Scientific Computing Lab MA – 322 Lab – 7

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1)

The absolute error is calculated at x = 0.25

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Question 1
Using Hermite Interpolation, Lagrange basis method to approximate f(0.25)

The approximate value of f(0.25) = 1.189069761182
Exact value of f(0.25) = 1.189069931112
Absolute error = 1.699303e-07
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Using Hermite Interpolation, Newton's Divided Differences method to approximate f(0.25)

Newton's Divided Differences Table for Hermite Interpolation
0.86199480      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000
0.86199480      0.15536240      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000
0.95802009      0.19205058      0.07337636      0.00000000      0.00000000      0.00000000      0.00000000      0.00000000
0.95802009      0.23269654      0.08129192      0.01583112      0.00000000      0.00000000      0.00000000      0.00000000
1.09861230      0.28118442      0.09697576      0.01568384      -0.00014728     0.00000000      0.00000000      0.00000000
1.09861230      0.33333333      0.10429782      0.01464412      -0.00103972     -0.00089244     0.00000000      0.00000000
1.29437670      0.39152880      0.11639094      0.01209312      -0.00255100     -0.00100752     -0.00007672     0.00000000
1.29437670      0.45186776      0.12067792      0.00857396      -0.00351916     -0.00096816     0.00002624      0.00006864

The approximate value of f(0.25) = 1.189069761182
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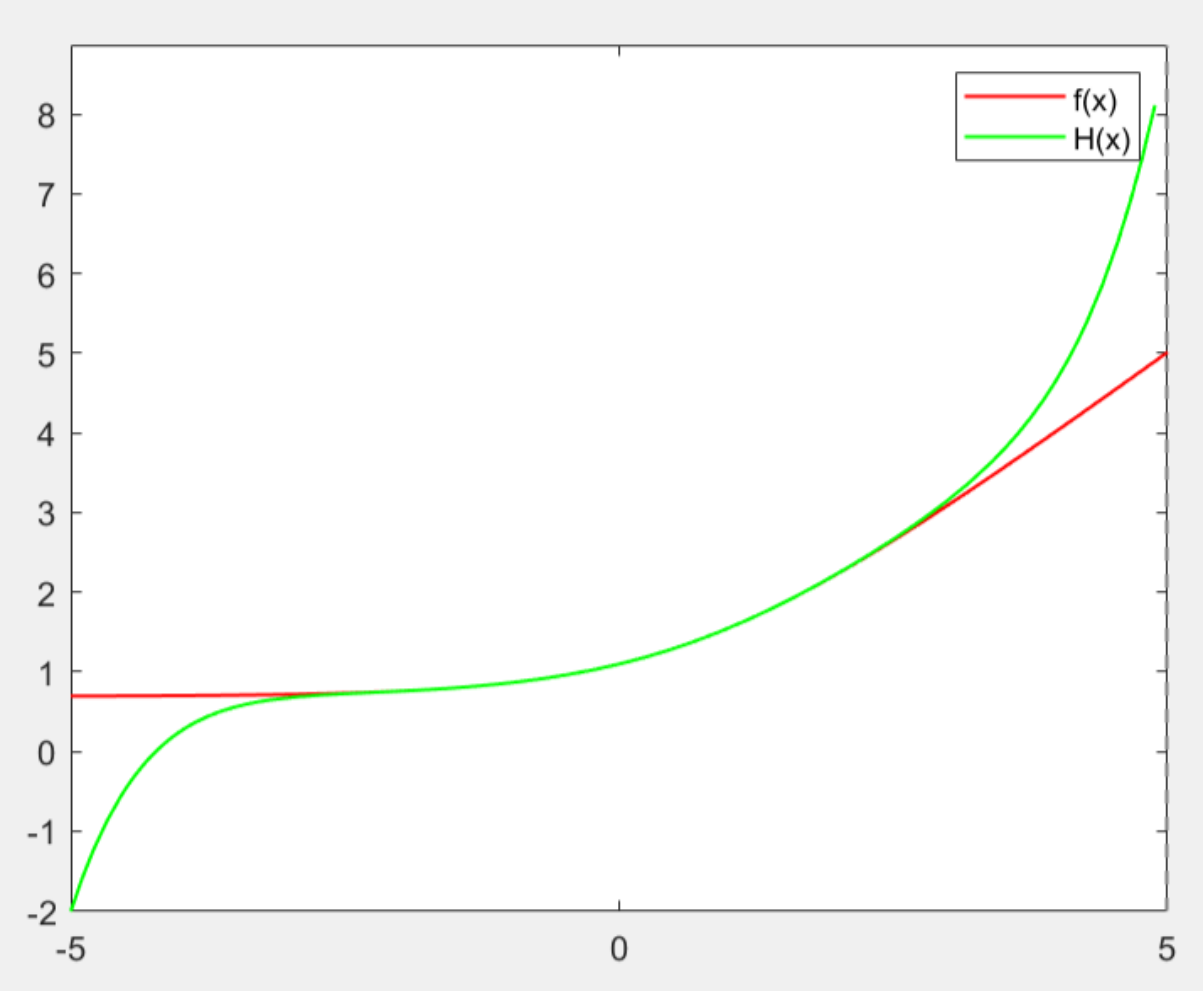
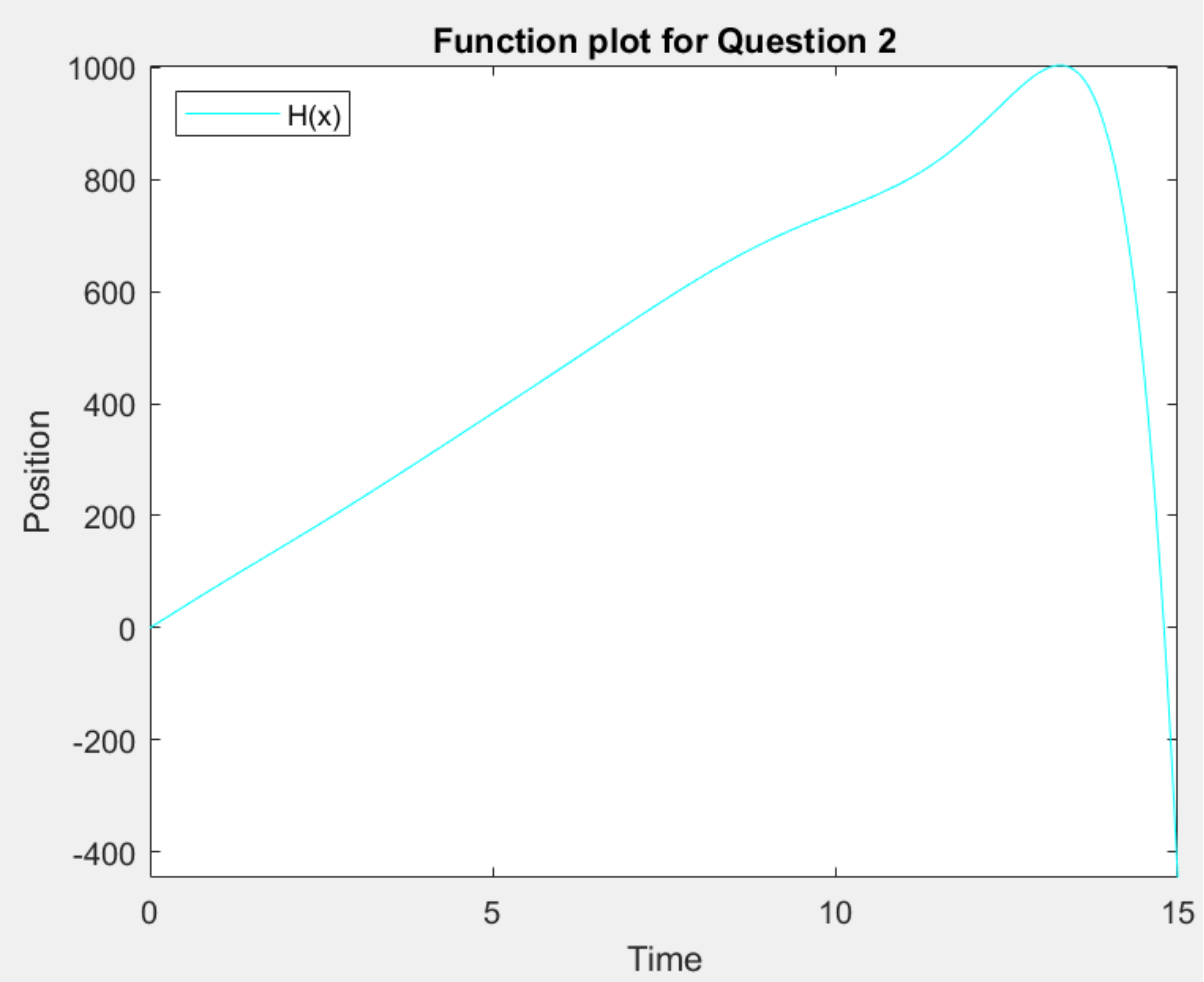
The plot of the function f(x) and interpolating polynomial H(x) is as follows:



2)

Taking the function f (t) as distance and f'(t) as speed, we can approximate the given data with the help of Hermite interpolation.



a)

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The approximate value of f(10.00) = 742.502839098771
So, The approximate position of the car at t = 10 seconds is 742.502839098771 feet
Approximate speed of the car at t = 10 seconds is 48.381736363981 feet per second
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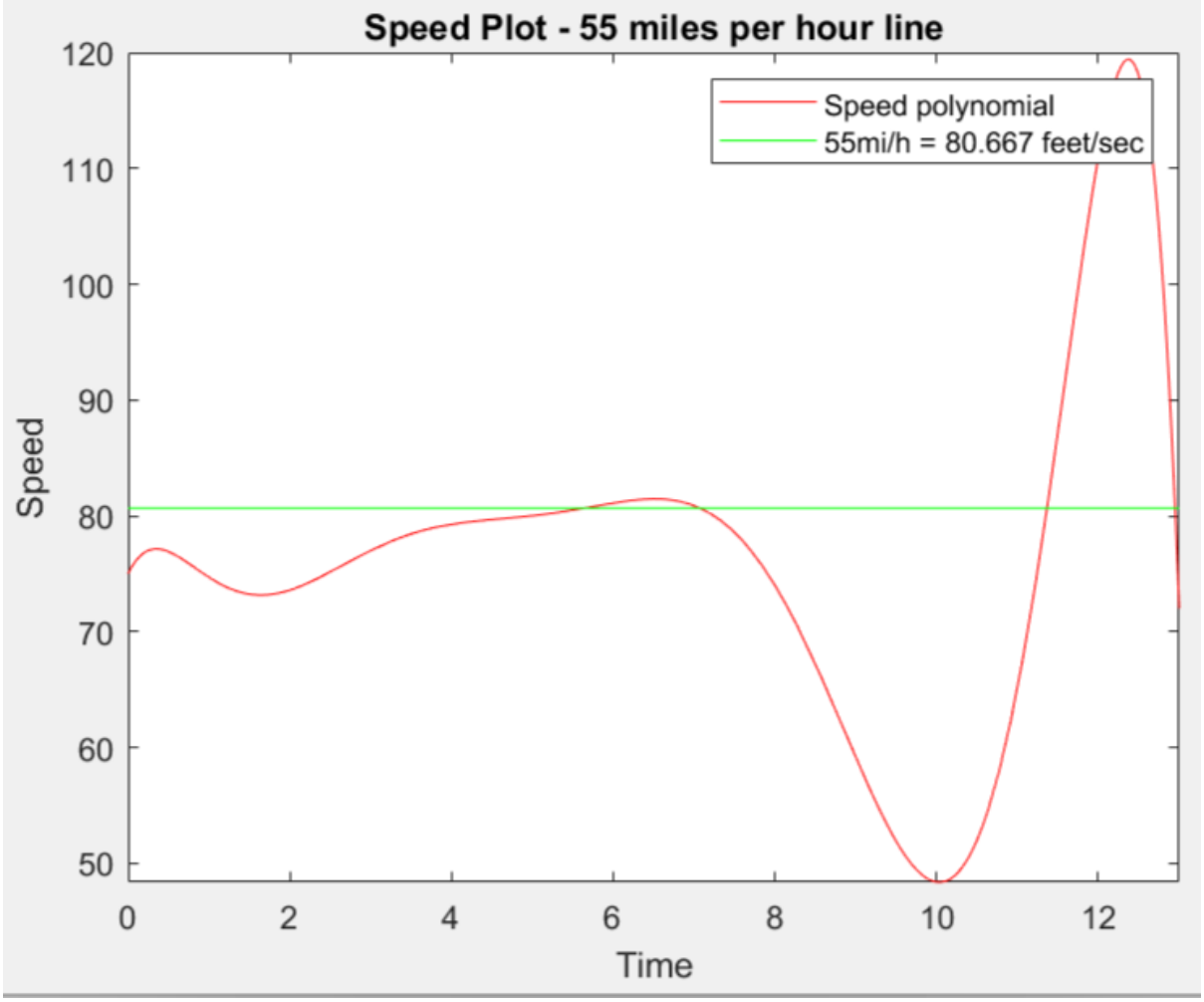
b) and c)

We can observe the speed v/s time graph from the derivative of obtained H (x)



To predict the maximum speed of car, we can observe the speed-time graph. We see that the maxima in speed time graph is obtained somewhere between t = 11 sec and t = 13 sec.

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Question 2 part b and part c
55 mi/h = 80.6667 feet/sec
Yes, the car exceeds the 55 mi/h speed limit on the road
The car crosses 55 mi/h for the first time at 5.65 second
The predicted maximum speed for the car = 119.4173365161 feet per second
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3)

a)

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Question 3 part a
Exact value of f(0.43) = 2.3631606937
Exact value of f'(0.43) = 4.7263213874
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Using Natural Cubic Spline Interpolation

Approximate Value of f(0.43) = 2.477722372475
Approximate Value of f'(0.43) = 3.436563656918

The error in f(x) at x = 0.43 is 0.1145616788
The error in f'(x) at x = 0.43 is 1.2897577305

Using Clamped Cubic Spline Interpolation

Approximate Value of f(0.43) = 2.362071013974
Approximate Value of f'(0.43) = 4.751931537828

The error in f(x) at x = 0.43 is 0.0010896797
The error in f(x) at x = 0.43 is 0.0256101504
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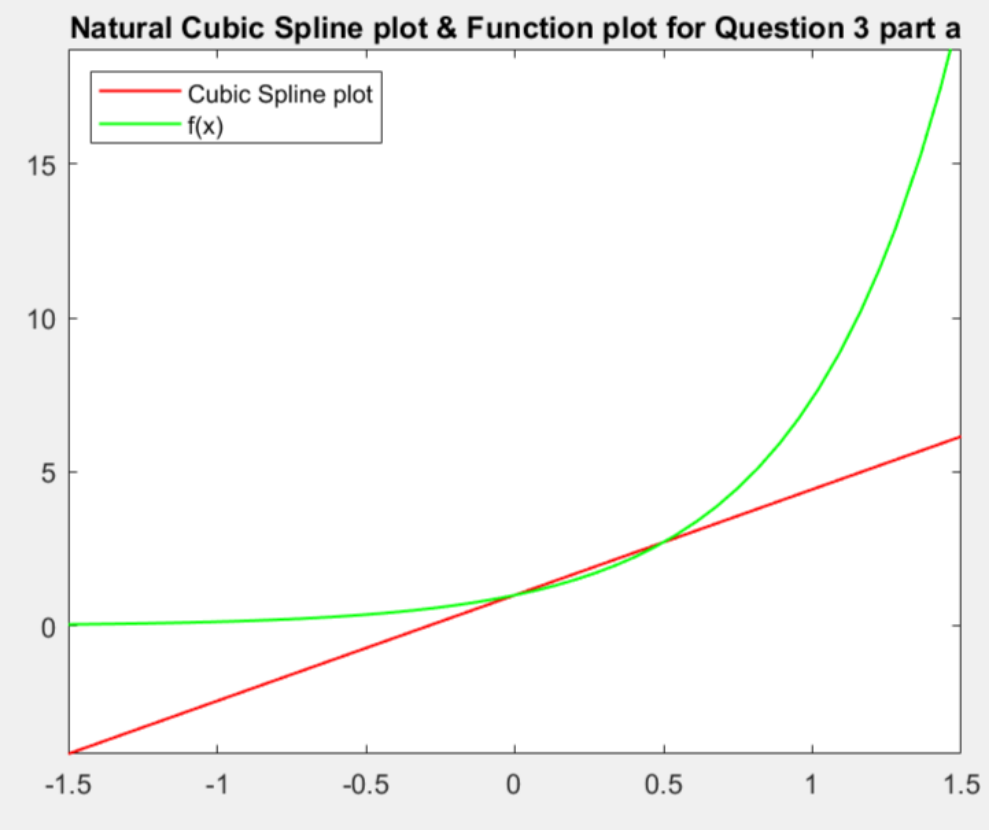

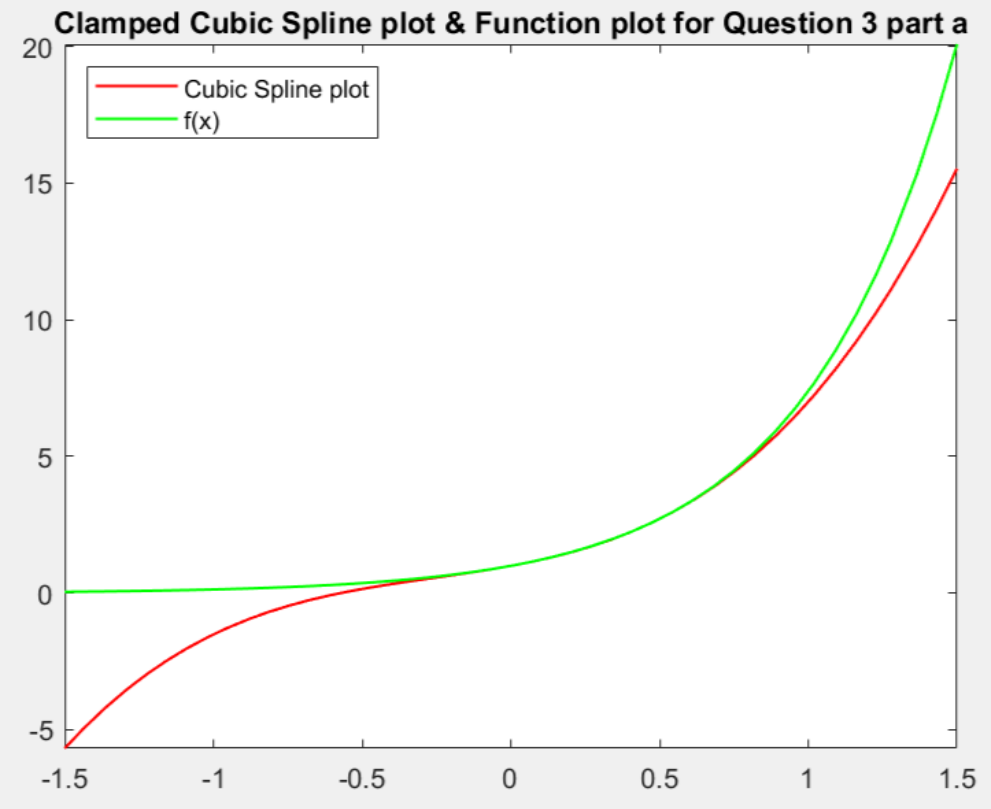
Clamped Cubic Spline plot & Function plot for Question 3 part a
Cubic Spline plot
f(x)
20
15
10
5
0
-5
-1.5
-1
-0.5
0
0.5
1
1.5

b)

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Question 3 part b
Exact value of f(0.25) = 1.1890699311
Exact value of f1(0.25) = 0.3909913152


Using Natural Cubic Spline Interpolation

Approximate Value of f(0.25) is: 1.196494529043
Approximate Value of f1(0.25) is: 0.391528961499

The error in f(x) at x = 0.25 is 0.0074245979
The error in f(x) at x = 0.25 is 0.0005376463

Using Clamped Cubic Spline Interpolation

Approximate Value of f(0.25) is: 1.189086439759
Approximate Value of f1(0.25) is: 0.390994418446

The error in f(x) at x = 0.25 is: 0.0000165086
The error in f(x) at x = 0.25 is: 0.0000031033
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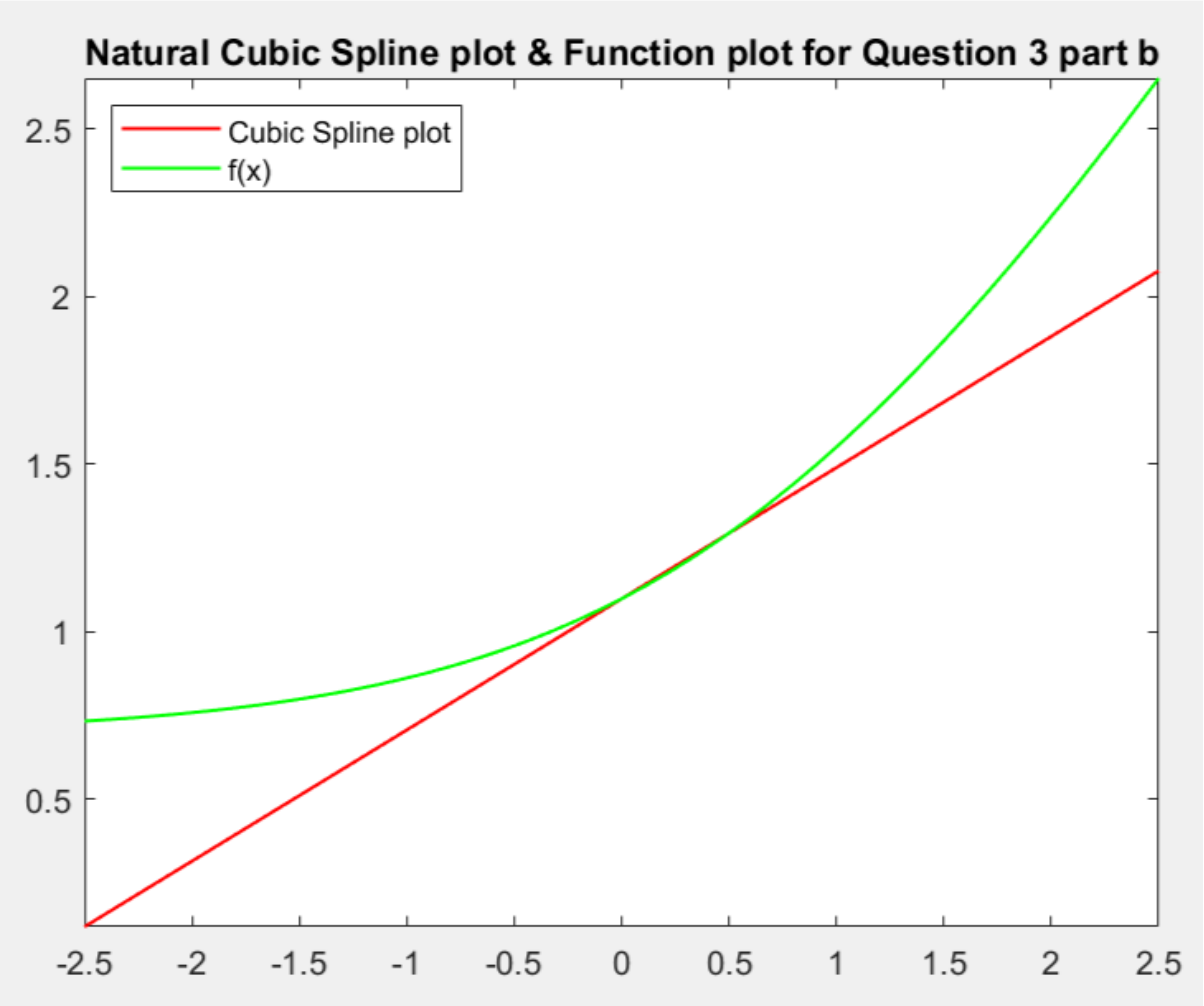
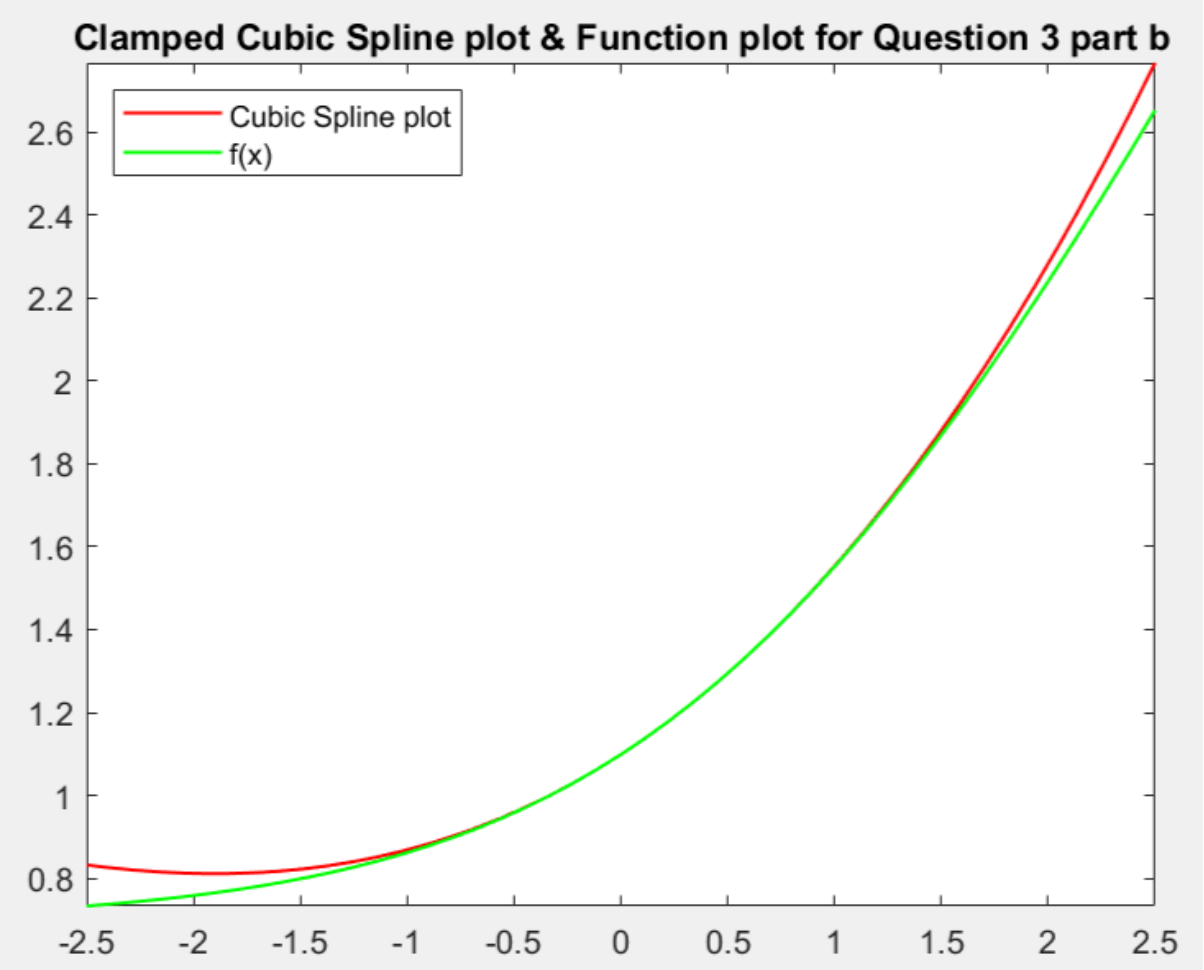

Natural Cubic Spline plot & Function plot for Question 3 part b
Cubic Spline plot
f(x)
2.5
2
1.5
1
0.5
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5



4)

S(x) = clamped cubic spline interpolated polynomial

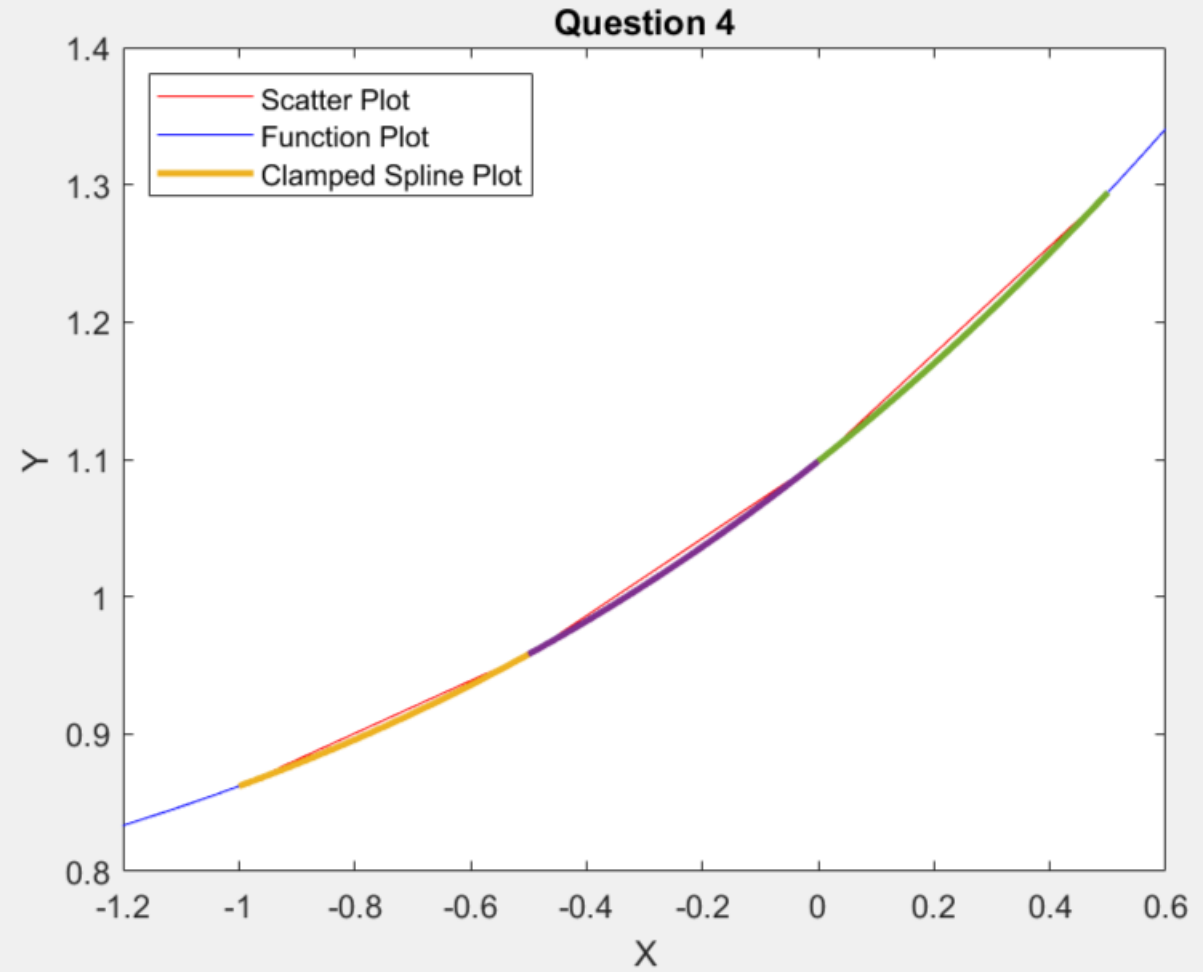
The obtained S(x) = $S_1(x)$, if -1 <= x <= -0.5

$S_2(x)$, if -0.5 <= x <= 0

$S_3(x)$, if 0 <= x <= 0.5

where, $Si(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$ for i = 1,2,3 for the nodes {x1, x2, x3, x4}.

```
The coefficients of the spline on the subintervals are:
     a(i)          b(i)          c(i)          d(i)
  0.86199480    0.15536240    0.06537475    0.01600323
  0.95802009    0.23273957    0.08937959    0.01502024
  1.09861230    0.33338433    0.11190995    0.00875797
```



5)

a)

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Question 5
The coefficients of the spline on the subintervals are:
     a(i)         b(i)           c(i)           d(i)
 0.00000000  73.88796680   0.00000000   0.12355924
225.00000000  77.22406639   1.11203320  -0.11203320
383.00000000  80.32780083   0.43983402  -0.18303366
623.00000000  78.02489627  -1.20746888   0.08049793

Estimated distance travelled at t = 10 seconds using natural cubic spline interpolation = 774.863900414938 feet
Estimated speed using natural cubic spline interpolation at t = 10 seconds = 74.160988603010 feet/sec
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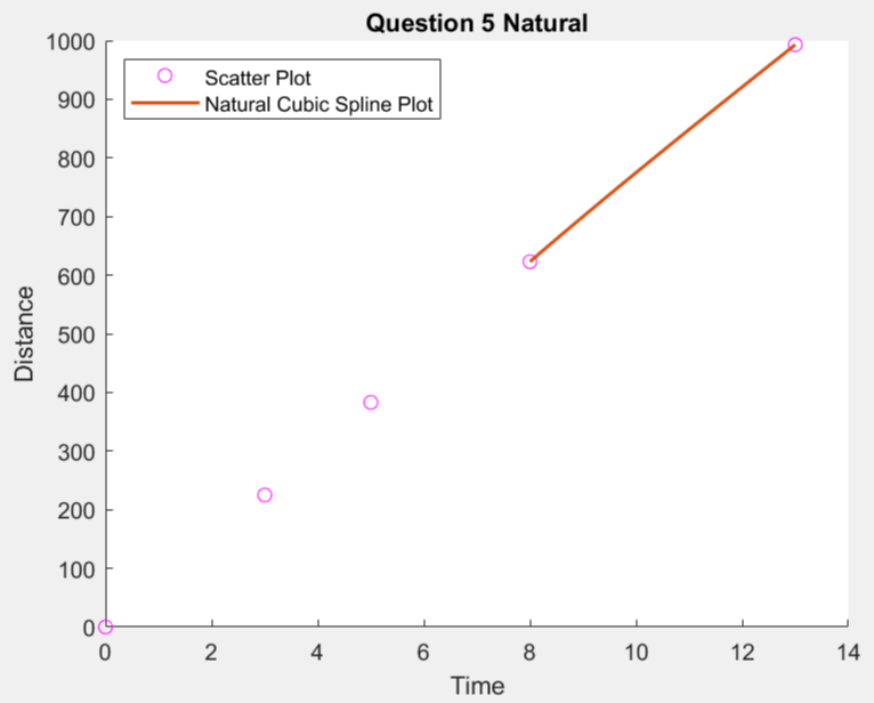

b)

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The coefficients of the spline on the subintervals are:
     a(i)          b(i)           c(i)          d(i)
  0.00000000   75.00000000   -0.65929204    0.21976401
225.00000000   76.97787611    1.31858407   -0.15376106
383.00000000   80.40707965    0.39601770   -0.17723697
623.00000000   77.99778761   -1.19911504    0.07991150

Estimated distance travelled at t = 10 seconds using Clamped cubic Spline interpolation is 774.838407079646 feet
Estimated speed using Clamped cubic Spline interpolation at t = 10 seconds = 74.160258287466 feet/sec
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