Scientific Computing Lab **MA – 322** **Lab – 2**

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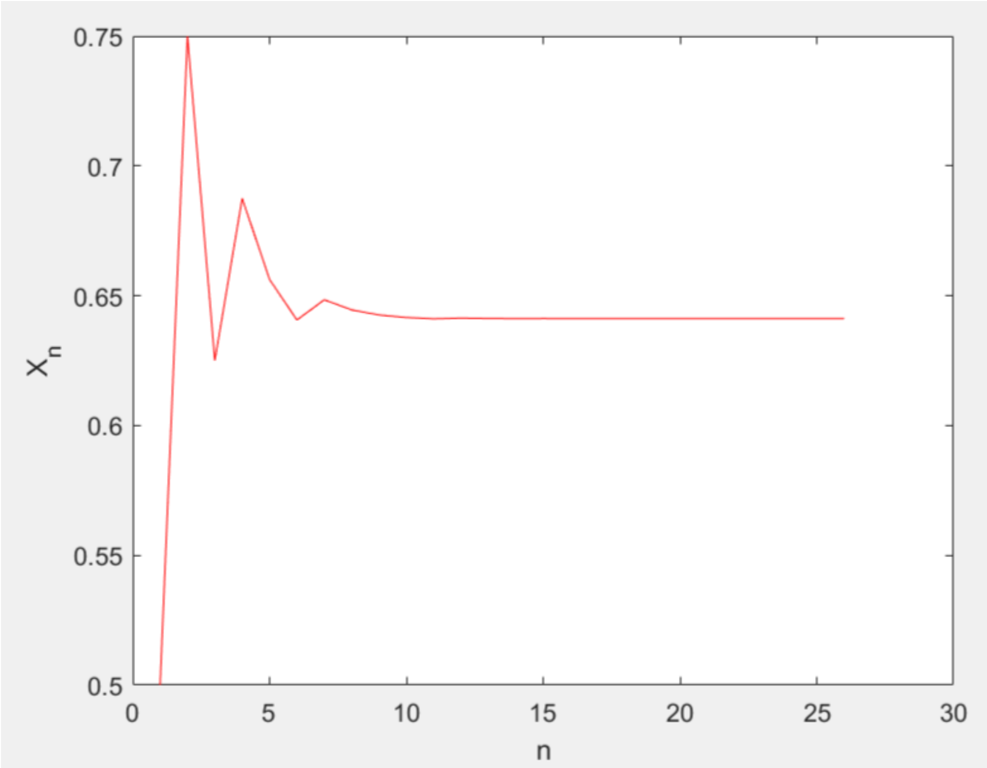
Branch – Mathematics and Computing

1)

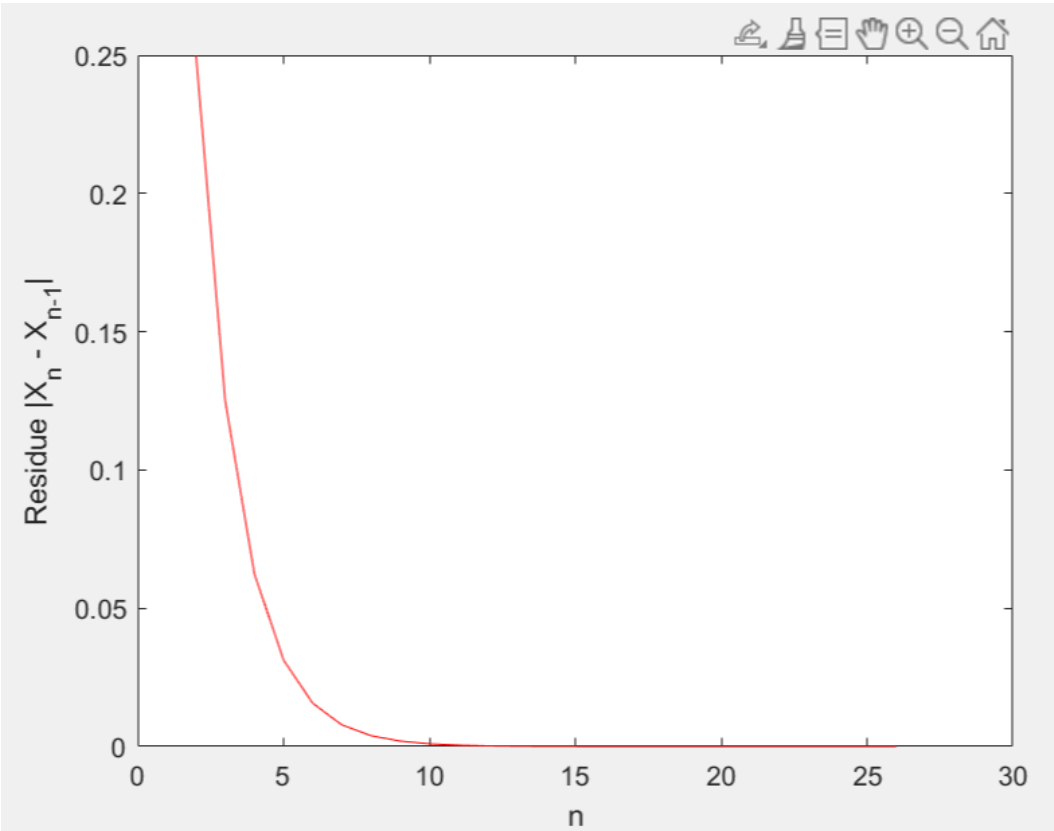
```
Bisection Method for Q1
Root found after 26 iterations.
Root is: 0.64119
```

No. of iterations	Approximate solution	Error $|X_n - X_{n-1}|$
1	0.500000000000000	--
2	0.750000000000000	0.250000000000000
3	0.625000000000000	0.125000000000000
4	0.687500000000000	0.062500000000000
5	0.656250000000000	0.031250000000000
6	0.640625000000000	0.015625000000000
7	0.648437500000000	0.007812500000000
8	0.644531250000000	0.003906250000000
9	0.642578125000000	0.001953125000000
10	0.641601562500000	0.000976562500000
11	0.641113281250000	0.000488281250000
12	0.641357421875000	0.000244140625000
13	0.641235351562500	0.000122070312500
14	0.641174316406250	0.000061035156250
15	0.641204833984375	0.000030517578125
16	0.641189575195312	0.000015258789062
17	0.641181945800781	0.000007629394531
18	0.641185760498047	0.000003814697266
19	0.641183853149414	0.000001907348633
20	0.641184806823730	0.000000953674316
21	0.641185283660889	0.000000476837158
22	0.641185522079468	0.000000238418579
23	0.641185641288757	0.000000119209290
24	0.641185700893402	0.000000059604645
25	0.641185730695724	0.000000029802322
26	0.641185745596886	0.000000014901161

Solution Plot:



N versus Residue Plot

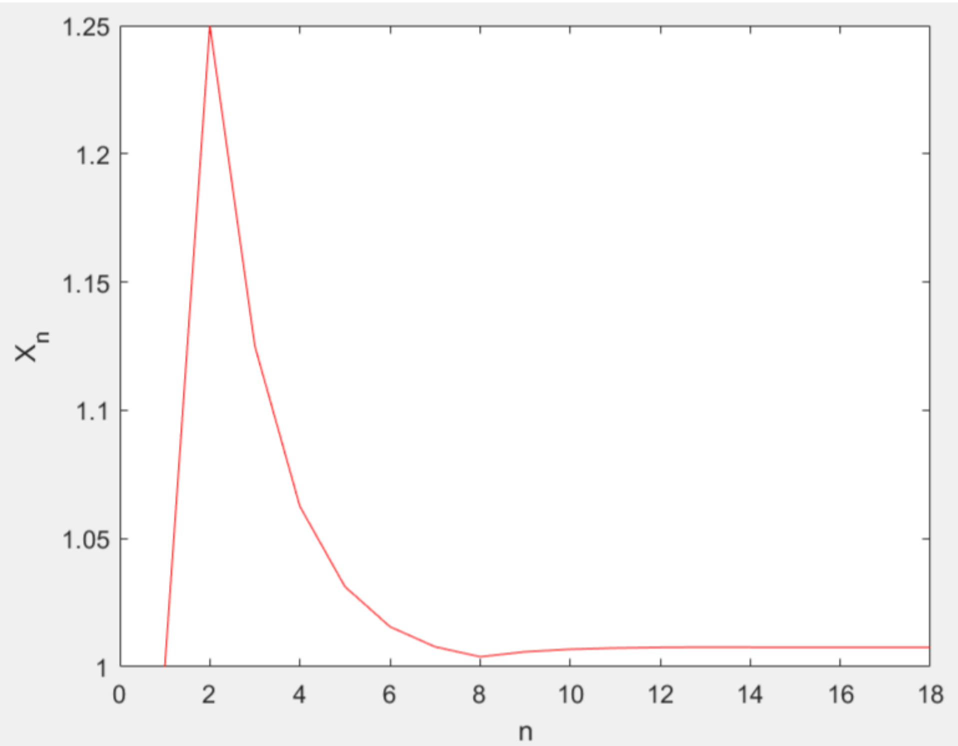


2)

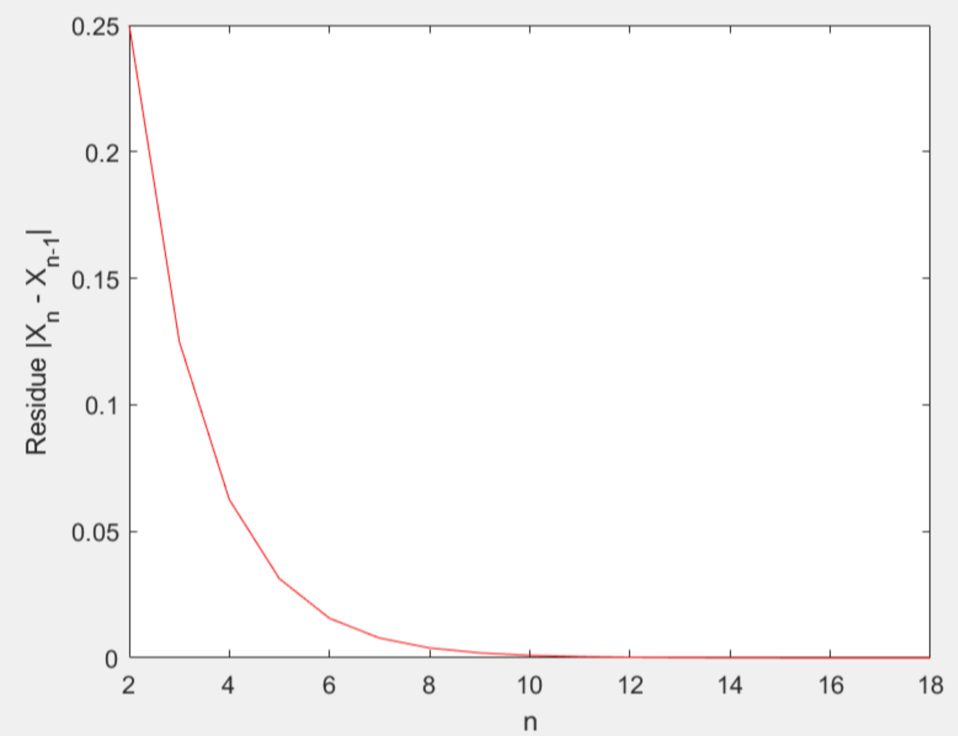
a)

```
Bisection Method for Q2 part a
Root found after 18 iterations.
Root is: 1.0076
No. of iterations        Approximate solution        Error |X_n - X_n-1|
    1                    1.000000000000000           --
    2                    1.250000000000000           0.250000000000000
    3                    1.125000000000000           0.125000000000000
    4                    1.062500000000000           0.062500000000000
    5                    1.031250000000000           0.031250000000000
    6                    1.015625000000000           0.015625000000000
    7                    1.007812500000000           0.007812500000000
    8                    1.003906250000000           0.003906250000000
    9                    1.005859375000000           0.001953125000000
    10                   1.006835937500000           0.000976562500000
    11                   1.007324218750000           0.000488281250000
    12                   1.007568359375000           0.000244140625000
    13                   1.007690429687500           0.000122070312500
    14                   1.007629394531250           0.000061035156250
    15                   1.007598876953125           0.000030517578125
    16                   1.007614135742188           0.000015258789062
    17                   1.007621765136719           0.000007629394531
    18                   1.007625579833984           0.000003814697266
```

Solution Plot



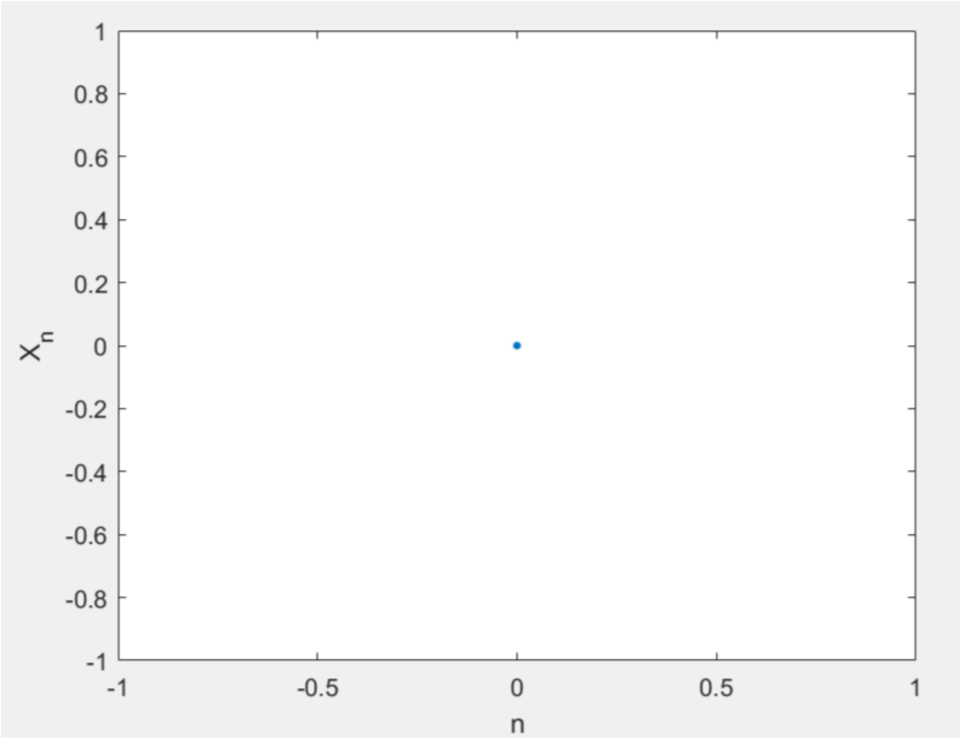
N versus Residue Plot



b)

```
Bisection Method for Q2 part b
Root found after 0 iterations.
Root is: 0
Staring point, a, is a root
No. of iterations        Approximate solution        Error |X_n - X_n-1|
    0                    0.000000000000000           --
```

Solution Plot

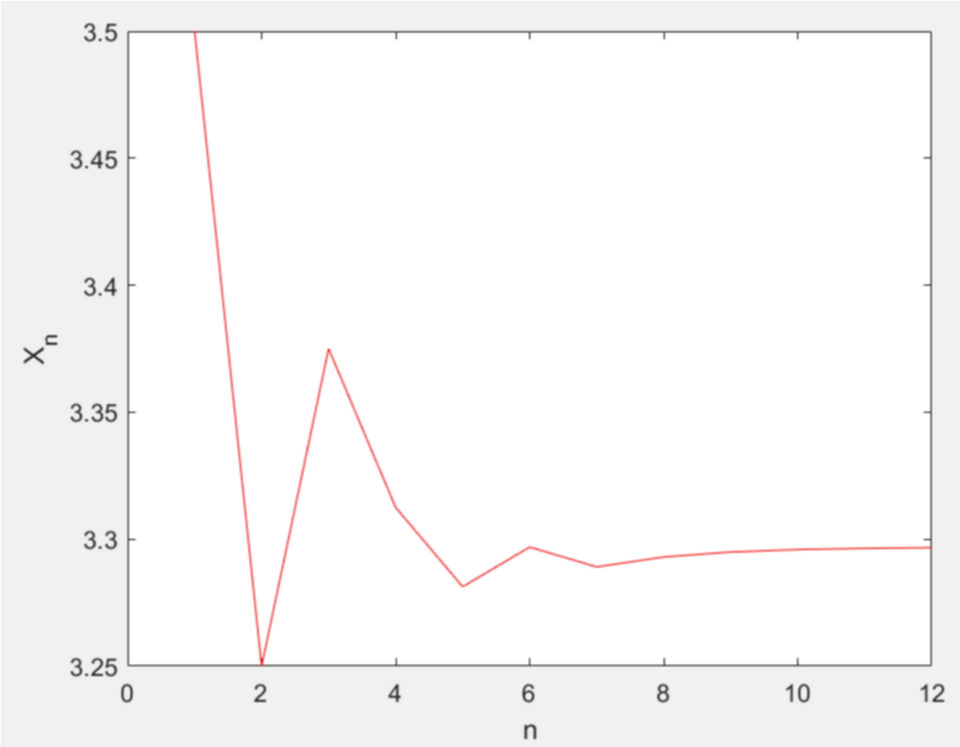


There is no Residue since we found the root at the starting point only, so only Solution Plot is plotted.

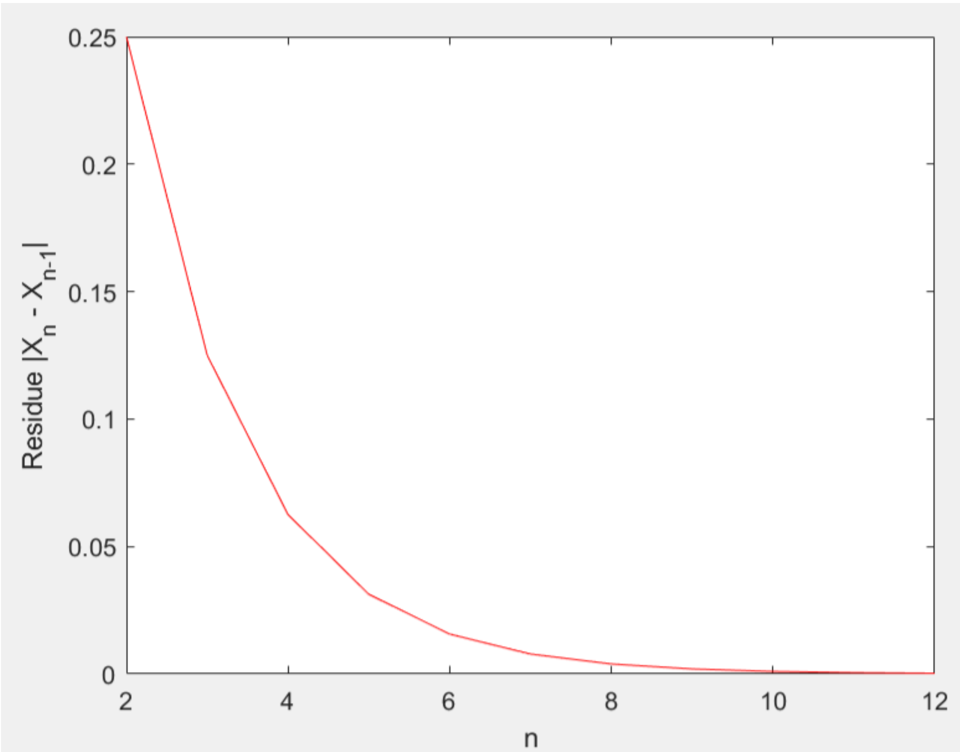
c)

```
Bisection Method for Q2 part c
Root found after 12 iterations.
Root is: 3.2966
No. of iterations        Approximate solution        Error |X_n - X_n-1|
    1                    3.500000000000000           --
    2                    3.250000000000000           0.250000000000000
    3                    3.375000000000000           0.125000000000000
    4                    3.312500000000000           0.062500000000000
    5                    3.281250000000000           0.031250000000000
    6                    3.296875000000000           0.015625000000000
    7                    3.289062500000000           0.007812500000000
    8                    3.292968750000000           0.003906250000000
    9                    3.294921875000000           0.001953125000000
    10                   3.295898437500000           0.000976562500000
    11                   3.296386718750000           0.000488281250000
    12                   3.296630859375000           0.000244140625000
```

Solution Plot



N versus Residue Plot



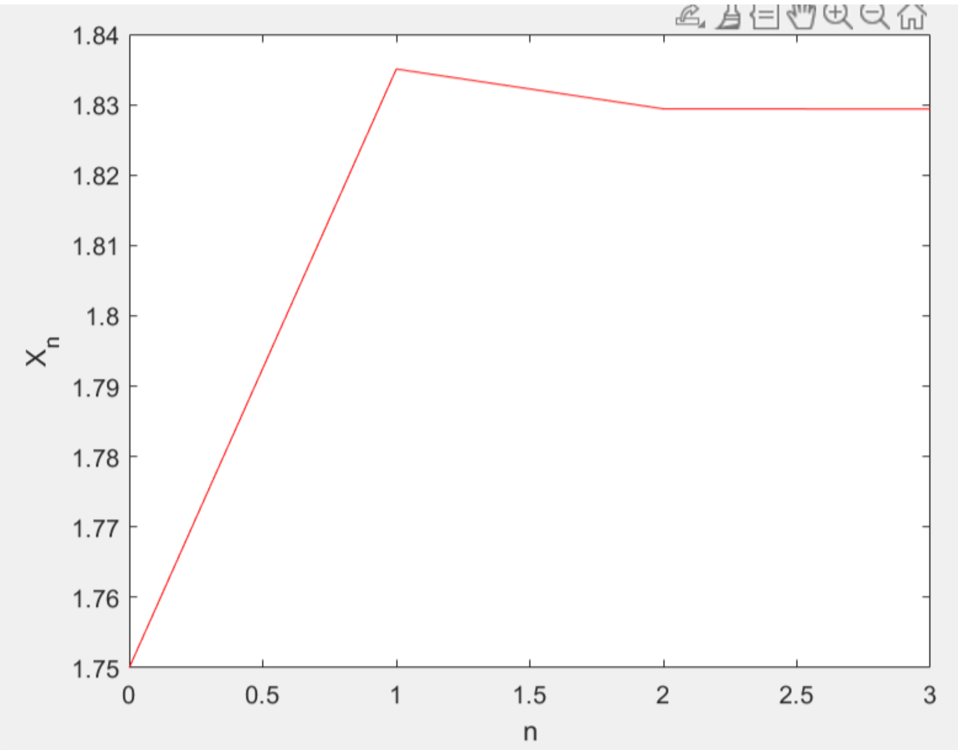
3)

a)

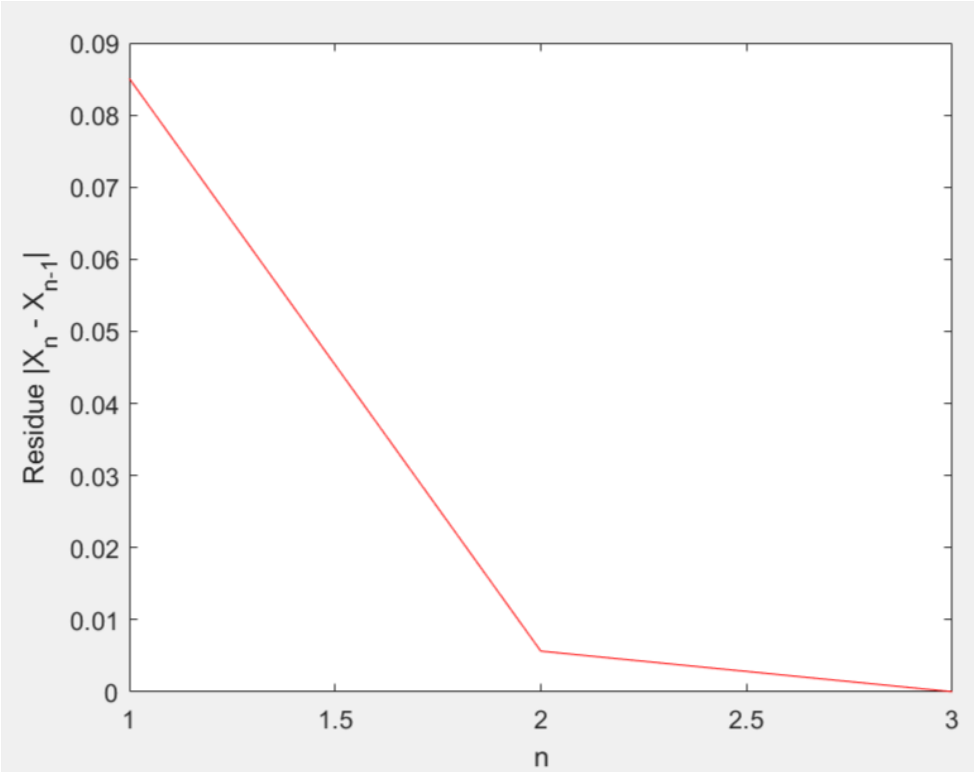
Initial Approximation $x_0 = 1.75$

```
Newton Method for Q3 part a
Root found after 3 iterations.
Root is: 1.8294
No. of iterations          Approximate solution          Error |X_n - X_n-1|
     0                     1.750000000000000             --
     1                     1.835067118239919             0.085067118239919
     2                     1.829410544363042             0.005656573876877
     3                     1.829383602542360             0.000026941820682
```

Solution Plot



N versus Residue Plot

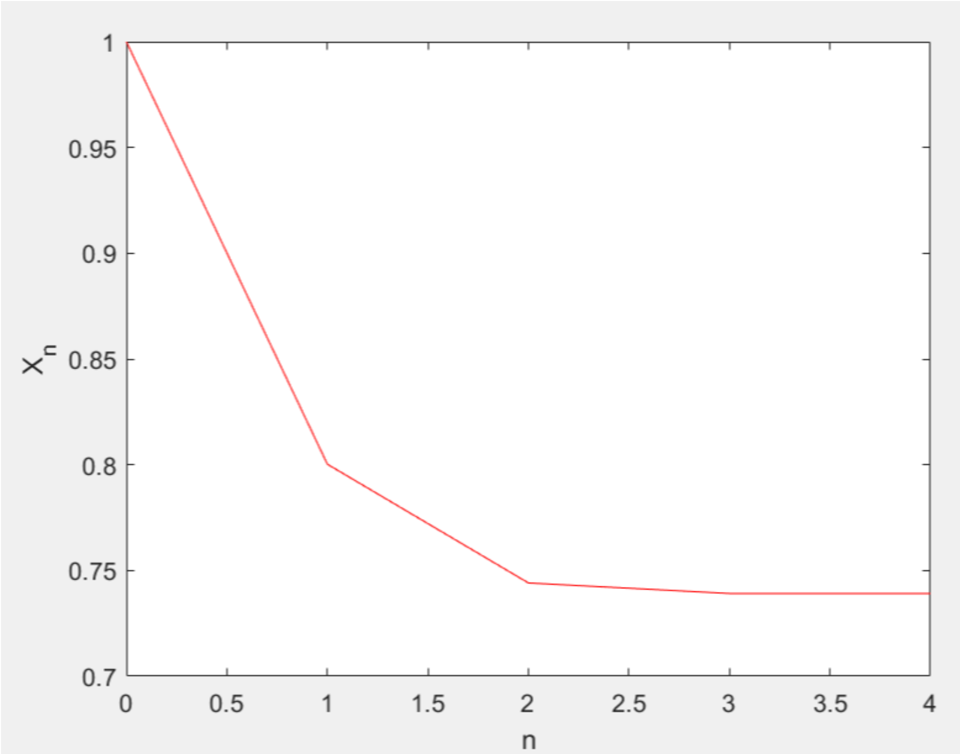


b)

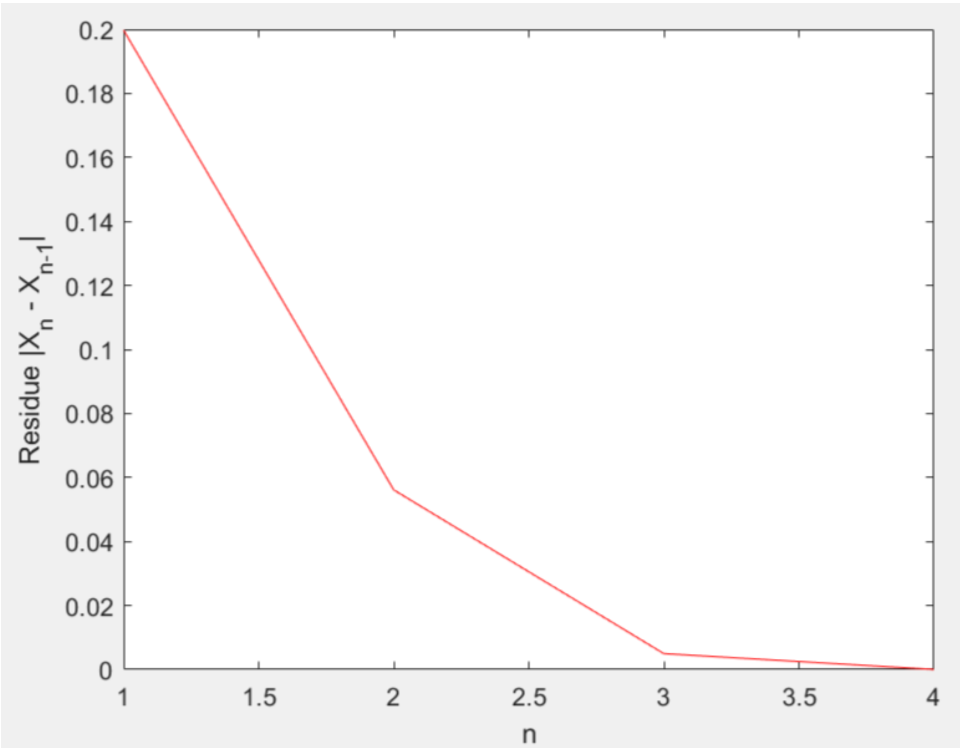
Initial Approximation $x_0 = 1$

```
Newton Method for Q3 part b
Root found after 4 iterations.
Root is: 0.73909
No. of iterations        Approximate solution         Error |X_n - X_n-1|
    0                    1.000000000000000            --
    1                    0.800232943226195            0.199767056773805
    2                    0.744094398494345            0.056138544731850
    3                    0.739124068356762            0.004970330137582
    4                    0.739085135600735            0.000038932756027
```

Solution Plot



N versus Residue Plot

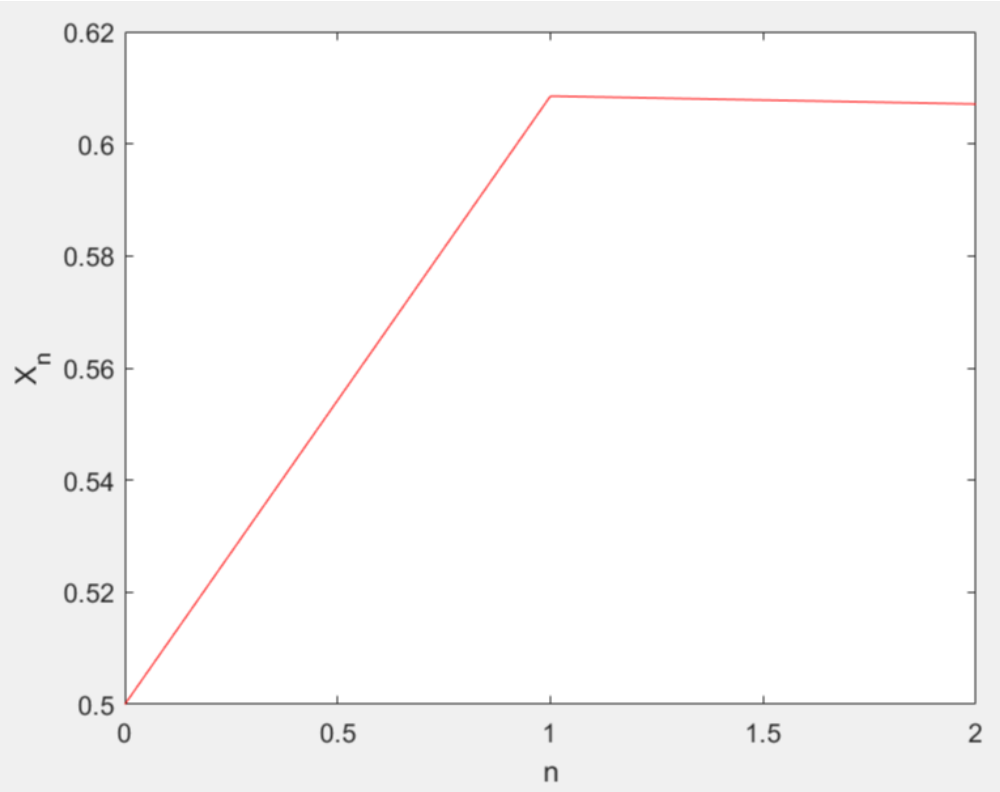


c)

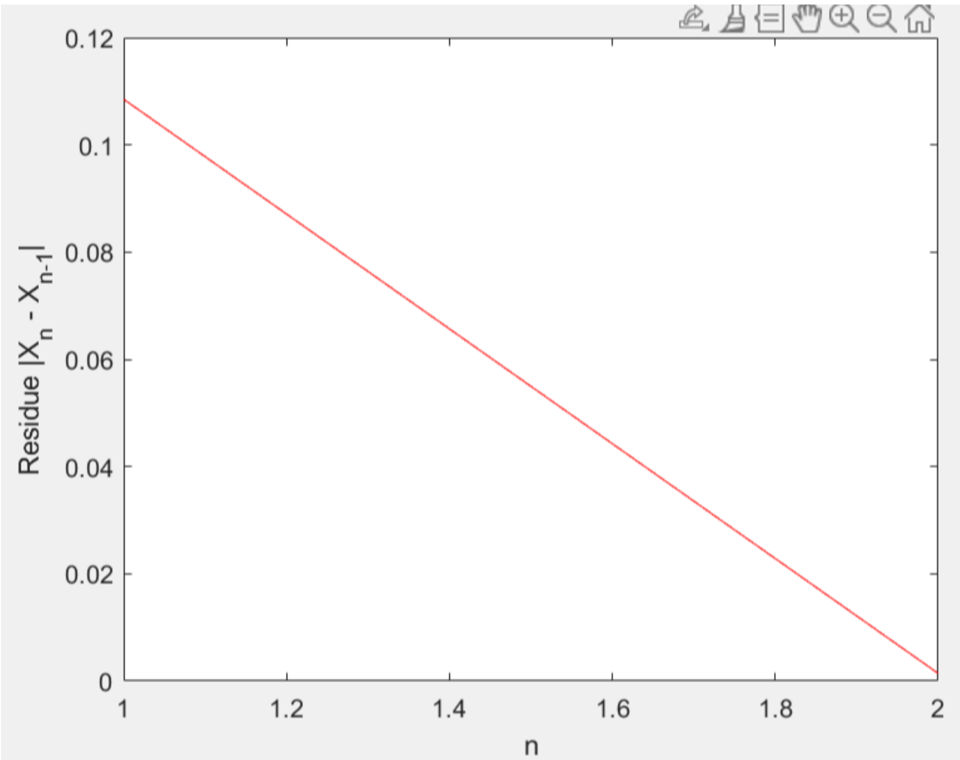
Initial Approximation $x_0 = 0.5$

```
Newton Method for Q3 part c
Root found after 2 iterations.
Root is: 0.6071
No. of iterations         Approximate solution         Error |X_n - X_n-1|
     0                    0.500000000000000            --
     1                    0.608518649903870            0.108518649903870
     2                    0.607101878810108            0.001416771093762
```

Solution Plot



N versus Residue Plot

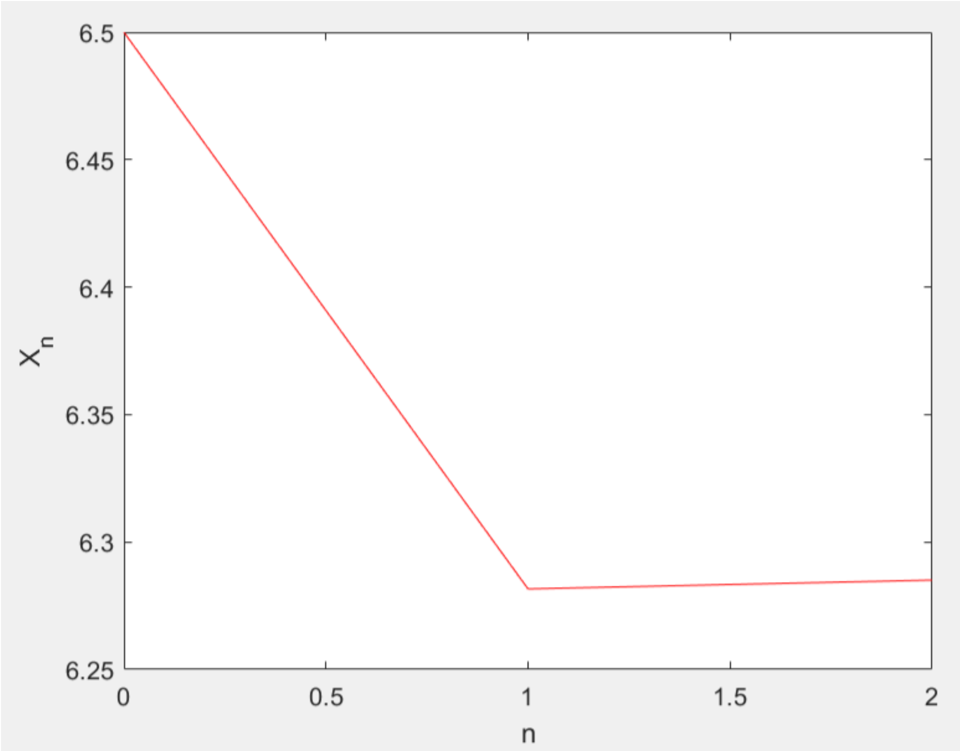


d)

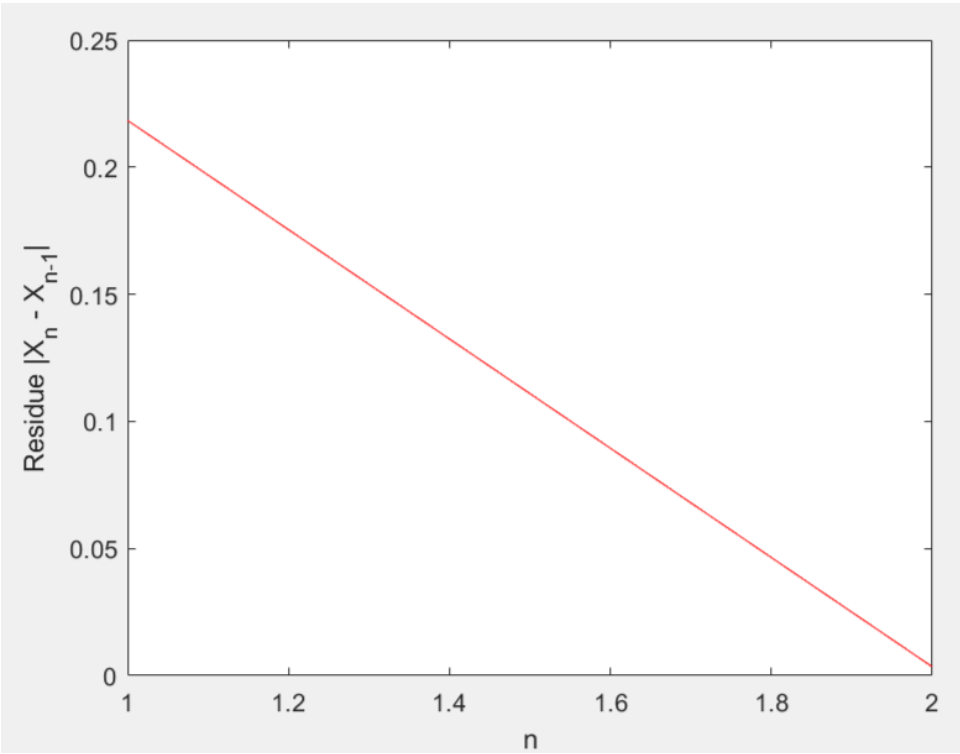
Initial Approximation $x_0 = 6.5$

```
Newton Method for Q3 part d
Root found after 2 iterations.
Root is: 6.285
No. of iterations       Approximate solution        Error |X_n - X_n-1|
    0                   6.500000000000000           --
    1                   6.281598506973284           0.218401493026716
    2                   6.285049264874215           0.003450757900931
```

Solution Plot



N versus Residue Plot

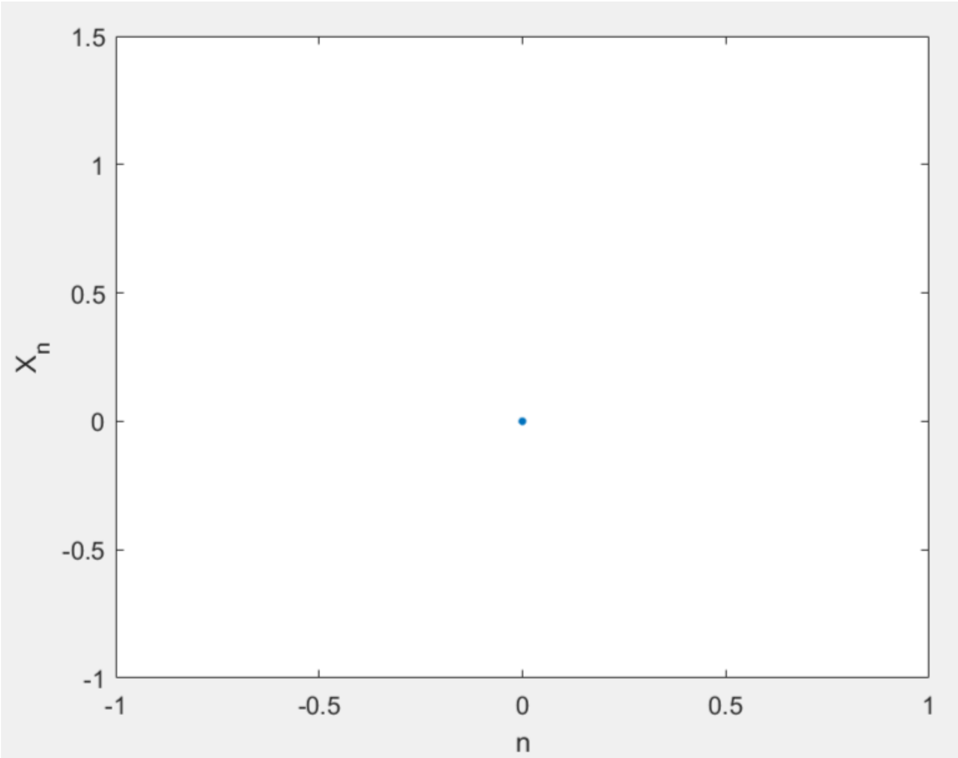


4)

Initial Approximation $x_0 = 0.0001$

```
Newton Method for Q4
No iterations were required, the root is :0.0001
No. of iterations        Approximate solution        Error |X_n - X_n-1|
    0                    0.000100000000000           --
```

Solution Plot



There is no Residue since we found the root at the starting point only, so only Solution Plot is plotted.

The given point $x_0 = 0.0001$ is a very small input to the function $f(x) = \exp(-1 / x^2)$. Thus, the value of the function at x_0 is equal to 0, due to computational limits, and hence, there are no more iterations.

At $x_0 = 0.0001$, which is our initial approximation, the function f(x) takes the value of $\exp(-10^8)$, which is an extremely small number, much smaller than the computer's value of epsilon (the smallest positive number), and thus evaluates to 0. Since this initial value of $f(x_0)$ is evaluated as 0 and hence, less than the tolerance value, so the initial approximation is printed as the final answer as the required root.

It is not possible to go below 0.00005 starting from $x_0 = 0.0001$ as the value of f(x) becomes even smaller than $\exp(-10^8)$, which is evaluated as 0 and hence, the iterations stop. At any $x_0 < 0.0001$ also, the function converges to 0 at that point x_0 itself. Hence, we won't make any progress even if we start with $x_0 = 0.00005$. We cannot go below 0.00005

5)

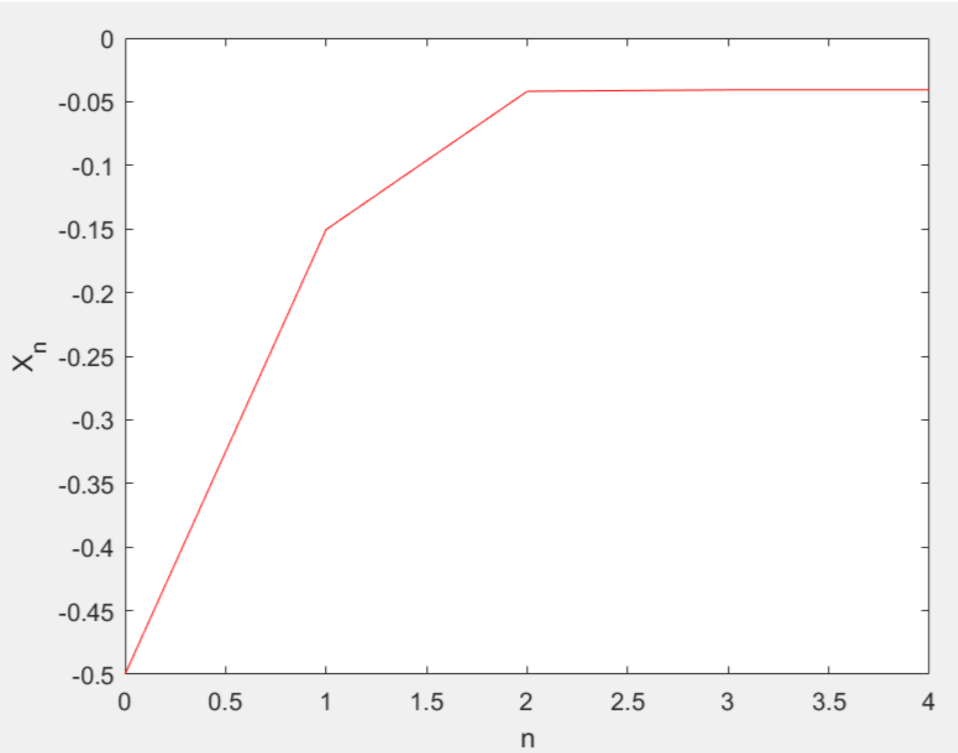
a) Using Newton's Method:

(i) For the real zero in [-1, 0]

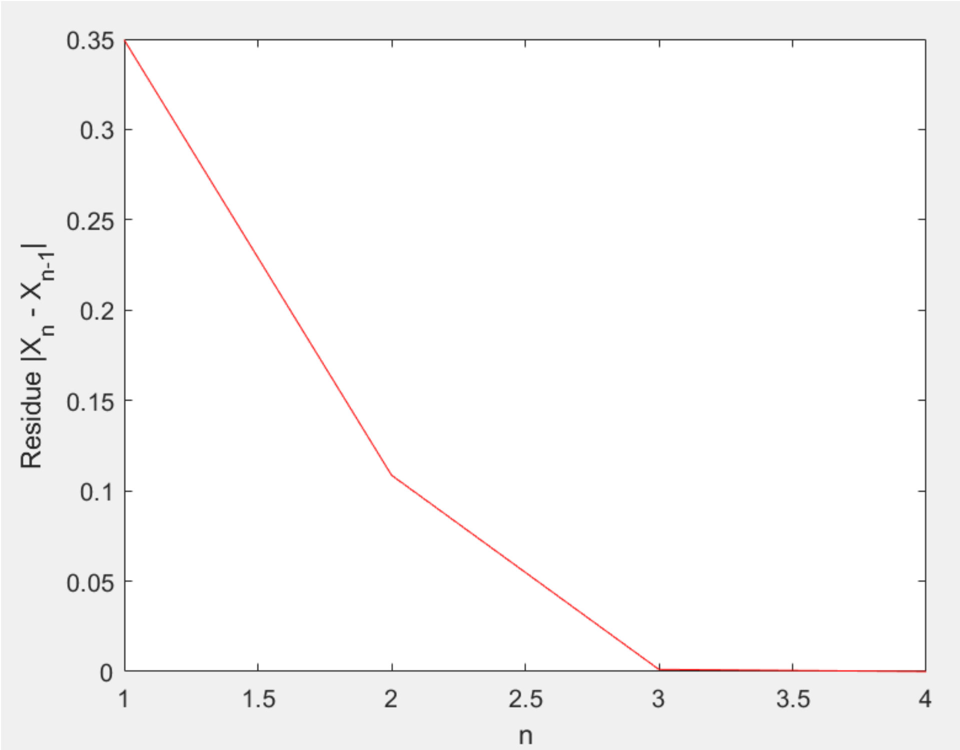
Initial Approximation x_0 = -0.5

```
Newton Method for Q5 for the root between -1 and 0
Root found after 4 iterations.
Root is: -0.040659
No. of iterations        Approximate solution        Error |X_n - X_n-1|
     0                   -0.500000000000000          --
     1                   -0.150452488687783          0.349547511312217
     2                   -0.041816813948870          0.108635674738912
     3                   -0.040659343497329          0.001157470451541
     4                   -0.040659288315759          0.000000055181570
```

Solution Plot



N versus Residue Plot

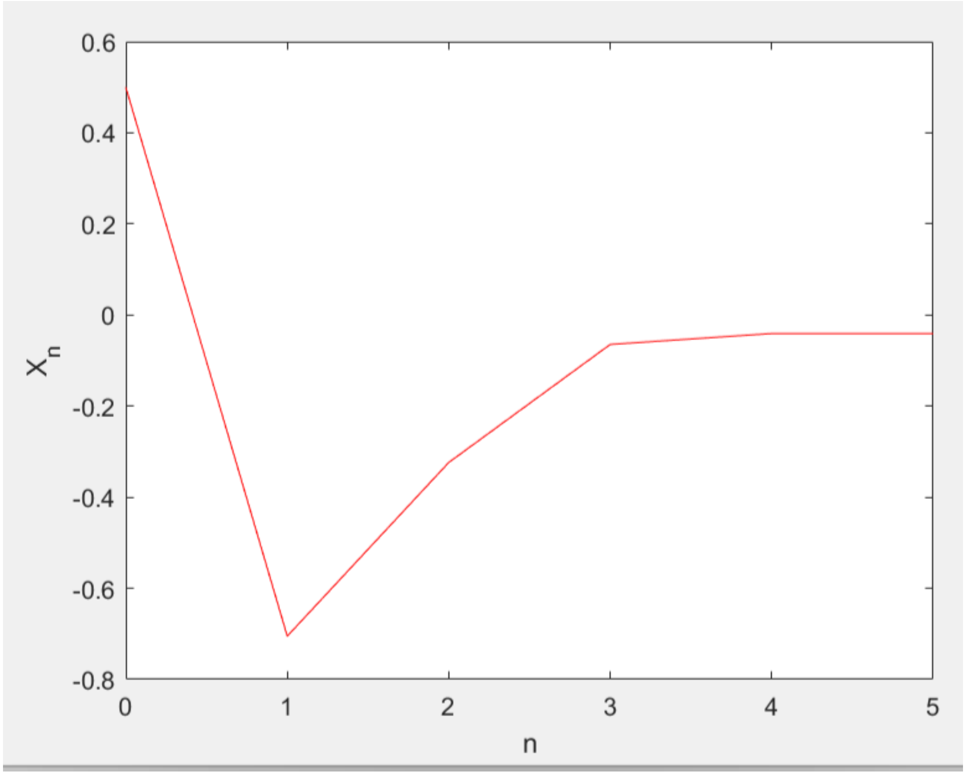


(ii) For the real zero in [0, 1]

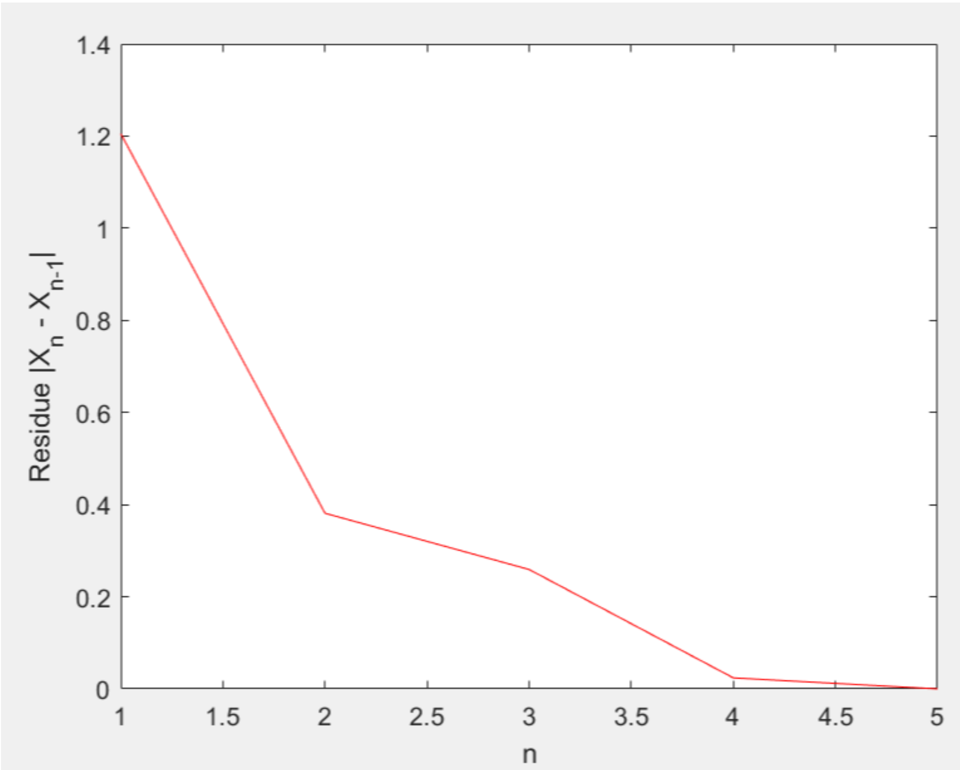
Initial Approximation $x_0 = 0.5$

```
Newton Method for Q5 for the root between 0 and 1
Root found after 5 iterations.
Root is: -0.040659
No. of iterations        Approximate solution        Error |X_n - X_n-1|
    0                    0.500000000000000           --
    1                    -0.705089820359281          1.205089820359281
    2                    -0.323791114230475          0.381298706128807
    3                    -0.064603131030575          0.259187983199900
    4                    -0.040686151151956          0.023916979878619
    5                    -0.040659288345335          0.000026862806621
```

Solution Plot



N versus Residue Plot



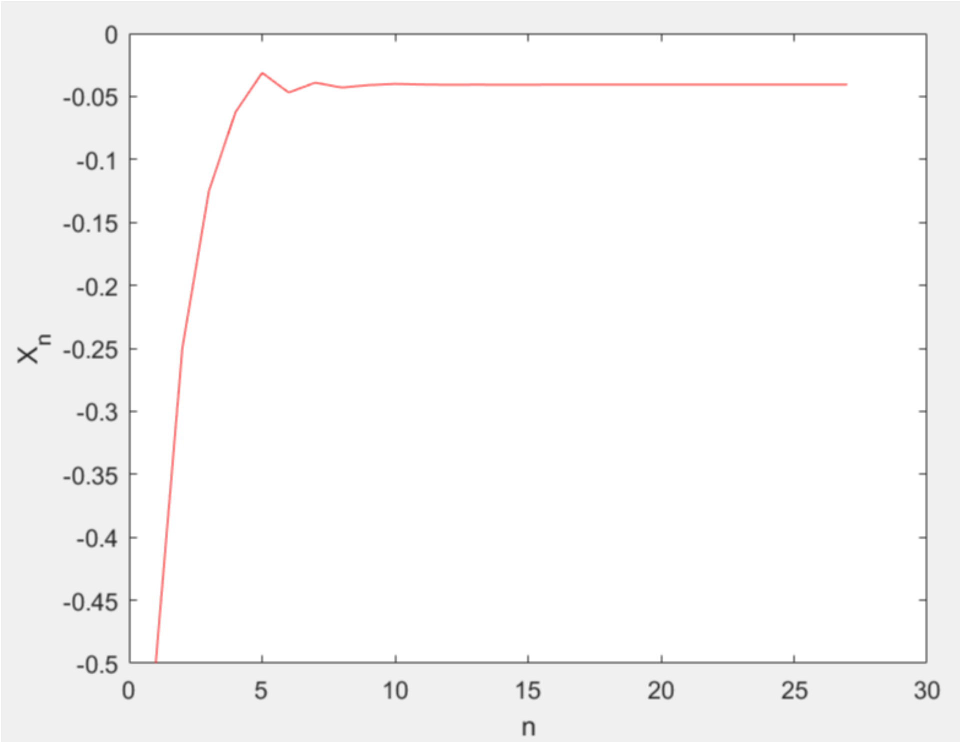
b) Using Bisection Method:

(i) For the real zero in [-1, 0]

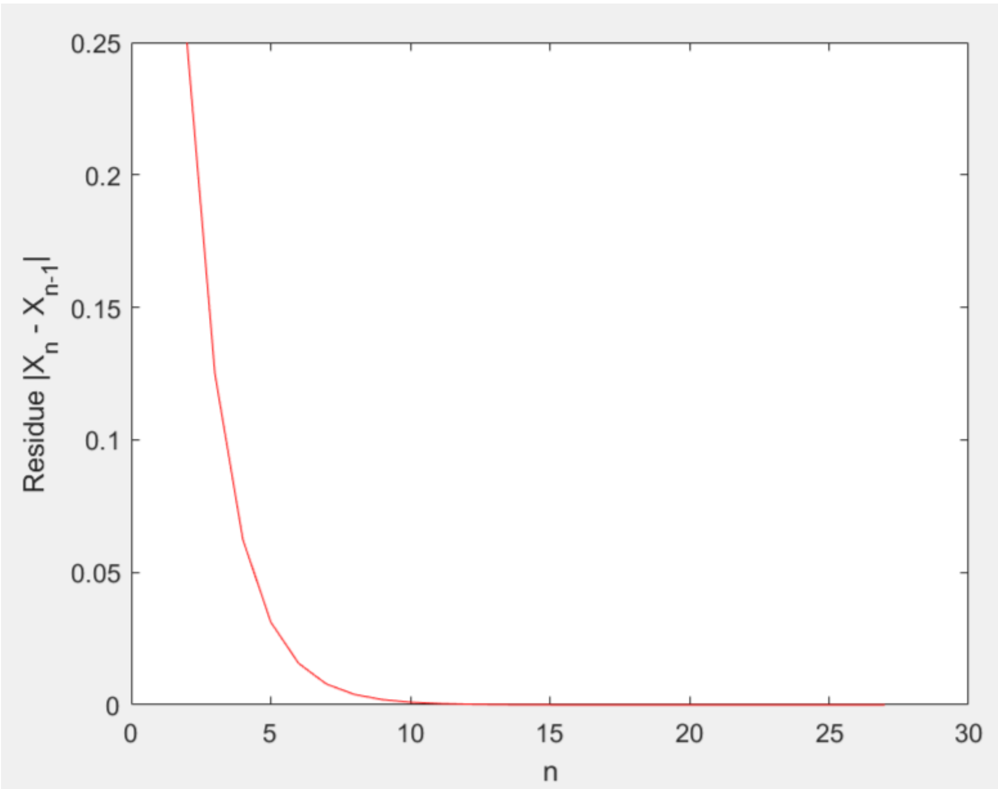
```
Bisection Method for Q5 for the root between -1 and 0
Root found after 27 iterations.
Root is: -0.040659
```

No. of iterations	Approximate solution	Error |X_n - X_n-1|
1	-0.500000000000000	--
2	-0.250000000000000	0.250000000000000
3	-0.125000000000000	0.125000000000000
4	-0.062500000000000	0.062500000000000
5	-0.031250000000000	0.031250000000000
6	-0.046875000000000	0.015625000000000
7	-0.039062500000000	0.007812500000000
8	-0.042968750000000	0.003906250000000
9	-0.041015625000000	0.001953125000000
10	-0.040039062500000	0.000976562500000
11	-0.040527343750000	0.000488281250000
12	-0.040771484375000	0.000244140625000
13	-0.040649414062500	0.000122070312500
14	-0.040710449218750	0.000061035156250
15	-0.040679931640625	0.000030517578125
16	-0.040664672851562	0.000015258789062
17	-0.040657043457031	0.000007629394531
18	-0.040660858154297	0.000003814697266
19	-0.040658950805664	0.000001907348633
20	-0.040659904479980	0.000000953674316
21	-0.040659427642822	0.000000476837158
22	-0.040659189224243	0.000000238418579
23	-0.040659308433533	0.000000119209290
24	-0.040659248828888	0.000000059604645
25	-0.040659278631210	0.000000029802322
26	-0.040659293532372	0.000000014901161
27	-0.040659286081791	0.000000007450581

Solution Plot



N versus Residue Plot

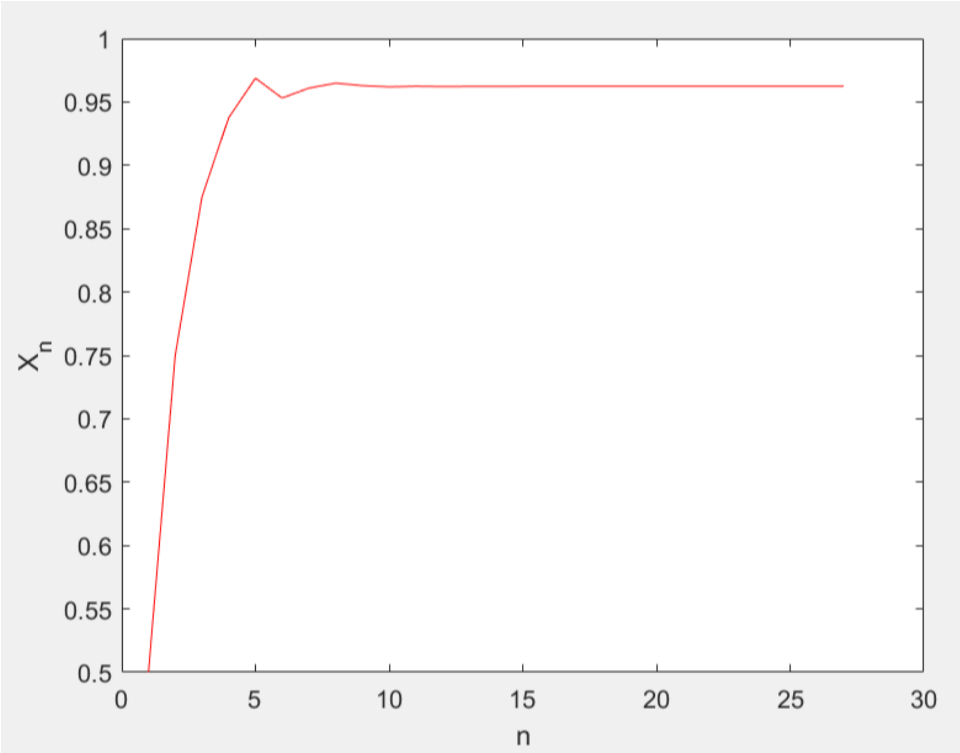


(ii) For the real zero in [0, 1]

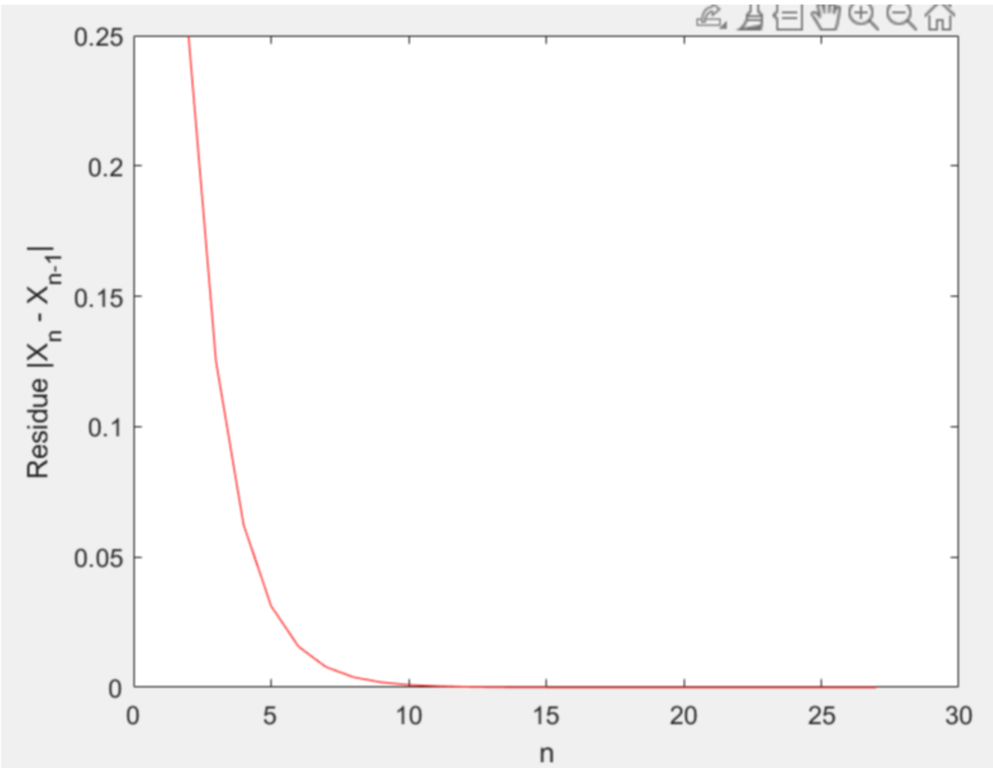
```
Bisection Method for Q5 for the root between 0 and 1
Root found after 27 iterations.
Root is: 0.9624
```

No. of iterations	Approximate solution	Error |X_n - X_n-1|
1	0.500000000000000	--
2	0.750000000000000	0.250000000000000
3	0.875000000000000	0.125000000000000
4	0.937500000000000	0.062500000000000
5	0.968750000000000	0.031250000000000
6	0.953125000000000	0.015625000000000
7	0.960937500000000	0.007812500000000
8	0.964843750000000	0.003906250000000
9	0.962890625000000	0.001953125000000
10	0.961914062500000	0.000976562500000
11	0.962402343750000	0.000488281250000
12	0.962158203125000	0.000244140625000
13	0.962280273437500	0.000122070312500
14	0.962341308593750	0.000061035156250
15	0.962371826171875	0.000030517578125
16	0.962387084960938	0.000015258789062
17	0.962394714355469	0.000007629394531
18	0.962398529052734	0.000003814697266
19	0.962396621704102	0.000001907348633
20	0.962397575378418	0.000000953674316
21	0.962398052215576	0.000000476837158
22	0.962398290634155	0.000000238418579
23	0.962398409843445	0.000000119209290
24	0.962398469448090	0.000000059604645
25	0.962398439645767	0.000000029802322
26	0.962398424744606	0.000000014901161
27	0.962398417294025	0.000000007450581

Solution Plot



N versus Residue Plot



Observations:

We observe that in Newton's method starting with the midpoints of the intervals as the initial approximations in [-1,0] as well as in [0,1], the sequence $\{X_n\}$ converges to the same root which is the negative root whose value is -0.04065929 approximately.

However, while using the Bisection method, we observe that starting in the interval [-1,0], the sequence $\{X_n\}$ converges to the negative root whose value is -0.04065929 approximately while starting in the interval [0,1], the sequence $\{X_n\}$ converges to the positive root whose value is 0.9623984 approximately.

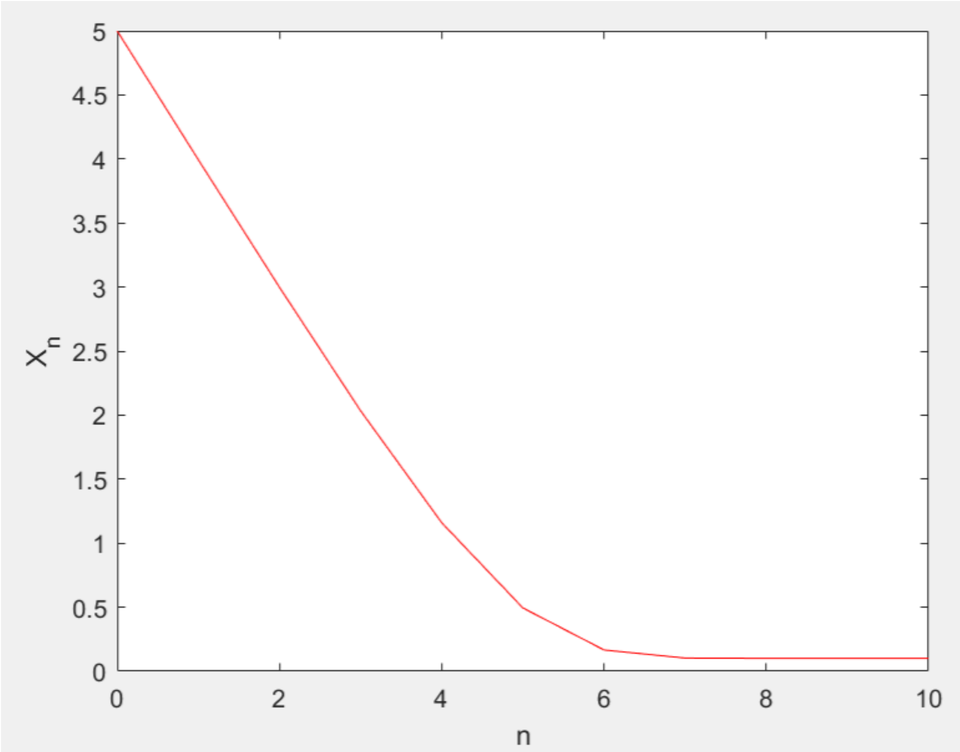
Since at x = 0.5, the value of f(x) is negative as well as the value of df(x)/dx, that is, f'(x), is also negative, so the iterates start to move towards the negative root in Newton's method. Hence, we get the negative root using Newton's method even if we start with the initial approximation of 0.5.

6)

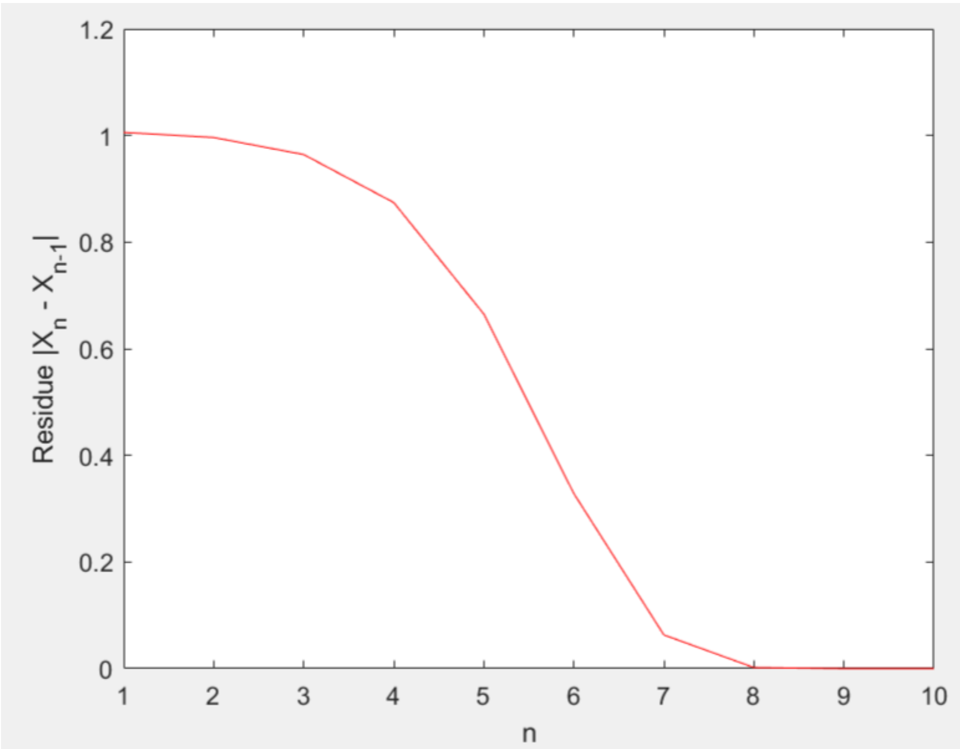
Initial Approximation $x_0 = 5$

```
Newton Method for Q6
Root found after 10 iterations.
Root is: 0.101
No. of iterations        Approximate solution        Error |X_n - X_n-1|
    0                    5.000000000000000           --
    1                    3.994242955993561           1.005757044006439
    2                    2.997988865033202           0.996254090960358
    3                    2.033725390018712           0.964263475014490
    4                    1.159536595423912           0.874188794594800
    5                    0.494389899481718           0.665146695942195
    6                    0.166098216755200           0.328291682726518
    7                    0.102958152950362           0.063140063804838
    8                    0.100999740427971           0.001958412522391
    9                    0.100997929687296           0.000001810740674
    10                   0.100997929685750           0.000000000001546
```

Solution Plot



N versus Residue Plot

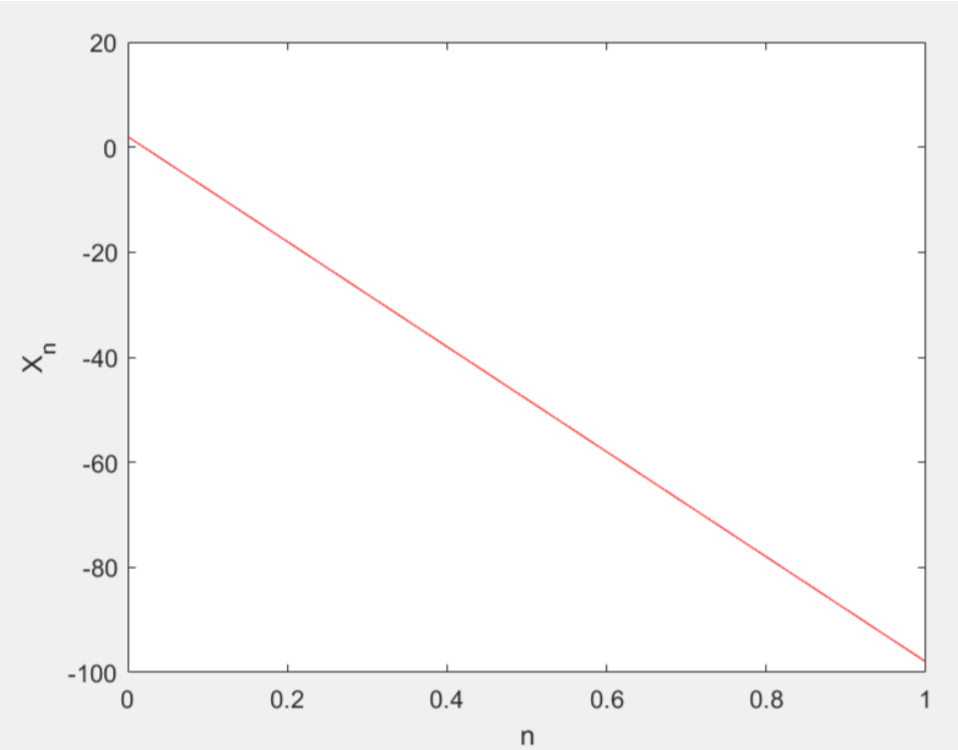


7)

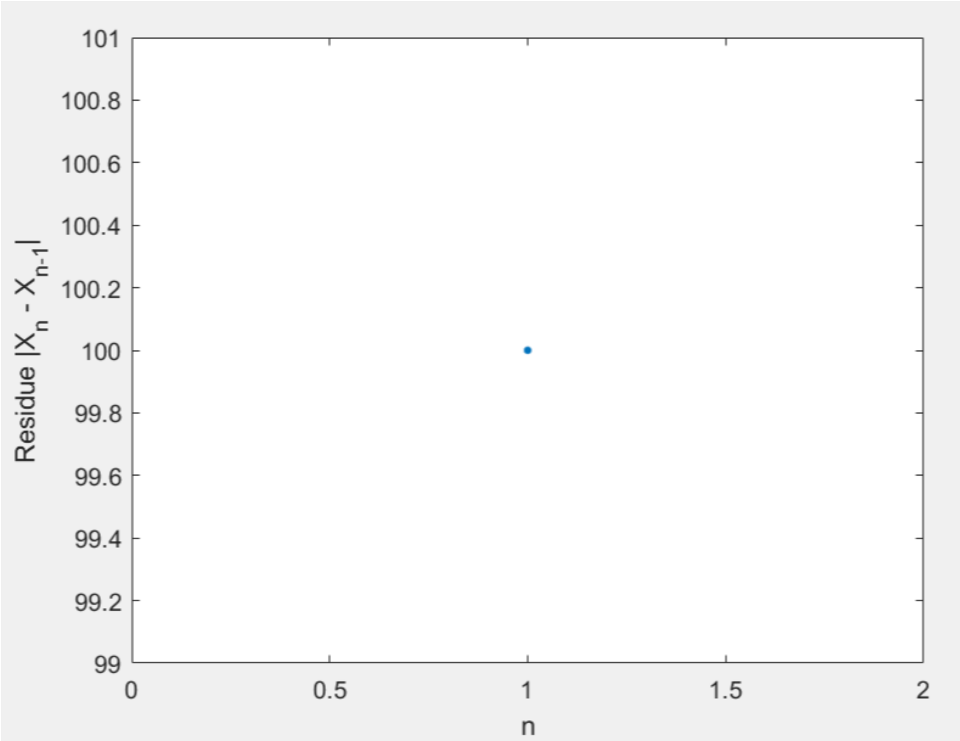
Initial Approximation $x_0 = 2$

```
Newton Method for Q7
Root found after 1 iterations.
Root is: -98
No. of iterations          Approximate solution          Error |X_n - X_n-1|
    0                      2.000000000000000             --
    1                      -98.000000000000000           100.000000000000000
```

Solution Plot



N versus Residue Plot



On applying the formula of Newton's method and carrying out the calculations, in the first iteration itself, the value of X_n becomes −98 at n=1, that is, $X_0 = 2$ and $X_1 = -98$. Since −98 is a root of p(x) as p(-98) = 0, so p(x) becomes 0 at X_1 and hence, no further iterations will take place. We obtain the root as −98.