Monte Carlo Simulation MA – 323 Lab – 5

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Question 1:

1. Consider the multivariate normal distribution $\boldsymbol{X} \sim N_2(\boldsymbol{\mu}, \Sigma)$.

   (a) Taking $\boldsymbol{\mu} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 2a \\ 2a & 4 \end{pmatrix}$ and for each of the four values of $a = -0.5, 0, 0.5, 1$, generate 10,000 sample from the distribution of $\boldsymbol{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2(\boldsymbol{\mu}, \Sigma)$.

   (b) For the cases a, plot the 2-dimensional histogram (please see https://plotly.com/python/2D-Histogram/) based on $10,000$ simulated values of $\boldsymbol{X}$.

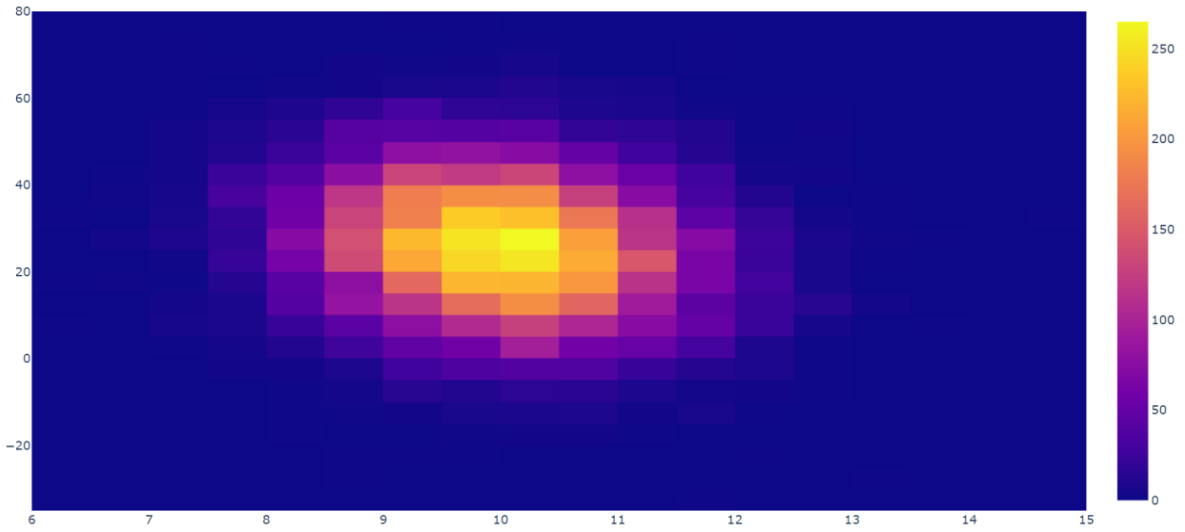
   (c) Also, draw the contour plots of the actual densities on the histograms.

N = number of samples generated = 10,000

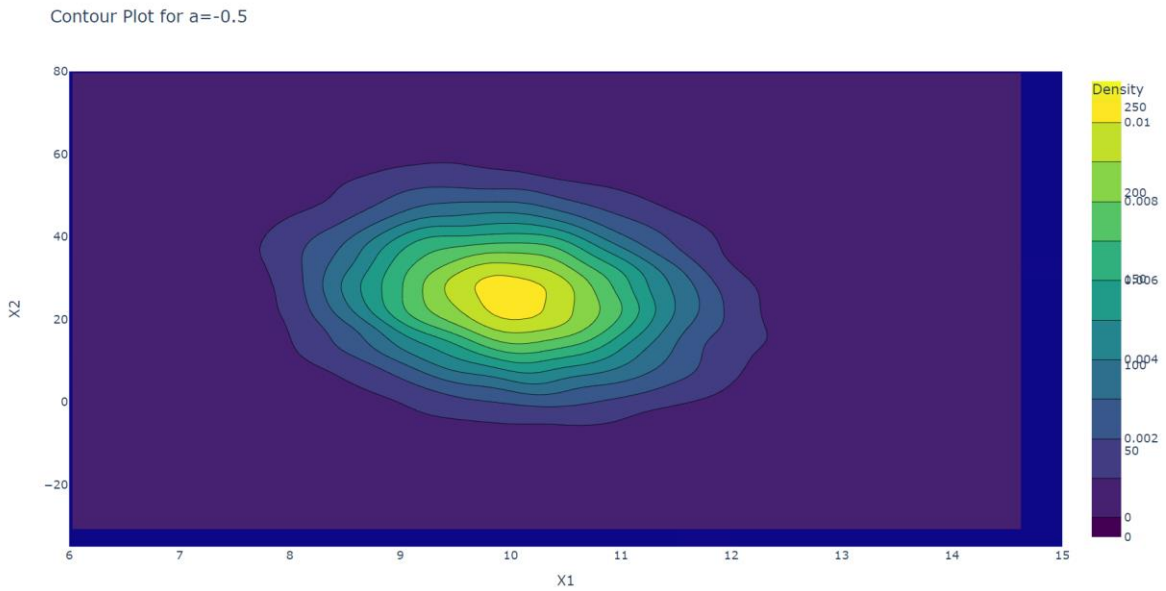
a = correlation coefficient between X_1 and X_2

For **a = -0.5,**

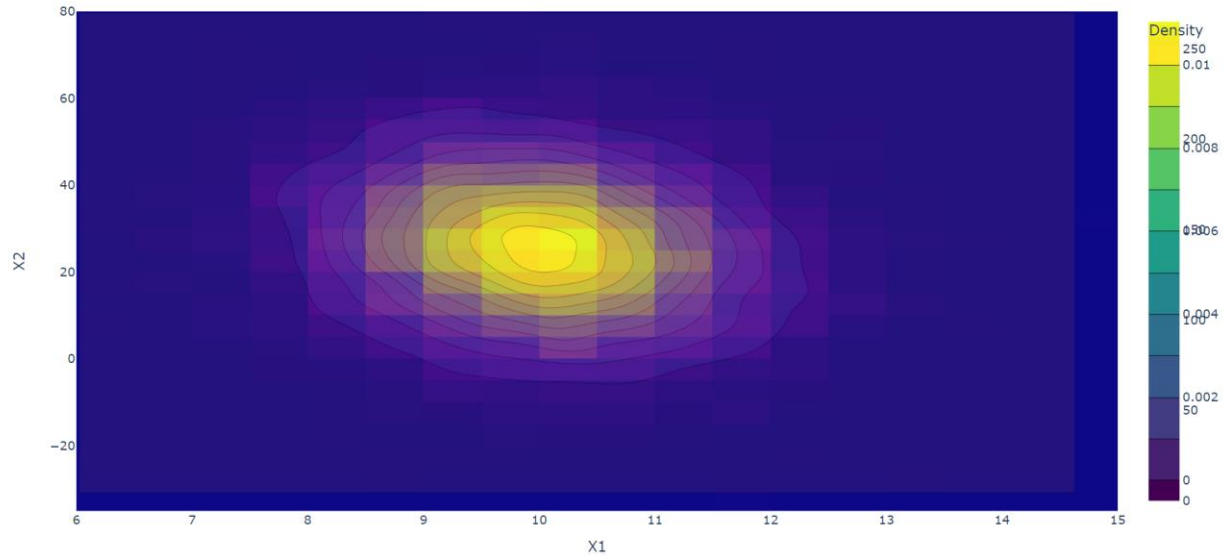
2-dimensional histogram is:



Contour plot is:

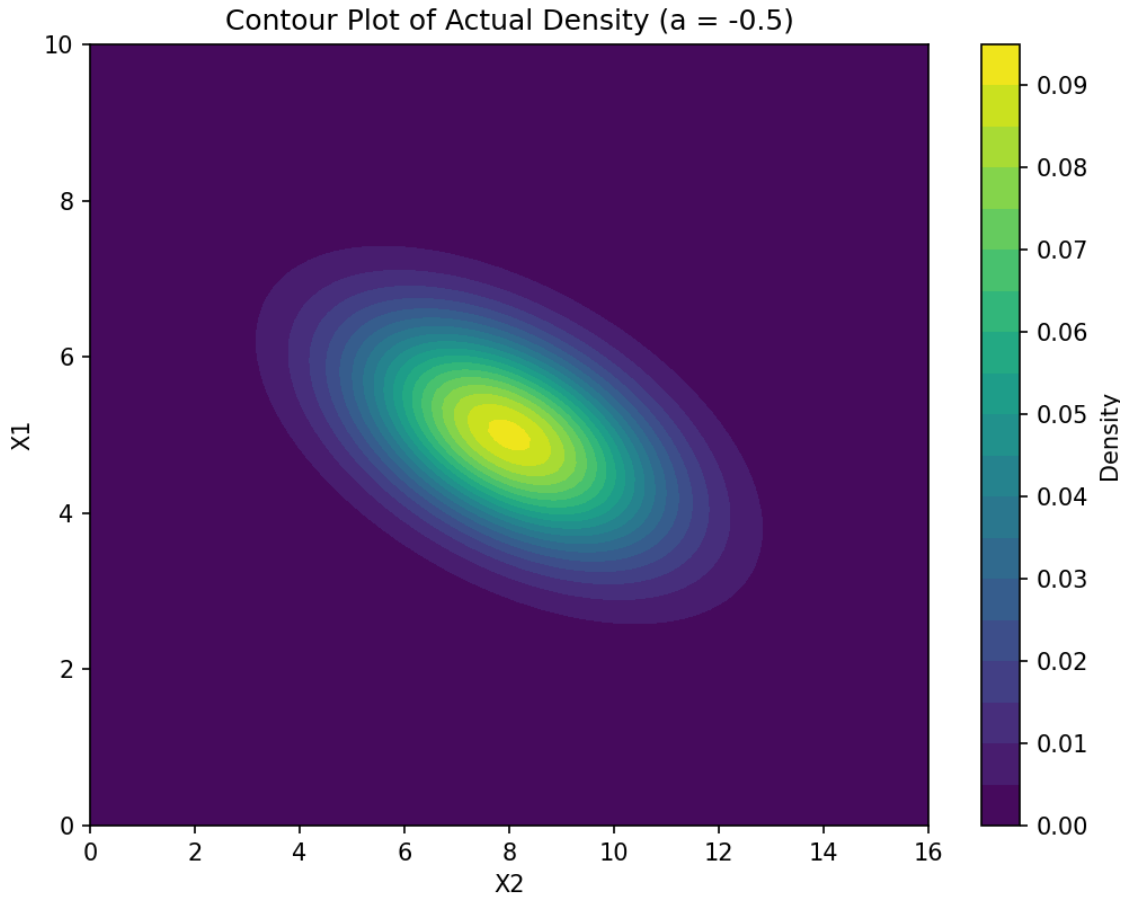


Contour plot on the histogram is:



Theoretical Contour Plot is:

(Generated from another sample, so values may differ but shape can be observed)

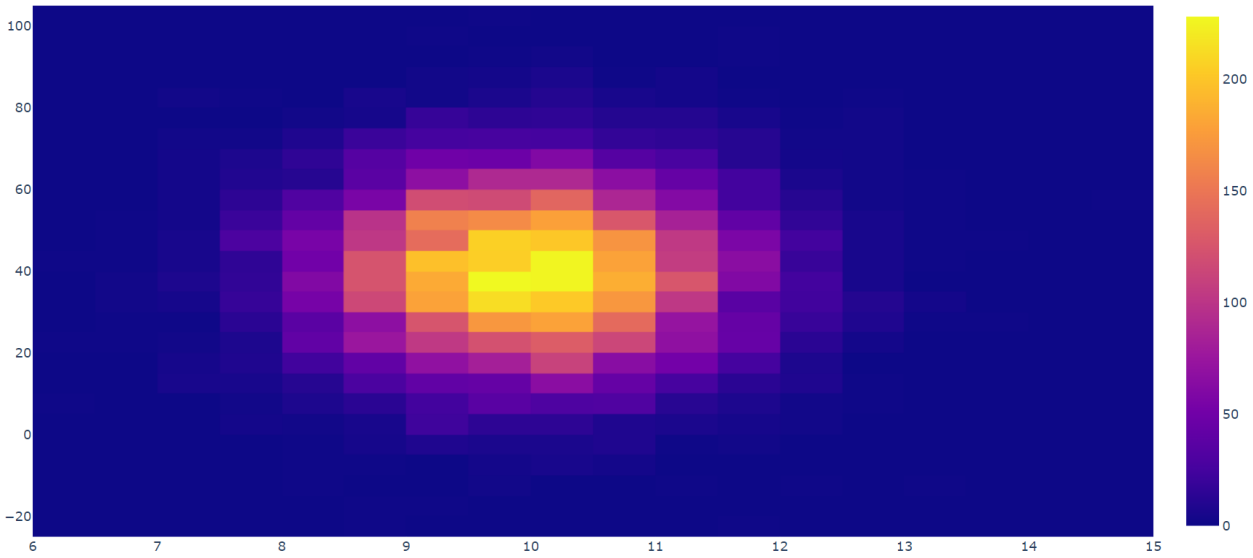


Observations:

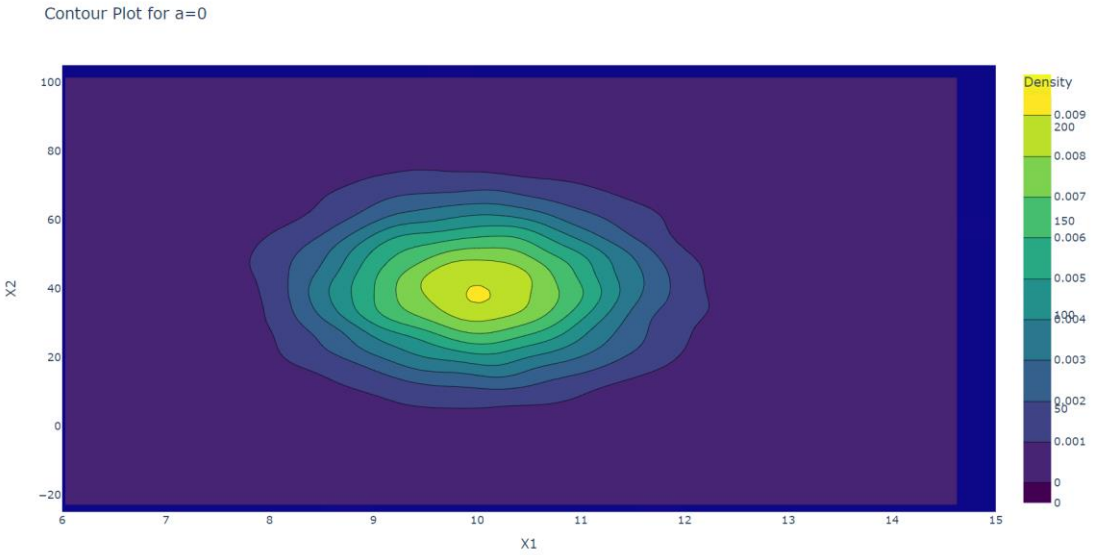
1. Since correlation coefficient (= a = -0.5) between the two marginal distributions X1 and X2 is negative, we can see that the spread of values is in direction parallel to the line y = -x on X-Y plane, since the random numbers on the two axes are negatively correlated. So, if X_1 increases then X_2 decreases and vice-versa.

For **a = 0,**

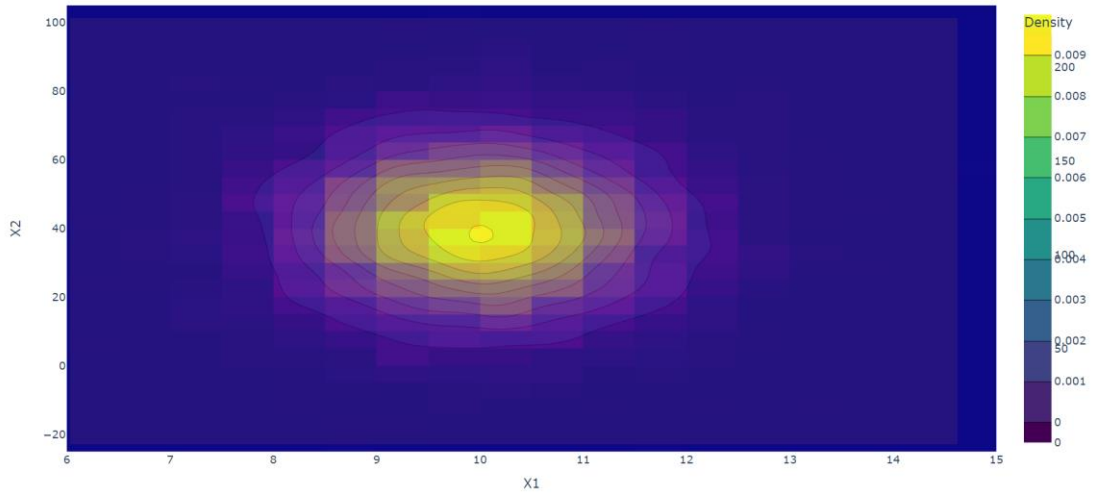
2-dimensional histogram is:



Contour plot is:

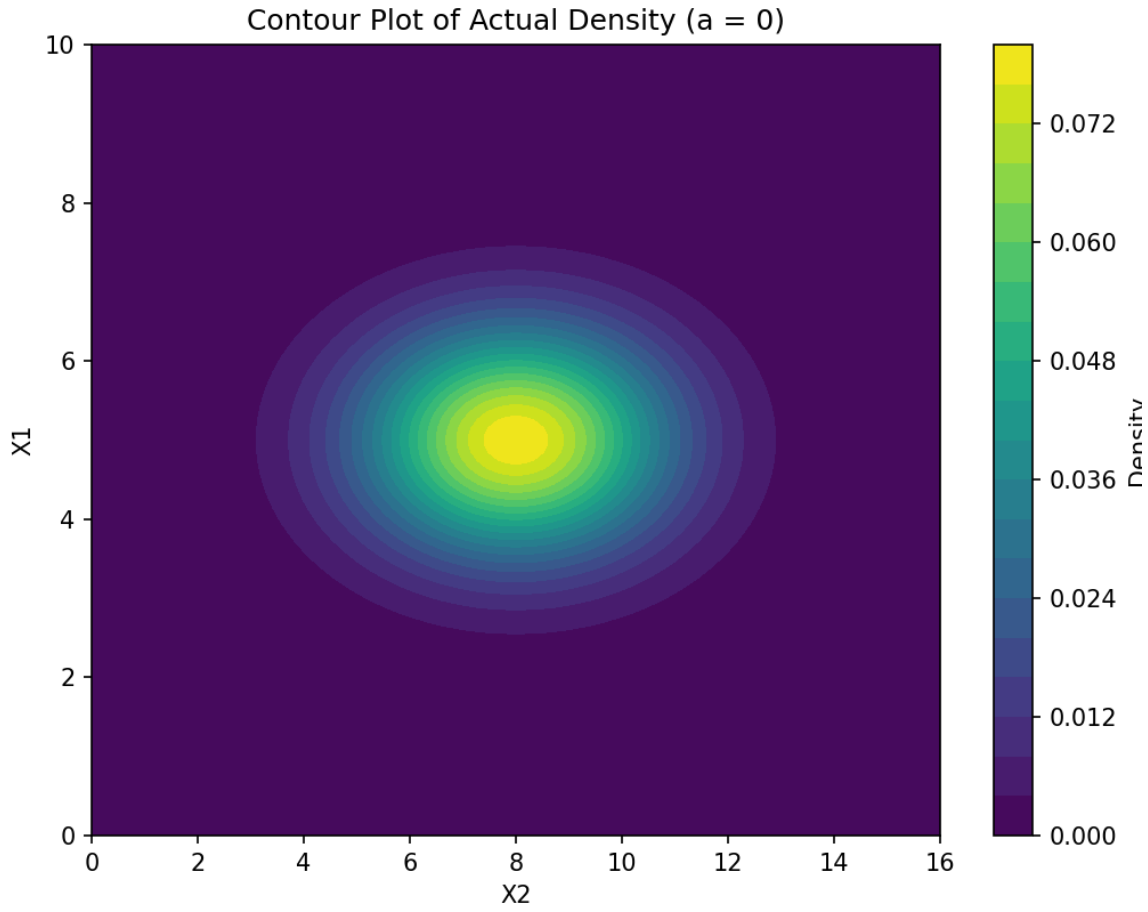


Contour plot on the histogram is:



Theoretical Contour Plot is:

(Generated from another sample, so values may differ, but shape can be observered.)

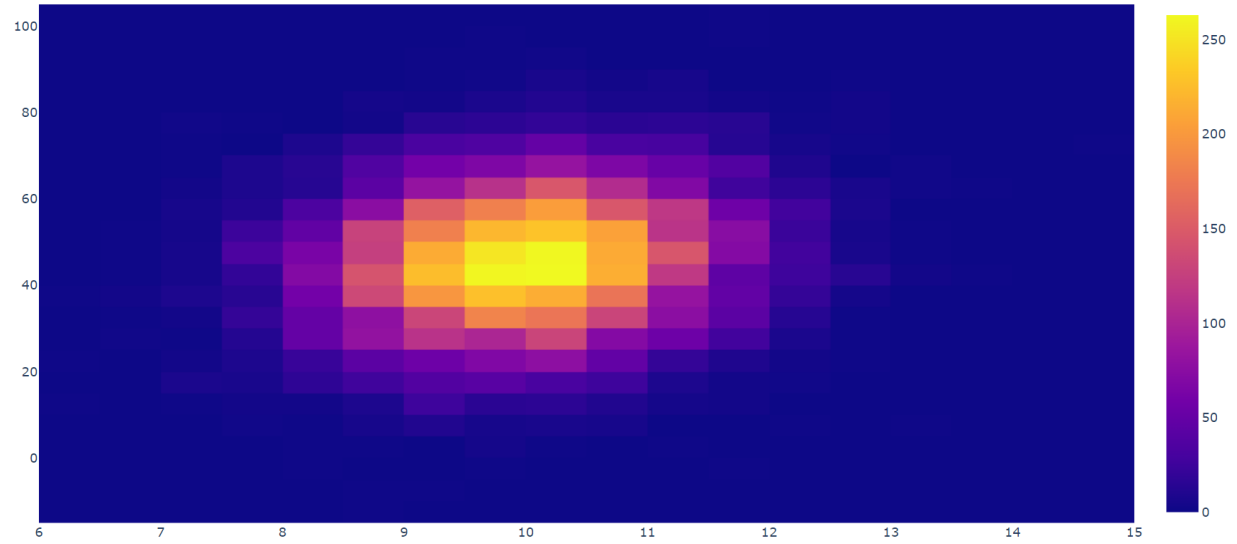


Observations:

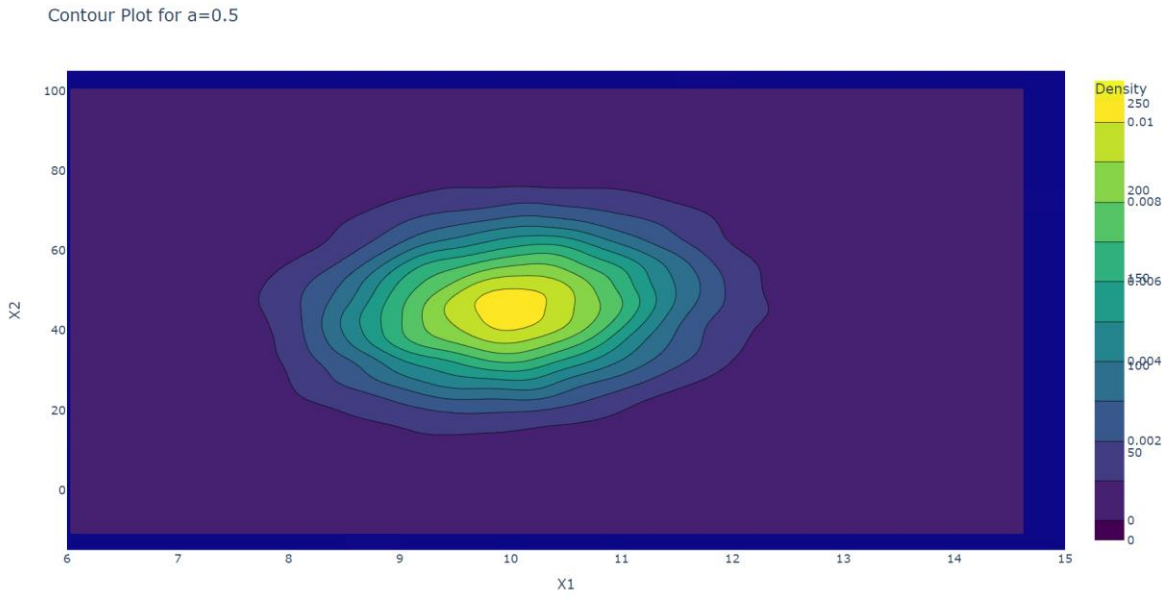
1. Since correlation coefficient (= a = 0) between the two marginal distributions is zero, we can see that the spread of values is uniform. It is not in a specific direction like along the line y = x or y = -x. The marginal distributions X_1 and X_2 are independent of each other so the spread of the values is like this, normal to each other, independent of each other.

For **a = 0.5,**

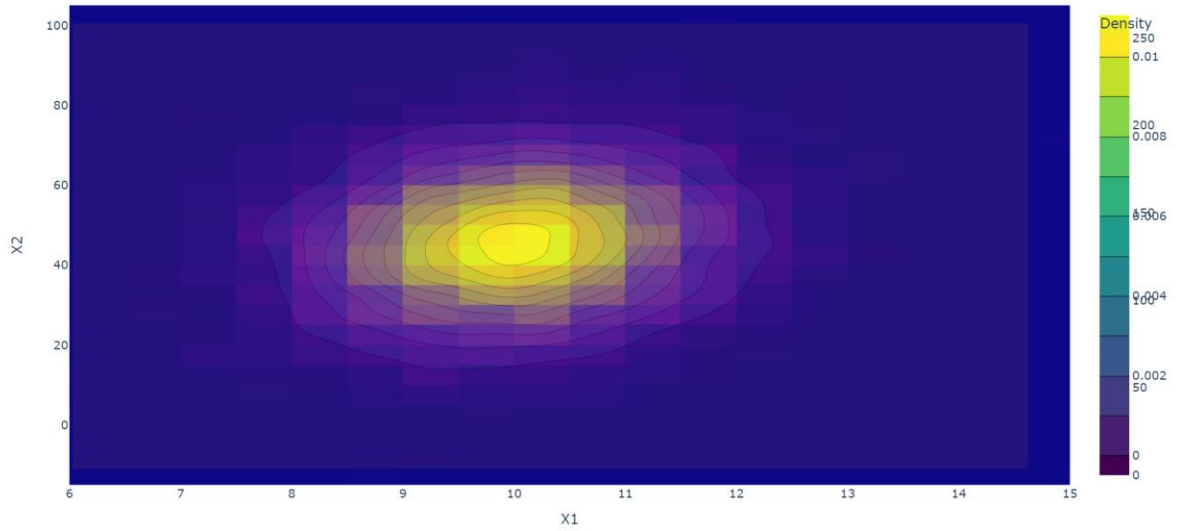
2-dimensional histogram is:



Contour plot is:

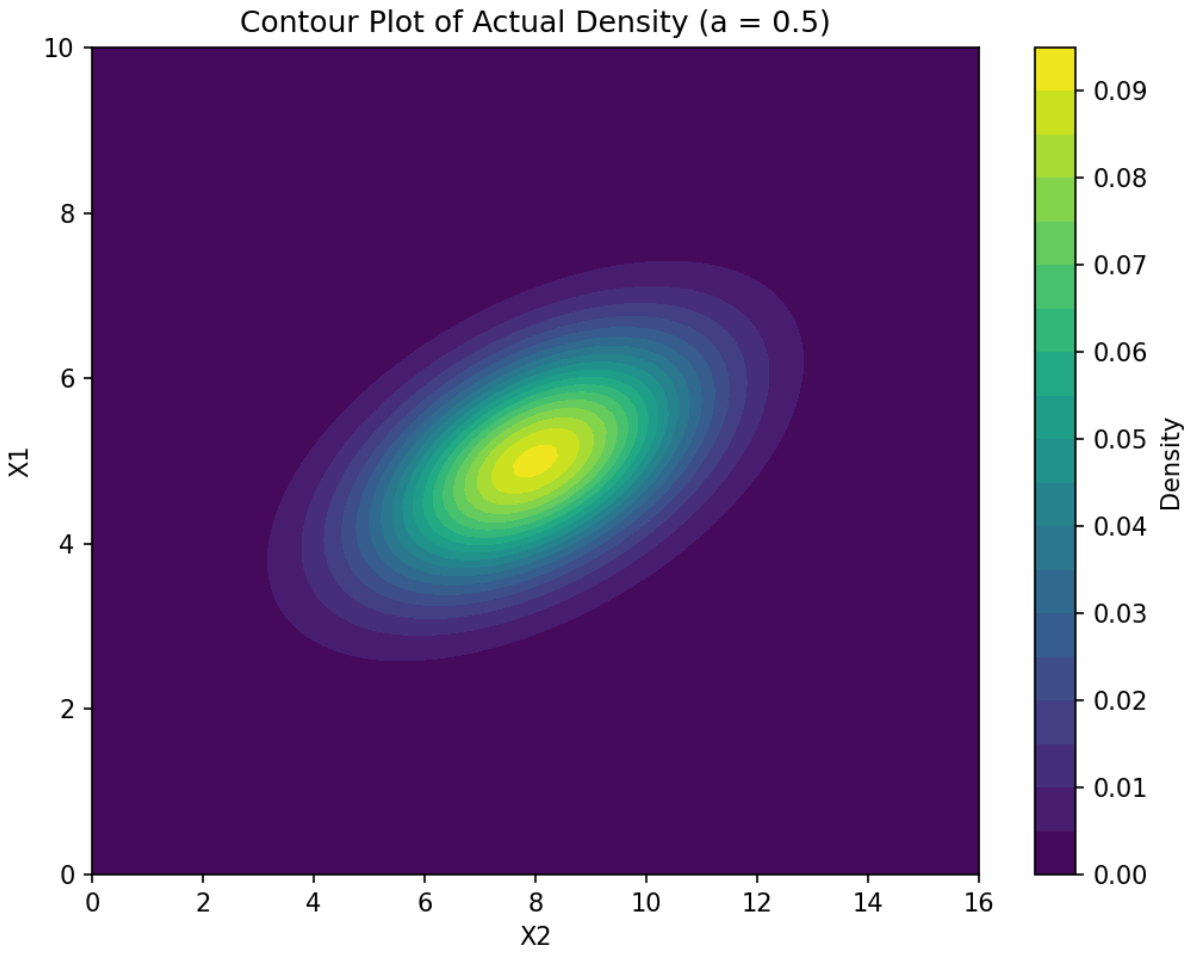


Contour plot on the histogram is:



Theoretical Contour Plot is:

(Generated from another sample, so values may differ but shape can be observed)

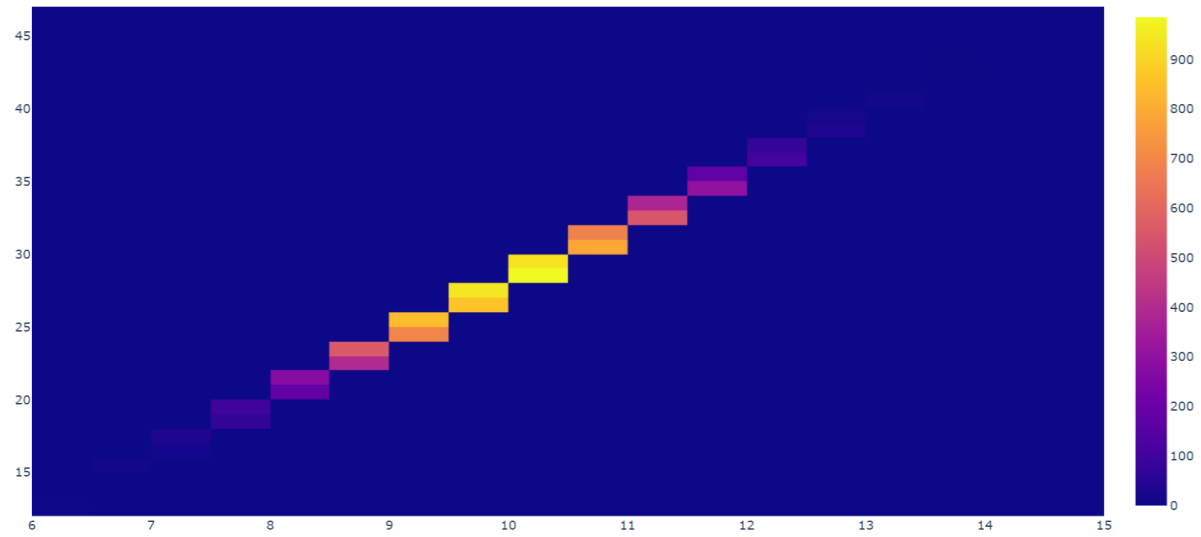


Observations:

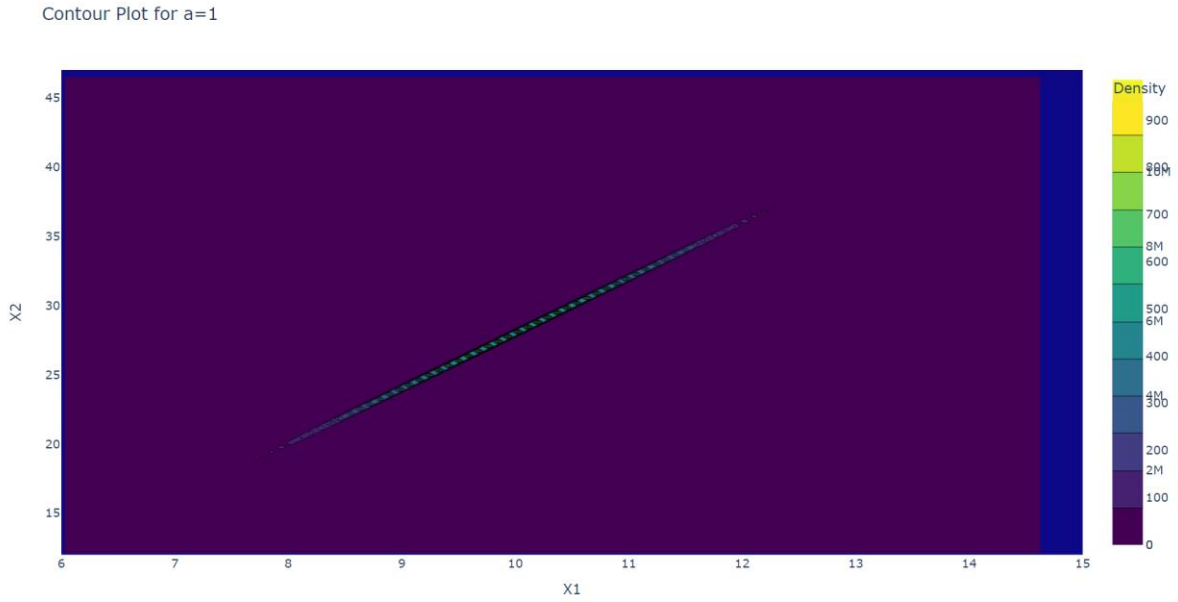
1. Since correlation coefficient (= a = 0.5) between the two marginal distributions is positive, we can see that the spread of values is in direction parallel to the line y = x on X-Y plane, since the random numbers on the two axes are positively correlated. So, if X_1 increases then X_2 also increases and vice-versa.

For **a = 1,**

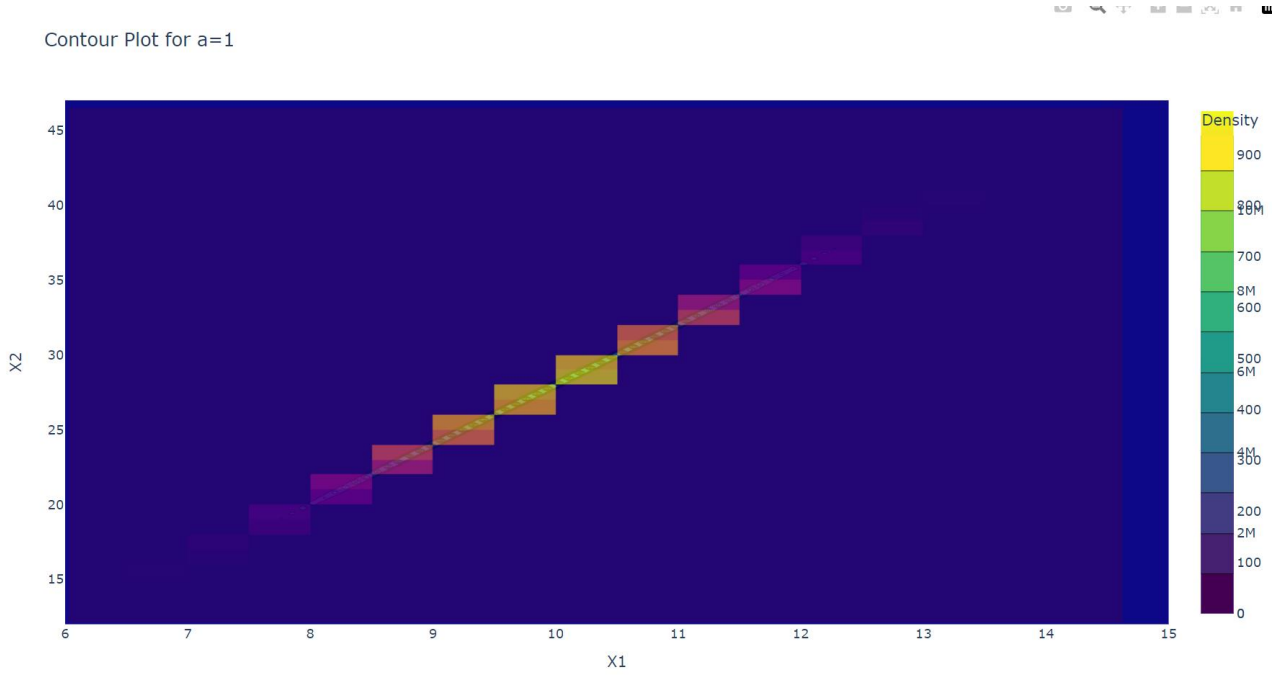
2-dimensional histogram is:



Contour plot is:



Contour plot on the histogram is:



Observations:

1. When correlation coefficient a = 1, the determinant of the variance-covariance matrix becomes zero. So, it becomes a singular matrix and hence, the inverse of the variance-covariance matrix doesn't exist. So, the PDF of the Normal Distribution of this Random Variable couldn't be written in that special form. So, we cannot obtain its theoretical contour plot.
2. The generated contour plot for the case where a = 1 appears as a straight line because the covariance matrix for this case has a determinant of zero, indicating that the two variables are perfectly linearly dependent. In other words, they are collinear, and there is no variation or spread in the data in the direction orthogonal to the line formed by the variables. In practical terms, when a = 1, it implies that the two variables X_1 and X_2 are perfectly correlated with each other, and all data points lie along a straight line in the two-dimensional space defined by these variables. Consequently, the contour plot appears as a straight line because there is no variation or density away from this line. So, the 2D-Histogram is also almost a straight line.

Overall Observations:

1. We observe that the actual density and the generated/simulated density for all the cases roughly match each other.