<u>Monte Carlo Simulation</u> <u>MA – 323</u> <u>Lab – 6</u>

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1. Use the following Monte Carlo estimator to approximate the expected value $I = E[\exp(\sqrt{U})]$ where, $U \sim U(0,1)$:

$$I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i, \text{ where } Y_i = \exp(\sqrt{U_i}) \text{ with } U_i \sim U(0,1).$$

Take the values of M to be $10^2, 10^3, 10^4$ and 10^5. Determine the 95% confidence interval for I for all the four values of M that you have taken. What is the exact value of I? Compare the exact value of I with the estimated values of I for different values of M.

I = E [exp (√ U)] where, U ~ U (0, 1)

Exact Value of I = 2.0000000000000004 (Using scipy.integrate's numerical integration)

For M = 100,

Estimated Value of I = 1.972233262076766

95% Confidence Interval for I = (1.891727203945577, 2.052739320207955)

For M = 1000,

Estimated Value of I = 1.9916605509837473

95% Confidence Interval for I = (1.963971252807752, 2.0193498491597426)

For M = 10000,

Estimated Value of I = 1.9955339717944796

95% Confidence Interval for I = (1.9868681633511696, 2.0041997802377898)

For M = 100000,

Estimated Value of I = 2.000954539253838

95% confidence interval for I = (1.998219964607142, 2.0036891139005344)

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For M = 100:
Estimated Value of I: 1.972233262076766
95% Confidence Interval: (1.891727203945577,2.052739320207955)

For M = 1000:
Estimated Value of I: 1.9916605509837473
95% Confidence Interval: (1.963971252807752,2.0193498491597426)

For M = 10000:
Estimated Value of I: 1.9955339717944796
95% Confidence Interval: (1.9868681633511696,2.0041997802377898)

For M = 100000:
Estimated Value of I: 2.000954539253838
95% Confidence Interval: (1.998219964607142,2.0036891139005344)

Exact Value of I: 2.0000000000000004
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Observations:

1) As the value of M increases, that is, when we take a greater number of samples, the estimated value of I approaches to the exact value of I, that is, the error in the estimation of the value of I decreases as M increases.