<u>Financial Engineering Lab</u> <u>MA – 374</u> <u>Lab – 7</u>

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Branch – Mathematics and Computing

<u>Question 1</u>:

1. Write a single program to compute the prices of European call and put options at time t for $0 \le t \le T$ in the classical BSM framework. Denote the call and put prices by $C(t, s)$ and $P(t, s)$ respectively, with s being the price of an underlying asset.

The prices of European call and put options at time t for 0 ≤t ≤T in the classical BSM framework is calculated by solving the Black-Scholes-Merton Partial Differential Equation.

$$C(t,s) = sN(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$P(t,s) = Ke^{-r(T-t)}N(-d_2) - sN(-d_1)$$

$$d_1 = \frac{ln\left(\frac{s}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{ln\left(\frac{s}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$N(x) = \textit{Cumulative Standard Normal Distribution of } x$$

$$C(t,s) = \textit{Price of European Call Option}$$

$$P(t,s) = \textit{Price of European Put Option}$$

The program to compute the prices of European call and put options at time t for 0 ≤t ≤T in the classical BSM framework is as follows:

```python
from scipy.stats import norm
import math
def ClassicalBlackScholesMertonModel(s, t, T, K, r, sigma):
    if t == T:
        return max(0, s - K), max(0, K - s)
    d2 = (math.log(s/K) + (r - 0.5 * sigma * sigma) * (T - t) ) / ( sigma * math.sqrt(T - t))
    d1 = (math.log(s/K) + (r + 0.5 * sigma * sigma) * (T - t) ) / ( sigma * math.sqrt(T - t))
    PriceOfPutOption = K * math.exp( -r * (T - t) ) * norm.cdf(-d2) - s * norm.cdf(-d1)
    PriceOfCallOption = s * norm.cdf(d1) - K * math.exp( -r * (T - t) ) * norm.cdf(d2)
    return PriceOfCallOption, PriceOfPutOption

K = 1
t = 0
sigma = 0.6
T = 1
r = 0.05
s = 1.5
C, P = ClassicalBlackScholesMertonModel(s, t, T, K, r, sigma)
print(f"Using Model paramaters as: s = {s}, t = {t}, T = {T}, K = {K}, r = {r}, sigma = {sigma}\n")
print("Price of European Put Option =", P)
print("Price of European Call Option =", C)
```

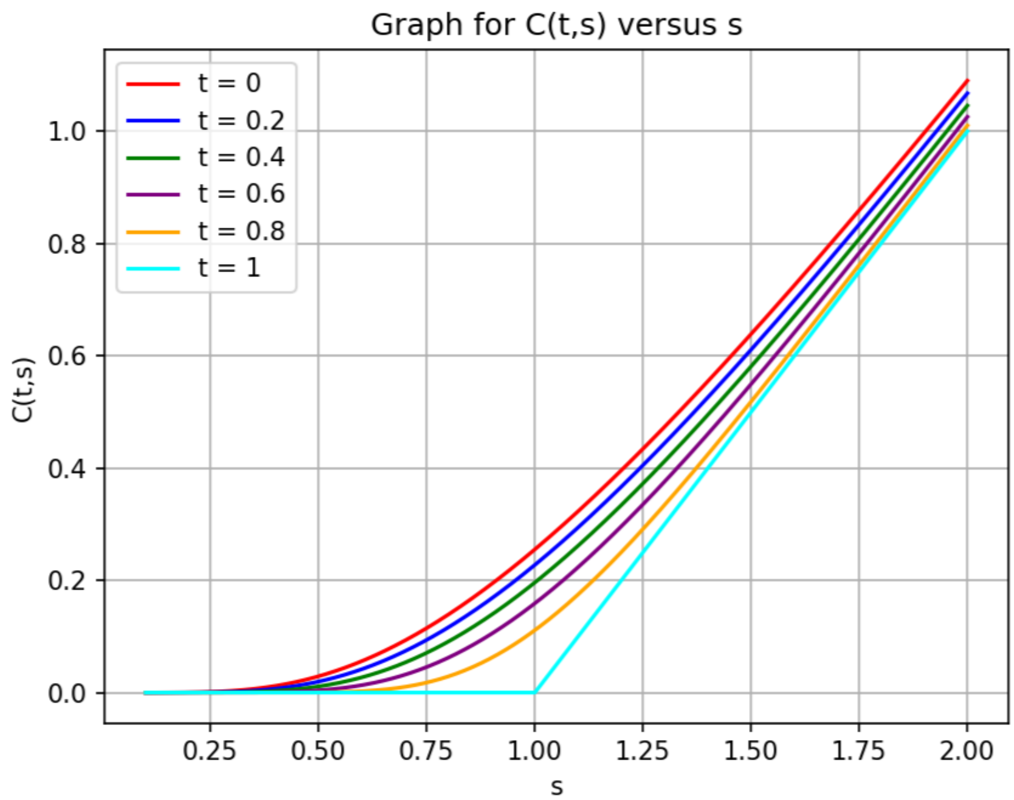
```
Using Model paramaters as: s = 1.5, t = 0, T = 1, K = 1, r = 0.05, sigma = 0.6

Price of European Put Option = 0.09015864336212953
Price of European Call Option = 0.6389292188614155
```

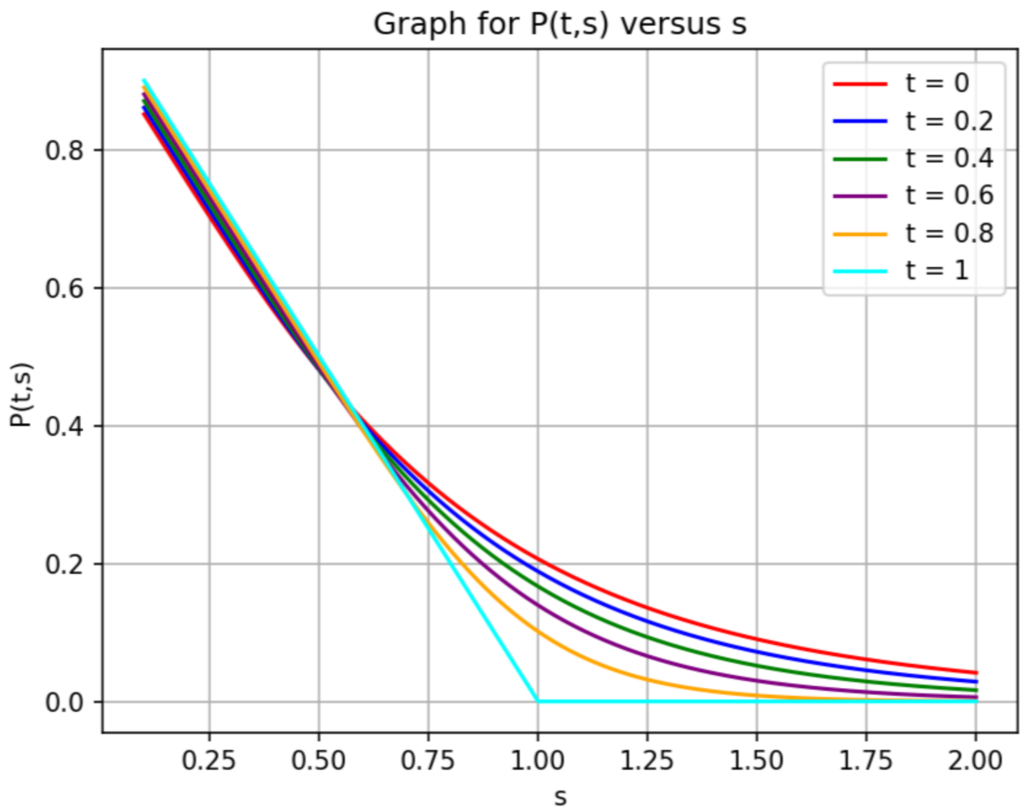
Question 2:

2. Assume $T = 1, K = 1, r = 0.05, \sigma = 0.6$. Plot, in a single graph, $C(t, s)$ as a function of s alone for $t = 0, 0.2, 0.4, 0.6, 0.8, 1$. Do a similar plot for $P(t, s)$ as a function of s. Now, show the same information in a 3-dimensional form, i.e., as a function both t and s.

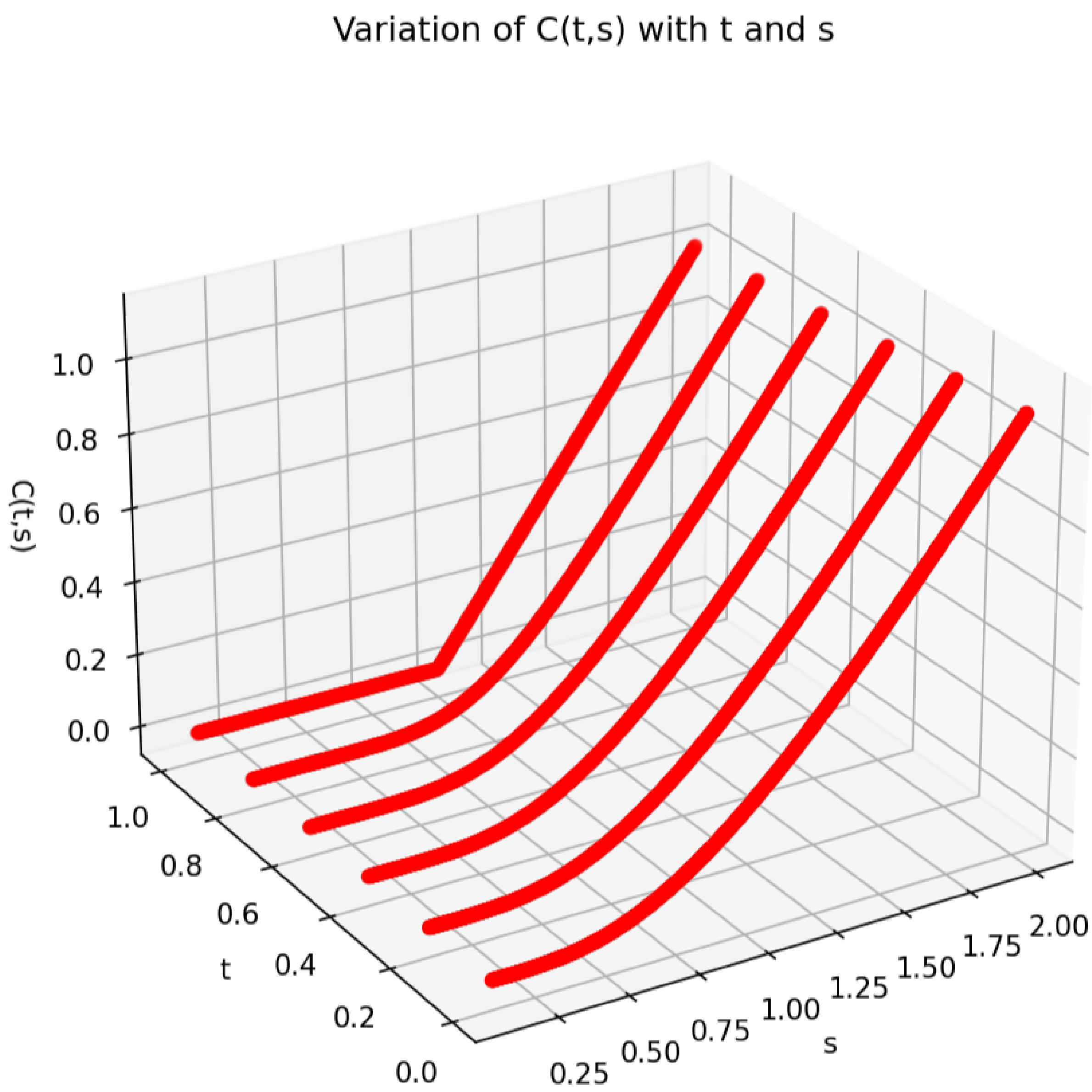
The graph of C(t,s) as a function of s is as follows:



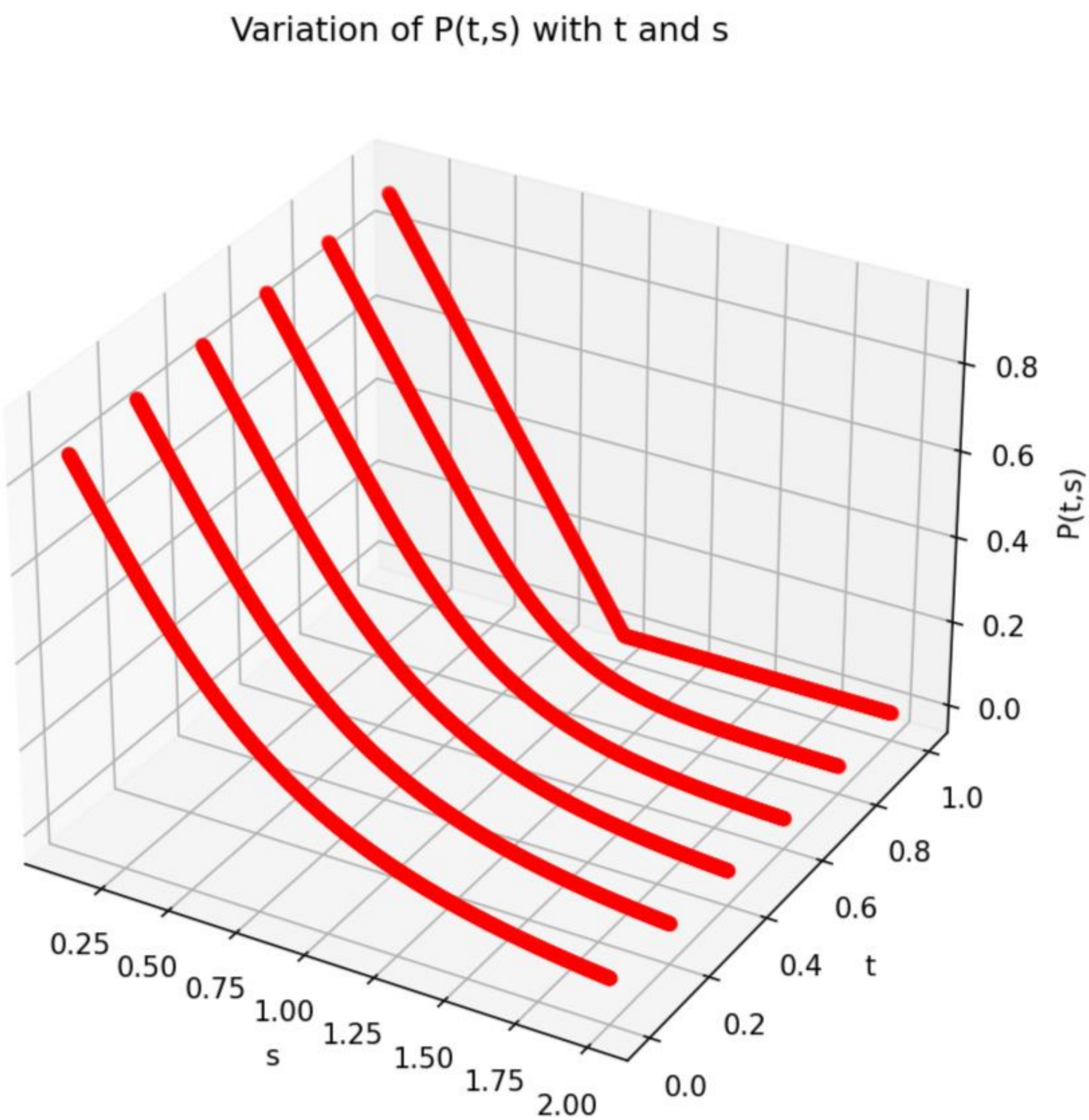
The graph of P(t,s) as a function of s is as follows:



The graph of C(t,s) as a function of both t and s is as follows:



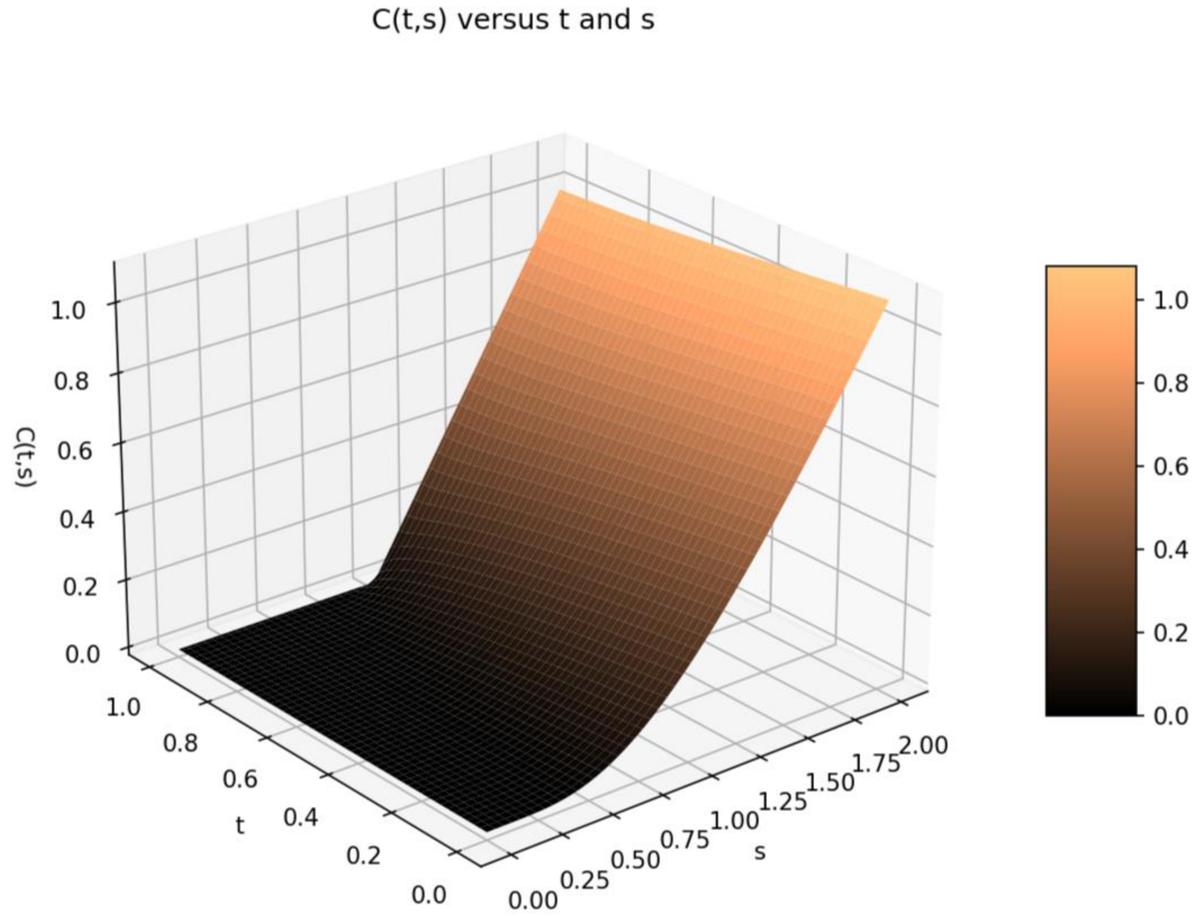
The graph of P(t,s) as a function of both t and s is as follows:



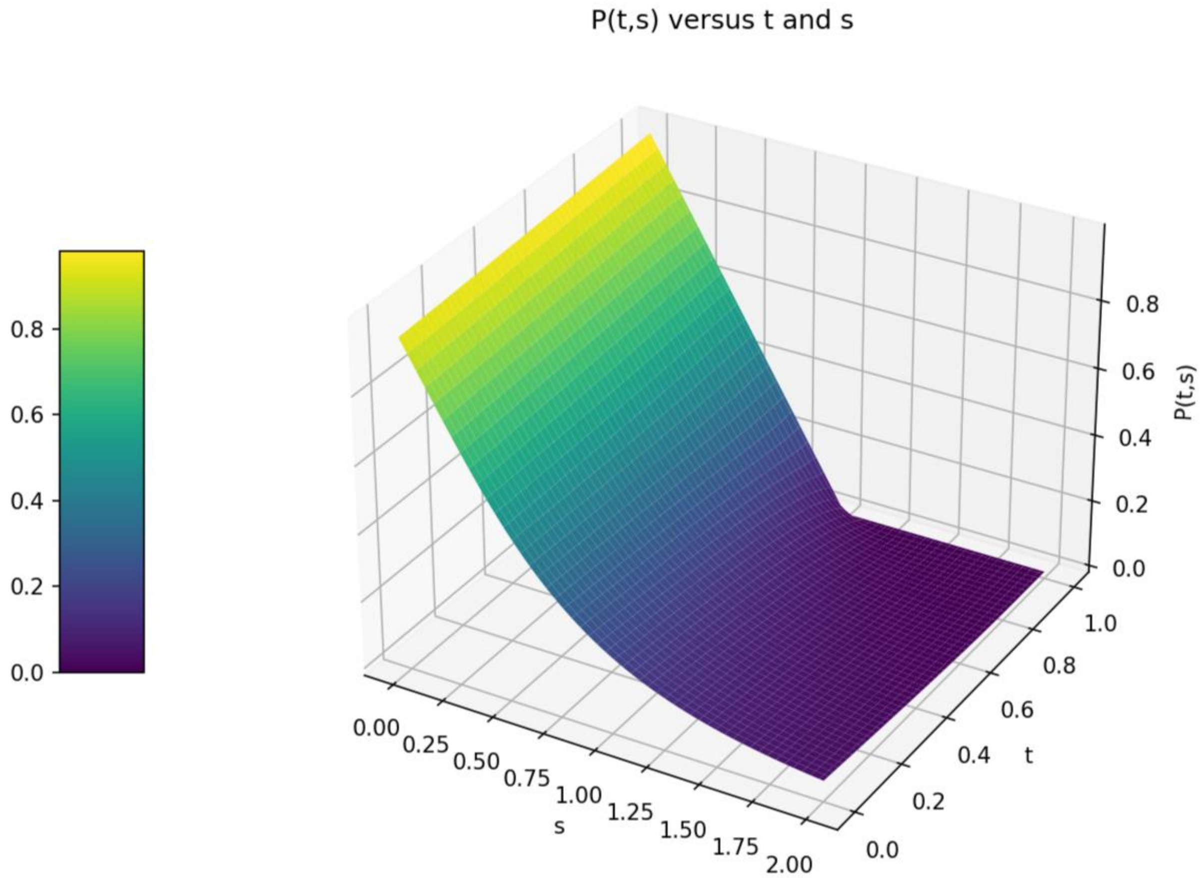
Question 3:

3. Plot $C(t, s)$ and $P(t, s)$ as a smooth surface above the (t, s)-plane.

The plot of C(t, s) as a smooth surface above the (t, s)-plane is as follows:



The plot of P(t, s) as a smooth surface above the (t, s)-plane is as follows:



Question 4:

4. Study the sensitivity of both the functions C and P as a function of model parameters. If required, you may assume different parameter values as opposed to the one given above. Present your results in the form of tables and graphs (both in two and three dimensional).

Put all your observations in the report.

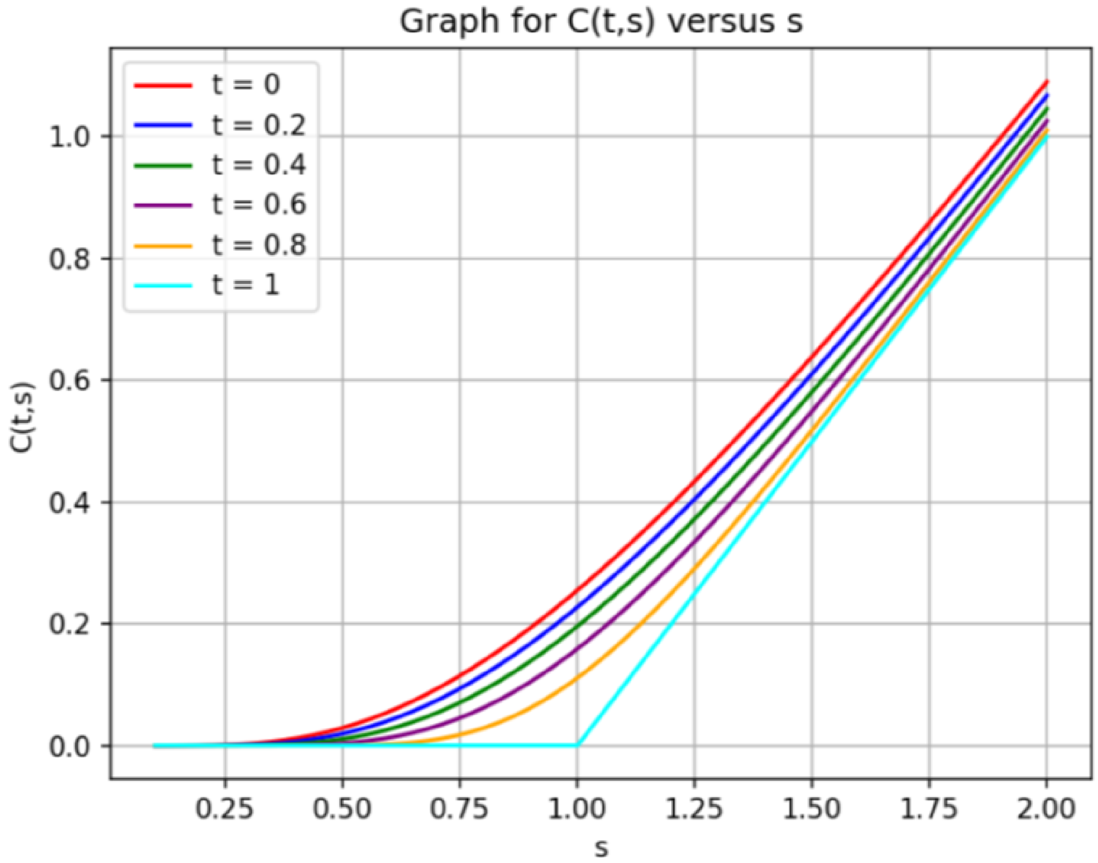
When some particular values of parameters are required, those values are taken from the following:

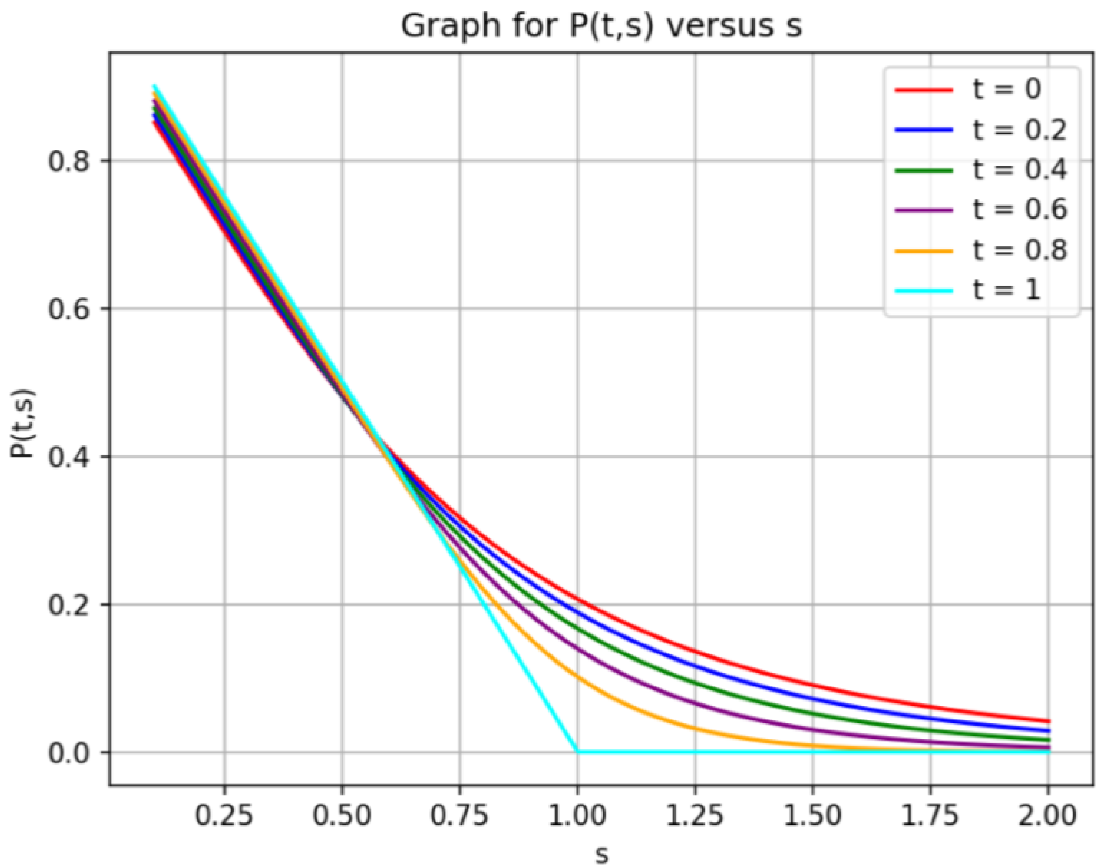
$$s = 0.8,\ t = 0,\ T = 1,\ K = 1,\ r = 0.05,\ \sigma = 0.6$$

The other parameter values are varied accordingly.

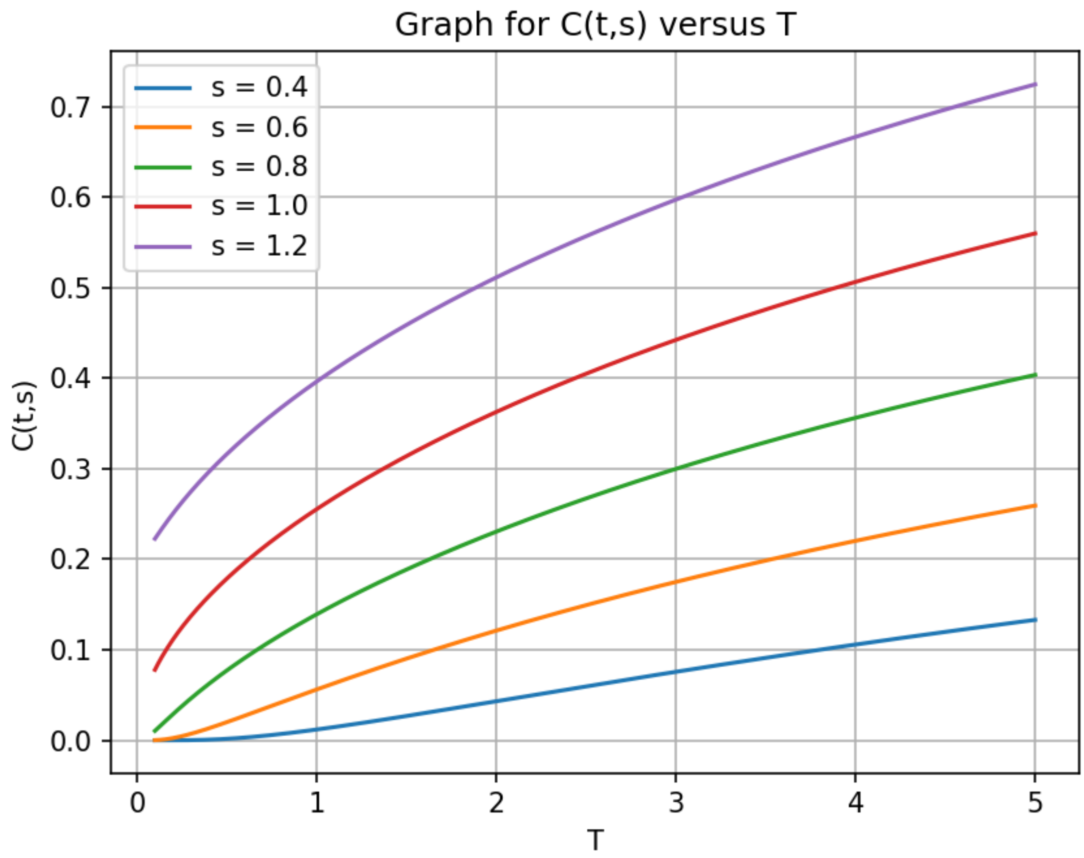
Variation of C and P with s, the price of underlying asset:

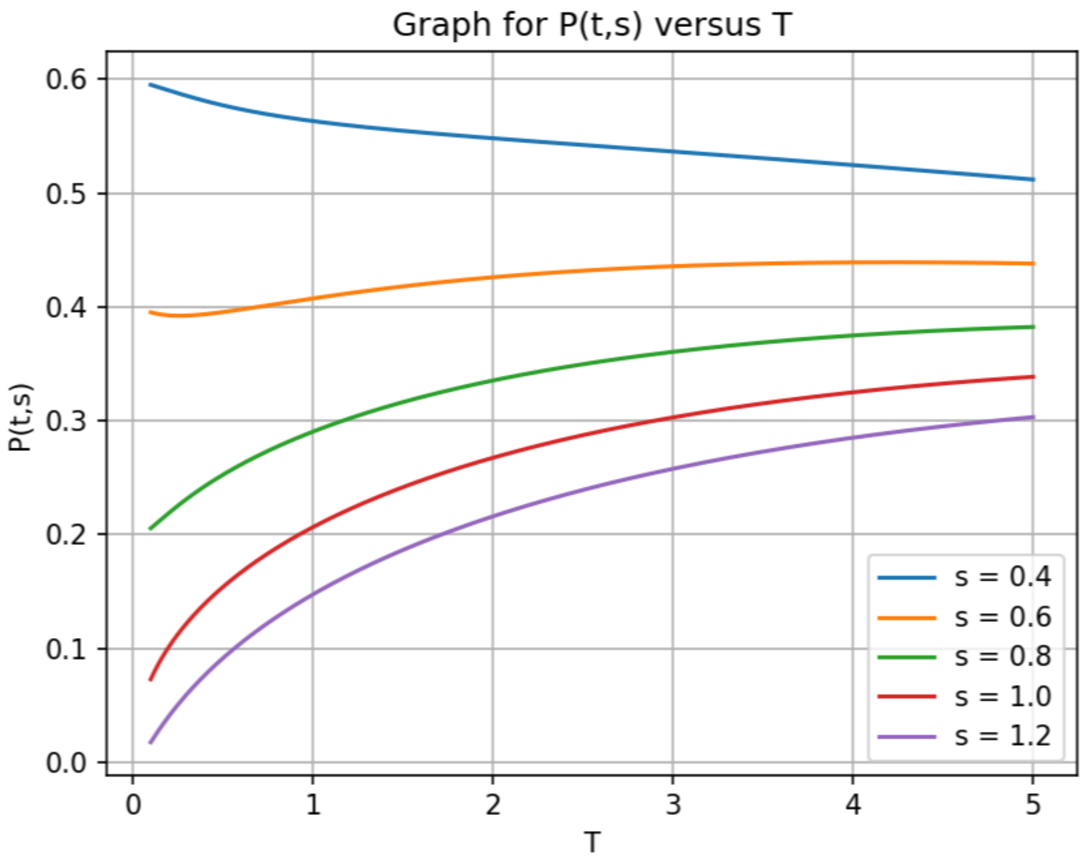
Already done in Question numbers 2 and 3.





<u>Variation of C and P with maturity time T</u>:

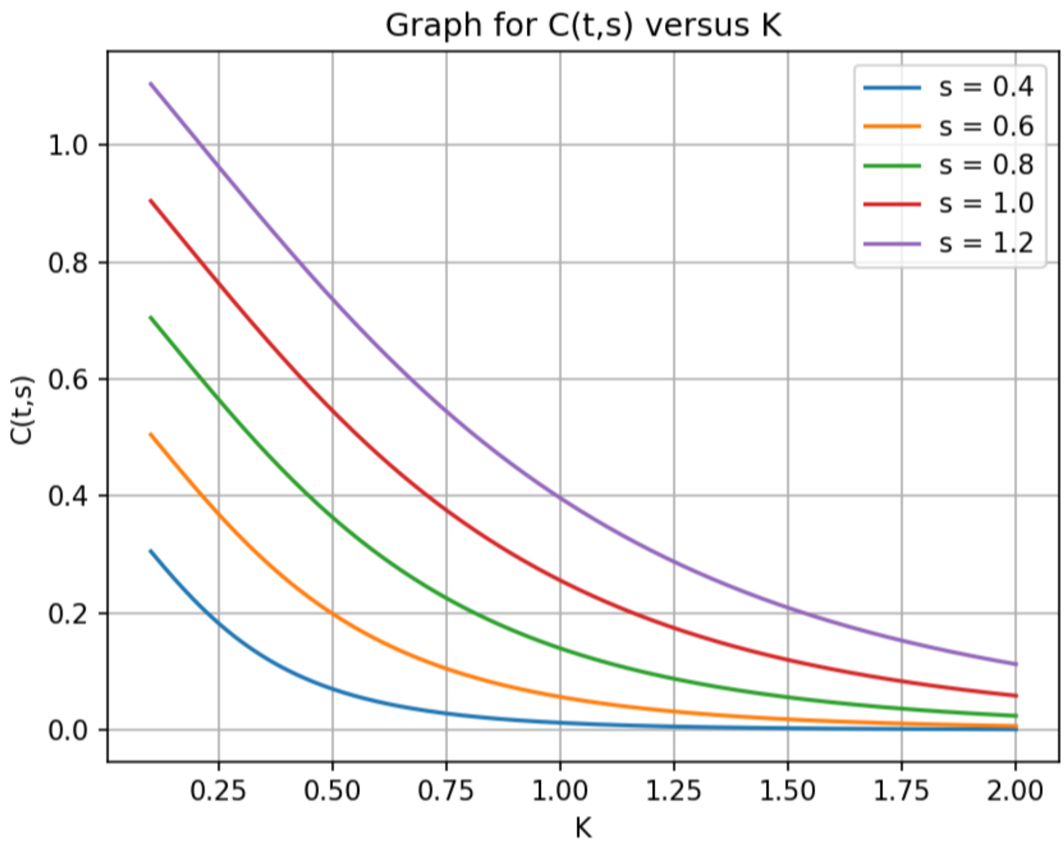


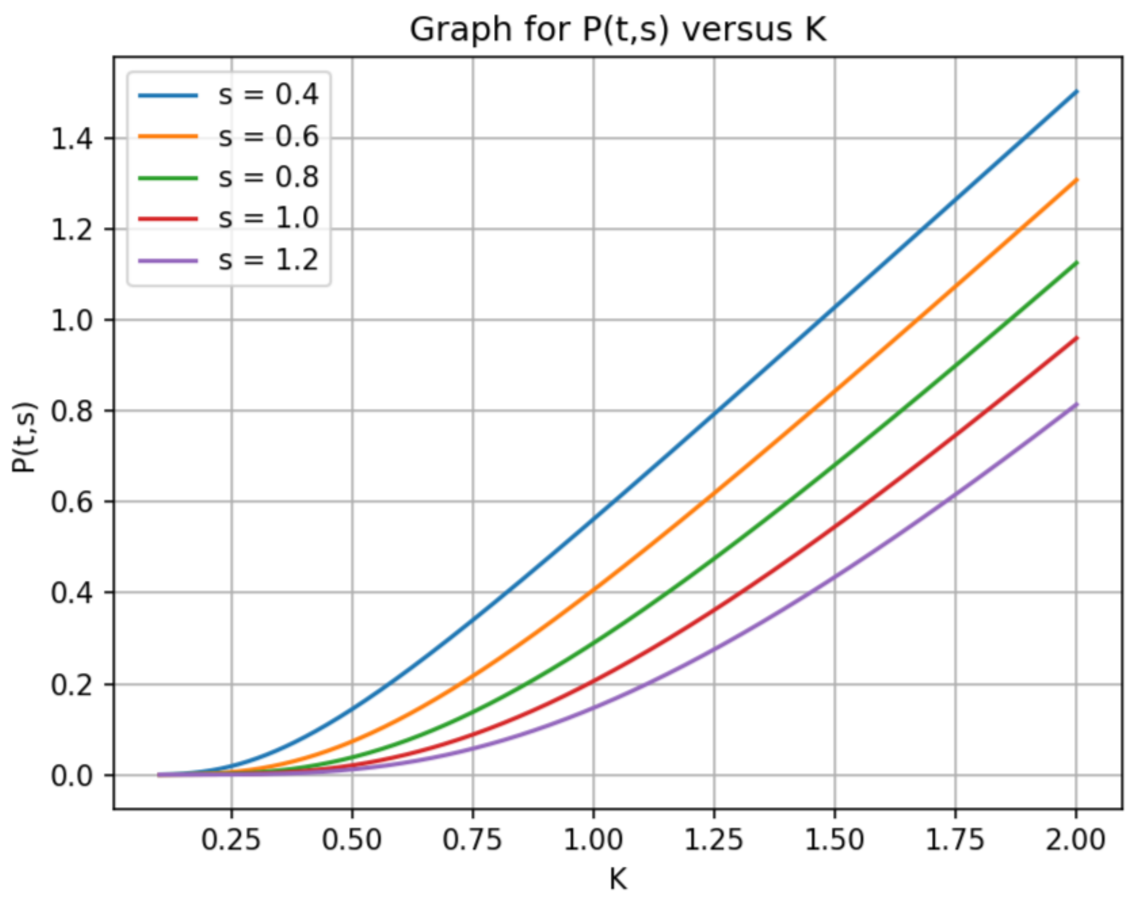


With parameter values as: $s = 0.8$, $t = 0$, $K = 1$, $r = 0.05$ and $\sigma = 0.6$, Variation of C(t,s) and P(t,s) with T

Serial Number	T	C(t,s)	P(t,s)
1	0.1	0.0104876	0.2055
2	0.590982	0.0897777	0.260661
3	1.08196	0.147911	0.29525
4	1.57295	0.195092	0.319458
5	2.06393	0.235421	0.33737
6	2.55491	0.270874	0.350952
7	3.04589	0.302594	0.361329
8	3.53687	0.331314	0.369225
9	4.02786	0.357547	0.375138
10	4.51884	0.381665	0.379429

<u>Variation of C and P with strike price K</u>:



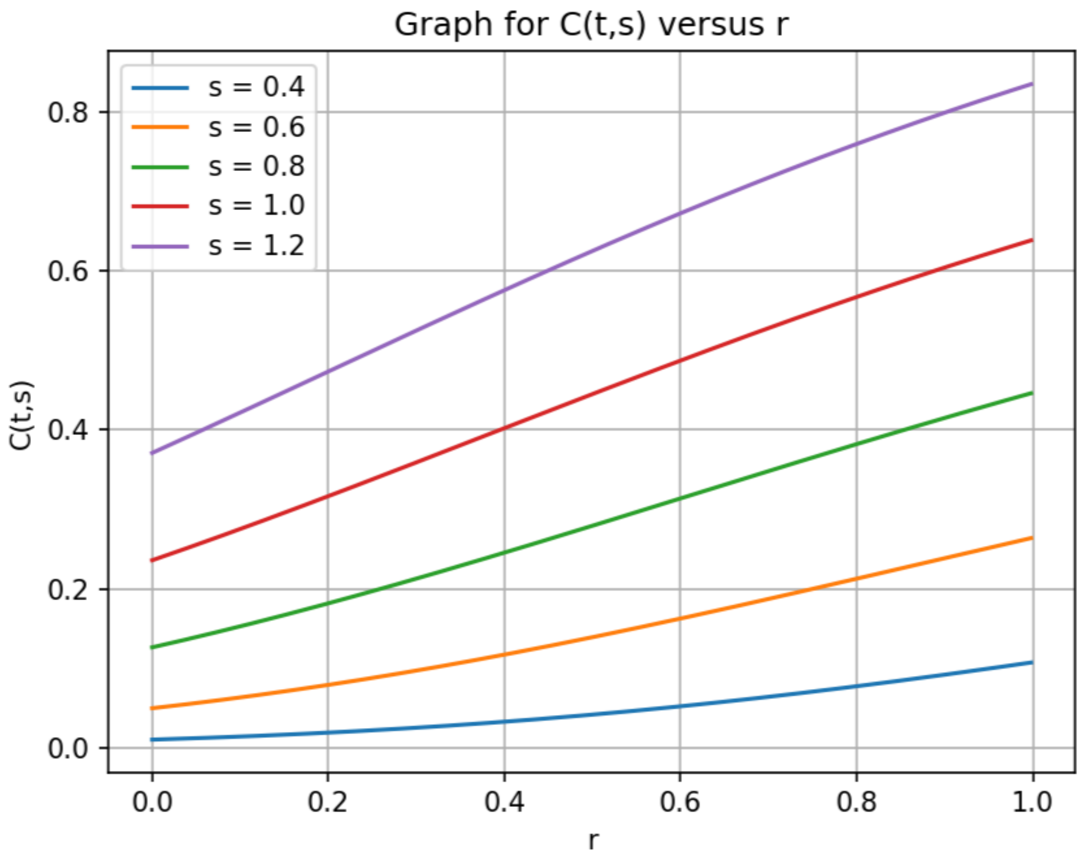


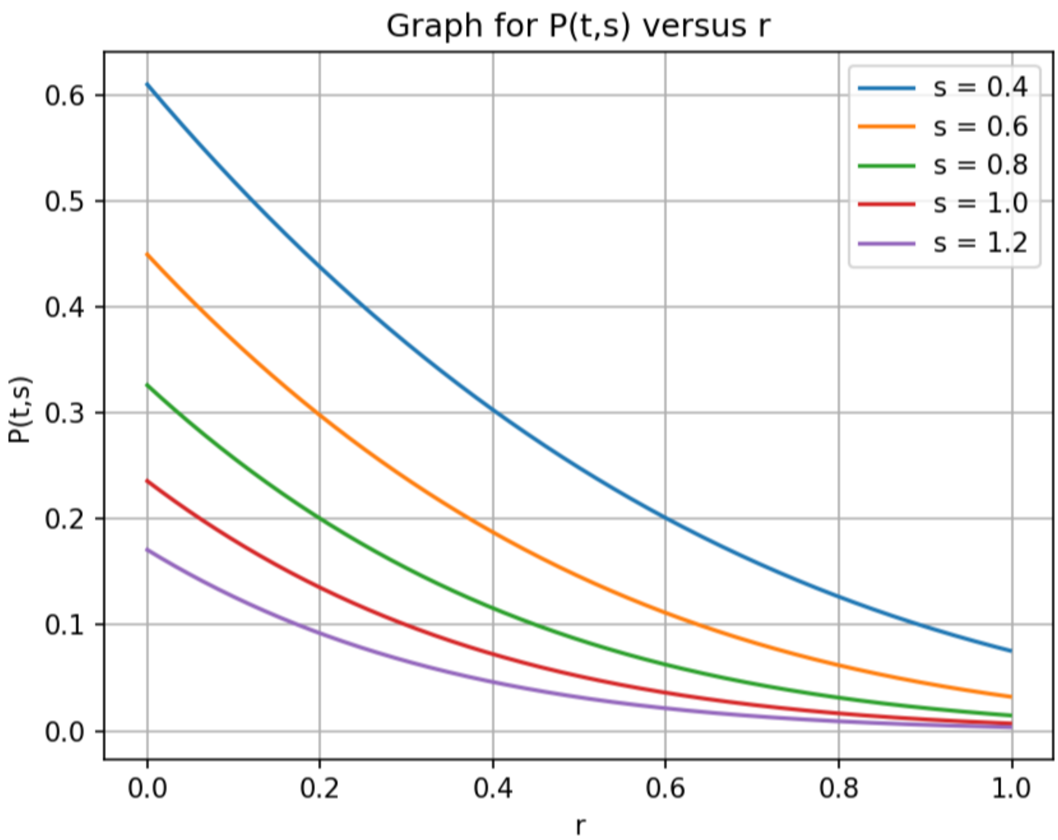
With parameter values as: s = 0.8, t = 0, T = 1, r = 0.05 and σ = 0.6,

Variation of C(t,s) and P(t,s) with K

Serial Number	K	C(t,s)	P(t,s)
1	0.1	0.704885	7.64124e-06
2	0.290381	0.527951	0.00417013
3	0.480762	0.376097	0.0334112
4	0.671142	0.261841	0.100251
5	0.861523	0.181355	0.200861
6	1.0519	0.126098	0.3267
7	1.24228	0.0883989	0.470096
8	1.43267	0.0625982	0.625392
9	1.62305	0.0448062	0.788695
10	1.81343	0.0324182	0.957403

Variation of C and P with rate of interest r:



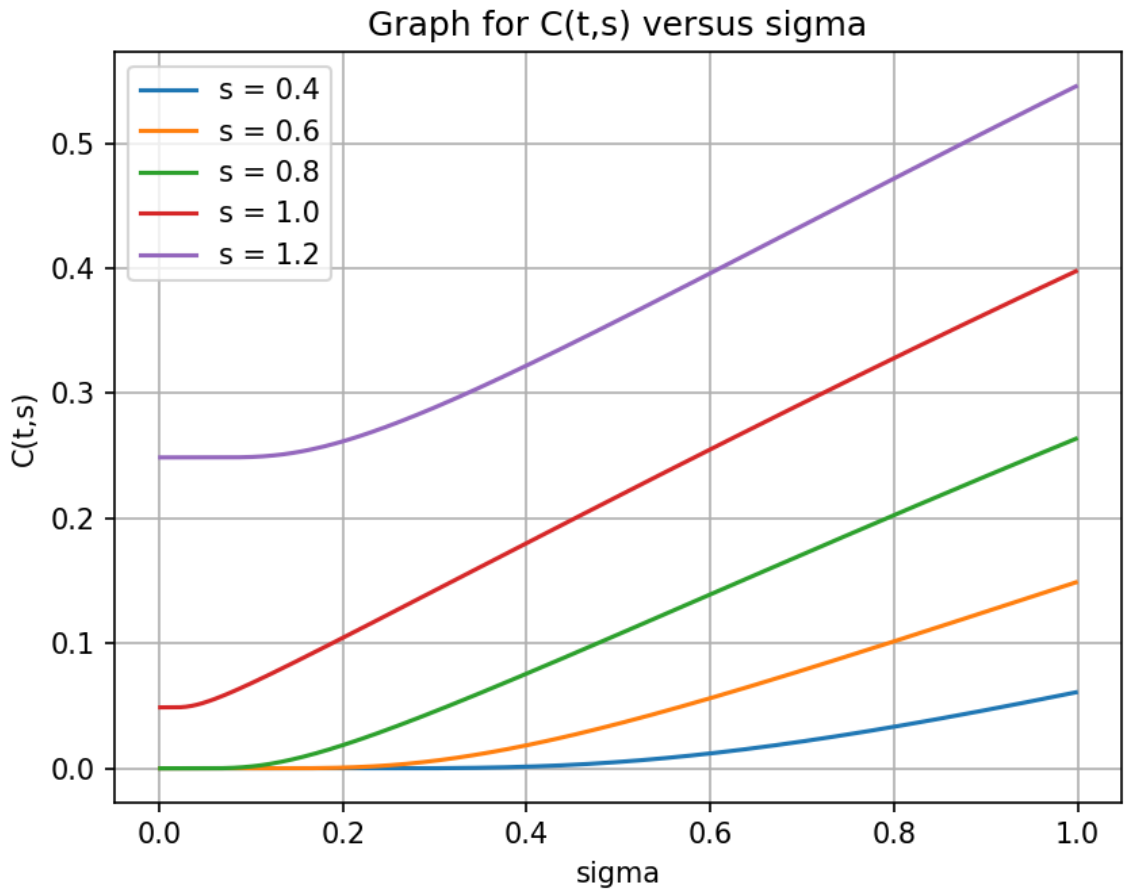


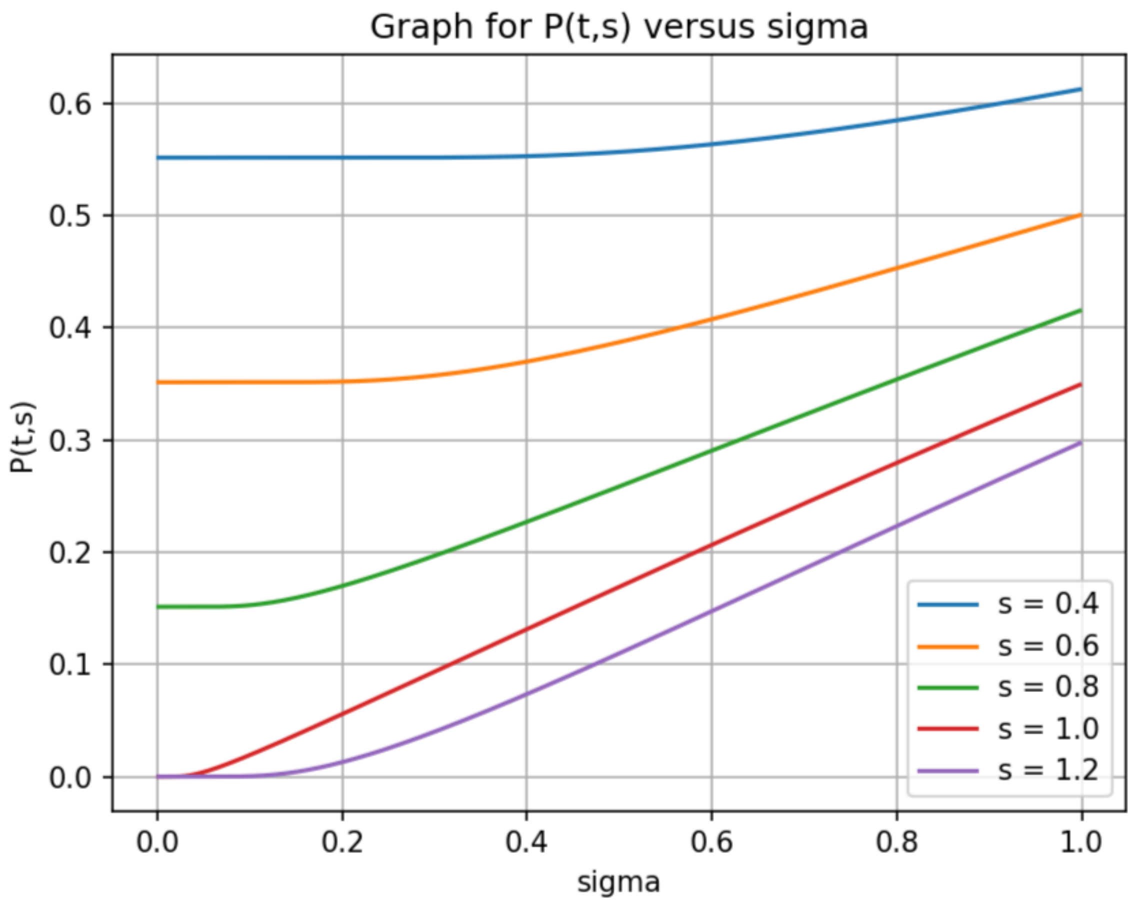
With parameter values as: s = 0.8, t = 0, T = 1, K = 1 and σ = 0.6,

Variation of C(t,s) and P(t,s) with r

Serial Number	r	C(t,s)	P(t,s)
1	0	0.126249	0.326249
2	0.1	0.152689	0.257526
3	0.2	0.181639	0.20037
4	0.3	0.212714	0.153533
5	0.4	0.24544	0.11576
6	0.5	0.279282	0.0858129
7	0.6	0.313685	0.0624963
8	0.7	0.348098	0.0446838
9	0.8	0.382015	0.0313436
10	0.9	0.414987	0.0215565

<u>Variation of C and P with Volatility σ</u>:


Graph for C(t,s) versus sigma
s = 0.4
s = 0.6
s = 0.8
s = 1.0
s = 1.2
C(t,s)
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.2
0.4
0.6
0.8
1.0
sigma

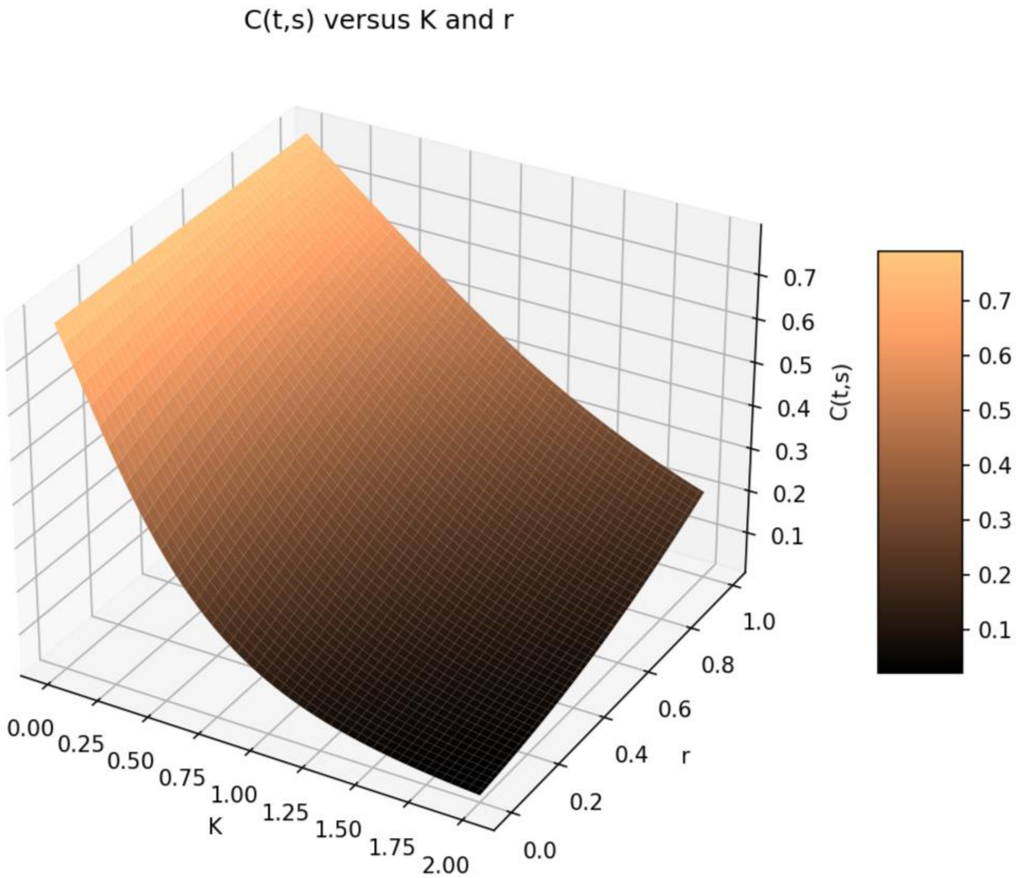

Graph for P(t,s) versus sigma
s = 0.4
s = 0.6
s = 0.8
s = 1.0
s = 1.2
P(t,s)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.0
0.2
0.4
0.6
0.8
1.0
sigma

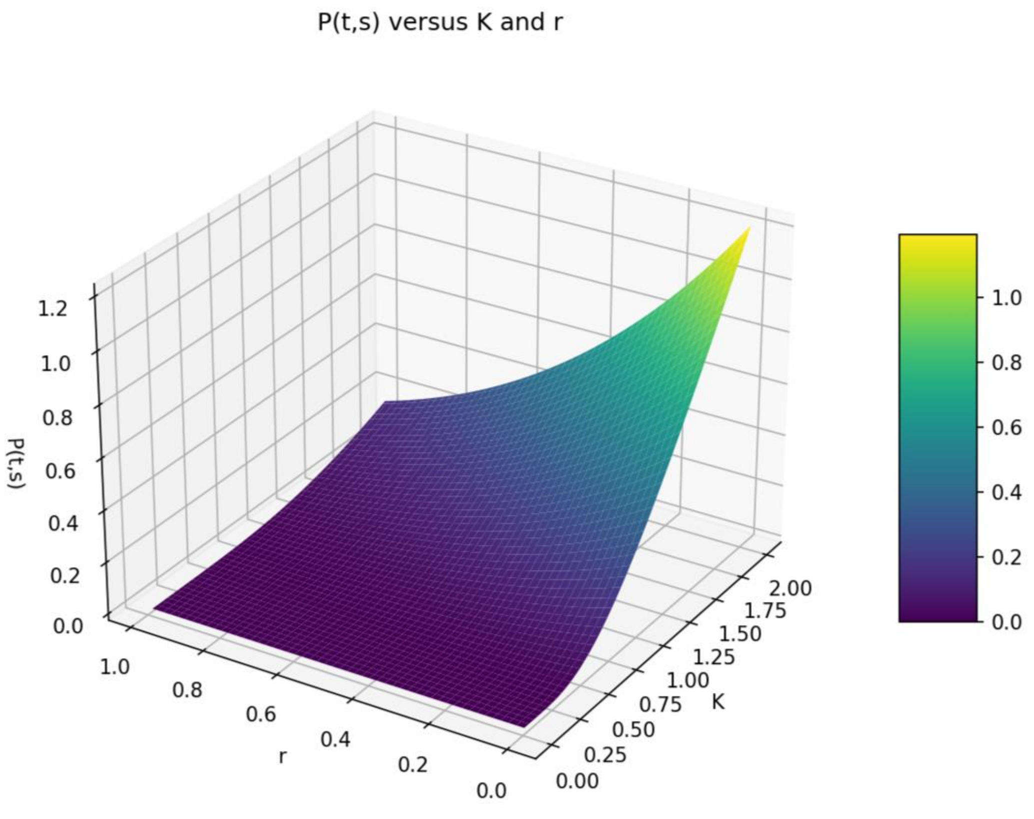
With parameter values as: s = 0.8, t = 0, T = 1, K = 1 and r = 0.05,

Variation of C(t,s) and P(t,s) with sigma

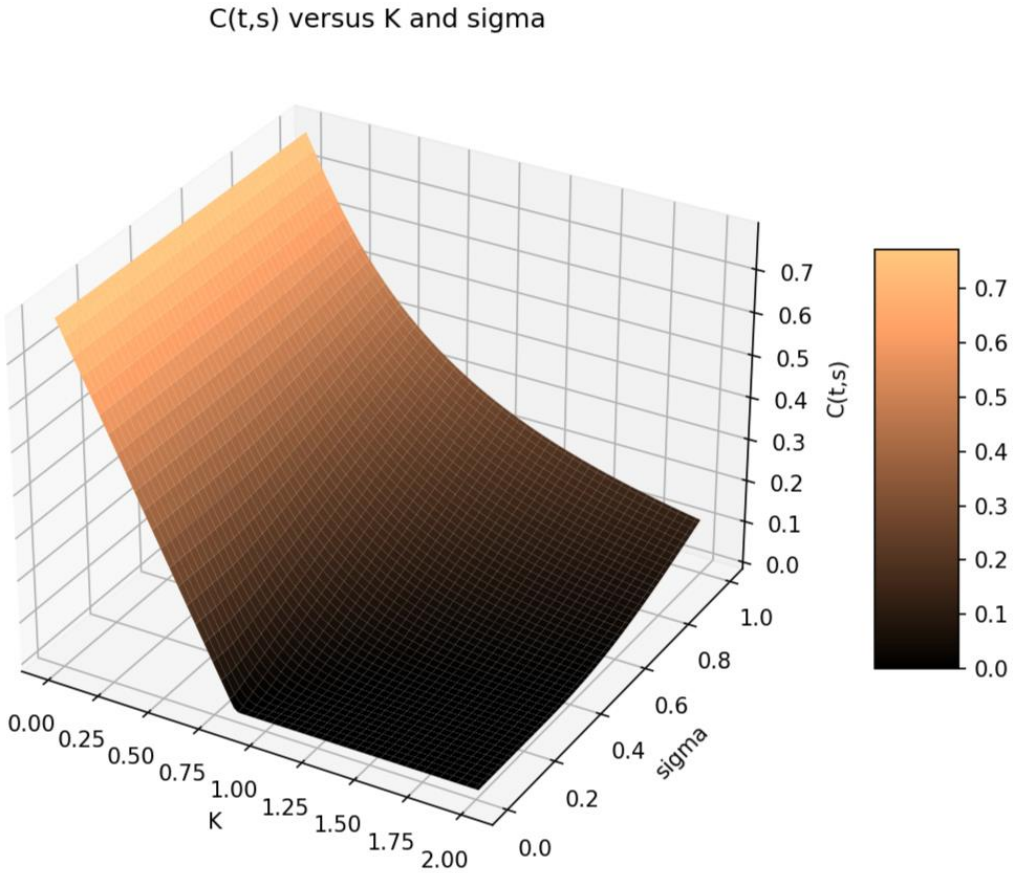
Serial No.	sigma	C(t,s)	P(t,s)
1	0.001	0	0.151229
2	0.1009	0.00154652	0.152776
3	0.2008	0.0187849	0.170014
4	0.3007	0.0457362	0.196966
5	0.4006	0.0759687	0.227198
6	0.5005	0.10742	0.258649
7	0.6004	0.139262	0.290492
8	0.7003	0.171081	0.32231
9	0.8002	0.202627	0.353856
10	0.9001	0.233733	0.384963

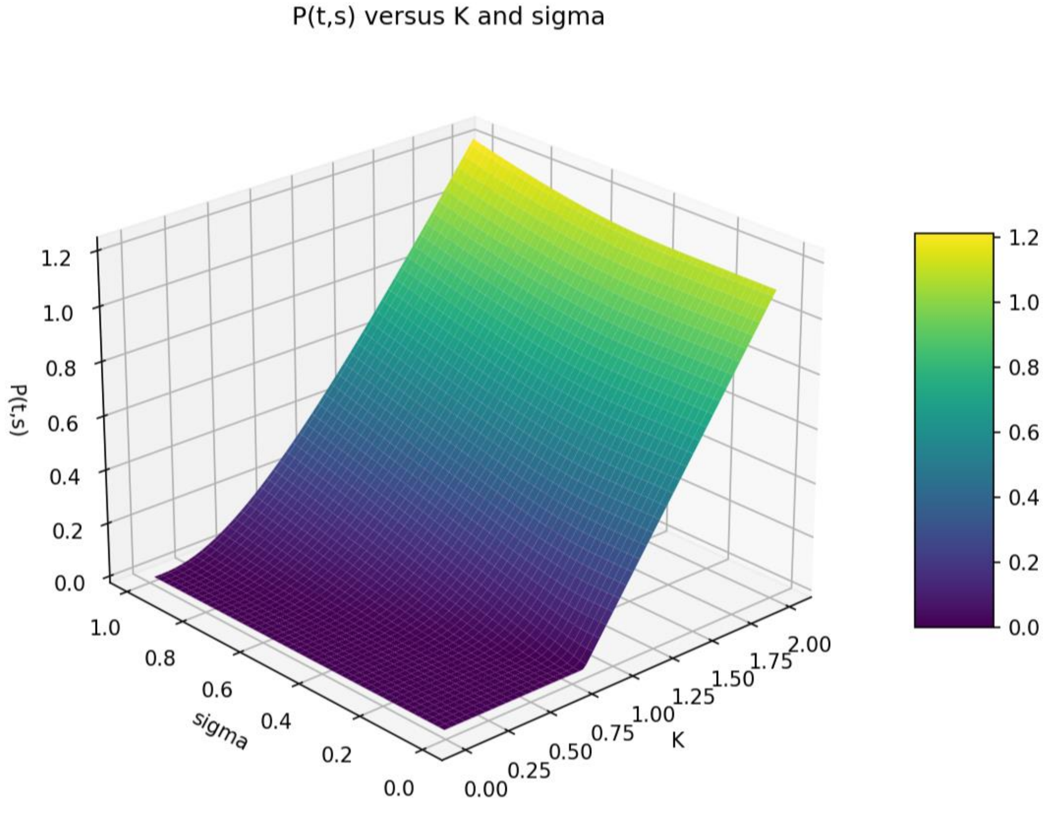
Variation of C and P with K and r:



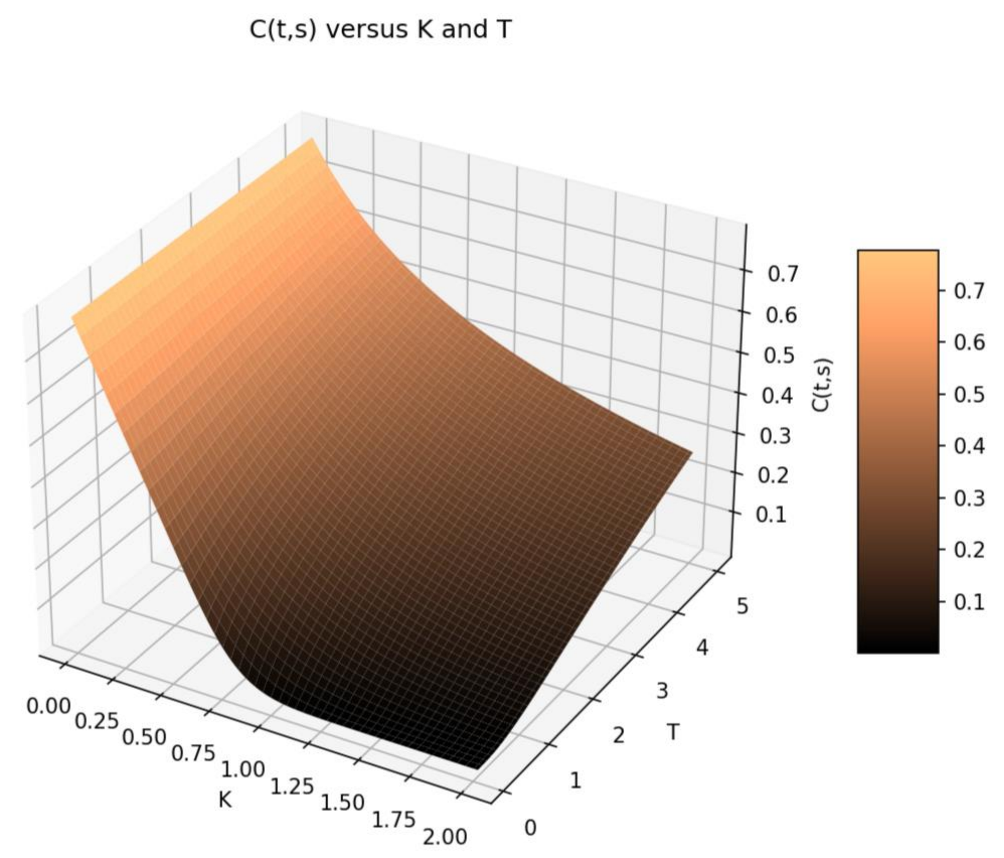


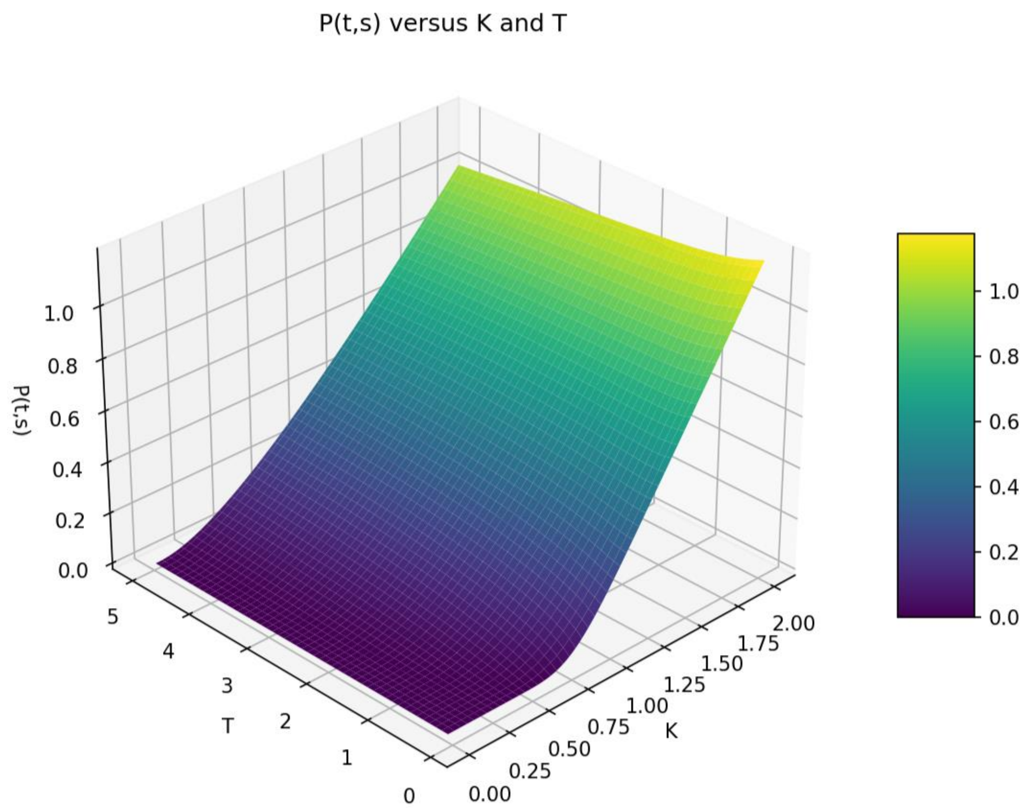
Variation of C and P with K and σ:



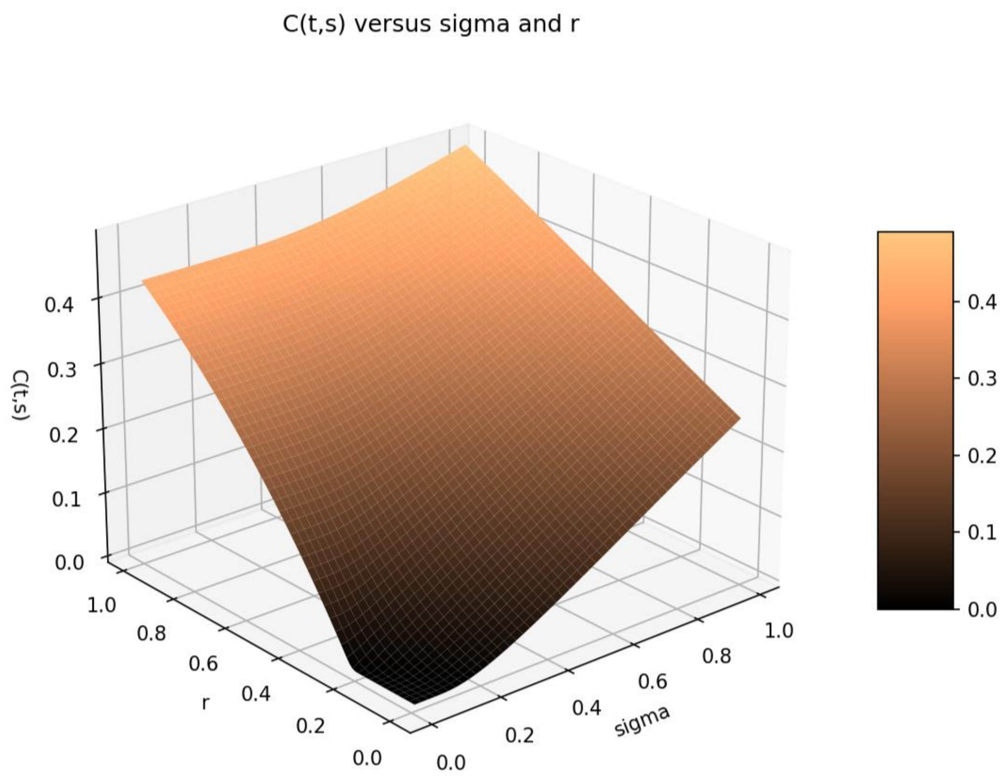


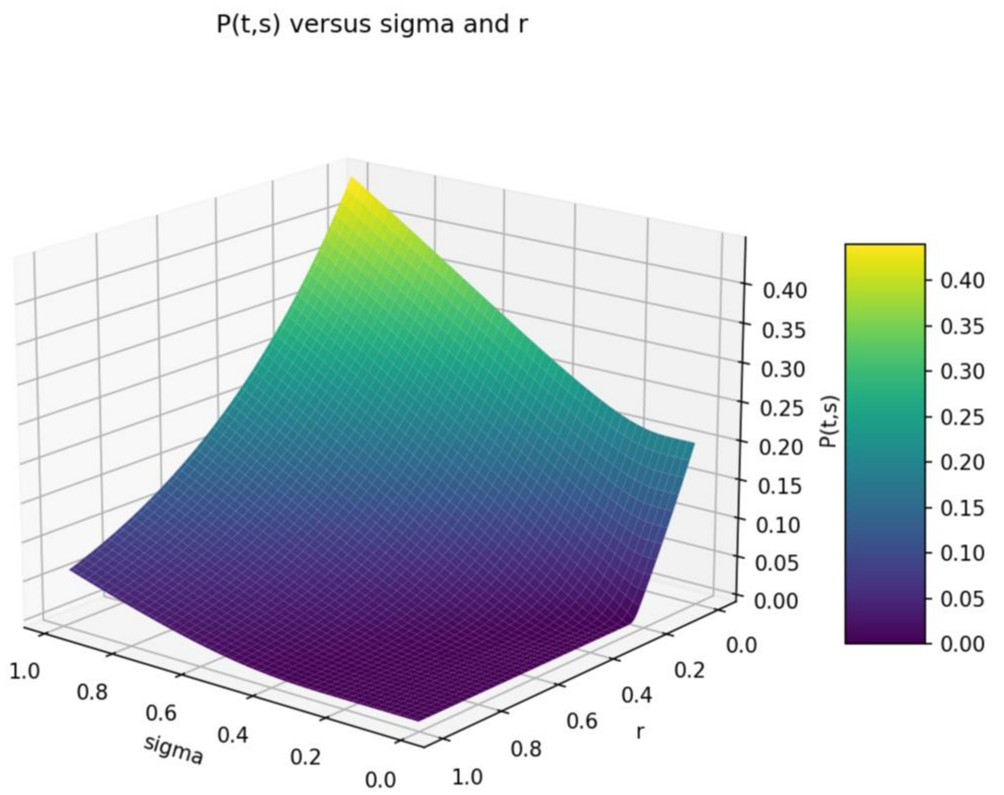
<u>Variation of C and P with K and T</u>:



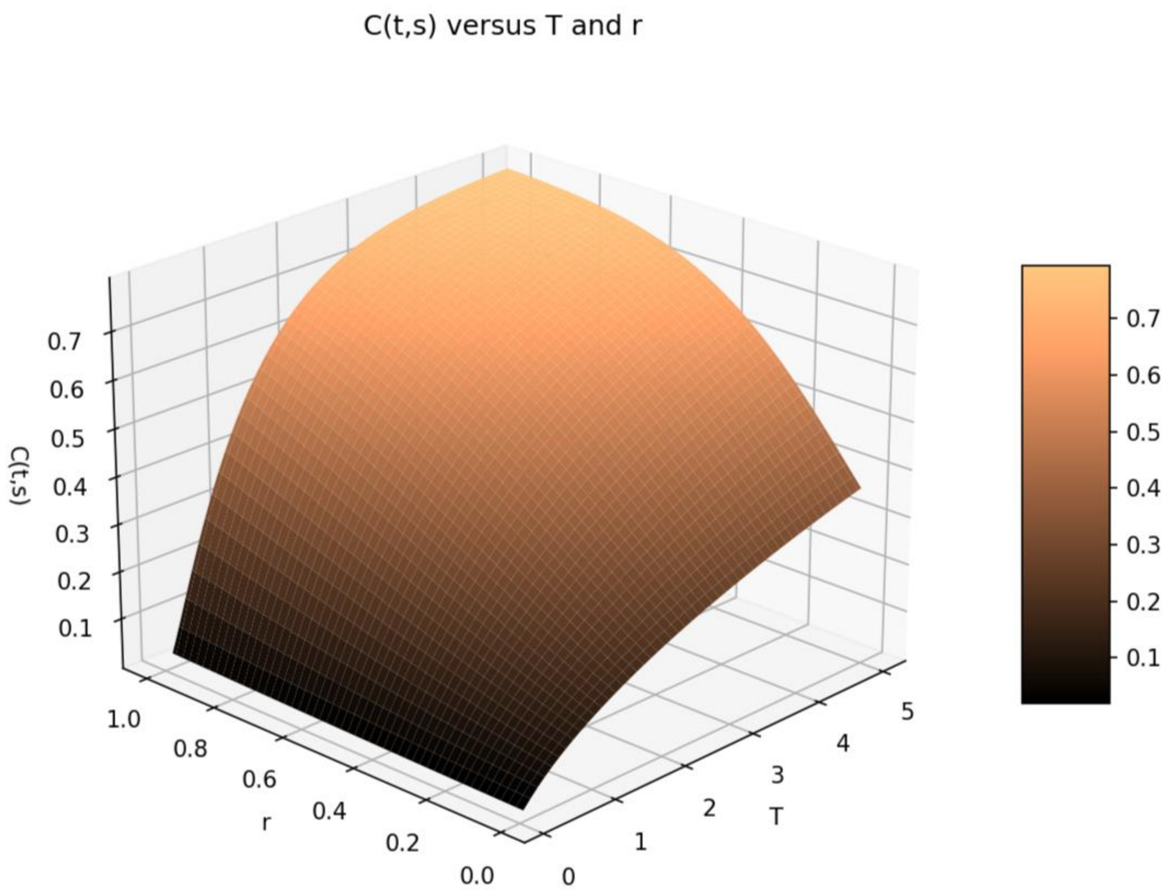


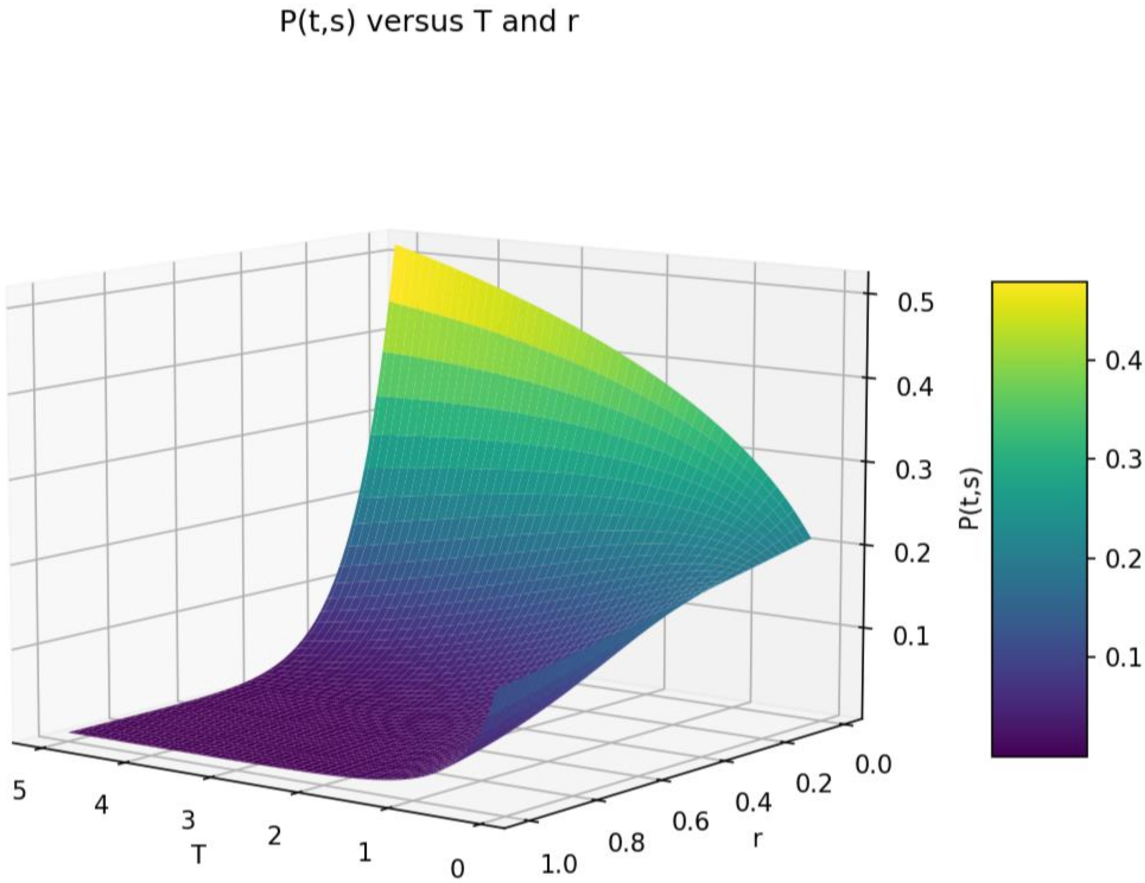
<u>Variation of C and P with r and σ</u>:



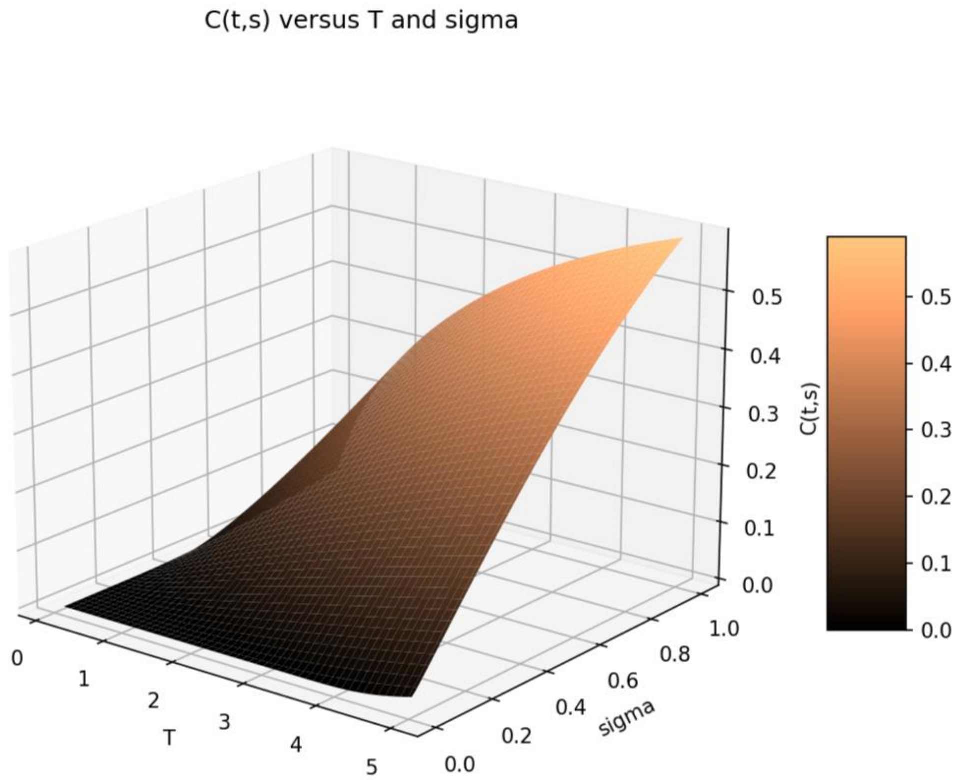


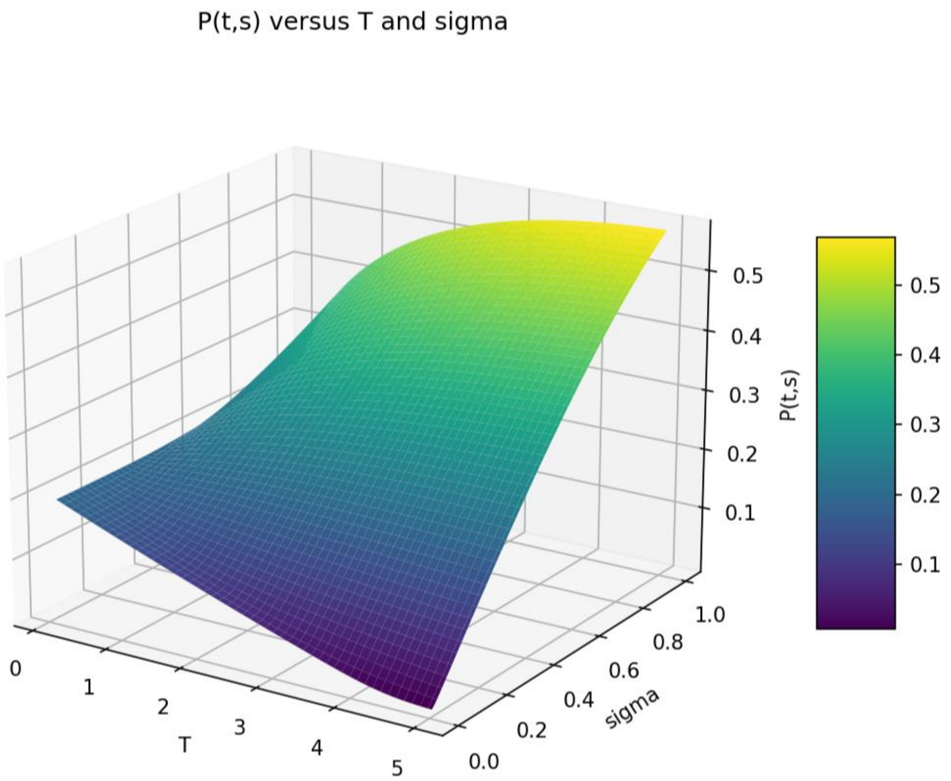
<u>Variation of C and P with T and r</u>:


C(t,s) versus T and r
C(t,s)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
r
0.0
0.2
0.4
0.6
0.8
1.0
T
0
1
2
3
4
5


P(t,s) versus T and r
P(t,s)
0.1
0.2
0.3
0.4
0.5
T
0
1
2
3
4
5
r
0.0
0.2
0.4
0.6
0.8
1.0

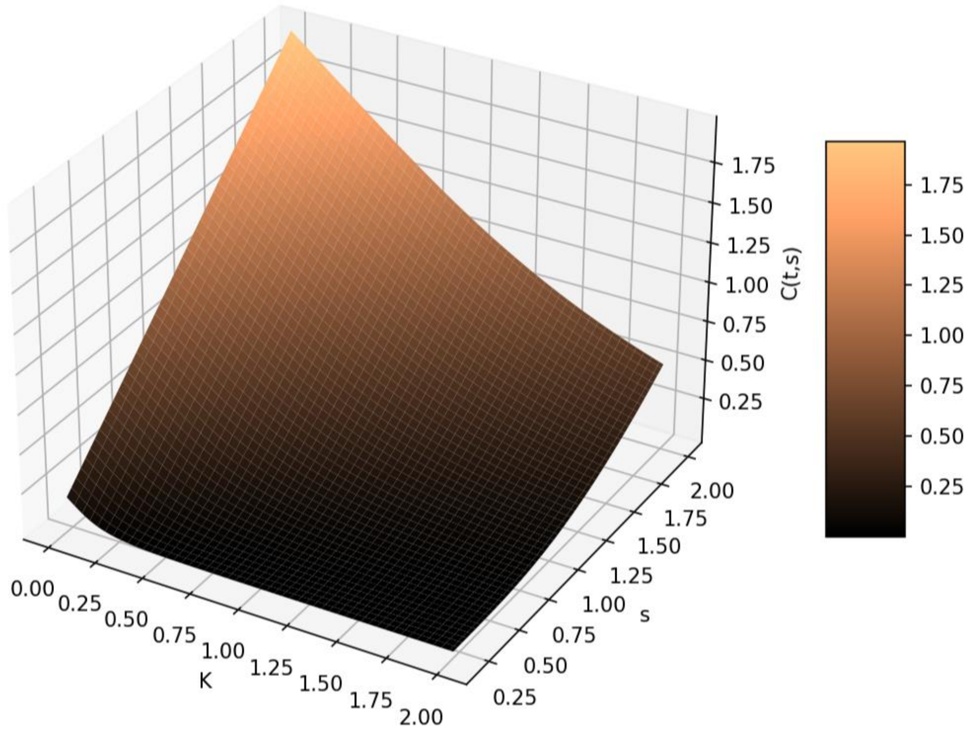
Variation of C and P with T and σ:



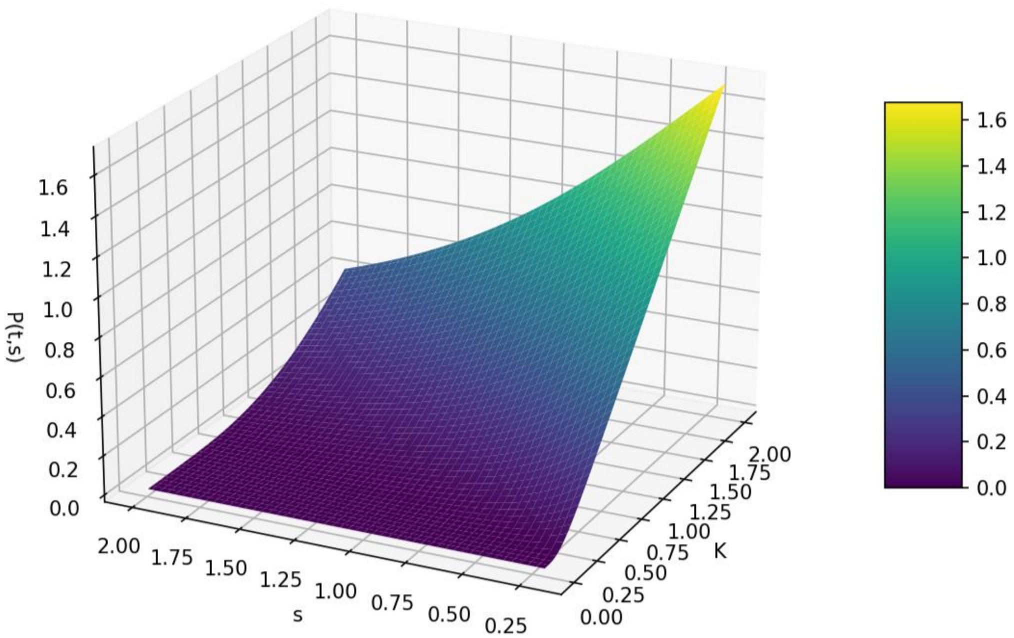


Variation of C and P with K and s:

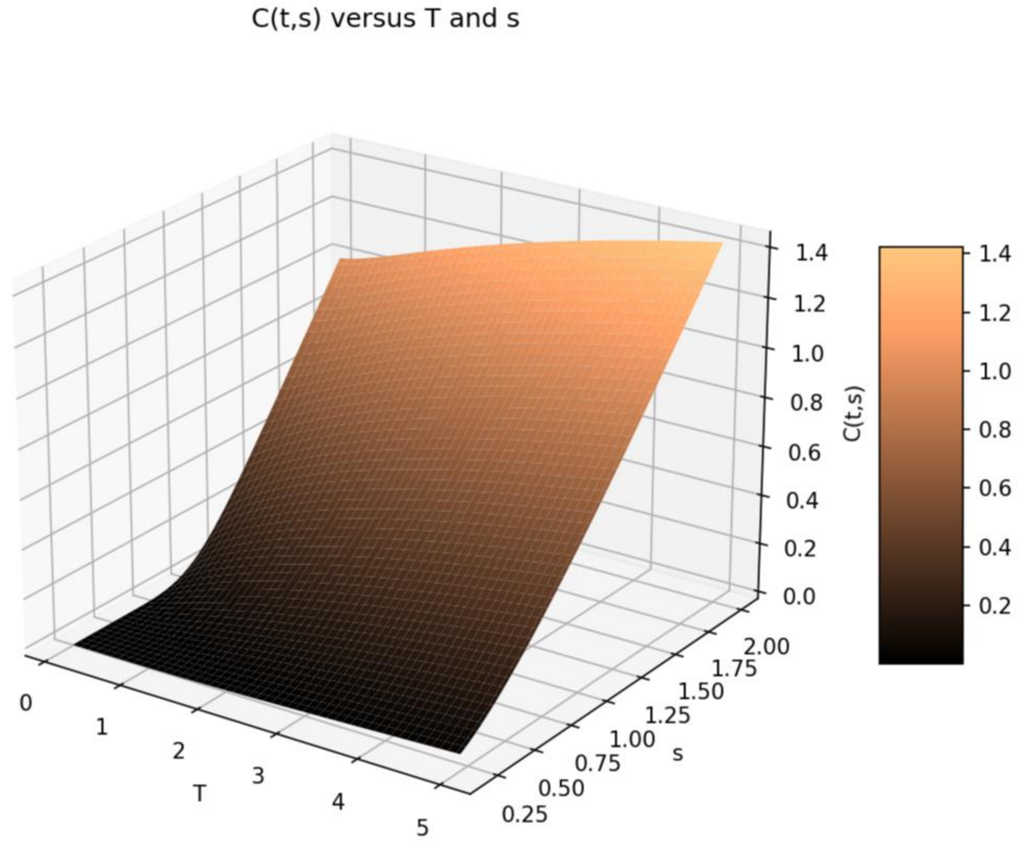
C(t,s) versus K and s

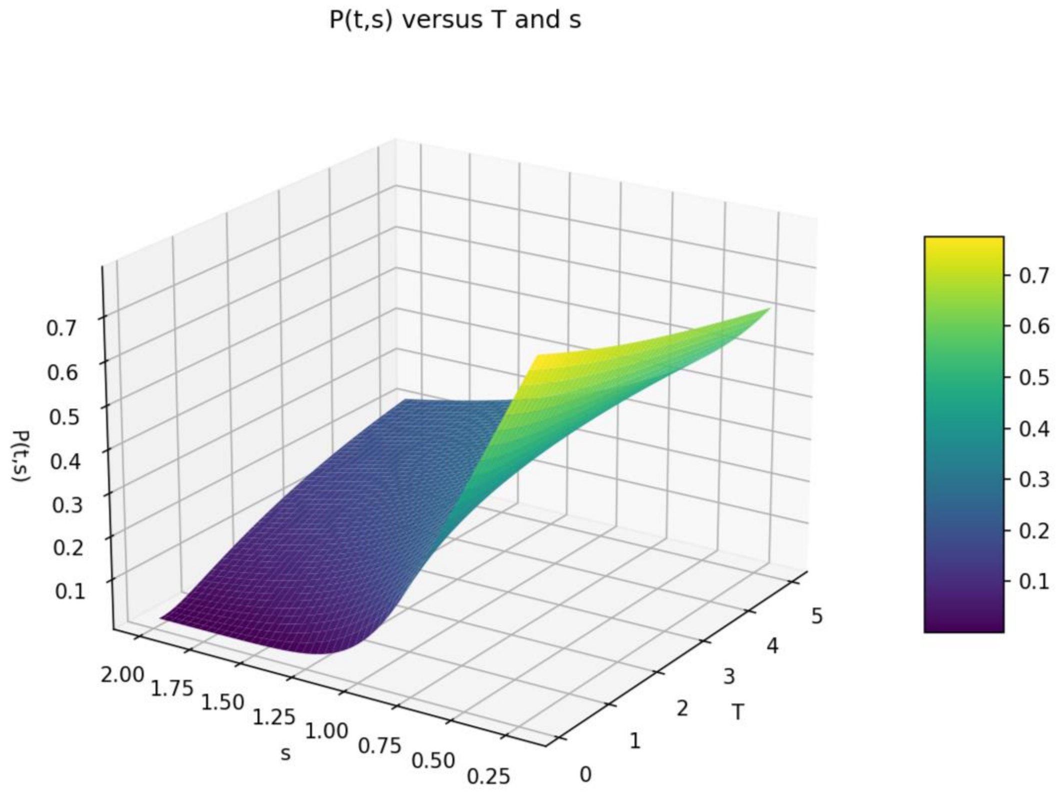


P(t,s) versus K and s

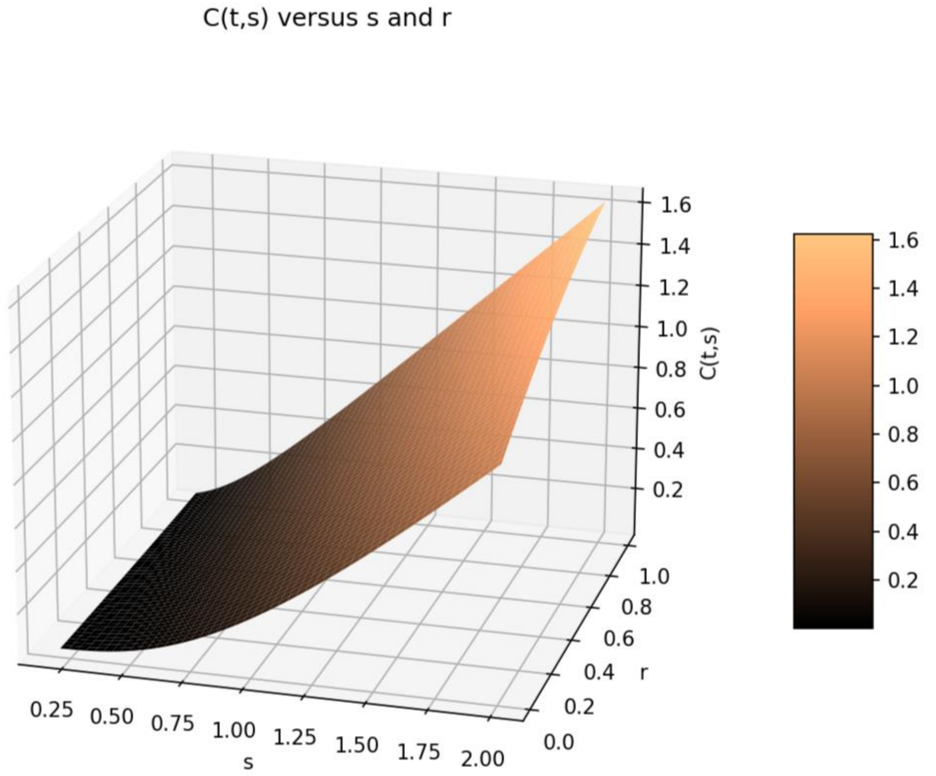


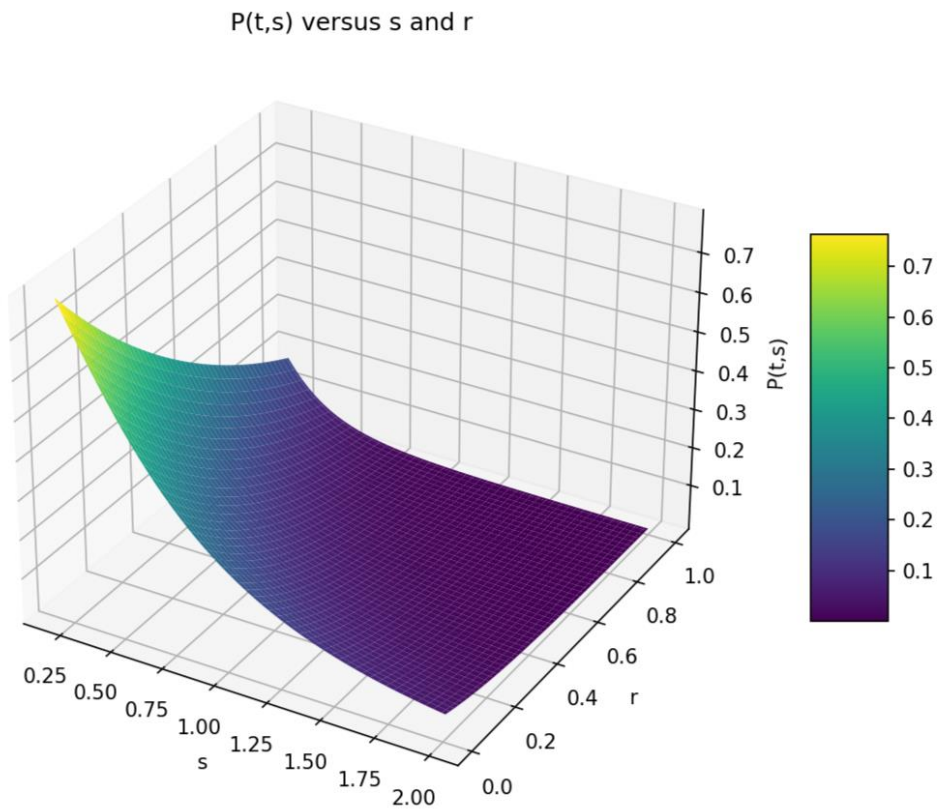
<u>Variation of C and P with T and s</u>:





Variation of C and P with s and r:





Variation of C and P with s and σ: