Financial Engineering Lab MA – 374 Lab – 4

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Question 1:

1. Consider three assets with the following mean return vector and dispersion (variance-covariance) matrix:

$$\mu = \begin{bmatrix} 0.1 & 0.2 & 0.15 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 0.005 & -0.010 & 0.004 \\ -0.010 & 0.040 & -0.002 \\ 0.004 & -0.002 & 0.023 \end{bmatrix}.$$

(a) Construct and plot the Markowitz efficient frontier using the above data.

(b) Tabulate the weights, return and risk of the portfolios for 10 different values on the efficient frontier.

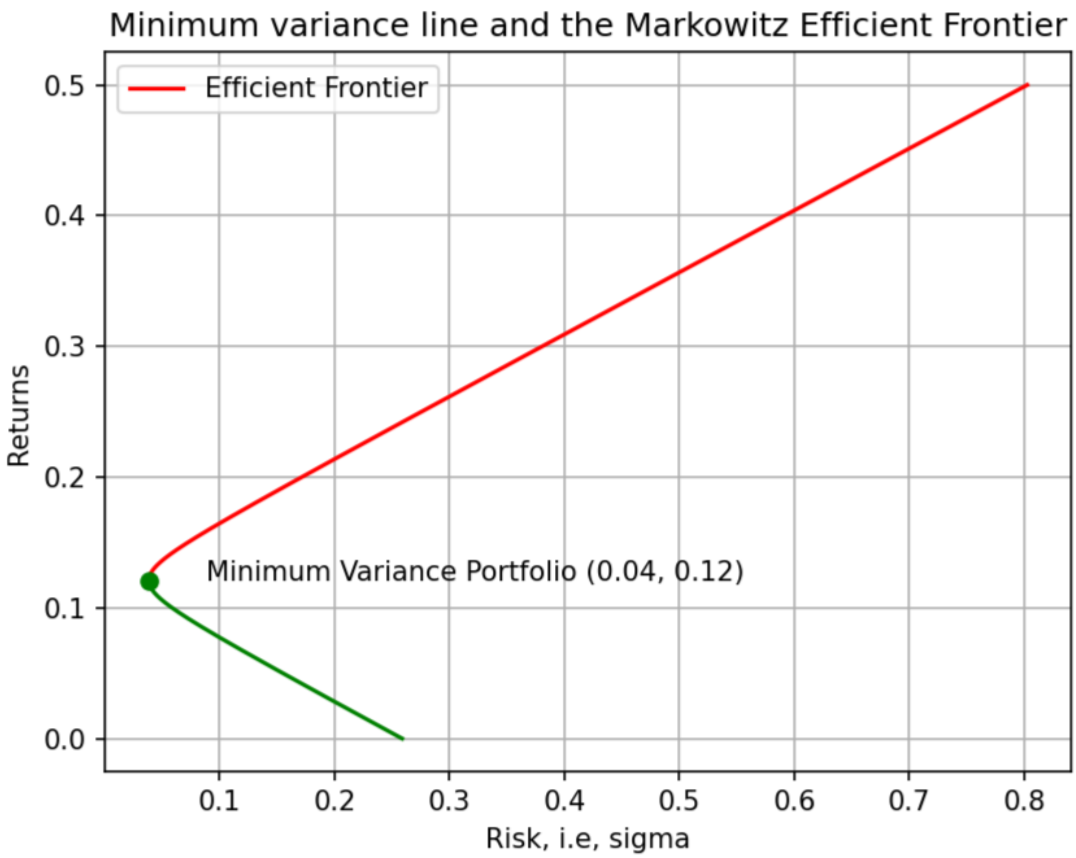
(c) For a 15 % risk, what is the maximum and minimum return and the corresponding portfolios ?

(d) For a 18 % return, what is the minimum risk portfolio ?

(e) Assuming the risk free return $\mu_{rf} = 10\%$, compute the market portfolio. Also determine and plot the Capital Market Line.

(f) Find two portfolios (consisting of both risky and risk free assets) with the risk at 10% and 25%.

a) The Markowitz efficient frontier is shown in the following plot:



The efficient frontier represents a collection of portfolios that offer the maximum expected return for a given level of risk, specifically characterized by lower standard deviation. In essence, it delineates a boundary where portfolios with higher returns and lower risk exist.

b) The weights, risk and return of the portfolios for ten different values on the efficient frontier are tabulated in the following table:

Index	Weights	Return	Risk
1.	[1.83550649, -0.1653936, -0.67011288]	0.04995499549954996	0.02405612017613421
2.	[1.11983859, 0.11903851, -0.2388771]	0.09995999599959997	0.0034570647912315974
3.	[0.40417069, 0.40347062, 0.19235869]	0.14996499649964998	0.005229455948986979
4.	[-0.3114972, 0.68790274, 0.62359447]	0.19996999699969997	0.029373293649400157
5.	[-1.0271651, 0.97233485 , 1.05483025]	0.24997499749975	0.0758885778924713
6.	[-1.742833 , 1.25676696 , 1.48606604]	0.29997999799979996	0.14477530867820082
7.	[-2.4585009, 1.54119907 , 1.91730182]	0.34998499849985	0.23603348600658714
8.	[-3.17416879, 1.82563119 , 2.34853761]	0.3999899989999	0.34966310987763205
9.	[-3.88983669, 2.1100633 , 2.77977339]	0.44999499949995003	0.4856641802913356

10. [-4.60550459, 2.39449541 , 3.21100917] 0.5 0.6440366972476959

c) For a 15% risk,

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Question 1 Part (c)

Minimum return = 0.052455245524552455
Weights of the portfolio for Minimum return = [ 1.79972309 -0.151172   -0.64855109]

Maximum return = 0.1895689568956896
Weights of the portfolio for Maximum return = [-0.16263828  0.62874086  0.53389743]
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d) For a 18% return, the minimum risk portfolio is as follows:

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Question 1 Part (d)

Minimum risk for 18% return = 13.056827100982627 %
Weights of the portfolio = [-0.02568807  0.57431193  0.45137615]
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e) Assuming the risk-free return μ_{rf} = 10%, the market portfolio is:

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Question 1 Part (e)

Weights of Market Portfolio = [0.59375  0.328125 0.078125]
Return = 0.13671875
Risk = 5.081128919221594 %
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The equation of the capital market line is:

$$\mu = \frac{\mu_M - \mu_{rf}}{\sigma_M}\sigma + \mu_{rf}$$

where,

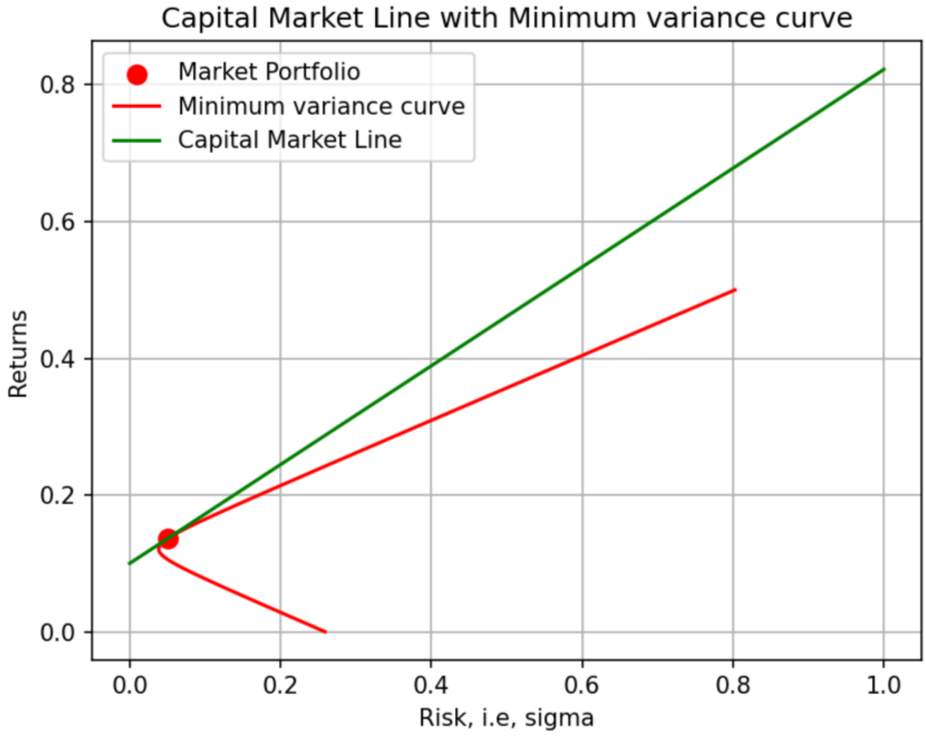
μ_{rf} = risk-free return

μ_M = return corresponding to market portfolio

σ_M = risk corresponding to market portfolio

On putting the values, we get the equation of Capital market line as:

$$\mu = 0.72\sigma + 0.1$$



f)

The required portfolio with risk at 10% is:

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Risk = 10.0 %
Risk-Free Weights = -0.9680665771282883
Risky Weights = [1.16853953 0.64577185 0.1537552 ]
Returns = 0.17226494462892933
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The required portfolio with risk at 25% is:

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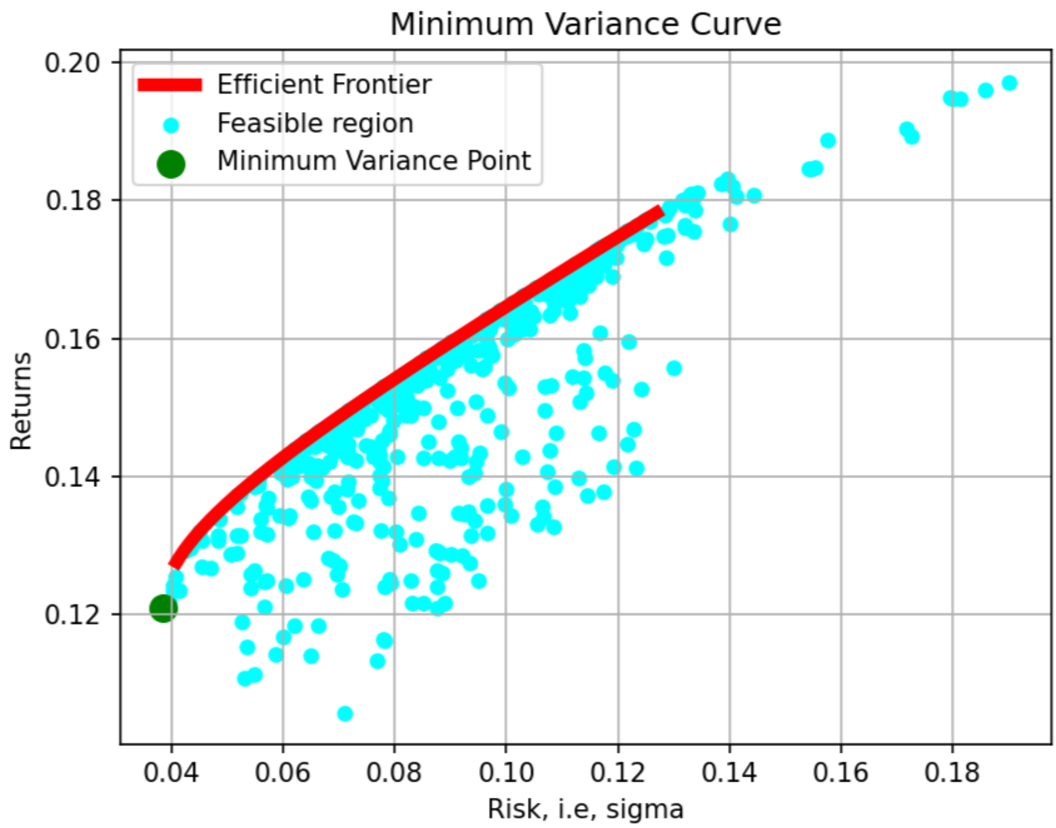
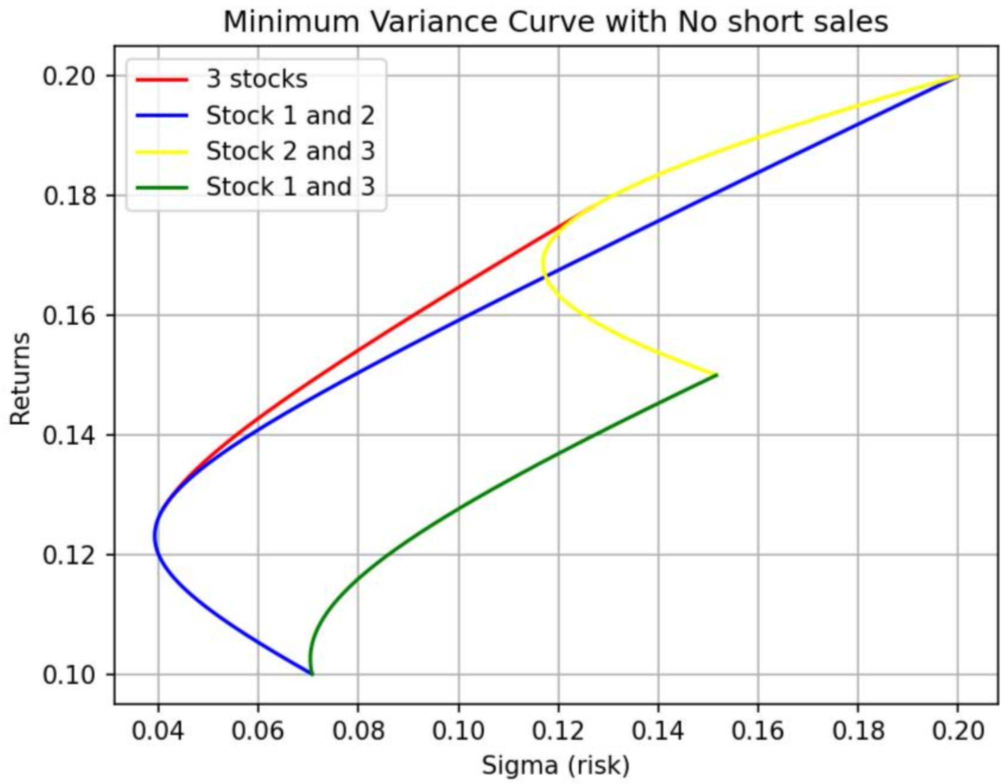
Risk = 25.0 %
Risk-Free Weights = -3.920166442820721
Risky Weights = [2.92134883 1.61442961 0.384388  ]
Returns = 0.280662361572323
```

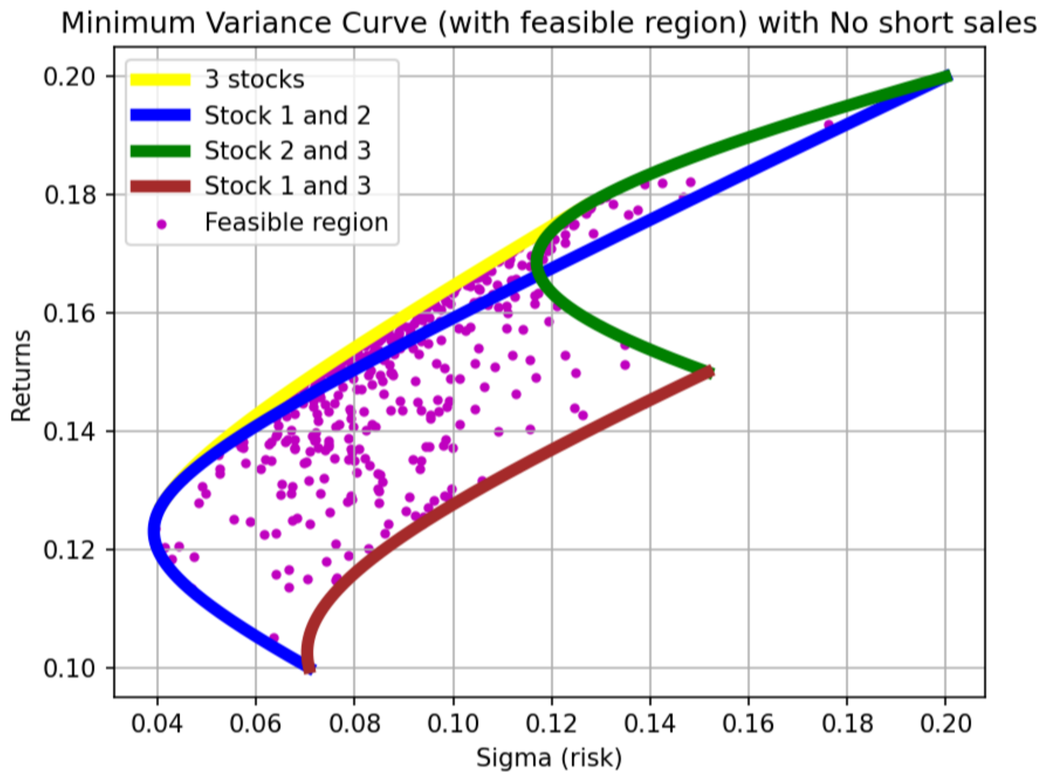
Question 2:

2. Consider the same data given in Problem 1. Now, construct the minimum variance curve (and efficient frontier) and the feasible region (in the risk-return plot) assuming that short sales are not allowed. In the same plot, also indicate the minimum variance curves (there are three of those) if you consider any two out of three securities at a time. Also, in another graph, plot the weights corresponding to the minimum variance curve (and write the equation that these weights satisfy). *[Note: Look at Capinski for more information.]*

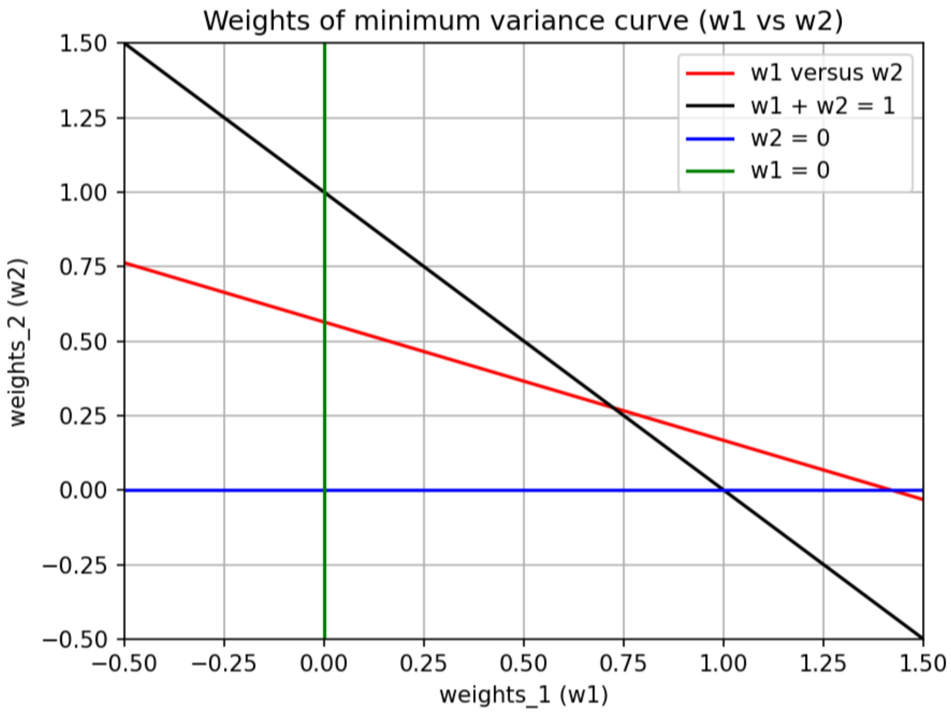
Assuming that the weights are non-negative, that is, short sales are not allowed, the various plots required in the question are as follows:







The plots for the weights corresponding to the minimum variance curve are:

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Equation of line w1 versus w2 is:
w2 = -0.40 w1 + 0.56
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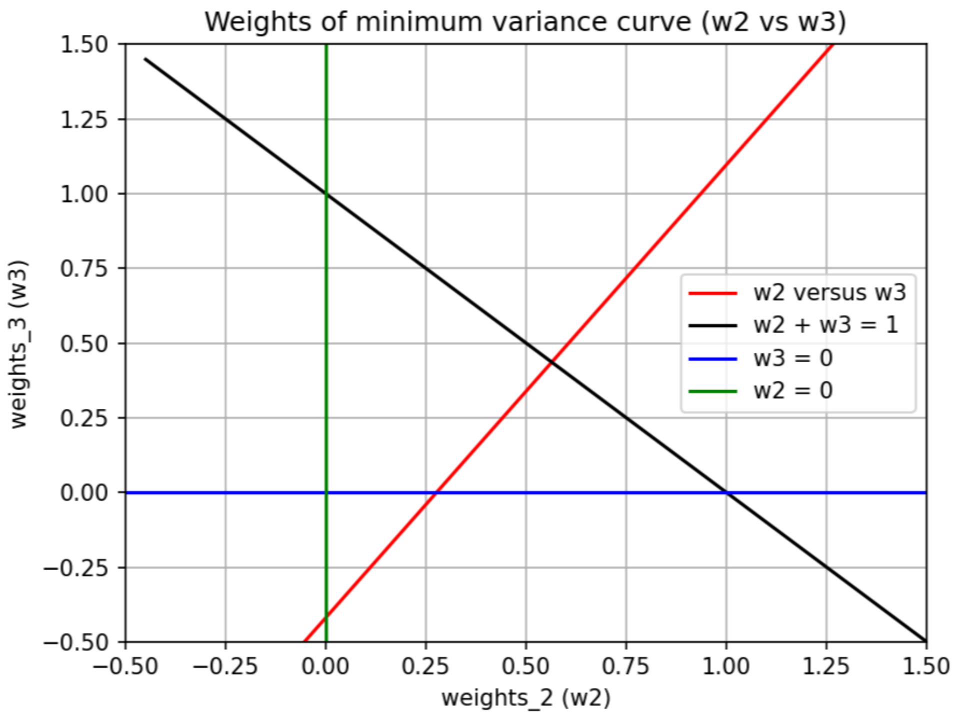

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Equation of line w2 versus w3 is:
w3 = 1.52 w2 + -0.42
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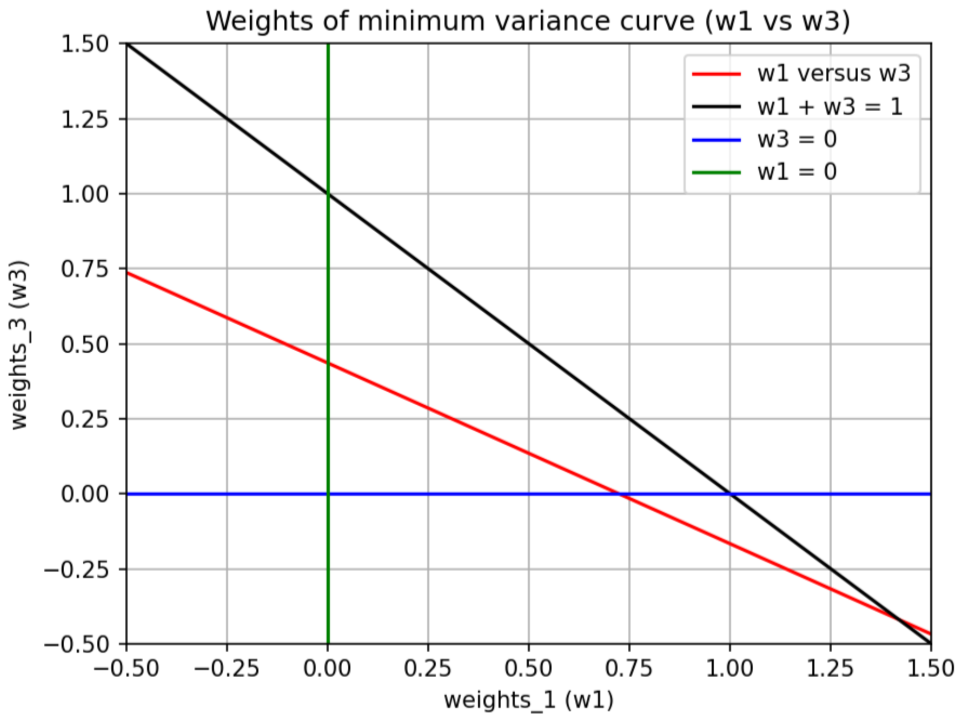


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Equation of line w1 versus w3 is:
w3 = -0.60 w1 + 0.44
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Question 3:

3. Obtain data (from online resources) of monthly prices for 10 stocks each with 60 data points all taken at the same duration. Put this data and it's details in a single Excel/CSV file. Using the data and assuming 5% (*change this, if required*) risk free return:

   (a) Construct and plot the Markowitz efficient frontier.

   (b) Determine the market portfolio.

   (c) Determine and plot the Capital Market Line.

   (d) Determine and plot the Security Market Line for all the 10 stocks.

The data of the monthly prices for 10 stocks has been collected from online resources for the period between 01/02/2019 (February 2019) to 01/01/2024 (January 2024). (total 60 data points, 5 years data, so 12 * 5 = 60 data points).

The companies whose monthly stock prices are considered are as follows: Apple, Google, Amazon, Microsoft, Tesla, Nvidia, PayPal, IBM, Cisco and JP Morgan Chase.

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StockPricesDataArray = ['AAPL', 'GOOGL', 'AMZN', 'MSFT', 'TSLA', 'NVDA','PYPL', 'IBM', 'CSCO', 'JPM']
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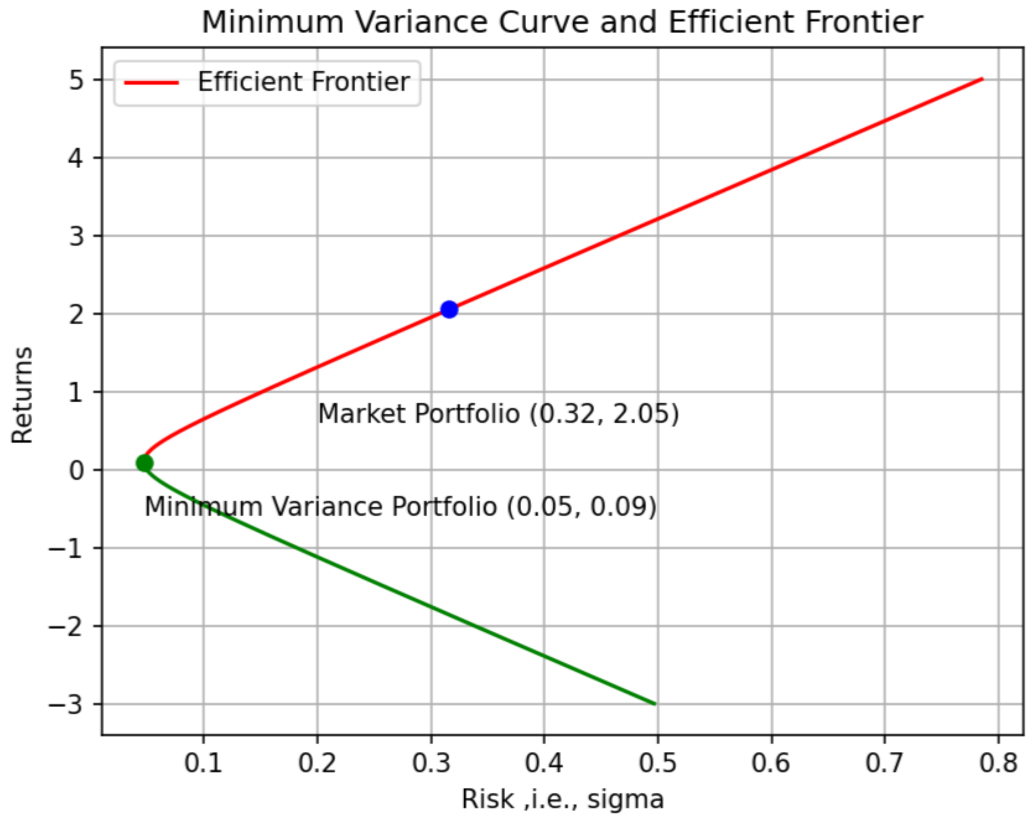
The monthly return was obtained as the difference in stock prices between the stock prices at the beginning of two consecutive months. Then, the annual return was calculated accordingly.

a) The plot of the Markowitz efficient frontier is drawn below:



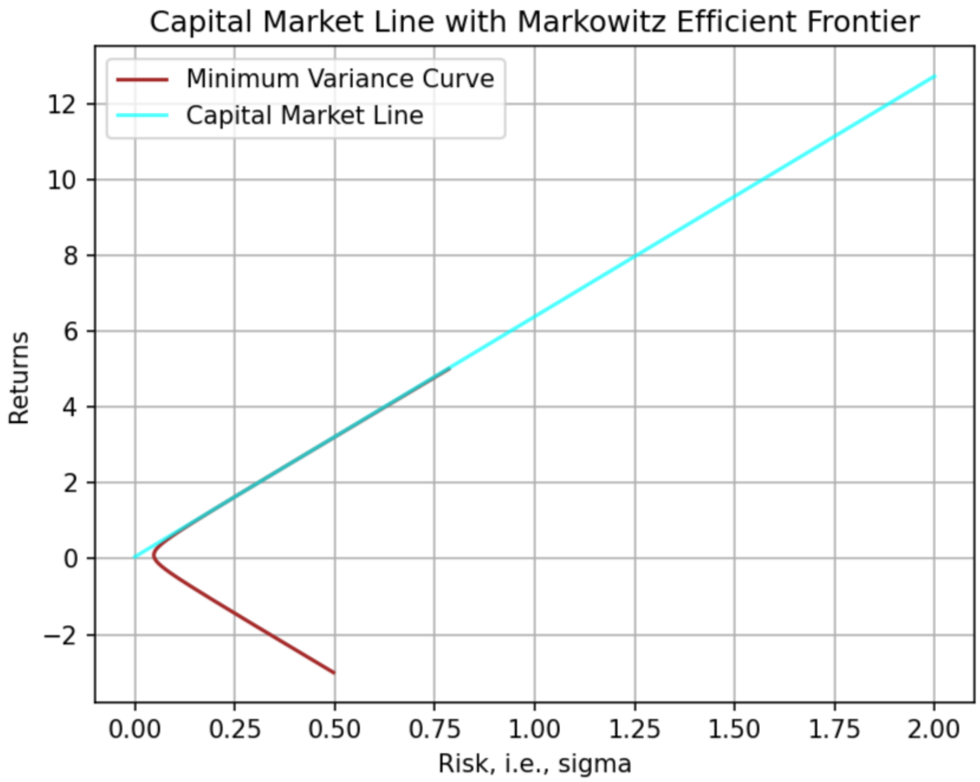
b) The market portfolio is:

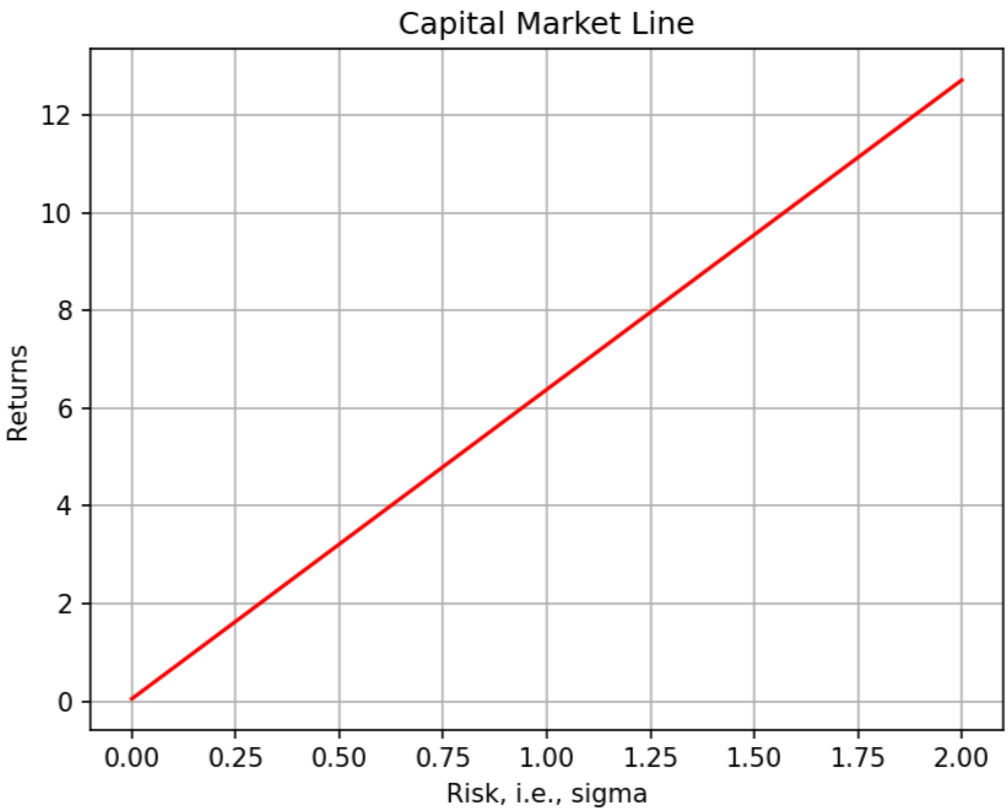
Market Portfolio Weights = [1.420507685472, 0.188262831062, -2.933886890194, 3.101708635950, 0.382610962595, 1.395922786459, -0.815839240010, -0.002396260083, -1.486878577970, -0.250011933282]

Return = 2.051729787457427

Risk = 31.61016740366777 %

c) The equation of the Capital Market Line is: $y = 6.33\,x + 0.05$





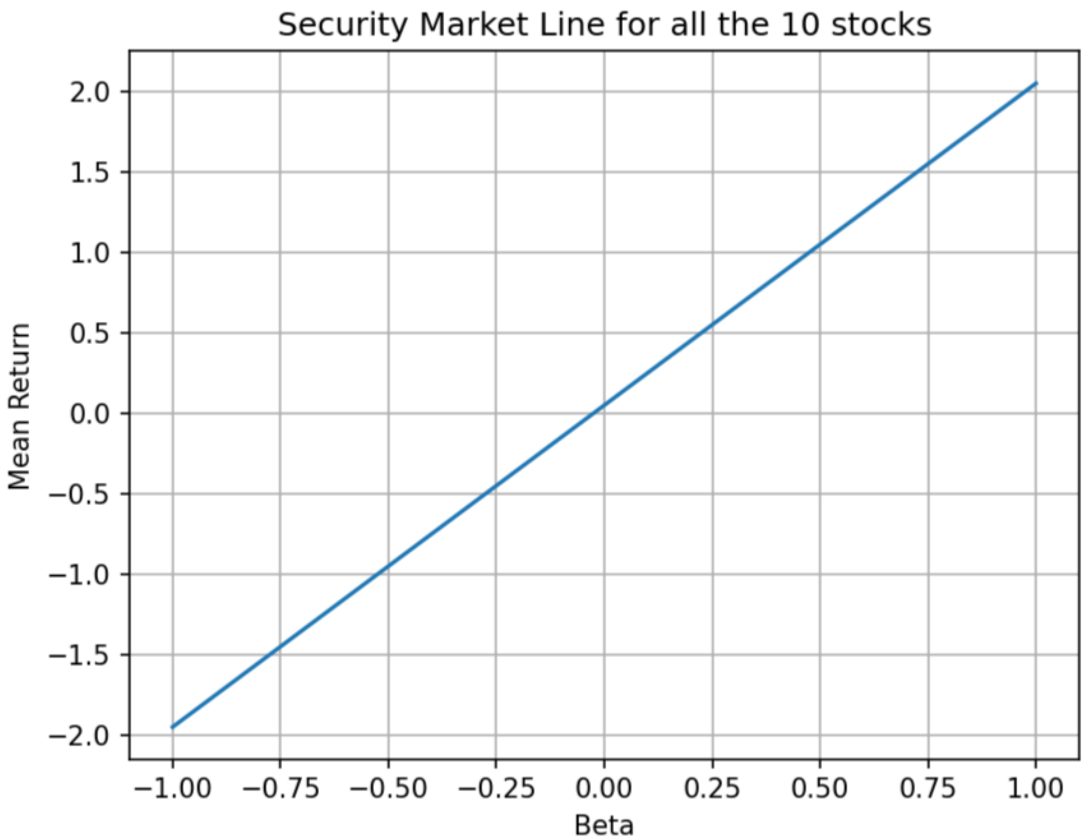
d) The equation of the Security Market Line is obtained by using the following formula:

$$\mu = (\mu_M - \mu_{rf})\beta + \mu_{rf}$$

Where, μ_{rf} = risk-free return and μ_M = return corresponding to market portfolio

So, plugging in the values, the equation of the Security Market Line is:

$$\mu = 2\beta + 0.05$$



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Question 3 Part (c)


Equation of CML is:
y = 6.33 x + 0.05



Question 3 Part (d)

Equation of Security Market Line is:
mu = 2.00 beta + 0.05
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