<u>Monte Carlo Simulation</u> <u>MA – 323</u> <u>Lab – 3</u>

Name – Rasesh Srivastava

Roll Number – 210123072

Branch – Mathematics and Computing

Question 1:

1. Implement the acceptance rejection method to generate samples from a distribution with PDF

$$f(x) = 20x(1-x)^3 \quad \text{for} \quad 0 < x < 1.$$

Please use the smallest value of c such that $f(x) \leq cg(x)$ for your choice of g.

(a) What is the average of number of iterations needed to generate a random number and why?

(b) Generate 10000 random numbers from the distribution. Compute the sample mean and compare it with expectation of the PDF f.

(c) What is the approximate value of $P(0.25 \leq X \leq 0.75)$ based on the generated sample in the part (b)? What is the exact value of the probability? Compare them.

(d) Keep a count of number of iterations needed to generate each of the random numbers in part (b). Compute the average of all these values and compare it with the value obtained in part (a).

(e) Draw the histogram of the sample obtained in part (b). Also, draw the PDF f on the same plot. Compare them.

(f) Repeat parts (a)–(d) above with two values of c higher than the smallest value that you have chosen. What are your observations ?

We take Uniform (0,1) as the known density function of g(x), g(x) = 1 for all x in the range [0,1].

The smallest value of c is **2.109375**

Since g(x) = 1, ∀ x in [0, 1], we need to find maximum value of f(x) in [0, 1]. It can be easily derived by taking derivative of f(x) and finding the maxima in the given interval.

a) Theoretically, Average number of iterations needed to generate a random number = c = 2.109375 (if we are taking the smallest value of c)

The average number of iterations needed to generate a random number using the acceptance-rejection method depends on the relationship between the probability distributions defined by f(x) and g(x), as well as the chosen value of c.

The acceptance-rejection method involves generating random numbers from g(x) and comparing them to a scaled version of f(x) to accept or reject the samples. The value of c is crucial because it determines the envelope that bounds f(x) and scales g(x) so that it encloses f(x) throughout the interval of interest.

The acceptance-rejection method involves generating samples from a proposal distribution g(x) and accepting these samples with a probability proportional to f(x)/c. g(x), where f(x) is the target distribution's PDF and c is the scaling constant. If we choose c correctly, then the acceptance probability becomes f(x)/f(x) =1.

When c is chosen correctly, the average number of iterations is equal to c, which is the scaling factor that balances the envelope c· g(x) with f(x).

While the theoretical average number of iterations is c, in practice, due to randomness, variations in the distribution, and other factors, we observe values close to c rather than exactly c.

b) (As I have not seeded my random number generator, the Sample Mean may vary when code is executed at different times, but it will be close to the Theoretical Mean always)

c	Sample Mean	Theoretical Mean
2.109375	0.3349944919154887	0.333333
3	0.335068752855454	0.333333
4	0.33582515653361916	0.333333

c) P (0.25 ≤ X ≤ 0.75)

(As I have not seeded my random number generator, the Approximate Value of P (0.25 ≤ X ≤ 0.75) may vary when code is executed at different times, but it will be close to the Theoretical Exact Value always).

c	Approximate Value	Exact Value
2.109375	0.6225	0.61718
3	0.621	0.61718
4	0.6252	0.61718

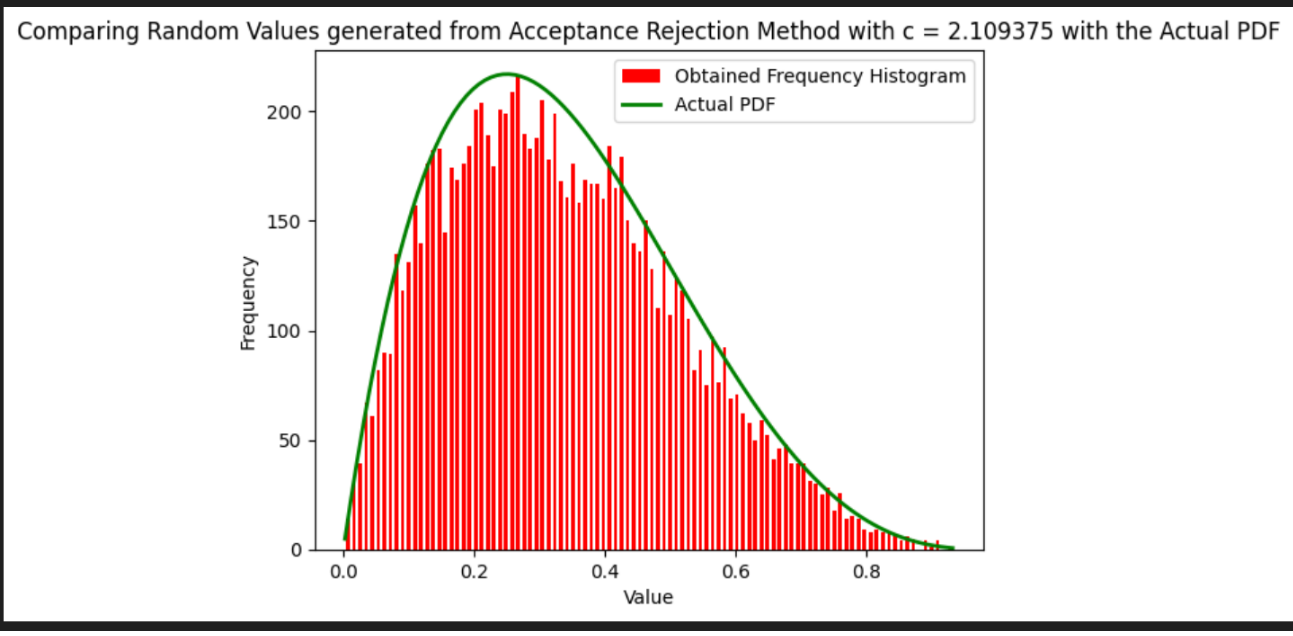
d) Value obtained in part (a) = Theoretical Average Number of iterations needed to generate a random number = c

(As I have not seeded my random number generator, the average number of iterations needed to generate random numbers may vary when code is executed at different times, but it will be close to the theoretical c value chosen always)

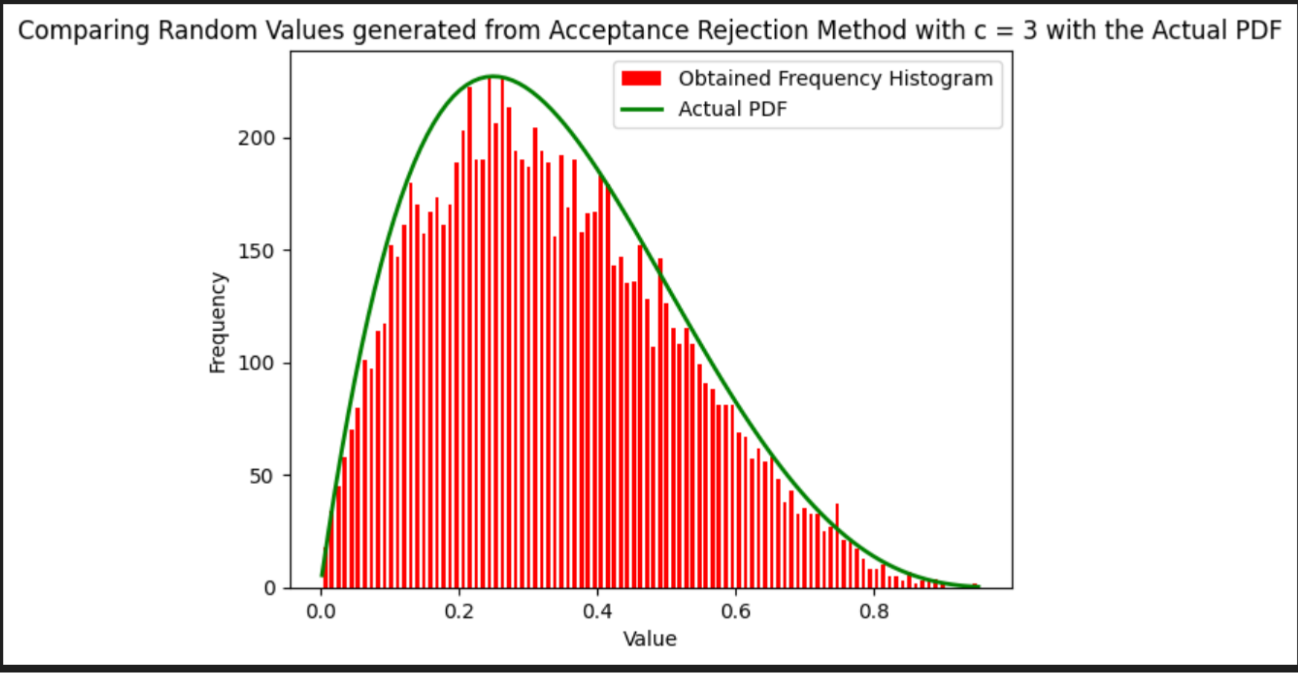
c	Average Number of Iterations needed to generate random numbers in part (b)
2.109375	2.1224
3	2.9788
4	4.003

e)

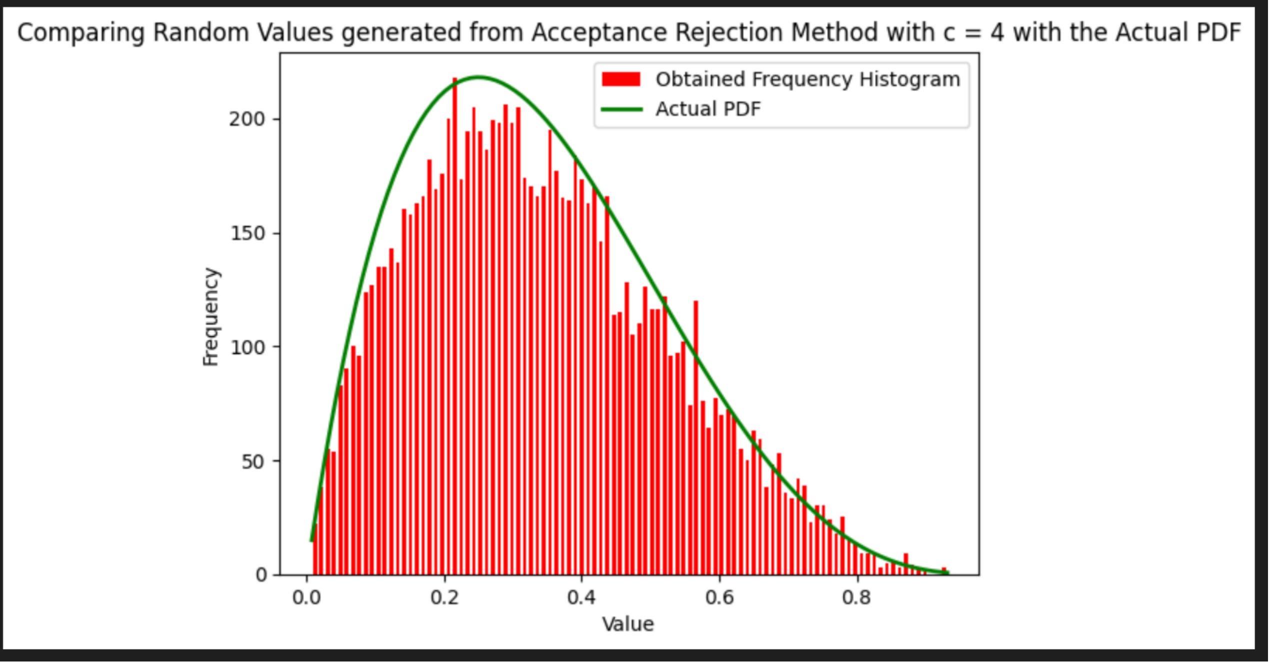
```
Value of c = 2.109375
Sample Mean of the random numbers obtained = 0.3335723615789062
Theoretical Mean of the function f(x) = Expectation of PDF f = 0.3333333333333333
Approximate(Experimental) value of P(0.25 ≤ X ≤ 0.75) based on the generated sample = 0.621
Actual Probability of P(0.25 ≤ X ≤ 0.75) = 0.61718
Average Number of Iterations = 2.1262
```



```
Value of c = 3
Sample Mean of the random numbers obtained = 0.33478161281745794
Theoretical Mean of the function f(x) = Expectation of PDF f = 0.3333333333333333
Approximate(Experimental) value of P(0.25 ≤ X ≤ 0.75) based on the generated sample = 0.6247
Actual Probability of P(0.25 ≤ X ≤ 0.75) = 0.61718
Average Number of Iterations = 3.0193
```




```
Value of c = 4
Sample Mean of the random numbers obtained = 0.3394857449296429
Theoretical Mean of the function f(x) = Expectation of PDF f = 0.3333333333333333
Approximate(Experimental) value of P(0.25 ≤ X ≤ 0.75) based on the generated sample = 0.6274
Actual Probability of P(0.25 ≤ X ≤ 0.75) = 0.61718
Average Number of Iterations = 3.9783
```



f) Smallest Value of c = 2.109375,

I have chosen c = 3 and 4, as the two values of c higher than the smallest value of c.

Observations:

- As we can see that when the value of c deviates (increases) from the smallest possible value of c, the errors in the random numbers generated increase, and the random numbers generated deviates more from the distribution for which we are generating them. This deviation is more notable as the value of c becomes more and more greater than its smallest value.
- The deviation from the expected result is more when we increase the value of c.

Question 2:

2. Implement acceptance rejection method to generate random number from the PDF

$$f(x) \propto x^{\alpha-1}e^{-x} \quad \text{for} \quad 0 < x < 1.$$

Generate 10000 random numbers form the above PDF. Please mention the rejection constant and dominating PDF in the report.

Rejection Constant: A = $\frac{1}{\alpha} + \frac{1}{e}$

{ c = $\frac{A}{\Gamma(\alpha)}$, where $\Gamma(\alpha)$ = 1. (Since we are taking f(x) = $x^{\alpha-1} e^{-x}$) }

Dominating PDF: g(x) = $\frac{x^{\alpha-1}}{\frac{1}{\alpha}+\frac{1}{e}}$ for 0 < x < 1

Since 0 < x < 1, so we're not mentioning the Dominating PDF for x >= 1.