UNIT-3

Numerical Differential & Numerical Integration

Numerical Differential

1. **Newton's forward formula of derivative** [x = ~~mean~~ to start of table]

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\right]$$

2. **Newton's Backward formula of Derivative** [x = ~~mean~~ to end of table]

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\Delta^4 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \cdots\right]$$

Ques) Given that-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	
1.0	7.989						
		0.414					
1.1	8.403		-0.036				
		0.378		0.006			
1.2	8.781		-0.03		-0.002		
		0.348		0.004		0.001	
1.3	9.129		-0.026		-0.001		0.002
		0.322		0.003		0.003	
1.4	9.451		-0.023		0.002		
		0.299		0.005			
1.5	9.750		-0.018				
		0.281					
1.6	10.031						

find dy/dx & $\frac{d^2y}{dx^2}$ at $x = 1.1$ also find dy/dx for $x = 1.6$

by Newton's Forward formula

- $\left(\frac{dy}{dx}\right)_{x=1.1} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0\right]$

taking, $x_0 = 1.1$, $y_0 = 8.403$, $h = 0.1$ $\therefore \frac{1}{h} = 10$

$$= \frac{1}{0.1}\left[0.378 - \left(\frac{1}{2}(-0.03)\right) + \left(\frac{1}{3} \times 0.004\right) - \left[\frac{1}{4}(-0.001)\right] + \left(\frac{1}{5} \times 0.003\right)\right)$$

$= 10 \times 0.3925166$

$= 3.925166$

- $\left(\frac{d^2y}{dx^2}\right)_{x=1.1} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$

$$= \frac{1}{0.01}\left[-0.030 - 0.004 + \frac{11}{12} \times (-0.001) - \frac{5}{6}(0.003)\right]$$

$$= 100 \times \left(-0.030 - 0.004 + (-9.1666 \times 10^{-4}) - (0.0025)\right]$$

$= -3.7416$

by Newton's Backward

$$\left(\frac{dy}{dx}\right)_{x=1.6} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \frac{1}{5}\nabla^5 y_n + \frac{1}{6}\nabla^6 y_n\right]$$

$$= \frac{1}{0.1}\left[0.281 + \frac{1}{2}(-0.018) + \frac{1}{3}(0.005) + \frac{1}{4}(0.002) + \frac{1}{5}(0.003) + \frac{1}{6}(0.002)\right]$$

$= 10 \times 0.2751$

$= 2.751$

Ques: from the following table find $y'(0)$ $y''(0)$

Given that

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

Sol:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	4	4	3	-18	40	-72
1	8	7	-15	22	-32	
2	15	-8	7	-10		
3	7	-1	-3			
4	6	-4				
5	2					

$\Rightarrow h = 1$

by NFA

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0\right]$$

$$= \left[4 - \frac{1}{2}\times 3 + \frac{1}{3}(-18) - \frac{1}{4}\times 40 + \frac{1}{5}(-72)\right]$$

$$= -27.9$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

$$= \frac{1}{\;}\left[3 - (-18) + \frac{11}{12}\times 40 - \frac{5}{6}(-72)\right]$$

$$= 117.67$$

Ques: find the first & second ~~derivative~~ derivative of a func^n ~~give at~~ 12-04-23

$x = 1.1 \rightarrow$ lies b/n x_1 & x_2

Given that -

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	0					
		0.128				
1.2	0.128		0.288			
		0.416		0.048		
1.4	0.544		0.336		0	
		0.752		0.048		0
1.6	1.296		0.384		0	
		1.136		0.048		
1.8	2.432		0.432			
		1.568				
2.0	4.00					

by Newton's forward interpolation formula -

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$\therefore h = 0.2$

$$\Rightarrow y = 0 + u(0.128) + \frac{1}{2}(u^2 - u)\,0.288 + \frac{1}{6}(u^3 - 3u^2 + 2u)(0.048) \quad -① $$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{h}\left[0.128 + 0.144(2u-1) + 0.008(3u^2 - 6u + 2)\right] \quad -②$$

$$\because \frac{du}{dx} = \frac{1}{h}$$

for $x = 1.1$,

$$u = \frac{x-a}{h} = \frac{1.1 - 1.0}{0.2} = 0.5 \quad , h = 0.2$$

$$\frac{dy}{dx} = \frac{1}{0.2}\left[0.128 + 0 + (-0.002)\right]$$

$$\frac{dy}{dx} = 0.63 \quad ③$$

from ②

$$\frac{d^2y}{dx} = \frac{1}{h^2}\left[0 + 0.144\times 2 + 0.008\,(6u - 6)\right]$$

$$= \frac{1}{0.04}\left[0.288 - 0.024\right]$$

$$= 25 \times 0.264$$

$$= 6.6$$

Stirling's formula of derivative -

13-04-23

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) - \frac{1}{6}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{1}{12}\Delta^4 y_{-2} + \frac{1}{90}\Delta^6 y_{-3} + \cdots\right]$$

Ques: A slider in a machine moves along a fixed ~~string~~ straight rod it's distance x cm along the rod is given for various values of time t second. find the velocity and acceleraⁿ of the slider when t= 0.3 seconds.

t	x	Δx	$\Delta^2 x$	$\Delta^3 x$	$\Delta^4 x$	$\Delta^5 x$	$\Delta^6 x$
0	y_{-3} 30.13						
		1.49					
0.1	y_{-2} 31.62		-0.24				
		1.25		-0.24			
0.2	y_{-1} 32.87		-0.48		0.26		
		0.77		0.02		-0.27	
0.3	y_0 33.64		-0.46		-0.01		0.29
		0.31		0.01		0.02	
0.4	33.95		-0.45		0.01		
		-0.14		0.02			
0.5	33.81		-0.43				
		-0.57					
0.6	33.24						

by Stirling's formula of derivative

$$\text{velocity} = \left(\frac{dx}{dt}\right)_{t=0.3} = \frac{1}{h}\left[\left(\frac{\Delta x_0 + \Delta x_{-1}}{2}\right) - \frac{1}{6}\left(\frac{\Delta^3 x_{-1} + \Delta^3 x_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 x_{-2} + \Delta^5 x_{-3}}{2}\right) \cdots\right]$$

$h = 0.1$

$$= \frac{1}{0.1}\left[\left(\frac{0.31+0.77}{2}\right) - \frac{1}{6}\left(\frac{0.01+0.07}{2}\right) + \frac{1}{30}\left(\frac{-0.27+0.02}{2}\right)\right]$$

$$= 5.33$$

acceleration $= \dfrac{d^2x}{dt^2} = \dfrac{1}{h^2}\left[\Delta^2 x_{-1} - \dfrac{1}{12}\Delta^4 x_{-2} + \dfrac{1}{90}\Delta^6 x_3\right]$

$$= \frac{1}{0.01}\left[-0.46 - \frac{1}{12}(-0.01) + \frac{1}{90} \times 0.29\right]$$

$$= -45.6$$

Bessel's Formula of derivative -

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{4}\left(\Delta^2 y_0 + \Delta^2 y_{-1}\right) + \frac{1}{12}\Delta^3 y_{-1} + \frac{1}{24}\left(\Delta^4 y_{-1} + \Delta^4 y_{-2}\right) - \frac{1}{120}\Delta^5 y_{-2} - \frac{1}{240}\left(\Delta^6 y_{-2} + \Delta^6 y_{-3}\right) + \cdots\right]$$

Ques: Use Bessel's formula and find $f'(7.5)$ Given that -

x	$f(x)$ y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
7.47	y_{-3} 0.193						
		0.002					
7.48	y_{-2} 0.195		-0.001				
		0.003		-0.001			
					0		
7.49	y_{-1} 0.198		0			0.003	
				-0.001			-0.010
		0.003			0.003		
7.50	y_0 0.201		-0.001				
		0.002		0.002		-0.007	
7.51	0.203		0.001		-0.004		
		0.003		-0.002			
7.52	0.206		-0.001				
		0.002					
7.53	0.208						

by Bessel's formula -

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{4}\left(\Delta^2 y_0 + \Delta^2 y_{-1}\right) + \frac{1}{12}\Delta^3 y_{-1} + \frac{1}{24}\left(\Delta^4 y_{-1} + \Delta^4 y_{-2}\right) - \frac{1}{120}\Delta^5 y_{-2} - \frac{1}{240}\left(\Delta^6 y_{-2} + \Delta^6 y_{-3}\right) + \cdots\right]$$

$h = 0.01$

$$\left(\frac{dy}{dx}\right)_{x=7.5} = \frac{1}{0.01}\left[0.002 - \frac{1}{4}(-0.001+0.001) + \frac{1}{12}\times 0.002 + \frac{1}{24}(0.003-0.004) - \frac{1}{170}(-0.007) - \frac{1}{240}(0-0.010)\right]$$

$$= 0.002225 \times 100$$

$$= 0.2225$$

Ques: find $f'(10)$ from the following data

x	$f(x)$ y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
3	-13				
		$\frac{23-(-13)}{5-3} = 18$			
			$\frac{146-18}{11-3} = 16$		
5	23			$\frac{40-16}{27-3} = 1$	
		$\frac{899-23}{11-5} = 146$			0
			$\frac{1026-146}{27-5} = 40$		
11	899			$\frac{69-40}{34-5} = 1$	
		$\frac{17315-899}{27-11} = 1026$			
			~~17315-899~~ $\frac{2613-1026}{34-11} = 69$		
27	17315				
		$\frac{17315-35606}{34-27} = 2613$			
34	35606				

by Newton's divided diff. form.

$$y = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_1 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0$$

$$y = -13 + (x-3)18 + (x-3)(x-5)16 + (x-3)(x-5)(x-11).1$$

$$y = -13 + 18(x-3) + 16(x^2-8x+15) + (x^3-19x^2+103x-165)$$

$$\frac{dy}{dx} = f'(x) = 18(1) + 16(2x-8) + (3x^2-38x+103)$$

at $x=10$

$$= 18 + 16\times 12 + 300 - 380 + 103$$

$$= 233$$

17-04-23

Numerical Integration

$$I = \int_a^b f(x)\,dx$$

- We divide the interval (a, b) in 'n' sub-interval,

we get, $I = \int_{x_0}^{x_0+nh} f(x)\,dx$

where, $x_0 = a$

$x_0 + nh = b$

$$h = \frac{b-a}{n}$$

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1. Trapezoidal rule -

$$\boxed{\frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+ \cdots + y_{n-1})\right]}$$

Note: There is no rule for no. of sub-intervals.

2. Simpson's $\frac{1}{3}$ rule -

$$\boxed{\frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots) + 2(y_2+y_4+\cdots)\right]}$$

Note: No. of subintervals must be a multiple of 2.

3. Simpson's $\frac{3}{8}$ rule -

$$\boxed{\frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+y_7 \cdots\right] + 2(y_3+y_6+\cdots)]}$$

Note: No. of subintervals must be a multiple of 3

4. Boole's rule -

$$\boxed{\int_{x_0}^{x_0+nh} f(x)dx = \frac{2h}{45}\left[7y_0 + 32y_1 + 12y_2 + 32y_3 + 14y_4 + 32y_5 + 12y_6 + \cdots\right]}$$

Note: No. of sub-interval must be a multiple of 4.

5. Weddle's rule -

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6 + \cdots\right]$$

Note | No. of sub-intervals must be multiple of 6.

Ques: Evaluate:

$$\int_0^1 \frac{dx}{1+x^2}$$ by

(i) Trapezoidal rule, taking $h=\frac{1}{4}$

(ii) Simpson's $\frac{1}{3}$ rule, taking $h=\frac{1}{4}$

(iii) Simpson's $\frac{3}{8}$ rule, taking $h=\frac{1}{6}$

(iv) Weddle's rule, taking $h=\frac{1}{6}$

also find 'π' in each case-

Sol (i) Trapezoidal rule -

$(h=\frac{1}{4})$ x:	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$y=f(x)=\frac{1}{1+x^2}$:	1	$\frac{16}{17}$	$\frac{4}{5}$	$\frac{16}{25}$	$\frac{1}{2}$
	$\downarrow$ y_0	$\downarrow$ y_1	$\downarrow$ y_2	$\downarrow$ y_3	$\downarrow$ y_4

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$= \frac{1}{8}\left[\left(1+\frac{1}{2}\right)+2\left(\frac{16}{17}+\frac{4}{5}+\frac{16}{25}\right)\right]$$

$$= 0.782794$$

for value of π,

$$\int_0^1 \frac{dx}{1+x^2} = 0.782794$$

$$(\tan^{-1}x)_0^1 = 0.782794$$

$$\Rightarrow \frac{\pi}{4} = 0.782794$$

$$\pi = 3.131176$$

(ii) Simpson's $\frac{1}{3}$ rule -

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{1}{12}\left[\left(1+\frac{1}{2}\right)+4\left(\frac{16}{17}+\frac{16}{25}\right)+2\left(\frac{4}{5}\right)\right]$$

$$= 0.785392$$

for value of π

$$\int_0^1 \frac{dx}{1+x^2} = 0.785392$$

$$(\tan^{-1}x)_0^1 = 0.785392$$

$$\frac{\pi}{4} = 0.785392$$

$$\pi = 4 \times 0.785392$$

$$\pi = 3.141568$$

(iii) Simpson's 3/8 rule -

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2)+2(y_3)\right]$$

$\left(h=\frac{1}{6}\right)$ x : 0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ 1

$y=f(x)=\frac{1}{1+x^2}$: 1 —

$\left(h=\frac{1}{6}\right)$ x : 0 $\frac{1}{6}$ $\frac{1}{3}$ $\frac{4}{3}$ $\frac{7}{3}$

(iii) Simpson's 3/8 rule

$(h=\frac{1}{6})$ x :	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1
$y=f(x)=\frac{1}{1+x^2}$:	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\Big[(y_0+y_n) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\Big]$$

$$= \frac{3}{8}\times\frac{1}{6}\left[\left(1+\frac{1}{2}\right) + 3\left(\frac{36}{37}+\frac{9}{10}+\frac{9}{13}+\frac{36}{61}\right) + 2\left(\frac{4}{5}\right)\right]$$

$= 0.785395$

For value of π

$$\int_0^1 \frac{dx}{1+x^2} = 0.785395$$

$$(\tan^{-1}x)_0^1 = 0.785395$$

$$\frac{\pi}{4} = 0.785395$$

$$\pi = 3.14158$$

(iv) Weddle's rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{10}\Big[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\Big]$$

$$= \frac{3}{10}\times\frac{1}{6}\left[1 + 5\times\frac{36}{37} + \frac{9}{10} + 6\times\frac{4}{5} + \frac{9}{13} + 5\times\frac{36}{61} + \frac{1}{2}\right]$$

$= 0.785399$

For value of π

$$\int_0^1 \frac{dx}{1+x^2} = 0.785399$$

$$(\tan^{-1}x)_0^1 = 0.785399$$

$$\frac{\pi}{4} = 0.785399$$

$$\pi = 3.141599$$

18-04-23

Ques: the velocity V (km/min) of a bike which starts from ~~at~~ rest is given at fixed intervals of time 't' (min) as follows —

→ bike starts from rest

t :	0	2	4	6	8	10	12	14	16	18	20
V :	0	10	18	25	29	32	20	11	5	2	0
	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}

estimate approximately the distance covered in 20 mins.

$$\text{Distance} = \int_0^{20} v\,dt$$

by Simpson's 1/3 rule.

$$= \frac{h}{3}\left[(V_0 + V_{10}) + 4(V_1 + V_3 + V_5 + V_7 + V_9) + 2(V_2 + V_4 + V_6 + V_8)\right]$$

$$= \frac{2}{3}\left[4(10+25+32+11+2) + 2(18+29+20+5)\right]$$

$$= 309.333$$

Ques: Evaluate : $\int_0^6 \frac{dx}{1+x^2}$ by using —

(i) Trapezoidal rule

(ii) Simpson's 1/3 rule

(iii) Simpson's 3/8 rule

(iv) Weddle's rule

Sol: Taking no. of subintervals $n = 6$.

$$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

$(h=1)$	x :	0	1	2	3	4	5	6
$y=f(x)=\frac{1}{1+x^2}$	:	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$	$\frac{1}{26}$	$\frac{1}{37}$
		y_0	y_1	y_2	y_3	y_4	y_5	y_6

by trapezoidal $\int$

$$\frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{1}{2}\left[\left(1+\frac{1}{37}\right) + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} + \frac{1}{26}\right)\right]$$

$= \cancel{1.81079}$ / 1.410798

(ii) Simpson's $\frac{1}{3}$ rule –

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

~~1.36617~~

$$= \frac{1}{3}\left[\left(1+\frac{1}{37}\right) + 4\left(\frac{1}{2}+\frac{1}{10}+\frac{1}{26}\right) + 2\left(\frac{1}{5}+\frac{1}{17}\right)\right]$$

$$= 1.366173$$

(iii) Simpson's $\frac{3}{8}$ rule –

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3}{8}h\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$= \frac{3}{8}\left[\left(1+\frac{1}{37}\right) + 3\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{12}\right) + 2\left(\frac{1}{10}\right)\right]$$

$$= 1.35708$$

(iv) Weddle's rule –

$$\int_0^6 \frac{dx}{1+x^2} = \frac{3h}{10}\left[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6\right]$$

$$= \frac{3}{10}\left[1+\frac{5}{2}+\frac{1}{5}+6\times\frac{1}{10}+\frac{1}{17}+5\times\frac{1}{26}+\frac{1}{37}\right]$$

$$= 1.3735$$

Ques: Evaluate 6 $\int_0^4 e^x dx$ by Simpson's 1/3 rule –

$n = 4$

$$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$$

$(h=1)$ x:	0	1	2	3	4
$y=f(x)=e^x$:	1	2.7182	7.3890	20.0855	54.5981

<u>by Simpson's 1/3 rule</u>,

$$\int_0^4 e^x dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$\cancel{= \frac{1}{3}\left[(1+20.0855) + 4(2.7182) + 2(7.3890)\right]}$$

$$\cancel{= \frac{1}{3}\left[(1+54.5981) + 4(2.7182) + 2(7\right.}$$

$$= \frac{1}{3}\left[(1+54.5981) + 4(2.7182+20.0855) + 2(7.3890)\right]$$

$$= 53.86386$$

19-04-23