

Scientific Computing Lab MA – 322 Lab – 11

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For each part of all the questions, the five plots are as follows:

- (i) t versus actual and approximate solutions
- (ii) t versus absolute error
- (iii) N versus Order of Convergence
- (iv) loglog plot of t versus absolute error
- (v) $\log(E_N)$ versus $\log(N)$

Where, $N = (b-a)/h$ = Number of intervals

Order of convergence = $\log_2(E_N/E_{2N})$, where E_N and E_{2N} are the maximum errors obtained during the computation for that specific value of N

1)

The following methods are implemented:

- (i) Implicit-Euler Method
- (ii) Second-order Runge-Kutta method with:
 - $c_2 = 1/2$ (Improved Tangent Method)
 - $c_2 = 2/3$ (Optimal Method)
 - $c_2 = 1$ (Euler-Cauchy Method)

(iii) Fourth-order Runge-Kutta method with:

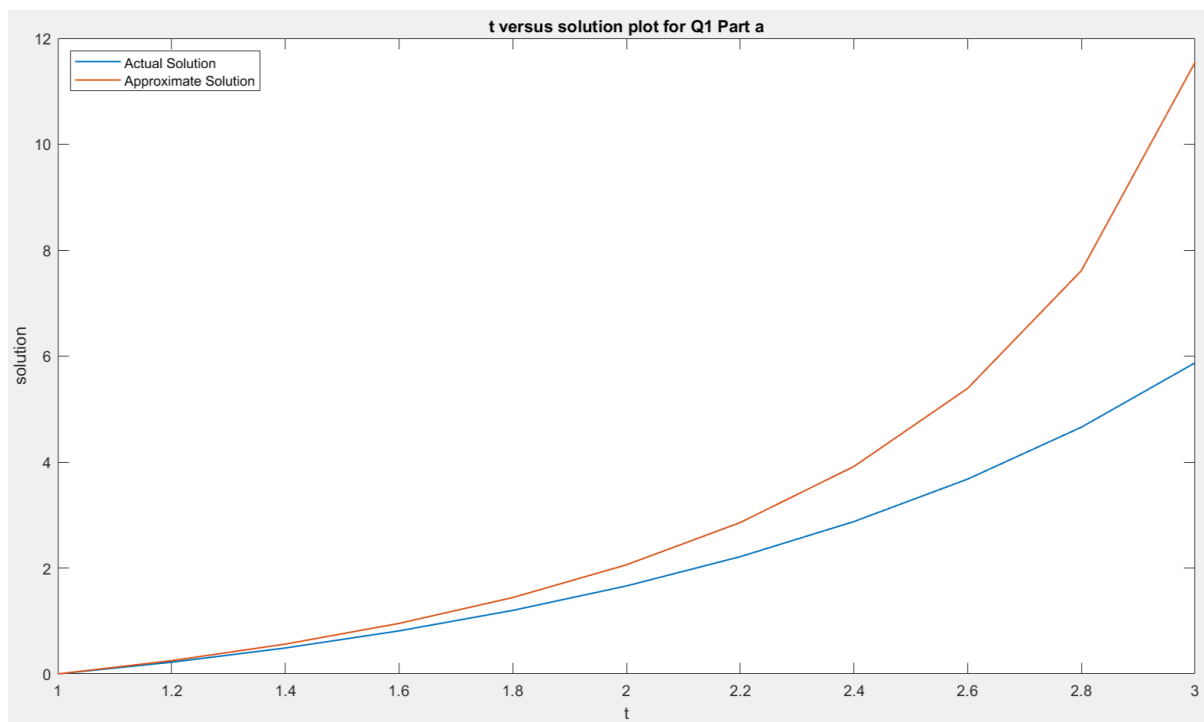
- $c_2 = 1/2, c_3 = 1/2, c_4 = 1$ (Classical Method)
- $c_2 = 1/3, c_3 = 2/3, c_4 = 1$ (Kutta Method)

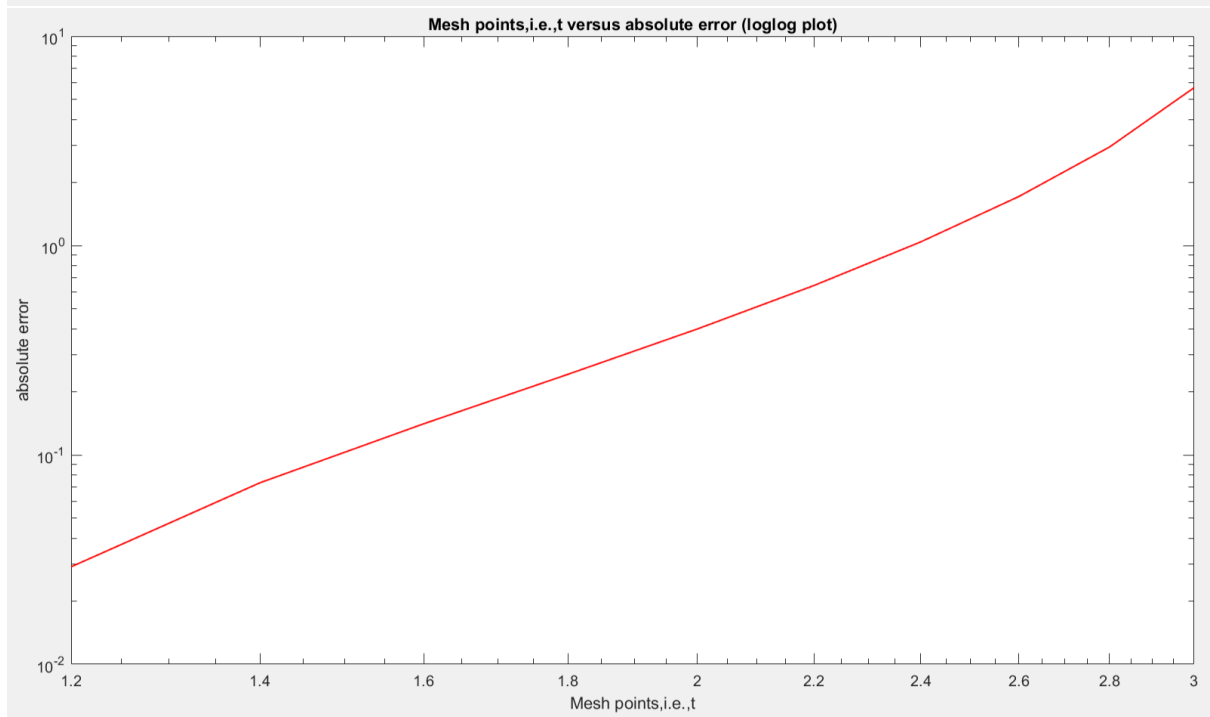
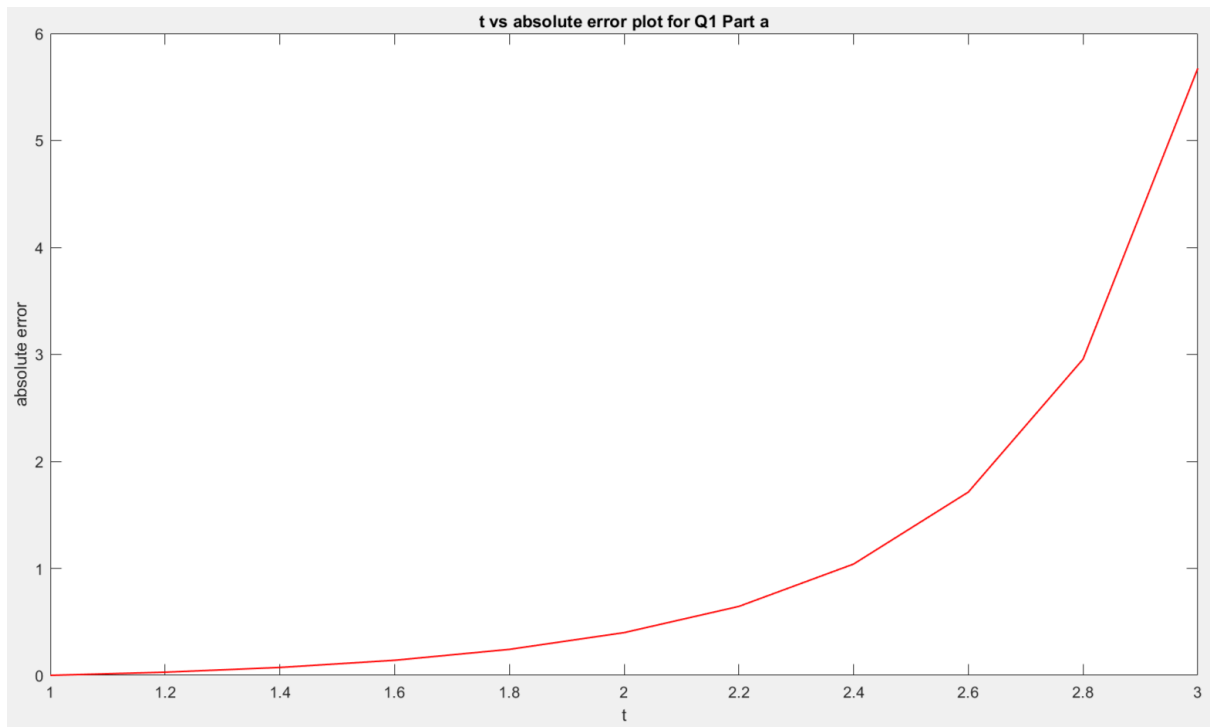
a)

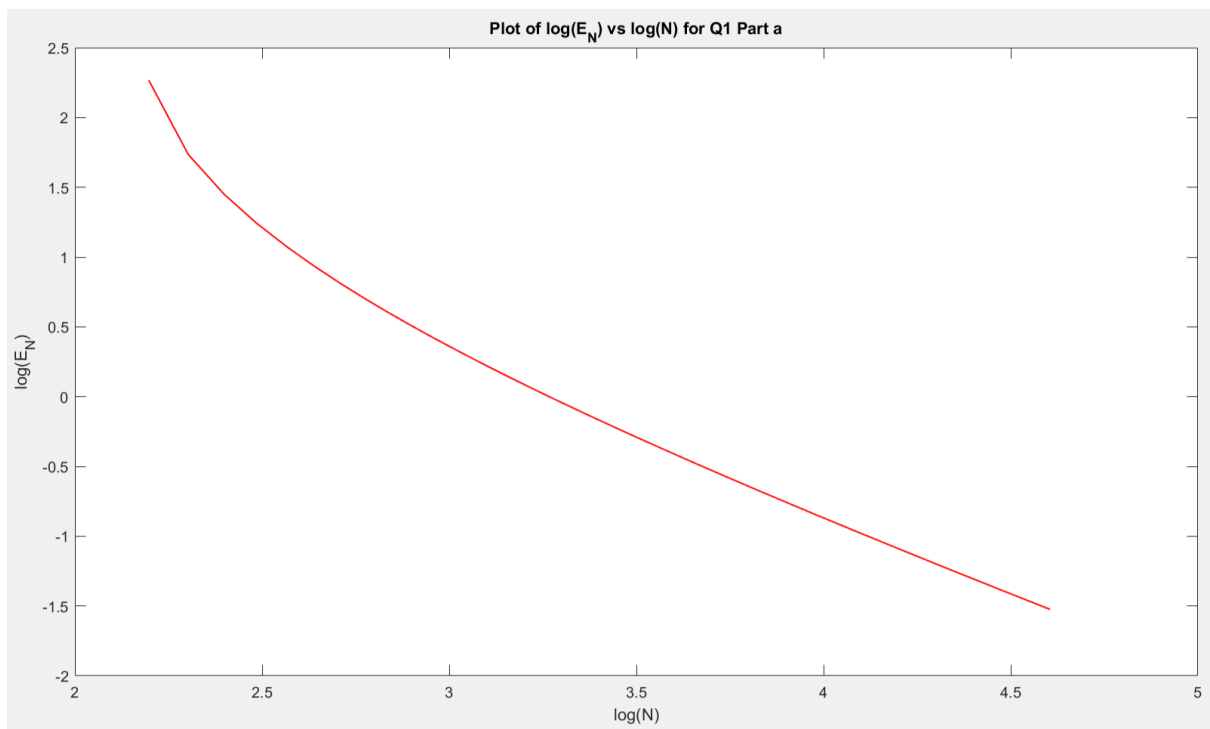
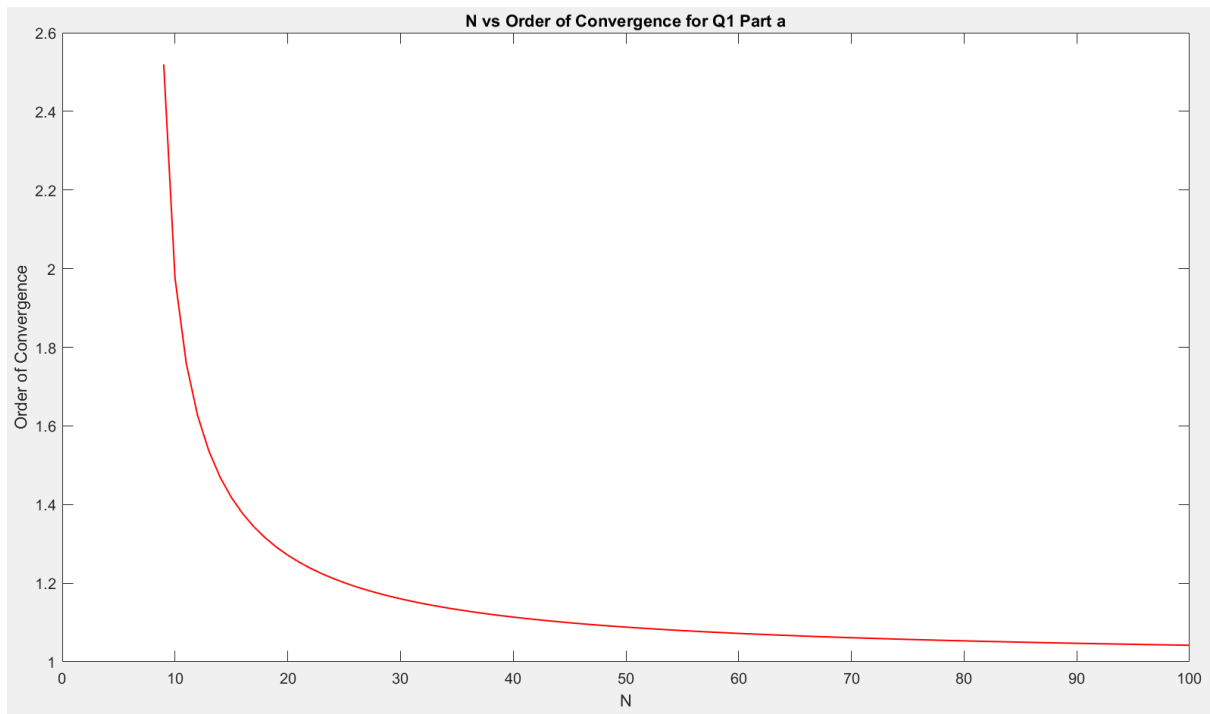
Question 1 Part a

Using Implicit-Euler method for Q1 Part a

t	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.000000	0.000000	0.000000
1.200000	0.250454	0.221243	0.029212
1.400000	0.563305	0.489682	0.073624
1.600000	0.953529	0.812753	0.140776
1.800000	1.442150	1.199439	0.242711
2.000000	2.060475	1.661282	0.399194
2.200000	2.857734	2.213502	0.644232
2.400000	3.916828	2.876551	1.040277
2.600000	5.391610	3.678475	1.713135
2.800000	7.614685	4.658665	2.956020
3.000000	11.548058	5.874100	5.673958

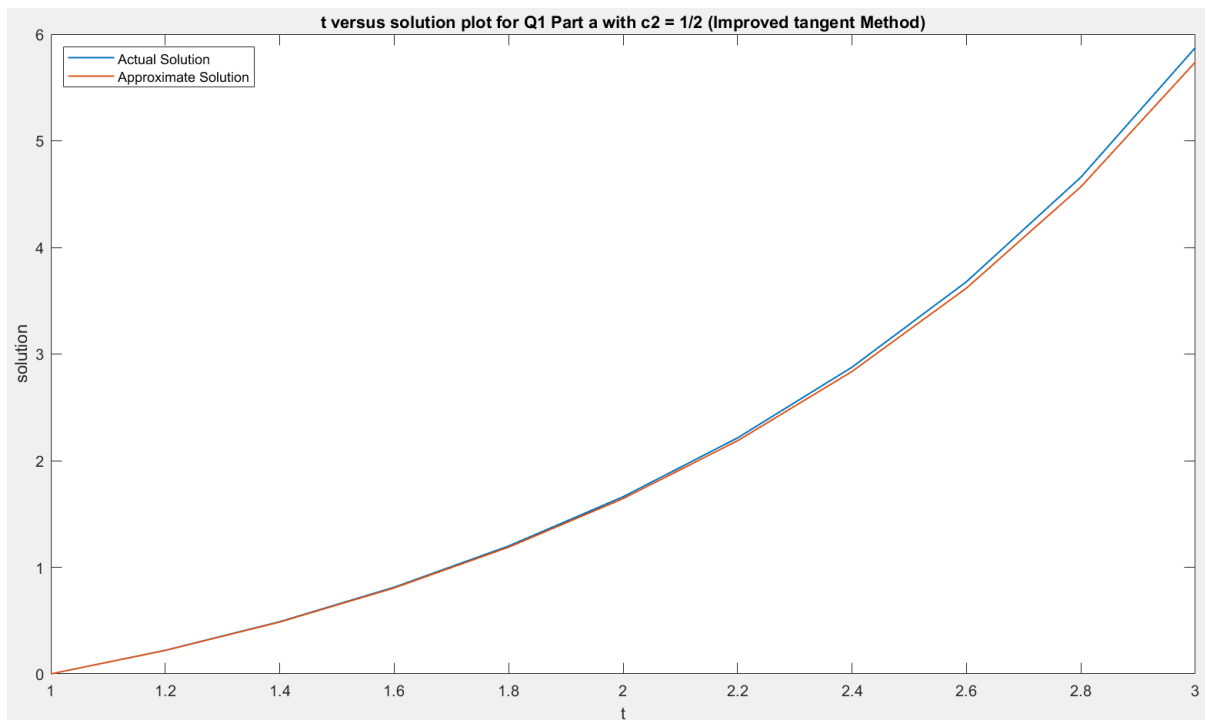


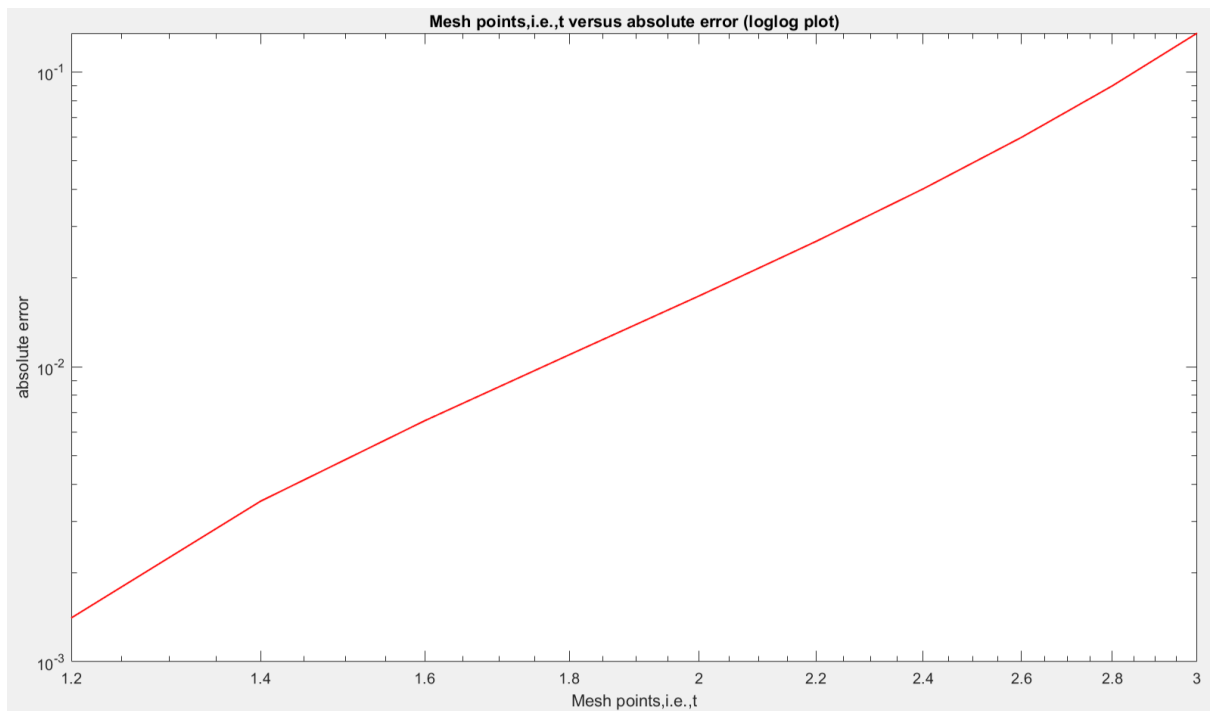
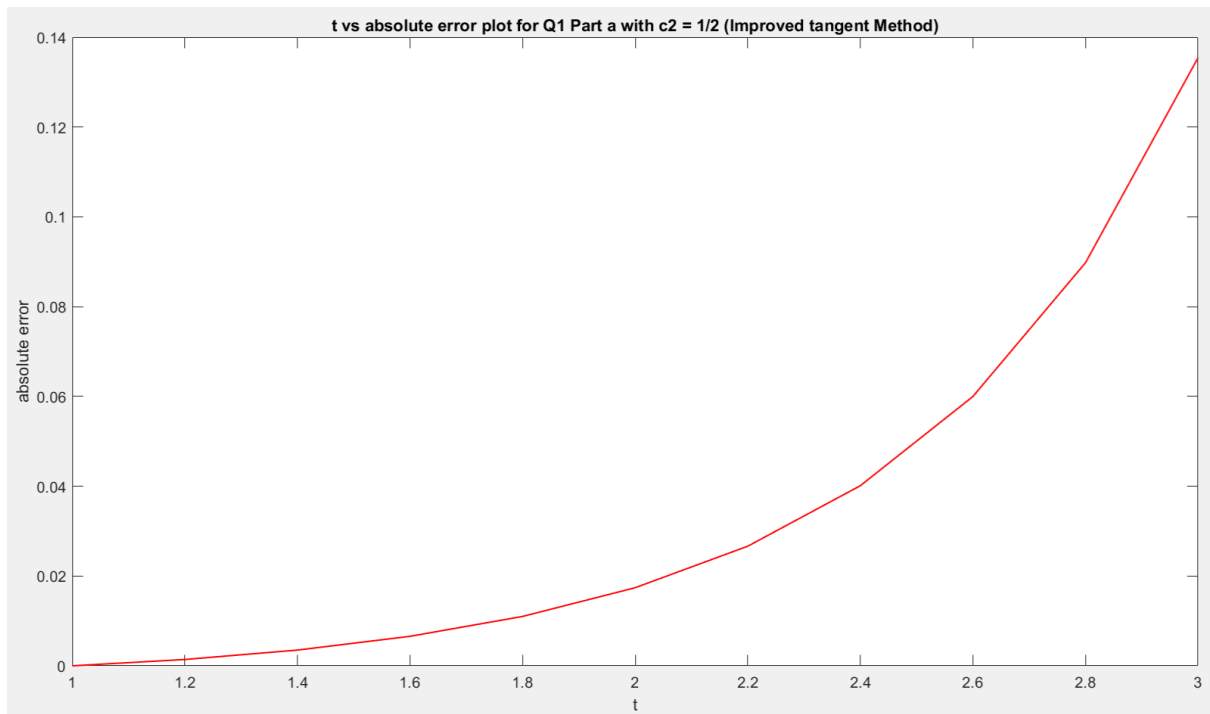


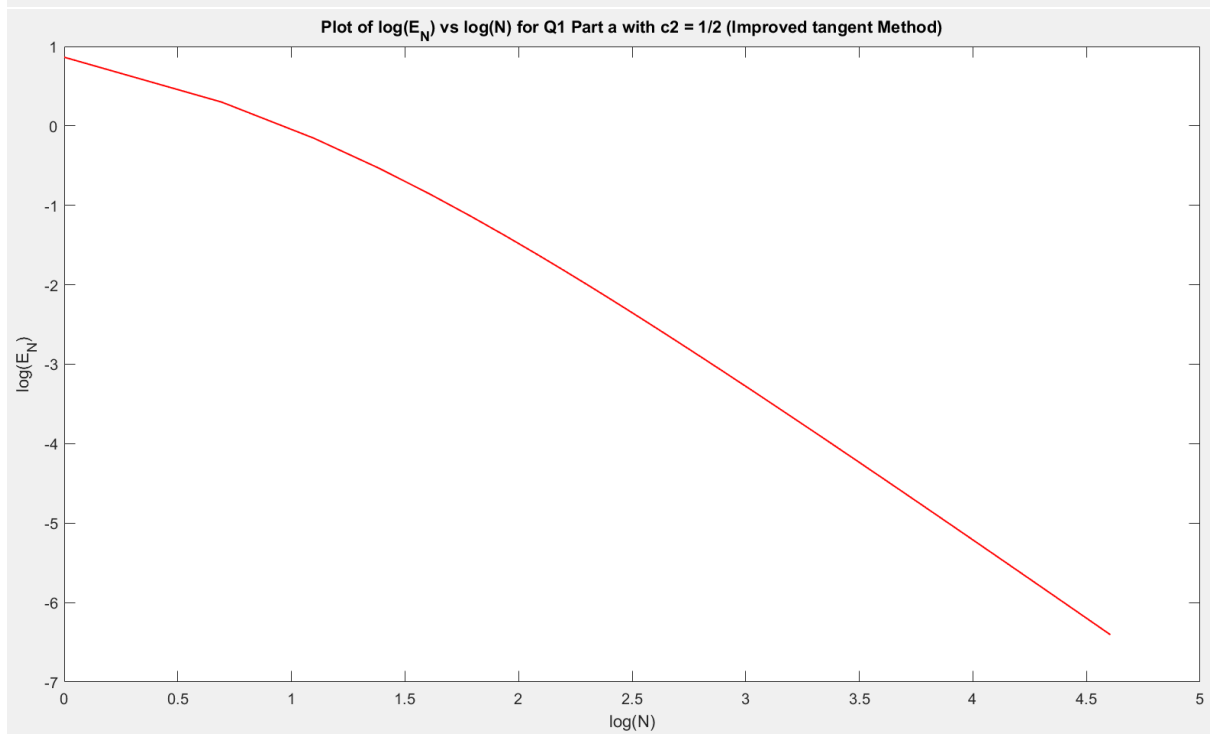
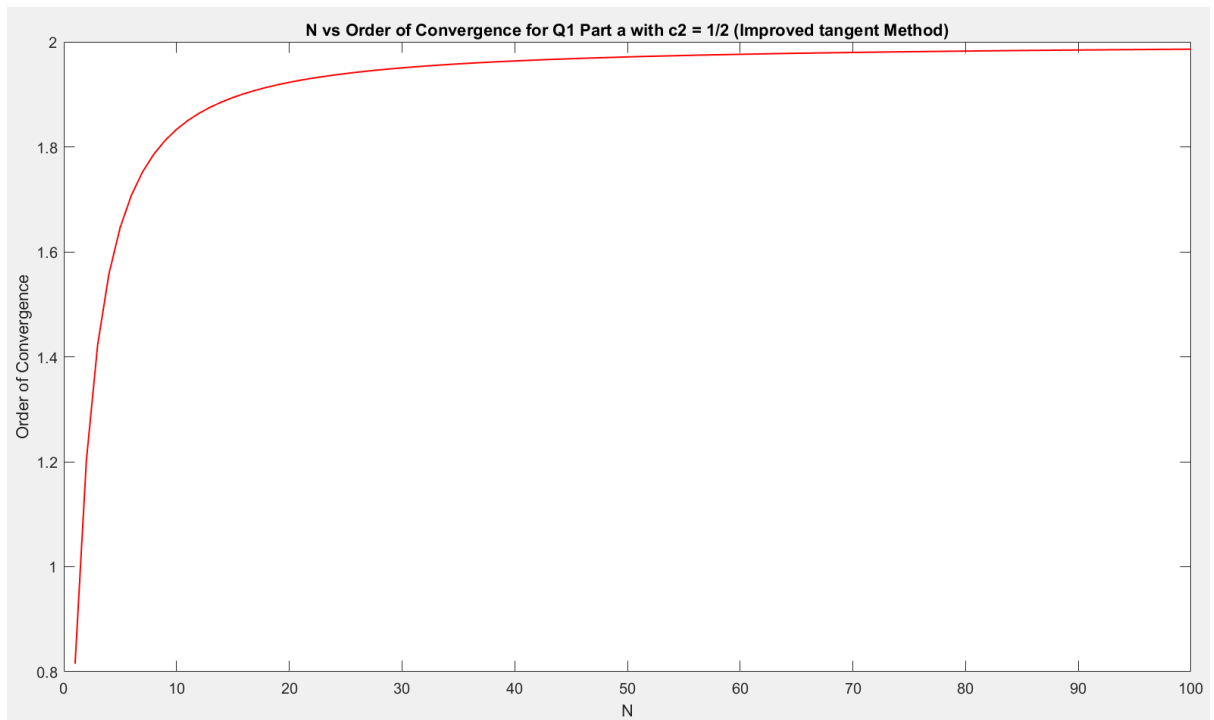


Using Second-order Runge-Kutta Method for Q1 Part a with $c_2 = 1/2$ (Improved tangent Method)

t	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.000000	0.000000	0.000000
1.200000	0.219835	0.221243	0.001408
1.400000	0.486177	0.489682	0.003505
1.600000	0.806185	0.812753	0.006568
1.800000	1.188439	1.199439	0.010999
2.000000	1.643889	1.661282	0.017393
2.200000	2.186861	2.213502	0.026641
2.400000	2.836436	2.876551	0.040116
2.600000	3.618493	3.678475	0.059983
2.800000	4.568894	4.658665	0.089771
3.000000	5.738647	5.874100	0.135453

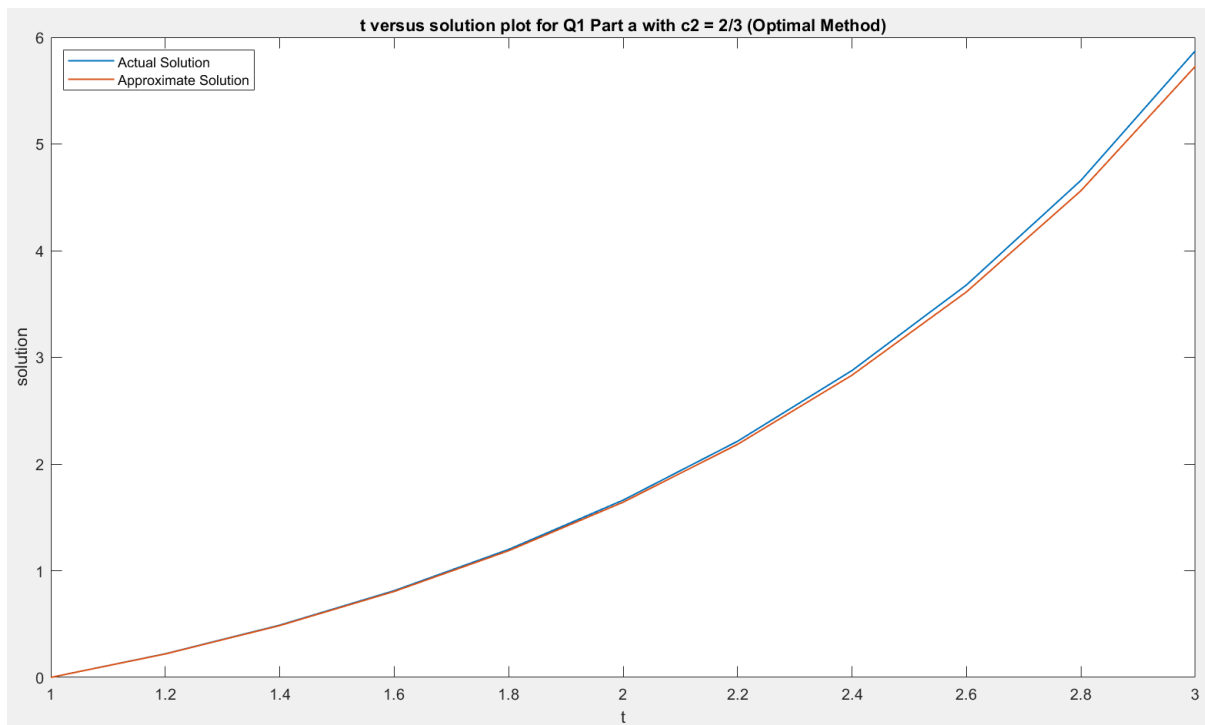


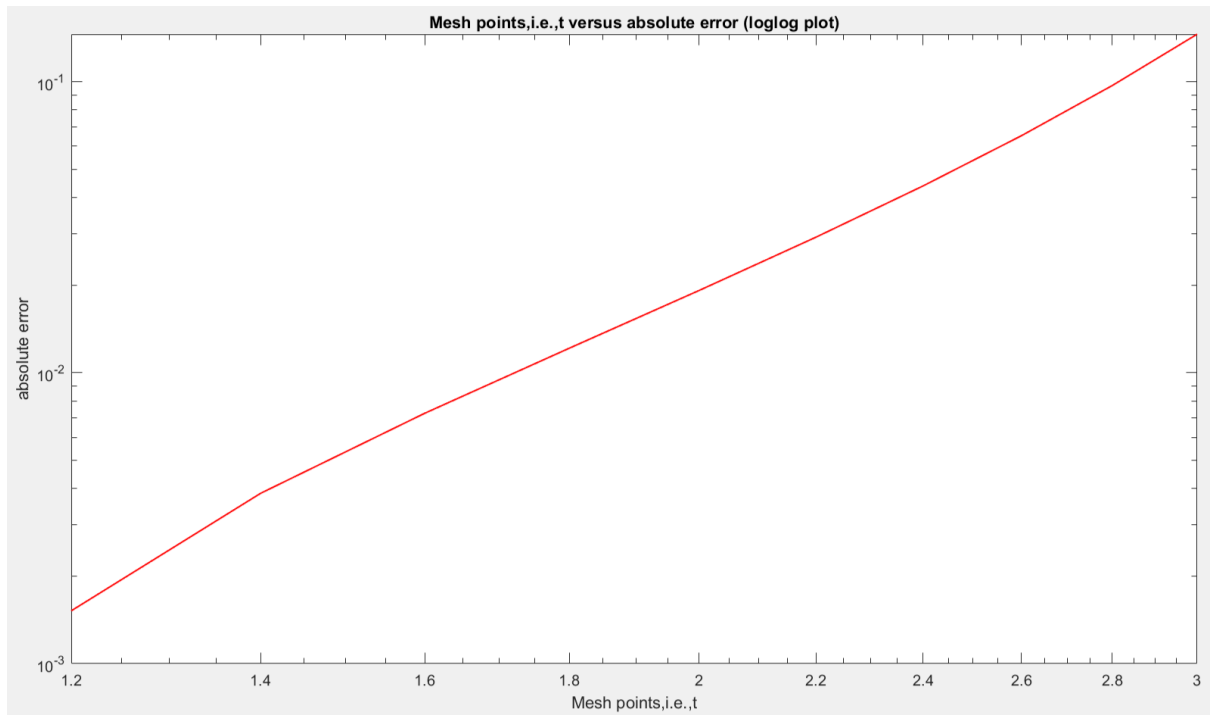
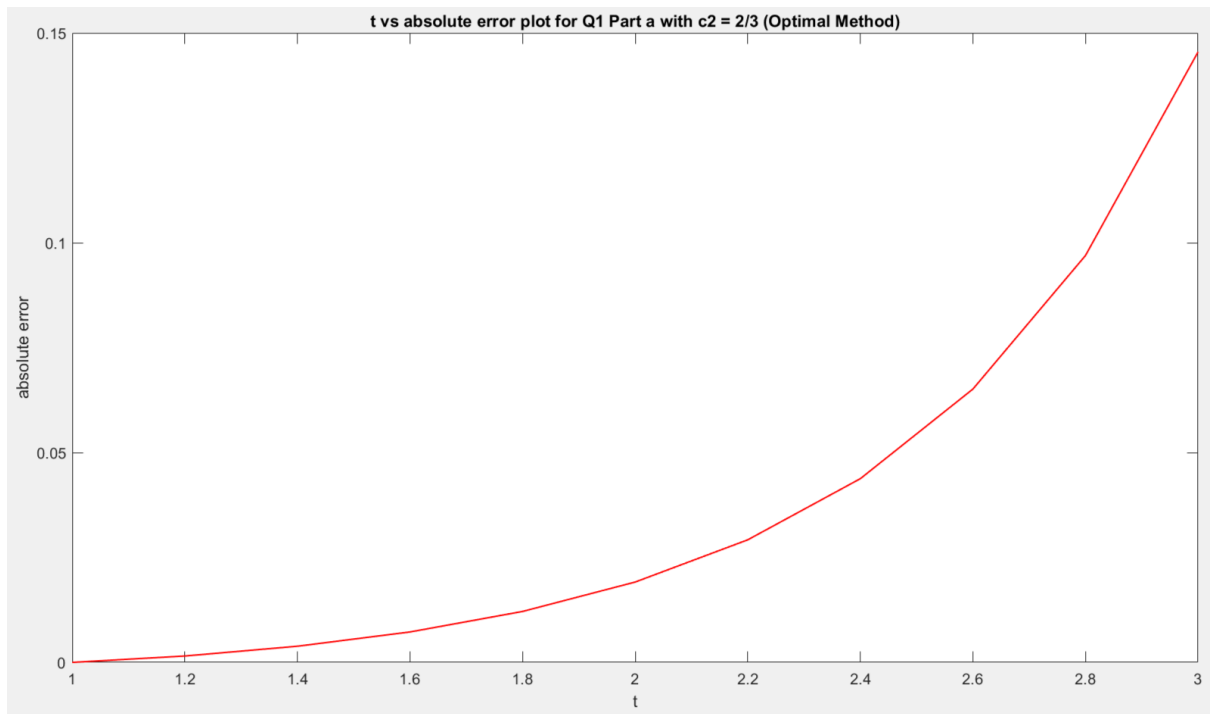


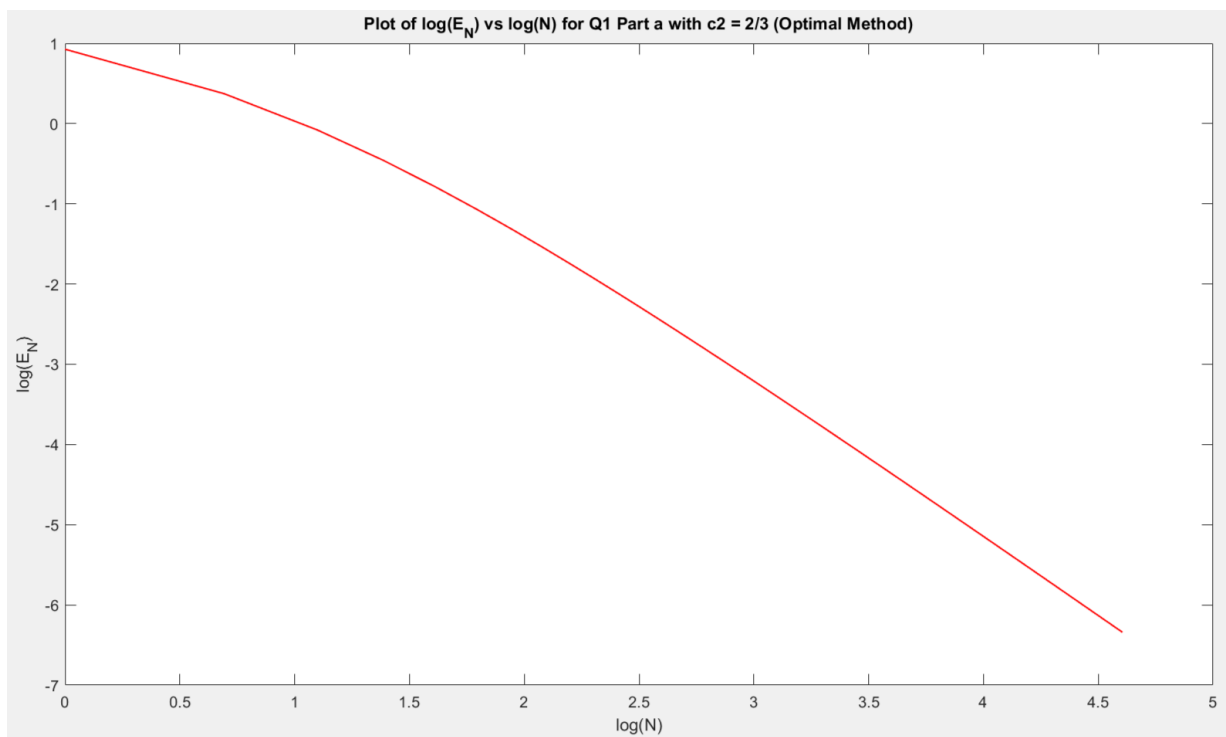
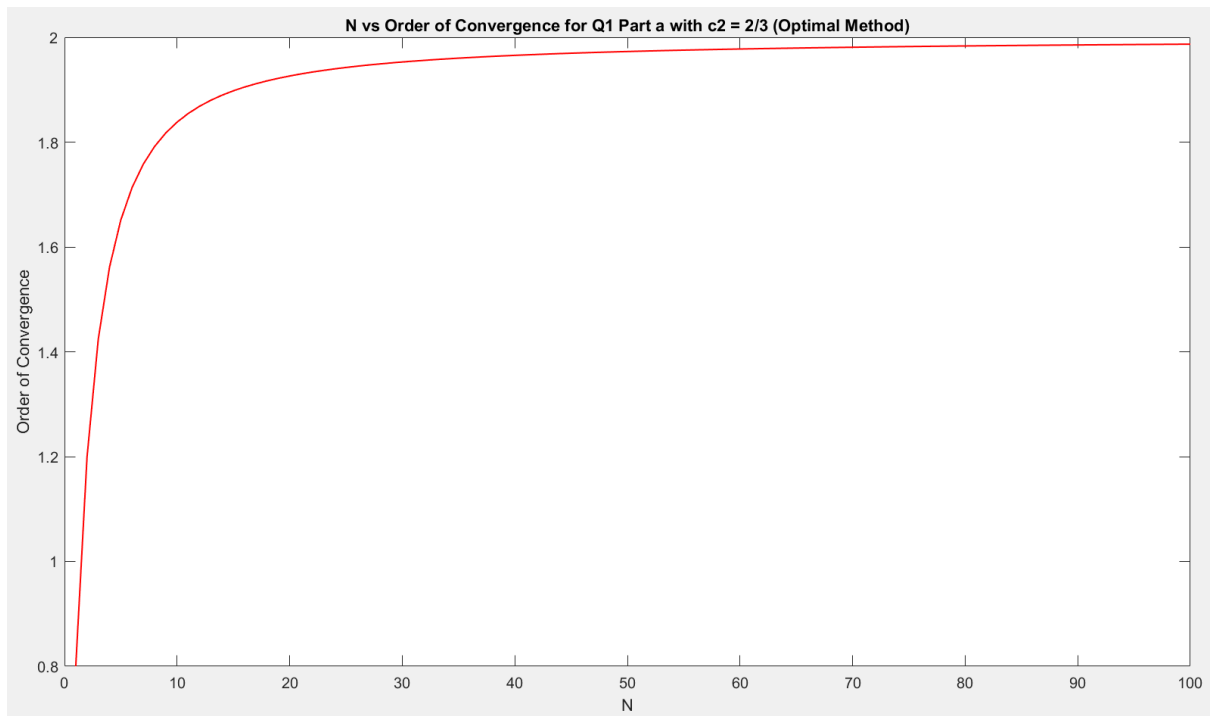


Using Second-order Runge-Kutta Method for Q1 Part a with $c_2 = 2/3$ (Optimal Method)

t	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.000000	0.000000	0.000000
1.200000	0.219723	0.221243	0.001520
1.400000	0.485831	0.489682	0.003850
1.600000	0.805502	0.812753	0.007251
1.800000	1.187299	1.199439	0.012139
2.000000	1.642139	1.661282	0.019143
2.200000	2.184295	2.213502	0.029207
2.400000	2.832773	2.876551	0.043779
2.600000	3.613343	3.678475	0.065133
2.800000	4.561706	4.658665	0.096959
3.000000	5.728625	5.874100	0.145475

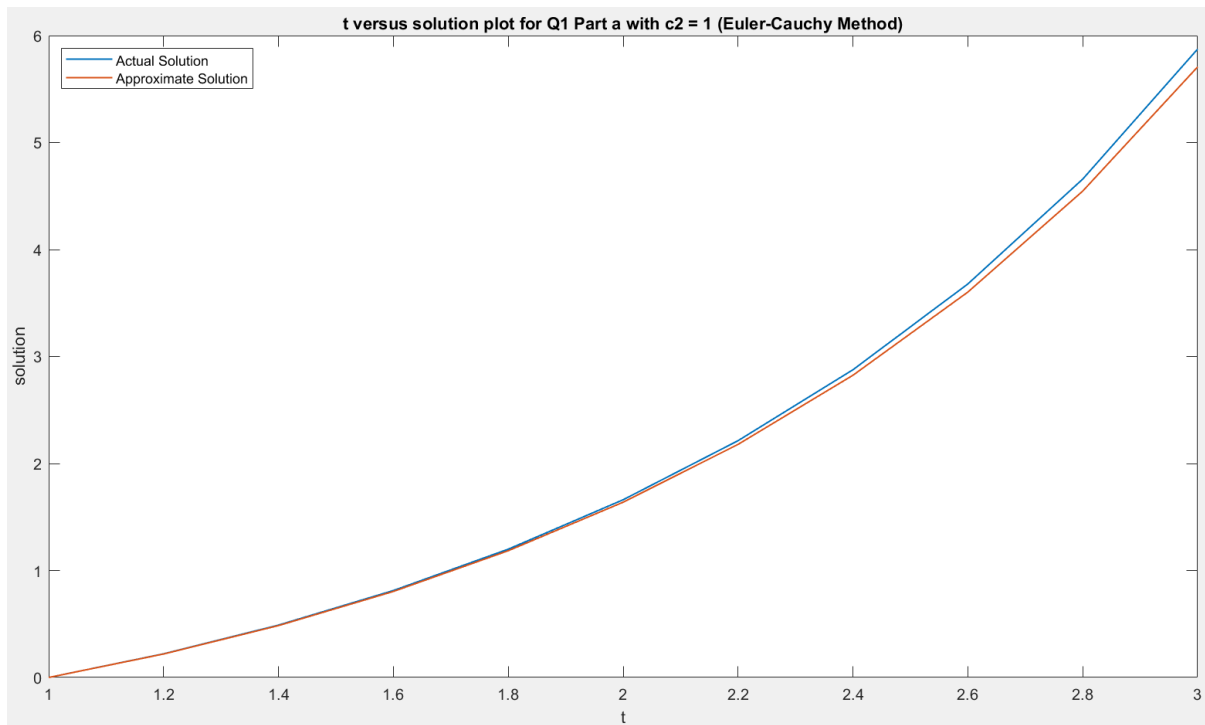


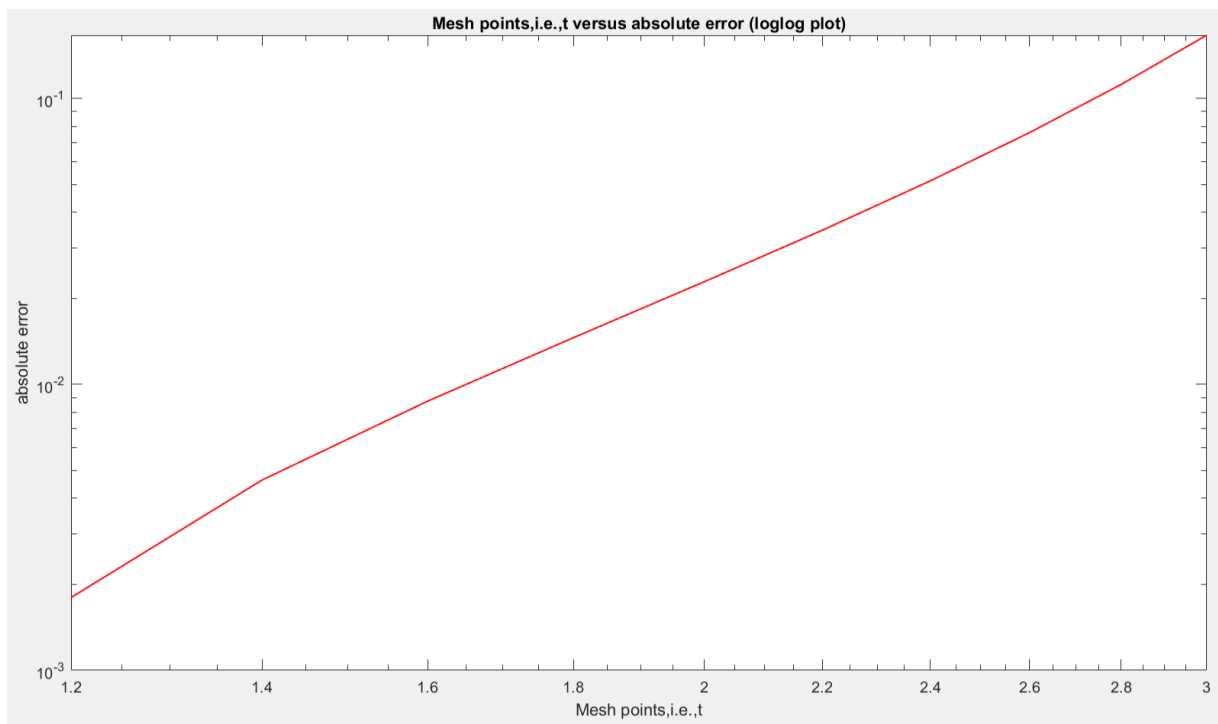
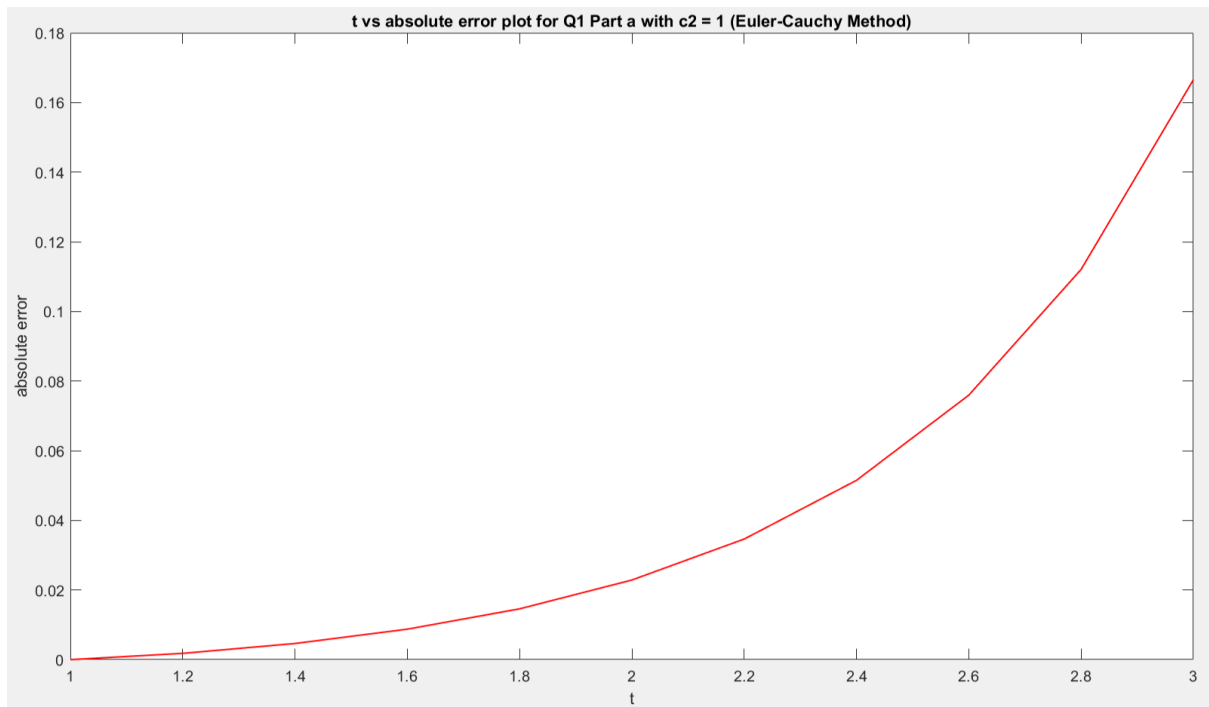


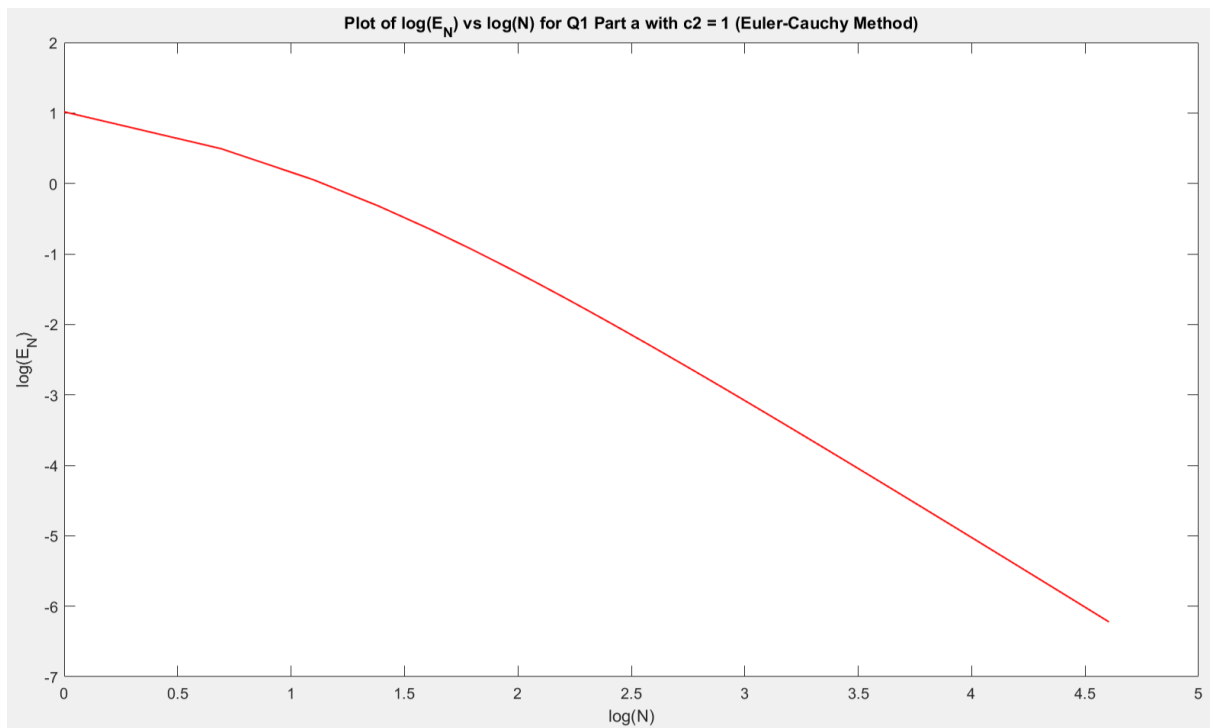
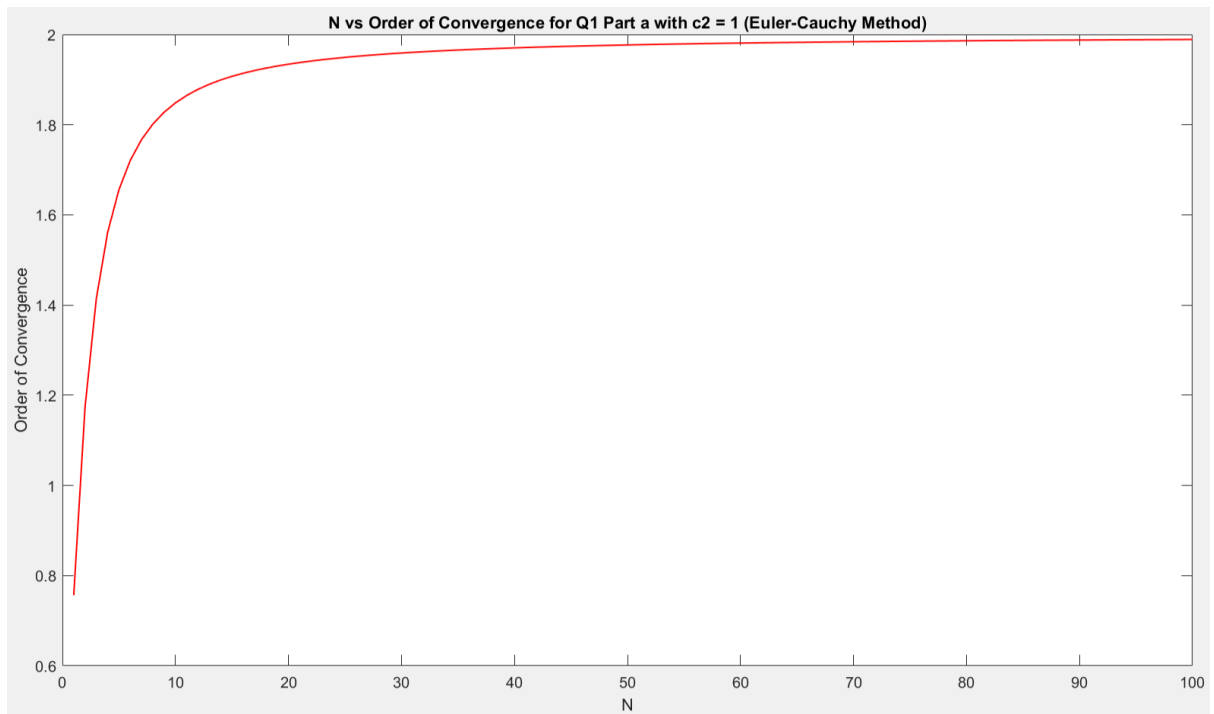


Using Second-order Runge-Kutta Method for Q1 Part a with $c_2 = 1$ (Euler-Cauchy Method)

t	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.000000	0.000000	0.000000
1.200000	0.219444	0.221243	0.001798
1.400000	0.485049	0.489682	0.004632
1.600000	0.804012	0.812753	0.008741
1.800000	1.184856	1.199439	0.014583
2.000000	1.638423	1.661282	0.022859
2.200000	2.178877	2.213502	0.034625
2.400000	2.825065	2.876551	0.051486
2.600000	3.602525	3.678475	0.075951
2.800000	4.546614	4.658665	0.112052
3.000000	5.707570	5.874100	0.166530

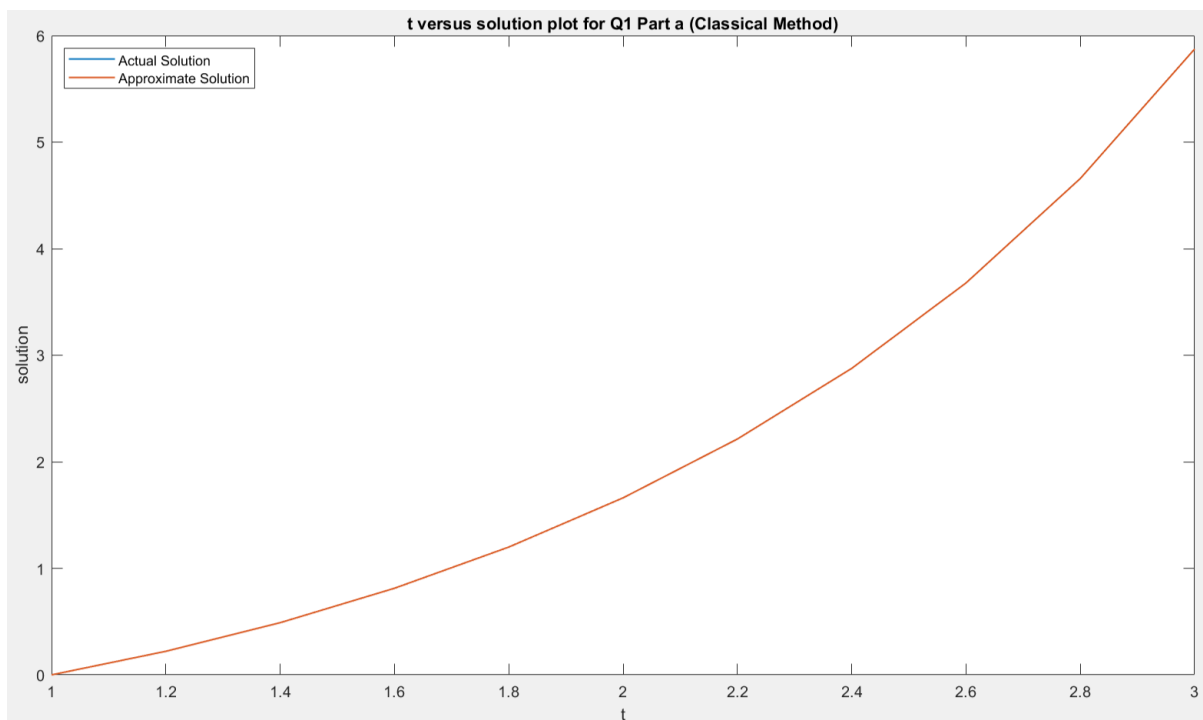


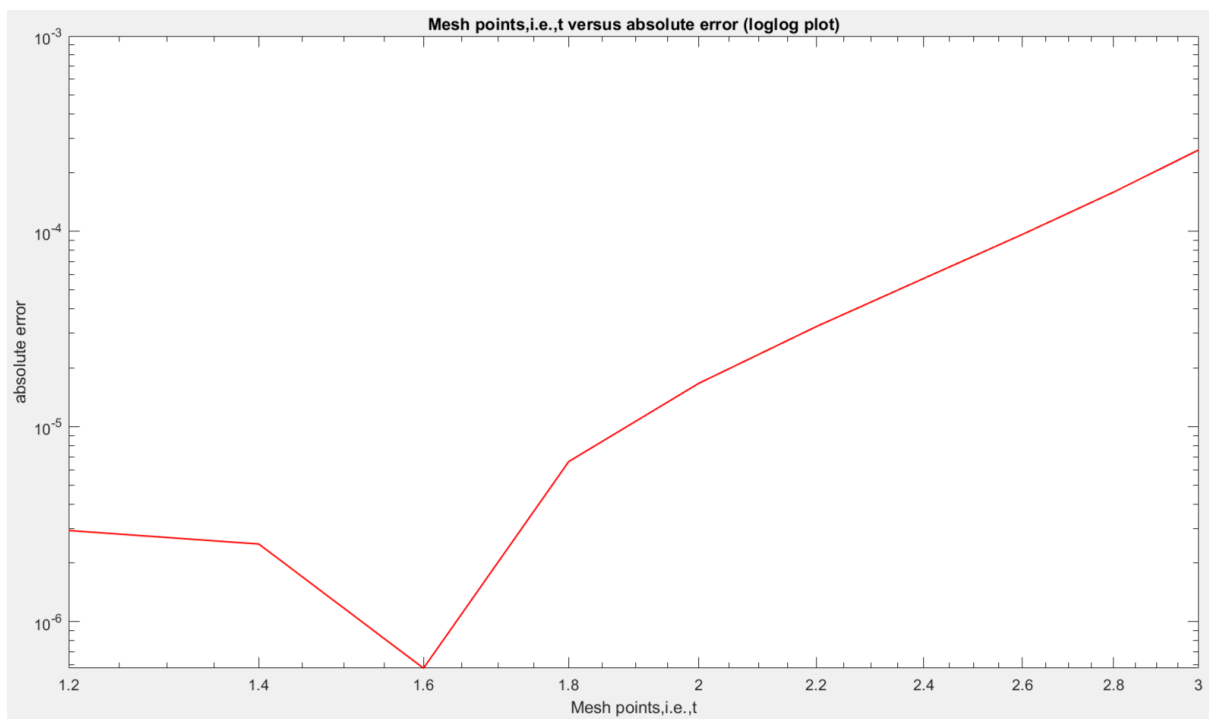
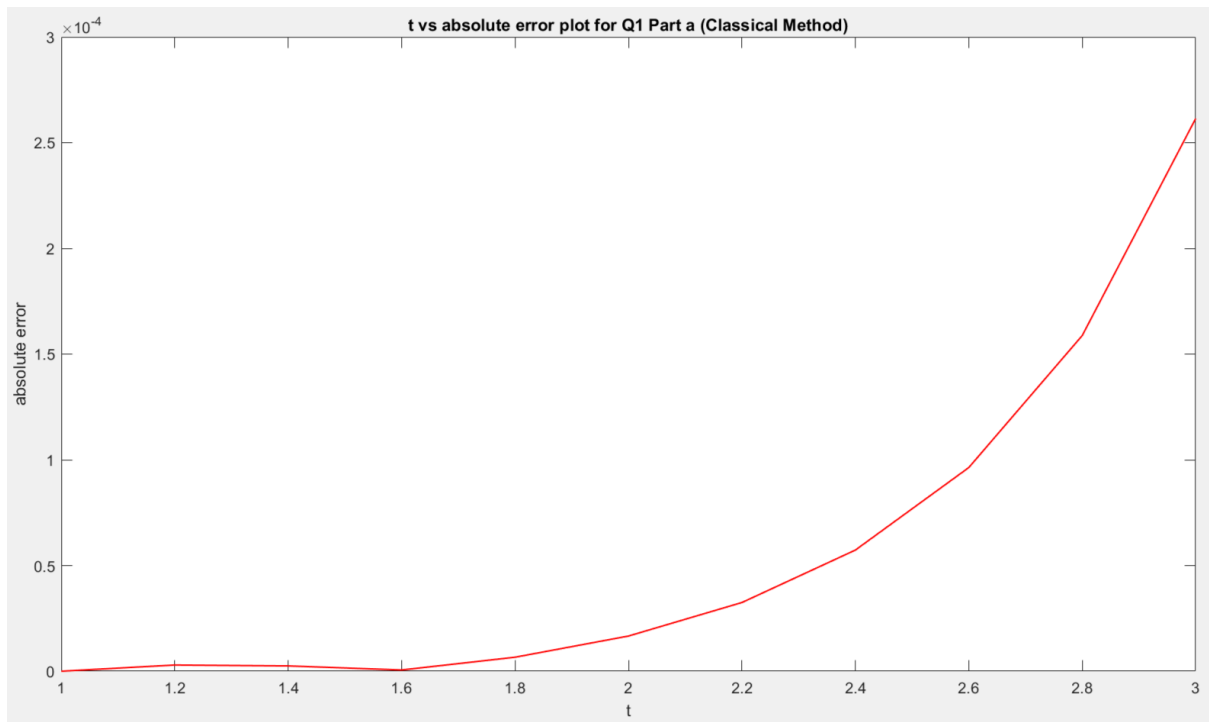


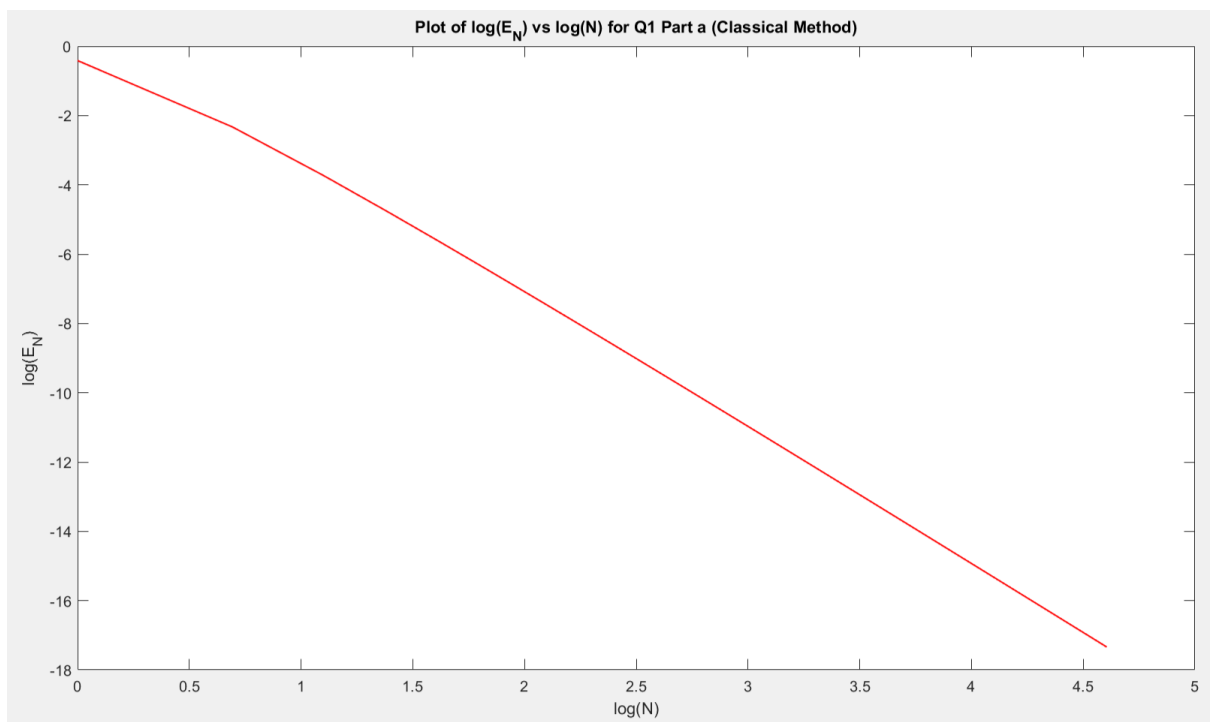
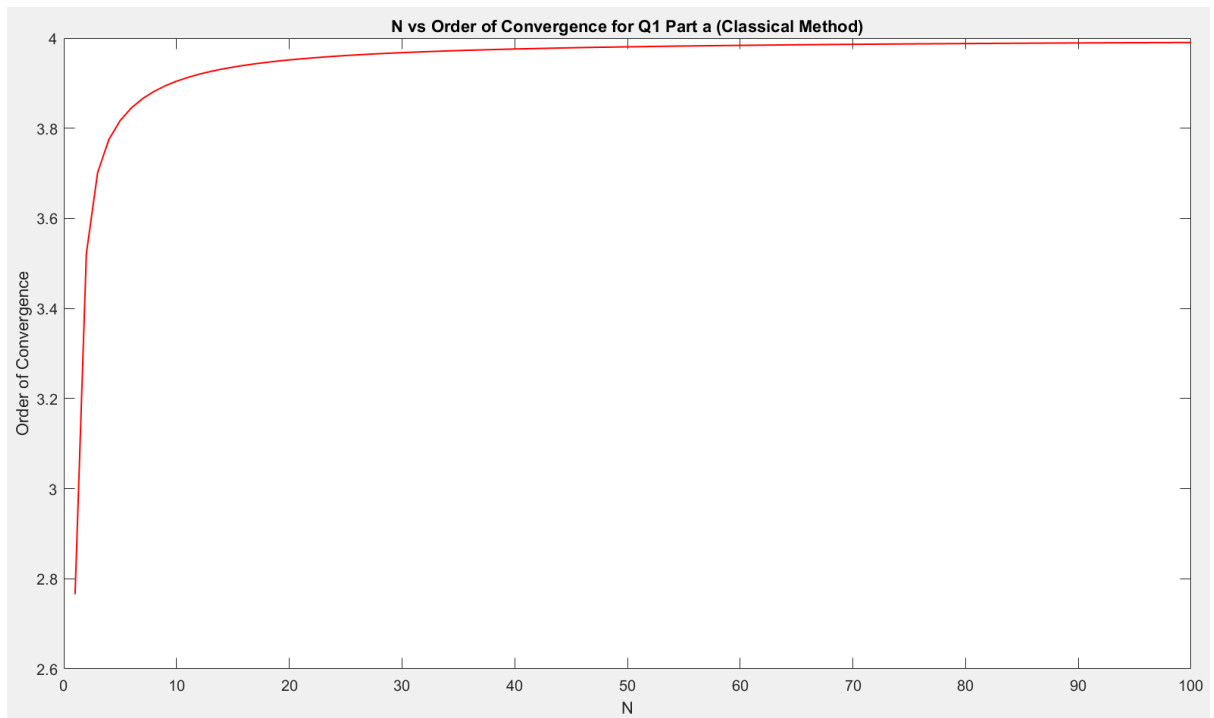


Using Fourth-order Runge-Kutta Method for Q1 Part a (Classical Method)

t	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.000000	0.000000	0.000000
1.200000	0.221246	0.221243	0.000003
1.400000	0.489684	0.489682	0.000003
1.600000	0.812752	0.812753	0.000001
1.800000	1.199432	1.199439	0.000007
2.000000	1.661265	1.661282	0.000017
2.200000	2.213469	2.213502	0.000032
2.400000	2.876494	2.876551	0.000057
2.600000	3.678379	3.678475	0.000096
2.800000	4.658506	4.658665	0.000159
3.000000	5.873839	5.874100	0.000261

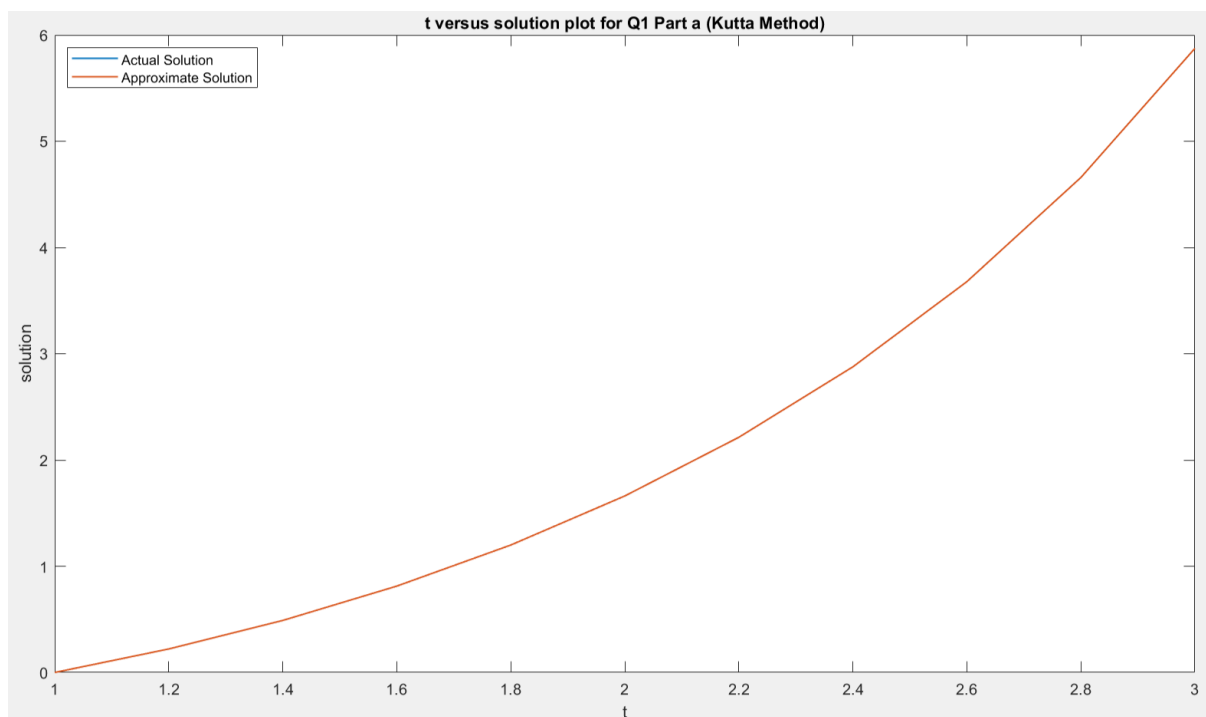


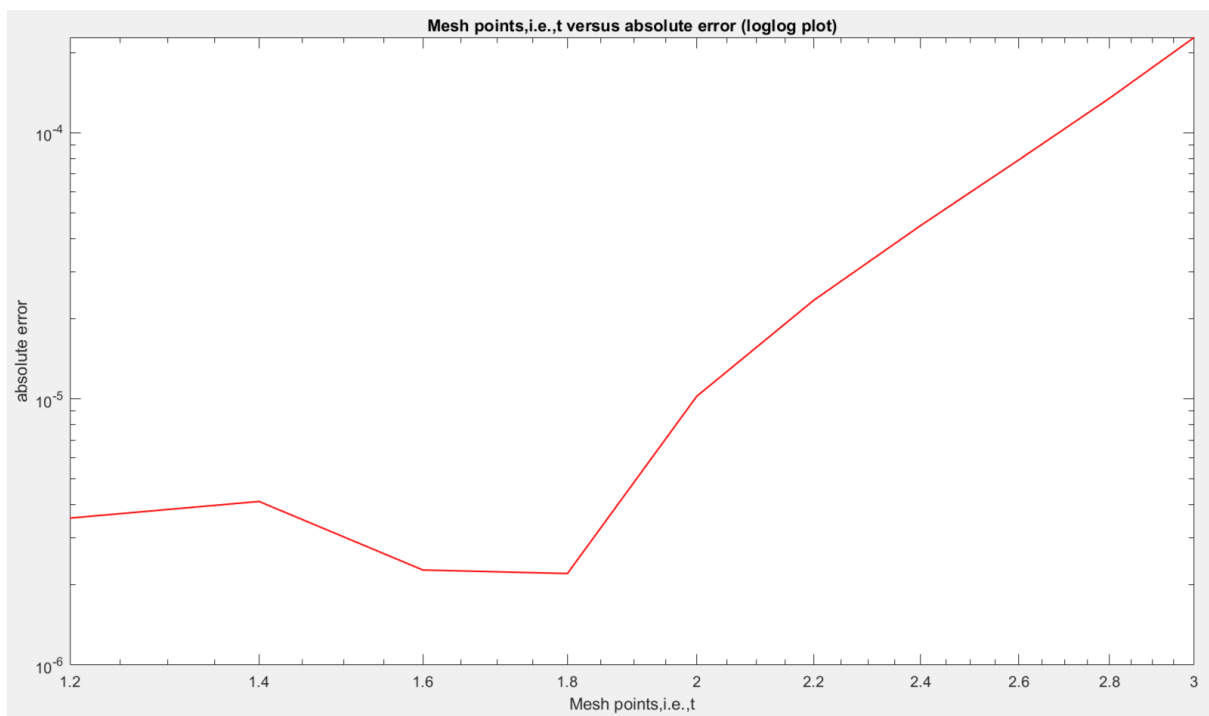
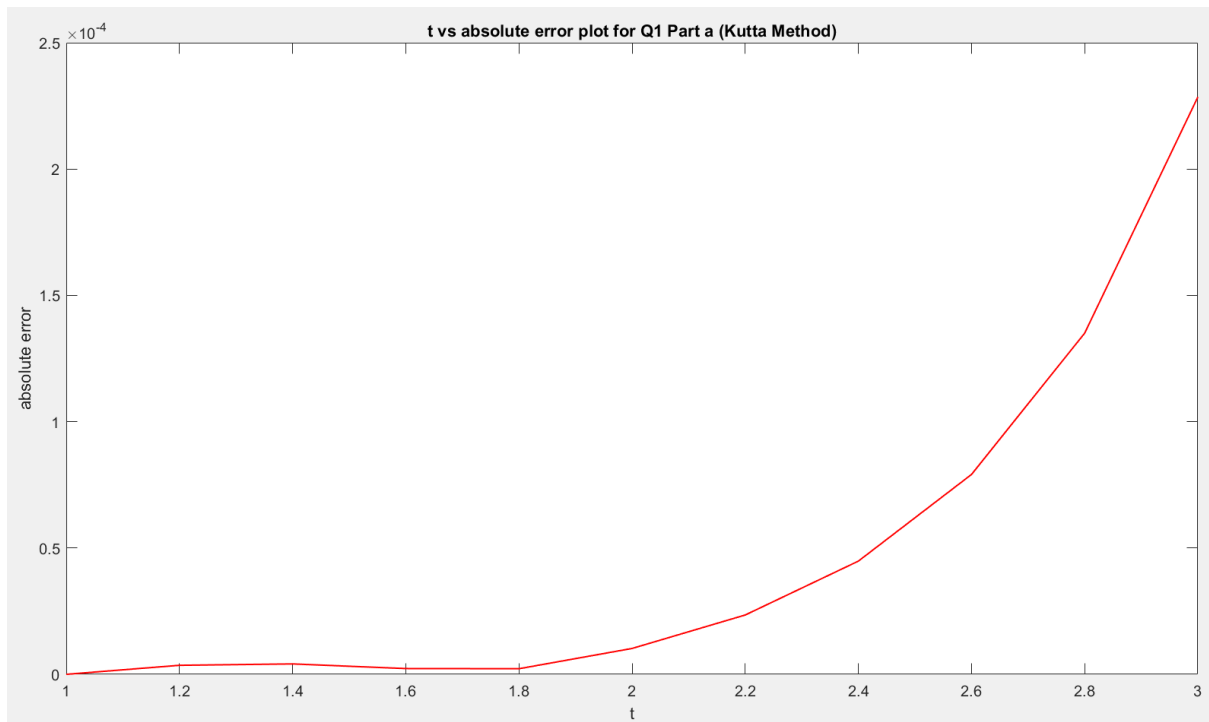


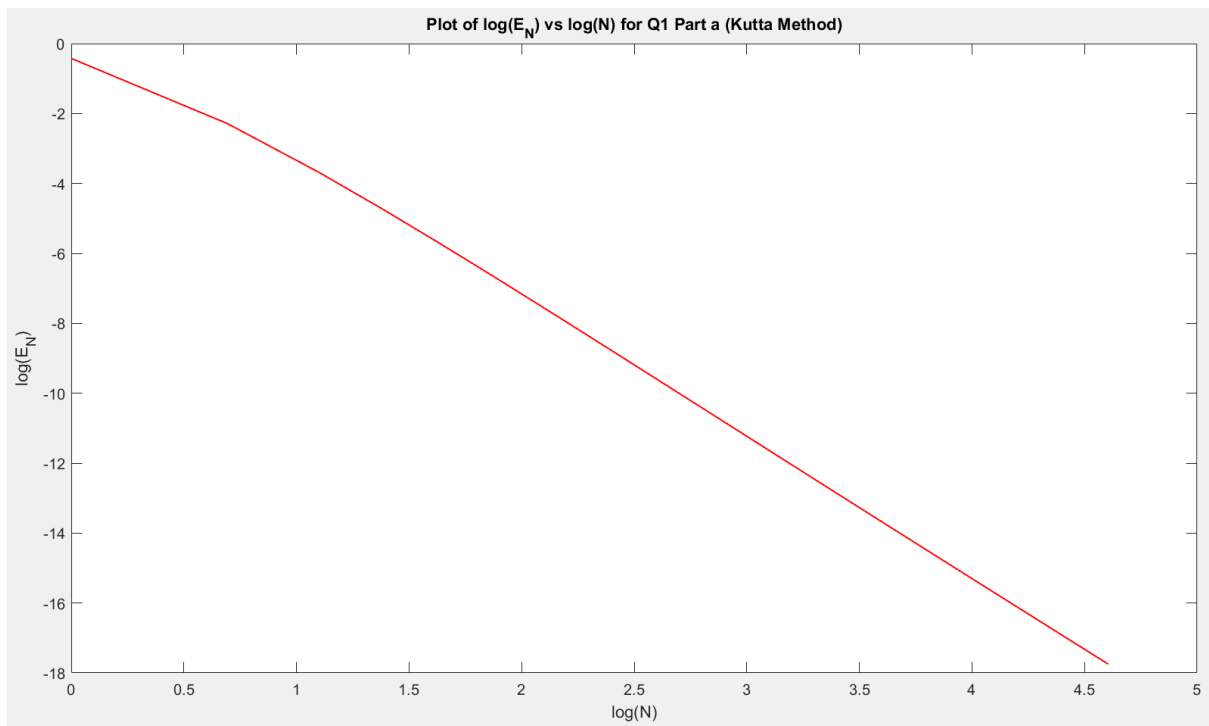
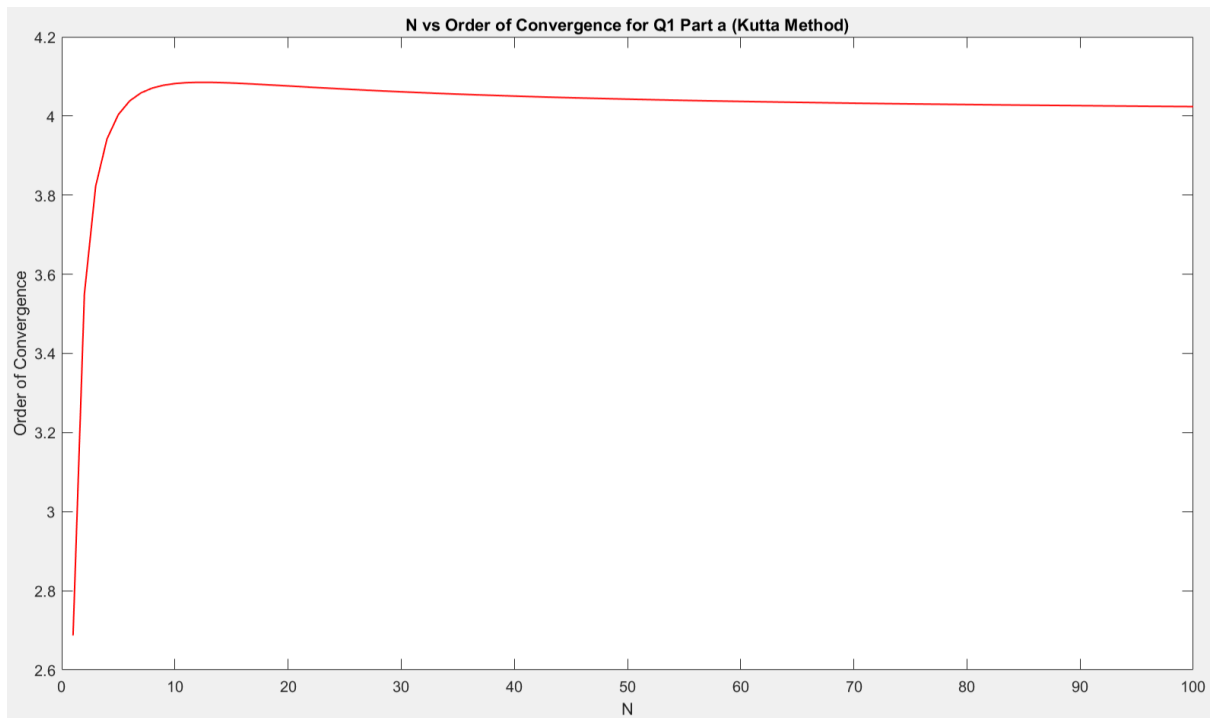


Using Fourth-order Runge-Kutta Method for Q1 Part a (Kutta Method)

t	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.000000	0.000000	0.000000
1.200000	0.221246	0.221243	0.000004
1.400000	0.489686	0.489682	0.000004
1.600000	0.812755	0.812753	0.000002
1.800000	1.199436	1.199439	0.000002
2.000000	1.661272	1.661282	0.000010
2.200000	2.213478	2.213502	0.000023
2.400000	2.876507	2.876551	0.000045
2.600000	3.678396	3.678475	0.000079
2.800000	4.658530	4.658665	0.000135
3.000000	5.873871	5.874100	0.000228





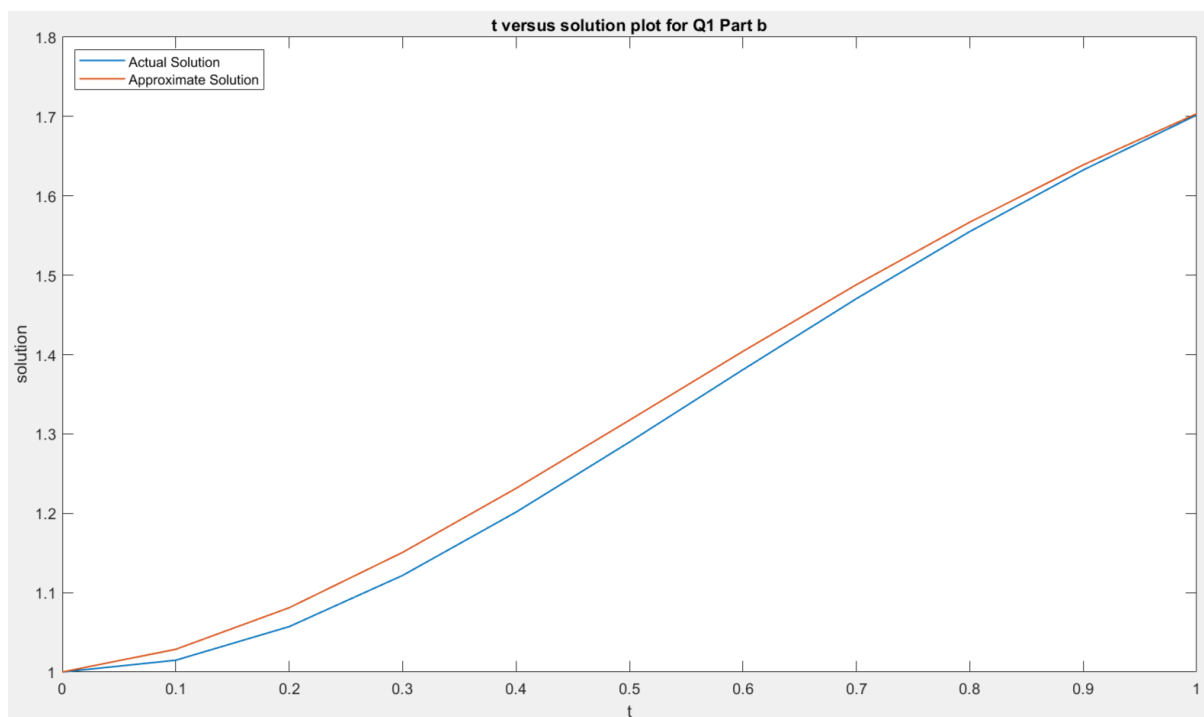


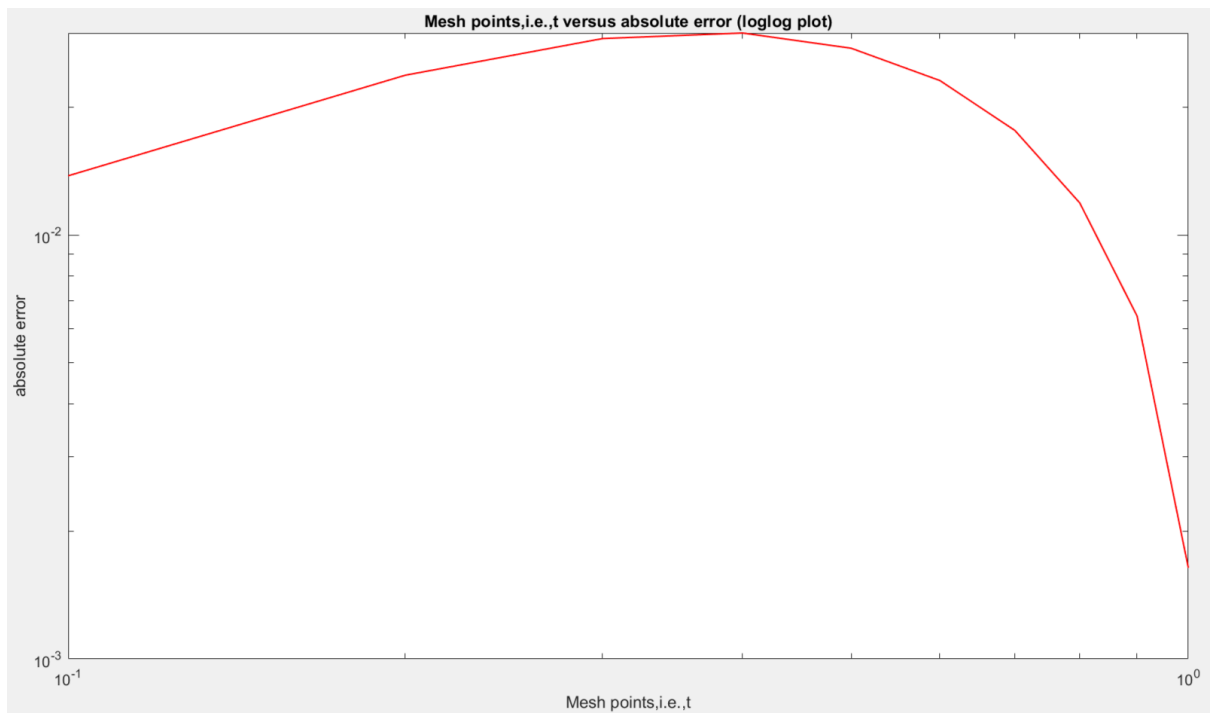
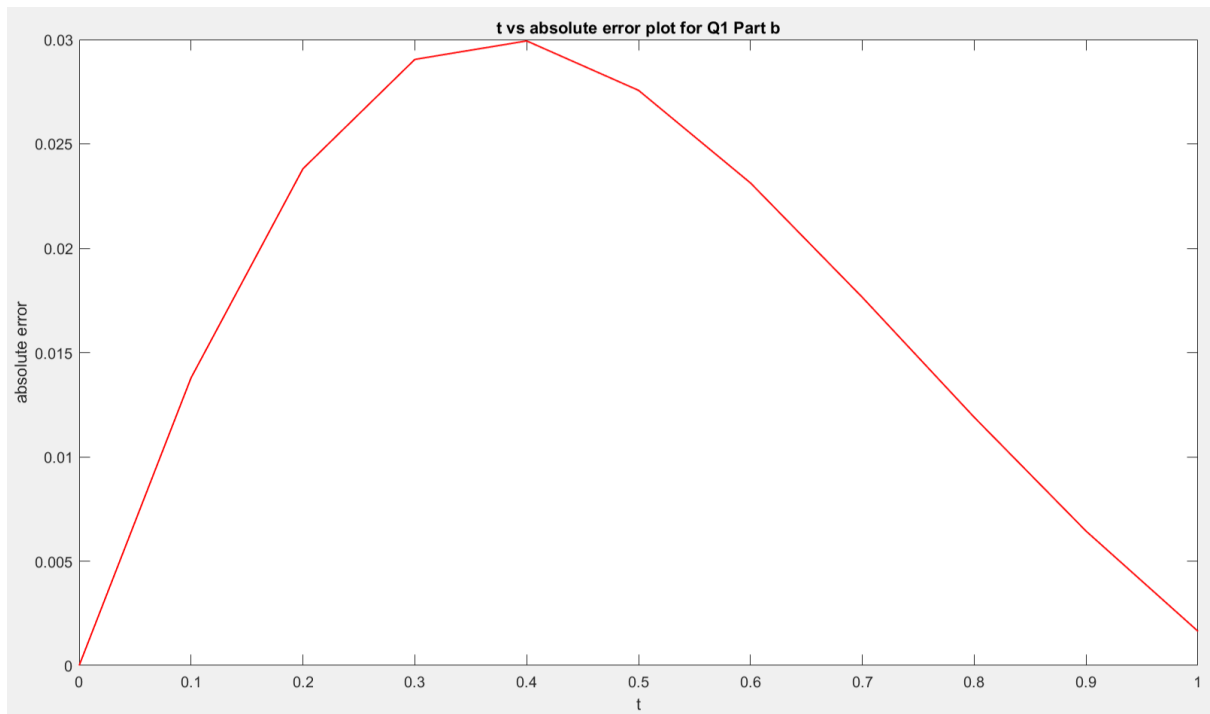
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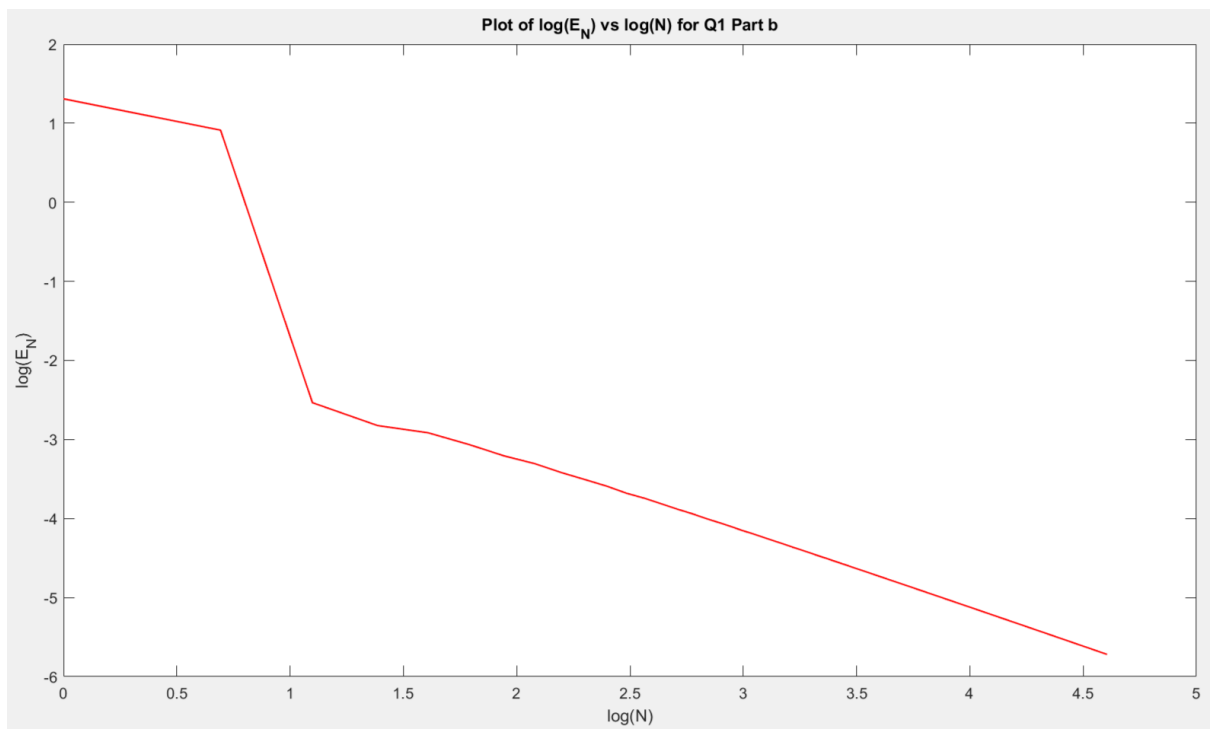
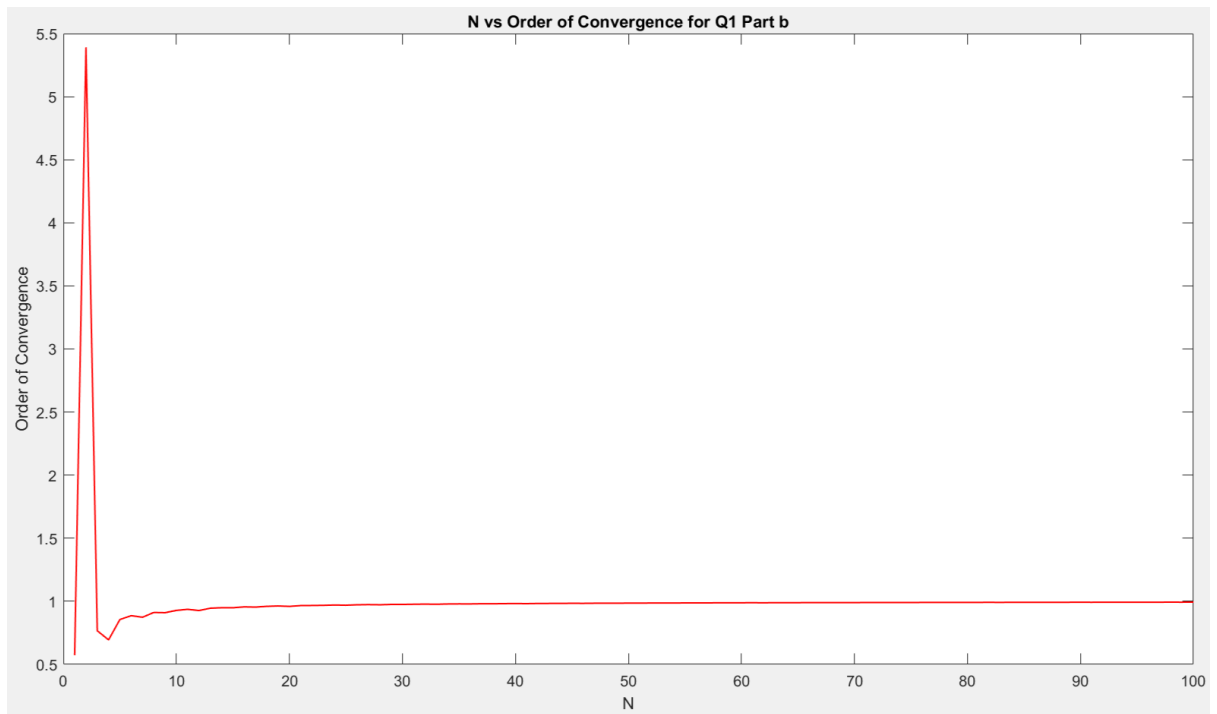
Question 1 Part b

Using Implicit-Euler method for Q1 Part b

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.028602	1.014815	0.013786
0.200000	1.080988	1.057181	0.023807
0.300000	1.150746	1.121698	0.029048
0.400000	1.231421	1.201486	0.029935
0.500000	1.317370	1.289805	0.027564
0.600000	1.404059	1.380931	0.023128
0.700000	1.488059	1.470415	0.017644
0.800000	1.566927	1.555031	0.011895
0.900000	1.639052	1.632613	0.006438
1.000000	1.703510	1.701870	0.001640

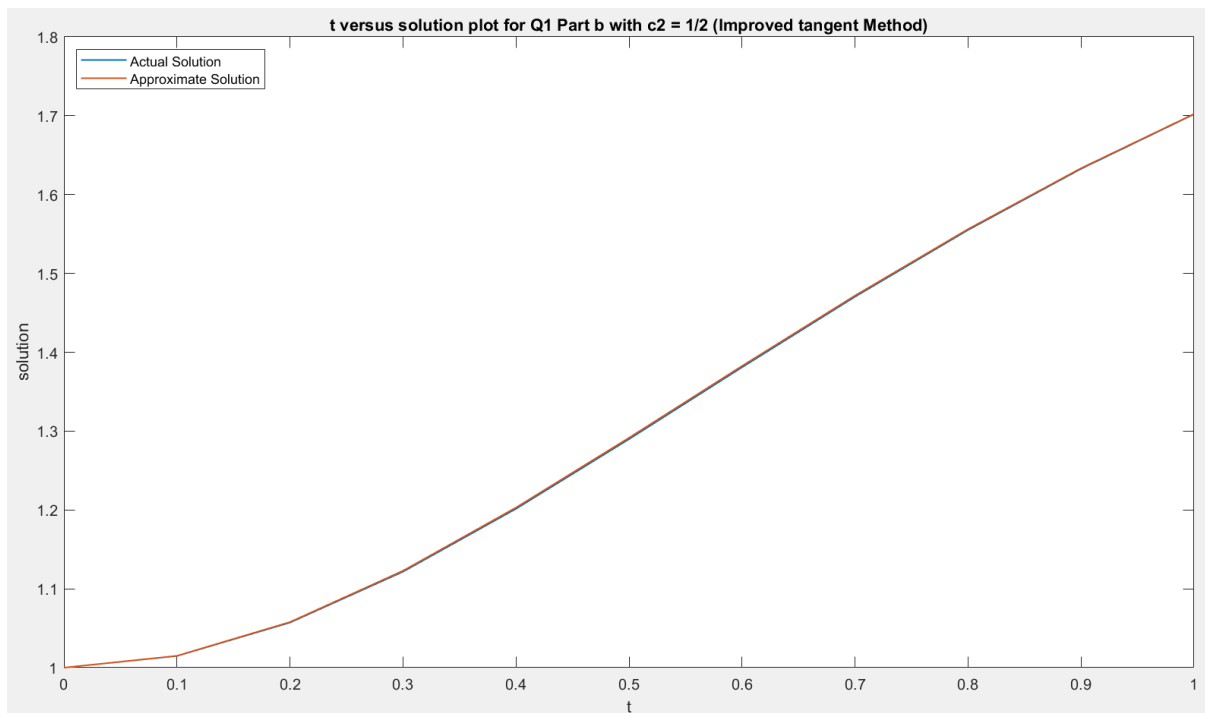


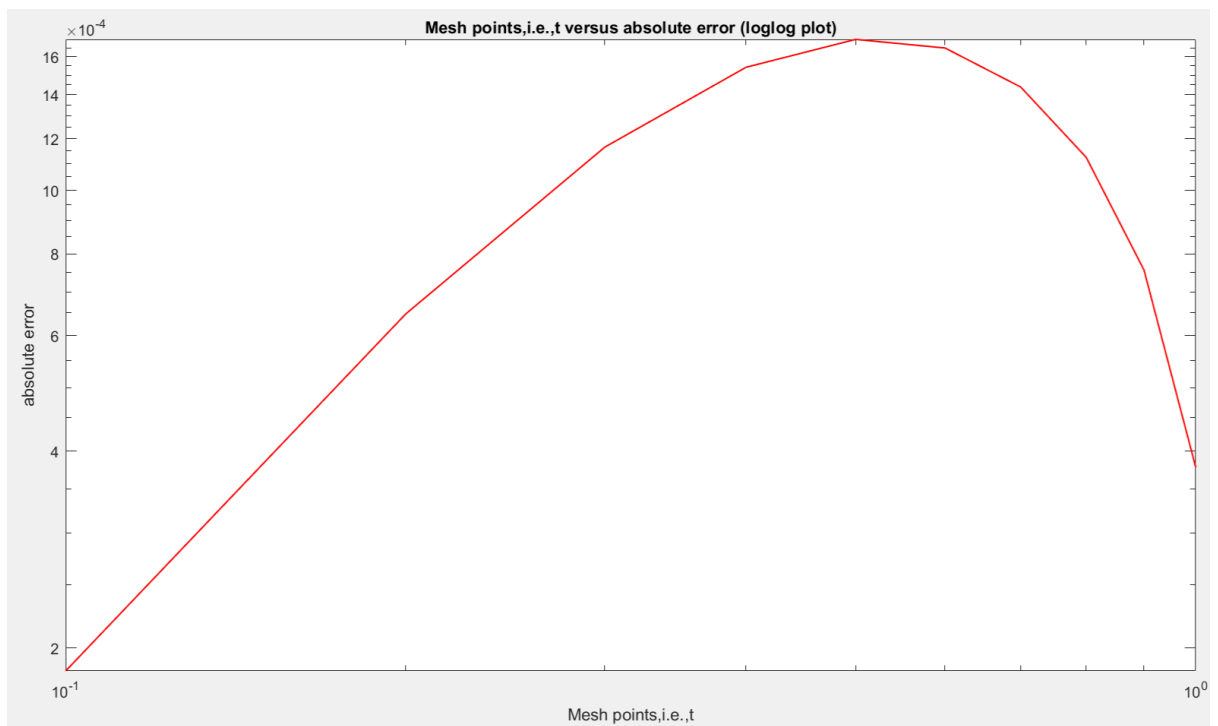
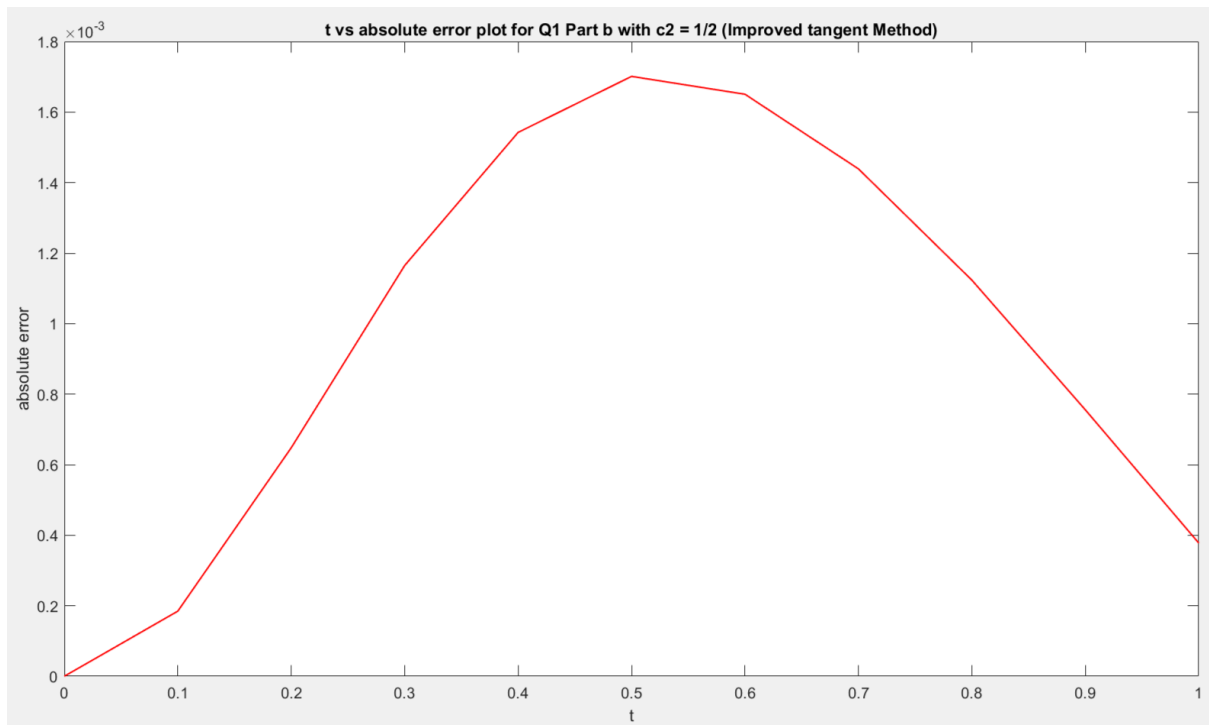


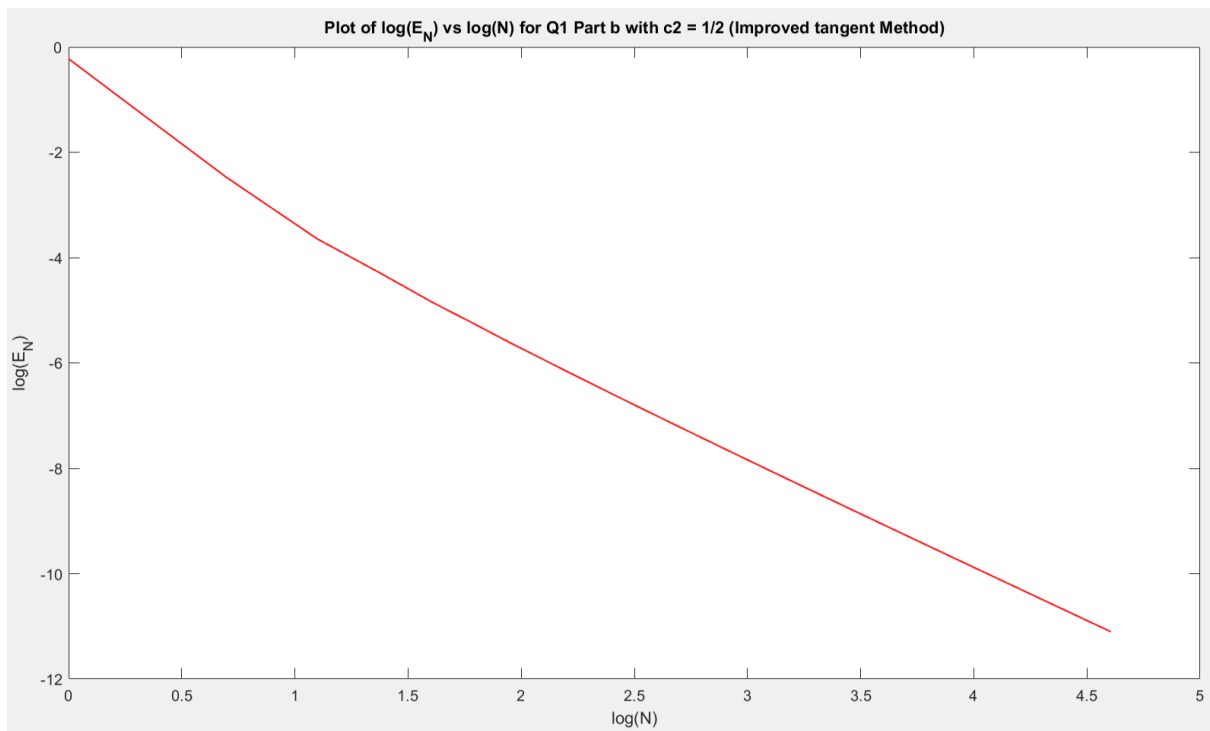
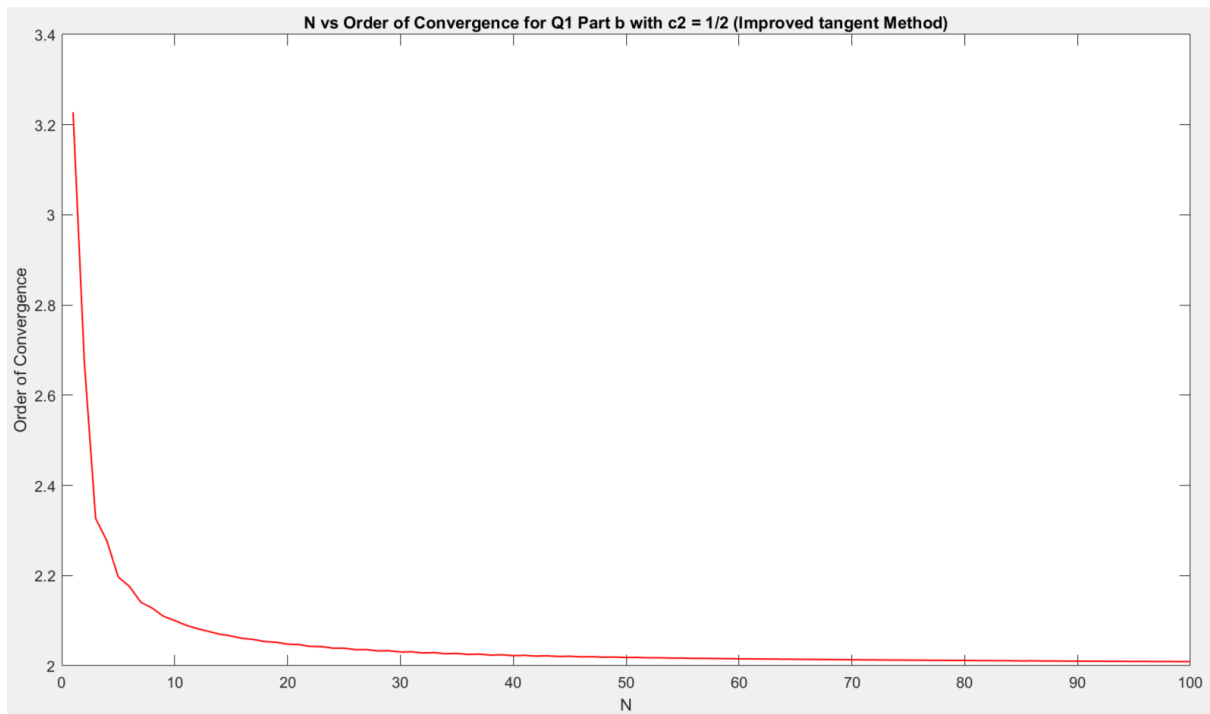


Using Second-order Runge-Kutta Method for Q1 Part b with $c_2 = 1/2$ (Improved tangent Method)

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.015000	1.014815	0.000185
0.200000	1.057829	1.057181	0.000648
0.300000	1.122863	1.121698	0.001165
0.400000	1.203028	1.201486	0.001542
0.500000	1.291506	1.289805	0.001701
0.600000	1.382582	1.380931	0.001650
0.700000	1.471854	1.470415	0.001439
0.800000	1.556155	1.555031	0.001123
0.900000	1.633369	1.632613	0.000756
1.000000	1.702248	1.701870	0.000378

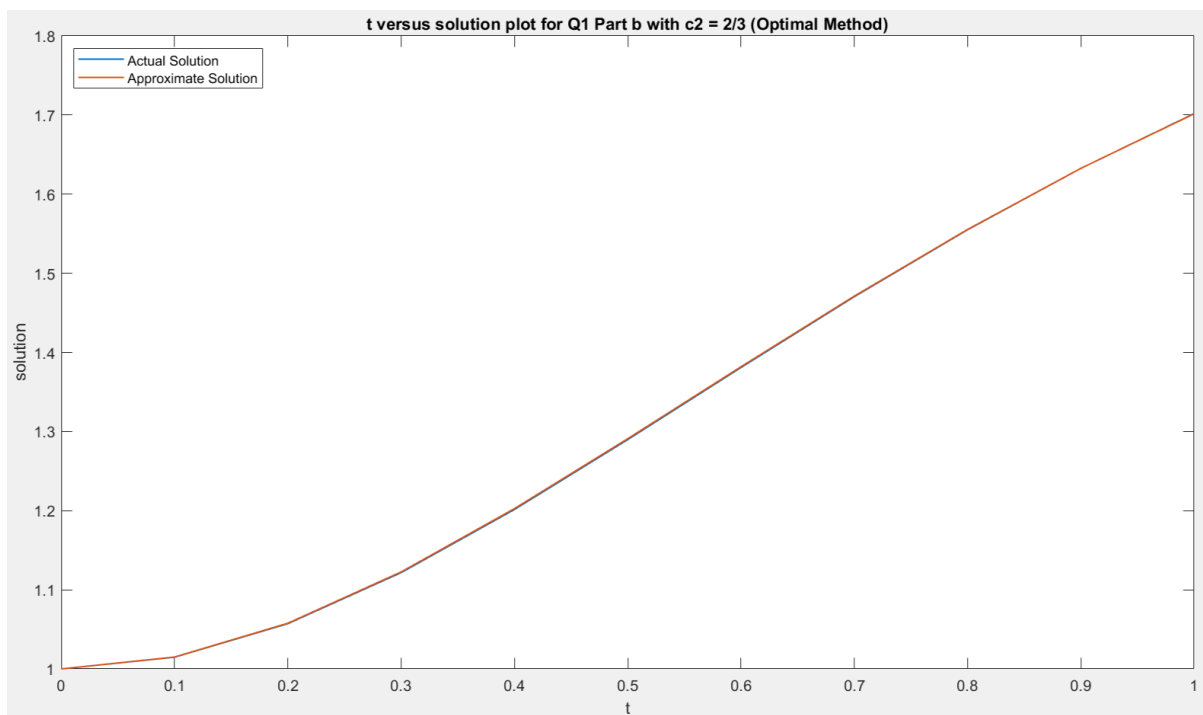


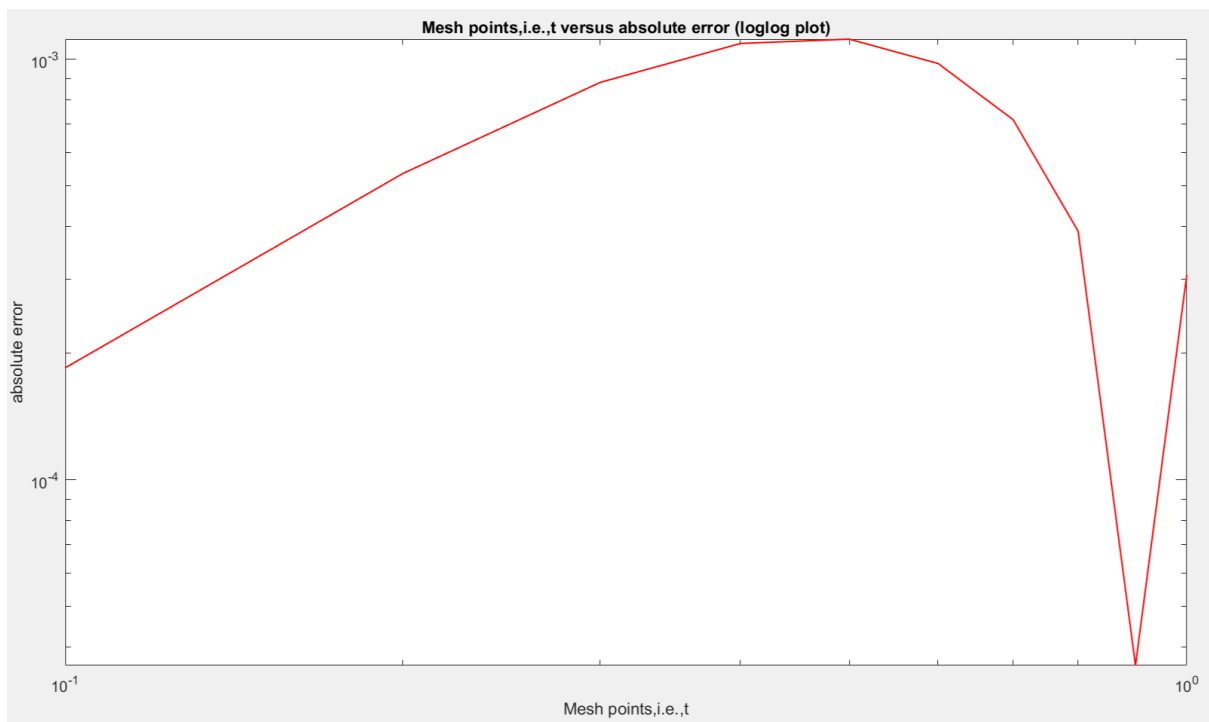
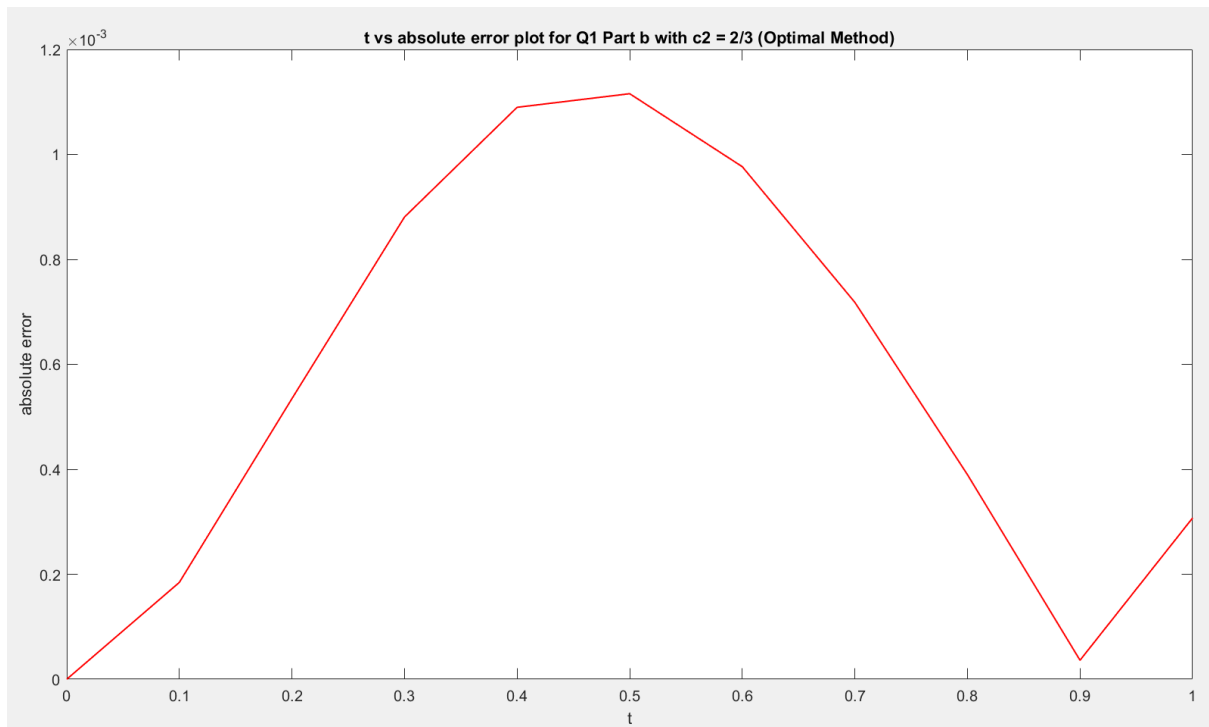


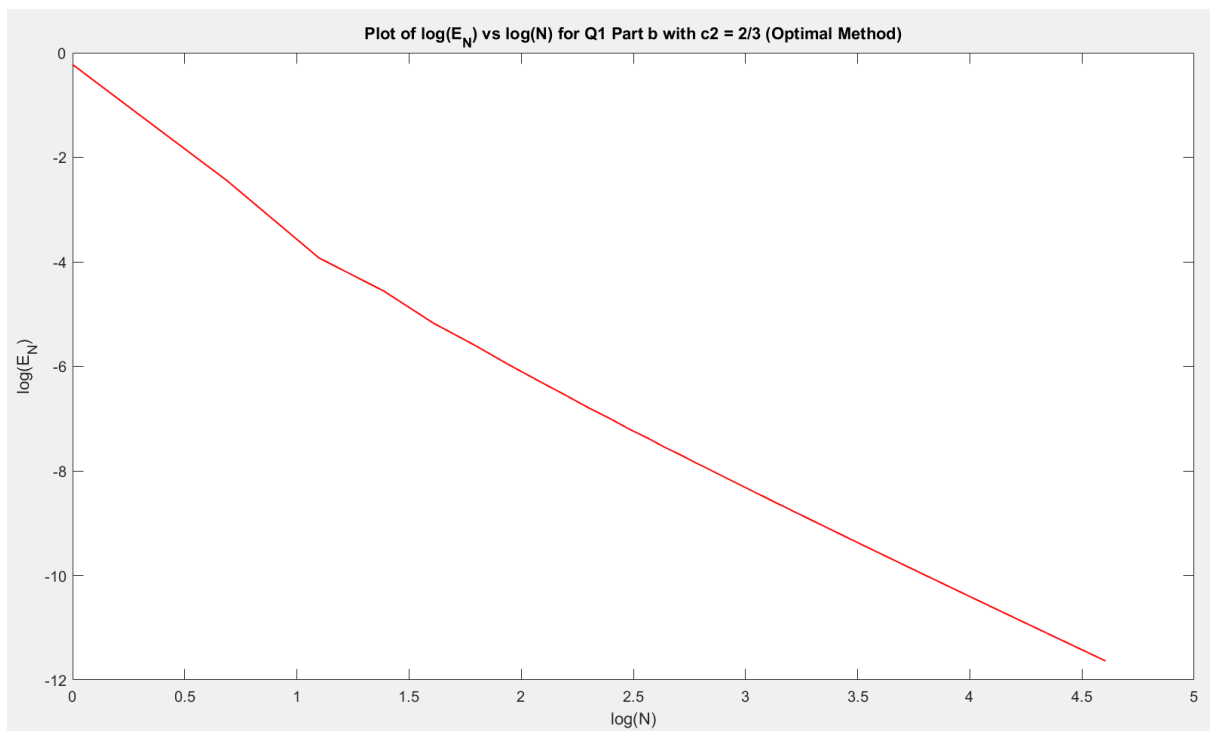
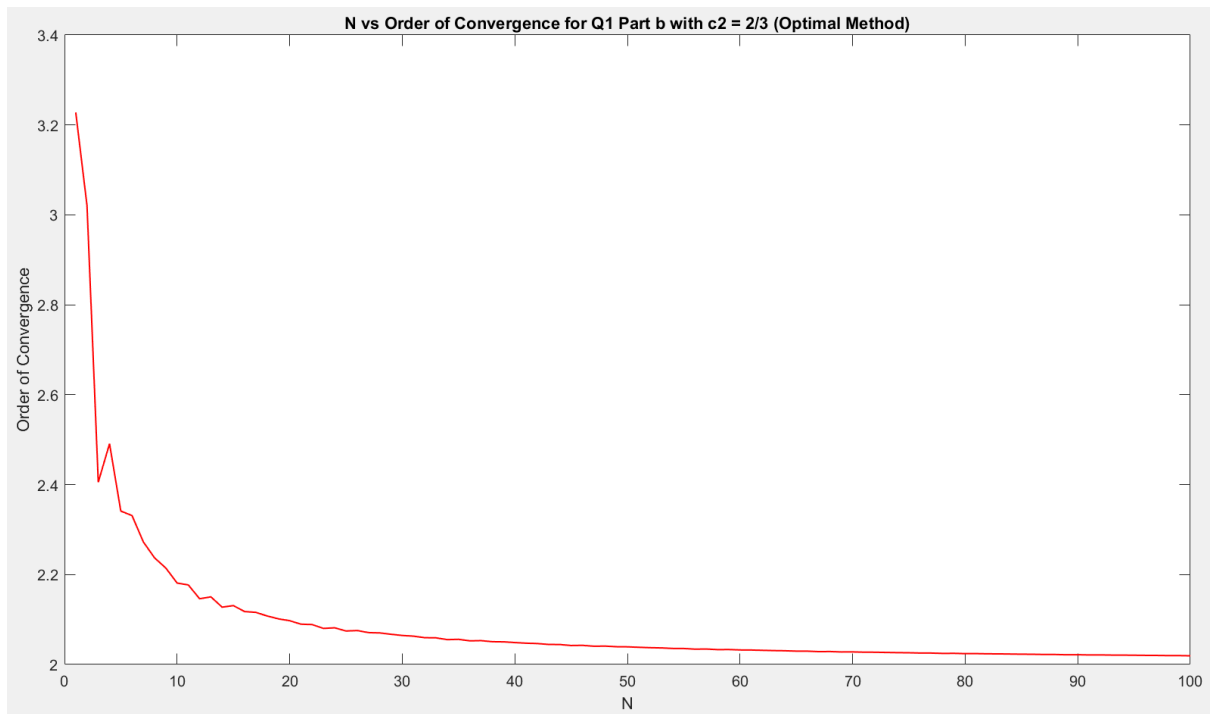


Using Second-order Runge-Kutta Method for Q1 Part b with $c_2 = 2/3$ (Optimal Method)

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.015000	1.014815	0.000185
0.200000	1.057716	1.057181	0.000535
0.300000	1.122579	1.121698	0.000881
0.400000	1.202576	1.201486	0.001090
0.500000	1.290921	1.289805	0.001116
0.600000	1.381908	1.380931	0.000977
0.700000	1.471133	1.470415	0.000718
0.800000	1.555421	1.555031	0.000390
0.900000	1.632649	1.632613	0.000036
1.000000	1.701563	1.701870	0.000307

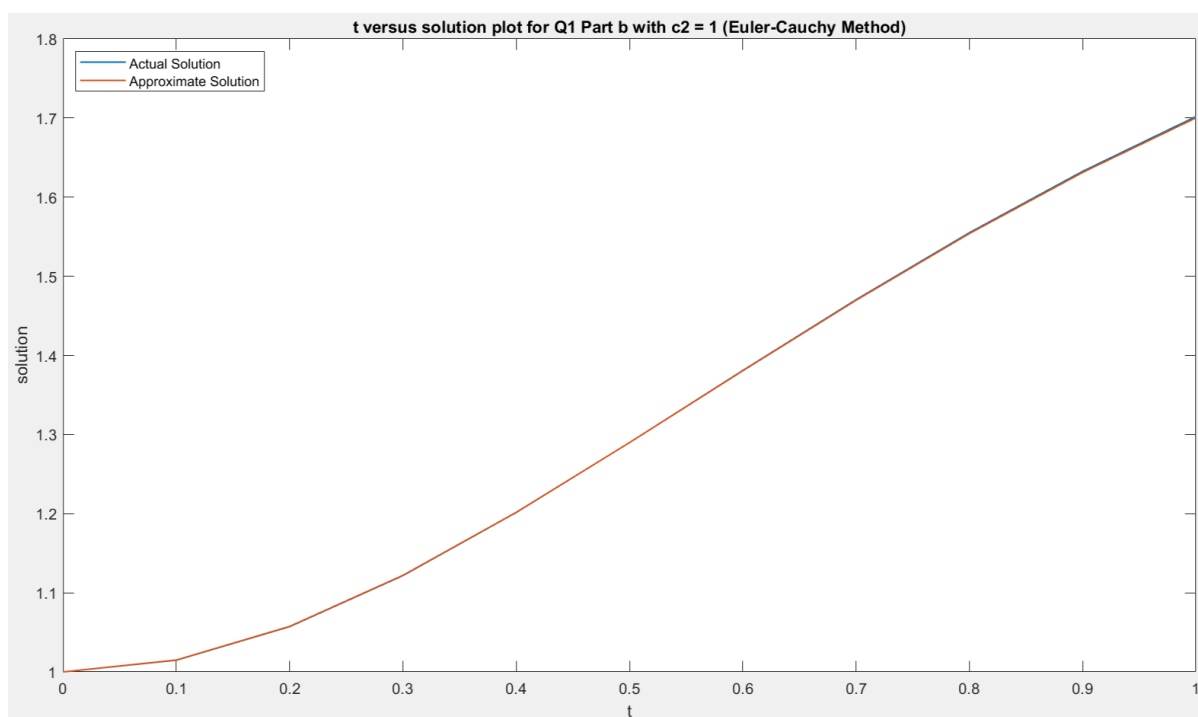


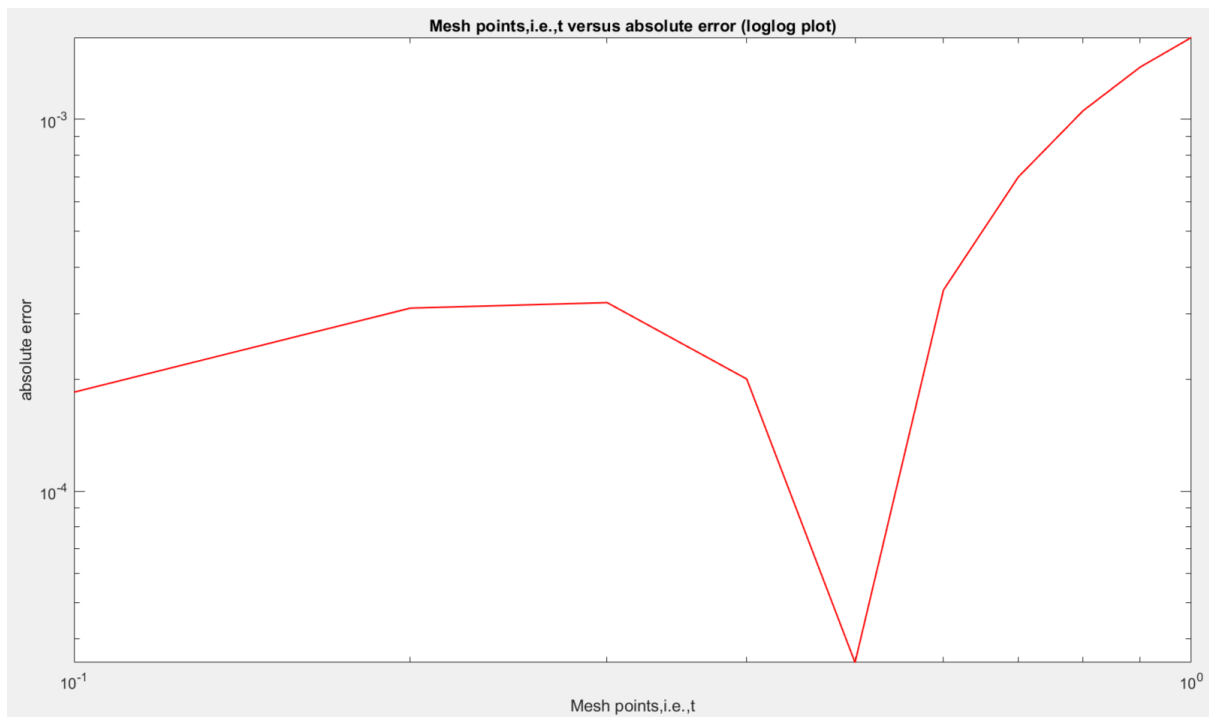
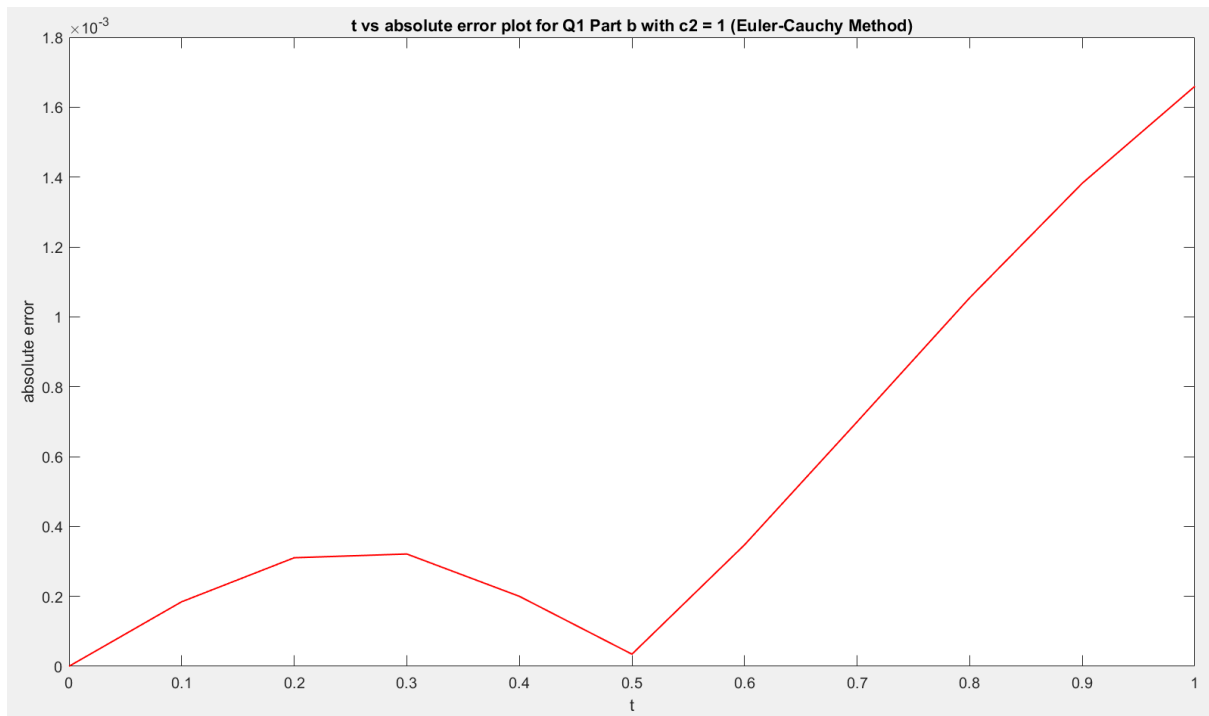


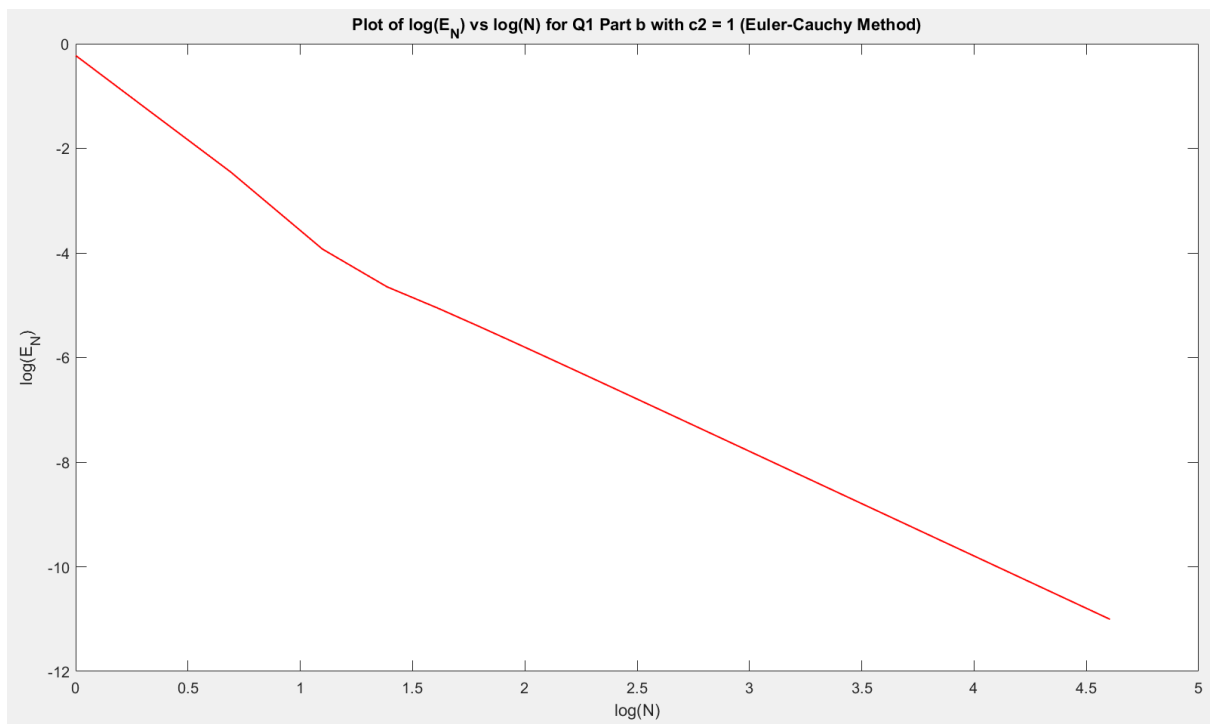
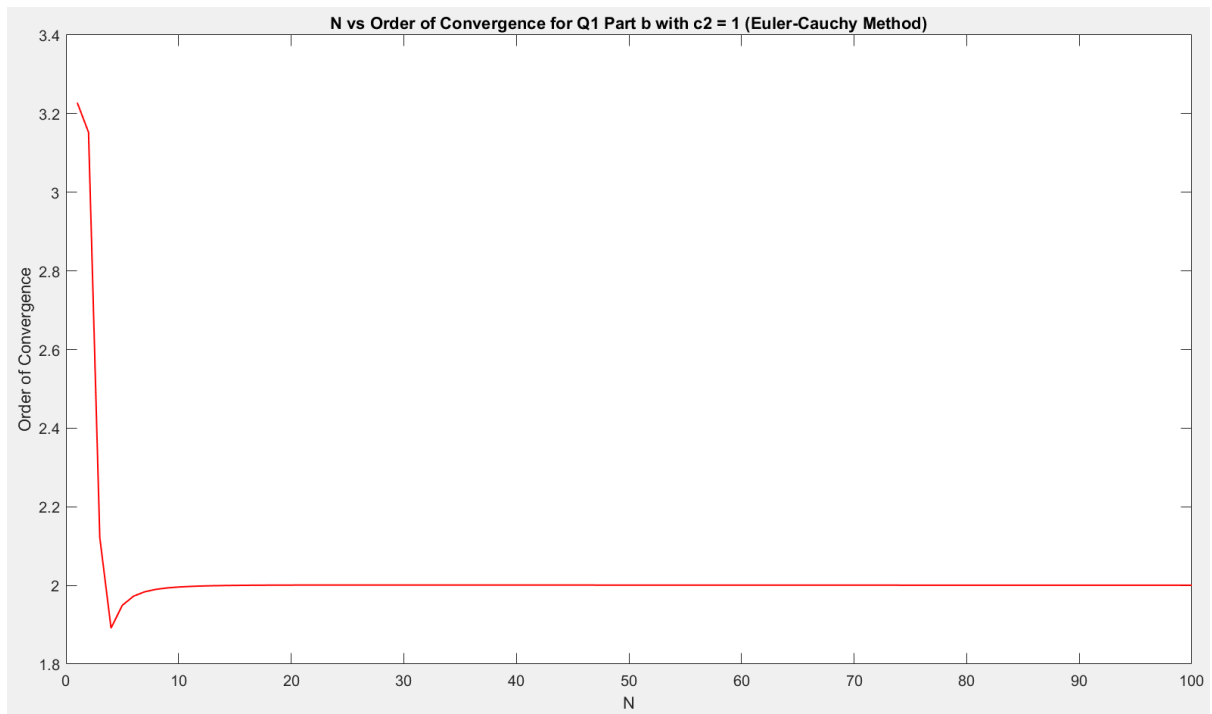


Using Second-order Runge-Kutta Method for Q1 Part b with $c_2 = 1$ (Euler-Cauchy Method)

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.015000	1.014815	0.000185
0.200000	1.057492	1.057181	0.000311
0.300000	1.122019	1.121698	0.000321
0.400000	1.201686	1.201486	0.000200
0.500000	1.289771	1.289805	0.000035
0.600000	1.380584	1.380931	0.000348
0.700000	1.469716	1.470415	0.000700
0.800000	1.553977	1.555031	0.001055
0.900000	1.631231	1.632613	0.001382
1.000000	1.700210	1.701870	0.001660

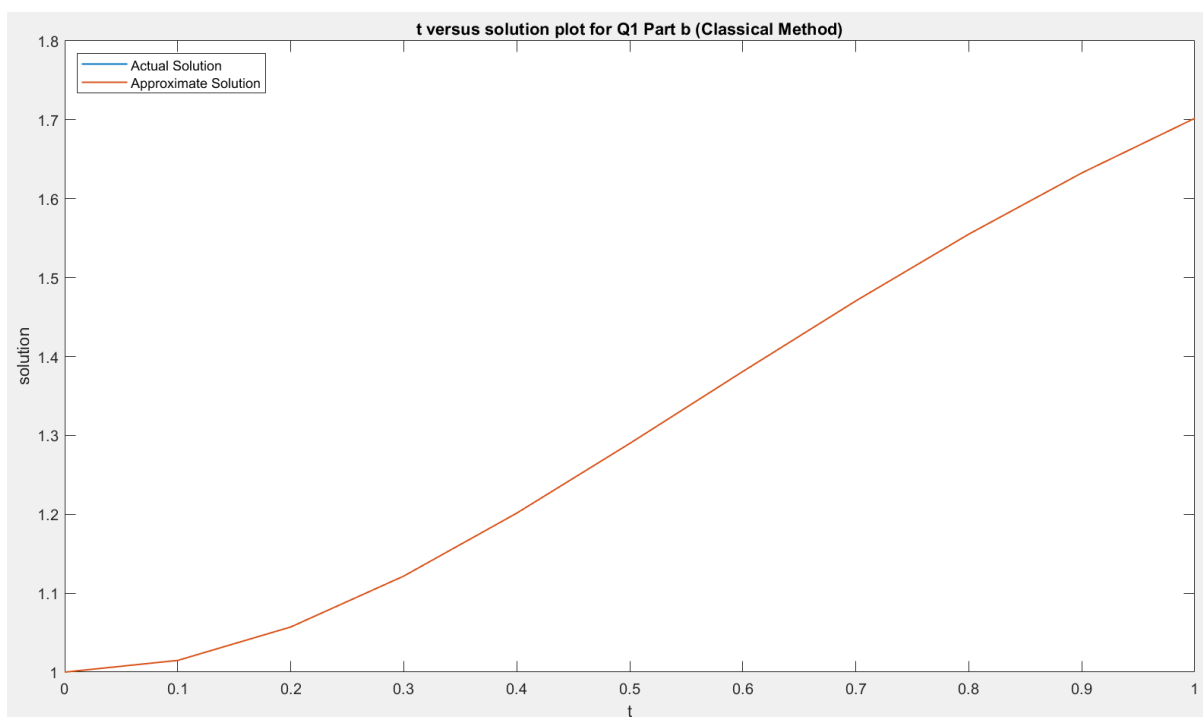


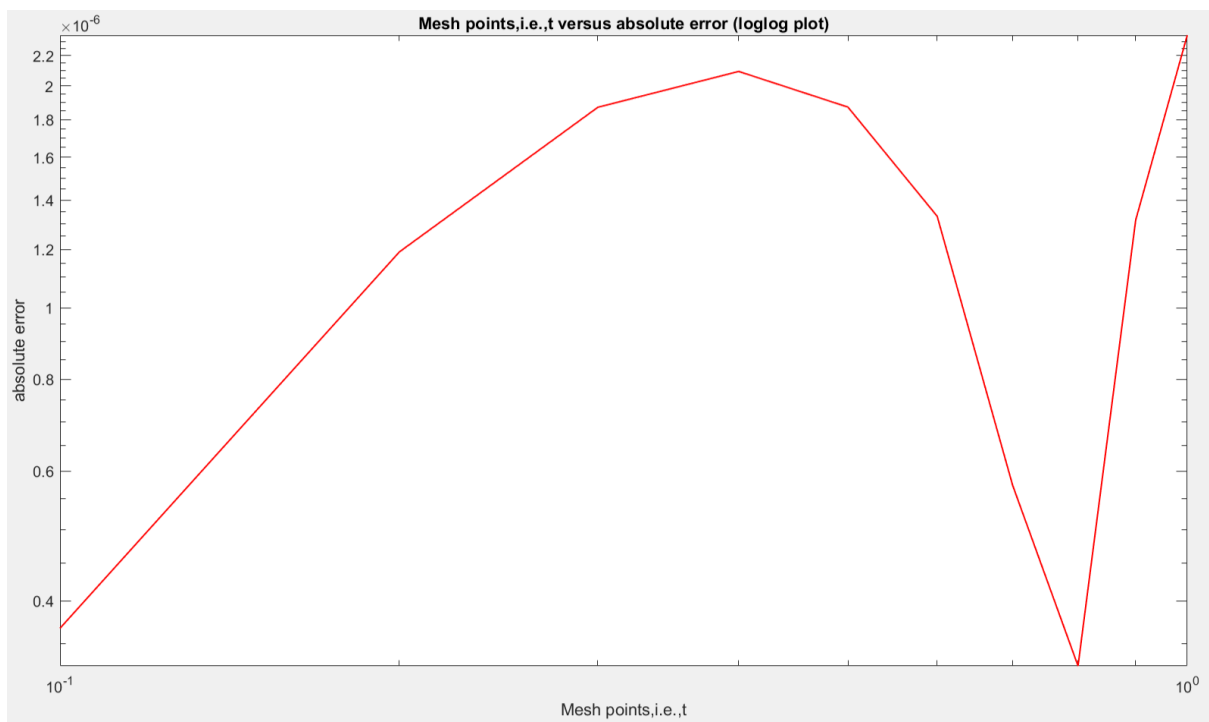
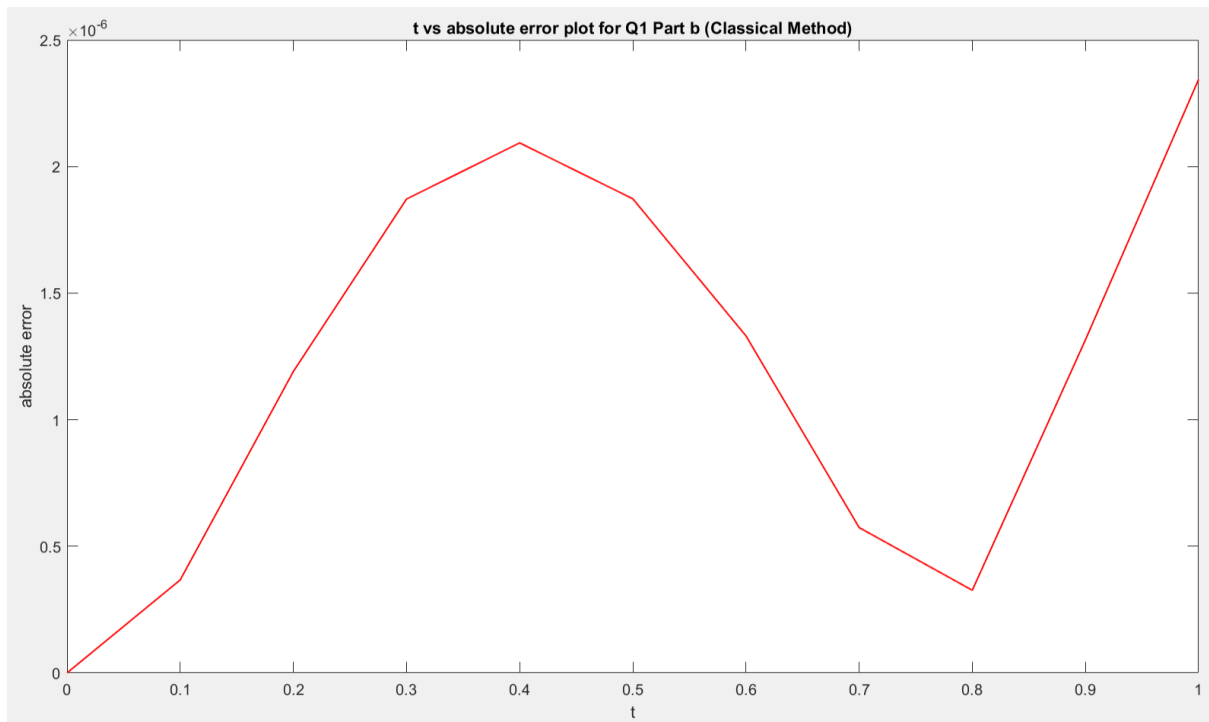


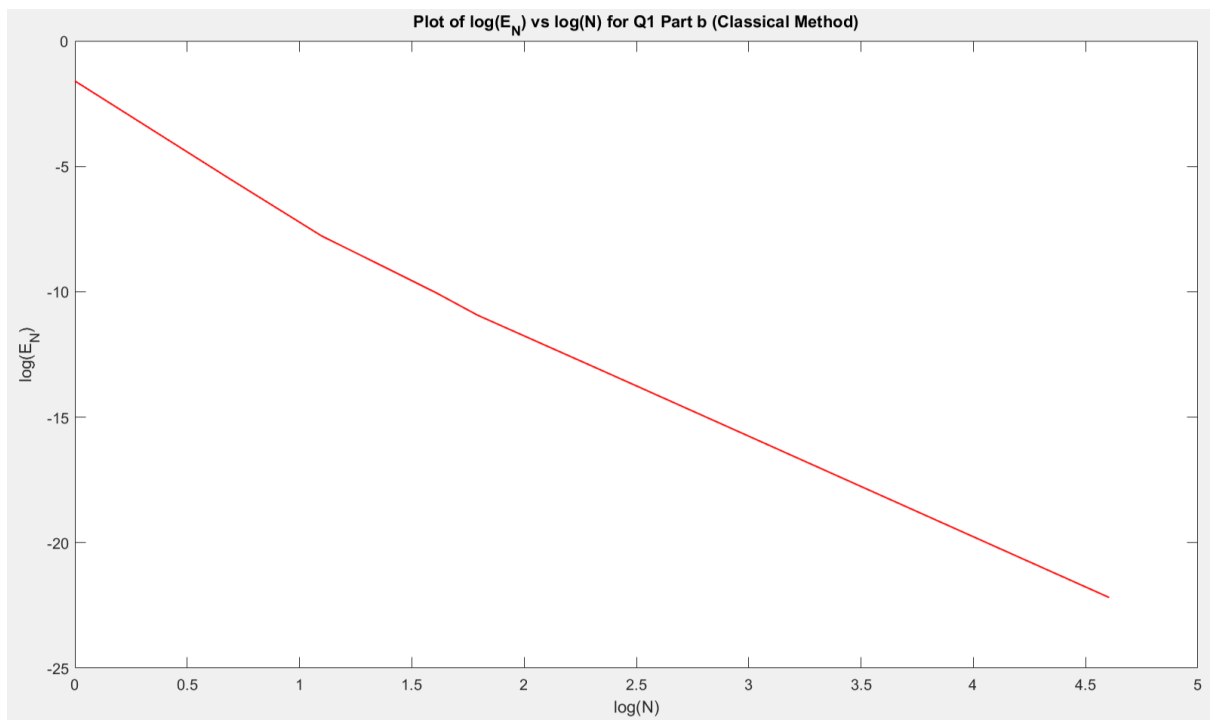
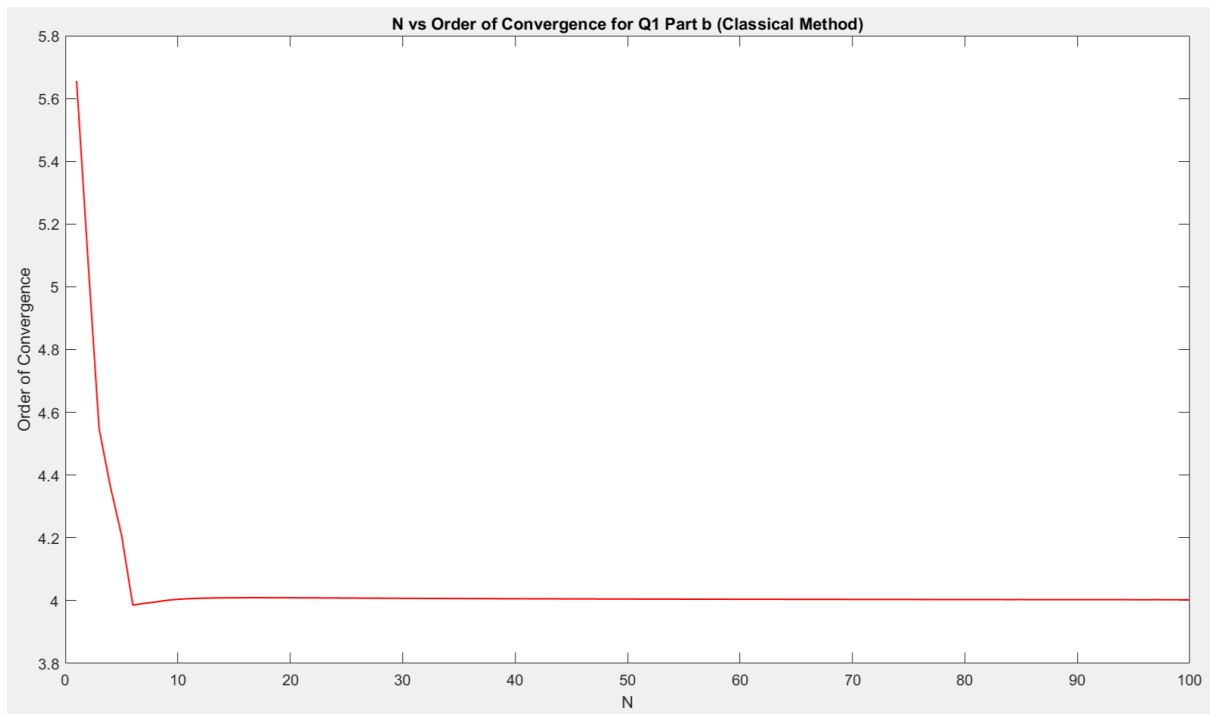


Using Fourth-order Runge-Kutta Method for Q1 Part b (Classical Method)

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.014816	1.014815	0.000000
0.200000	1.057182	1.057181	0.000001
0.300000	1.121700	1.121698	0.000002
0.400000	1.201488	1.201486	0.000002
0.500000	1.289807	1.289805	0.000002
0.600000	1.380933	1.380931	0.000001
0.700000	1.470416	1.470415	0.000001
0.800000	1.555031	1.555031	0.000000
0.900000	1.632612	1.632613	0.000001
1.000000	1.701868	1.701870	0.000002

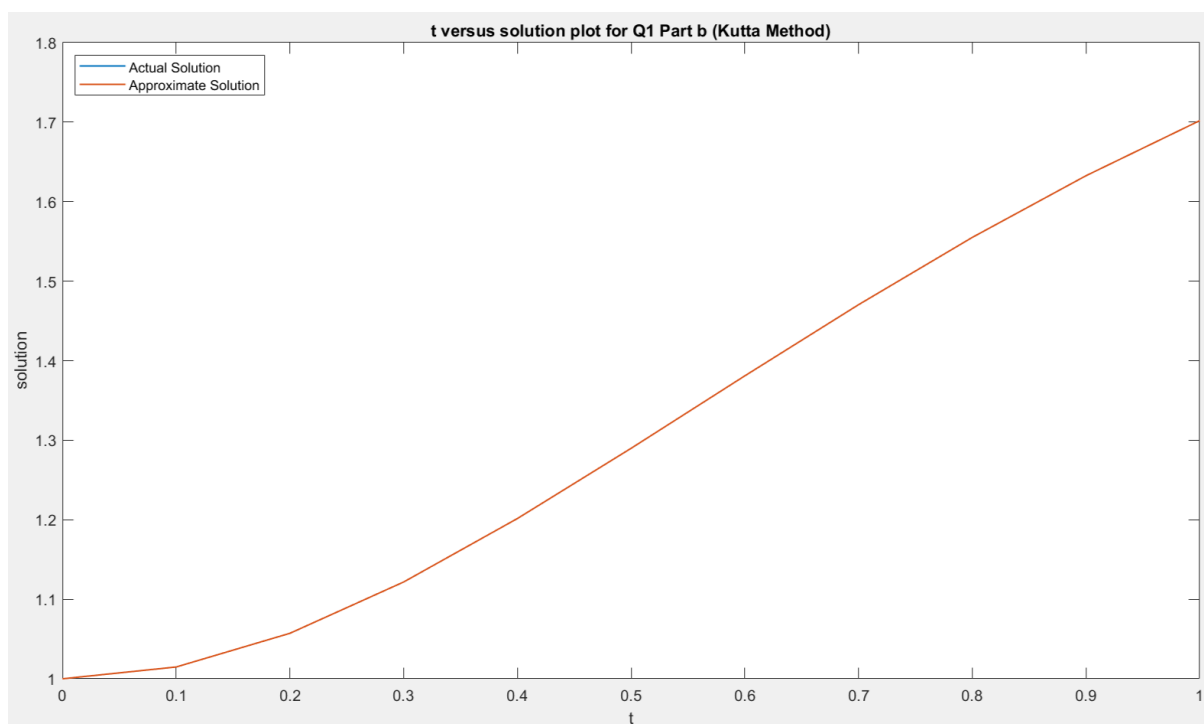


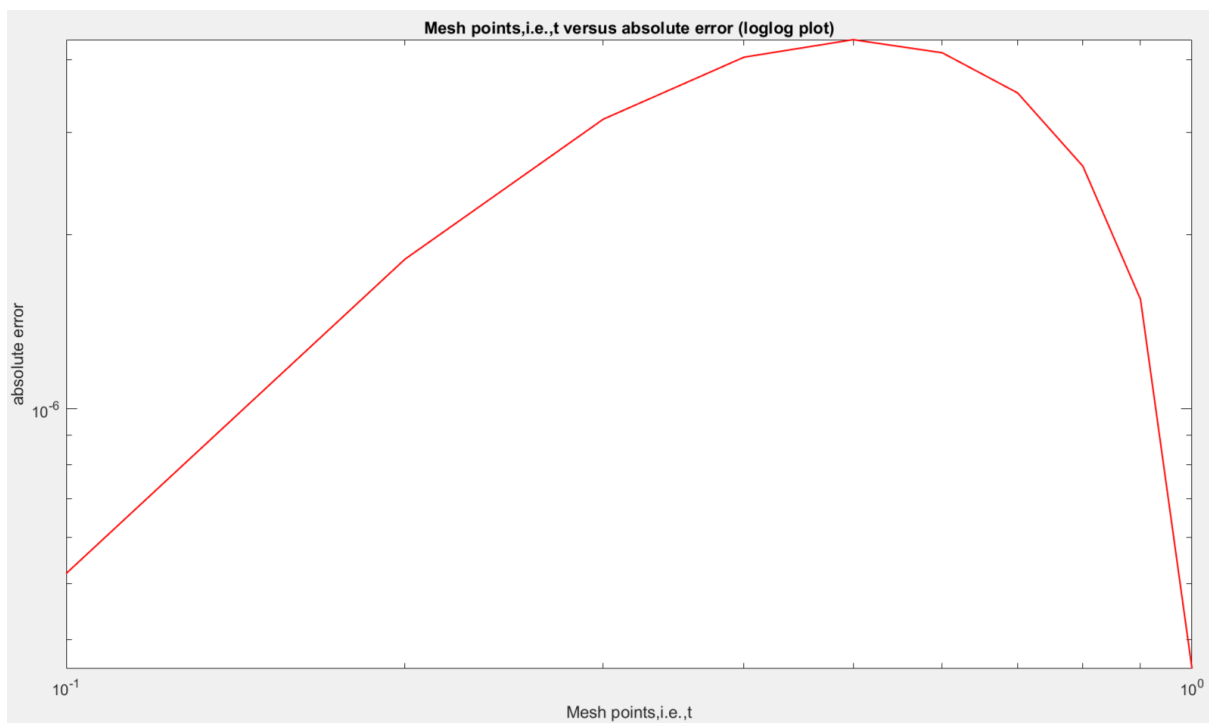
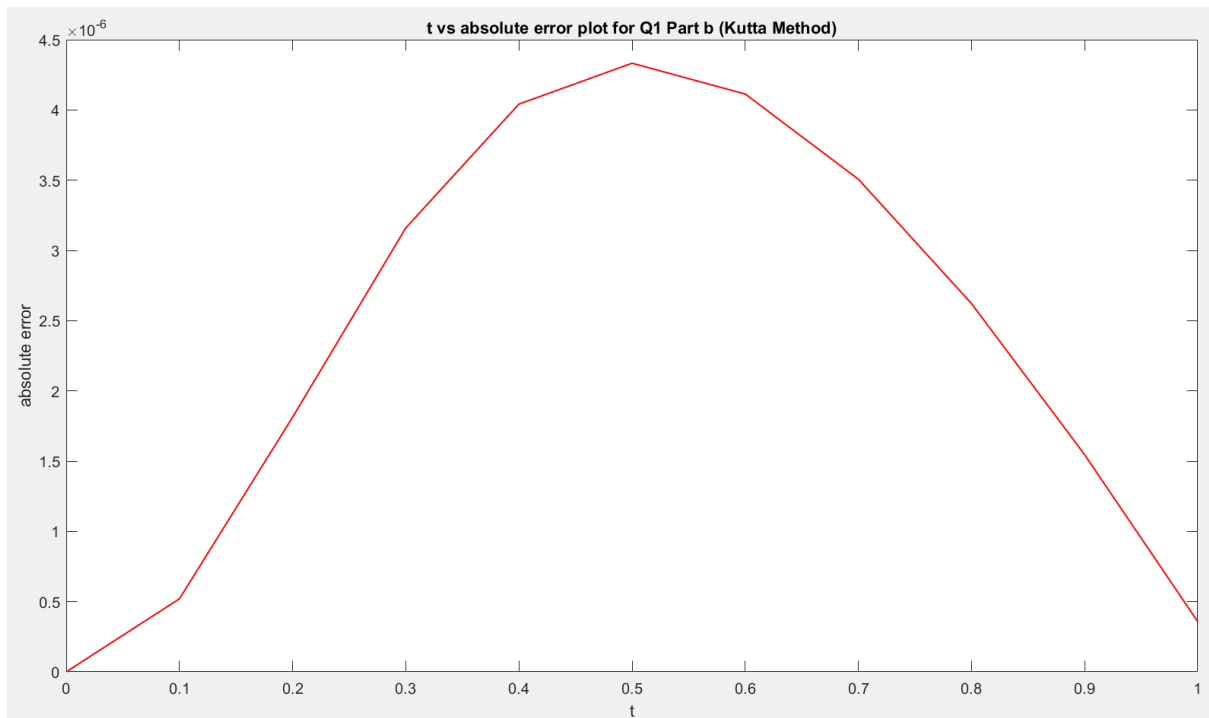


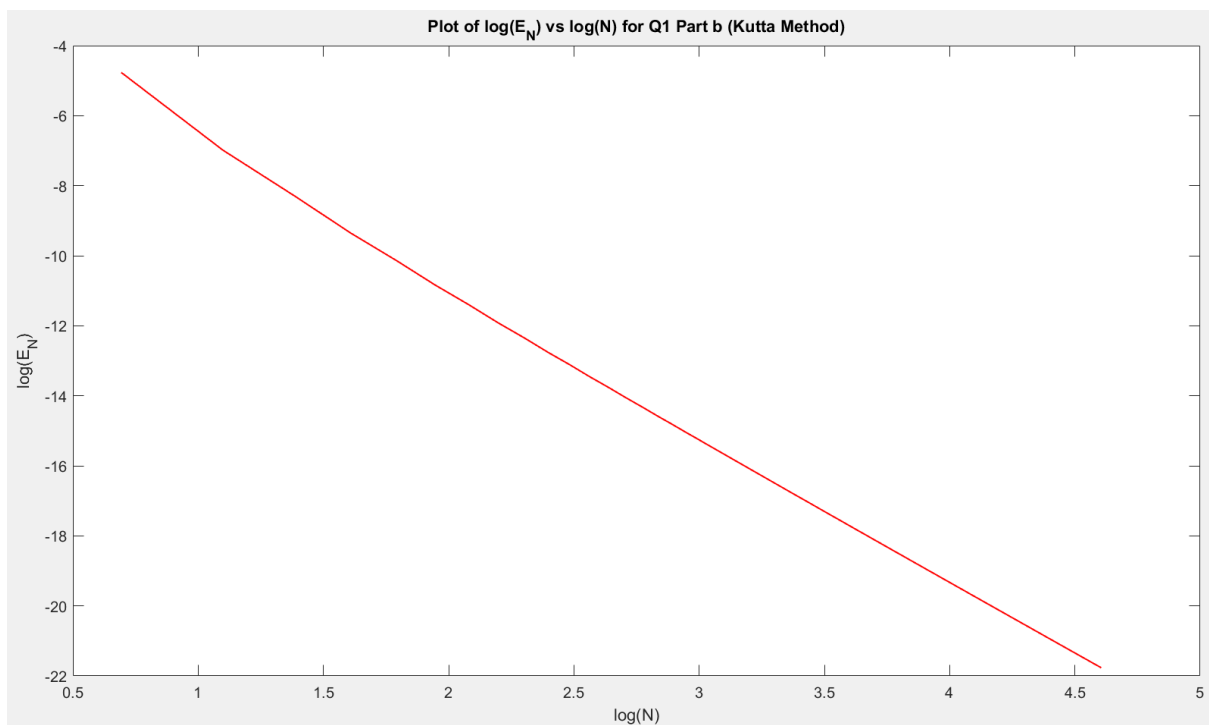
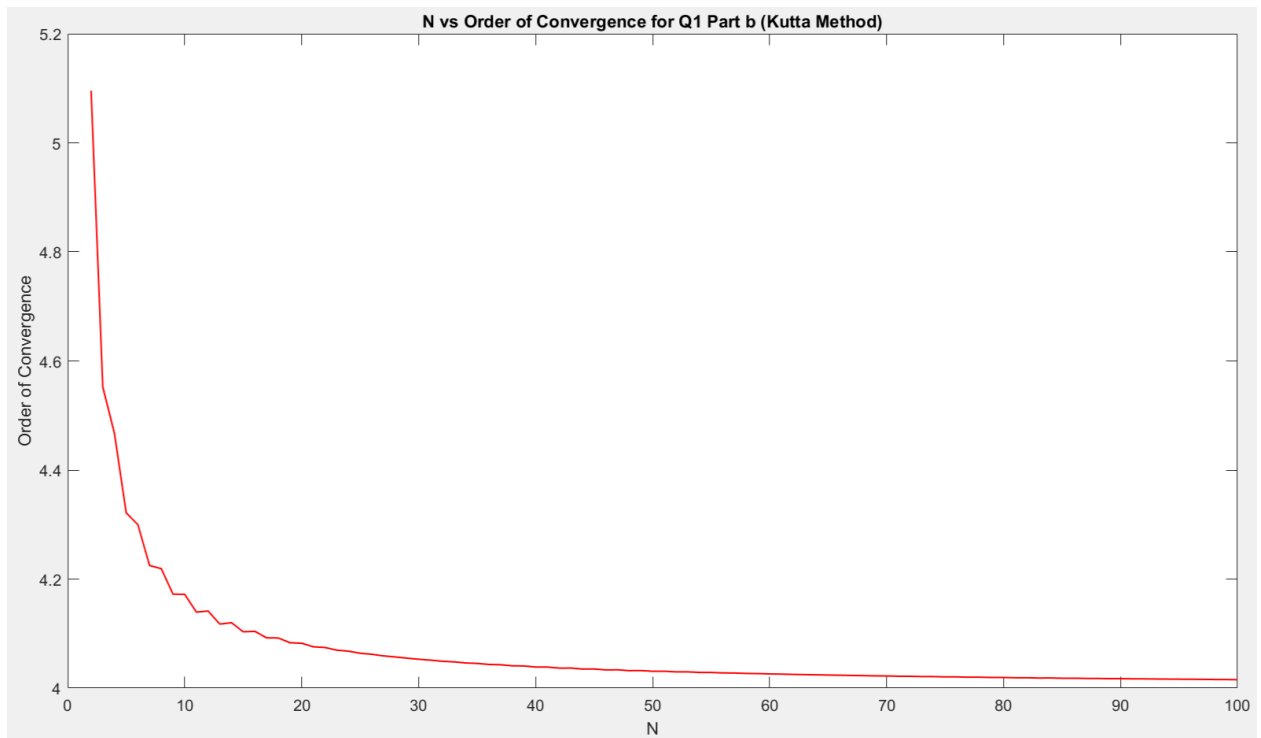


Using Fourth-order Runge-Kutta Method for Q1 Part b (Kutta Method)

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.014816	1.014815	0.000001
0.200000	1.057183	1.057181	0.000002
0.300000	1.121701	1.121698	0.000003
0.400000	1.201490	1.201486	0.000004
0.500000	1.289810	1.289805	0.000004
0.600000	1.380935	1.380931	0.000004
0.700000	1.470419	1.470415	0.000004
0.800000	1.555034	1.555031	0.000003
0.900000	1.632615	1.632613	0.000002
1.000000	1.701870	1.701870	0.000000







The graphs for actual and approximate solutions are almost coinciding in the case of Fourth Order Runge-Kutta methods. For the given initial-value problems, performance of: Implicit-Euler Method < Second-order Runge-Kutta Methods < Fourth-order Runge-Kutta Methods, which is as expected.

2)

The following methods are implemented:

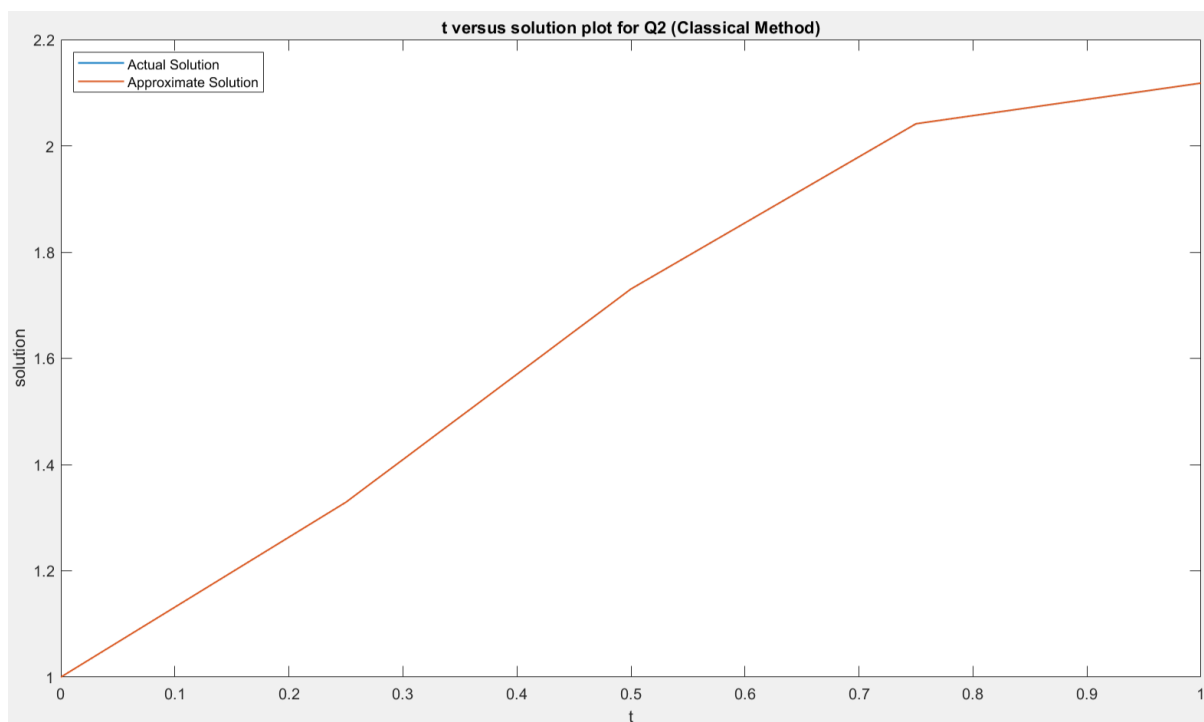
(i) Fourth-order Runge-Kutta method with:

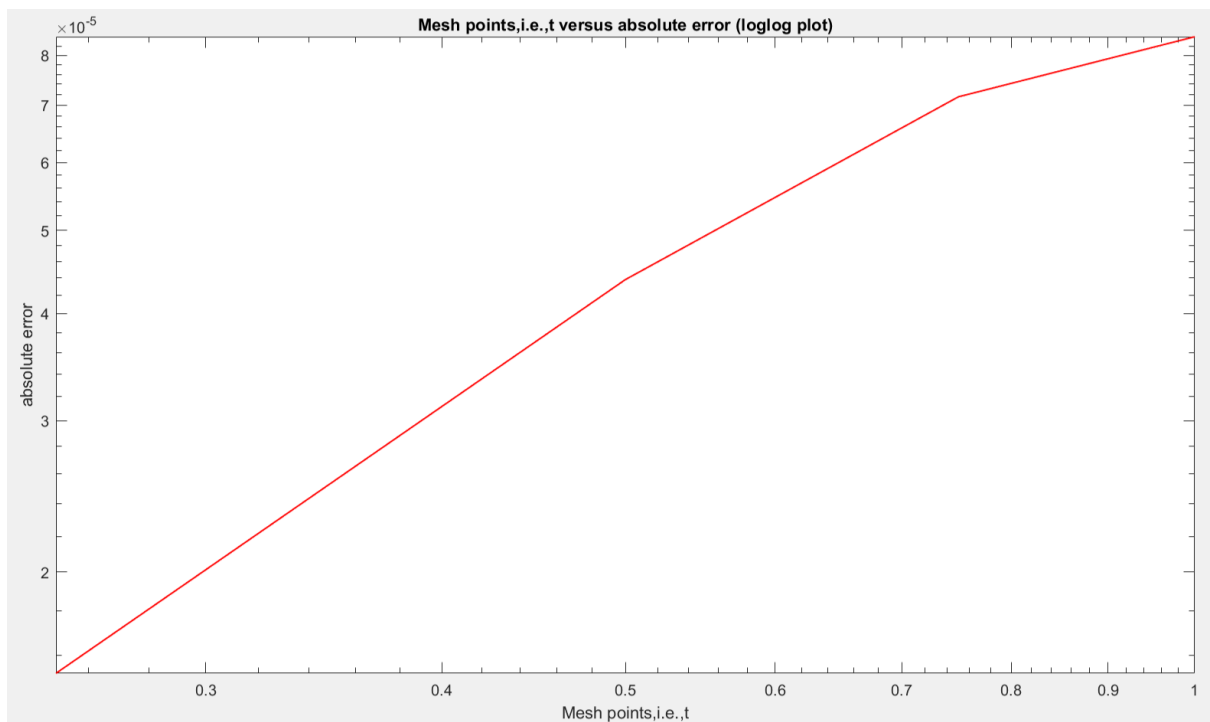
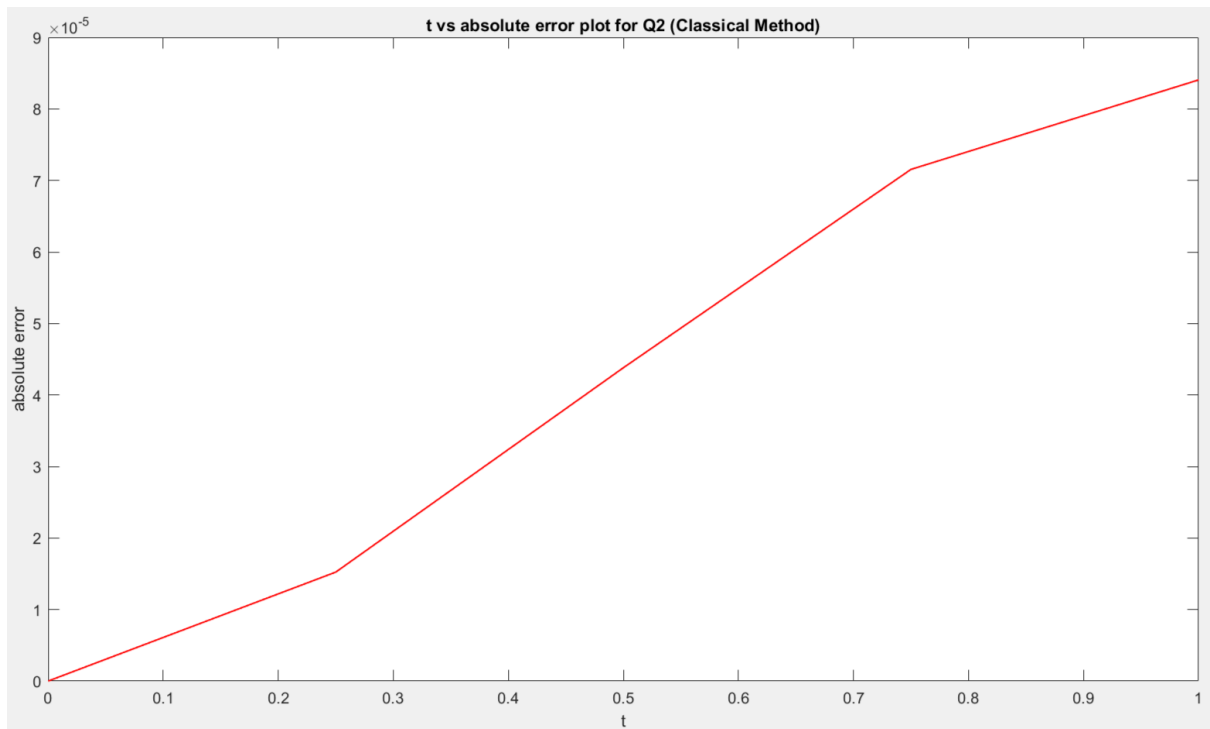
- $c_2 = 1/2, c_3 = 1/2, c_4 = 1$ (Classical Method)
- $c_2 = 1/3, c_3 = 2/3, c_4 = 1$ (Kutta Method)

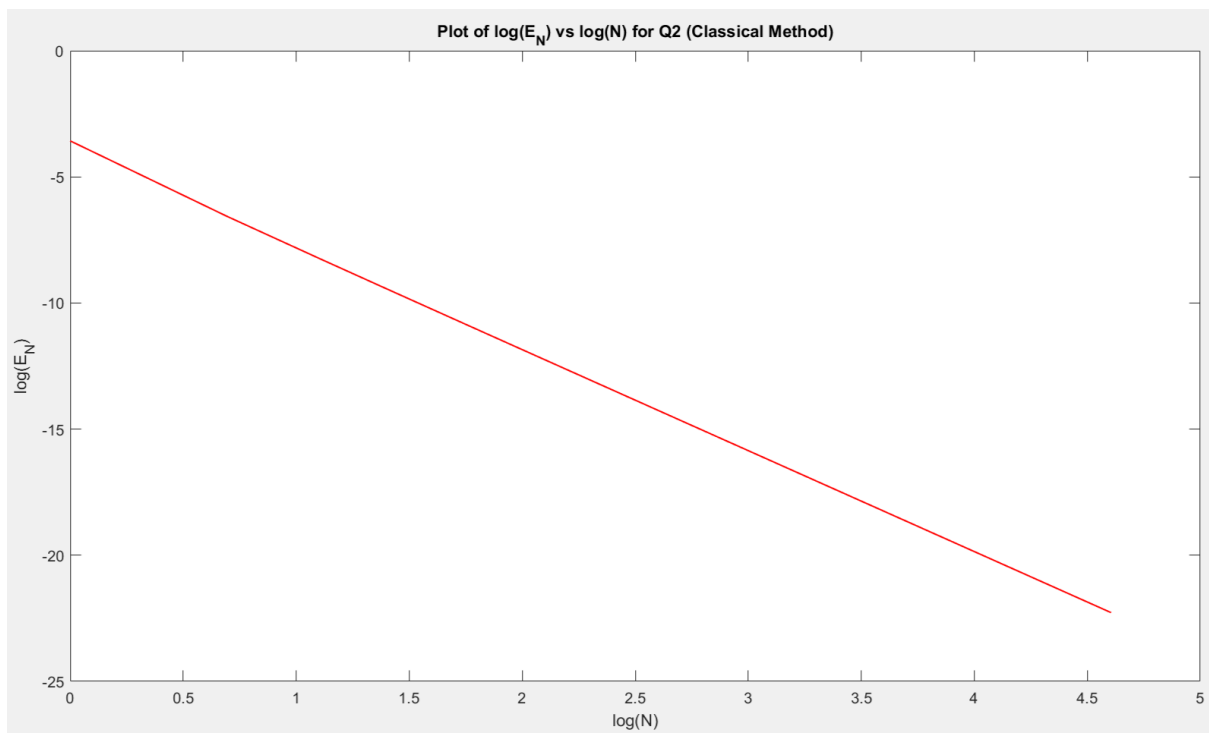
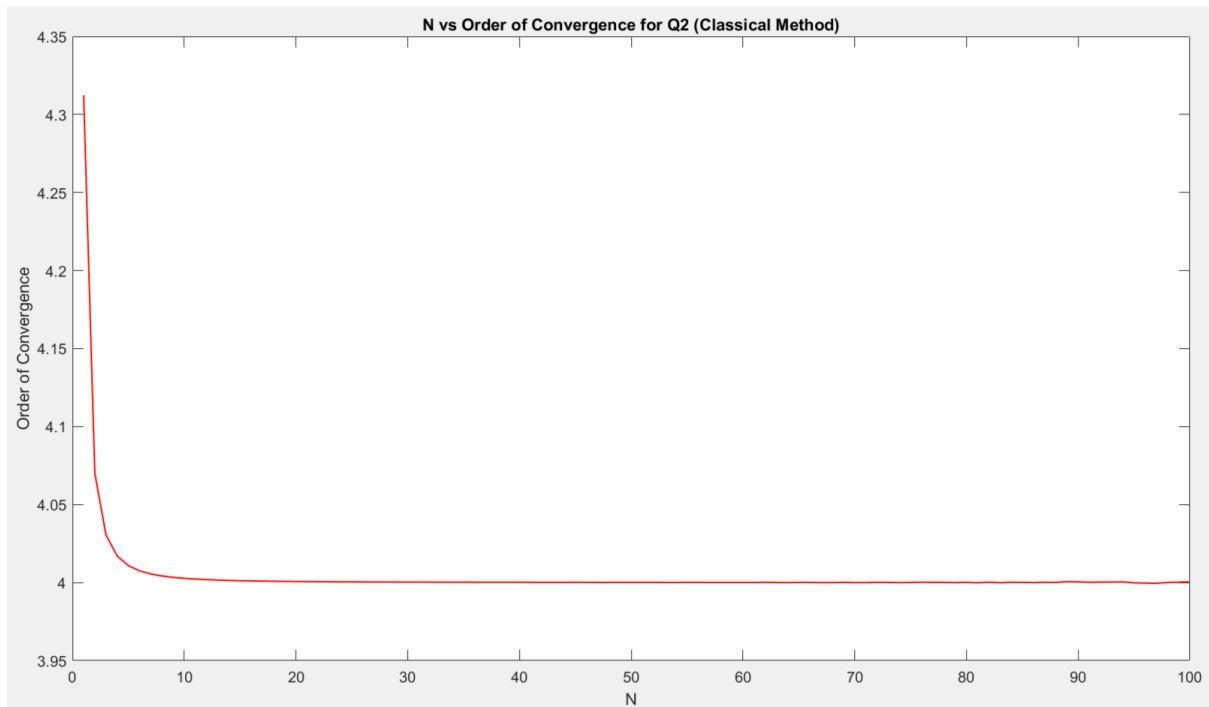
Question 2

Using Fourth-order Runge-Kutta Method for Q2 (Classical Method)

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.250000	1.329165	1.329150	0.000015
0.500000	1.730534	1.730490	0.000044
0.750000	2.041544	2.041472	0.000072
1.000000	2.118064	2.117980	0.000084

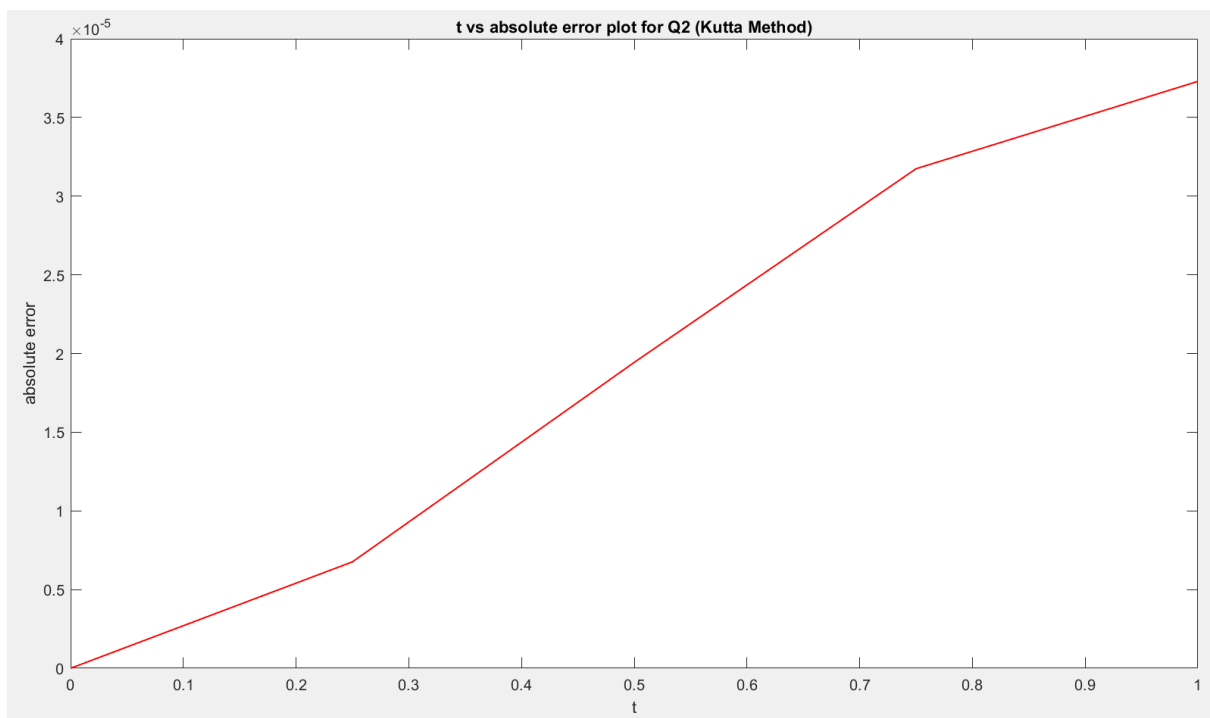
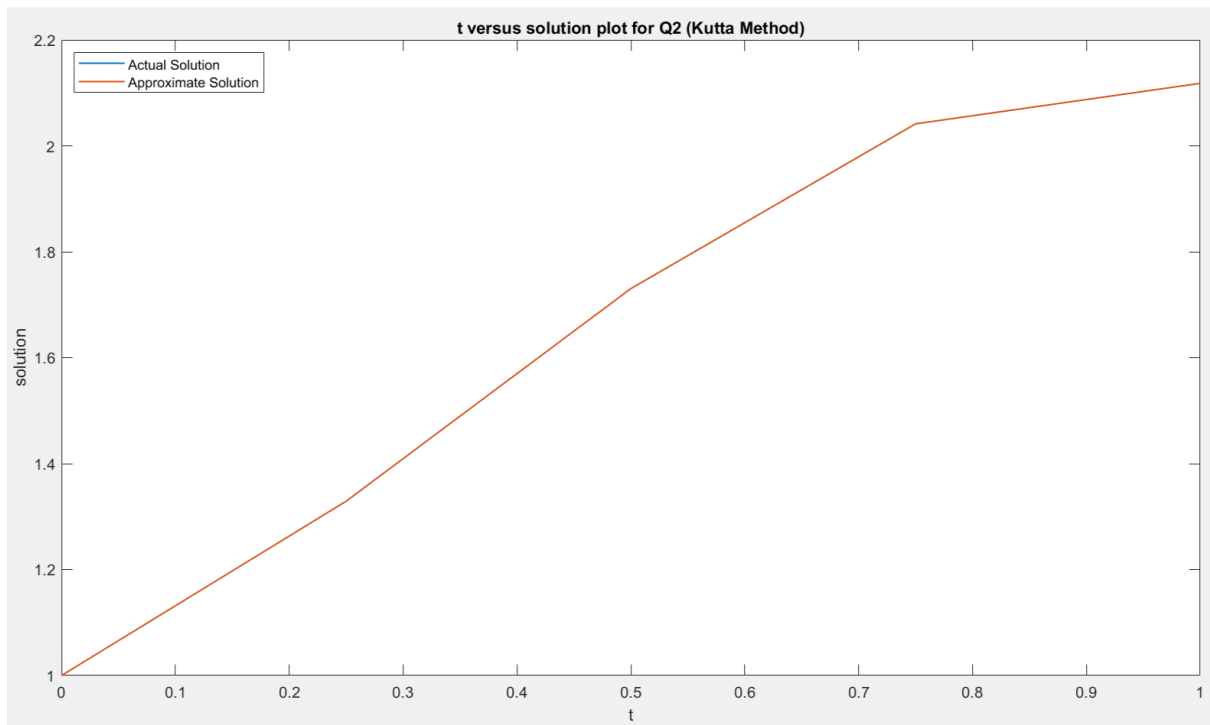


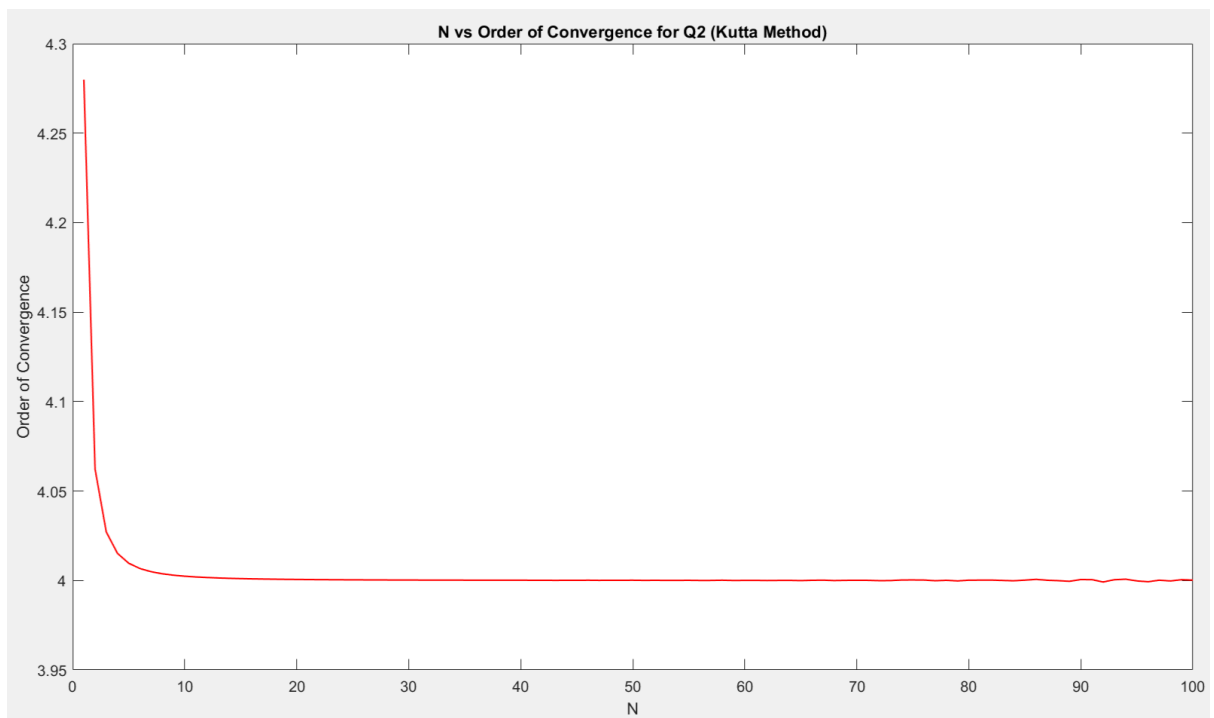
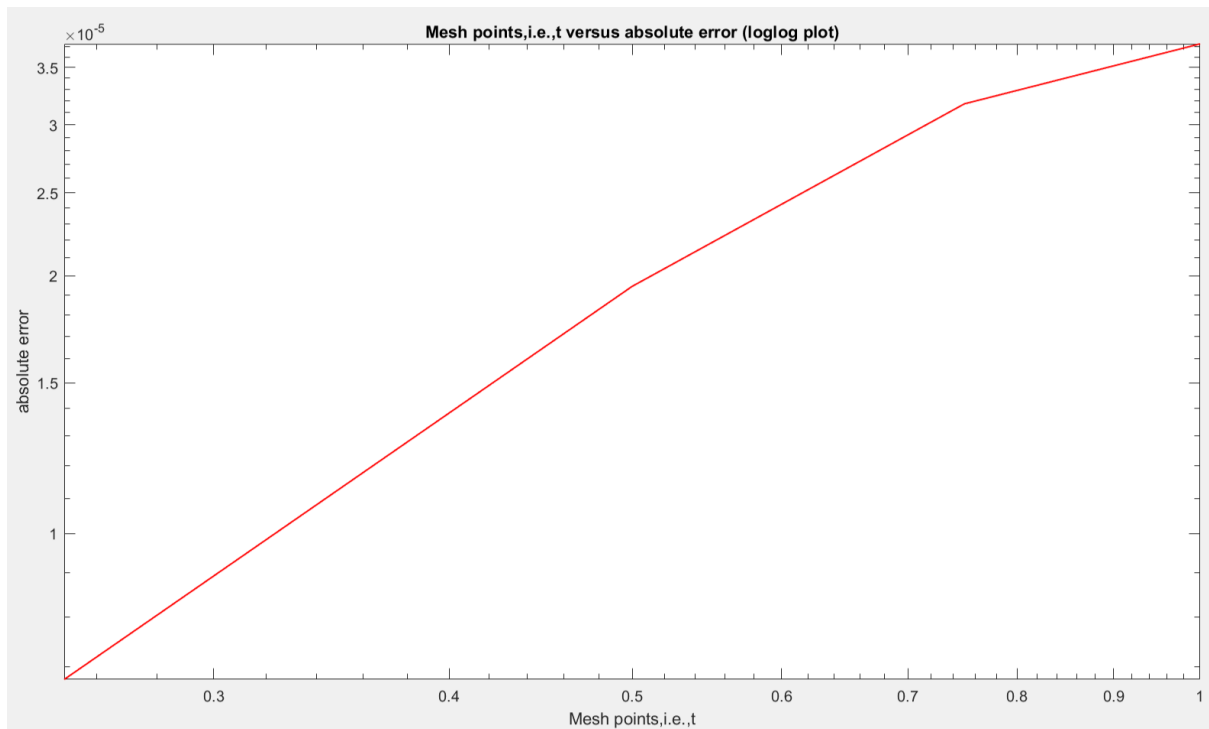


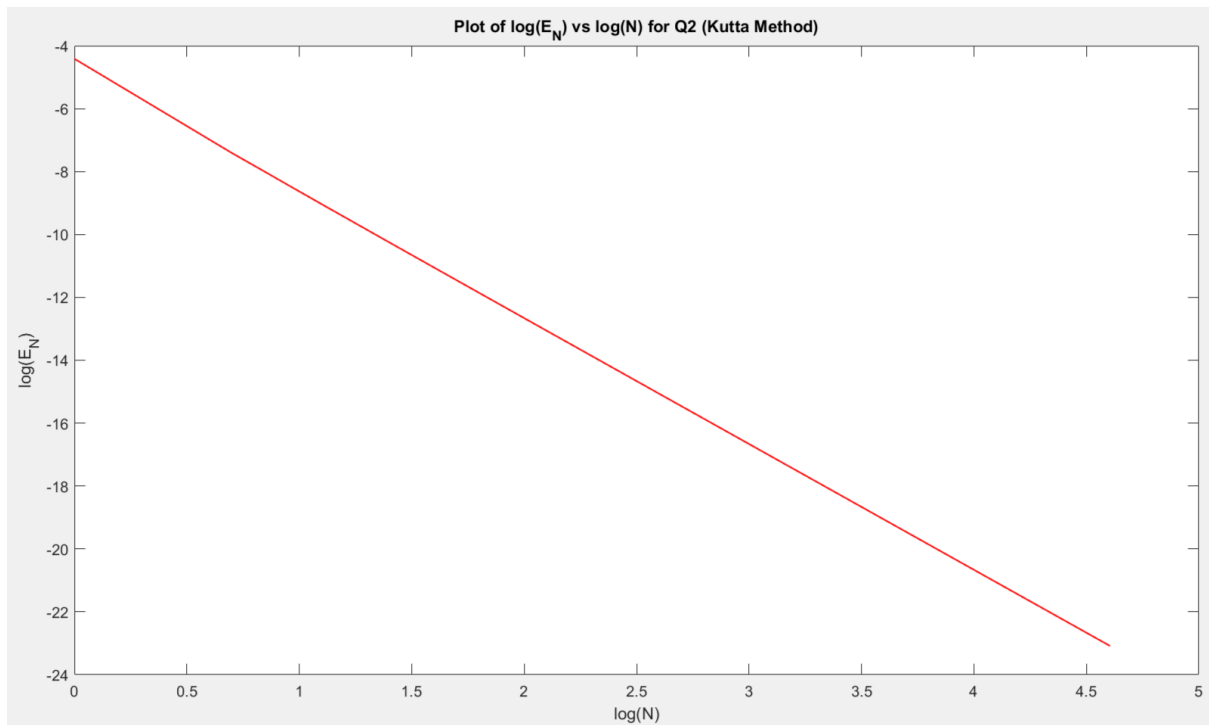


Using Fourth-order Runge-Kutta Method for Q2 (Kutta Method)

t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.250000	1.329157	1.329150	0.000007
0.500000	1.730509	1.730490	0.000019
0.750000	2.041504	2.041472	0.000032
1.000000	2.118017	2.117980	0.000037







Both Classical and Kutta methods exhibit similar performance, with little discernible difference between them. Kutta method is giving slightly better estimates.