Financial Engineering Lab MA – 374 Lab – 10

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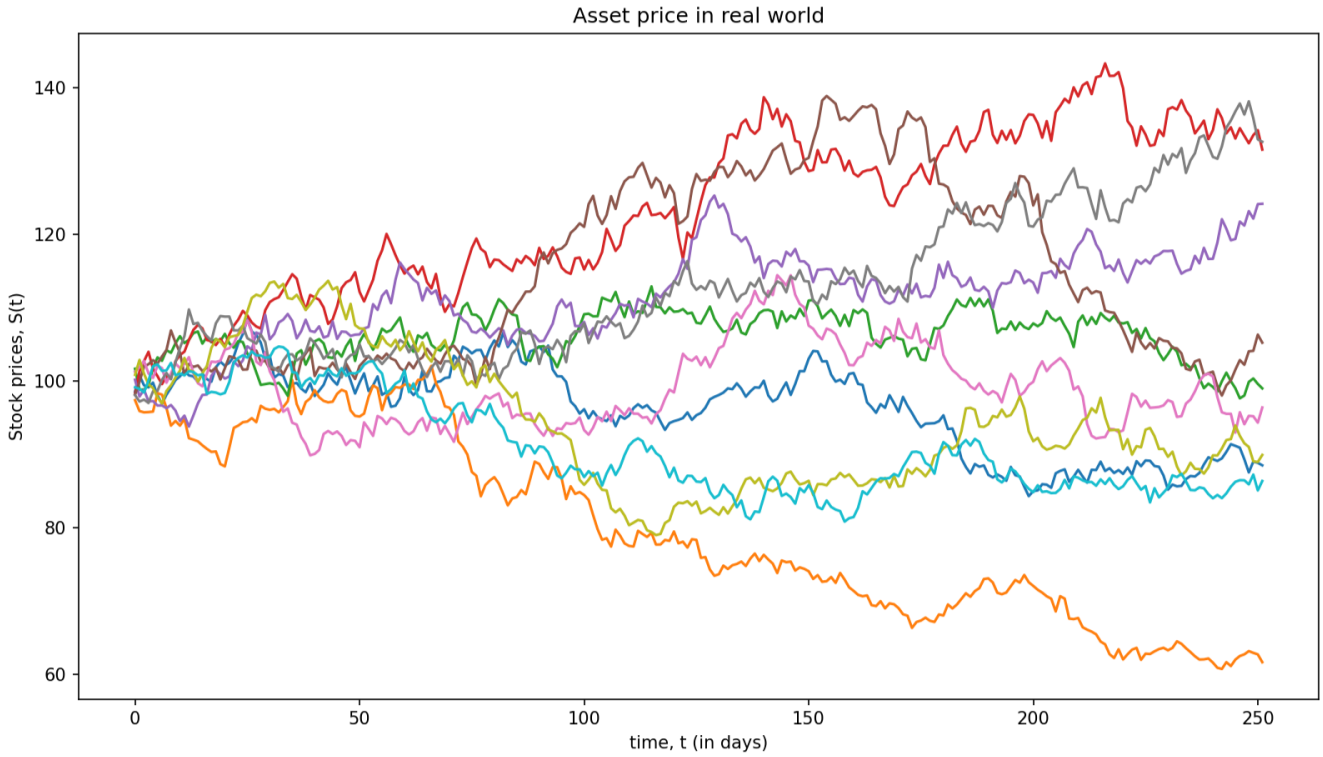
Branch – Mathematics and Computing

Question 1:

1. Consider an asset which follows a geometric Brownian motion (GBM) with drift $\mu = 10\%$ and volatiltiy $\sigma = 20\%$. Assume that the risk free rate is $r = 5\%$. The initial asset price at time $t = 0$ is $S(0) = 100$.
   Simulate 10 different paths of the asset price making use of the GBM, in both the real and the risk-neutral worlds.
   Now compute the price of a six month fixed-strike Asian option with a strike price of 105 (using arithmetic average). Do the pricing for both call and put options, using Monte Carlo simulation.
   Repeat the above exercise with strike price $K = 110$ and $K = 90$. How do your results compare ?
   Now do a sensitivity analysis of the option prices.

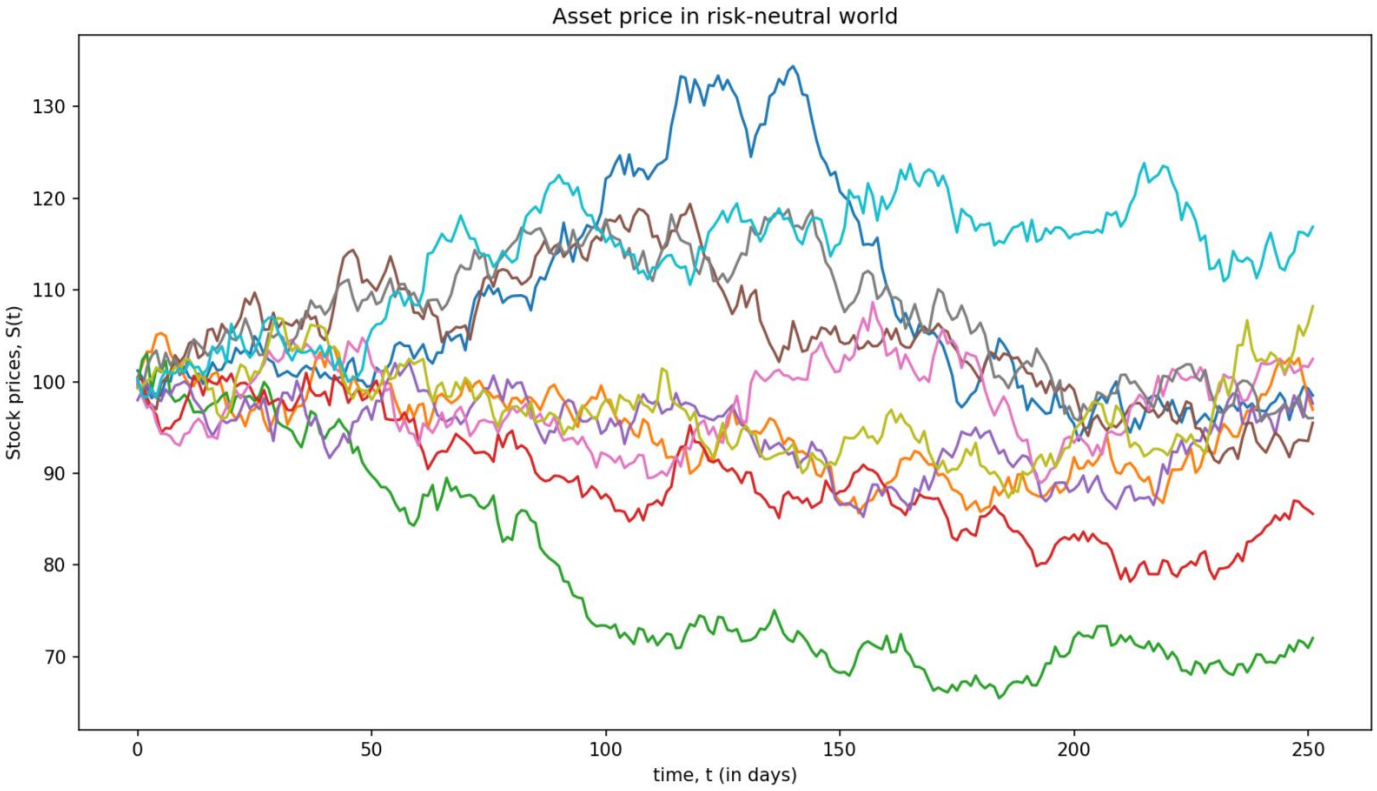
10 different paths of the asset price making use of GBM in real world are as follows:



where the evolution of the asset price in real world is given by the following differential equation:

$$dS(\mathrm{t}) = \mu S(\mathrm{t})dt + \sigma S(\mathrm{t})dW(t)$$

10 different paths of the asset price making use of GBM in risk-neutral world are as follows:



where the evolution of the asset price in the risk-neutral world is given by the following differential equation:

$$dS(t) = rS(t)dt + \sigma S(t)d\widetilde{W}(t)$$

where $\widetilde{W}(t)$ *is a Brownian motion under risk – neutral probability measure*

The prices of a six-month fixed-strike Asian option with various strike prices are as follows:

1) For K = 90,

```
For K = 90,
Price of Asian Call Option = 10.902886043690051
Variance in Price of Asian Call Option = 57.11516949695493

Price of Asian Put Option = 0.325501645380826O4
Variance in Price of Asian Put Option = 1.8015876743021508
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2) For K = 105,

```
For K = 105,
Price of Asian Call Option = 1.80004001656251 83
Variance in Price of Asian Call Option = 14.372311207618779

Price of Asian Put Option = 5.519400431507837
Variance in Price of Asian Put Option = 31.643362586066814
```

3) For K = 110,

```
For K = 110,
Price of Asian Call Option = 0.65038642660944 65
Variance in Price of Asian Call Option = 5.242431544735323

Price of Asian Put Option = 9.57536576047394
Variance in Price of Asian Put Option = 48.47287290549498
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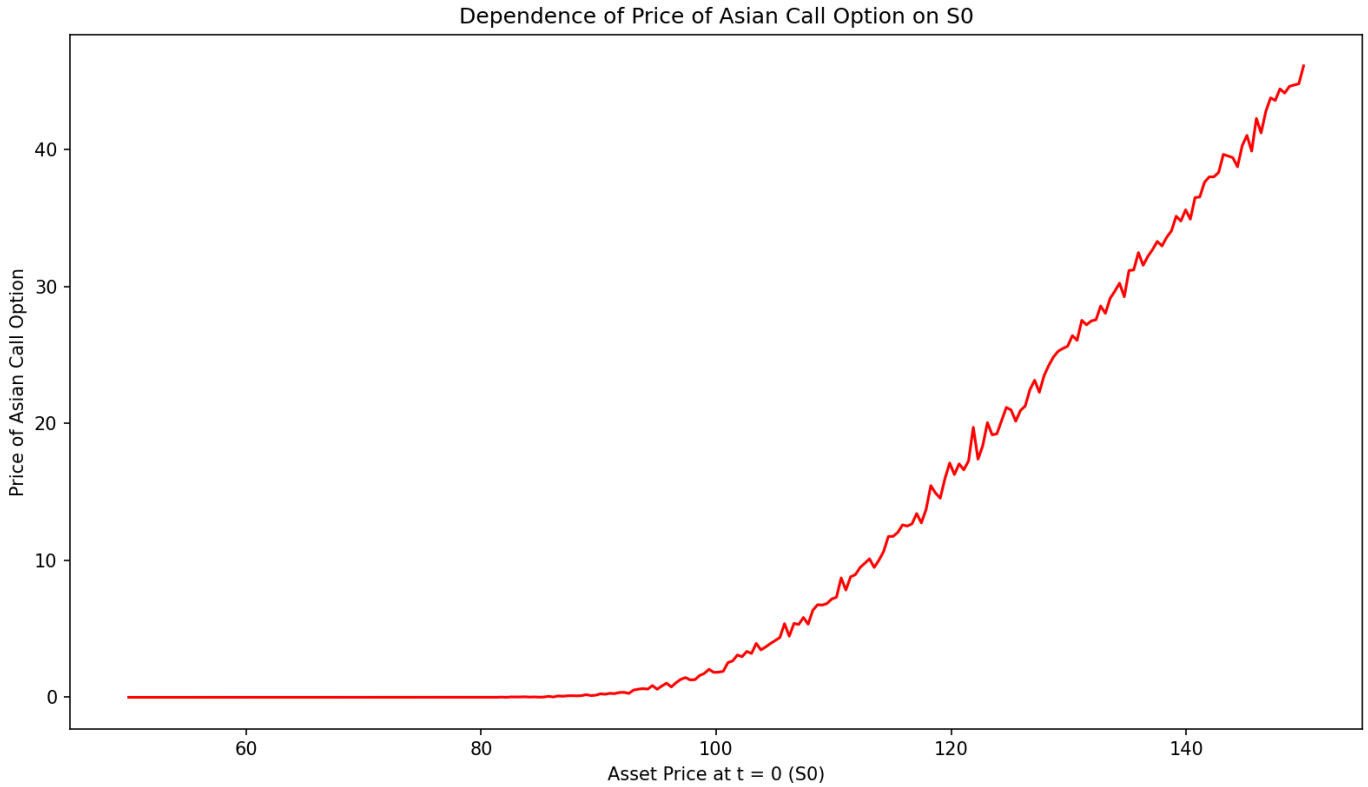
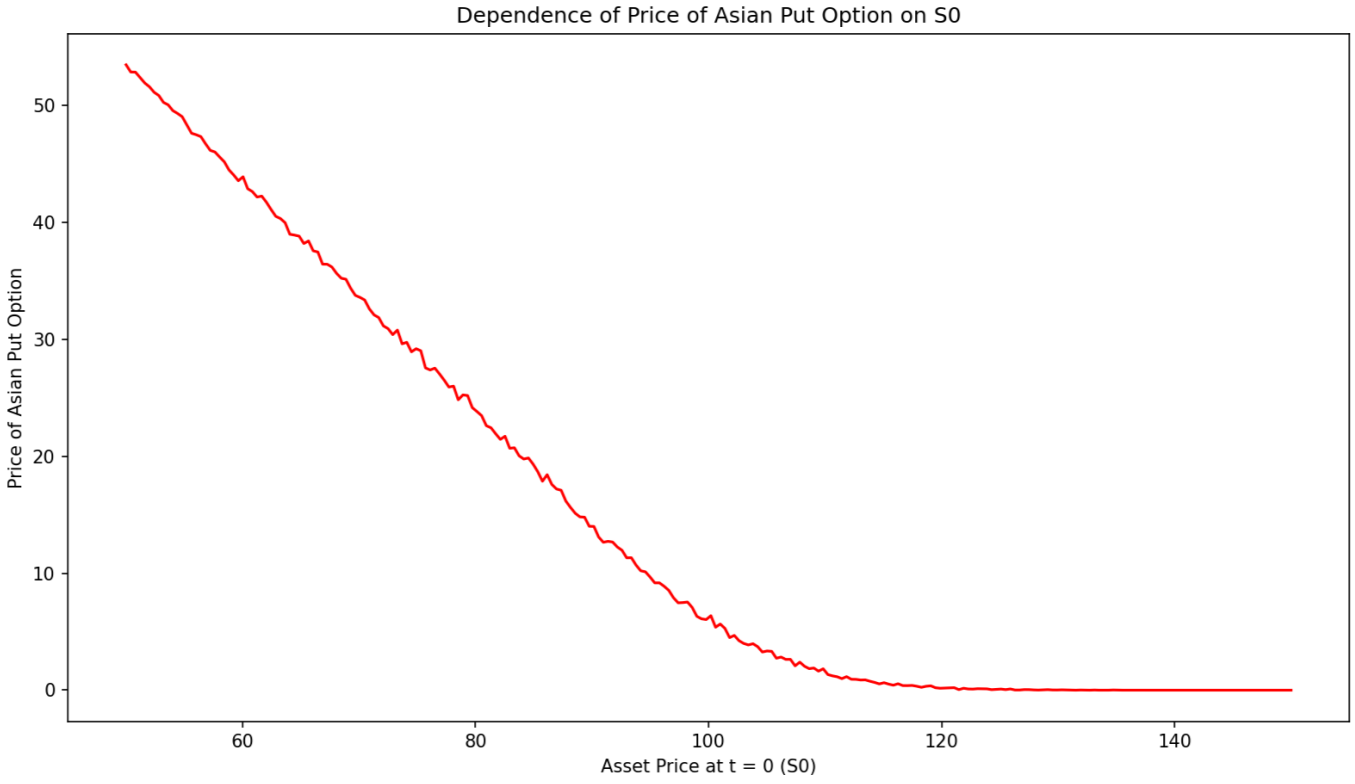
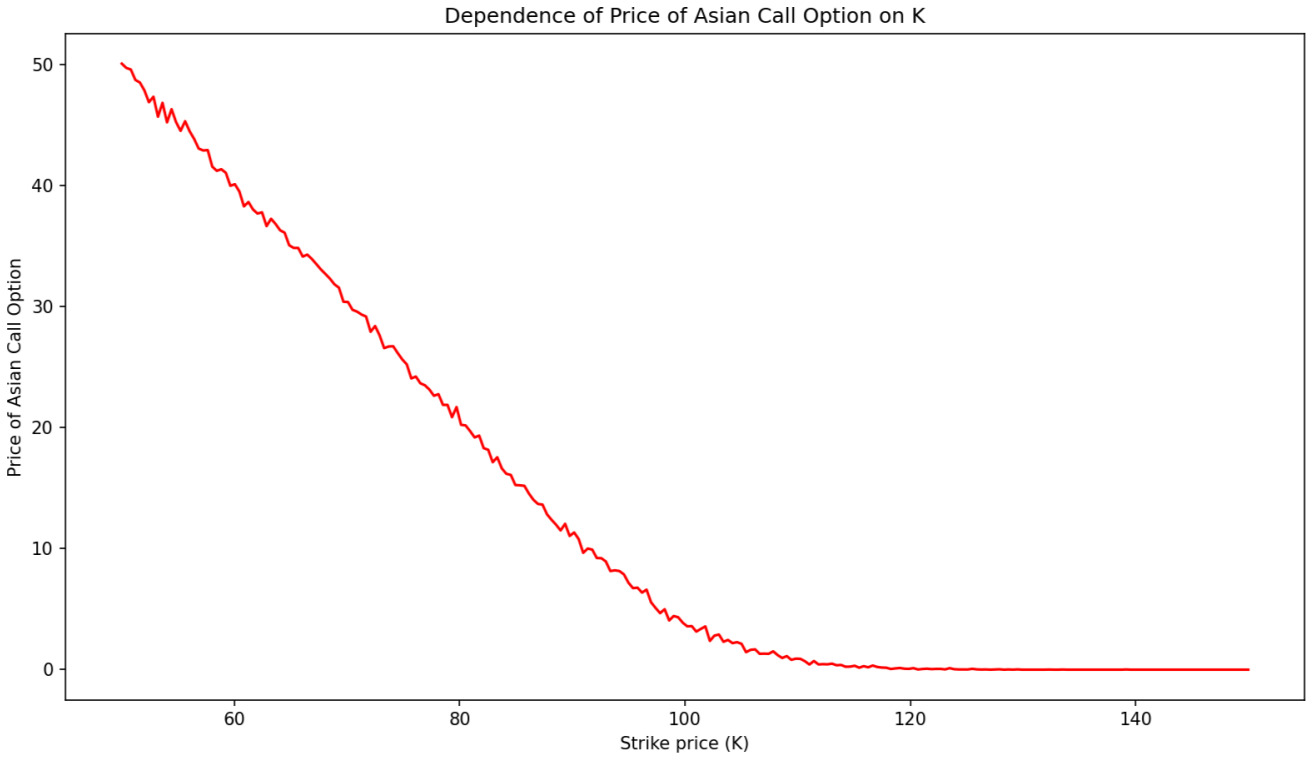
Sensitivity Analysis:

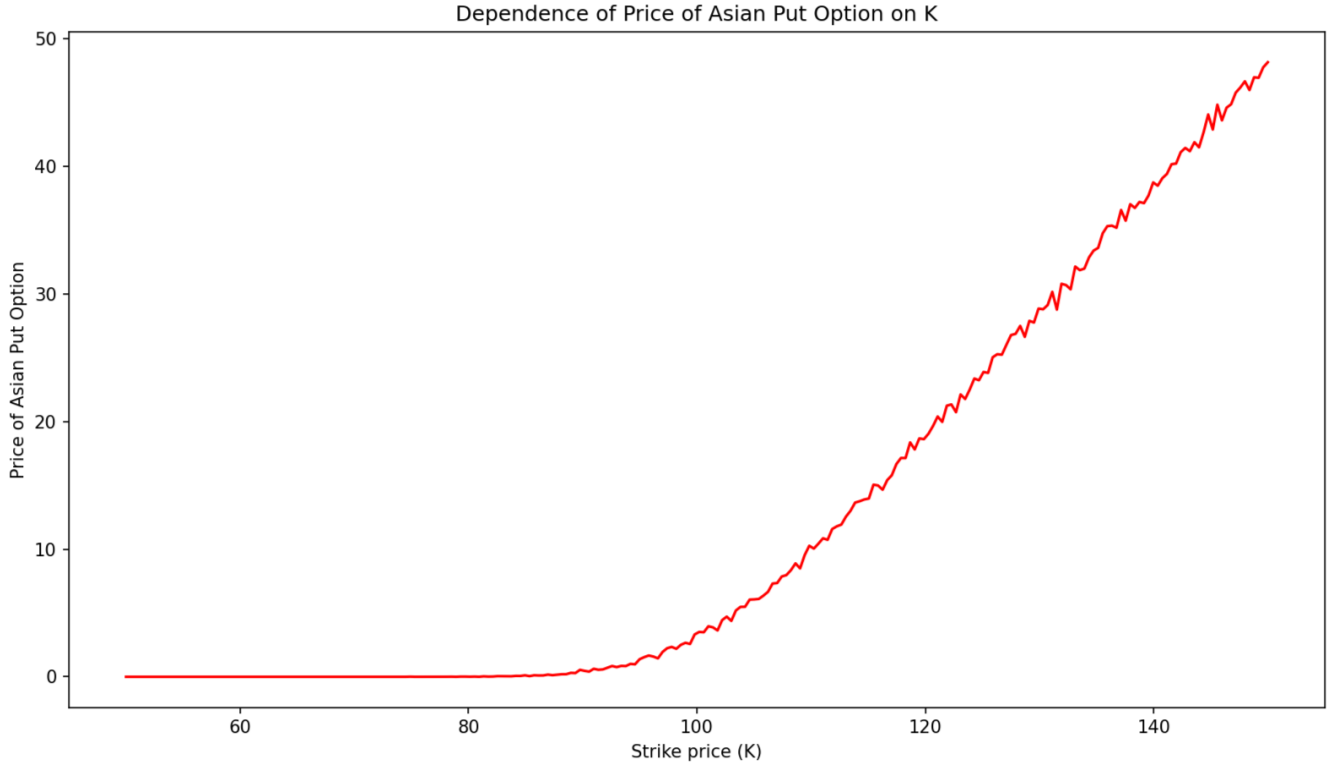
1) Variation of Asian Option Prices with S_0:



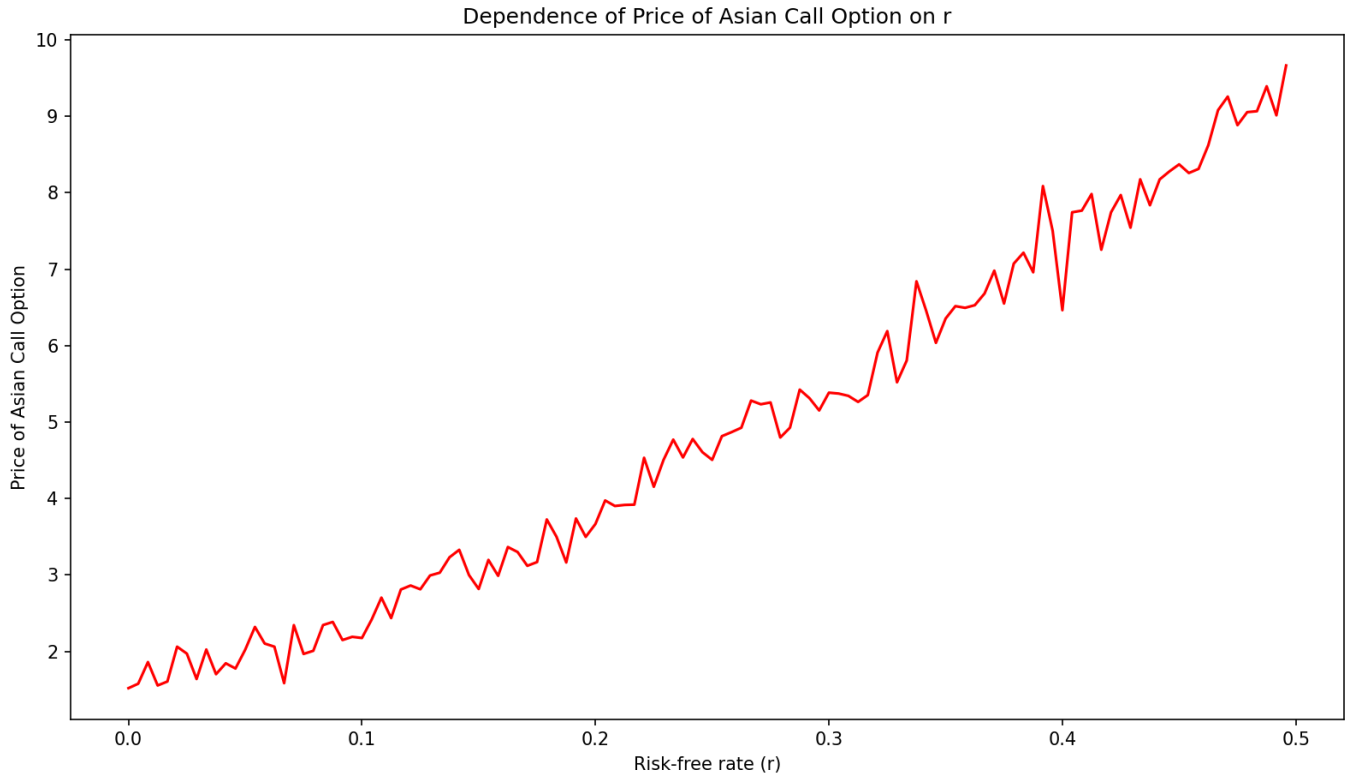


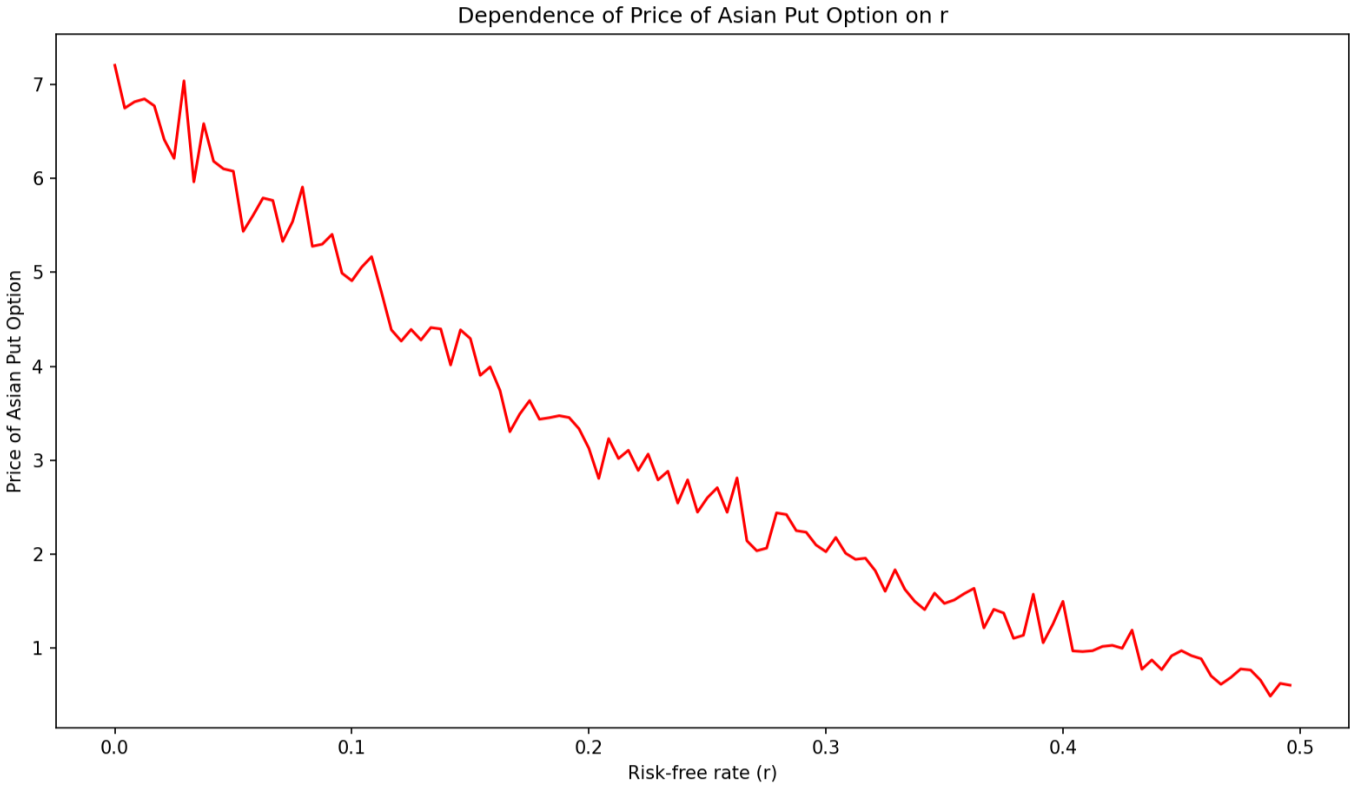
2) Variation of Asian Option Prices with K:



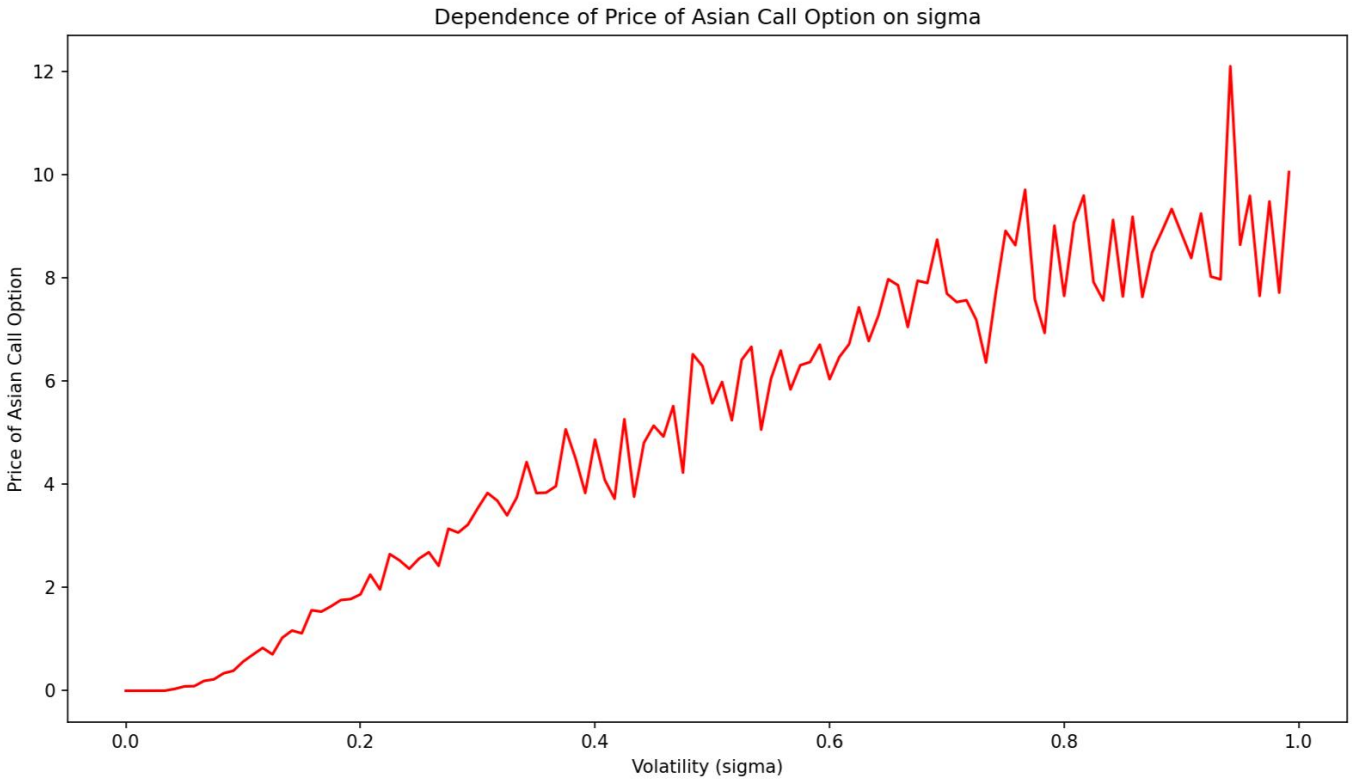


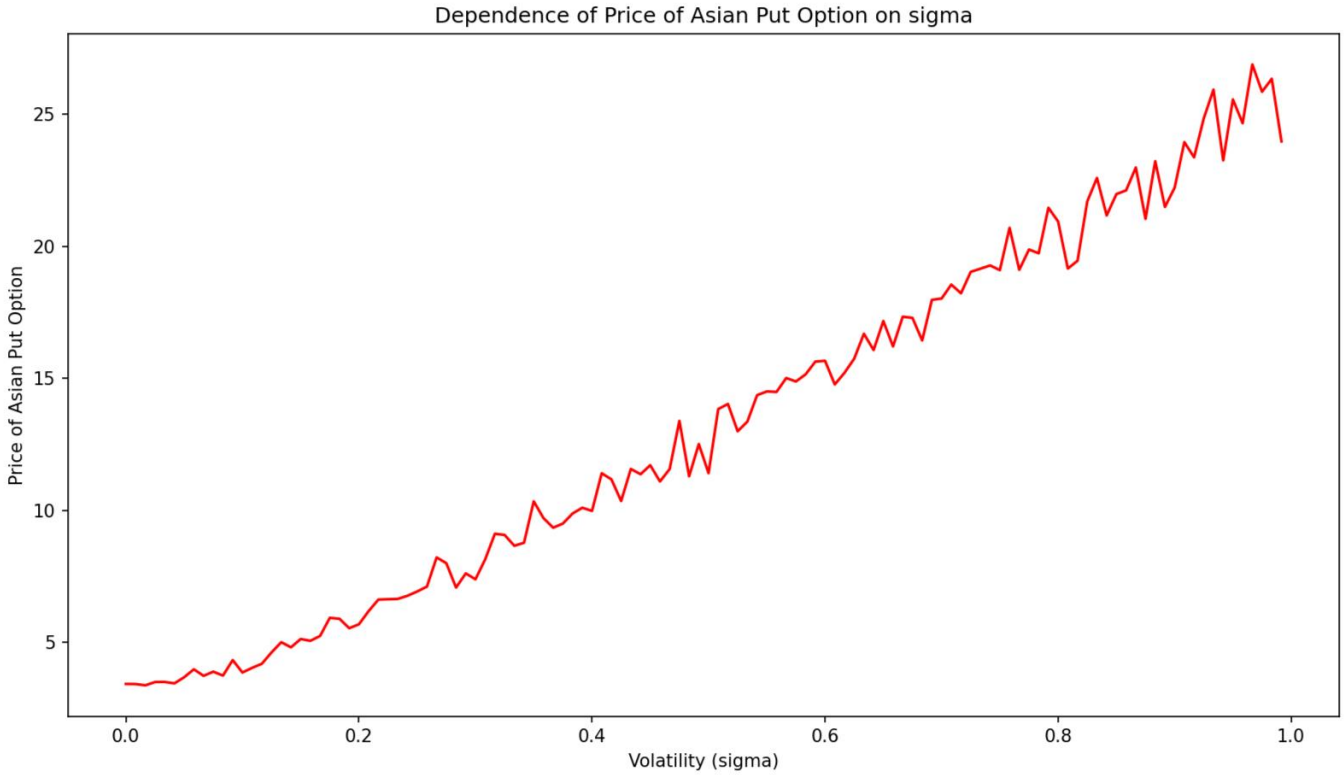
3) Variation of Asian Option Prices with r:





4) Variation of Asian Option Prices with σ:


Dependence of Price of Asian Call Option on sigma
Price of Asian Call Option
Volatility (sigma)
0
2
4
6
8
10
12
0.0
0.2
0.4
0.6
0.8
1.0


Dependence of Price of Asian Put Option on sigma
Price of Asian Put Option
Volatility (sigma)
5
10
15
20
25
0.0
0.2
0.4
0.6
0.8
1.0

Observations:

- The price of both Asian put and Asian call option increases with an increase in the volatility.
- The price of the Asian call option increases while that of the Asian put option decreases, with an increase in risk-free interest rate, r.
- The price of the Asian call option decreases while that of the Asian put option increases, with an increase in the strike price, K.
- The price of the Asian call option increases while that of the Asian put option decreases, with an increase in the initial asset price, S_0.
- There appears to be some fluctuations in the plots, which we try to minimize by employing the method of Control Variates as variance reduction technique.

Question 2:

2. Compute the prices of the Asian options given above by employing variance reduction techniques also and compare your results.

The prices of a six-month fixed-strike Asian option with various strike prices, after employing variance reduction technique are as follows:

1) For K = 90,

```
For K = 90,
Price of Asian Call Option = 10.711532882760373
Variance in Price of Asian Call Option = 49.057881593464565

Price of Asian Put Option = 0.301451041486337 56
Variance in Price of Asian Put Option = 1.535571795417376
```

2) For K = 105,

```
For K = 105,
Price of Asian Call Option = 1.71525416038597 08
Variance in Price of Asian Call Option = 11.888634361942788

Price of Asian Put Option = 5.495994862244111
Variance in Price of Asian Put Option = 23.361012446699213
```

3) For K = 110,

```
For K = 110,
Price of Asian Call Option = 0.5697748784065564
Variance in Price of Asian Call Option = 4.228139900856122

Price of Asian Put Option = 9.727843803922243
Variance in Price of Asian Put Option = 36.765873609109434
```

Observations:

The prices of both Asian call and Asian put options, obtained with and without the application of variance reduction techniques, demonstrate similar values. The corresponding variances are juxtaposed in the following tables for comparison:

1) For Asian Call Option:

Strike Price (K)	Variance (without reduction)	Variance (with reduction)
90	57.11516949695493	49.057881593464565
105	14.372311207618779	11.888634361942788
110	5.242431544735323	4.228139900856122

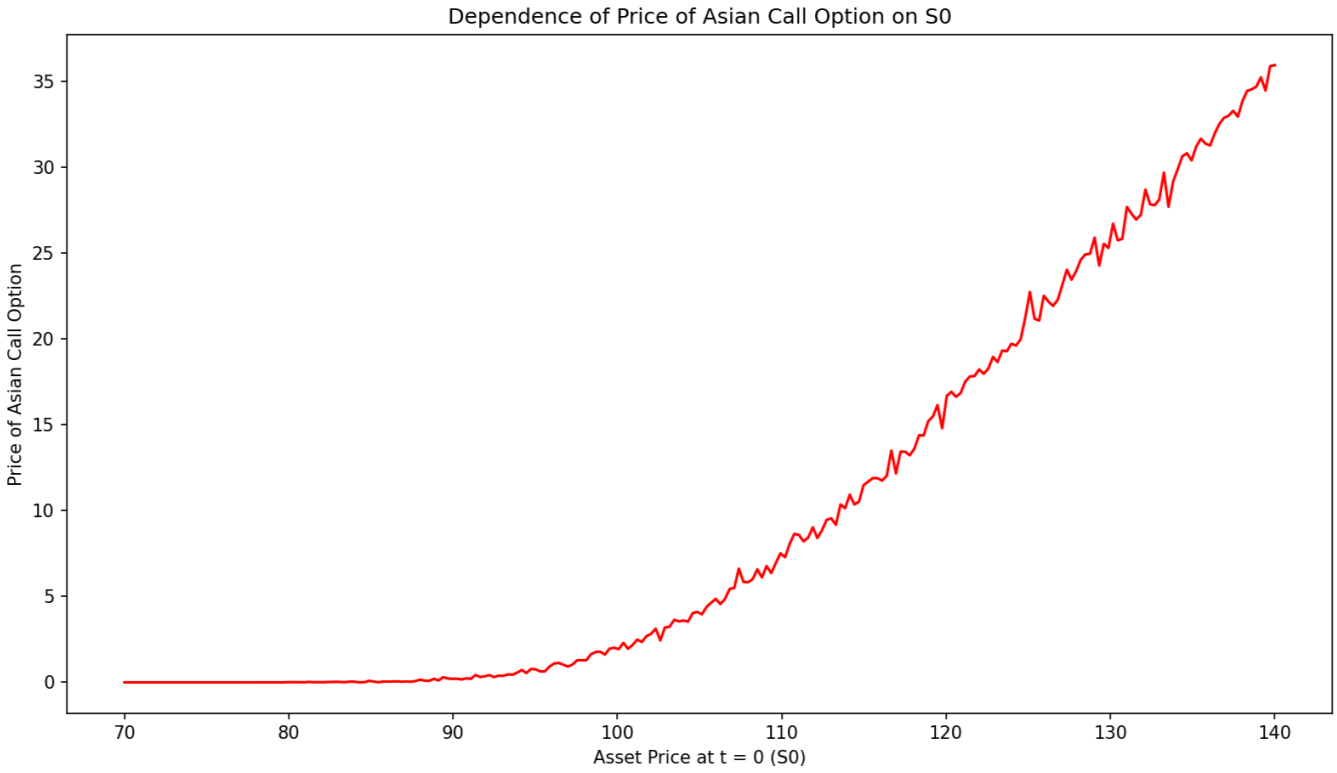
2) For Asian Put Option:

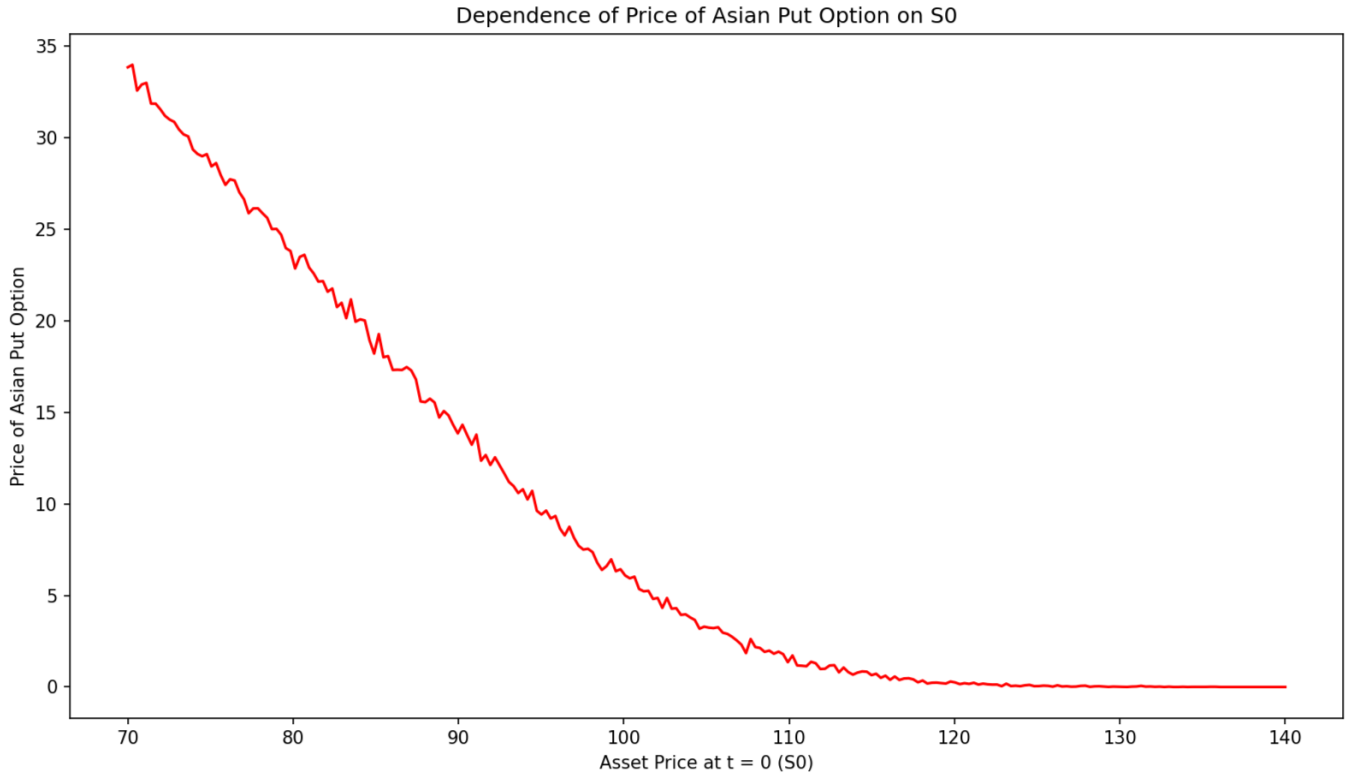
Strike Price (K)	Variance (without reduction)	Variance (with reduction)
90	1.8015876743021508	1.535571795417376
105	31.643362586066814	23.361012446699213
110	48.47287290549498	36.765873609109434

So, from these tables, we can observe that the variance reduction technique is successful, and we have reduced the variance in calculating the Asian option prices.

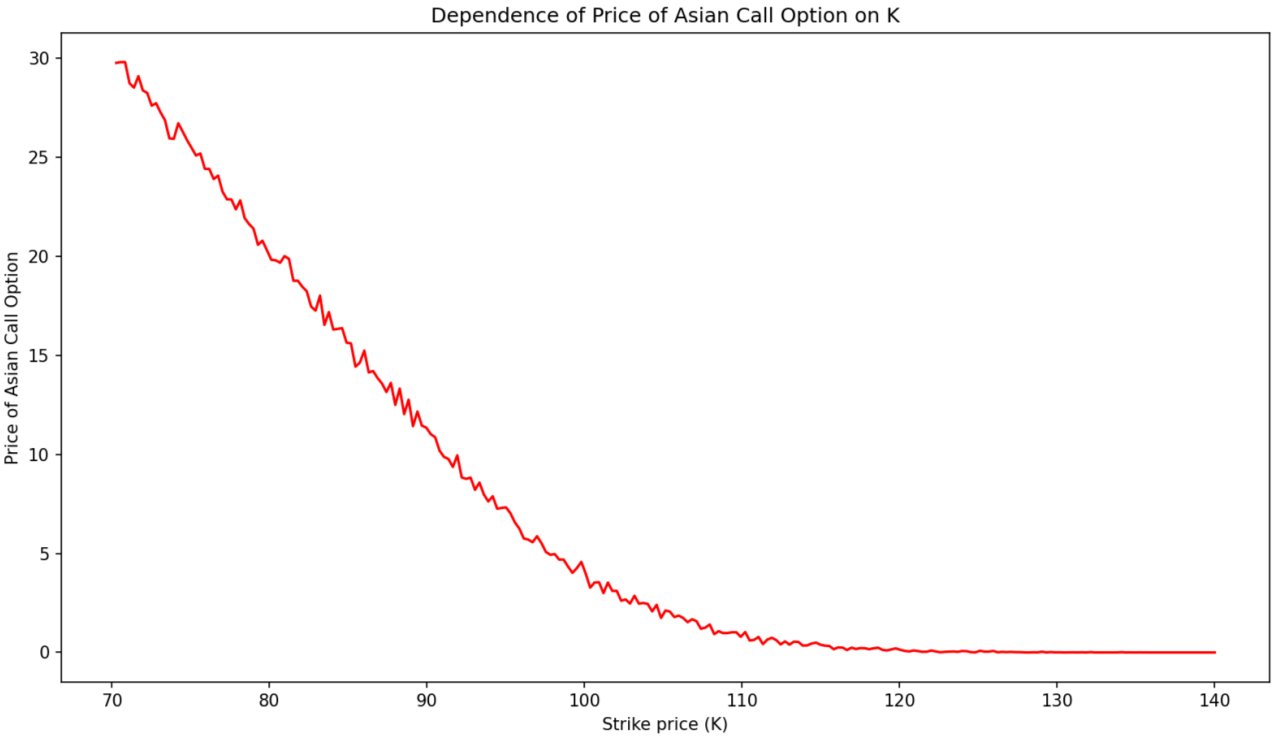
Sensitivity Analysis after employing Variance Reduction Technique:

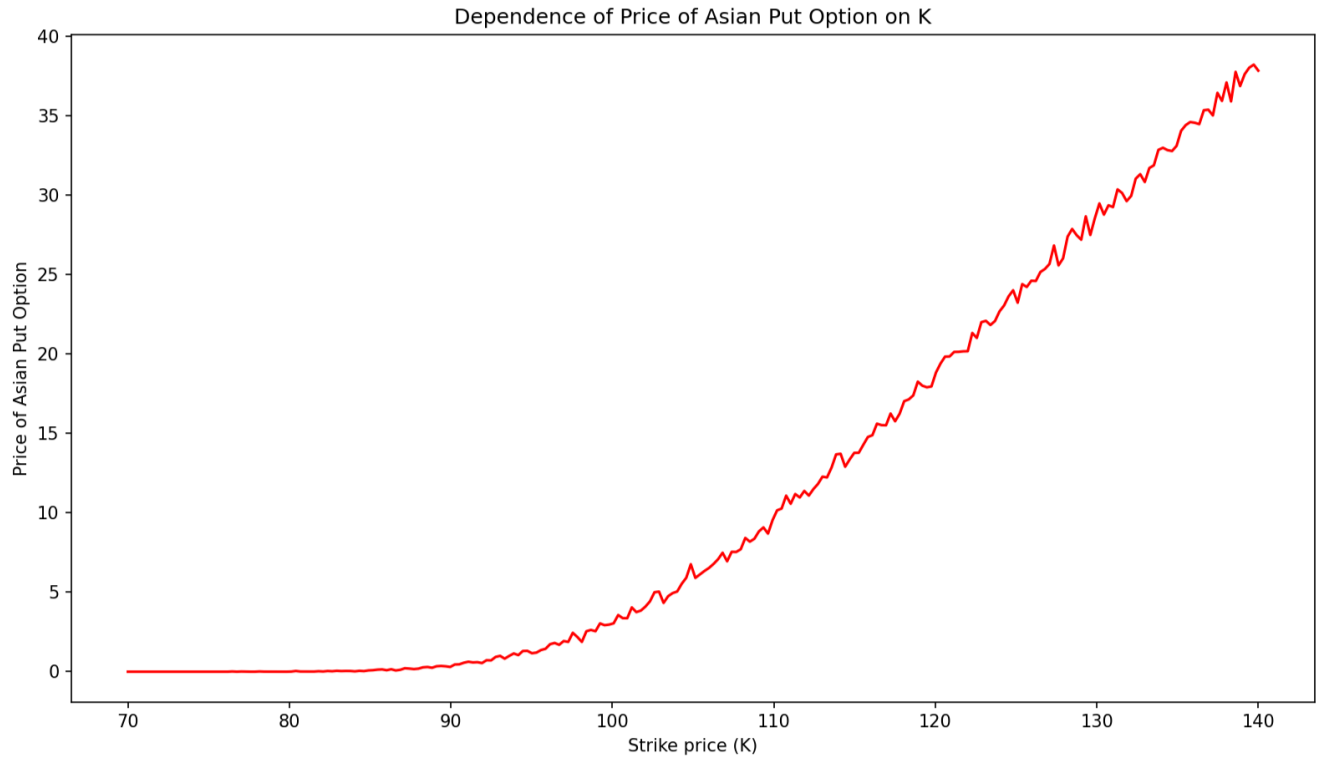
1) Variation of Asian Option Prices with S0:



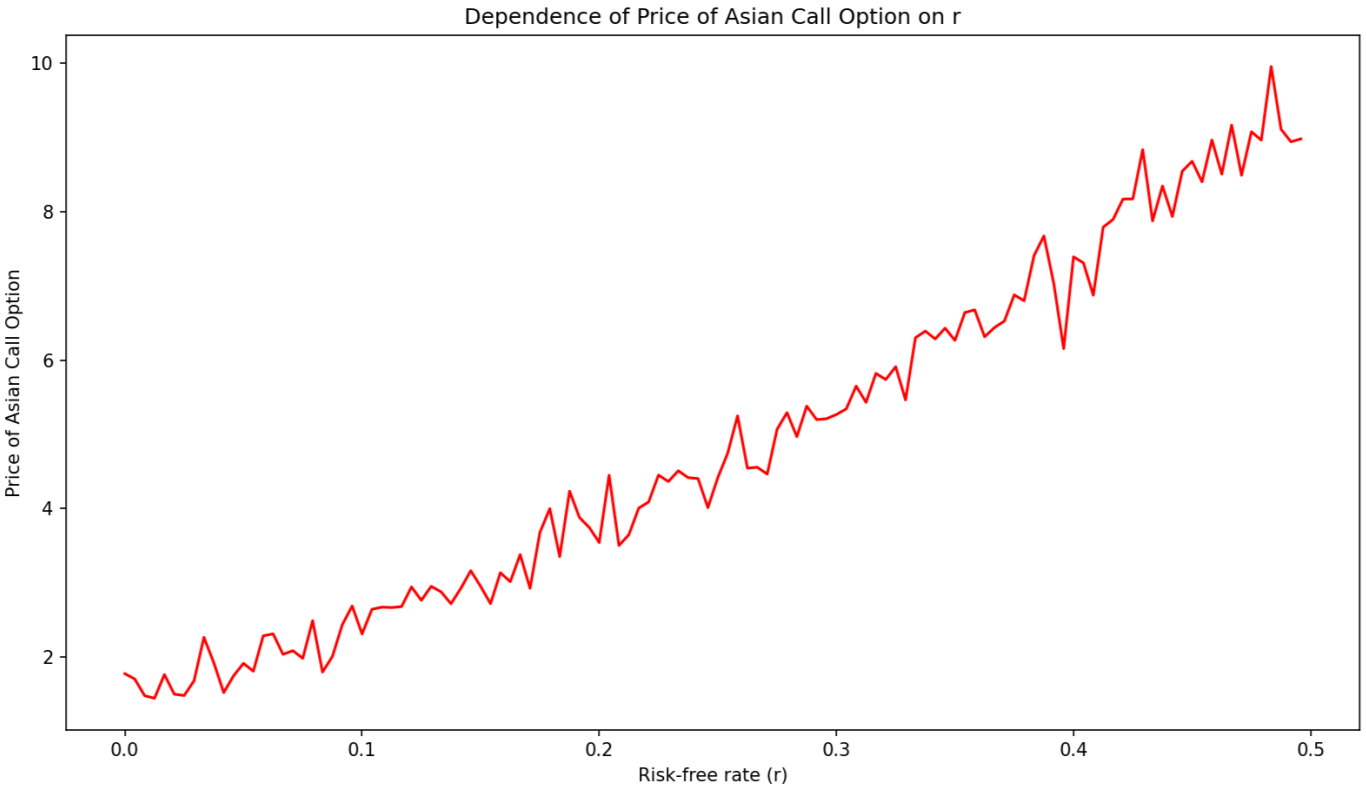


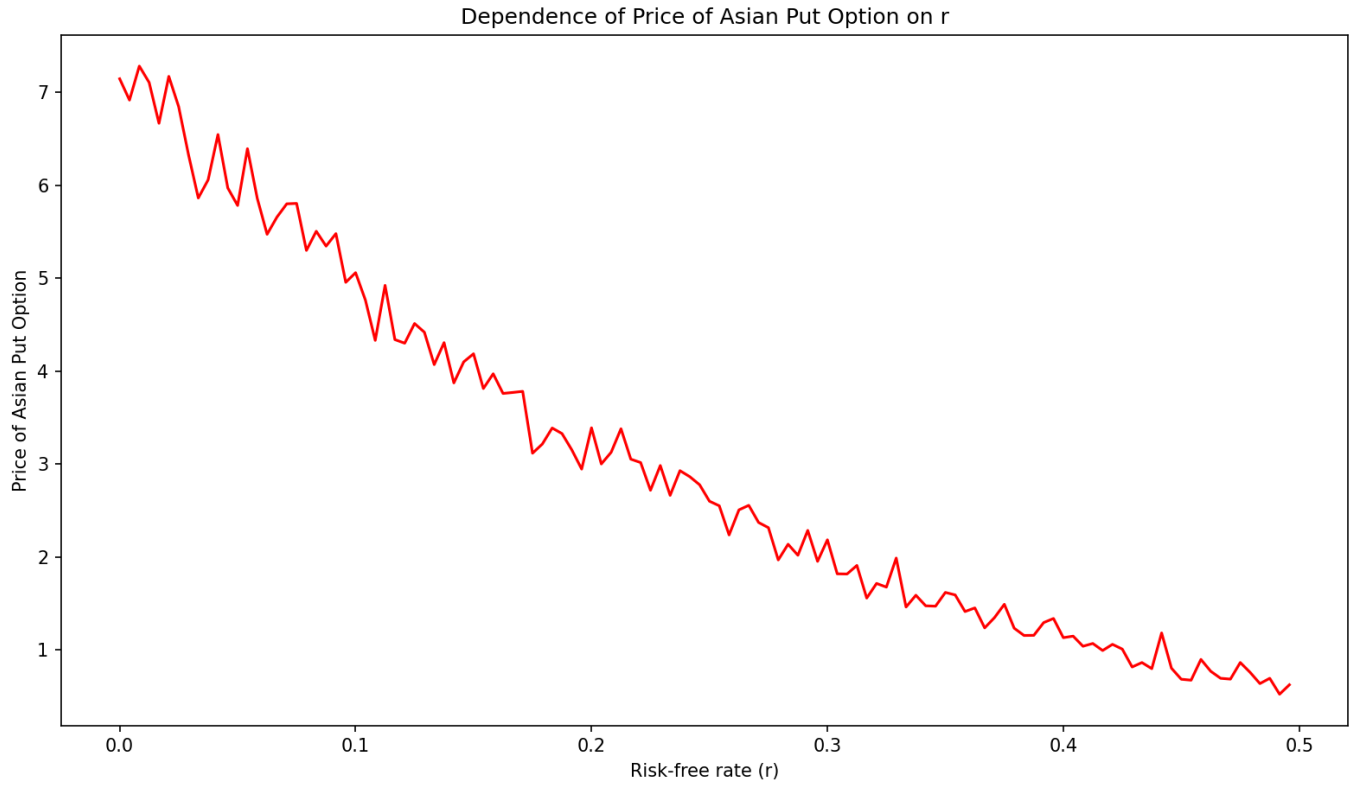
2) Variation of Asian Option Prices with K:



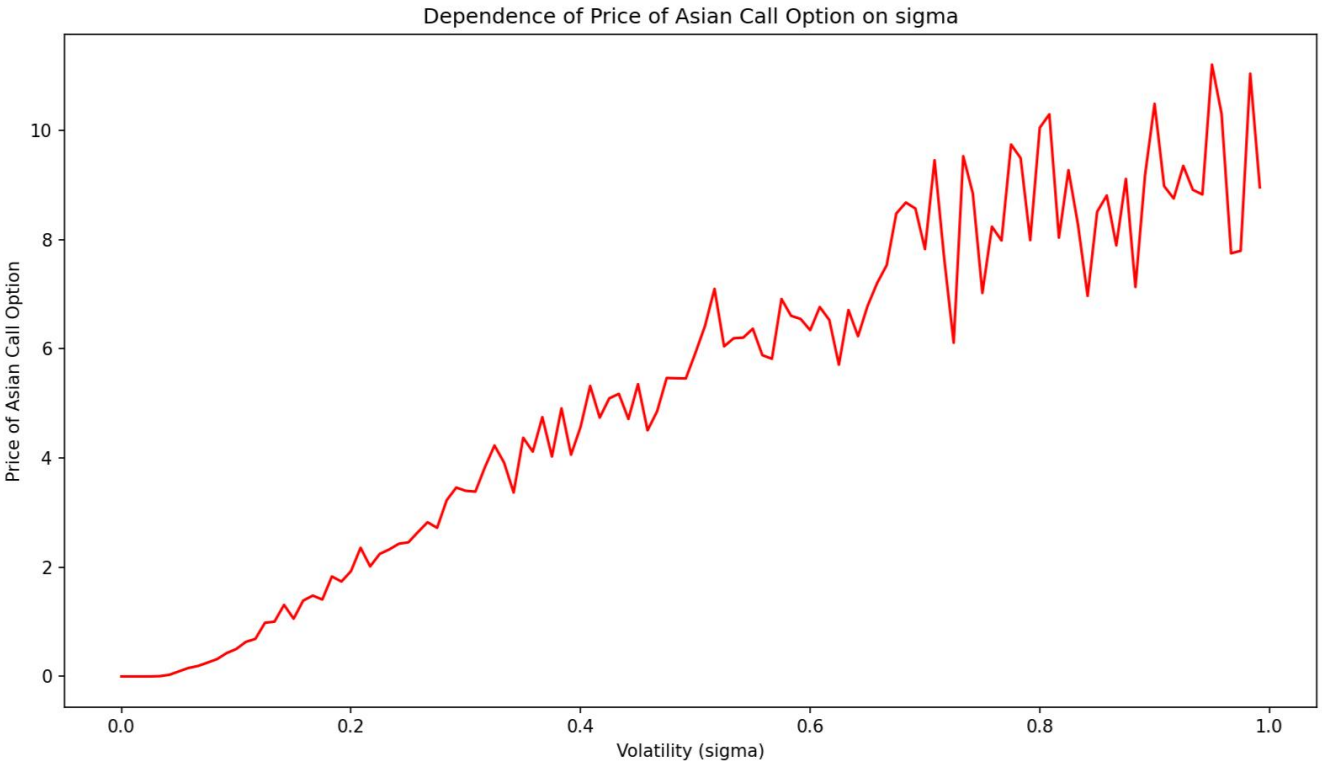


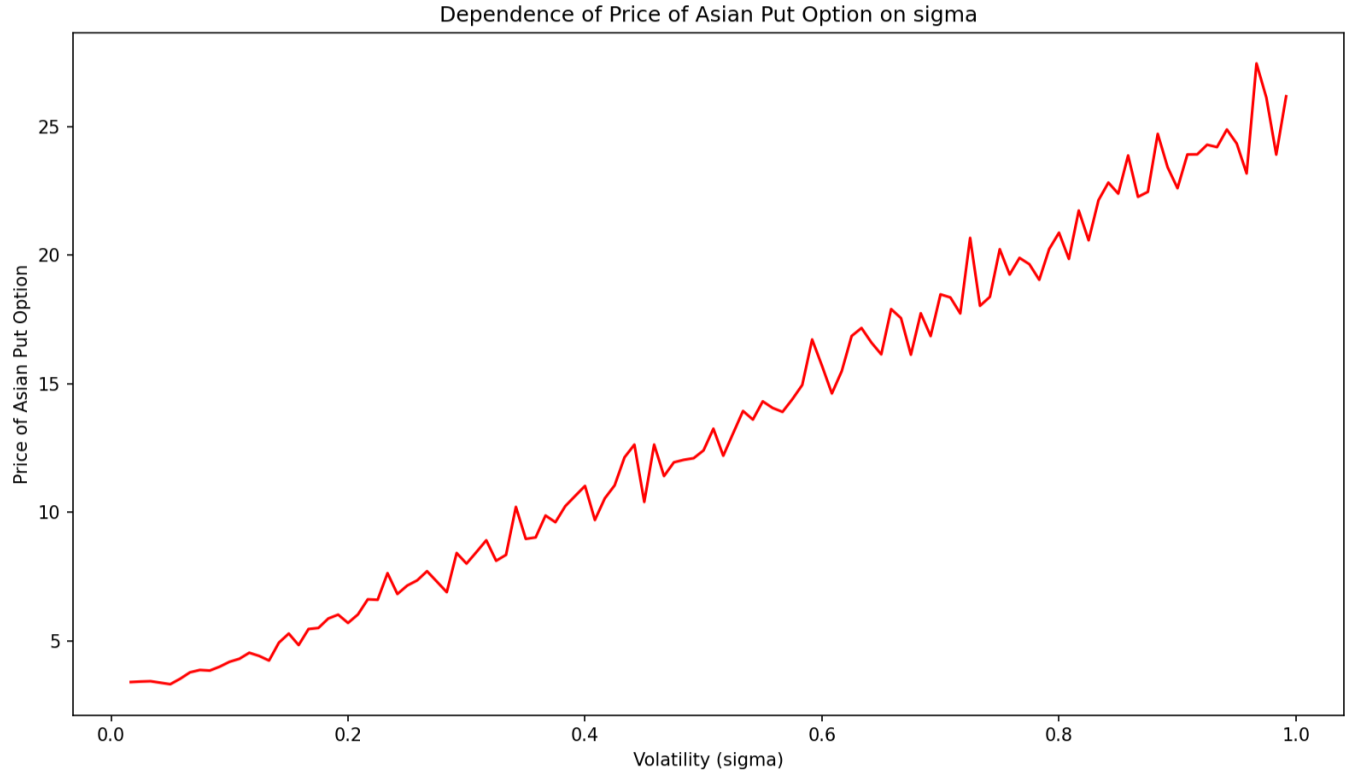
3) Variation of Asian Option Prices with r:





4) Variation of Asian Option Prices with σ:





The method of Control Variates is used as the variance reduction technique. Control Variates, as a variance reduction technique, leverages information regarding estimation errors of known quantities to minimize errors in estimating unknown quantities.

Observations:

- On careful analysis, we can observe that the fluctuations in the plots are lesser than the case when variance reduction technique was not applied. So, the method of Control Variates for variance reduction achieves its goal.
- The nature of the plots matches our expectations.
- We have quantitatively demonstrated that variance reduction is achieved by using variance reduction technique. This claim is supported by the constructed plots.