Monte Carlo Simulation MA – 323 Lab – 10

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Question 1:

1. Generate the first 25 values of the Van der Corput sequence $x_1, x_2, \ldots, x_{25}$ using the radical inverse function $x_i := \phi_2(i)$ and list them in your report. Next, generate the first 1000 values of this sequence and plot the overlapping pairs (x_i, x_{i+1}) as a two dimensional plot. What do you observe ?

```
x_1 = 0.5
x_2 = 0.25
x_3 = 0.75
x_4 = 0.125
x_5 = 0.625
x_6 = 0.375
x_7 = 0.875
x_8 = 0.0625
x_9 = 0.5625
x_10 = 0.3125
x_11 = 0.8125
x_12 = 0.1875
x_13 = 0.6875
x_14 = 0.4375
x_15 = 0.9375
x_16 = 0.03125
x_17 = 0.53125
x_18 = 0.28125
x_19 = 0.78125
x_20 = 0.15625
x_21 = 0.65625
x_22 = 0.40625
x_23 = 0.90625
x_24 = 0.09375
x_25 = 0.59375
```

The first 25 values of the Van der Corput sequence using the radical function xi = ∅_2(i) are:

x_1 = 0.5

$x_2 = 0.25$

$x_3 = 0.75$

$x_4 = 0.125$

$x_5 = 0.625$

$x_6 = 0.375$

$x_7 = 0.875$

$x_8 = 0.0625$

$x_9 = 0.5625$

$x_{10} = 0.3125$

$x_{11} = 0.8125$

$x_{12} = 0.1875$

$x_{13} = 0.6875$

$x_{14} = 0.4375$

$x_{15} = 0.9375$

$x_{16} = 0.03125$

$x_{17} = 0.53125$

$x_{18} = 0.28125$

$x_{19} = 0.78125$

$x_{20} = 0.15625$

$x_{21} = 0.65625$

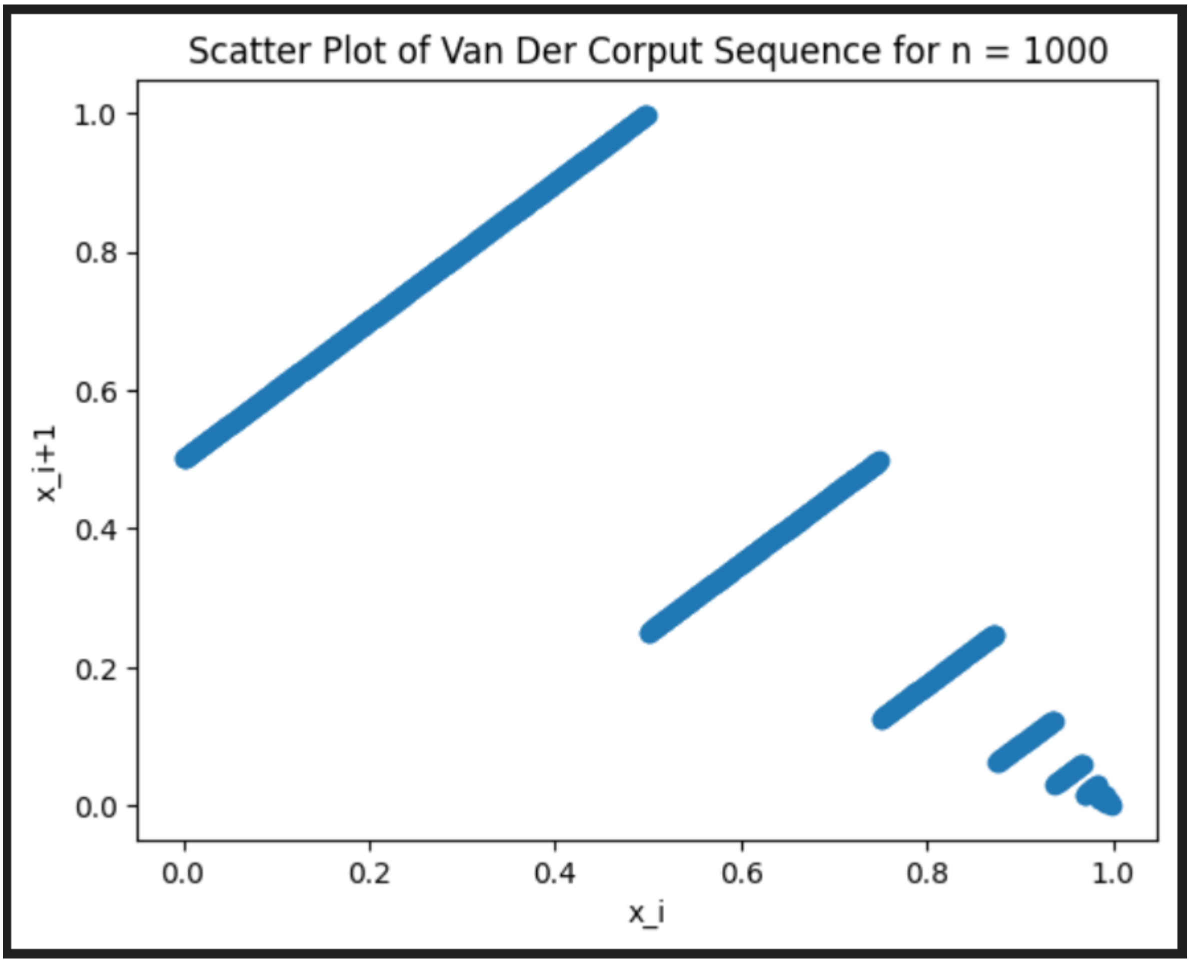
$x_{22} = 0.40625$

$x_{23} = 0.90625$

$x_{24} = 0.09375$

$x_{25} = 0.59375$

The plot for the first 1000 values in the form of (x_i, x_{i+1}) is:



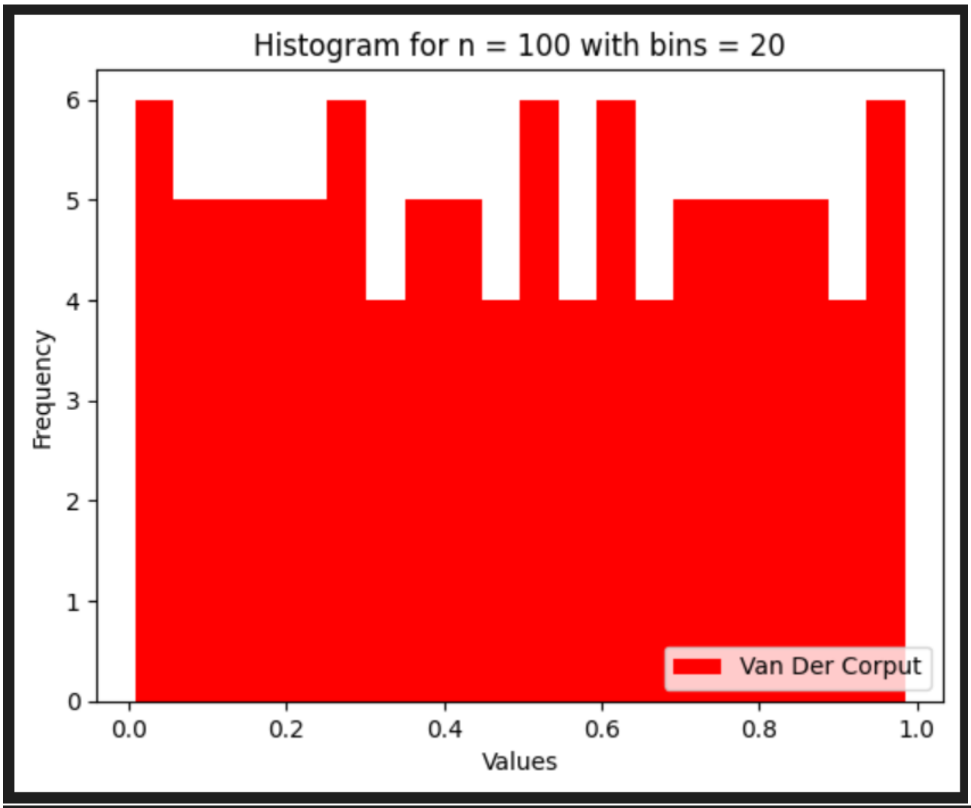
Observations:

1) The plot is not completely random, and a specific pattern can be observed from it. The points are not uniformly distributed.
2) After plotting the graph of (x_i,x_i+1) for 1000 values, we observe that the points are in some sort of line, the number of lines is equal to greatest integer of $\log_2$(max value) because when we increase the numbers by 1, then, in the case of base 2, there is some bit of the number which is changed, and there are 10 (that is, from 0 to 9) possible bits in this case, then there is a different change for each of them when they change. So, there are 9 lines each corresponding to each bit. Also, I found that this plot helps in what is known as "Spectral Test".

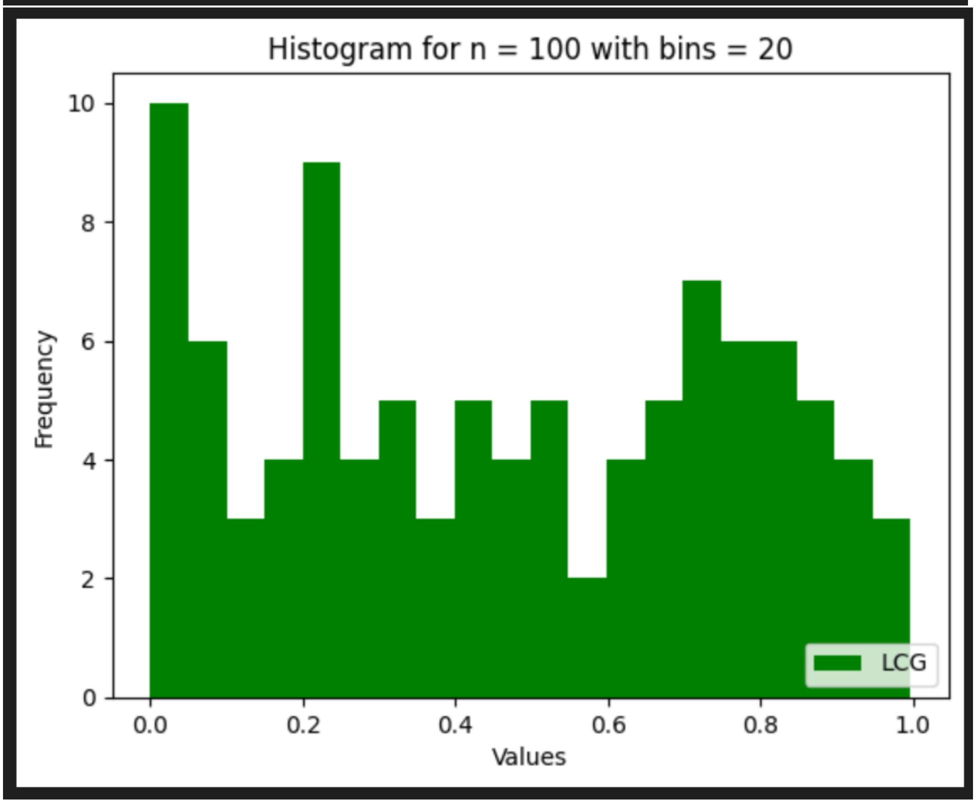
Question 2:

2. Generate first 100 and 100000 values of Van der Corput sequence and plot histogram for both the cases. Compare these plots with the histogram of 100 and 100000 values generated by an LCG, by plotting the sampled distributions in two graphs side by side for both the cases. Specify the LCG that you have used.

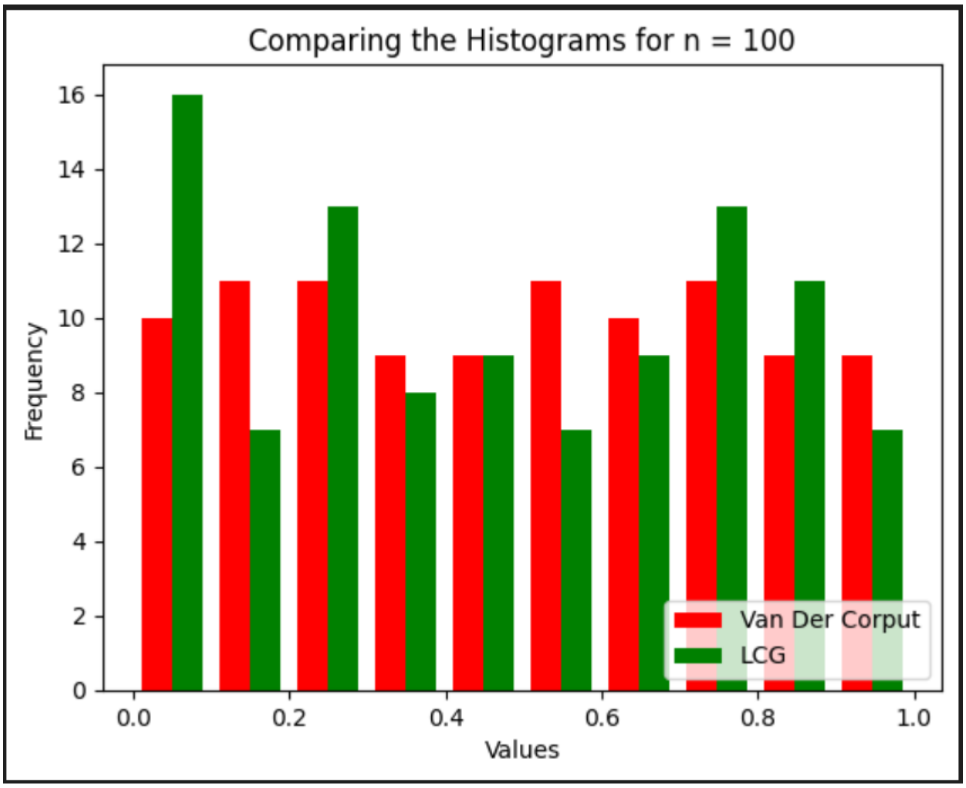
For n = 100,



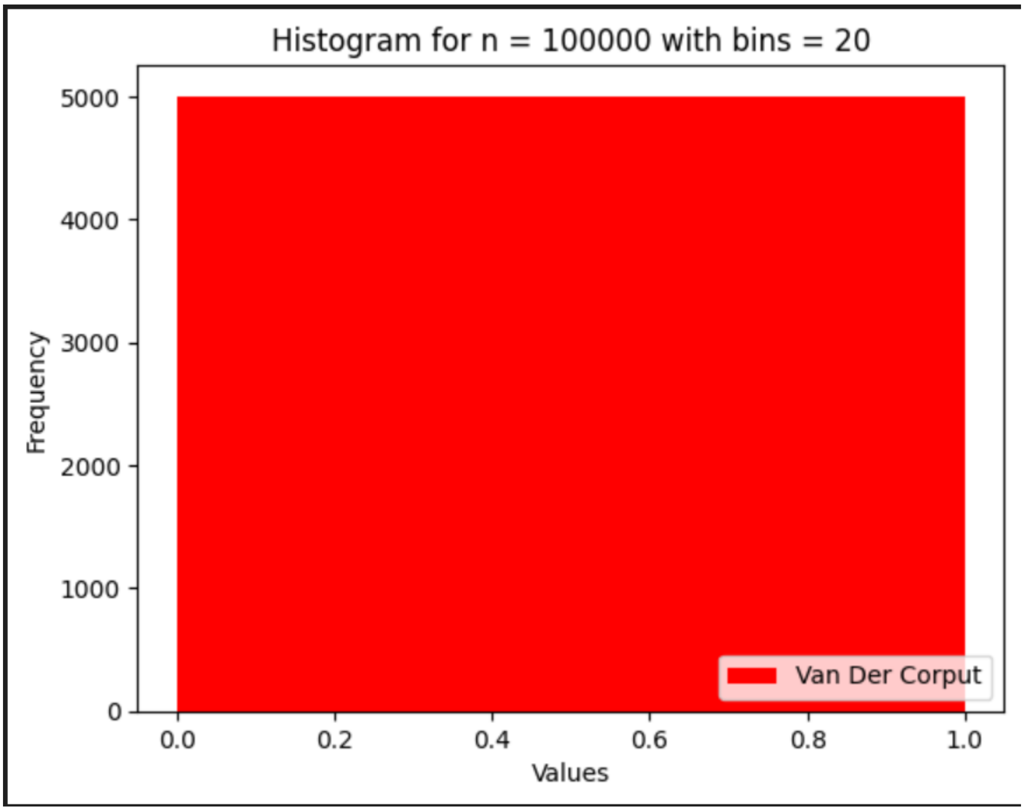
Histogram for n = 100 with bins = 20
Frequency
Values
Van Der Corput
0
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
1.0

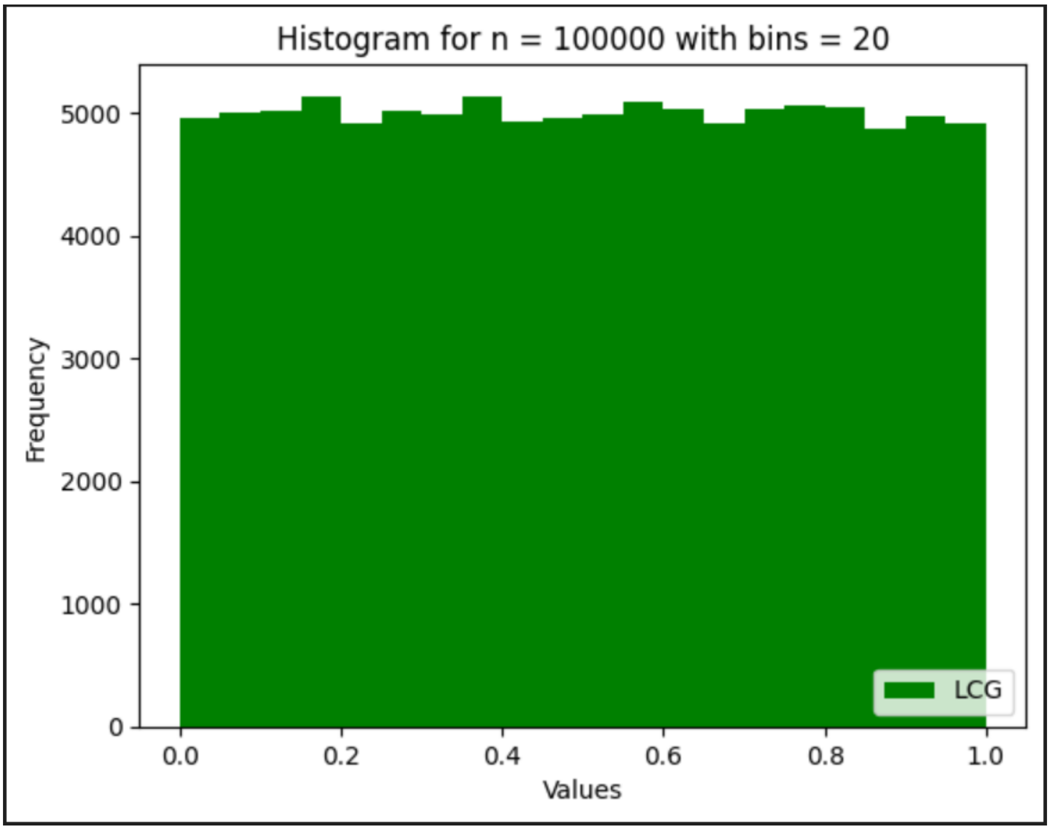


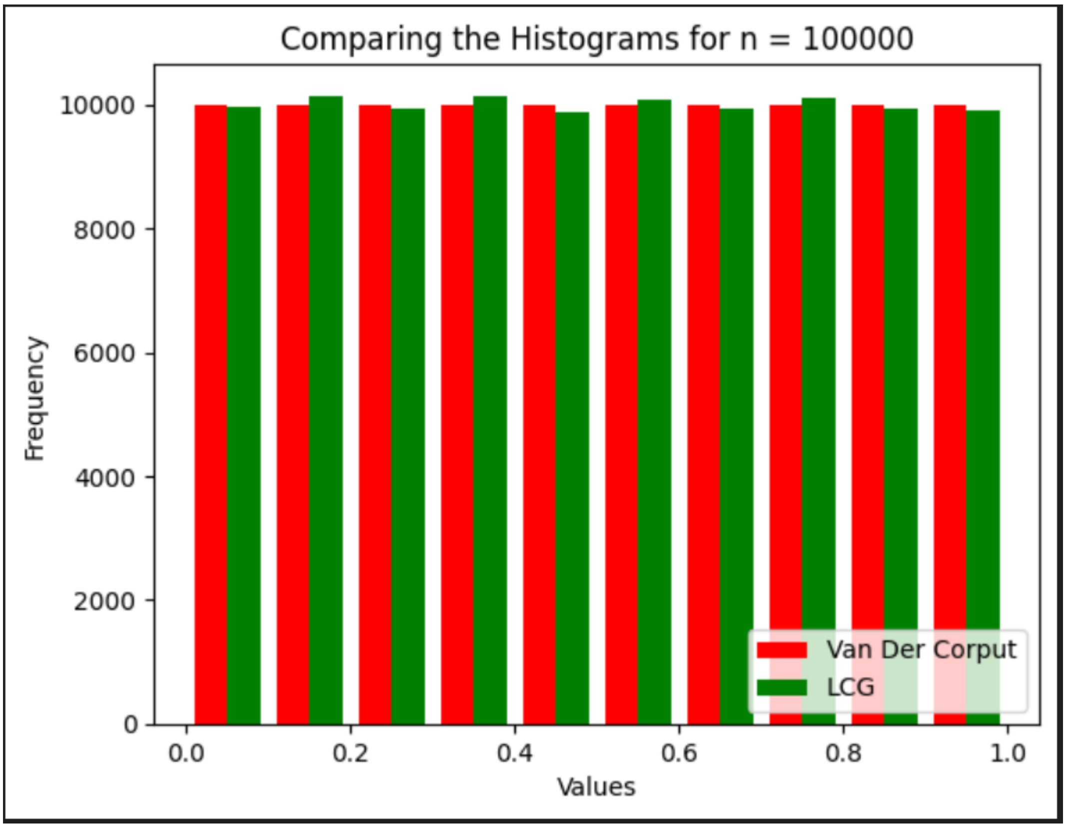
Histogram for n = 100 with bins = 20
Frequency
Values
LCG
0
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0


Comparing the Histograms for n = 100
Frequency
Values
Van Der Corput
LCG
0
2
4
6
8
10
12
14
16
0.0
0.2
0.4
0.6
0.8
1.0

For n = 100000,


Histogram for n = 100000 with bins = 20
Frequency
Values
Van Der Corput
0
1000
2000
3000
4000
5000
0.0
0.2
0.4
0.6
0.8
1.0





Linear Congruence Generator Used:

Seed $x_0 = 3$

Constant to be added = c = 8

m = 1000000007

a = 12

The LCG used is with seed value 3, the multiplicative factor, a is 12, the constant to be added, c is 8, and the number from which mod is taken, m is 1000000007.

$x_{i+1} = (a * x_i + c) \bmod m$

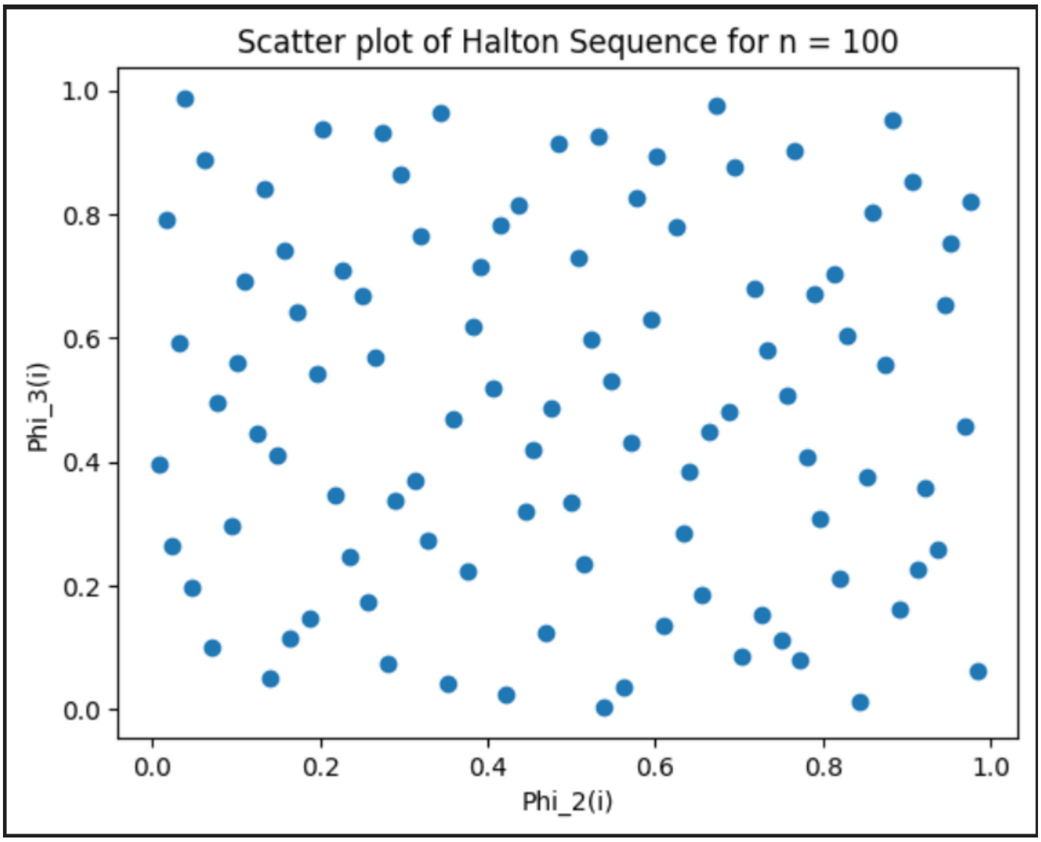
$u_{i+1} = (x_{i+1})/m$ = the value finally generated

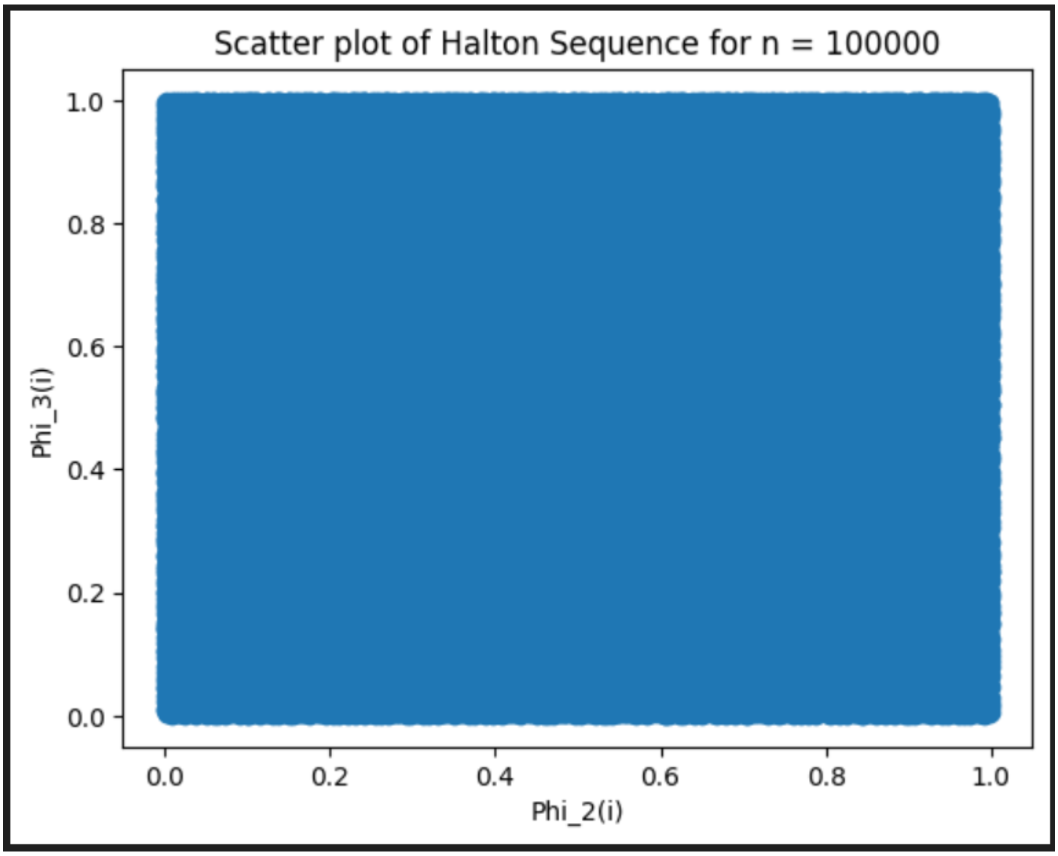
Observations:

1) For a small value of n i.e. 100, there is some significant variation in LCG, but Van Der Corput gives much less variation.
2) For n=100000, almost all the bars are of same size in Van Der Corput, but there is a slight variation in LCG, so we can say that if we want almost perfectly uniform numbers, then Van Der Corput is to be chosen over LCG.
3) We can observe that the plot for the Van der Corput sequence is more uniformly distributed than that of the LCG used.
4) In both the plots of Van der Corput sequence, the proportional of points belonging to a fixed interval is almost proportional to the length of the interval.

Question 3:

3. Generate the Halton sequence $x_i = (\phi_2(i), \phi_3(i))$ (as points in $\mathbb{R}^2$) and plot the first 100 and 100000 values. What are your observations?





Observations:

1) The points are uniformly scattered all over the cartesian plane, which is also evident from the fact that Halton sequence is generated using Van Der Corput sequence, and in Van Der Corput sequence, numbers are generated very uniformly, so the base 2 numbers give uniformity in x-coordinates and base 3 numbers generated give uniformity in y-coordinates.
2) We can observe that the points are more uniformally and equi-distantly located in the R 2 plane. They completely cover the whole region.
3) So, this sequence generates the required set of points which are used in the Quasi-Monte Carlo simulation.
4) For n=100, the plot is less dense but shows no identifiable pattern.
5) For n=100000, the unit square is completely filled.
6) Both the plots fill the whole hypercube in R^2, which is the unit square, because 2 and 3 are relatively prime to each other.
7) Halton Sequence mimics Uniform distribution and it is a low-discrepancy sequence.
8) In both the plots, the density of the points appears to be uniform.