ASEN 5050
SPACEFLIGHT DYNAMICS
Clohessy-Wiltshire / Hill Equations

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10/1/2013

Announcements

- Samantha Krening from JPL will be here to say a few words.
- Homework #4 is due today. Homework #5 will be released later today (I'll send another email)

- No quiz tomorrow

- Reading: Chapter 2, 6

- Space News:
  – DANDE launched!
  – Cygnus docked with the ISS (note the use of CW/H equations…)
  – Juno's Earth flyby is coming up: Oct 9th!

Credit: NASA/JPL-Caltech

HW 2 reflections

- Problem 1 showed several frequent errors. If you got points knocked off, go figure out why and fix it! Check the answers Josh provided on your HW.
- It is correct to specify an angle outside of the range 0 – 360 deg, but in so doing, be aware of quadrant checks and make sure you're doing quadrant checks properly.
- Velocity in AU/year? Correct – sure, but please specify in SI units because that's how I make the answer sheets. Though it's great to consider alternative units and mention those answers, too.

Quiz #8

Question 1 (1 point)

We made several assumptions when deriving the Clohessy-Wiltshire / Hill Equations. Pick the answer that includes two of these assumptions for the case where we're modeling a rendezvous of two satellites.

- The two satellites are in non-circular orbits and the rendezvous takes about one orbital period's worth of time.
- The two satellites are in ~circular orbits and their relative distance is much smaller than their position vectors.
- The two satellites are in ~circular orbits and one of those orbits is much larger than the other.
- The two satellites are in ~circular orbits that are equatorial.

Quiz #8

Question 2 (1 point)

Recall from the lecture that the solution to the Clohessy-Wiltshire / Hill Equations, assuming neither satellite is thrusting at all, looks like this (where "xdd" = x-double-dot, i.e., acceleration of x, "yd" = y-dot, i.e., velocity of y, etc):

xdd - 2*omega*yd - 3*omega^2*x = 0

ydd + 2*omega*xd = 0

zdd + omega^2*z = 0

Look at the third equation: zdd + omega^2*z = 0

This equation describes what sort of solution?

- ○ Simple harmonic motion. I.e., the satellite's motion in the z-axis oscillates about the reference.
- ○ An exponential decay process, i.e., the satellite's motion in the z-axis asymptotically approaches the reference.
- ○ A linear solution, i.e., the satellite's motion in the z-axis approaches the reference in a linear fashion.

Quiz #8

Question 3 (1 point)

If you had a satellite in a circular, polar orbit (inclination = 90 deg), and you wanted to change the value of the satellite's right ascension of ascending node using the least amount of fuel, where would you perform the maneuver? Assume the Earth's gravity is a point-mass, i.e., two-body equations of motion.

- ○ Over one of the poles.
- ○ At either point when the satellite is crossing the equator.
- ○ Over a latitude of +/- 45 deg.
- ○ It can't be done.

Quiz #8

Question 4 (1 point)

Let's say you have a satellite in the following orbit:

hp: 300 km (perigee altitude)

ha: 1600 km (apogee altitude)

inclination: 0 deg

You wish to raise the satellite's inclination to 10 degrees using the least amount of fuel. Where is the best place to perform the maneuver?

- Anywhere, since the orbit is equatorial.
- At the perigee point.
- $\boxed{\text{At the apogee point.}}$
- Not enough information.

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Prox Ops

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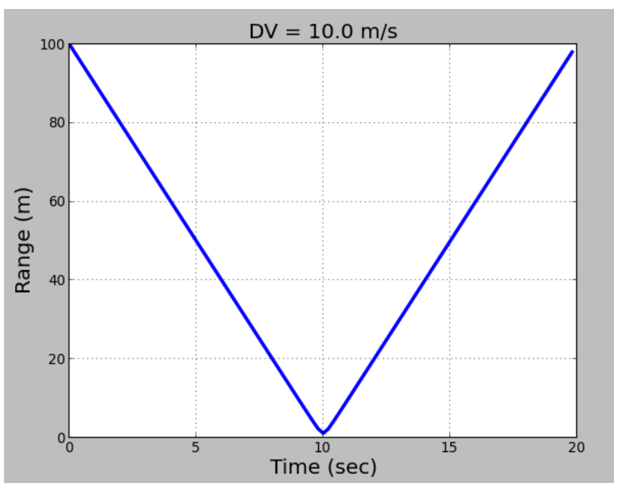
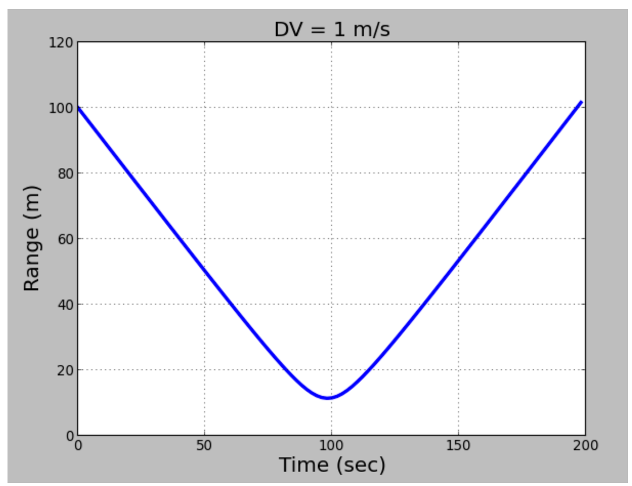
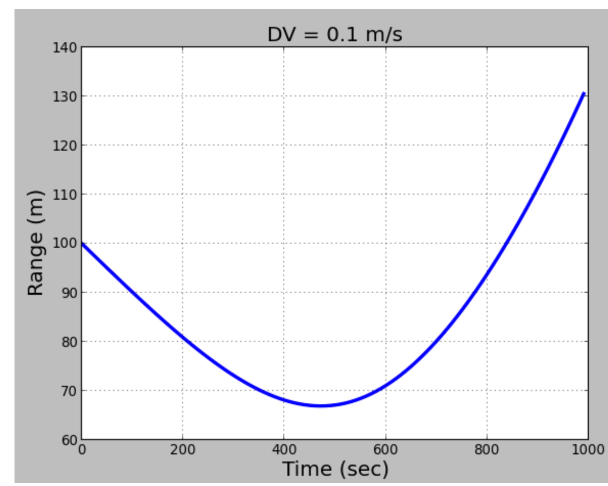
In-Space Prox Ops

- Say you want to close a distance of 100 meters, where the target is in front of us in our orbit.
  - Let’s consider what would happen if we just executed a maneuver in the along-track direction.
  - What happens?







CW / Hill Equations

- Clohessy/Wiltshire (1960)
- And Hill (1878)
- (notice the timeline here – when did Sputnik launch? Gemini was in need of this!)

Coordinate Systems

Satellite Coordinate System (RSW) -- (Radial-Transverse-Normal)

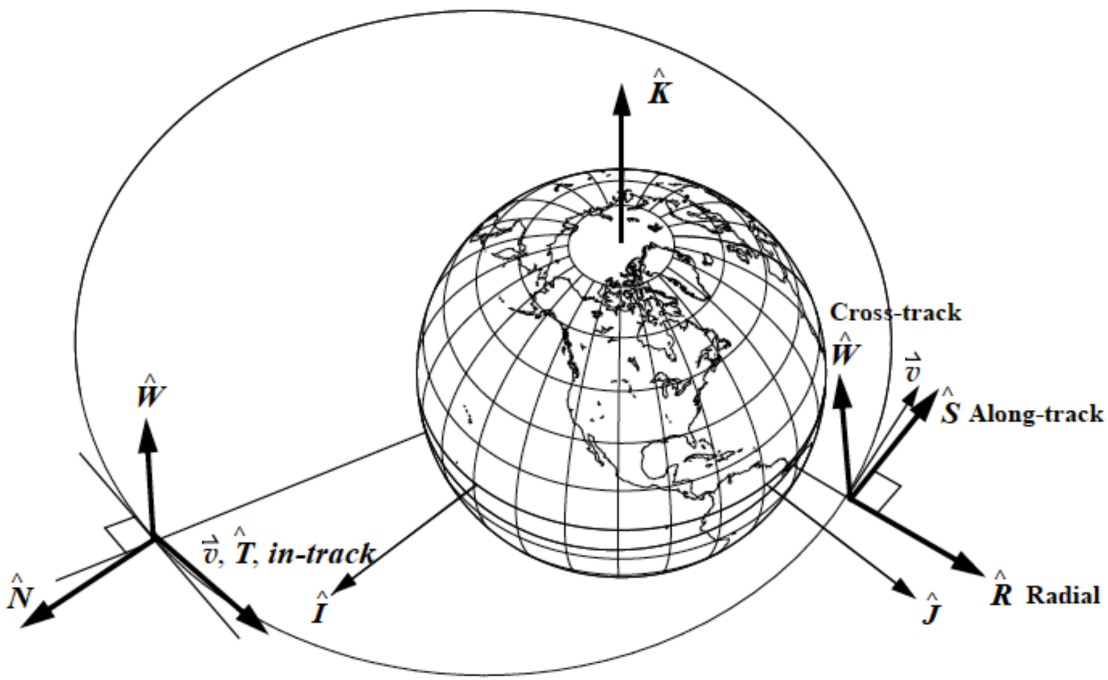


Figure 3-15. Satellite Coordinate Systems, RSW and NTW. These coordinate systems move with the satellite. The R axis points out from the satellite along the geocentric radius vector, the W axis is normal to the orbital plane (*not* usually aligned with the K axis), and the S axis is normal to the position vector and positive in the direction of the velocity vector. The S axis is aligned with the velocity vector *only* for circular orbits. In the NTW system, the T axis is always parallel to the velocity vector. The N axis is normal to the velocity vector and is *not* aligned with the radius vector, except for circular orbits, and at apogee and perigee in elliptical orbits.

Coordinate Transformations

To convert between IJK and PQW:

$$\vec{r}_{IJK} = ROT3(-\Omega)ROT1(-i)ROT3(-\omega)\vec{r}_{PQW}$$

$$\vec{r}_{PQW} = ROT3(\omega)ROT1(i)ROT3(\Omega)\vec{r}_{IJK}$$

To convert between PQW and RSW:

$$\vec{r}_{RSW} = ROT3(\nu)\vec{r}_{PQW}$$

$$\vec{r}_{PQW} = ROT3(-\nu)\vec{r}_{RSW}$$

Thus, RSW → IJK is:

$$\vec{r}_{IJK} = ROT3(-\Omega)ROT1(-i)ROT3(-u)\vec{r}_{RSW}$$

where $u = \nu + \omega$

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

Use RSW coordinate system *(may be different from NASA)*

Target satellite has two-body motion:

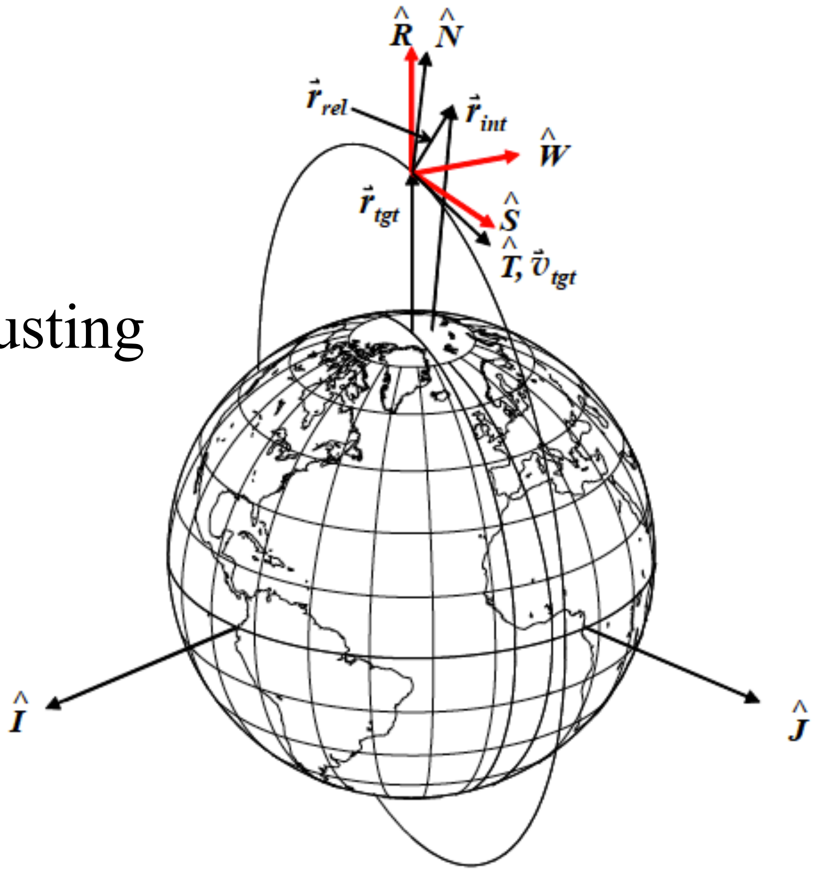
$$\ddot{\vec{r}}_{tgt} = -\frac{\mu\, \vec{r}_{tgt}}{r_{tgt}^{3}}$$

The interceptor is allowed to have thrusting

$$\ddot{\vec{r}}_{int} = -\frac{\mu\, \vec{r}_{int}}{r_{int}^{3}} + \vec{F}$$

Then $\vec{r}_{rel} = \vec{r}_{int} - \vec{r}_{tgt} \quad \Rightarrow \quad \ddot{\vec{r}}_{rel} = \ddot{\vec{r}}_{int} - \ddot{\vec{r}}_{tgt}$

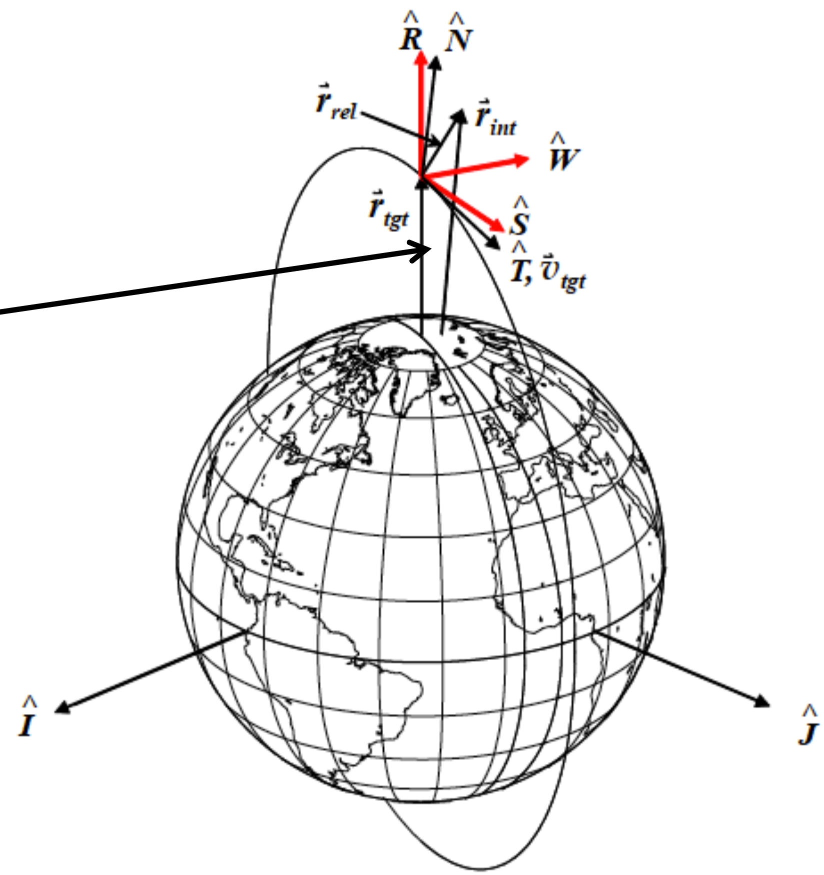
So, $\ddot{\vec{r}}_{rel} = -\dfrac{\mu\, \vec{r}_{int}}{r_{int}^{3}} + \vec{F} + \dfrac{\mu\, \vec{r}_{tgt}}{r_{tgt}^{3}}$



Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

Need more information to solve this:

$$\ddot{\vec{r}}_{rel} = -\frac{\mu\,\vec{r}_{int}}{r_{int}^{3}} + \vec{F} + \frac{\mu\,\vec{r}_{tgt}}{r_{tgt}^{3}}$$



Consider the oblique triangle

We can use the cosine law to find:

$$\frac{\vec{r}_{int}}{r_{int}^{3}} = \frac{\vec{r}_{tgt} + \vec{r}_{rel}}{\left(\underbrace{r_{tgt}^{2} + 2\vec{r}_{tgt}\cdot\vec{r}_{rel} + r_{rel}^{2}}_{r_{\text{int}}^{2} = r_{tgt}^{2} - 2 r_{tgt} r_{rel}\cos\alpha + r_{rel}^{2}}\right)^{3/2}}$$

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

Now, $$\frac{\vec{r}_{int}}{r_{int}^3} = \frac{\vec{r}_{tgt} + \vec{r}_{rel}}{\left(\underbrace{r_{tgt}^2 - 2\vec{r}_{tgt}\cdot\vec{r}_{rel} + \cancelto{0}{r_{rel}^2}}\right)^{3/2}} \qquad r_{int}^2 = r_{tgt}^2 + r_{rel}^2 - 2r_{tgt}r_{rel}\cos\alpha$$

If r_{rel}^2 is small relative to r_{tgt}^2 then

$$\frac{\vec{r}_{int}}{r_{int}^3} = \frac{\vec{r}_{tgt} + \vec{r}_{rel}}{r_{tgt}^3}\left\{\frac{1}{\left(1 + \dfrac{2\vec{r}_{tgt}\cdot\vec{r}_{rel}}{r_{tgt}^2}\right)^{3/2}}\right\}$$

Simplify using binomial series $(1+x)^n = 1 + nx + n(n-1)x^2/2! + \ldots$

$$\frac{\vec{r}_{int}}{r_{int}^3} = \frac{\vec{r}_{tgt} + \vec{r}_{rel}}{r_{tgt}^3}\left\{1 - \frac{3}{2}\left(\frac{2\vec{r}_{tgt}\cdot\vec{r}_{rel}}{r_{tgt}^2}\right) + \ldots\right\}$$

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

Substituting in:

$$\ddot{\vec{r}}_{rel} = -\mu\left\{\frac{\vec{r}_{tgt}+\vec{r}_{rel}}{r_{tgt}^{3}}\left\{1-\frac{3}{2}\left(\frac{2\vec{r}_{tgt}\cdot\vec{r}_{rel}}{r_{tgt}^{2}}\right)+\ldots\right\}\right\}+\vec{F}+\frac{\mu\,\vec{r}_{tgt}}{r_{tgt}^{3}}$$

$$= -\frac{\mu}{r_{tgt}^{3}}\left\{-\frac{3\vec{r}_{tgt}}{2}\left(\frac{2\vec{r}_{tgt}\cdot\vec{r}_{rel}}{r_{tgt}^{2}}\right)+\vec{r}_{rel}-\underbrace{\frac{3\vec{r}_{rel}}{2}\left(\frac{2\vec{r}_{tgt}\cdot\vec{r}_{rel}}{r_{tgt}^{2}}\right)}_{\sim 0}\right\}+\vec{F}$$

$$= -\frac{\mu}{r_{tgt}^{3}}\left\{-\frac{3}{2}\frac{\vec{r}_{tgt}}{r_{tgt}}\left(\frac{2\vec{r}_{tgt}\cdot\vec{r}_{rel}}{r_{tgt}}\right)+\vec{r}_{rel}\right\}+\vec{F}$$

$$\frac{\vec{r}_{tgt}}{r_{tgt}} = \hat{R} \qquad \hat{R}\cdot\vec{r}_{rel} = x$$

$$\ddot{\vec{r}}_{rel} = -\frac{\mu}{r_{tgt}^{3}}\left\{-3x\hat{R}+\vec{r}_{rel}\right\}+\vec{F}$$

Inertial acceleration in the target frame (a rotating frame)

We'd prefer the acceleration and frame to be consistent! Let's keep cranking.

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

Recall:

$$\ddot{\vec{r}}_I = \ddot{\vec{r}}_R + \dot{\vec{\omega}}_R \times \vec{r}_R + 2\vec{\omega}_R \times \dot{\vec{r}}_R + \vec{\omega}_R \times \left(\vec{\omega}_R \times \vec{r}_R\right)$$

Coriolis Acceleration

We have this

We want this

Centripetal Acceleration

Acceleration due to a
changing frame rate
(zero for circular orbits)

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

0 *for circular orbit*

$$\ddot{\vec{r}}_I = \ddot{\vec{r}}_R + \cancel{\dot{\vec{\omega}}_R} \times \vec{r}_R + 2\vec{\omega}_R \times \dot{\vec{r}}_R + \vec{\omega}_R \times \left(\vec{\omega}_R \times \vec{r}_R\right)$$

$$\ddot{\vec{r}}_R = \ddot{\vec{r}}_I - \dot{\vec{\omega}}_R \times \vec{r}_R - 2\vec{\omega}_R \times \dot{\vec{r}}_R - \vec{\omega}_R \times \left(\vec{\omega}_R \times \vec{r}_R\right)$$

So, $\ddot{\vec{r}}_{rel\,R} = \ddot{\vec{r}}_{rel\,I} - \dot{\vec{\omega}}_R \times \vec{r}_{rel} - 2\vec{\omega}_R \times \dot{\vec{r}}_{rel} - \vec{\omega}_R \times \left(\vec{\omega}_R \times \vec{r}_{rel}\right)$

$$\omega = \sqrt{\frac{\mu}{r_{tgt}^3}} \qquad \dot{\vec{\omega}}_R \times \vec{r}_{rel} = \begin{bmatrix} \hat{R} & \hat{S} & \hat{W} \\ 0 & 0 & \dot{\omega} \\ x & y & z \end{bmatrix} = -\dot{\omega} y \hat{R} + \dot{\omega} x \hat{S}$$

$$\vec{\omega}_R \times \dot{\vec{r}}_{rel} = -\omega \dot{y} \hat{R} + \omega \dot{x} \hat{S} \qquad \vec{\omega}_R \times \left(\vec{\omega}_R \times \vec{r}_{rel}\right) = -\omega^2 x \hat{R} - \omega^2 y \hat{S}$$

$$\ddot{\vec{r}}_{rel\ R} = -\frac{\mu}{r_{tgt}^3}\left\{\vec{r}_{rel} - 3x\hat{R}\right\} + \vec{F} + \dot{\omega} y \hat{R} - \dot{\omega} x \hat{S} - 2\left(-\omega \dot{y} \hat{R} + \omega \dot{x} \hat{S}\right)$$
$$+ \omega^2 x \hat{R} + \omega^2 y \hat{S}$$

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

If we assume circular motion, $\frac{\mu}{r_{tgt}^3} = \omega^2, \qquad \dot{\omega} = 0,$

$$\ddot{\vec{r}}_{rel\ R} = -\omega^2 \left\{ x\hat{R} + y\hat{S} + z\hat{W} - 3x\hat{R} \right\} + \vec{F} + 2\omega\dot{y}\hat{R} - 2\omega\dot{x}\hat{S} + \omega^2 x\hat{R} + \omega^2 y\hat{S}$$

Thus,

$$\boxed{\begin{aligned} \ddot{x} - 2\omega\dot{y} - 3\omega^2 x &= f_x \\ \ddot{y} + 2\omega\,\dot{x} &= f_y \\ \ddot{z} + \omega^2 z &= f_z \end{aligned}}$$

<u>CW or Hill's Equations</u>

Assume $\vec{F} = 0$ (good for impulsive ΔV maneuvers, not for continuous thrust targeting).

Assumptions

- Please Take Note:
  - We've assumed a lot of things
  - We've assumed that the relative distance is very small
  - We've assumed circular orbits
  - These equations do fall off as you break these assumptions!

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

The above equations can be solved (see book: Algorithm 48) leaving:

$$x(t) = \frac{\dot{x}_0}{\omega}\sin\omega t - \left(3x_0 + \frac{2\dot{y}_0}{\omega}\right)\cos\omega t + \left(4x_0 + \frac{2\dot{y}_0}{\omega}\right)$$

$$y(t) = \left(6x_0 + \frac{4\dot{y}_0}{\omega}\right)\sin\omega t + \frac{2\dot{x}_0}{\omega}\cos\omega t - \left(6\omega x_0 + 3\dot{y}_0\right)\, t + \left(y_0 - \frac{2\dot{x}_0}{\omega}\right)$$

$$z(t) = z_0\cos\omega t + \frac{\dot{z}_0}{\omega}\sin\omega t$$

$$\dot{x}(t) = \dot{x}_0\cos\omega t + \left(3\omega x_0 + 2\dot{y}_0\right)\sin\omega t$$

$$\dot{y}(t) = \left(6\omega x_0 + 4\dot{y}_0\right)\cos\omega t - 2\dot{x}_0\sin\omega t - \left(6\omega x_0 + 3\dot{y}_0\right)$$

$$\dot{z}(t) = -z_0\omega\sin\omega t + \dot{z}_0\cos\omega t$$

So, given $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$ of interceptor, can compute $x, y, z,$ $\dot{x}, \dot{y}, \dot{z}$ of interceptor at future time.

Applications

- What can we do with these equations?
- Estimate where the satellite will go after executing a small maneuver.
- Rendezvous and prox ops!
- Examples. First from the book.

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

▼ **Example 6-14. Scenario Using the Hubble Space Telescope.**

GIVEN: The Hubble Space Telescope is about to be released from the Space Shuttle, which is in a circular orbit at 590 km altitude. The relative velocity (from the Space Shuttle bay) of the ejection is 0.1 m/s down, 0.04 m/s backwards, and 0.02 m/s to the right.

FIND: Position and velocity of the Hubble Space Telescope after 5 and 20 minutes.

First, convert the given data to the correct sign convention for the relative motion definitions. The initial velocities are then

$$\dot{x}_o = -0.1 \qquad \dot{y}_o = -0.04 \qquad \dot{z}_o = -0.02 \quad \text{m/s}$$

Next, find the angular rate of the target (Shuttle). In a 590 km altitude orbit,

$$\omega = \sqrt{\frac{\mu}{r_{tgt}^3}} = \sqrt{\frac{398{,}600.5}{6968.1363^3}} = 0.001\,085\,4 \text{ rad/s}$$

The initial position of 0.0 simplifies the equations in Algorithm 48 to

$$x(t) = \frac{\dot{x}_o}{\omega}\operatorname{SIN}(\omega t) - \frac{2\dot{y}_o}{\omega}\operatorname{COS}(\omega t) + \frac{2\dot{y}_o}{\omega}$$

$$y(t) = \frac{4\dot{y}_o}{\omega}\operatorname{SIN}(\omega t) + \frac{2\dot{x}_o}{\omega}\operatorname{COS}(\omega t) - 3\dot{y}_o t - \frac{2\dot{x}_o}{\omega}$$

$$z(t) = \frac{\dot{z}_o}{\omega}\operatorname{SIN}(\omega t)$$

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

$$\dot{x}(t) = \dot{x}_o \cos(\omega t) + 2\dot{y}_o \sin(\omega t)$$

$$\dot{y}(t) = 4\dot{y}_o \cos(\omega t) - 2\dot{x}_o \sin(\omega t) - 3\dot{y}_o$$

$$\dot{z}(t) = \dot{z}_o \cos(\omega t)$$

Substituting the time of 5 minutes (300^S) yields

$$x(5) = -33.345 \text{ m} \qquad \dot{x}(5) = -0.12 \text{ m/s}$$

$$y(5) = -1.473 \text{ m} \qquad \dot{y}(5) = 0.03 \text{ m/s}$$

$$z(5) = -5.894 \text{ m} \qquad \dot{z}(5) = -0.02 \text{ m/s}$$

and, for the case of 20 minutes (1200^S),

$$x(20) = -143.000 \text{ m} \qquad \dot{x}(20) = -0.10 \text{ m/s}$$

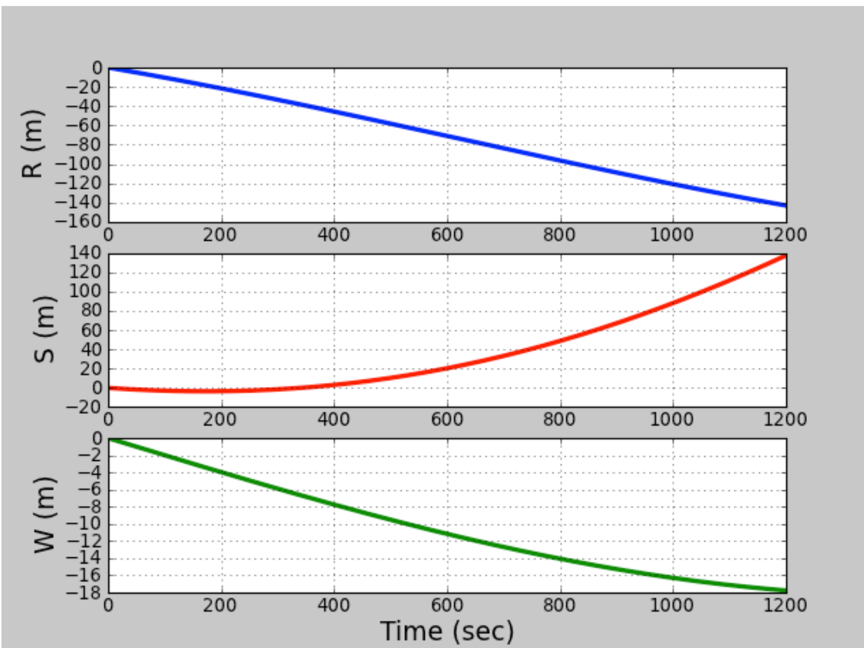
$$y(20) = 137.279 \text{ m} \qquad \dot{y}(20) = 0.27 \text{ m/s}$$

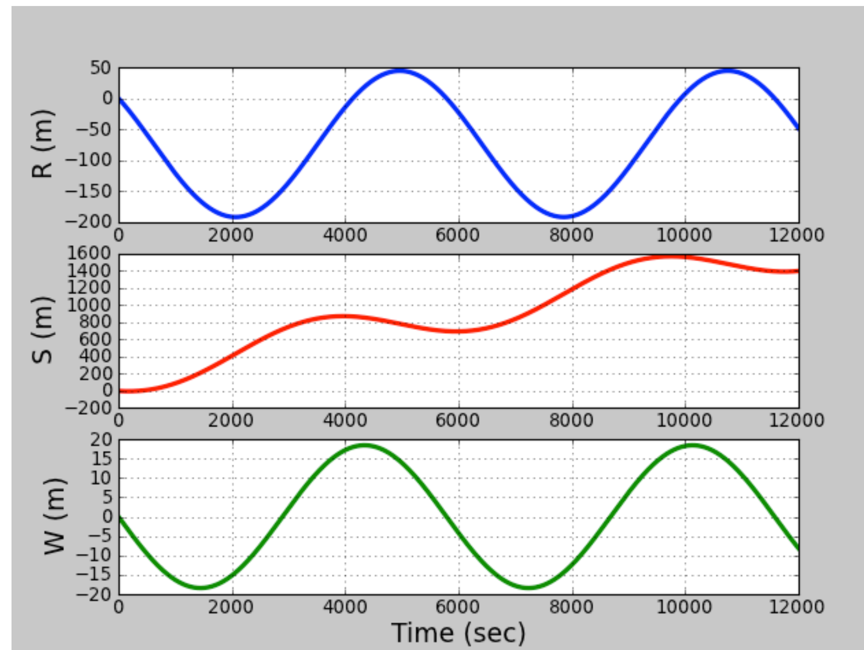
$$z(20) = -17.766 \text{ m} \qquad \dot{z}(20) = -0.01 \text{ m/s}$$

▲ Notice how both the position and velocity change dramatically.

Hubble's Drift from Shuttle

- RSW Coordinate Frame



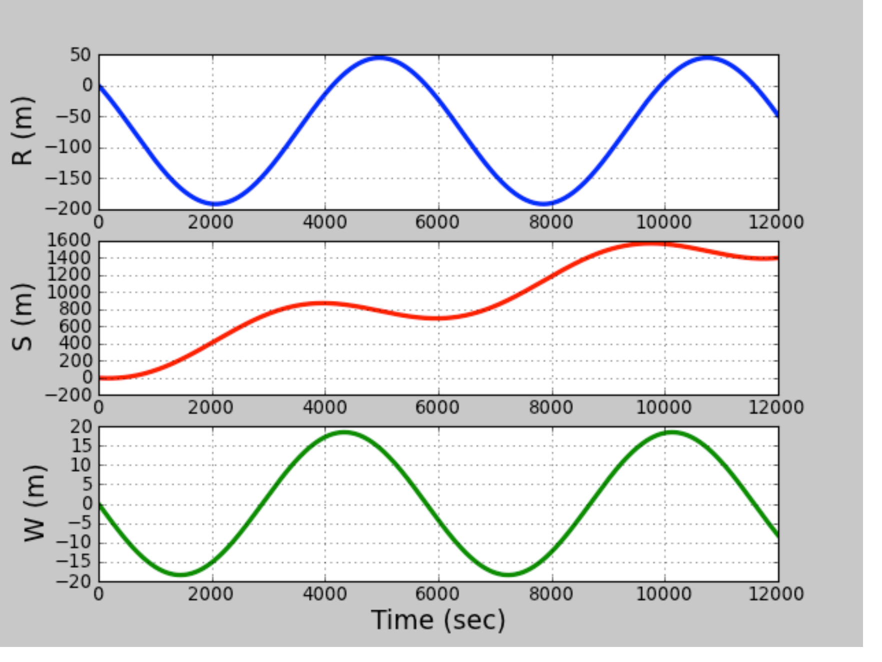


Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

$$x(t) = \frac{\dot{x}_0}{\omega}\sin\omega t - \left(3x_0 + \frac{2\dot{y}_0}{\omega}\right)\cos\omega t + \left(4x_0 + \frac{2\dot{y}_0}{\omega}\right)$$

$$y(t) = \left(6x_0 + \frac{4\dot{y}_0}{\omega}\right)\sin\omega t + \frac{2\dot{x}_0}{\omega}\cos\omega t - \left(6\omega x_0 + 3\dot{y}_0\right)\ t + \left(y_0 - \frac{2\dot{x}_0}{\omega}\right)$$

$$z(t) = z_0\cos\omega t + \frac{\dot{z}_0}{\omega}\sin\omega t$$

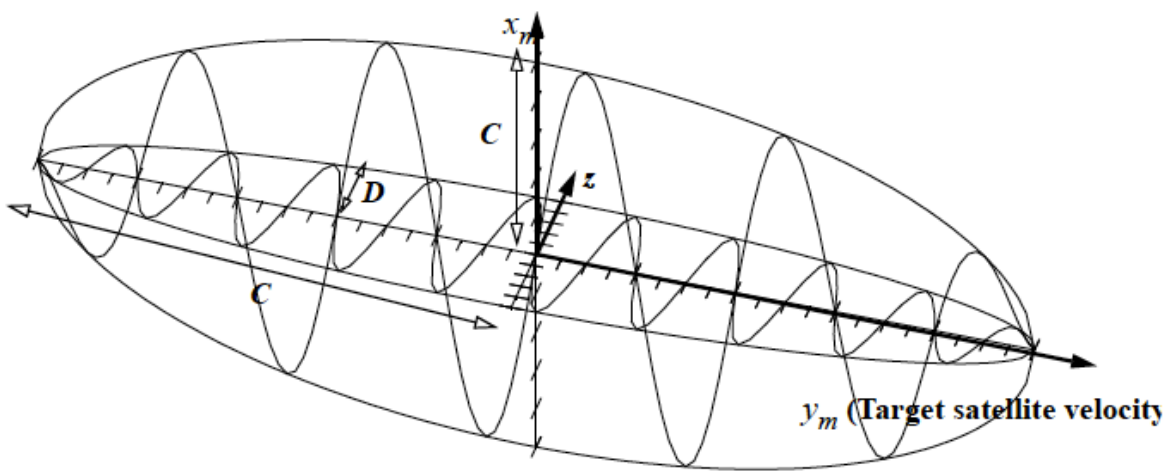


Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

$$x(t) = \frac{\dot{x}_0}{\omega}\sin\omega t - \left(3x_0 + \frac{2\dot{y}_0}{\omega}\right)\cos\omega t + \left(4x_0 + \frac{2\dot{y}_0}{\omega}\right)$$

$$y(t) = \left(6x_0 + \frac{4\dot{y}_0}{\omega}\right)\sin\omega t + \frac{2\dot{x}_0}{\omega}\cos\omega t - \left(6\omega x_0 + 3\dot{y}_0\right)\, t + \left(y_0 - \frac{2\dot{x}_0}{\omega}\right)$$

$$z(t) = z_0\cos\omega t + \frac{\dot{z}_0}{\omega}\sin\omega t$$



Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

We can also determine ΔV needed for rendezvous. Given x_0, y_0, z_0, we want to determine $\dot{x}_0, \dot{y}_0, \dot{z}_0$ necessary to make x=y=z=0. Set first 3 equations to zero, and solve for $\dot{x}_0, \dot{y}_0, \dot{z}_0$.

$$\dot{x}_0 = -\frac{\omega x_0 (4 - 3\cos\omega t) + 2(1 - \cos\omega t)\dot{y}_0}{\sin\omega t}$$

$$\dot{y}_0 = \frac{(6x_0(\omega t - \sin\omega t) - y_0)\omega\sin\omega t - 2\omega x_0(4 - 3\cos\omega t)(1 - \cos\omega t)}{(4\sin\omega t - 3\omega t)\sin\omega t + 4(1 - \cos\omega t)^2}$$

$$\dot{z}_0 = -z_0\omega\cot\omega t$$

Assumptions:

1. Satellites only a few km apart
2. Target in circular orbit
3. No external forces (drag, etc.)

Clohessy-Wiltshire (CW) Equations
(Hill's Equations)

▼ **Example 6-15. Solving for Velocity in Hill's Equation.**

GIVEN: A satellite in circular orbit at 590 km altitude. Suppose we continue Example 6-14; after 10 minutes, we decide to retrieve the Hubble Space Telescope.

FIND: Δv to rendezvous with the Shuttle in 5 and 15 minutes.

Examining Eq. (6-66) reveals quantities similar to those in Example 6-14:

$\omega = 0.001\,085\,4$ rad/s

First, we find the position of the Hubble Space Telescope after 10 minutes (600^s). Using Algorithm 48,

$x(600) = -70.933$ m, $y(600) = 20.357$ m, and $z(600) = -11.170$ m

Now, solve for the required velocity to rendezvous [Eq. (6-66)] in 5 and 15 minutes (300^s and 900^s), but be sure to calculate $\dot{y}_o$ first because you need it to calculate $\dot{x}_o$.

At t = 5 minutes (300 seconds),	At t = 15 minutes (900 seconds),
$\dot{y}_o = 0.0135$ m/s	$\dot{y}_o = 0.0753$ m/s
$\dot{x}_o = 0.2742$ m/s	$\dot{x}_o = 0.1356$ m/s
$\dot{z}_o = 0.0359$ m/s	$\dot{z}_o = 0.0082$ m/s

Notice how much the required velocity drops as the allowable time to rendezvous increases.

This problem has a practical application—rescuing astronauts who become detached from the space station. Williams and Baughman (1994) show how these calculations are used for designing self-rescue modules for astronauts. ▲

Comparing Hill to Keplerian Propagation

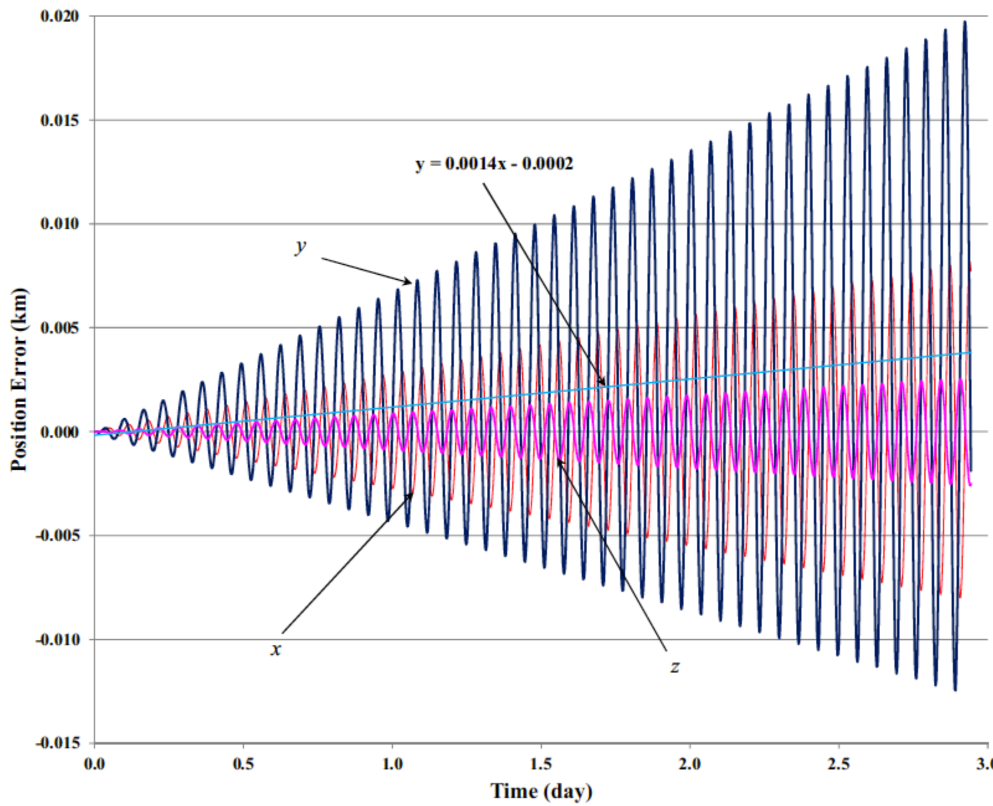


Figure 6-36. Comparing Hill's and Keplerian Motion—Circular. This figure shows the differences (averaging about 1.4 m /day) of simulations using Hill's and Keplerian propagations. The straight line is a least squares fit to the data in km. The period of the target is about 95 minutes.

Deviation:
100 meters
1 cm/s

Position Error:
~1.4 meters/day

Comparing Hill to High-Fidelity, e=0.15

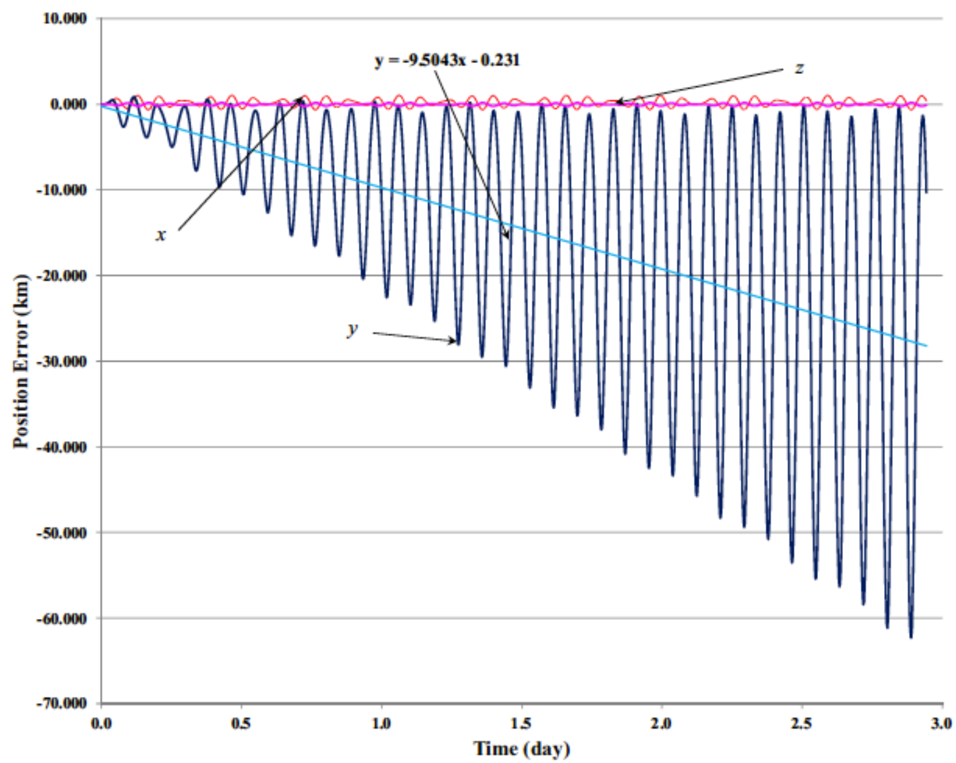


Figure 6-37. Comparing Hill's and Numerically Integrated Motion—LEO Eccentric. This figure shows the component differences resulting from numerically propagating (70 × 70 gravity, drag, third-body, solar radiation pressure) the target and interceptor satellites. Although the eccentricity is only 0.15, the error from Hill's is actually larger than the HEO case.

Generally returns to 0.0 near one apsis

Position Error: ~9.5 km/day

Comparing Hill to High-Fidelity, e=0.73

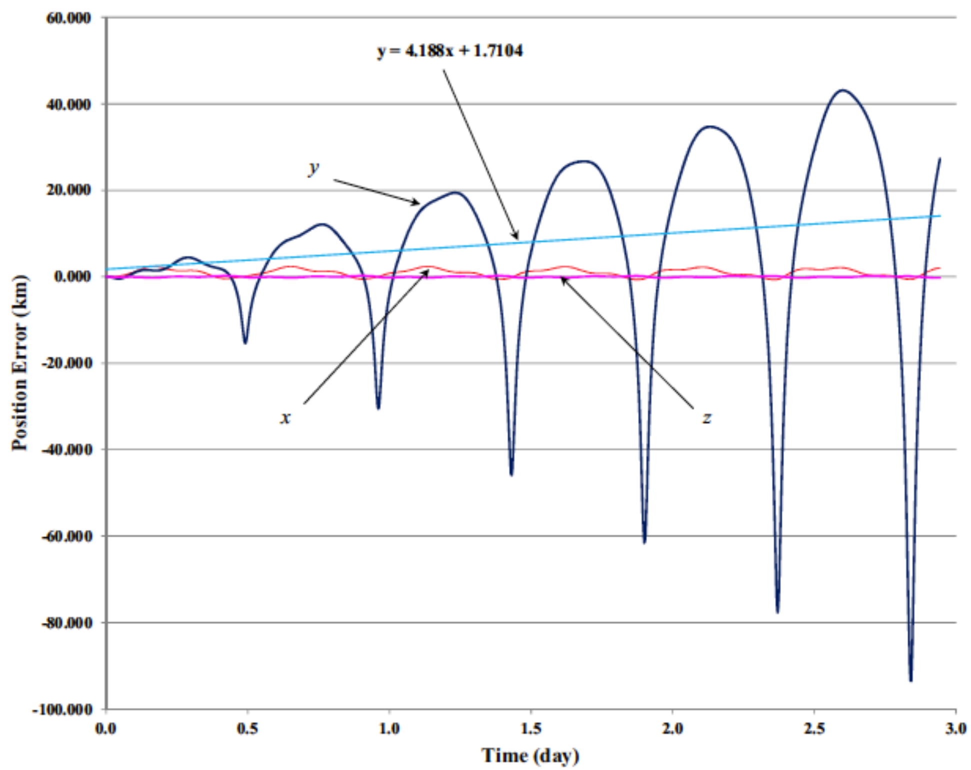


Figure 6-38. Comparing Hill's and Numerically Integrated Motion—Highly Elliptical. This figure shows the component differences (RSW) resulting from numerically propagating (70 × 70 gravity, tides, drag, third-body, solar radiation pressure) the target and interceptor satellites and converting to an equivalent Hill's representation, compared to a Hill's propagation. The straight line now shows about 4.18 km /day.

Generally returns to 0.0

Position Error: ~4.2 km/day

Analyzing Prox Ops

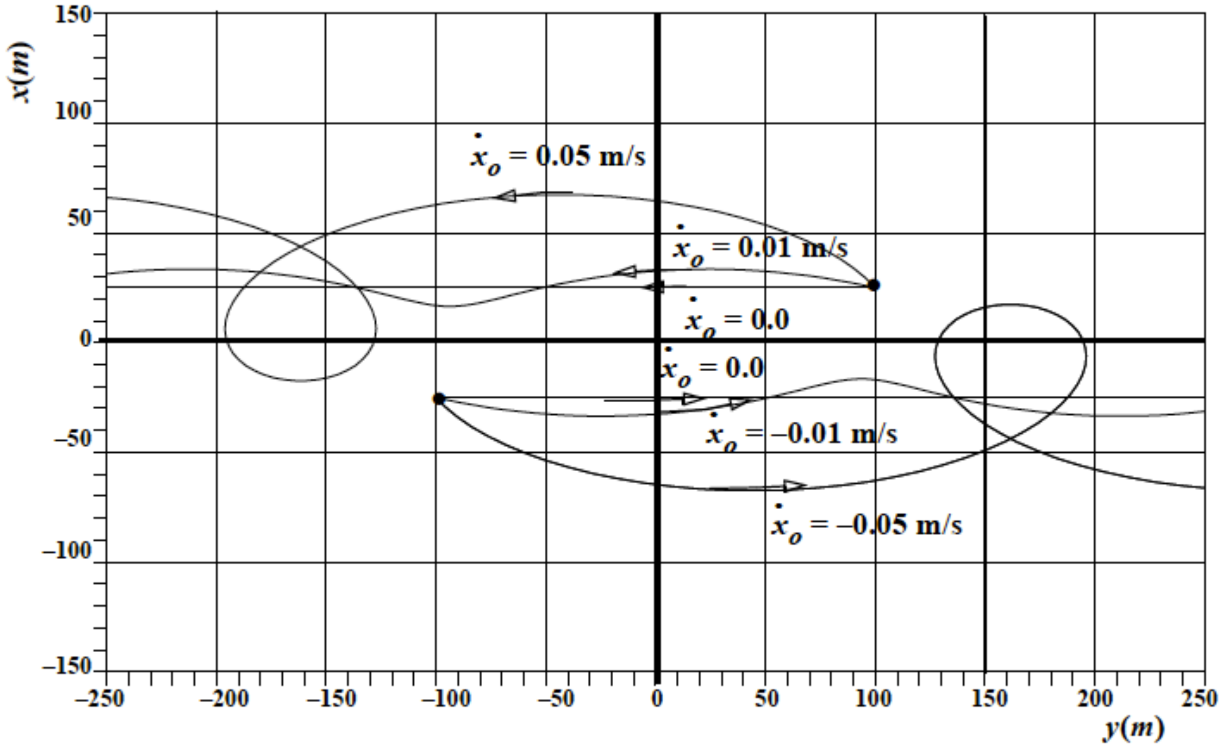


Figure 6-27. Interceptor Motion for x_o and y_o Displacements, $\dot{x}_o$ and $\dot{y}_o$ Variations. By varying the interceptor's initial conditions, several interesting motions are possible, including straight lines — $\dot{y}_o$ = 0.044 15 m/s for each x_o negative case, and $\dot{y}_o$ = –0.044 15 m/s for each x_o positive case. Note, these straight lines are not physically possible for an extended period of time.

Analyzing Prox Ops

- What happens if you just change x0?
  - Radial displacement?

Analyzing Prox Ops

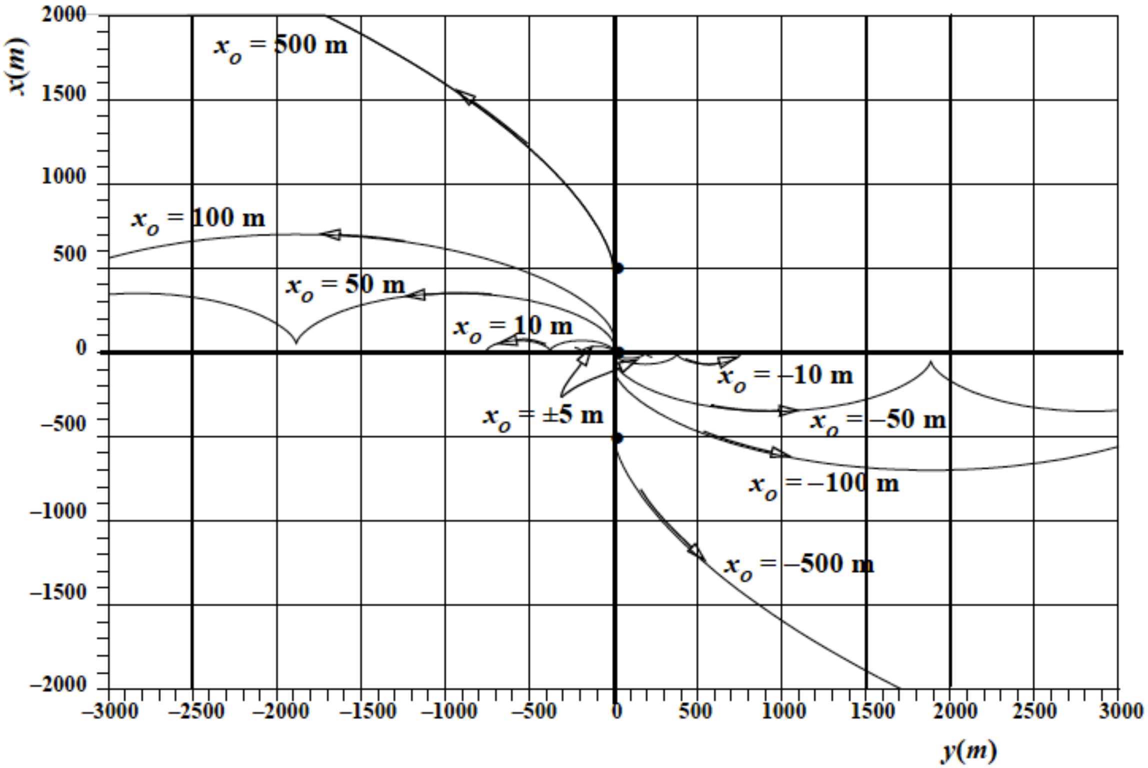


Figure 6-28. Motion of the Interceptor for Various x_o Displacements. Notice how all displacements above the satellite move to the left (lower velocity causes a lagging effect), whereas those below move to the right (higher velocity causes a lead). In addition, as the satellite's initial location gets farther from the target, the motion away from the target increases.

Analyzing Prox Ops

- What happens if you just change vx0?
  - Radial velocity change?

Analyzing Prox Ops

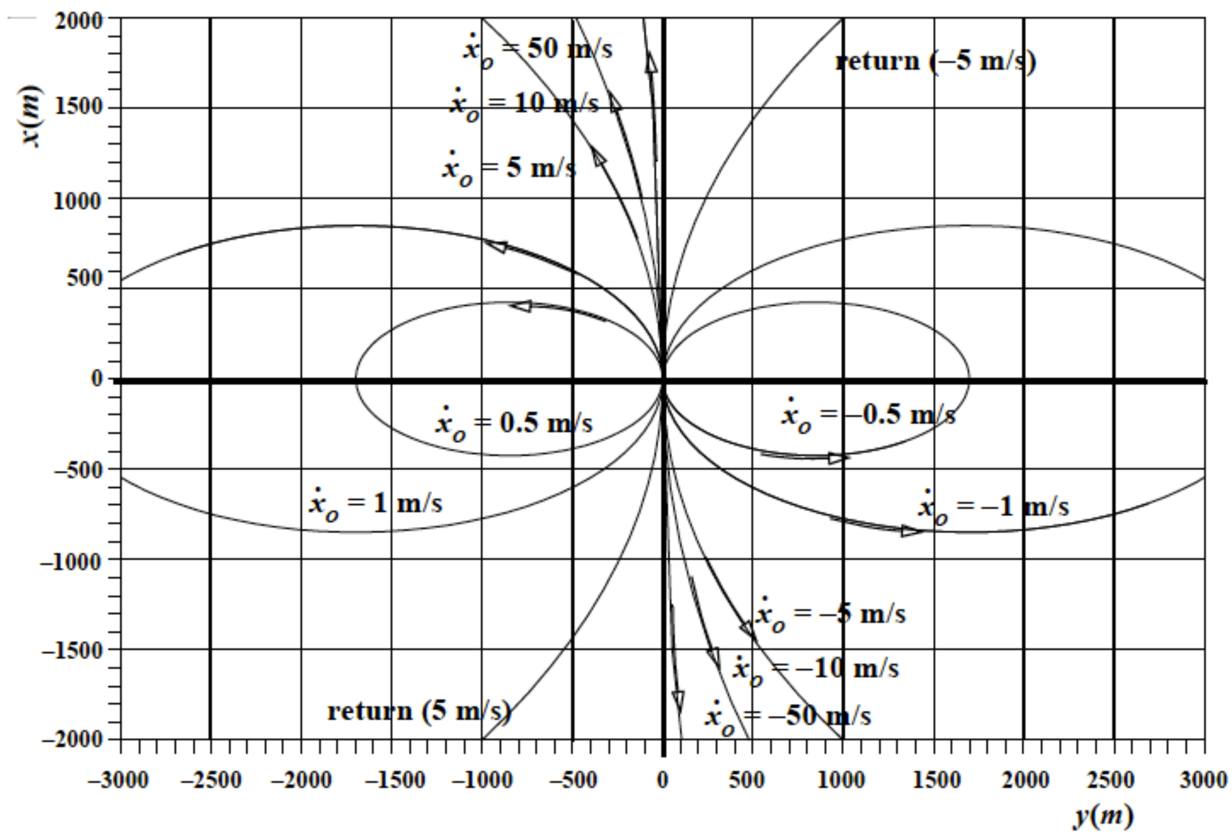


Figure 6-29. Motion of the Interceptor for $\dot{x}_o$ Variations. All motions which result from variations in the *x* velocity become periodic motions about the target. Notice each path *will* return to the target in one period.

Analyzing Prox Ops

- What happens if you just change vy0?
  - Tangential velocity change?

Analyzing Prox Ops

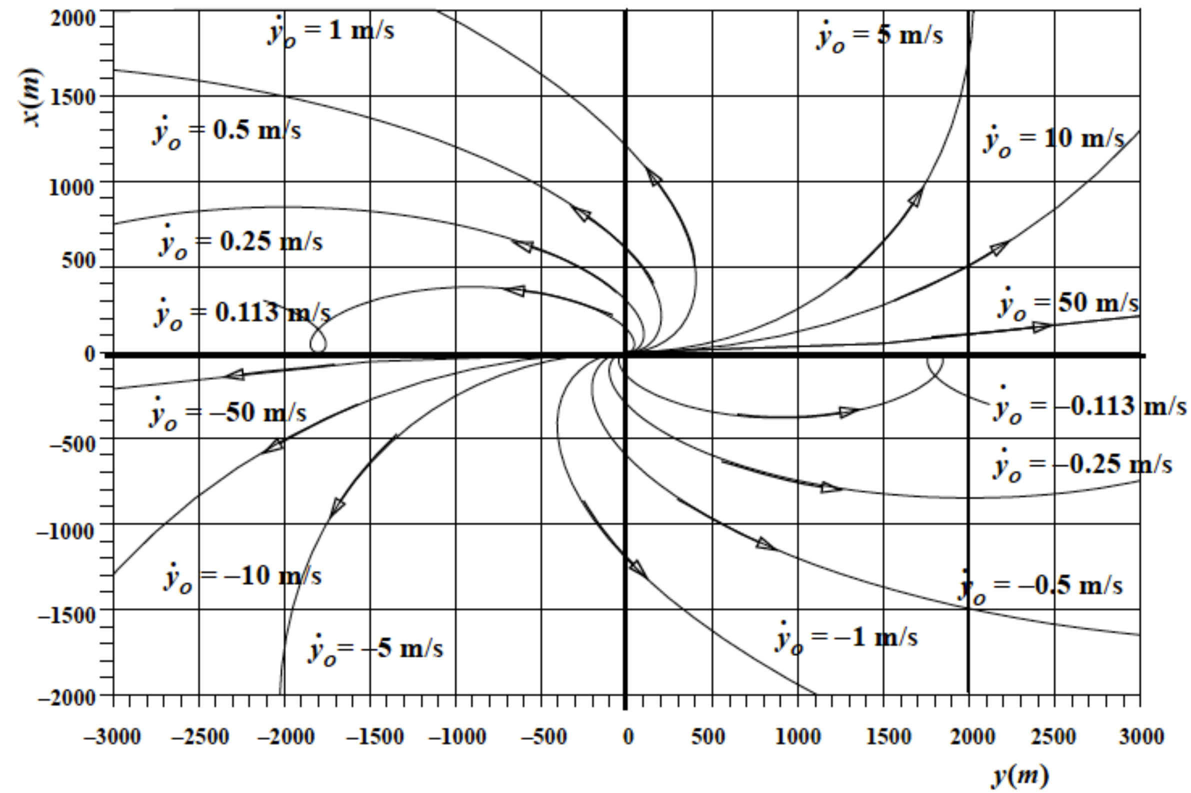


Figure 6-30. Motion of the Interceptor for $\dot{y}_o$ Variations. All positive y velocity changes cause the interceptor to travel left, whereas negative velocities cause motion to the right. Although $\dot{y}_o$ = 50 m/s appears to travel to the right, it will curve back around to the left over time.

Analyzing Prox Ops

- What happens if you just change vz0?
  - Out-of-plane velocity change?

Analyzing Prox Ops

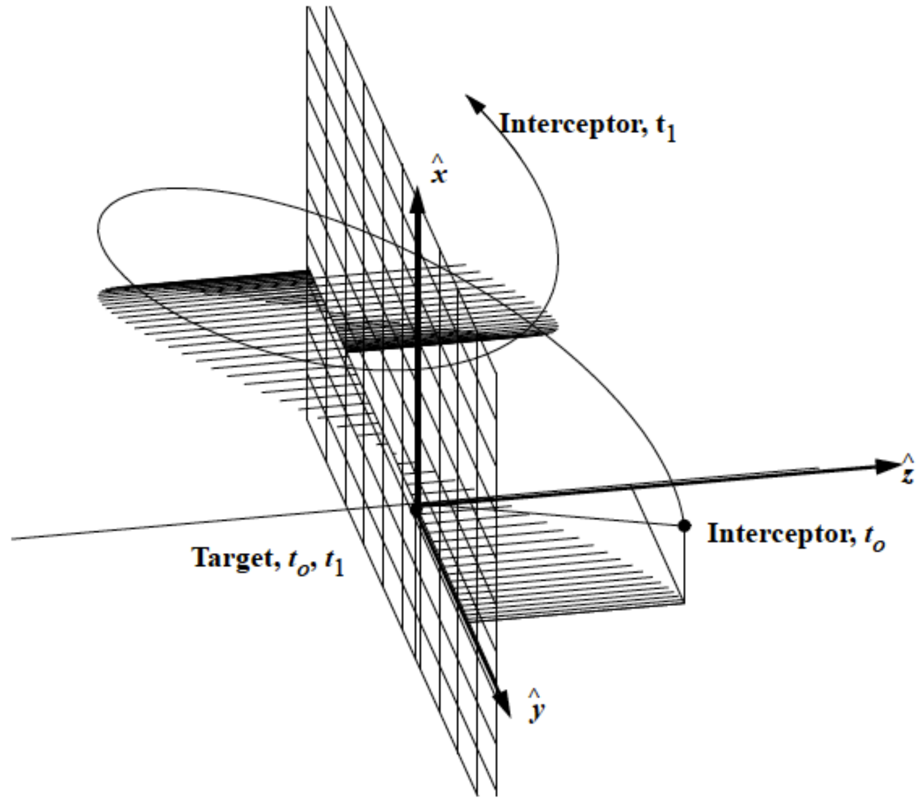


Figure 6-33. Relative Motion of the Interceptor with a *z* Variation. Adding a z variation to any of the other motions described simply adds a three-dimensional component. In this example, the interceptor goes above and then below the target satellite.

Announcements

- Samantha Krening from JPL will be here to say a few words.
- Homework #4 is due today. Homework #5 will be released later today (I'll send another email)

- No quiz tomorrow

- Reading: Chapter 2, 6

- Space News:
  - DANDE launched!
  - Cygnus docked with the ISS (note the use of CW/H equations…)
  - Juno's Earth flyby is coming up: Oct 9th!

Credit: NASA/JPL-Caltech