Lesson 2: Space Mission Geometry

Dr. Andrew Ketsdever

MAE 5595

Space Mission Geometry

- Must specify an appropriate coordinate system
  - Earth fixed
  - Spacecraft fixed
  - Other
- Coordinate system must have a specified
  - Origin
  - Fixed point

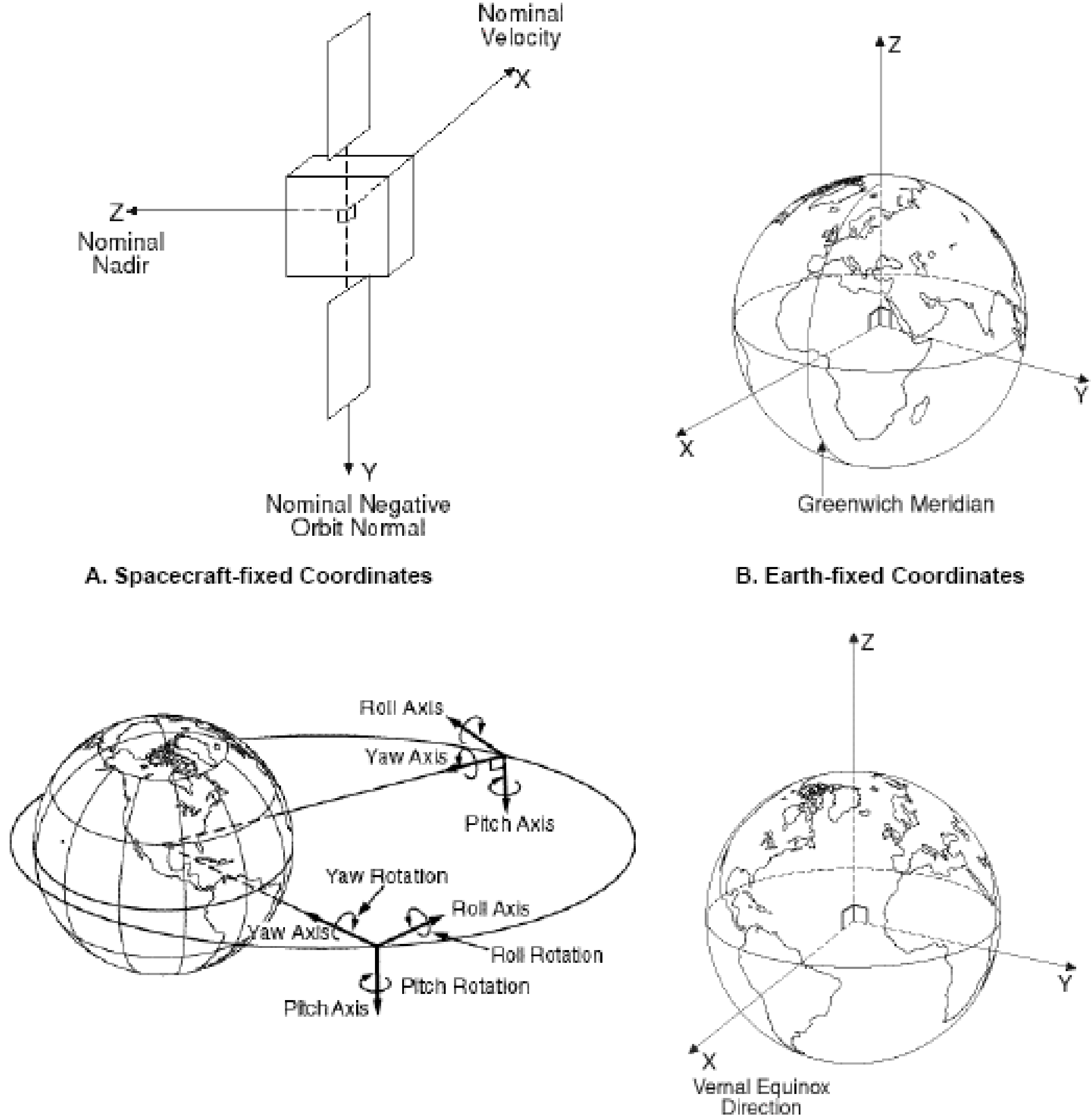
TABLE 5-1. Common Coordinate **Systems Used in Space Applications.** Also see Fig. 5-1.

Coordinate Name	Fixed with Respect to	Center	Z-axis or Pole	X-axis or Ref. Point	Applications
Celestial (Inertial)	Inertial space*	Earth† or spacecraft	Celestial pole	Vernal equinox	Orbit analysis, astronomy, inertial motion
Earth-fixed	Earth	Earth	Earth pole = celestial pole	Greenwich meridian	Geolocation, apparent satellite motion
Spacecraft-fixed	Spacecraft	Defined by engineering drawings	Spacecraft axis toward nadir	Spacecraft axis in direction of velocity vector	Position and orientation of spacecraft instruments
Local Horizontal‡	Orbit	Spacecraft	Nadir	Perpendicular to nadir toward velocity vector	Earth observations, attitude maneuvers
Ecliptic	Inertial space	Sun	Ecliptic pole	Vernal equinox	Solar system orbits, lunar/solar ephemerides

* Actually rotating slowly with respect to inertial space. See text for discussion.

† Earth-centered inertial coordinates are frequently called *GCI (Geocentric Inertial)*.

‡ Also called *LVLH (Local Vertical/Local Horizontal)*, *RPY (Roll, Pitch, Yaw)*, or *Local Tangent Coordinates*.



A. Spacecraft-fixed Coordinates

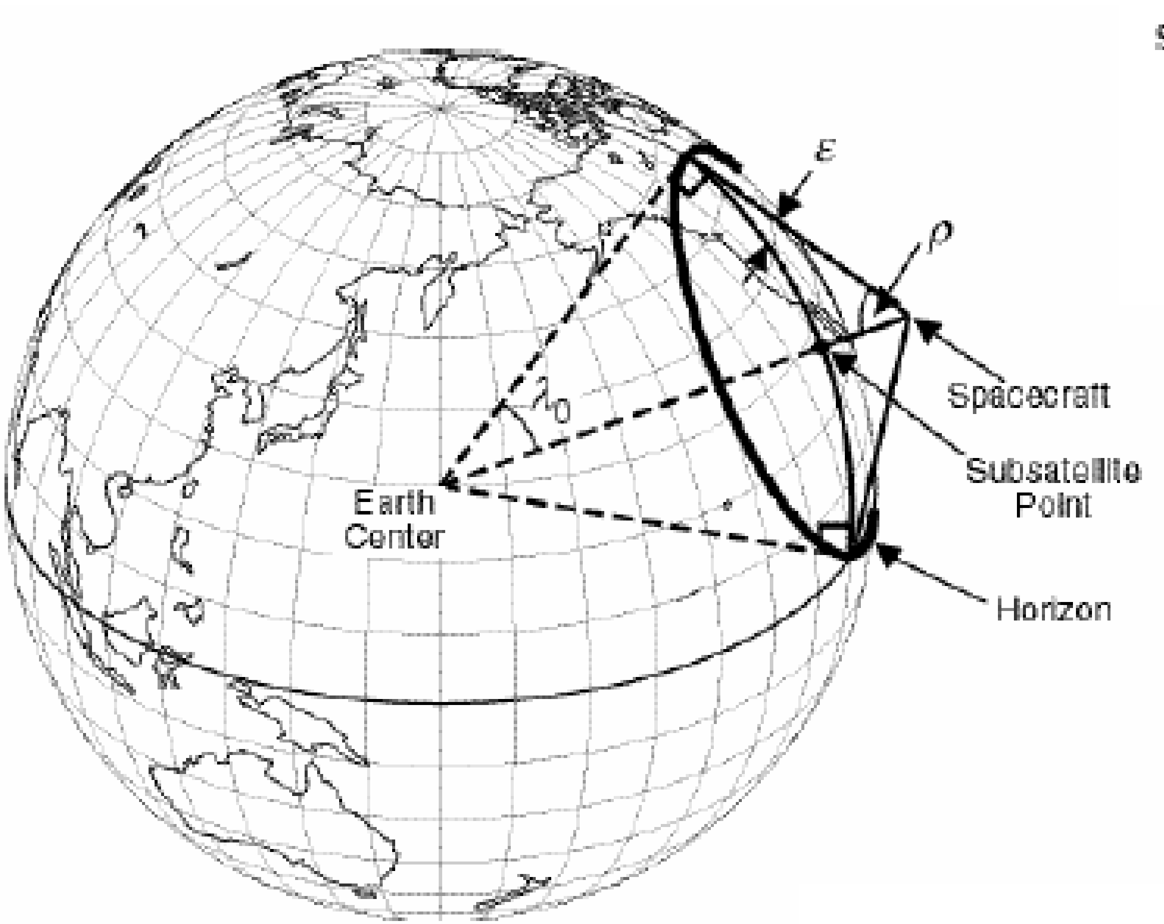
B. Earth-fixed Coordinates

C. Roll, Pitch, and Yaw (RPY) Coordinates

D. Celestial Coordinates

g. 5-1. **Coordinate Systems in Common Use.** See Table 5-1 for characteristics.

Earth Geometry Viewed from Space

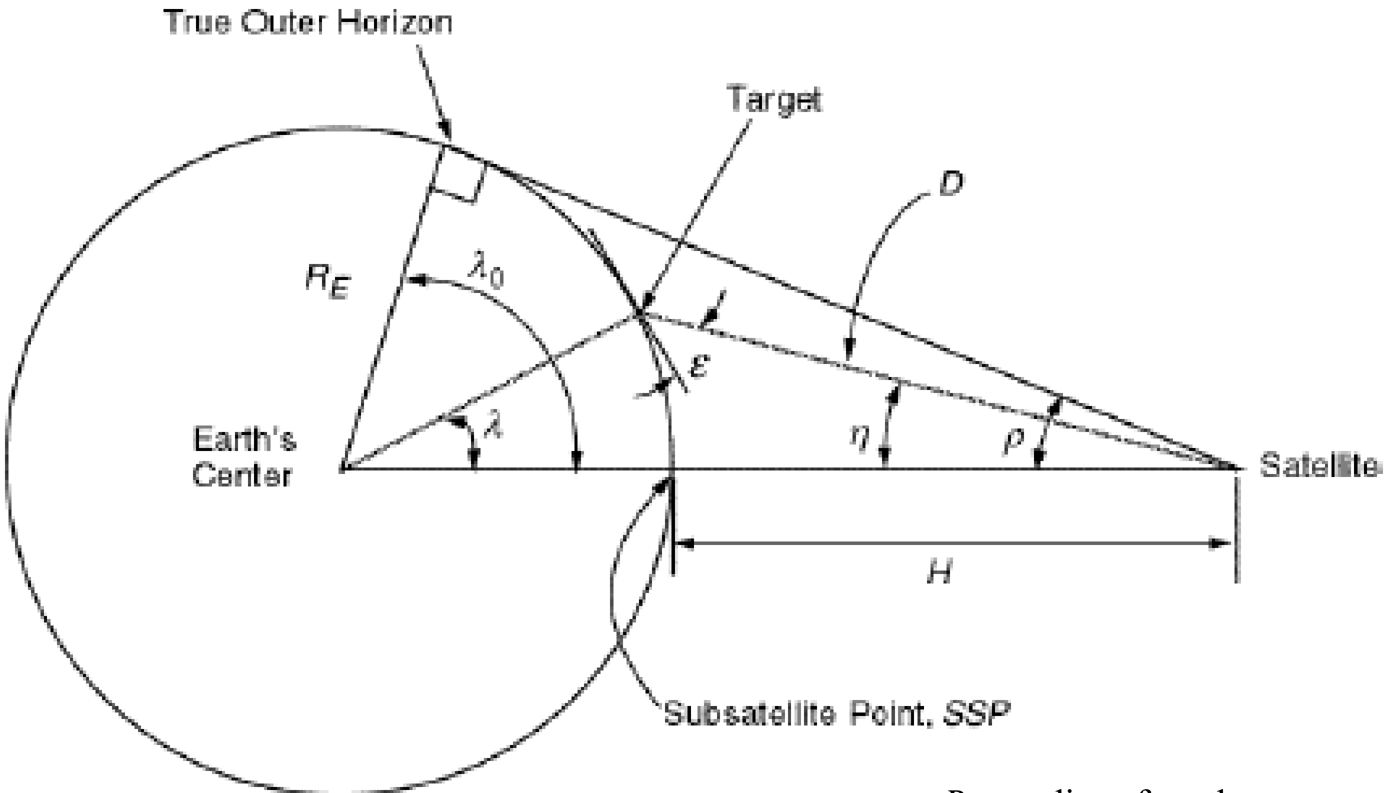


$$\sin \rho = \cos \lambda_0 = \frac{R_E}{R_E + H}$$

$$\rho + \lambda_0 = 90 \text{ deg}$$

$$D_{max} = [(R_E + H)^2 - R_E^{\ 2}]^{1/2} = R_E \tan \lambda_0$$

Mission Viewing Geometry



R_E = radius of earth

H = altitude of satellite

D = slant range to target

η = angle from nadir

λ = earth central angle (Lat, Long, or any combination)

ρ = angular earth radius (defines boundary of footprint)

λ_0 = maximum earth angle seen by satellite

ε = satellite elevation angle or grazing angle.

$$\sin \rho = \cos \lambda_0 = \frac{R_\oplus}{R_\oplus + H}$$

$$\tan \eta = \frac{\sin \rho \sin \lambda}{1 - \sin \rho \cos \lambda}$$

$$\cos \varepsilon = \frac{\sin \eta}{\sin \rho}$$

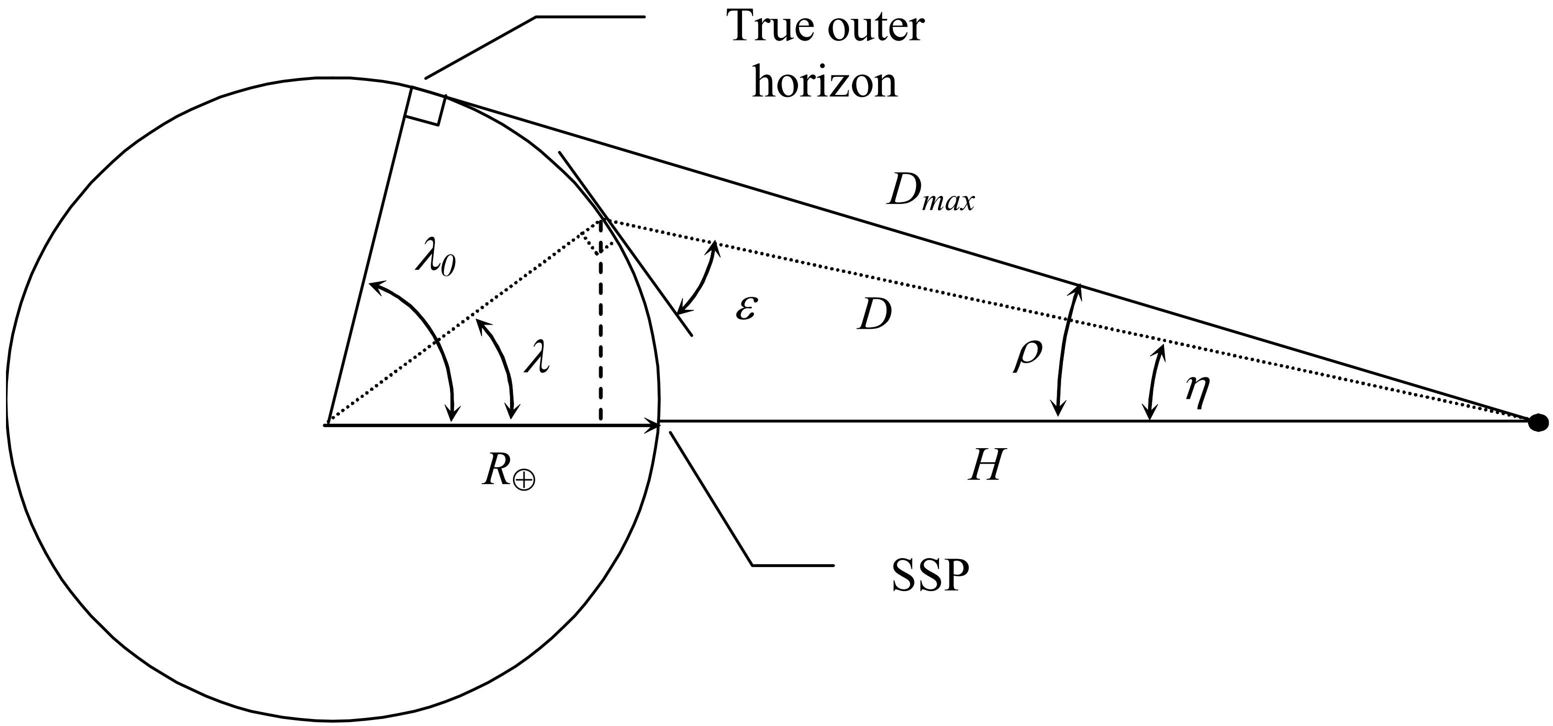
$$\eta + \lambda + \varepsilon = \frac{\pi}{2}$$

$$R_E \sin \lambda = D \sin \eta$$

$$D_{\max} = \sqrt{\left(R_\oplus + H\right)^2 - R_\oplus^2}$$

$$D = \sqrt{R_\oplus^2 + \left(R_\oplus + H\right)^2 - 2R_\oplus\left(R_\oplus + H\right)\cos \lambda}$$

Special Cases

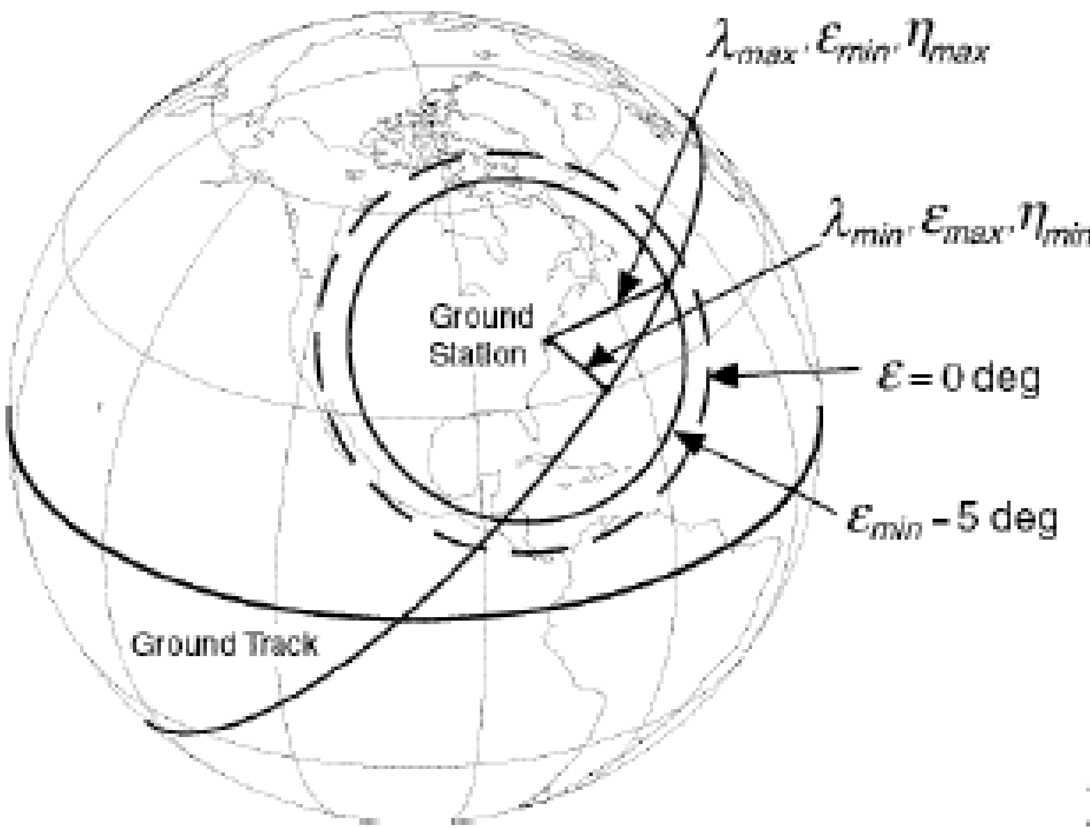


Note,

- If point of interest is SSP: $\eta = 0, \varepsilon = 90^\circ, \lambda = 0$

- If point of interest is true outer horizon: $\eta = \rho, \varepsilon = 0, \lambda = 90^\circ - \rho$

Satellite Motion for an Earth Observer



$$\sin \eta_{max} = \sin \rho \cos \varepsilon_{min}$$

$$\lambda_{max} = 90 \text{ deg} - \varepsilon_{min} - \eta_{max}$$

$$D_{max} = R_E \frac{\sin \lambda_{max}}{\sin \eta_{max}}$$

Circular, LEO

Total Time in View

For a LEO satellite, assume brief pass (ignore Earth's rotation). The time in view, *T*, for the satellite is given by:

$$T = \left(\frac{Period}{180^o}\right)\cos^{-1}\left(\frac{\cos\lambda_{\max}}{\cos\lambda_{\min}}\right)$$

$$T_{\max} = Period\left(\lambda_{\max}/180\right)$$

$$\lambda_{\max} = 90^\circ - \varepsilon_{\min} - \eta_{\max}$$

$$\sin\eta_{\max} = \sin\rho\cos\varepsilon_{\min}$$

$$\sin\lambda_{\min} = \sin\left(lat_{pole}\right)\sin\left(lat_{GS}\right) + \cos\left(lat_{pole}\right)\cos\left(lat_{GS}\right)\cos\left(\Delta long\right)$$

where:

$lat_{pole} = 90^\circ - i$

lat_{GS} = latitude of ground station

$\Delta long$ = difference in longitude between orbit pole and ground station

$long_{pole} = L_{node} - 90^\circ$

L_{node} = longitude of Earth (geographic, not inertial) where orbit ascending node at time of pass

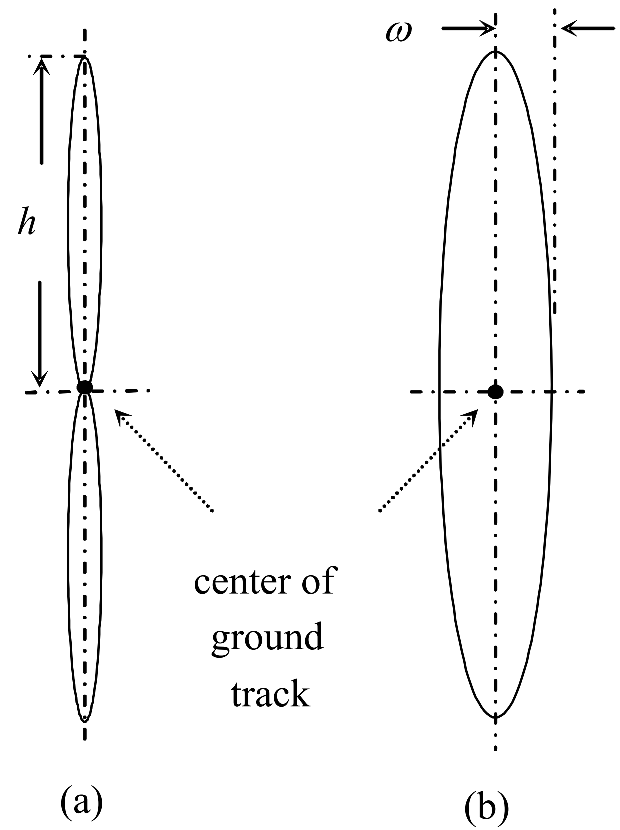
Satellite Slew Rate

- Maximum Slew Rate for a Satellite

$$\dot{\theta}_{\max} = \frac{V_{sat}}{D_{\min}} = \frac{2\pi\left(R_E + H\right)}{Period D_{\min}}$$

$$D_{\min} = R_E \left(\frac{\sin \lambda_{\min}}{\sin \eta_{\min}} \right)$$

Satellite Motion for an Earth Observer



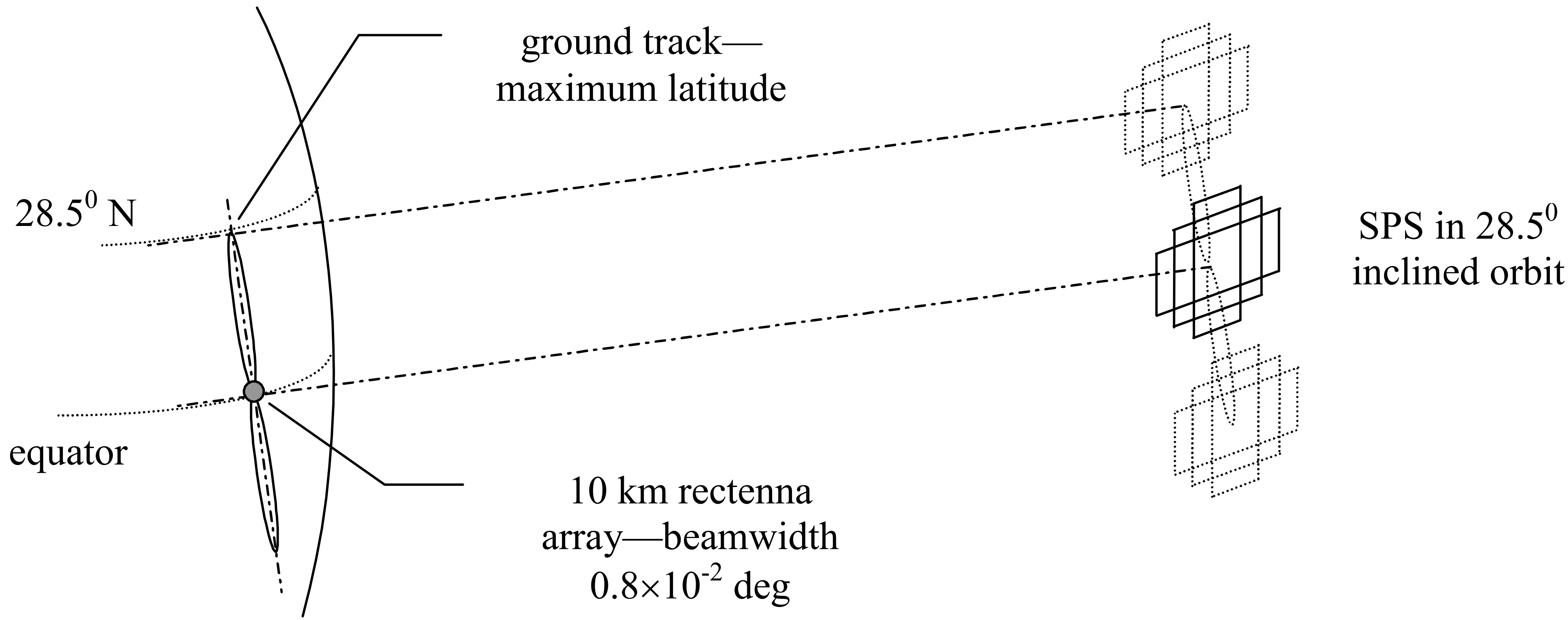
Dominant factor:

(a) Inclination

(b) Eccentricity

GEO

GEO Ground Track

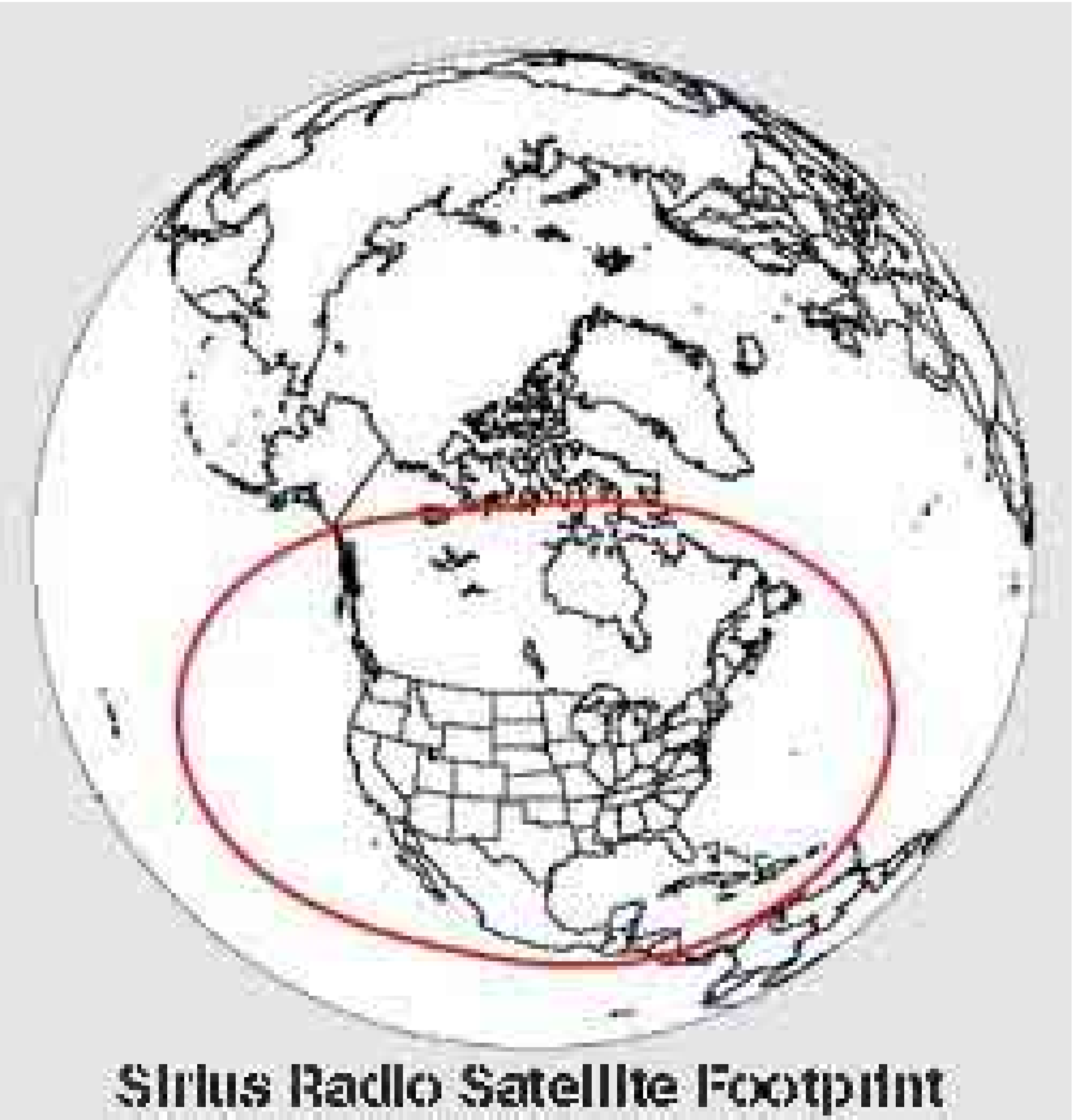


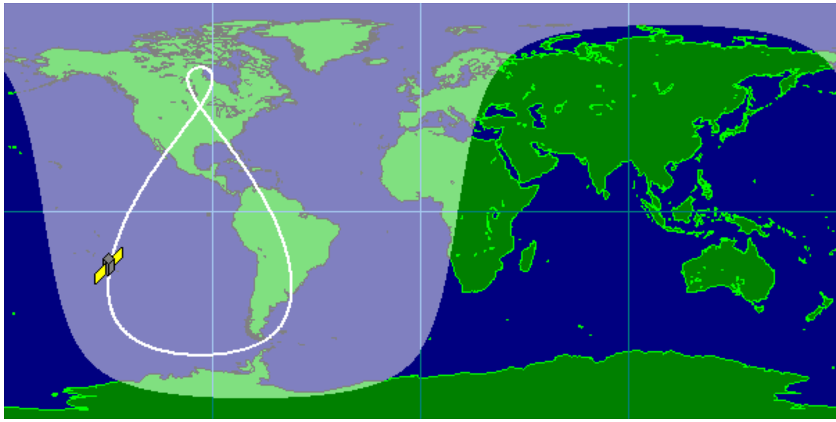
Sirius Radio Satellite Constellation

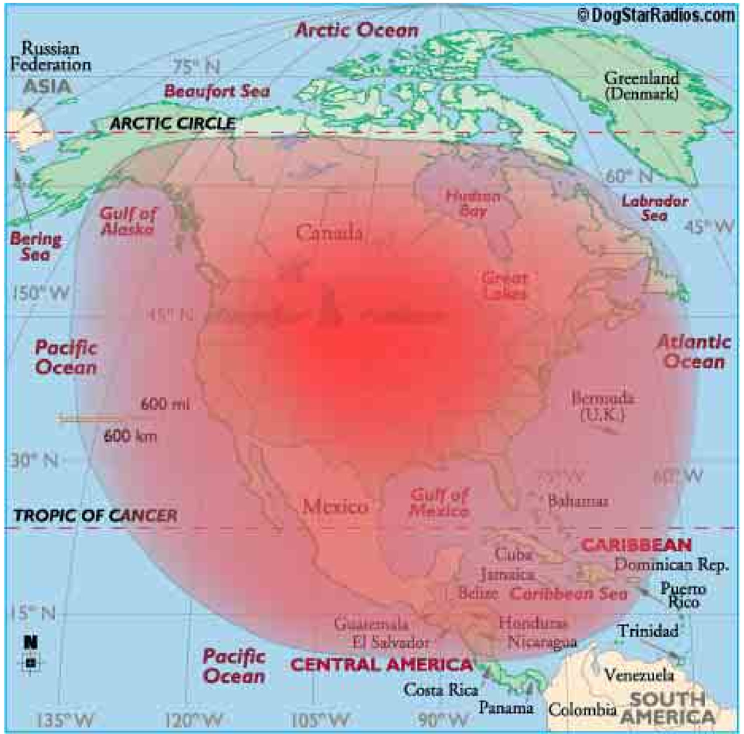
- Mission:
  - Continuous radio broadcasts over North America
  - Radio reception to remote areas
- What mission geometry would you choose?

- Geosynchronous
- 60 deg inclination
- Highly eccentric
- Apogee over N. Hemisphere

Sirius Constellation



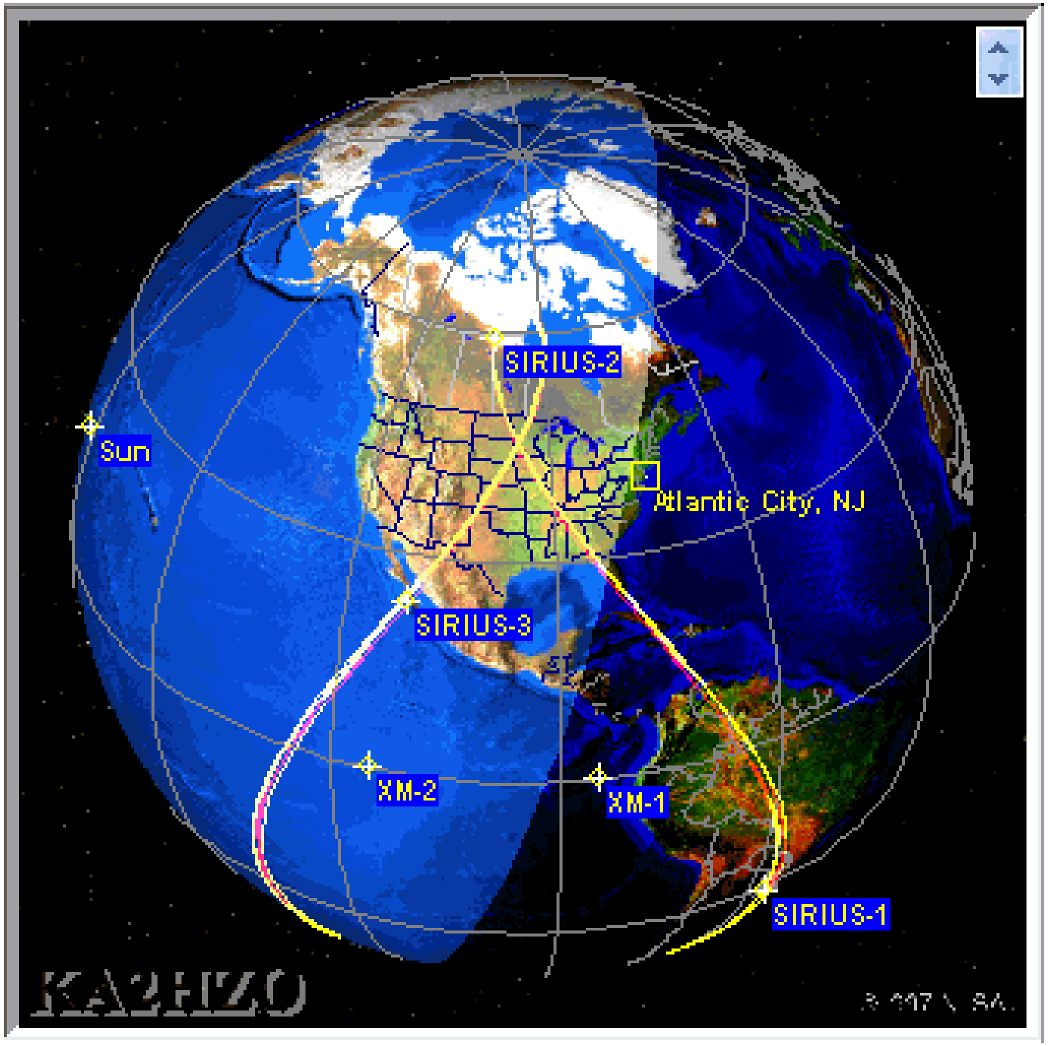




How Did XM Do It?

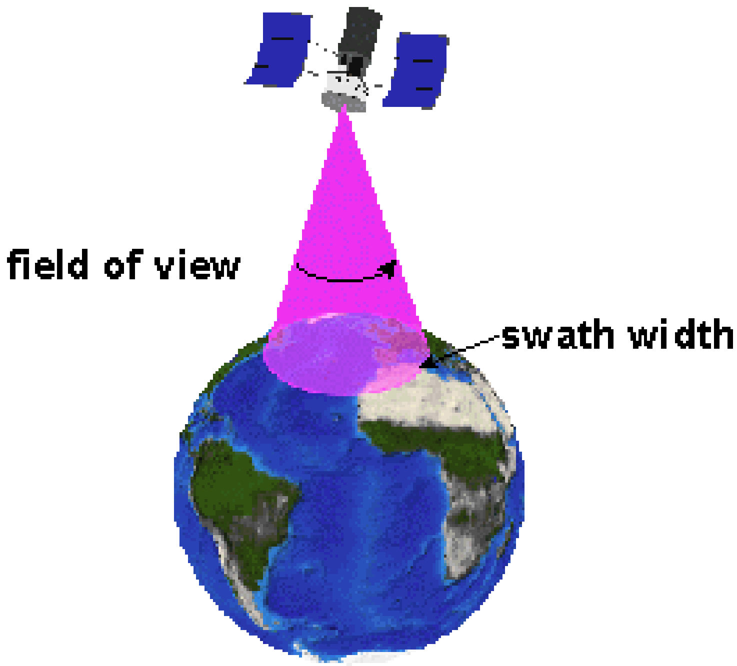
- Named "Rock" and "Roll," XM Radio’s two Boeing HS 702 satellites were placed in parallel geostationary orbit (GEO), 35,764 km above Earth.
- The first XM satellite, "Rock," was launched on March 18, 2001, with "Roll" following on May 8th.

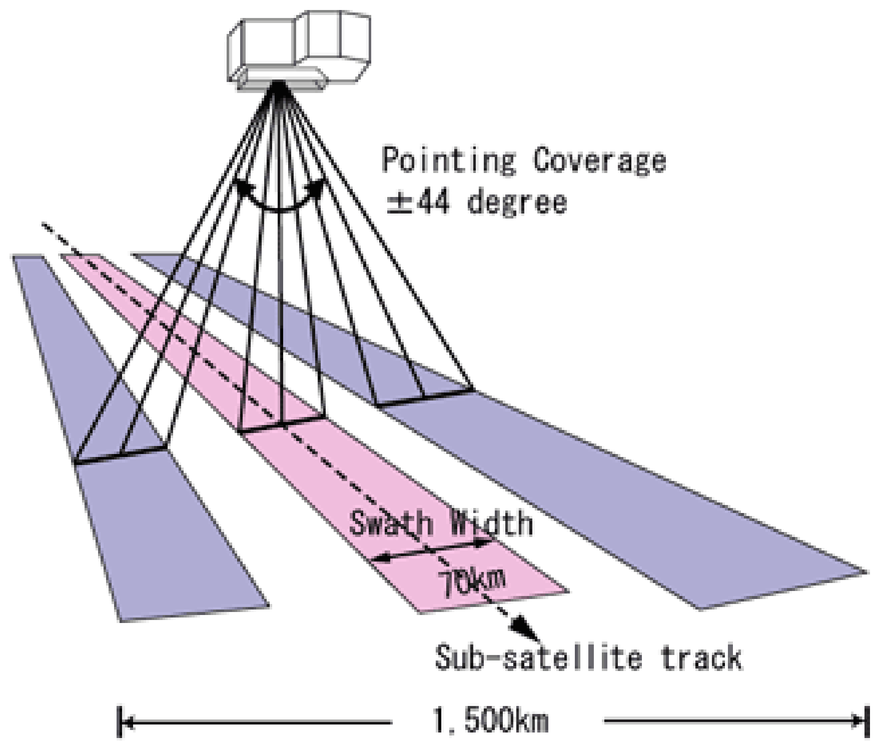
Sirius and XM Constellations



http://www.dogstarradios.com/sirasasecoma.html

Swath Width


field of view
swath width


Pointing Coverage
±44 degree
Swath Width
70km
Sub-satellite track
1,500km

Eclipse

- Knowing the mission geometry leads to a simple calculation for the eclipse time for a satellite (SMAD has a more complicated analysis)

$$\rho = \sin^{-1}\left(\frac{R_E}{R_E + H}\right)$$

$$TE = \frac{2\rho}{360^\circ} xPeriod$$