Analysis of Algorithm

Amypo Technologies

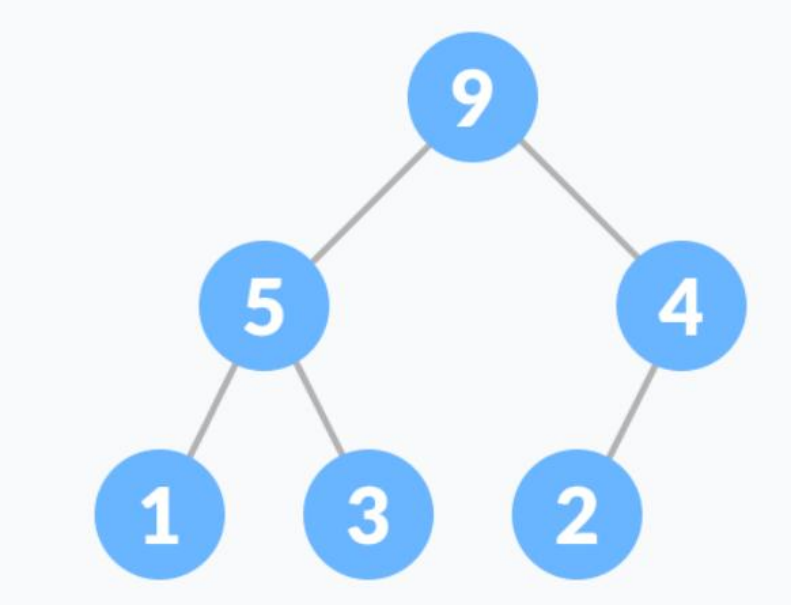
Agenda

- Heaps
- Binary Heap
- Heap Sort

Heaps

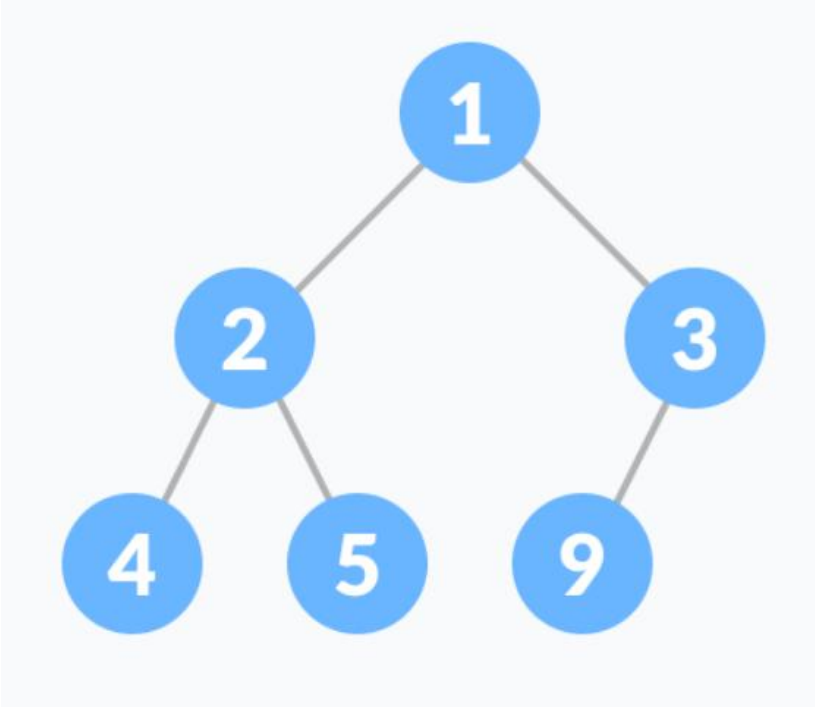
Heap is a complete binary tree that satisfies **the heap property**, where any given node is :

- always greater than its child node/s and the key of the root node is the largest among all other nodes. This property is also called **max heap property**.



Heaps

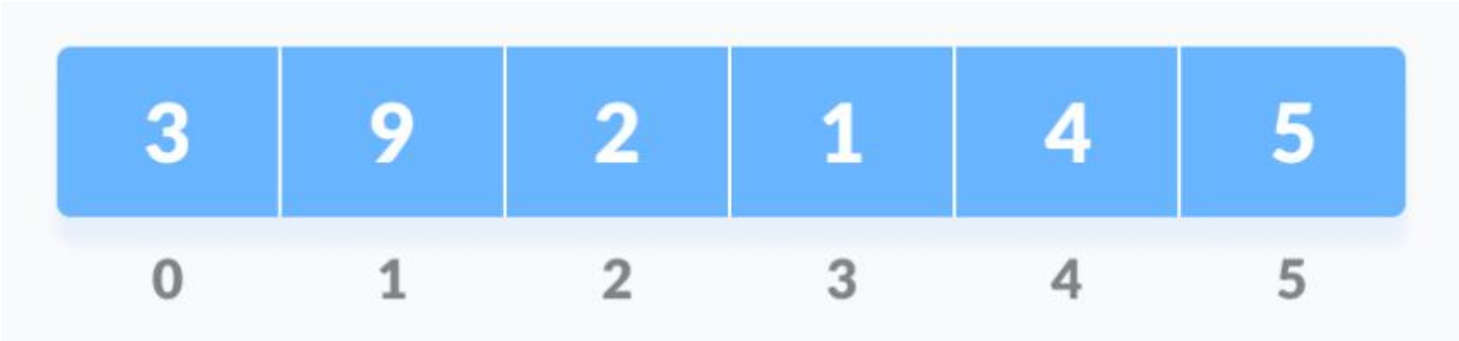
- always smaller than the child node/s and the key of the root node is the smallest among all other nodes. This property is also called **min heap property**.



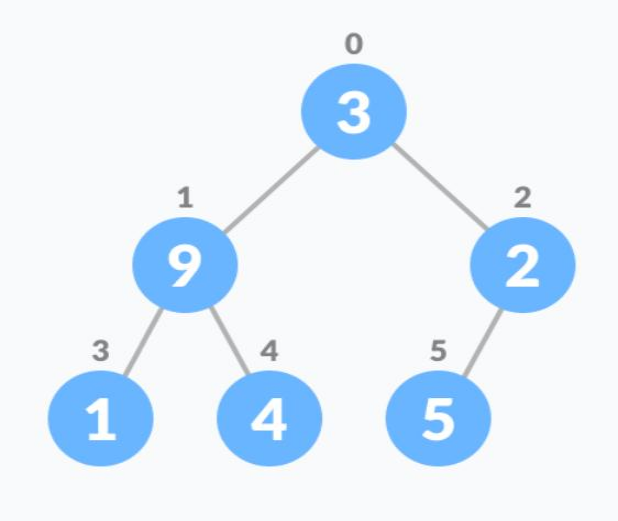
Heap Operations

- **Heapify**
- Heapify is the process of creating a heap data structure from a binary tree. It is used to create a Min-Heap or a Max-Heap.

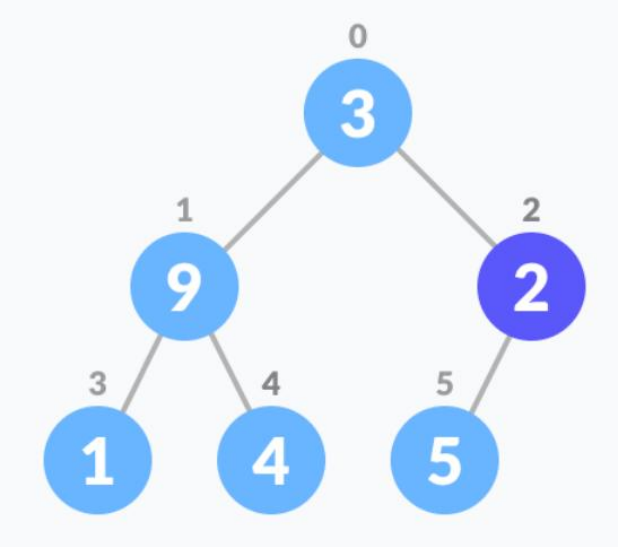
1.Let the input array be



2.Create a complete binary tree from the array



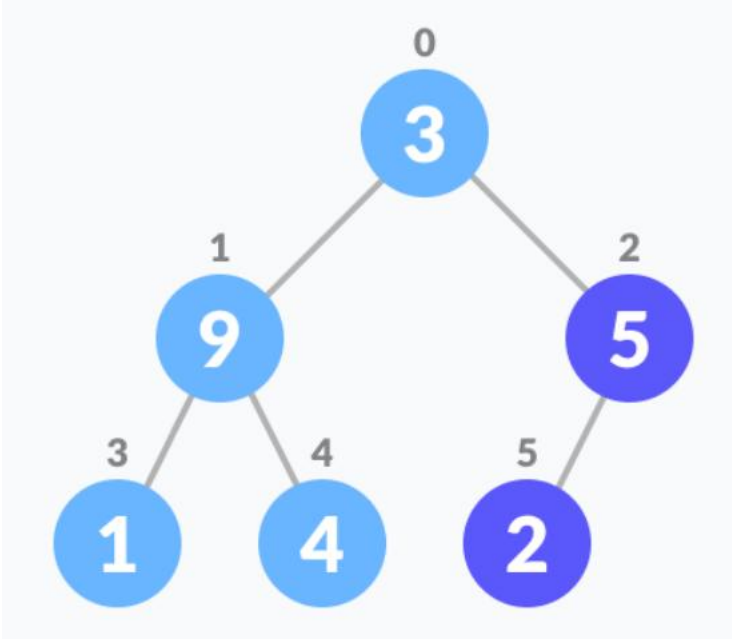
3.Start from the first index of non-leaf node whose index is given by n/2 - 1.



4.Set current element i as largest.

5.The index of left child is given by 2i + 1 and the right child is given by 2i + 2.
If leftChild is greater than currentElement (i.e. element at ith index), set leftChildIndex as largest.
If rightChild is greater than element in largest, set rightChildIndex as largest.

6.Swap largest with currentElement



7. Repeat steps 3-7 until the subtrees are also heapified.

Algorithm

```
Heapify(array, size, i)
set i as largest
leftChild = 2i + 1
rightChild = 2i + 2
if leftChild > array[largest]
set leftChildIndex as largest
if rightChild > array[largest]
set rightChildIndex as largest
swap array[i] and array[largest]
```

To create a Max-Heap:

MaxHeap(array, size)

loop from the first index of non-leaf node down to zero

call heapify

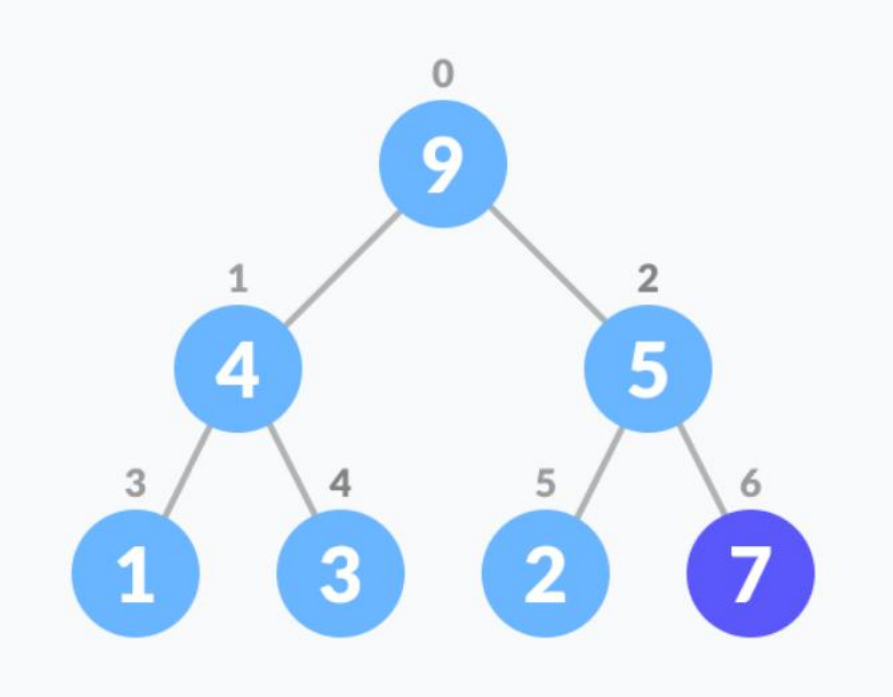
For Min-Heap, both leftChild and rightChild must be larger than the parent for all nodes.

Insert Element into Heap

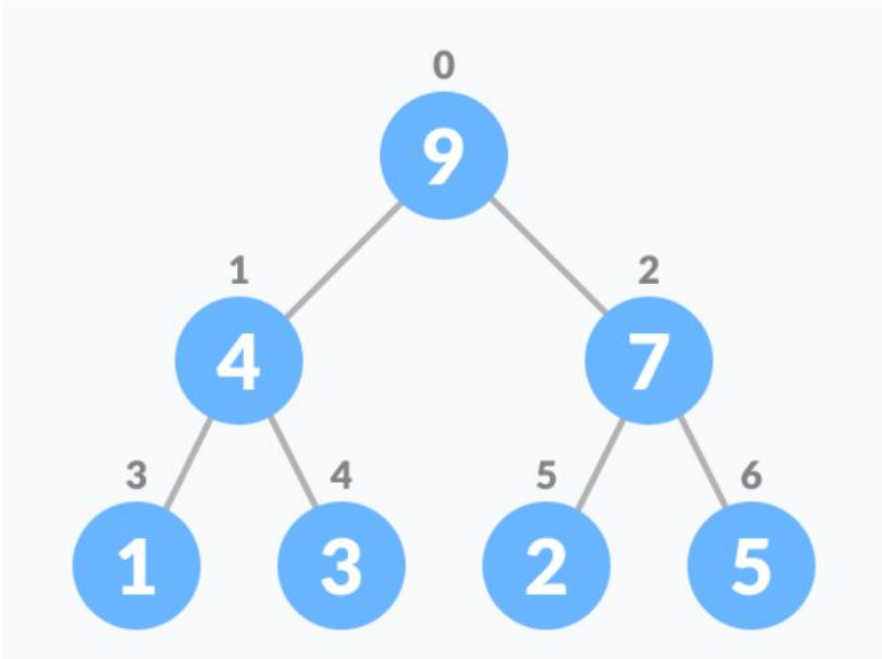
- Algorithm for insertion in Max Heap

If there is no node,
create a newNode.
else (a node is already present)
insert the newNode at the end (last node from left to right.)
heapify the array

1. Insert the new element at the end of the tree.



2. Heapify the tree.

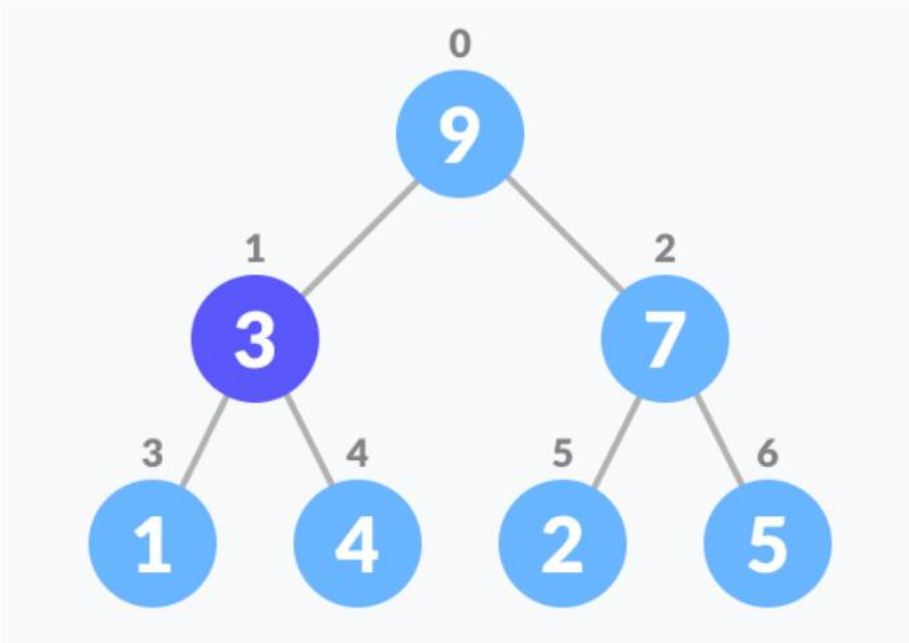


Delete Element from Heap

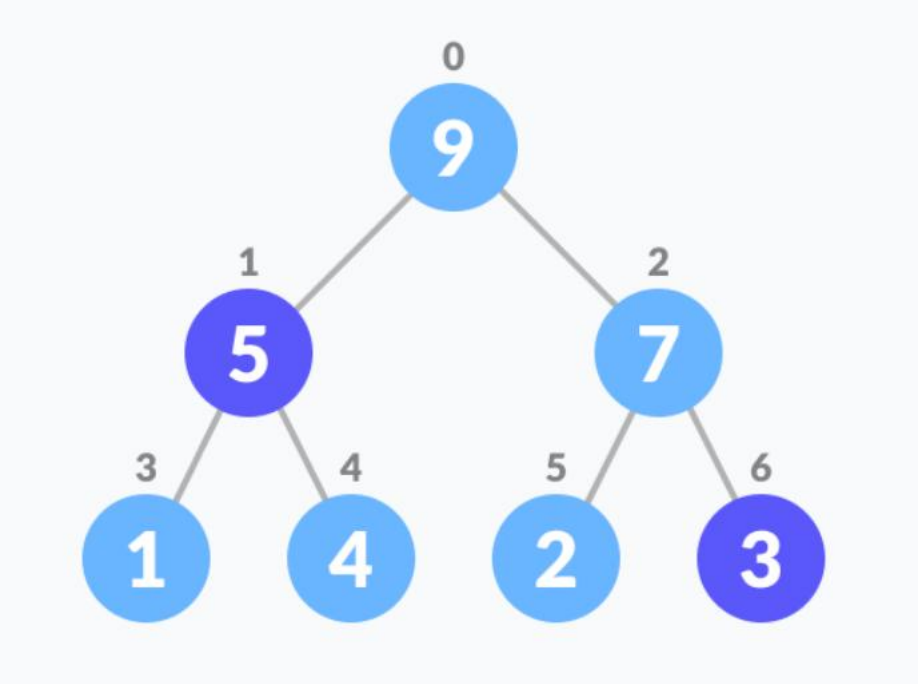
- Algorithm for deletion in Max Heap

```
If nodeToBeDeleted is the leafNode
remove the node
Else swap nodeToBeDeleted with
the lastLeafNode
remove noteToBeDeleted
heapify the array
```

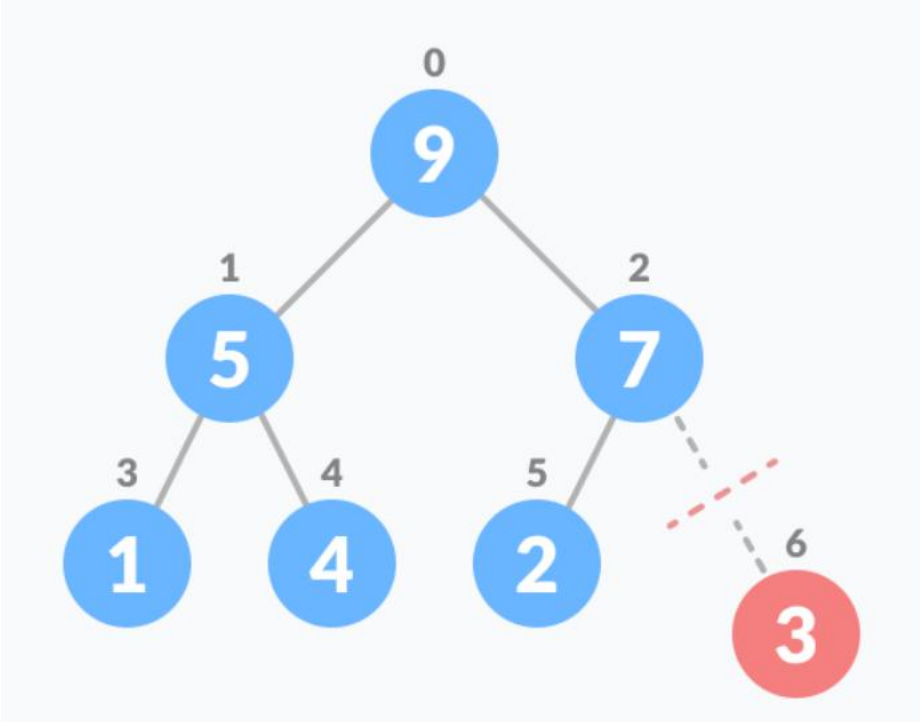
1.Select the element to be deleted.



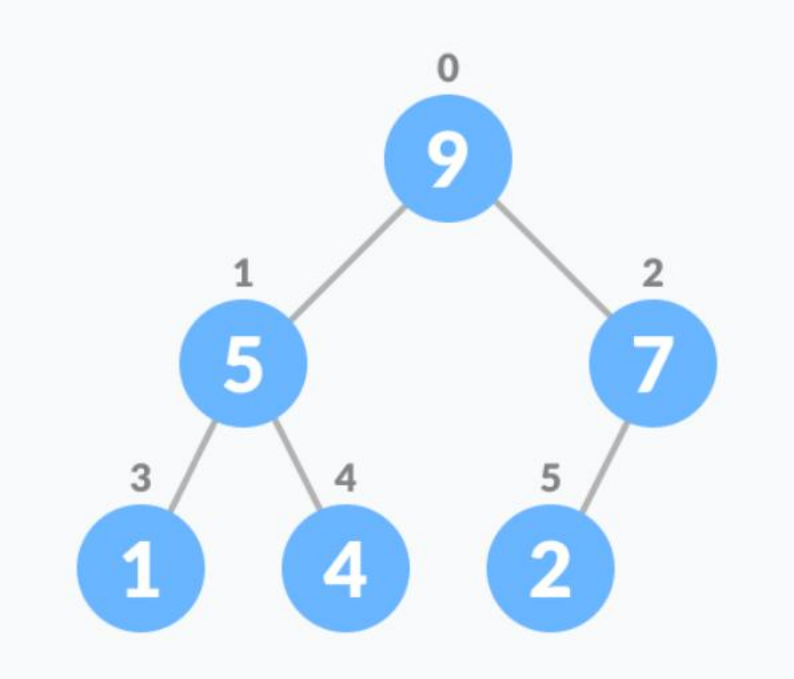
2.Swap it with the last element.



3. Remove the last element.



4. Heapify the tree



- For Min Heap, above algorithm is modified so that both childNodes are greater smaller than currentNode.

Peek (Find max/min)

- Peek operation returns the maximum element from Max Heap or minimum element from Min Heap without deleting the node.
- For both Max heap and Min Heap

```
return rootNode
```

Extract-Max/Min

- Extract-Max returns the node with maximum value after removing it from a Max Heap whereas Extract-Min returns the node with minimum after removing it from Min Heap.

Example

```
#include <iostream>
#include <vector>
using namespace std;
void swap(int *a, int *b)    {
  int temp = *b;
  *b = *a;
  *a = temp;         }
void heapify(vector<int> &hT, int i)    {
  int size = hT.size();
  int largest = i;
  int l = 2 * i + 1;
  int r = 2 * i + 2;
  if (l < size && hT[l] > hT[largest])
    largest = l;
  if (r < size && hT[r] > hT[largest])
    largest = r;
if (largest != i)  {
    swap(&hT[i], &hT[largest]);
    heapify(hT, largest);
  }       }
```

```
void insert(vector<int> &hT, int newNum) {
  int size = hT.size();
  if (size == 0)     {
    hT.push_back(newNum);
  }
  else     {
    hT.push_back(newNum);
    for (int i = size / 2 - 1; i >= 0; i--)       {
      heapify(hT, i);
    }       }          }
void deleteNode(vector<int> &hT, int num)    {
  int size = hT.size();
  int i;
  for (i = 0; i < size; i++)      {
    if (num == hT[i])
      break;
  }
  swap(&hT[i], &hT[size - 1]);
 hT.pop_back();
  for (int i = size / 2 - 1; i >= 0; i--)      {
    heapify(hT, i);
  }     }
void printArray(vector<int> &hT)    {
  for (int i = 0; i < hT.size(); ++i)
    cout << hT[i] << " ";
  cout << "\n";
}
```

```
int main()
{
  vector<int> heapTree;
  insert(heapTree, 3);
  insert(heapTree, 4);
  insert(heapTree, 9);
  insert(heapTree, 5);
  insert(heapTree, 2);
  cout << "Max-Heap array: ";
  printArray(heapTree);
  deleteNode(heapTree, 4);
  cout << "After deleting an element: ";
  printArray(heapTree);
}
```

Applications

- Heap is used while implementing a priority queue.
- Dijkstra's Algorithm
- Heap Sort