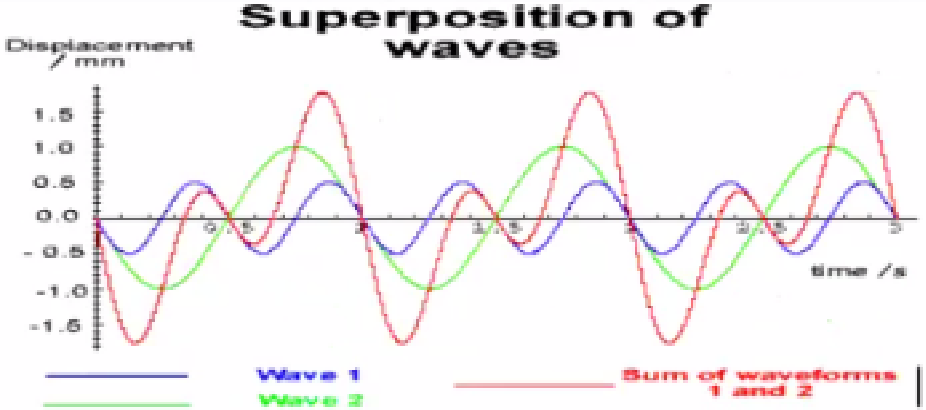
The principle of superposition of Wave:

The principle of superposition states that "***when two or more waves simultaneously pass through a point, the disturbance at the point is given by the vector sum of the disturbance each wave would produce in absence of the other wave***".



To understand superposition, we consider two waves passing through a point in a medium. Let y_1 be the displacement of a particle at that point due to the first wave in the absence of second wave and y_2 be the displacement at that point due to the second wave in the absence of the first wave. The resultant displacement (R) at that point when both the waves act simultaneously is

$$R = y_1 \pm y_2$$

The ‘+’ sign is used when both displacements are in same direction.

Progressive Wave

Periodic disturbance propagation from one layer to another layer of a wide medium and advances continuously in the forward direction is known as Progressive Wave.

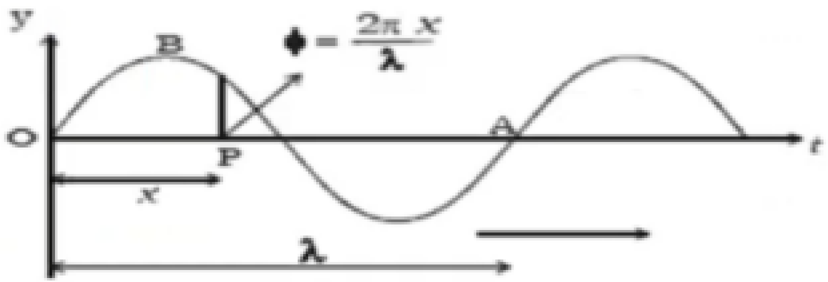
Characteristics of progressive wave

1. Every particle of the medium executes periodic motion.
2. The amplitude of each particle of the medium is same, but there exits phase difference between them.
3. The distance between two successive crests of a transverse wave and distance between a compression and rarefaction is a wavelength.
4. The changes in pressure and density of the medium are similar in case of progressive waves.
5. In a progressive wave, the particle of the medium wave attains a stationary position.
6. The equation of a progressive wave is,

$$y = A \sin \frac{2\pi}{\lambda}(vt - x)$$

Equation of a plane progressive wave

Let us assume that a progressive wave travels from the origin O along the positive direction of X axis (Fig.). The displacement of a particle at a given instant is



Plane Progressive wave

$$y = a \sin \omega t \qquad \ldots\ldots (1)$$

Where *a* is the amplitude of the vibration of the particle and $\omega = 2\pi n$.

The displacement of the particle P at a distance x from O at a given instant is given by,

$$y = a \sin (\omega t - \varphi) \qquad \ldots\ldots (2)$$

If the two particles are separated by a distance λ, they will differ by a phase of 2π. Therefore, the phase φ of the particle P at a distance X is $\varphi = (2\pi/\lambda)\, x$

$$y = a \sin (\omega t - 2\pi x/\lambda) \qquad \ldots\ldots (3)$$

Since $\omega = 2\pi n = 2\pi\, (v/\lambda)$, the equation is given by,

$$y = a \sin [(2\pi vt/\lambda) - (2\pi x/\lambda)]$$

$$y = a \sin 2\pi/\lambda\, (vt - x) \qquad \ldots\ldots (4)$$

If the wave travels in opposite direction, the equation becomes,

$$y = a \sin 2\pi/\lambda\,(vt + x) \qquad \ldots\ldots (5)$$

Stationary Wave

The resultant wave produce by the superposition of two progressive waves having same wavelength and amplitude, travelling in opposite direction is called Stationary Waves.

Characteristics of stationary Wave

1) Stationary waves are produced when two identical progressive waves travelling along the same straight line but in opposite direction are superposed.
2) Crests and trough or compression and rarefraction do not progress forward through the medium, but simply appear and disappear at the same plane alternately.
3) The point where the amplitude is zero is called nodes and where it is maximum is called antinodes.
4) All the particles, excepts those at the nodes, execute simple harmonic motion.
5) The distance between two adjacent nodes or antinodes is equal to half of the wavelength.
6) There is no propagation of energy in a stationary wave.
7) Stationary waves are produced both by transverse & longitudinal waves.

Equation of a standing wave

The phenomenon can be demonstrated mathematically by deriving the equation for the sum of two oppositely moving waves:

A harmonic wave traveling to the right along the x-axis is described by the equation

$$y_1 = a \sin \frac{2\pi}{\lambda} (vt - x) \qquad \text{---------(1)}$$

An identical harmonic wave traveling to the left is described by the equation

$$y_2 = a \sin \frac{2\pi}{\lambda} (vt + x) \qquad \text{---------(2)}$$

Where: **a** is the amplitude of the wave, ω (called the angular frequency and measured in radians per second) is 2π times the frequency (in hertz), λ is the wavelength of the wave (in *meters*)

So the equation of the resultant wave y will be the sum of y_1 and y_2:

$y = y_1 + y_2$ =to simplify using the trigonometric sum-to-product identity

$$y = a \sin \frac{2\pi}{\lambda} (vt - x) + a \sin \frac{2\pi}{\lambda} (vt + x)$$

$$y = 2a \sin \frac{2\pi}{\lambda} \left(\frac{vt - x + vt + x}{2}\right) \text{Cos} \frac{2\pi}{\lambda} \left(\frac{vt - x - vt - x}{2}\right)$$

$$\sin a + \sin b = 2 \sin \left(\frac{a+b}{2}\right) \cos \left(\frac{a-b}{2}\right)$$

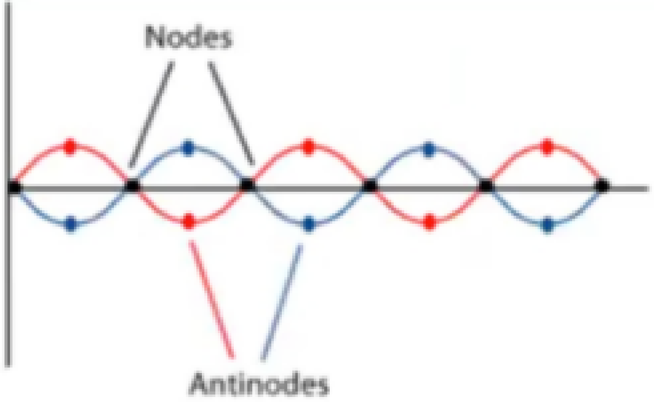
$$y = 2a \sin \frac{2\pi}{\lambda} vt \quad \text{Cos} \frac{2\pi}{\lambda} (-x)$$

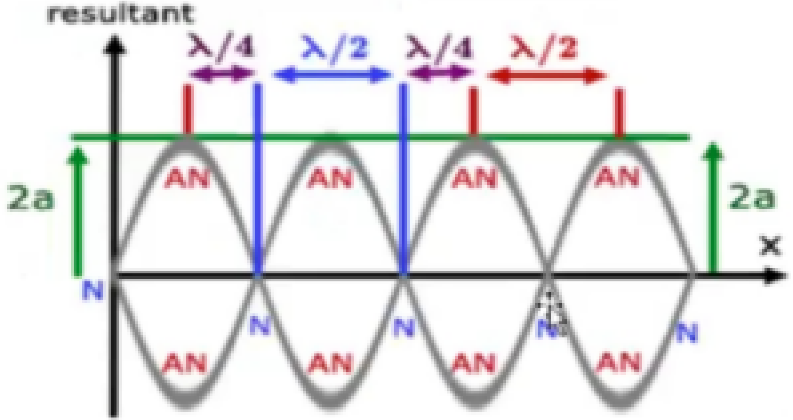
$$y = 2a \cos \frac{2\pi}{\lambda} x \ \sin \frac{2\pi}{\lambda} vt$$

$$y = A \sin \frac{2\pi}{\lambda} vt$$

$$\text{Where, } A = 2a \cos \frac{2\pi}{\lambda} x$$

This describes a wave that oscillates in time, but has a spatial dependence that is stationary; at any point x the amplitude of the oscillations is constant with value $2a\sin\left(\frac{2\pi vt}{\lambda}\right)$.At locations which are *even* multiples of a quarter wavelengths called the nodes, the amplitude is always zero.





<u>**Antinode:**</u> The points on the stationary wave where the particles of the medium vibrate with maximum amplitude are called antinode.

<u>**Node:**</u> The points on the stationary wave where particles of the medium remain static i.e. the displacement of the particles are zero are called nodes.

The distance between two **NODES** or between two **ANTINODES** is half a wavelength.

The distance between a **NODE** and the next **ANTINODES** is one quarter of a wavelength.