Analysis of Algorithm

Amypo Technologies Pvt Ltd

Concepts:

- Minimum Spanning Tree
- Prims Algorithm
- Kruskal Algorithm

Minimum Spanning Tree

A minimum spanning tree (MST) is a subset of the edges of a connected, undirected graph that connects all the vertices with the most negligible possible total weight of the edges. A minimum spanning tree has precisely n-1 edges, where n is the number of vertices in the graph.

Prim’s Algorithm

The algorithm starts with an empty spanning tree. The idea is to maintain two sets of vertices. The first set contains the vertices already included in the MST, and the other set contains the vertices not yet included. At every step, it considers all the edges that connect the two sets and picks the minimum weight edge from these edges. After picking the edge, it moves the other endpoint of the edge to the set containing MST.

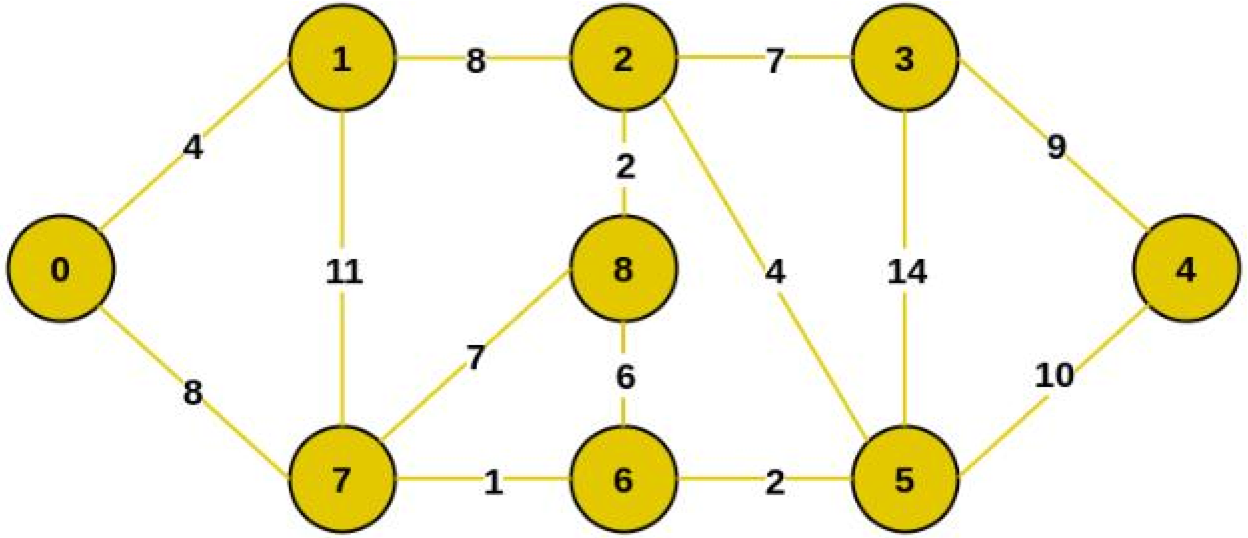
Algorithm

Step 1: Determine an arbitrary vertex as the starting vertex of the MST.
Step 2: Follow steps 3 to 5 till there are vertices that are not included in the MST (known as fringe vertex).
Step 3: Find edges connecting any tree vertex with the fringe vertices.
Step 4: Find the minimum among these edges.
Step 5: Add the chosen edge to the MST if it does not form any cycle.
Step 6: Return the MST and exit

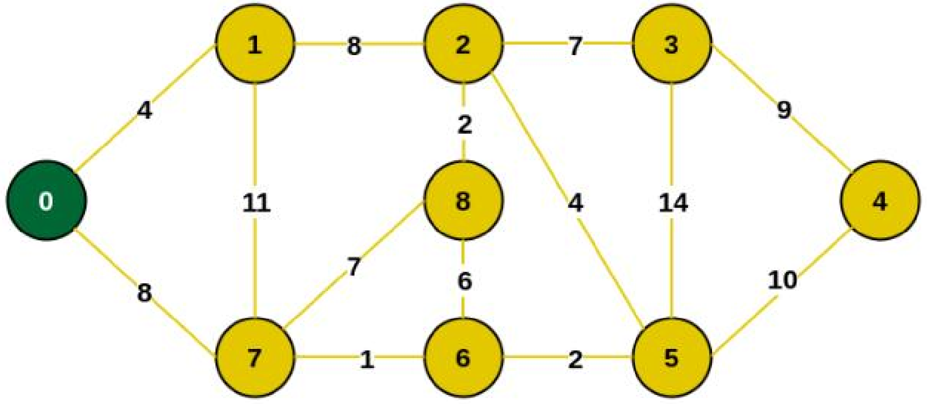
Note: For determining a cycle, we can divide the vertices into two sets [one set contains the vertices included in MST and the other contains the fringe vertices.]

Illustration:

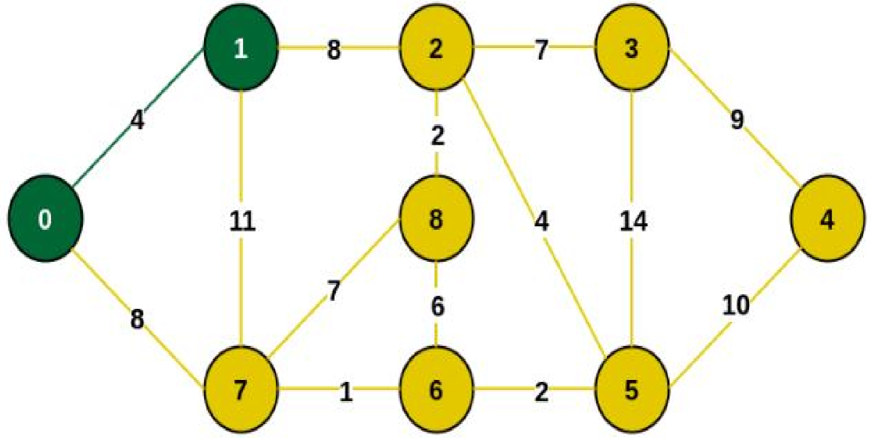
Consider the following graph as an example for which we need to find the Minimum Spanning Tree (MST).



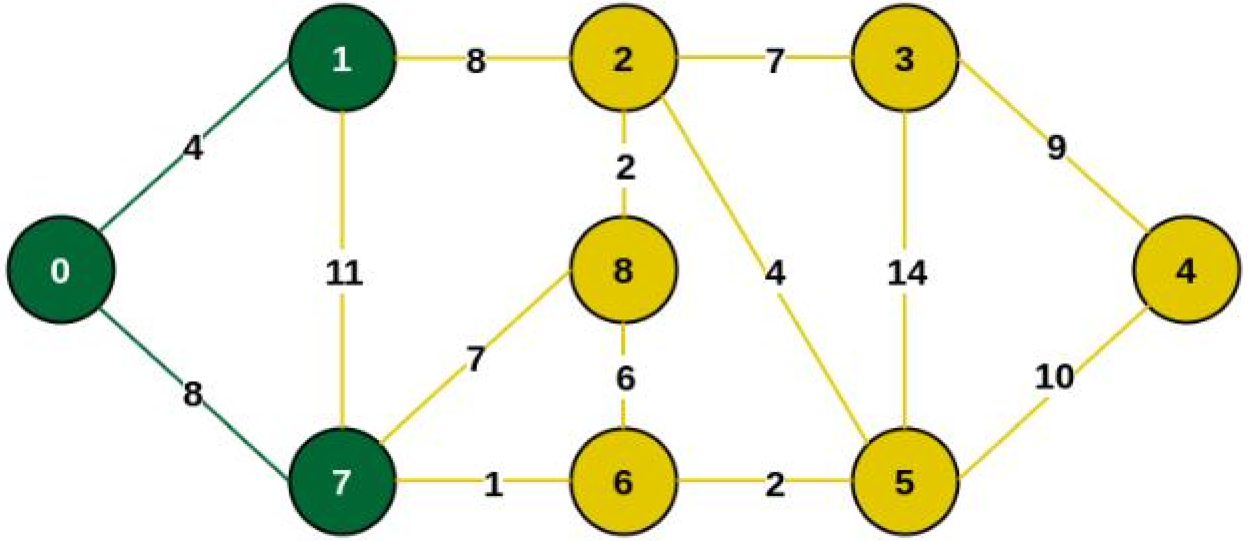
Step 1: *Firstly, we select an arbitrary vertex that acts as the starting vertex of the Minimum Spanning Tree. Here we have selected vertex **0** as the starting vertex.*



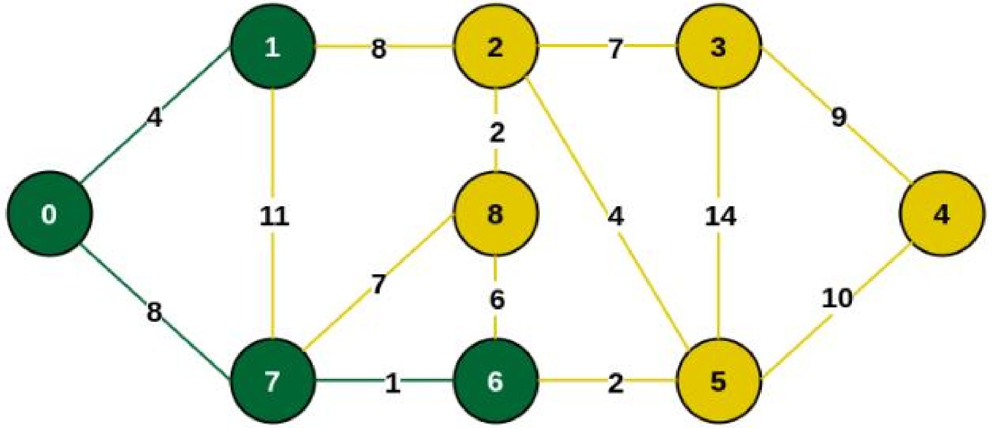
Step 2: *All the edges connecting the incomplete MST and other vertices are the edges {0, 1} and {0, 7}. Between these two the edge with minimum weight is {0, 1}. So include the edge and vertex 1 in the MST.*



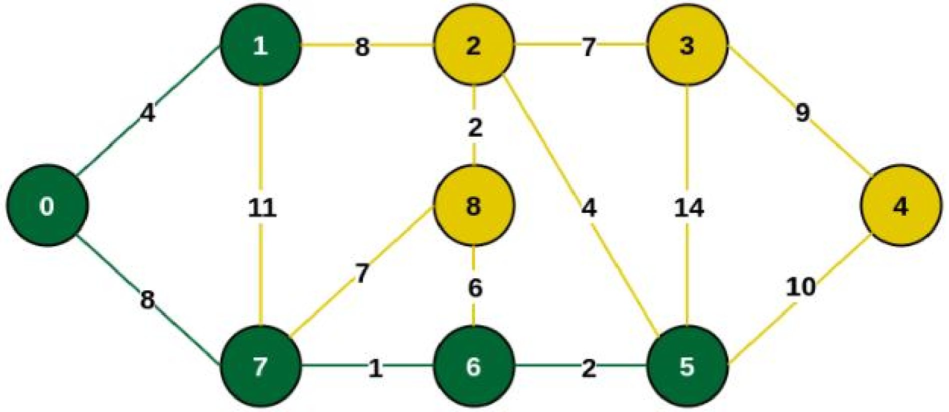
Step 3: *The edges connecting the incomplete MST to other vertices are {0, 7}, {1, 7} and {1, 2}. Among these edges the minimum weight is 8 which is of the edges {0, 7} and {1, 2}. Let us here include the edge {0, 7} and the vertex 7 in the MST. [We could have also included edge {1, 2} and vertex 2 in the MST].*



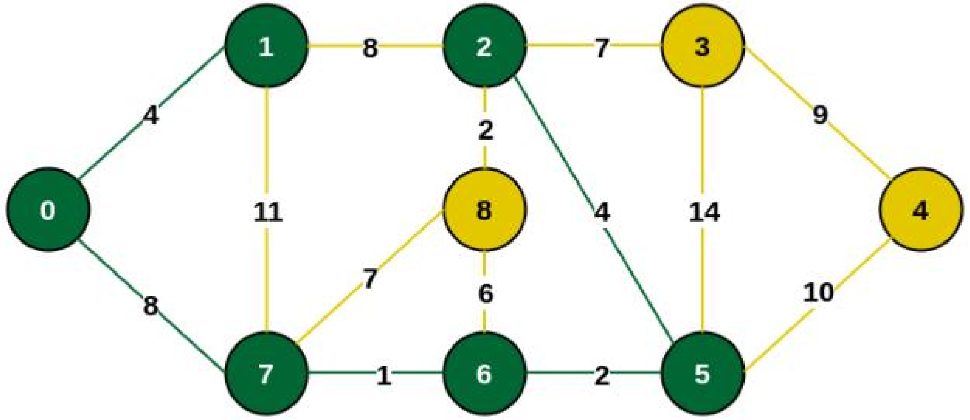
Step 4: *The edges that connect the incomplete MST with the fringe vertices are {1, 2}, {7, 6} and {7, 8}. Add the edge {7, 6} and the vertex 6 in the MST as it has the least weight (i.e., 1).*



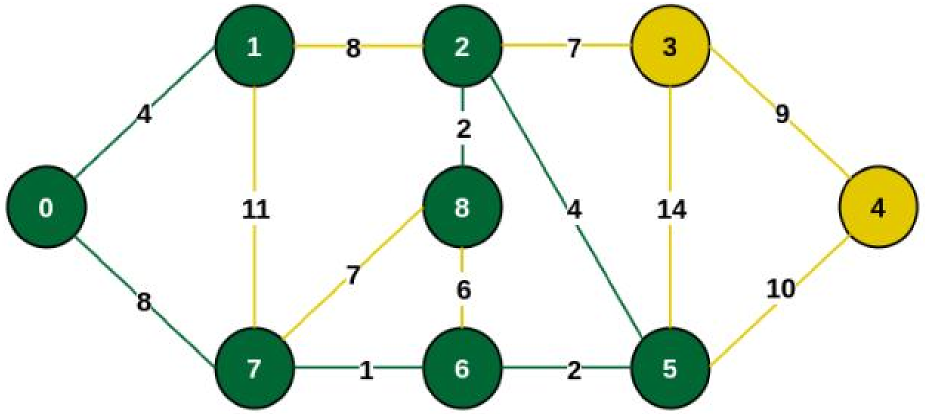
Step 5: *The connecting edges now are {7, 8}, {1, 2}, {6, 8} and {6, 5}. Include edge {6, 5} and vertex 5 in the MST as the edge has the minimum weight (i.e., 2) among them.*



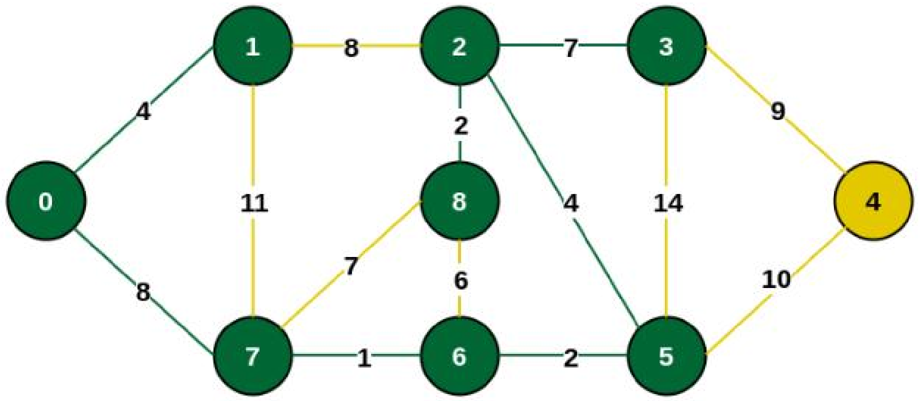
Step 6: *Among the current connecting edges, the edge {5, 2} has the minimum weight. So include that edge and the vertex 2 in the MST.*



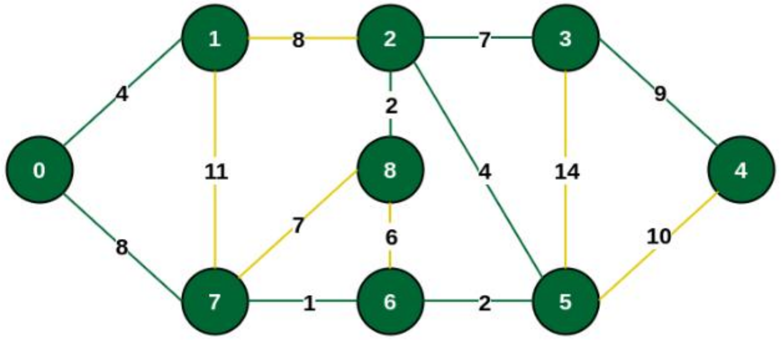
Step 7: *The connecting edges between the incomplete MST and the other edges are {2, 8}, {2, 3}, {5, 3} and {5, 4}. The edge with minimum weight is edge {2, 8} which has weight 2. So include this edge and the vertex 8 in the MST.*



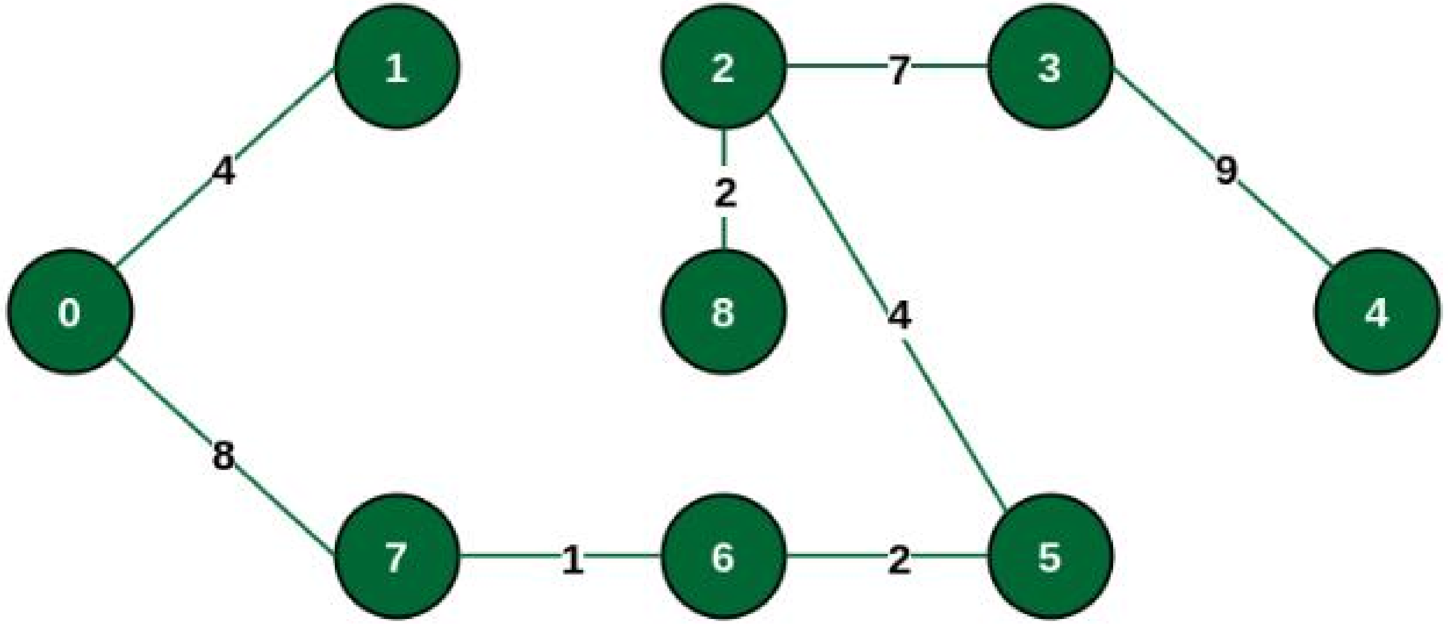
Step 8: *See here that the edges {7, 8} and {2, 3} both have same weight which are minimum. But 7 is already part of MST. So we will consider the edge {2, 3} and include that edge and vertex 3 in the MST.*



Step 9: *Only the vertex 4 remains to be included. The minimum weighted edge from the incomplete MST to 4 is {3, 4}.*



*The final structure of the MST is as follows and the weight of the edges of the MST is (4 + 8 + 1 + 2 + 4 + 2 + 7 + 9) = **37**.*



Kruskal Algorithm

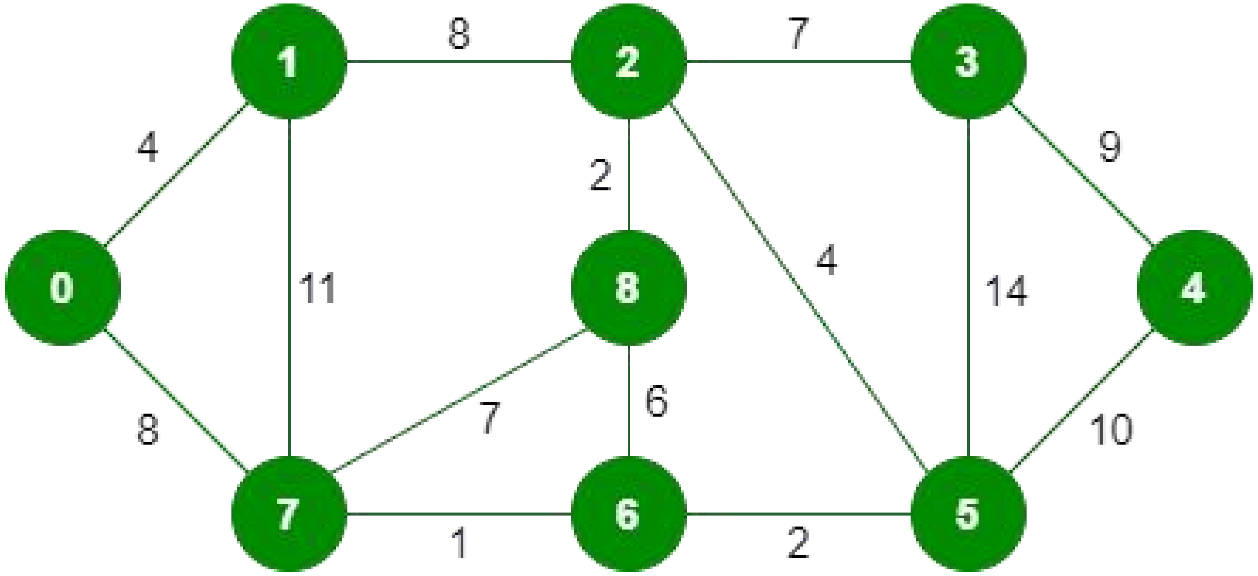
In Kruskal’s algorithm, sort all edges of the given graph in increasing order. Then it keeps on adding new edges and nodes in the MST if the newly added edge does not form a cycle. It picks the minimum weighted edge at first and the maximum weighted edge at last. Thus we can say that it makes a locally optimal choice in each step in order to find the optimal solution.

Algorithm:

- Sort all the edges in non-decreasing order of their weight.
- Pick the smallest edge. Check if it forms a cycle with the spanning tree formed so far. If the cycle is not formed, include this edge. Else, discard it.
- Repeat step#2 until there are (V-1) edges in the spanning tree.

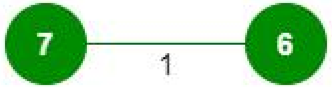
Illustration

Input Graph:

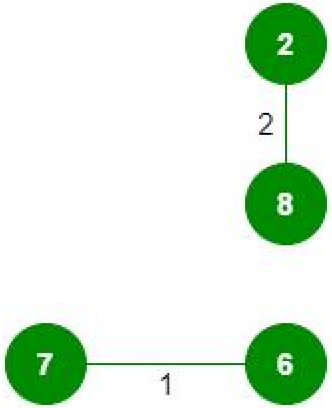


The graph contains 9 vertices and 14 edges. So, the minimum spanning tree formed will be having (9 – 1) = 8 edges.

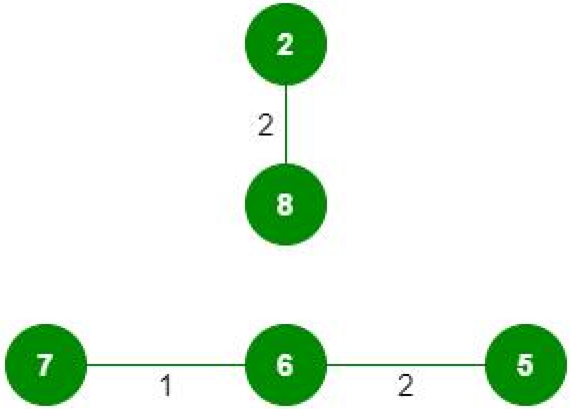
Step 1: *Pick edge 7-6. No cycle is formed, include it.*



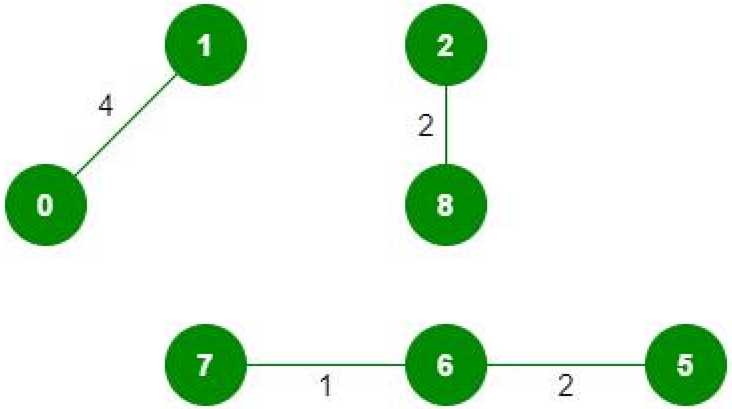
Step 2: *Pick edge 8-2. No cycle is formed, include it.*



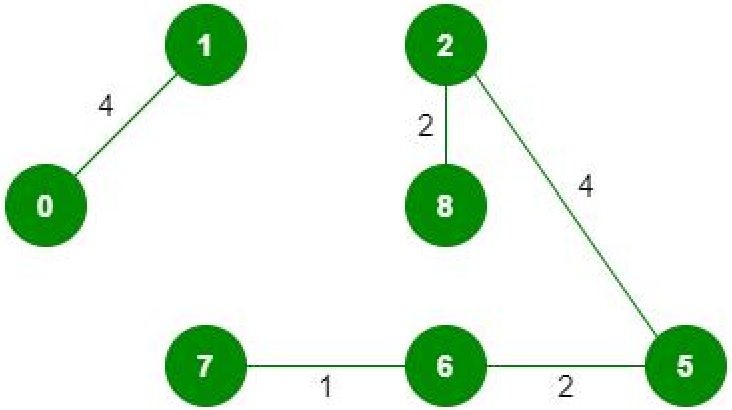
Step 3: *Pick edge 6-5. No cycle is formed, include it.*



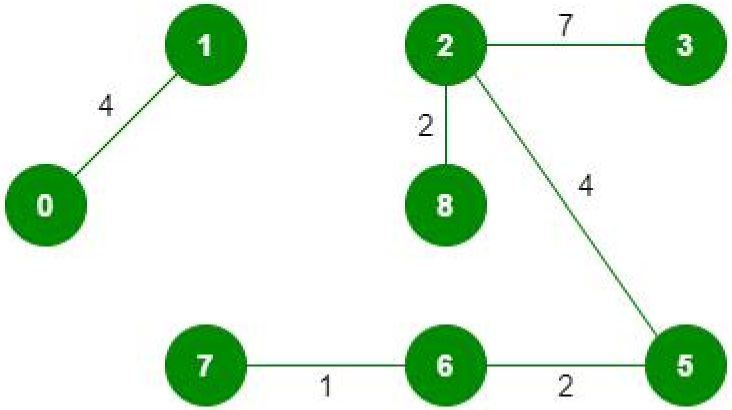
Step 4: *Pick edge 0-1. No cycle is formed, include it.*



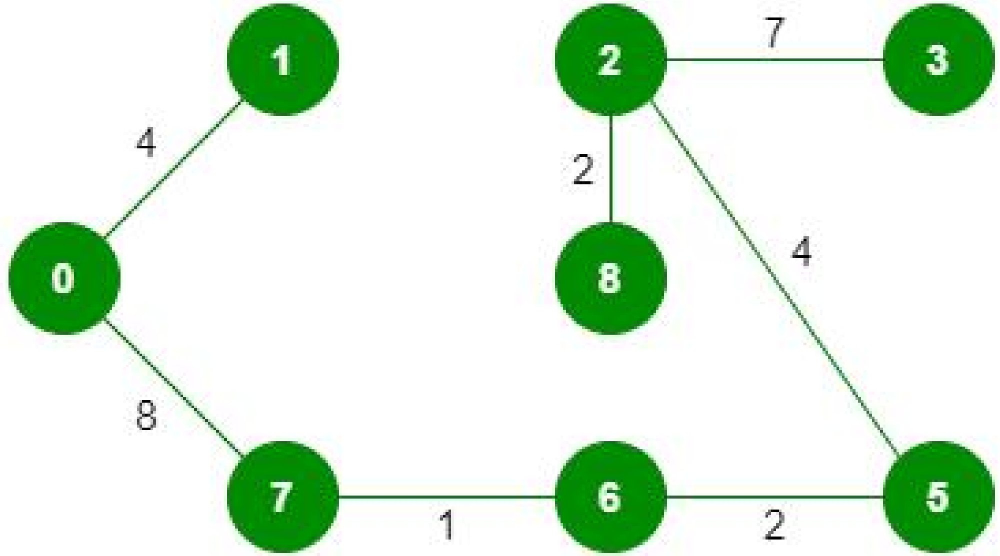
Step 5: *Pick edge 2-5. No cycle is formed, include it.*



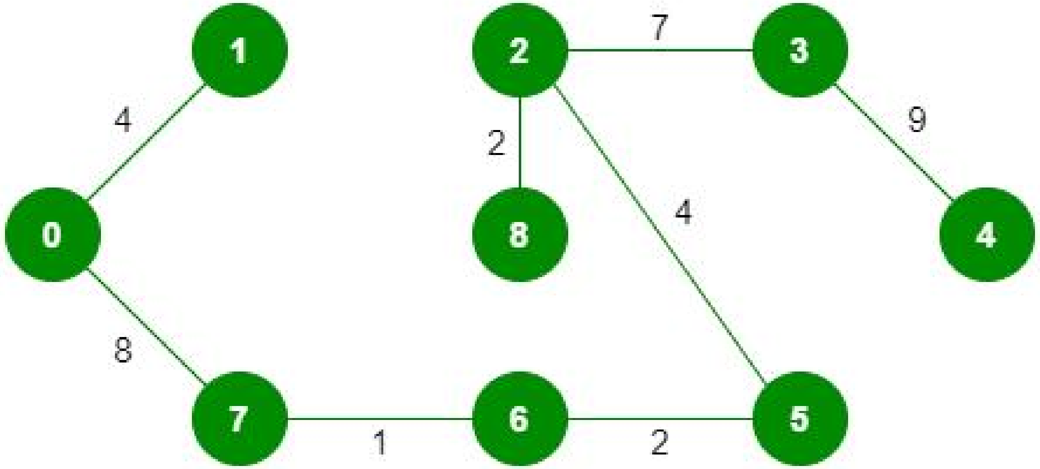
Step 6: *Pick edge 8-6. Since including this edge results in the cycle, discard it. Pick edge 2-3: No cycle is formed, include it.*



Step 7: *Pick edge 7-8. Since including this edge results in the cycle, discard it. Pick edge 0-7. No cycle is formed, include it.*



Step 8: *Pick edge 1-2. Since including this edge results in the cycle, discard it. Pick edge 3-4. No cycle is formed, include it.*



Note: *Since the number of edges included in the MST equals to (V – 1), so the algorithm stops here*