<u>Monte Carlo Simulation</u>   <u>MA – 323</u>   <u>Lab – 7</u>

Name – Rasesh Srivastava

Roll Number – 210123072

Branch – Mathematics and Computing

<u>Question 1</u>:

1. Consider the expectation $I = E[\exp(\sqrt{U})]$ where, $U \sim U(0,1)$. Use the following antithetic method to approximate I:

$$\widehat{I}_M = \frac{1}{M}\sum_{i=1}^{M/2} \widehat{Y}_i \text{ where } \widehat{Y}_i = \exp(\sqrt{U_i}) + \exp(\sqrt{1-U_i}) \text{ with } U_i \sim U(0,1).$$

Take the values of M to be $10^2, 10^3, 10^4$ and 10^5. Determine the 95% confidence interval for I_M for all the four values of M that you have taken.

I = E [exp (√ U)] where, U ~ U (0, 1)

Actual Value of I = 2 ($\int_0^1 \exp(\sqrt{x})\,dx = 2$)

It is calculated by taking the integral of exp(sqrt(x)) with limits from 0 to 1.

For M = 100,

Estimated Value of I = 1.9930161300230471

95% Confidence Interval for I = (1.9824862654143782, 2.003545994631716)

For M = 1000,

Estimated Value of I = 1.9980843243353144

95% Confidence Interval for I = (1.9951848623546362, 2.0009837863159925)

For M = 10000,

Estimated Value of I = 1.999877417024356

95% Confidence Interval for I = (1.9989581894176025, 2.0007966446311096)

For M = 100000,

Estimated Value of I = 1.9999567501250843

95% confidence interval for I = (1.999669273369198, 2.0002442268809704)

```
For M = 100,
Estimated Value of I = 1.9930161300230471
95% Confidence Interval: (1.9824862654143782, 2.003545994631716)

For M = 1000,
Estimated Value of I = 1.9980843243353144
95% Confidence Interval: (1.9951848623546362, 2.0009837863159925)

For M = 10000,
Estimated Value of I = 1.999877417024356
95% Confidence Interval: (1.9989581894176025, 2.0007966446311096)

For M = 100000,
Estimated Value of I = 1.9999567501250843
95% Confidence Interval: (1.999669273369198, 2.0002442268809704)

Actual Value of I = 2
```

Observations:

1. As the value of M increases, that is, when we take a greater number of samples, the estimated value of I approaches to the exact value of I, that is, the error in the estimation of the value of I decreases as M increases.
2. As the value of M increases, that is, when we take a greater number of samples, the lower bound of the 95% confidence interval increases and approaches towards the actual value of I, that is, 2 and the upper bound of the 95% confidence interval decreases and approaches towards the actual value of I, that is, 2.

Question 2:

2. Present the results that you have obtained in Question 1 of Lab 06 and Question 1 of Lab 07 in a tabular form. Your table must consist of the values of $\widehat{I}_M$ (using two methods), 95% confidence intervals for I (from two methods), and the ratio of widths of both the intervals. How do the values of I_M and $\widehat{I}_M$ compare with the actual value of I?

Actual Value of I = 2 ($\int_0^1 \exp(\sqrt{x})\, dx = 2$)

It is calculated by taking the integral of exp(sqrt(x)) with limits from 0 to 1.

```
Comparison of the results of Question 1 of Lab 6 and Question 1 of Lab 7 in a tabular form:
        M       I_M  I_M using Antithetic Method  \
0     100  1.960903                     2.002870
1    1000  1.988475                     2.000169
2   10000  2.005029                     1.999793
3  100000  2.001313                     1.999829

             95% Confidence Interval for I  \
0   (1.874969408782346, 2.0468372483732935)
1  (1.9619602860598633, 2.0149889510933785)
2   (1.9964424354588663, 2.01361602665882)
3     (1.998584624620874, 2.00404159465956)

  95% Confidence Interval for I using Antithetic Method  Ratio of the Widths
0             (1.995348588266023, 2.01039060431981)            11.425851
1            (1.997369909260367, 2.002967478042018)             9.473517
2           (1.9988864280096155, 2.000699635644898)             9.471387
3           (1.999540452485819, 2.0001178512663493)             9.450955

Actual value of I = 2
```

M	I_M (Lab 6)	I_M (Lab 7)	95% Confidence Interval (Lab 6)	95% Confidence Interval (Lab 7)	Ratio of Widths of the 95% Confidence Intervals (Lab 6 width / Lab 7 width values)
100	1.960903	2.002870	(1.874969408782346, 2.0468372483732935)	(1.995348588266023, 2.01039060431981)	11.425851
1000	1.988475	2.000169	(1.9619602860598633, 2.0149889510933785)	(1.997369909260367, 2.002967478042018)	9.473517
10000	2.005029	1.999793	(1.9964424354588663, 2.01361602665882)	(1.9988864280096155, 2.000699635644898)	9.471387
100000	2.001313	1.999829	(1.998584624620874, 2.00404159465956)	(1.999540452485819, 2.0001178512663493)	9.450955

If width of 95% Confidence Interval of Lab 6/width of 95% Confidence Interval of Lab 7 = x, then the ratio of the widths = x : 1.

Observations:

1. The Antithetic method is better than the other simple Monte Carlo estimator method used in Lab 6 assignment because it improves the width of the 95% confidence interval approximately by a factor of 10 since the ratio of the widths of the 95% confidence intervals of Lab 6 and Lab 7 is approximately 10 : 1, so the Antithetic method reduces the Variance.
2. Also, in the Antithetic method, we are calculating uniform random variable only M/2 times compared to M times in the method that was used in Assignment 6. So, the Antithetic method used in Lab 7 assignment is better than the method used in Lab 6 assignment.
3. As the value of M increases, that is, when we take a greater number of samples, the estimated value of I approaches to the exact value of I, that is, the error in the estimation of the value of I decreases as M increases. For most of the values of M, the value of I_M_hat is closer to 2 (the actual value of I) than the value of I_M.