Monte Carlo Simulation MA – 323 Lab – 9

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Question 1:

1. The problem is based on Project Evaluation and Review Technique (PERT), a project planning tool. Consider the software project described in following table. It has 10 tasks (activities), indexed by $j = 1, \ldots, 10$. The project is completed when all of the tasks are completed. A task can begin only after all of its predecessors have been completed. The project starts at time 0. Task j starts at time

j	Task	Predecessors	Mean time (in Days)
1	Planning	None	4
2	Database Design	1	4
3	Module Layout	1	2
4	Database Capture	2	5
5	Database Interface	2	2
6	Input Module	3	3
7	Output Module	3	2
8	GUI Structure	3	3
9	I/O Interface Implementation	5,6,7	2
10	Final Testing	4,8,9	2

S_j, takes time T_j and ends at time $E_j = S_j + T_j$. Any task j with no predecessors (here only task 1) starts at $S_j = 0$. The start time for a task with predecessors is the maximum of the ending times of its predecessors. For example, $S_4 = E_2$ and $S_9 = \max\{E_5, E_6, E_7\}$. The project as a whole ends at time E_{10}. Assume that T_j are independent exponentially distributed random variables with means (say θ_j) given in the final column of the table.

(a) Write E_{10} in terms of T_j, $j = 1, 2, \ldots, 10$.

(b) Find an approximate value of mean of E_{10} using a simple Monte Carlo. You may take $n = 10000$. Save the generated sample as it will be used.

(c) Plot a histogram of generated values of E_{10}. Comment on the shape (mainly skewness) of the histogram.

(d) Assume that there will be a severe penalty should the project miss a deadline in 70 days. Find an approximate value of the probability that the project miss the deadline using a simple Monte Carlo. You may use the same sample that you generated in (1b). Also calculate the standard deviation. Comment on the performance.

(e) Find an approximate value of the probability that the project miss the deadline using importance sampling technique. To write q, take T_j are independent exponential with mean $\lambda_j = 4\theta_j$. Compute the standard deviation and effective sample size.

(f) Find an approximate value of the probability that the project miss the deadline using importance sampling technique. Here to write q, take T_j are independent exponential with mean λ_j, where $\lambda_j = \kappa\theta_j$ for $j = 1, 2, 4, 10$ and $\lambda_j = \theta_j$ for $j \notin \{1, 2, 4, 10\}$. Take $\kappa = 3.0, 4.0, 5.0$. Compute the standard deviation and effective sample size for each values of κ.

(g) Compare results that you obtained in 1e and 1f.

(h) Obtain the confidence interval for the probability taking the value of κ that has minimum effective sample size among 3.0, 4.0, 5.0.

(a)

$$E_{10} = S_{10} + T_{10} = max\{E_4, E_8, E_9\} + T_{10}$$
$$E_{10} = max\{S_4 + T_4, S_8 + T_8, S_9 + T_9\} + T_{10}$$
$$E_{10} = max\{E_2 + T_4, E_3 + T_8, max\{E_5, E_6, E_7\} + T_9\} + T_{10}$$
$$E_{10} = max\{S_2 + T_2 + T_4, S_3 + T_3 + T_8, max\{S_5 + T_5, S_6 + T_6, S_7 + T_7\} + T_9\} + T_{10}$$
$$E_{10} = max\{E_1 + T_2 + T_4, E_1 + T_3 + T_8, max\{E_2 + T_5, E_3 + T_6, E_3 + T_7\} + T_9\} + T_{10}$$
$$E_{10} = max\{T_1 + T_2 + T_4, T_1 + T_3 + T_8, max\{S_2 + T_2 + T_5, S_3 + T_3 + T_6, S_3 + T_3 + T_7\} + T_9\} + T_{10}$$
$$E_{10} = max\{T_1 + T_2 + T_4, T_1 + T_3 + T_8, max\{E_1 + T_2 + T_5, E_1 + T_3 + T_6, E_1 + T_3 + T_7\} + T_9\} + T_{10}$$
$$E_{10} = max\{T_1 + T_2 + T_4, T_1 + T_3 + T_8, max\{T_1 + T_2 + T_5, T_1 + T_3 + T_6, T_1 + T_3 + T_7\} + T_9\} + T_{10}$$
$$E_{10} = max\{T_1 + T_2 + T_4, T_1 + T_3 + T_8, max\{T_2 + T_5, T_3 + T_6, T_3 + T_7\} + T_1 + T_9\} + T_{10}$$
$$E_{10} = max\{T_2 + T_4, T_3 + T_8, max\{T_2 + T_5, T_3 + T_6, T_3 + T_7\} + T_9\} + T_1 + T_{10}$$

So,

$$E_{10} = max\{T_2 + T_4, T_3 + T_8, max\{T_2 + T_5, T_3 + T_6, T_3 + T_7\} + T_9\} + T_1 + T_{10}$$

(b)

Taking n = 10000, an approximate value of mean of E_{10} using a simple Monte Carlo = 18.193634942542122

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Taking n = 10000, The sample mean of E_10 using simple Monte Carlo = 18.193634942542122
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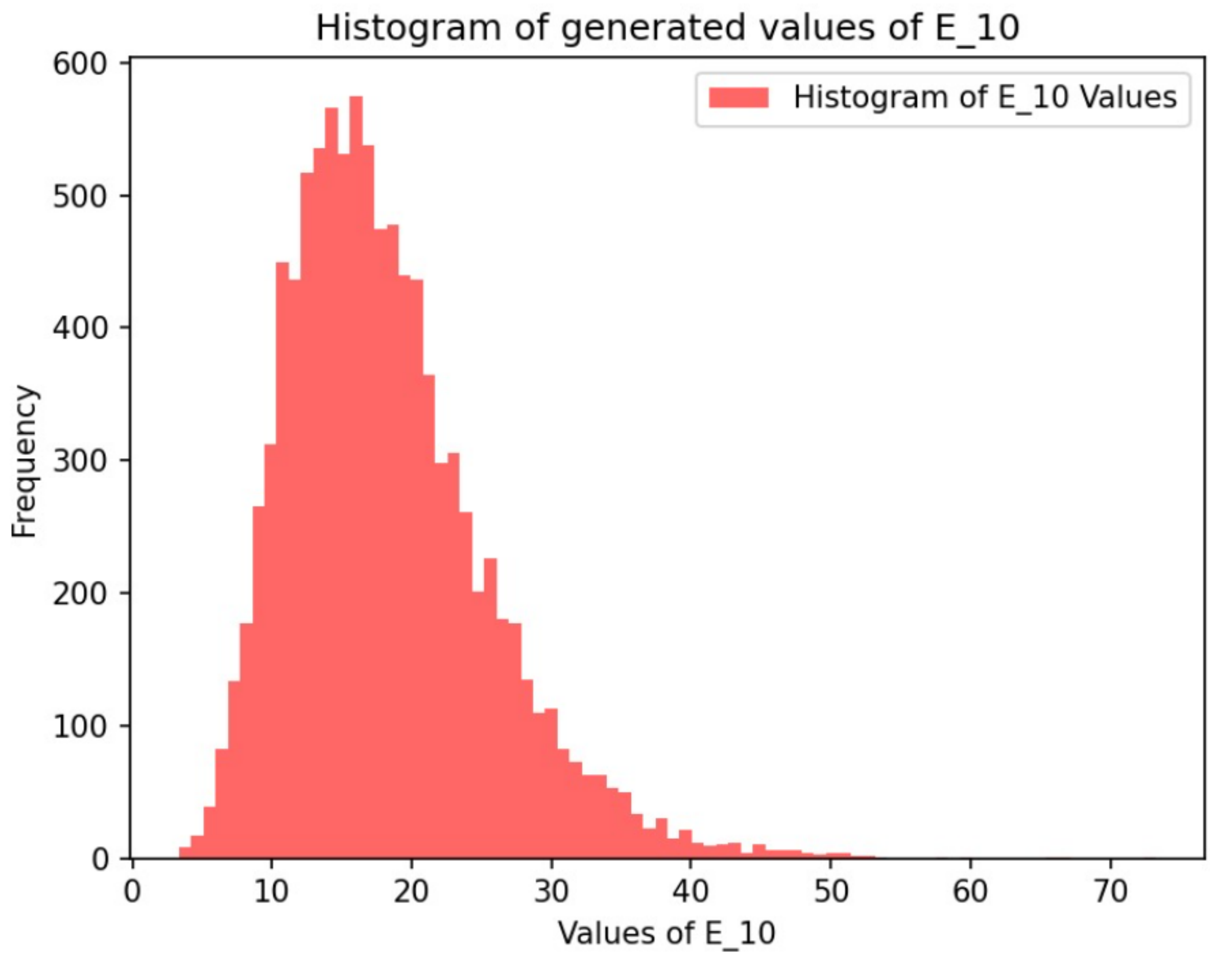
(c)

The Histogram is positively skewed, which means it is very much shifted to the left, as most of the values obtained in E_{10} are closer to 18, which is also very near to its mean. The exponential distribution is right-skewed, meaning that longer task durations are less likely but can significantly impact the project completion time.

In a positively skewed distribution, the tail on the right-hand side is longer than the left-hand side, and the mean is typically greater than the median. The skewness of the histogram suggests that, on average, projects are more likely to be completed earlier than the mean E10, but there is a chance of experiencing longer project durations due to the variability in task times.



(d)

Approximate value of the probability that the project misses the deadline using a simple Monte Carlo = 0.0001 (which is approximately 0, if we run the code many times, mostly this value is 0.0, but sometimes, this value comes out to be 0.0001 because I have not seeded the random number generator)

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Approximate value of the probability that the project misses the deadline using simple Monte Carlo = 0.0001
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Standard Deviation of E_{10} = 7.2035718489332465

Standard Deviation of Probability = 0.010000000000002247

The mean of E_{10} is 18.19 approximately and severe penalty imposed is very rare, mostly the probability is 0 but sometimes it comes out to be 0.0001, so overall, the probability is very low. The Standard Deviation of the E_{10} values is approximately equal to the square root of the variance of the obtained values.

It's important to consider that the accuracy of this estimate depends on the number of simulations (n) and the randomness of the generated samples. While this method provides a reasonable approximation of the probability, it may still have some level of uncertainty.

In practical project management, where there could be real-world consequences for missing a deadline, it's essential to ensure the reliability of such estimates. If the stakes are high, it might be worth exploring more advanced techniques, such as importance sampling or variance reduction methods, to obtain more precise and consistent estimates.

Overall, the simple Monte Carlo method is a good starting point for estimating the probability of missing the project deadline, but in critical situations, further analysis and validation of the results might be necessary.

(e)

Approximate value of the probability that the project misses the deadline using importance sampling technique = 0.4654

Standard Deviation = 29.459476175124006

Effective Sample Size = 588.6842958646648

(f)

For $\kappa = 3.0$,

Sample Mean of E_{10} = 72.37741228526815

Approximate value of the probability that the project misses the deadline using importance sampling technique = 0.4652

Standard Deviation = 28.826745317122437

Effective Sample Size = 614.8104897687065

For $\kappa = 4.0$,

Sample Mean of E_{10} = 72.71589888075601

Approximate value of the probability that the project misses the deadline using importance sampling technique = 0.472

Standard Deviation = 28.332732281628285

Effective Sample Size = 636.4372274506549

For $\kappa = 5.0$,

Sample Mean of E_{10} = 73.36674790805135

Approximate value of the probability that the project misses the deadline using importance sampling technique = 0.4802

Standard Deviation = 29.193700769582346

Effective Sample Size = 599.4516866663289

(g)

	Part (e)	Part (f) with κ = 3.0	Part (f) with κ = 4.0	Part (f) with κ = 5.0
Approximate value of the probability that the project misses the deadline	0.4654	0.4652	0.472	0.4802
Standard Deviation	29.459476175124006	28.826745317122437	28.332732281628285	29.193700769582346
Effective Sample Size	588.6842958646648	614.8104897687065	636.4372274506549	599.4516866663289

(h)

the value of κ that has minimum effective sample size among 3.0, 4.0, 5.0, is κ = 5.0

For κ = 5.0, 95% confidence interval = (72.57208157319685, 74.05990032715614)

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k = 5 has the minimum effective sample size.
99% Confidence interval (72.57208157319685, 74.05990032715614)
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