

Disjoint Set Union (C++)

Sets can make friends too!

#csspre

Online

Disjoint Set

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Group of sets with no common elements.

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$\{1, 3, 4\}$ $\{5, 6\}$ $\{2, 7, 11\}$

$\{12, 15\}$ $\{9, 13\}$

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No sets overlap

Valid ✓

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$\{4, 15\}$ $\{9, 13\}$

No sets overlap

Valid ✓

Disjoint Set

Group of sets with no common elements.

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 $\{12, 15\}$ $\{9, 13\}$

No sets overlap

Valid ✓

$\{1, 3, 4\}$ $\{5, 6\}$ $\{2, 9, 11\}$
 $\{4, 15\}$ $\{9, 13\}$

Sets have common element(s)

Invalid ✗

Disjoint Set Union

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Things it does:

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1. `Find(u)` // which set `u` belongs to

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1. `Find(u)` // which set `u` belongs to
2. `Union(u, v)` // merge sets of `u` and `v`

Some trivial operations...

3. `MakeSet(u)` // initialize `u` as a set
4. `IsSameSet(u, v)` // check if `u` and `v` belongs to same set (`isFriend(u, v)`)

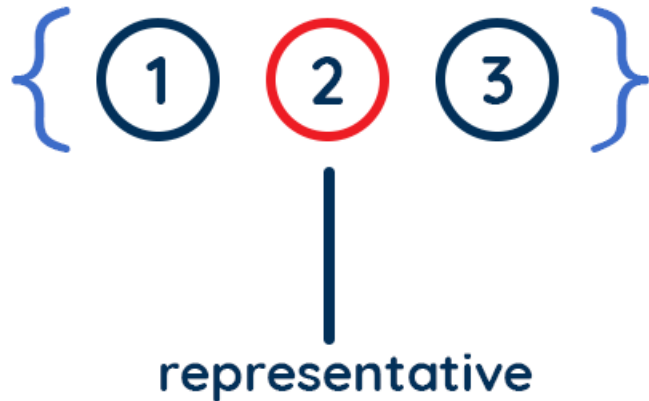
Disjoint Set Union

How it does:

Disjoint Set Union

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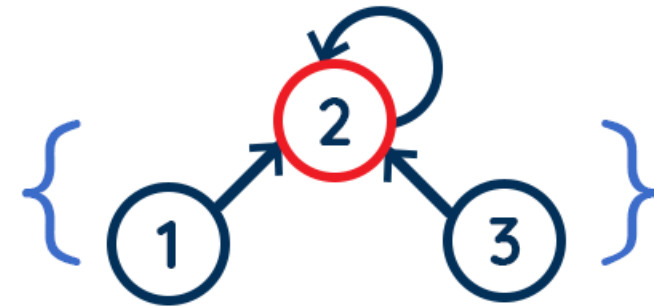
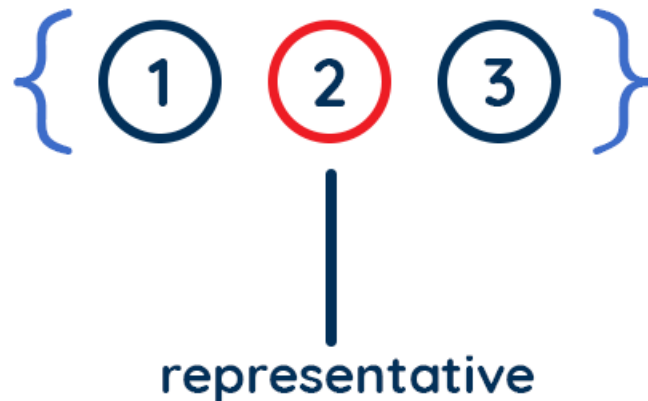
1. [Identification] : Every set has a representative
(one of the elements of the set)



Disjoint Set Union

How it does:

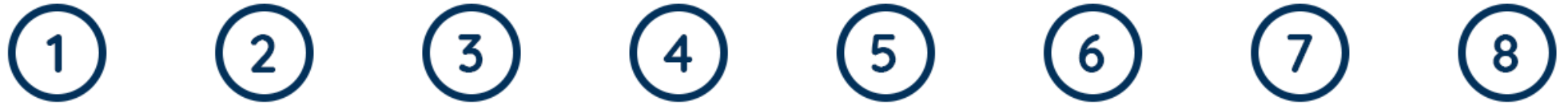
1. [Identification] : Every set has a representative (one of the elements of the set)
2. [Relation] : Elements are connected via parent-child relation



2 is parent of 1, 3 and itself

Simulation

Simulation



Simulation



initially, every element is parent of itself

Simulation



parent[] =

1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

Simulation



`Union(1, 5)`

Simulation



Union(1, 5)

```
p = Find(1) // find parent/representative of 1's set  
q = Find(5) // find parent/representative of 5's set
```

Simulation



Union(1, 5)

```
p = Find(1) // find parent/representative of 1's set
```

```
q = Find(5) // find parent/representative of 5's set
```

If the parents (p and q) are not same, they are in different set.
In this case, make p the parent of q (or vice versa).

Simulation



`Union(1, 5)`

`p = Find(1)`

Simulation



`Union(1, 5)`

`p = Find(1) = 1`

Simulation



`Union(1, 5)`

`p = Find(1) = 1`

`q = Find(5)`

Simulation



`Union(1, 5)`

`p = Find(1) = 1`

`q = Find(5) = 5`

Simulation

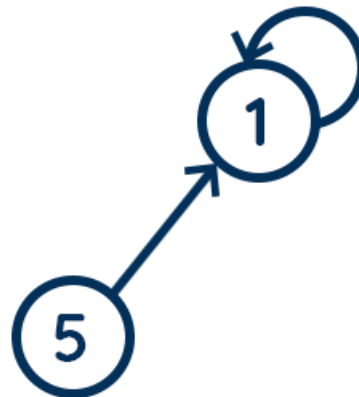


Union(1, 5)

`p = Find(1) = 1`

`q = Find(5) = 5`

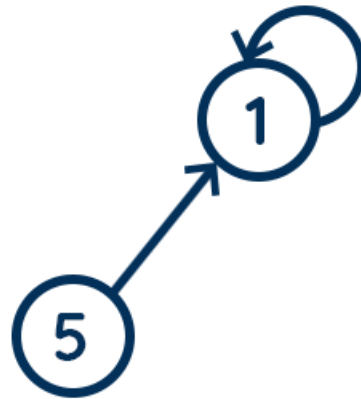
`parent[q] = p //merge`



Simulation



`Union(2, 8)`

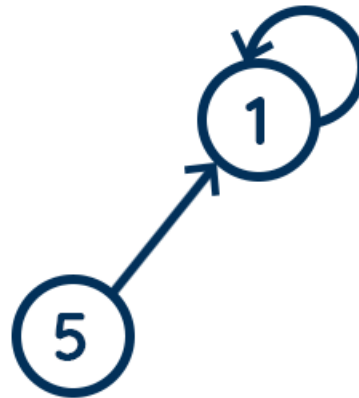


Simulation



Union(2, 8)

$p = 2$
 $q = 8$



Simulation

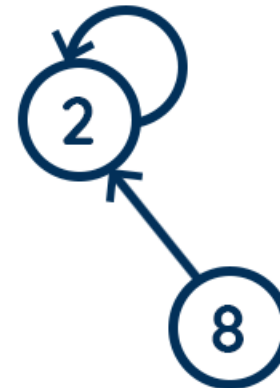
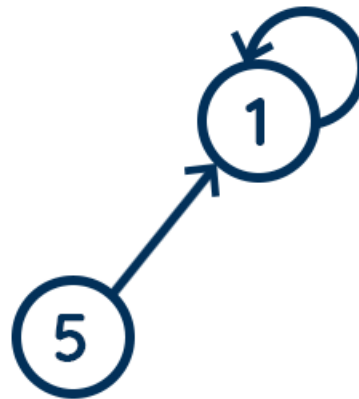


Union(2, 8)

`p = 2`

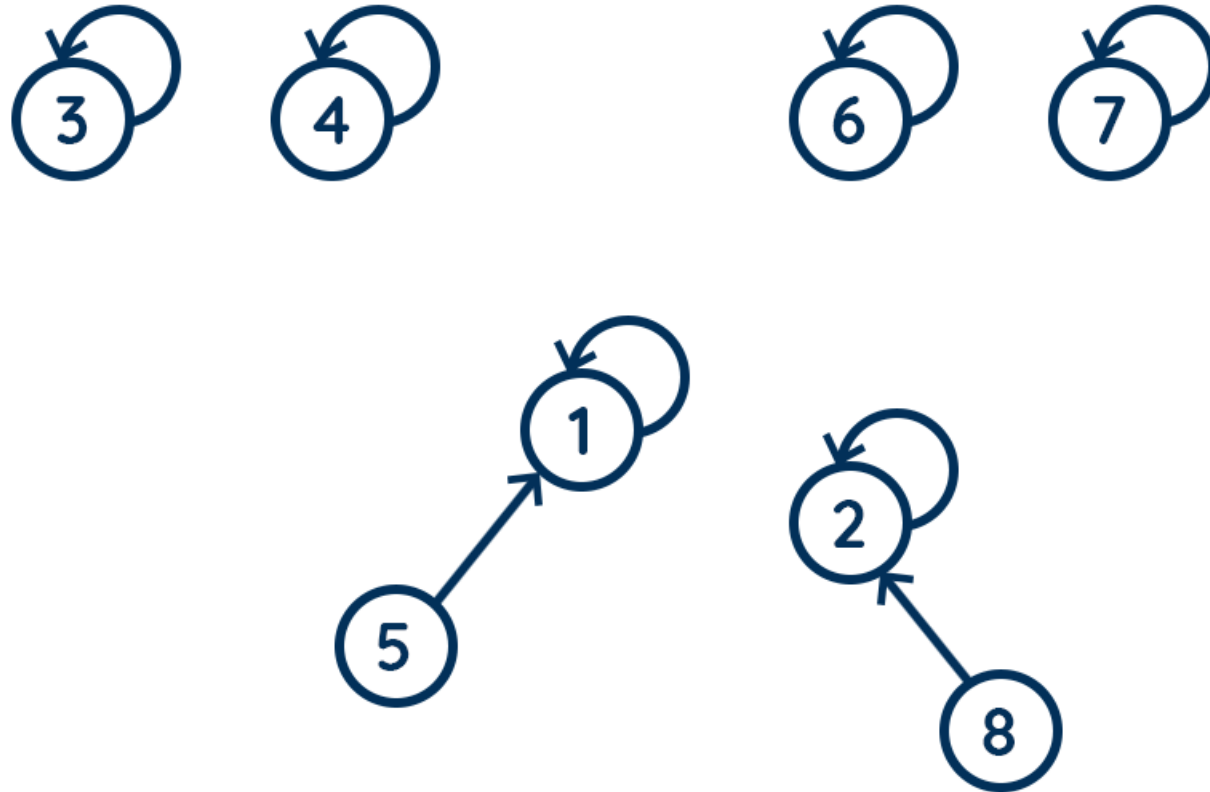
`q = 8`

`parent[8] = 2 //merge`



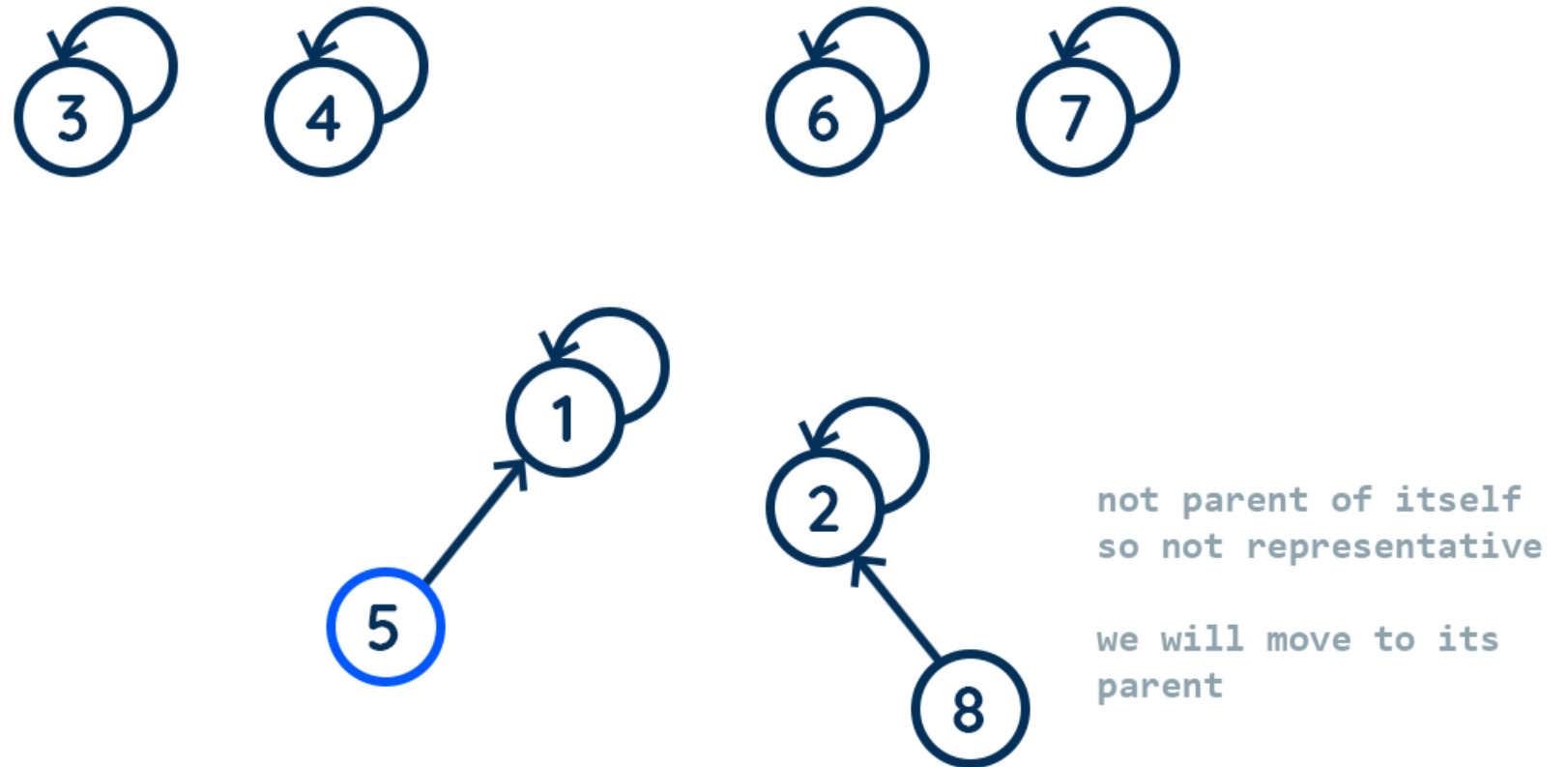
Simulation

Union(5, 8)



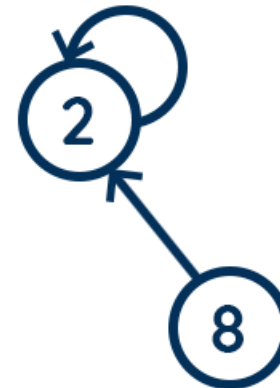
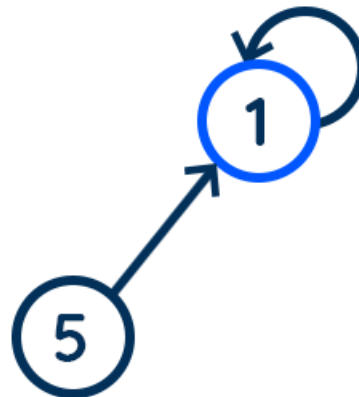
Simulation

Union(5, 8)



Simulation

Union(5, 8)

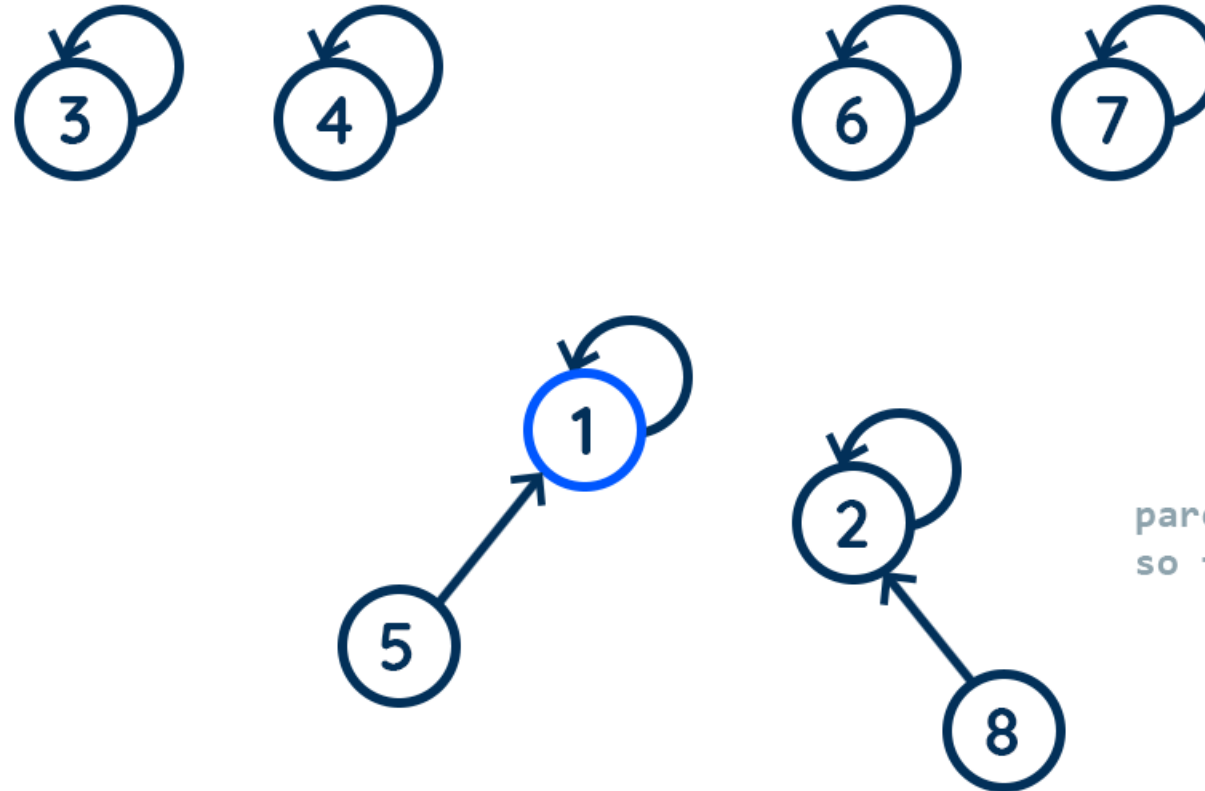


parent of itself
so found representative

Simulation

Union(5, 8)

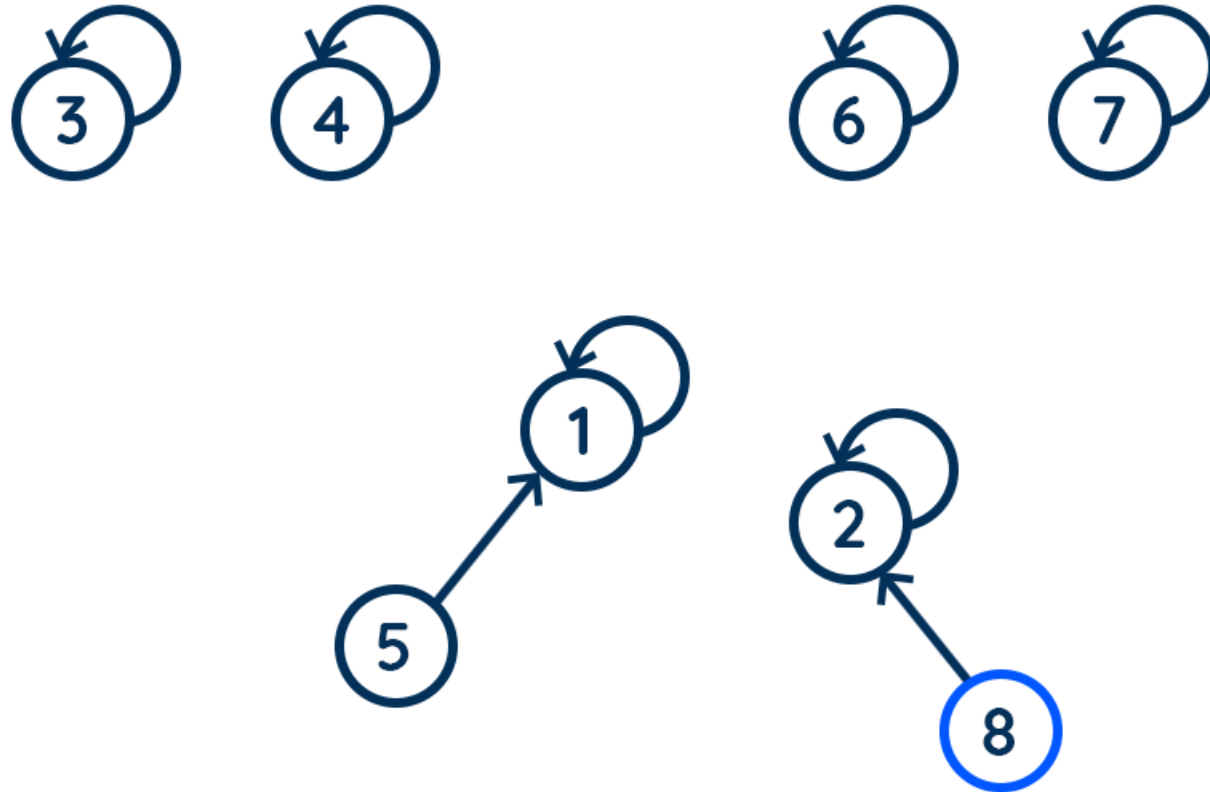
p = 1



Simulation

Union(5, 8)

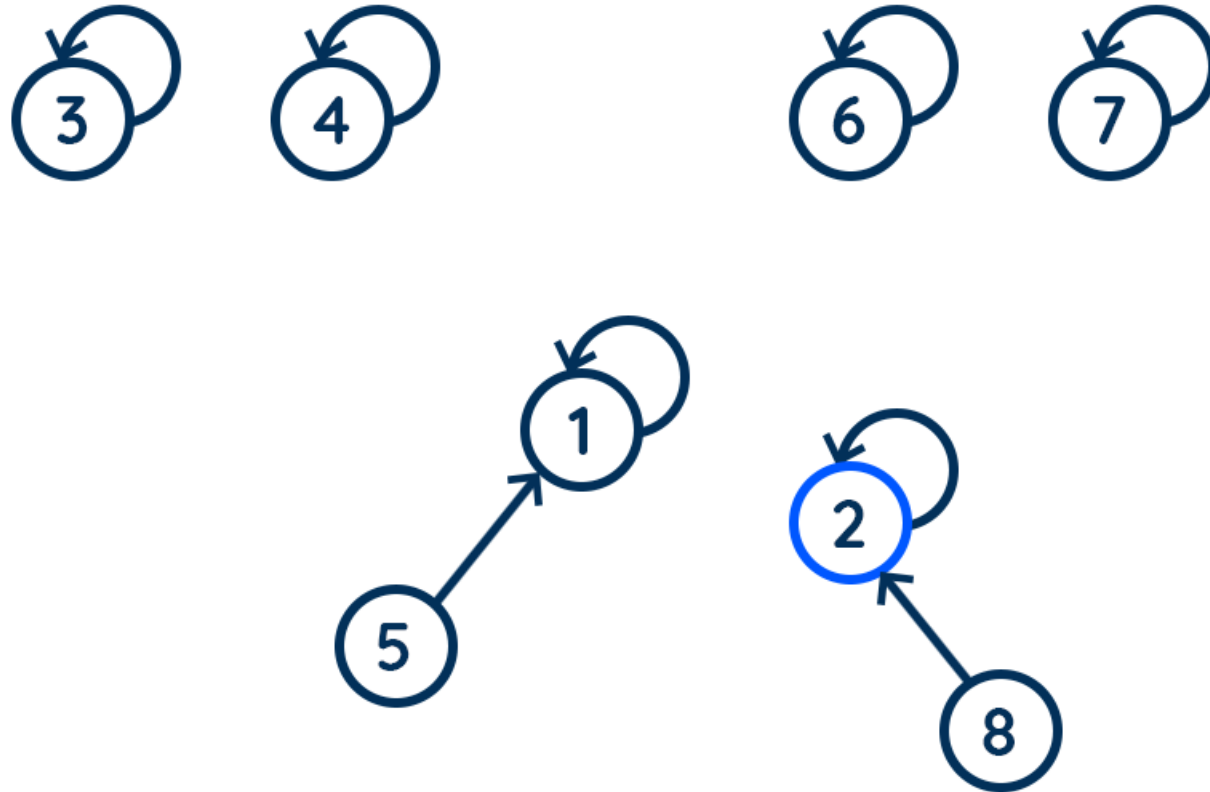
p = 1



Simulation

Union(5, 8)

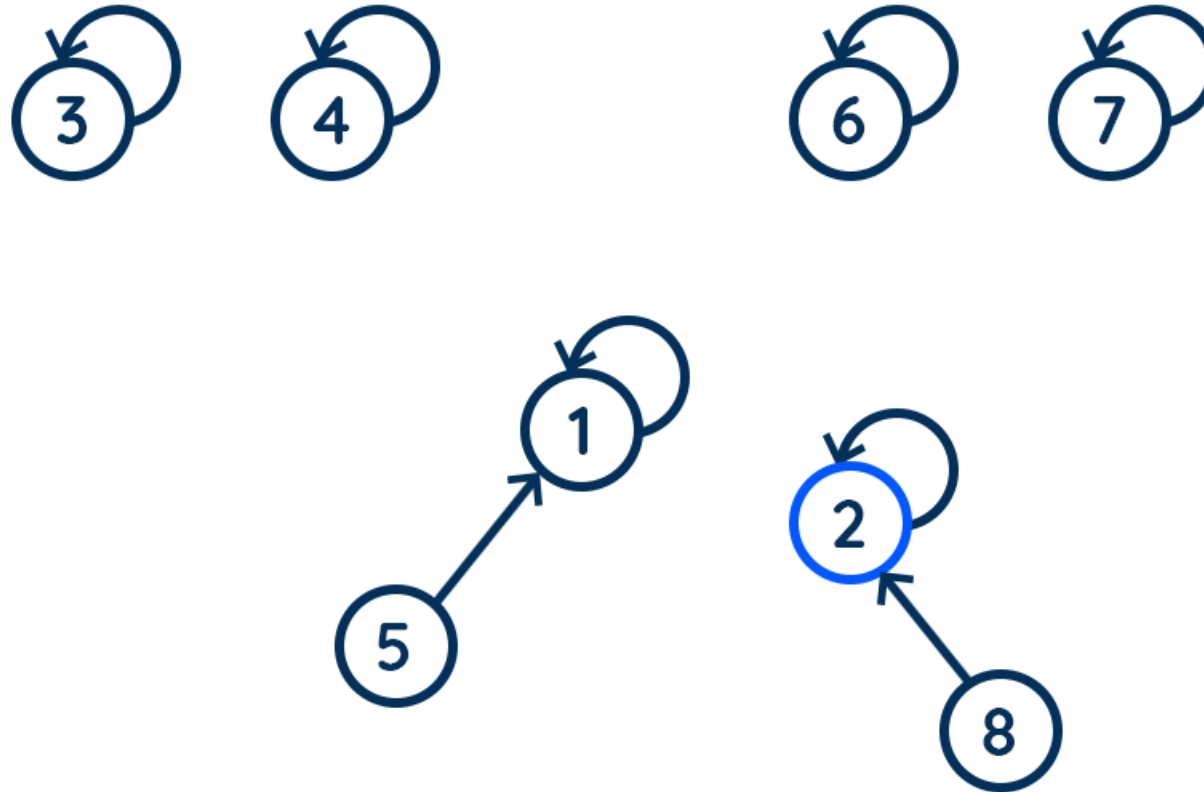
p = 1



Simulation

Union(5, 8)

p = 1
q = 2



Simulation

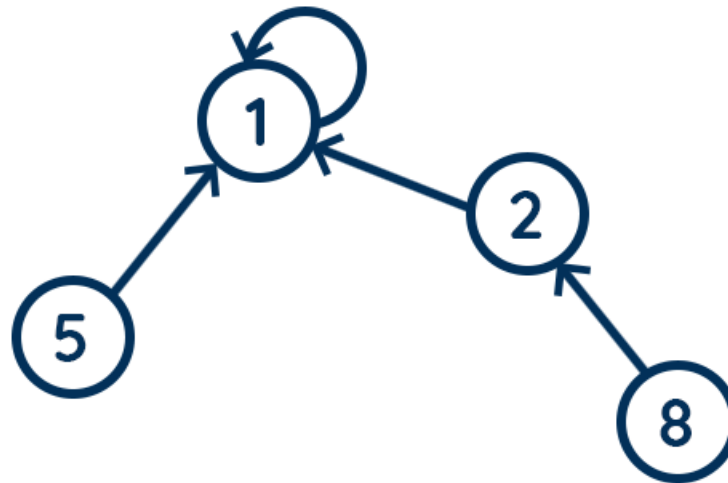


Union(5, 8)

$p = 1$

$q = 2$

$\text{parent}[2] = 1$



Optimization

Optimization

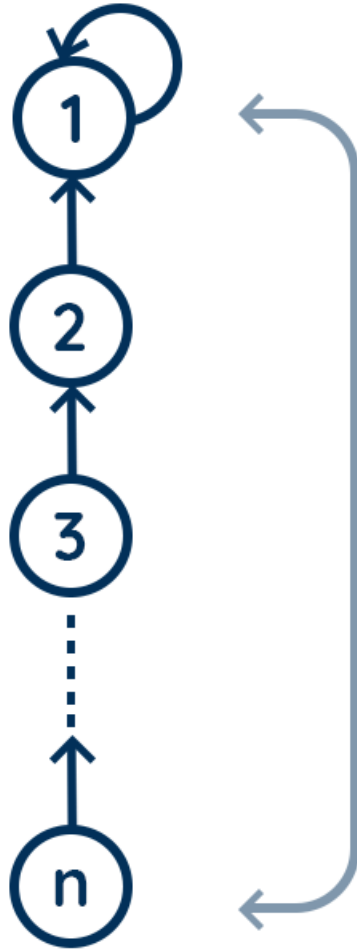


Optimization



Find(n) //returns parent of n's set

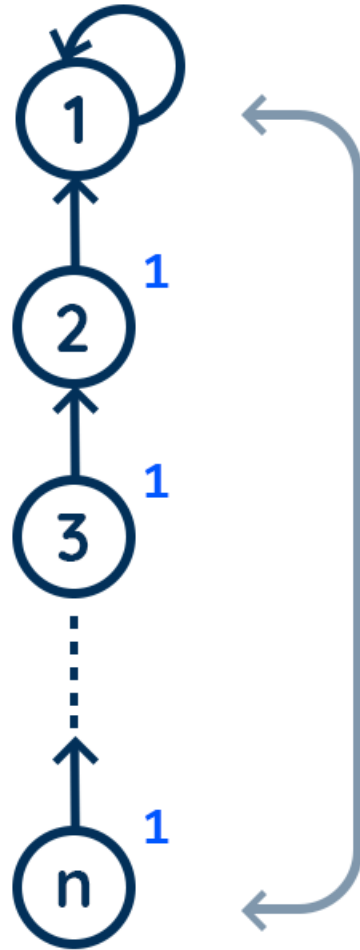
Optimization



Find(n) //returns parent of n's set

traverses length n
every time we call Find(n)

Optimization

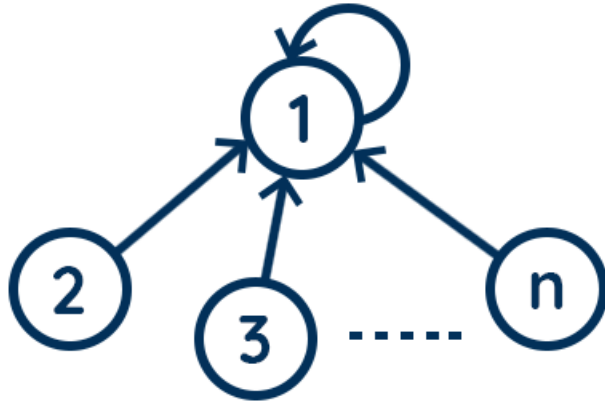


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we can set
parent[u] = Find(parent[u])

Optimization



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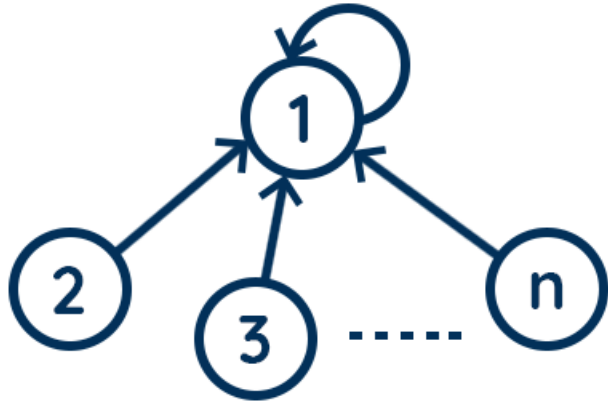
traverses length n
every time we call Find(n)

we can set

parent[u] = Find(parent[u])

so next time we need to know
parent of u we have to call
once!

Optimization



this technique is called
path compression

Find(n) //returns parent of n's set

traverses length n
every time we call Find(n)

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parent[u] = Find(parent[u])

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