

Auto-Correlation Test (Example)

0.12, 0.01, 0.23, 0.28, 0.89, 0.31, 0.64, 0.28, 0.83, 0.93, 0.99, 0.15, 0.33, 0.35, 0.91, 0.41, 0.60, 0.27, 0.75, 0.88, 0.68, 0.49, 0.05, 0.43, 0.95, 0.58, 0.19, 0.36, 0.69, 0.87. ($\alpha=0.025$, Test No=3 at position 3rd, 8th, 13th are auto correlated) $N=30$

S1: Define Hypothesis,

S2: $i=3$, lag $m=5$

S3: Find **M**, $i+(M+1)m \leq N$

$$3+(M+1)5 \leq 30$$

$$\rightarrow (M+1)5 \leq 27$$

$$\rightarrow (M+1) \leq 5.4$$

$$\rightarrow M \leq 4.4 \text{ [So, } M = \max(4, 3, 2 \dots 0)]$$

M= 4

$$\textbf{S4: } \hat{\rho}_{im} = \frac{1}{M+1} \cdot \left[\sum_{k=0}^M R_{i+km} \cdot R_{i+[k+1]m} \right] - 0.25$$

$$\hat{\rho}_{35} = \frac{1}{4+1} \left[\sum_{k=0}^4 R_{3+5k} \cdot R_{3+5[k+1]} \right] - 0.25$$

$$= \frac{1}{5} [R_3 \cdot R_8 + R_8 \cdot R_{13} + R_{13} \cdot R_{18} + R_{18} \cdot R_{23} + R_{23} \cdot R_{28}] - 0.25$$

$$= \frac{1}{5} [0.23 \cdot 0.28 + 0.28 \cdot 0.33 + 0.33 \cdot 0.27 + 0.27 \cdot 0.05 + 0.05 \cdot 0.36]$$

$$= \frac{1}{5} (0.2774) - 0.25$$

$$= 0.05548 - 0.25$$

$$= -0.19452, \text{ This is the estimator}$$

$$\textbf{S5: } \sigma_{\hat{\rho}_m} = \frac{\sqrt{13(4)+7}}{12(4+1)} = \frac{\sqrt{52+7}}{60} = 0.128$$

$$\textbf{S6: } Z_0 = \frac{\hat{\rho}_{im}}{\sigma_{\hat{\rho}_m}} = \frac{-0.19452}{0.128} = \textbf{-1.51}$$

S7: $Z_{0.025} = 1.96$

S8: $-Z_{\alpha/2} \leq Z_0 \leq +Z_{\alpha/2}$

$-1.96 \leq -1.51 \leq 1.96$, H_0 is accepted.