# <u>Scientific Computing Lab</u> <u>MA – 322</u> <u>Lab – 6</u>

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**Branch –** Mathematics and Computing

1)

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Question 1
f(x) = exp(x)

Using Newton forward-difference interpolation,
The Forward Difference Table is:
2.71830      1.76340      1.13680      0.76360
4.48170      2.90020      1.90040      0.00000
7.38190      4.80060      0.00000      0.00000
12.1825      0.00000      0.00000      0.00000

The approximate value of f(2.25) = 9.4969250000
--------------------------------------------------
Using Newton backward-difference interpolation,
The Backward Difference Table is:
2.71830      0.00000      0.00000      0.00000
4.48170      1.76340      0.00000      0.00000
7.38190      2.90020      1.13680      0.00000
12.1825      4.80060      1.90040      0.76360

The approximate value of f(2.25) = 9.4969250000
--------------------------------------------------
Exact value of f(2.25) = 9.4877358364
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2)

a)

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Question 2 part a

Using Newton forward-difference formula,
Constructing interpolating polynomial of degree 1
The Forward Difference Table is:
1.00000000       0.64872000
1.64872000       0.00000000

The obtained interpolating polynomial is: p(x) = (8109*x)/3125 + 1
The approximate value of f(0.43) = 2.1157984000
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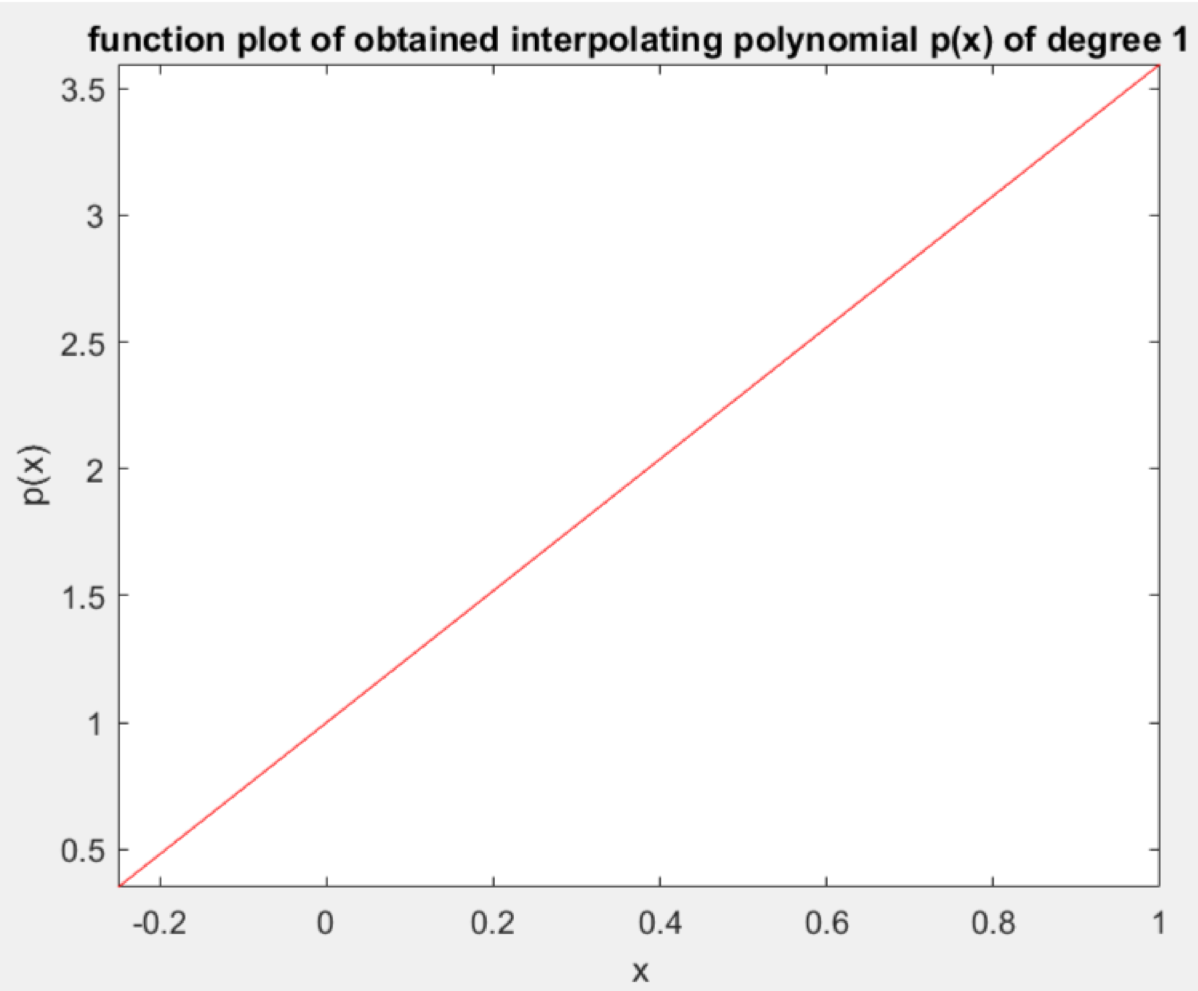

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Using Newton forward-difference formula,
Constructing interpolating polynomial of degree 2
The Forward Difference Table is:
1.00000000      0.64872000      0.42084000
1.64872000      1.06956000      0.00000000
2.71828000      0.00000000      0.00000000

The obtained interpolating polynomial is: p(x) = (8109*x)/3125 + (10521*x*(4*x - 1))/12500 + 1
The approximate value of f(0.43) = 2.3763825280
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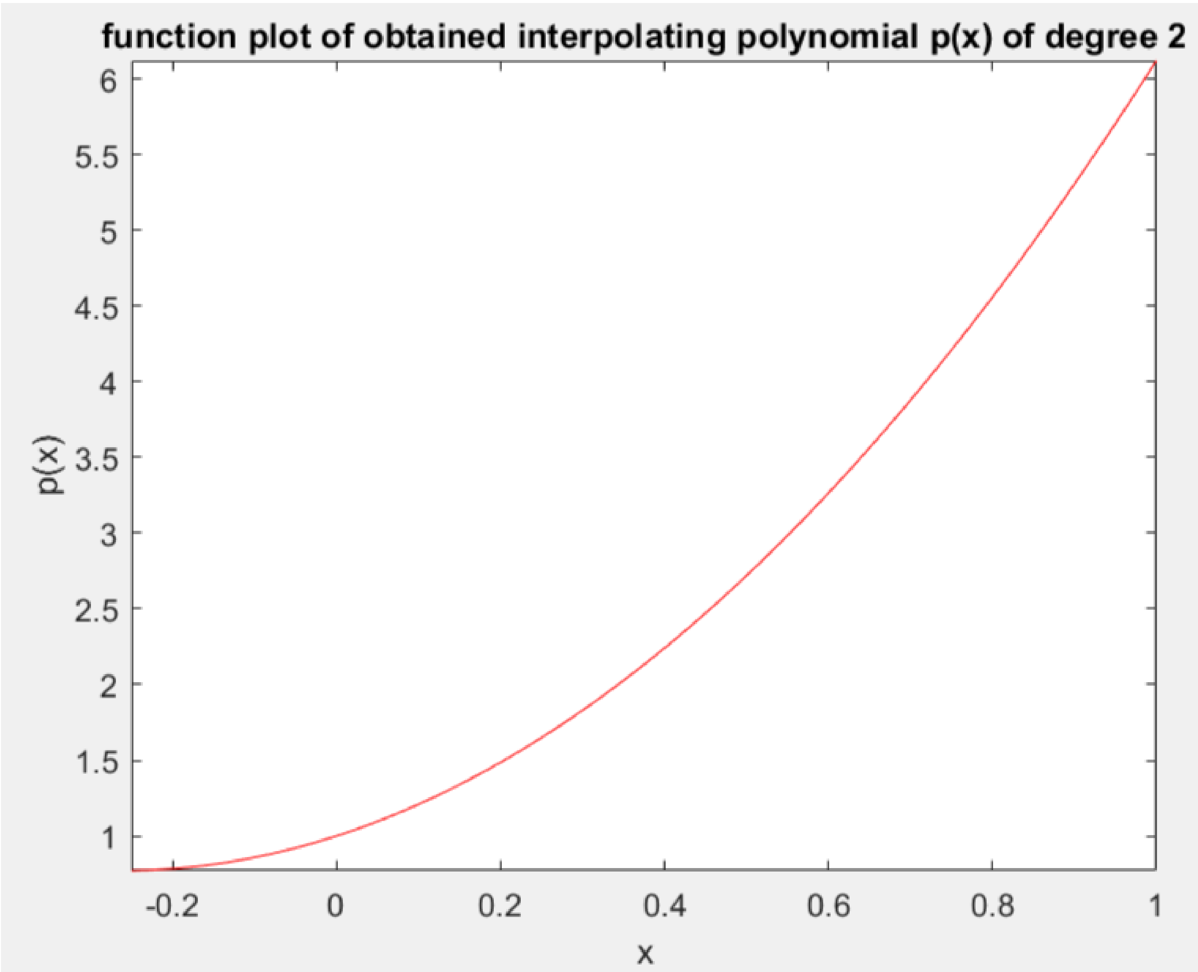


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Using Newton forward-difference formula,
Constructing interpolating polynomial of degree 3
The Forward Difference Table is:
1.00000000      0.64872000      0.42084000      0.27301000
1.64872000      1.06956000      0.69385000      0.00000000
2.71828000      1.76341000      0.00000000      0.00000000
4.48169000      0.00000000      0.00000000      0.00000000

The obtained interpolating polynomial is: p(x) = (8109*x)/3125 + (10521*x*(4*x - 1))/12500 + (27301*x*(4*x - 1)*(4*x - 2))/150000 + 1
The approximate value of f(0.43) = 2.3606047341
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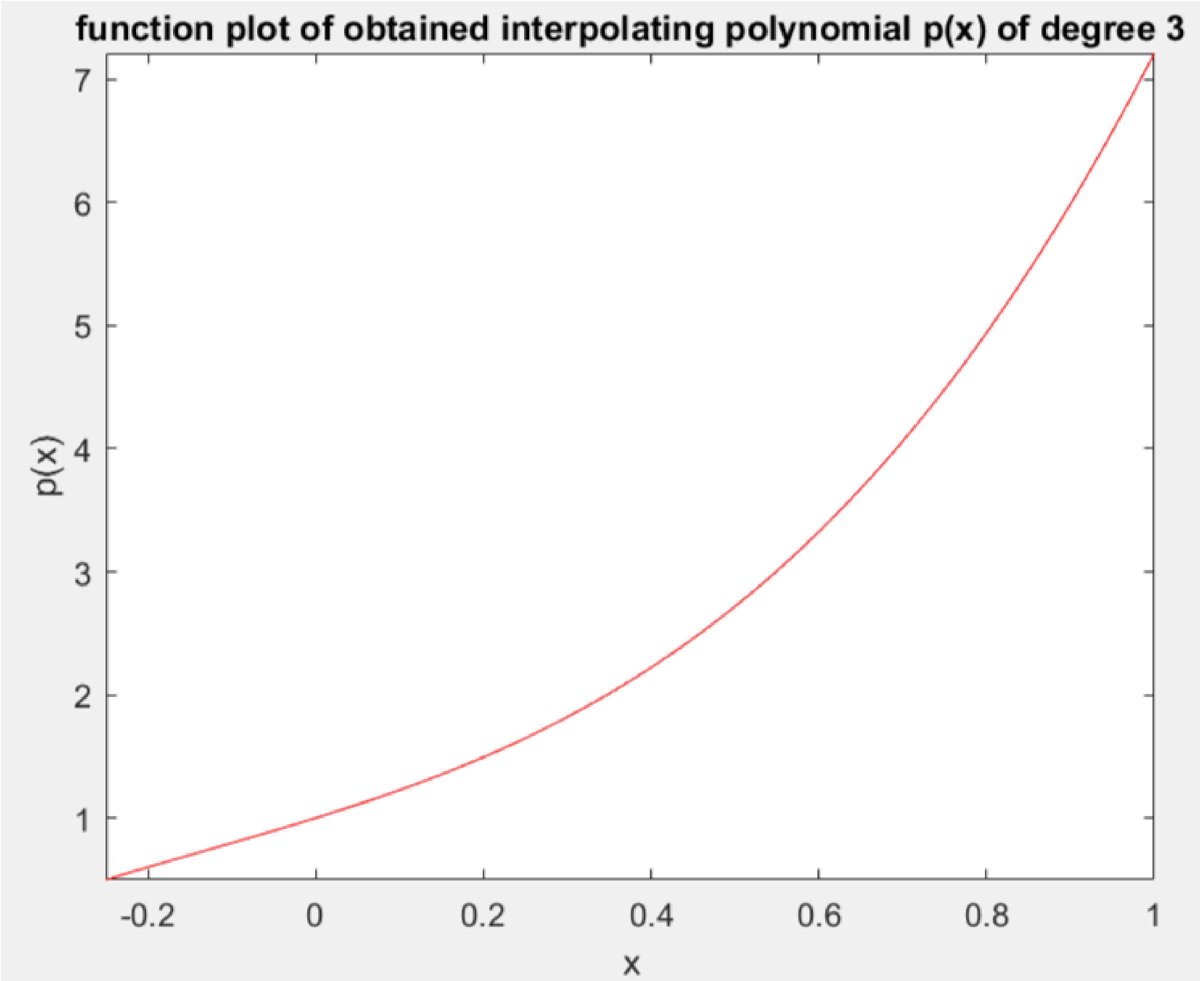
function plot of obtained interpolating polynomial p(x) of degree 3
p(x)
x
7
6
5
4
3
2
1
-0.2
0
0.2
0.4
0.6
0.8
1

b)

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Question 2 part b

Using Newton forward-difference formula,
Constructing interpolating polynomial of degree 1
The Forward Difference Table is:
-0.07181250     0.04706250
-0.02475000     0.00000000

The obtained interpolating polynomial is: p(x) = (753*x)/4000 + 111/1600
The approximate value of f(-1/3) = 0.0066250000
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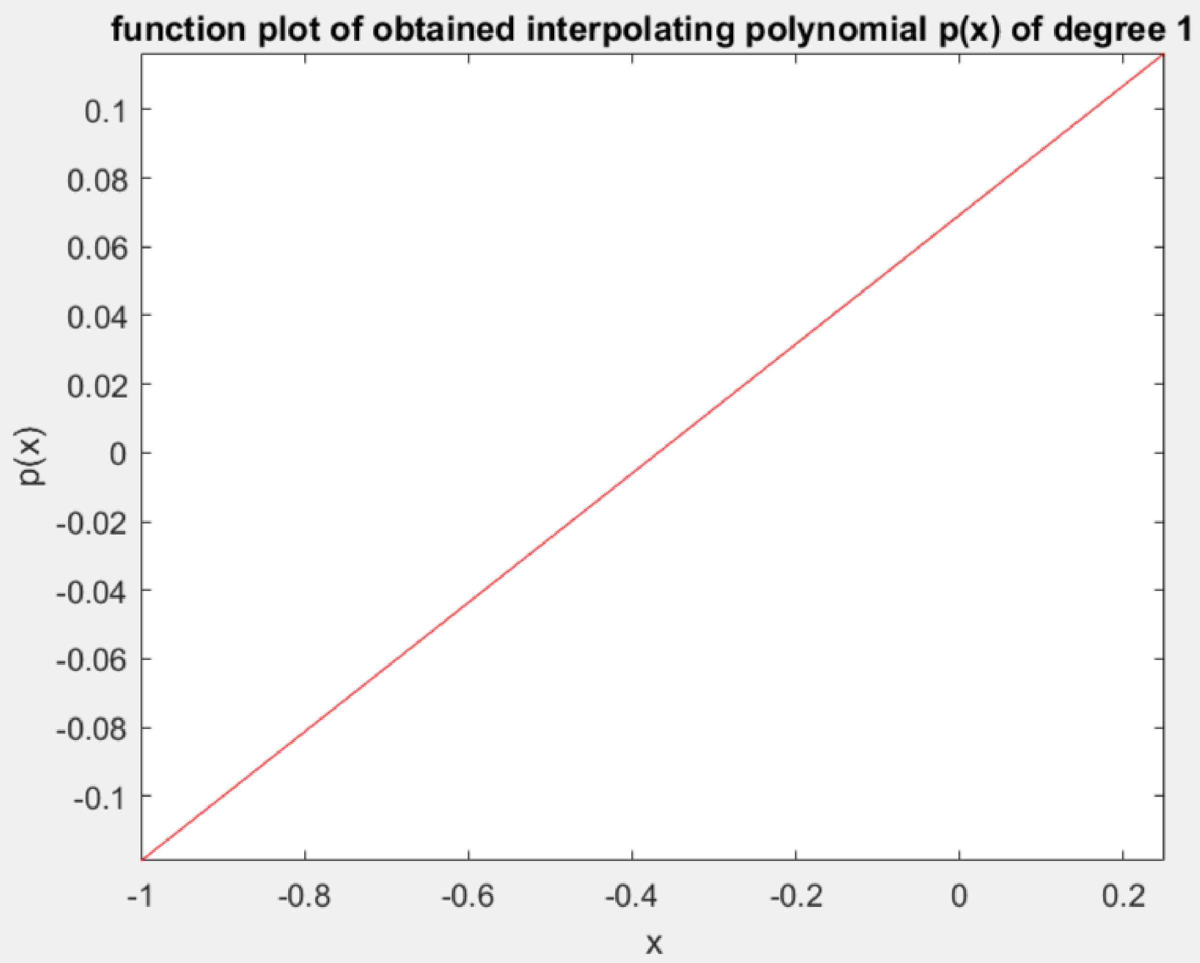

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Using Newton forward-difference formula,
Constructing interpolating polynomial of degree 2
The Forward Difference Table is:
-0.07181250      0.04706250      0.31262500
-0.02475000      0.35968750      0.00000000
0.33493750       0.00000000      0.00000000

The obtained interpolating polynomial is: p(x) = (753*x)/4000 + (2501*(4*x + 2)*(4*x + 3))/16000 + 111/1600
The approximate value of f(-1/3) = 0.1803055556
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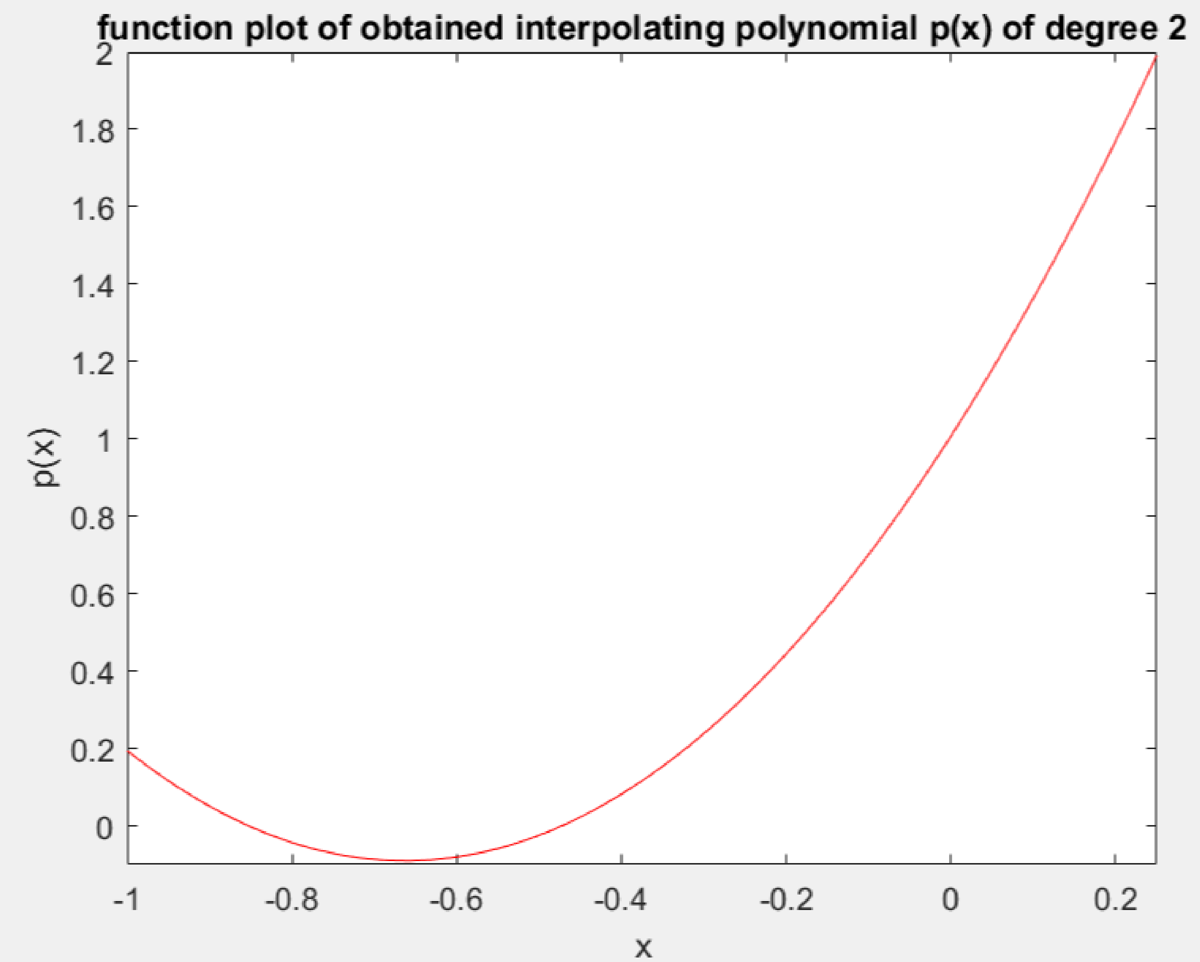


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Using Newton forward-difference formula,
Constructing interpolating polynomial of degree 3
The Forward Difference Table is:
-0.07181250      0.04706250      0.31262500      0.09375000
-0.02475000      0.35968750      0.40637500      0.00000000
0.33493750       0.76606250      0.00000000      0.00000000
1.10100000       0.00000000      0.00000000      0.00000000

The obtained interpolating polynomial is: p(x) = (753*x)/4000 + (2501*(4*x + 2)*(4*x + 3))/16000 + ((4*x + 1)*(4*x + 2)*(4*x + 3))/64 + 111/1600
The approximate value of f(-1/3) = 0.1745185185
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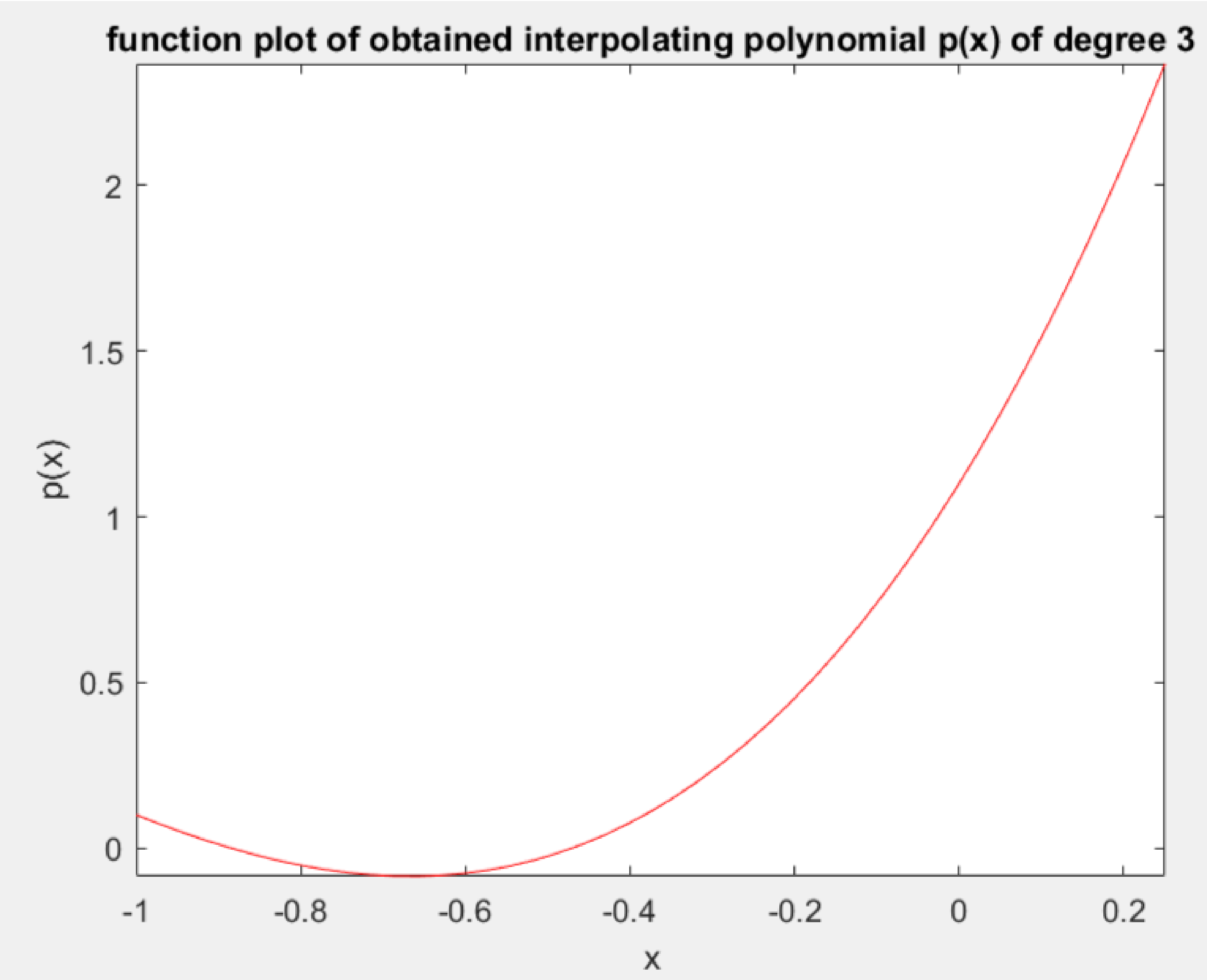


3)

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Question 3

Taking P_n as Newton forward-difference interpolating polynomial,
For n = 1, The approximate value of f(1 + sqrt(10)) = 0.0384615385. So, y_1 = 0.0384615385
For n = 2, The approximate value of f(1 + sqrt(10)) = 0.3336709492. So, y_2 = 0.3336709492
For n = 3, The approximate value of f(1 + sqrt(10)) = 0.1166052060. So, y_3 = 0.1166052060
For n = 4, The approximate value of f(1 + sqrt(10)) = -0.3717596394. So, y_4 = -0.3717596394
For n = 5, The approximate value of f(1 + sqrt(10)) = -0.0548918740. So, y_5 = -0.0548918740
For n = 6, The approximate value of f(1 + sqrt(10)) = 0.6059346282. So, y_6 = 0.6059346282
For n = 7, The approximate value of f(1 + sqrt(10)) = 0.1902492330. So, y_7 = 0.1902492330
For n = 8, The approximate value of f(1 + sqrt(10)) = -0.5133526169. So, y_8 = -0.5133526169
For n = 9, The approximate value of f(1 + sqrt(10)) = -0.0668173424. So, y_9 = -0.0668173424
For n = 10, The approximate value of f(1 + sqrt(10)) = 0.4483348123. So, y_10 = 0.4483348123
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Taking P_n as Newton backward-difference interpolating polynomial,
For n = 1, The approximate value of f(1 + sqrt(10)) = 0.0384615385. So, y_1 = 0.0384615385
For n = 2, The approximate value of f(1 + sqrt(10)) = 0.3336709492. So, y_2 = 0.3336709492
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For n = 9, The approximate value of f(1 + sqrt(10)) = -0.0668173424. So, y_9 = -0.0668173424
For n = 10, The approximate value of f(1 + sqrt(10)) = 0.4483348123. So, y_10 = 0.4483348123
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Taking P_n as Lagrange Interpolant,
For n = 1, The approximate value of f(1 + sqrt(10)) = 0.0384615385. So, y_1 = 0.0384615385
For n = 2, The approximate value of f(1 + sqrt(10)) = 0.3336709492. So, y_2 = 0.3336709492
For n = 3, The approximate value of f(1 + sqrt(10)) = 0.1166052060. So, y_3 = 0.1166052060
For n = 4, The approximate value of f(1 + sqrt(10)) = -0.3717596394. So, y_4 = -0.3717596394
For n = 5, The approximate value of f(1 + sqrt(10)) = -0.0548918740. So, y_5 = -0.0548918740
For n = 6, The approximate value of f(1 + sqrt(10)) = 0.6059346282. So, y_6 = 0.6059346282
For n = 7, The approximate value of f(1 + sqrt(10)) = 0.1902492330. So, y_7 = 0.1902492330
For n = 8, The approximate value of f(1 + sqrt(10)) = -0.5133526169. So, y_8 = -0.5133526169
For n = 9, The approximate value of f(1 + sqrt(10)) = -0.0668173424. So, y_9 = -0.0668173424
For n = 10, The approximate value of f(1 + sqrt(10)) = 0.4483348123. So, y_10 = 0.4483348123

Exact value of f(1 + sqrt(10)) = 0.0545715835
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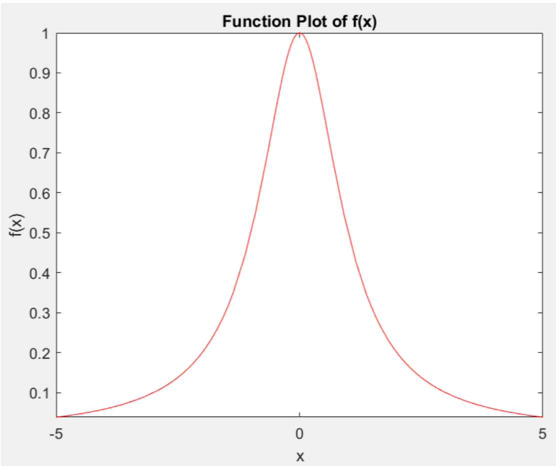


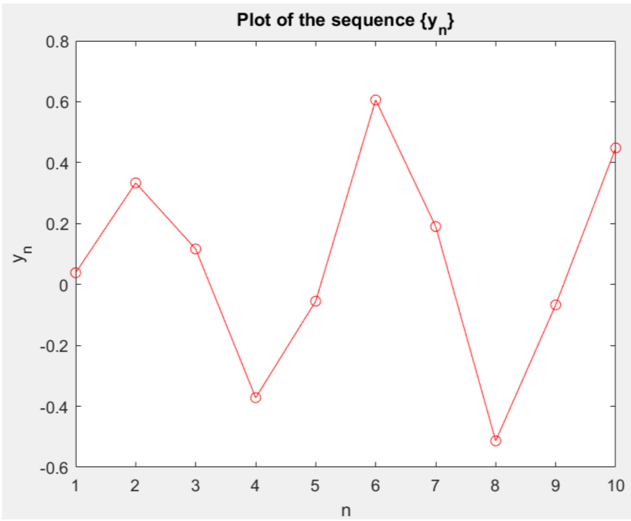


We get the same sequence of $\{y_n\}$ in all the three cases, that is, when we take $P_n$ as Lagrange interpolant, Newton forward-difference interpolant and Newton backward-difference interpolant.

No, the sequence $\{y_n\}$ does not appear to converge to f (1 + sqrt(10)). The sequence $\{y_n\}$ oscillates and diverges and does not converge to f(1+sqrt(10)).

The oscillating patterns evident in the graphs generated by Lagrange interpolation, Newton's forward interpolation, and Newton's backward interpolation can be attributed to a phenomenon known as Runge's phenomenon.

Runge's phenomenon manifests when equidistant interpolation points are utilized to approximate a function with polynomial interpolation. As the number of interpolation points increases, the discrepancies between adjacent points are magnified, resulting in pronounced oscillations or "wiggles" in the interpolated polynomial, particularly near the boundaries of the interpolation interval.

In our question, the following scenario arises:

- Higher Degree Polynomials: The interpolating polynomials employed in Lagrange interpolation, Newton's forward interpolation, and Newton's backward interpolation possess higher degrees, corresponding to the increasing number of interpolation points.
- Equidistant Interpolation Points: Interpolation points are evenly spaced within the range [-5, 5]. Consequently, as the quantity of interpolation points rises, the gap between neighbouring points diminishes.
- Runge's Phenomenon: The use of equidistant interpolation points with higher degree polynomials exacerbates the oscillations between adjacent points, resulting in the observed oscillatory behaviour or "wiggles" in the interpolated polynomial.

To address the oscillations induced by Runge's phenomenon, alternative approaches can be considered, such as employing non-equidistant interpolation points such as Chebyshev nodes or utilizing interpolation methods which are less susceptible to Runge's phenomenon, such as spline interpolation or rational function interpolation.

Furthermore, it is important to note that augmenting the number of interpolation points may not invariably lead to improved approximations due to the amplification of oscillations. Striking a balance between the quantity of interpolation points and the desired accuracy of the approximation is crucial.