Scientific Computing Lab MA – 322 Lab – 8

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To calculate the exact value of the integral, I have used the inbuilt integral(f, a, b) function in MATLAB.

Absolute Error = | Exact Value – Approximate Value of the integral |

1) Using Rectangle Rule,

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Question 1

Part (a)
Exact value of integral = 0.1937500000
Using Rectangle Rule, Approximate value of integral = 0.0312500000
Absolute Error = 0.1625000000

Part (b)
Exact value of integral = -0.2670627852
Using Rectangle Rule, Approximate value of integral = -0.2500000000
Absolute Error = 0.0170627852

Part (c)
Exact value of integral = -0.7339691751
Using Rectangle Rule, Approximate value of integral = -0.4000000000
Absolute Error = 0.3339691751

Part (d)
Exact value of integral = 2.5886286325
Using Rectangle Rule, Approximate value of integral = 0.0000000000
Absolute Error = 2.5886286325

Part (e)
Exact value of integral = -0.0203767960
Using Rectangle Rule, Approximate value of integral = 0.2431952925
Absolute Error = 0.2635720885
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2)

Using Midpoint Rule,

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Question 2

Part (a)
Exact value of integral = 0.1937500000
Using Midpoint Rule, Approximate value of integral = 0.1582031250
Absolute Error = 0.0355468750

Part (b)
Exact value of integral = -0.2670627852
Using Midpoint Rule, Approximate value of integral = -0.2666666667
Absolute Error = 0.0003961186

Part (c)
Exact value of integral = -0.7339691751
Using Midpoint Rule, Approximate value of integral = -0.6753246753
Absolute Error = 0.0586444998

Part (d)
Exact value of integral = 2.5886286325
Using Midpoint Rule, Approximate value of integral = 1.8039147773
Absolute Error = 0.7847138552

Part (e)
Exact value of integral = -0.0203767960
Using Midpoint Rule, Approximate value of integral = -0.0118952585
Absolute Error = 0.0084815375
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Using Trapezoidal Rule,

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Question 2

Part (a)
Exact value of integral = 0.1937500000
Using Trapezoidal Rule, Approximate value of integral = 0.2656250000
Absolute Error = 0.0718750000

Part (b)
Exact value of integral = -0.2670627852
Using Trapezoidal Rule, Approximate value of integral = -0.2678571429
Absolute Error = 0.0007943576

Part (c)
Exact value of integral = -0.7339691751
Using Trapezoidal Rule, Approximate value of integral = -0.8666666667
Absolute Error = 0.1326974916

Part (d)
Exact value of integral = 2.5886286325
Using Trapezoidal Rule, Approximate value of integral = 4.1432596552
Absolute Error = 1.5546310227

Part (e)
Exact value of integral = -0.0203767960
Using Trapezoidal Rule, Approximate value of integral = -0.0370242527
Absolute Error = 0.0166474567
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Using Simpson's Rule,

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Question 2

Part (a)
Exact value of integral = 0.1937500000
Using Simpson's one-third Rule, Approximate value of integral = 0.1940104167
Absolute Error = 0.0002604167

Part (b)
Exact value of integral = -0.2670627852
Using Simpson's one-third Rule, Approximate value of integral = -0.2670634921
Absolute Error = 0.0000007068

Part (c)
Exact value of integral = -0.7339691751
Using Simpson's one-third Rule, Approximate value of integral = -0.7391053391
Absolute Error = 0.0051361640

Part (d)
Exact value of integral = 2.5886286325
Using Simpson's one-third Rule, Approximate value of integral = 2.5836964032
Absolute Error = 0.0049322293

Part (e)
Exact value of integral = -0.0203767960
Using Simpson's one-third Rule, Approximate value of integral = -0.0202715899
Absolute Error = 0.0001052061
```

3)

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Question 3
Exact value of pi = 3.1415926536
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Using Rectangle Rule,

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Exact value of integral = 3.1415926536
Using Rectangle Rule, Approximate value of integral = 4.0000000000
Absolute Error = 0.8584073464
```

Using Trapezoidal Rule,

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Exact value of integral = 3.1415926536
Using Trapezoidal Rule, Approximate value of integral = 3.0000000000
Absolute Error = 0.1415926536
```

Using Simpson’s one-third Rule,

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Exact value of integral = 3.1415926536
Using Simpson's one-third Rule, Approximate value of integral = 3.1333333333
Absolute Error = 0.0082593203
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Using Simpson's three-eighth Rule,

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Exact value of integral = 3.1415926536
Using Simpson's three-eighth Rule, Approximate value of integral = 3.1384615385
Absolute Error = 0.0031311151
```

The worst approximation is given by Rectangle Rule, then Trapezoidal Rule, then Simpson's one-third Rule and finally, the best approximation is given by Simpson's three-eighth Rule.

We observe that the Simpson's one-third and Simpson's three-eight rules are giving an approximate value significantly closer to the actual value of the integral, and relatively lesser error than other methods.

Absolute Error in Rectangle Rule > Trapezoidal Rule > Simpson's one-third Rule > Simpson's three-eighth Rule.

We can get better approximations of the integral value by using the composite versions of these methods.

4)

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Question 4
Using Composite Trapezoidal Rule, Approximate value of integral = 7.1250000000
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5)

For determining such n and h, we iterate over n starting from 1, take h = (b-a)/n and consider n+1 equally spaced points $x_0, x_1, \ldots, x_n$ with $x_i = a + ih$, $i = 0, 1, \ldots, n$. Now we apply the given composite rules on these points and find an estimate of the integral, and check if the Absolute Error is less than or equal to the required tolerance of 10^{-5}. We repeat this process until the tolerance condition is satisfied and then break the loop to get the required value of n and h.

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Question 5
Part (a) Composite Trapezoidal Rule
Exact value of integral = 0.6362943611
For the error to be within 10^(-5), n = 77, h = 0.01298701

For n = 77 and h = 0.01298701, Approximate Value of the Integral = 0.6363041034
Absolute Error = 0.0000097423

Part (b) Composite Simpson's Rule
Exact value of integral = 0.6362943611
For the error to be within 10^(-5), n = 3, h = 0.33333333

For n = 3 and h = 0.33333333, Approximate Value of the Integral = 0.6362975008
Absolute Error = 0.0000031397

Part (c) Composite Midpoint Rule
Exact value of integral = 0.6362943611
For the error to be within 10^(-5), n = 54, h = 0.01851852

For n = 54 and h = 0.01851852, Approximate Value of the Integral = 0.6362844569
Absolute Error = 0.0000099043
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6) Distance = integral of speed with respect to time

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Question 6
Using Composite Trapezoidal Rule, Approximate length of the track = 9855.00 feet
Using Composite Simpson's Rule, Approximate length of the track = 9858.00 feet
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