UNIT-1

21/07/23

1. Bisection method or Bolzano method –

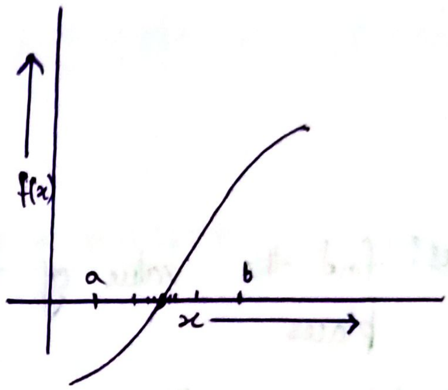
$f(x) = 0$

Step 1: We put the given eqⁿ in the form $f(x) = 0$.

Step 2: find two points a & b s.t, $f(a) \cdot f(b) < 0$

e.g. $f(a) < 0$; $f(b) > 0$

Step 3: Calculate – $\boxed{x_1 = \frac{a+b}{2}}$



① Case 1: If $f(x_1) < 0$

then, $x_2 = \frac{x_1 + b}{2}$

② Case 2: If $f(x_1) > 0$

then, $x_2 = \frac{a + x_1}{2}$

and proceed in this ways until we find the root of desired accuracy.

Ques 6 find the root of eqⁿ $x^3 - 4x - 9 = 0$ by using bisection method correct upto 3 decimal places.

$\Rightarrow$ $x^3 - 4x - 9 = 0$

$f(x) = x^3 - 4x - 9$

$f(0) = -9$

$f(1) = -12$

$f(2) = -9$

$f(3) = 6$] — sign get change ∴ so, solⁿ is somewhere in b/w.

$f(2.7) = -0.11$

$f(2.72) = 0.24$

let, ∴ $a = 2.7$ $\quad f(a) < 0$

$b = 2.72$ $\quad f(b) > 0$

I - iteraⁿ $\quad x_1 = \frac{a+b}{2} = \frac{2.7 + 2.72}{2} = 2.71$

∵ $f(x_1) > 0$

II - iteration $\quad x_2 = \frac{a + x_1}{2} = \frac{2.7 + 2.71}{2} = 2.705$ ∵ $f(x_2) < 0$

III - iteration - $x_3 = \frac{x_2 + x_1}{2} = \frac{2.705 + 2.71}{2} = \overline{2.7075}$

$\therefore f(x) > 0$

IV - iteration - $x_4 = \frac{2.705 + 2.7075}{2} = \overline{2.70625}$

$\therefore f(x) < 0$

V - iteration - $x_5 = \frac{2.70625 + 2.7075}{2} = \overline{2.706875}$

Values matches upto 3 decimal place

$\therefore f(x) < 0$

VI - iteration - $x_6 = \frac{2.706875 + 2.7075}{2} = 2.7071875$

$\therefore f(x) > 0$

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Ques6 find the value of $\sqrt{12}$ by using bisection method upto 4 decimal places.

$\Rightarrow$ let, $x = \sqrt{12}$

$x^2 = 12$

$x^2 - 12 = 0$

so, $f(x) = x^2 - 12$ $\quad \therefore \sqrt{12} = 3.46$

let us take, $a = 3.4$, $b = 3.5$

I - iteration - $x_1 = \frac{a+b}{2} = \overline{3.45}$ $\quad$ ~~f(x) > 0~~ $\therefore f(x) < 0$

II - iteration - $x_2 = \frac{a + x_1}{2} = \frac{3.4 + 3.45}{2} = 3.425$

$f(x) < 0$

III - iteration - $x_3 = \frac{x_2 + x_1}{2} = \frac{3.425 + 3.45}{2} = 3.4375$

$f(x) < 0$

IV - iteration - $x_4 = \frac{x_3 + x_1}{2} = \frac{3.4375 + 3.45}{2} = 3.44375$

$f(x) < 0$

V - iteration - $x_5 = 3.44375 + 3.45$

VI iteration $x_6 = \frac{x_5 + x_1}{2} = \frac{3.446875 + 3.45}{2}$

II - iteration - $x_2 = \frac{x_1 + b}{2} = \frac{3.45 + 3.5}{2} = 3.475$

$\therefore f(x) > 0$

III - iteration - $x_3 = \frac{x_1 + x_2}{2} = \frac{3.45 + 3.475}{2} = 3.4625$

$\therefore f(x) < 0$

IV - iteration - $x_4 = \frac{x_3 + x_2}{2} = \frac{3.4625 + 3.475}{2} = 3.46875$

$\therefore f(x) > 0$

V - iteration - $x_5 = \frac{x_3 + x_4}{2} = \frac{3.4625 + 3.46875}{2} = 3.465625$

$\therefore f(x) > 0$

VI - iteration - $x_6 = \frac{x_3 + x_5}{2} = \frac{3.4625 + 3.465625}{2} = 3.4640625$

$\therefore f(x) < 0$

VII - iteration - $x_7 = \frac{x_6 + x_5}{2} = \frac{3.4640625 + 3.465625}{2}$

$= 3.46484375$

$\therefore f(x) > 0$

VIII - iteration - $x_8 = \frac{x_6 + x_7}{2} = \frac{3.4640625 + 3.46484375}{2} = 3.464453125$

$\therefore f(x) > 0$

IX - iteration - $x_9 = \frac{x_6 + x_8}{2} = \frac{3.4640625 + 3.464453125}{2} = 3.464257813$

$\therefore f(x) > 0$

X - iteration - $x_{10} = \frac{x_6 + x_9}{2} = \frac{3.4640625 + 3.464257813}{2} = 3.464160157$

$\therefore f(x) > 0$

XI - iteration - $x_{11} = \frac{x_6 + x_{10}}{2} = \frac{3.4640625 + 3.464160157}{2} = 3.464111329$

$\therefore f(x) > 0$

$\Rightarrow \overline{3.464160157}$

Regular - Falsi Method or Method of False Position :

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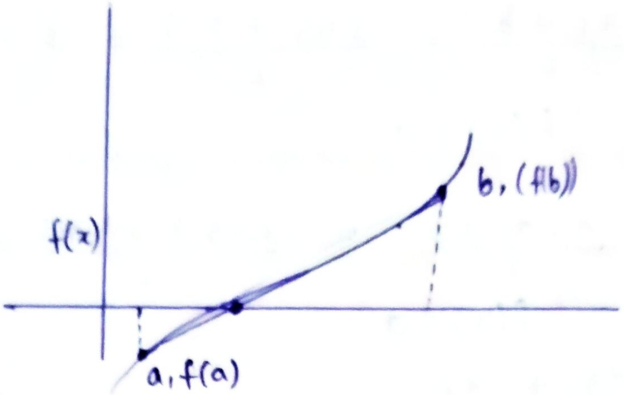

$f(x) = 0$

$$(y - f(a)) = \frac{f(b) - f(a)}{b - a}(x - a)$$

$y = 0 \quad x = x_2$

$$-f(a) = \frac{f(b) - f(a)}{(b - a)}(x_2 - a)$$

$$x_2 = a - \frac{f(a)\,[b - a)}{f(b) - f(a)}$$

$$x_2 = \frac{af(b) - \cancel{af(a)} - bf(a) + \cancel{af(a)}}{f(b) - f(a)}$$

$$\boxed{x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

Ques: find real root of the eq^n $x^3 - 2x - 5 = 0$ by regular falsi method correct to 4 decimal places.

Sol^n

$f(x) = x^3 - 2x - 5$

$f(0) = -5$

$f(1) = -6$

$f(2) = -1$

$f(3) = 16$

$f(2.2) = 1.248$

$f(2.1) = 0.061$

$f(2.08) = -0.16108$

$a = 2.08 \qquad f(a) = -0.16108$

$b = \cancel{0.06}\ 2.1 \qquad f(b) = 0.061$

I - iteration -

$$x_1 = \frac{af(b) - b(fa)}{f(b) - f(a)} = \frac{((2.08)(0.061) - 2.1(-0.16108))}{(0.061 - (-0.16108))}$$

$$x_1 = 2.094506$$

$$f(x_1) = -0.000502$$

this is negative.

II - iteration -

$$x_2 = \frac{x_1(fb) - b\,f(x_1)}{f(b) - f(x_1)} = \frac{2.094506(0.061) - 2.1(-0.000502)}{((0.061 - (-0.000502)))}$$

$$x_2 = 2.094550844$$

Ques: find the real root of the eq^n $x.\log_{10}x = 1.2$ by using regular falsi method up to 4 decimal places.

Sol: $f(x) = x\log_{10}x - 1.2$

$f(1) = -1.2$

$f(2) = -0.5974$

$f(3) = 0.231363$

$f(2.6) = -0.12$

~~f(2.26~~ $f(2.62) = -0.1$

$f(2.65) = -0.07$

$f(2.7) = -0.035317$

$f(2.8) = 0.052042$

$a = 2.7$ $\quad f(a) = -0.03531$

$b = 2.8$ $\quad f(b) = 0.052042$

I - iteration -

$$x_1 = \frac{af(b) - b(fa)}{f(b) - f(a)} =$$

$= \cancel{2.2999}\ 2.74036$

$f(x_1) = -0.000194$

II - iteration - $x_2 = \dfrac{x_1 f(b) - b(f(x_1))}{f(b) - f(x_1)}$

$= 2.740644$

~~2.740639~~

$f(x_2) = -0.00001827$

III - iteration -

$$x_3 = \frac{x_2 (f(b)) - b \cdot f(x_2)}{f(b) - f(x_2)}$$

$$= 2.740646$$

$$f(x_3) = -8.370721 \times 10^{-8}$$

The real root of eq^n:

$= 2.740646$

Iteration Method -

(i) We put given eq^n $f(x) = 0$ in the form of $x = \phi(x)$ and we can apply iteration method if $|\phi'(x)| < 1$, $x \in [a, b]$ where the interval $[a, b]$ has sol^n of given eq^n.

Ques: find real root of the eq^n $\cos x = 3x - 1$ correct up to 3 d.p by using iteration method.

Sol: The given eq^n can be written as $\cos x - 3x + 1 = 0$

$$f(x) = \cos x - 3x + 1$$
$$f(0) = 2$$
$$f(1) = -1.45$$

The root of the eq^n lies b/w $[0, 1]$

now, we can write $x = \frac{1}{3}[\cos x + 1]$ — (i)

let $\phi(x) = \frac{1}{3}[\cos x + 1]$

$\phi'(x) = -\frac{1}{3} \sin x \Rightarrow |\phi'(x)| = \frac{1}{3}|\sin x|$

Clearly $|\phi'(x)| < 1 \ \forall \ x \in [0, 1]$

I iteran — taking $x_0 = \square$

$$x_1 = 0.66666$$

II iteran — taking $x_1 = 0.666666$

$$x_2 = 0.59529$$

III iteraⁿ $x_3 = 0.60932$ IV iteration $x_4 = 0.60667$

V iteraⁿ $x_5 = 0.60718$ VI iteration $x_6 = \overline{0.60708}$

Ques: find real root $2x - \log x = 7$ correct 4 decimal places by using iteration method.

Solⁿ $2x - \log_{10} x = 7$

$2x - \log_{10} x - 7 = 0$

$f(x) = 2x - \log_{10} x - 7$

$f(1) = -5$ $f(3) = -1.47$

$f(2) = -3.3$ $f(4) = 0.39$] lies b/w [3,4]

now $x = \frac{1}{2}[\log_{10} x + 7]$ — (1)

$\phi(x) = \frac{1}{2}[\log_{10} x + 7]$

$\phi'(x) = \frac{1}{2}[\frac{1}{x}][\log_{10} e]$ | $\log_{10} e = 0.4342$

$|\phi'(x)| = \left|\frac{1}{2x} \log_{10} e\right|$

Clearly $|\phi'(x)| < 1 \quad \forall x \in [3,4]$

so, iteration method can be applies

$x = \frac{1}{2}[\log_{10} x + 7]$ — (1)

I iteraⁿ $x_0 = 3$

$x_1 = 3.738560$

II iteraⁿ $x_2 = 3.786352$

III iteraⁿ $x_3 = 3.789110$

IV iteraⁿ $x_4 = 3.789268$

V $x_5 = \overline{3.789277}$

Newton - Raphson Method -

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Ques: Use Newton - Raphson method and find real root of eq^n $3x = \cos x + 1$ correct upto 4 d.p.

$$3x = \cos x + 1$$

$$f(x) = 3x - \cos x - 1$$

$$f(0) = -2$$

$$f(1) = 1.459$$

exact root lies b/w 0 & 1 ∴ [0,1]

$$f'(x) = 3 + \sin x$$

NR - Method form -

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{3x_n - \cos x_n - 1}{3 + \sin x_n}$$

$$= \frac{\cancel{3x_n} + x_n \sin x_n - \cancel{3x_n} + \cos x_n + 1}{3 + \sin x_n}$$

$$x_{n+1} = \frac{x_n \sin x_n + \cos x_n + 1}{3 + \sin x_n} \quad -(1)$$

→ taking $x_0 = 1$

$$x_1 = \frac{x_0 \sin x_0 + \cos x_0 + 1}{3 + \sin x_0}$$

I iteration $x_1 = \frac{1 \times \sin 1 + \cos 1 + 1}{3 + \sin 1} = 0.6200159$

II iteration ⇒ $x_2 = \frac{x_1 \cdot \sin x_1 + \cos x_1 + 1}{3 + \sin x_1} = 0.6071206$

3rd iteration $x_3 = \frac{x_2 \sin x_2 + \cos x_2 + 1}{3 + \sin x_2} = 0.6071016.$

Ques: Use Newt - Iterative method and find root of the eqn

$x \log_{10} x = 1.2$ correct to 5 d.p.

$f(x) = x \log_{10} x - 1.2$

$f(1) = -1.2$

$f(2) = -0.59$

$f(3) = 0.23$

The ex. root lies b/w 2 & 3 ∴ [2,3]

$f'(x) = \log_{10} x + x \frac{1}{x} \log_{10} e$

$f'(x) = \log_{10} x + 0.43429$

by Newton's formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_2 - \frac{(x_n \log_{10} x_n - 1.2)}{\log_{10} x_n + 0.43429}$$

$$x_{n+1} = \frac{\cancel{x_n \log_{10} x_n} + 0.43429 x_n - \cancel{x_n \log_{10} x_n} + 1.2}{\log_{10} x_n + 0.43429}$$

$$x_{n+1} = \frac{0.43429 x_n + 1.2}{\log_{10} x_n + 0.43429} \quad - (1)$$

taking, $x_0 \Rightarrow$

I iteran $x_1 = \frac{0.43429 \times x_0 + 1.2}{\log_{10} x_0 + 0.43429} = 2.74079903$

$x_2 = \frac{0.43429 x_1 + 1.2}{\log_{10} x_1 + 0.43429} = 2.74064609$

$x_3 = \frac{0.43429 x_2 + 1.2}{\log_{10} x_2 + 0.43429} = \overline{2.7406460}$

Ques: By using N. formula find the value of $\sqrt{12}$ correct to 4 d.p.

$\Rightarrow$ let, $x = \sqrt{12}$

$x^2 = 12$

$x^2 - 12 = 0$

$f(x) = x^2 - 12$

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^2 - 12)}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + 12}{2x_n}$$

$$x_{n+1} = \frac{x_n^2 + 12}{2x_n} \quad -(1)$$

take $x_0 = 3.$

I iteraⁿ -

$$x_1 = \frac{x_0^2 + 12}{2x_0}$$

$$= 3.5$$

II iteraⁿ

$$x_2 = \frac{x_1^2 + 12}{2x_1}$$

$$= \frac{(3.5)^2 + 12}{2 \times 3.5} = 3.464285714$$

III iteraⁿ

$$x_3 = \frac{x_2^2 + 12}{2x_2}$$

$$= 3.464016200$$

IV $x_4 = \frac{x_3^2 + 12}{2x_3} = \underline{3.46402}$

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Approximate numbers :

There are two types of numbers exact & ~~appr~~ approximate.

- Exact no's are like - 2, 5, $\frac{7}{2}$, 6.43 etc. but there are some no's s.a - $\frac{4}{3} = 1.3333\ldots$ $\sqrt{2} = 1.414213\ldots$. it can't be expressed by finite no. of digits these may be approximated by the no. - 1.333 and 1.4142 etc, which are called approximate no's.

Significant Digits -

The no. of digits used to express a no. are called significant digits.

for ex :- The following the no's has two significant digits only -

0.0014, 0.016, 1600.

- If we can remove the zeroes by using multiple of 10's then those are non-significant

Rounding off -

There are some no's with large no. of digits that can be reduced to a desirable limit by rounding off.

ex : round off upto two decimal places -

i.e. 3.1457981 $\Rightarrow$ 3.14
(even)

4.1357632 $\Rightarrow$ 4.14
(odd)

5.6324317 $\Rightarrow$ 5.63

2.1478325 $\Rightarrow$ 2.15

Errors -

In any numerical computation we come across the following types of errors -

1. Inherent Errors :

The errors which are already present in the statement of a problem are called inherent ~~E~~ errors.

2. Rounding Errors :

These errors arise from the process of rounding off the no. ~~of~~ during the computation, such types of errors are unavoidable in most of the calculation.

3. Truncation error:

These errors are caused by using approximate results or on replacing an infinite process by a finite one.

ex: $f(x) = 1 + x + x^2 + x^3 + \cdots$

$f(x) = 1 + x + x^2 + x^3$

4. Absolute error:

If x = exact value of a variable and x' = approx. value of variable then absolute error is given by –

$$\boxed{E_a = |x - x'|}$$

5. Relative Error:

The relative error is defined as –

$$\boxed{E_r = \left|\frac{x - x'}{x}\right|}$$

6. Percentage error:

Percentage error is given by –

$$\boxed{E_p = \frac{x - x'}{x} \times 100}$$

If 'u' is a func^n of x & y $u = f(x, y)$ then error in u resulting from δx and δy, in the values of x & y is given by

$$\boxed{\delta u = \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y.}$$

Ques: If $u = \frac{4x^2y^3}{z^4}$ & errors in x, y & z be 0.001 complete the maximum error in u when $x = y = z = 1$

Sol^n: $\frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta x$

$$\delta u = \frac{8xy^3}{z^4}\delta x + \frac{12x^2y}{z^4}\delta y - \frac{16x^2y^3}{z^5}\cdot\delta z$$

$$= \frac{8xy^3}{z^4}\delta x + \frac{12x^2y}{z^4}\delta y - \frac{16x^2y^3}{z^5}\delta z$$

for max error

in 4

Taking $\delta x = 0.001$ $\delta y = 0.001$ $\delta z = -0.001$

$x = y = z = 1$

$$\delta u = 8 \times 0.001 + 12 \times 0.001 + 16 \times 0.001$$
$$= 0.036$$

Order of convergence or Rate of Convergance -

let x is exact root of the equation $f(x) = 0$ & e_r error of r^{th} iteration,

$$e_r = \alpha - x_r$$

then $e_{r+1} = \alpha - x_{r+1}$

then rate of convergence will be p if

$$\boxed{\frac{e_{r+1}}{(e_r)^p} < |k|} \Rightarrow e_{r+1} < |k| (e_r)^p$$

- **Roots of algebric equation**

→ Graeffe's root squaring Method -

Case 1 - When all roots are real.

find all roots of the equation : $x^3 - 2x^2 - 5x + 6 = 0$

$$f(x) = x^3 - 2x^2 - 5x + 6$$

There are 2 changes in sign so by Descartes rule there will be 2 positive roots

Now $f(-x) = -x^3 - 2x^2 + 5x + 6$ $\qquad -x^3 + 5x = 2x^2 - 6$

There is only 1 negative root $\qquad -x(x^2 - 5) = 2x^2 - 6$

$\Rightarrow x^3 - 5x = 2x^2 - 6$ $\quad [x^3 - 2x^2 - 5x + 6 = 0]$

$x(x^2 - 5) = 2x^2 - 6$

Squaring on both side

$$x^2 (x^2 - 5)^2 = (2x^2 - 6)^2$$
$$x^2 (x^4 - 10x^2 + 25) = 4(x^4 + 9 - 6x^2)$$

Putting $x^2 = y$ $\qquad y^2 + 9 - 6y$

$y(y-5)^2 = 4(y-3)^2$ $\qquad y^2 - 3y - 3y + 9$

$y(y^2 - 10y + 25) = 4(y^2 + 9 - 6y)$ $\qquad y(y-3) - 3(y-3)$

$$y^3 - 10y^2 + 25y = 4y^2 + 36 - 24y$$
$$y^3 + 49y = 14y^2 + 36$$
$$y(y^2 + 49) = (14y^2 + 36)$$

Squaring,

$$y^2(y^2+49)^2 = (14y^2+36)^2$$

Putting $y^2 = z$

$$z(z+49)^2 = (14z+36)^2$$

$$z^3 + 98z^2 + 2401z = 196z^2 + 1296 + 1008z$$

$$z^3 + 1393z = 98z^2 + 1296$$

$$z(z^2+1393) = 98z^2 + 1296$$

Squaring,

$$z^2(z^2+1393)^2 = (98z^2+1296)^2$$

putting $z^2 = u$

$$u(u+1393)^2 = (98u+1296)^2$$

$$u(u^2 + 1940449 + 2786u) = 9604u^2 + 1679616 + 254016u$$

$$u^3 + 1940449u + 2786u^2 = 9604u^2 + 1679616 + 254016u$$

$$u^3 + 1686433u = 6818u^2 + 1679616$$

$$u(u^2 + 1686433) = 6818u^2 + 1679616$$

$$u^3 - 6818u^2 + 1686433u - 1679616 = 0$$

$$x_1 = -C_1 = 6818$$

$$x_2 = \frac{-C_2}{C_1} = 247.3504$$

$$x_3 = \frac{-C_3}{C_2} = 0.996$$

$$\alpha_1 = (x_1)^{1/8} = 3.01444 \simeq 3$$

$$\alpha_2 = (x_2)^{1/8} = 1.99142 \simeq 2$$

$$\alpha_3 = (x_3)^{1/8} = 0.99499 \simeq 1$$

Ques: Apply Graeffe's method and find all roots of the eqn

$$x^4 - 3x + 1 = 0 \quad (1)$$

for (+)ive roots $\quad f(x) = x^4 - 3x + 1$

There are 2 changes in sign so, there will be two positive real roots.

for (-)ive roots

$$f(-x) = x^4 + 3x + 1$$

There is no change in sign, so there will be no negative real root

There will be two complex roots.

$\boxed{\text{Note}}$ If α_r and α_{r+1} form a complex pair then the coefficient of x^{n-r} in successive squaring would fluctuate both magnitude and sign

from (1),

$$x^4 + 1 = 3x$$

Sq. both side,

$$((x^2)^2 + 1)^2 = 9x^2 \qquad \text{Put } x^2 = y$$

$$(y^2 + 1)^2 = 9y$$

$$y^4 + 2y^2 + 1 = 9y$$

again sq. both sides

$$[(y)^2 + 2y^2 + 1]^2 = 81y^2, \quad \text{Put } y^2 = z$$

$$[z^2 + 2z + 1]^2 = 81z$$

$$z^4 + 4z^2 + 1 + 4z^3 + 4z + 2z^2 = 81z$$

$$z^4 + 4z^3 + 6z^2 + 4z + 1 = 81z$$

$$(z^4 + 6z^2 + 1) = -4z^3 + 77z$$

$$z^4 + 6z^2 + 1 = -z(4z^2 - 77)$$

$$(z^2)^2 + 6z^2 + 1 = -z(4z^2 - 77)$$

Sq. on both side

$$[(z^2)^2 + 6z^2 + 1]^2 = z^2(4z^2 - 77)^2$$

put $z^2 = u$

$$(u^2 + 6u + 1)^2 = u(4u - 77)^2$$

$$u^4 + 36u^2 + 1 + 12u^3 + 12u + 2u^2 = u(16u^2 + 5929 - 616u)$$

$$u^4 + 12u^3 + 38u^2 + 12u + 1 = 16u^3 - 616u^2 + 5929u$$

$$u^4 + 12u^3 - 16u^3 + 38u^2 + 616u^2 + 12u - 5929u + 1 = 0$$

$$u^4 - 4u^3 + 654u^2 - 5917u + 1 = 0$$

The coefficient C_1 & C_4 are almost same, so, α_1 and α_4 will be real roots and α_2 and α_3 are pair of complex roots

$$\Rightarrow \quad \alpha_1^8 = -C_1 = 4$$

$$\alpha_1 = (4)^{1/8} = 1.1892$$

$$\Rightarrow \quad \alpha_4^8 = \frac{-C_4}{C_3} = 0.000169$$

$$\alpha_4 = 0.3376$$

let the pair of complex root is $\alpha \pm i\beta$

$$\beta = \sqrt{\rho_2^2 - \alpha^2}$$

and $\rho_2^{2(2^3)} = \frac{C_{2+1}}{C_{2-1}} = \frac{5917}{4}$ { Here in, 2^3, 3 is the times of squaring done

$$\rho_2^{16} = 1479.25$$

$$\rho_2 = 1.5782$$

from eq^n (1)

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = \frac{-b}{a} = 0$$

$$1.1892 + 2\alpha + 0.3376 = 0$$

$$\alpha = -\frac{1}{2}\left[1.1892 + 0.3376\right]$$

$$= -0.7634$$

$$\beta = \sqrt{\rho_2^2 - \alpha^2} = 1.381$$

Complex roots are –

$$-0.7634 \pm 1.381\, i$$

Real roots are –

$$1.1892 \; \& \; 0.3376$$