Financial Engineering Lab MA – 374 Lab – 2

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Branch – Mathematics and Computing

Question 1:

1. Determine the initial prices of an European call and an European put option in the binomial model with the following data :

$$S(0) = 100; K = 100; T = 1; M = 100; r = 8\%; \sigma = 30\%.$$

Use the following two sets of u and d for your program.

(a) Set 1 : $u = e^{\sigma\sqrt{\Delta t}}$; $d = e^{-\sigma\sqrt{\Delta t}}$.

(b) Set 2 : $u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$; $d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}$.

Here $\Delta t = \frac{T}{M}$, with M being the number of subintervals in the time interval $[0, T]$. Use the continuous compounding convention in your calculations (i.e., both in $\tilde{p}$ and in the pricing formula).

Now, for each set (of u, d), carry out a sensitivity analysis of the initial price as follows: Plot the initial prices of the call and put options by varying one of the parameters at a time (as given below) while keeping the other parameters fixed (as given above):

(a) $S(0)$.

(b) K.

(c) r.

(d) σ.

(e) M (Do this for three values of K, $K = 95, 100, 105$).

You may also do plots in 3-D also (by considering two parameters at a time).

Using binomial model with continuous compounding convention:

For Set 1:

The initial price of the European Call Option = 15.68176045159868

The initial price of the European Put Option = 7.993395090262697

For Set 2:

The initial price of the European Call Option = 15.736778626185817

The initial price of the European Put Option = 8.048413264849804

```
No arbitrage exists for M = 100
For Set 1,
Initial Price of European Call Option = 15.68176045159868
Initial Price of European Put Option = 7.993395090262697

No arbitrage exists for M = 100
For Set 2,
Initial Price of European Call Option = 15.73677862618581 7
Initial Price of European Put Option = 8.048413264849804
```

No-arbitrage Condition:

The code checks for the arbitrage possibilities using the following conditions necessary for the market to be arbitrage free -

$$d < e^{rt} < u$$

That is, for no arbitrage opportunity to exist, following relations must hold true:

$$d < R < u$$

where,

$$R = e^{rt}$$

$$d = e^{-\sigma\sqrt{t} + \left(r - \frac{\sigma^2}{2}\right)t}$$

$$u = e^{\sigma\sqrt{t} + \left(r - \frac{\sigma^2}{2}\right)t}$$

$$t = \frac{T}{M}$$

R = net rate

Since continuous compounding is followed and the final amount in continuous compounding is present value times e^{rt}, so R = e^{rt}.

Binomial Pricing Algorithm:

$$\Delta t = \frac{T}{M}$$

(Using the continuous compounding convention)

At time $t = t_i$ ($= i * \Delta t$), there are $i + 1$ possible option prices, that is,

$$S_n^i = d^{i-n} u^n S_0 \,, \quad 0 \leqslant n \leqslant i$$

Since continuous compounding convention is used, the probability p of an upward return in price is:

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

$R = e^{r\Delta t}$

At time t = T, we are calculating the price of the options using the payoff functions of the call and put options respectively.

$$C_n^M = max\left(S_n^M - K,\, 0\right),\ 0 \leqslant n \leqslant M$$
$$P_n^M = max\left(K - S_n^M,\, 0\right),\ 0 \leqslant n \leqslant M$$

Where C_n^M / P_n^M is the n^{th} possible price of the call / put option respectively for the M^{th} interval.

Initial Call Option Price = C_0^0 and Initial Put Option Price = P_0^0, these 2 are the required values.

So, we continuously apply Backward Induction to find out the option prices at t = 0 by using following relations:

$$C_n^i = (1 - p)C_{n+1}^{i+1} + pC_n^{i+1} \,,\ 0 \leqslant n \leqslant i \text{ and } 0 \leqslant i \leqslant M-1$$

$$P_n^i = (1 - p)P_{n+1}^{i+1} + pP_n^{i+1} \,,\ 0 \leqslant n \leqslant i \text{ and } 0 \leqslant i \leqslant M-1$$

and hence, we get the required initial option prices information.

Sensitivity Analysis:

Sensitivity Analysis of option price variance with S_0, K, M, r, σ are done by plotting 2-D and 3-D plots.

Cox – Ross – Rubinstein Formula:

$$C = \frac{1}{(1+R)^M} \sum_{k=0}^{M} \binom{M}{k} \tilde{p}^{\,k} \left(1-\tilde{p}\right)^{M-k} \left(S_0(1+u)^M(1+d)^{M-k} - K\right)^+$$

Where C = Price of European Call Option and $p = \tilde{p}\,\frac{1+u}{1+R}$

Put Call Parity Equation:

$$C + Ke^{-rT} = P + S_0$$

K = Strike Price

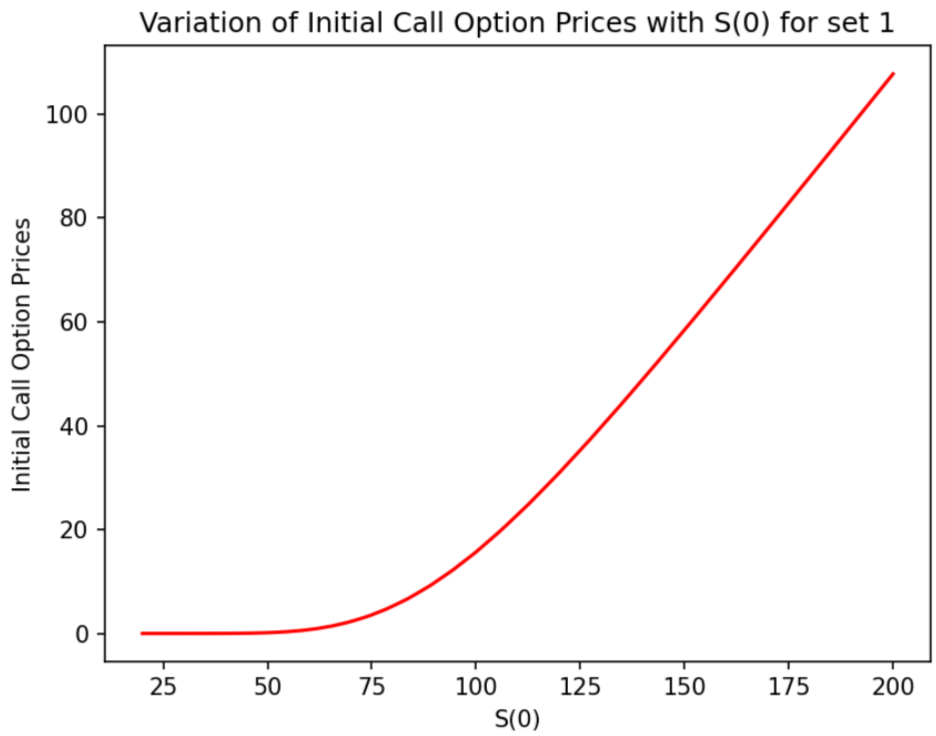
T = Maturity Time

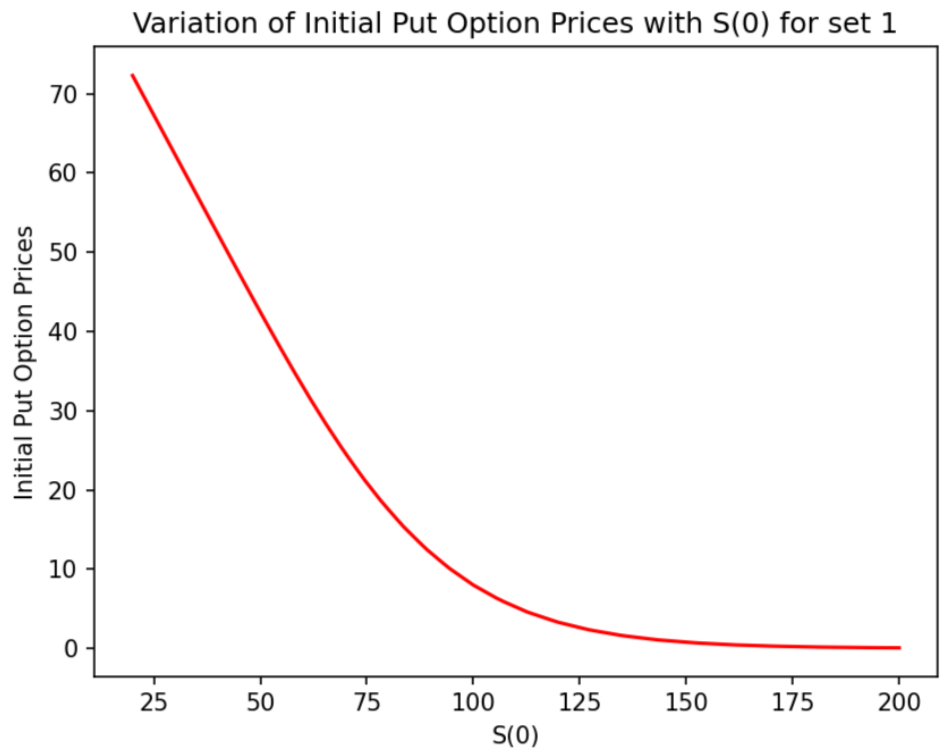
C = Price of European Call option

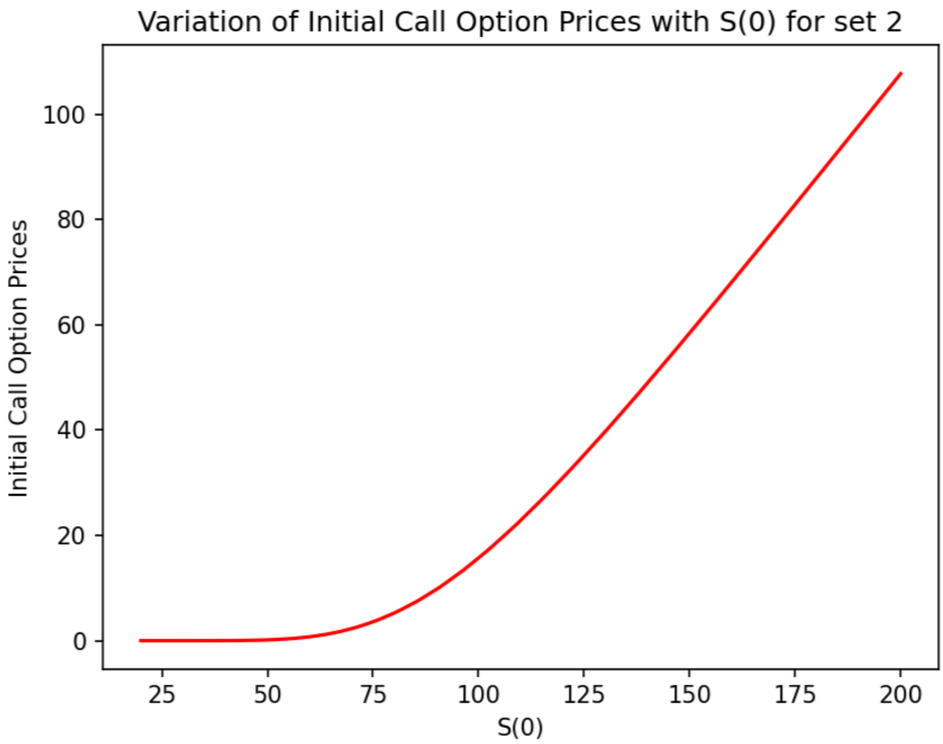
P = Price of European Put Option

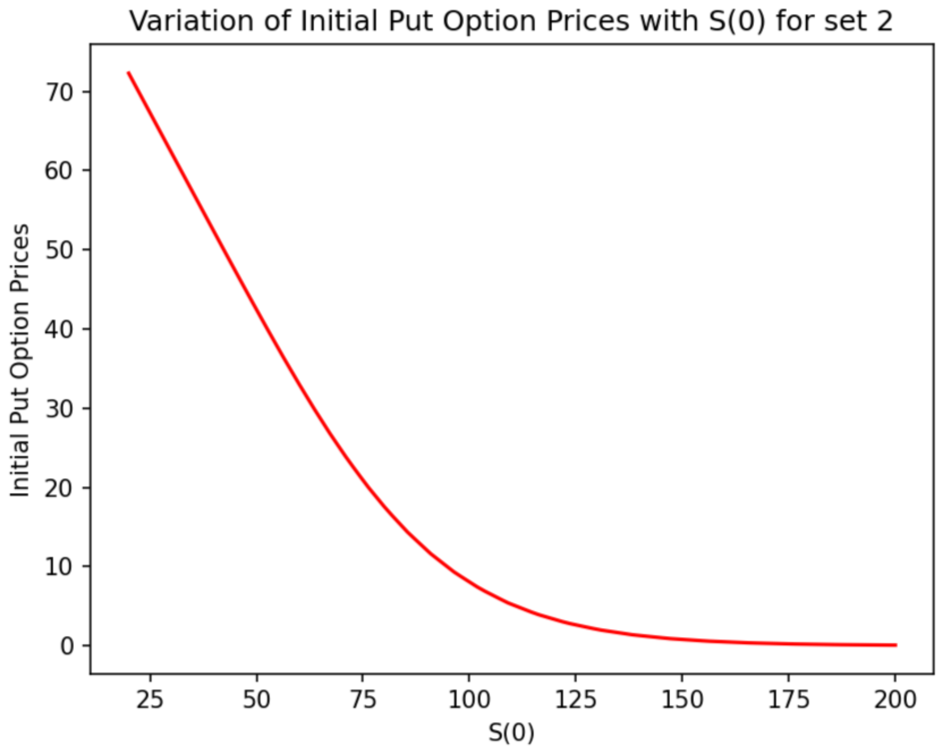
Note – While plotting with respect to M considering only a single variable, then I have plotted the graphs for all the three given values of K, that is, K = 95, 100 and 105 but while plotting the graphs taking 2 parameters at a time, graphs of only 1 value of K are shown because the graphs are approximately the same for all the 3 values of K.

1) Variation with S(0):

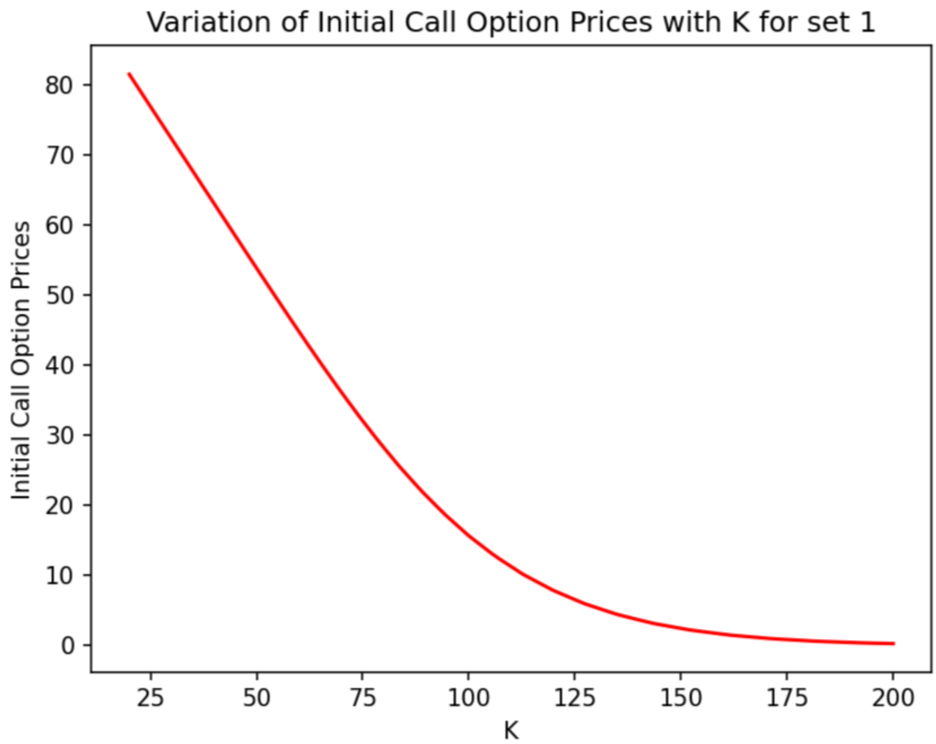

Variation of Initial Call Option Prices with S(0) for set 1
Initial Call Option Prices
S(0)
0
20
40
60
80
100
25
50
75
100
125
150
175
200

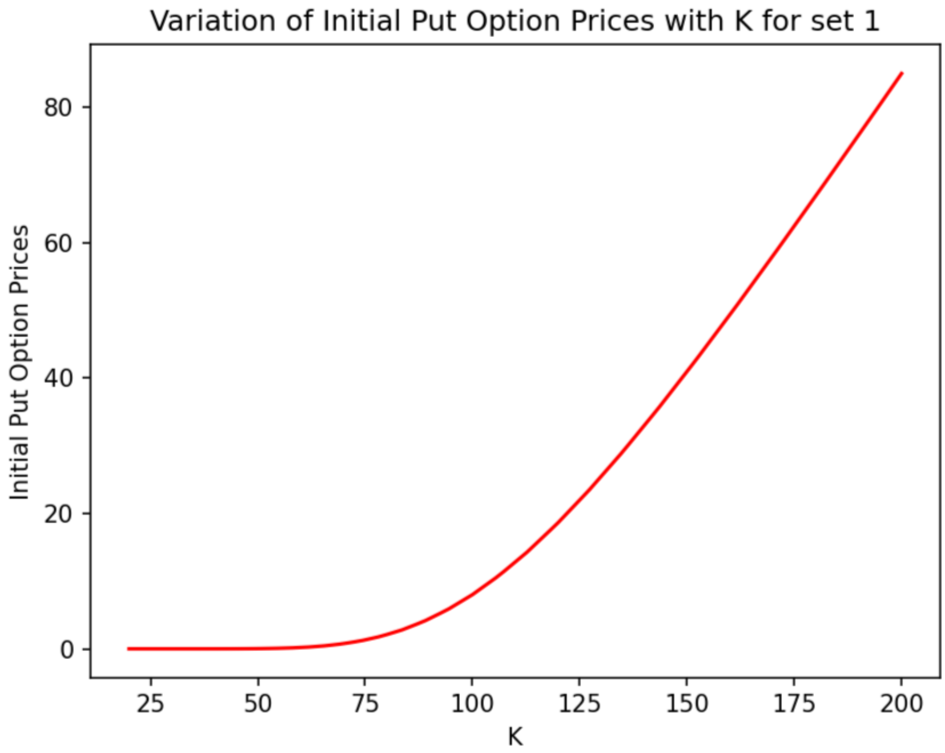

Variation of Initial Put Option Prices with S(0) for set 1
Initial Put Option Prices
S(0)
0
10
20
30
40
50
60
70
25
50
75
100
125
150
175
200

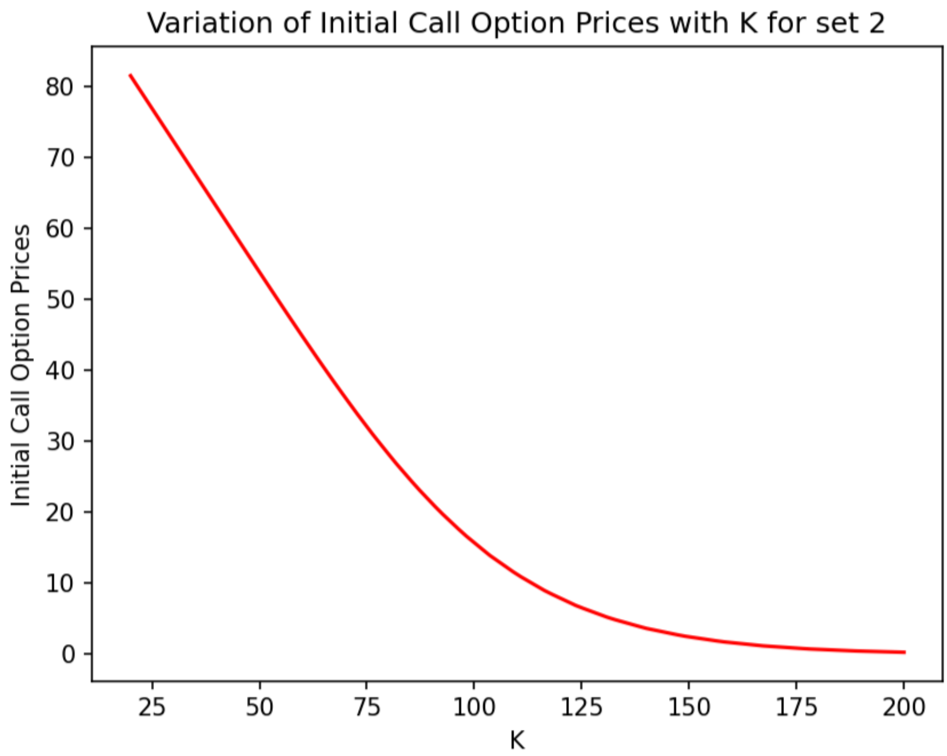


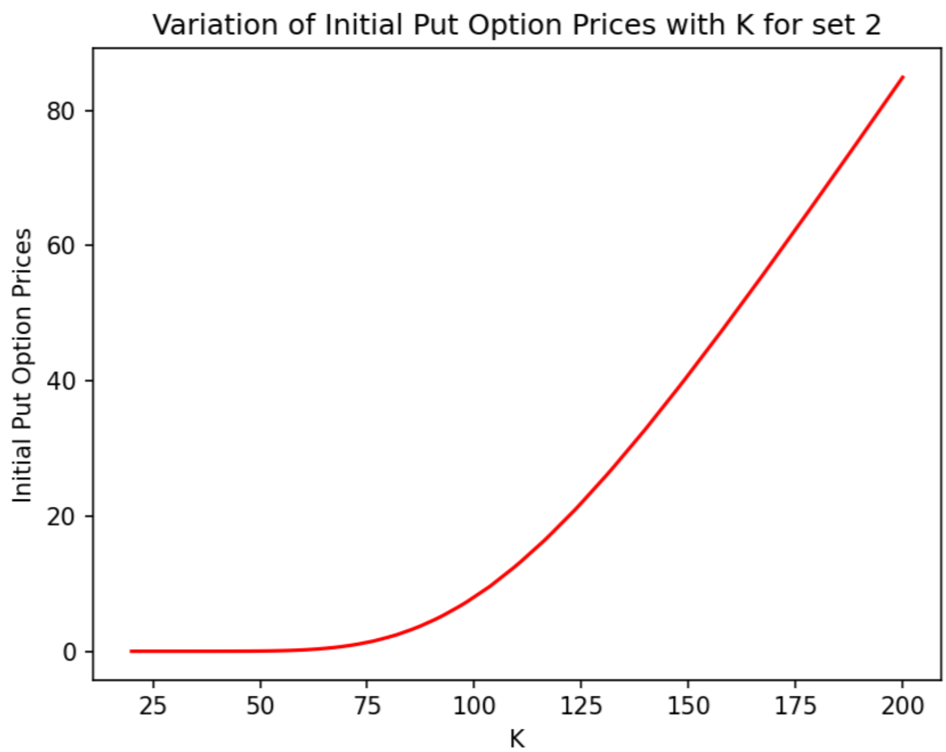


2) Variation with K:

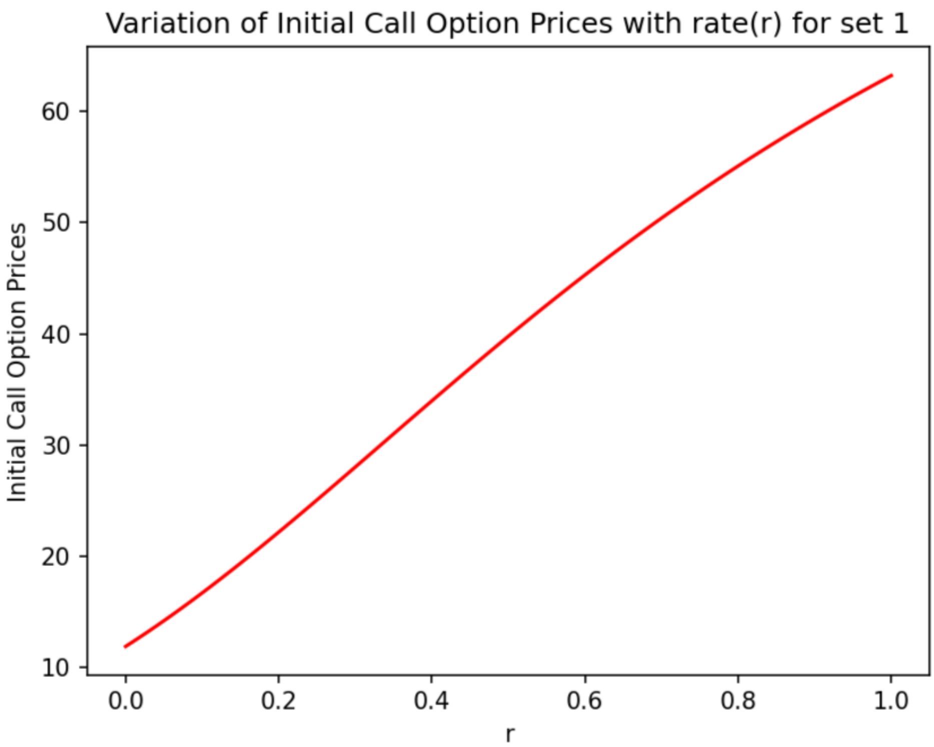

Variation of Initial Call Option Prices with K for set 1
Initial Call Option Prices
K
0
10
20
30
40
50
60
70
80
25
50
75
100
125
150
175
200

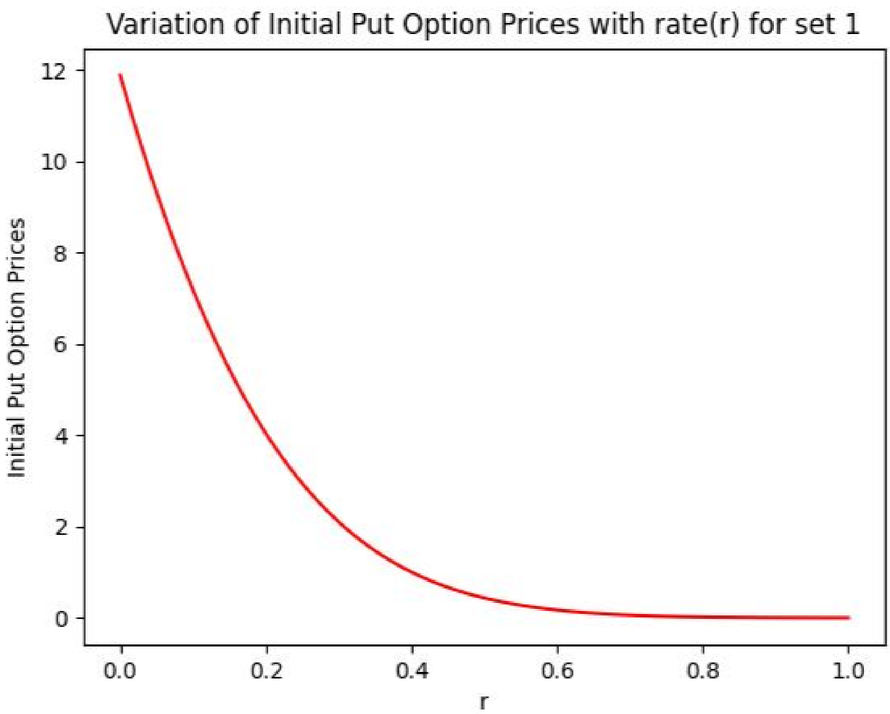

Variation of Initial Put Option Prices with K for set 1
Initial Put Option Prices
K
0
20
40
60
80
25
50
75
100
125
150
175
200

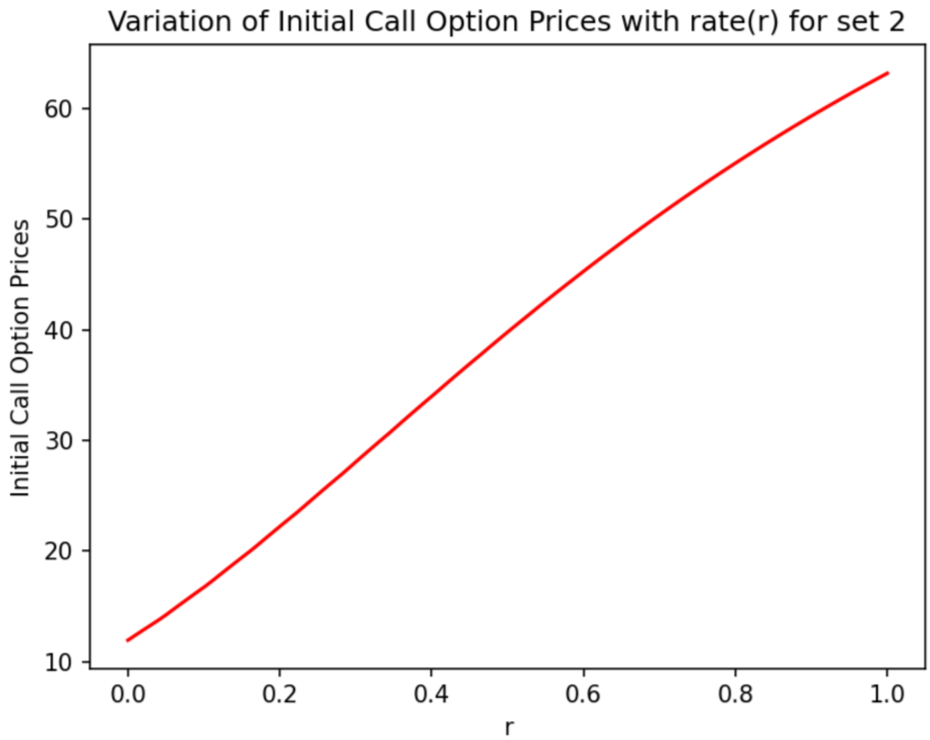


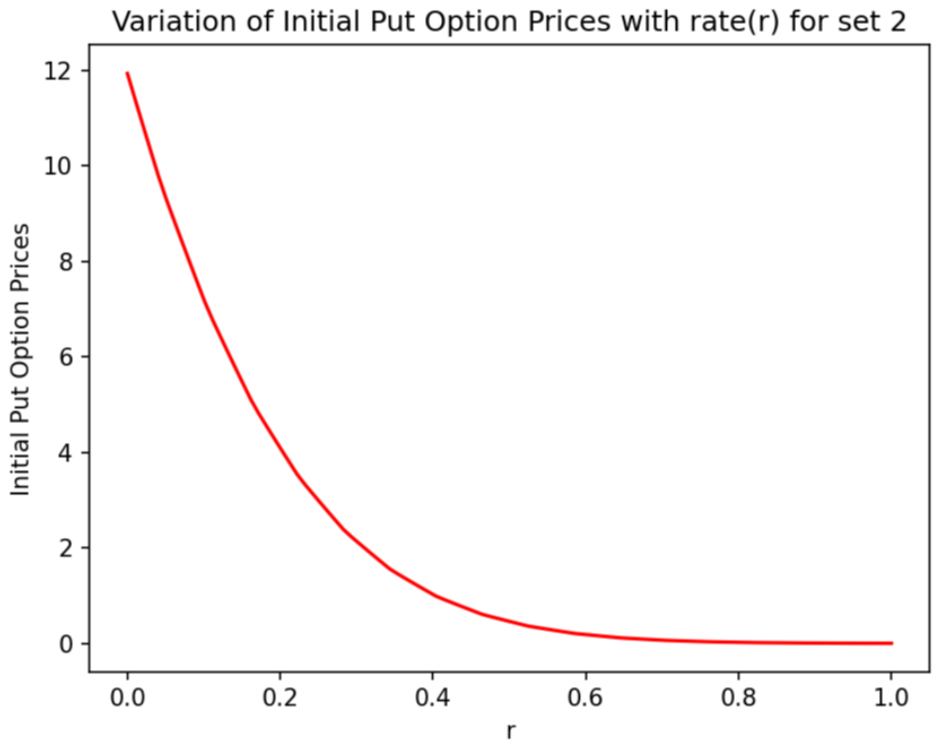


3) Variation with r:

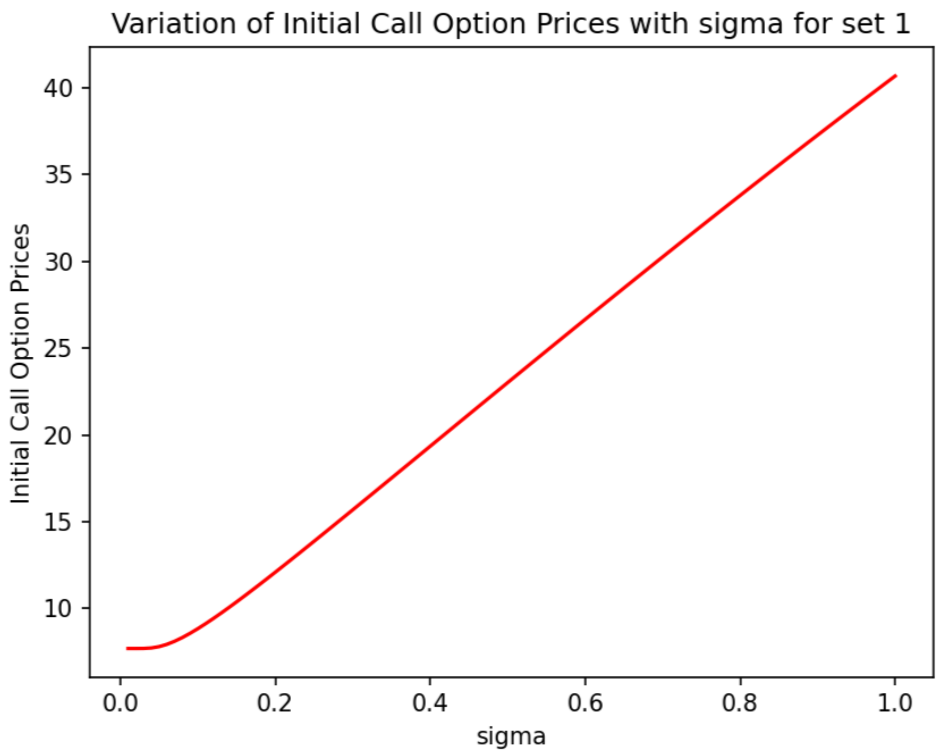

Variation of Initial Call Option Prices with rate(r) for set 1
Initial Call Option Prices
r
0.0
0.2
0.4
0.6
0.8
1.0
10
20
30
40
50
60

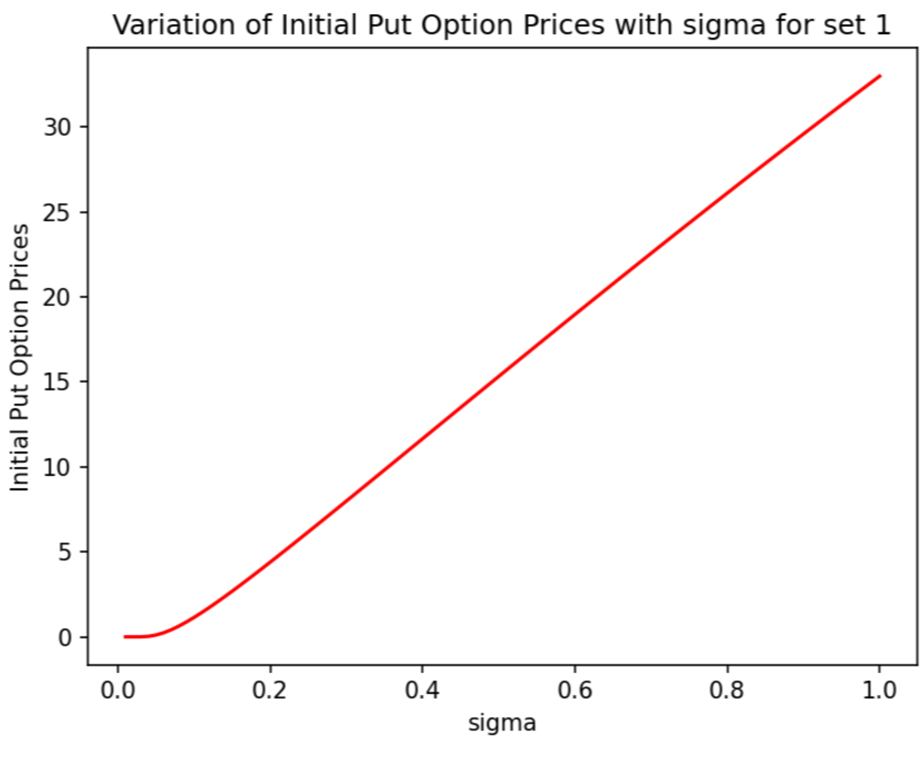

Variation of Initial Put Option Prices with rate(r) for set 1
Initial Put Option Prices
r
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
12

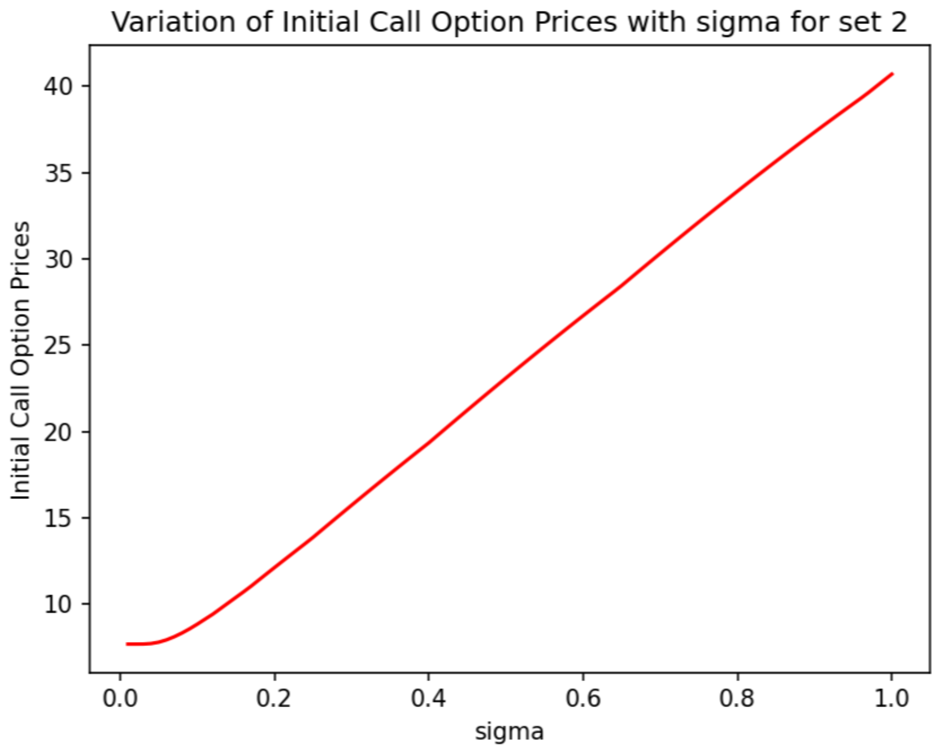


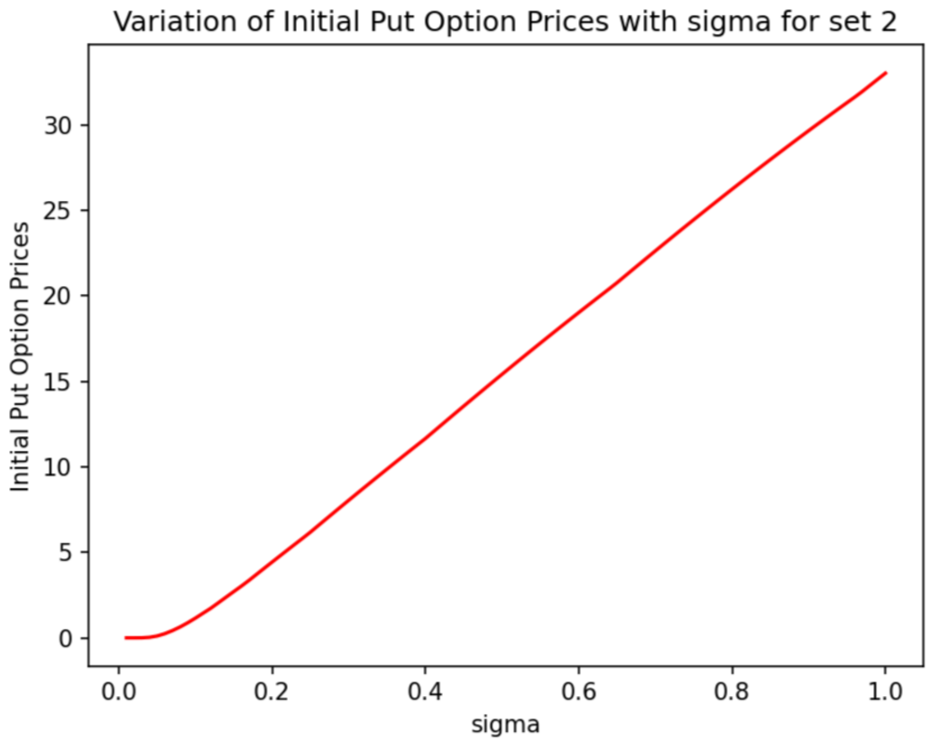


4) Variation with σ:


Variation of Initial Call Option Prices with sigma for set 1
Initial Call Option Prices
sigma
10
15
20
25
30
35
40
0.0
0.2
0.4
0.6
0.8
1.0

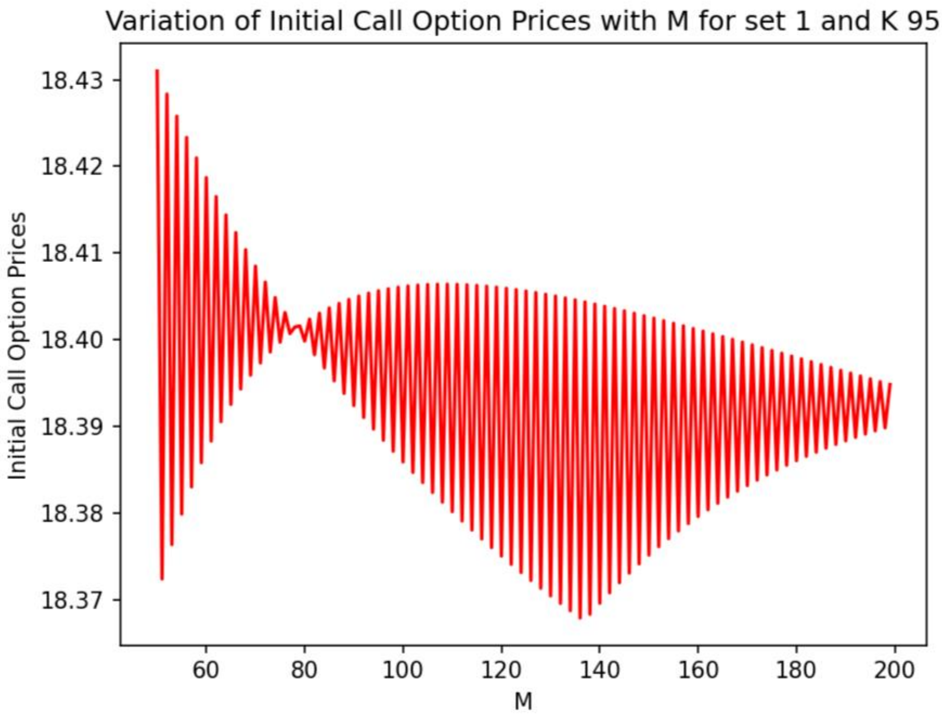

Variation of Initial Put Option Prices with sigma for set 1
Initial Put Option Prices
sigma
0
5
10
15
20
25
30
0.0
0.2
0.4
0.6
0.8
1.0

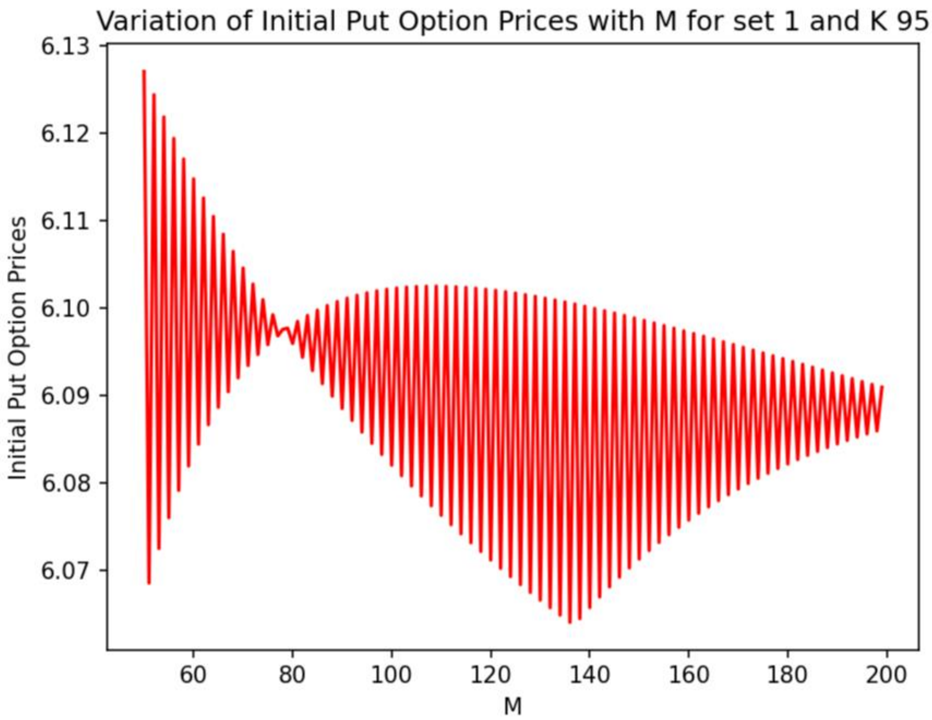


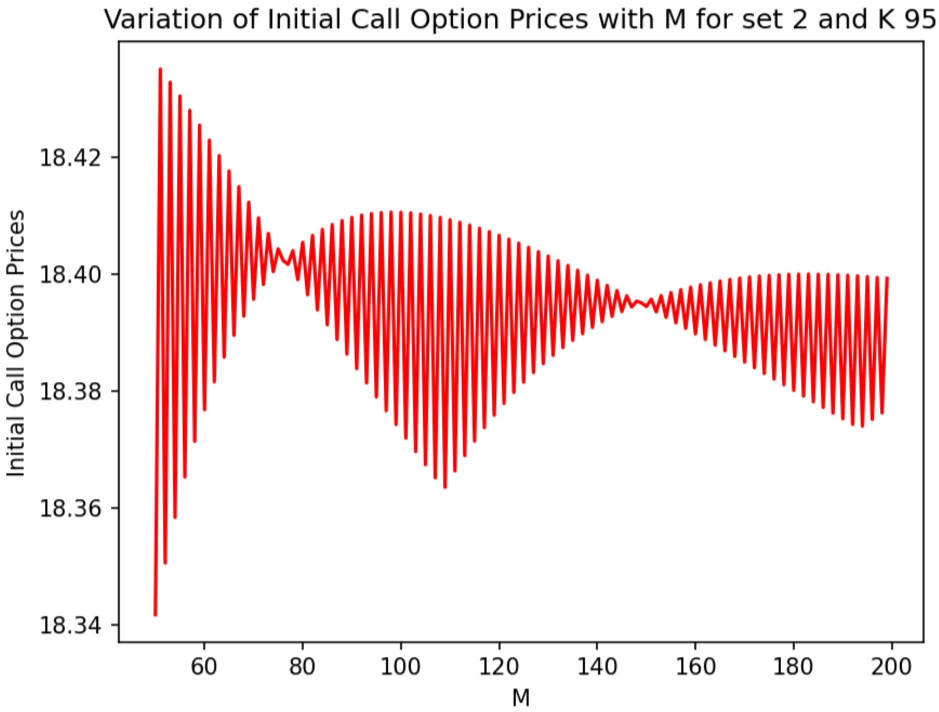


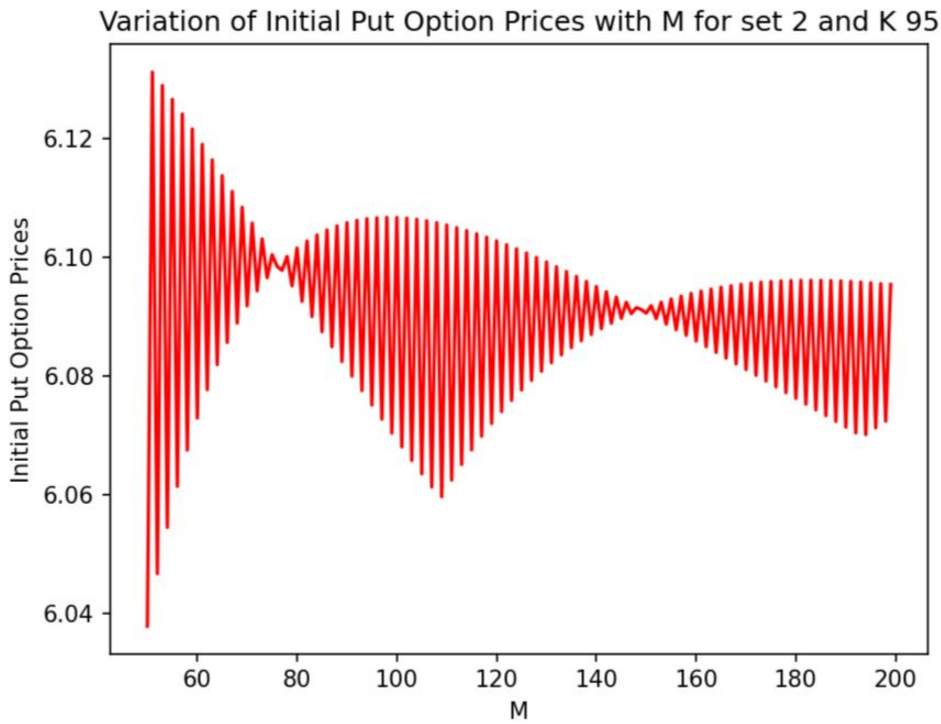
5) Variation with M:

a) For K = 95,

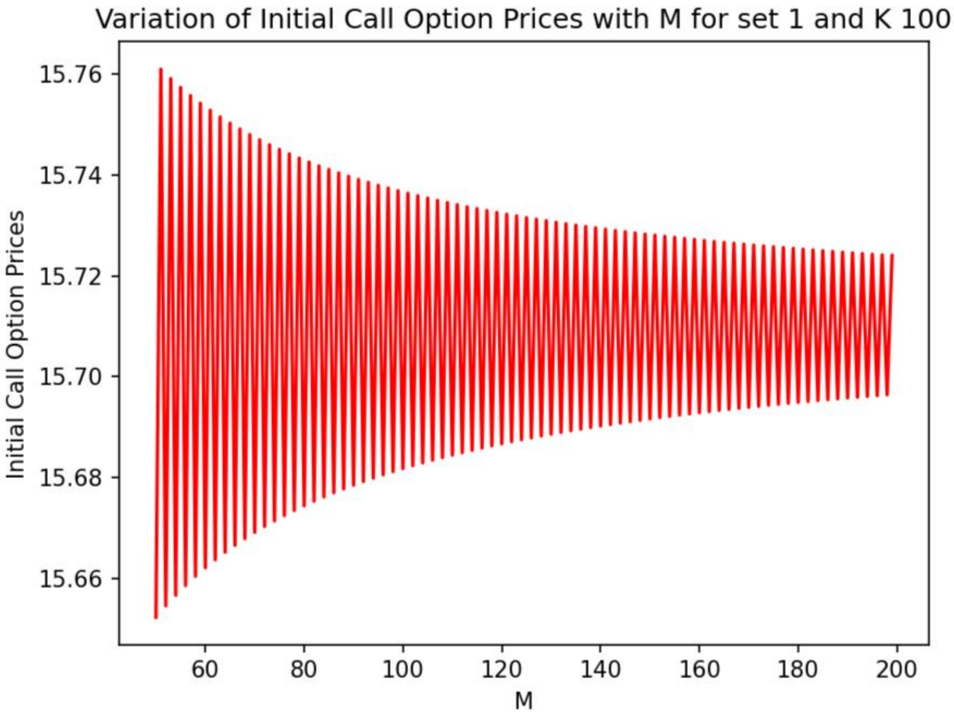

Variation of Initial Call Option Prices with M for set 1 and K 95
Initial Call Option Prices
18.43
18.42
18.41
18.40
18.39
18.38
18.37
60
80
100
120
140
160
180
200
M

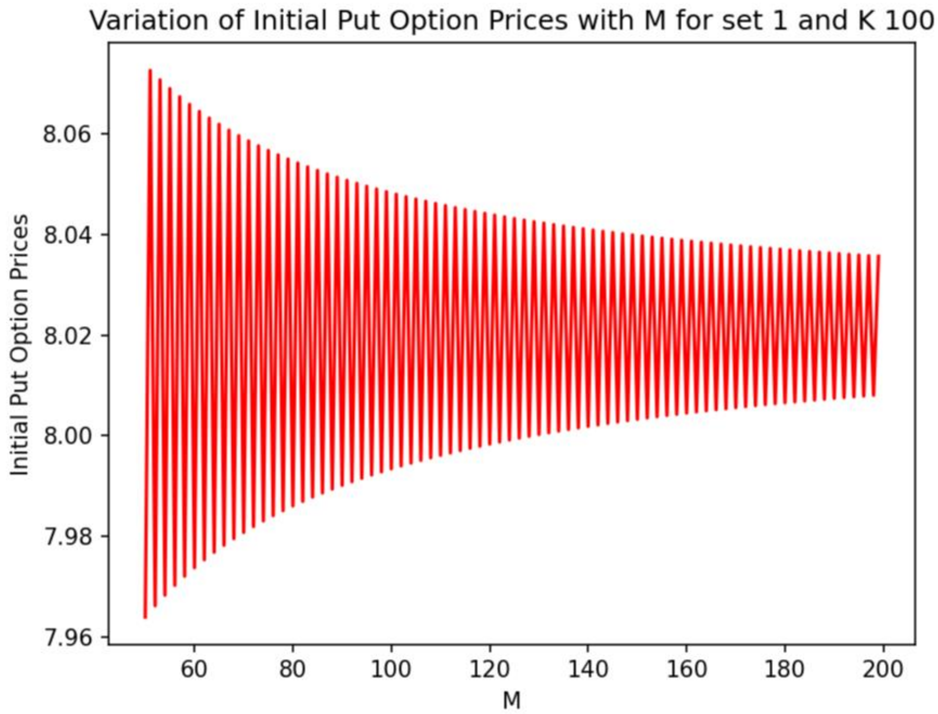

Variation of Initial Put Option Prices with M for set 1 and K 95
Initial Put Option Prices
6.13
6.12
6.11
6.10
6.09
6.08
6.07
60
80
100
120
140
160
180
200
M

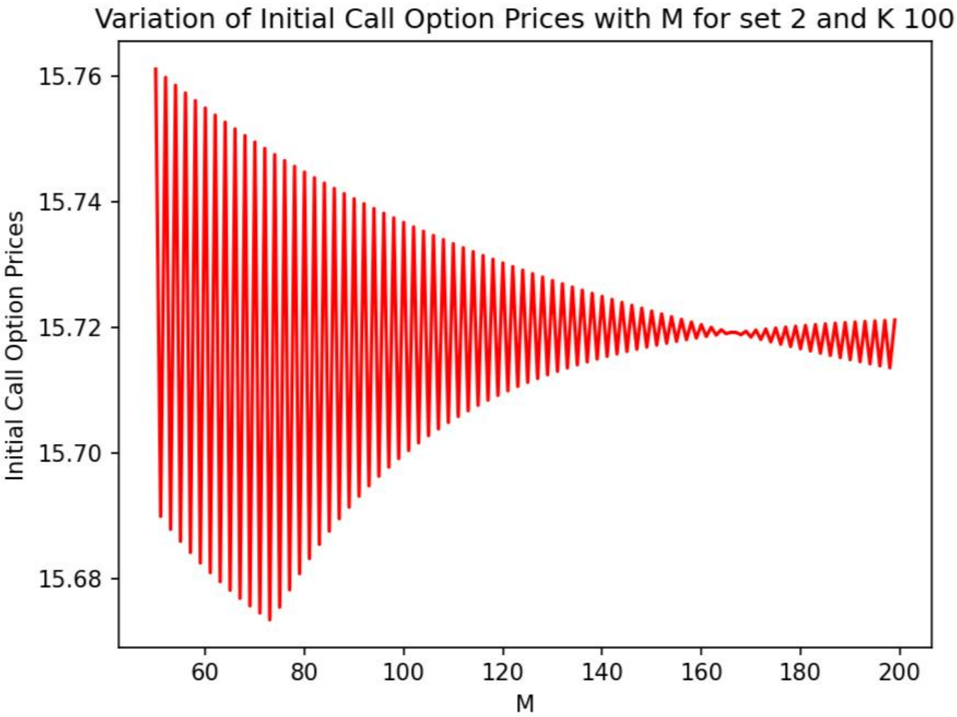

Variation of Initial Call Option Prices with M for set 2 and K 95
Initial Call Option Prices
18.42
18.40
18.38
18.36
18.34
60
80
100
120
140
160
180
200
M

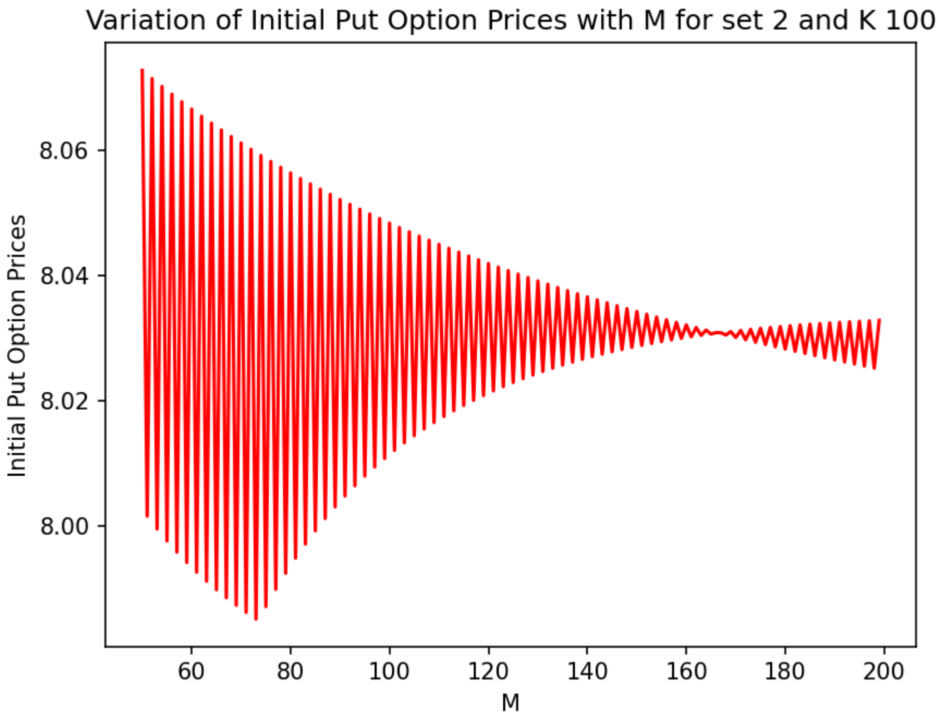

Variation of Initial Put Option Prices with M for set 2 and K 95
Initial Put Option Prices
6.12
6.10
6.08
6.06
6.04
60
80
100
120
140
160
180
200
M

b) For K = 100,

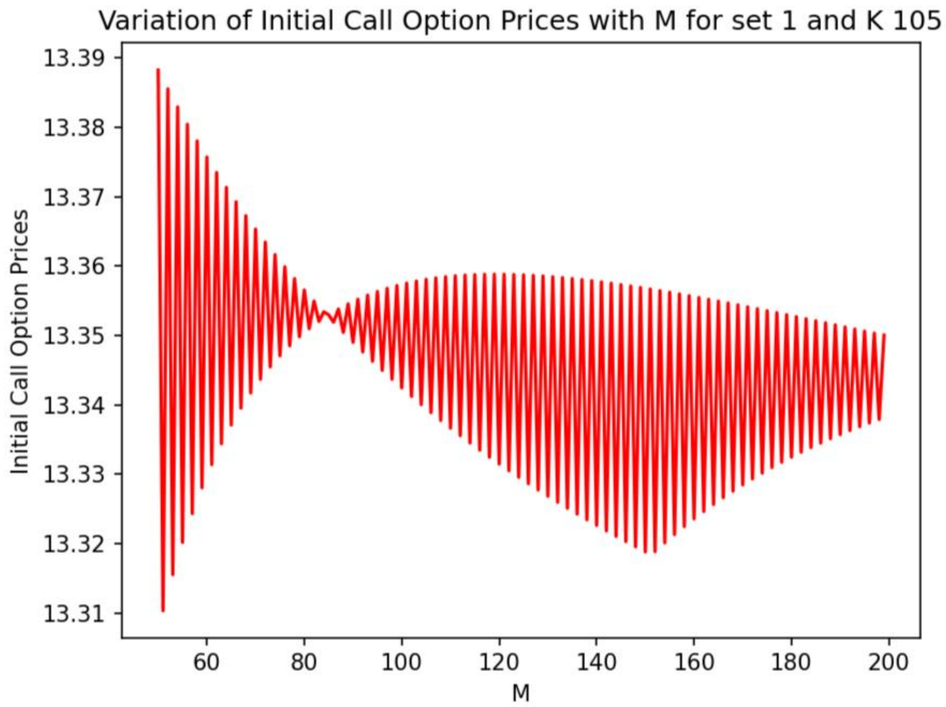

Variation of Initial Call Option Prices with M for set 1 and K 100
Initial Call Option Prices
15.76
15.74
15.72
15.70
15.68
15.66
60
80
100
120
140
160
180
200
M

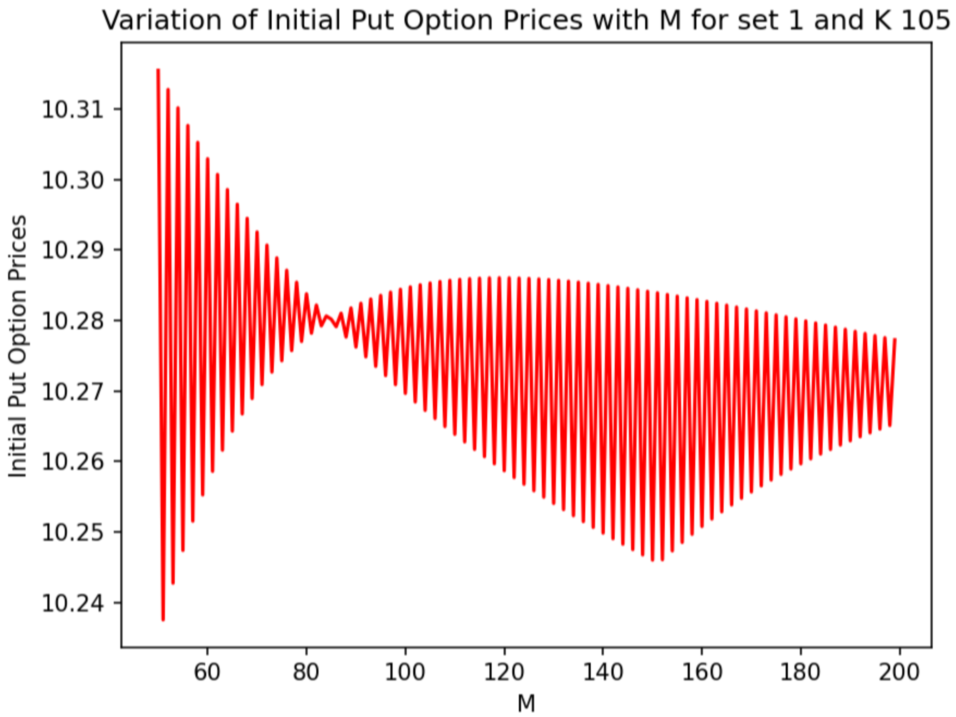

Variation of Initial Put Option Prices with M for set 1 and K 100
Initial Put Option Prices
8.06
8.04
8.02
8.00
7.98
7.96
60
80
100
120
140
160
180
200
M

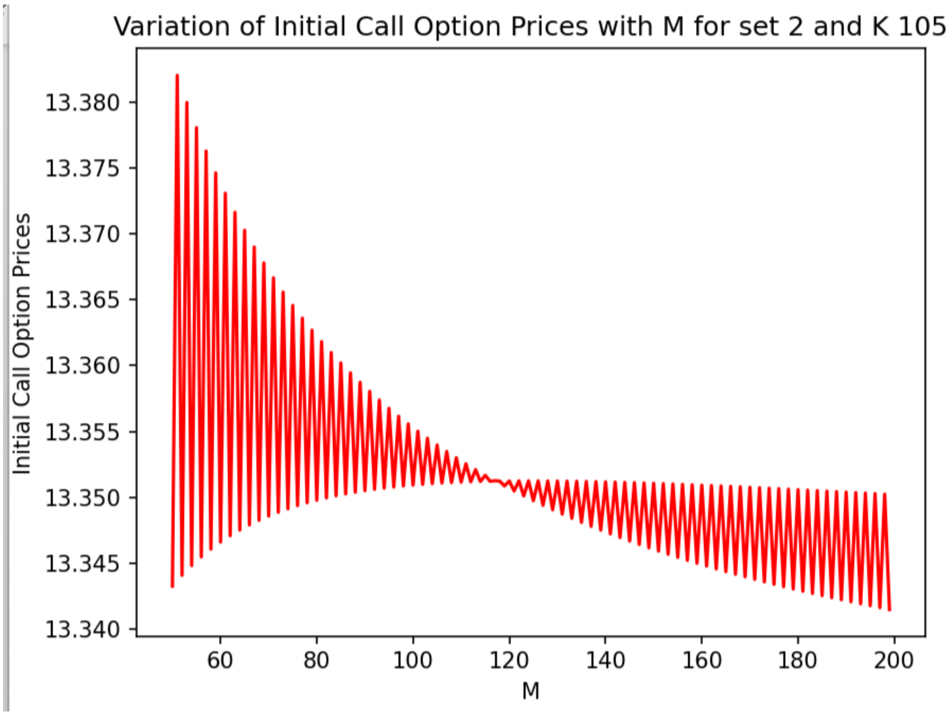

Variation of Initial Call Option Prices with M for set 2 and K 100
Initial Call Option Prices
15.76
15.74
15.72
15.70
15.68
60
80
100
120
140
160
180
200
M

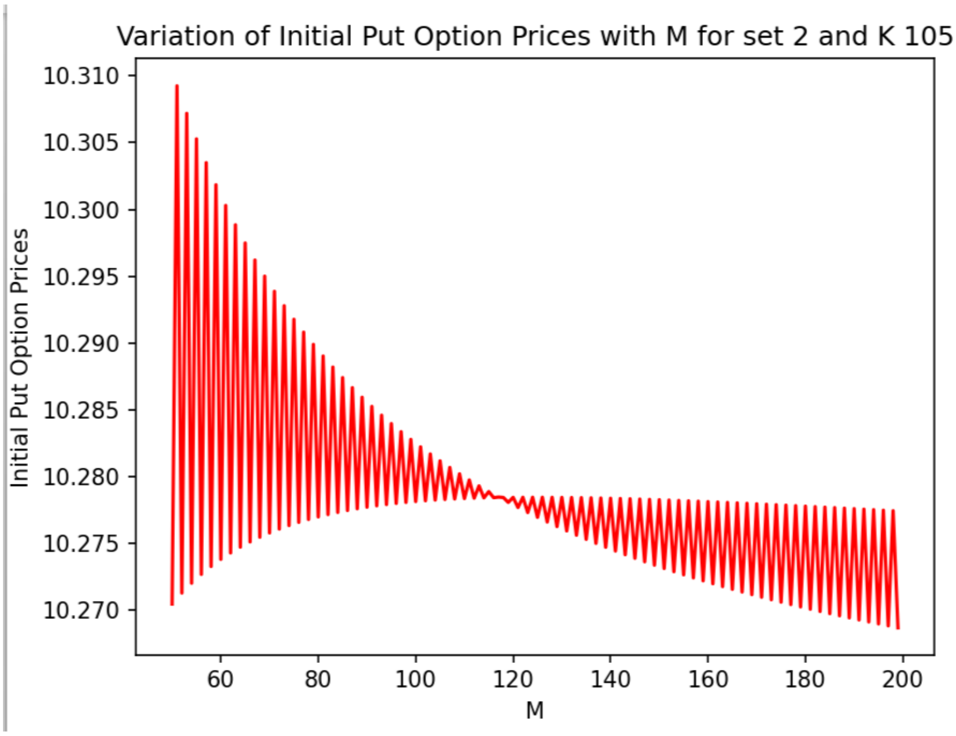

Variation of Initial Put Option Prices with M for set 2 and K 100
Initial Put Option Prices
8.06
8.04
8.02
8.00
60
80
100
120
140
160
180
200
M

c) For K = 105,


Variation of Initial Call Option Prices with M for set 1 and K 105
Initial Call Option Prices
13.39
13.38
13.37
13.36
13.35
13.34
13.33
13.32
13.31
60
80
100
120
140
160
180
200
M

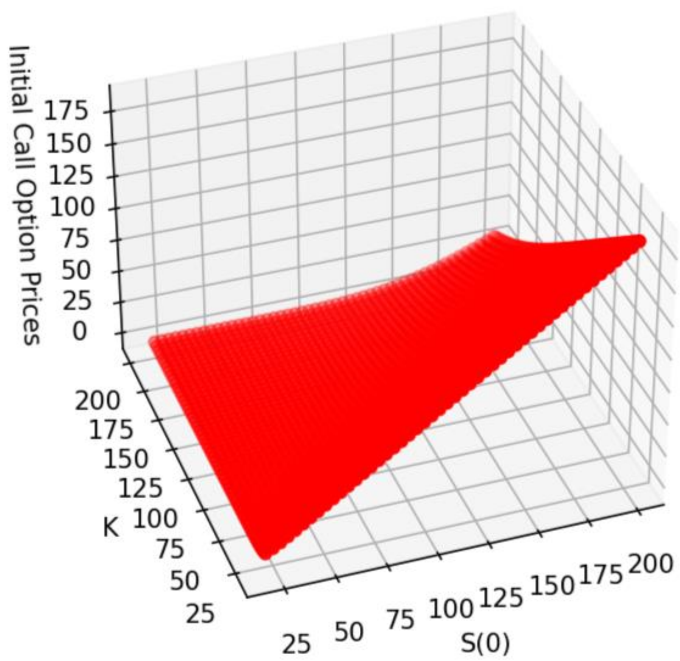

Variation of Initial Put Option Prices with M for set 1 and K 105
Initial Put Option Prices
10.31
10.30
10.29
10.28
10.27
10.26
10.25
10.24
60
80
100
120
140
160
180
200
M



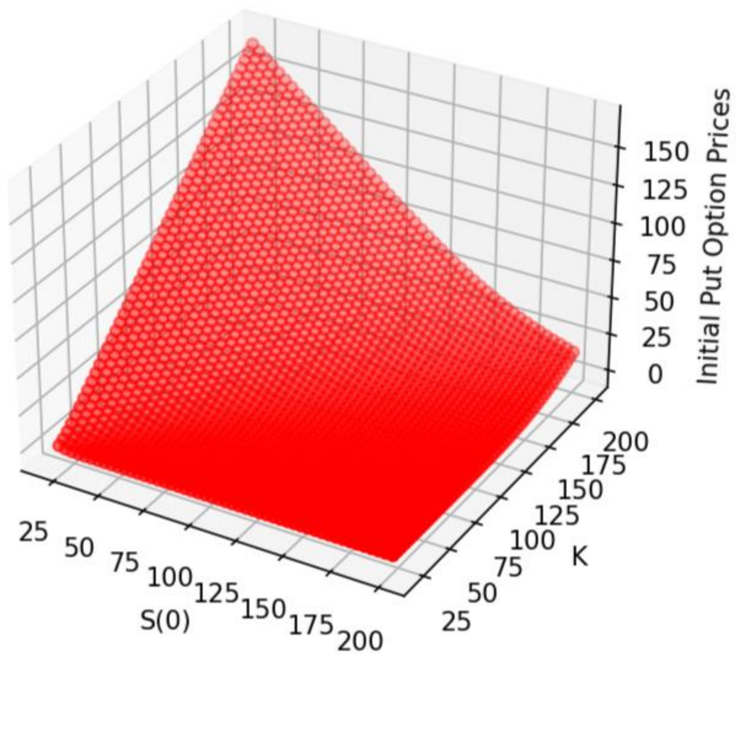


(i) Variation with S(0) and K:

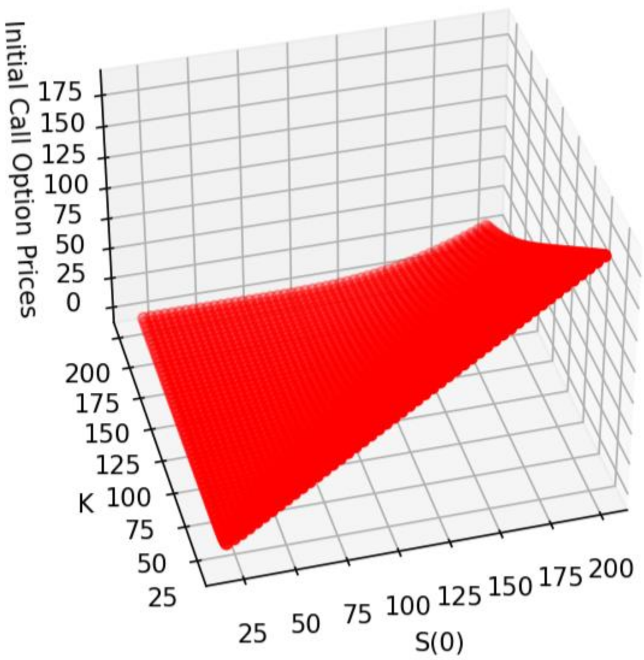
Variation of Initial Call Option Prices with S(0) and K for set 1



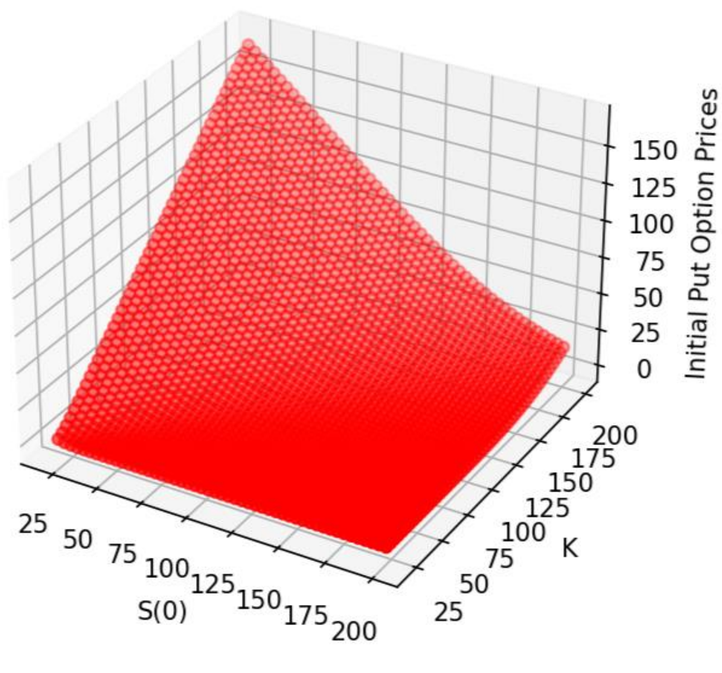
Variation of Initial Put Option Prices with S(0) and K for set 1



Variation of Initial Call Option Prices with S(0) and K for set 2

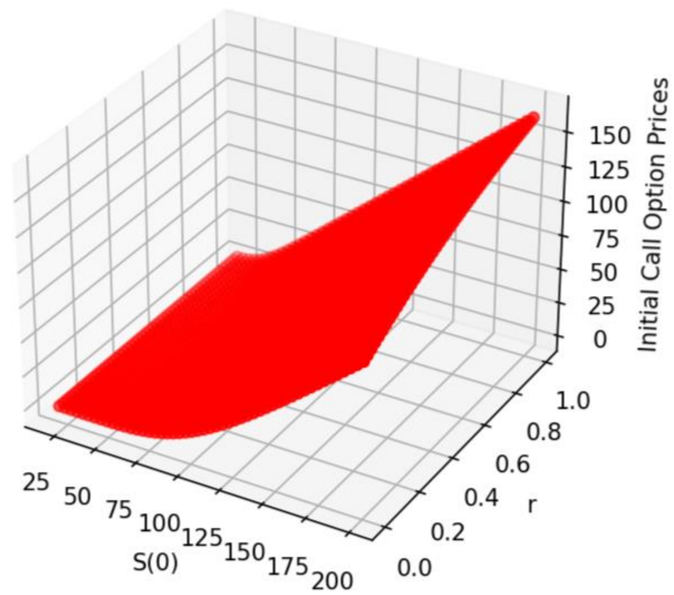


Variation of Initial Put Option Prices with S(0) and K for set 2

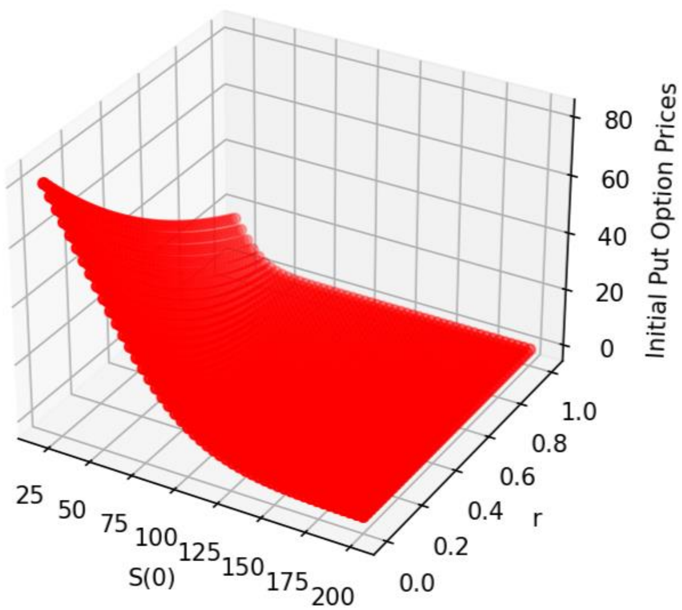


(ii) Variation with S(0) and r:

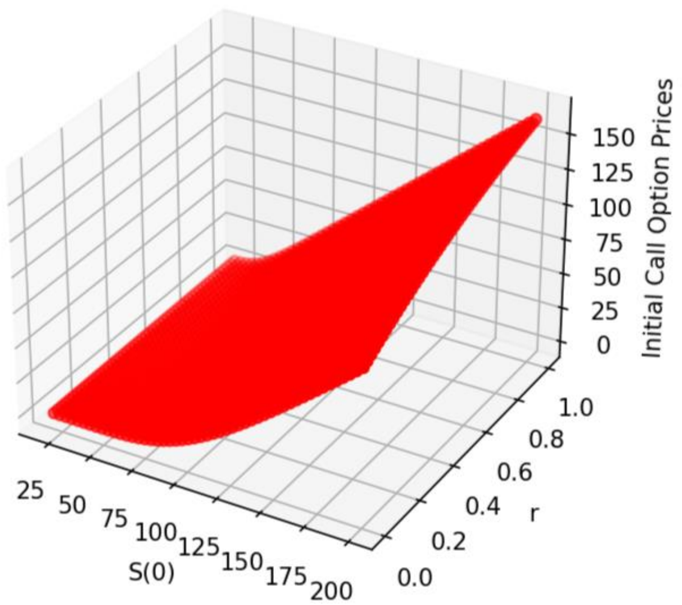
Variation of Initial Call Option Prices with S(0) and rate(r) for set 1



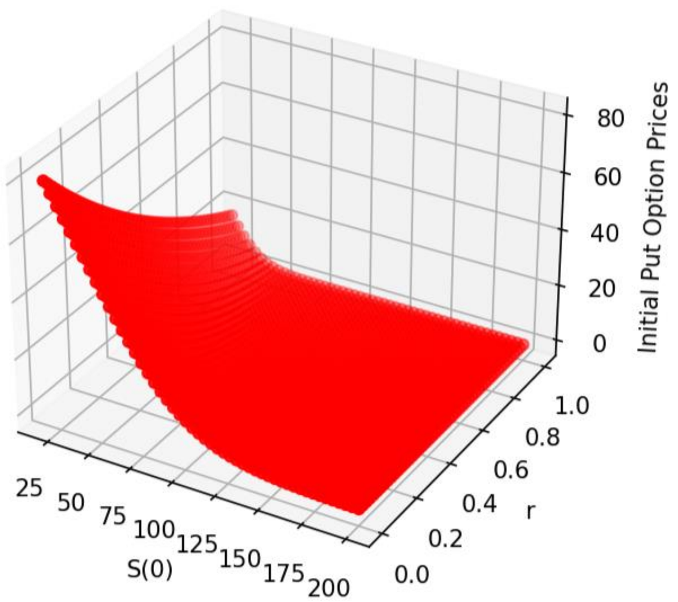
Variation of Initial Put Option Prices with S(0) and rate(r) for set 1



Variation of Initial Call Option Prices with S(0) and rate(r) for set 2

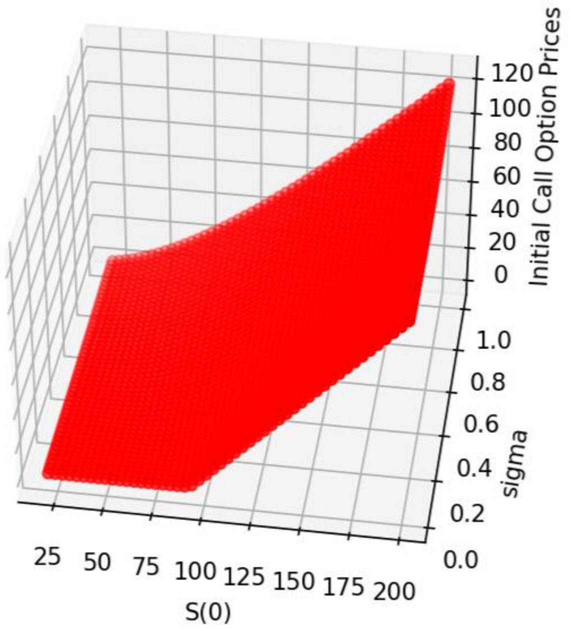


Variation of Initial Put Option Prices with S(0) and rate(r) for set 2

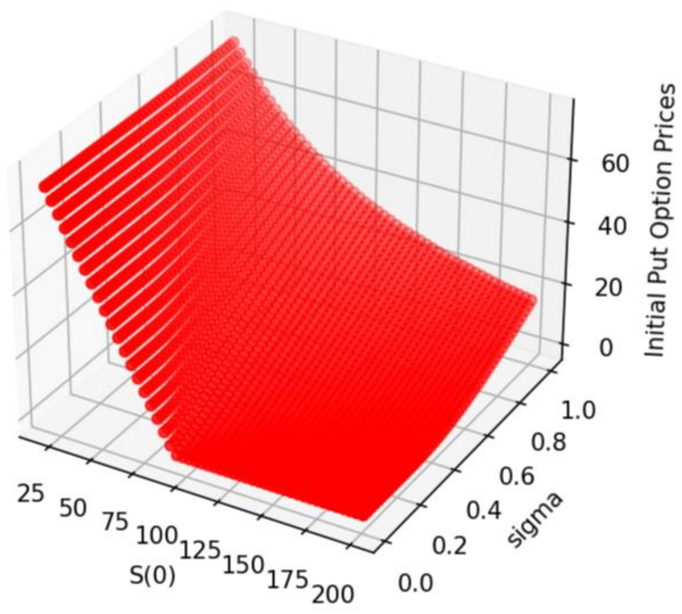


(iii) Variation with S(0) and σ:

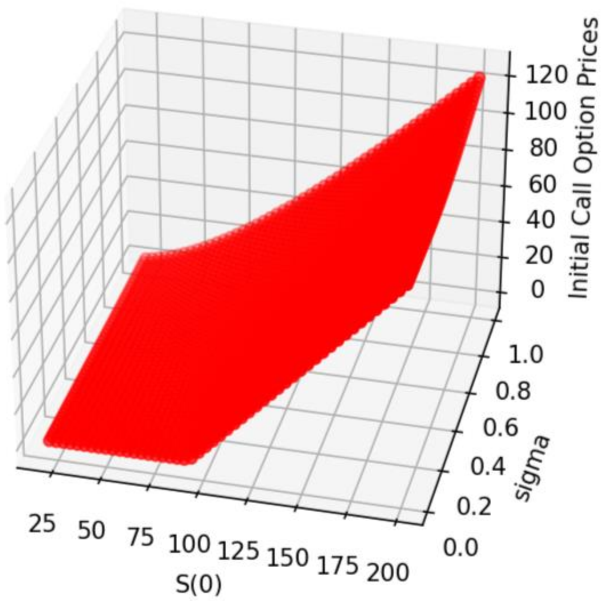
Variation of Initial Call Option Prices with S(0) and sigma for set 1



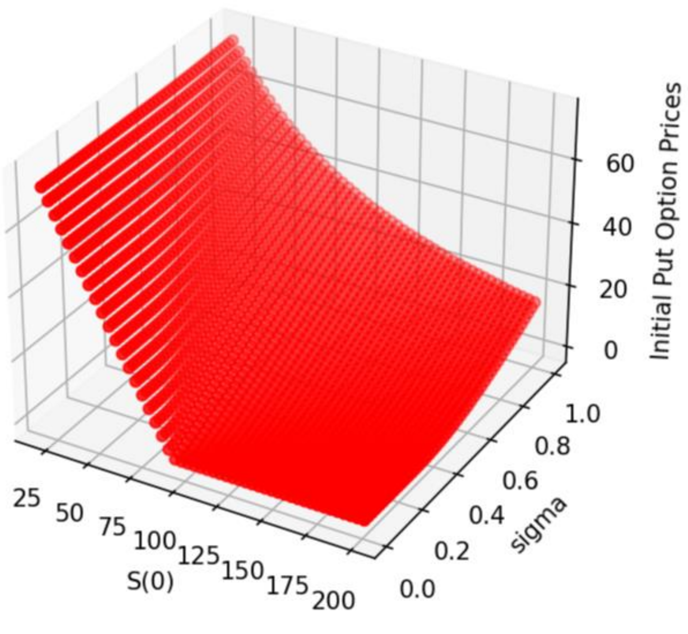
Variation of Initial Put Option Prices with S(0) and sigma for set 1



Variation of Initial Call Option Prices with S(0) and sigma for set 2

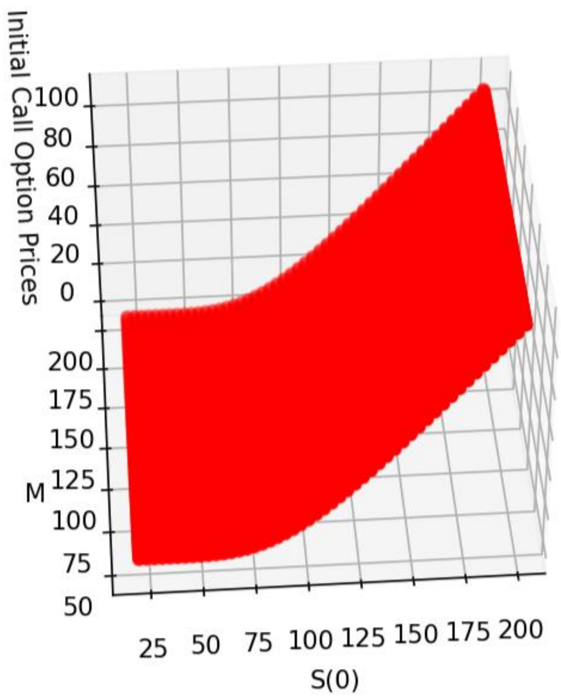


Variation of Initial Put Option Prices with S(0) and sigma for set 2

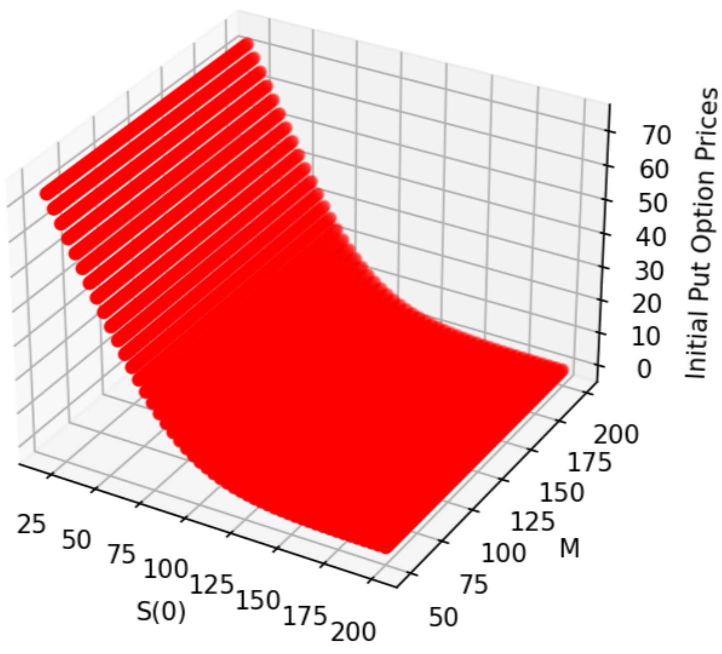


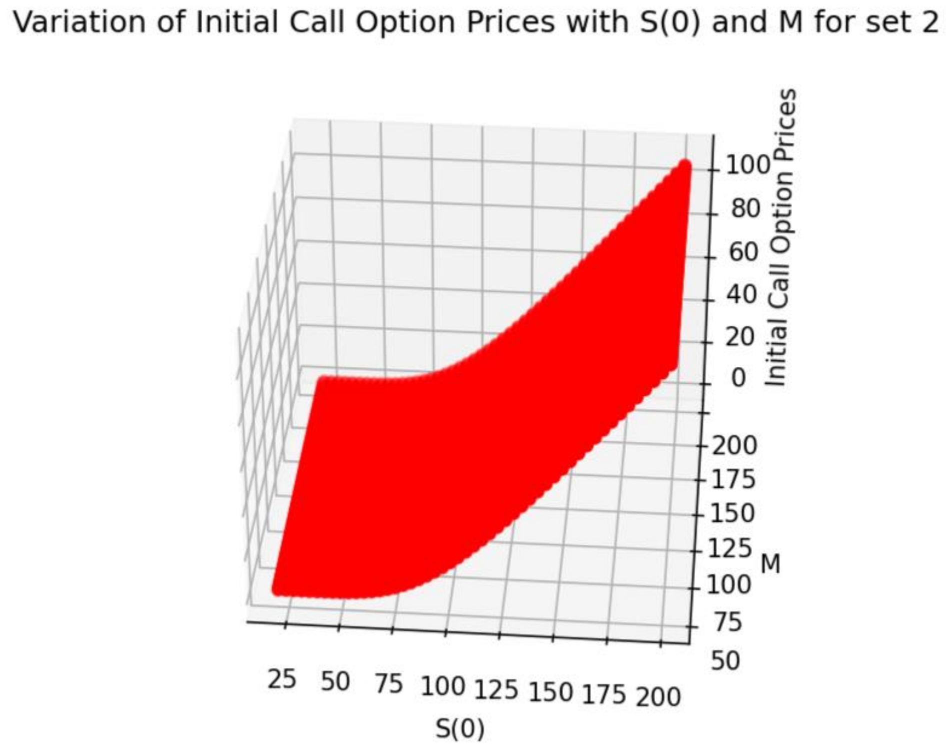
(iv) Variation with S(0) and M:

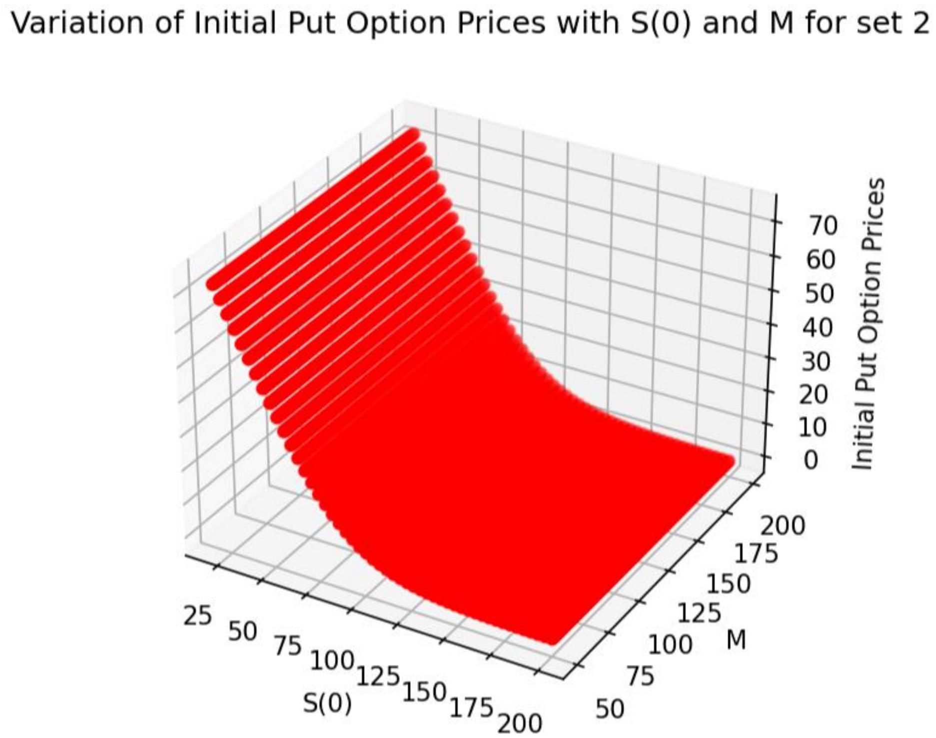
Variation of Initial Call Option Prices with S(0) and M for set 1



Variation of Initial Put Option Prices with S(0) and M for set 1

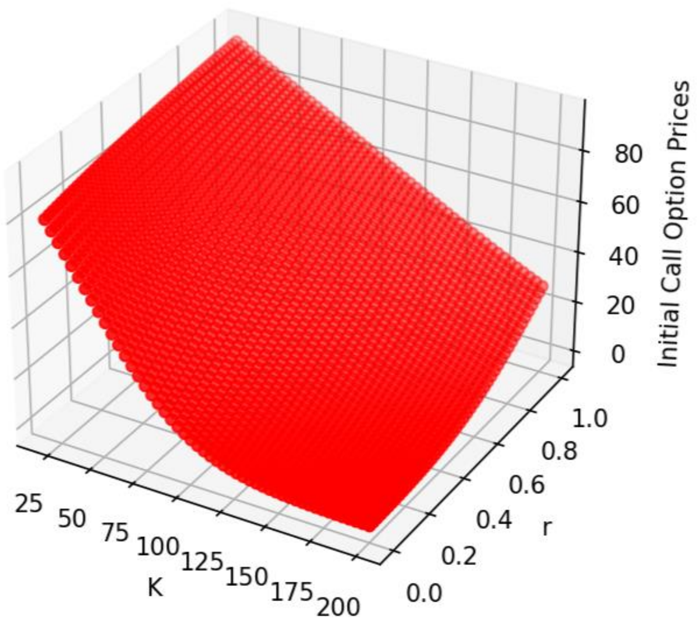




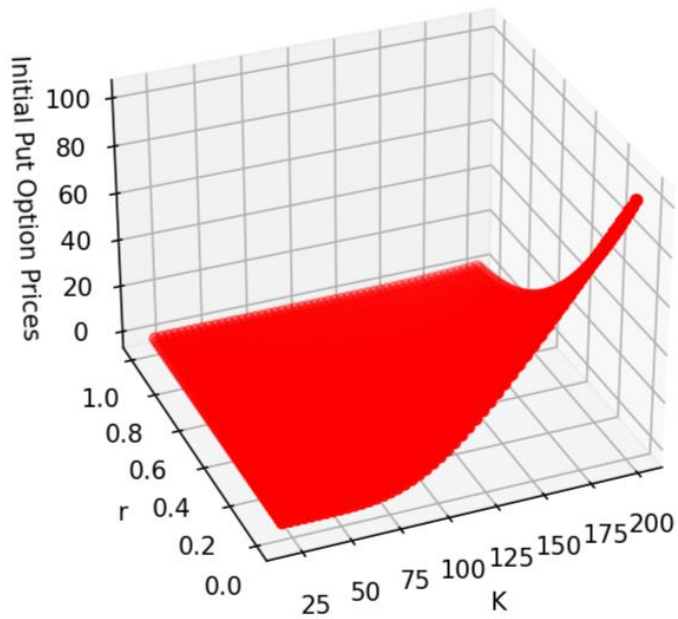


(v) Variation with K and r:

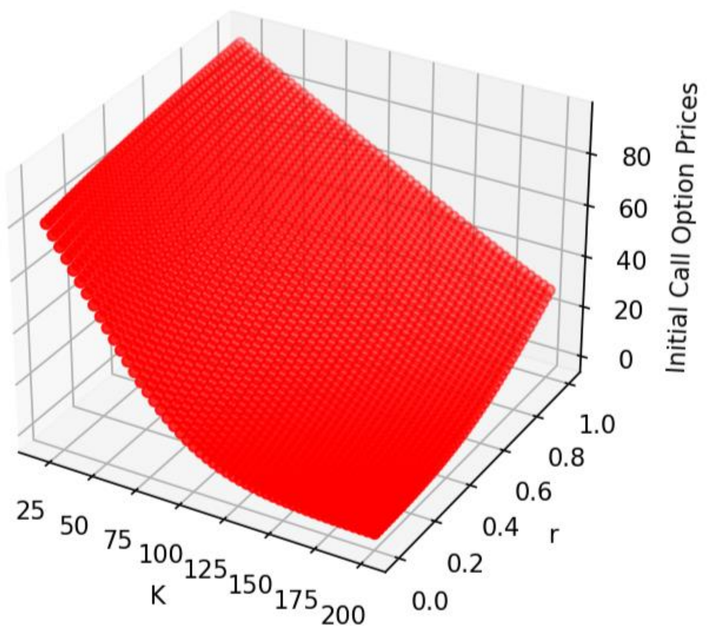
Variation of Initial Call Option Prices with K and rate(r) for set 1



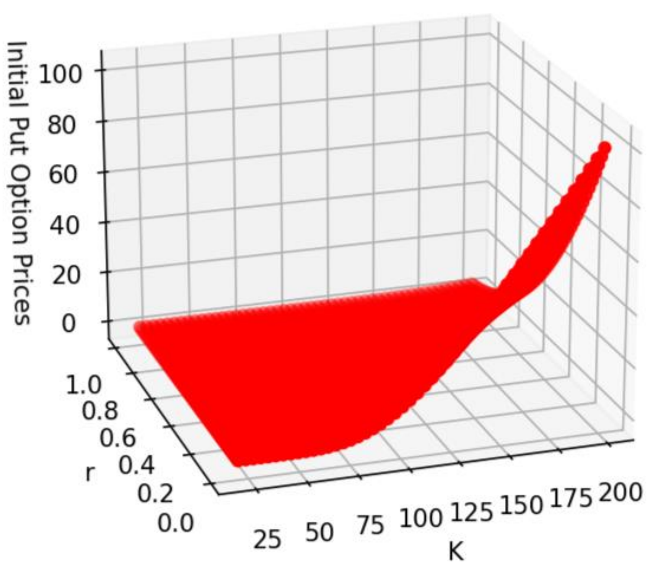
Variation of Initial Put Option Prices with K and rate(r) for set 1



Variation of Initial Call Option Prices with K and rate(r) for set 2

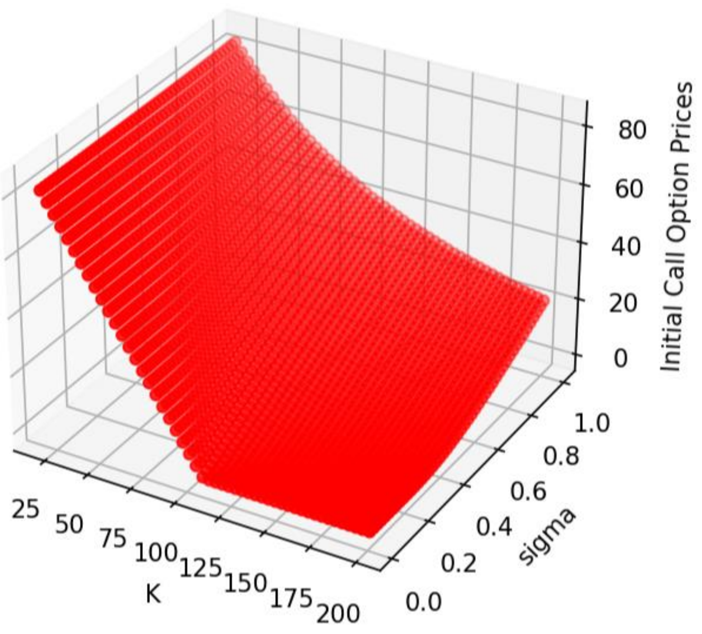


Variation of Initial Put Option Prices with K and rate(r) for set 2

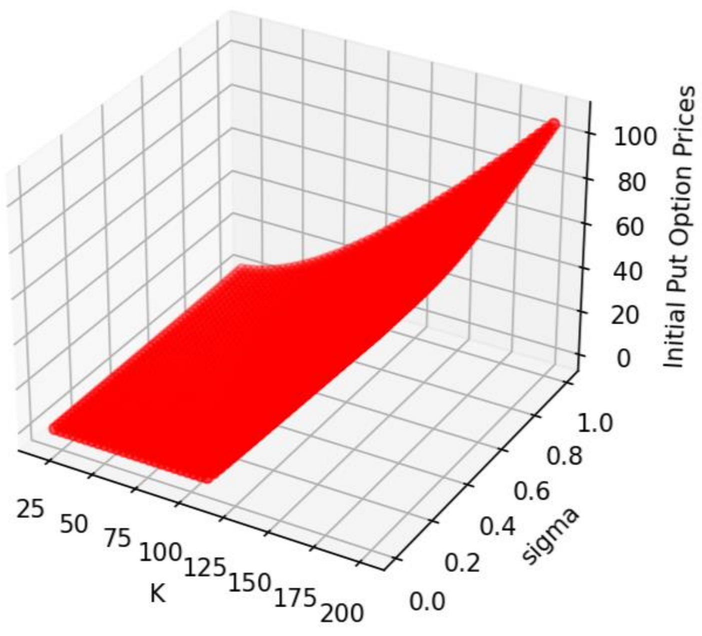


(vi) Variation with K and σ:

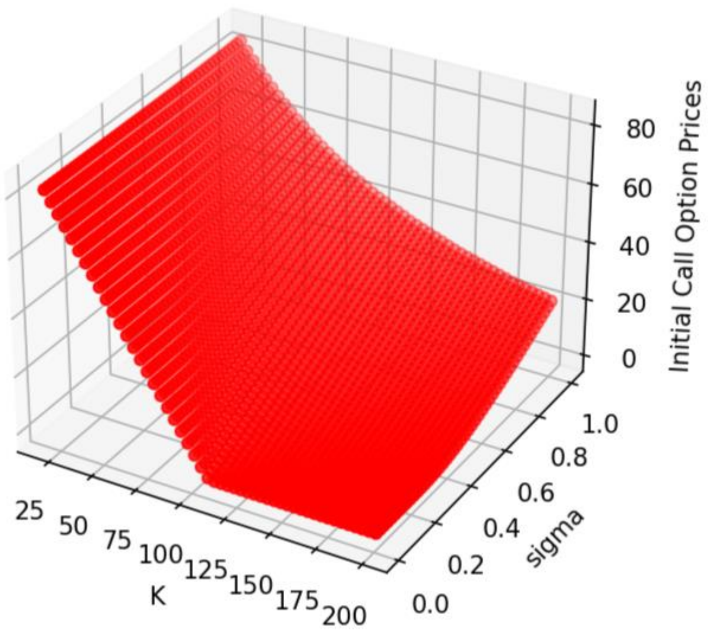
Variation of Initial Call Option Prices with K and sigma for set 1



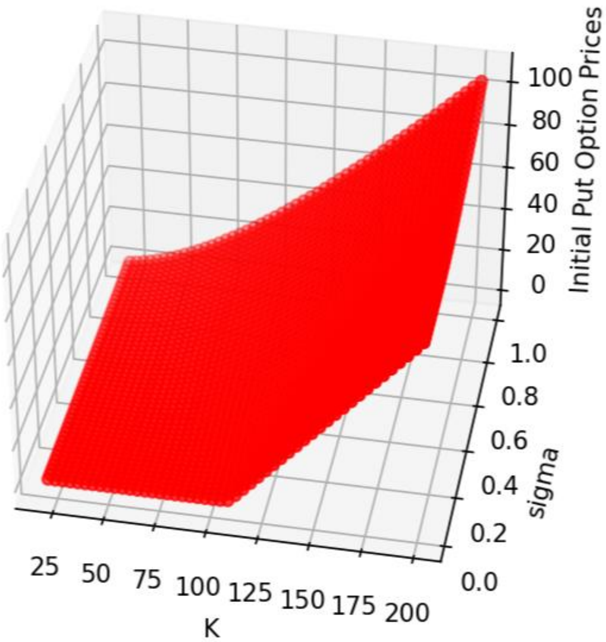
Variation of Initial Put Option Prices with K and sigma for set 1



Variation of Initial Call Option Prices with K and sigma for set 2

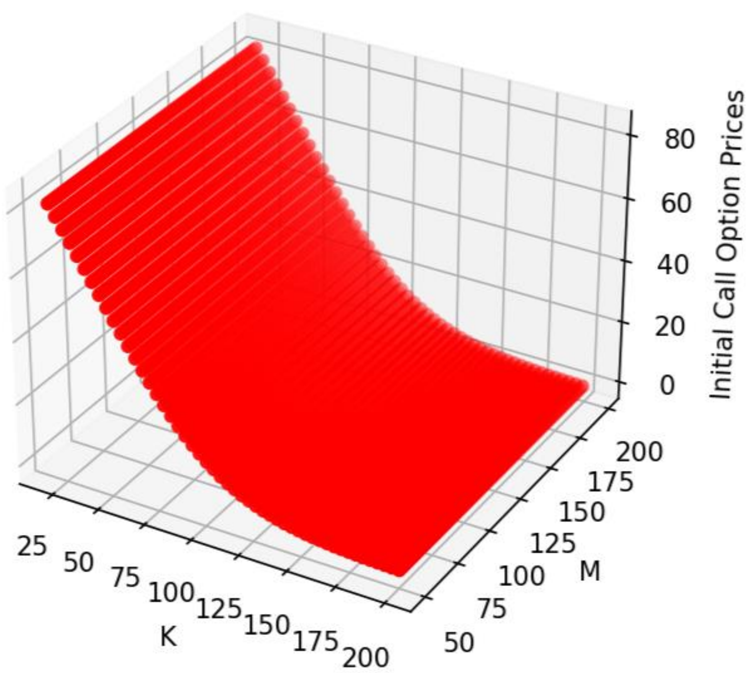


Variation of Initial Put Option Prices with K and sigma for set 2

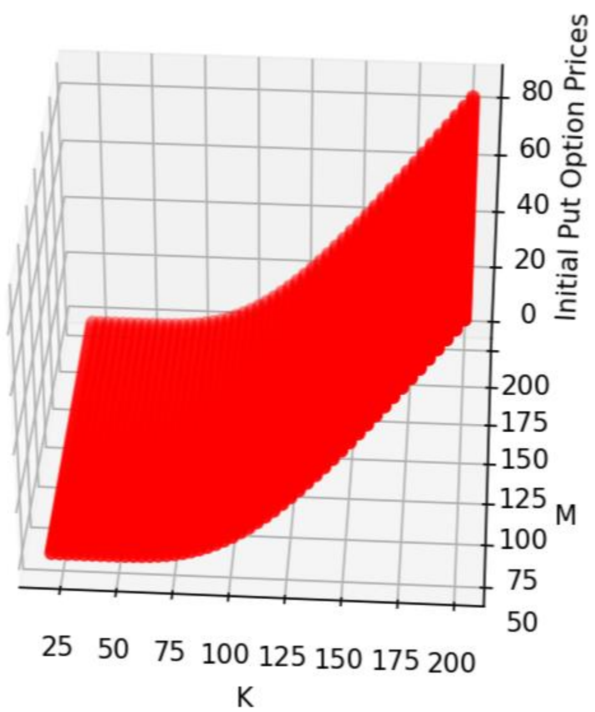


(vii) Variation with K and M:

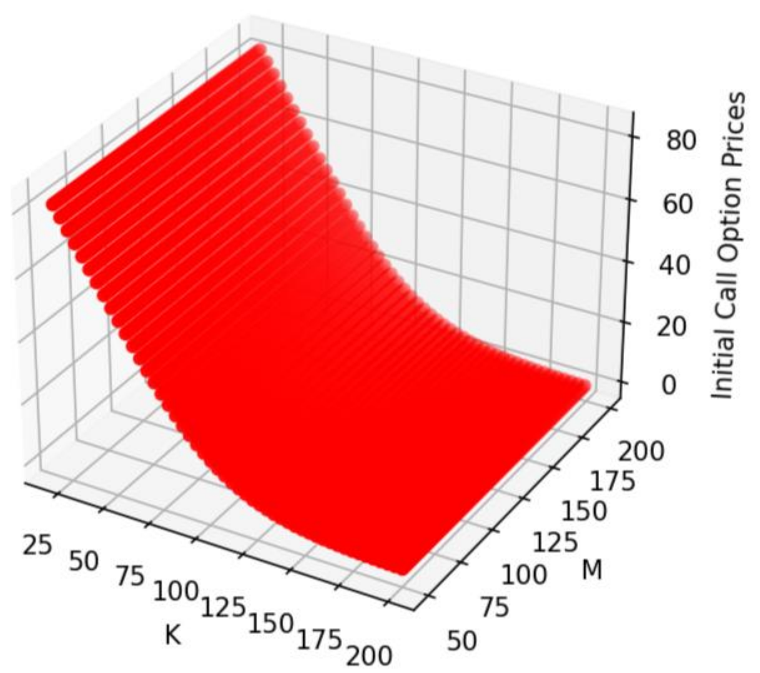
Variation of Initial Call Option Prices with K and M for set 1



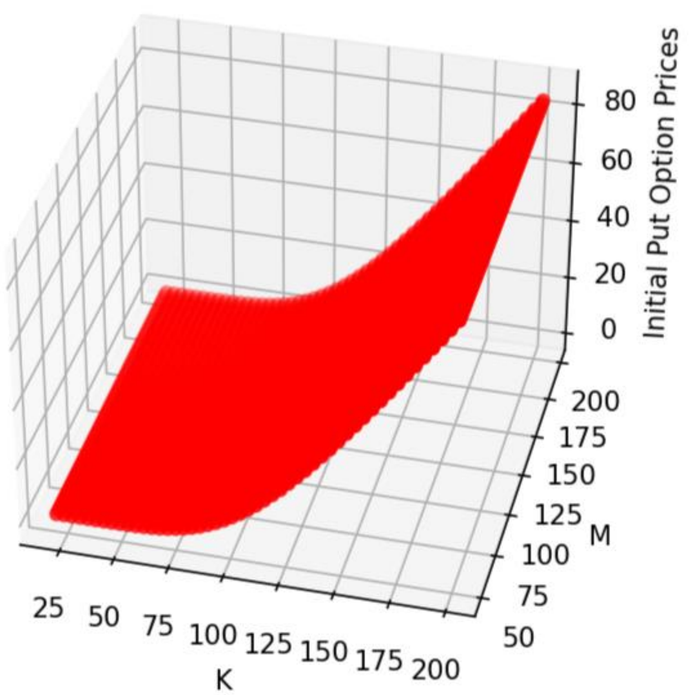
Variation of Initial Put Option Prices with K and M for set 1



Variation of Initial Call Option Prices with K and M for set 2

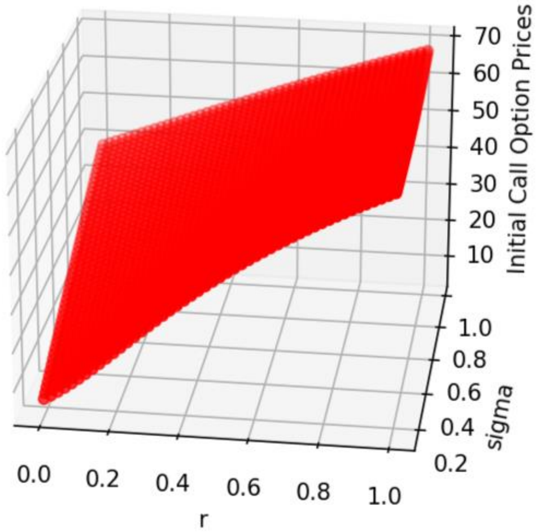


Variation of Initial Put Option Prices with K and M for set 2

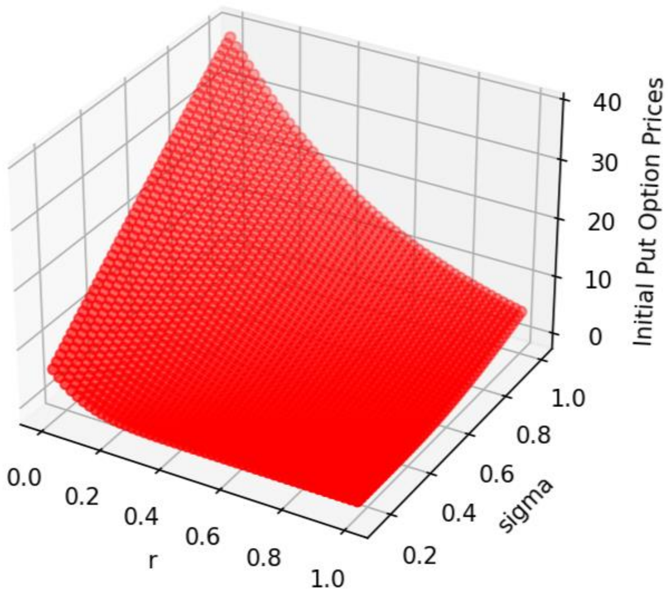


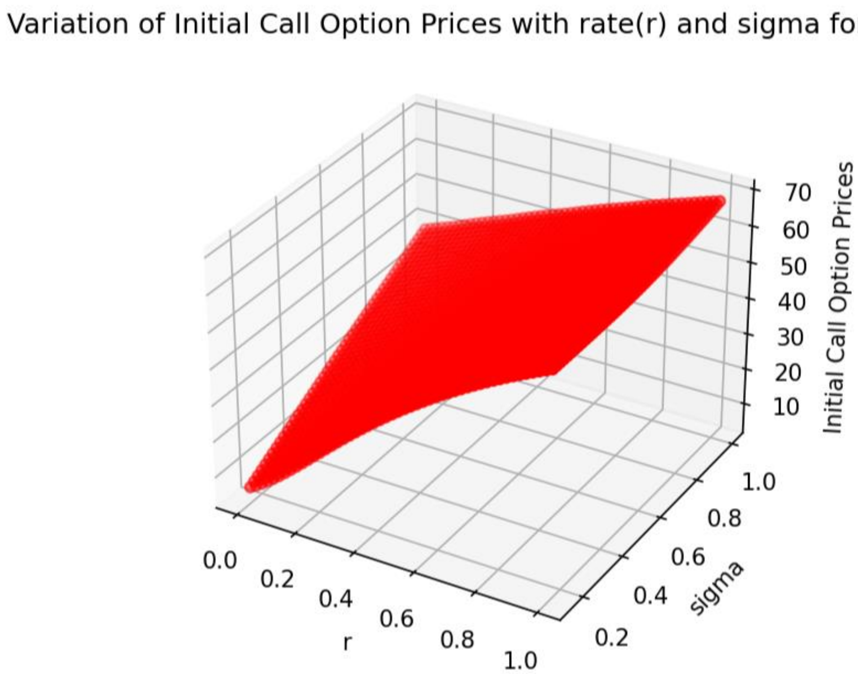
(viii) Variation with r and σ:

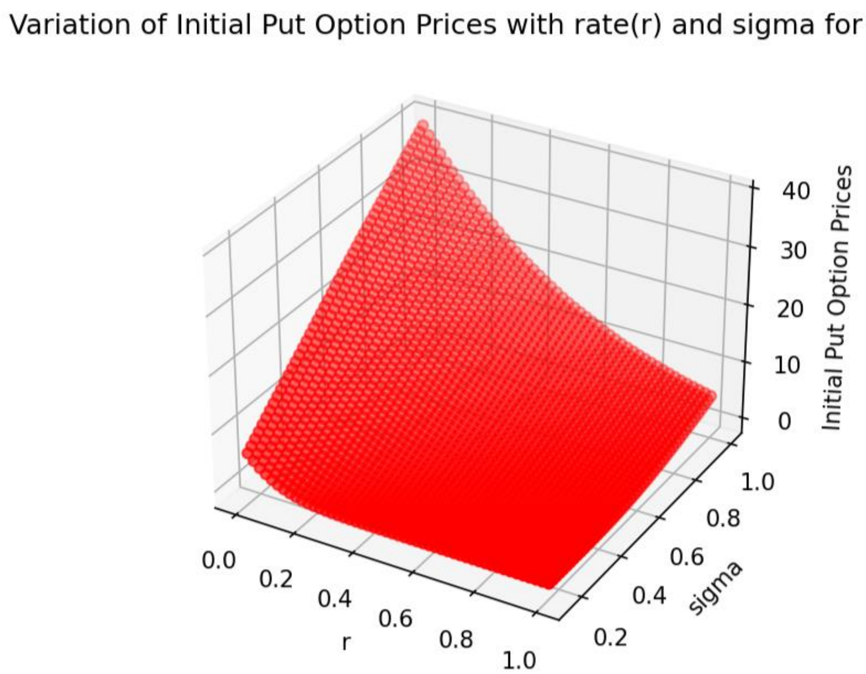
Variation of Initial Call Option Prices with rate(r) and sigma for set 1



Variation of Initial Put Option Prices with rate(r) and sigma for set 1

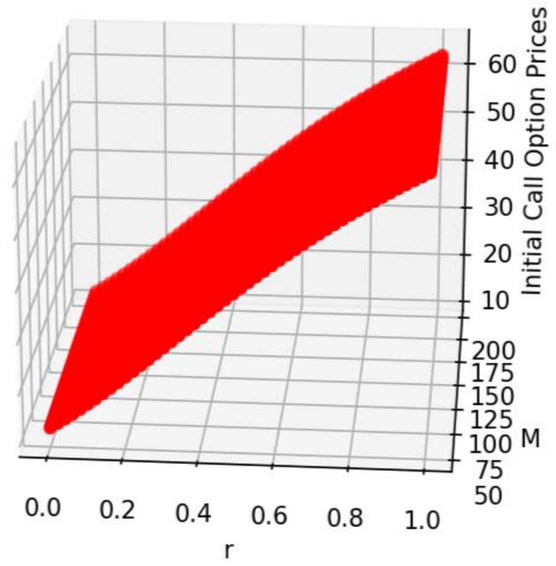




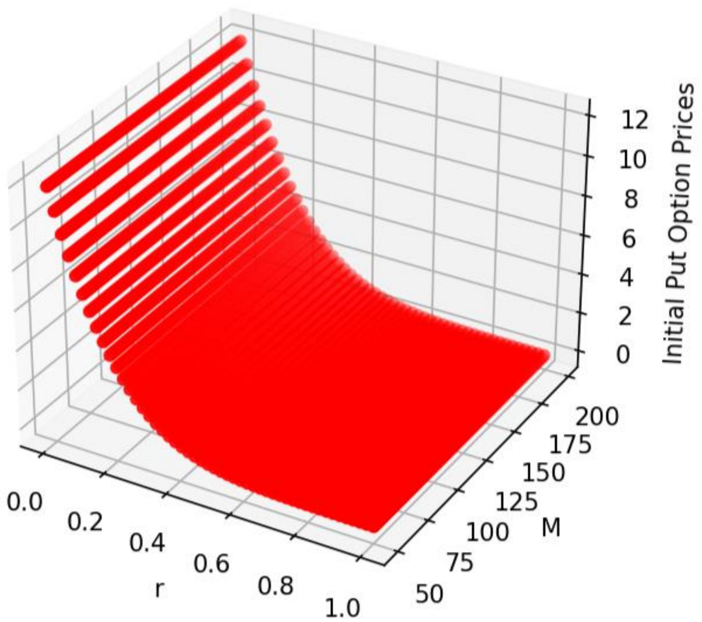


(ix) Variation with r and M:

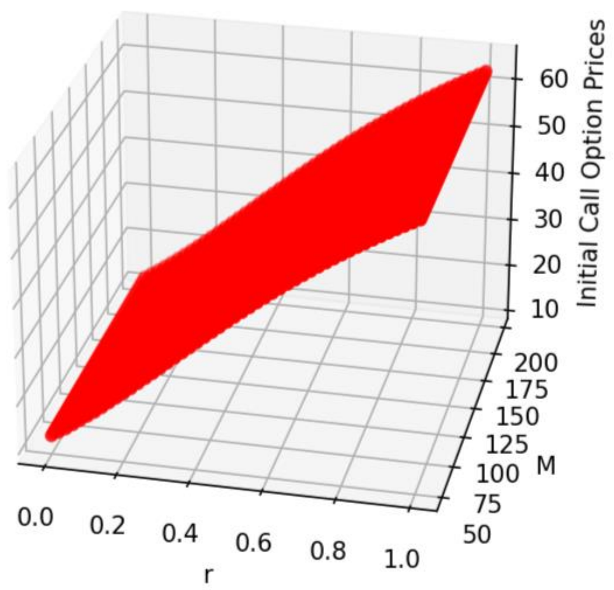
Variation of Initial Call Option Prices with rate(r) and M for set 1



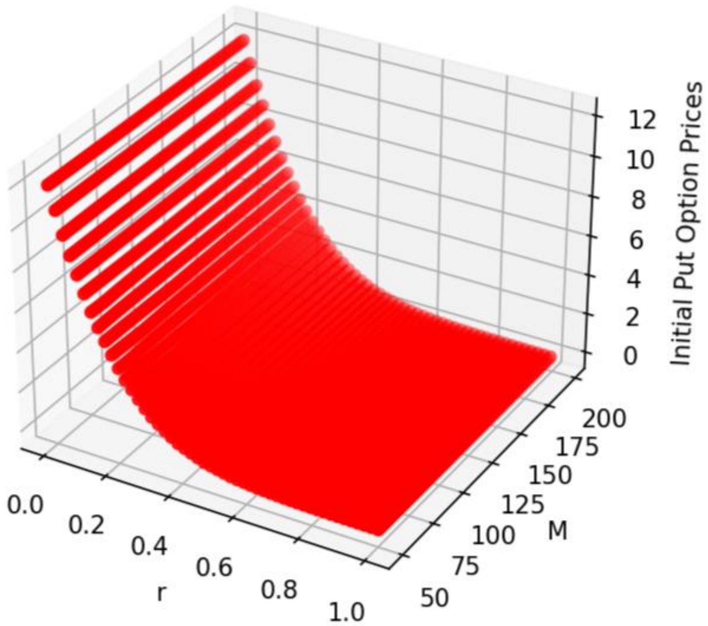
Variation of Initial Put Option Prices with rate(r) and M for set 1



Variation of Initial Call Option Prices with rate(r) and M for set 2

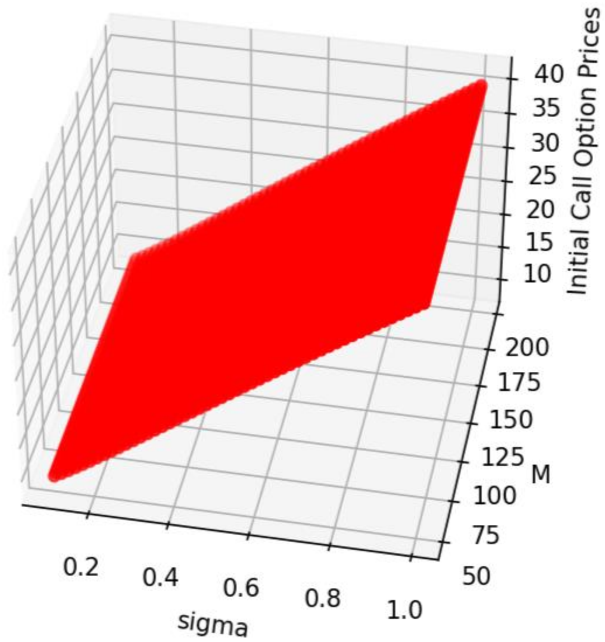


Variation of Initial Put Option Prices with rate(r) and M for set 2

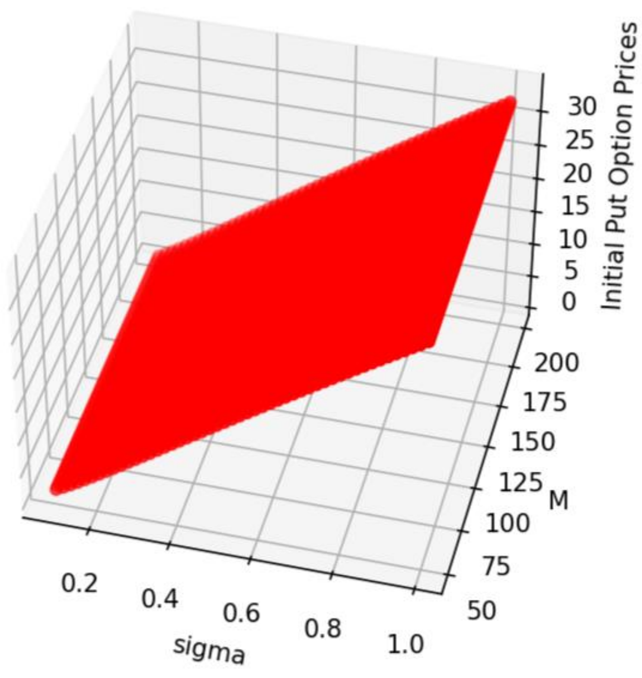


(x) Variation with σ and M:

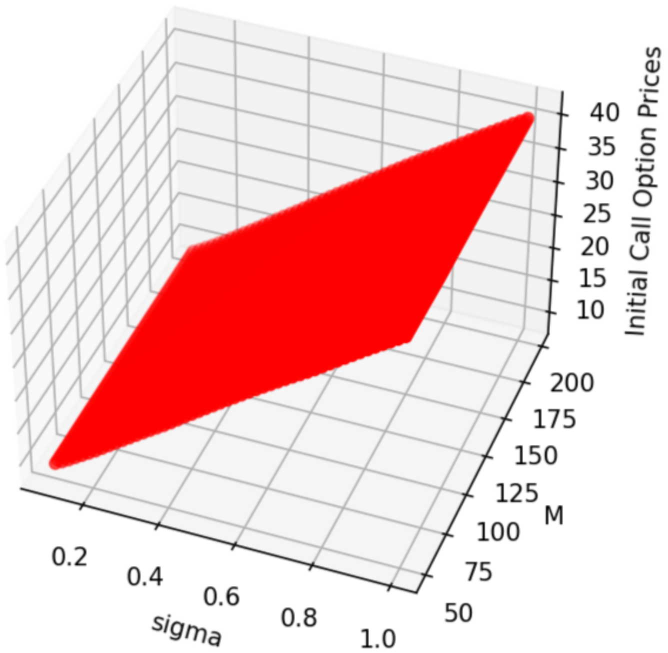
Variation of Initial Call Option Prices with sigma and M for set 1



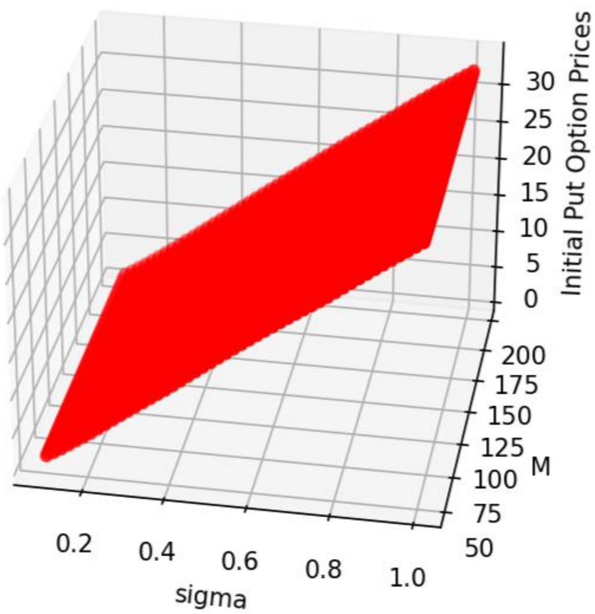
Variation of Initial Put Option Prices with sigma and M for set 1



Variation of Initial Call Option Prices with sigma and M for set 2



Variation of Initial Put Option Prices with sigma and M for set 2



Observations:

From the 2D Plots, we can observe that:

- The price of the Call Option has positive dependence on S(0), and the price of the Put Option price has negative dependence on S(0).
- The price of the Put Option has positive dependence on K, and the price of the Call Option price has negative dependence on K.
- The price of the Call Option has positive dependence on r, and the price of the Put Option price has negative dependence on r.

The same observations are expected theoretically from the Cox-Ross-Rubinstein Formula and the Put Call Parity Equation.

Question 2:

2. Now take any path-dependent derivative of your choice and do the above exercise for at least one set (of u, d).

Put all your observations in the report.

I have considered Asian call and put options, that is, the path-dependent derivative chosen is Asian Option. In this type of Options, the final value of call and put options is dependent on all the intermediate steps and then at last, the average value is taken. The value of M is taken as 10 in most cases because it is computationally very expensive and not feasible to use larger values of M since the computational cost grows exponentially.

For M = 10,

For Set 1:

The initial price of the Asian Call Option = 8.550121325598722

The initial price of the Asian Put Option = 4.752063868096397

For Set 2:

The initial price of the Asian Call Option = 8.559554720713114

The initial price of the Asian Put Option = 4.761497263210775

```
No arbitrage exists for M = 10
For Set 1,
Initial Price of Asian Call Option = 8.550121325598722
Initial Price of Asian Put Option = 4.752063868096397

No arbitrage exists for M = 10
For Set 2,
Initial Price of Asian Call Option = 8.559554720713114
Initial Price of Asian Put Option = 4.761497263210775
```

At the Maturity/Expiry time T,

$$Payoff\ of\ Asian\ Call\ Option = max\left(\frac{1}{N}\sum_{i=1}^{N} S(t_i) - K,\ 0\right)$$

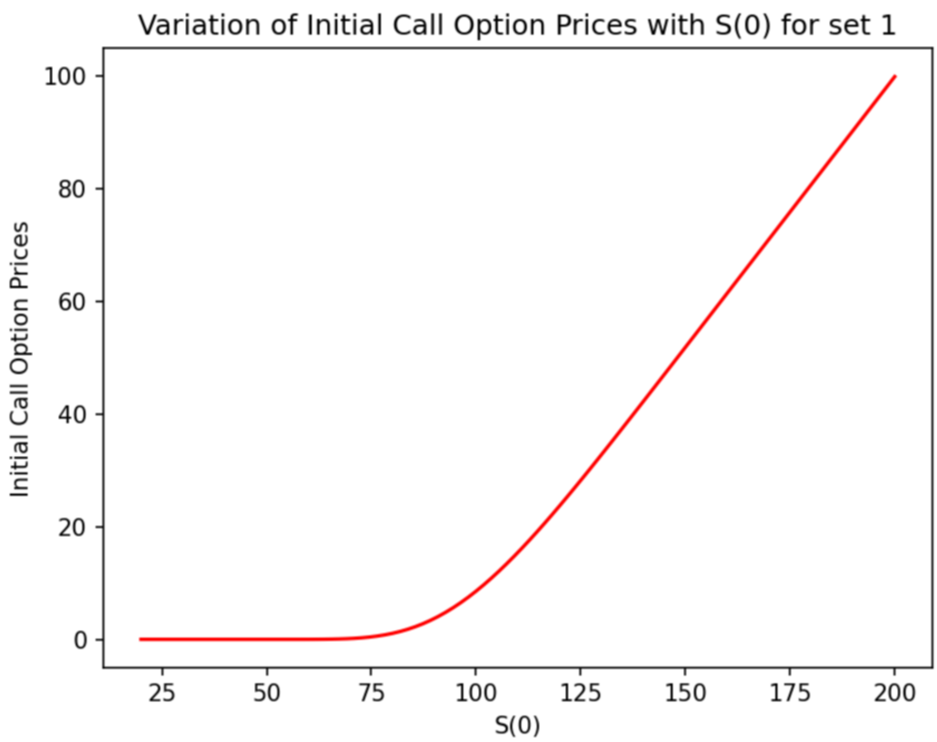
$$Payoff\ of\ Asian\ Put\ Option = max\left(K - \frac{1}{N}\sum_{i=1}^{N} S(t_i),\ 0\right)$$

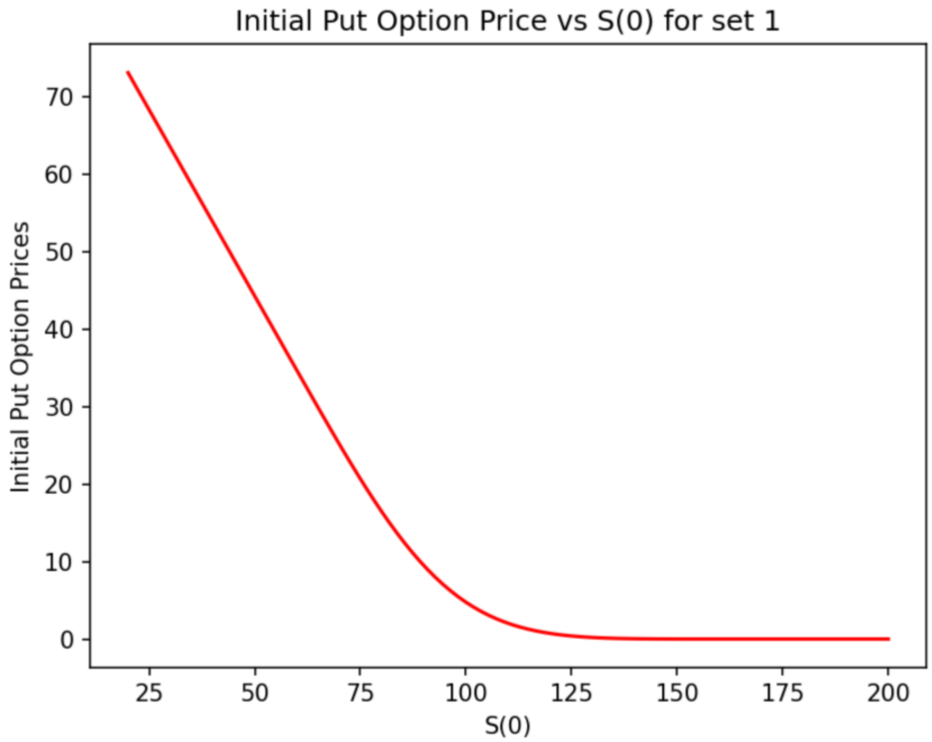
Sensitivity Analysis:

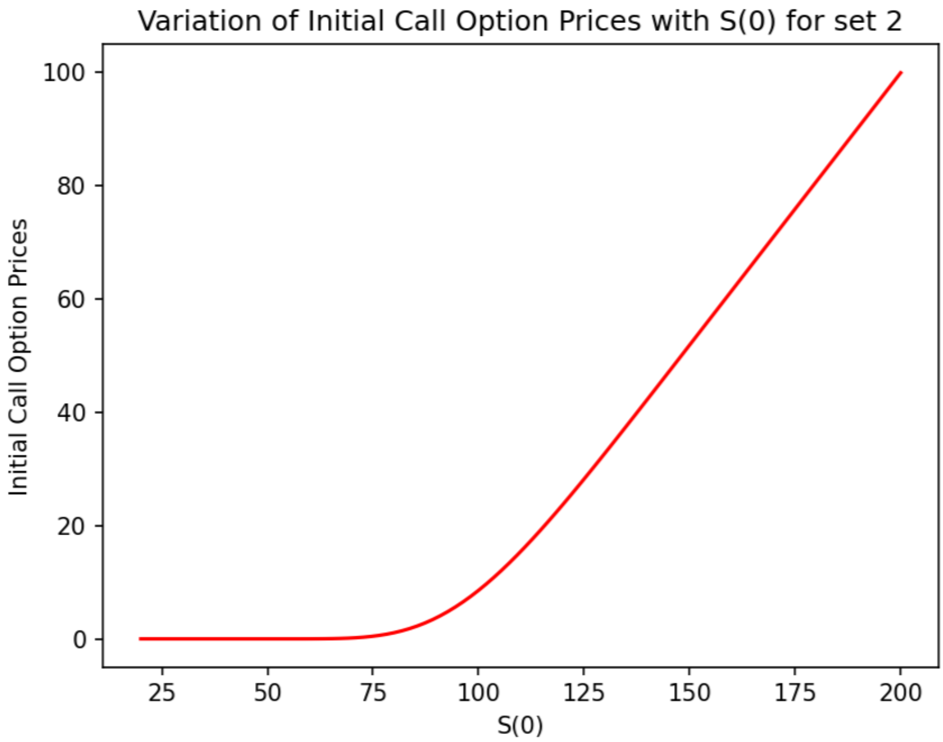
Sensitivity Analysis of option price variance with S_0, K, M, r, σ are done by plotting 2-D and 3-D plots.

Note – While plotting with respect to M considering only a single variable, then I have plotted the graphs for all the three given values of K, that is, K = 95, 100 and 105 but while plotting the graphs taking 2 parameters at a time, graphs of only 1 value of K are shown because the graphs are approximately the same for all the 3 values of K.

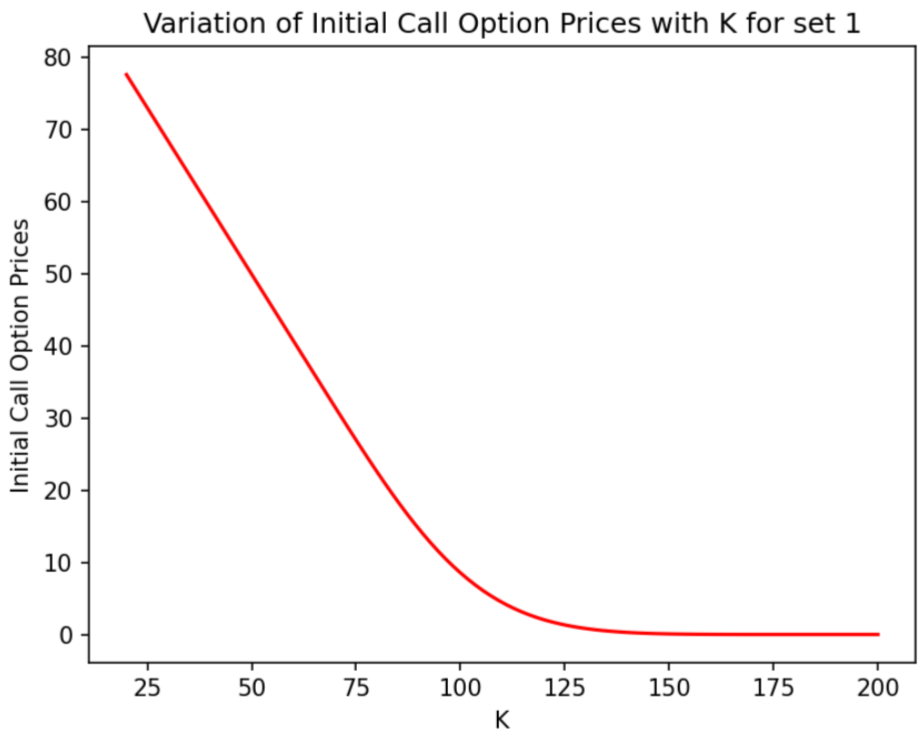
1) Variation with S(0):

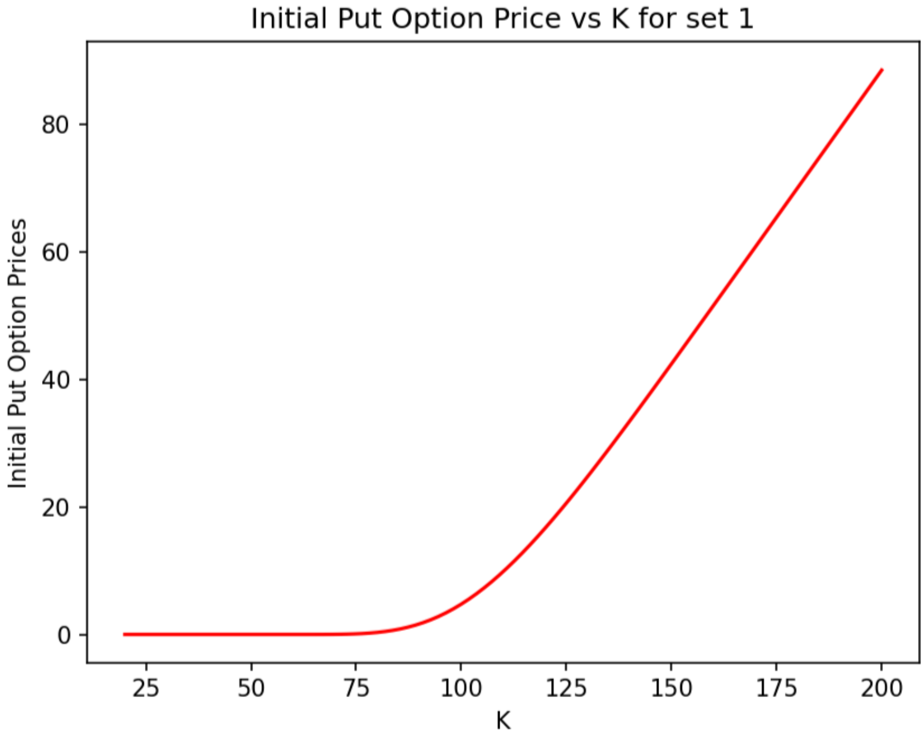


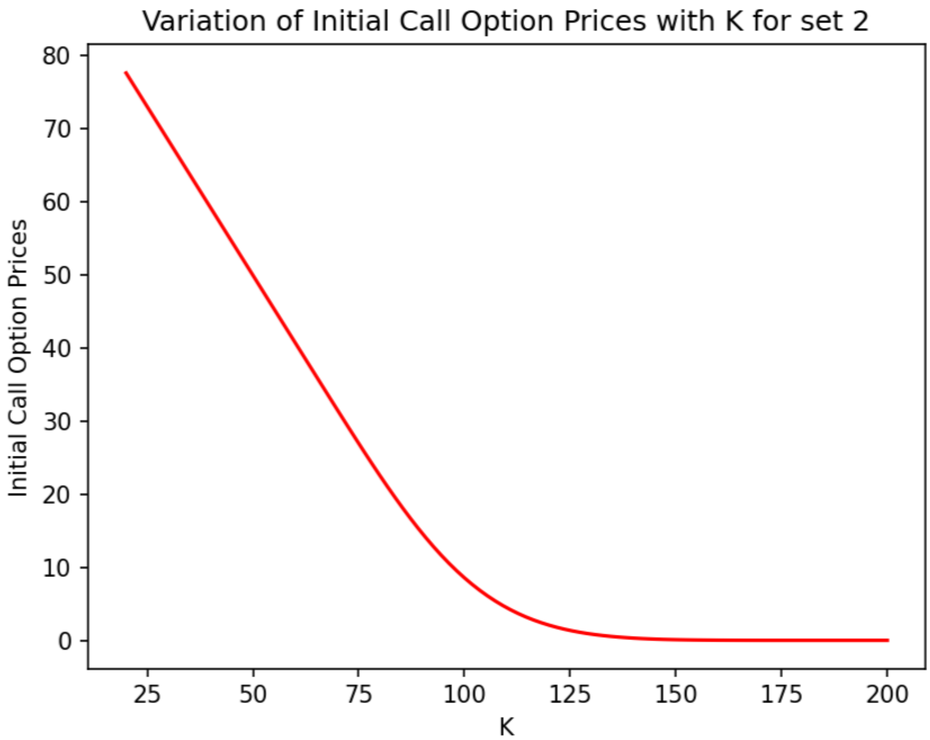

Initial Put Option Price vs S(0) for set 1
Initial Put Option Prices
70
60
50
40
30
20
10
0
25
50
75
100
125
150
175
200
S(0)

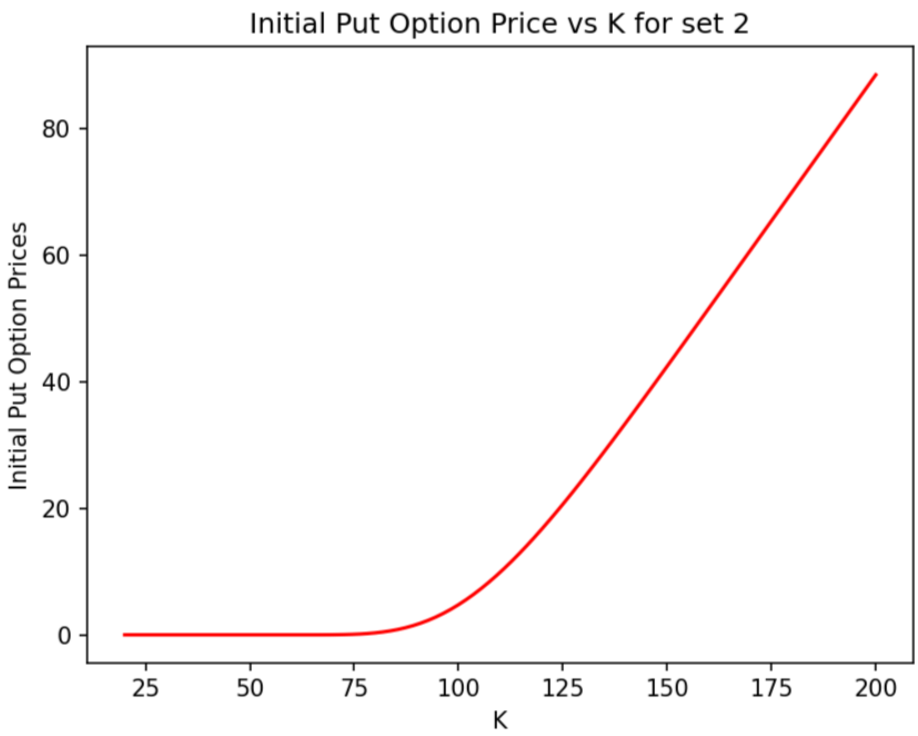

Variation of Initial Call Option Prices with S(0) for set 2
Initial Call Option Prices
100
80
60
40
20
0
25
50
75
100
125
150
175
200
S(0)

2) Variation with K:

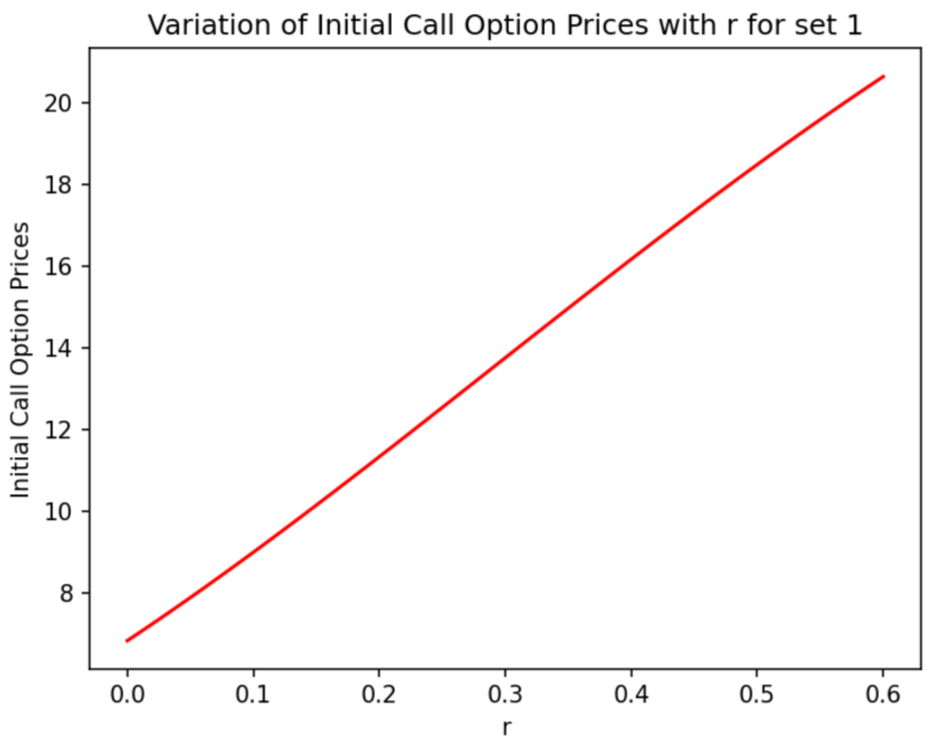


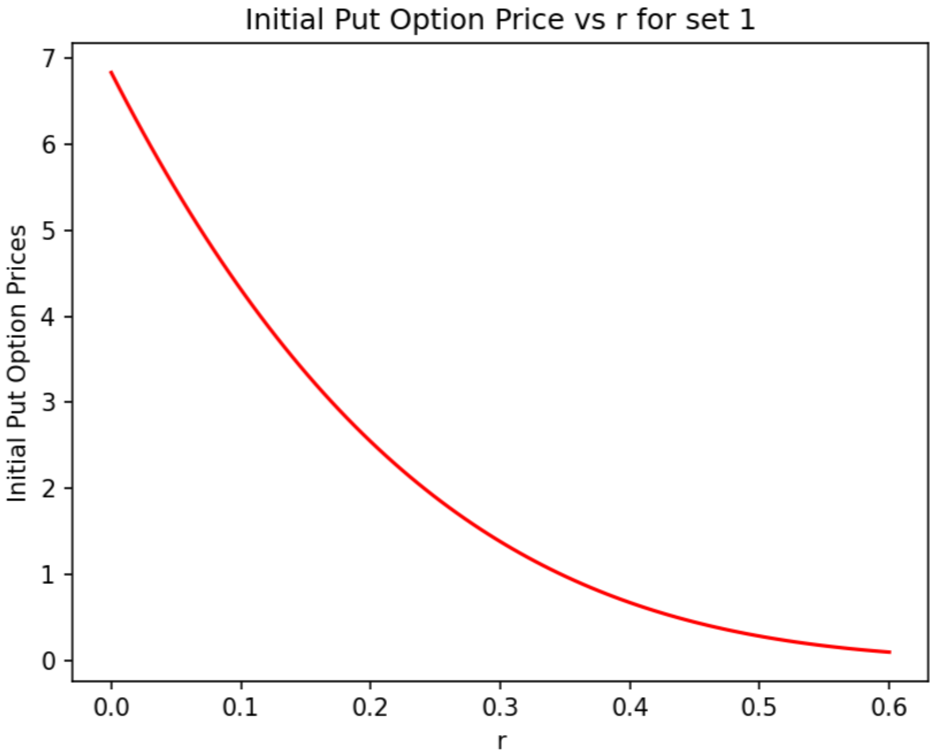

Initial Put Option Price vs K for set 1
Initial Put Option Prices
K
0
20
40
60
80
25
50
75
100
125
150
175
200

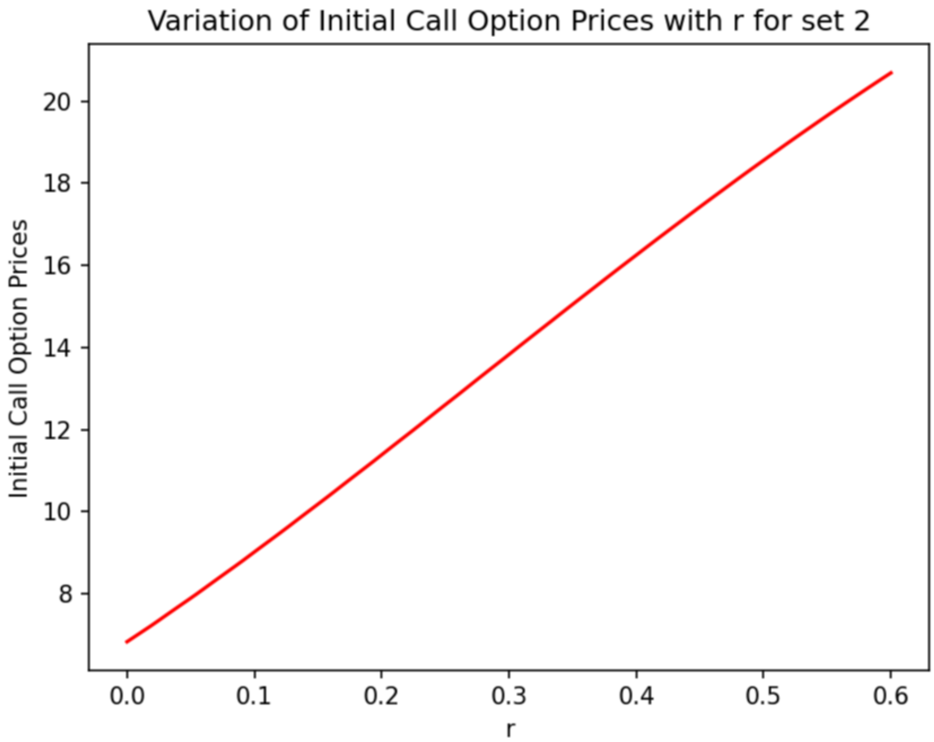

Variation of Initial Call Option Prices with K for set 2
Initial Call Option Prices
K
0
10
20
30
40
50
60
70
80
25
50
75
100
125
150
175
200

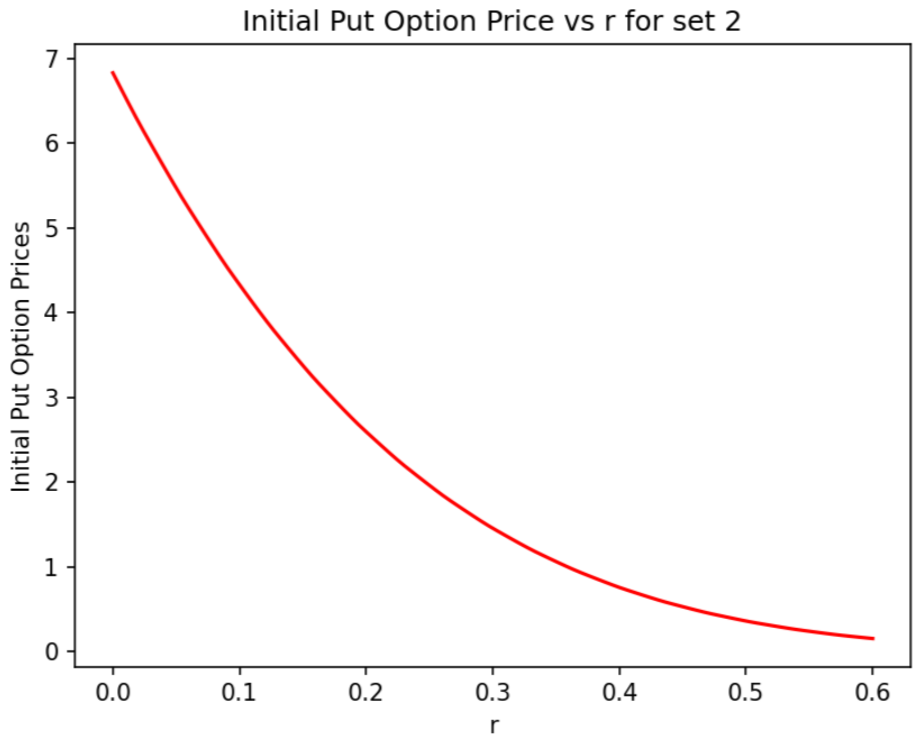


3) Variation with r:

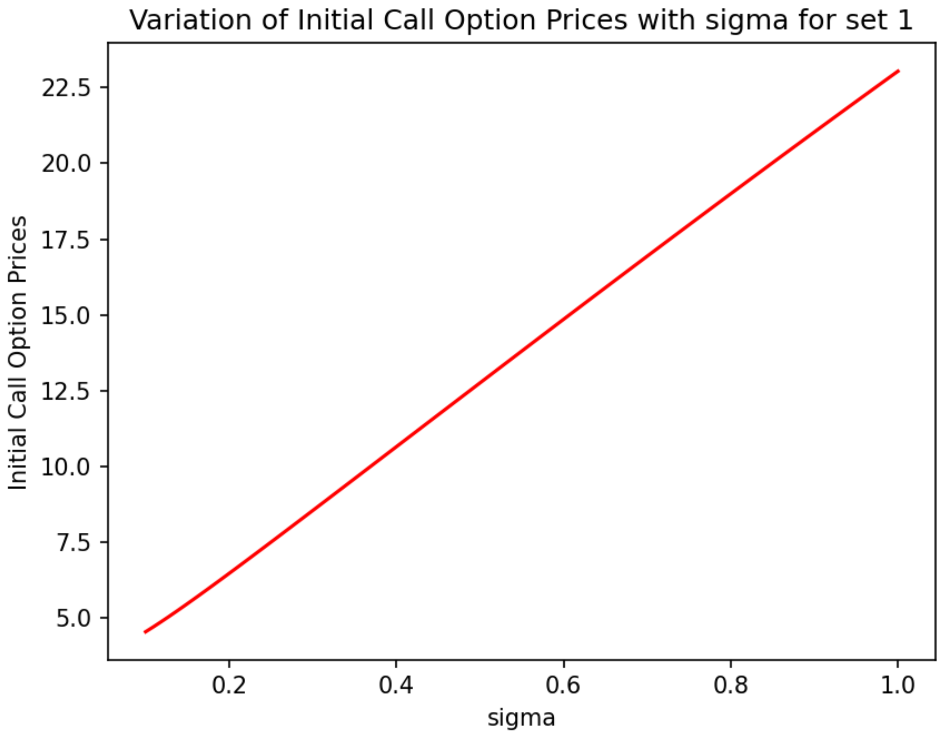


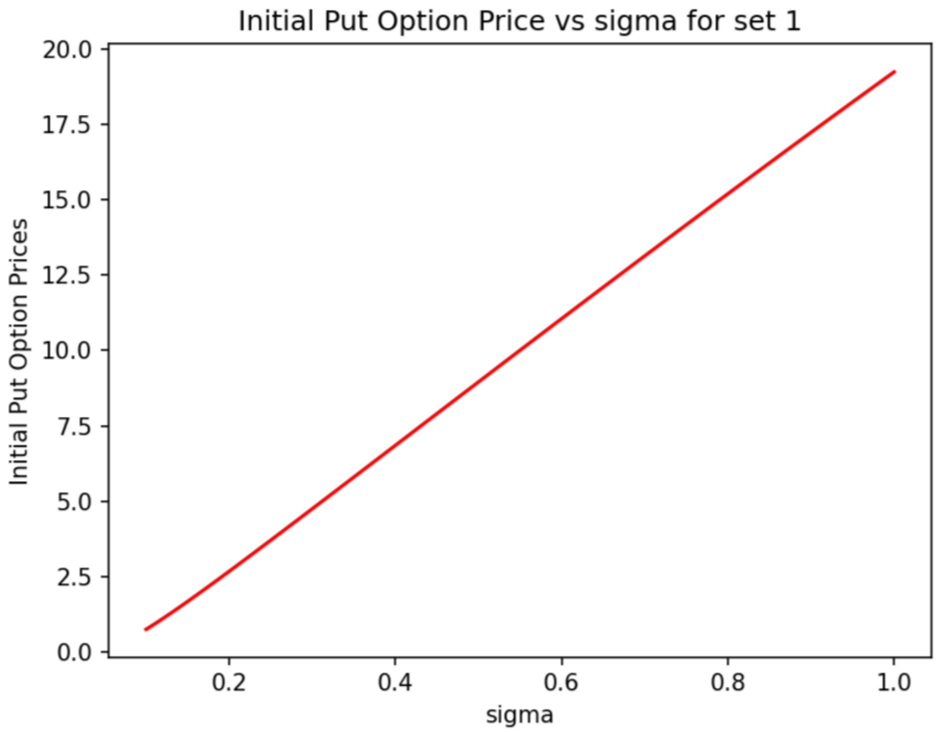

Initial Put Option Price vs r for set 1
Initial Put Option Prices
r
0
1
2
3
4
5
6
7
0.0
0.1
0.2
0.3
0.4
0.5
0.6

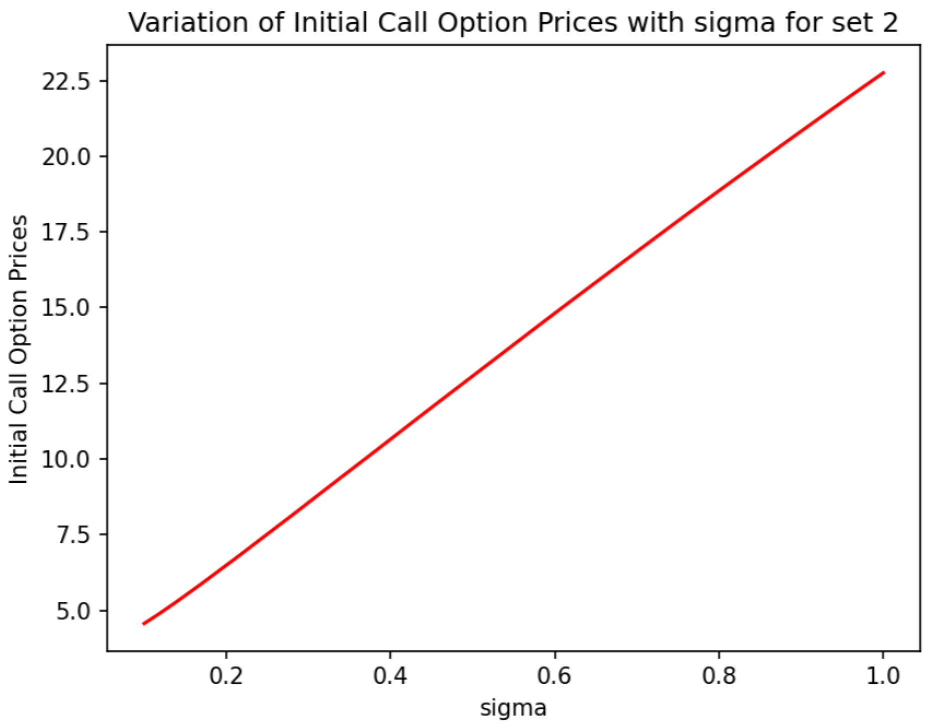

Variation of Initial Call Option Prices with r for set 2
Initial Call Option Prices
r
8
10
12
14
16
18
20
0.0
0.1
0.2
0.3
0.4
0.5
0.6

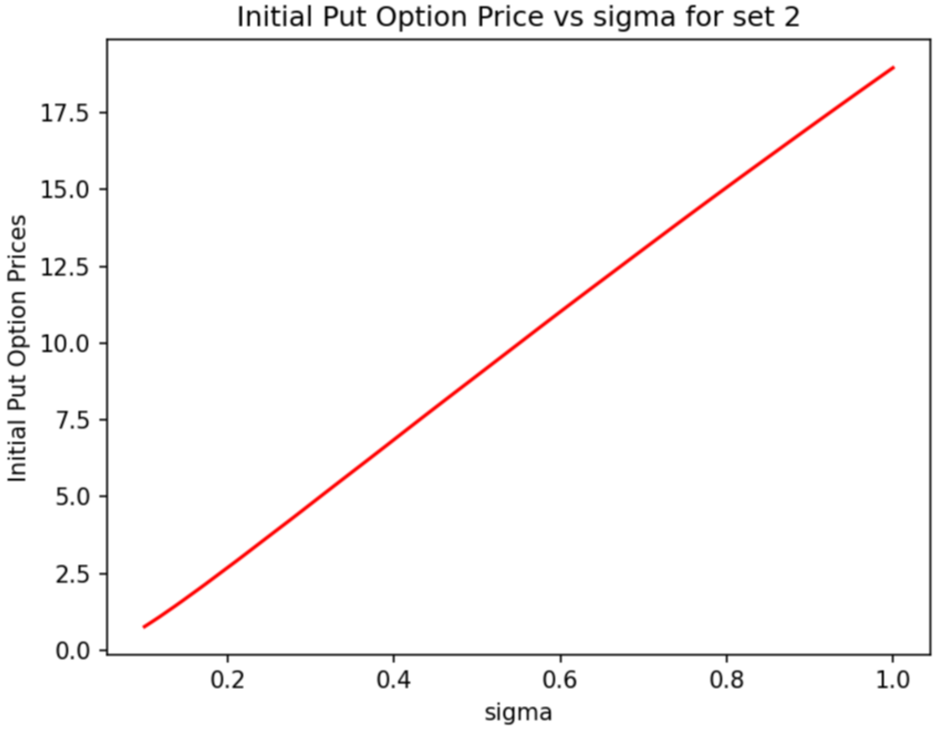


4) Variation with σ:

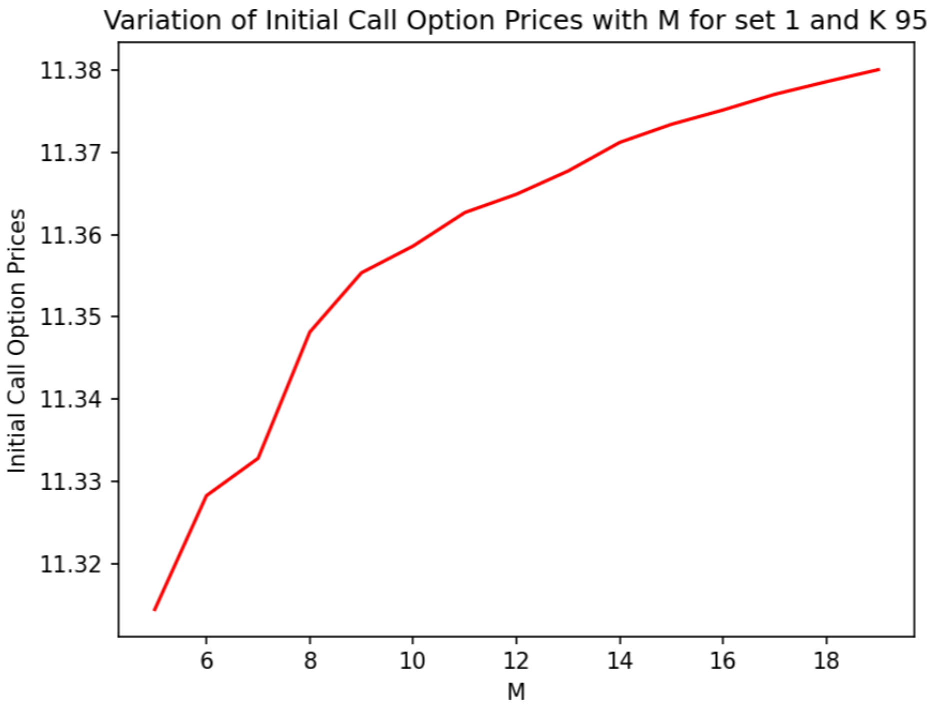


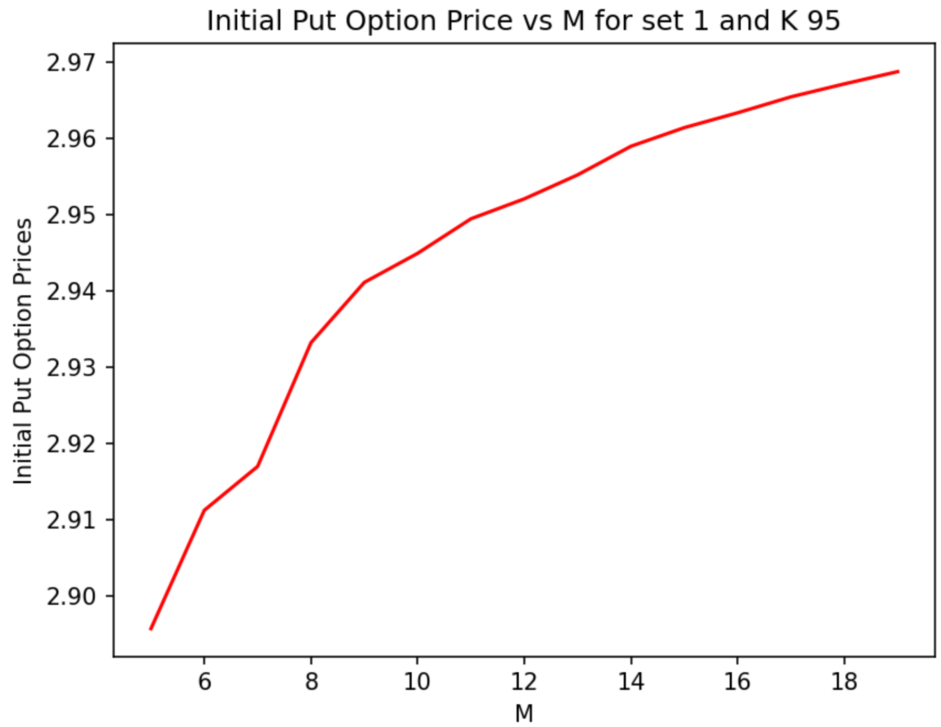

Initial Put Option Price vs sigma for set 1
Initial Put Option Prices
sigma
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0

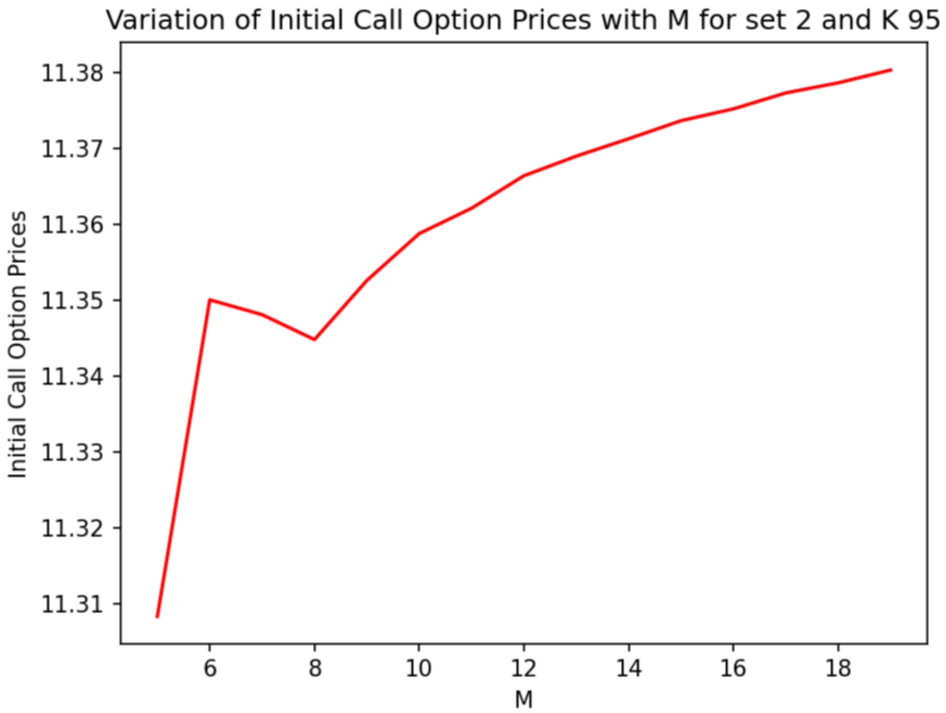

Variation of Initial Call Option Prices with sigma for set 2
Initial Call Option Prices
sigma
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
0.2
0.4
0.6
0.8
1.0

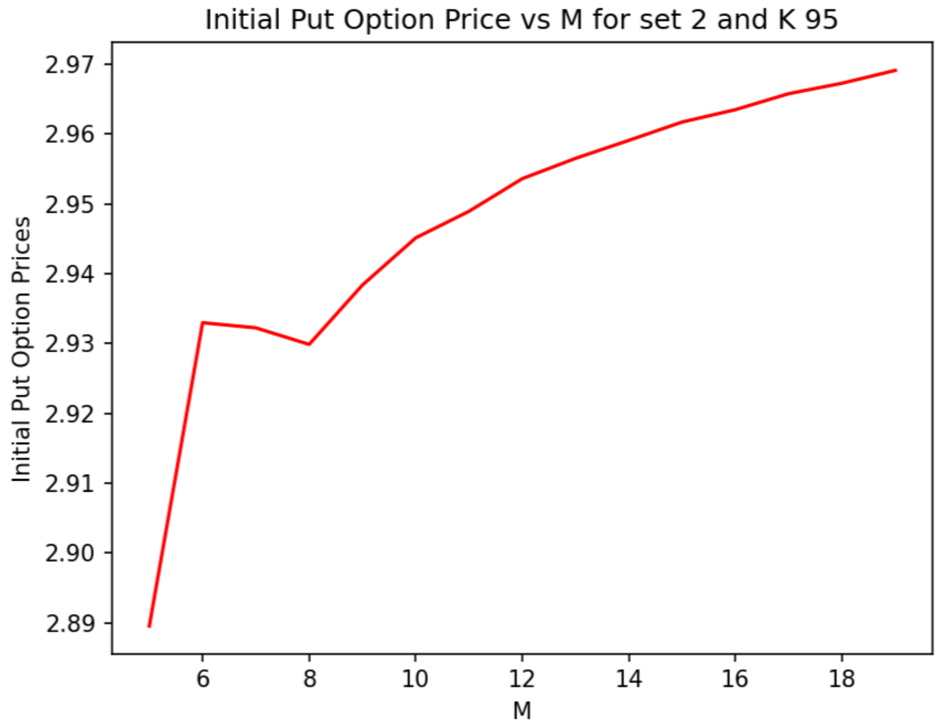


5) Variation with M:
a) For K = 95,



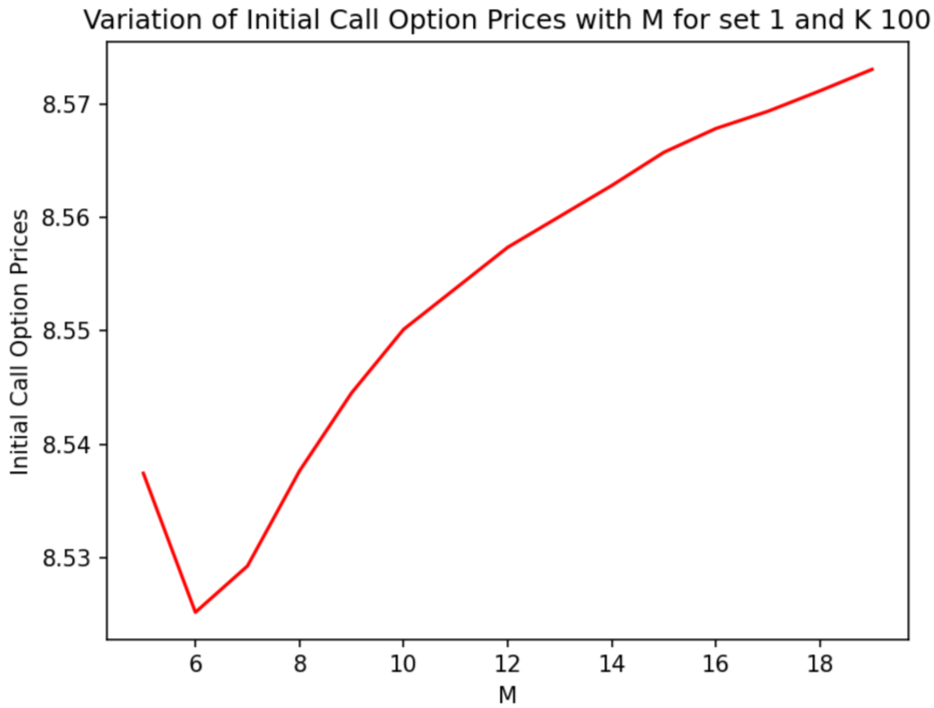

Initial Put Option Price vs M for set 1 and K 95
Initial Put Option Prices
2.97
2.96
2.95
2.94
2.93
2.92
2.91
2.90
6
8
10
12
14
16
18
M


Variation of Initial Call Option Prices with M for set 2 and K 95
Initial Call Option Prices
11.38
11.37
11.36
11.35
11.34
11.33
11.32
11.31
6
8
10
12
14
16
18
M



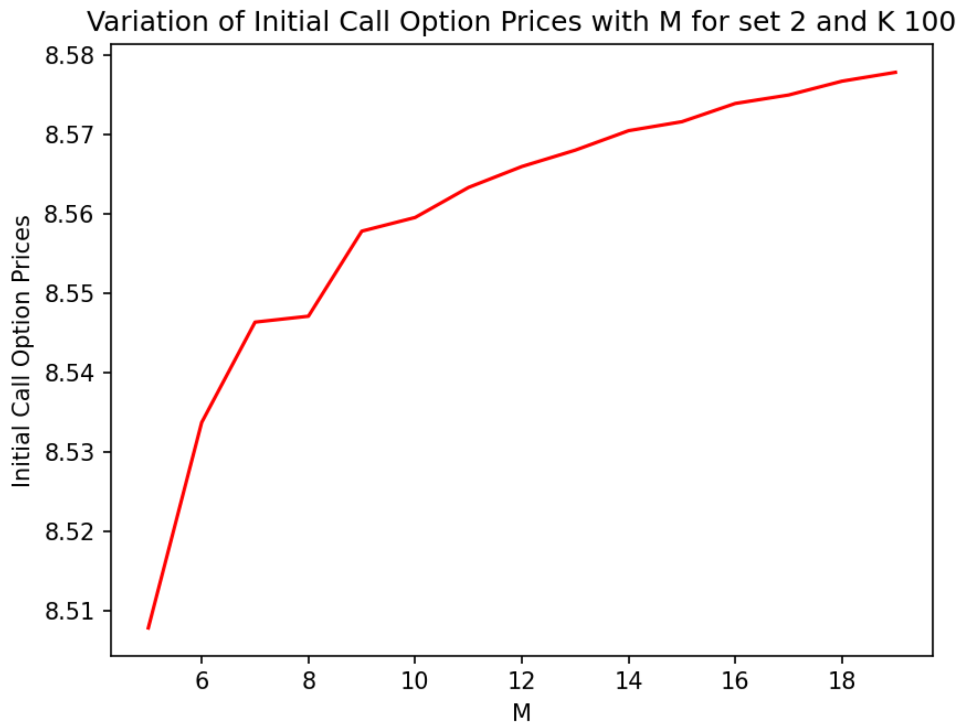
Initial Put Option Price vs M for set 2 and K 95
Initial Put Option Prices
2.97
2.96
2.95
2.94
2.93
2.92
2.91
2.90
2.89
6
8
10
12
14
16
18
M

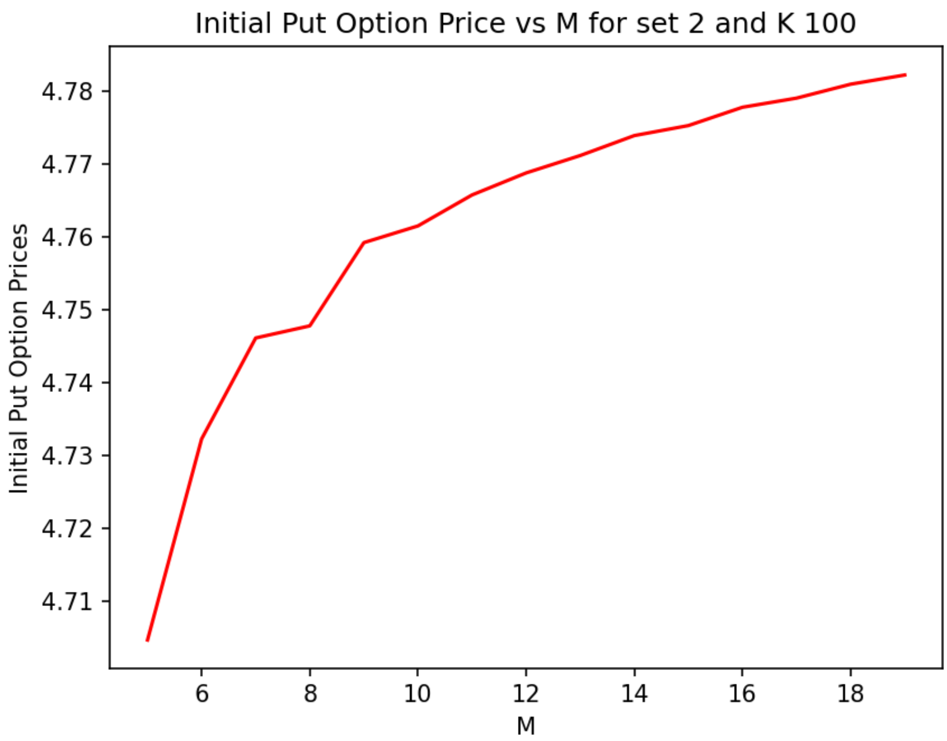
b) For K = 100,



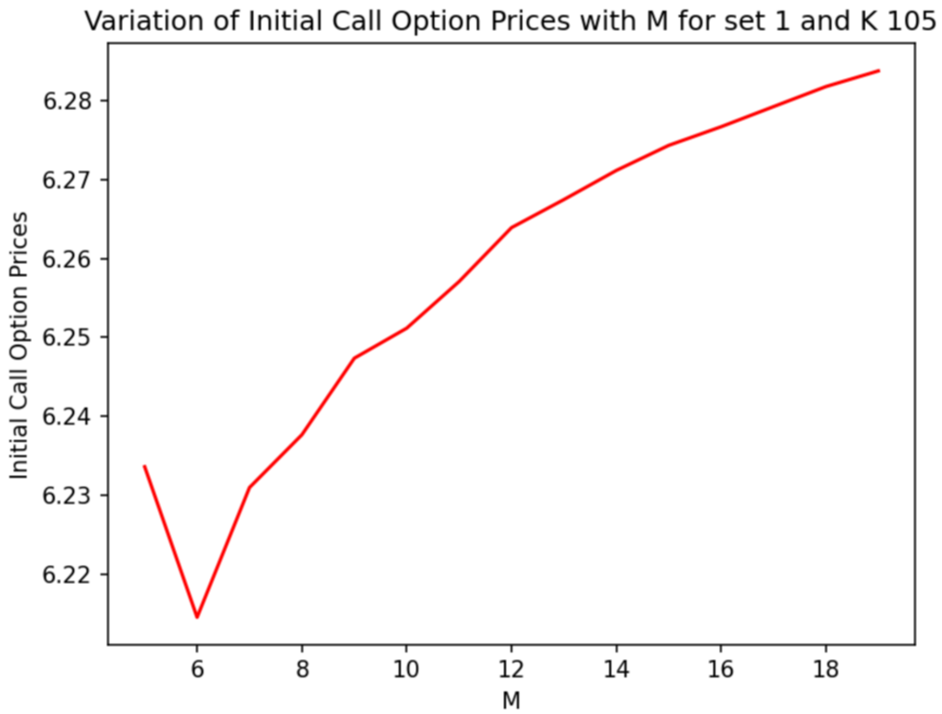
Variation of Initial Call Option Prices with M for set 1 and K 100
Initial Call Option Prices
8.57
8.56
8.55
8.54
8.53
6
8
10
12
14
16
18
M

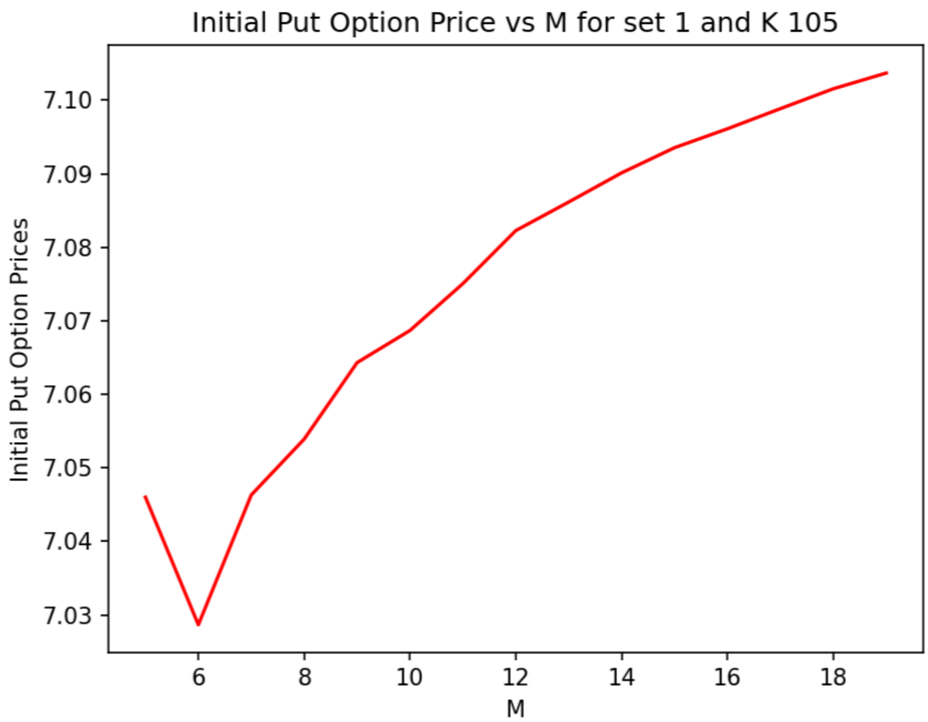

Initial Put Option Price vs M for set 1 and K 100
Initial Put Option Prices
4.78
4.77
4.76
4.75
4.74
4.73
6
8
10
12
14
16
18
M

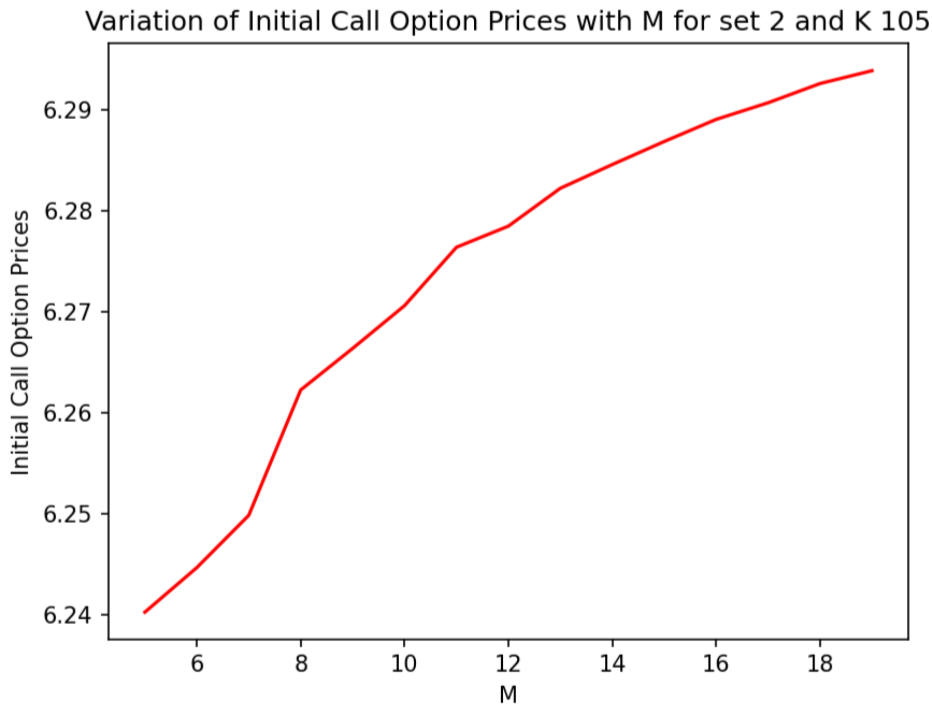

Variation of Initial Call Option Prices with M for set 2 and K 100
Initial Call Option Prices
8.58
8.57
8.56
8.55
8.54
8.53
8.52
8.51
6
8
10
12
14
16
18
M

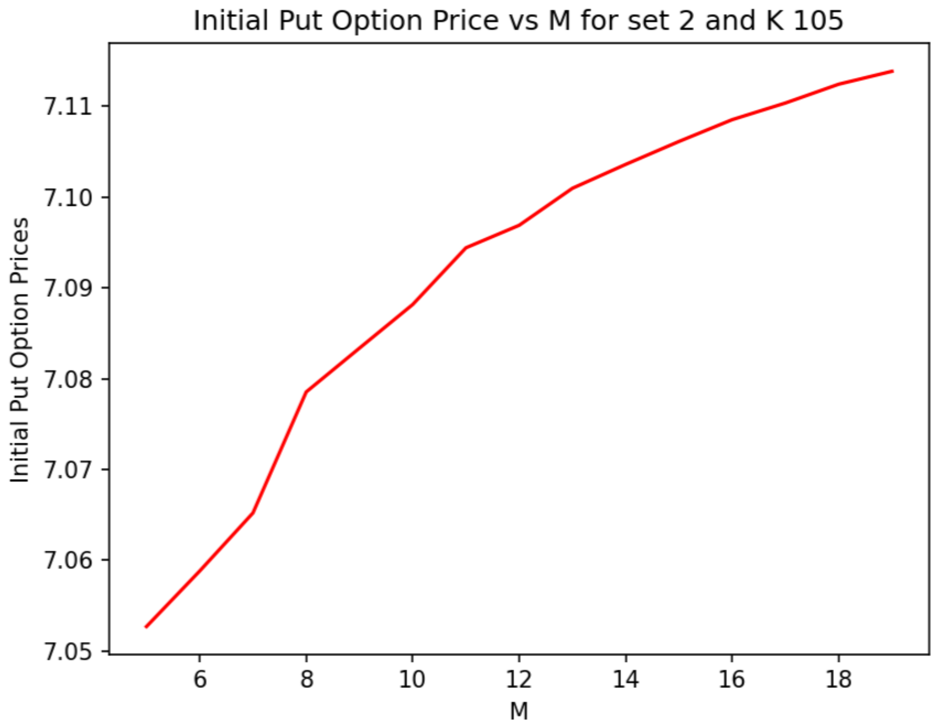

Initial Put Option Price vs M for set 2 and K 100
Initial Put Option Prices
4.78
4.77
4.76
4.75
4.74
4.73
4.72
4.71
6
8
10
12
14
16
18
M

c) For K = 105,

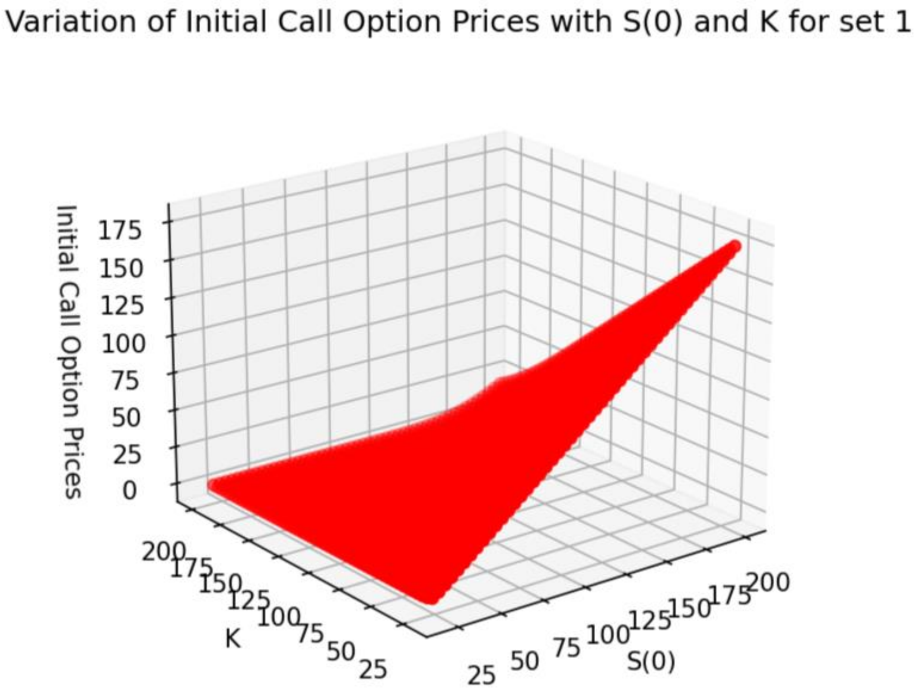

Variation of Initial Call Option Prices with M for set 1 and K 105
Initial Call Option Prices
6.28
6.27
6.26
6.25
6.24
6.23
6.22
6
8
10
12
14
16
18
M


Initial Put Option Price vs M for set 1 and K 105
Initial Put Option Prices
7.10
7.09
7.08
7.07
7.06
7.05
7.04
7.03
6
8
10
12
14
16
18
M

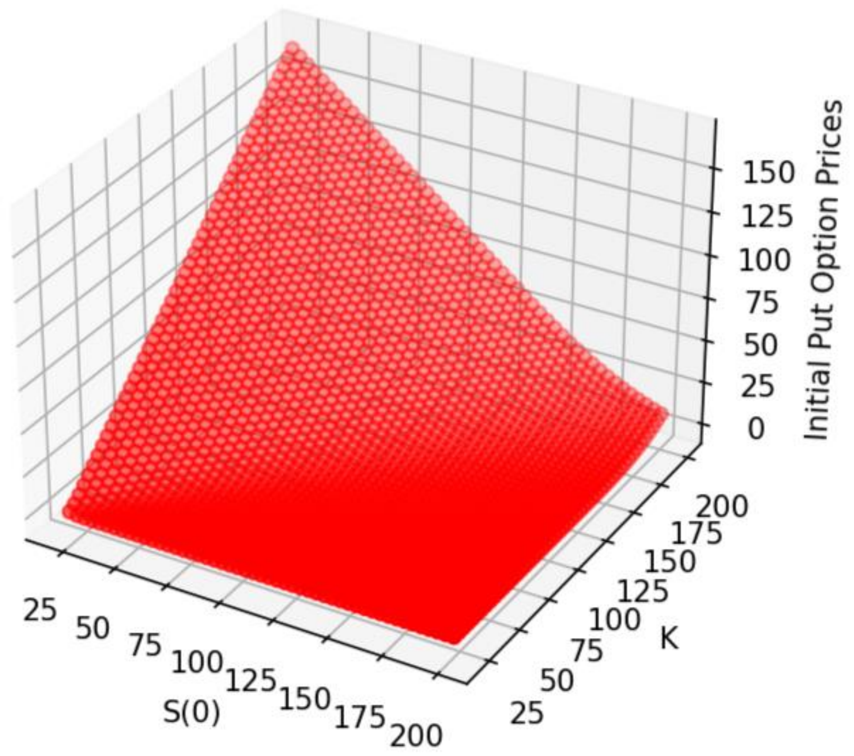

Variation of Initial Call Option Prices with M for set 2 and K 105
Initial Call Option Prices
6.29
6.28
6.27
6.26
6.25
6.24
6
8
10
12
14
16
18
M



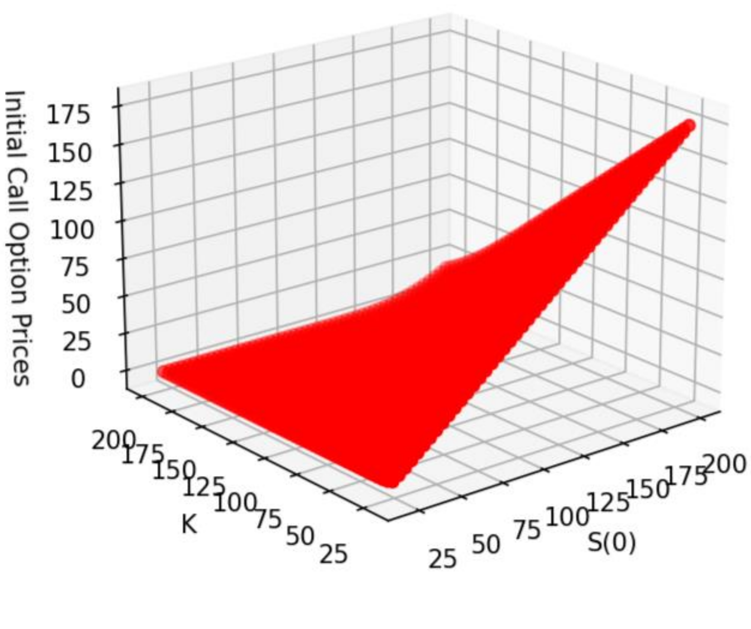
(i) Variation with S(0) and K:

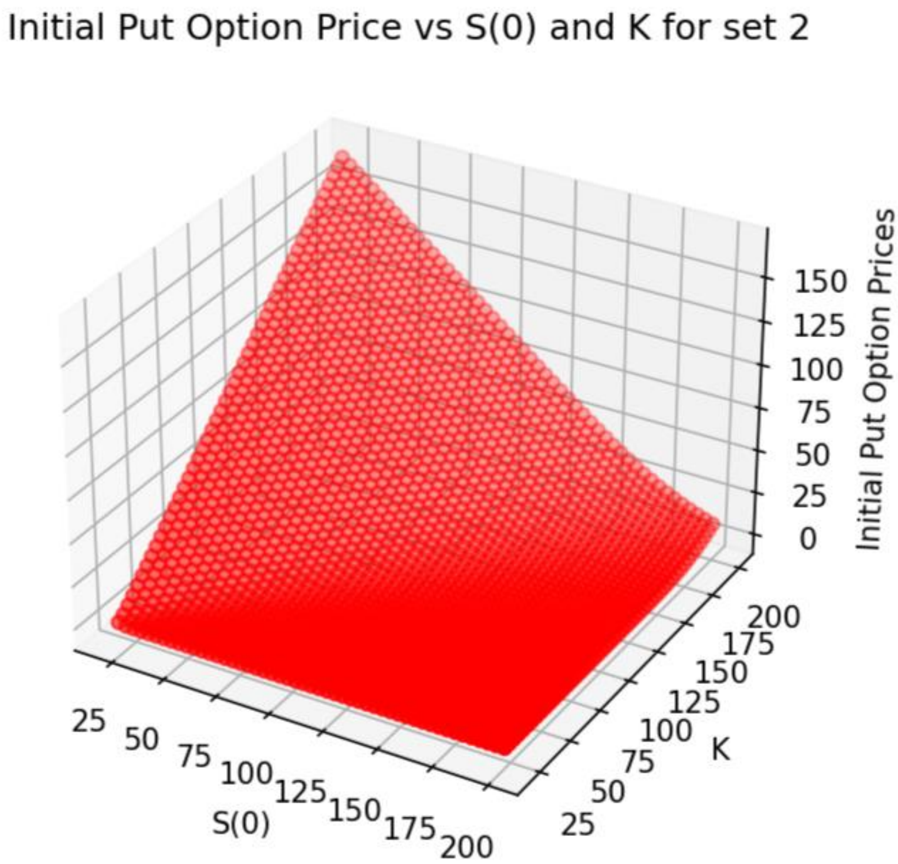


Initial Put Option Price vs S(0) and K for set 1

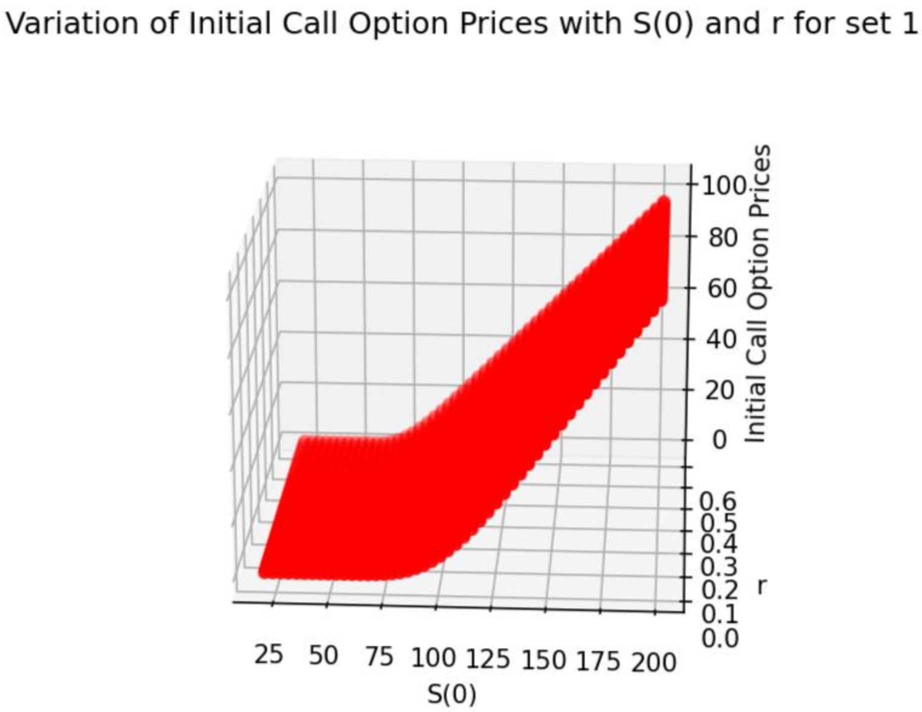


Variation of Initial Call Option Prices with S(0) and K for set 2

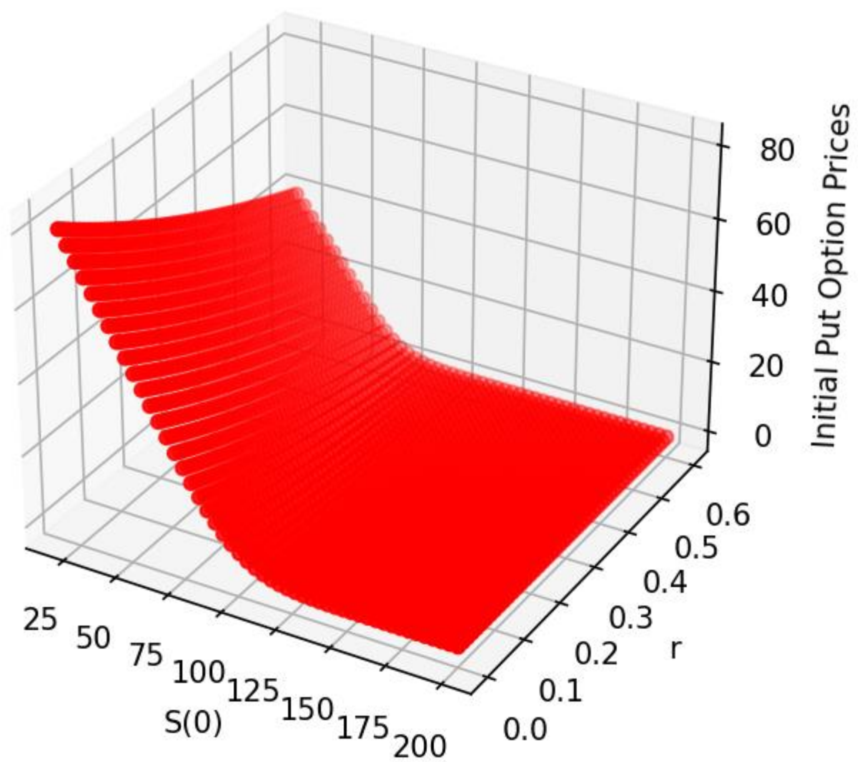




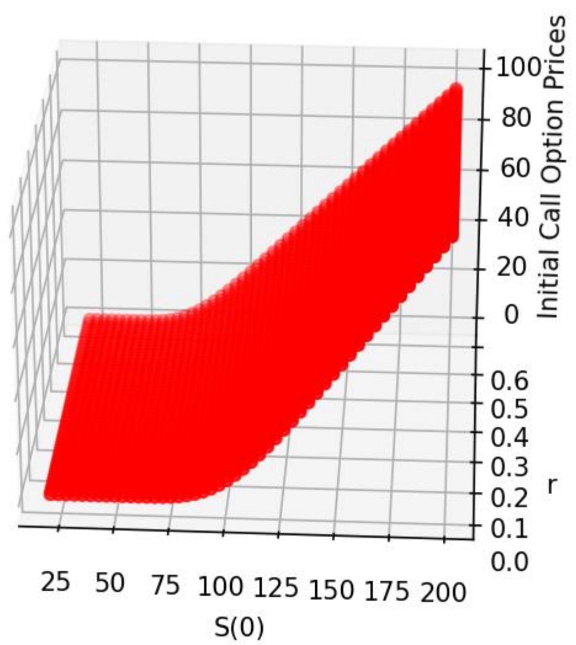
(ii) Variation with S(0) and r:



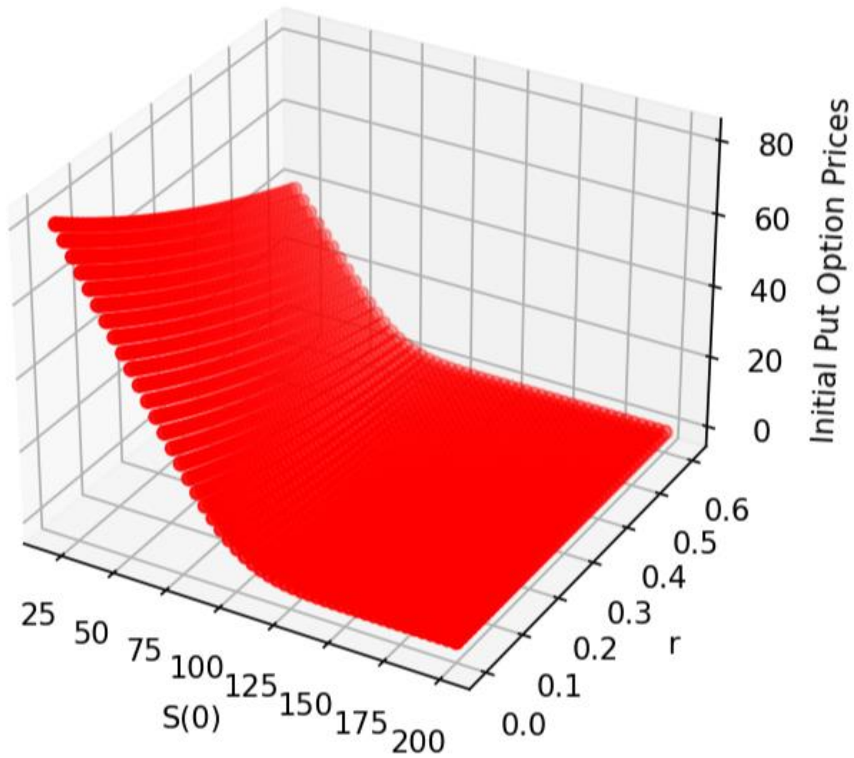
Initial Put Option Price vs S(0) and r for set 1



Variation of Initial Call Option Prices with S(0) and r for set 2

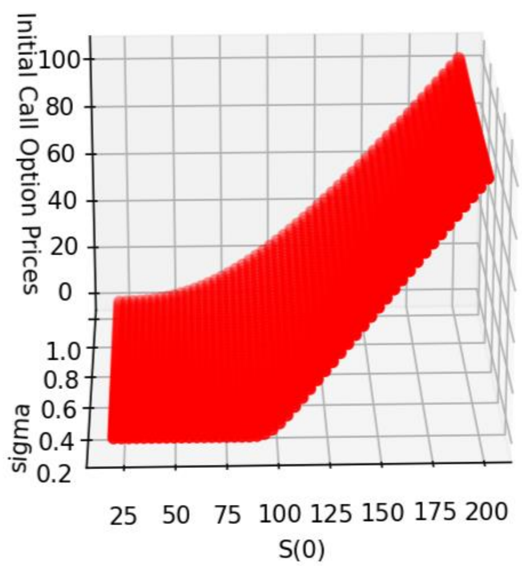


Initial Put Option Price vs S(0) and r for set 2

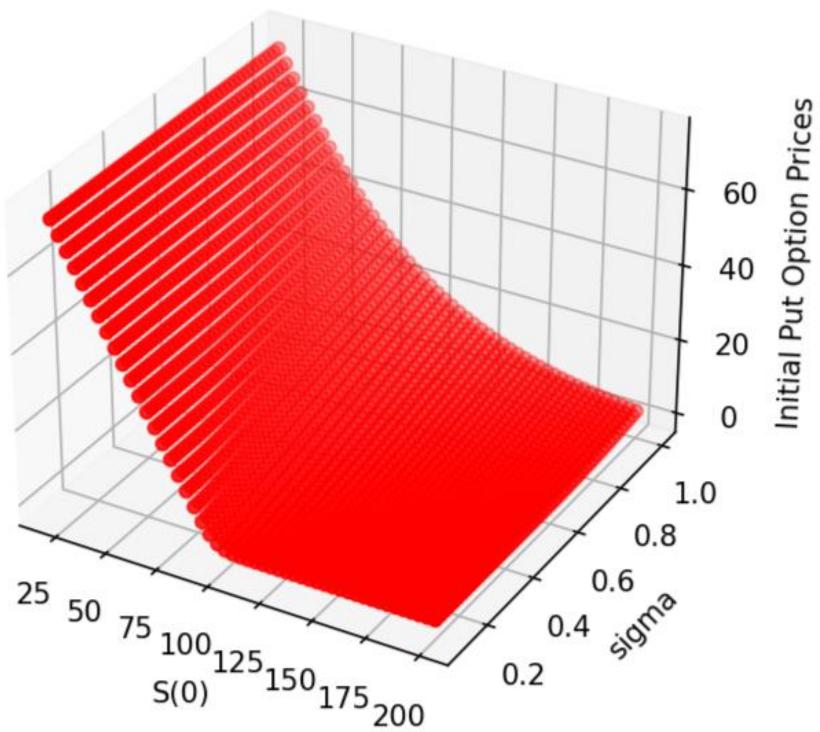


(iii) Variation with S(0) and σ:

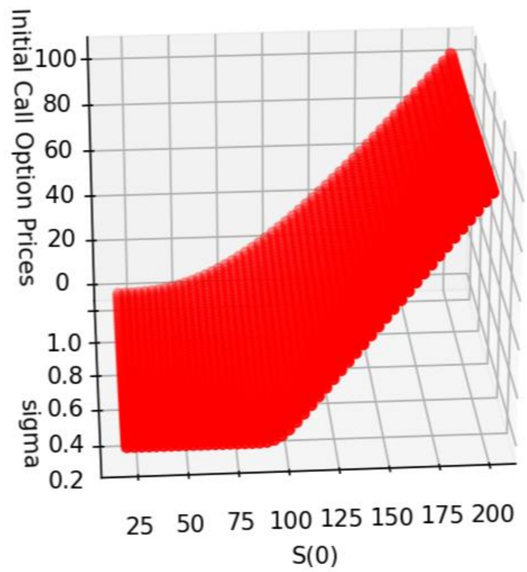
Variation of Initial Call Option Prices with S(0) and sigma for set 1



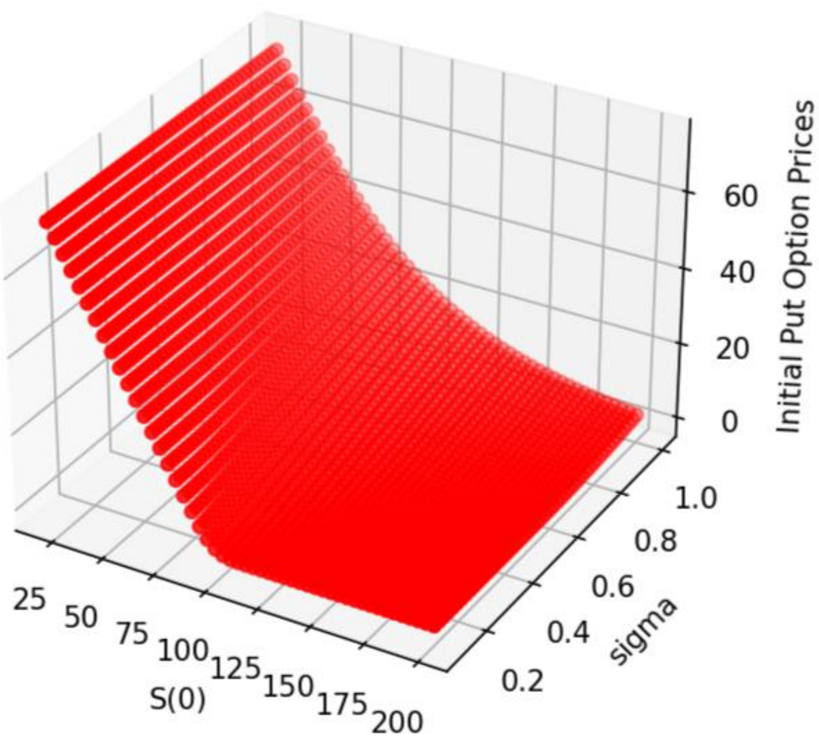
Initial Put Option Price vs S(0) and sigma for set 1



Variation of Initial Call Option Prices with S(0) and sigma for set 2

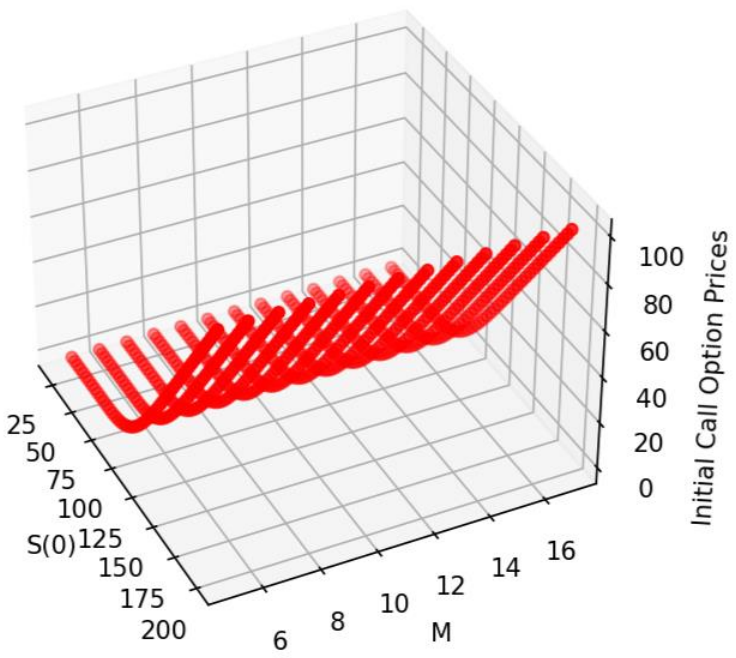


Initial Put Option Price vs S(0) and sigma for set 2

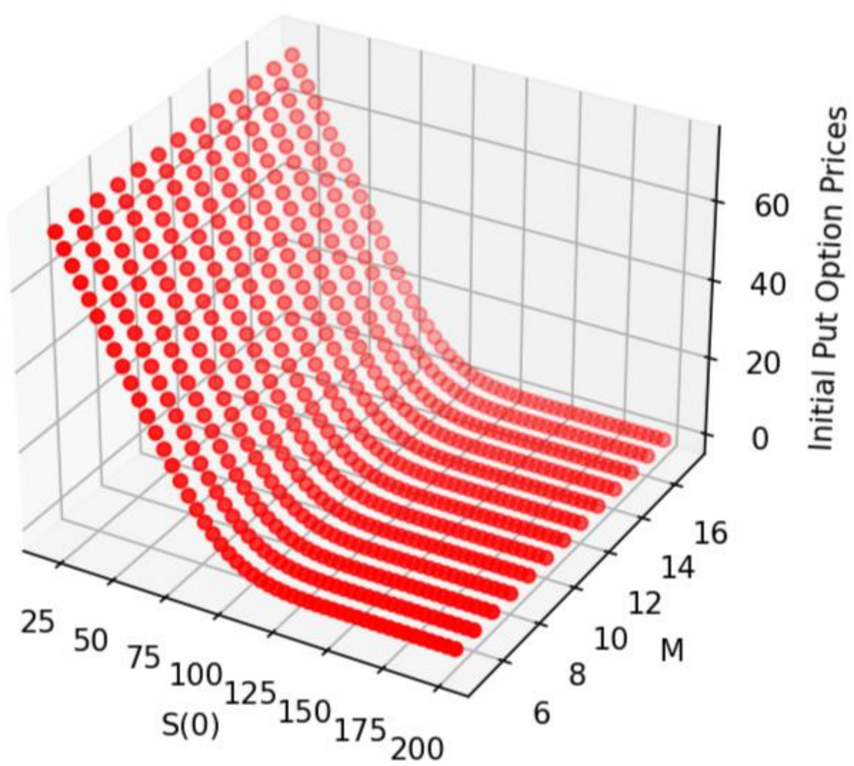


(iv) Variation with S(0) and M:

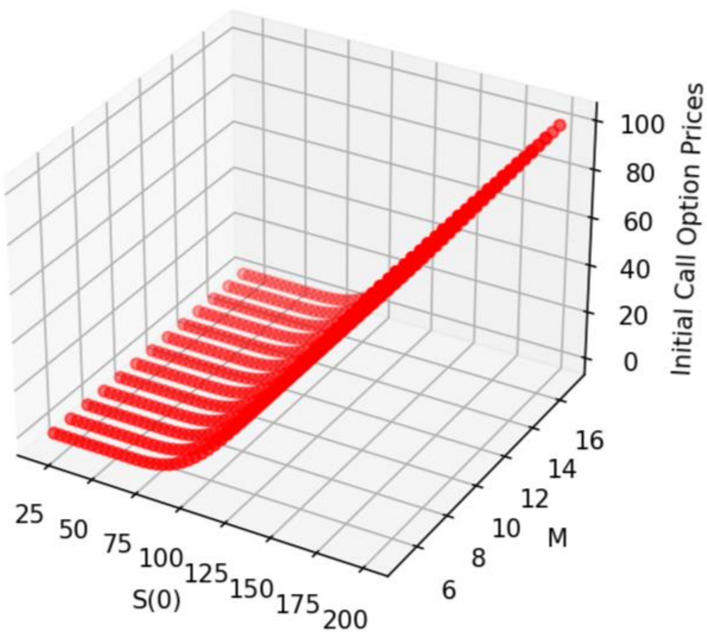
Variation of Initial Call Option Prices with S(0) and M for set 1

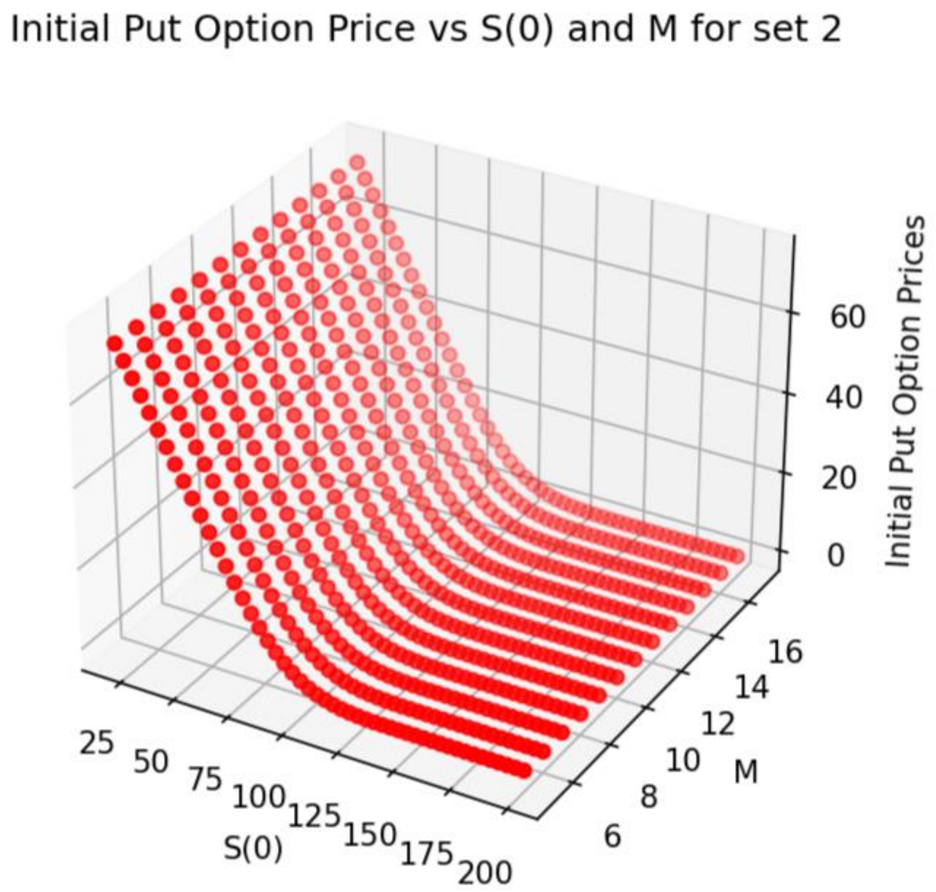


Initial Put Option Price vs S(0) and M for set 1



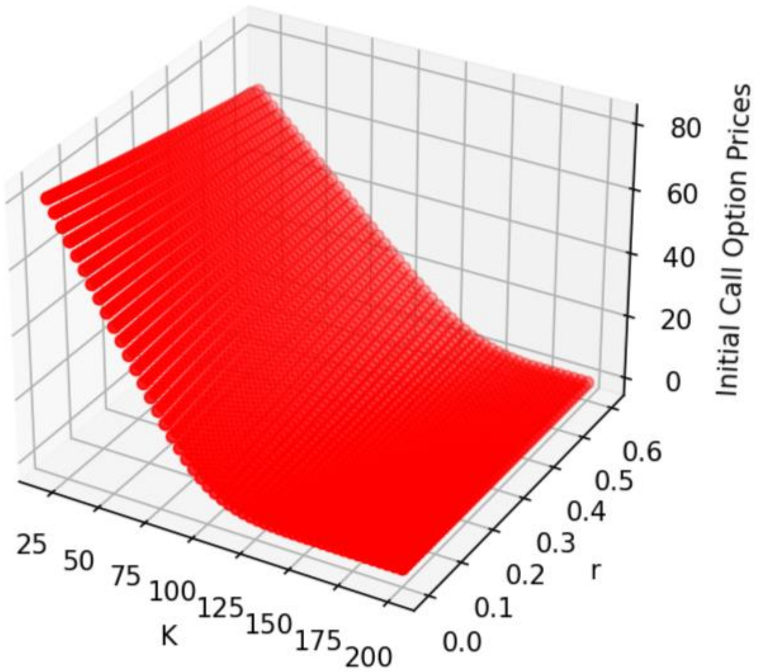
Variation of Initial Call Option Prices with S(0) and M for set 2



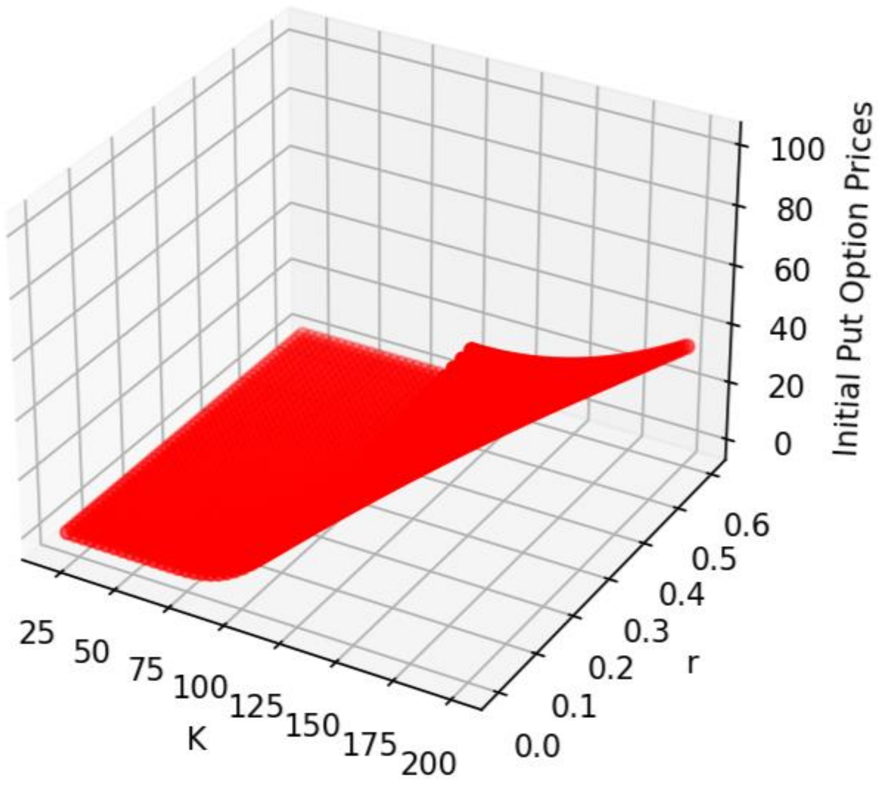


(v) Variation with K and r:

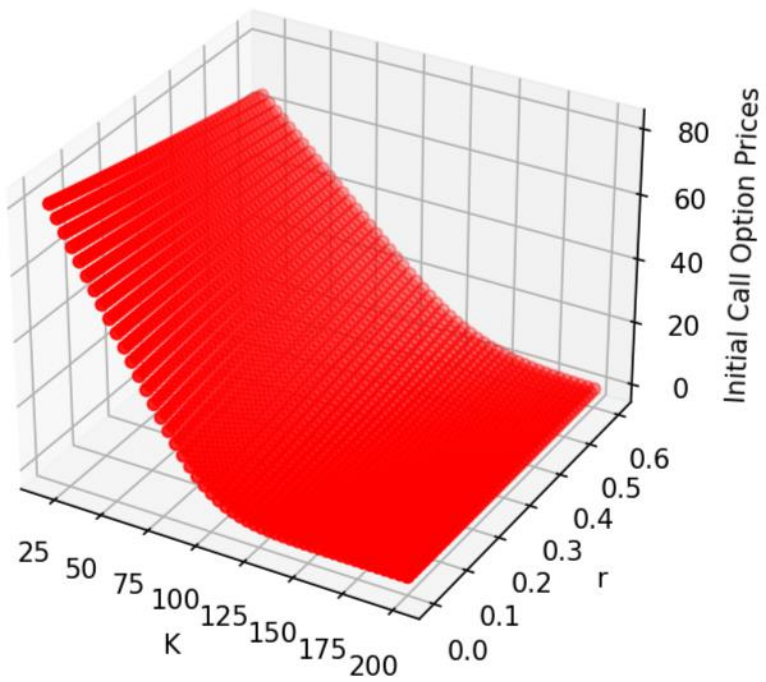
Variation of Initial Call Option Prices with K and r for set 1



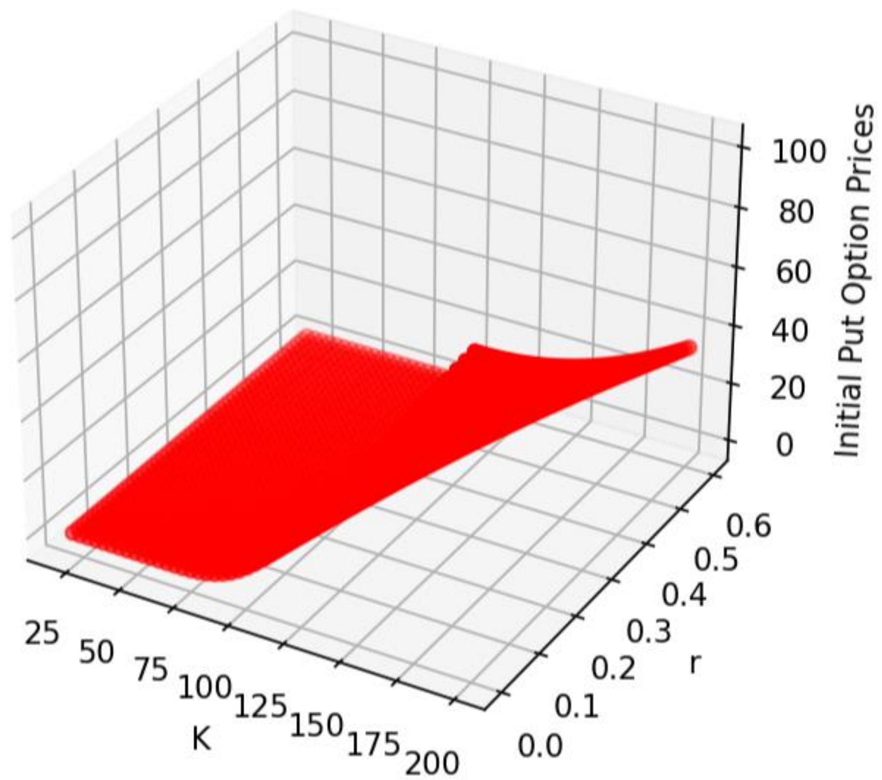
Initial Put Option Price vs K and r for set 1



Variation of Initial Call Option Prices with K and r for set 2

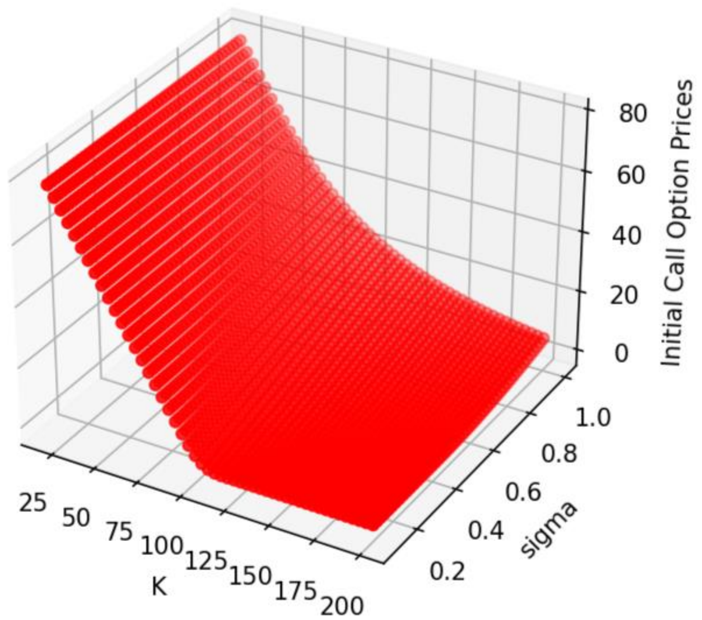


Initial Put Option Price vs K and r for set 2

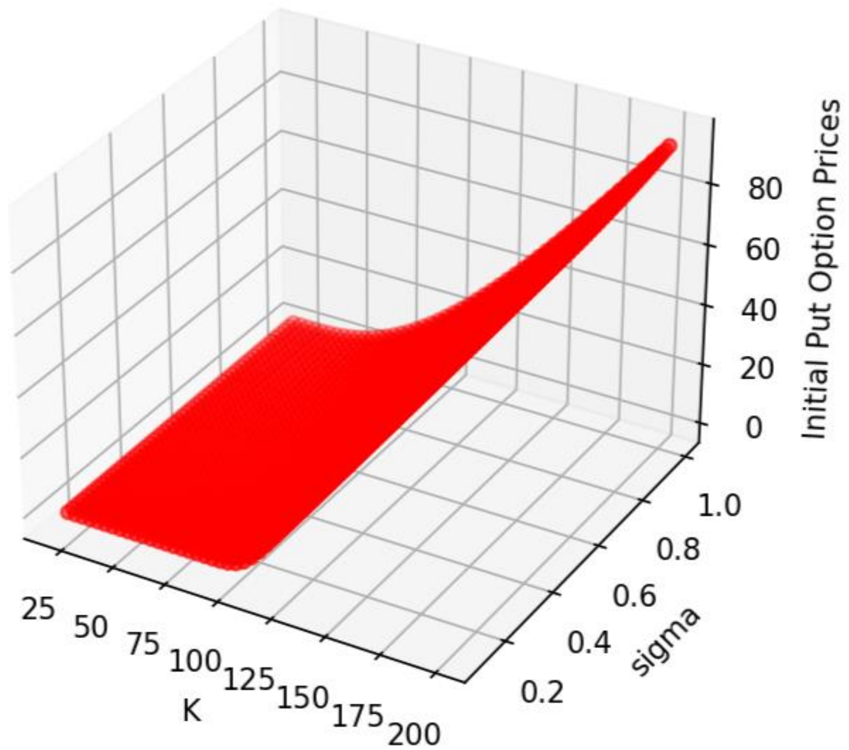


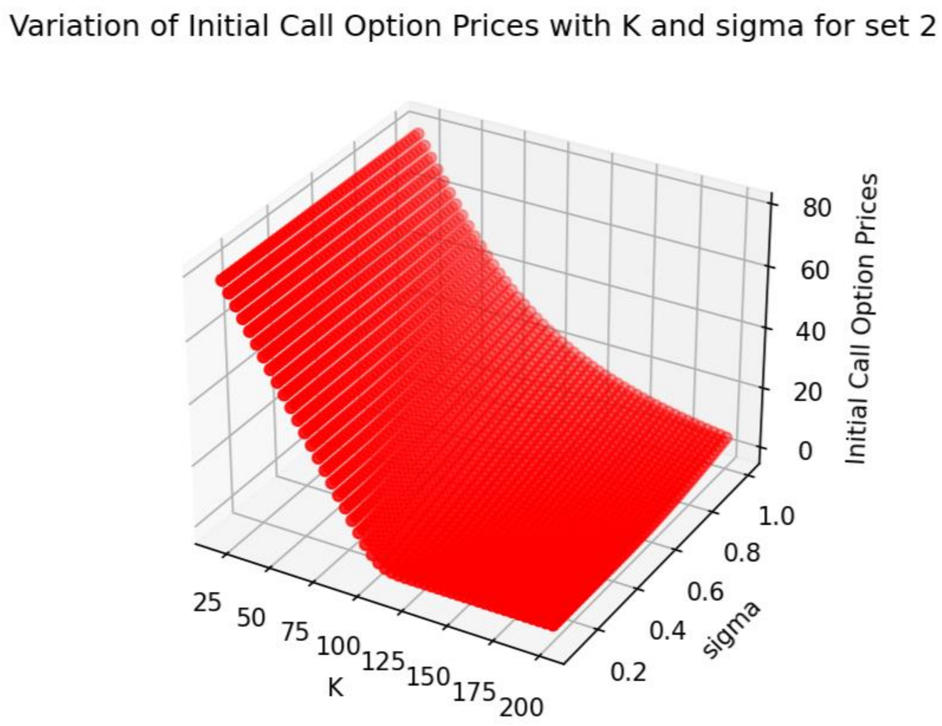
(vi) Variation with K and σ:

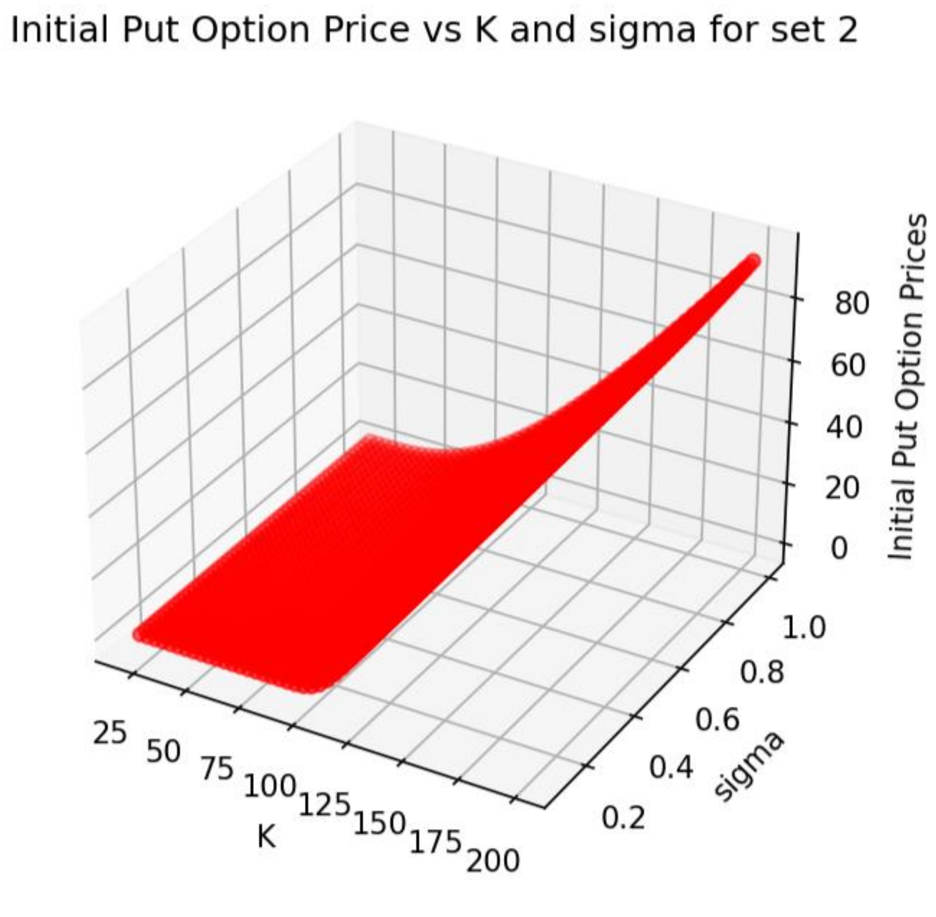
Variation of Initial Call Option Prices with K and sigma for set 1



Initial Put Option Price vs K and sigma for set 1

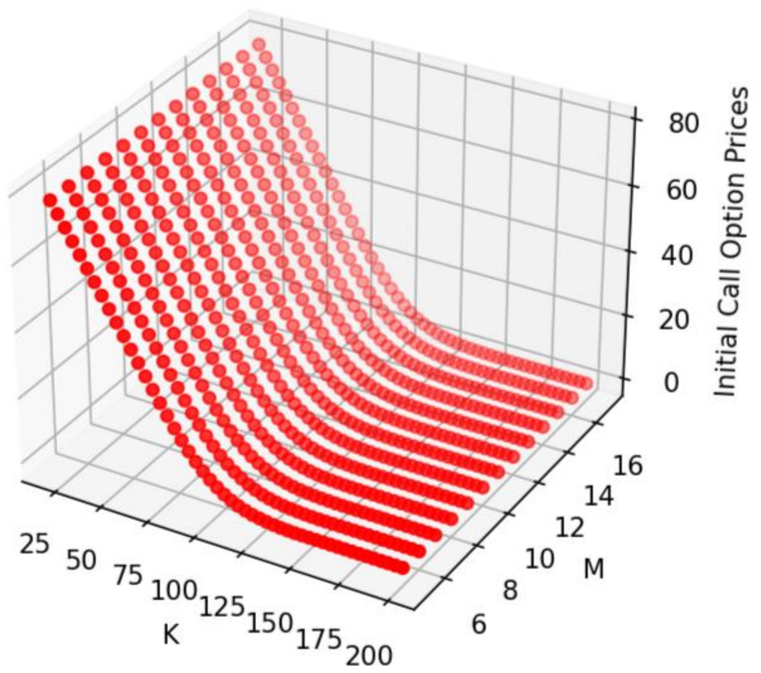




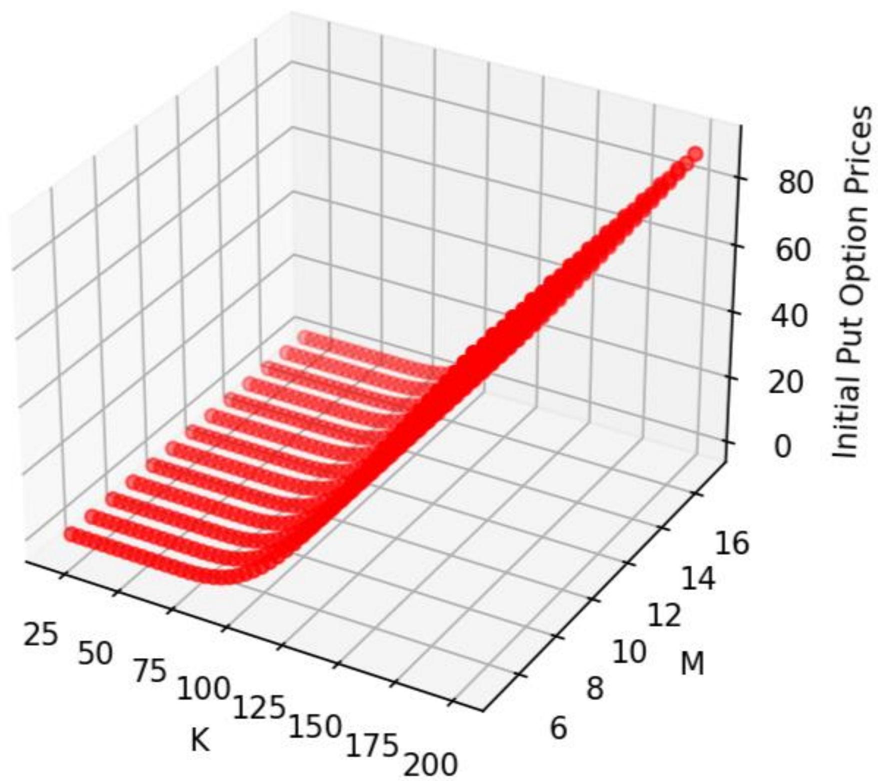


(vii) Variation with K and M:

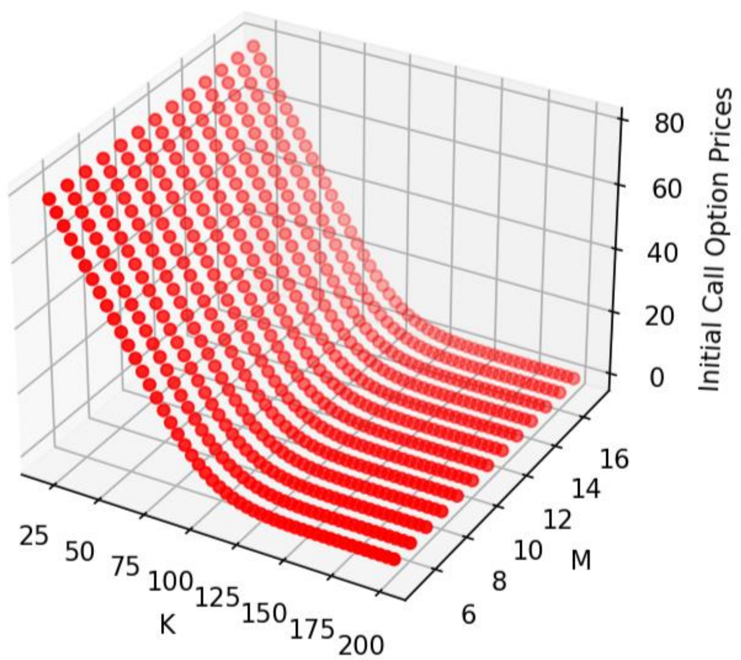
Variation of Initial Call Option Prices with K and M for set 1



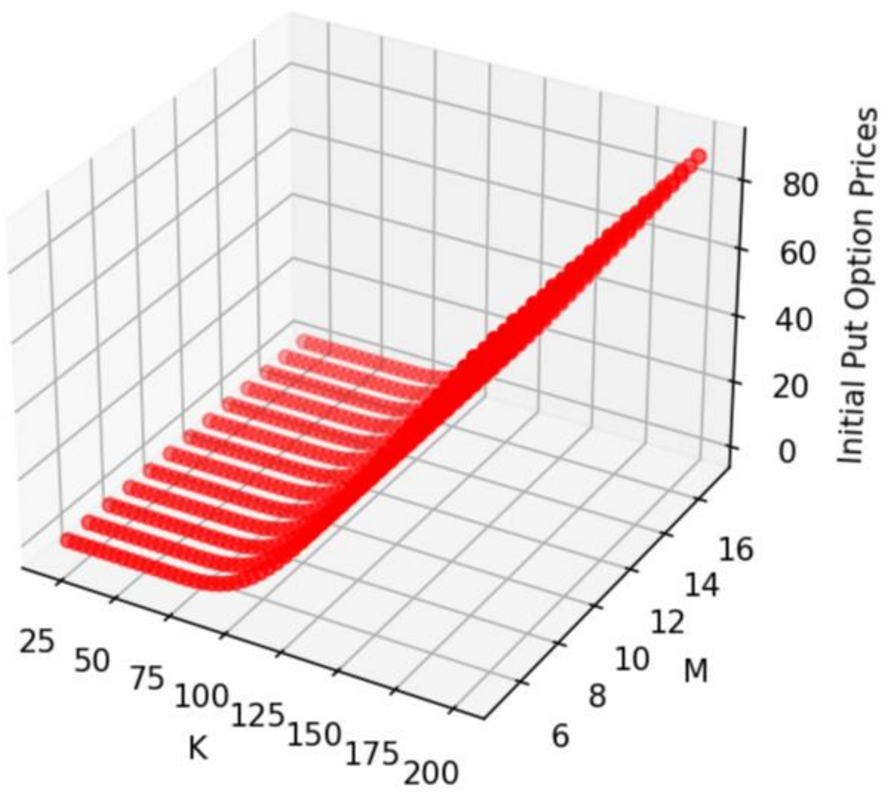
Initial Put Option Price vs K and M for set 1



Variation of Initial Call Option Prices with K and M for set 2

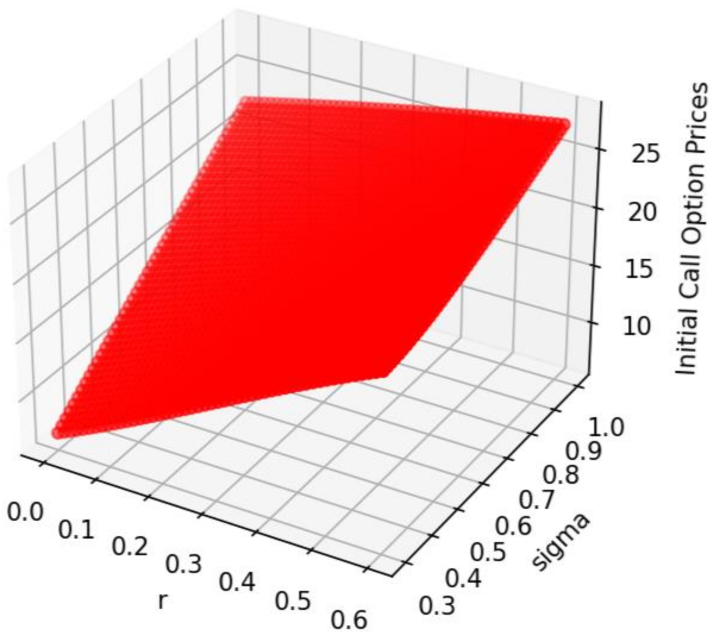


Initial Put Option Price vs K and M for set 2

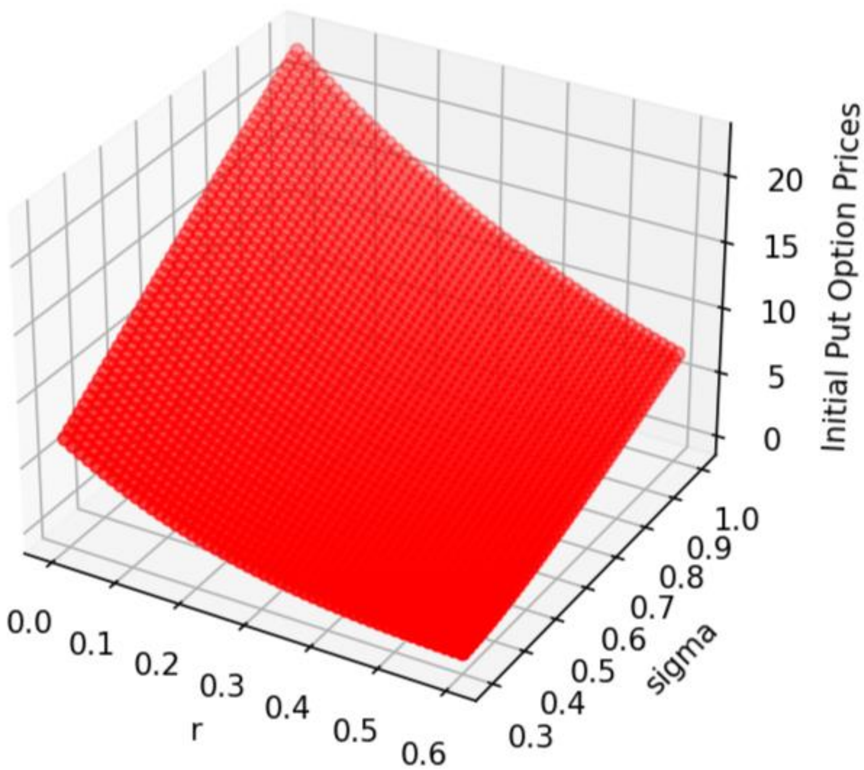


(viii) Variation with r and σ:

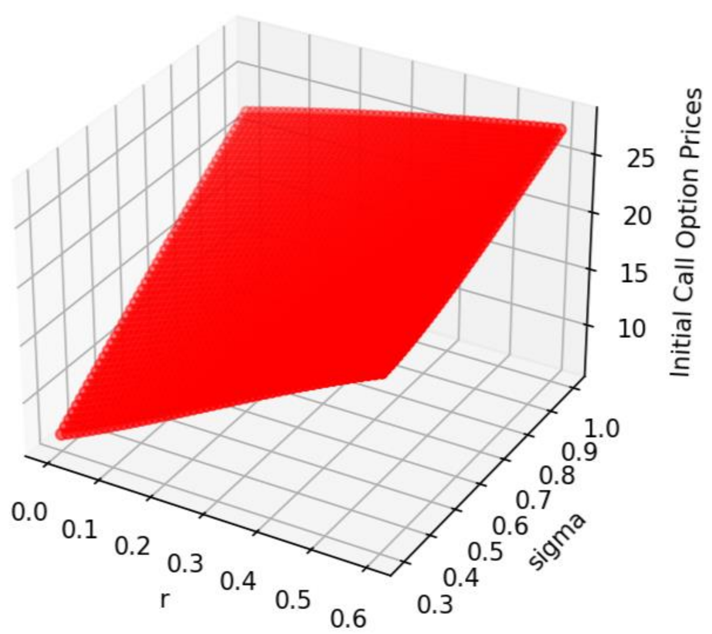
Variation of Initial Call Option Prices with r and sigma for set 1



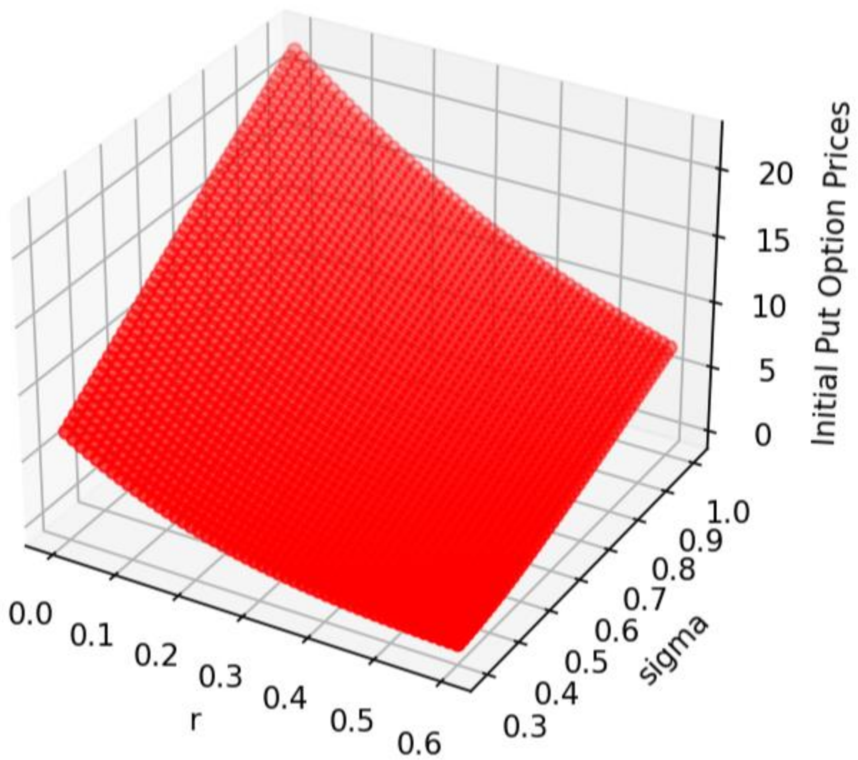
Initial Put Option Price vs r and sigma for set 1



Variation of Initial Call Option Prices with r and sigma for set 2

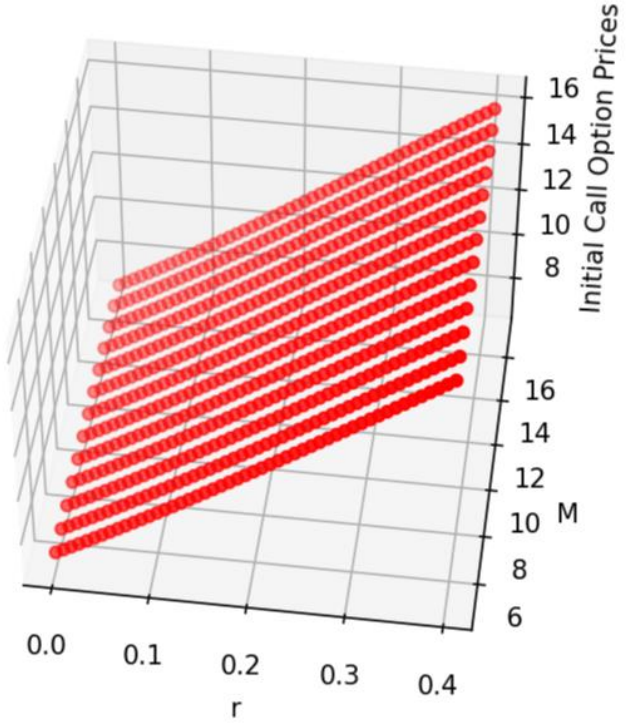


Initial Put Option Price vs r and sigma for set 2

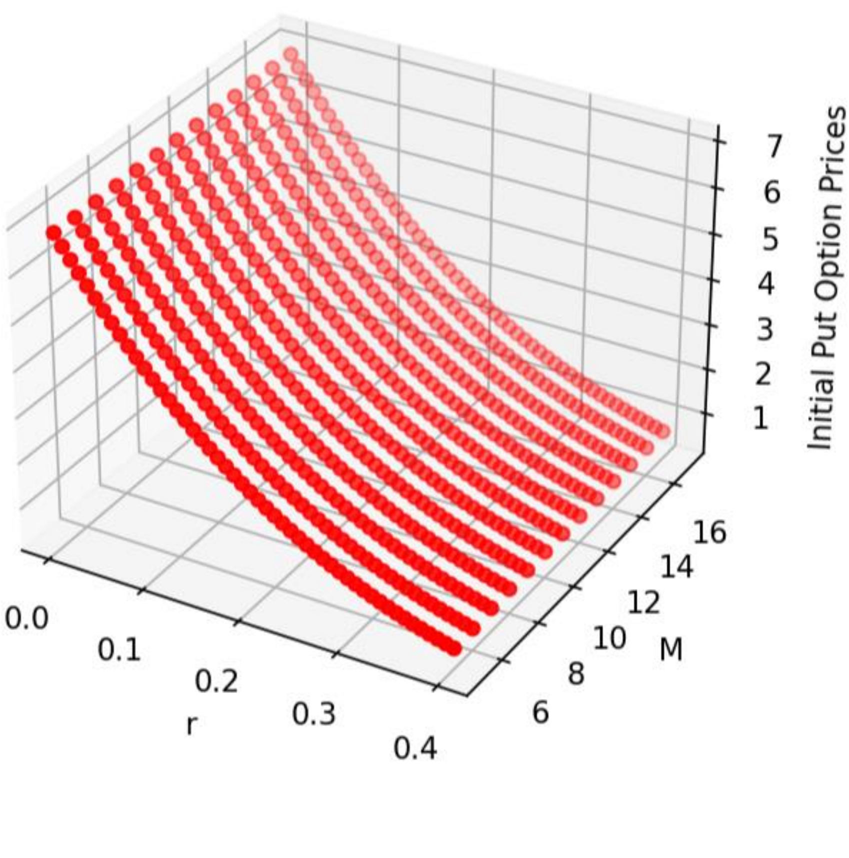


(ix) Variation with r and M:

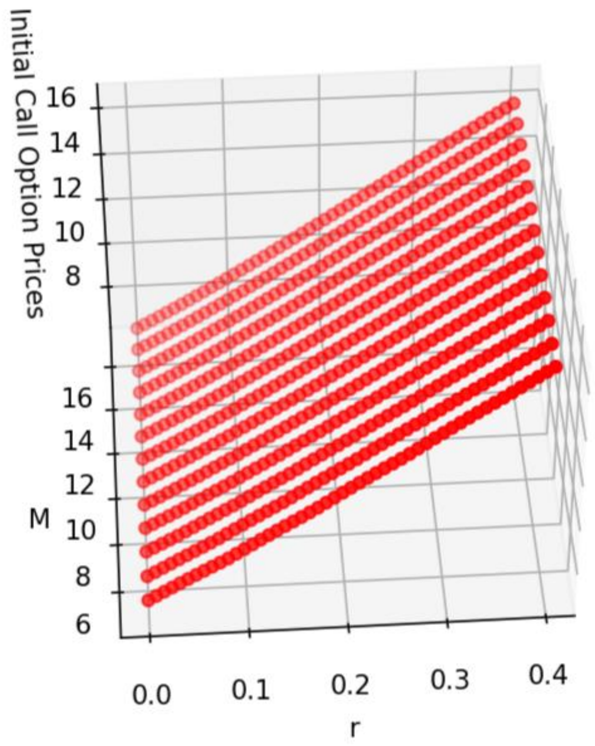
Variation of Initial Call Option Prices with r and M for set 1



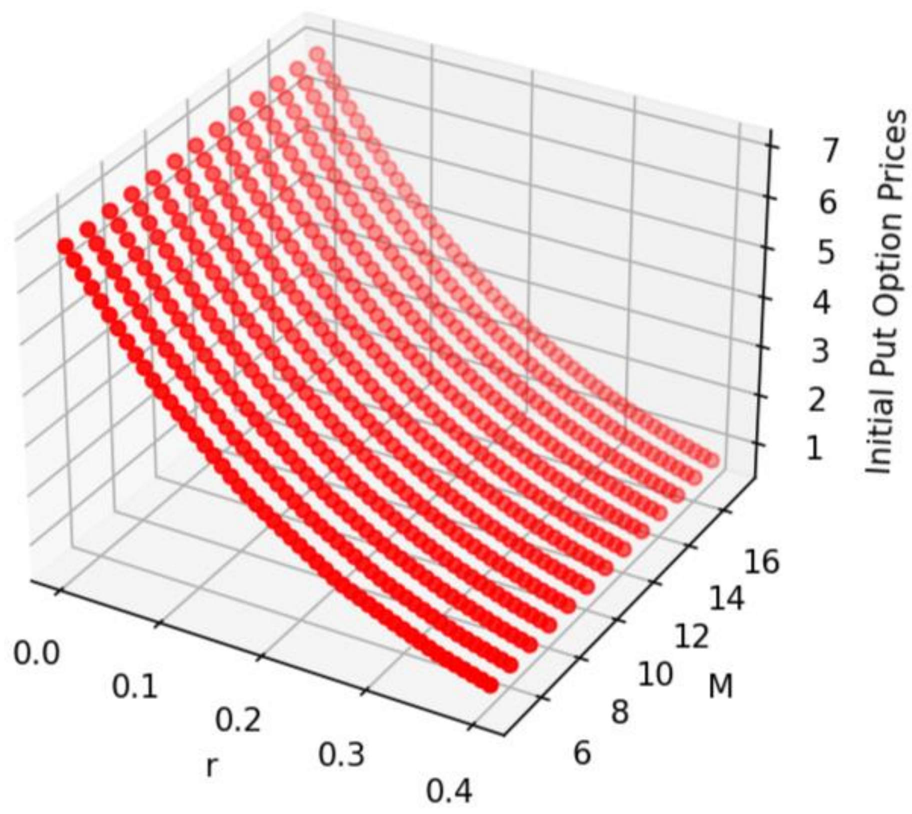
Initial Put Option Price vs r and M for set 1



Variation of Initial Call Option Prices with r and M for set 2

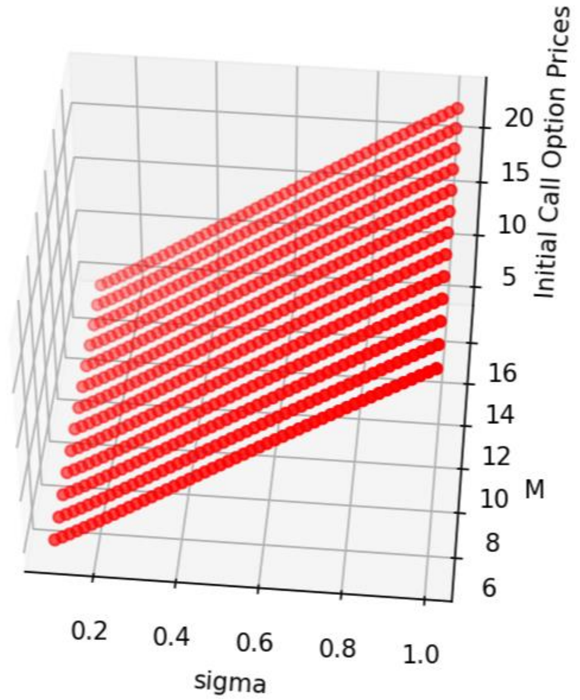


Initial Put Option Price vs r and M for set 2

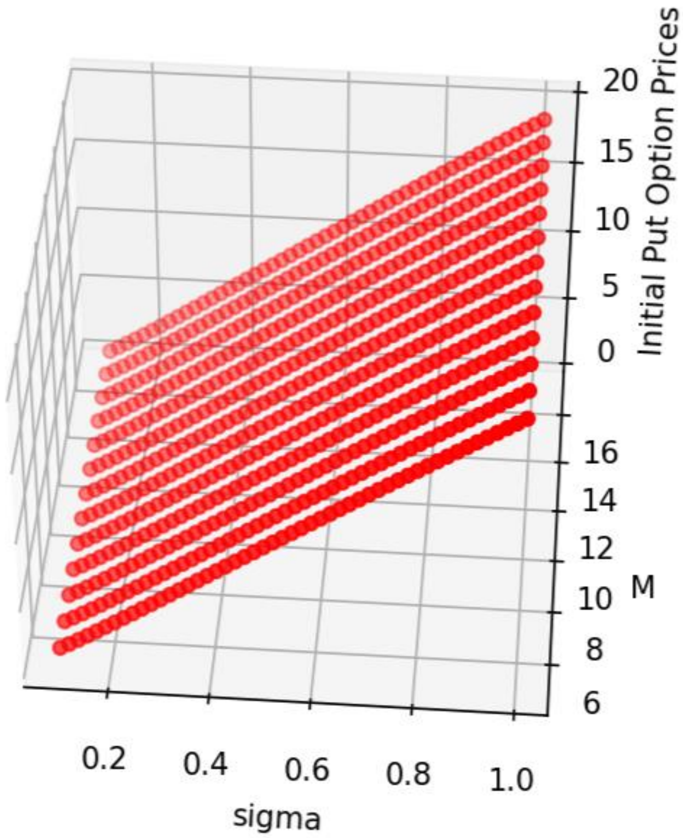


(x) Variation with σ and M:

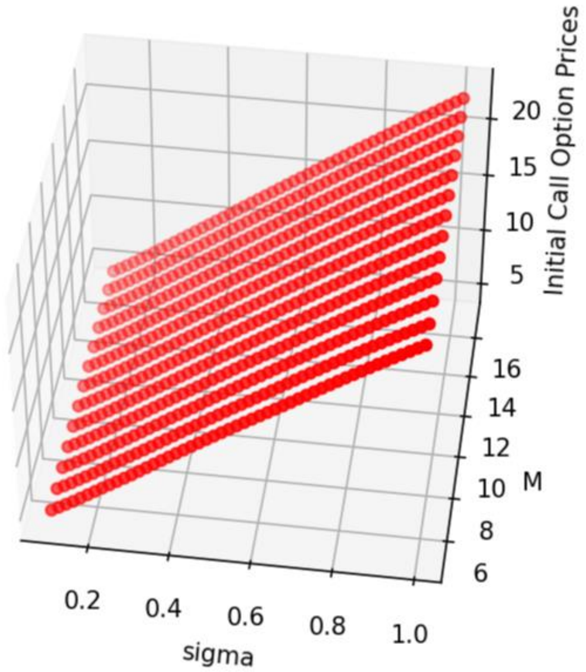
Variation of Initial Call Option Prices with sigma and M for set 1



Initial Put Option Price vs sigma and M for set 1



Variation of Initial Call Option Prices with sigma and M for set 2



Initial Put Option Price vs sigma and M for set 2