<u>Monte Carlo Simulation</u> <u>MA – 323</u> <u>Lab – 2</u>

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<u>Question 1</u>:

1. Consider the recursion:

$$U_{i+1} = (U_{i-17} - U_{i-5}).$$

In the event that $U_i < 0$, set $U_i = U_i + 1$.

(a) Use linear congruence generator to generate the first 17 values of U_i.

(b) Then generate the values of $U_{18}, U_{19}, \ldots, U_N$ for $N = 1000$, 10000, and 100000 based on the recursion above.

(c) For each N, plot histogram. What are your observations?

(d) For each N, plot (U_i, U_{i+1}). What are your observations?

a) The Linear Congruence Generator used to generate the 1st 17 values of U_i is of the type:

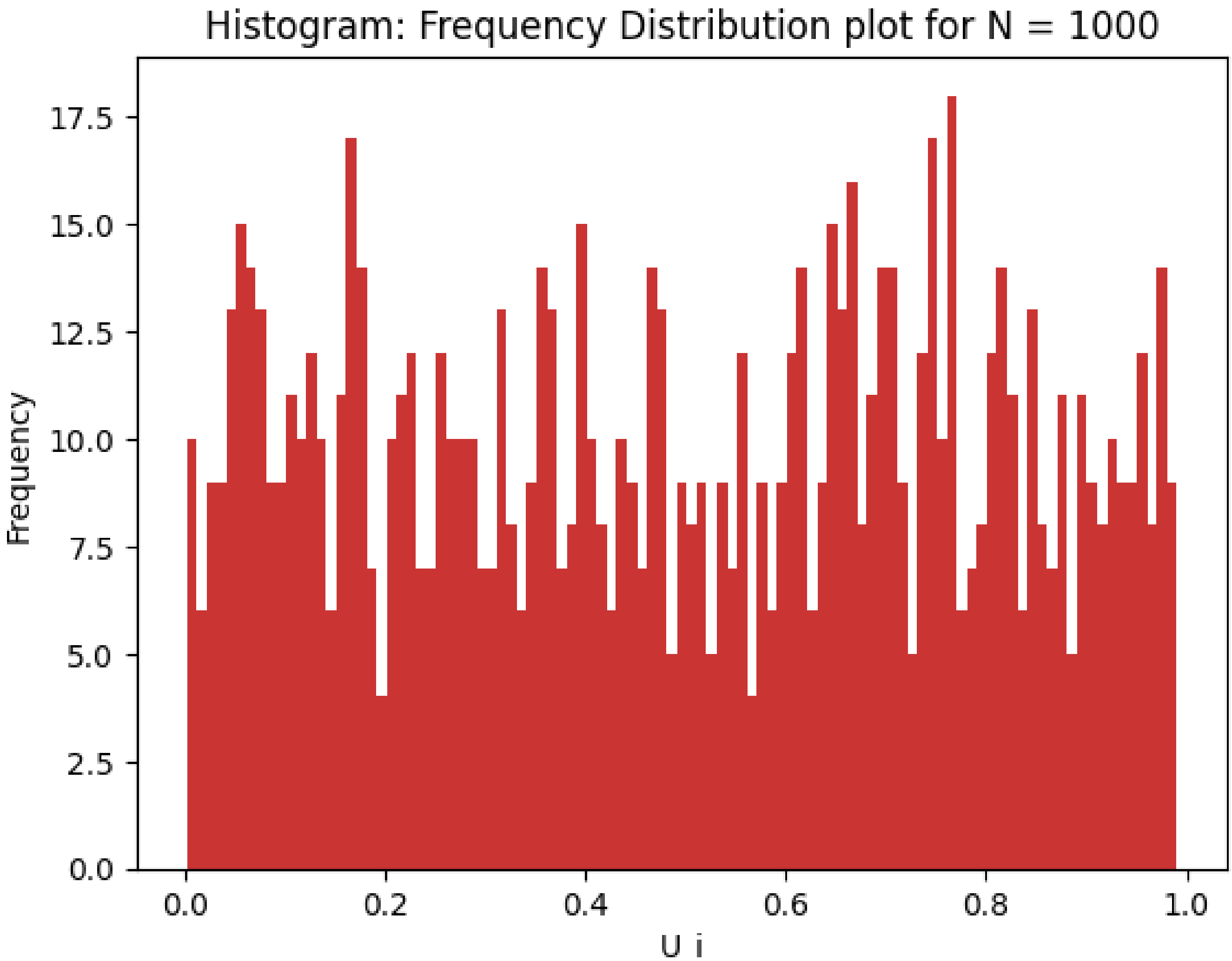
$$x_{i+1} = (ax_i + b) \bmod m$$

$$u_{i+1} = x_{i+1} / m$$

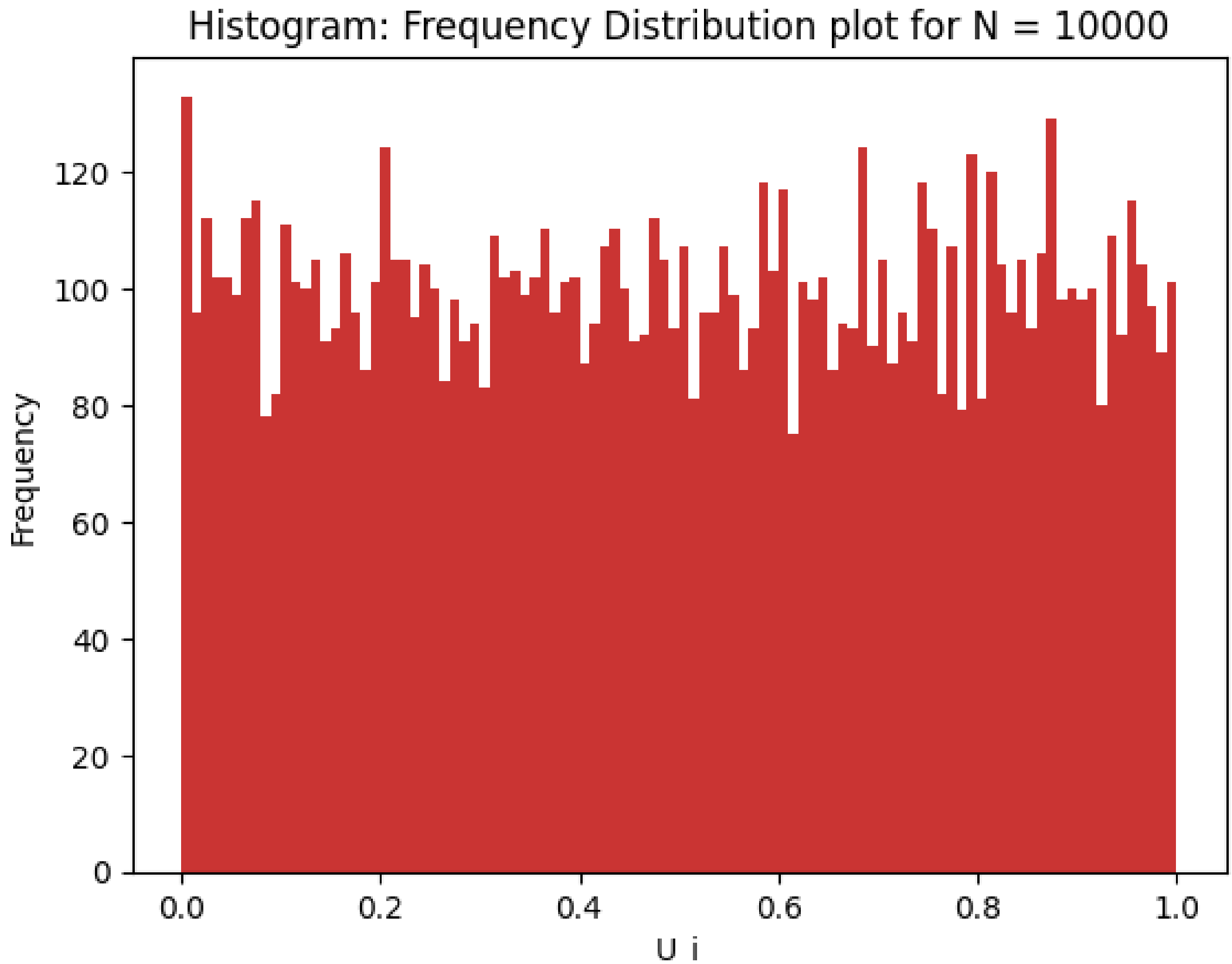
With m = 2048, b = 1, a = 1229

b) The values of U_i are generated for N = 1000, 10000 and 100000 based on the recursion above.

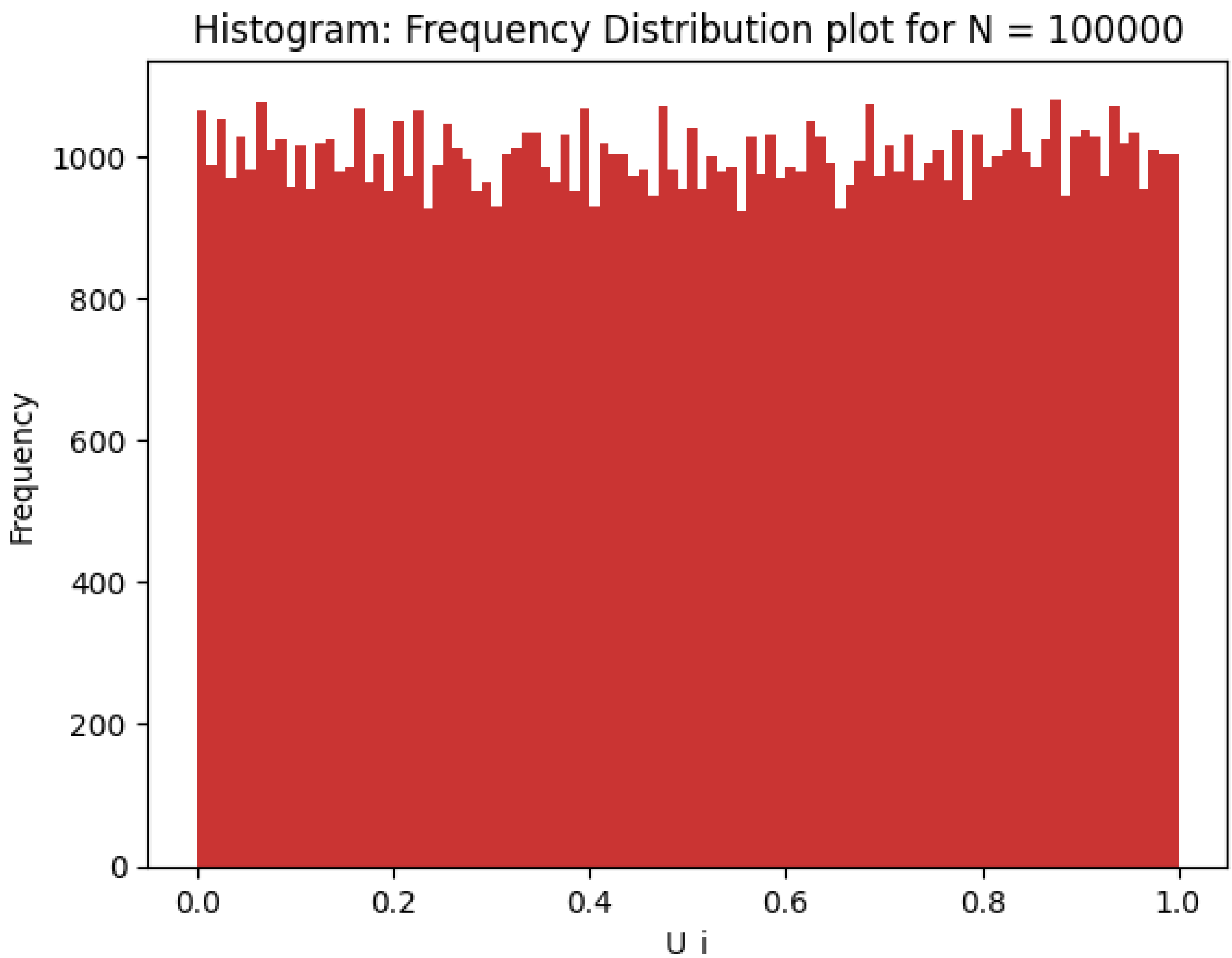
c) For N = 1000,


Histogram: Frequency Distribution plot for N = 1000
Frequency
U_i
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0.0
0.2
0.4
0.6
0.8
1.0

For N = 10000,


Histogram: Frequency Distribution plot for N = 10000
Frequency
U_i
0
20
40
60
80
100
120
0.0
0.2
0.4
0.6
0.8
1.0

For N = 100000,

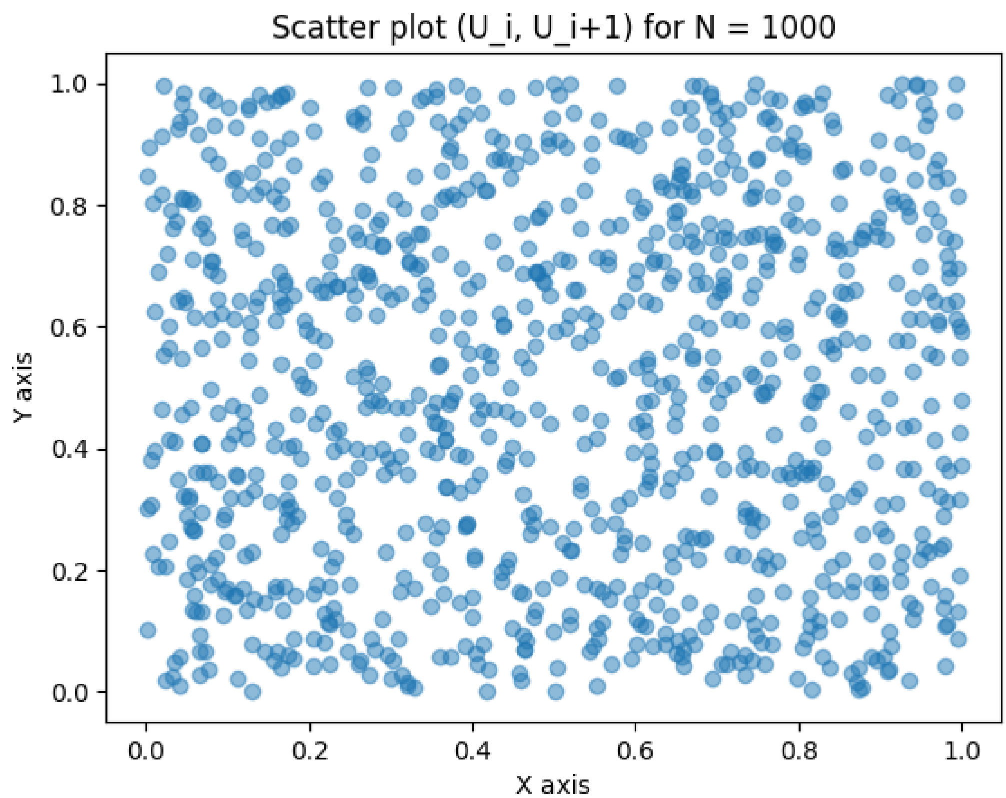


Observations:

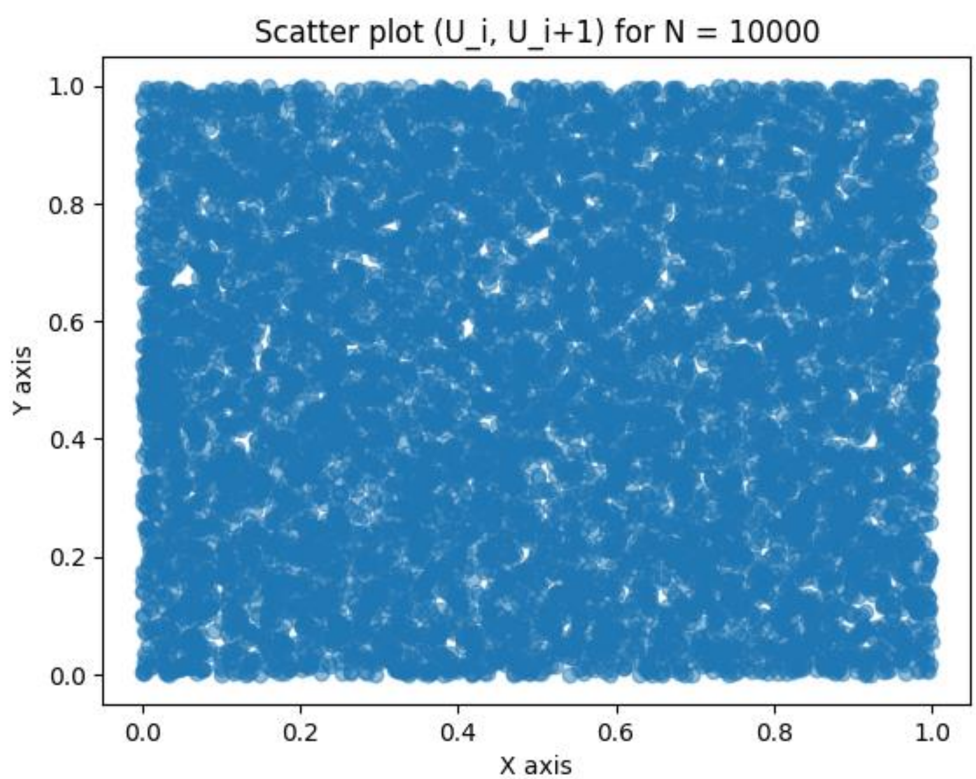
The Frequency Distribution Plots (Histograms) indicate that this random number generator follows the two properties of the ideal random number generator:

A) The Ui are mutually independent.
B) Each Ui is uniformly distributed between 0 and 1.

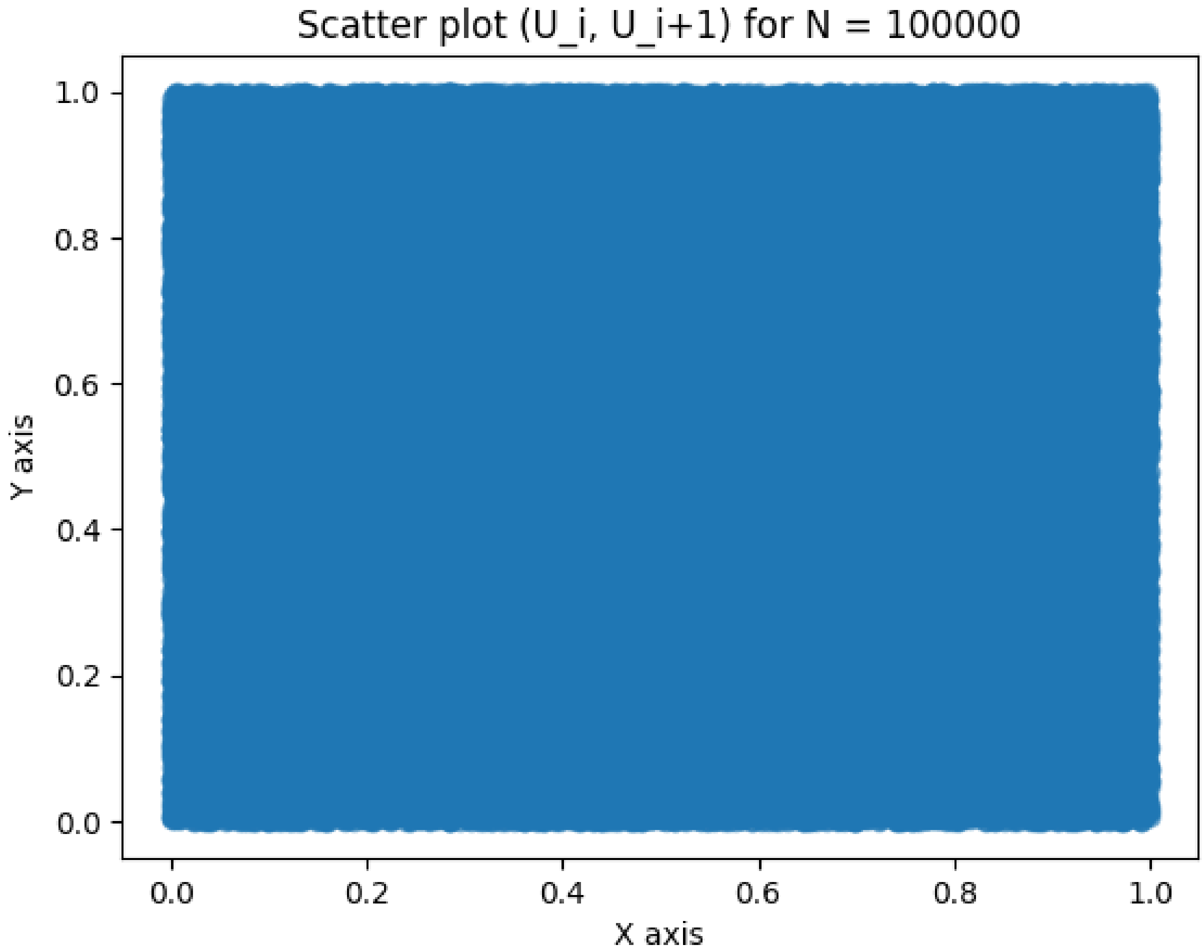
d) For N = 1000,


Scatter plot (U_i, U_i+1) for N = 1000
Y axis
X axis
0.0
0.2
0.4
0.6
0.8
1.0

For N = 10000,


Scatter plot (U_i, U_i+1) for N = 10000
Y axis
X axis
0.0
0.2
0.4
0.6
0.8
1.0

For N = 100000,



Observations:

The Scatter Plots indicate that the U_i 's don't follow any fixed particular pattern, so the U_i 's are almost completely random.

The frequencies of different numbers lying in intervals of the same length are almost the same. So, this given random number generator acts like a good random number generator.

Question 2:

2. Consider the exponential distribution with CDF

$$F(x) = 1 - e^{-x/\theta}\ ,\ x \geq 0,$$

where $\theta > 0$.

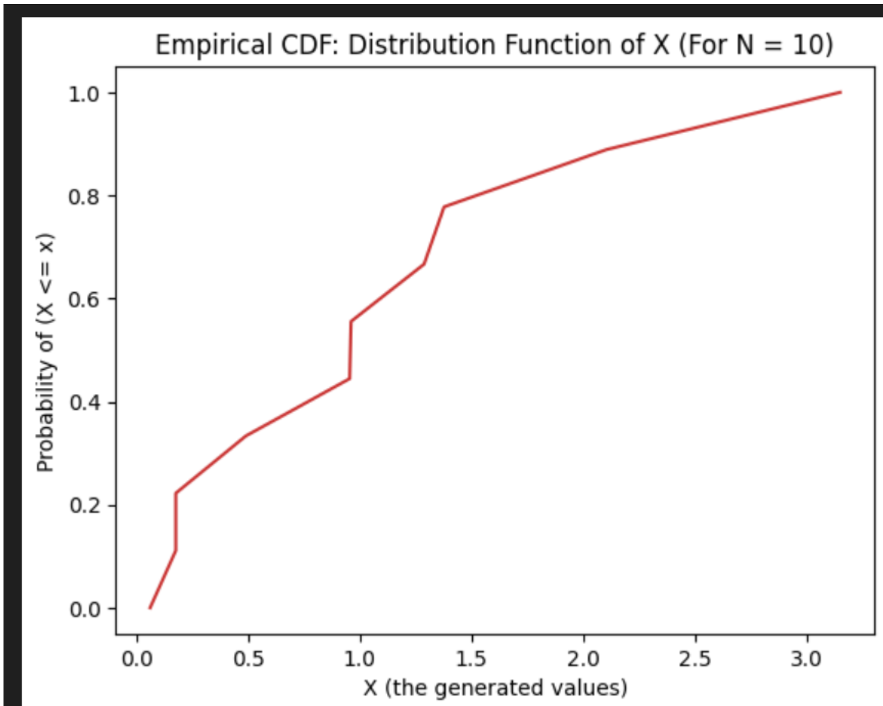
(a) Generate $X_1, X_2, \ldots, X_N$ from the above distribution for $N = 10, 100, 1000, 10000, 100000$.

(b) For each value of N, plot the empirical CDF of these generated values, and the actual distribution function (using the above formula).

(c) Provide the corresponding values of the sample mean and variance. Compare the values of mean and variance to see whether they converge to actual values.

Assumed Value of Mean (θ) = π / 3

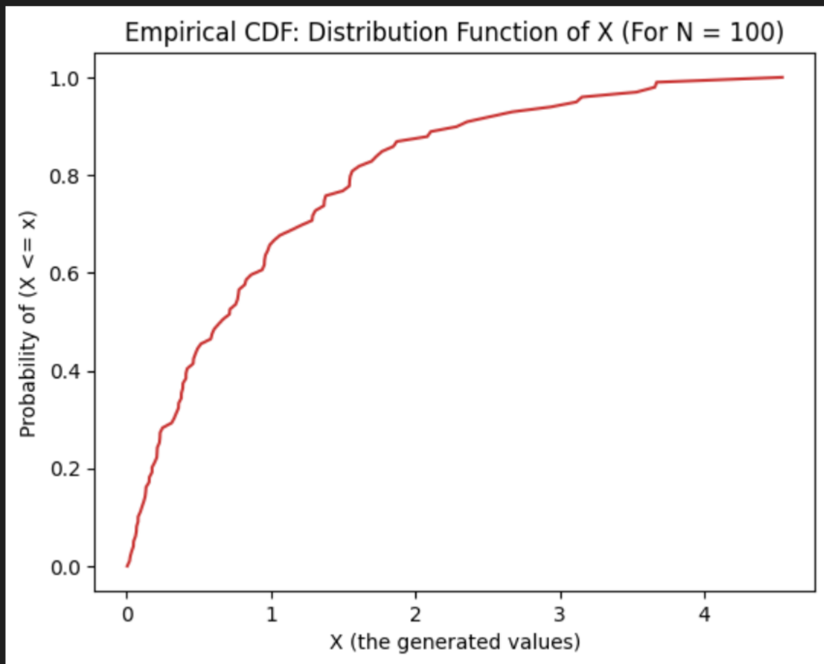
For N = 10 = Total Number of Values Generated,




```
Sample count (N) = 10, Sample Mean = 1.0754405507304425, Sample Variance = 0.8519120084525509
```

For N = 100 = Total Number of Values Generated,

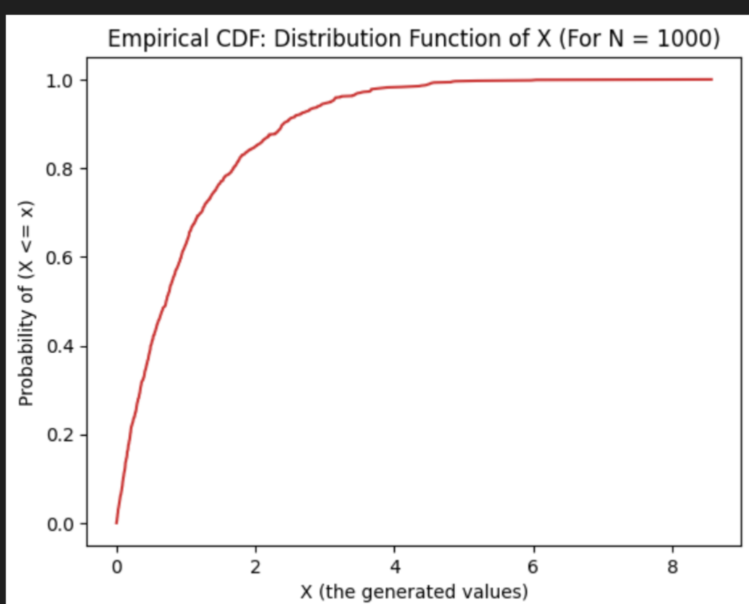
```
For N = 100,
```




```
Sample count (N) = 100, Sample Mean = 0.957921906443396, Sample Variance = 0.9184714135925798
```

For N = 1000 = Total Number of Values Generated,

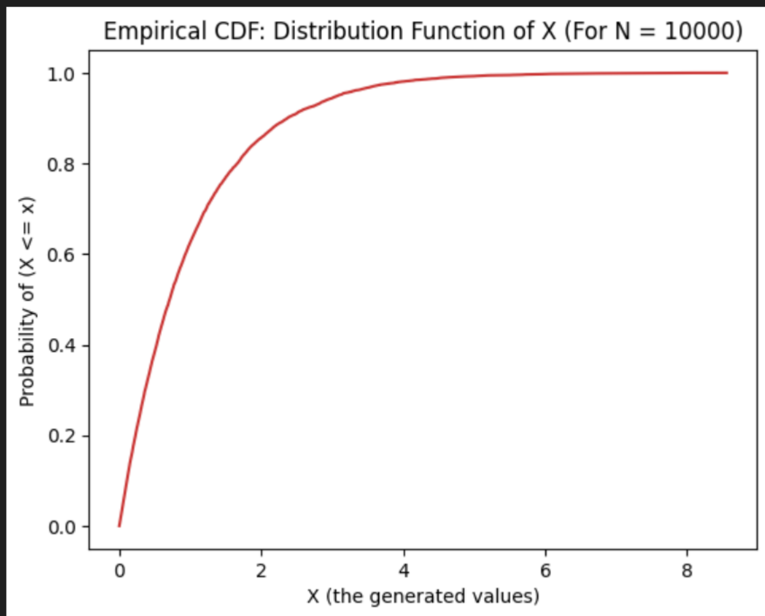
```
For N = 1000,
```




```
Sample count (N) = 1000, Sample Mean = 1.0184058472333088, Sample Variance = 1.0361217107779097
```

For N = 10000 = Total Number of Values Generated,

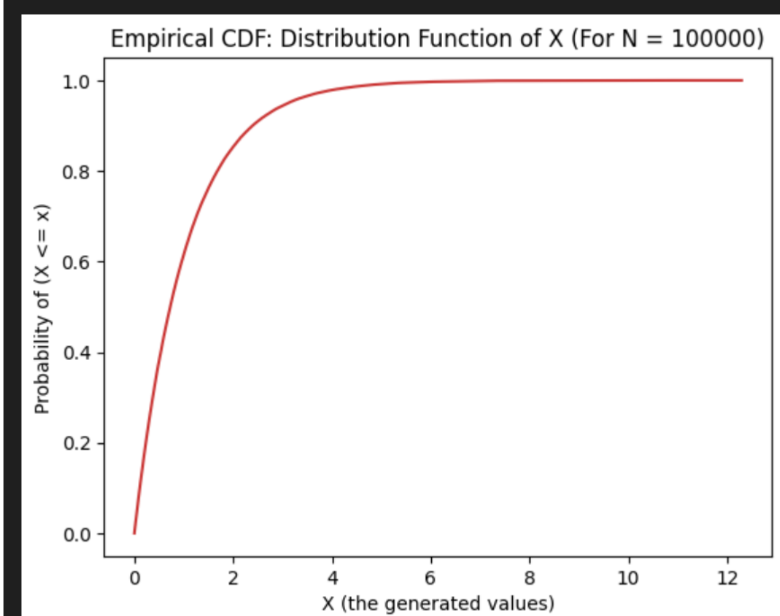
```
For N = 10000,
```




```
Sample count (N) = 10000, Sample Mean = 1.023634511648848, Sample Variance = 1.0411002497539281
```

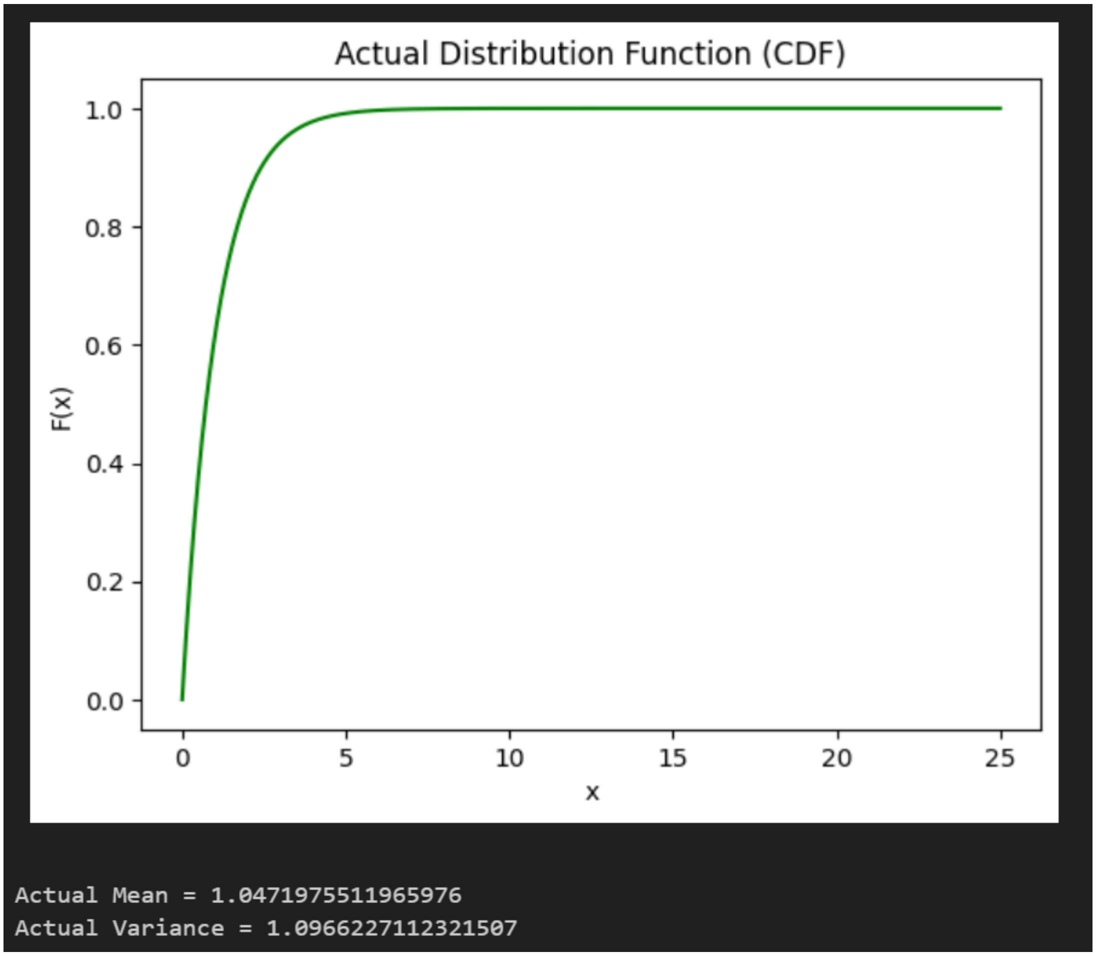
For N = 100000 = Total Number of Values Generated,

```
For N = 100000,
```




```
Sample count (N) = 100000, Sample Mean = 1.0429775116285227, Sample Variance = 1.0812563172102294
```

Actual Distribution Function:



Actual Mean = θ = 1.0471975511965976

Actual Variance = θ^2 = 1.0966227112321507

Sample Count (N)	Sample Mean	Sample Variance
10	1.0754405507304425	0.8519120084525509
100	0.957921906443396	0.9184714135925798
1000	1.0184058472333088	1.0361217107779097
10000	1.023634511648848	1.0411002497539281
100000	1.0429775116285227	1.0812563172102294

Observations:

- The distribution function of X is identical to the C.D.F. (F(x)) from which the random variable X was generated. This is because U is uniform distribution function on [0, 1] and F(x) is a continuous and strictly increasing function, so, $F^{-1}(U)$ will be a sample from F only. This demonstrates the Inverse Transform Method.
- As we increase N (i.e., the number of values generated), the mean of the generated values (X) converges to the actual mean & the variance of the generated values (X) converges to the actual variance. It is also clear from the empirical cumulative distribution function of X for different values of the sample count(N) which approaches the actual graph of the C.D.F. F(x) as the number of values generated(N) increases. This behavior results from the Law of Large Numbers.

Question 3:

3. Consider the Arcsin law with the distribution:

$$F(x) = \frac{2}{\pi} \text{ arc sin } \sqrt{x} \ , \ 0 \le x \le 1.$$

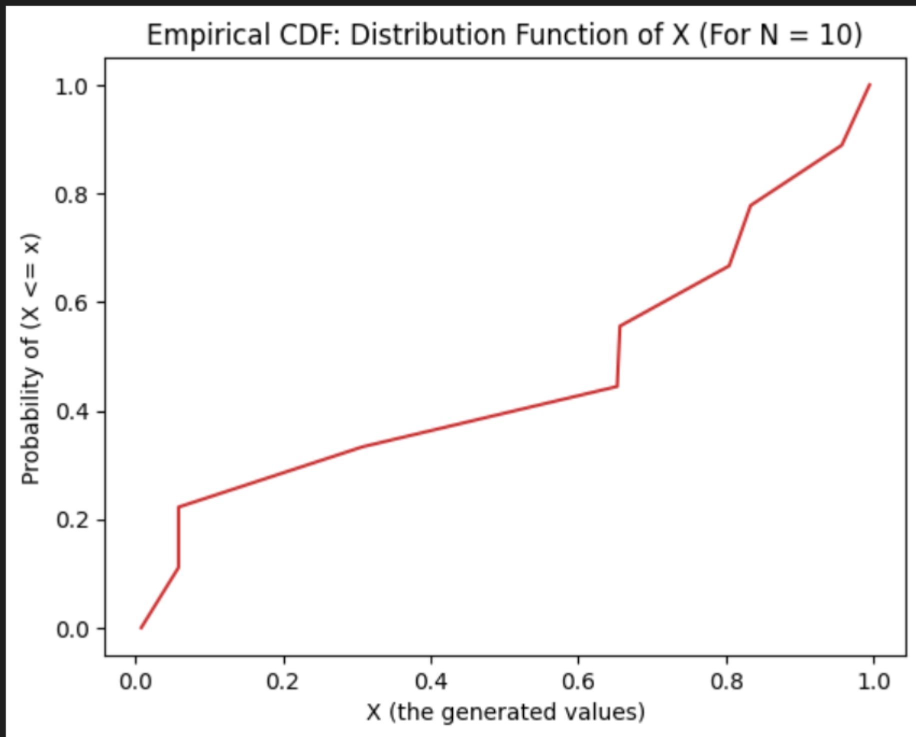
(a) Generate $X_1, X_2, \ldots, X_N$ from the above distribution for $N = 10, 100, 1000, 10000, 100000$.

(b) For each value of N, plot the empirical CDF of these generated values, and the actual distribution function (using the above formula).

(c) Provide the corresponding values of the sample mean and variance.

For N = 10 = Total Number of Values Generated,

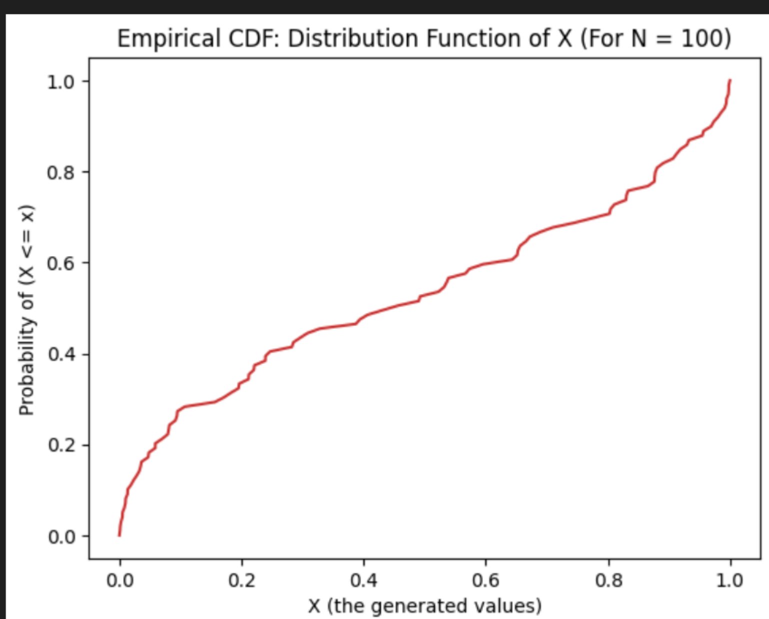
```
For N = 10,
```




```
Sample count (N) = 10, Sample Mean = 0.5330218847871611, Sample Variance = 0.13603351783787718
```

For N = 100 = Total Number of Values Generated,

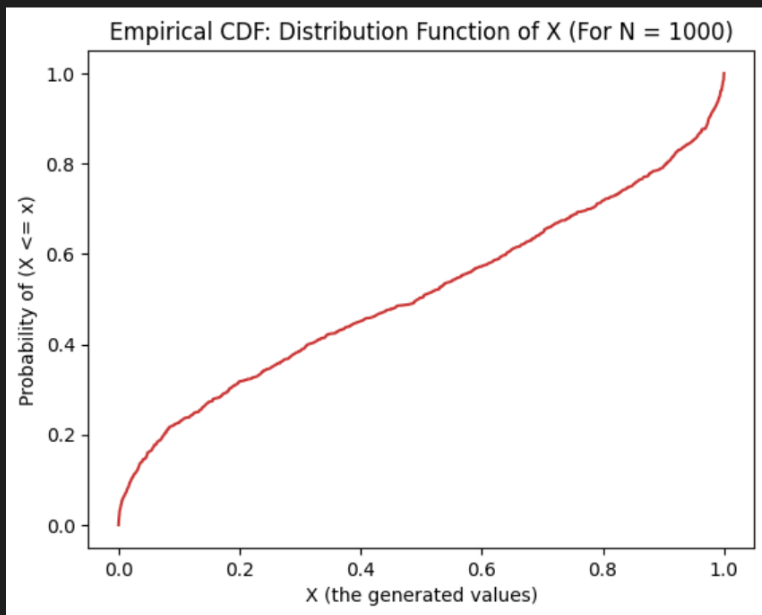
```
For N = 100,
```




```
Sample count (N) = 100, Sample Mean = 0.4649330111462657, Sample Variance = 0.13098678304171854
```

For N = 1000 = Total Number of Values Generated,

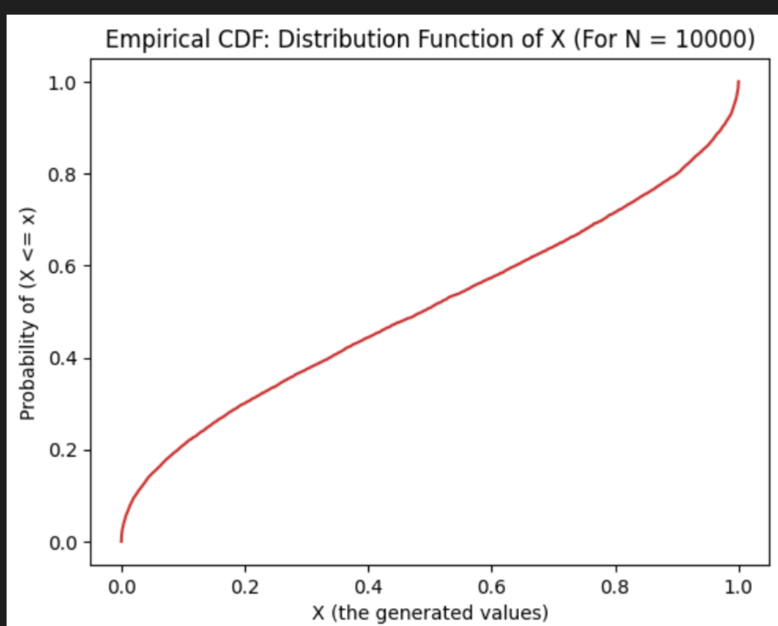
```
For N = 1000,
```




```
Sample count (N) = 1000, Sample Mean = 0.48833044199671294, Sample Variance = 0.12727982689751888
```

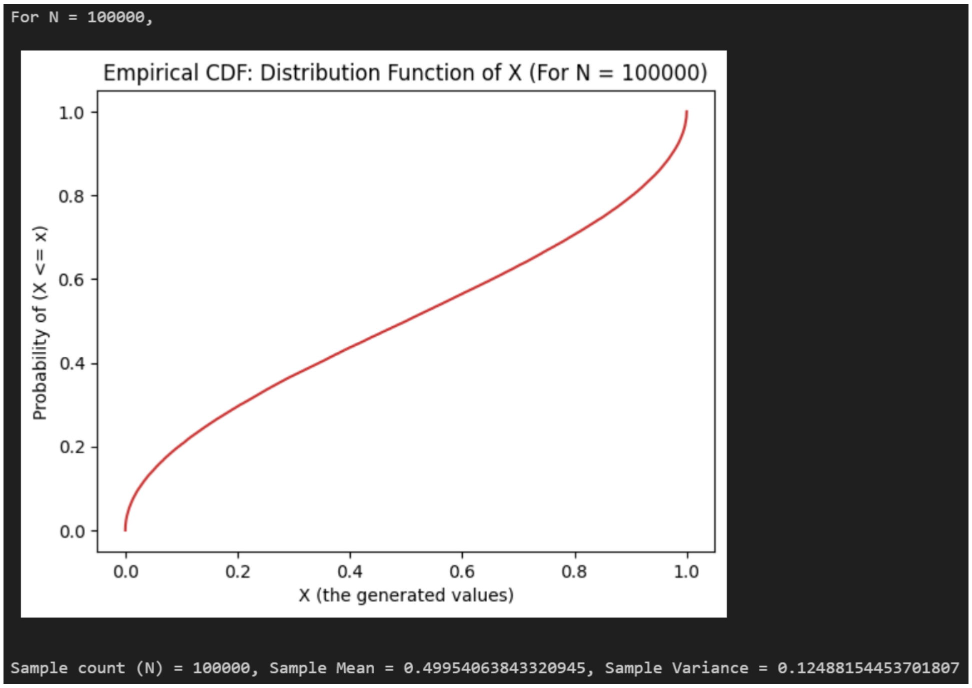
For N = 10000 = Total Number of Values Generated,

```
For N = 10000,
```

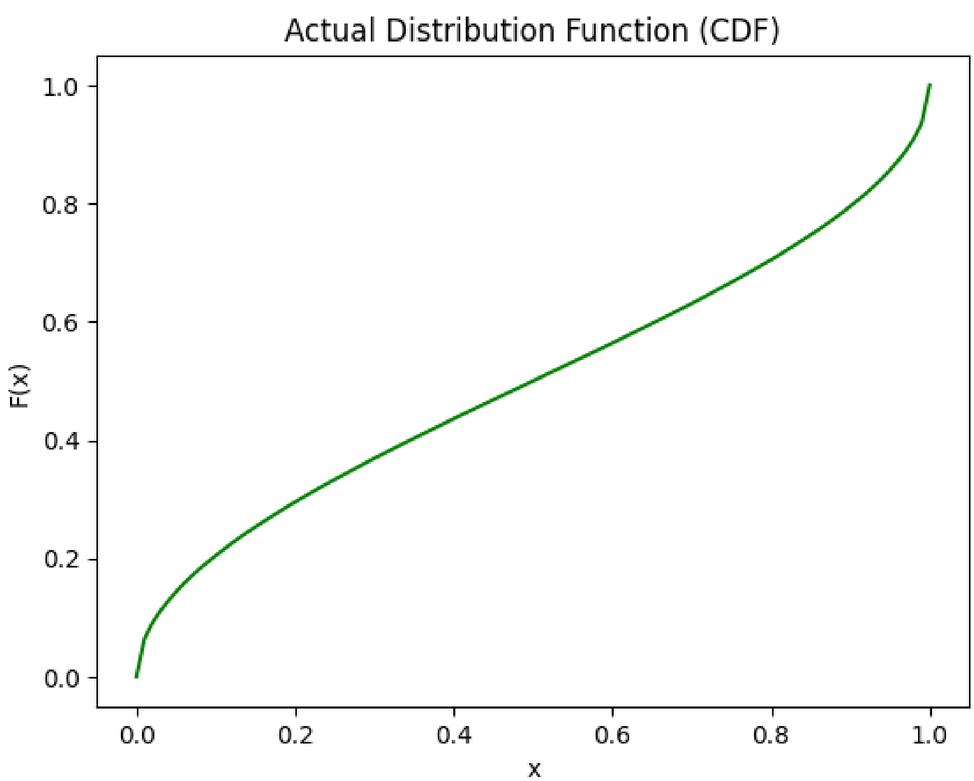



```
Sample count (N) = 10000, Sample Mean = 0.49285879887658973, Sample Variance = 0.12416196616477684
```

For N = 100000 = Total Number of Values Generated,



Actual Distribution Function:



Sample Count (N)	Sample Mean	Sample Variance
10	0.5330218847871611	0.13603351783787718
100	0.4649330111462657	0.13098678304171854
1000	0.48833044199671294	0.12727982689751888
10000	0.49285879887658973	0.12416196616477684
100000	0.49954063843320945	0.12488154453701807

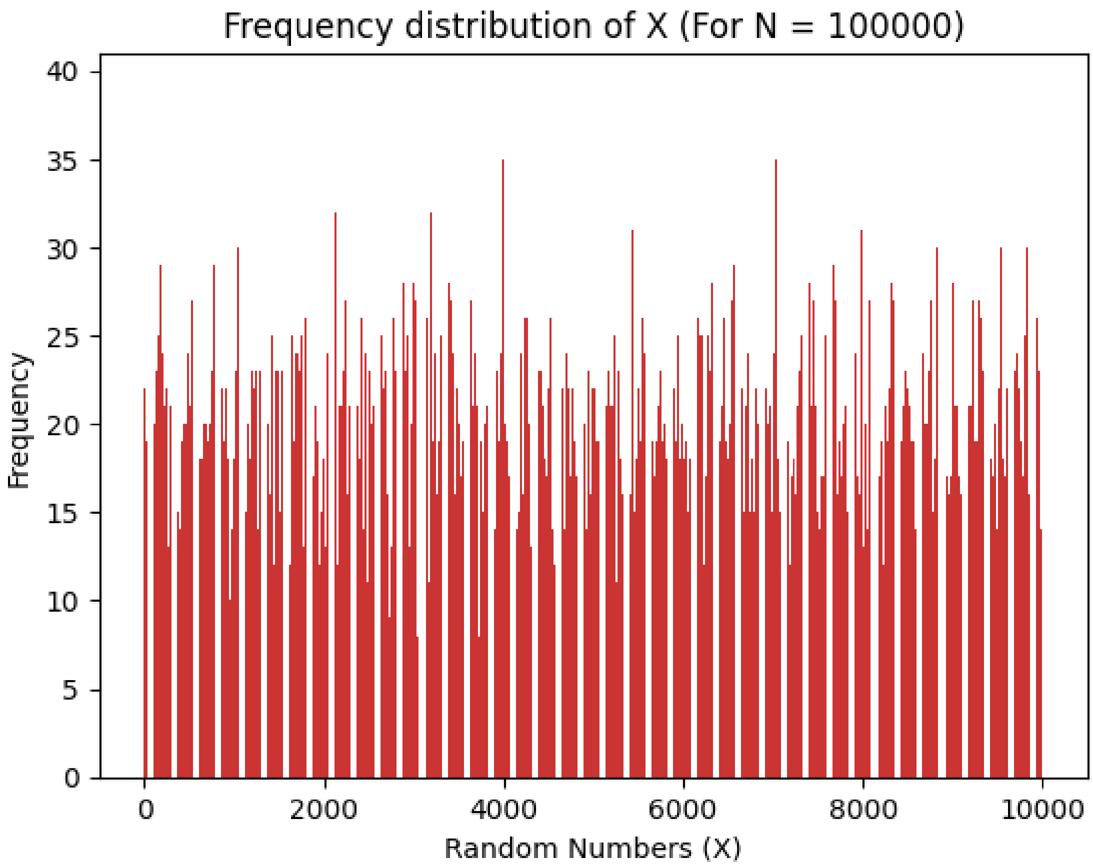
Observations:

- The empirical cumulative distribution functions of X approach the actual graph of the C.D.F. F(x) as the number of values generated(N) increases. This behavior results from the Law of Large Numbers.
- The distribution function of X is identical to the C.D.F. (F(x)) from which the random variable X was generated. This is because U is uniform distribution function on [0, 1] and F(x) is a continuous and strictly increasing function, so, $F^{-1}(U)$ will be a sample from F only. This demonstrates the Inverse Transform Method.

Question 4:

4. Using the algorithm to generate random variables from a discrete distribution, generate 100000 random numbers from a discrete uniform distribution on $\{1, 3, 5, \ldots, 9999\}$. Tabulate the frequency of each observed values.

Generated 100000 random numbers from the discrete uniform distribution on {1, 3, 5, ..., 9999}. The Frequency of each observed value is shown in the graph.


Frequency distribution of X (For N = 100000)
Frequency
Random Numbers (X)
0
5
10
15
20
25
30
35
40
0
2000
4000
6000
8000
10000