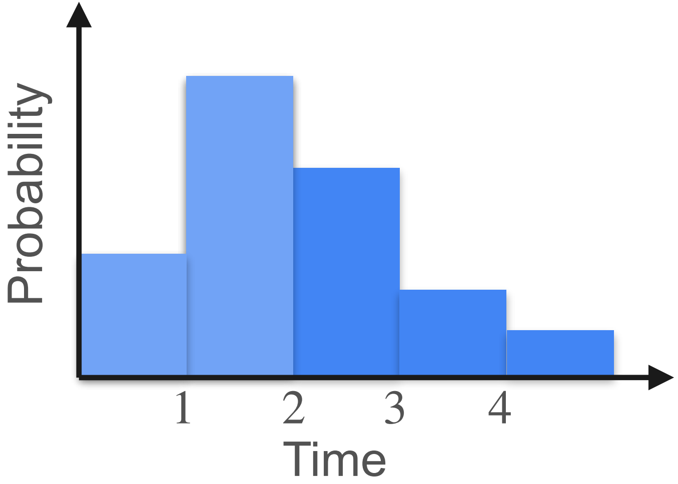

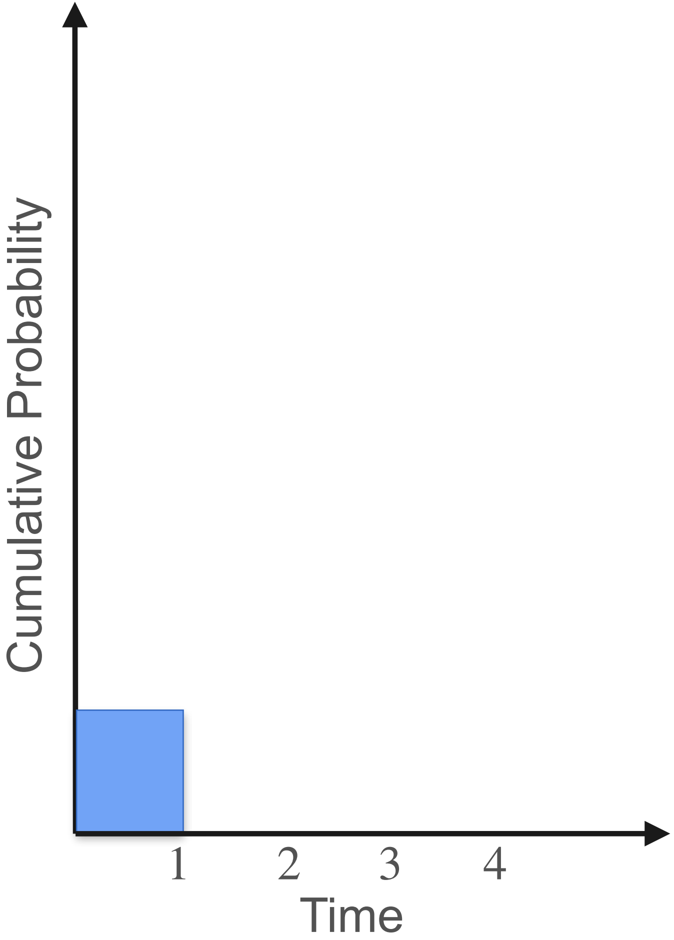

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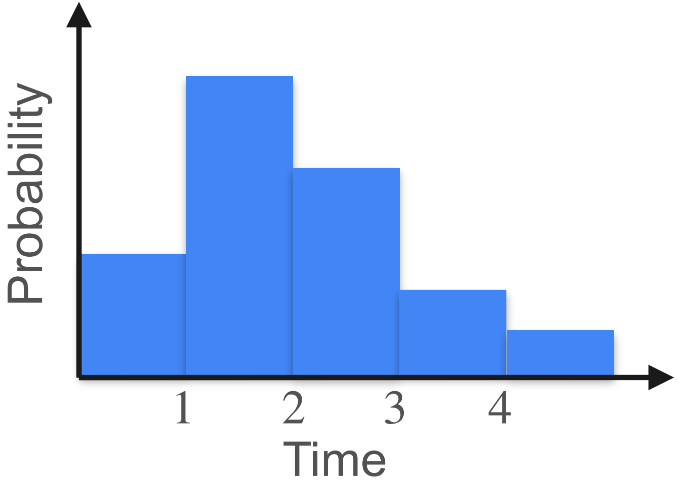
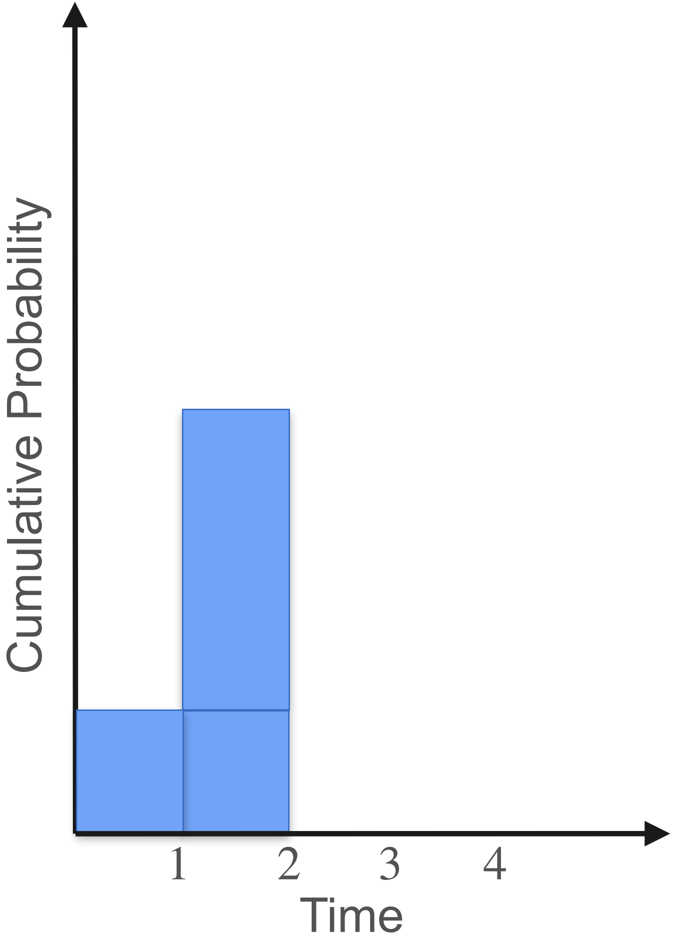
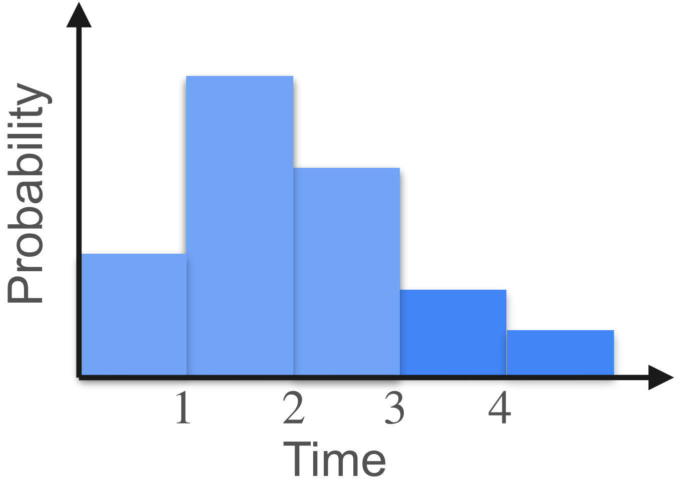
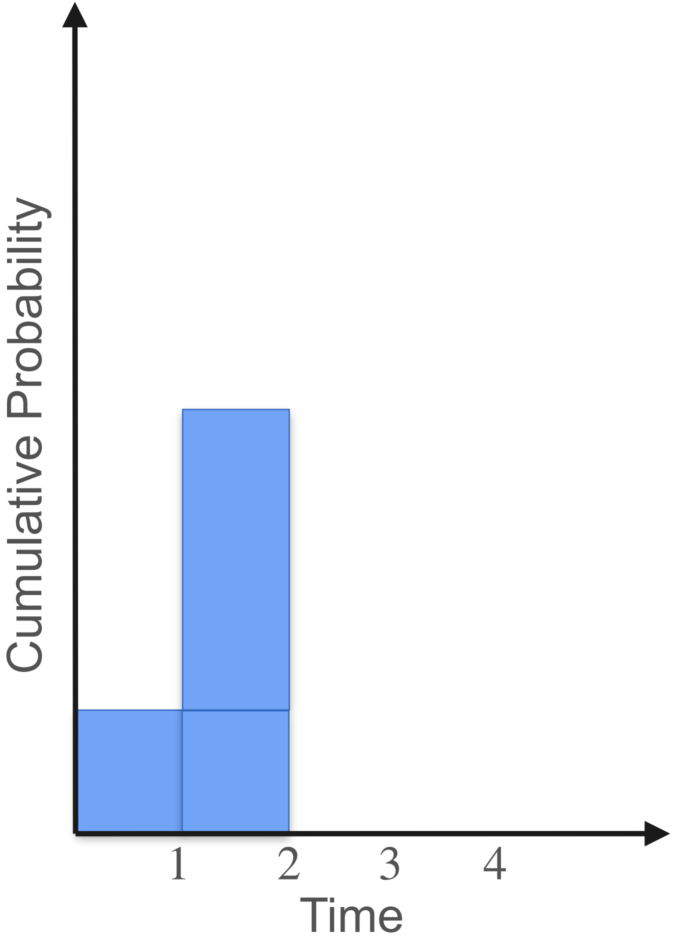




Cumulative Distribution





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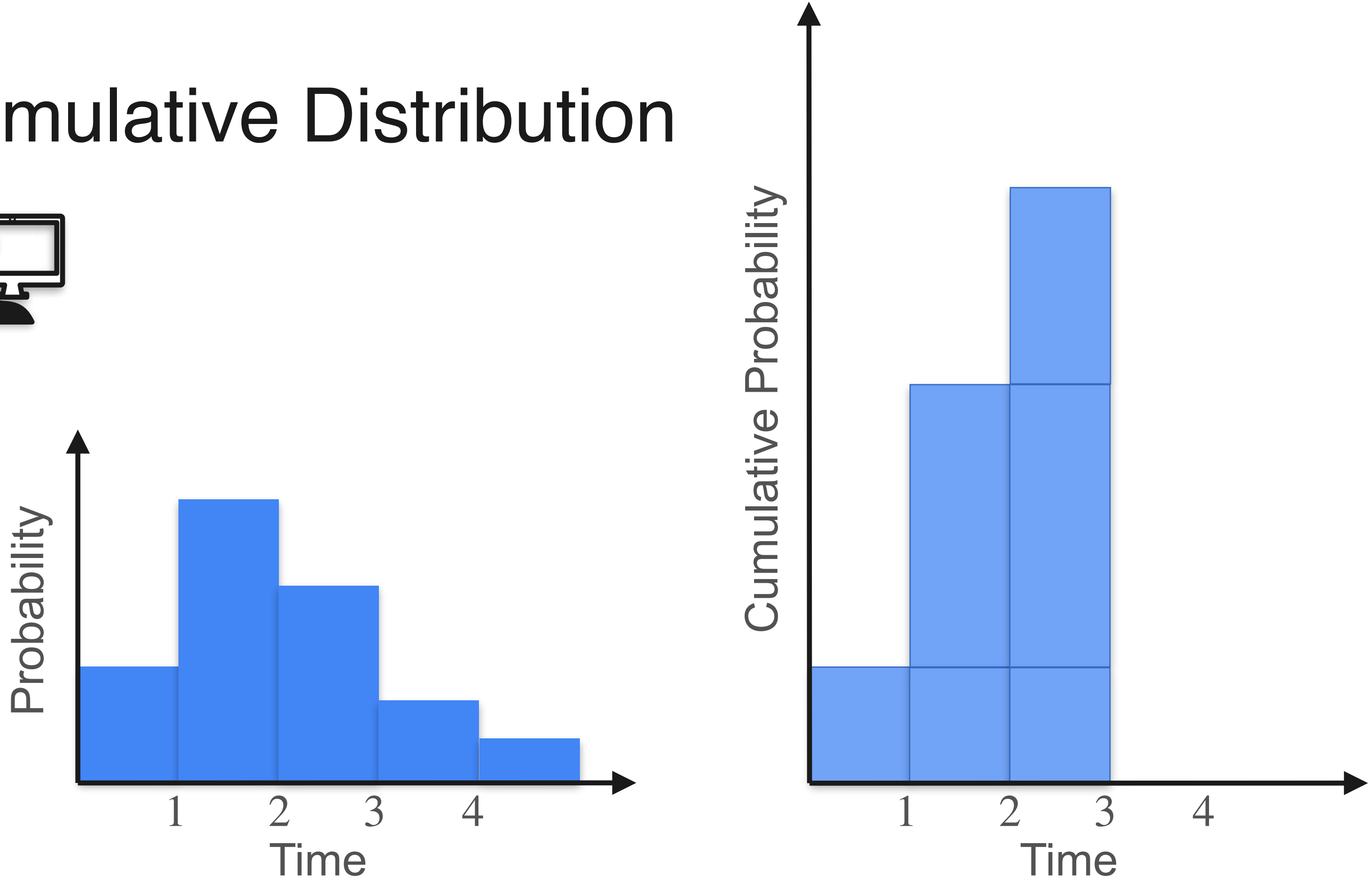

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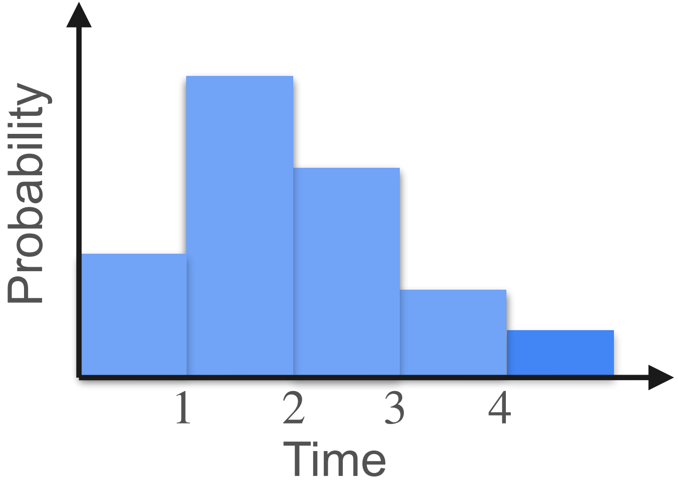

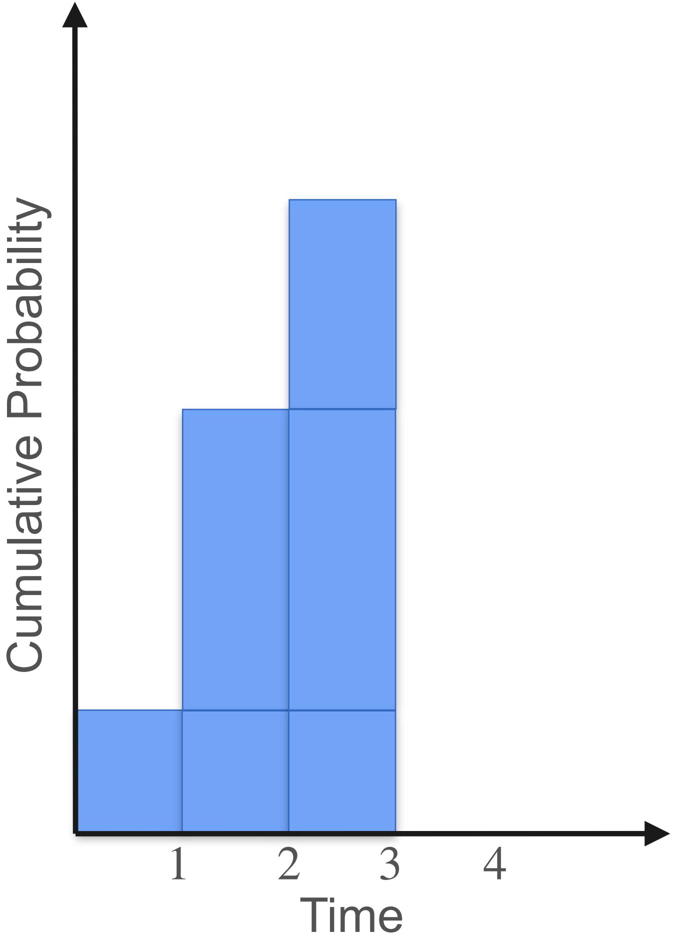

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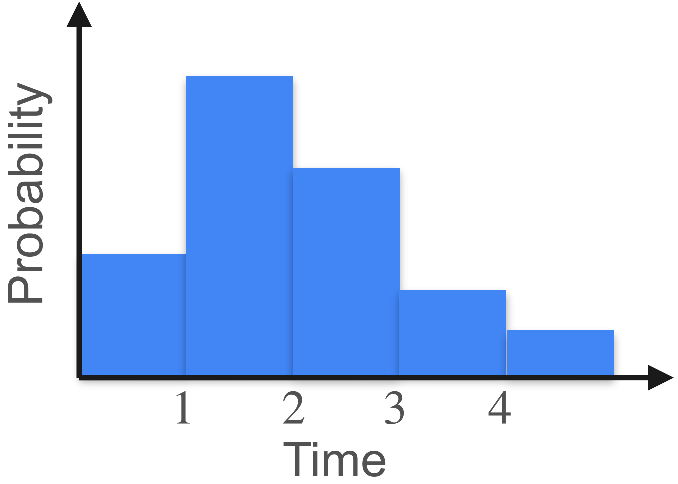

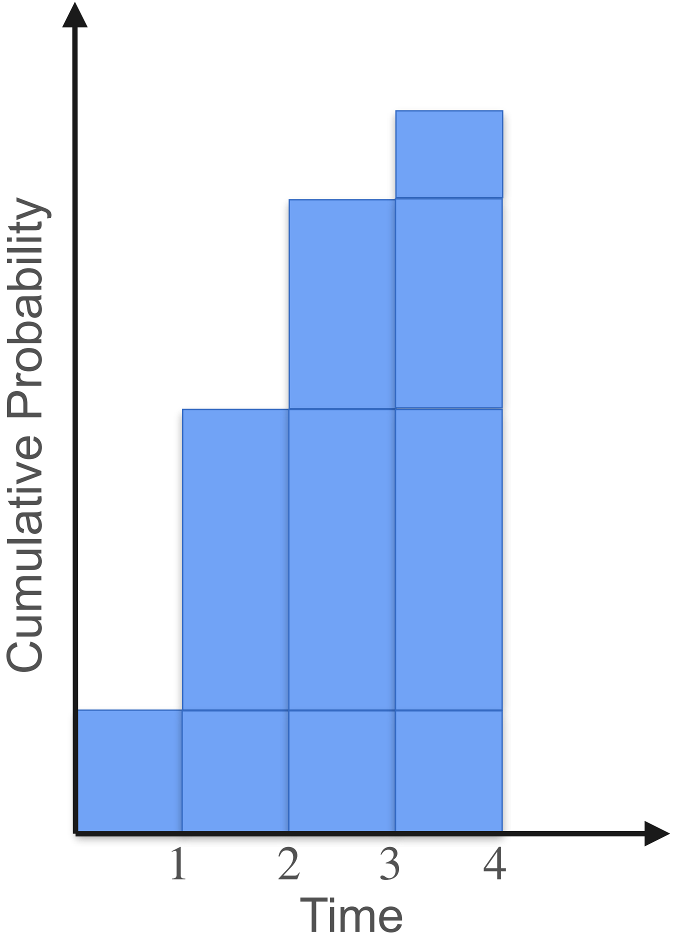

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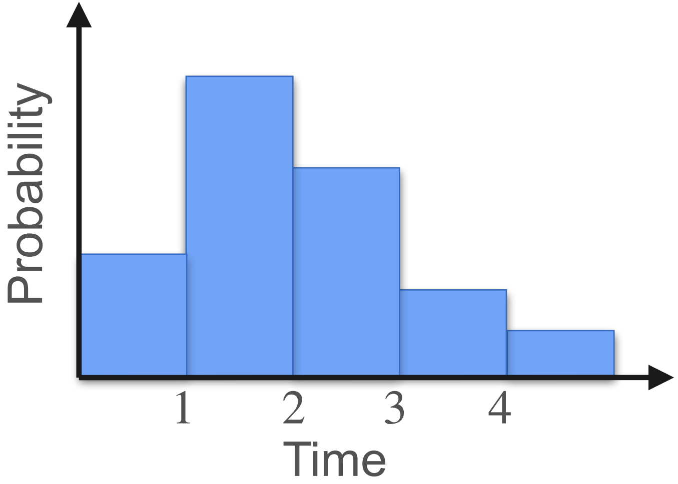
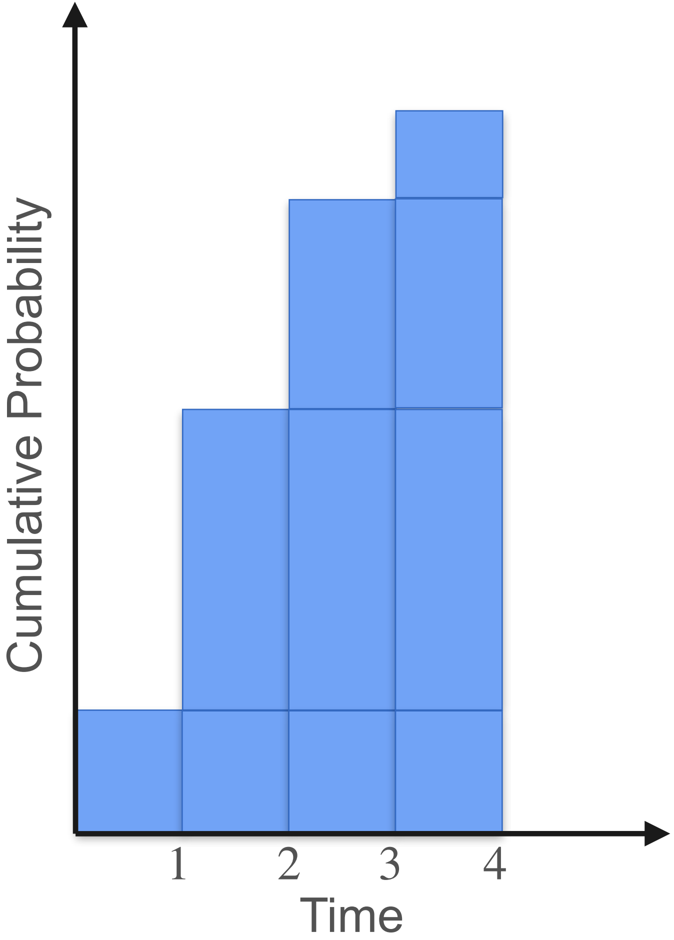
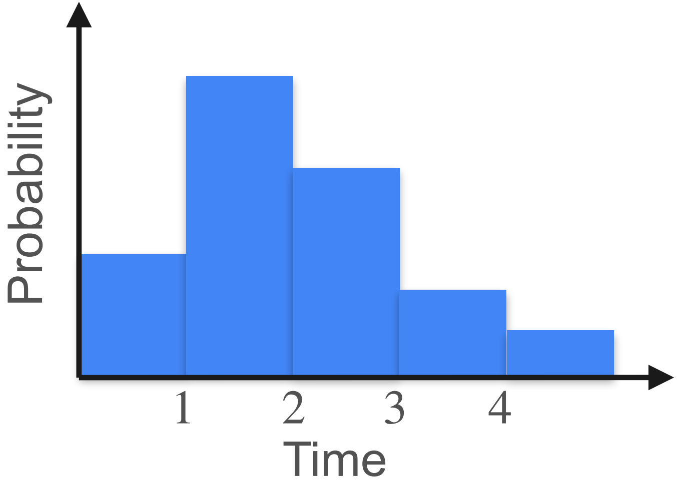
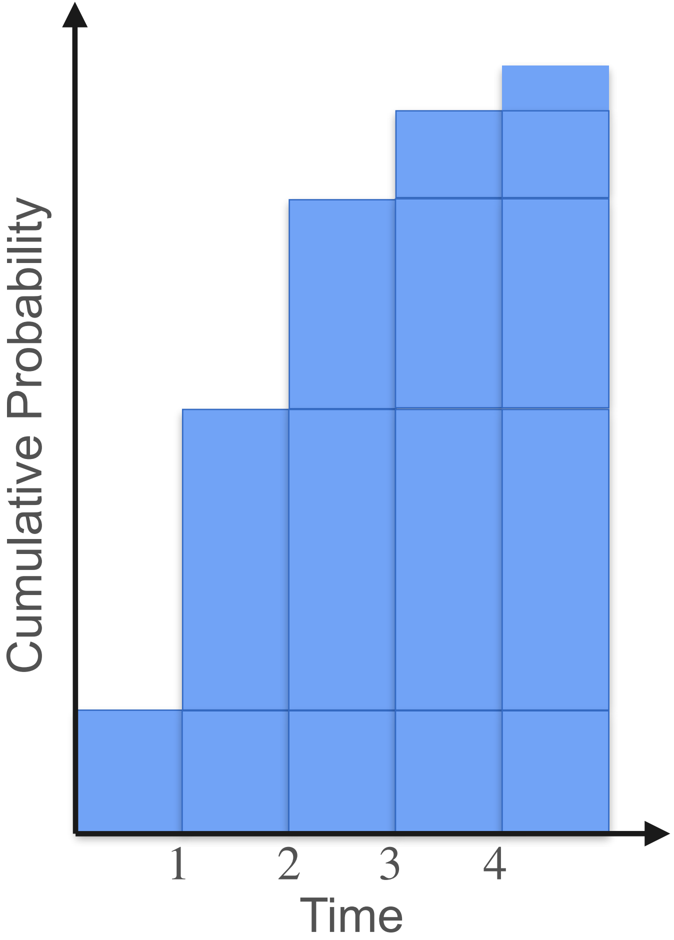




Cumulative Distribution





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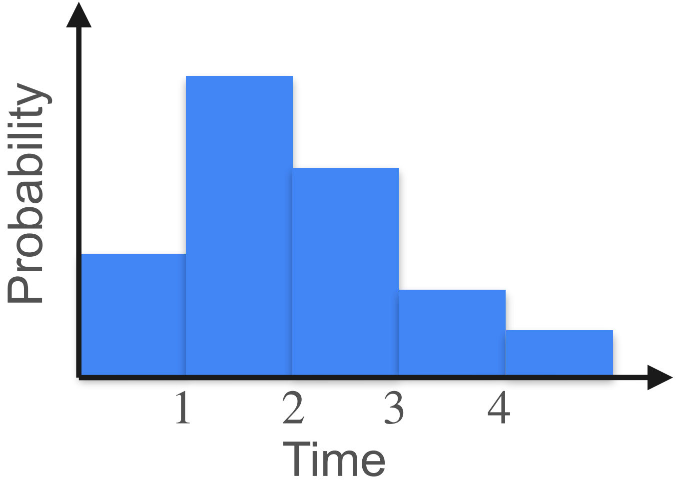
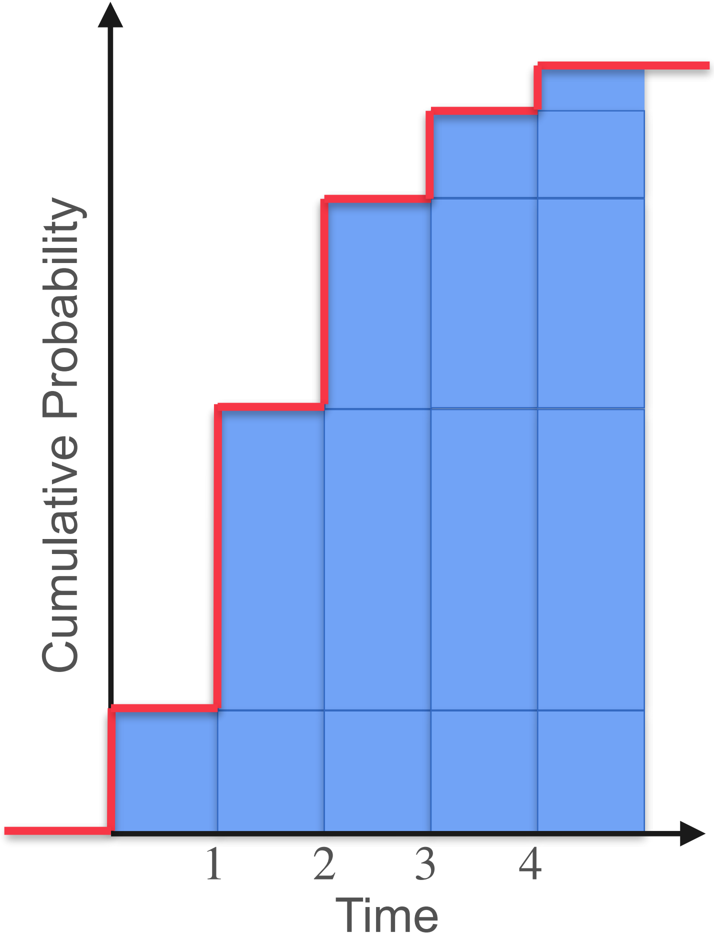
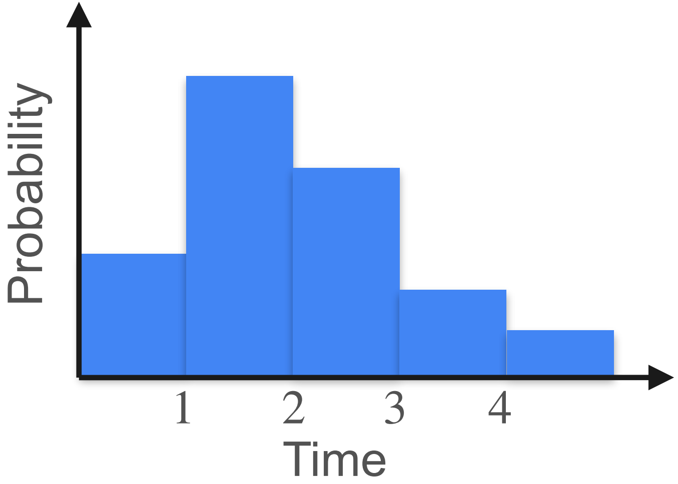
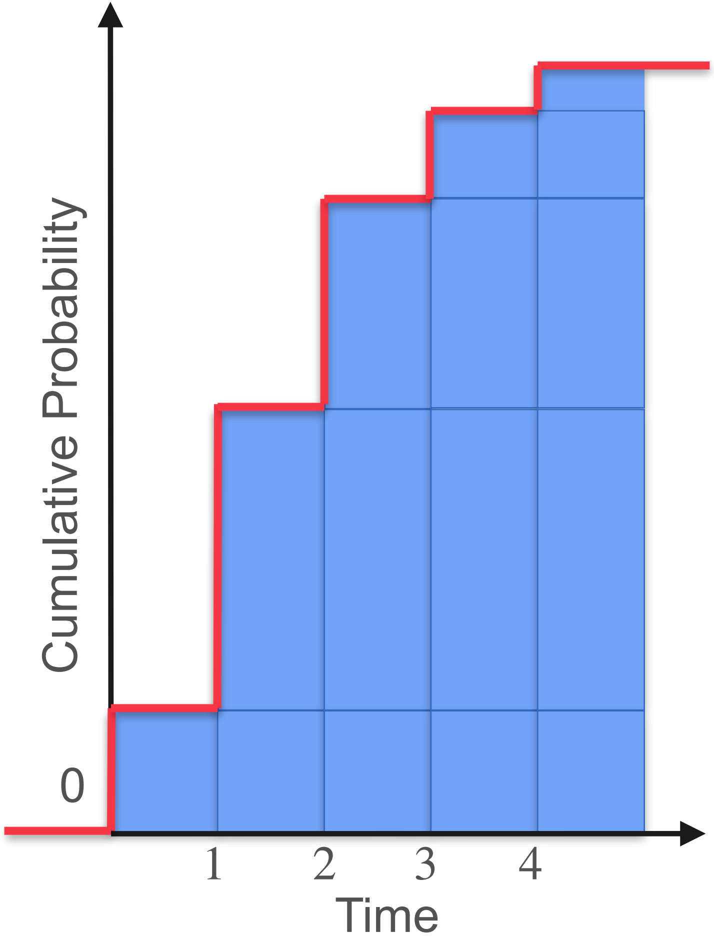




Cumulative Distribution





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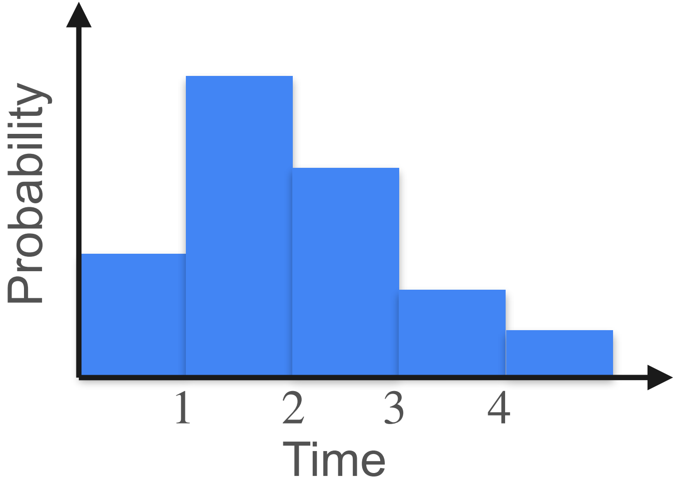
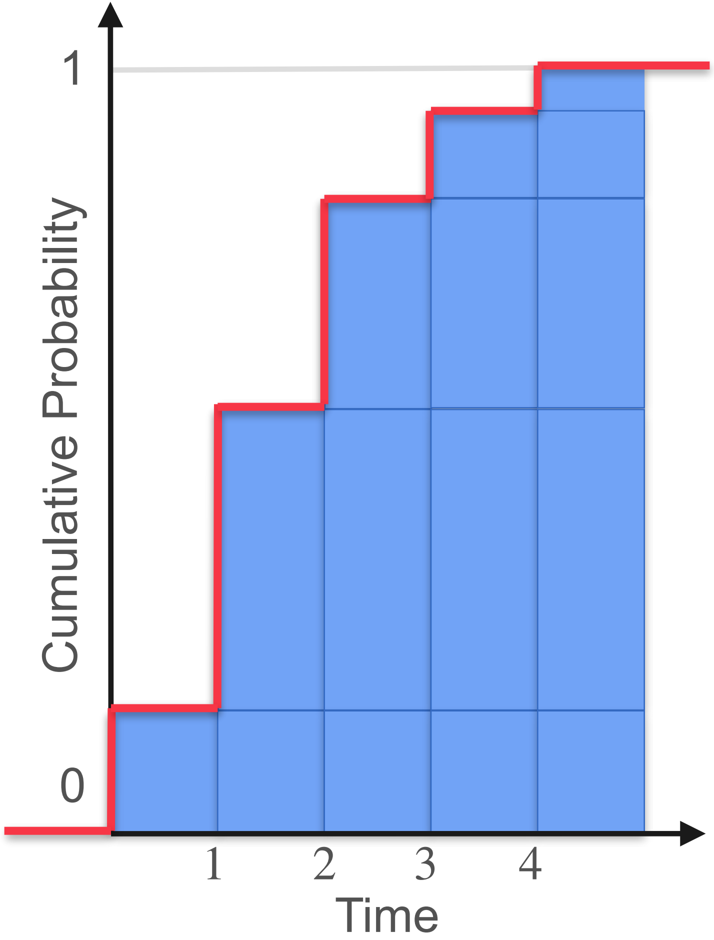
Cumulative Distribution





Cumulative Distribution

CDF: Cumulative distribution function

Cumulative Distribution

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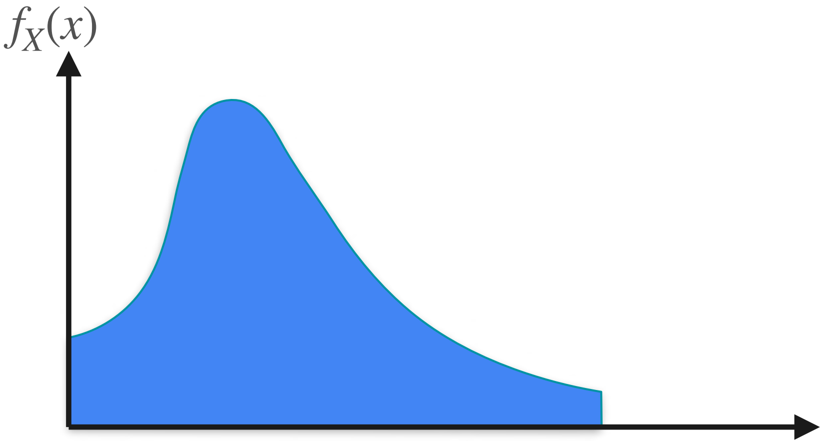
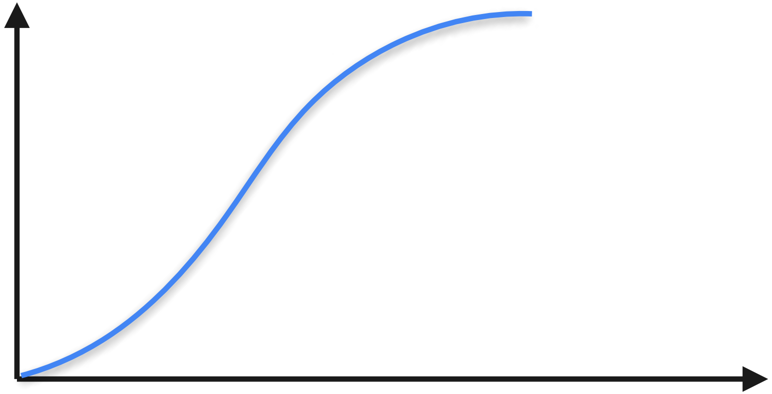
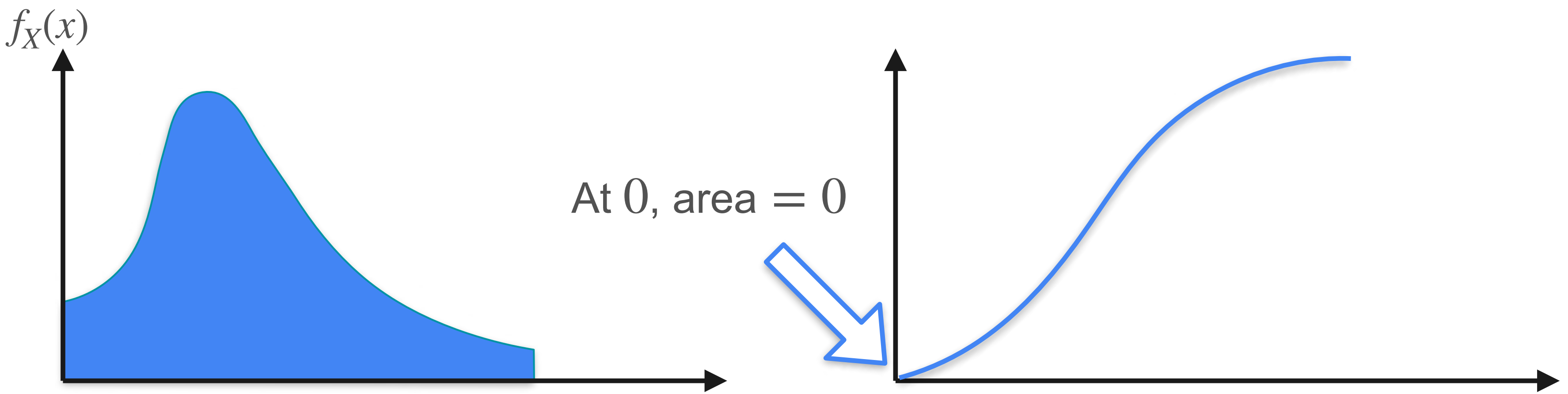




Cumulative Distribution

CDF: Cumulative distribution function



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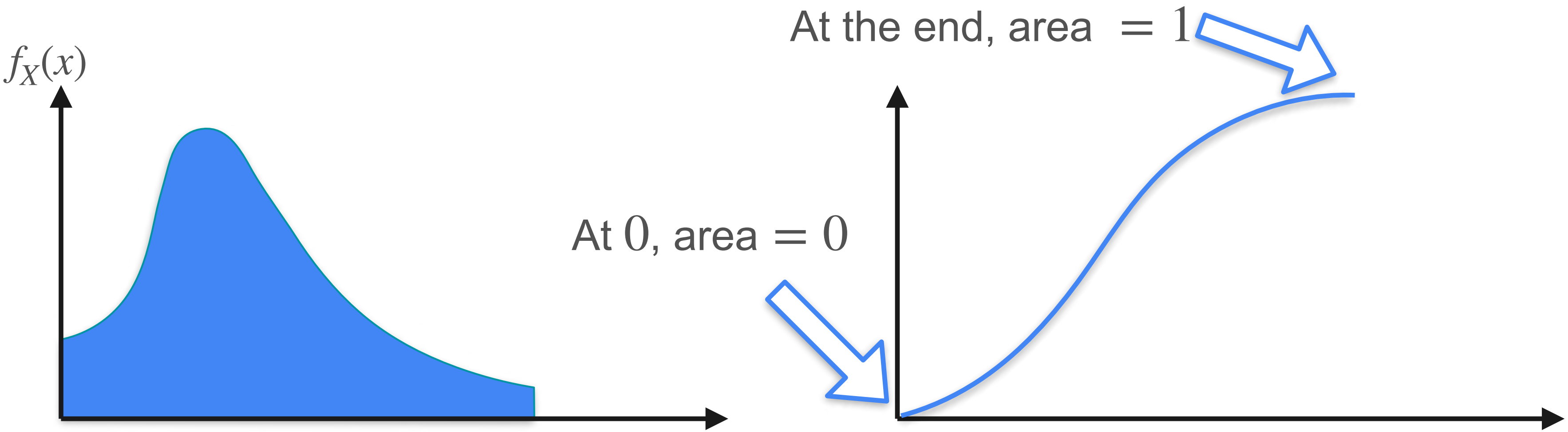
CDF: Cumulative distribution function



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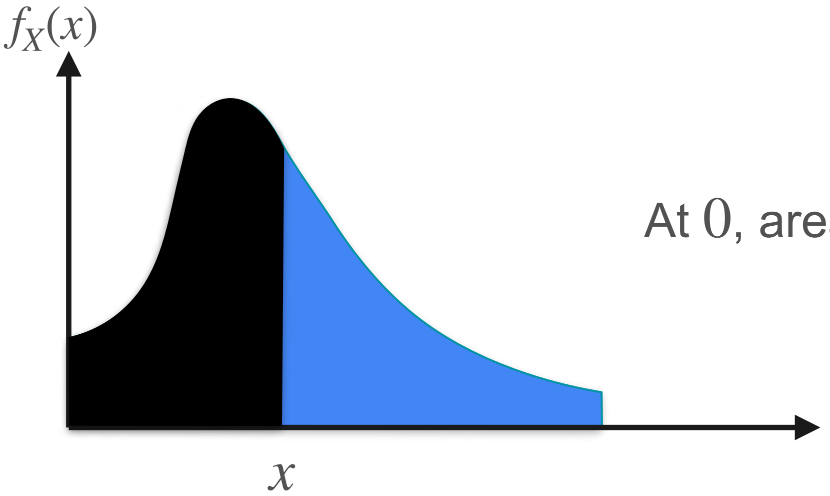

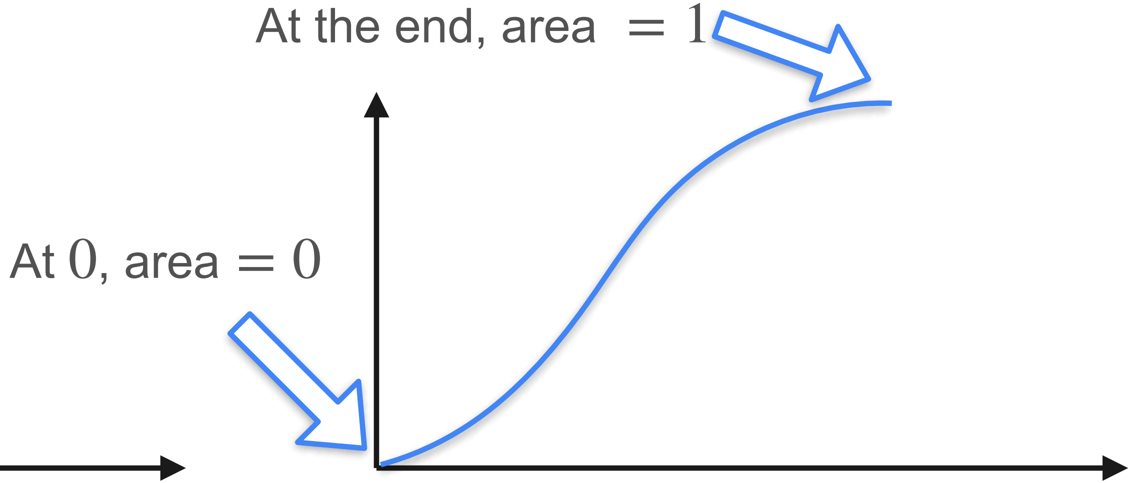

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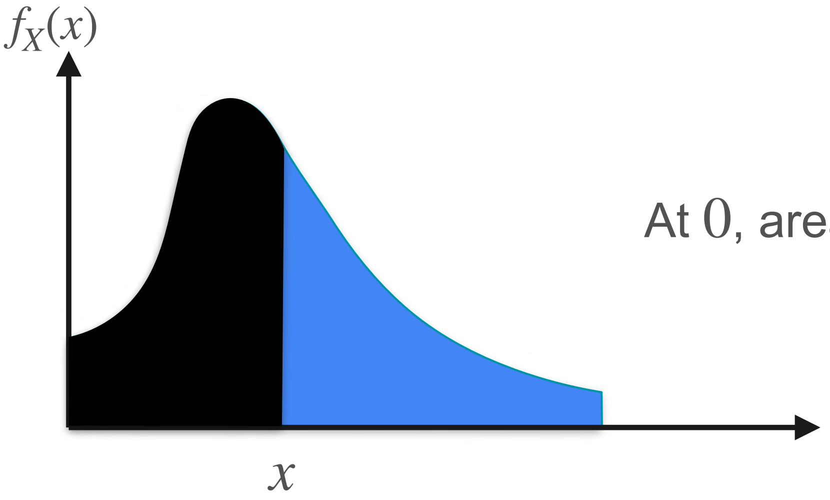
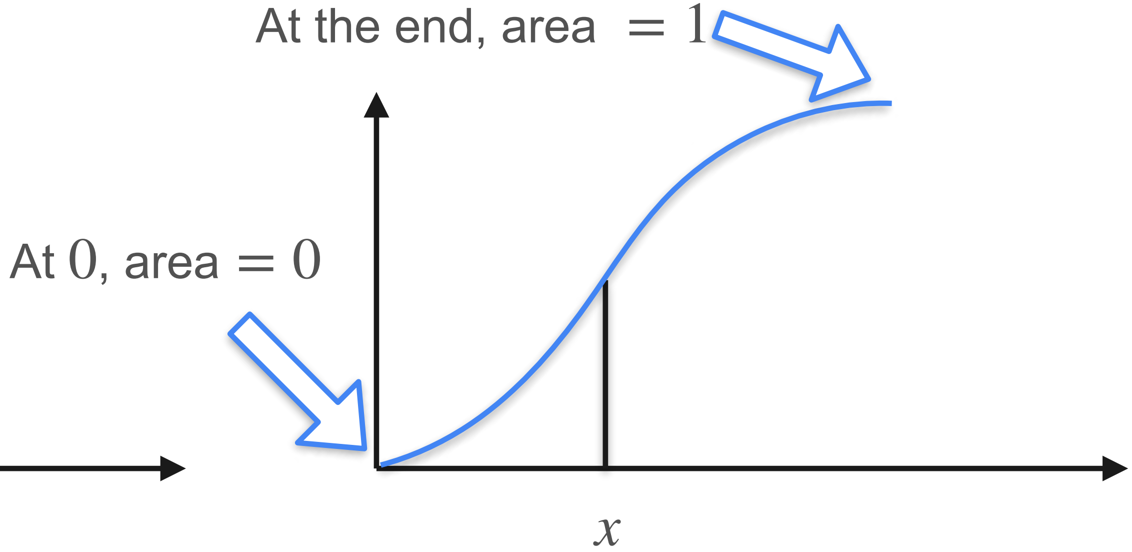
CDF: Cumulative distribution function





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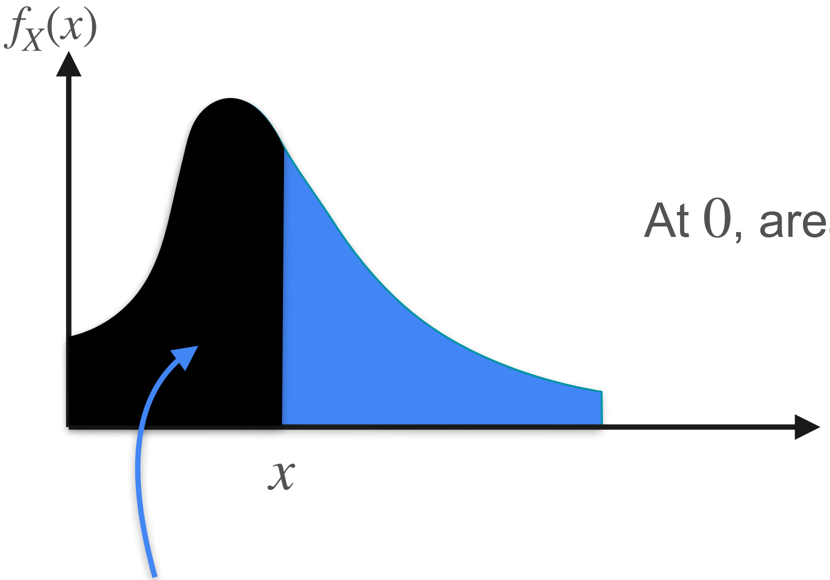

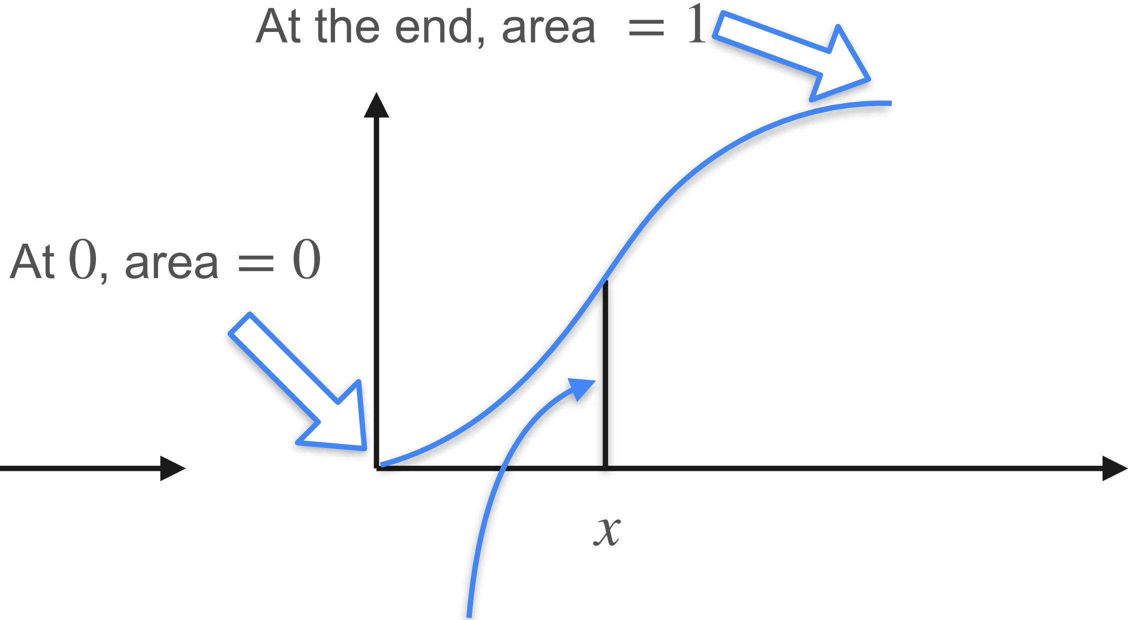

$\mathbf{P}$(less than or equal to 2 minutes) $= 0.5$ $\mathbf{P}$(less than or equal to 2 minutes) $= 0.5$

Cumulative Distribution Function: Formal Definition

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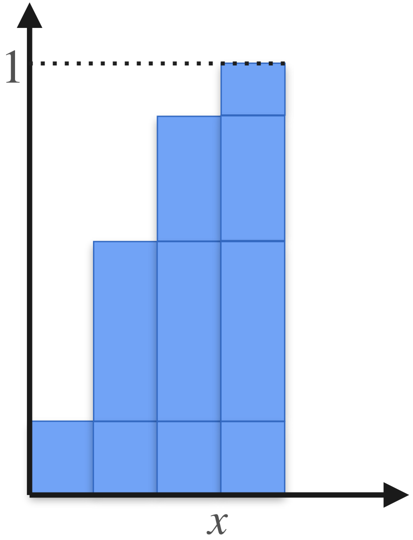
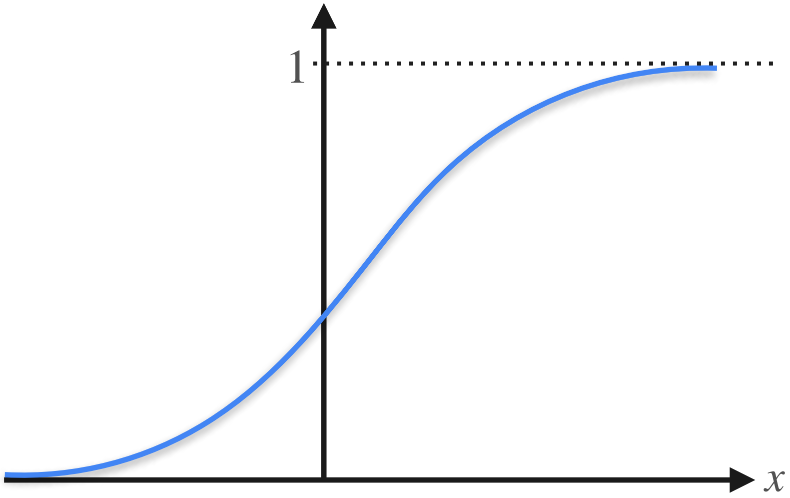
The CDF shows how much probability the variable has accumulated until a certain value

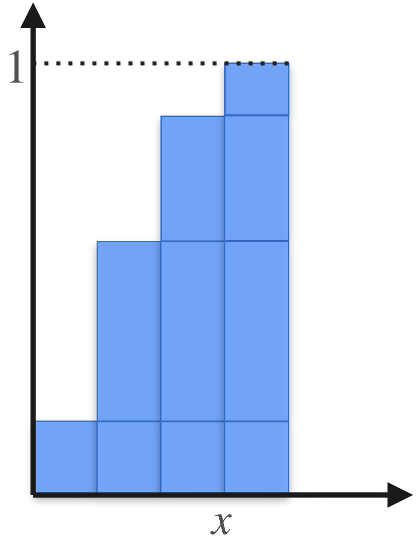


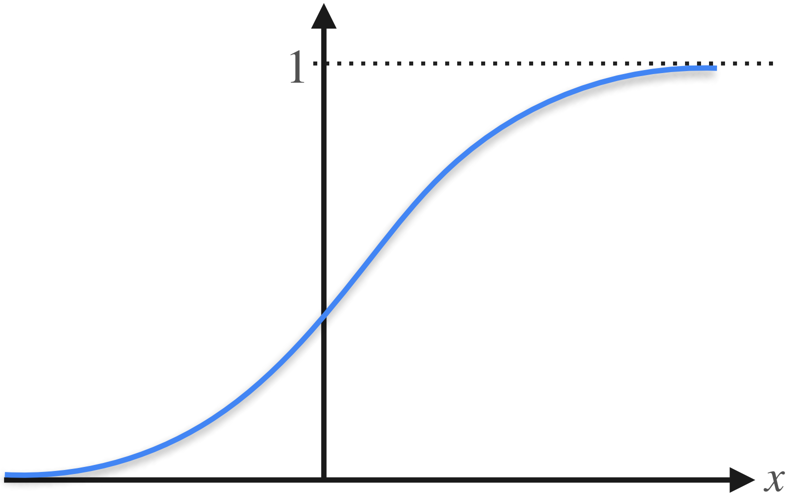


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $\text{CDF}(x) = \mathbf{P}(X \leq x)$

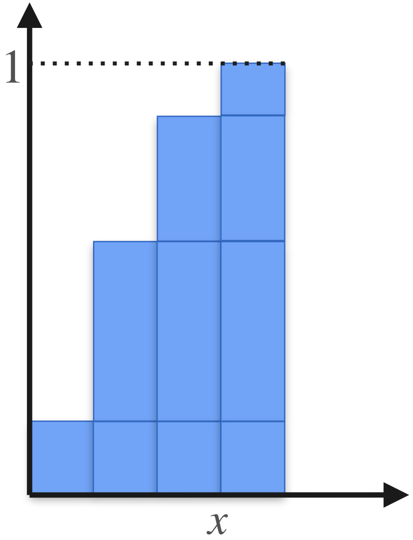


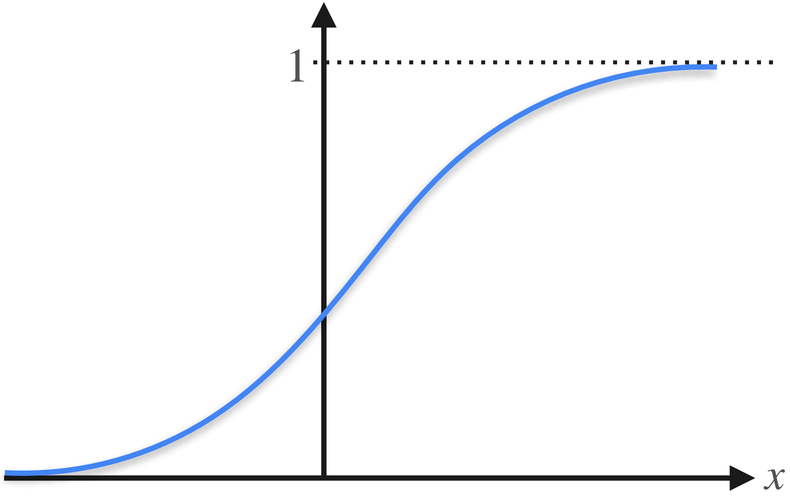


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $\text{CDF}(x) = \mathbf{P}(X \leq x)$ ← It is defined for every real number

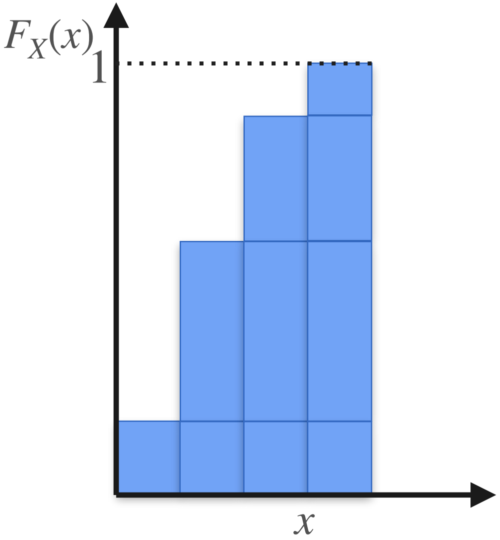


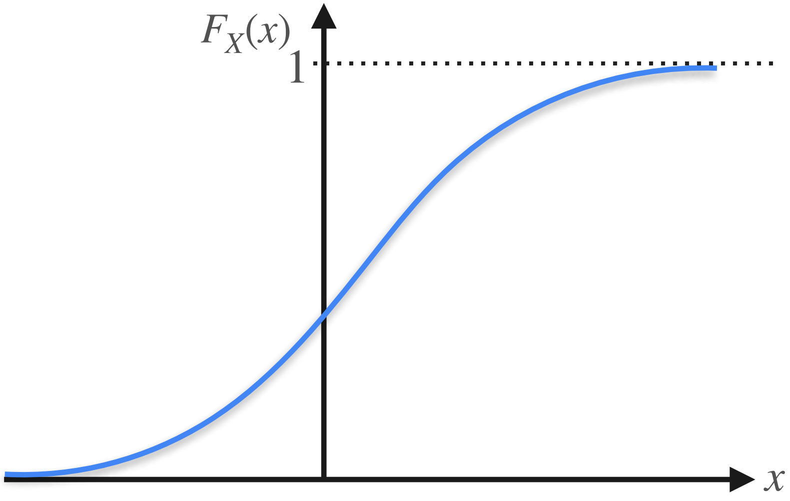


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $F_X(x) = \mathbf{P}(X \leq x)$ ← It is defined for every real number





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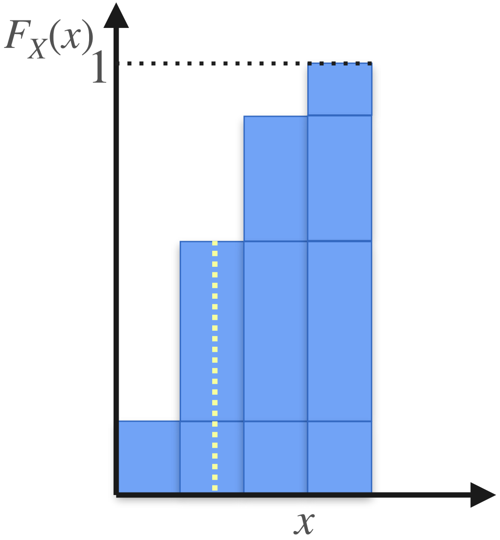

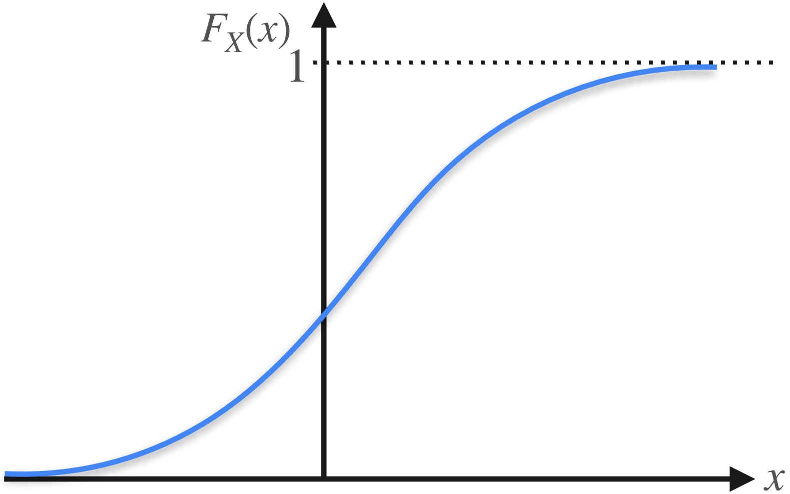

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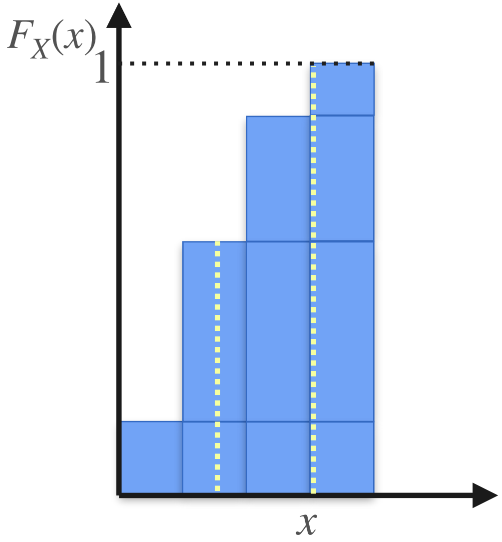

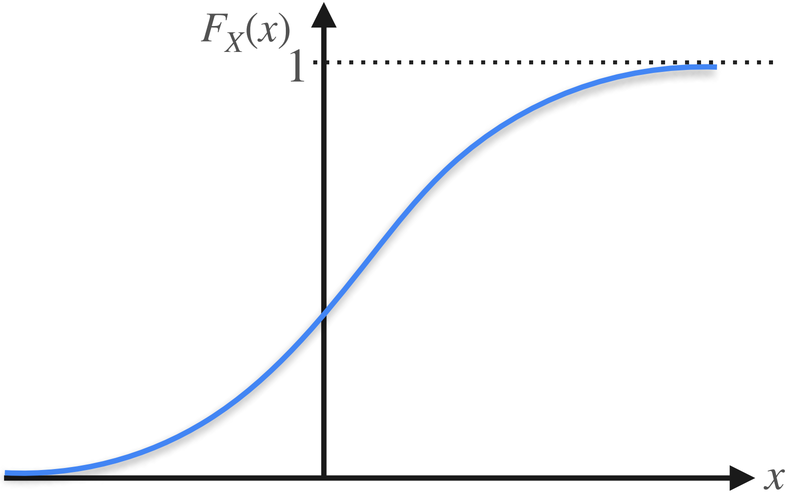

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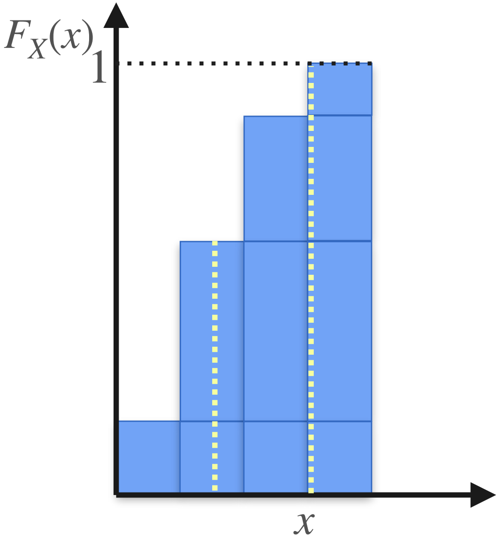

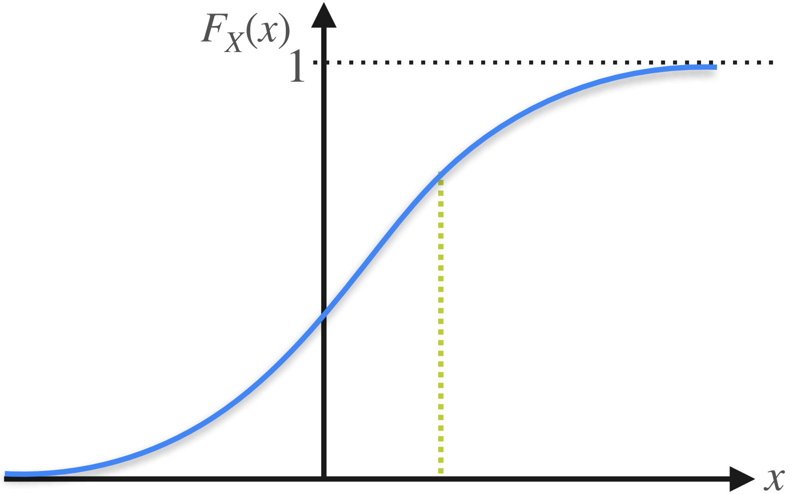

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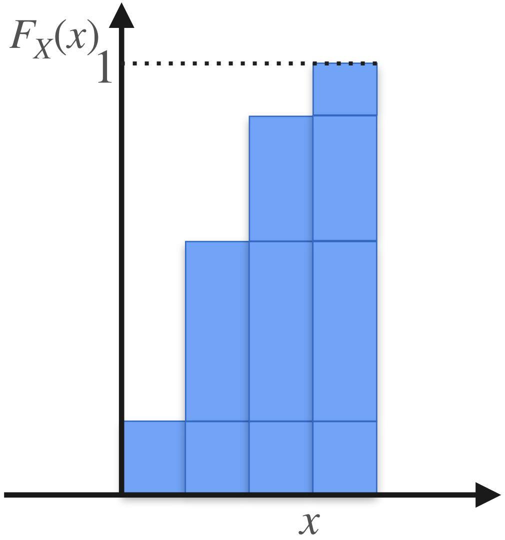

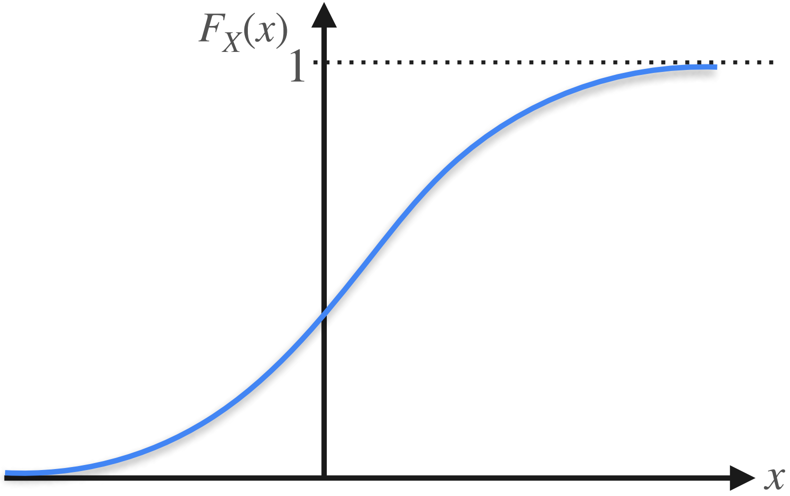

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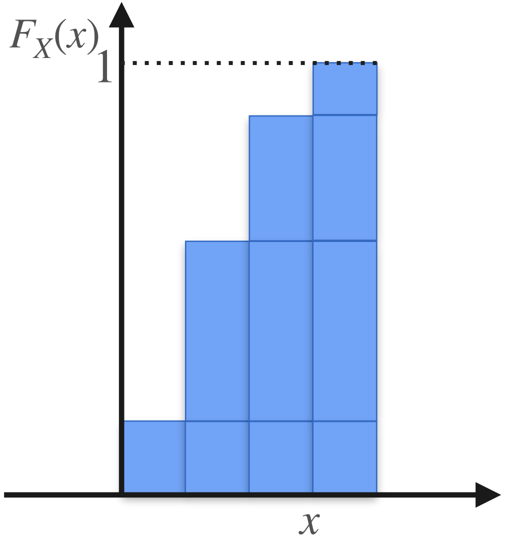

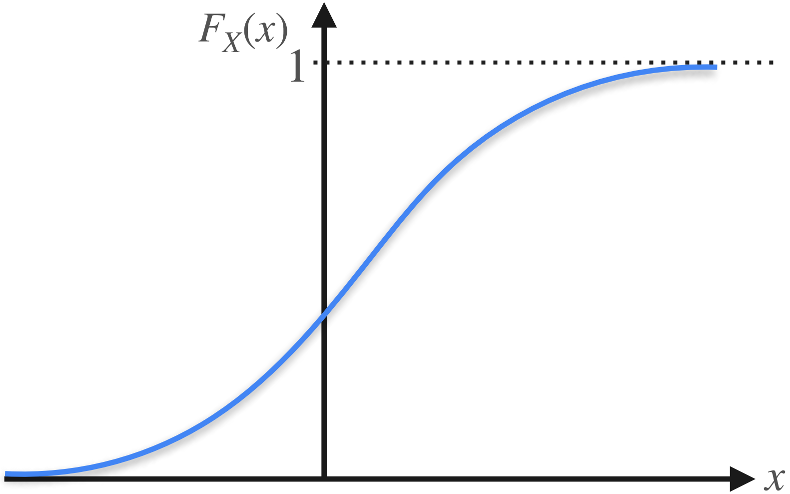

Properties

Cumulative Distribution Function: Formal Definition

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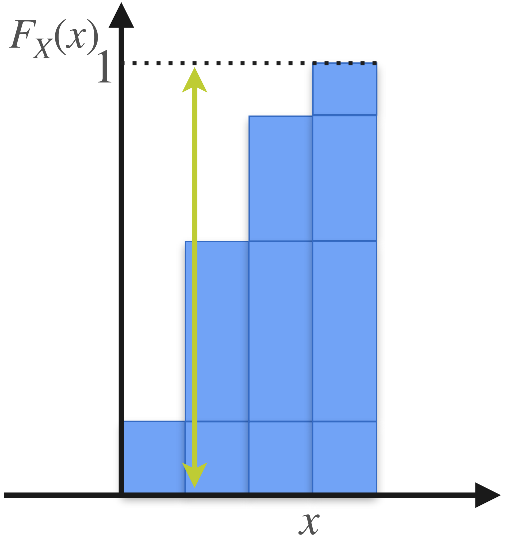

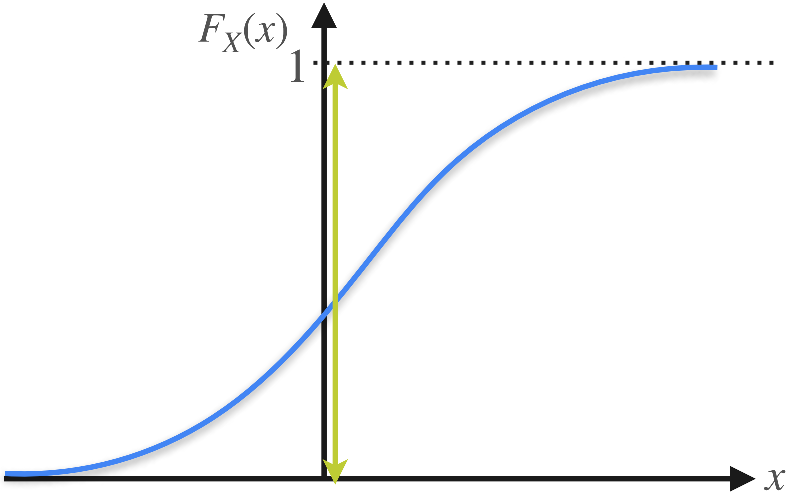

Properties

- $0 \leq F_X(x) \leq 1$

Cumulative Distribution Function: Formal Definition

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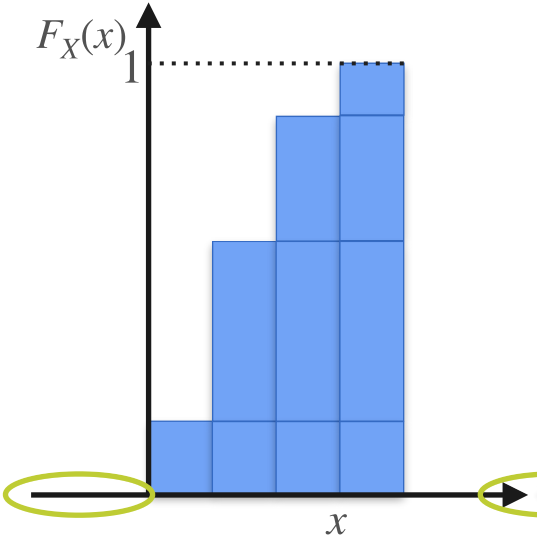
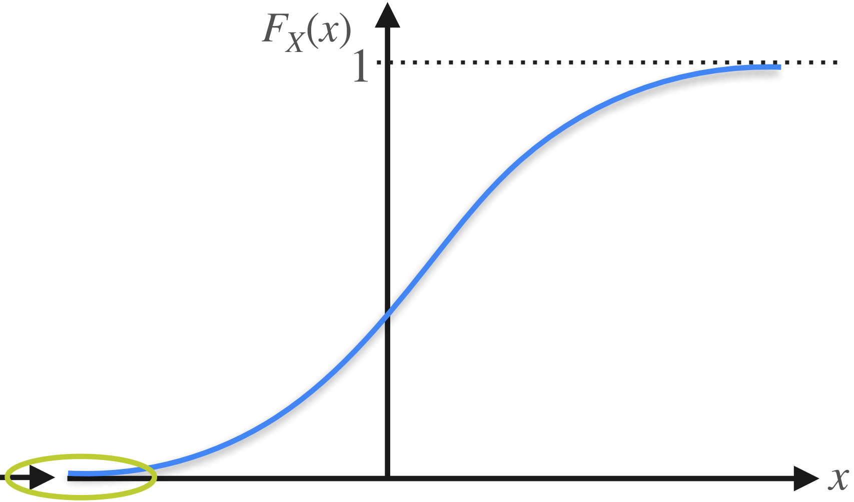




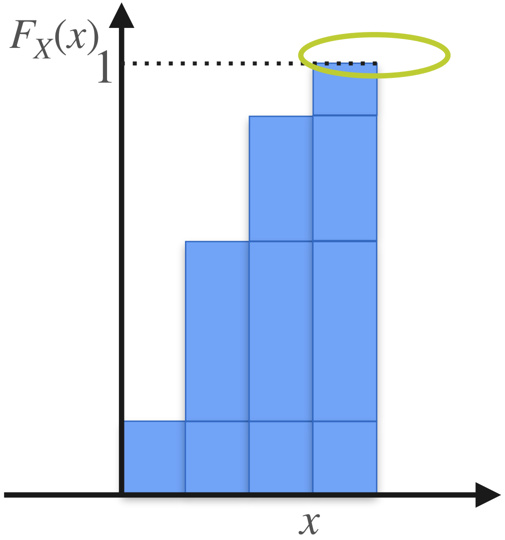
Properties

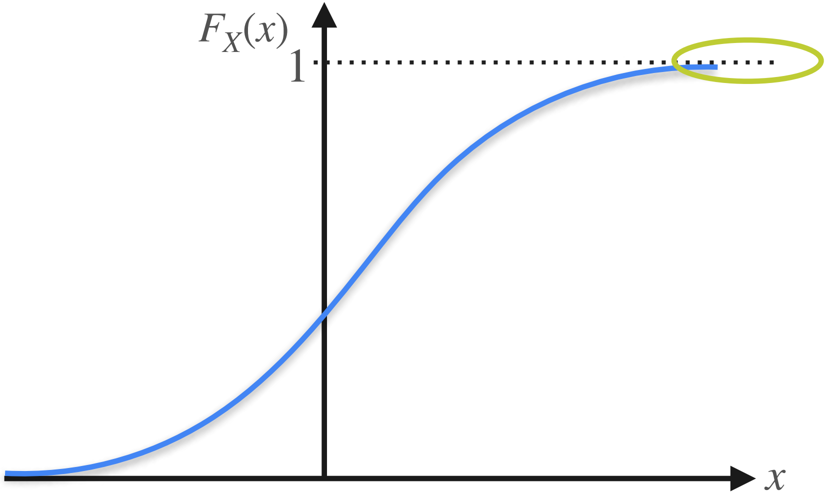
- $0 \leq F_X(x) \leq 1$
- Left “endpoint” is 0

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that $F_X(x) = \mathbf{P}(X \leq x)$ ← It is defined for every real number





Properties

- $0 \leq F_X(x) \leq 1$
- Left "endpoint" is 0
- Right "endpoint" is 1

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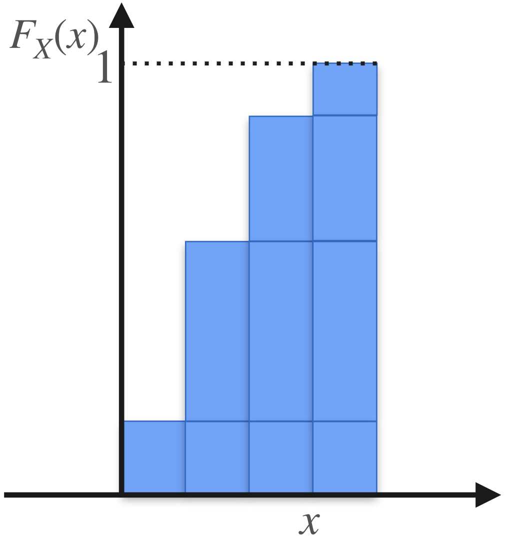
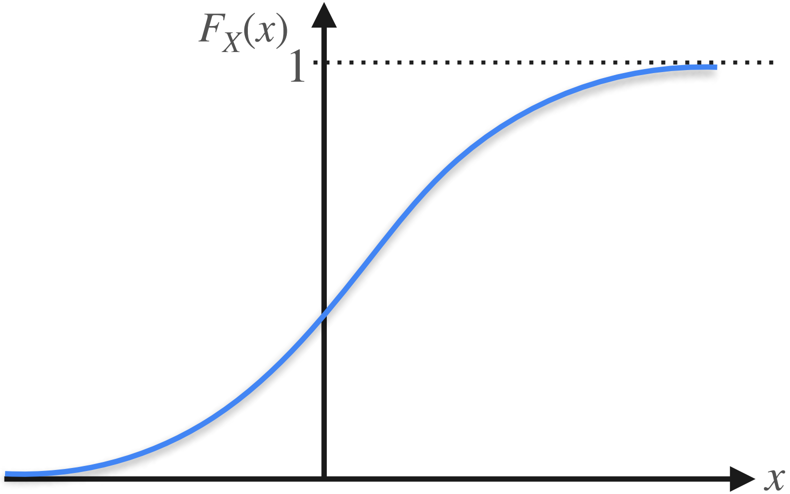


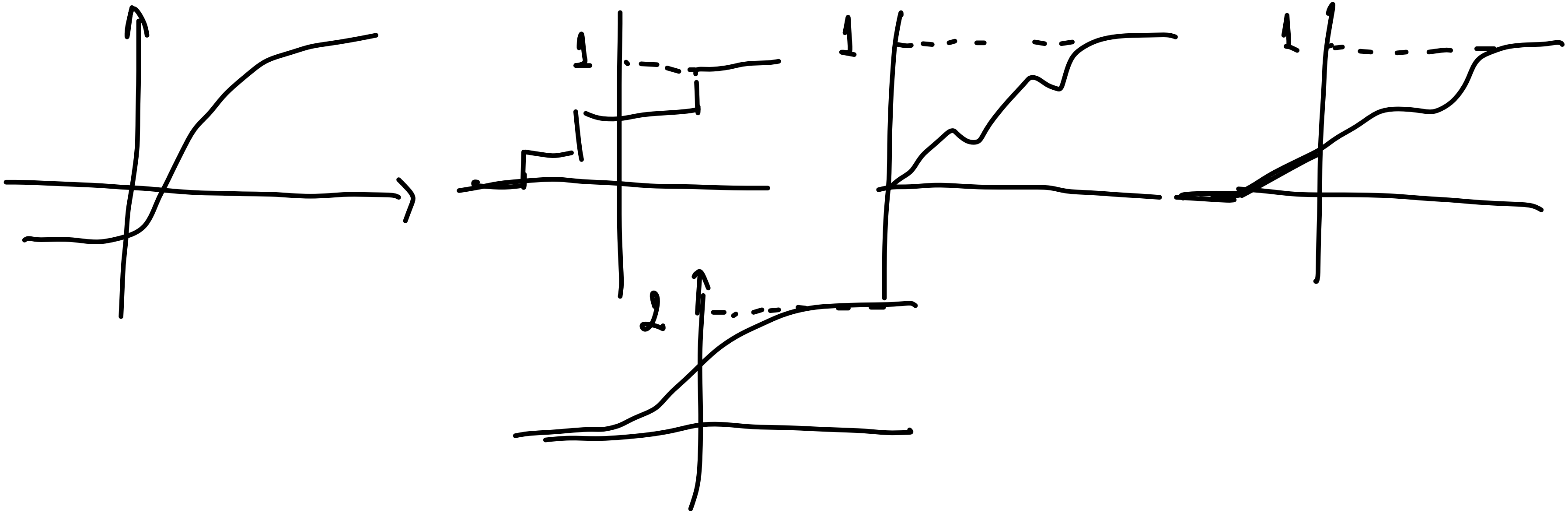


Properties

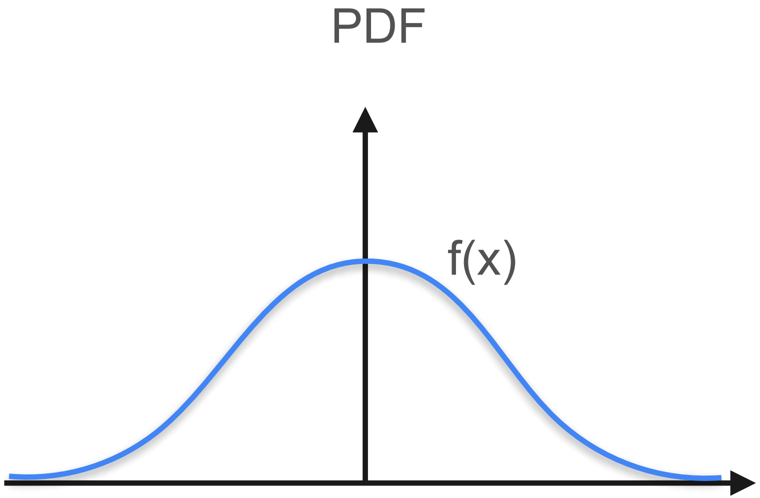
- $0 \leq F_X(x) \leq 1$
- Left "endpoint" is 0
- Right "endpoint" is 1
- **Never decreases**

Quiz

- Which of the following functions could be a CDF?

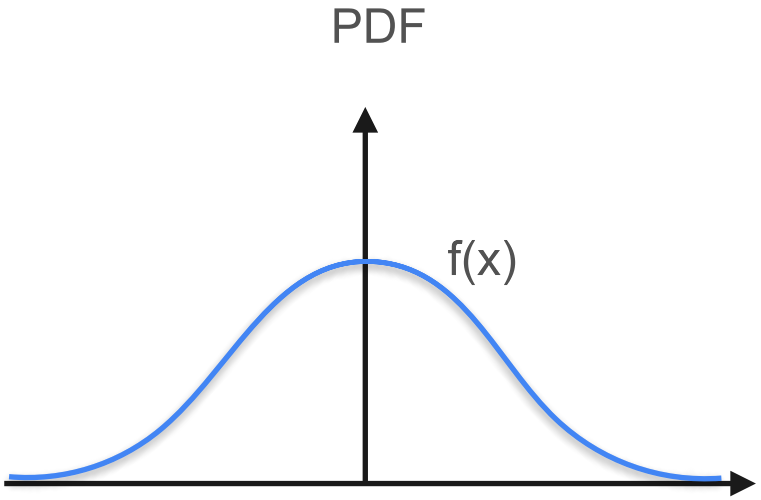


PDF and CDF Summary

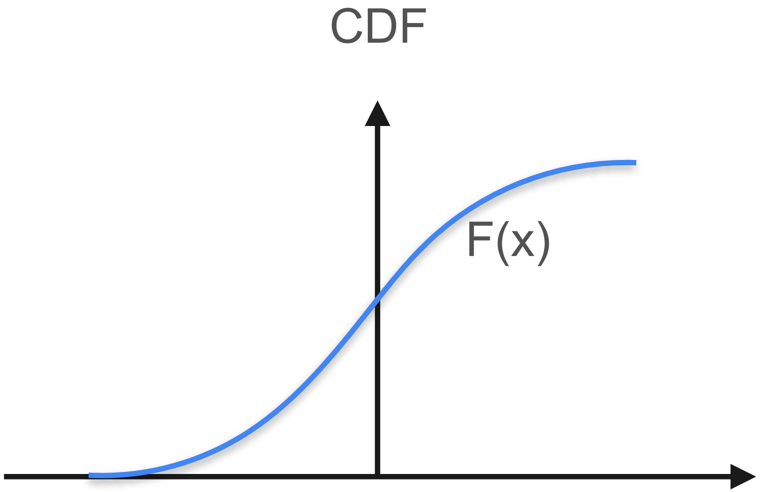


- area = 1
- Always positive

PDF and CDF Summary



- area = 1
- Always positive

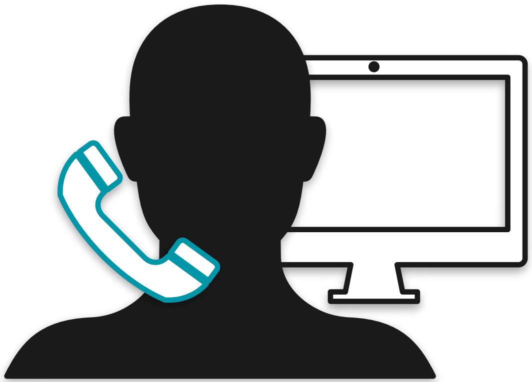


- left “endpoint” is 0
- right “endpoint" is 1
- (endpoints can be at infinity)
- Always positive and increasing

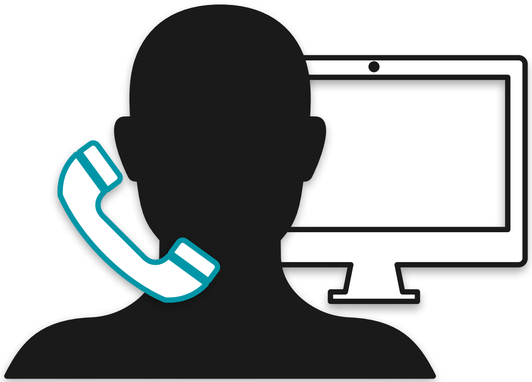
Probability Distributions

Uniform Distribution

Uniform Distribution: Motivation

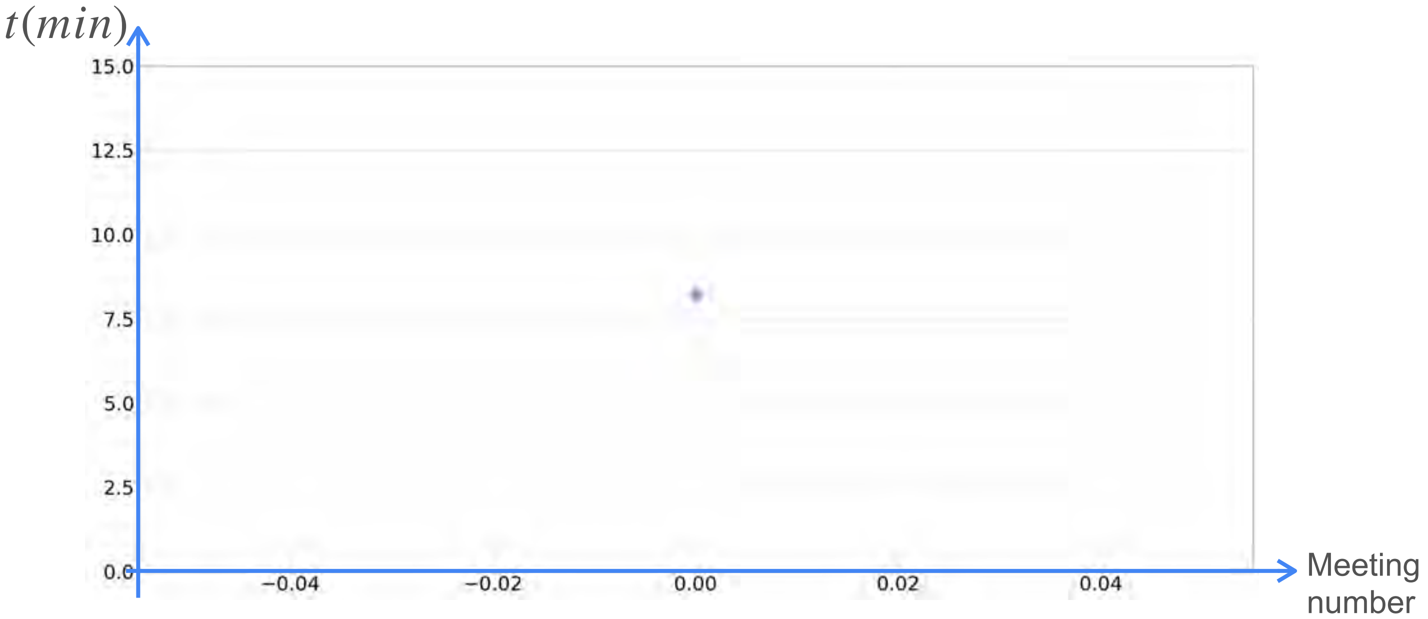


Uniform Distribution: Motivation



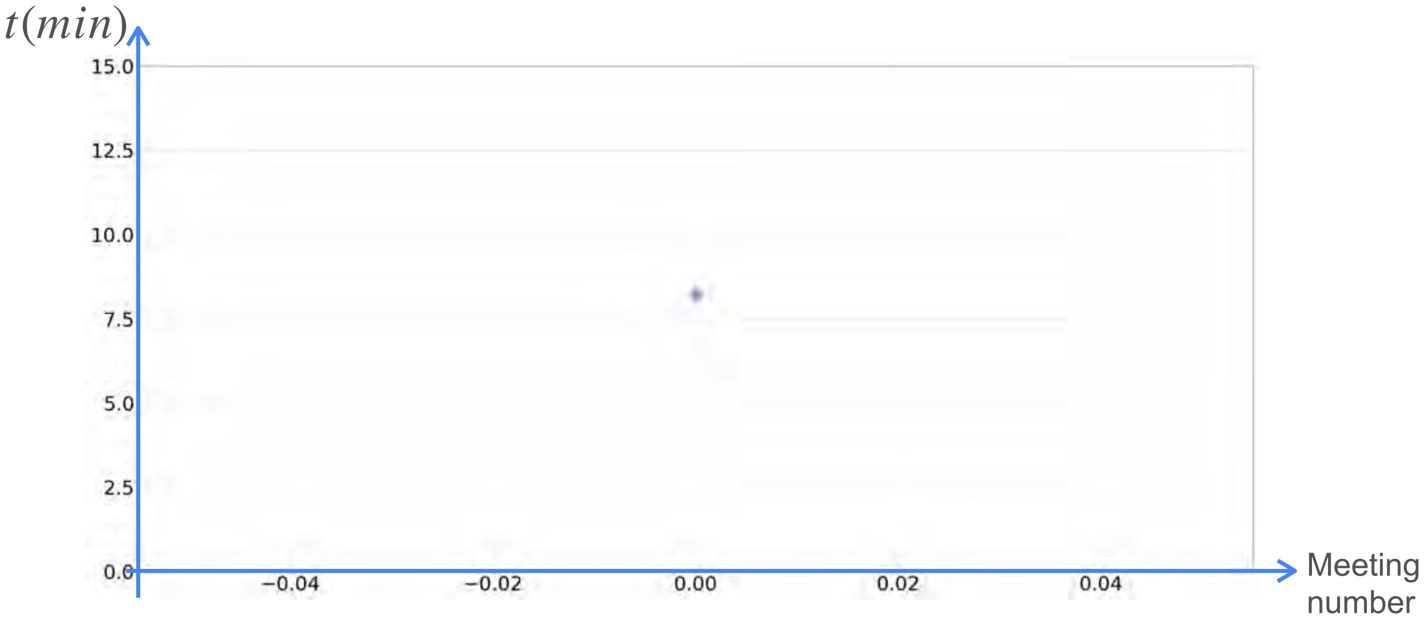
You’re calling a tech support line.
They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

Uniform Distribution: Motivation



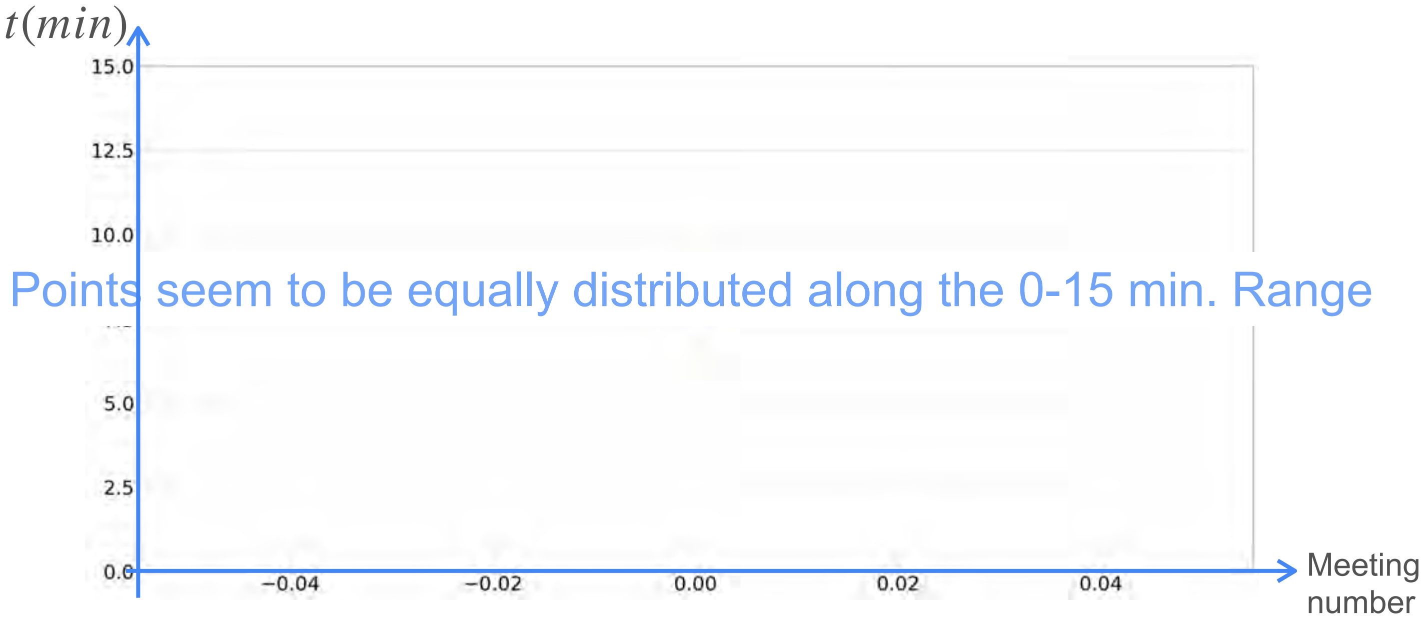
Uniform Distribution: Motivation

Last 200 times you called them, you took down notes of how long they took to respond



Uniform Distribution: Motivation

Last 200 times you called them, you took down notes of how long they took to respond



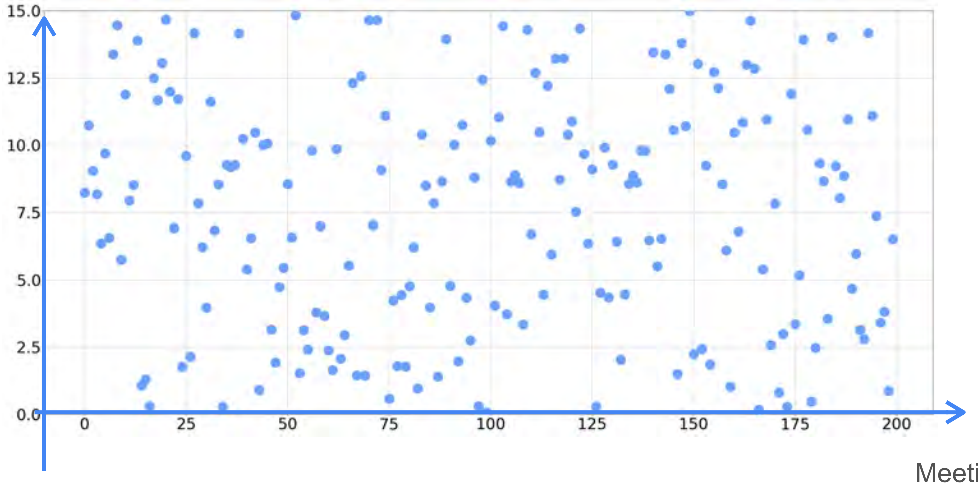
Points seem to be equally distributed along the 0-15 min. Range

Uniform Distribution: Motivation

$t(min)$

Uniform Distribution: Motivation

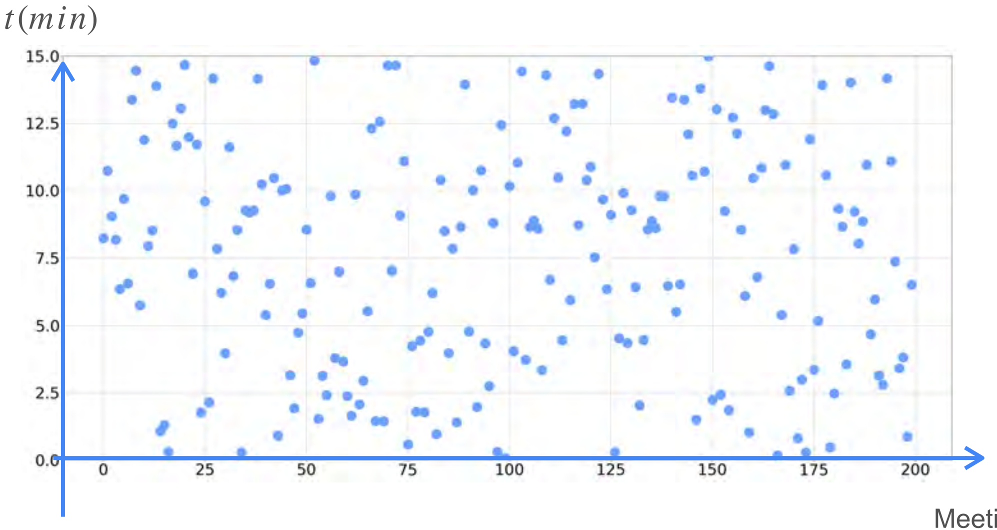
$t(min)$

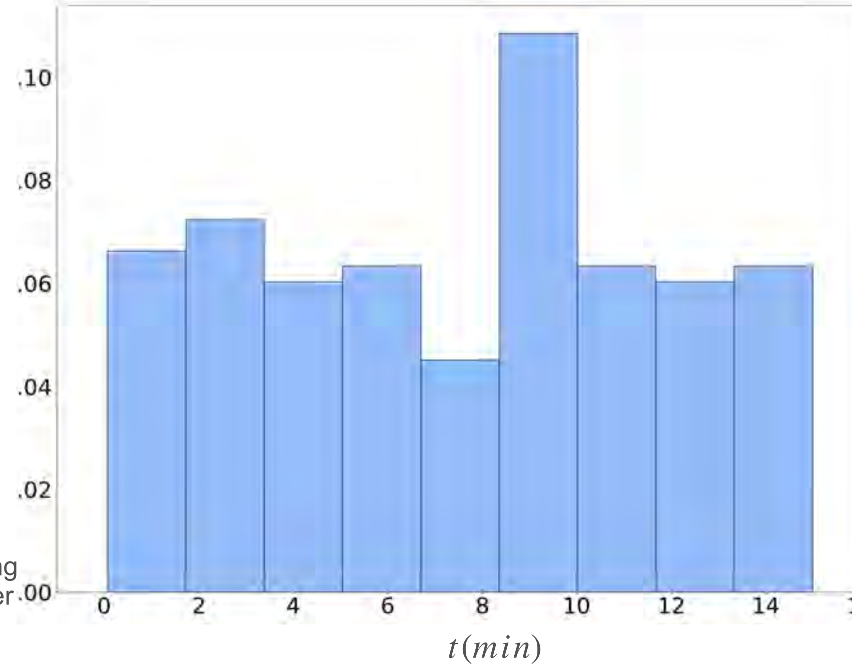


Meeting number

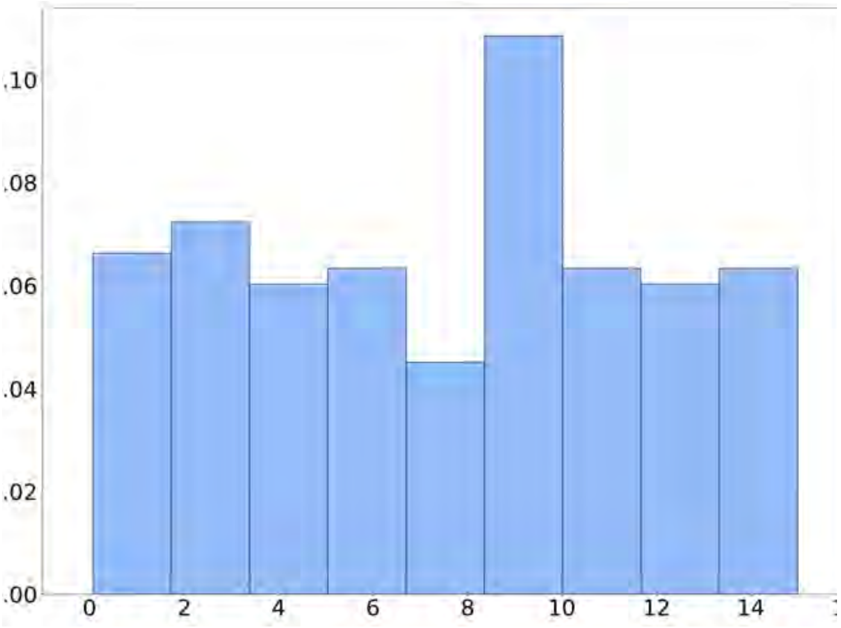
$t(min)$

Uniform Distribution: Motivation





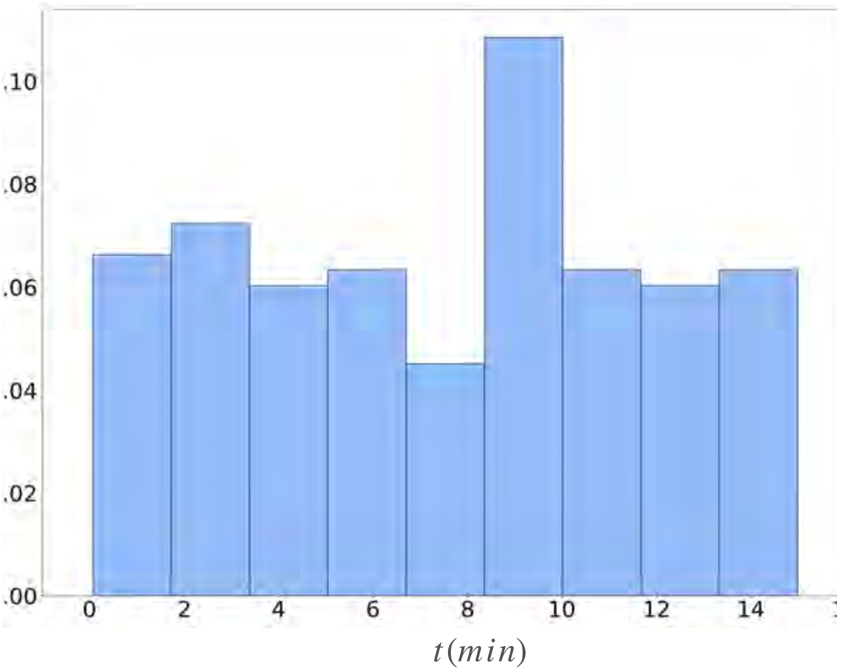
Uniform Distribution: Motivation



$t(min)$

Uniform Distribution: Motivation

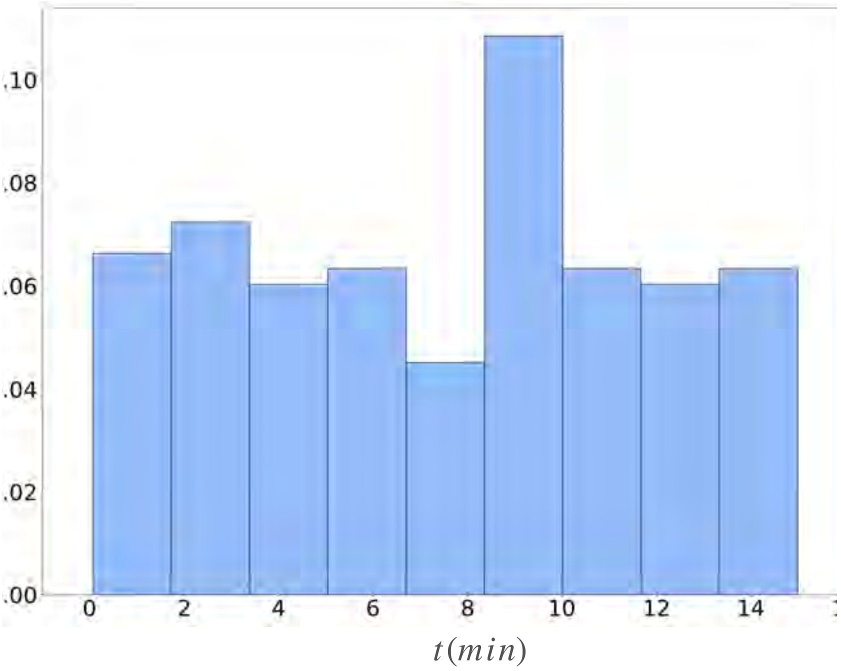
T: time (in minutes) you have to wait



Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.



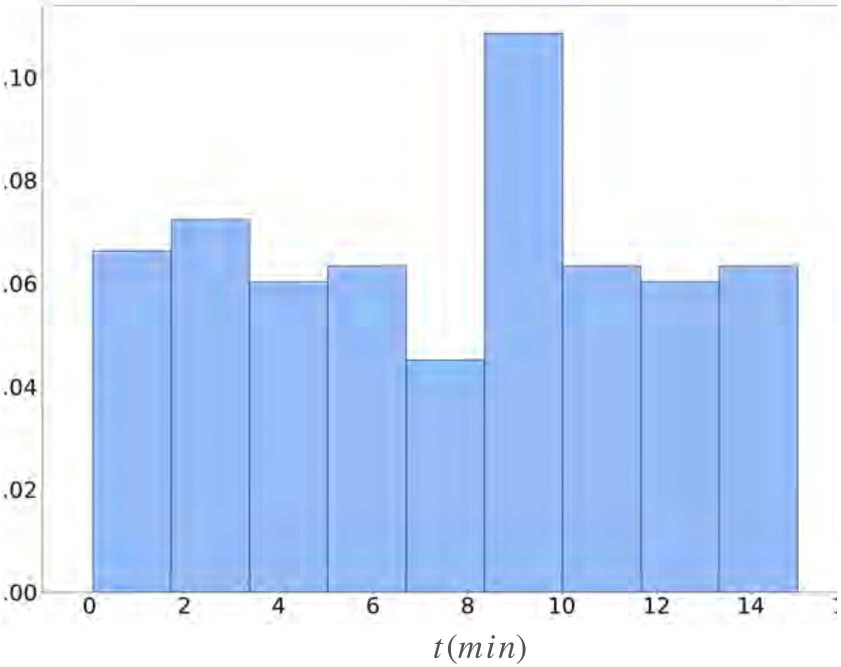
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

↓

The pdf must be constant for all values in the interval (0,15)



Uniform Distribution: Motivation

T: time (in minutes) you have to wait

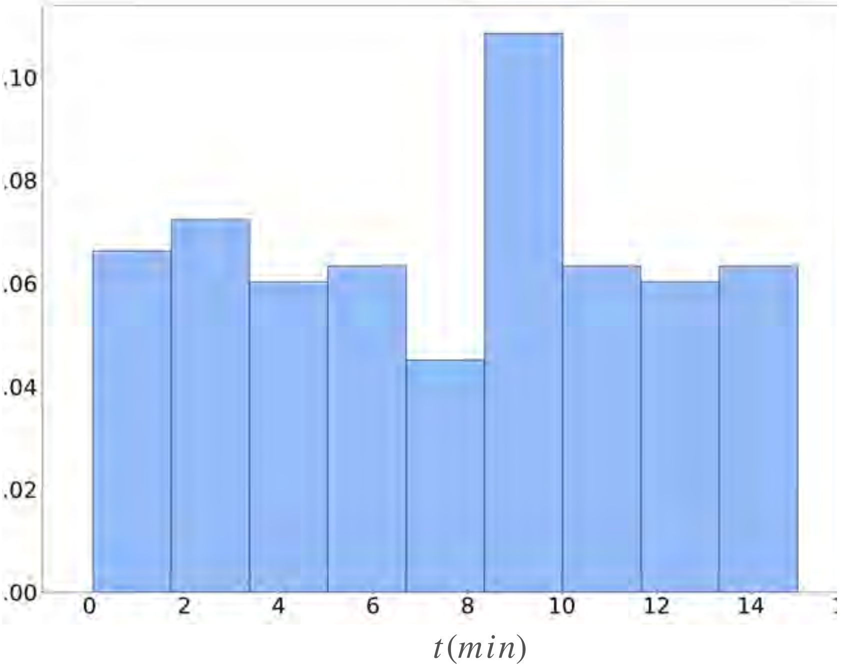
Any value between 0 and 15 minutes must have the same frequency of occurrence.

↓

The pdf must be constant for all values in the interval (0,15)

↓

Which constant?



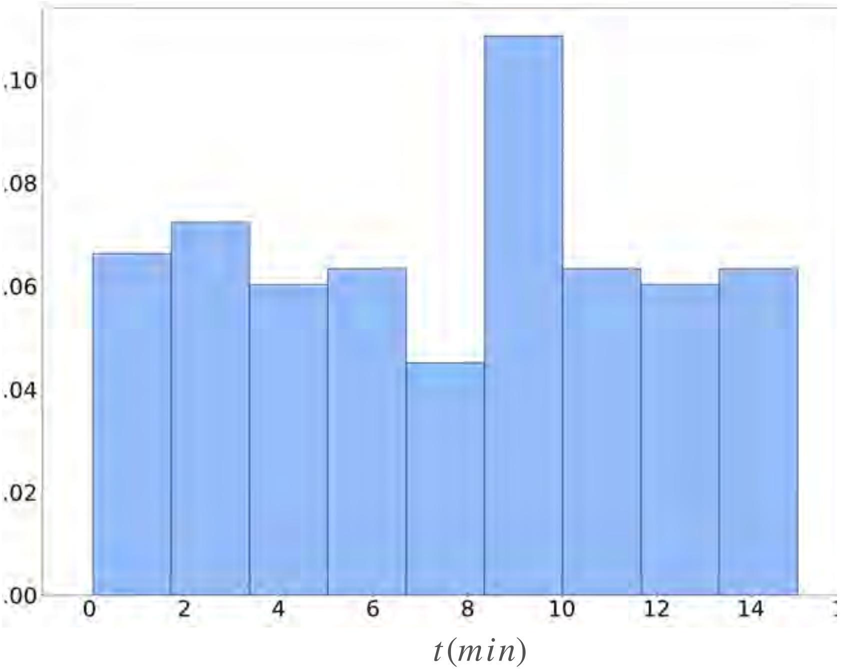
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant? $15 \times h = 1$



Uniform Distribution: Motivation

T: time (in minutes) you have to wait

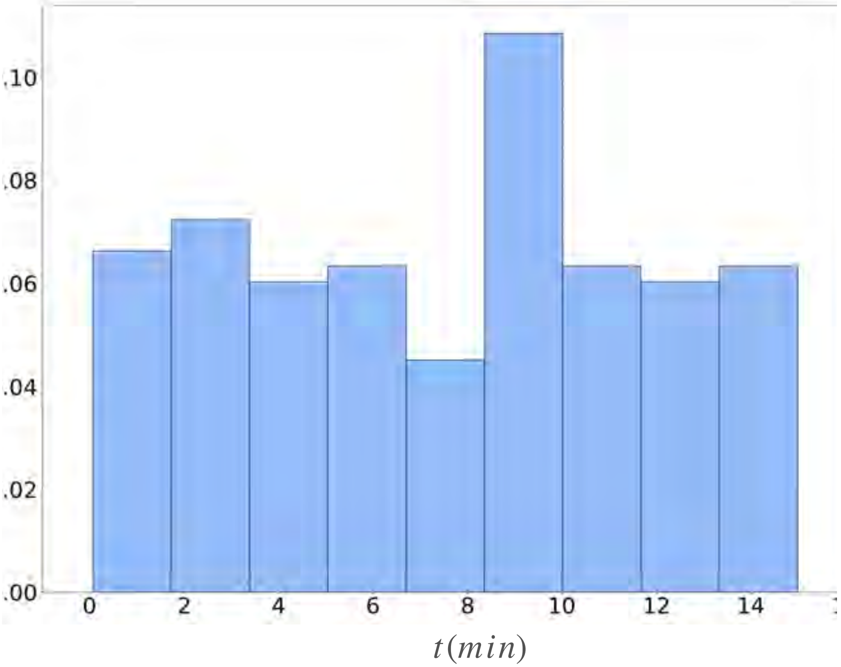
Any value between 0 and 15 minutes must have the same frequency of occurrence.

↓

The pdf must be constant for all values in the interval (0,15)

↓

Which constant? → $15 \times h = 1$ → $h = \frac{1}{15}$
$= 0.06$



Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

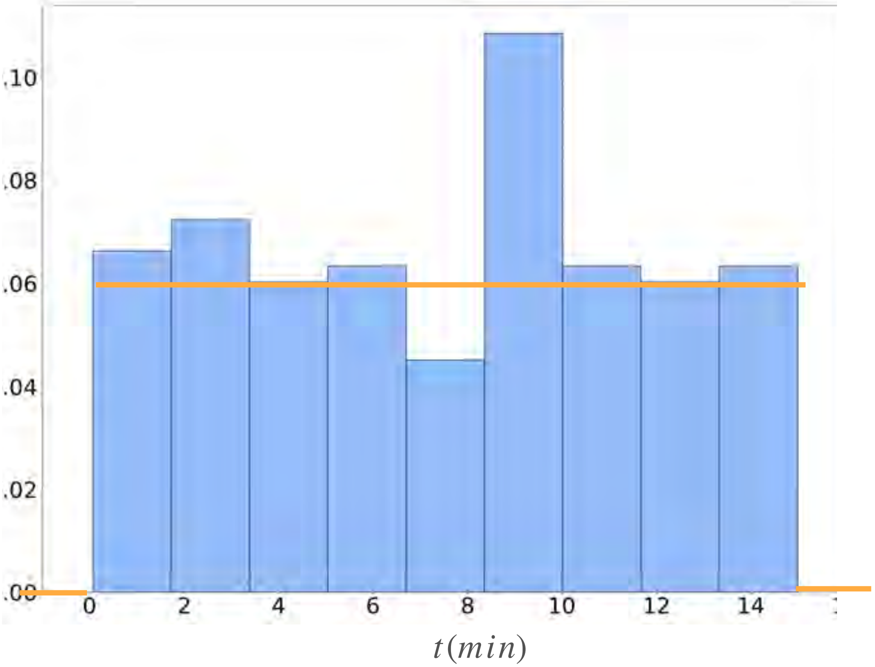
↓

The pdf must be constant for all values in the interval (0,15)

↓

Which constant? → $15 \times h = 1$ → $h = \frac{1}{15}$

$= 0.06$



Uniform Distribution: Model

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

- a: beginning of the interval

x

Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

Parameters:

- a: beginning of the interval
- b: end of the interval

x

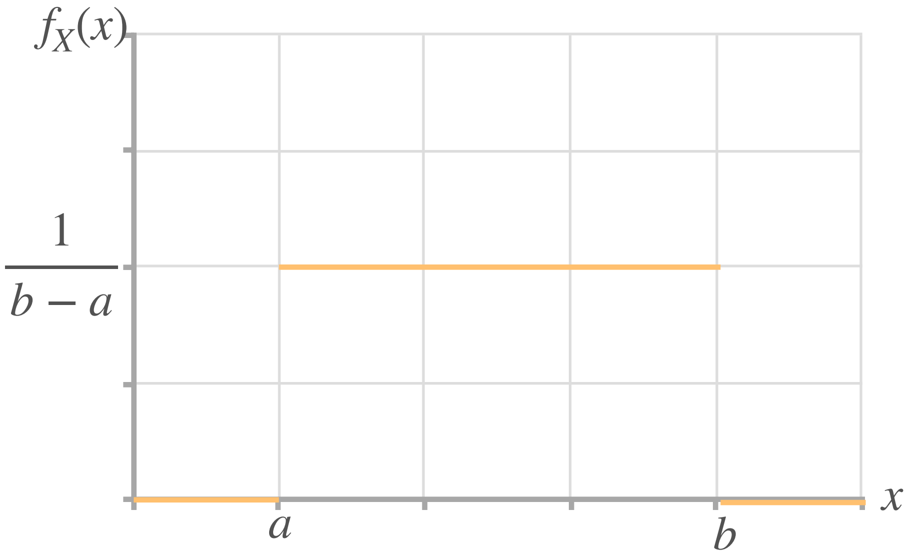
Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

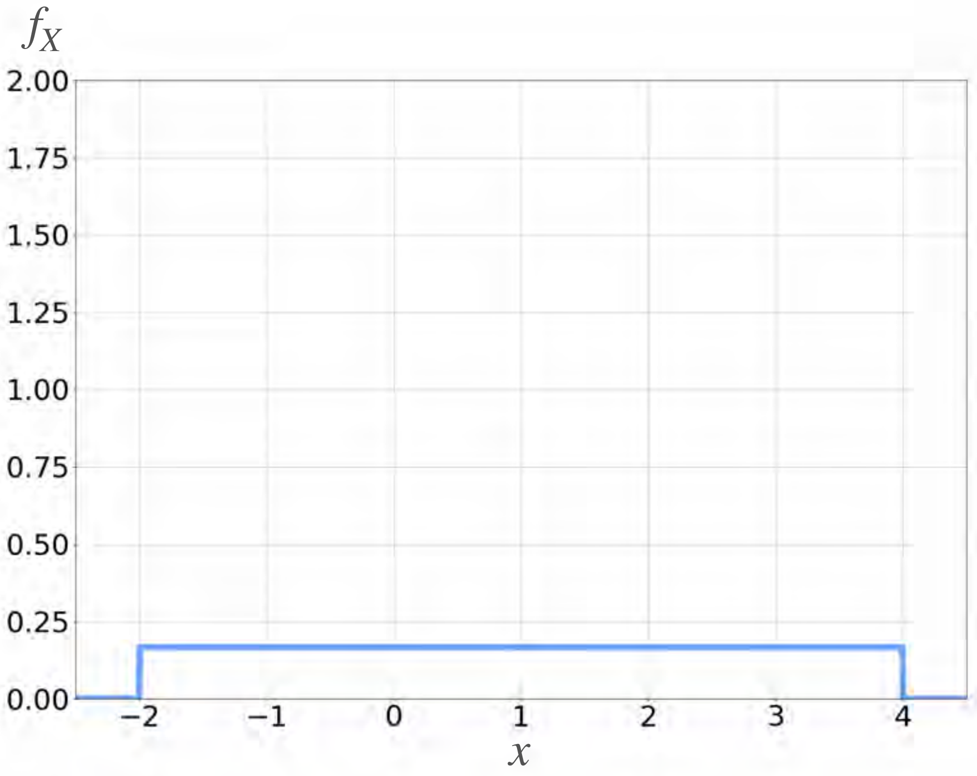
Parameters:

- a: beginning of the interval
- b: end of the interval

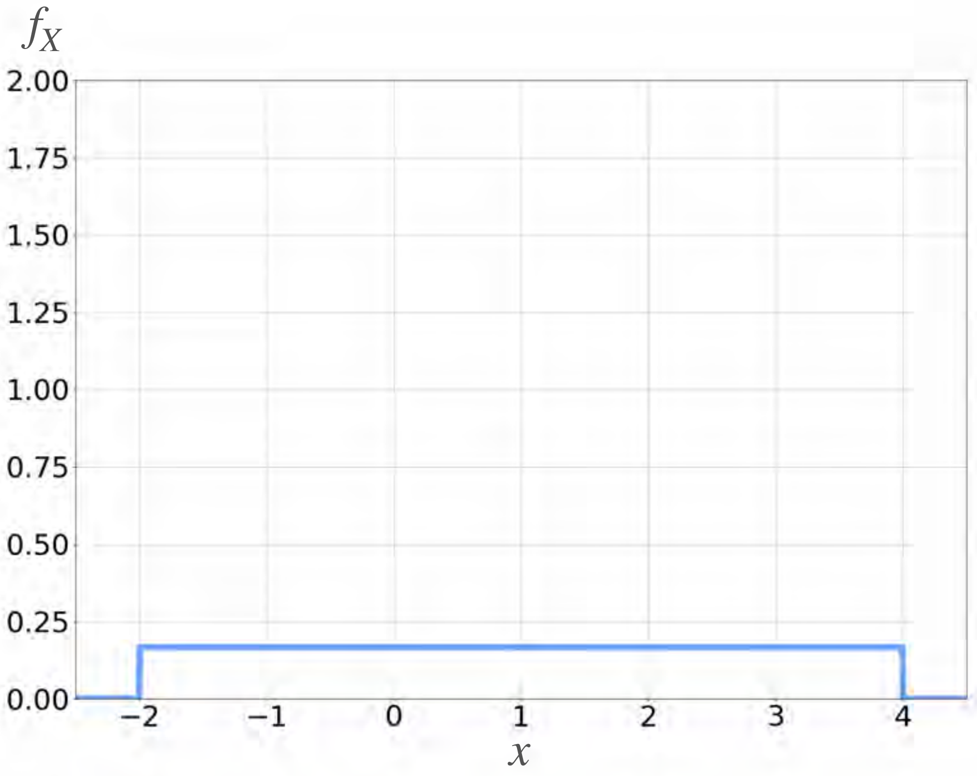
$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x \notin (a,b) \end{cases}$$



Uniform Distribution: PDF



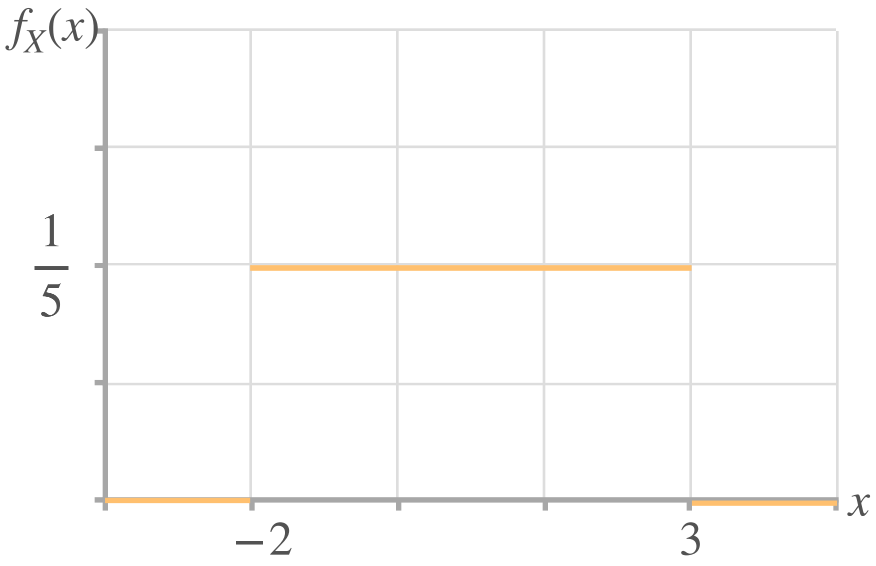
Uniform Distribution: PDF



Quiz

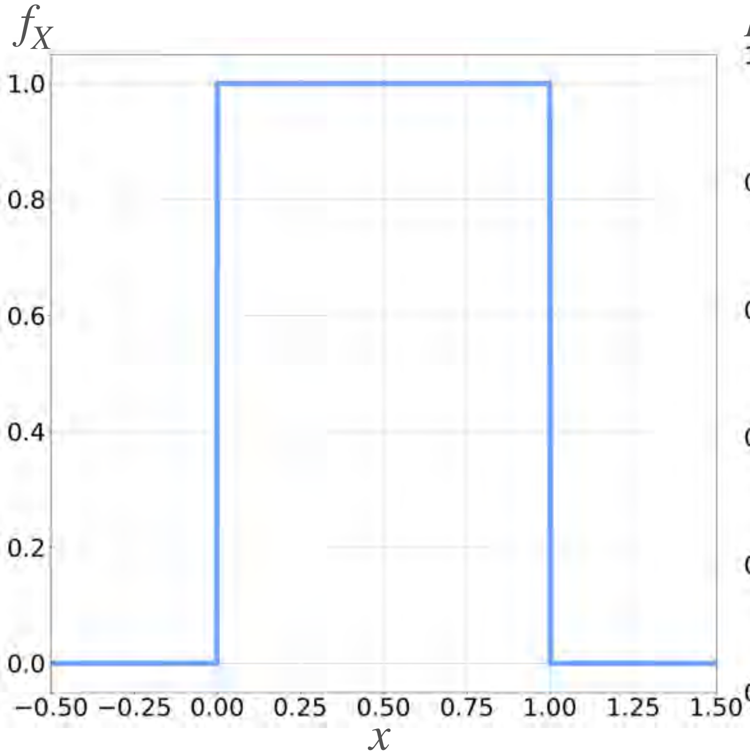
What is the probability of X being between -1 and 3?

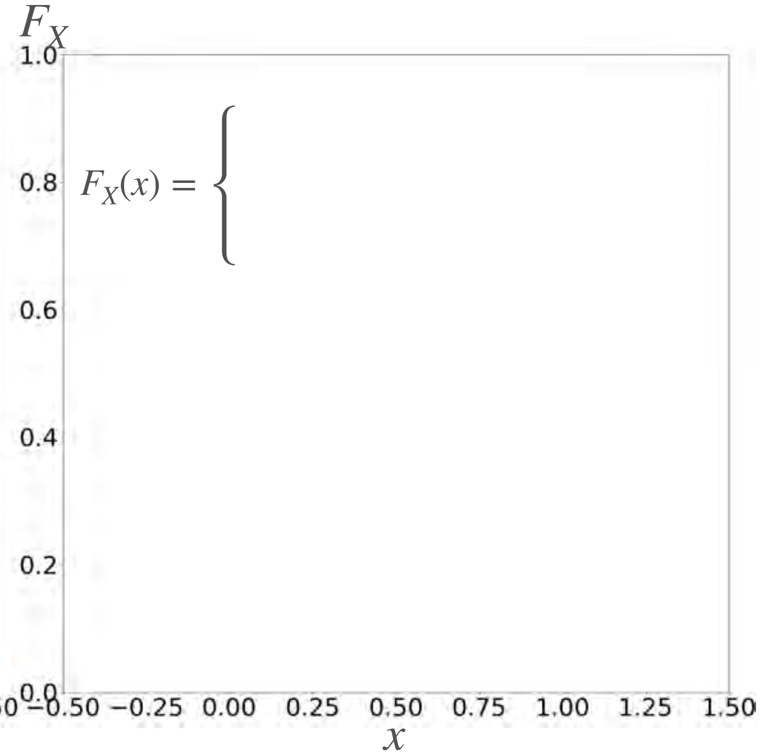
What is the probability of X being between -1 and 4?



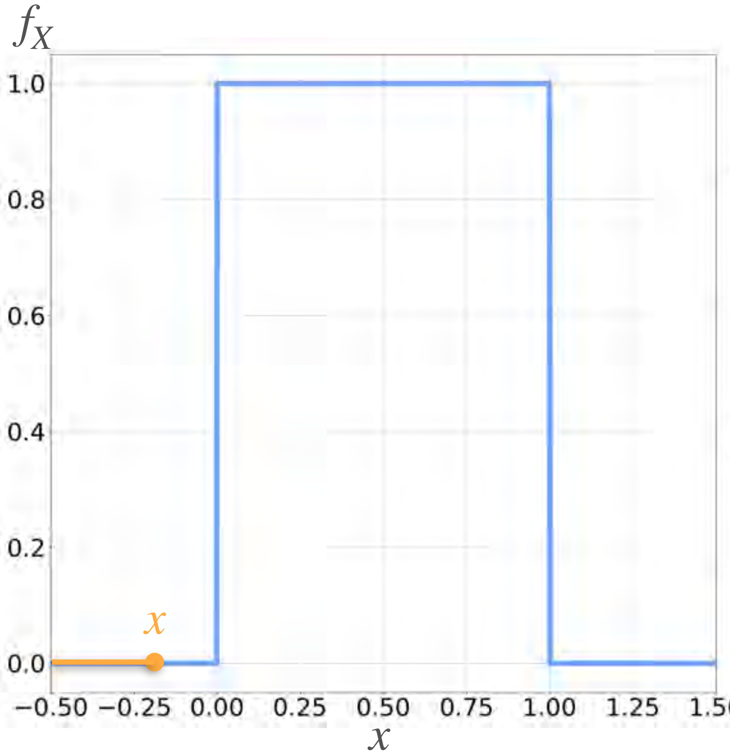
Uniform Distribution: CDF

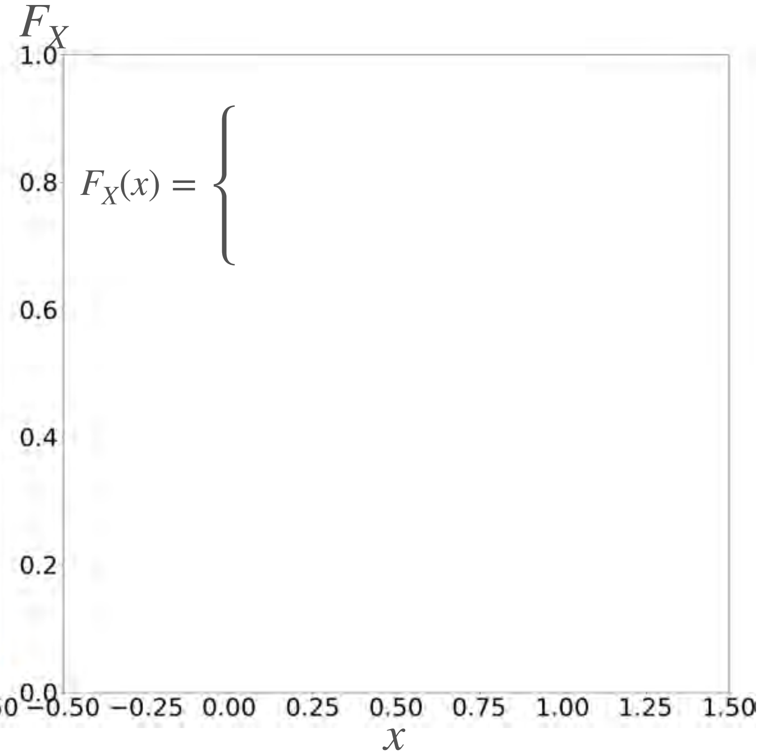
Uniform Distribution: CDF



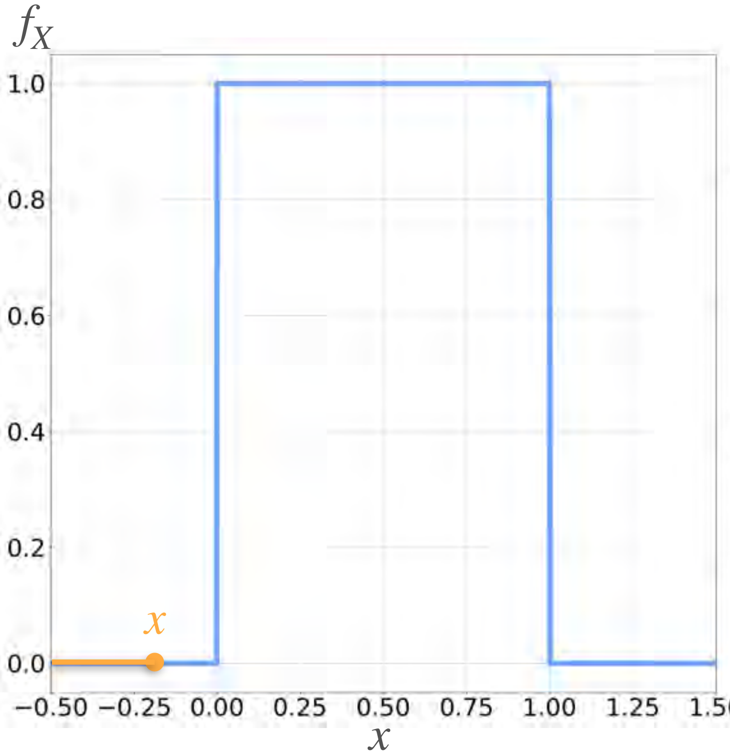


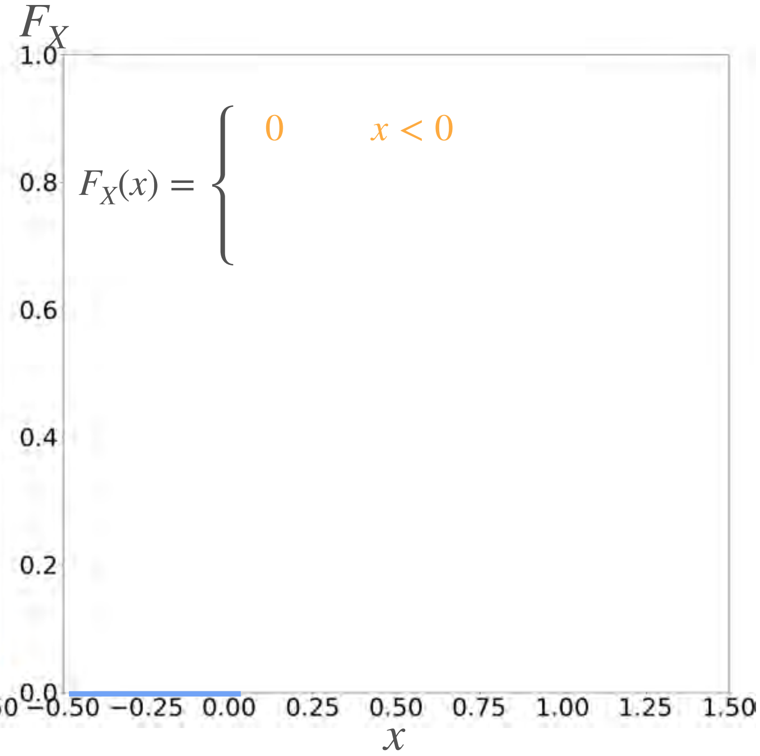
Uniform Distribution: CDF



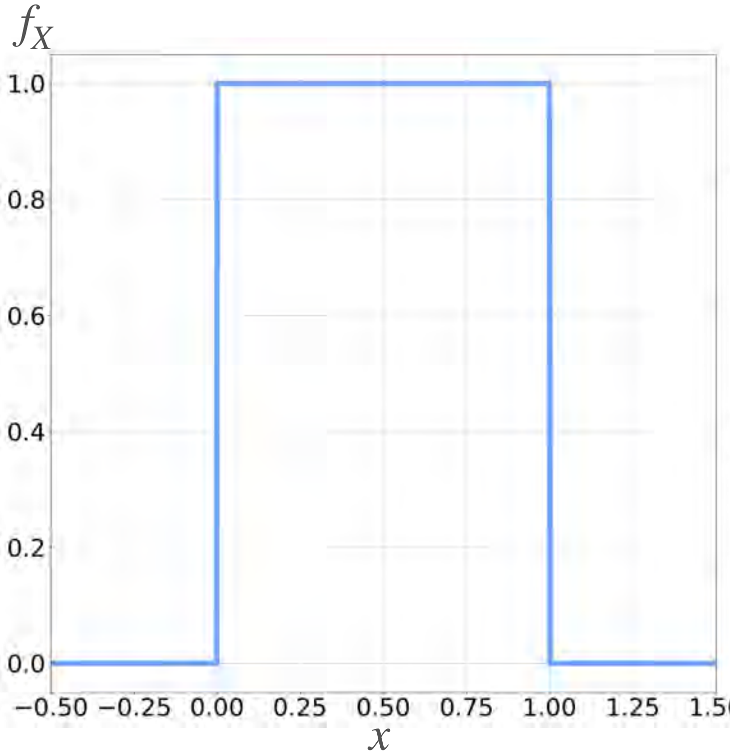


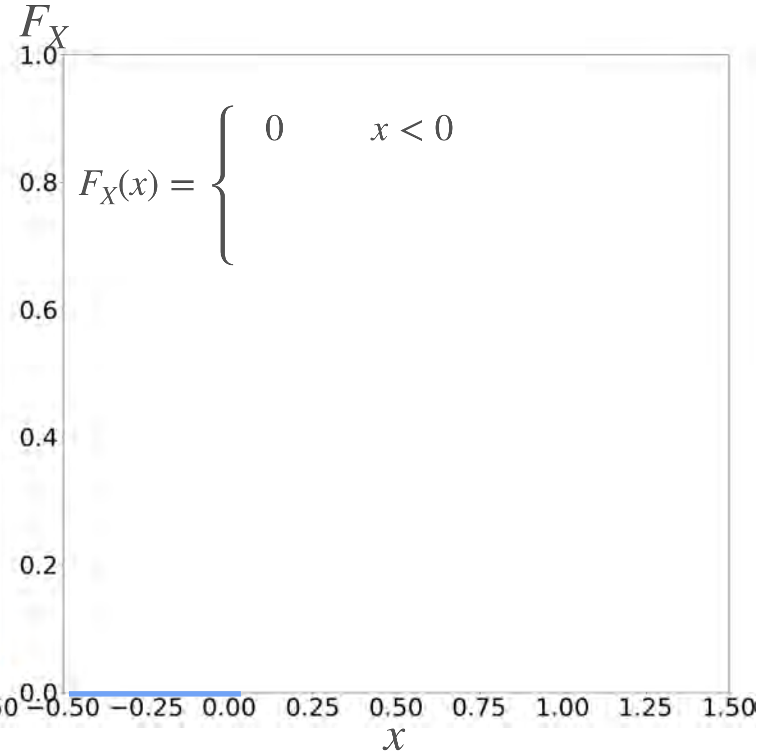
Uniform Distribution: CDF



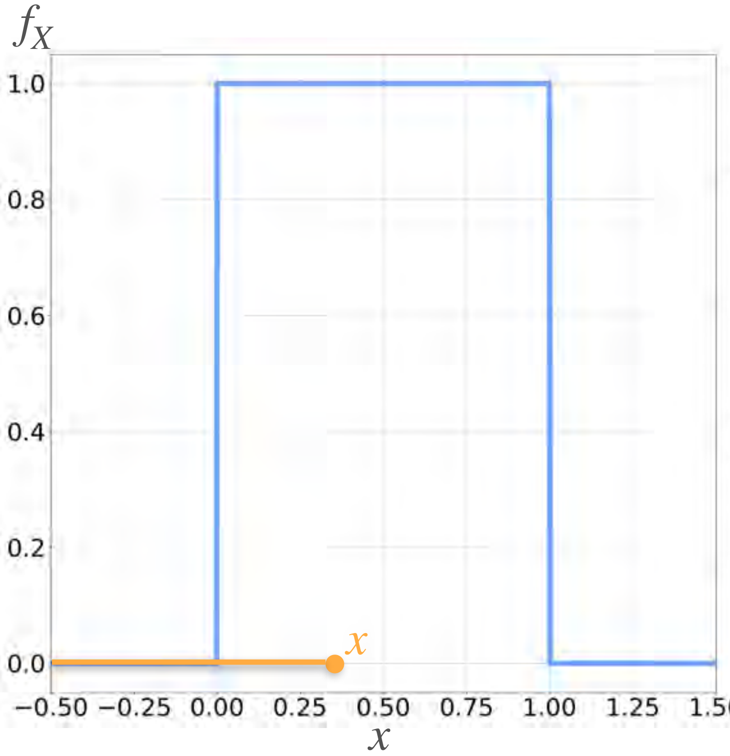


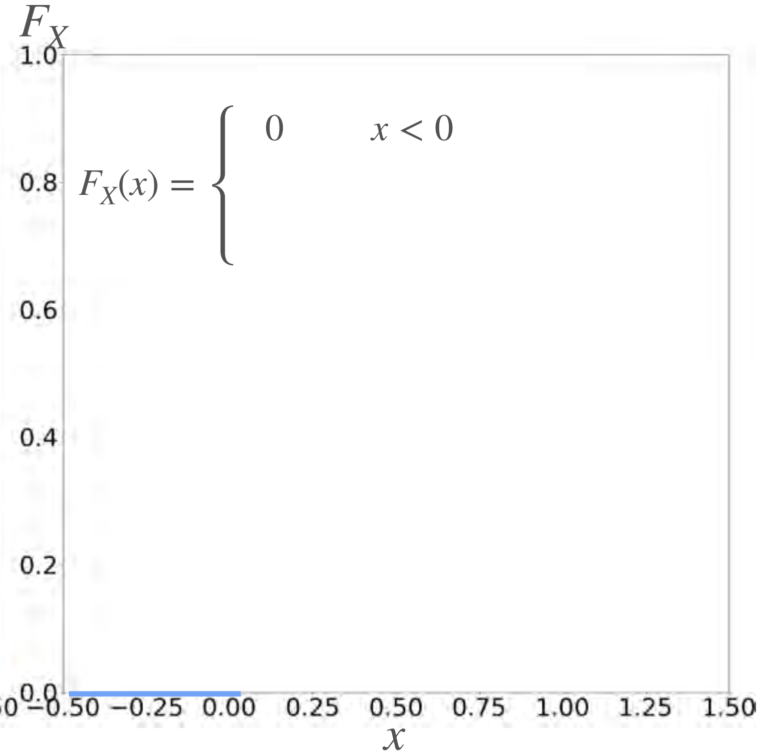
Uniform Distribution: CDF





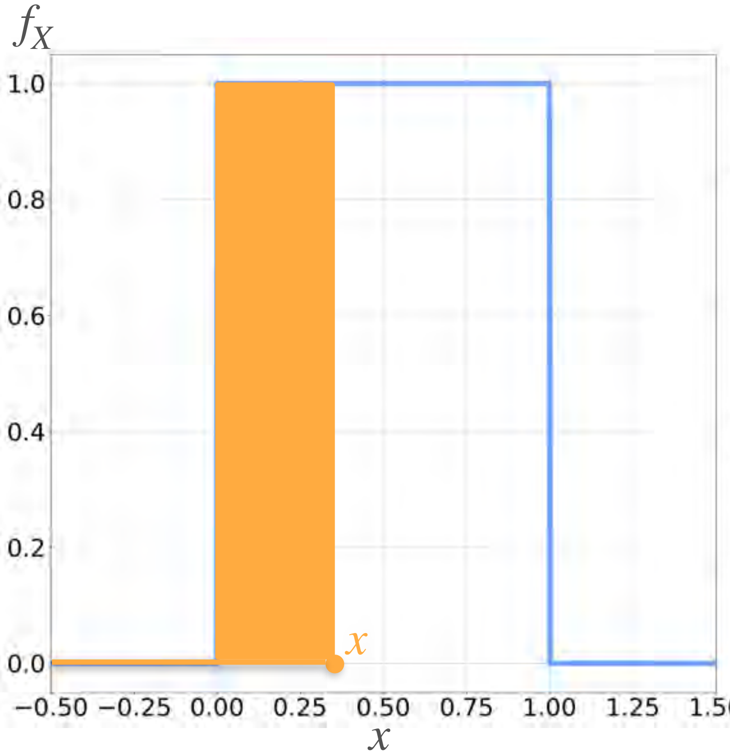
Uniform Distribution: CDF

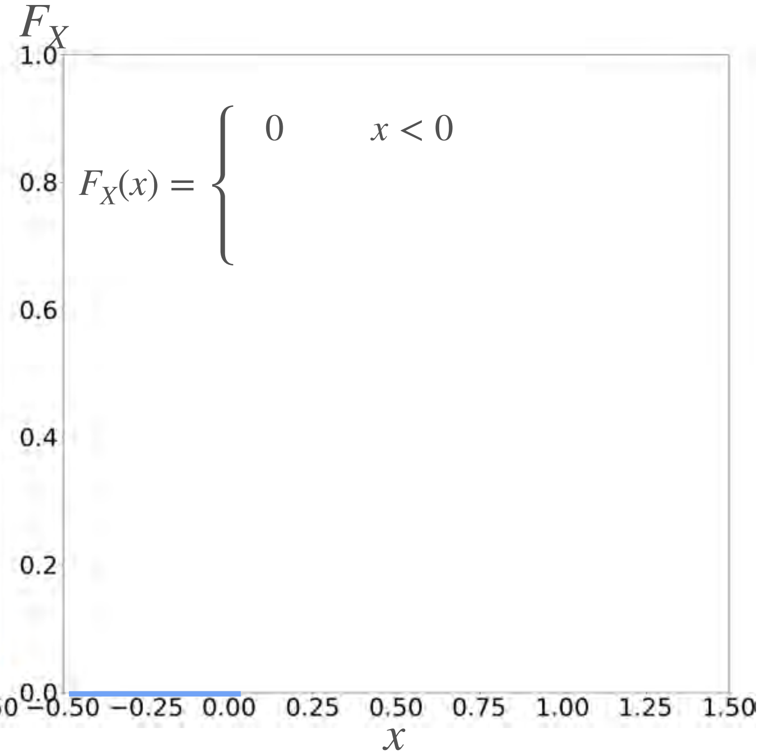




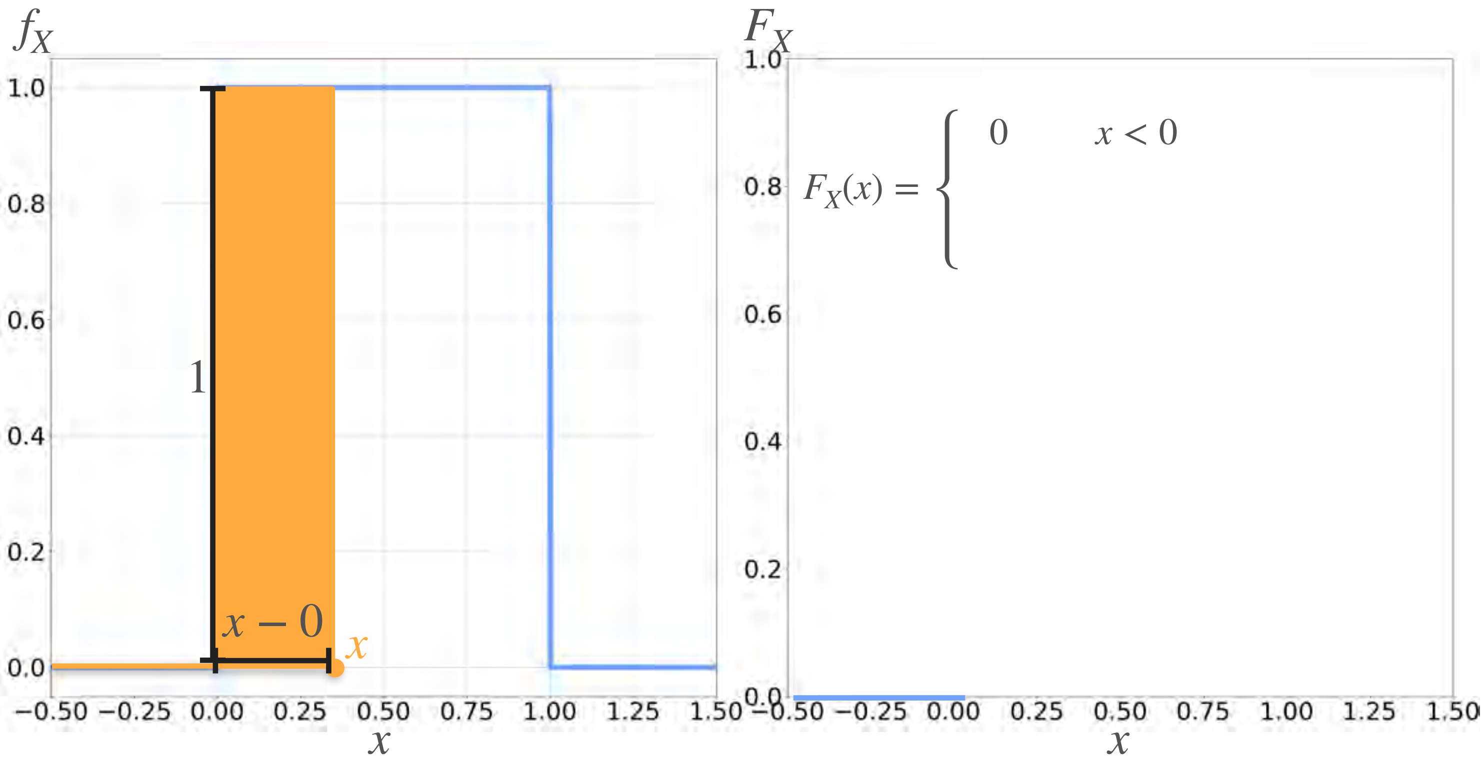
$$F_X(x) = \begin{cases} 0 & x < 0 \\ & \\ & \end{cases}$$

Uniform Distribution: CDF

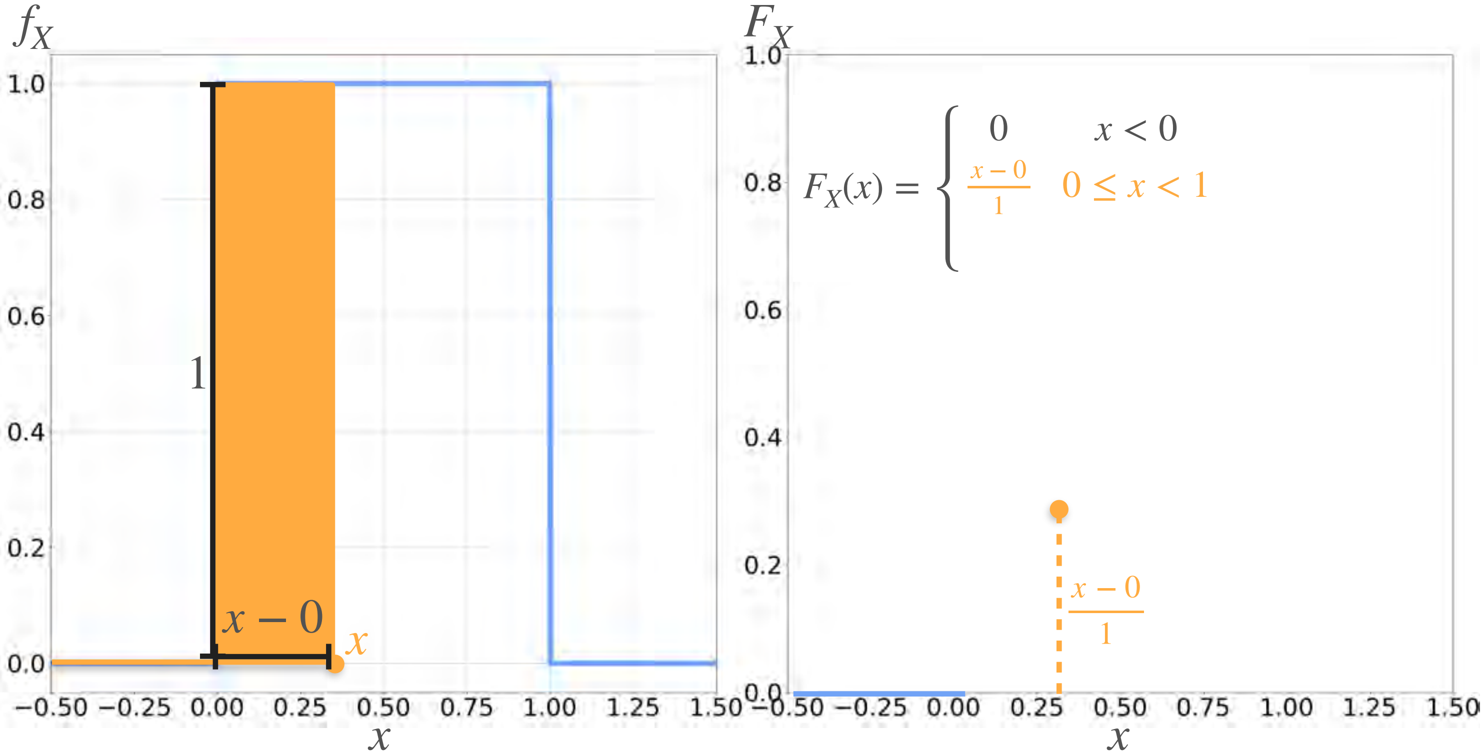




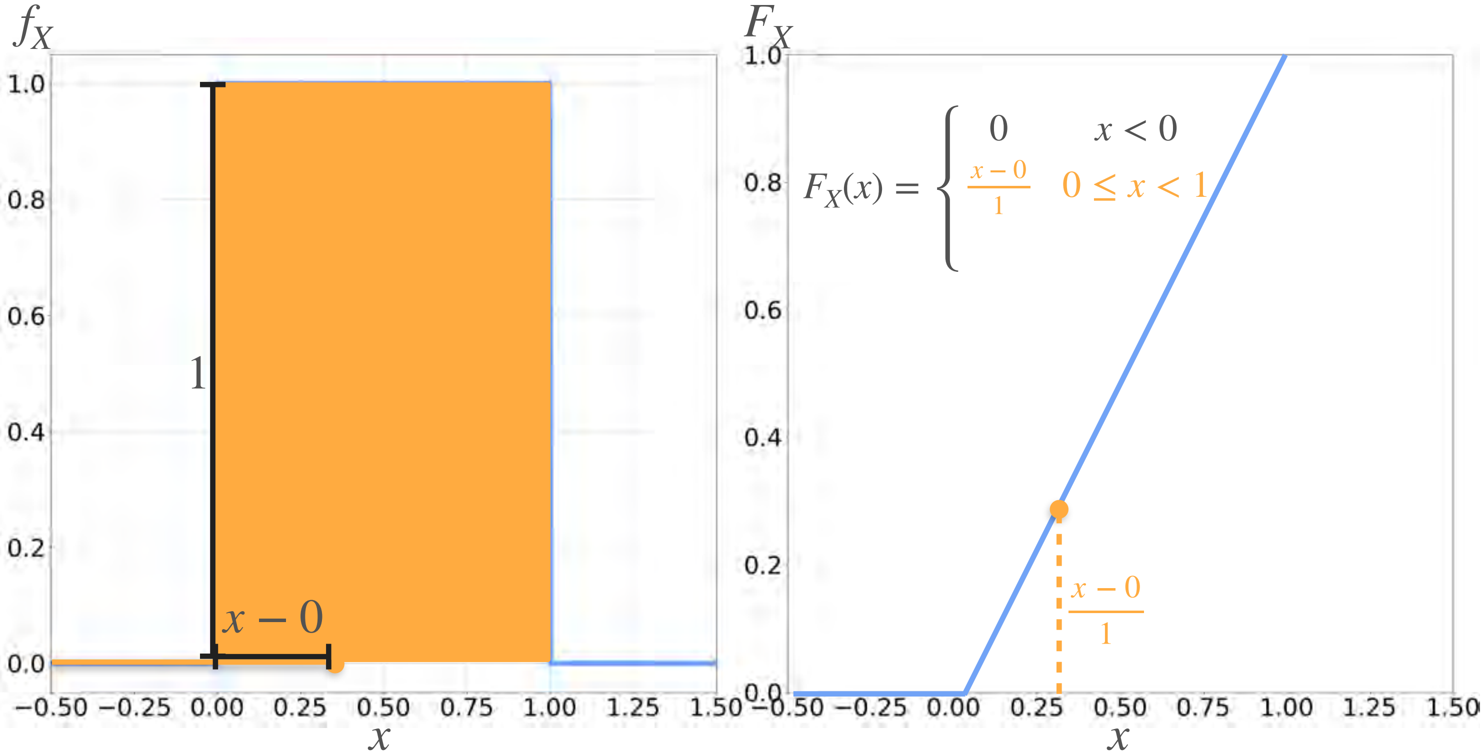
Uniform Distribution: CDF



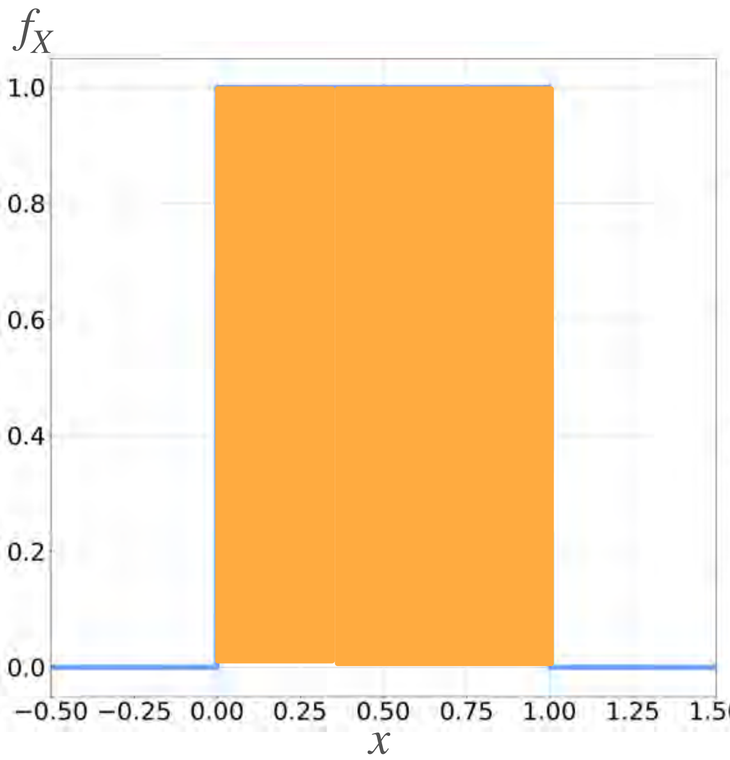
Uniform Distribution: CDF

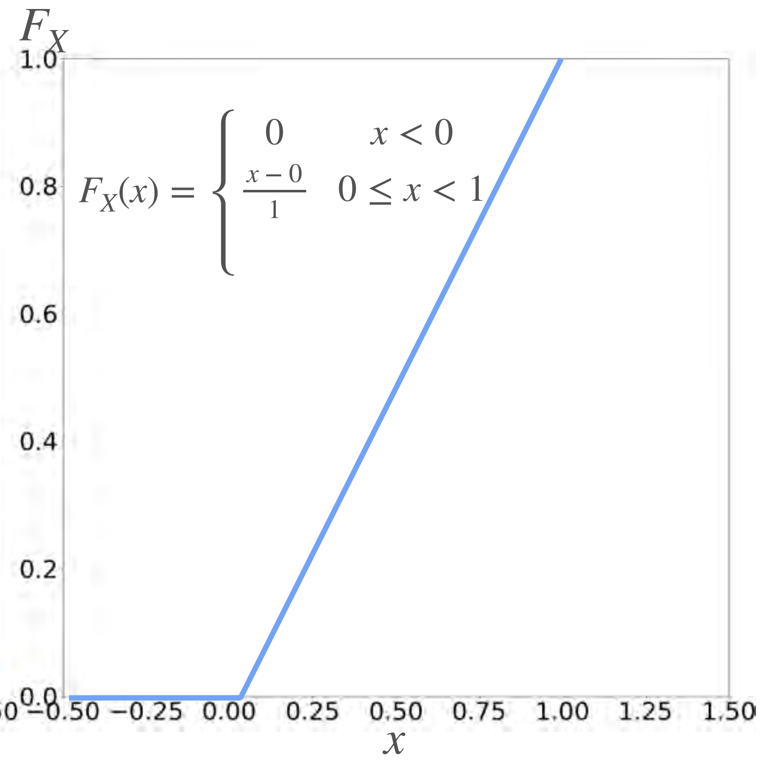


Uniform Distribution: CDF



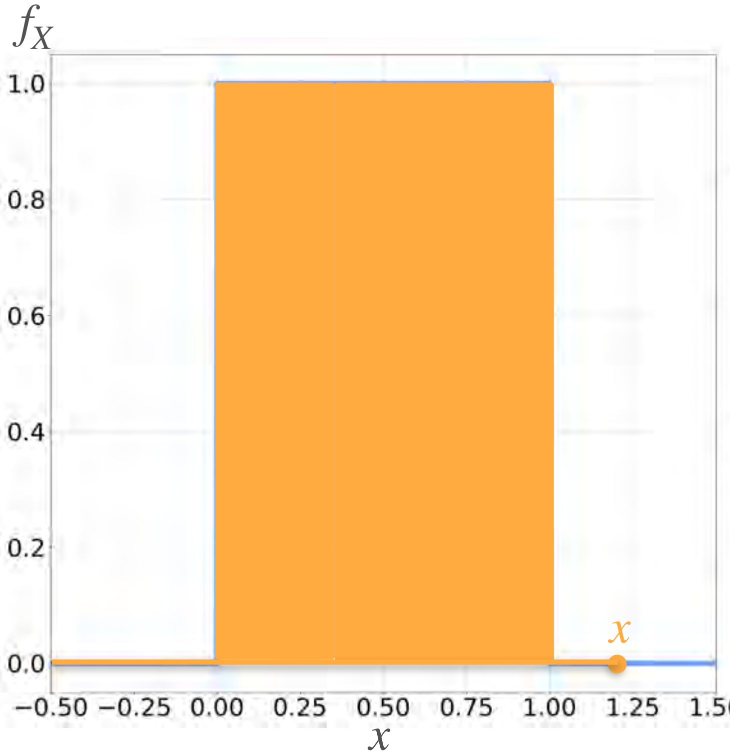
Uniform Distribution: CDF

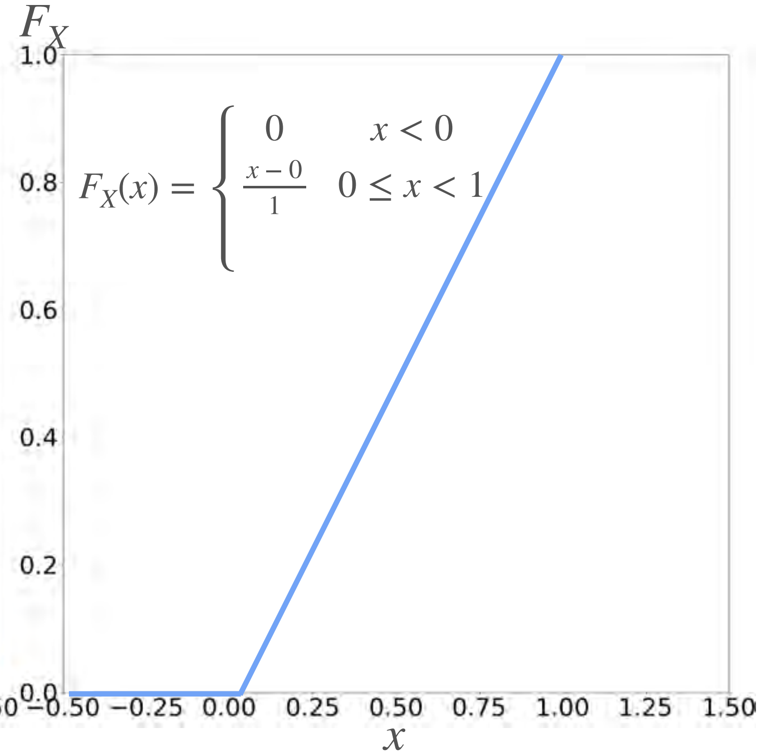




$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x-0}{1} & 0 \le x < 1 \end{cases}$$

Uniform Distribution: CDF





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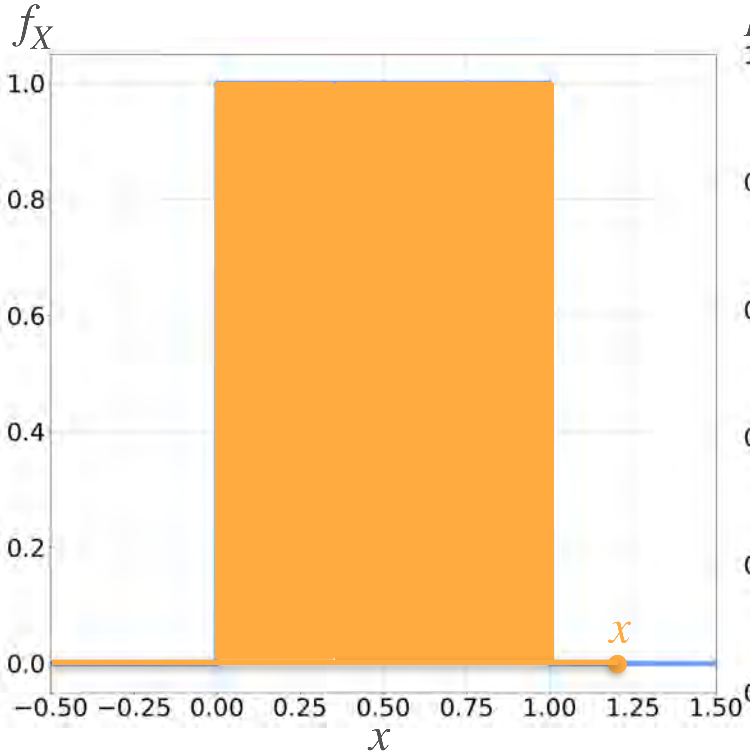
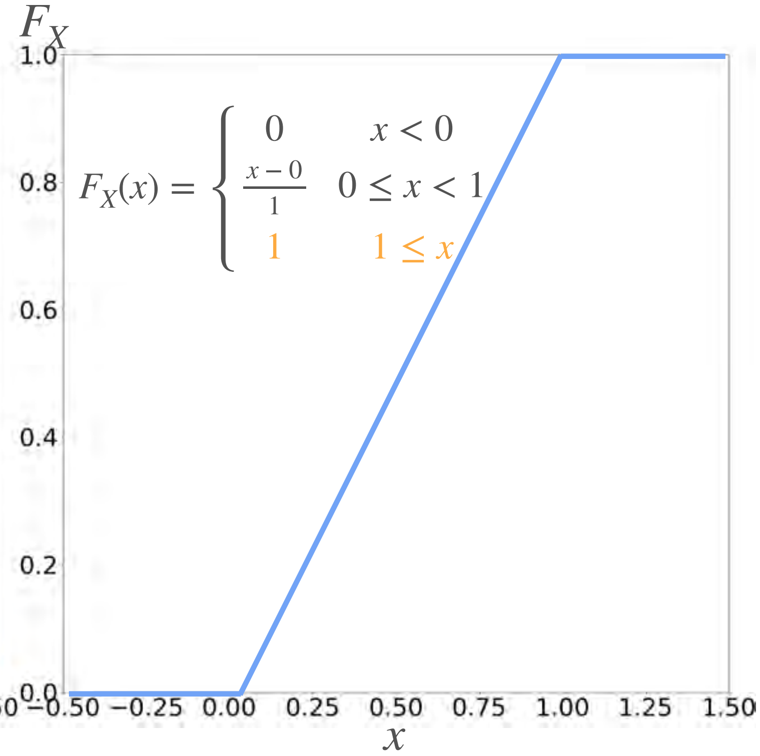
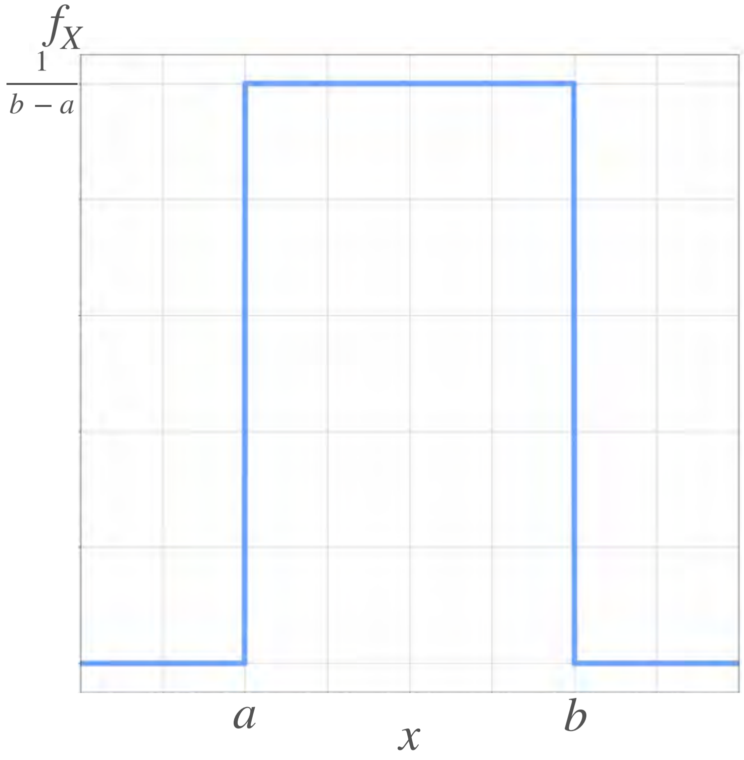
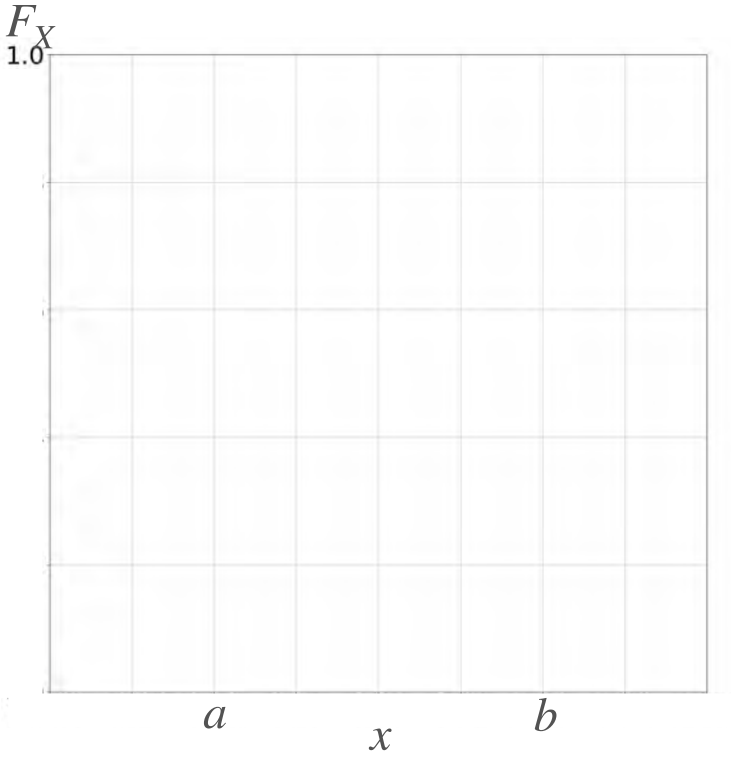
Uniform Distribution: CDF



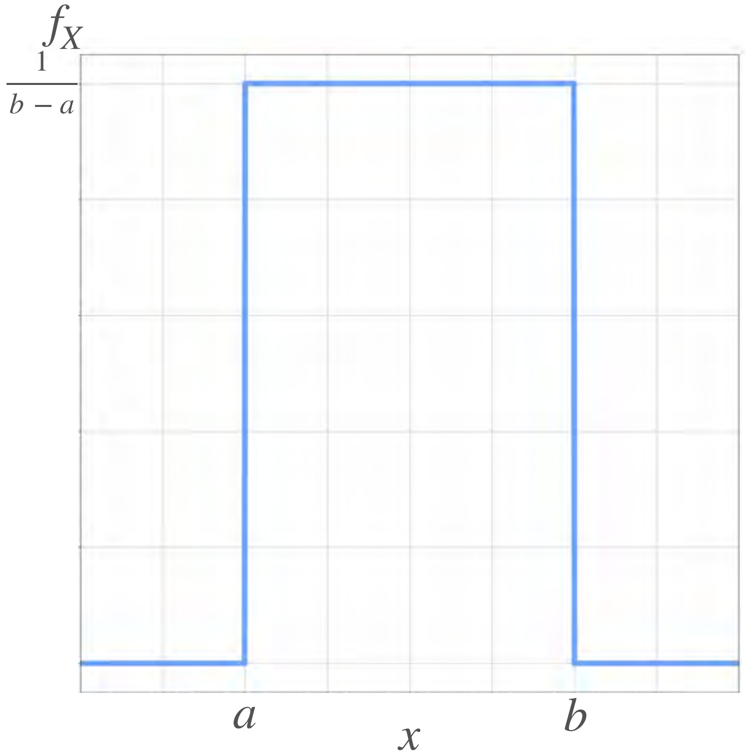


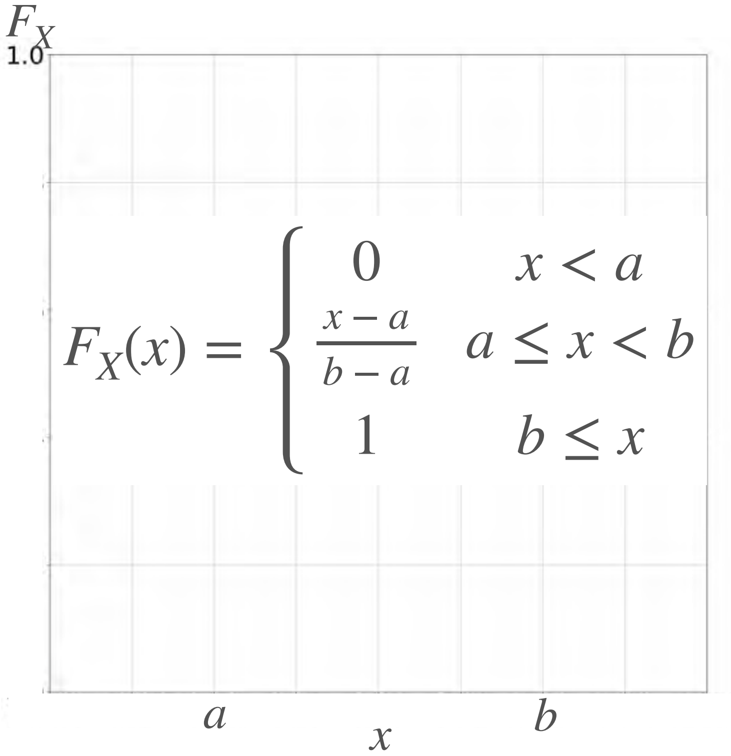
Uniform Distribution: CDF





Uniform Distribution: CDF





$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & b \le x \end{cases}$$

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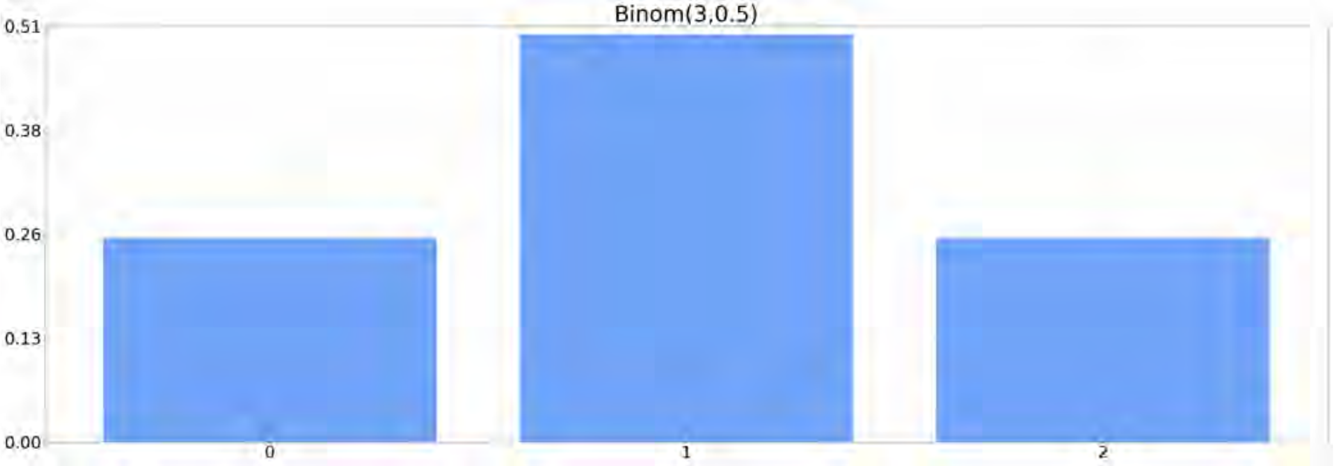
Probability Distributions

Normal distribution

Binomial Distribution With Very Large n

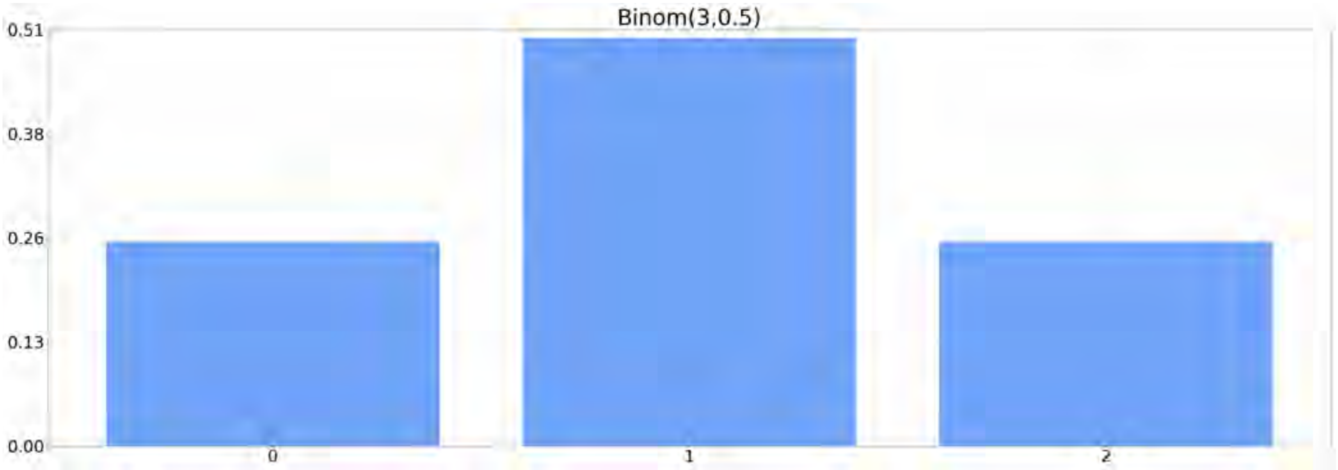
Binomial Distribution With Very Large n

Binomial Distribution With Very Large n



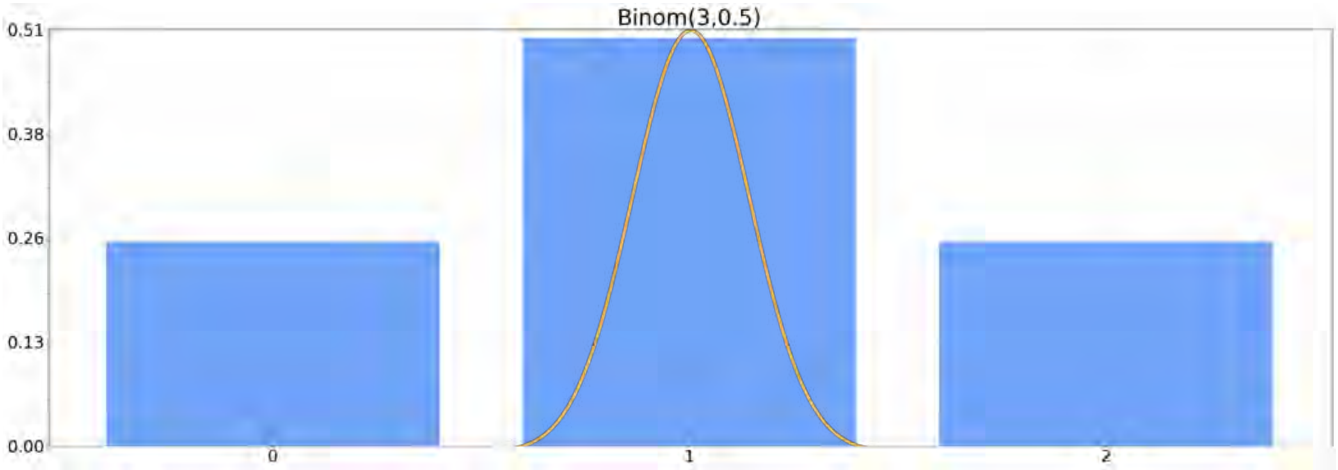
Number of throws = 2

Binomial Distribution With Very Large n



Number of throws $= n$

Binomial Distribution With Very Large n

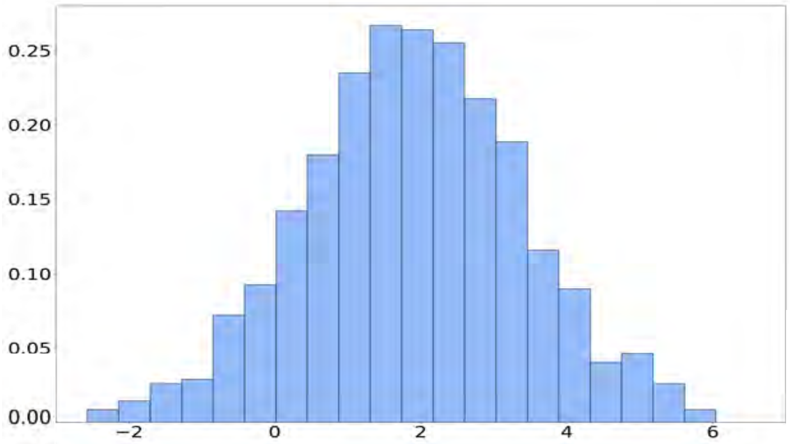


Number of throws $= n$

Bell Shaped Data

Bell Shaped Data

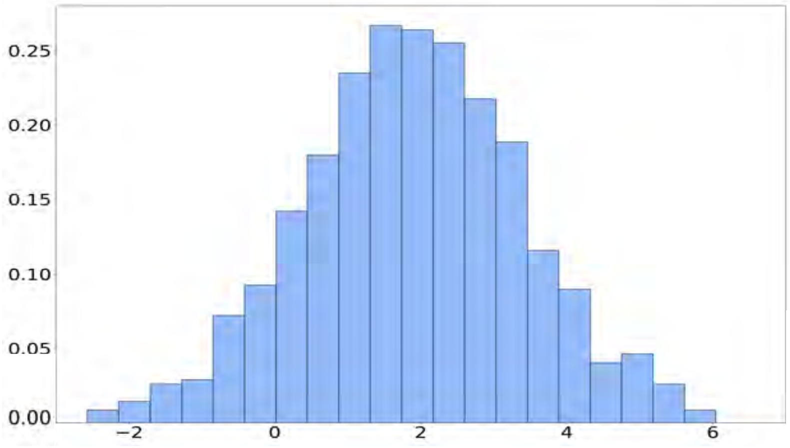
Data



Bell Shaped Data

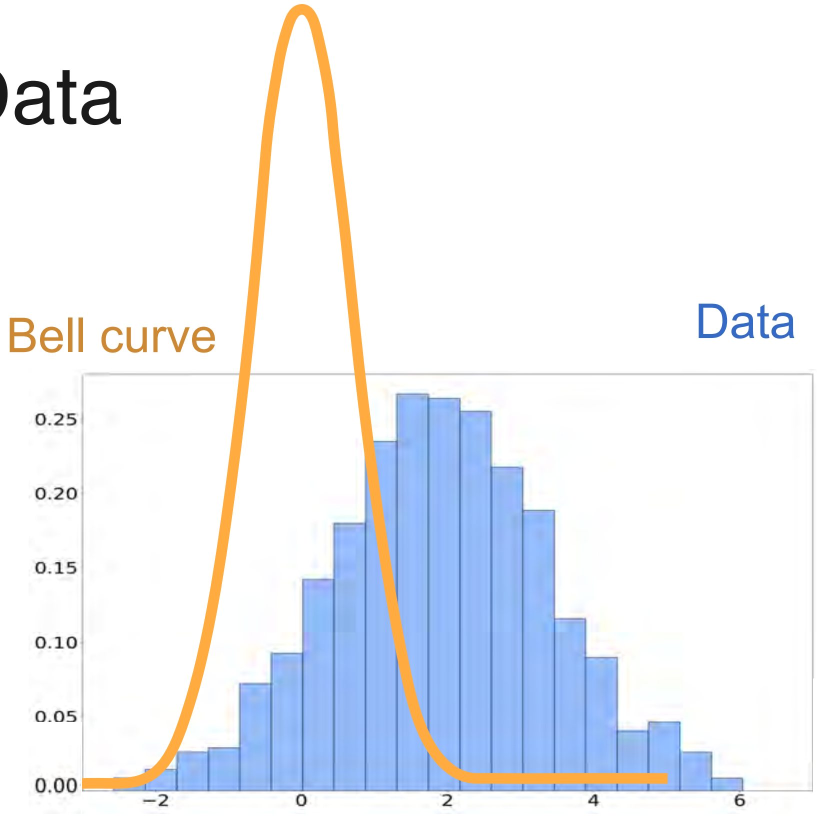
$$e^{-x^2}$$

Data



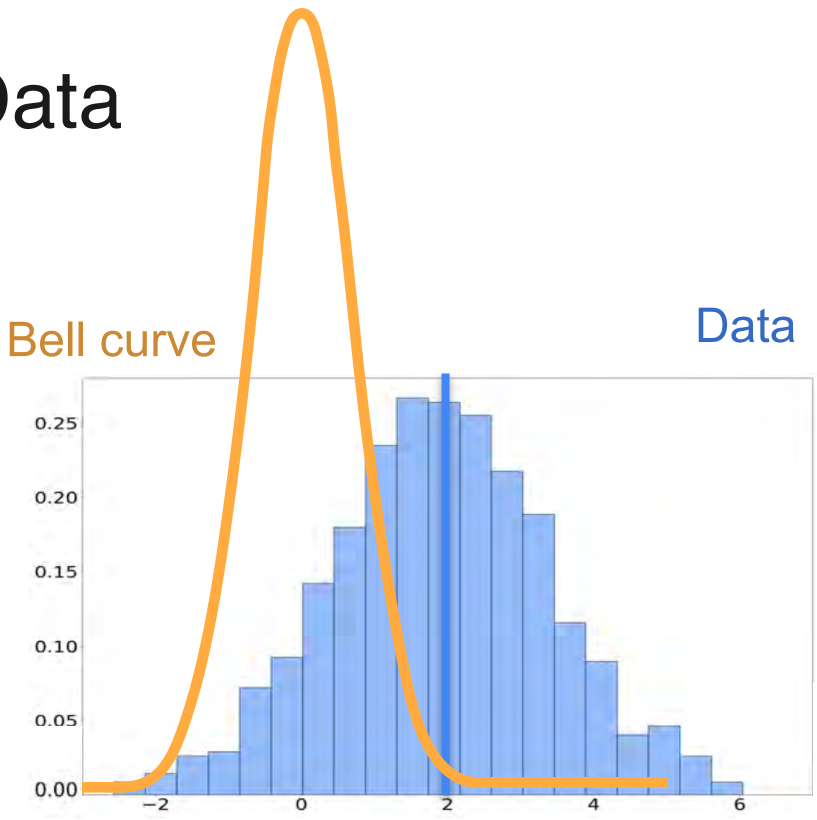
Bell Shaped Data

$$e^{-x^2}$$



Bell Shaped Data

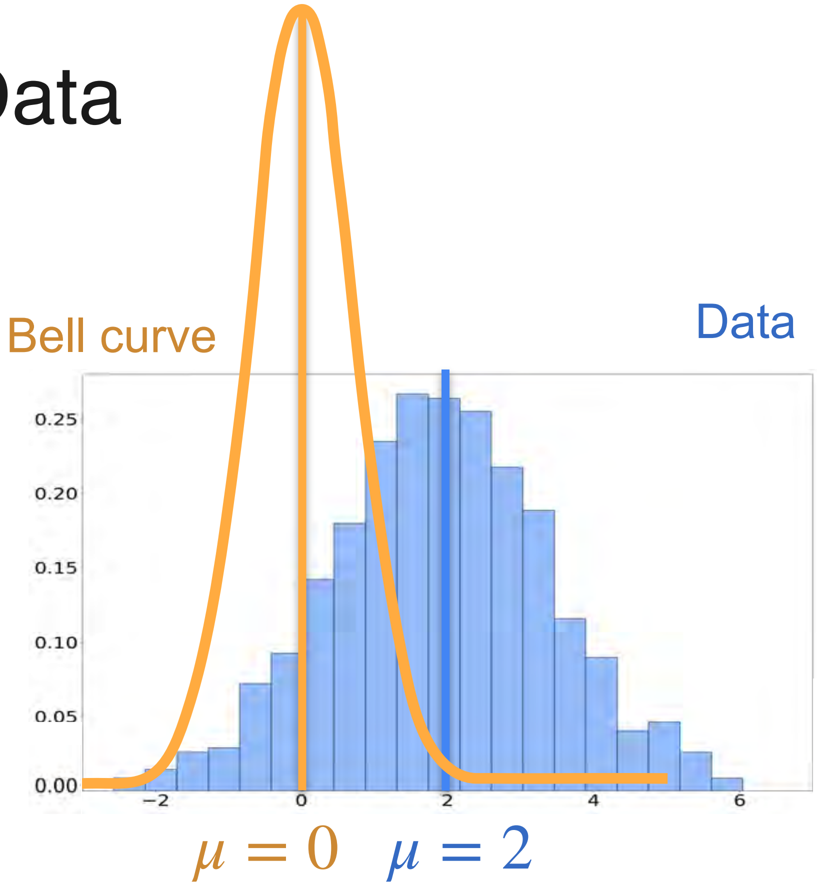
$$e^{-x^2}$$



$\mu = 2$

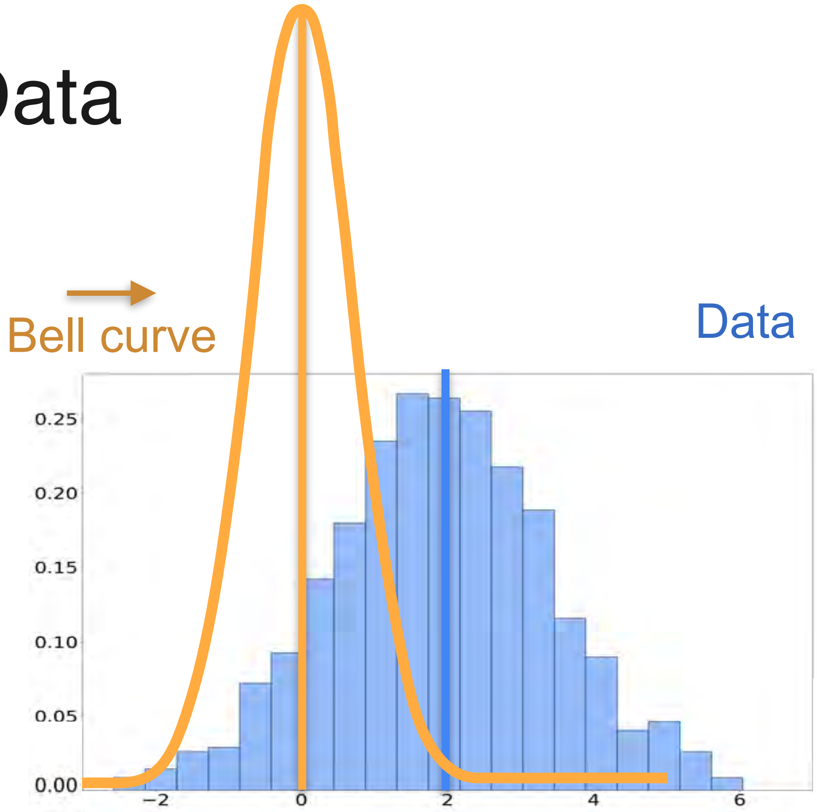
Bell Shaped Data

e^{-x^2}



Bell Shaped Data

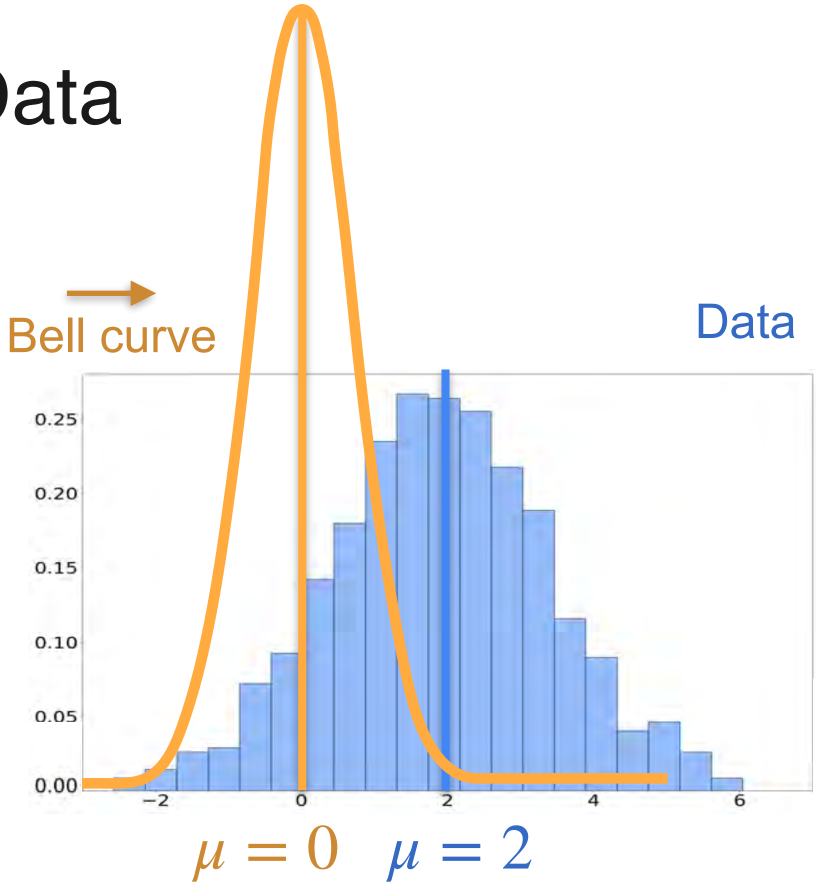
$$e^{-x^2}$$



$\mu = 0 \quad \mu = 2$

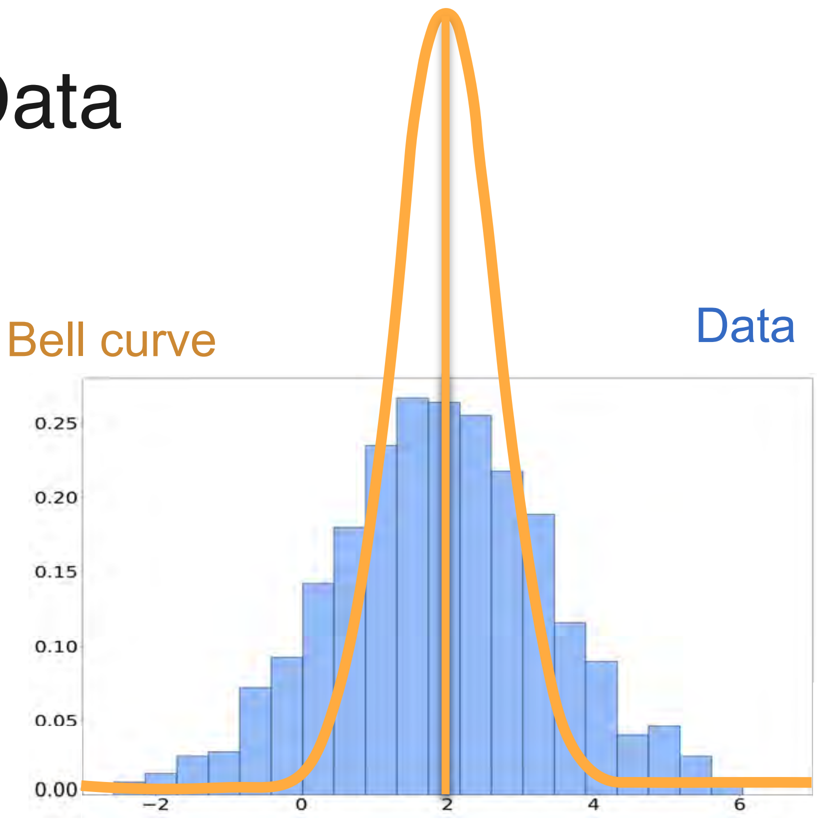
Bell Shaped Data

$$e^{-(x-2)^2}$$



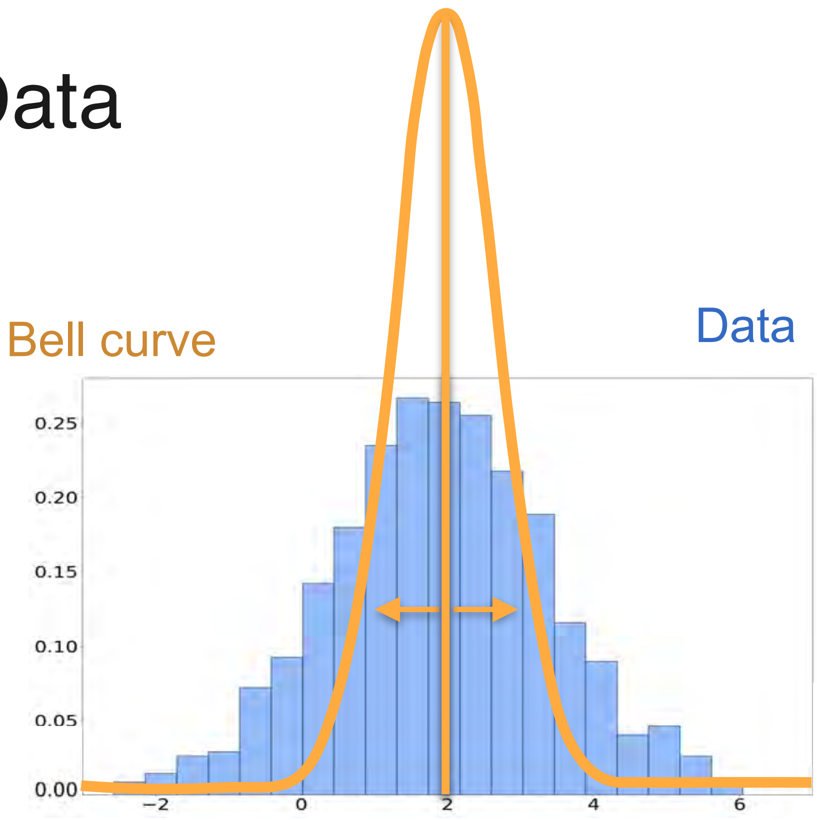
Bell Shaped Data

$$e^{-(x-2)^2}$$



Bell Shaped Data

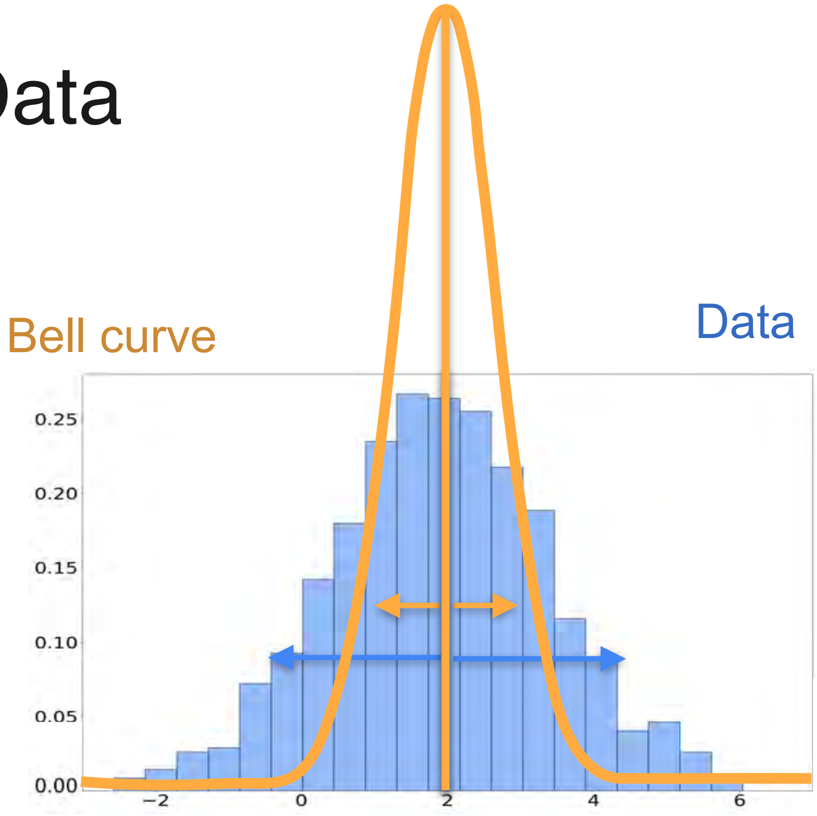
$$e^{-(x-2)^2}$$



$$\sigma = 1$$

Bell Shaped Data

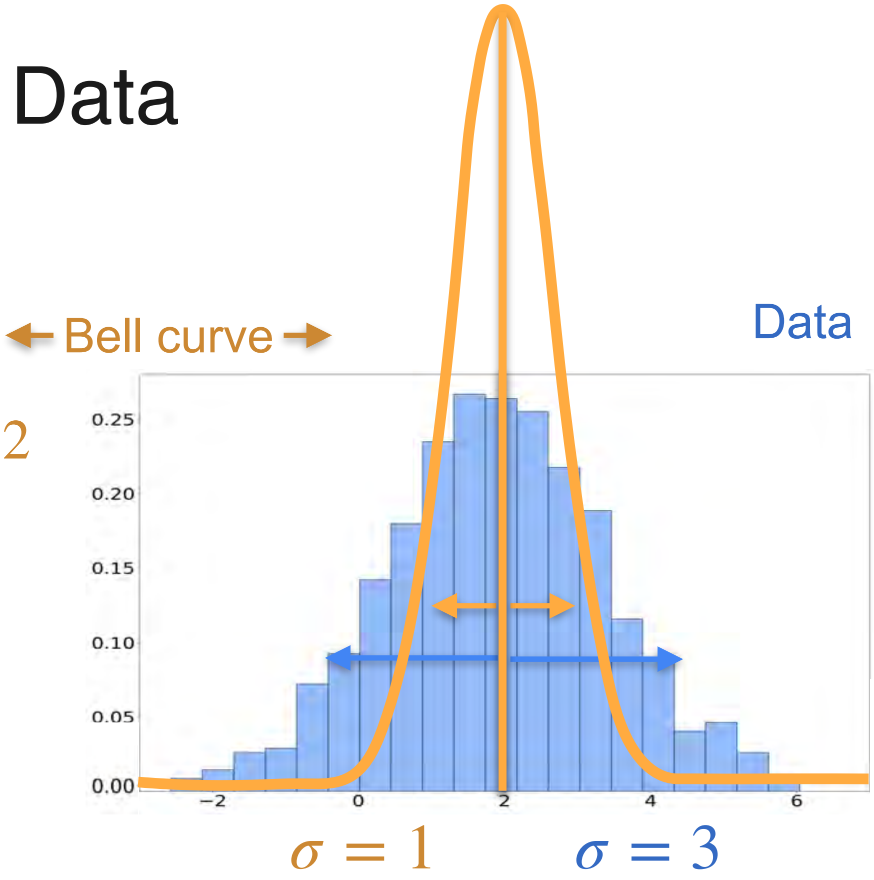
$$e^{-(x-2)^2}$$



$\sigma = 1$ $\sigma = 3$

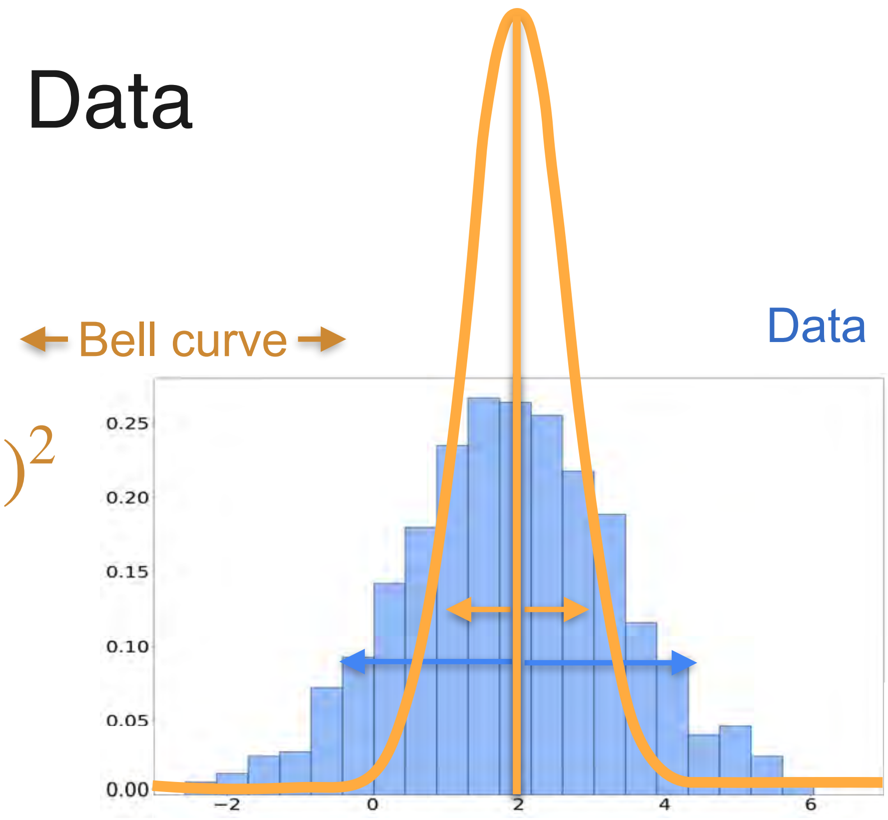
Bell Shaped Data

$$e^{-(x-2)^2}$$



Bell Shaped Data

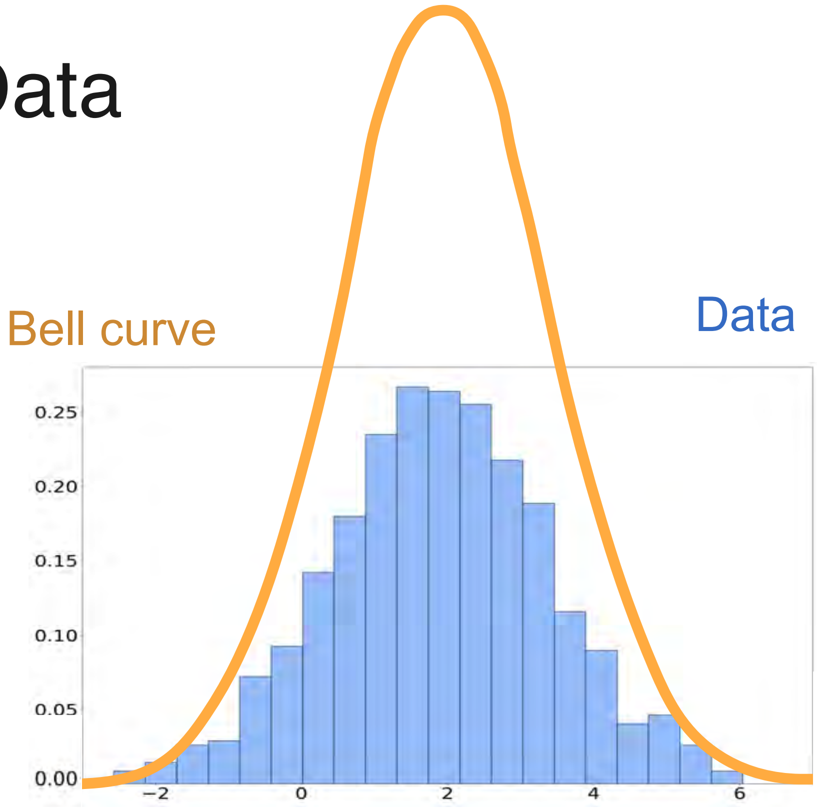
$$e^{-\left(\frac{x-2}{3}\right)^2}$$



$\sigma = 1$ $\sigma = 3$

Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



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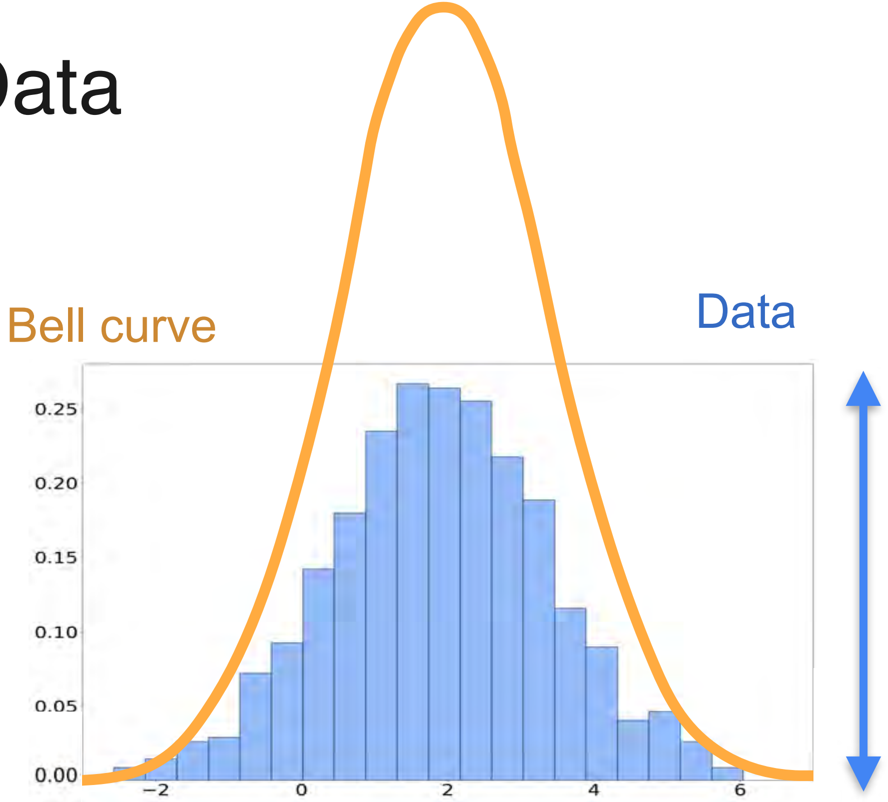

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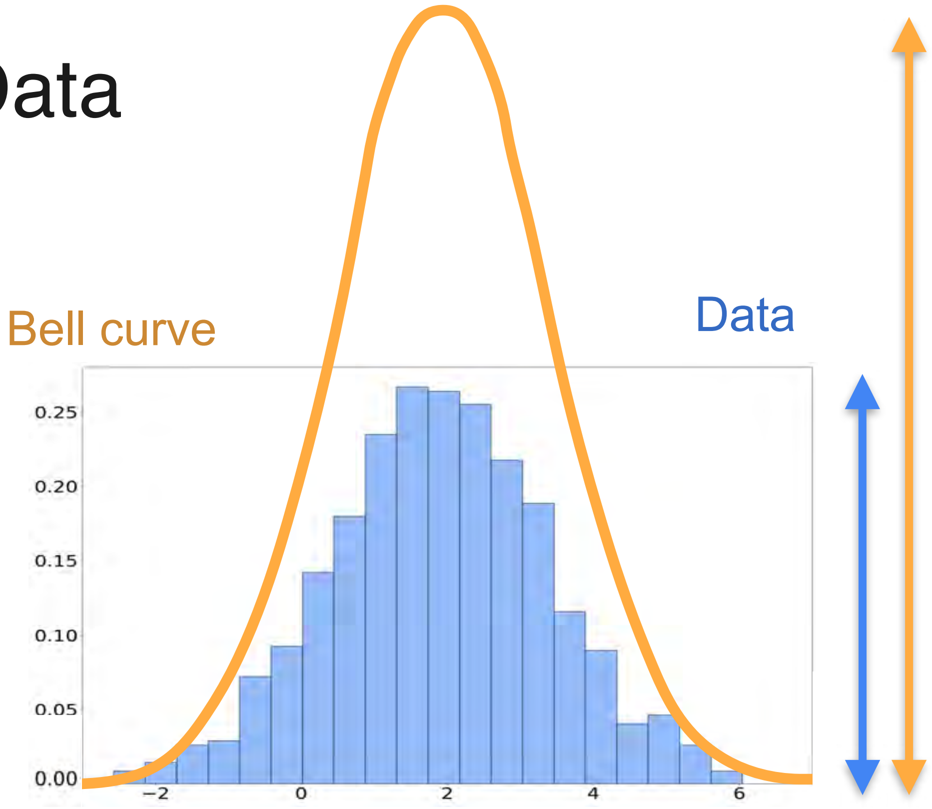

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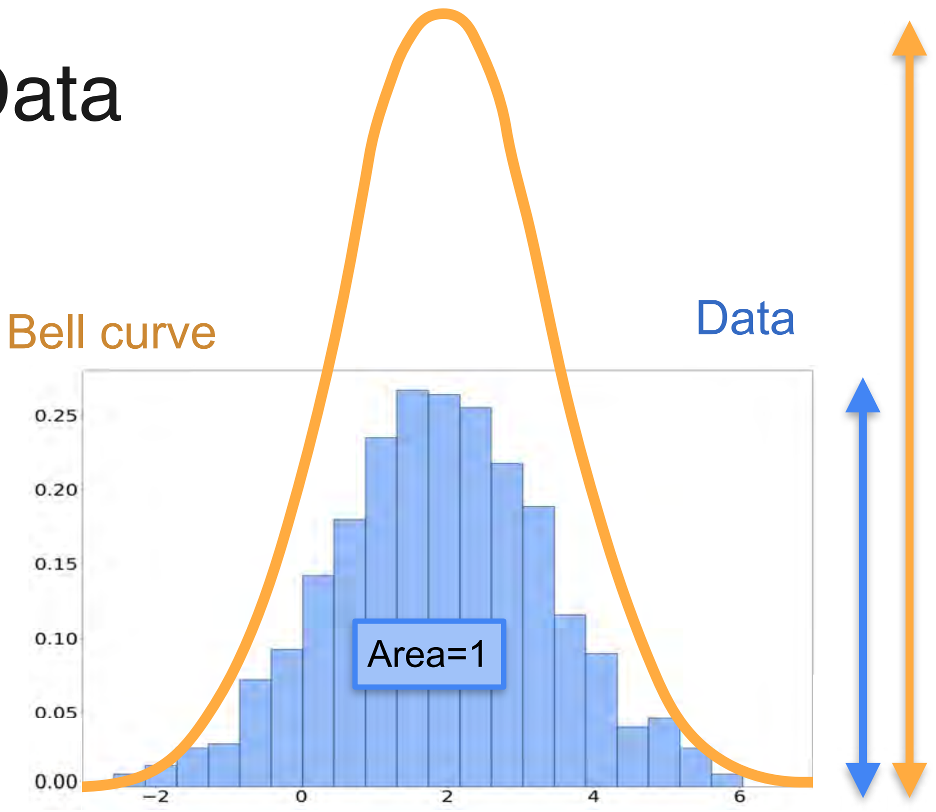
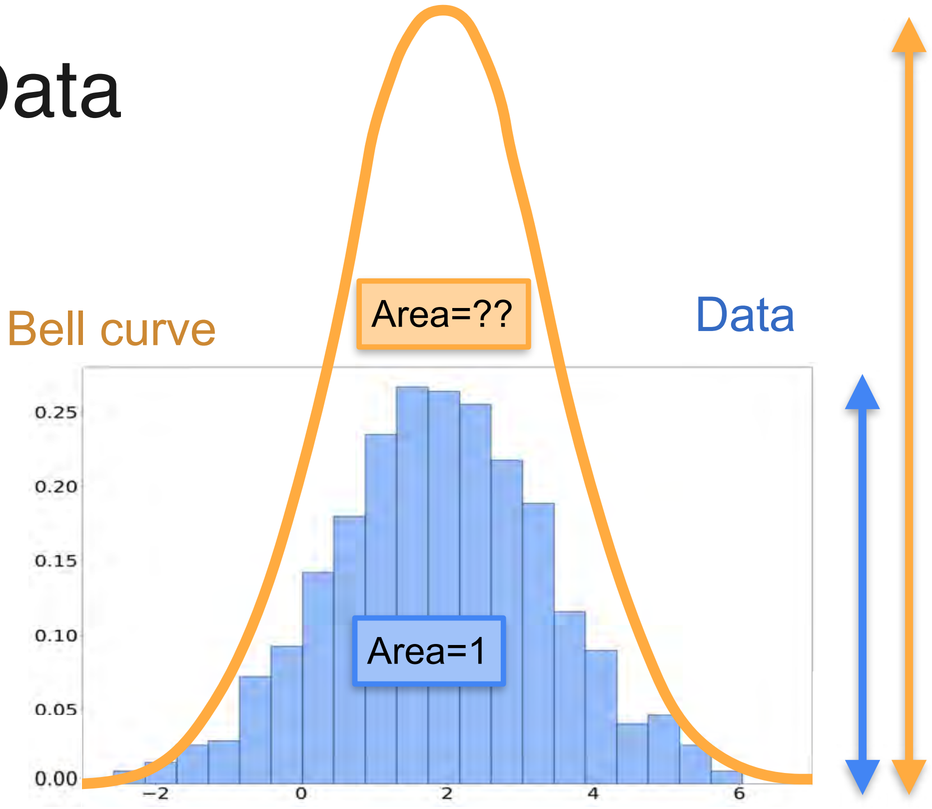
$$e^{-\left(\frac{x-2}{3}\right)^2}$$



Bell Shaped Data

$$e^{-\left(\frac{x-2}{3}\right)^2}$$



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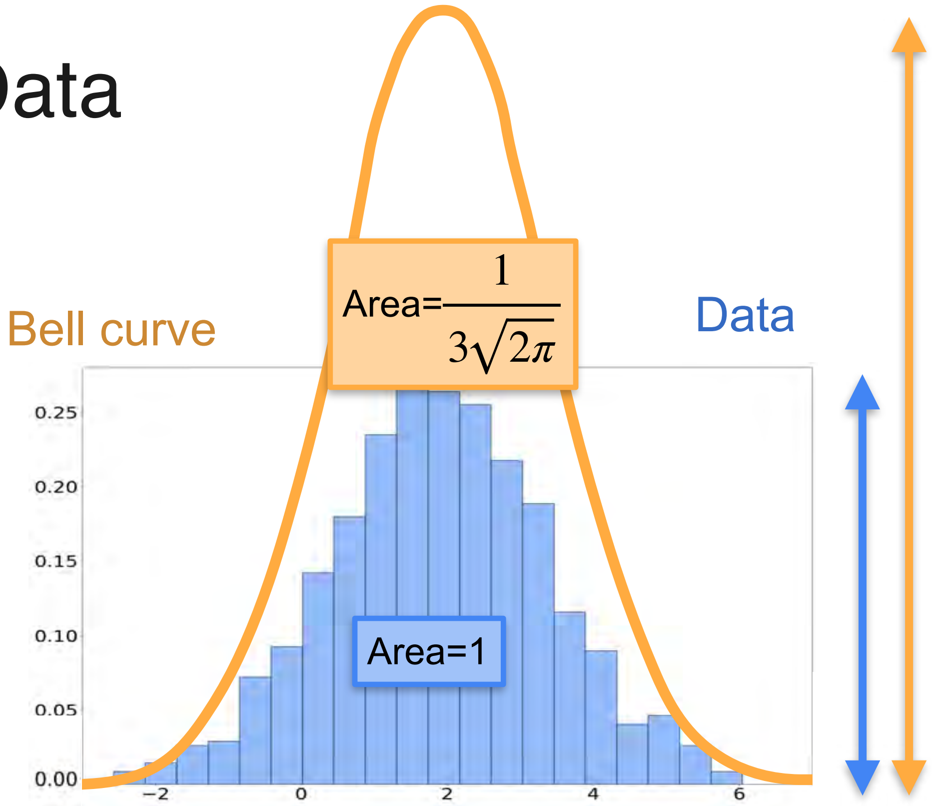
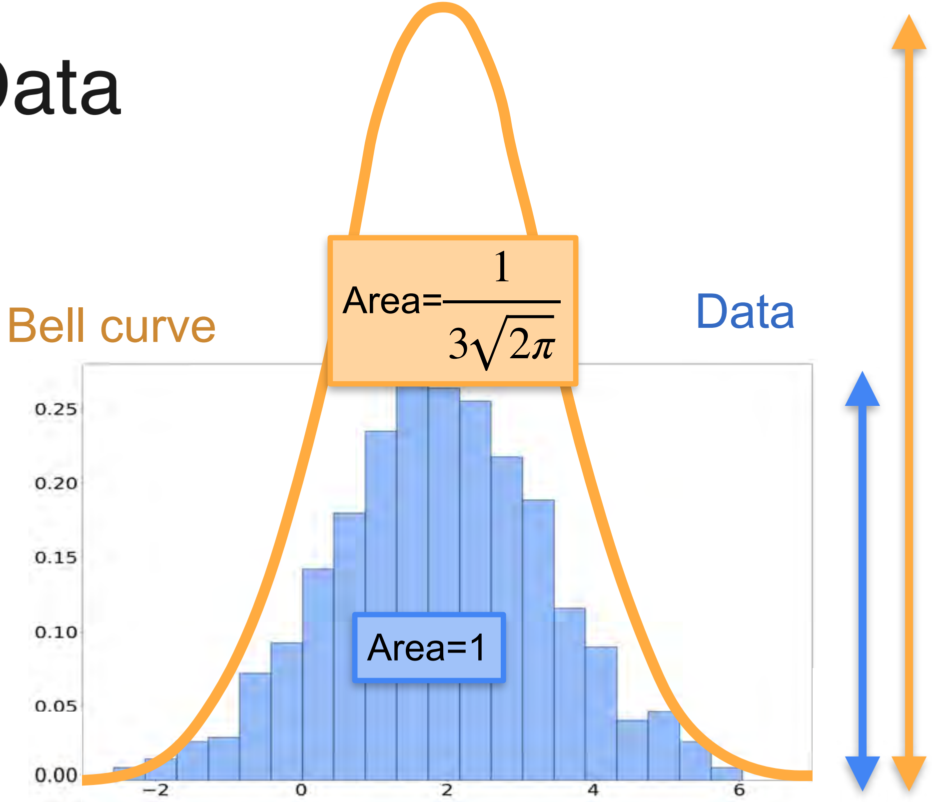


Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}}\, e^{-\left(\frac{x-2}{3}\right)^2}$$



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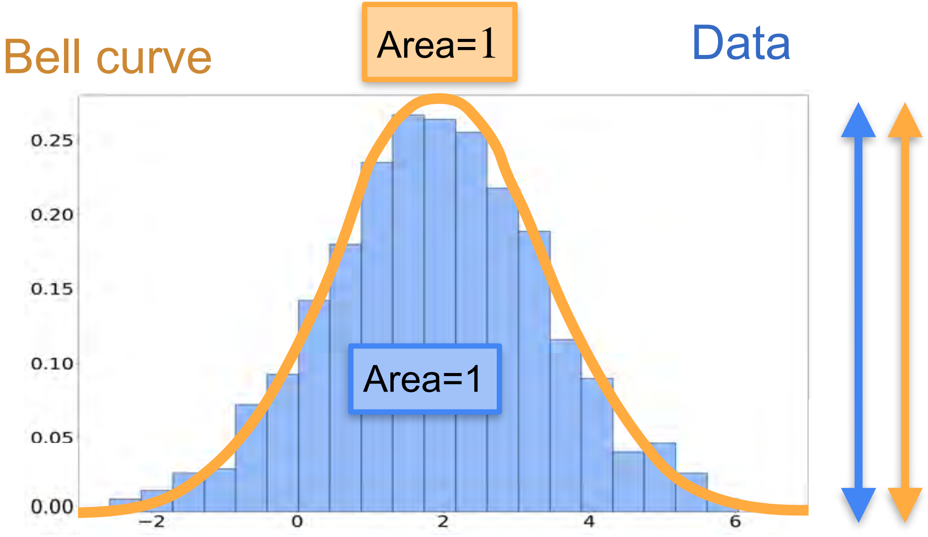
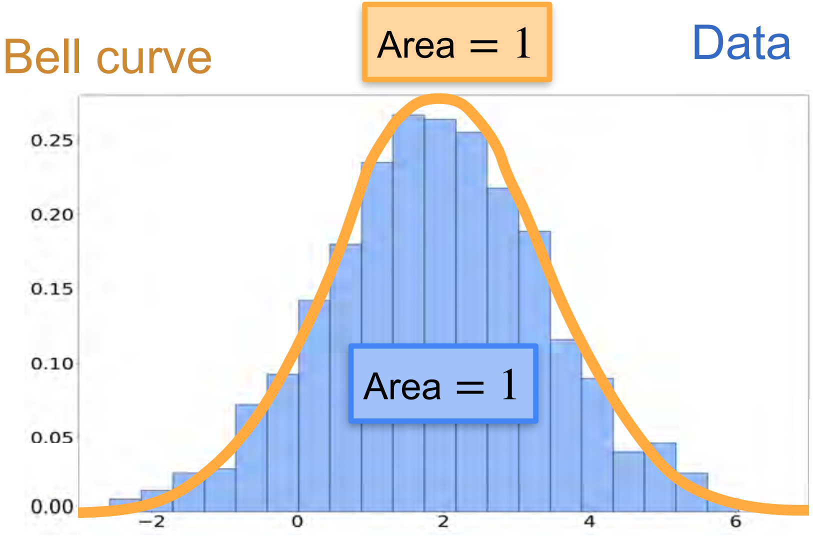
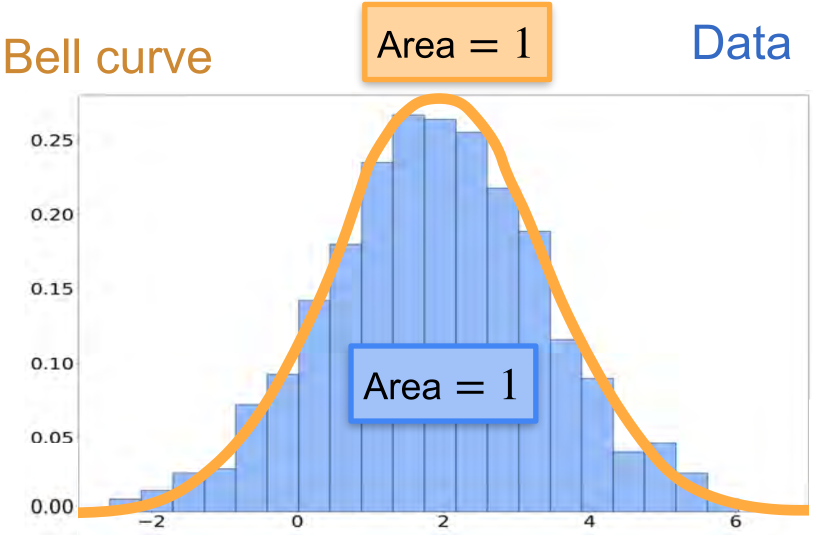


Bell Shaped Data



Bell Shaped Data

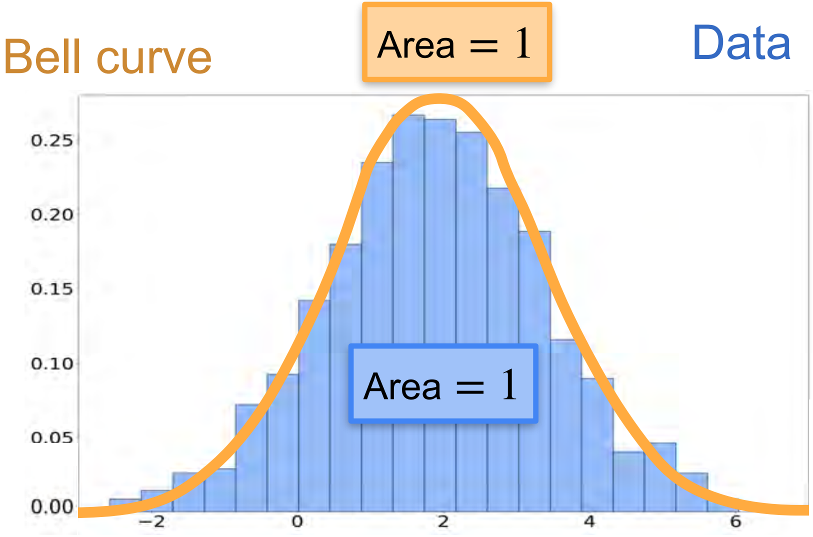
Mean = μ



Bell Shaped Data

Mean = μ

Standard deviation = σ

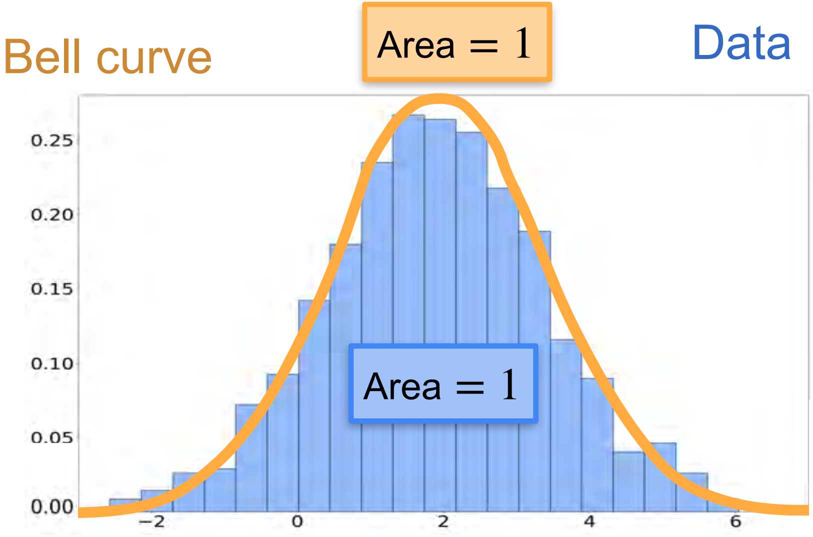


Bell Shaped Data

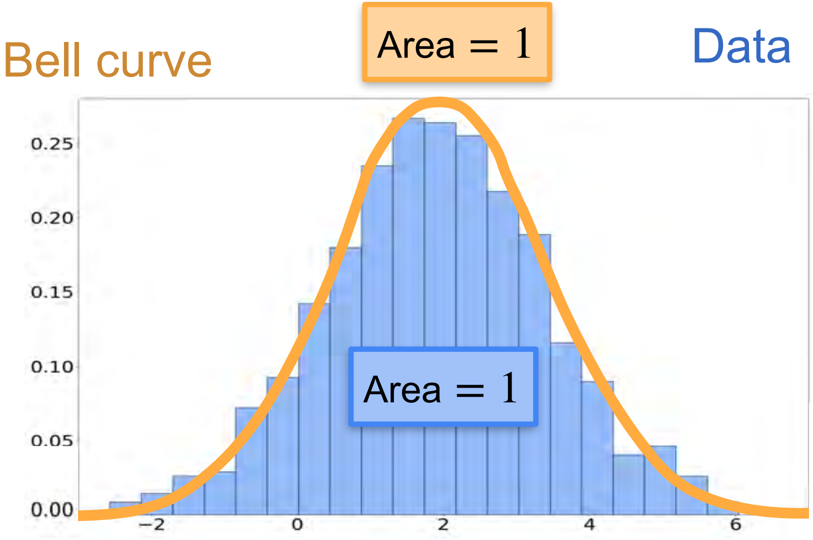
Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$



Bell Shaped Data



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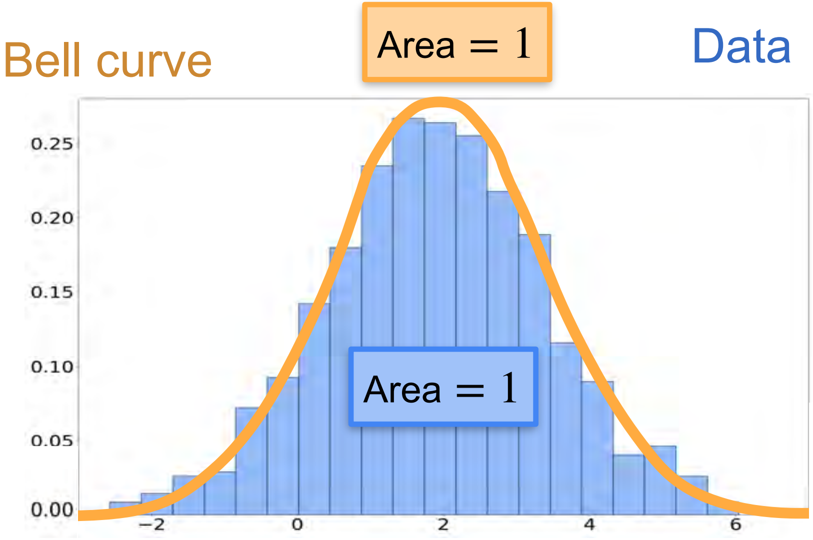
Mean = μ



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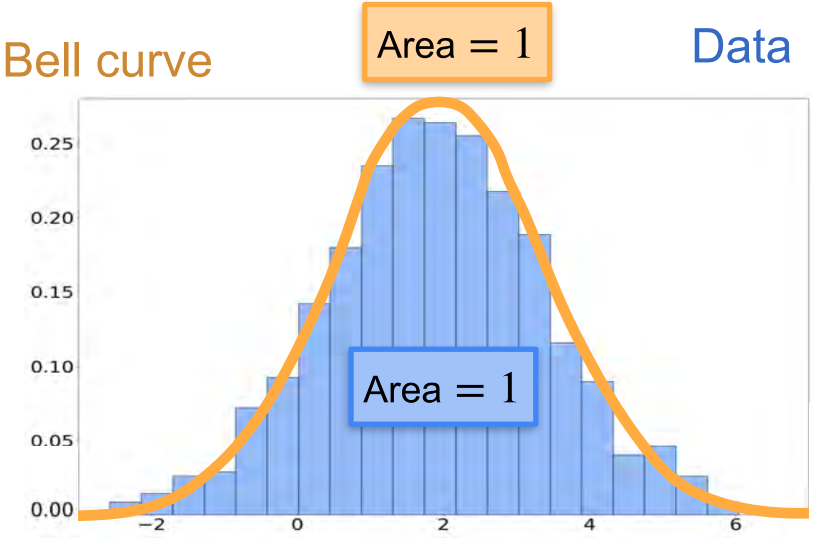

Bell Shaped Data

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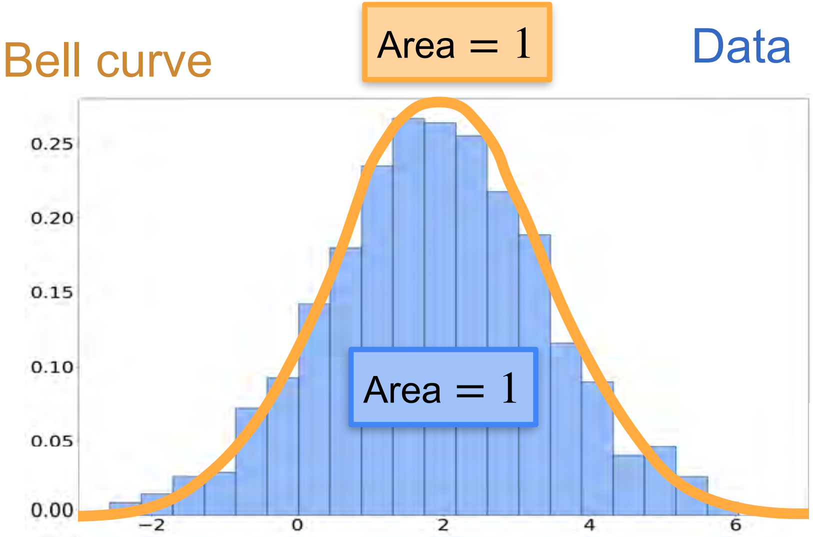


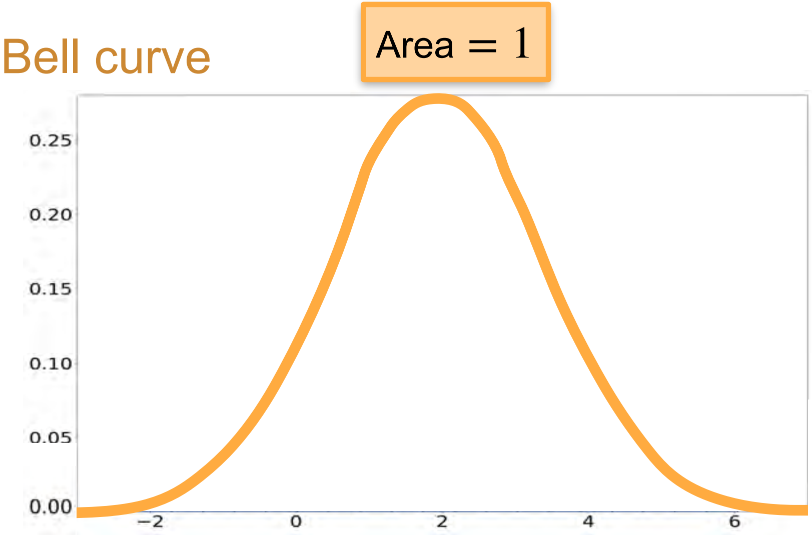
Normal Distribution

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\left(\frac{x-\mu}{\sigma}\right)^2}$$

Bell curve



Normal Distribution

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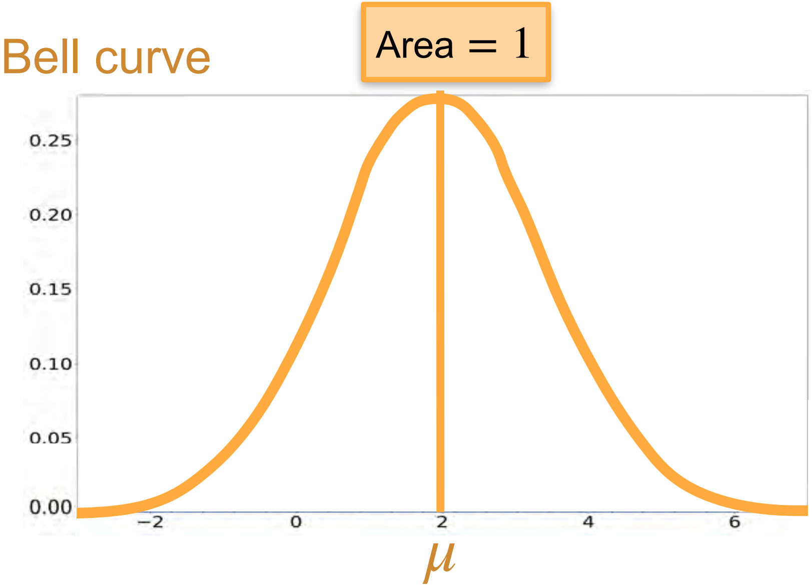

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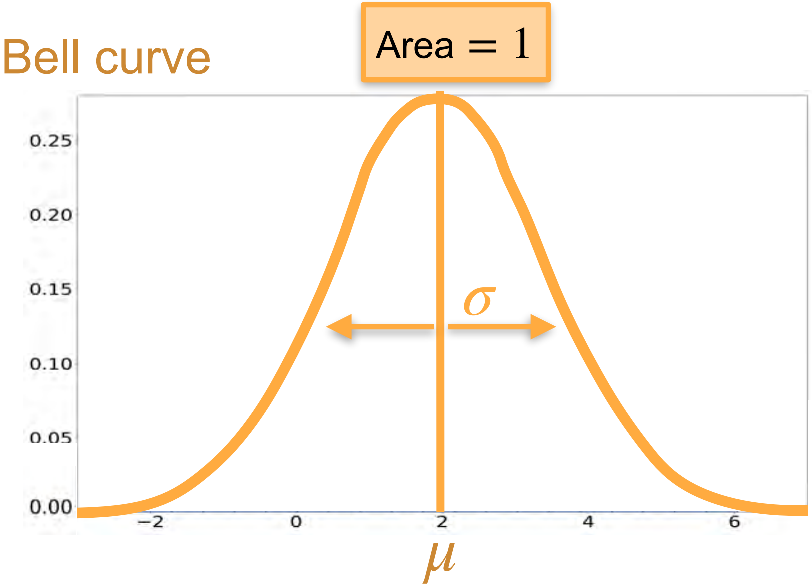
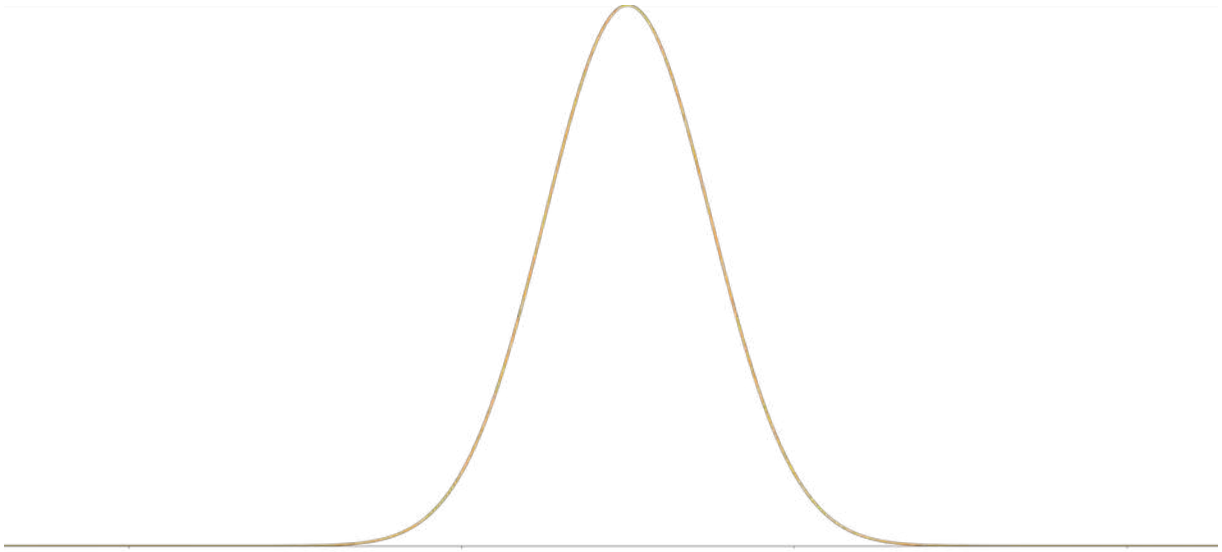


Normal Distribution

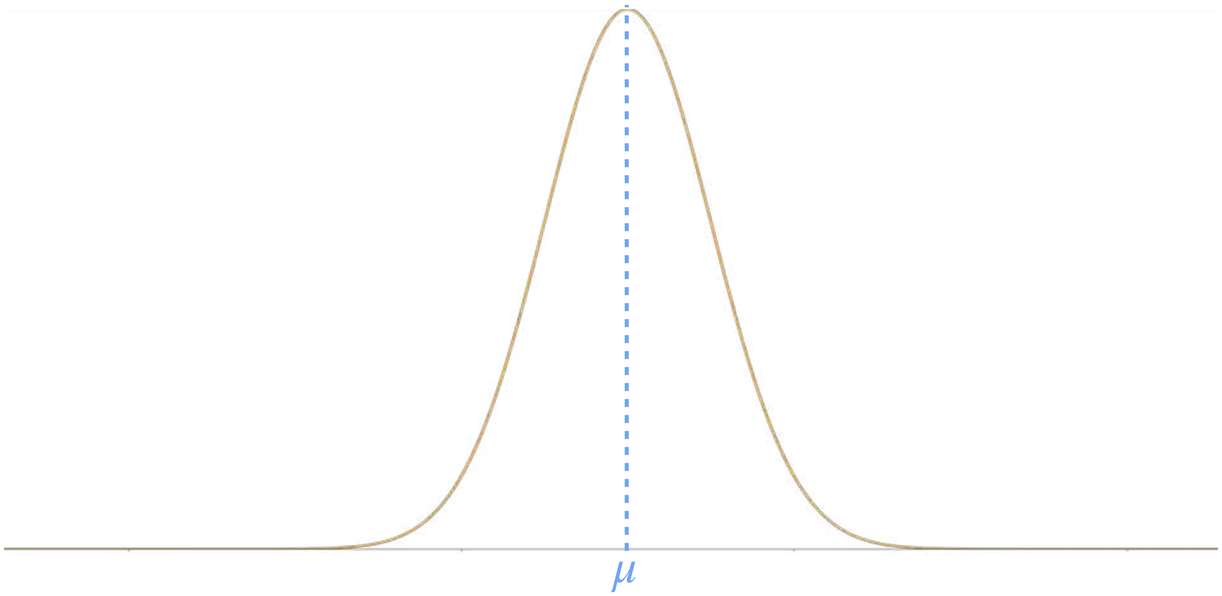
Normal Distribution



Normal Distribution

Parameters:

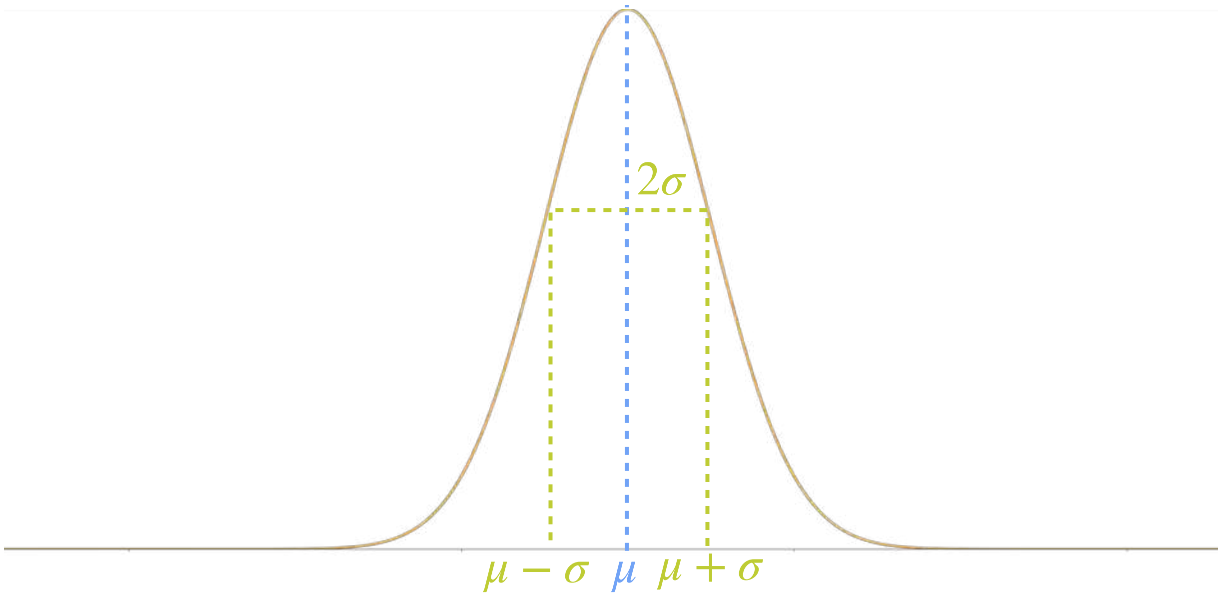
- μ: center of the bell



Normal Distribution

Parameters:

- μ: center of the bell
- σ: spread of the bell



Normal Distribution

Parameters:

- μ: center of the bell
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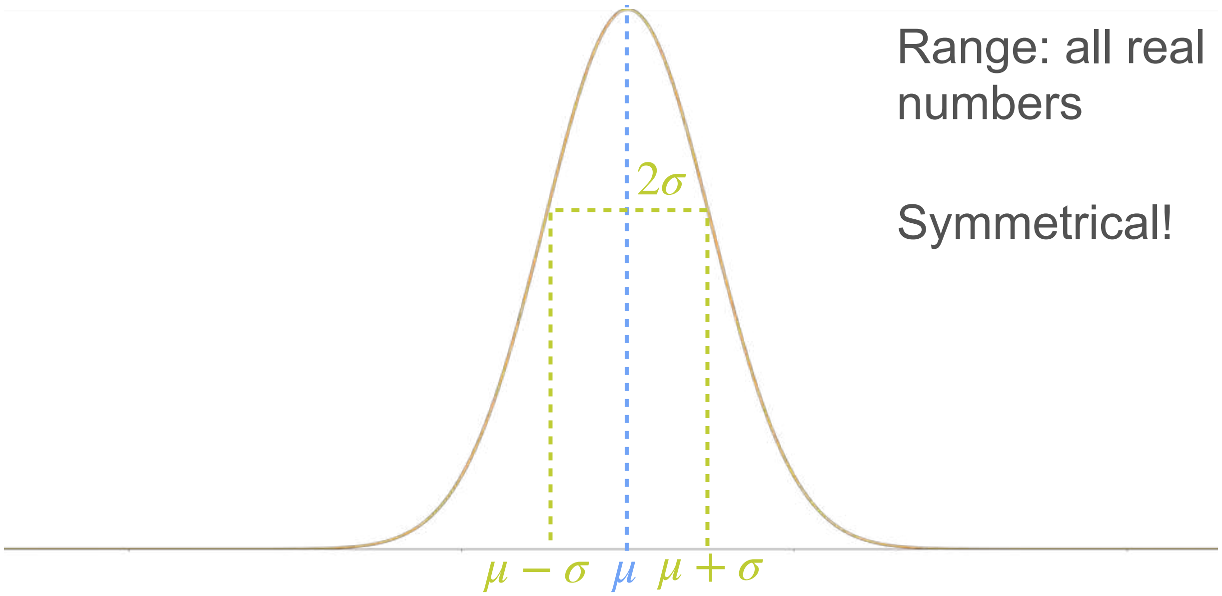


Range: all real numbers

Symmetrical!

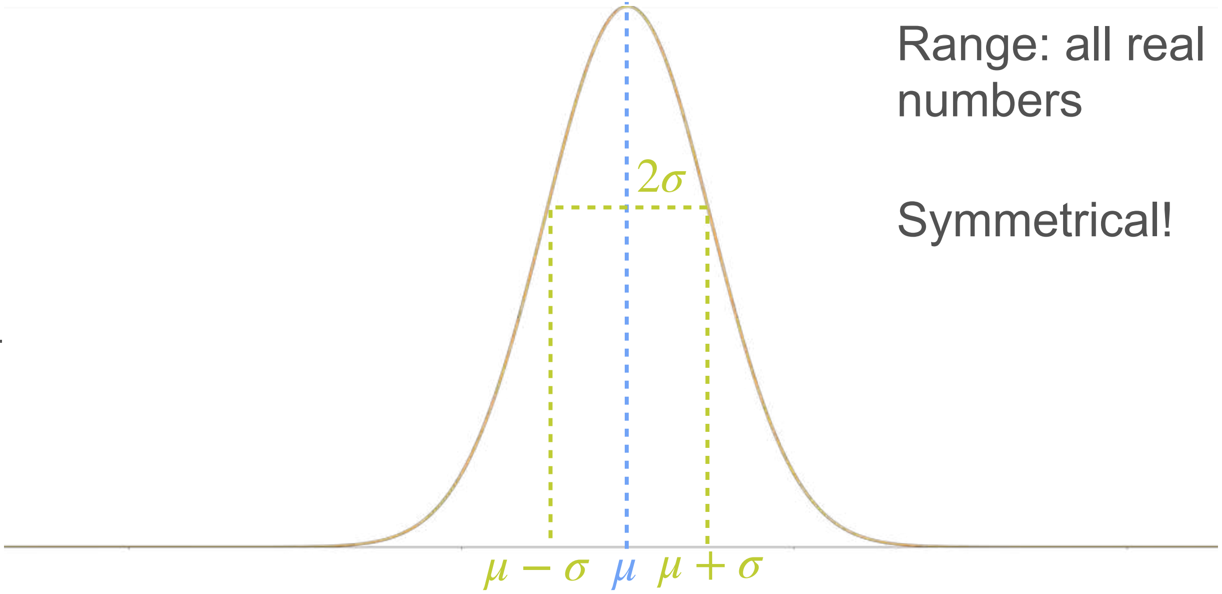
Normal Distribution

Parameters:

- μ: center of the bell
- σ: spread of the bell

$$f_X(x) = \boxed{\frac{1}{\sqrt{2\pi}\sigma}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Scaling constant



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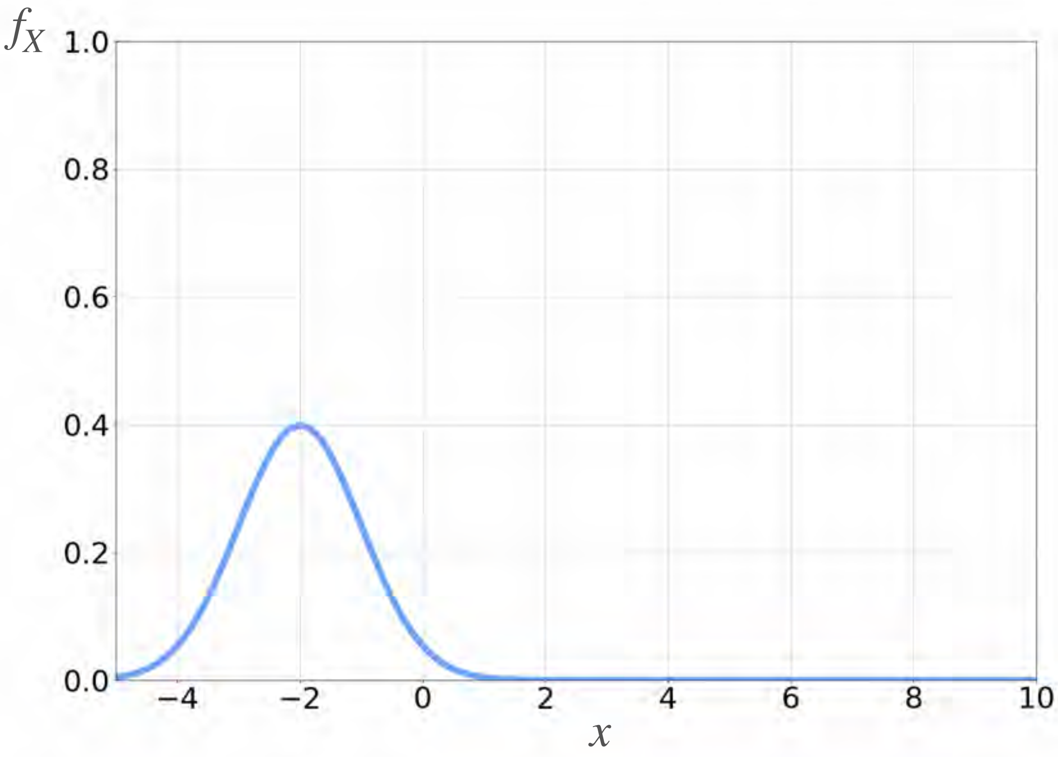

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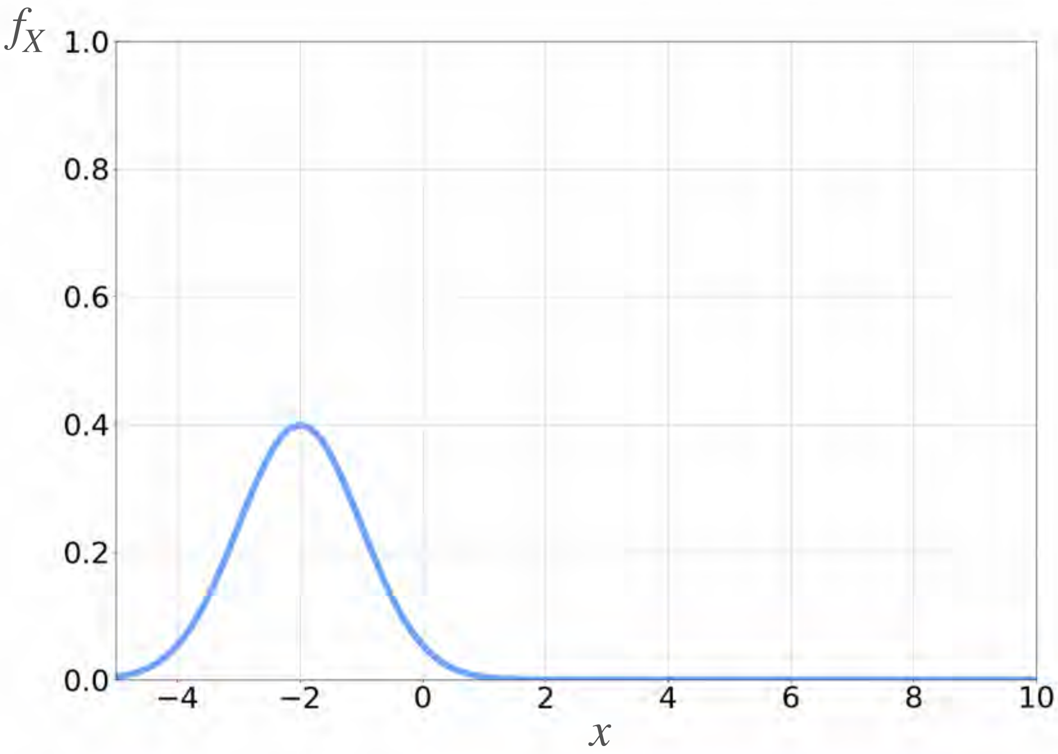

Normal Distribution - Notation

Parameters:

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$$X \sim \mathcal{N}(\mu, \sigma^2)$$

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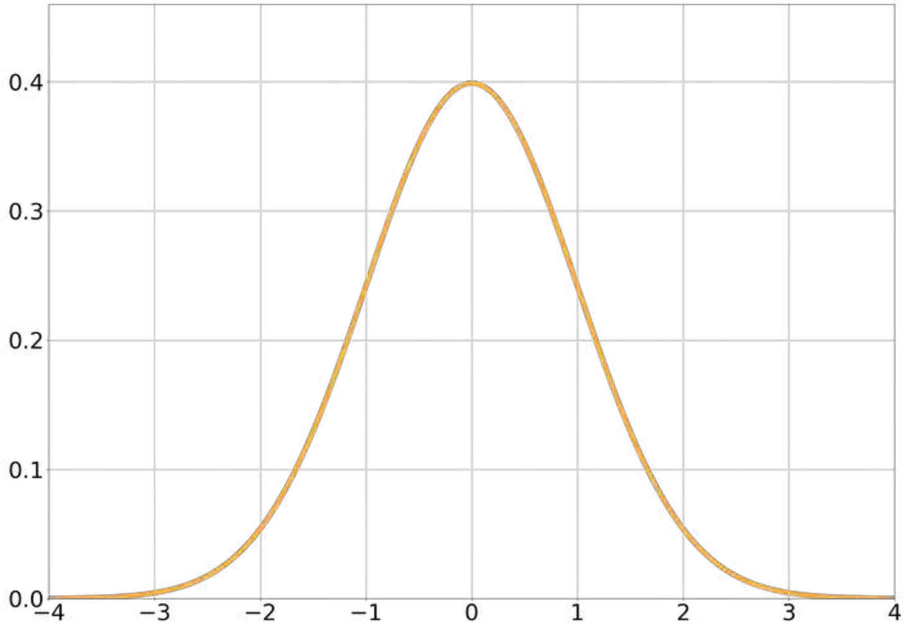
$$X \sim \mathcal{N}(\mu, \boxed{\sigma^2})$$

Standard Normal Distribution

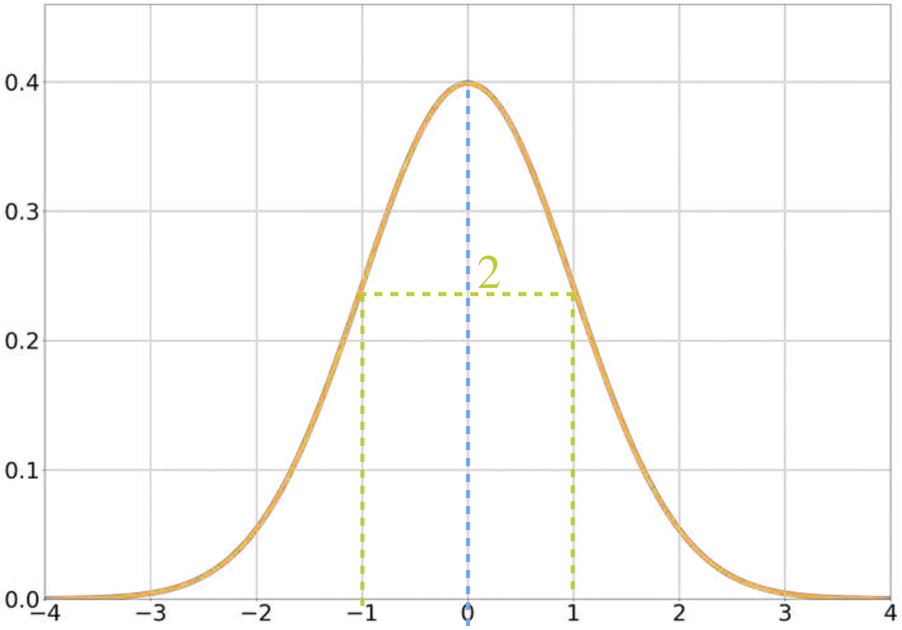


Standard Normal Distribution

Parameters:

- μ: 0
- σ: 1

$$X \sim \mathcal{N}(0, 1^2)$$



Standard Normal Distribution

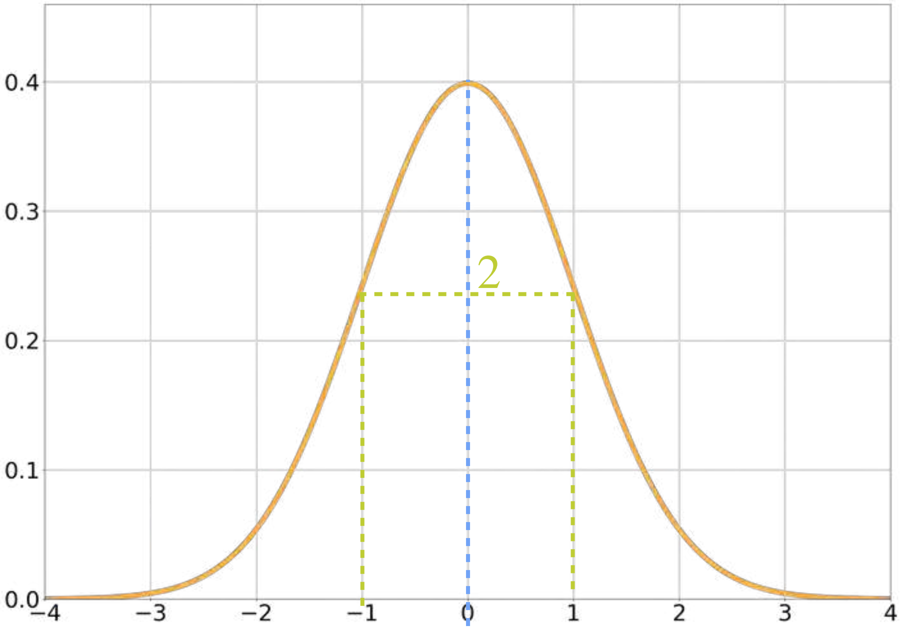
Parameters:

- μ: 0
- σ: 1

$$X \sim \mathcal{N}(0, 1^2)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}1} e^{-\frac{1}{2}\frac{(x-0)^2}{1^2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



Standardization

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X distributes normally with

$$\mu = 2, \ \sigma = 2.5$$

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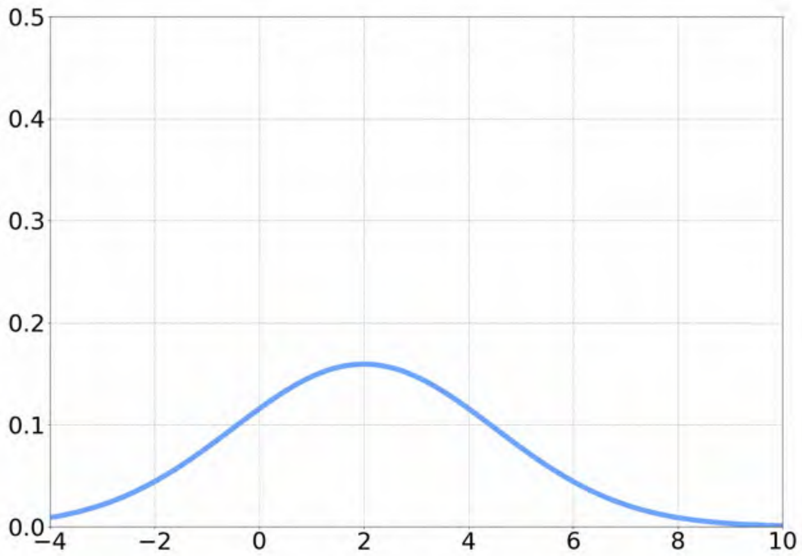
$$X - 2$$



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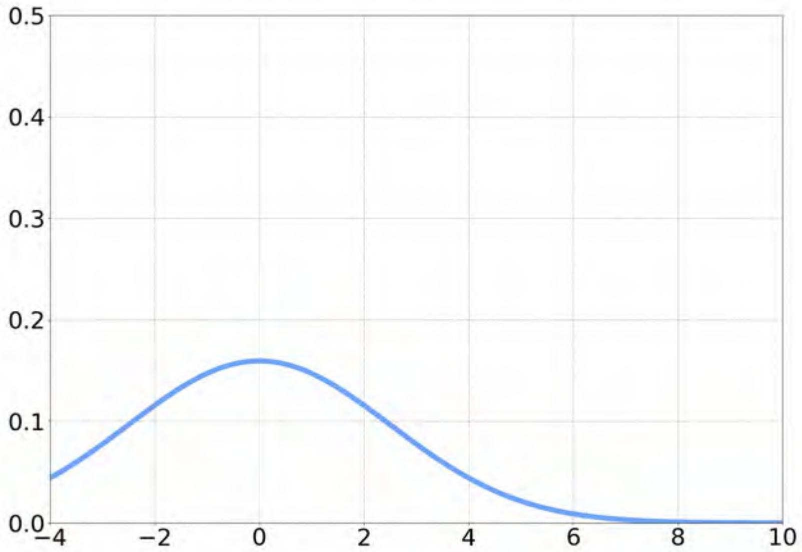
$$\frac{X - 2}{2.5}$$



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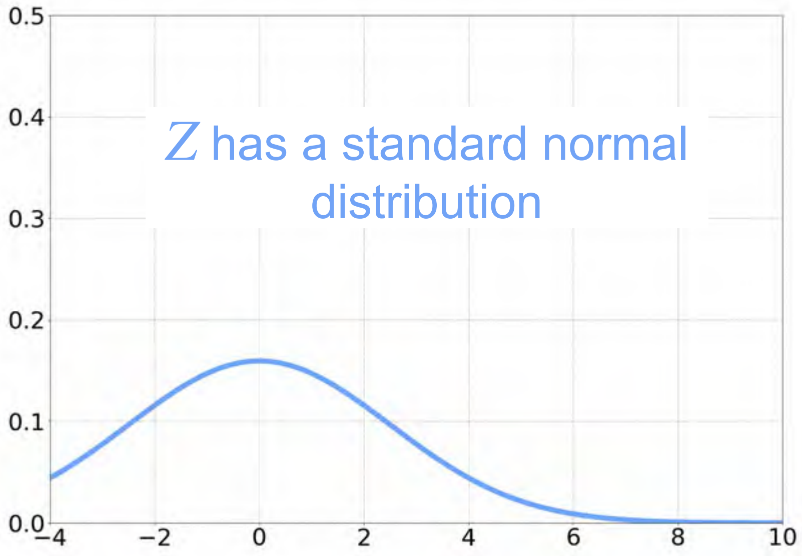
$$\mu = 2, \sigma = 2.5$$

$$Z = \frac{X - 2}{2.5}$$



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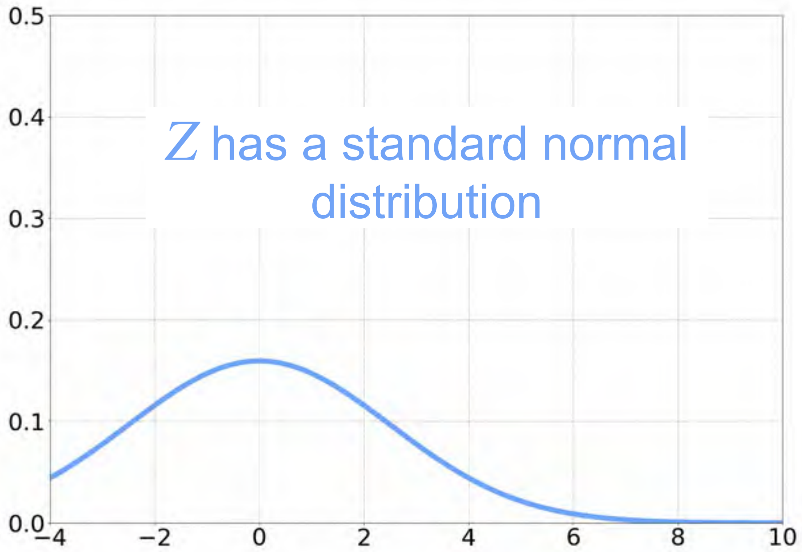


Standardization

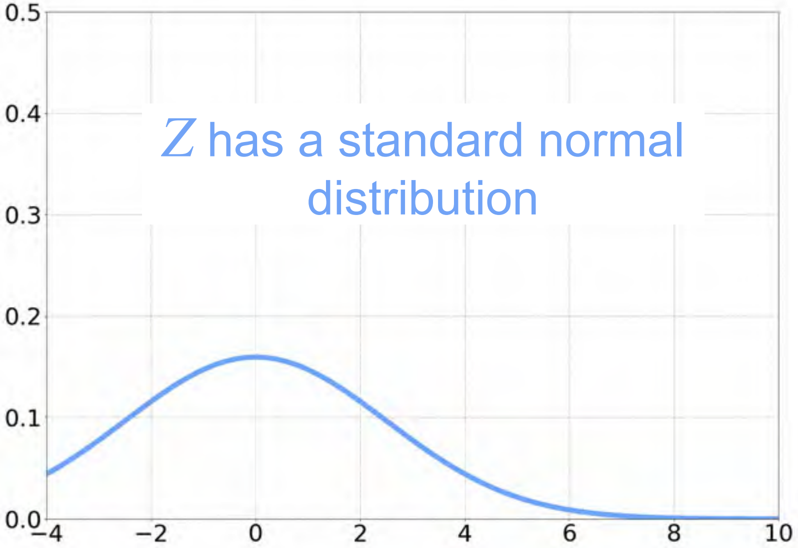
There's a really easy way to convert any normal distribution to the standard one!

X distributes normally with

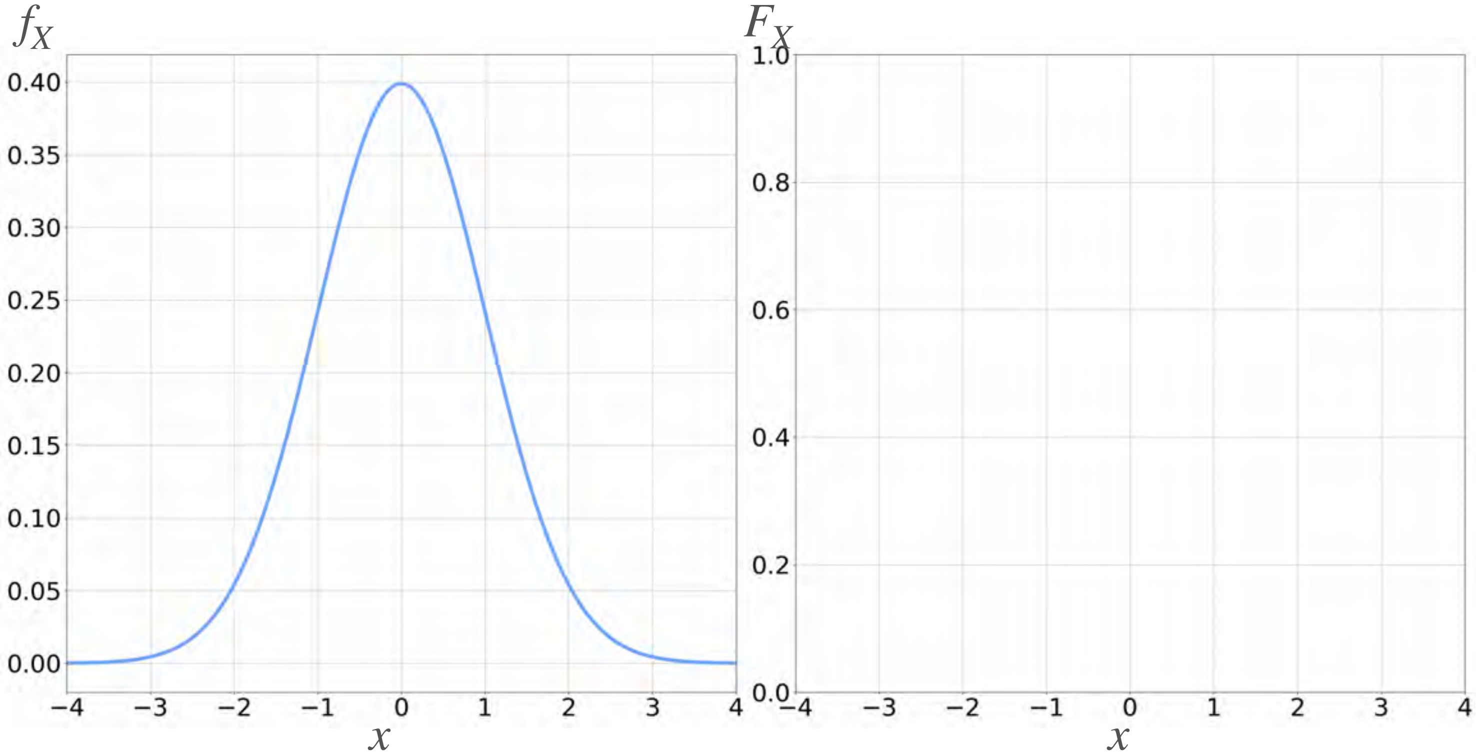
$$\mu = 2,\ \sigma = 2.5$$

$$Z = \frac{X - \mu}{\sigma}$$

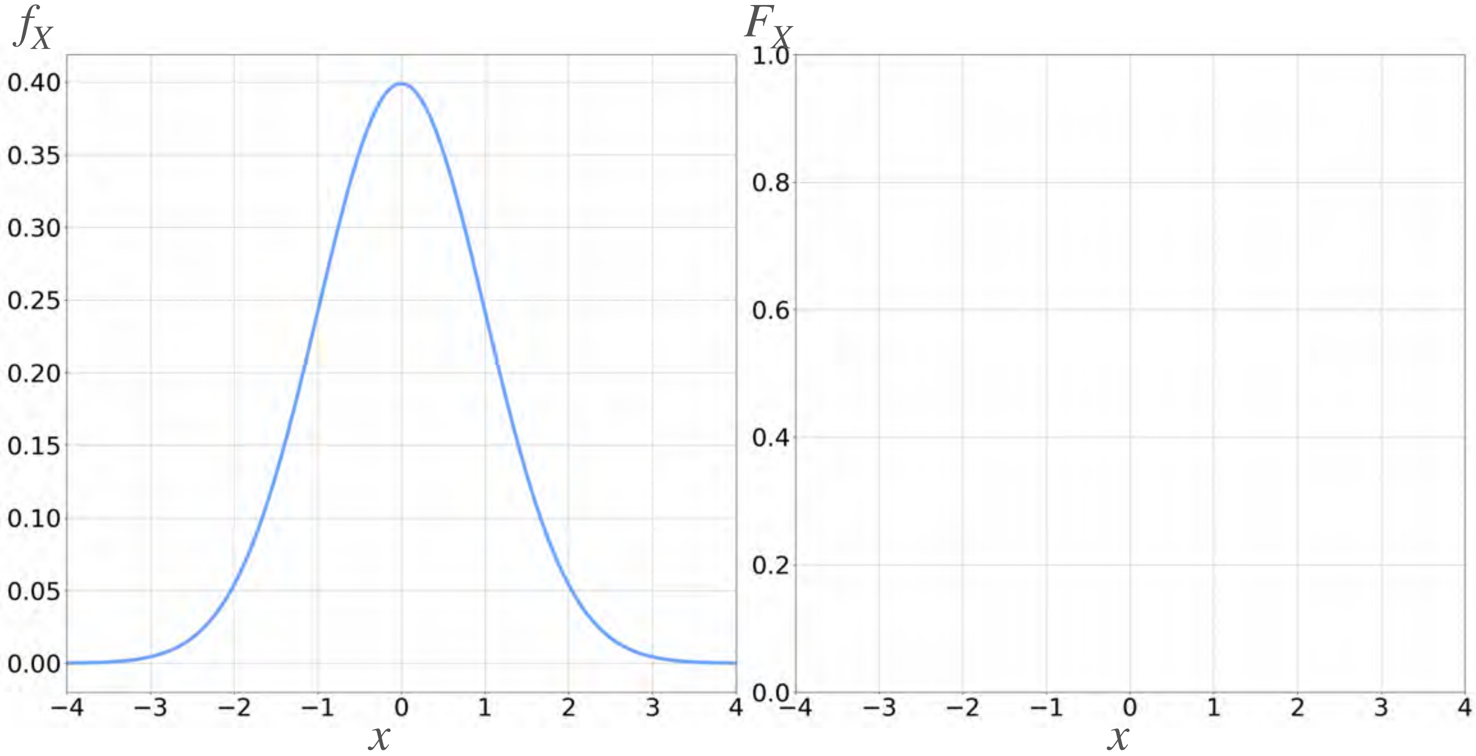
Standardization is crucial to compare variables of different magnitudes!



Normal Distribution: CDF



Normal Distribution: CDF



Normal Distribution: Computing Probabilities

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This math can't be done by hand

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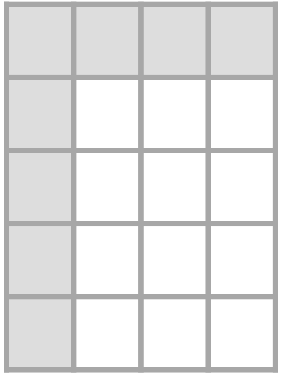

In the old days, people used tables of data

Now, you can use the help of some software to do the approximate area under the curve for you!

Normal Distribution: Applications

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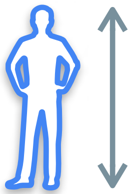
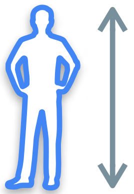


Height

Normal Distribution: Applications



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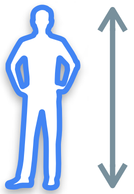
Normal Distribution: Applications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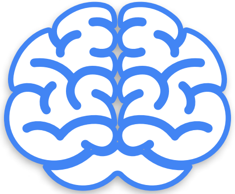
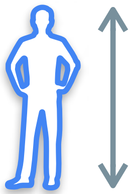
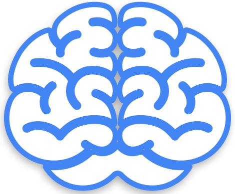

IQ

Normal Distribution: Applications



Height

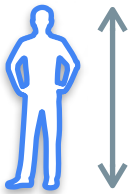
Weight



IQ

Noise in a communication channel

Normal Distribution: Applications



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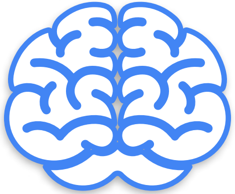
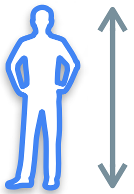


IQ

Noise in a communication channel

In general, characteristics that are the sum of many independent processes

Normal Distribution: Applications



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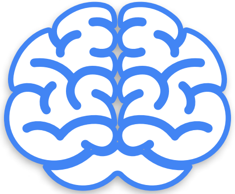


IQ

Noise in a communication channel

In general, characteristics that are the sum of many independent processes

Many models in ML are designed under the assumption that the variables follow a normal distribution

DeepLearning.AI

Probability Distributions

Chi-squared distribution

Chi-Square Distribution: Motivation

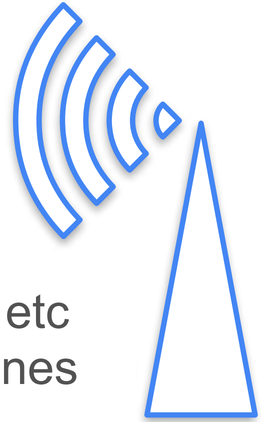
Chi-Square Distribution: Motivation

Message sent: 10010

Chi-Square Distribution: Motivation

Communication channel

Noise
Interference from other devices
Obstructions like walls, trees, etc.
Atmospheric conditions: rain, humidity, etc
Electrical interference, i.e. from power lines
Others



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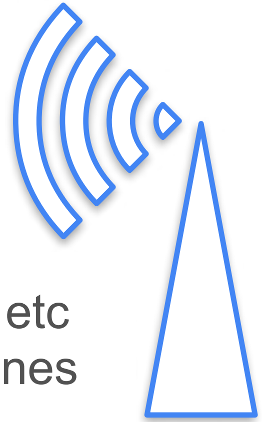
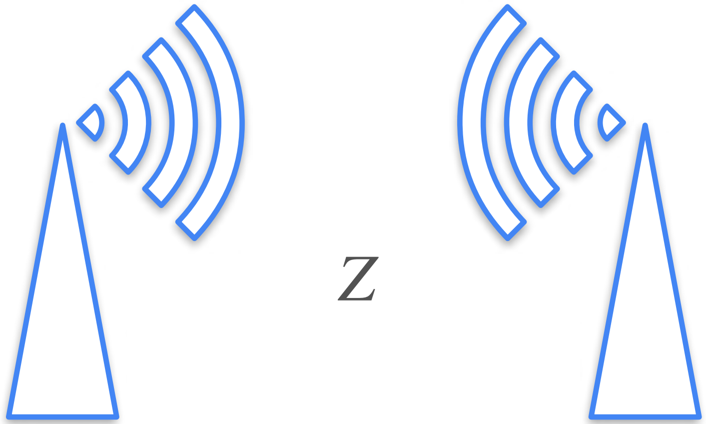


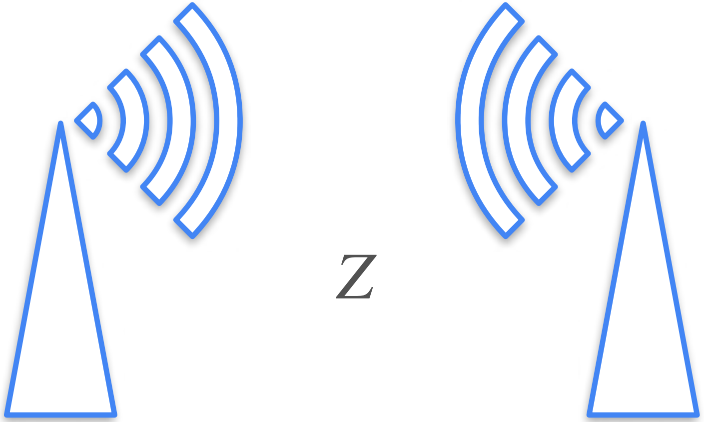
Message sent: 10010

Message received: 10010 $+Z$

Chi-Square Distribution: Motivation

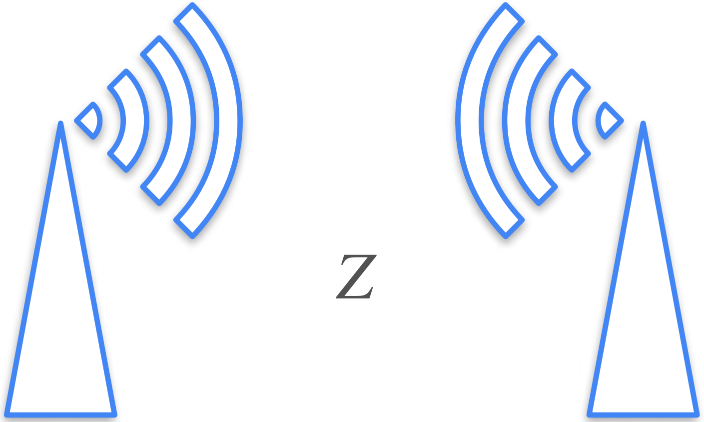


Chi-Square Distribution: Motivation



The communication channel has noise with a standard normal distribution

Chi-Square Distribution: Motivation



What is the **power** of the noise in the channel?

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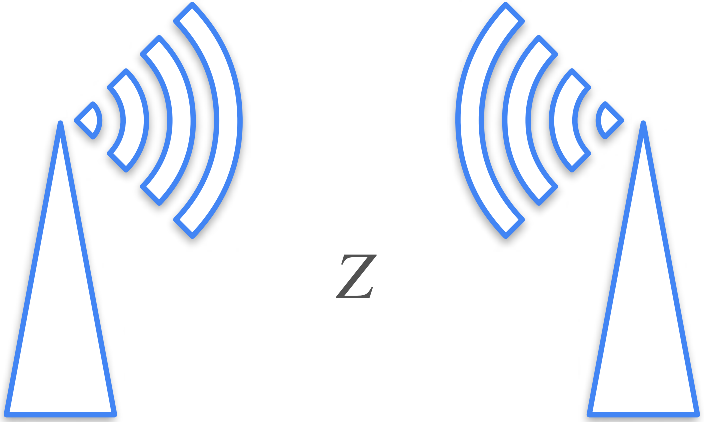

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$$W = Z^2$$

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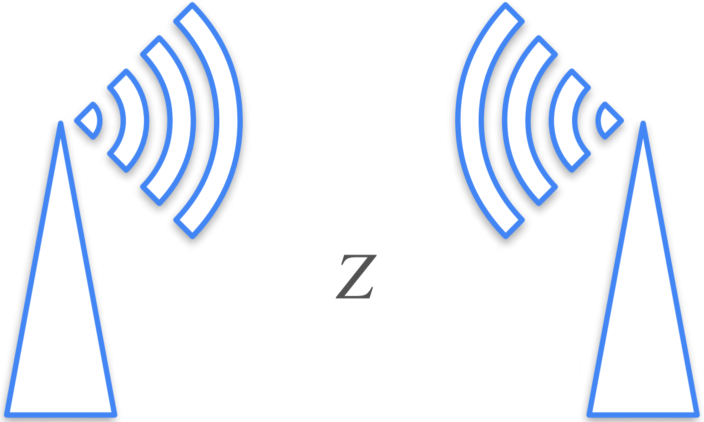


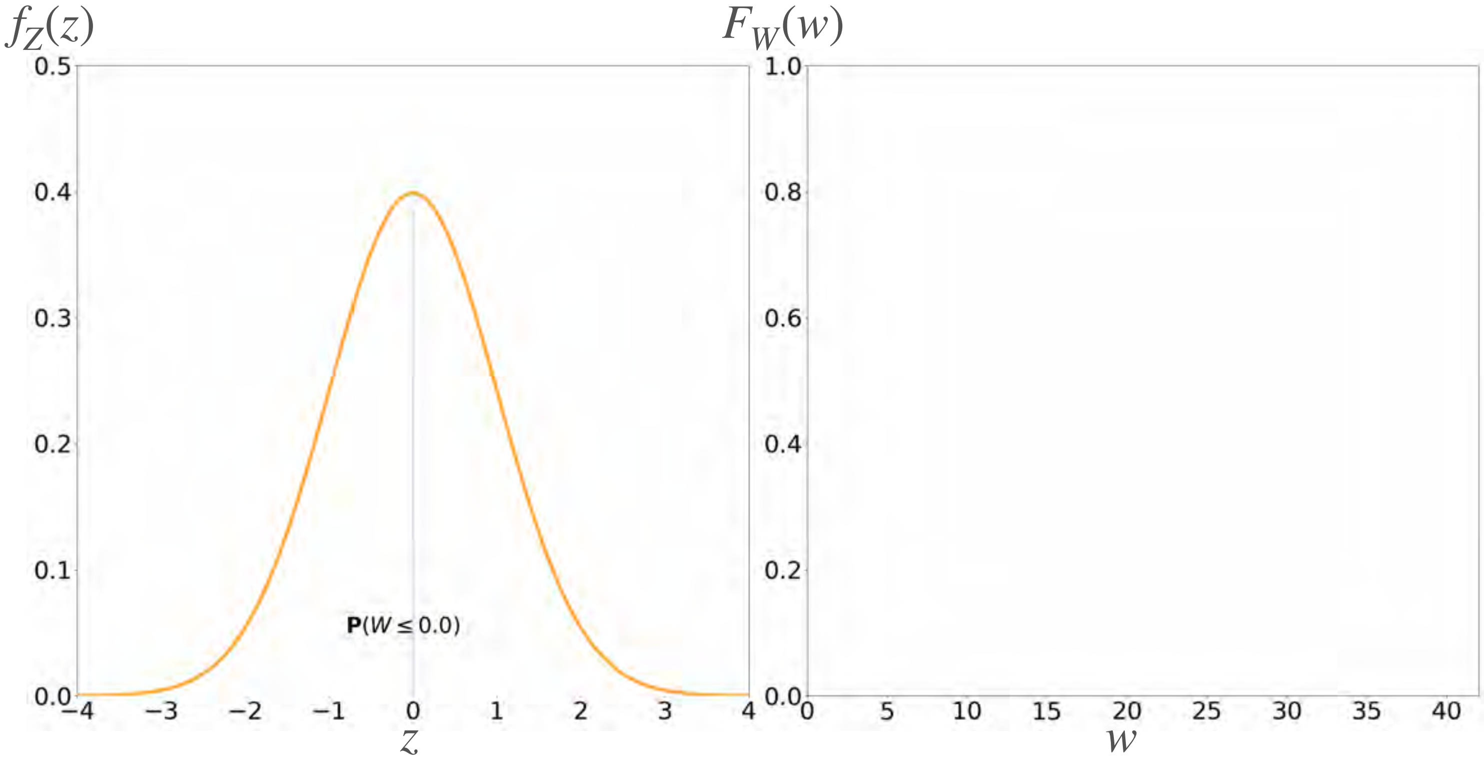
The communication channel has noise with a standard normal distribution

What is the **power** of the noise in the channel?

$$W = Z^2$$

What is the distribution of W?

Chi Square Distribution



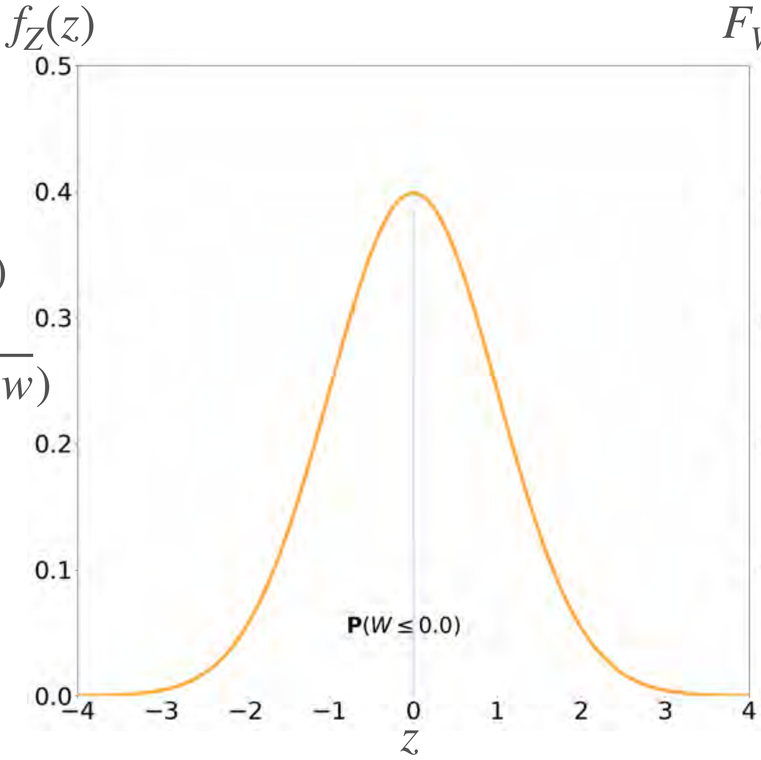
Chi Square Distribution

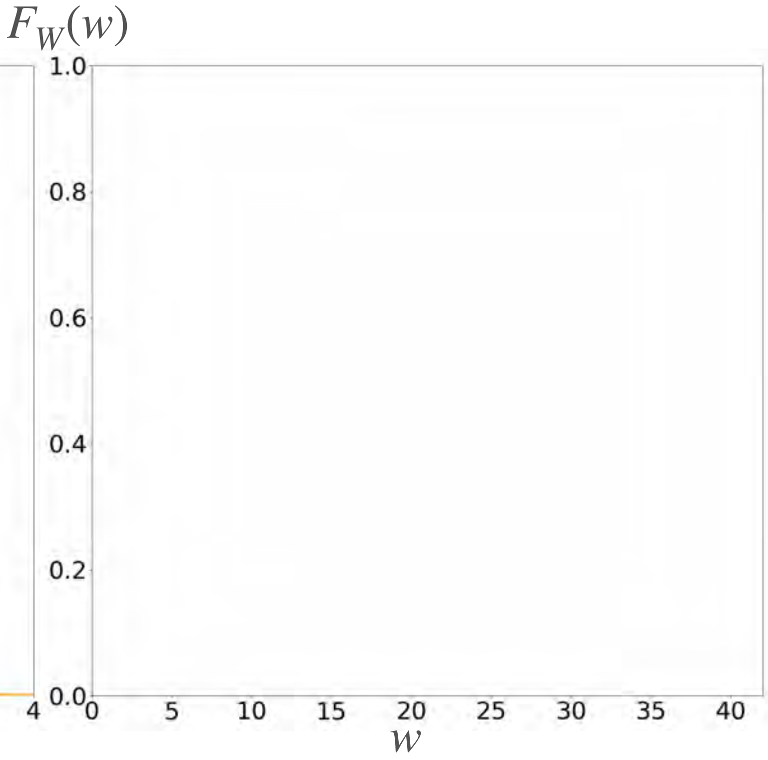
$$F_W(w) = \mathbf{P}(W \leq w)$$

$$= \mathbf{P}(Z^2 \leq w)$$

$$= \mathbf{P}(|Z| \leq \sqrt{w})$$

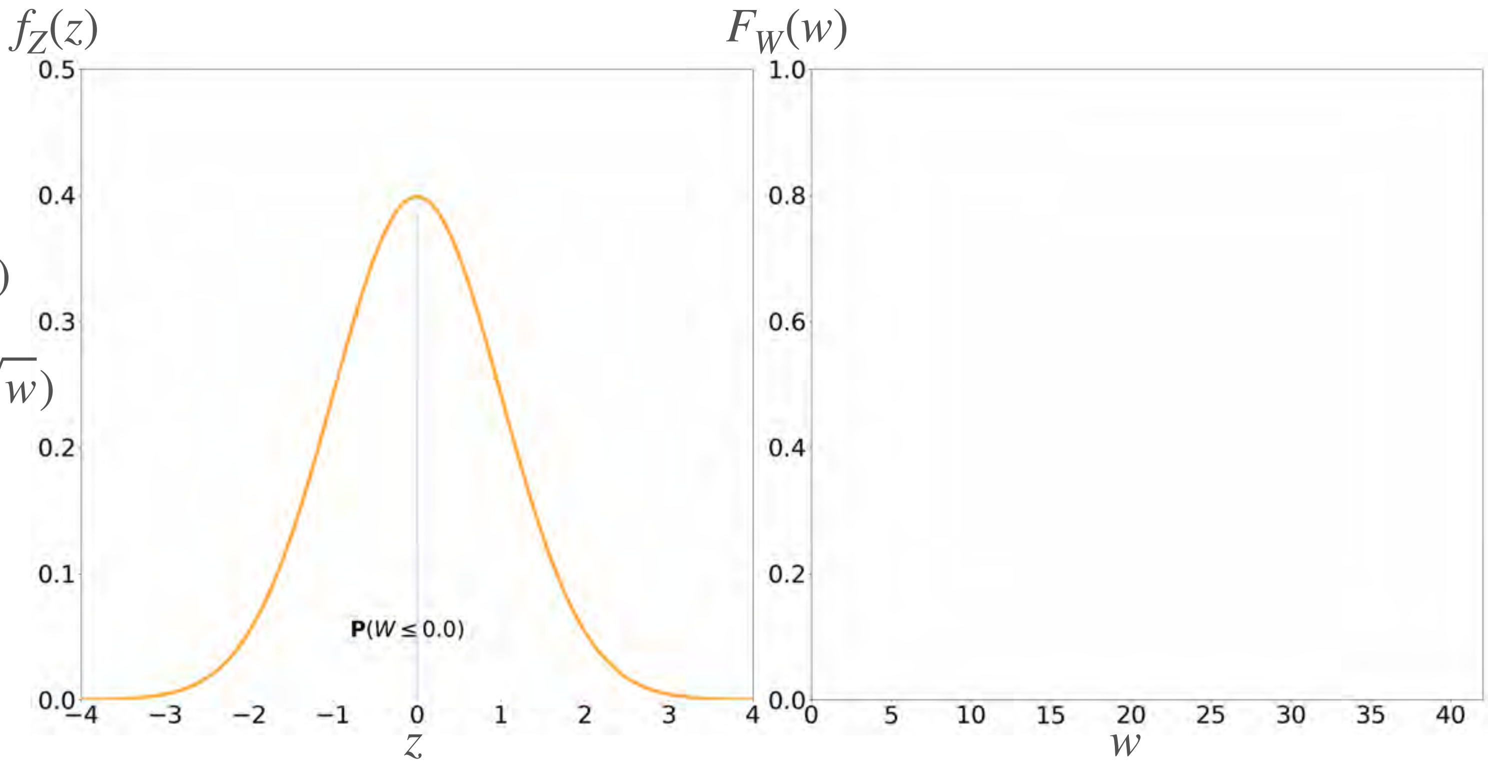
$$= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w})$$





Chi Square Distribution

$$F_W(w) = \mathbf{P}(W \leq w)$$
$$= \mathbf{P}(Z^2 \leq w)$$
$$= \mathbf{P}(|Z| \leq \sqrt{w})$$
$$= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w})$$



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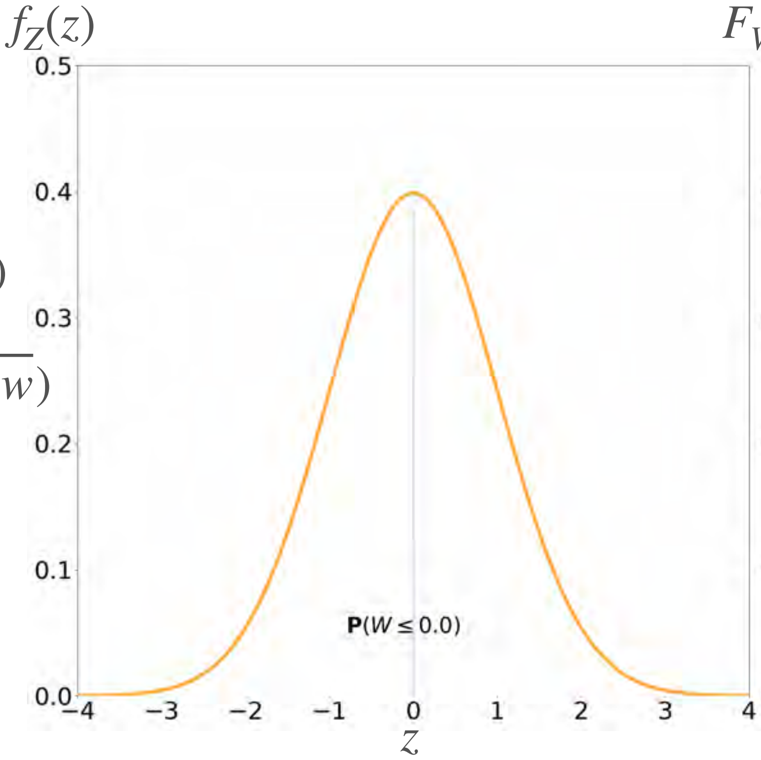
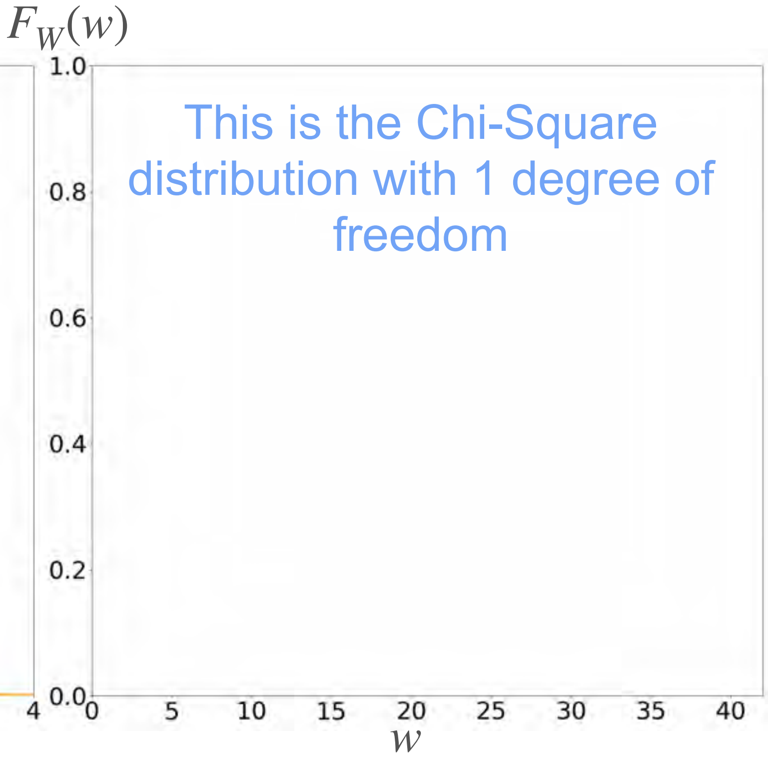
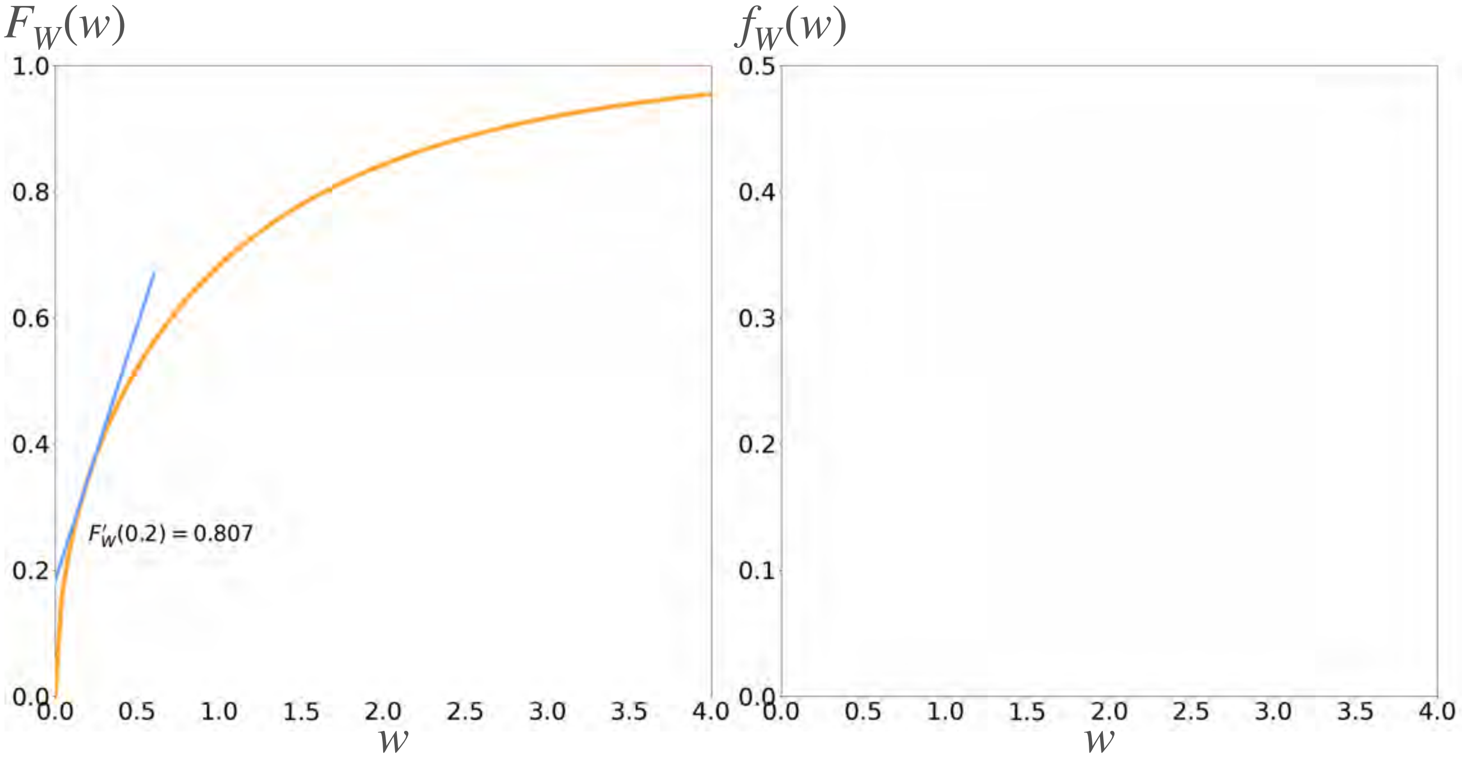


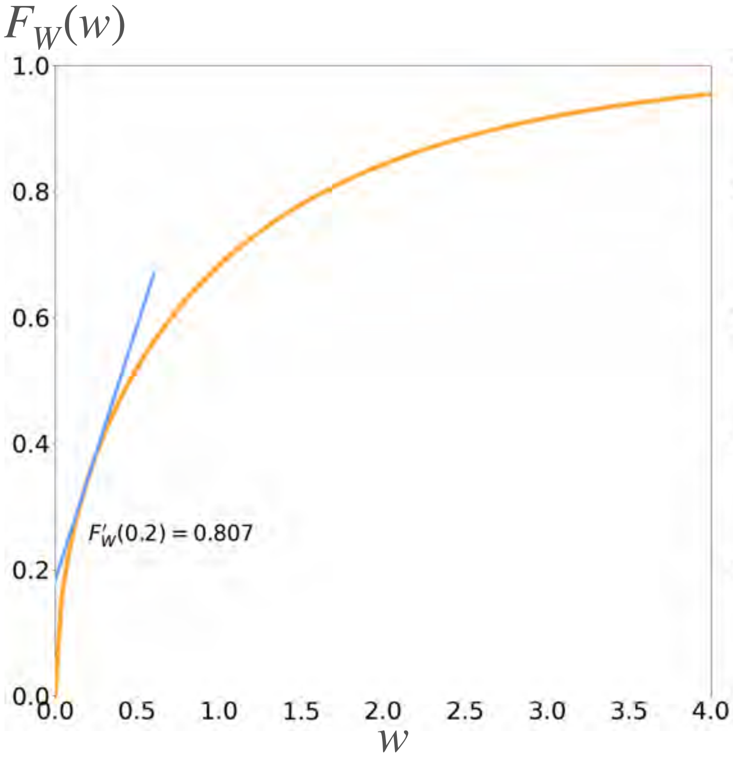


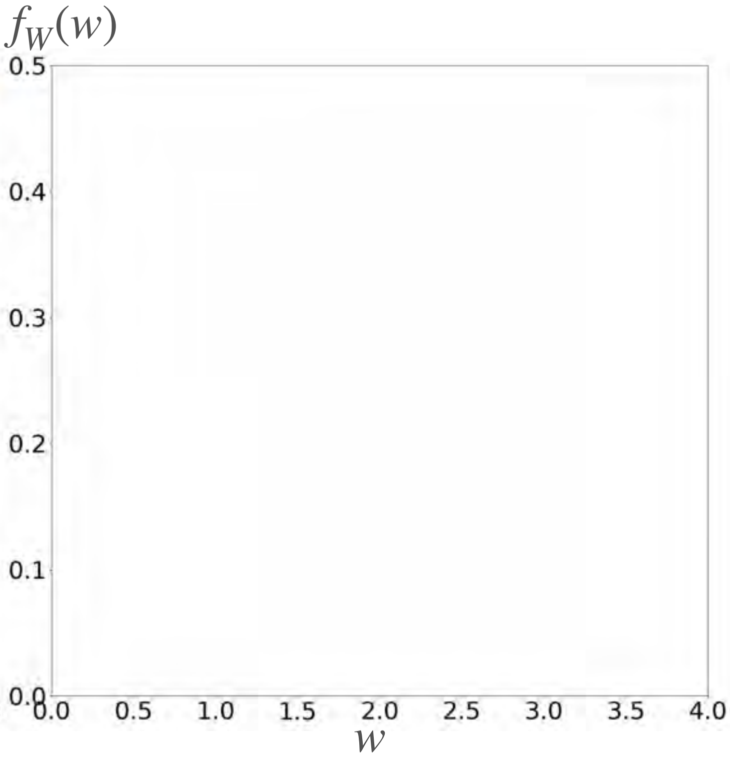
Chi Square Distribution



Chi Square Distribution

$$f_W(w) = F'_W(w)$$





Chi Square Distribution

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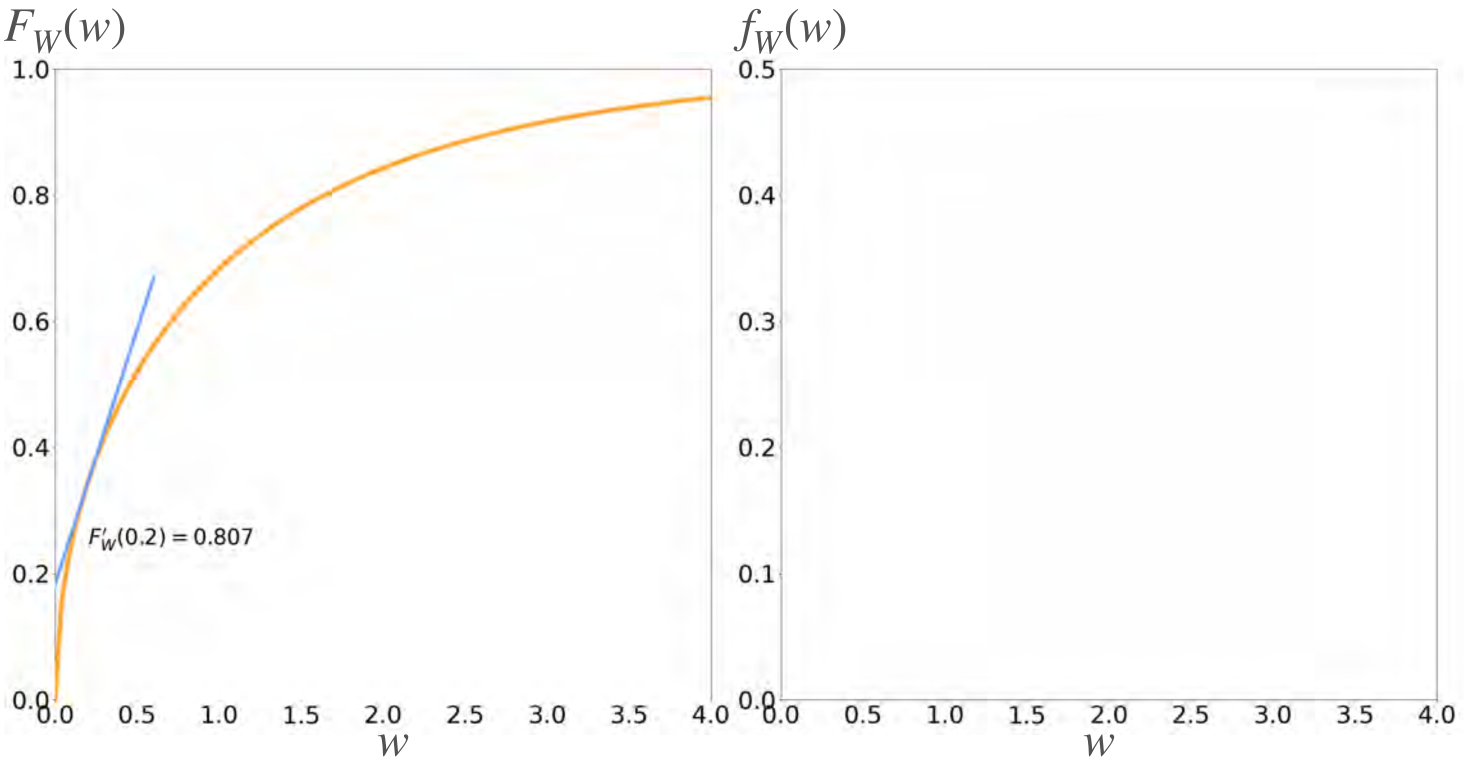


Chi-Square Distribution

Chi-Square Distribution

Accumulated power over 2 transmissions?

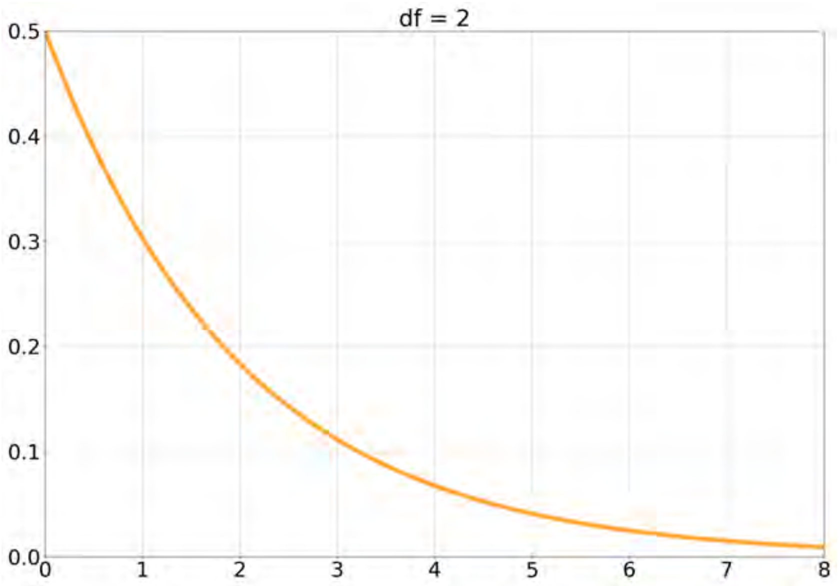
$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df



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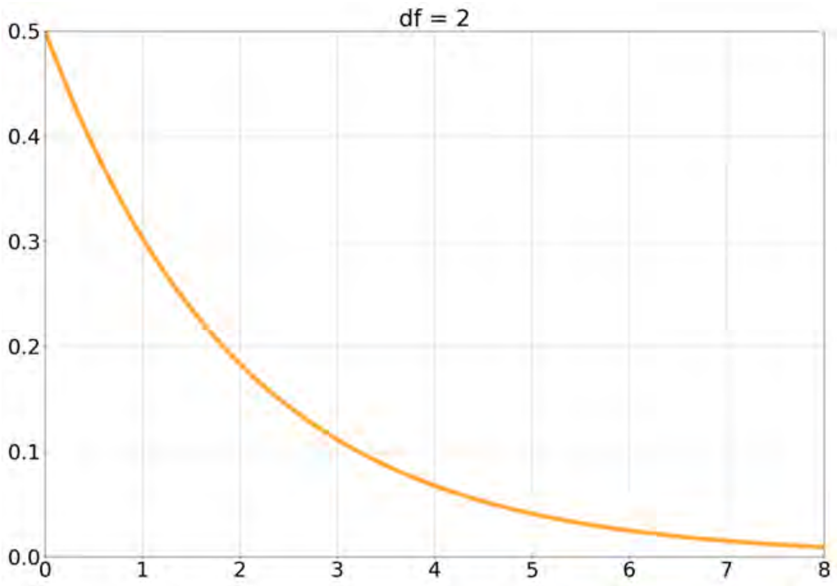
Chi-Square with 2 df



Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square Distribution

Accumulated power over 2 transmissions?

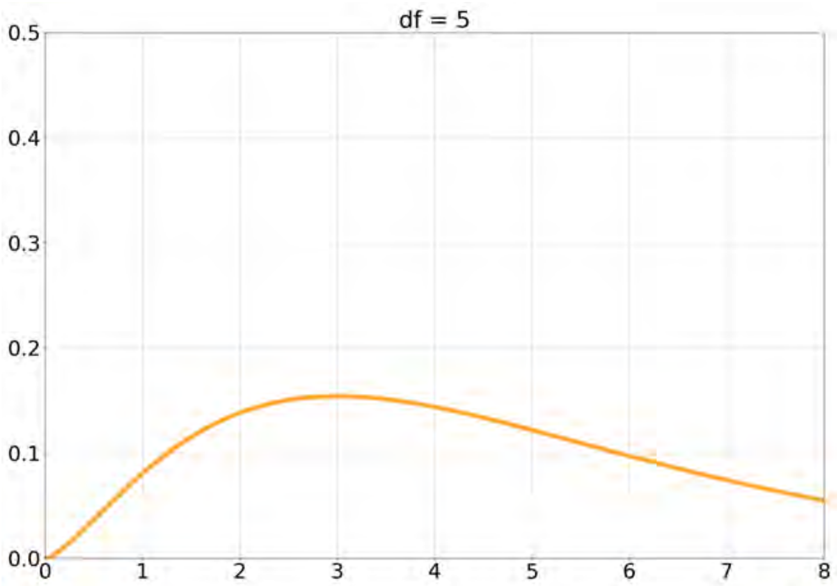
$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square with 5 df



Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

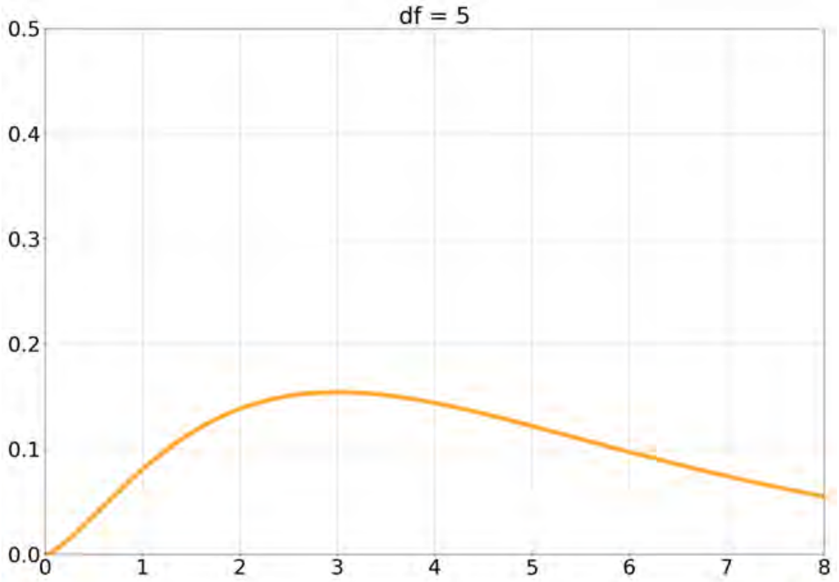
Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square with 5 df

Accumulated power over k transmissions?

$$W_k = \sum_{i=1}^{k} Z_i^2$$



Chi-Square Distribution

Accumulated power over 2 transmissions?

$$W_2 = Z_1^2 + Z_2^2$$

Chi-Square with 2 df

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2$$

Chi-Square with 5 df

Accumulated power over k transmissions?

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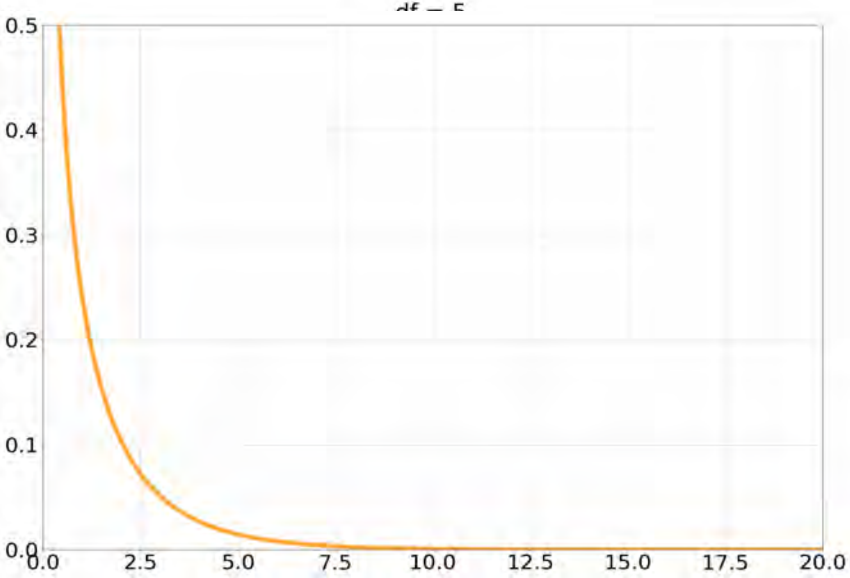
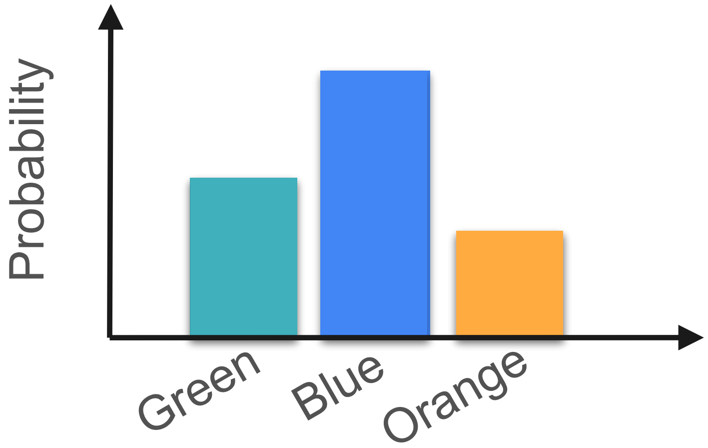


DeepLearning.AI

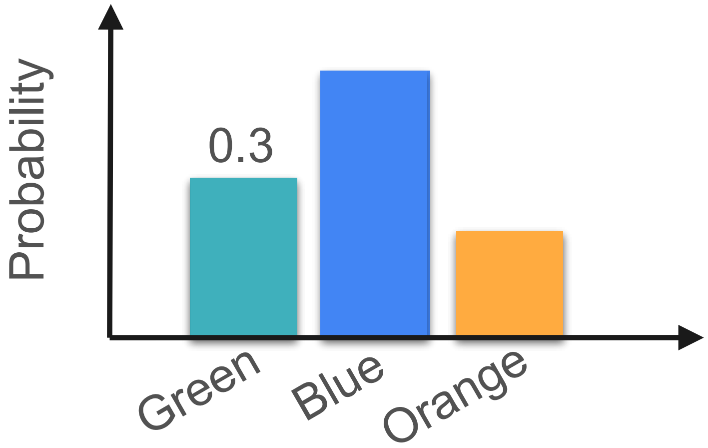
Probability Distributions

Sampling from a Distribution

Sampling From a Distribution



Sampling From a Distribution



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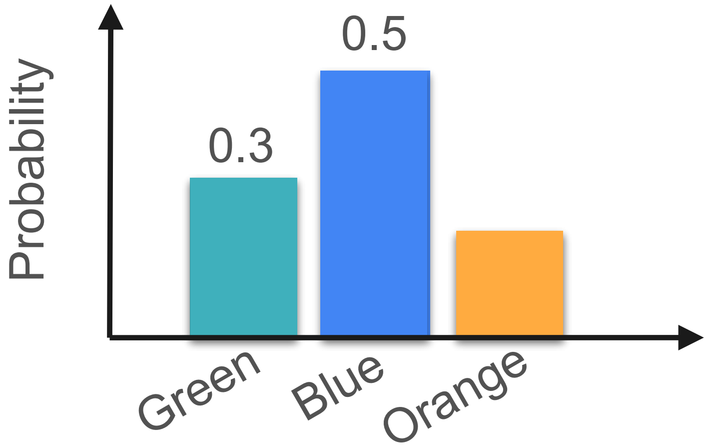

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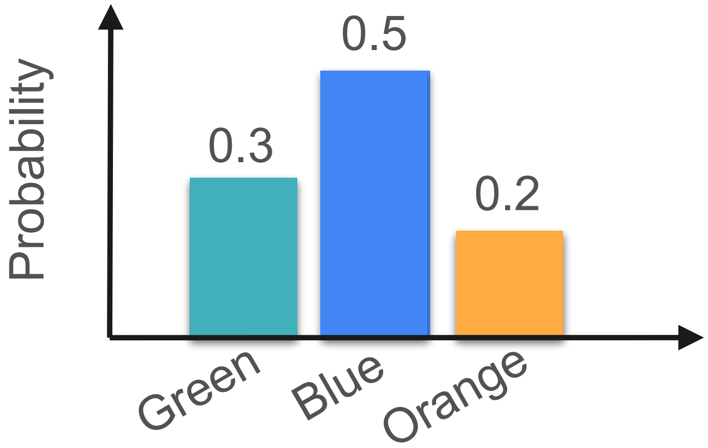

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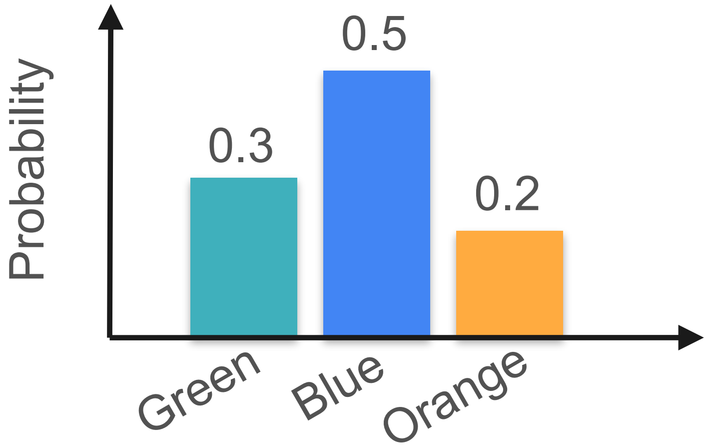

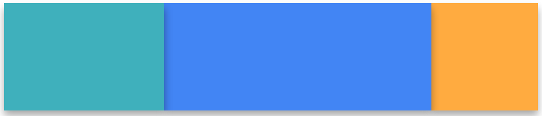

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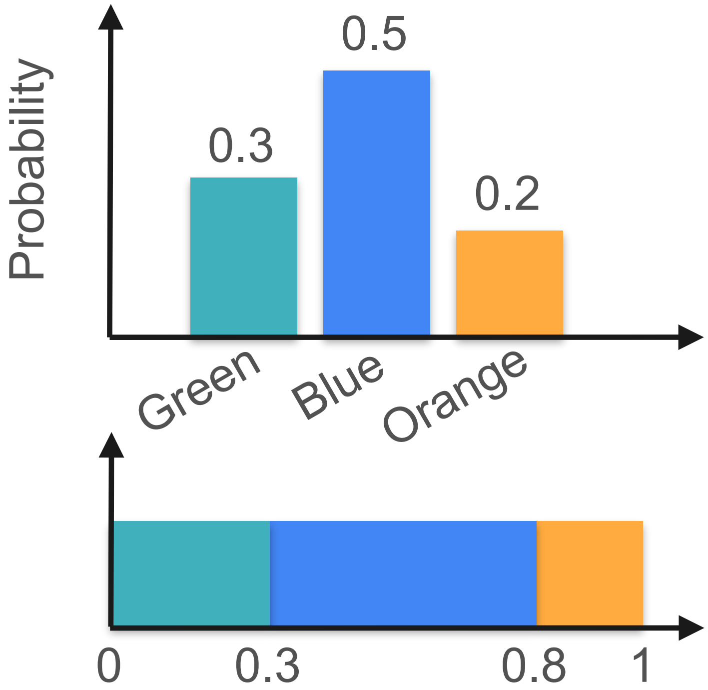
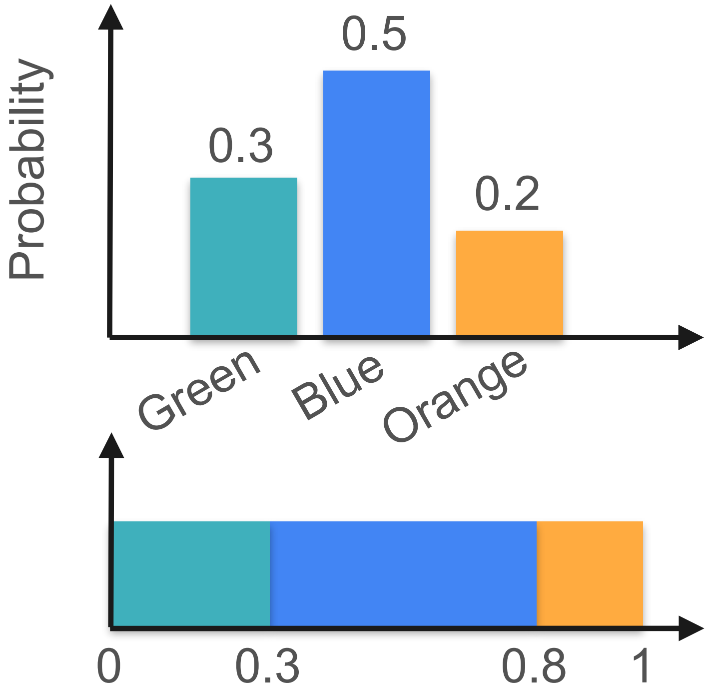
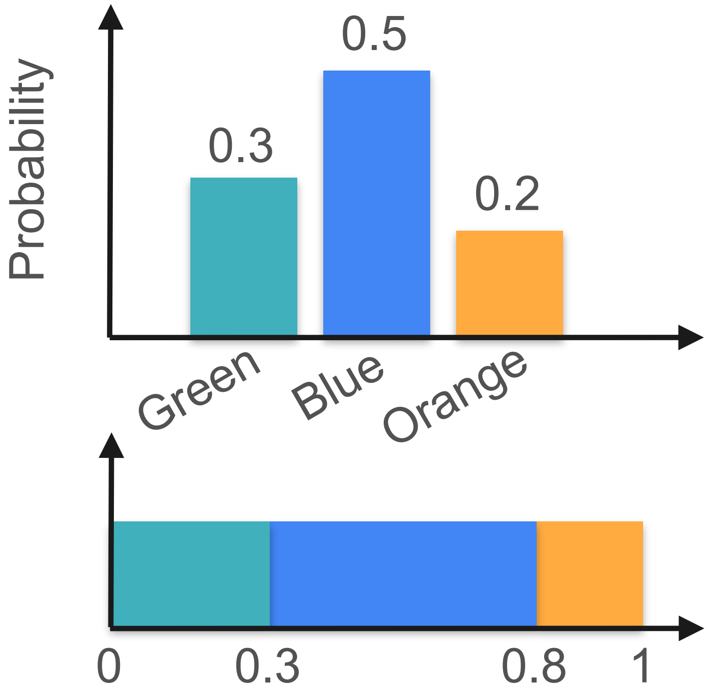


Sampling From a Distribution



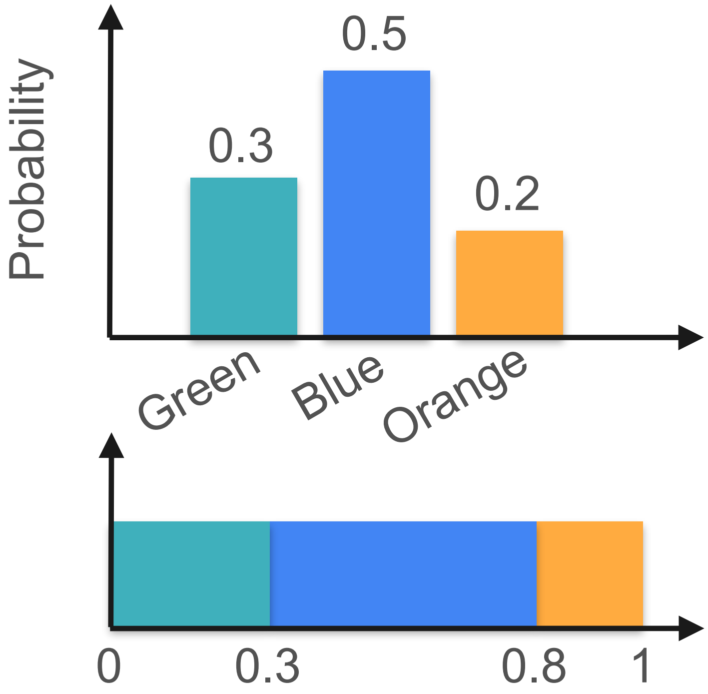
- **Step 1**: generate a random number between 0 and 1

Sampling From a Distribution



- **Step 1**: generate a random number between 0 and 1
- **Step 2**: find out which interval the number belongs to
  - [0, 0.3)
  - [0.3, 0.8)
  - [0.8, 1]

Sampling From a Distribution



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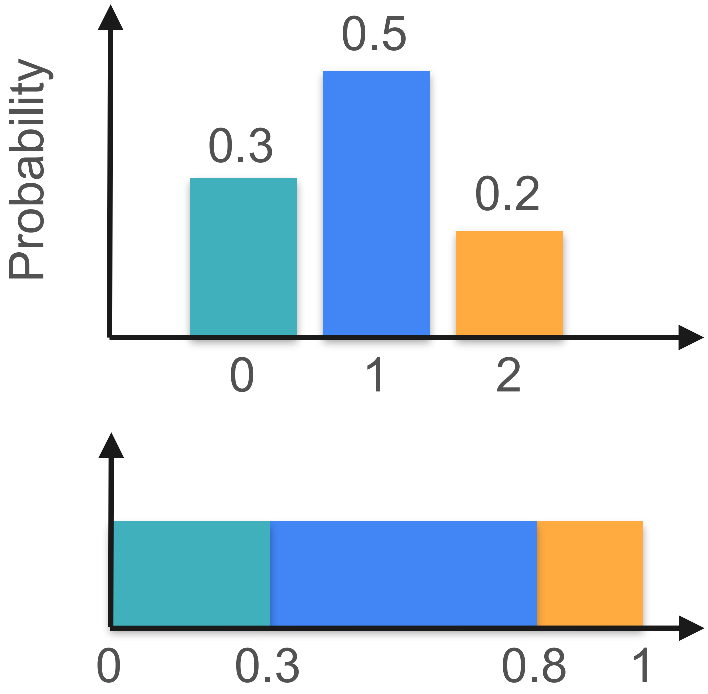
Sampling From a Distribution



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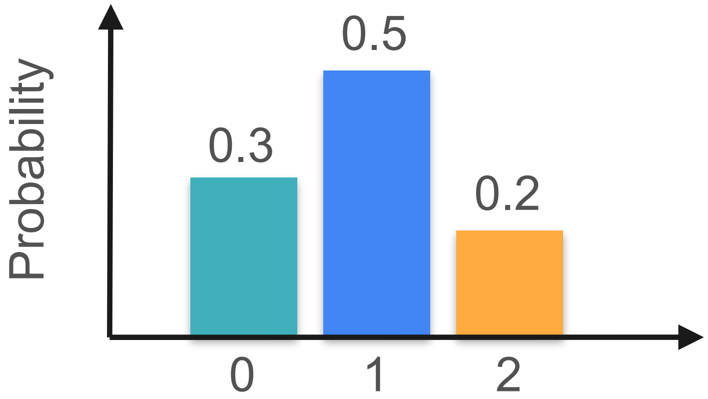

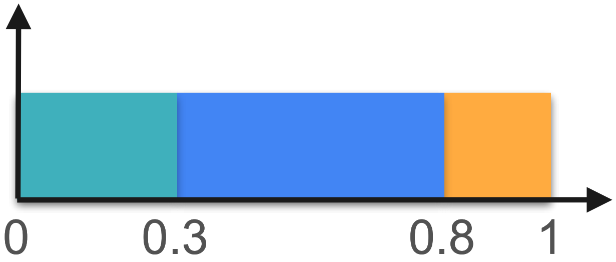

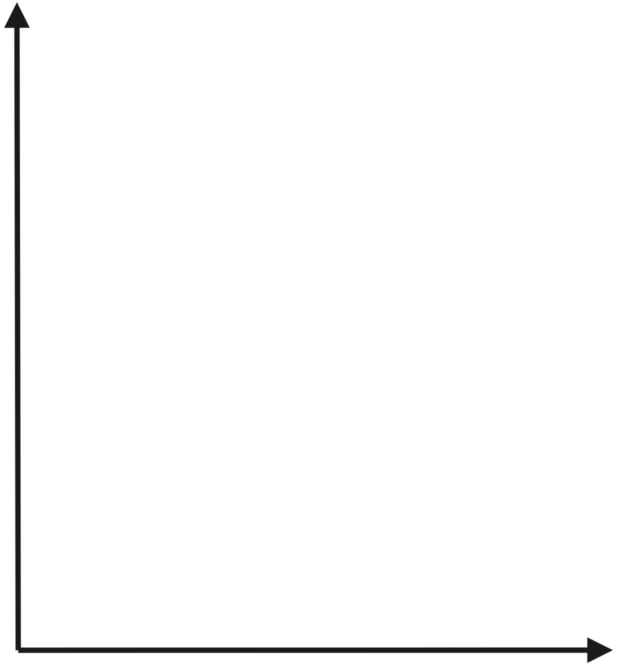

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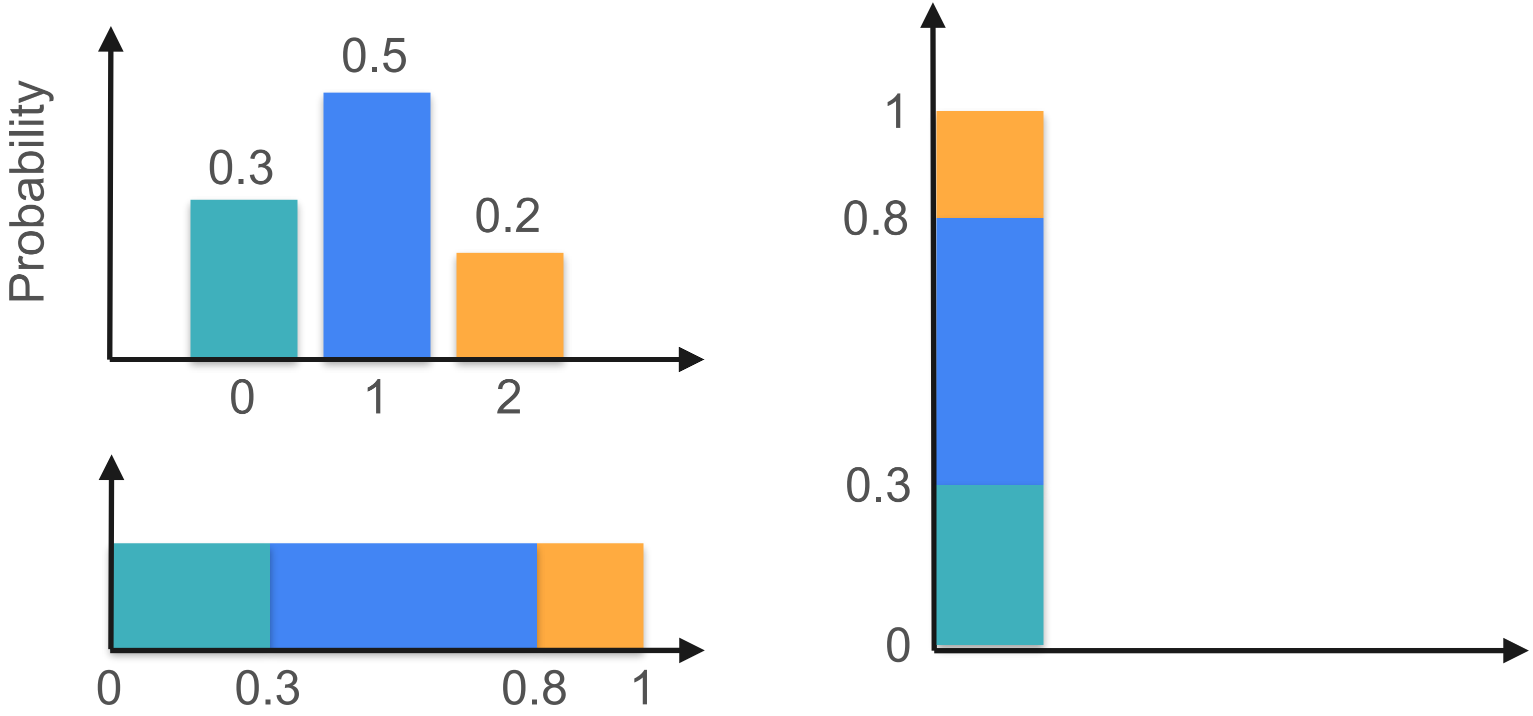

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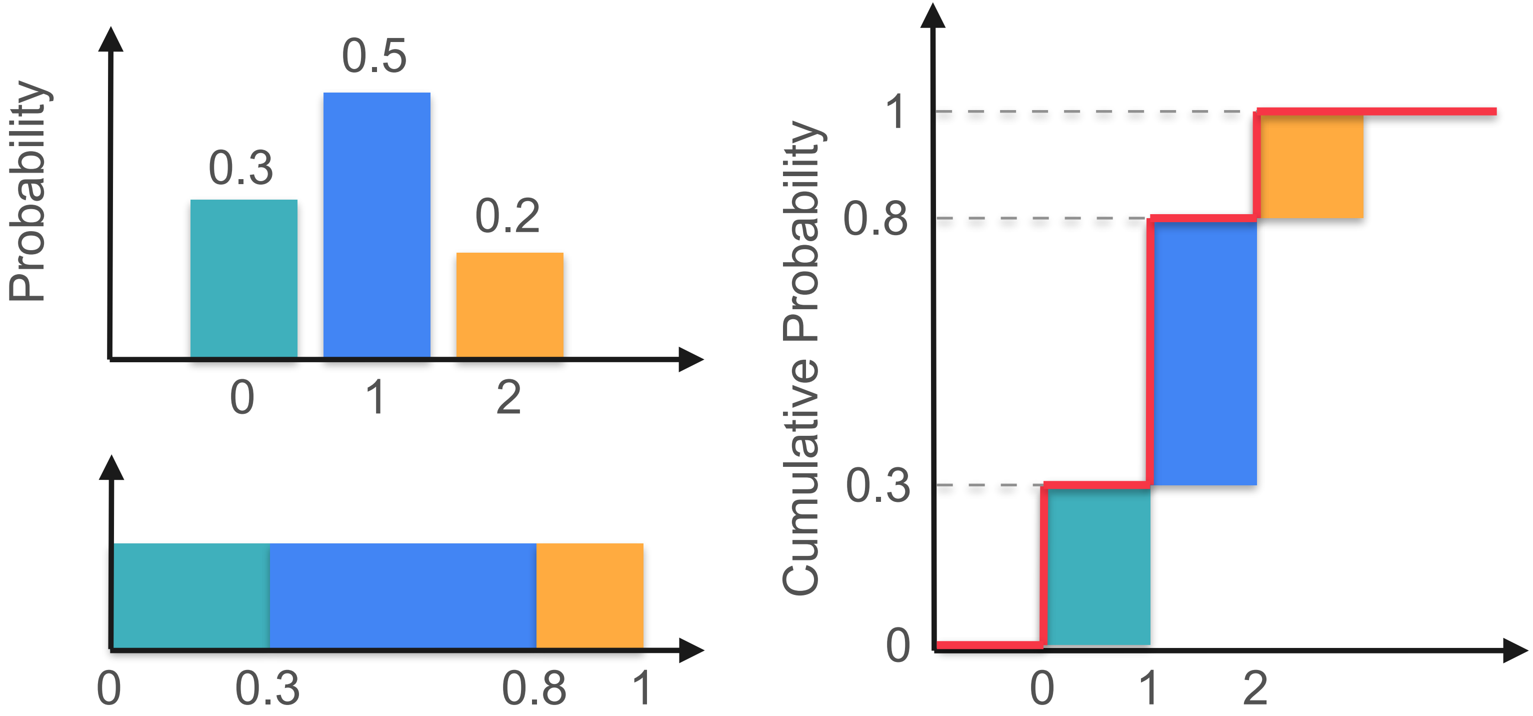

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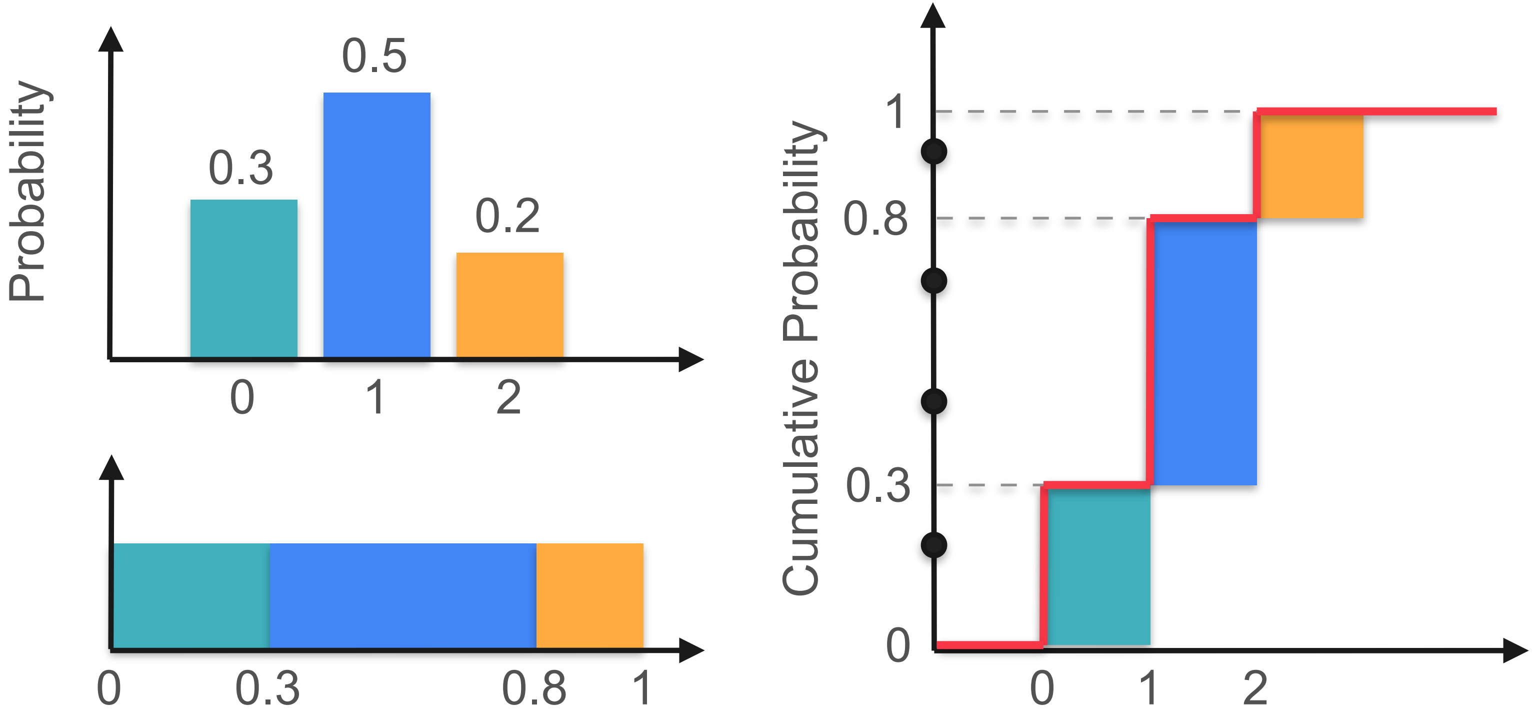

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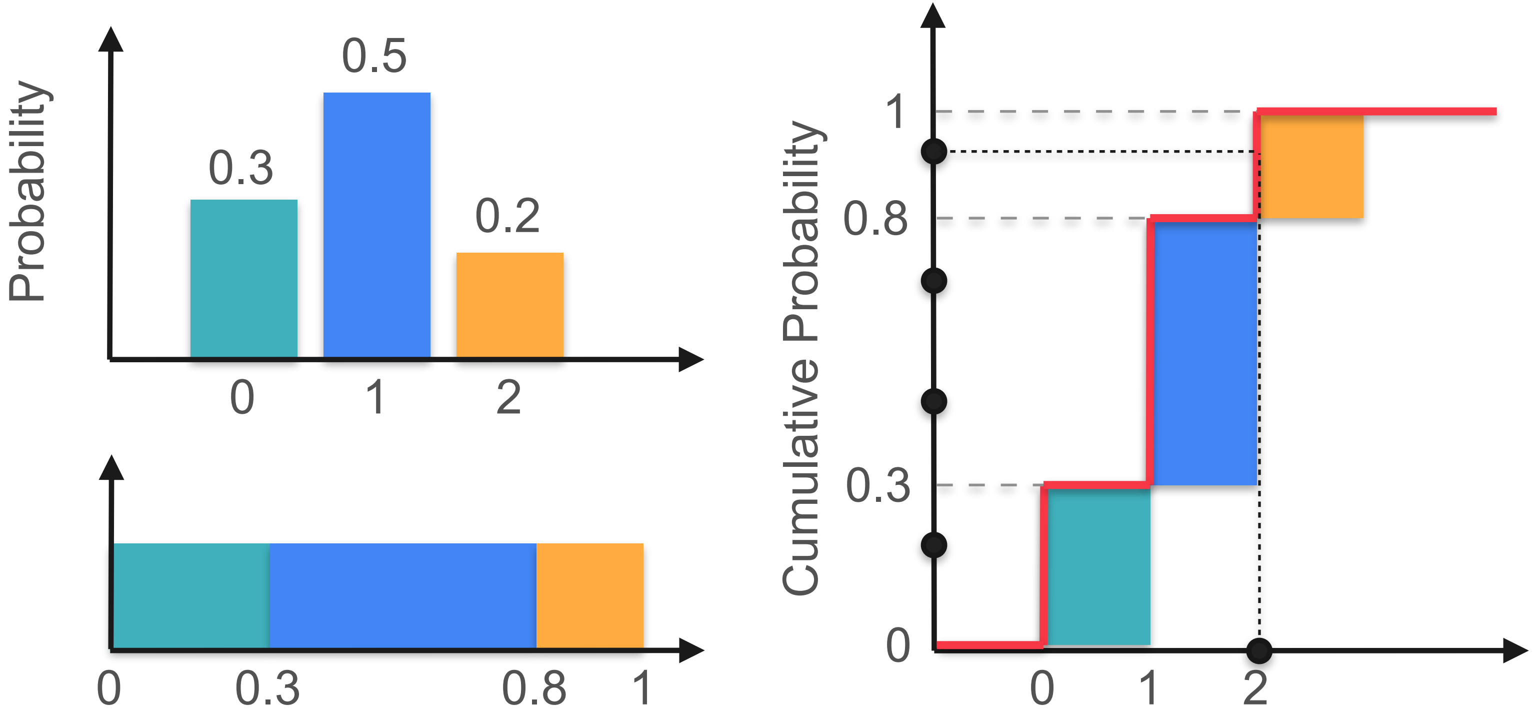

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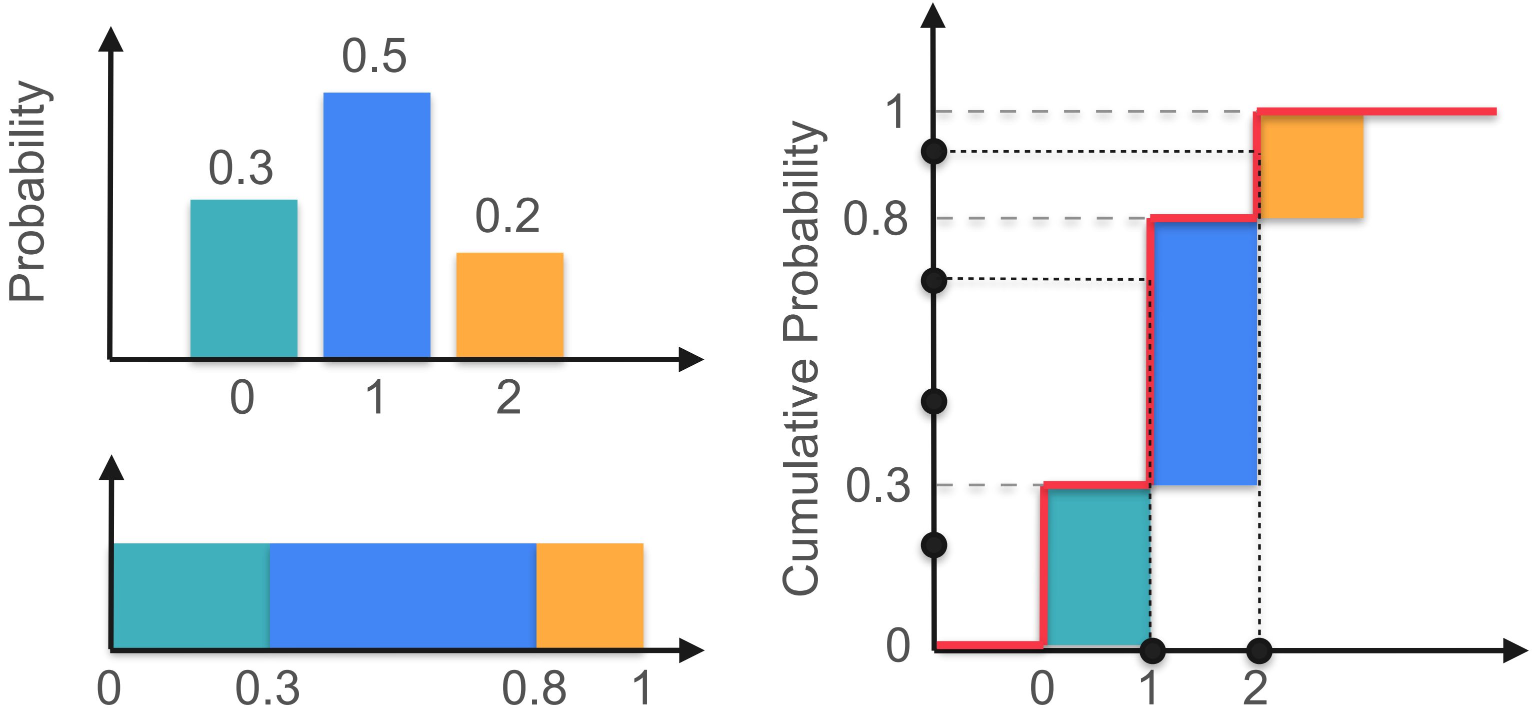

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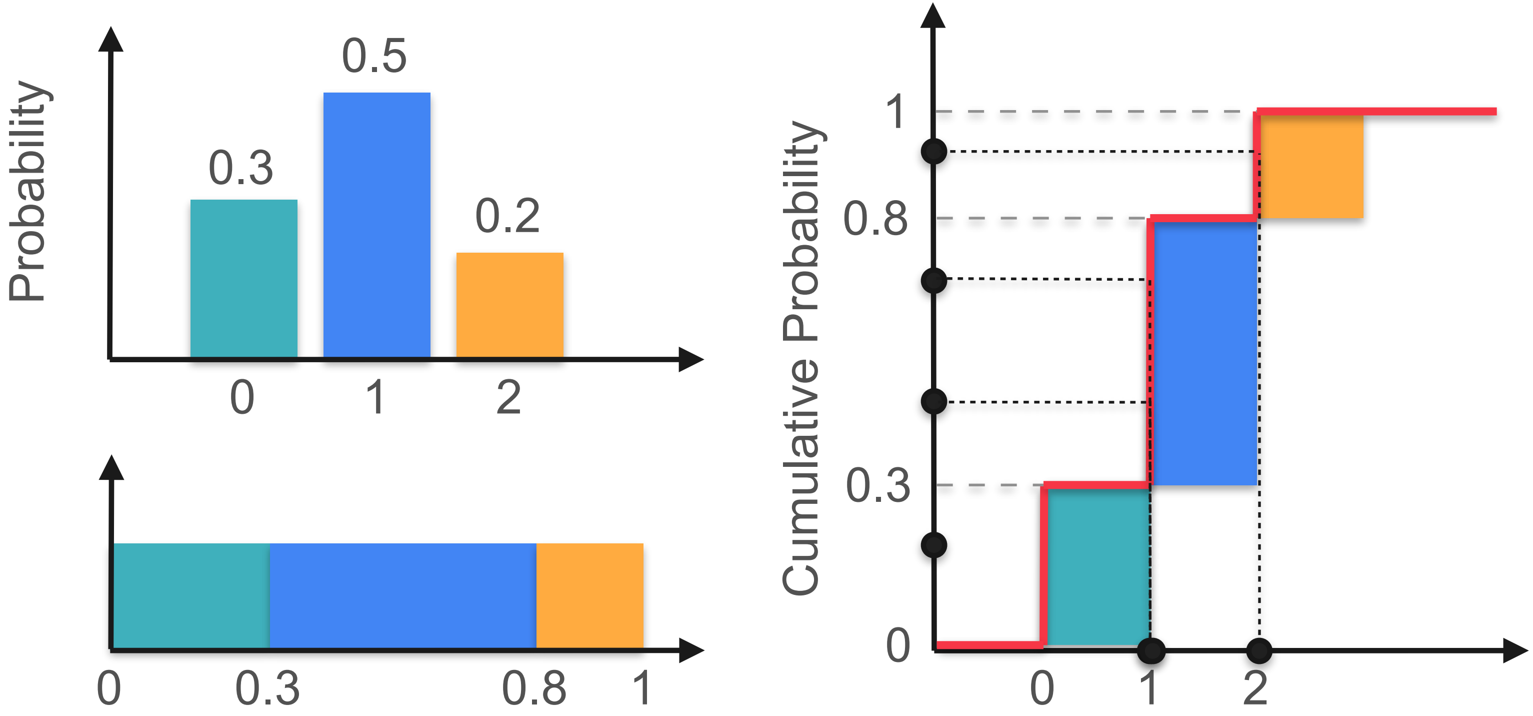

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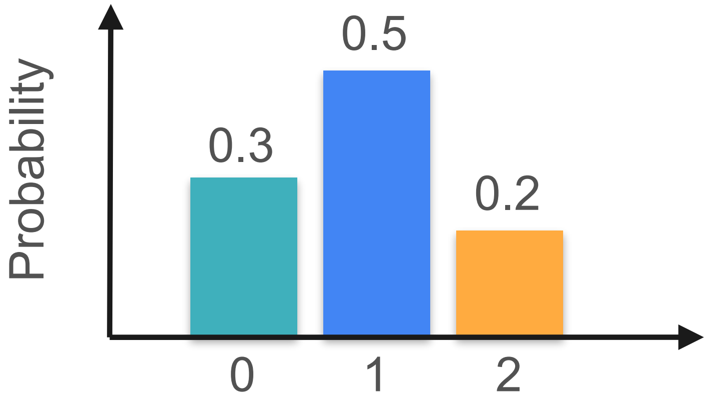
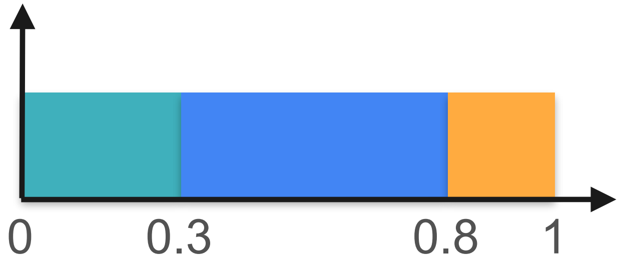
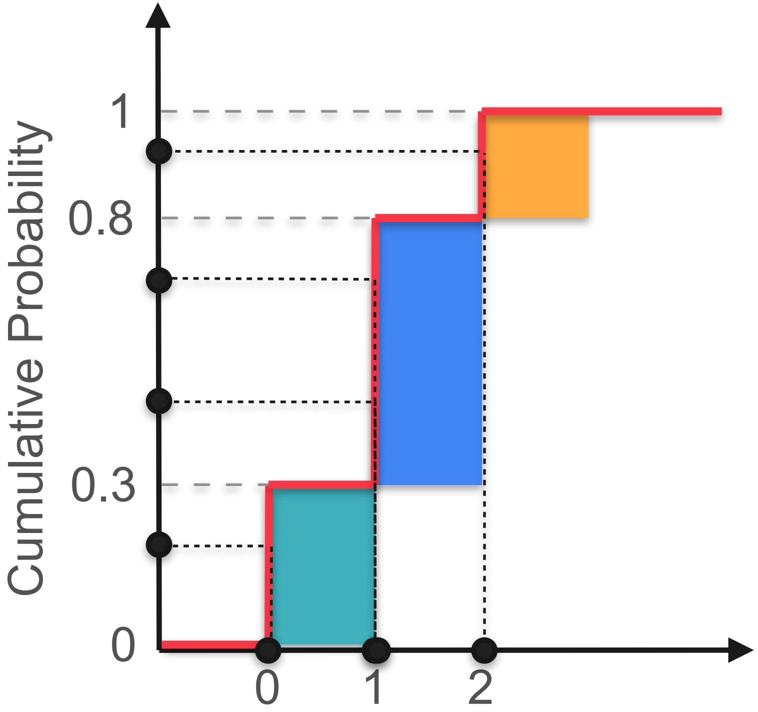
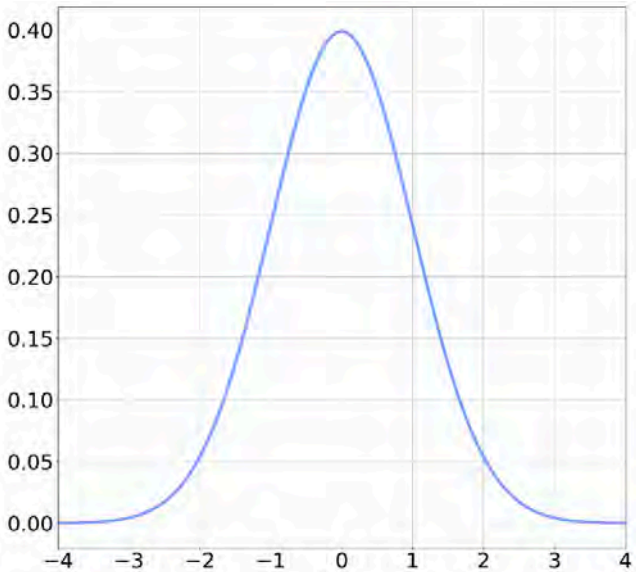






Sampling From a Normal Distribution



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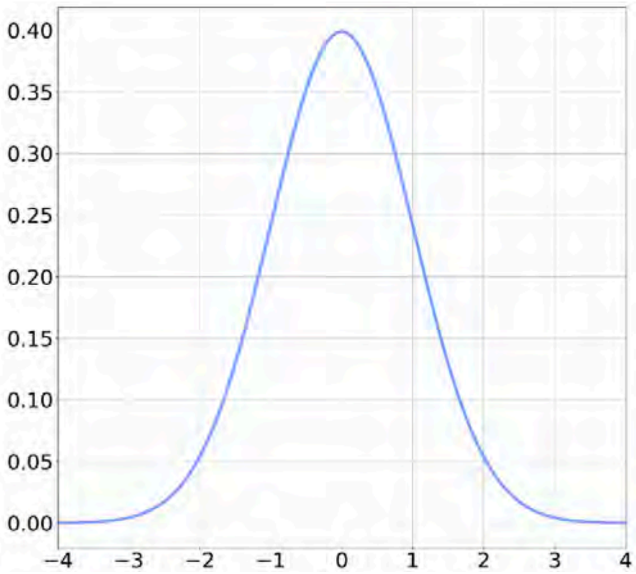

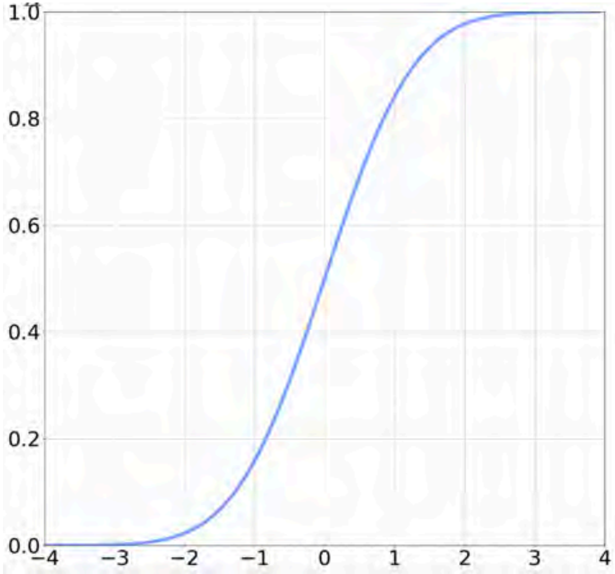

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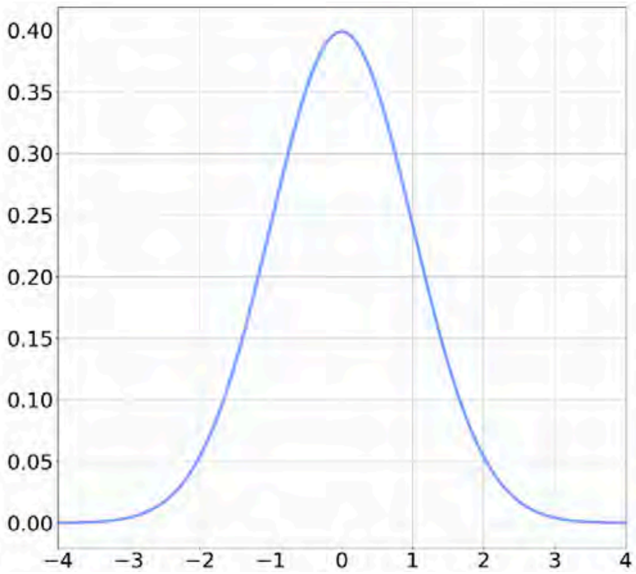

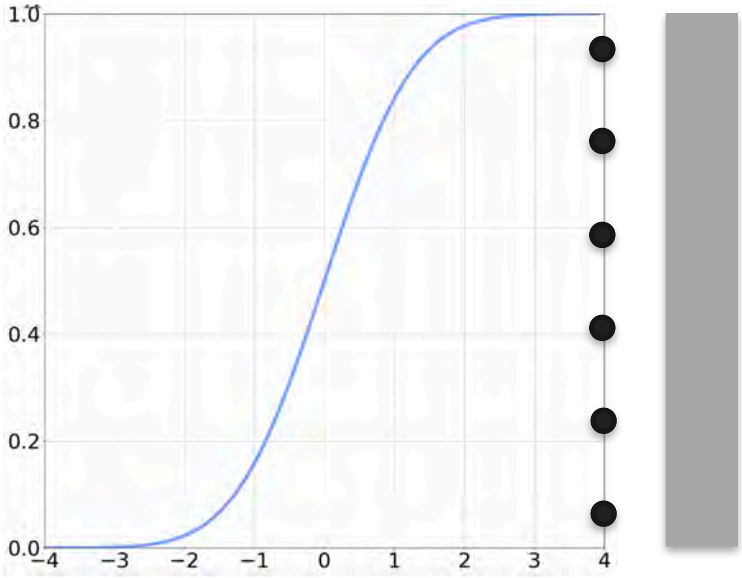

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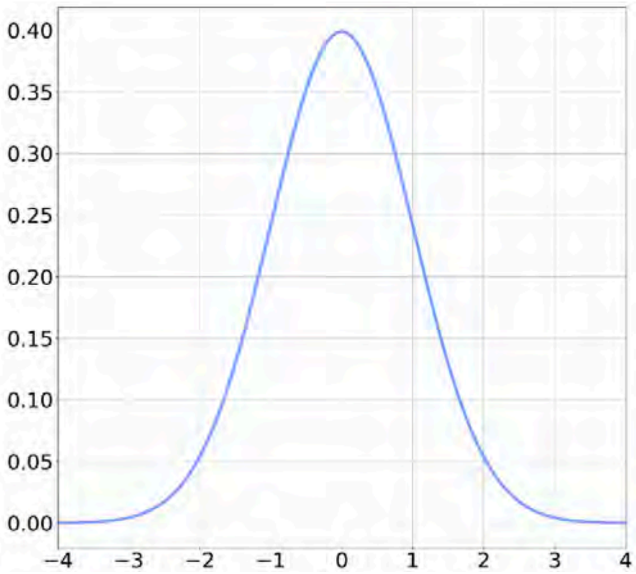

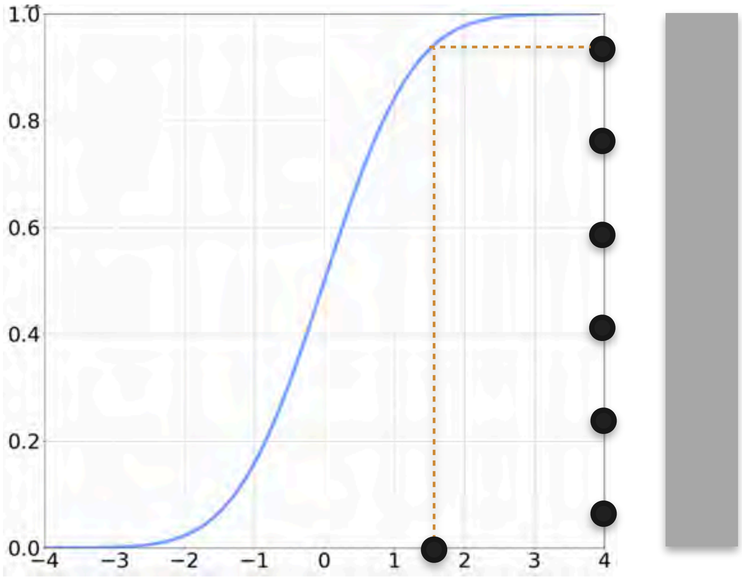

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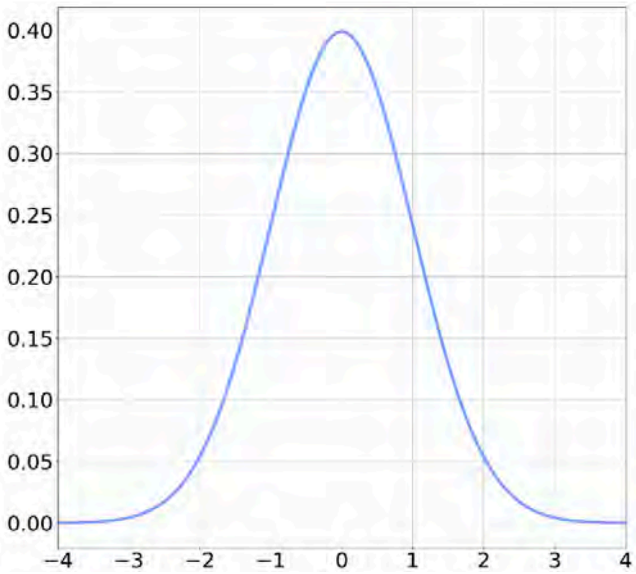

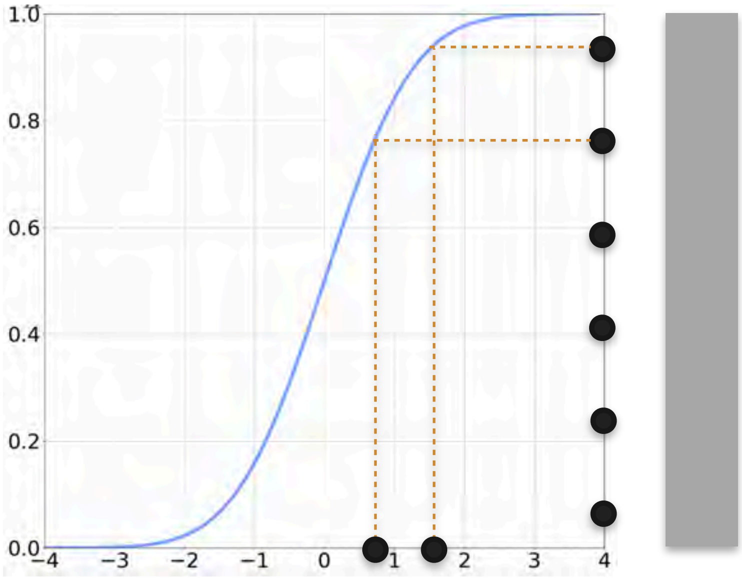

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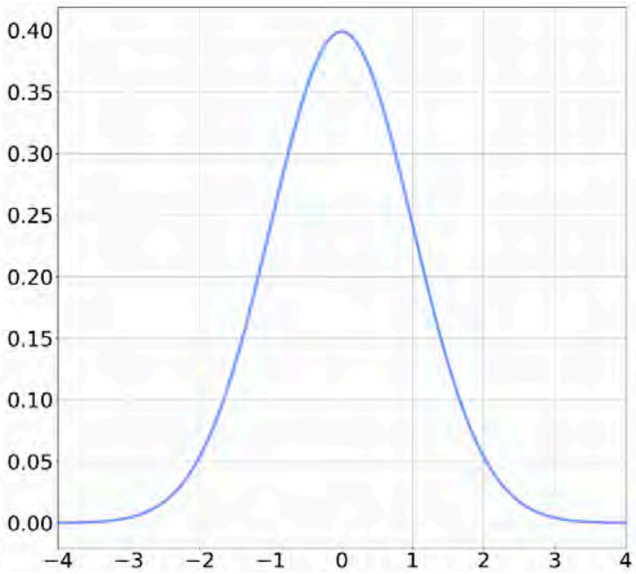

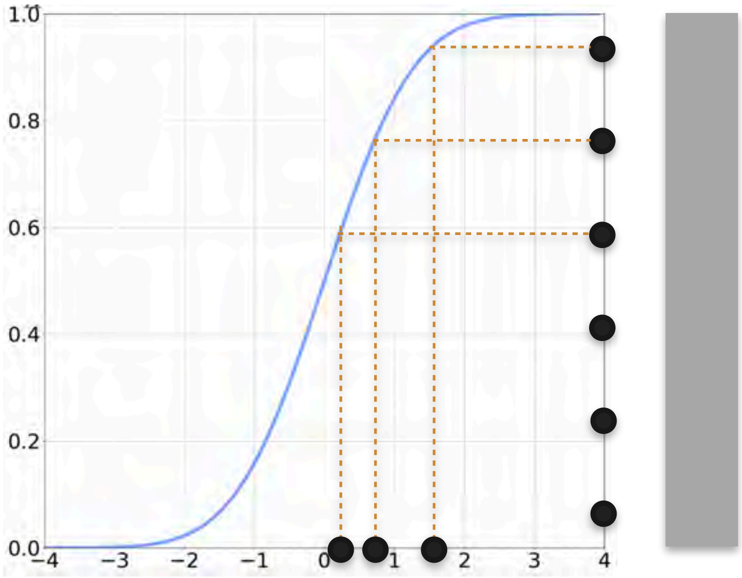

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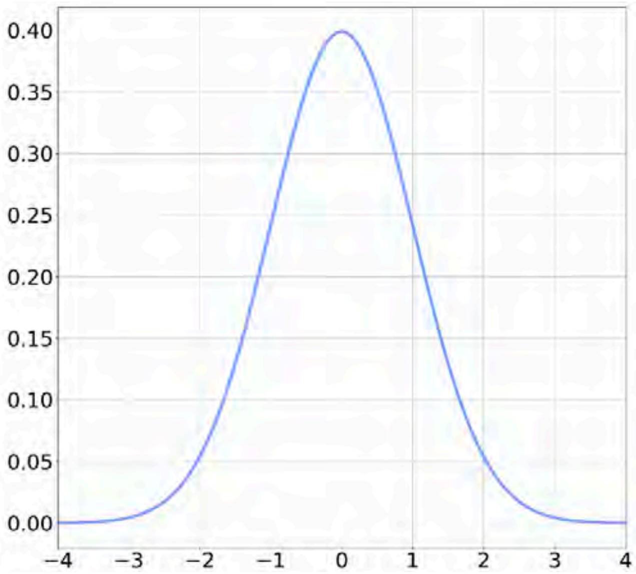

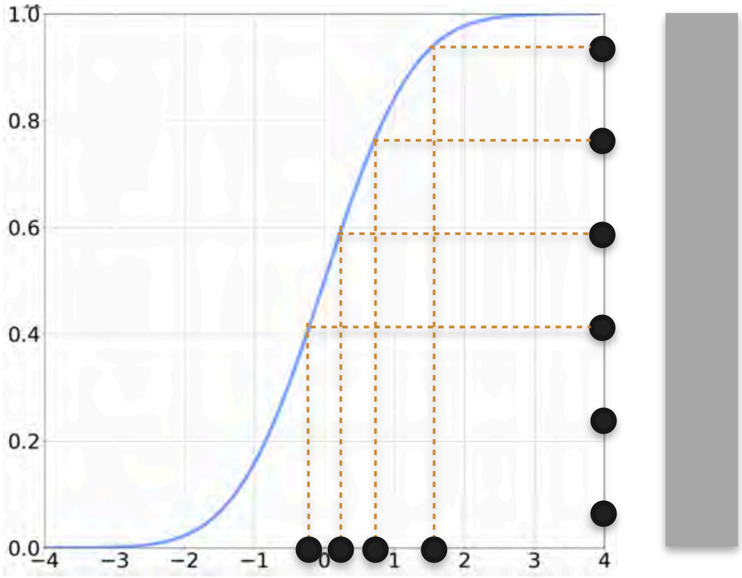

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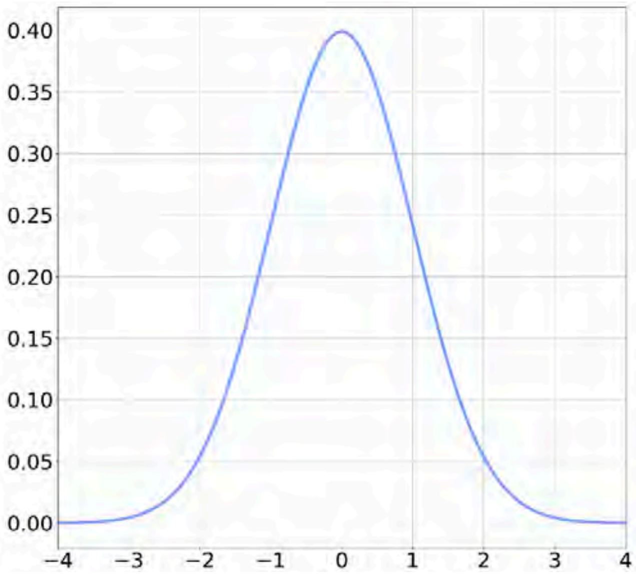

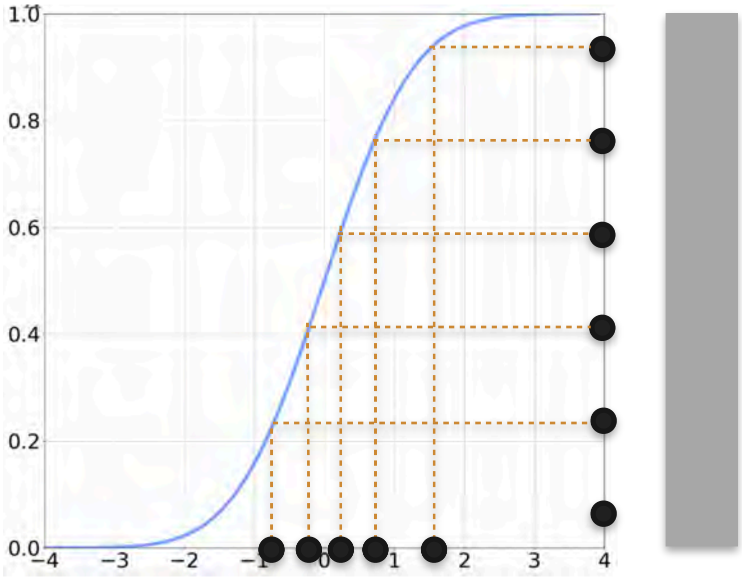

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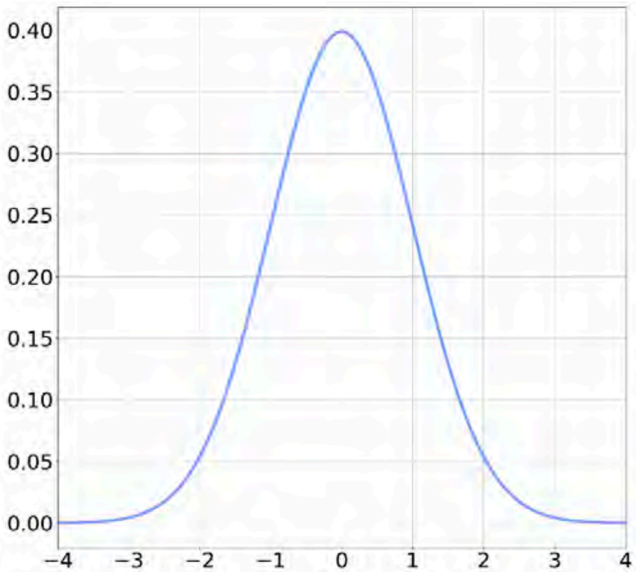

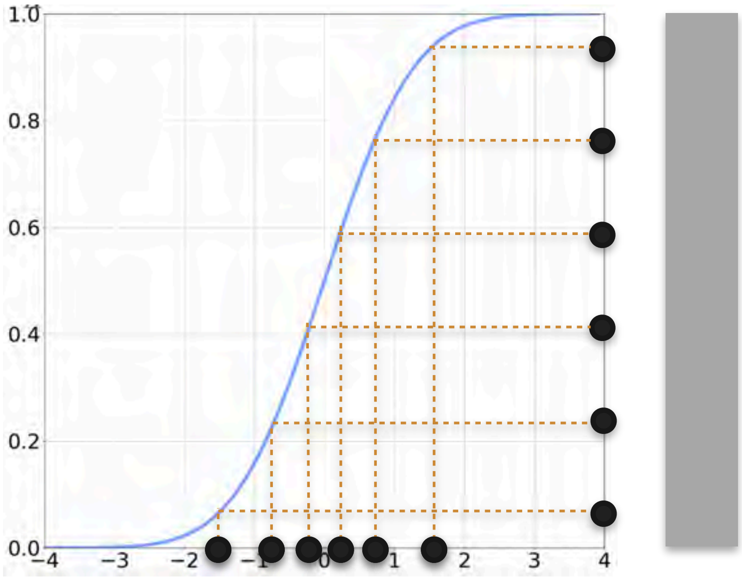

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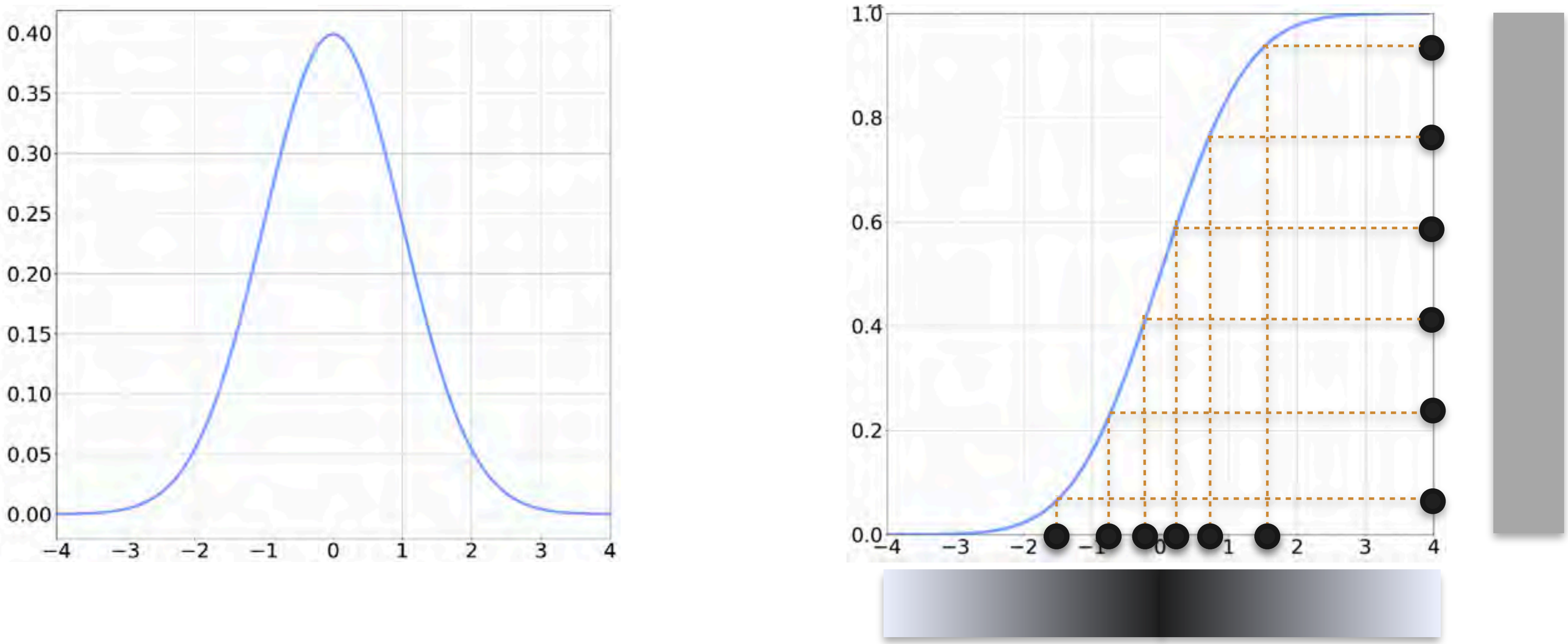

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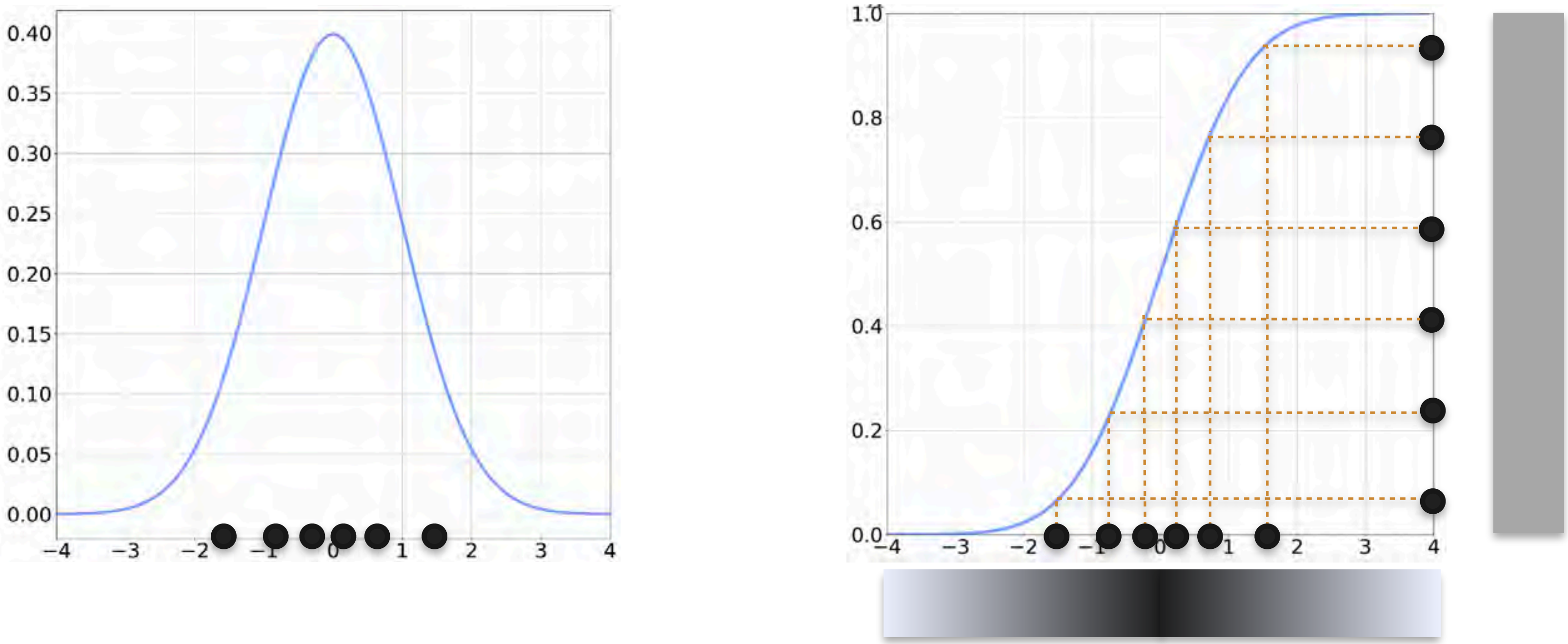

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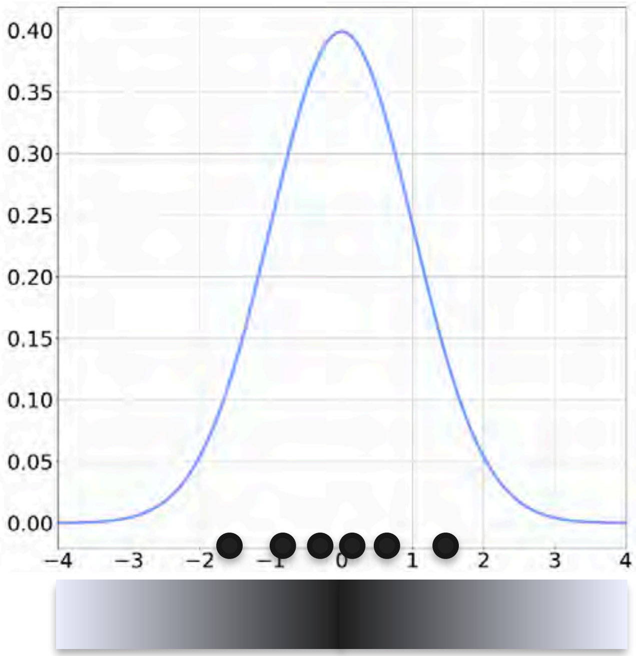

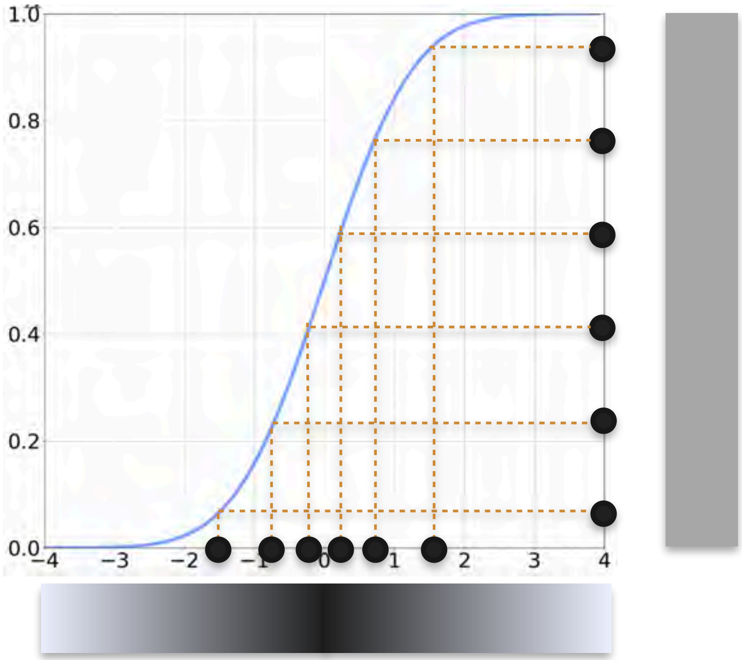

Probability Distributions

Conclusion