Monte Carlo Simulation MA – 323 Lab – 11

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Question 1:

1. Generate 5000 random numbers from the following Gaussian mixture PDF:

$$f(x) = \sum_{k=1}^{3} \pi_i \frac{1}{\sigma_i} \phi\left(\frac{x - \mu_i}{\sigma_i}\right), \quad x \in \mathbb{R},$$

where ϕ is the PDF of the standard normal distribution, $(\pi_1, \pi_2, \pi_3) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$, $(\mu_1, \mu_2, \mu_3) = (-1, 0, 1)$, and $(\sigma_1, \sigma_2, \sigma_3) = \left(\frac{1}{4}, 1, \frac{1}{2}\right)$. Find the average of the generated random numbers.

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The number of values generated is: 5000
The average of the generated random numbers is: -0.33903074995708027
The variance of the generated random numbers is: 0.9617461247111657
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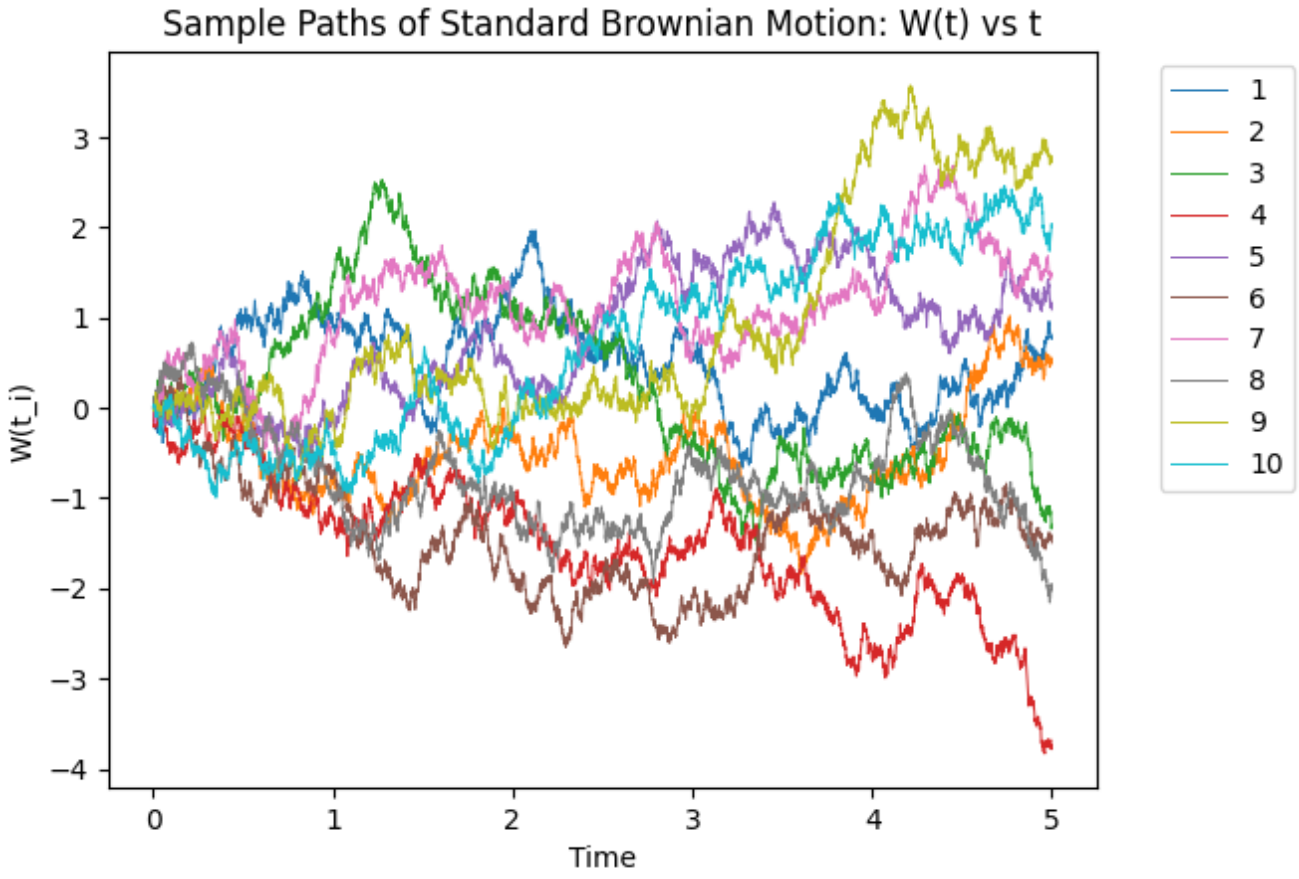
Average of the generated random numbers = -0.33903074995708027

Question 2:

2. Generate 10 sample paths for the standard Brownian Motion in the time interval $[0, 5]$ using the recursion

$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i}\, Z_{i+1},$$

with 5000 generated values for each of the paths. Plot all the sample paths in a single figure. Also estimate $E[W(2)]$ and $E[W(5)]$ from the 10 paths that you have generated.

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The Estimated Value of E(W(2)) is: -0.09242888713094978
The Estimated Value of E(W(5)) is: 0.007978055578923194
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Estimated value of E[W(2)] from the 10 paths that I have generated = -0.09242888713094978

Estimated value of E[W(5)] from the 10 paths that I have generated = 0.007978055578923194

Observations:

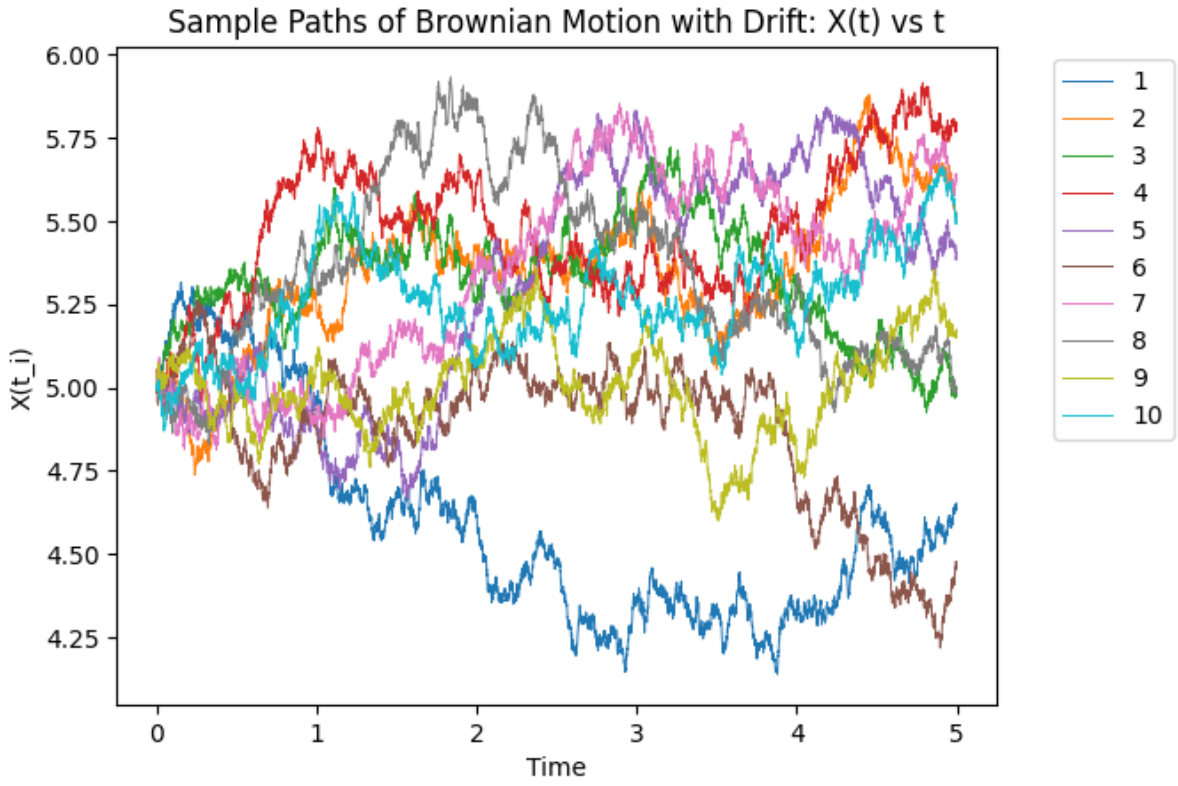
1) All the plots begin from 0 since W (0) = 0 as it is standard Brownian motion.
2) Estimated values of E[W(2)] and E[W(5)] are close to their actual value, that is, 0.

Question 3:

3. Repeat the above exercise with the following Brownian motion ($BM(\mu, \sigma^2)$) discretization

$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i}\, Z_{i+1}.$$

Take $X(0) = 5$, $\mu = 0.06$ and $\sigma = 0.3$.

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The Estimated Value of E(X(2)) is: 5.261201509673581
The Estimated Value of E(X(5)) is: 5.209172138615932
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Estimated value of E[X(2)] from the 10 paths that I have generated = 5.261201509673581

Estimated value of E[X(5)] from the 10 paths that I have generated = 5.209172138615932

Observations:

1) Brownian motion with drift μ and diffusion coefficient σ^2 can be used to model Stock prices. (From my knowledge of Financial Engineering).
2) These plots start from 5 since X(0) = 5