<u>Monte Carlo Simulation</u>   <u>MA – 323</u>   <u>Lab – 1</u>

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All the following problems are for the following general linear congruence generator:

$$x_{i+1} = (ax_i + b) \bmod m$$
$$u_{i+1} = x_{i+1}/m.$$

<u>Question 1</u>:

Generate the sequence of numbers x_i for $a = 6,\ b = 0,\ m = 11$, and x_0 ranging from 0 to 10. Also, generate the sequence of numbers x_i for $a = 3,\ b = 0,\ m = 11$, and x_0 ranging from 0 to 10. Observe the sequence of numbers generated and observe the repetition of values. Tabulate these for each group of values. How many distinct values appear before repetitions? Which, in your opinion, are the best choices and why?

-> **Sequence of x_i for a = 6, b = 0, and m = 11:**

Sequence / Seeds(x0)	1st value	2nd value	3rd value	4th value	5th value	6th value	7th value	8th value	9th value	10th value
0	0	0	0	0	0	0	0	0	0	0
1	6	3	7	9	10	5	8	4	2	1
2	1	6	3	7	9	10	5	8	4	2
3	7	9	10	5	8	4	2	1	6	3
4	2	1	6	3	7	9	10	5	8	4
5	8	4	2	1	6	3	7	9	10	5
6	3	7	9	10	5	8	4	2	1	6
7	9	10	5	8	4	2	1	6	3	7
8	4	2	1	6	3	7	9	10	5	8
9	10	5	8	4	2	1	6	3	7	9
10	5	8	4	2	1	6	3	7	9	10

When seed $x_0 = 0$, only 1 distinct value, i.e., 0 appears in the sequence x_i, which goes on repeating, and hence it has no randomness in it.

For seeds x_0 = 1 to 10, ***10* distinct values** from 1 to 10 appear in the sequence for each seed before the sequence starts to repeat itself, that is, Period Length = 10 = m-1 = maximum possible period length.

-> **Sequence of x_i for a = 3, b = 0, and m = 11:**

Sequence / Seeds(x0)	1st value	2nd value	3rd value	4th value	5th value
0	0	0	0	0	0
1	3	9	5	4	1
2	6	7	10	8	2
3	9	5	4	1	3
4	1	3	9	5	4
5	4	1	3	9	5
6	7	10	8	2	6
7	10	8	2	6	7
8	2	6	7	10	8
9	5	4	1	3	9
10	8	2	6	7	10

When seed x_0 = 0, only 1 distinct value, i.e., 0 appears in the sequence x_i, which goes on repeating, and hence it has no randomness in it.

For seeds x_0 = 1 to 10, ***5* distinct values** from 1 to 10 appear in the sequence for each seed before the sequence starts to repeat itself, that is, Period Length = 5.

Best Choice:

We know that the largest possible period length of a Linear Congruence Generator is m – 1, which is equal to 11 – 1 = 10 in this question. This value is achieved when a = 6 (We get the full period length of 10), whereas the period length for a = 3 is only 5.

So, the linear congruence generator with a = 6 is the better choice over a = 3 as it has higher period length (where b=0 and m=11). This is because there will be more randomness in the generated numbers as there are more numbers in the generated sequence.

And x_0 (seed) should be a non-zero value, as seed $x_0 = 0$ has no randomness in it.

Output Screenshot:

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The Linear Congruence Generator is:
x_i+1 = (a * x_i + b) mod m
u_i+1 = x_i+1 / m

Question 1 :

For a = 6, b = 0 and m = 11,

The Sequence of Numbers x_i with x_0 ranging from 0 to 10 are as follows:
seed x0 = 0, Sequence of Numbers = [0]
seed x0 = 1, Sequence of Numbers = [6, 3, 7, 9, 10, 5, 8, 4, 2, 1]
seed x0 = 2, Sequence of Numbers = [1, 6, 3, 7, 9, 10, 5, 8, 4, 2]
seed x0 = 3, Sequence of Numbers = [7, 9, 10, 5, 8, 4, 2, 1, 6, 3]
seed x0 = 4, Sequence of Numbers = [2, 1, 6, 3, 7, 9, 10, 5, 8, 4]
seed x0 = 5, Sequence of Numbers = [8, 4, 2, 1, 6, 3, 7, 9, 10, 5]
seed x0 = 6, Sequence of Numbers = [3, 7, 9, 10, 5, 8, 4, 2, 1, 6]
seed x0 = 7, Sequence of Numbers = [9, 10, 5, 8, 4, 2, 1, 6, 3, 7]
seed x0 = 8, Sequence of Numbers = [4, 2, 1, 6, 3, 7, 9, 10, 5, 8]
seed x0 = 9, Sequence of Numbers = [10, 5, 8, 4, 2, 1, 6, 3, 7, 9]
seed x0 = 10, Sequence of Numbers = [5, 8, 4, 2, 1, 6, 3, 7, 9, 10]

For a = 3, b = 0 and m = 11,

The Sequence of Numbers x_i with x_0 ranging from 0 to 10 are as follows:
seed x0 = 0, Sequence of Numbers = [0]
seed x0 = 1, Sequence of Numbers = [3, 9, 5, 4, 1]
seed x0 = 2, Sequence of Numbers = [6, 7, 10, 8, 2]
seed x0 = 3, Sequence of Numbers = [9, 5, 4, 1, 3]
seed x0 = 4, Sequence of Numbers = [1, 3, 9, 5, 4]
seed x0 = 5, Sequence of Numbers = [4, 1, 3, 9, 5]
seed x0 = 6, Sequence of Numbers = [7, 10, 8, 2, 6]
seed x0 = 7, Sequence of Numbers = [10, 8, 2, 6, 7]
seed x0 = 8, Sequence of Numbers = [2, 6, 7, 10, 8]
seed x0 = 9, Sequence of Numbers = [5, 4, 1, 3, 9]
seed x0 = 10, Sequence of Numbers = [8, 2, 6, 7, 10]
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Question 2:

Generate a sequence u_i, $i = 1, 2, \ldots, 10000$ with $m = 244944$, $a = 1597, 51749$ (choosing x_0 as per your choice). Then group the values in the ranges

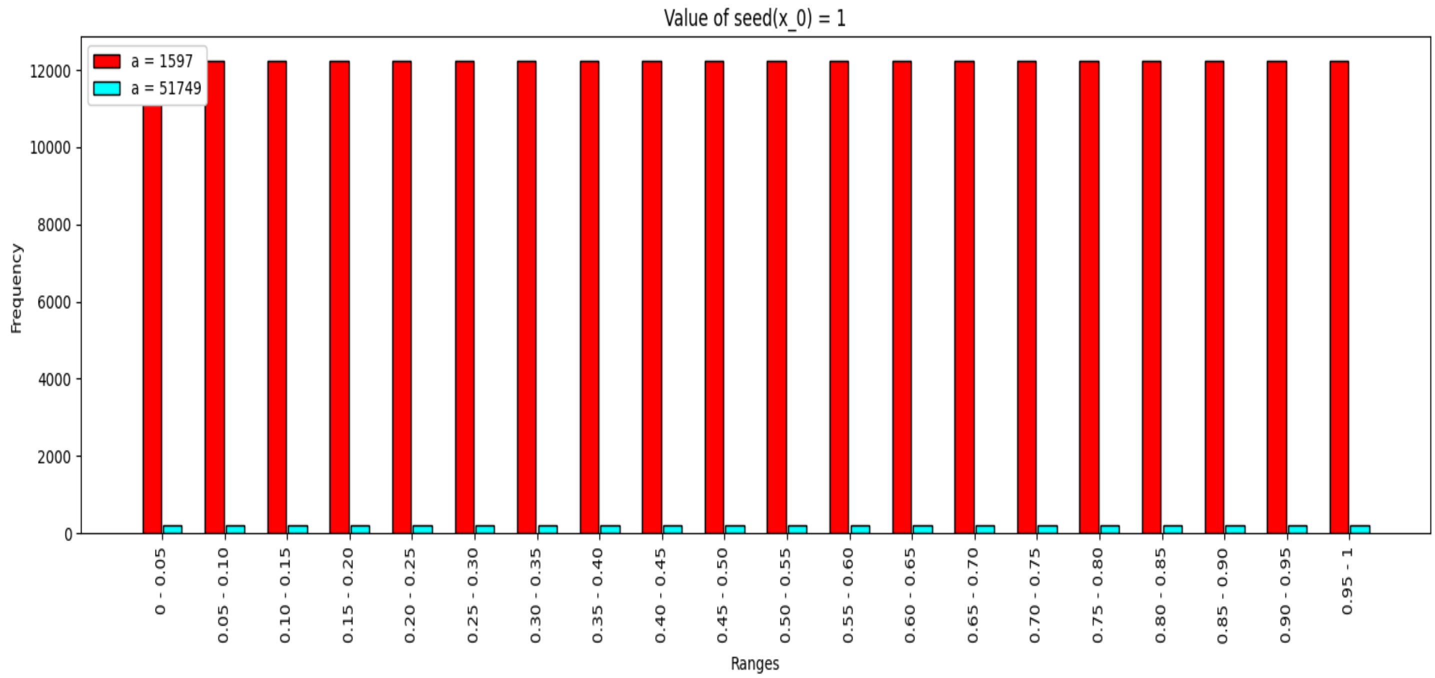
$$[0,\ 0.05),\ [0.05, 0.10),\ [0.10,\ 0.15),\ \ldots,\ [0.95,\ 1)$$

and observe their frequencies (*i.e.*, the number of values falling in each group). For 5 different x_0 values, tabulate the frequencies in each case, draw the bar diagrams for these data and put in your observations.

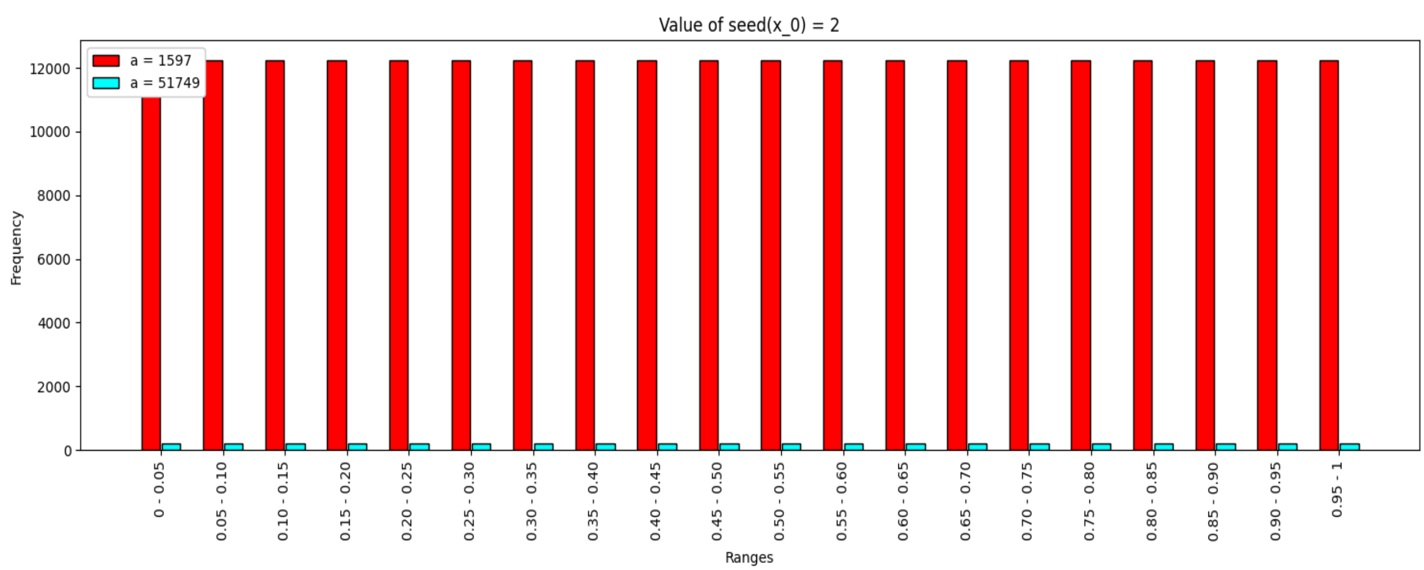
Table of Frequency of numbers(u_i) in different intervals as mentioned in the question:

	a = 1597, b = 1, m = 244944					a = 51749, b = 1, m = 244944				
Seeds / Frequency	$X_0 = 1$	$X_0 = 2$	$X_0 = 3$	$X_0 = 4$	$X_0 = 6$	$X_0 = 1$	$X_0 = 2$	$X_0 = 3$	$X_0 = 4$	$X_0 = 6$
0.00 – 0.05	12247	12247	12247	12247	12247	195	195	194	195	195
0.05 – 0.10	12247	12247	12247	12247	12247	194	194	193	194	194
0.10 – 0.15	12248	12248	12248	12248	12248	195	195	196	194	194
0.15 – 0.20	12247	12247	12247	12247	12247	193	193	193	195	194
0.20 – 0.25	12247	12247	12247	12247	12247	195	195	196	194	195
0.25 – 0.30	12247	12247	12247	12247	12247	195	195	194	195	195
0.30 – 0.35	12247	12247	12247	12247	12247	194	194	193	194	194
0.35 – 0.40	12247	12247	12247	12247	12247	194	194	196	194	194
0.40 – 0.45	12248	12248	12248	12248	12248	194	194	193	195	194
0.45 – 0.50	12247	12247	12247	12247	12247	195	195	196	194	195
0.50 – 0.55	12247	12247	12247	12247	12247	195	195	194	195	195
0.55 – 0.60	12247	12247	12247	12247	12247	194	194	193	194	194
0.60 – 0.65	12248	12248	12248	12248	12248	195	195	196	194	194
0.65 – 0.70	12247	12247	12247	12247	12247	193	193	193	195	194
0.70 – 0.75	12247	12247	12247	12247	12247	195	195	196	194	195
0.75 – 0.80	12247	12247	12247	12247	12247	195	195	194	195	195
0.80 – 0.85	12247	12247	12247	12247	12247	194	194	193	194	194
0.85 – 0.90	12247	12247	12247	12247	12247	194	194	196	194	194
0.90 – 0.95	12248	12248	12248	12248	12248	194	194	193	195	194
0.95 – 1.00	12247	12247	12247	12247	12247	195	195	196	194	195

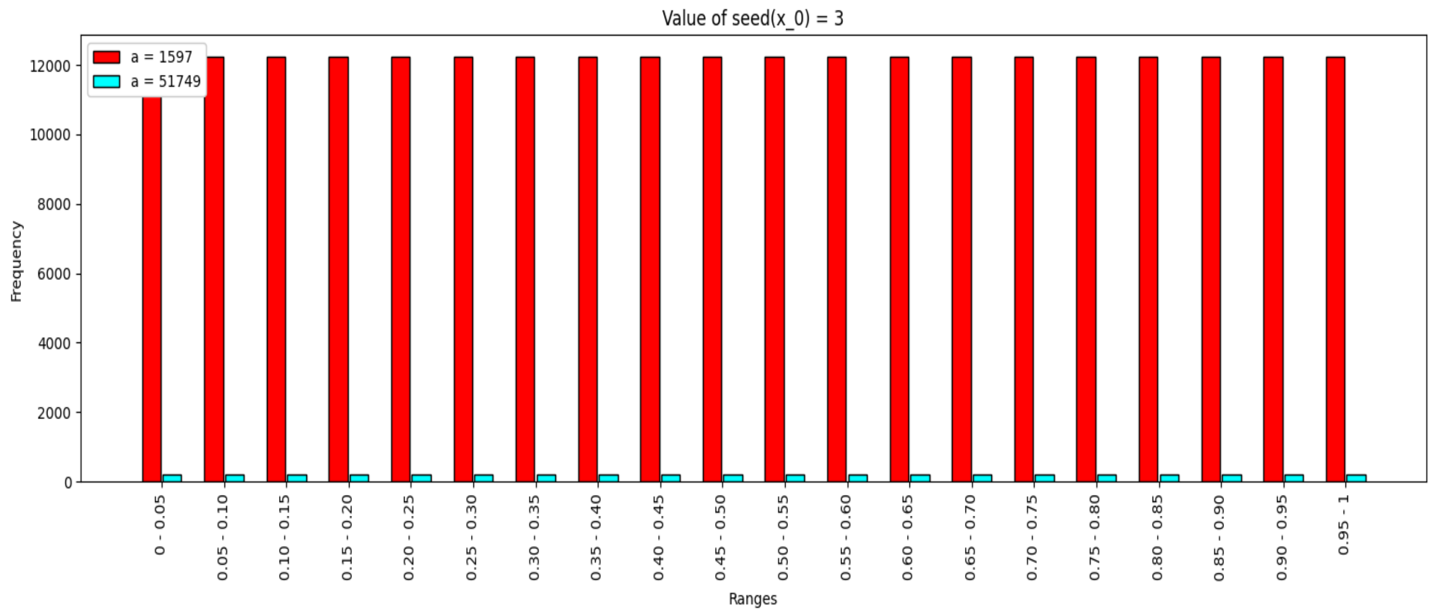
Bar Diagram for a = 1597,51749, m = 244944, b = 1:



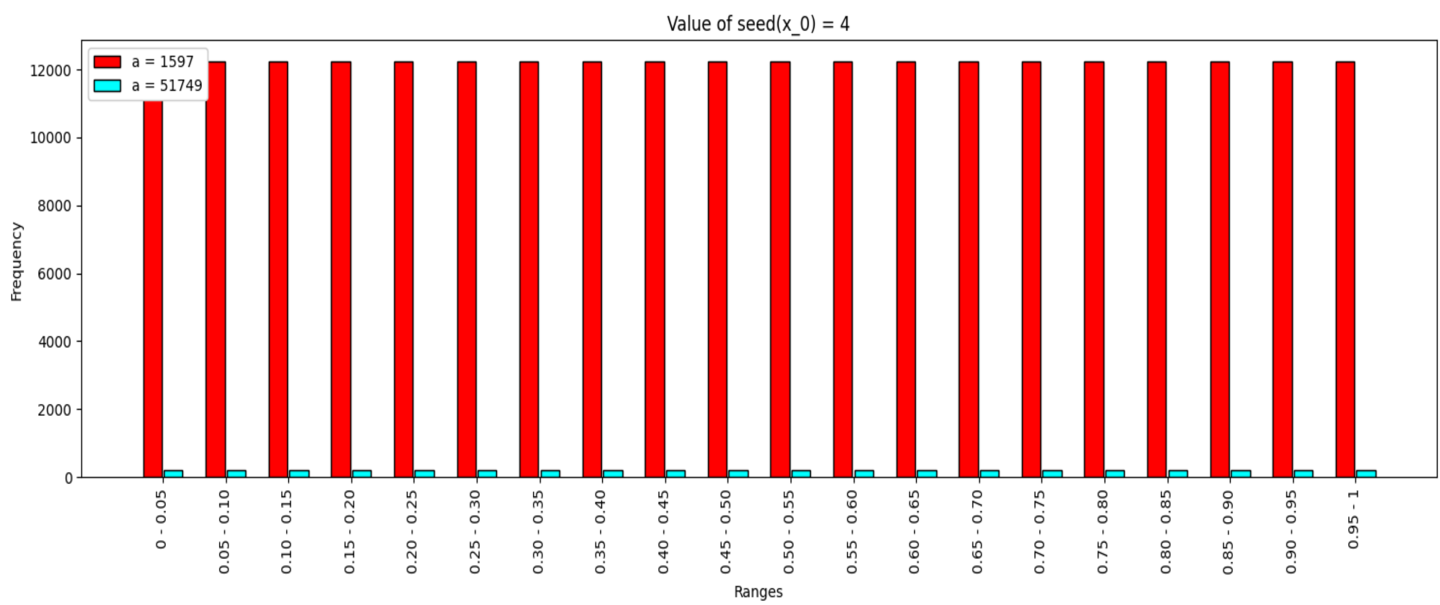
Bar Diagram for a = 1597, 51749, m = 244944, b = 1:



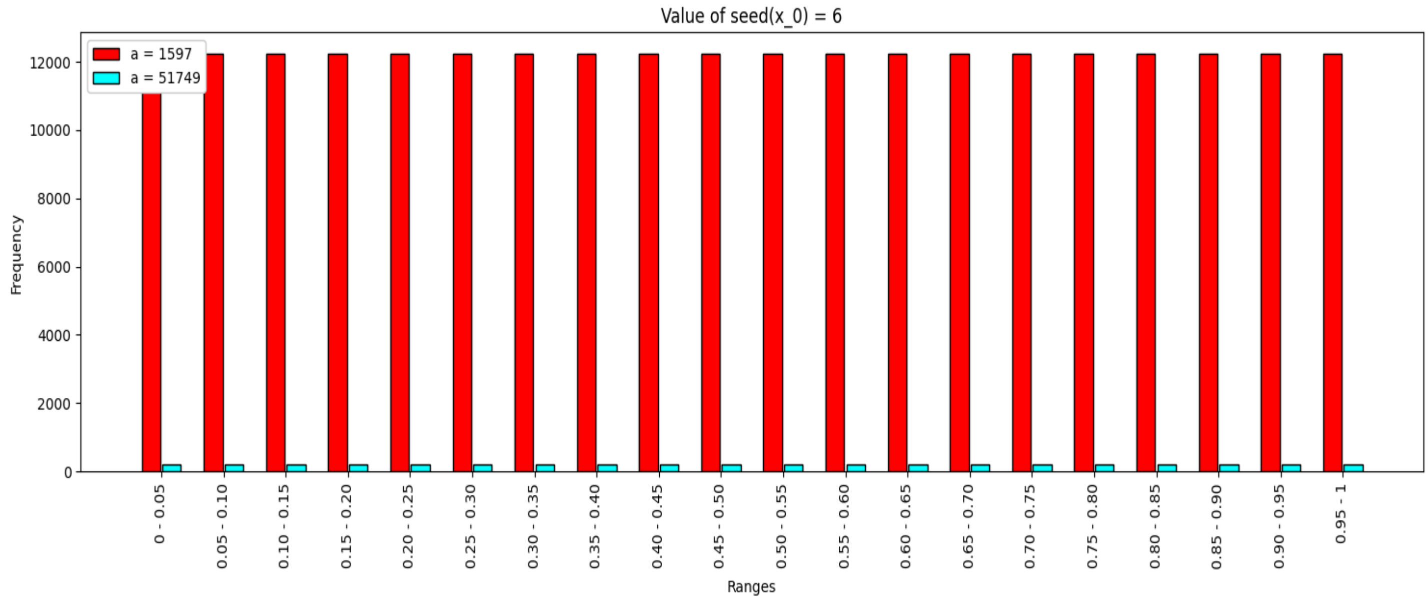
Bar Diagram for a = 1597, 51749, m = 244944, b = 1:



Bar Diagram for a = 1597, 51749, m = 244944, b = 1:



Bar Diagram for a = 1597, 51749, m = 244944, b = 1:



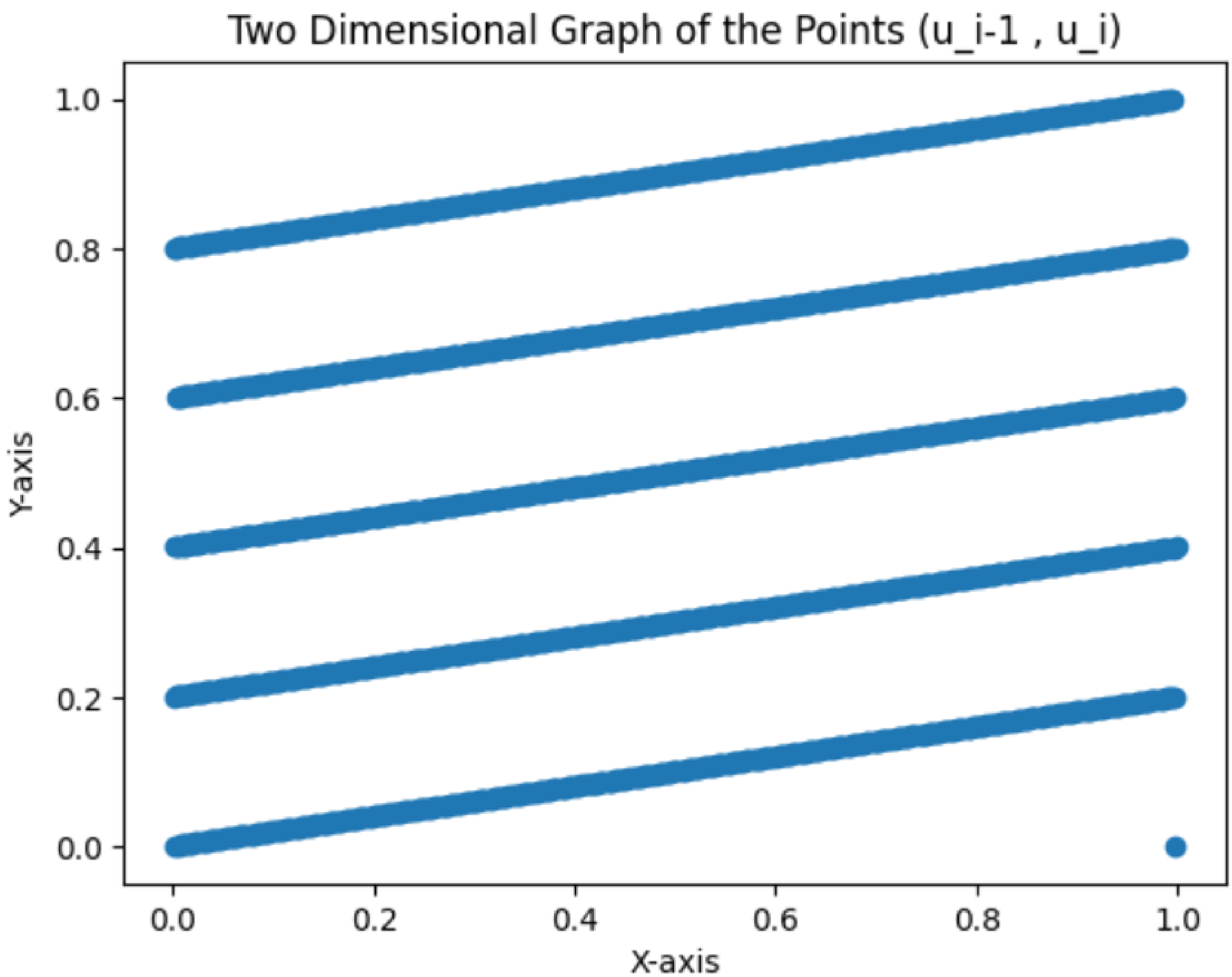
Observations:

- For different distinct values of the seed x_0, the frequencies, that is, the number of values falling in each group, are almost the same. Hence, the nature of the bar graphs is identical.
- When a = 1597, b = 1, m = 244944, Period Length of the Linear Congruence Generator = m-1, that is, the Linear Congruence Generator achieves its full period. But when a = 51749, b = 1, m = 244944, Period Length < m-1; So, the Linear Congruence Generator does not achieve its full period.
- The numbers are uniformly generated between 0 and 1. The frequencies of different numbers lying in intervals of the same length are almost the same. So, this Linear Congruence random number generator follows the property of generation of random numbers uniformly.

Question 3:

Generate a sequence u_i, $i = 1, 2, \ldots, 10000$ with $a = 1229$, $b = 1$, $m = 2048$. Plot in a two-dimensional graph the points (u_{i-1}, u_i), *i.e.*, the points (u_1, u_2), (u_2, u_3), (u_3, u_4), ...

Two – dimensional graph of the points (u_{i-1}, u_i) with seed x_0 = 1 and a=1229, b=1, and m=2048:



Observations:

- The two – dimensional plot contains 5 almost parallel lines originating at different Y – coordinates.
- There is an unusual/strange data point present at x = 1 (approximately). I think that this is present due to the precision issues while taking modulus operation in Python.

Output Screenshot:

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The Linear Congruence Generator is:
x_i+1 = (a * x_i + b) mod m
u_i+1 = x_i+1 / m

Question 3 :

Length =  2047
Sequence 1 =  [0.6005859375, 0.12060546875, 0.224609375, 0.04541015625, 0.8095703125, 0.96240234375, 0.79296875, 0.55908203125,
Sequence 2 =  [0.12060546875, 0.224609375, 0.04541015625, 0.8095703125, 0.96240234375, 0.79296875, 0.55908203125, 0.1123046875,
```