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UNIT - 4

Numerical Sol^n of Ordinary differential Eq^n

Methods 1.) Picard's Method

$$\frac{dy}{dx} = f(x, y)$$

at $x = x_0$, $y = y_0$

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\,dx$$

Ques Using Picard's Method obtain a sol^n upto 5^{th} approximation of the eq^n $\frac{dy}{dx} = x+y$ and at $x=0$, $y=1$

Then find value of y at $x=0.2$.

Sol^n Comparing with

$$\frac{dy}{dx} = f(x,y)$$

get $f(x,y) = x+y$

at $x_0 = 0$, $y_0 = 1$

by Picard's formula

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\,dx$$

$$y_n = 1 + \int_{0}^{x} (x + y_{n-1})\,dx$$

at $n=1$

$$y_1 = 1 + \int_0^x (x + y_0)\,dx$$

$$= 1 + \int_0^x (x+1)\,dx$$

$$= 1 + \left(\frac{x^2}{2} + x\right)_0^x$$

$$\boxed{y_1 = 1 + x + \frac{x^2}{2}}$$

at $n=2$

$$y_2 = 1 + \int_0^x (x + y_1)\,dx$$

$$= 1 + \int_0^x \left(x + 1 + x + \frac{x^2}{2}\right) dx$$

$$\boxed{y_2 = 1 + x + x^2 + \frac{x^3}{6}}$$

at $n=3$

$$y_3 = 1 + \int_0^x (x + y_2)\,dx$$

$$= 1 + \int_0^x \left(x + 1 + x + x^2 + \frac{x^3}{6}\right) dx$$

$$= 1 + \left(x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}\right)_0^x$$

$$\boxed{y_3 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}}$$

at $n=4$

$$y_4 = 1 + \int_0^x (x + y_3)\,dx$$

$$= 1 + \int_0^x \left(x + 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{24}\right) dx$$

$$y_4 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{120}$$

at $n=5$

$$y_5 = 1 + \int_0^x f(x, y_4)\, dx$$

$$= 1 + \int_0^x \left(1 + x + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{120}\right)$$

$$y_6 = 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \frac{x^5}{60} + \frac{x^6}{720}$$

$\Rightarrow$ at $x = 0.2$, $y = 1.249805$

Ques Use Picard's method to obtain sol^n of the diff. eq^n. $\frac{d^2y}{dx^2} = x^2 + y^2$ correct to 4 decimal places at $x = 0.4$, given that $y = 0$ when $x = 0$

Sol^n Comparing with

$$\frac{dy}{dx} = f(x, y)$$

get $f(x, y) = x^2 + y^2$

at $x_0 = 0$, $y_0 = 0$

by Picard's formula

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\, dx$$

$$= 0 + \int_0^x f(x, y_0)\, dx$$

$$= 0 + \int_0^x (x^2 + y_0^2)\, dx$$

$$= 0 + \int_0^x (x^2 + 0)\, dx$$

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$$y_n = \int_0^x (x^2 + y_{n-1}^2)\, dx$$

at $n=1$

$$y_1 = \int_0^x (x^2 + y_0^2)\, dx$$

$$= \int_0^x (x^2 + 0)\, dx$$

$$= \left(\frac{x^3}{3}\right)_0^x$$

$$\boxed{y_1 = \frac{x^3}{3}}$$

at $n=2$

$$y_2 = \int_0^x (x^2 + y_1^2)\, dx$$

$$= \int_0^x \left(x^2 + \left(\frac{x^3}{3}\right)^2\right) dx$$

$$= \int_0^x \left(x^2 + \frac{x^6}{9}\right) dx$$

$$y_2 = \frac{x^3}{3} + \frac{x^7}{63}$$

at $x = 0.4$, $y = 0.0213\,59$
$= 0.0214$

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Picard's method for the solⁿ of simultaneous differential eqⁿ.

The given eqⁿ is in the form

$$\frac{dy}{dx} = f(x,y,z)$$

$$\frac{dz}{dx} = \phi(x,y,z)$$

with at $x = x_0$

$y = y_0$ and $z = z_0$

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0, z_0)\, dx$$

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1, z_1)\, dx$$

$$z_1 = z_0 + \int_{x_0}^{x} \phi(x, y_0, z_0)\, dx$$

$$z_2 = z_0 + \int_{x_0}^{x} \phi(x, y_1, z_1)\, dx$$

Ques Use Picard's method and find the approximate value of y and z corresponding to $x = 0.1$ given that $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$

$y(0) = 2$, $z(0) = 1$

Solⁿ Comparing ~~with~~ we get,

$$f(x,y,z) = x + z$$

$$\phi(x,y,z) = x - y^2$$

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$x_0 = 0 , \quad y_0 = 2 , \quad z_0 = 1$

By Picard's Method

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0, z_0)\, dx$$

$$= 2 + \int_0^x (x + z_0)\, dx$$

$$= 2 + \int_0^x (x+1)\, dx$$

$$= 2 + \left(\frac{x^2}{2} + x\right)_0^x$$

$$\boxed{y_1 = 2 + x + \frac{x^2}{2}}$$

$$z_1 = z_0 + \int_{x_0}^{x} \phi(x, y_0, z_0)\, dx$$

$$= 1 + \int_0^x (x - y_0^2)\, dx$$

$$= 1 + \int_0^x (x-4)\, dx$$

$$= 1 + \left(\frac{x^2}{2} - 4x\right)_0^x$$

$$= 1 - 4x + \frac{x^2}{2}$$

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1, z_1)\, dx$$

$$= 2 + \int_0^x (x + z_1)\, dx$$

$$= 2 + \int_0^x \left(x + 1 - 4x + \frac{x^2}{2}\right) dx$$

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$$= 2 + \left(x - \frac{3x^2}{2} + \frac{x^3}{6}\right)_0^x$$

$$y_2 = 2 + x - \frac{3}{2}x^2 + \frac{x^3}{6}$$

$$z_2 = z_0 + \int_{x_0}^{x} \phi(x, y_1, z_1)\,dx$$

$$= z_0 + \int_{x_0}^{x} (x - y_1^2)\,dx$$

$$= 1 + \int_0^x \left(x - \left(2 + x + \frac{x^2}{2}\right)^2\right)dx$$

$$= 1 + \int_0^x \left(x - \left(4 + x^2 + \frac{x^4}{4} + 4x + x^3 + x^2\right)\right)dx$$

$$= 1 + \int_0^x \left[x - 4 - x^2 - \frac{x^4}{4} - 4x - x^2 - x^3\right]dx$$

$$= 1 + \left[-4x - \frac{3x^2}{2} - \frac{2x^3}{3} - \frac{x^4}{4} - \frac{x^5}{20}\right.$$

<u>at $x = 0.1$</u>

$y_1 = 2.105$ $\qquad z_1 = 0.605$

$y_2 = 2.08516$ $\qquad z_2 = 0.58430$

Method -2

Taylor's Series Method

$$y = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0'''$$
$$+ \frac{(x-x_0)^4}{4!}y_0^{(iv)} + \frac{(x-x_0)^5}{5!}y_0^{(v)} + \cdots$$

Ques Find by Taylor's series method the values of y and $x = 0.1$ and $x = 0.2$ upto 5 places of decimal given that $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$

Sol^n $y' = x^2y - 1$, $x_0 = 0$, $y_0 = 1$

$y_0 = 1$

$y' = x^2y - 1 \longrightarrow y_0' = -1$ putting $x = 0$

$y'' = 2xy + x^2y' \longrightarrow y_0'' = 0$

$y''' = 2y + 2xy' + 2xy' + x^2y'' \longrightarrow y_0''' = 2$

$y^{iv} = 2y' + 4y' + 4xy'' + 2xy'' + x^2y''' \longrightarrow y_0^{iv} = -6$

$y^v = 6y'' + 6y'' + 6xy''' + 2xy''' + x^2y^{iv}$, $y_0^v = 0$

By Taloy's Series Method

$$y = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0'''$$
$$+ \frac{(x-x_0)^4}{4!}y_0^{IV} + \frac{(x-x_0)^5}{5!}y_0^{v} + \cdots$$

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$$= 1 + (x-0)(-1) + \frac{(x-0)^2}{2!}(0) +$$

$$\frac{(x-0)^3}{3!}(2) + \frac{(x-0)^4}{4!}(-6) +$$

$$\frac{(x-0)^5}{5!}\cdot(0)$$

$$= 1 - x + 0 + \frac{2x^3}{6}$$

$$\frac{-6x^4}{24} + 0$$

at $\boxed{x = 0.1}$

$$y = 1 - (0.1) + \frac{2(0.1)^3}{6} - \frac{6(0.1)^4}{24}$$

$$y = 0.900308$$

at $\boxed{x = 0.2}$

$$y = 1 - (0.2) + \frac{2(0.2)^3}{6} - \frac{6(0.2)^4}{24}$$

$$= 0.802266$$

Ques Apply Taylor's series method to obtain approximate value of y at $x = 0.2$ for the diff. eq^n $\frac{dy}{dx} = 2y + 3e^x$ and $y(0) = 0$

upto 4 derivative

Sol^n $y' = 2y + 3e^x$, $x_0 = 0$, $y_0 = 0$

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$y_0 = 0$

$y' = 2y + 3e^x \longrightarrow y'_0 = 3$

$y'' = 2y' + 3e^x \longrightarrow y''_0 = 9$

$y''' = 2y'' + 3e^x \longrightarrow y'''_0 = 21$

$y^{iv} = 2y''' + 3e^x \longrightarrow y^{iv}_0 = 45$

$y^{v} = 2y^{iv} + 3e^x \longrightarrow y^{v}_0 = 93$

By Taylor's method

$$y = y_0 + (x-x_0)y'_0 + \frac{(x-x_0)^2}{2!}y''_0 + \frac{(x-x_0)^3}{3!}y'''_0 + \frac{(x-x_0)^4}{4!}y^{iv}_0 + \frac{(x-x_0)^5}{5!}y^{v}_0$$

$$= 0 + (x \times 3) + \frac{x^2 \times 9}{2} + \frac{x^3 \times \cancel{21}\,7}{\cancel{6}\,2} + \frac{x^4 \times 45}{24} + \frac{x^5 \times 93}{120}$$

$$= 0 + 3x + \frac{9x^2}{2} + \frac{7x^3}{2} + \frac{15x^4}{8} + \frac{31x^5}{40}$$

at $x = 0.2$

$= 0.811248$

Method - 3

Euler's Method

$$h = \frac{x_i - x_0}{n}$$

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$$

Ques. Use Euler's method and find the approximate value of y corresponding to x = 1. Given that —

$\frac{dy}{dx} = x+y$ and at $x=0$ & $y=1$

Sol^n $f(x,y) = x+y$ $\quad x_0 = 0,\ y_0 = 1$

taking $n = 10$, $h = \frac{x - x_0}{10} = \frac{1-0}{10} = 0.1$

	x	y	$f(x,y) = x+y$	$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$
x_0	0	1	1	$y_1 = 1 + 0.1(1) = 1.1$
x_1	0.1	1.1	1.2	$y_2 = 1.1 + 0.1(1.2) = 1.22$
x_2	0.2	1.22	1.42	$y_3 = 1.22 + 0.1(1.42) = 1.362$
x_3	0.3	1.362	1.662	$y_4 = 1.362 + 0.1(1.662) = 1.5282$
x_4	0.4	1.5282	1.9282	$y_5 = 1.5282 + 0.1(1.9282) = 1.72102$
x_5	0.5	1.72102	2.22102	$y_6 = 1.72102 + 0.1(2.22102) = 1.943122$
x_6	0.6	1.943122	2.543122	$y_7 = 1.943122 + 0.1(2.543122) = 2.1974342$
x_7	0.7	2.1974342	2.8974342	$y_8 = 2.1974342 + 0.1(2.8974342) = 2.48717762$ ~~2.8974342~~
x_8	0.8	2.48717762 ~~2.8974342~~	3.28717762	$y_9 =$ ~~2.887~~ 2.815895 ~~3.226151962~~
x_9	0.9	2.815895 ~~3.226151962~~	3.715895	$y_{10} = 3.187484$
x_1	1	$\boxed{3.187484}$		

Ques-5 $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y=1$ at $x=0$. find y for $x=0.1$

Soln $f(x,y) = \frac{y-x}{y+x}$, $x_0=0$, $y_0=1$ $x=0.1$

taking $n=5$, $h = \frac{0.1-0}{5} = 0.02$

x	y	$f(x,y) = \frac{y-x}{y+x}$	$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1})$
0	1	1	$y_1 = 1+0.02(1) = 1.02$
0.02	1.02	0.9615	$y_2 = 1.02+0.02(0.9615) = 1.0392$
0.04	1.0392	0.9258	$y_3 = 1.0577$
0.06	1.0577	0.8926	$y_4 = 1.0755$
0.08	1.0755	~~1.4495~~ 0.86157	$y_5 =$ ~~1.78399~~ 1.09273
0.1 ~~0.0~~	~~1.7839~~ 1.09273	~~0.8938~~	~~$y_6 = 1.801$~~

Modified Euler's Method

$$\frac{dy}{dx} = f(x,y)$$

at $x = x_0$, $y = y_0$

then ~~find~~ first find y_1 by odd formula

$$y_1 = y_0 + hf(x_0, y_0)$$

After that modify it

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,y_1)\right]$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1,(y_1)^{(1)})\right]$$

$\vdots$

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Take, $y_1 = y_1^{(k)}$

then find $y_2 = y_1 + hf(x_1, y_1)$

then Modify it —

$$y_2^{(1)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2)\right]$$

repeate this process.

Ques Use modified Euler's method and find an approximate value of y at $x = 0.3$ given that $\frac{dy}{dx} = x+y$

and $x=0$, $y=1$

Sol^n

$f(x, y) = x+y$

$x_0 = 0$, $y_0 = 1$

$x = 0.3$

taking $n = 3$

$$h = \frac{0.3 - 0}{3} = 0.1$$

For y_1

$y_1 = y_0 + hf(x_0, y_0)$

$y_1 = 1 + 0.1(x_0 + y_0)$

$y_1 = 1.1$

Now modify it —

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$$

$$= 1 + \frac{0.1}{2}\left[(0+1) + (0.1 + 1.1)\right]$$

$$= 1.11$$

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$$y_1^{(2)} = y_0 + \frac{h}{2}\left[\cancel{2y}\, f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + \frac{0.1}{2}\left[(0+1) + (0.1 + 1.11)\right]$$

$$= \cancel{Kd}\ 1.1105$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + \frac{0.1}{2}\left[(0+1) + (0.1 + 1.1105)\right]$$

$$= 1.1105$$

Taking $y_1 = 1.1105$

For y_2

$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_1, y_1)\right]$$

$$= 1.1105 + \frac{0.1}{2}\left[(0.1 + 1.1105) + 0.1 -\right.$$

$$y_2 = y_1 + hf(x_1, y_1)$$

$$= 1.1105 + 0.1\left[0.1 + 1.1105\right]$$

$$= 1.2315$$

$$y_2^{(1)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2)\right]$$

$$= 1.1105 + \frac{0.1}{2}\left[(0.1 + 1.1105) + (0.2 + 1.2315)\right]$$

$$= 1.2426$$

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$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$

$$= \cancel{1.0}\ 1.1105 + \frac{0.1}{2}\left[(0.1 + 1.1105) + (0.2 + 1.2426)\right]$$

$$= 1.2431$$

$$y_2^{(3)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(2)})\right]$$

$$= 1.1105 + \frac{0.1}{2}\left[(0.1 + 1.1105) + (0.2 + 1.2431)\right]$$

$$= 1.2431$$

Taking $y_2 = 1.2431$

For y_3

$$y_3 = y_2 + hf(x_2, y_2)$$
$$= 1.2431 + 0.1\,(0.2 + 1.2431)$$
$$= 1.3874$$

For $y_3^{(1)}$

$$y_3^{(1)} = y_2 + \frac{h}{2}\left[f(x_2, y_2) + f(x_3 + y_3)\right]$$

$$= 1.2431 + \frac{0.1}{2}\left[(0.2 + 1.2431) + (0.3 + 1.3874)\right]$$

$$= 1.3996$$

$$\cancel{y_3^{(2)} = y_2 + \frac{h}{2}\left[f(x_2 + y_2 + f(x_3 + y_3^{(1)})\right]}$$

$$= 1.2431 + \frac{0.1}{2}\left[(0.2 + 1.2431) + (0.3 + 1.3946) \cancel{+ 1.3874}\right]$$

$$= 1.3999$$

$$y_3^{(2)} = 1.2431 + \frac{0.1}{2}\left[(0.2 + 1.2431) + (0.3 + 1.3999)\right]$$

$$= 1.4002$$

$$y_3^{(3)} = 1.2431 + \frac{0.1}{2}\left[(0.2 + 1.2431) + (0.3 + 1.4002)\right]$$

$$= 1.4002$$

Milne's Predictor and Corrector Method

$\frac{dy}{dx} = f(x, y)$, at $x = x_0$, $y = y_0$

First we find y_1, y_2, y_3 correspon-ding to x_1, x_2, x_3 by using Picard's or Taylor's series method.

Then f_0, f_1, f_2, f_3 $\quad [f_0 = f(x_0, y_0)$

Then we use Milne's predictor formula.

$$y_4 = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]$$

also find $f_4 = f(x_4, y_4)$

After that we correct this value

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by corrector formula.

$$y_4 = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4\right]$$

and repeat the corrector formula untill y_4 remains unchange.

Ques Apply Milne's method to find the sol^n of diff. eq^n $y' = x - y^2$ in the range ~~$x \geq 0$~~ $0 \leq x \leq 1$ at $x = 0$, $y = 0$

Sol^n given : $f(x,y) = x - y^2$

$x_0 = 0$, $y_0 = 0$

by Picard's formula

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\, dx$$

$$y_1 = \int_0^x f(x, y_0)\, dx = \int_0^x (x - 0^2)\, dx$$

$$= \frac{x^2}{2}$$

$$y_2 = \int_0^x f(x, y_1)\, dx = \int_0^x \left(x - \frac{x^4}{4}\right) dx$$

$$= \frac{x^2}{2} - \frac{x^5}{20}$$

$$y_3 = \int_0^x f(x, y_2)\, dx = \int_0^x \left(x - \left(\frac{x^2}{2} - \frac{x^5}{2}\right)^2\right) dx$$

$$= \int_0^x \left[x - \frac{x^4}{4} + \frac{x^7}{20} - \frac{x^{10}}{400}\right] dx$$

$$\boxed{y = \frac{x^2}{2} - \frac{x^5}{20} + \frac{x^8}{160} - \frac{x^{11}}{4400}}$$ (1)

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taking $n = 5$

$h = \cancel{1-0} \quad \frac{x - x_0}{n} = \frac{1-0}{5} = 0.2$

	x	y	$f(x,y) = x - y^2$
$x_0 =$	0	$y_0 = 0$	$f_0 = 0$
$x_1 =$	0.2	$y_1 = 0.0199$	$f_1 = 0.1996$
$x_2 =$	0.4	$y_2 = 0.0794$	$f_2 = 0.3937$
$x_3 =$	0.6	$y_3 = 0.1762$	$f_3 = 0.5689$
$\cancel{x_4 =}$			

Now, at $x = 0.8$

$$y_4 = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]$$

after this at 0.8 & 1.0 we'll predict and correct

$$y_4 = 0.3049$$

$$f_4 = f(x_4, y_4) = x - y^2 = 0.8 - (0.3049)^2$$
$$= 0.7070$$

Now we use the ~~chara~~ corrector formula

$$y_4 = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4\right]$$

$$y_4 = \underline{0.3044}$$

$$f_4 = 0.8 - (0.3044)^2$$
$$f_4 = 0.7073$$

again

$$y_4 = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4\right]$$
$$y_4 = \underline{0.3045}$$

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taking $y_4 = 0.3045$, $f_4 = 0.7072$

now, at $x_5 = 1.0$

$$y_5 = y_1 + \frac{4h}{3}\left[2f_2 - f_3 + 2f_4\right]$$

$$= 0.4553$$

$$f_5 = f(x_5, y_5)$$
$$= 0.7927$$

by corrector's formula

$$y_5 = y_3 + \frac{h}{3}\left[f_3 + 4f_4 + f_5\right]$$

$$y_5 = 0.4555$$

$$f_5 = f(x_5, y_5) = 0.7925$$

again, $y_5 = y_3 + \frac{h}{3}\left[f_3 + 4f_4 + f_5\right]$

$$y_5 = 0.4555$$

Ques Solve by Milne's method given that $\frac{dy}{dx} = 1 + xy^2$, at $y(0) = 1$.

find y at $x = 0.4$

Given that :

x :	0.1	0.2	0.3
y :	1.105	1.223	1.355
$f(x,y)$:	1.1221	1.2991	1.5508

$h = 0.1$

at $x = 0.4$

$$y_4 = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]$$

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$$* = 1.5395$$

$$f_4 = f(x_4, y_4) = 1 + 0.4 \times (1.5395)^2$$
$$= 1.9480$$

Correcting :

again, $y_4 = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

$$= 1.5380$$

$$f_4 = 1.9461$$

$\rightarrow$ $y_4 = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

$$y_4 = \underline{1.5379}$$
$$f_4 = 1 + 0.4 \times (1.5379)^2$$
$$= 1.9460$$

$\rightarrow$ $y_4 = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4]$

$$y_4 = \underline{\underline{1.5379}}$$