Scientific Computing Lab   MA – 322   Lab – 13

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For each part of all the questions, the four plots are as follows:

(i) x versus actual and approximate solutions

(ii) x versus absolute error

(iii) N versus Order of Convergence

(iv) loglog plot of N versus absolute error

Where, N = (b-a)/h = Number of intervals

Order of convergence = $\log_2(E_N/E_{2N})$, where E_N and E_{2N} and the maximum errors obtained during the computation for that specific value of N

Absolute error = absolute value of the difference between the approximation and the actual solution

1)

Using Linear Shooting method, we got a system of 2 ODEs, which we solved using the fourth order Runge-Kutta method for system of equations.
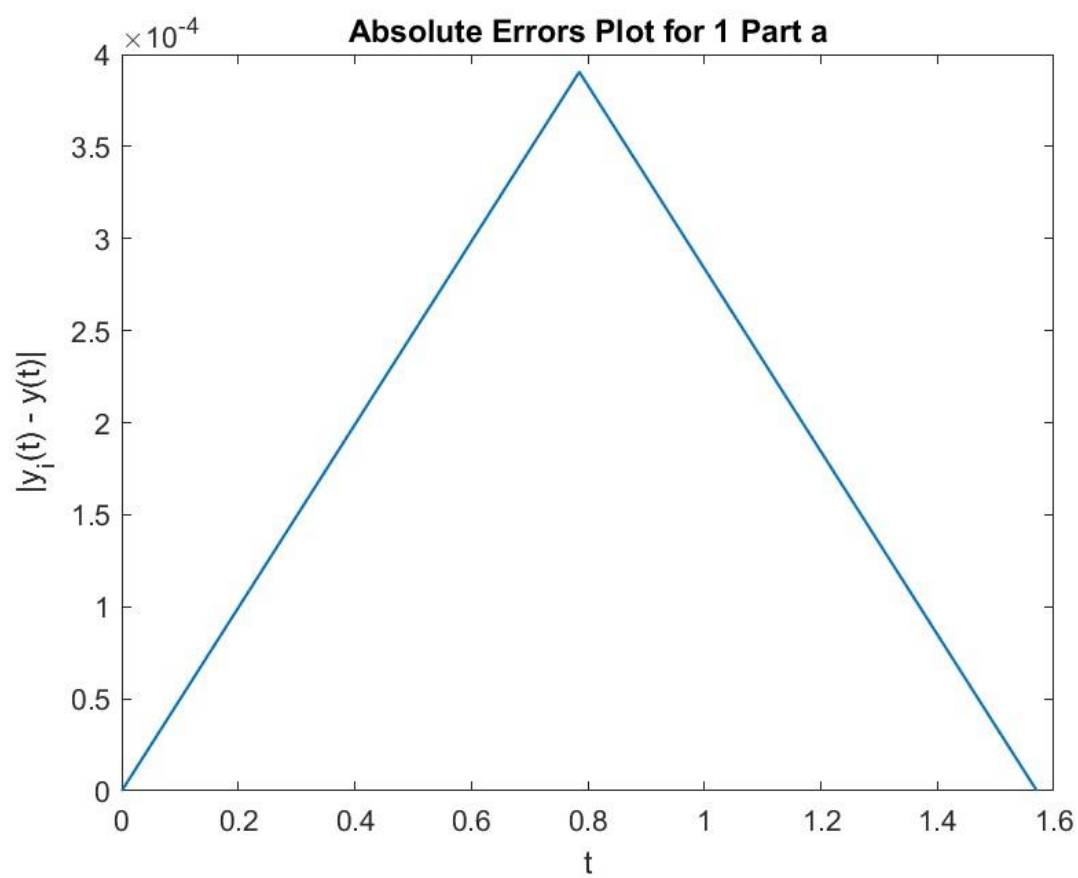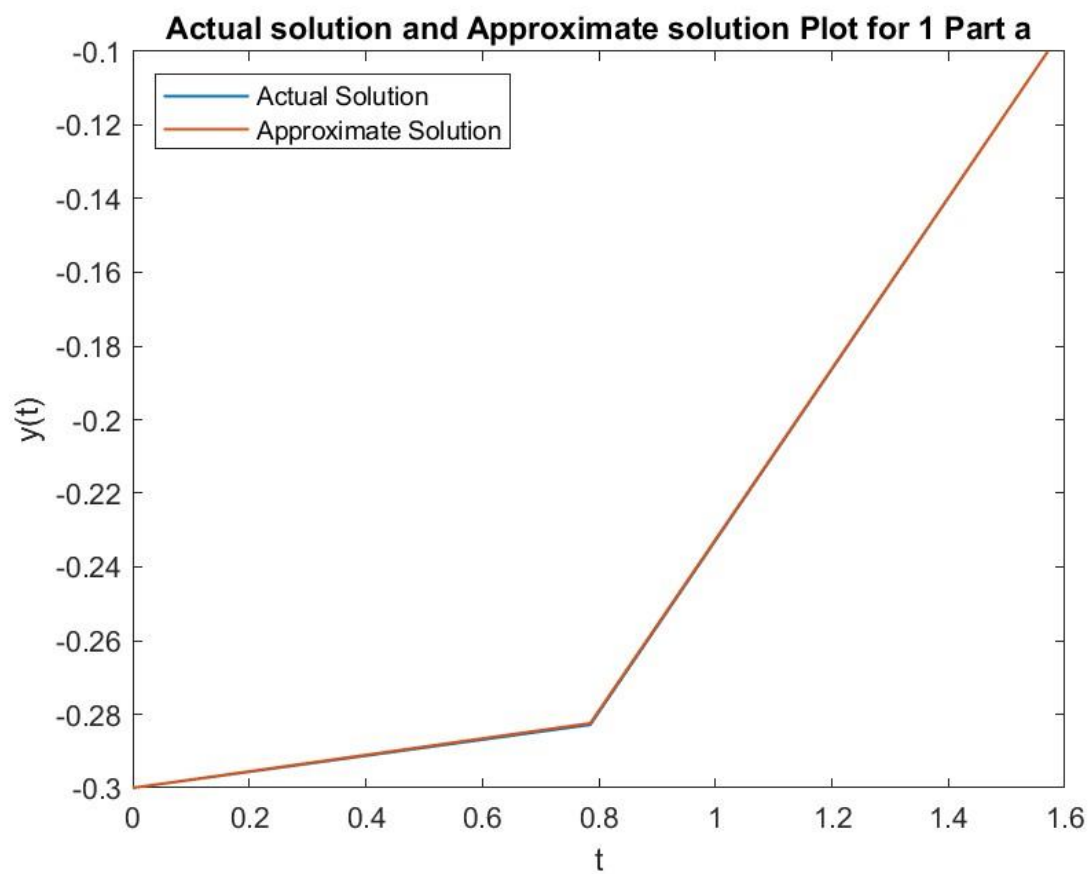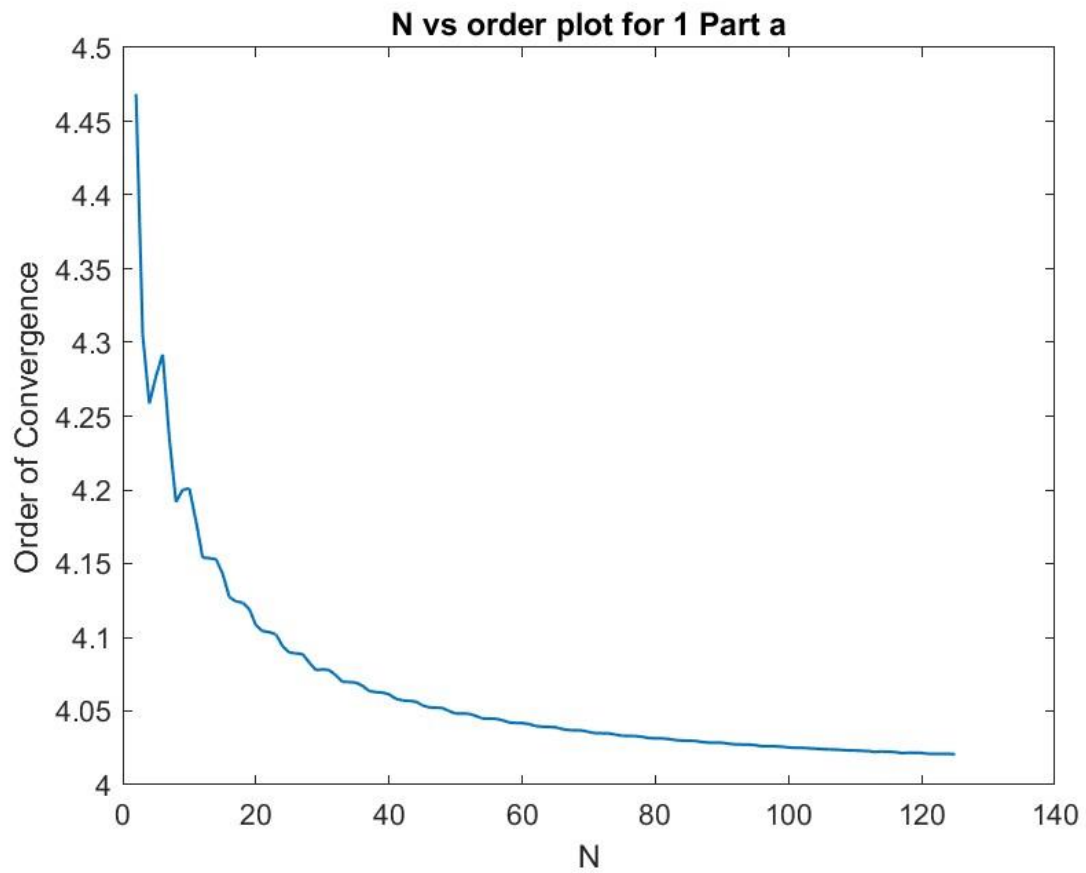
a)

```
Question 1
Using Linear Shooting Method with h = π/4,
For h = 0.785398
     x         Approximate Solution      Exact Solution    Absolute Error
0.000000          -0.3000000               -0.3000000          0.0000000
0.785398          -0.2824522               -0.2828427          0.0003905
1.570796          -0.1000000               -0.1000000          0.0000000
```
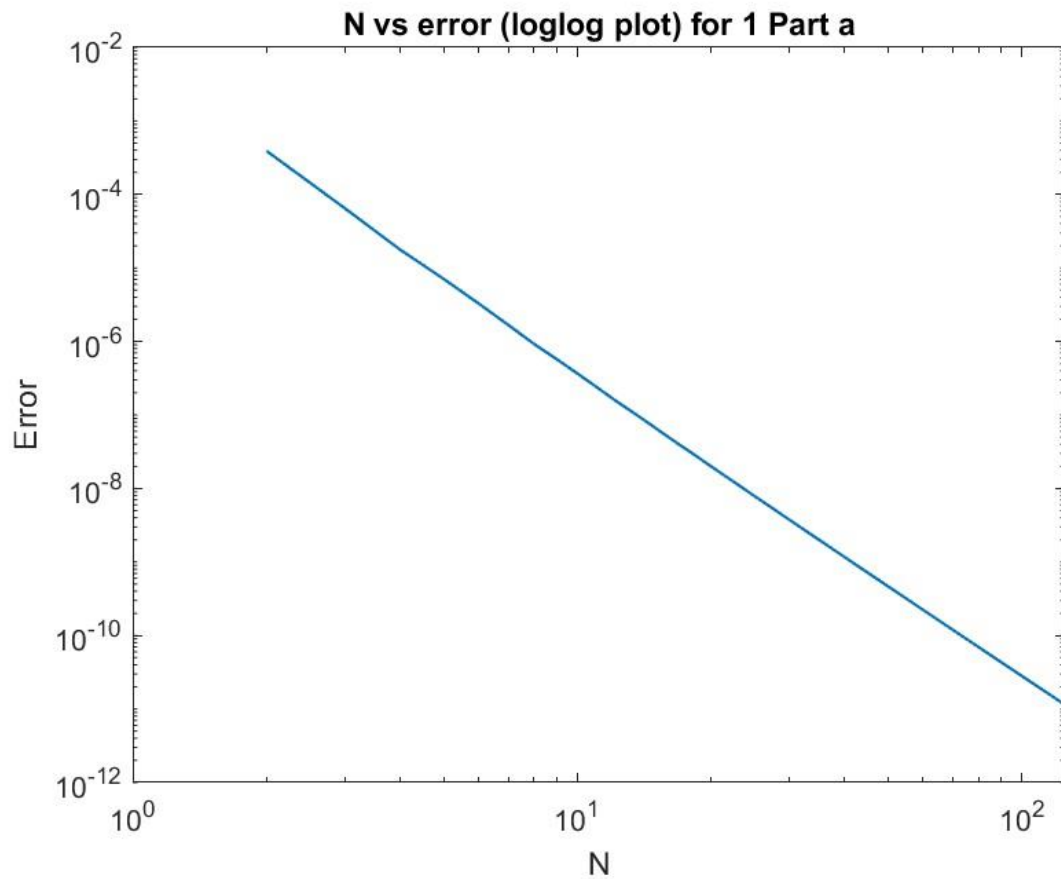
Actual solution and Approximate solution Plot for 1 Part a

Absolute Errors Plot for 1 Part a

Since the Runge-Kutta method is used, the order of convergence should converge to 4.

N vs order plot for 1 Part a
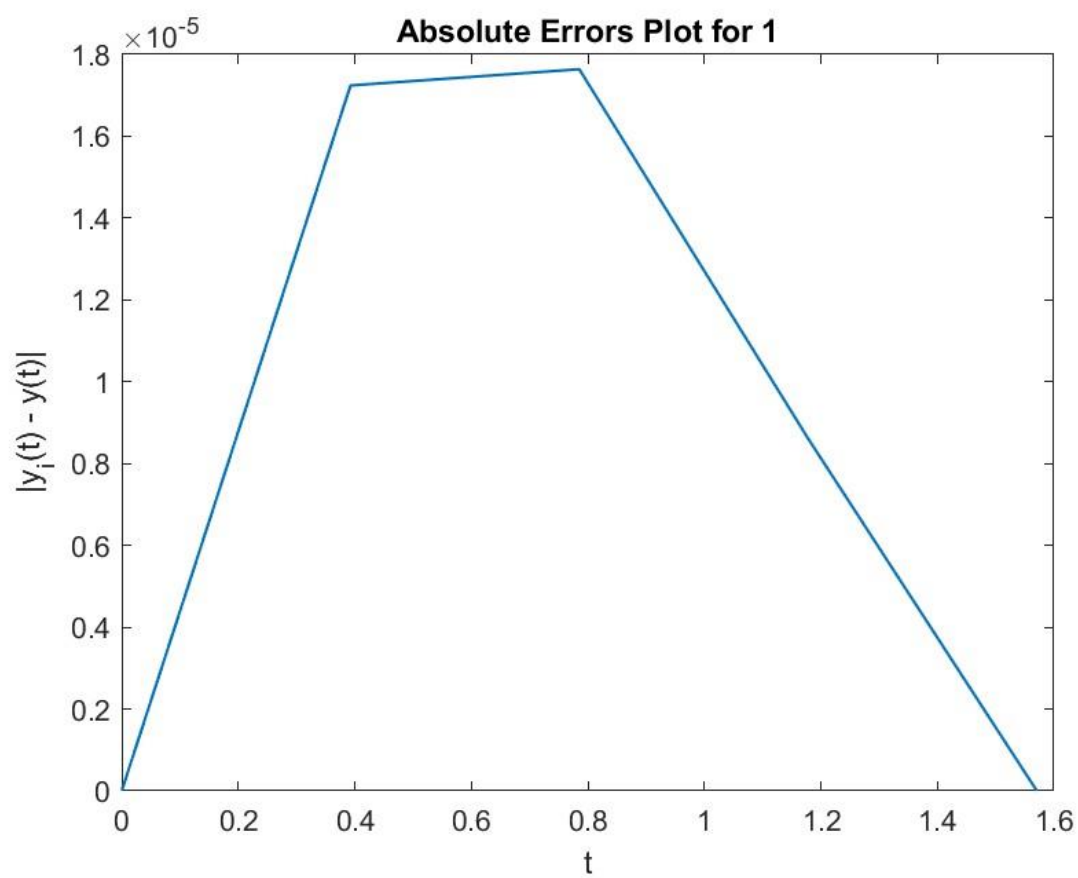
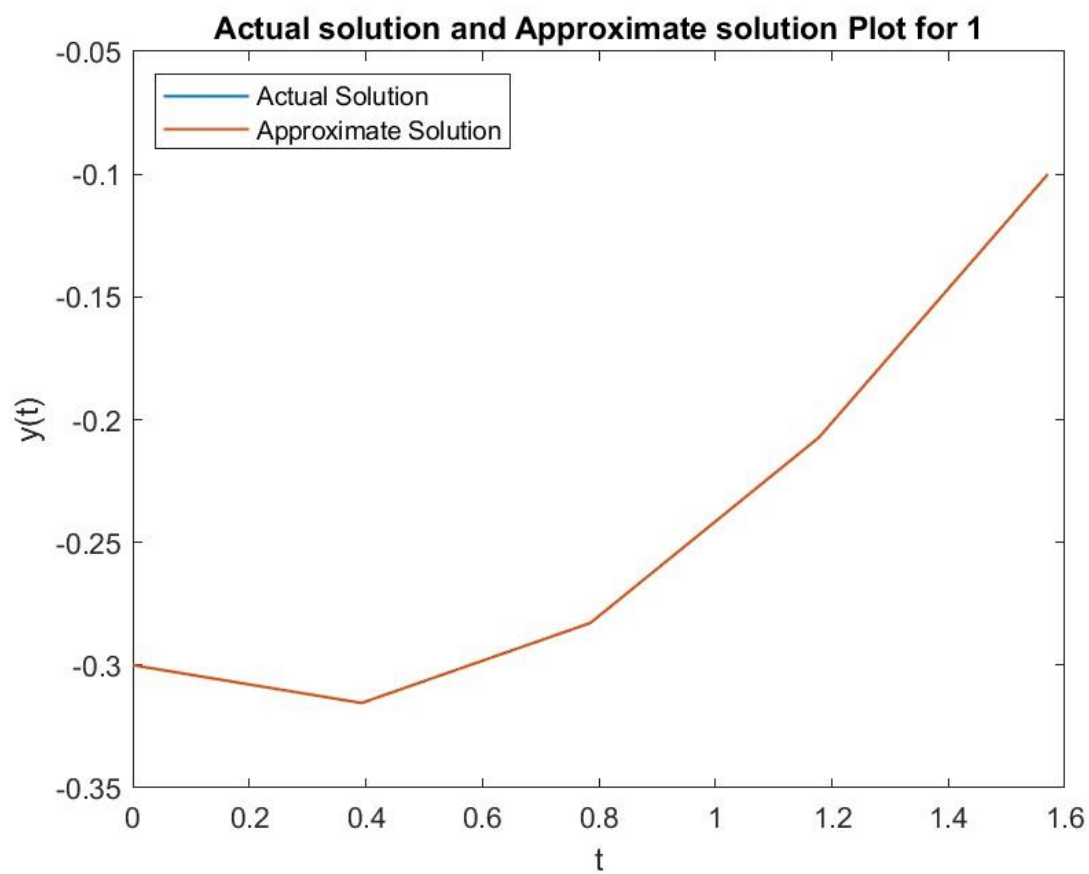N vs error (loglog plot) for 1 Part a

b)

```
Question 1
Using Linear Shooting Method with h = π/8,
For h = 0.392699
     x        Approximate Solution     Exact Solution   Absolute Error
0.000000          -0.3000000              -0.3000000        0.0000000
0.392699          -0.3154150              -0.3154322        0.0000172
0.785398          -0.2828251              -0.2828427        0.0000176
1.178097          -0.2071844              -0.2071930        0.0000086
1.570796          -0.1000000              -0.1000000        0.0000000
```

Actual solution and Approximate solution Plot for 1

Absolute Errors Plot for 1

Since the Runge-Kutta method is used, the order of convergence should converge to 4.

N vs order plot for 1

N vs error (loglog plot) for 1
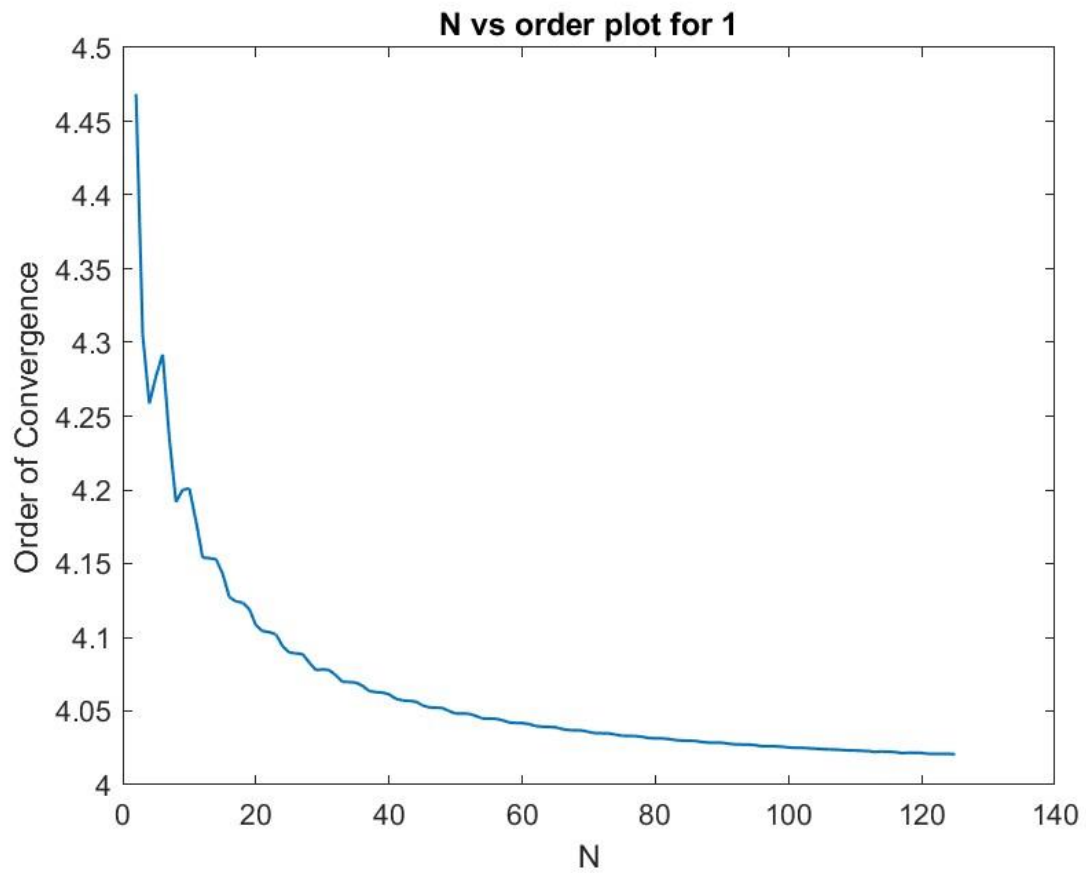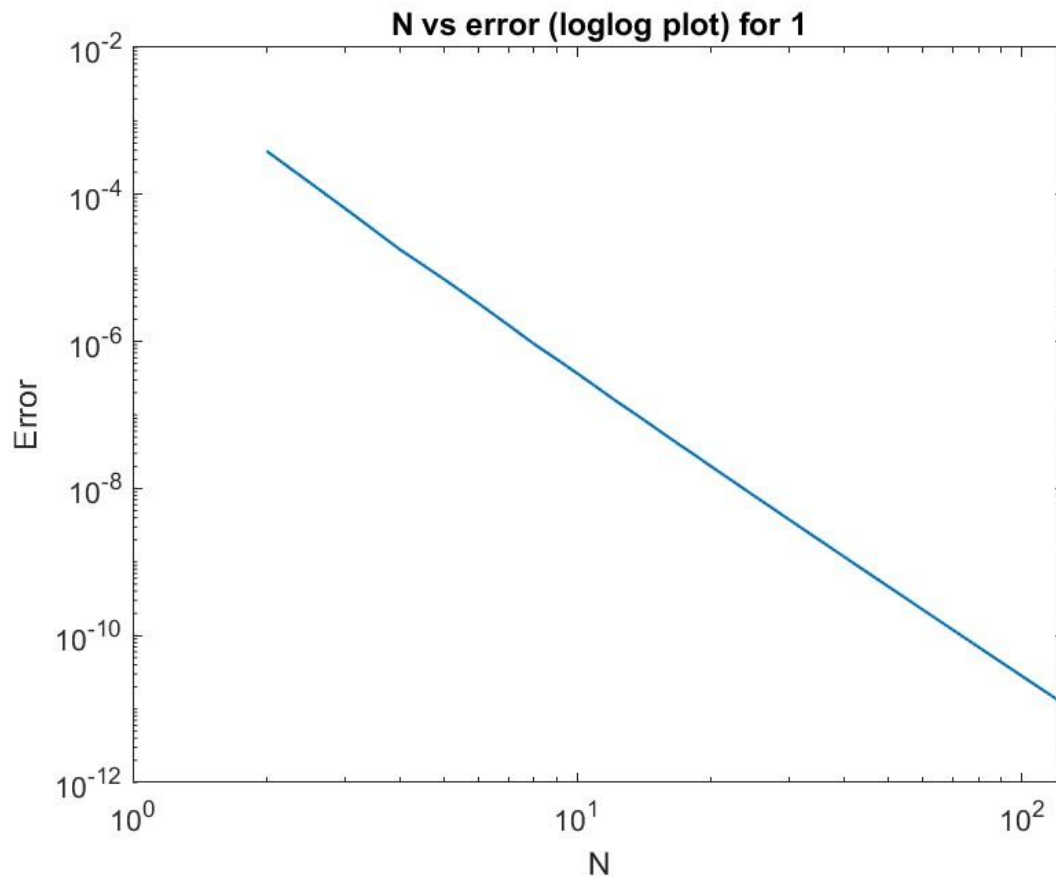
We can observe that the errors are very low.

2)

The six parts have in total 3 types of boundary conditions, which are as follows:

- For parts a and b, Dirichlet Conditions
- For parts c and d, Neumann Conditions
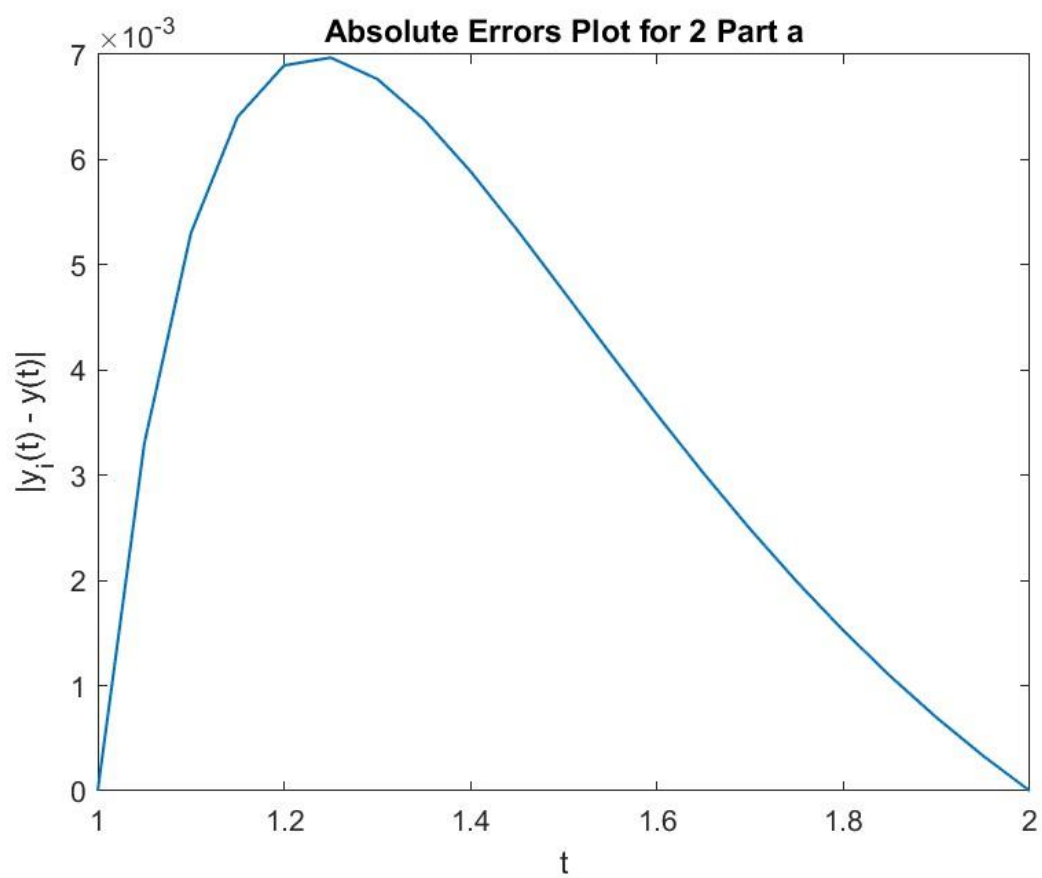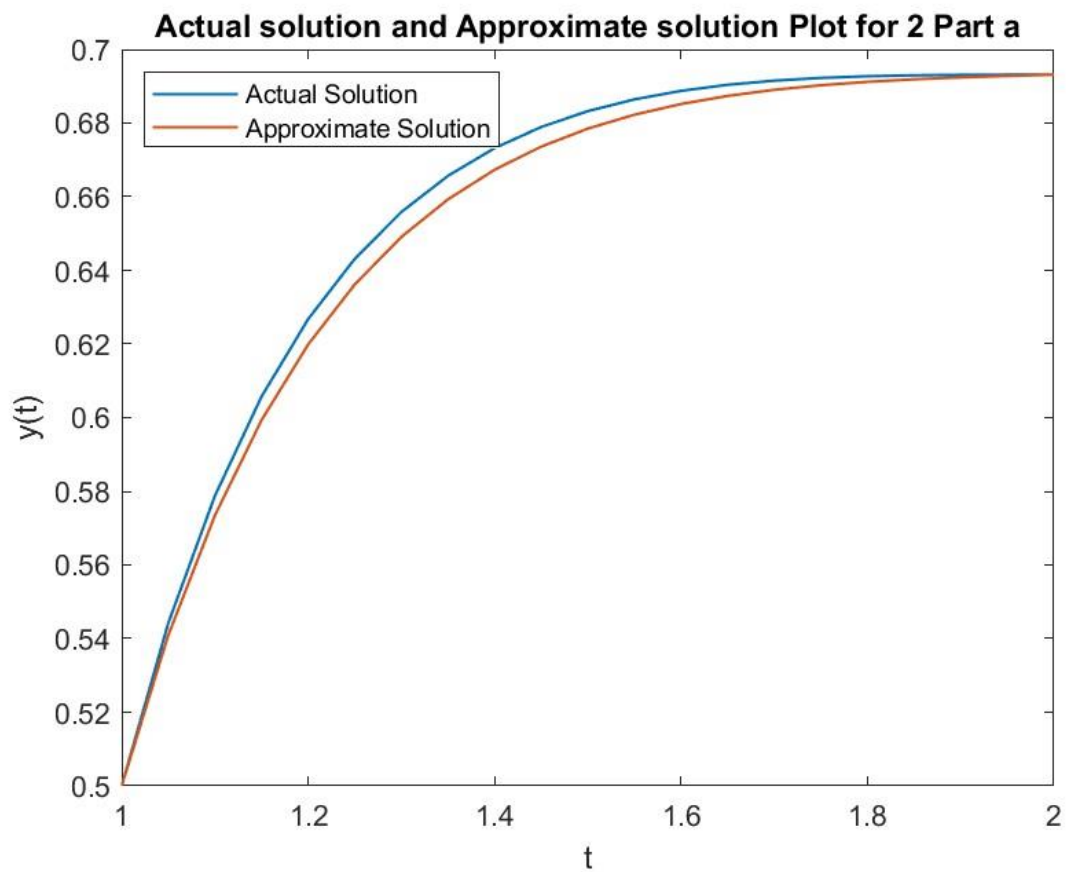- For parts e and f, Robin or Mixed Conditions

The second derivative is always replaced by the Central difference, while the first derivative is replaced by Forward difference, Backward Difference and Central Difference one by one.
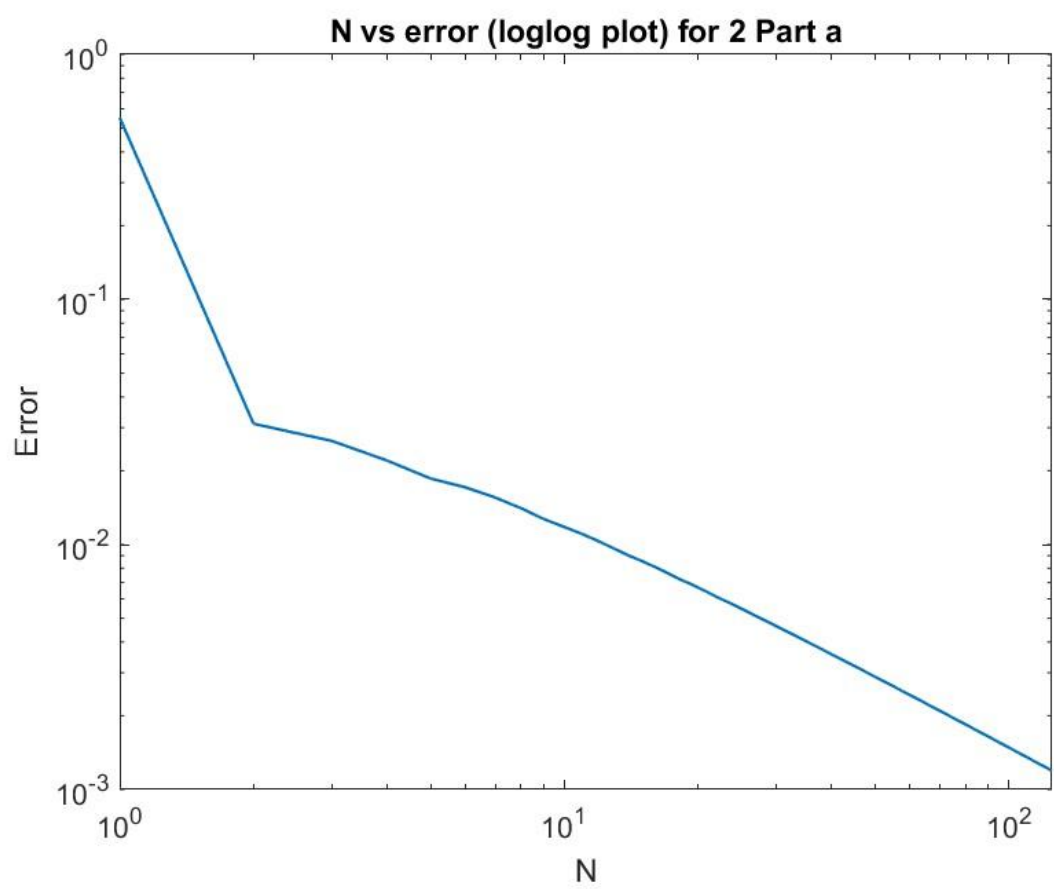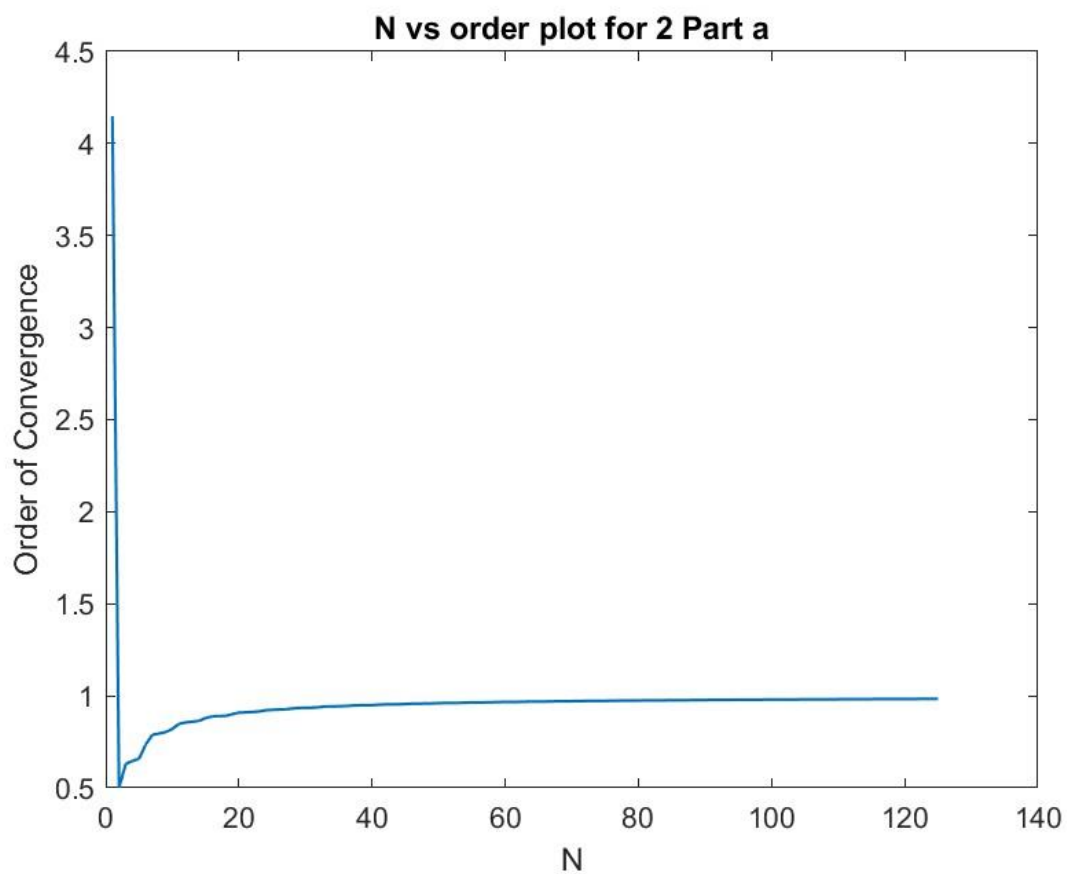
a)

Question 2 Part a
Using forward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.050000

x	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.5000000	0.5000000	0.0000000
1.050000	0.5409536	0.5442550	0.0033015
1.100000	0.5734796	0.5787813	0.0053016
1.150000	0.5993298	0.6057355	0.0064057
1.200000	0.6198702	0.6267660	0.0068958
1.250000	0.6361740	0.6431436	0.0069695
1.300000	0.6490897	0.6558554	0.0067657
1.350000	0.6592916	0.6656739	0.0063822
1.400000	0.6673189	0.6732069	0.0058880
1.450000	0.6736043	0.6789357	0.0053314
1.500000	0.6784966	0.6832429	0.0047463
1.550000	0.6822780	0.6864339	0.0041559
1.600000	0.6851774	0.6887536	0.0035762
1.650000	0.6873811	0.6903988	0.0030177
1.700000	0.6890409	0.6915279	0.0024870
1.750000	0.6902807	0.6922688	0.0019881
1.800000	0.6912020	0.6927249	0.0015230
1.850000	0.6918874	0.6929796	0.0010922
1.900000	0.6924049	0.6931004	0.0006955
1.950000	0.6928098	0.6931417	0.0003319
2.000000	0.6931472	0.6931472	0.0000000

Actual solution and Approximate solution Plot for 2 Part a

Absolute Errors Plot for 2 Part a

N vs order plot for 2 Part a

N vs error (loglog plot) for 2 Part a

Using backward difference for the first-order derivative and central difference for the second-order derivative,
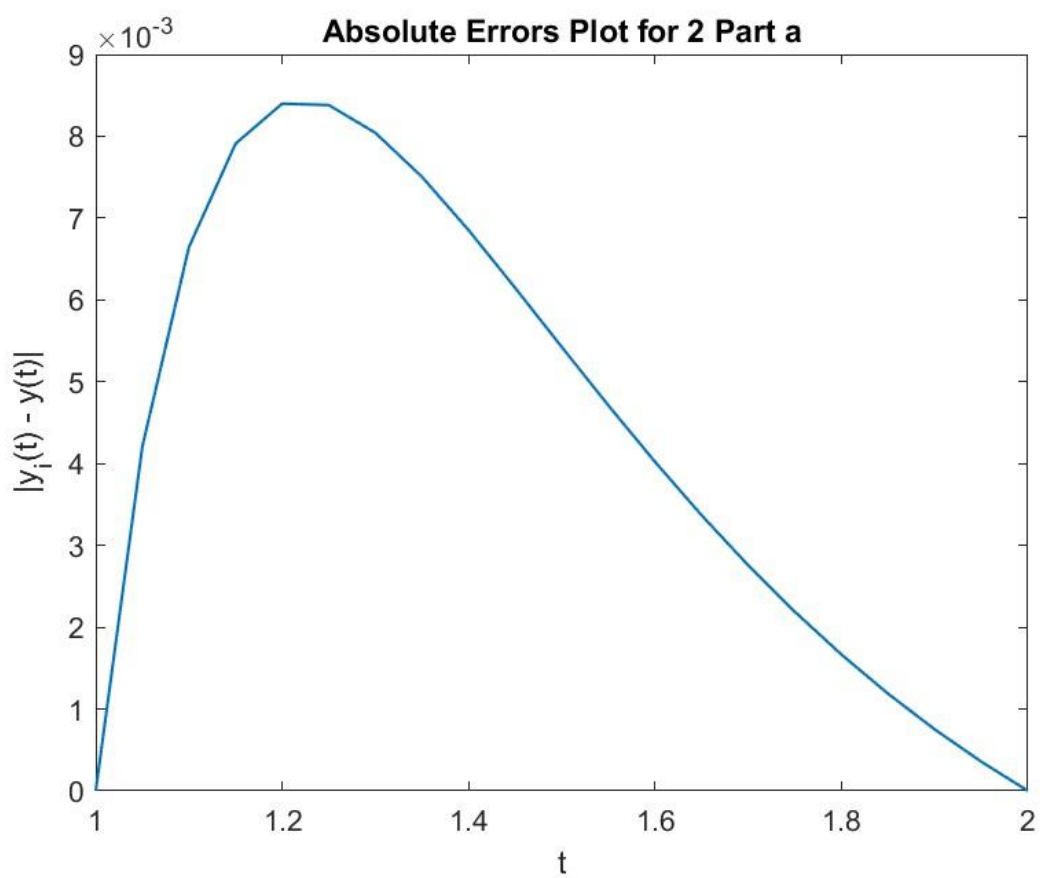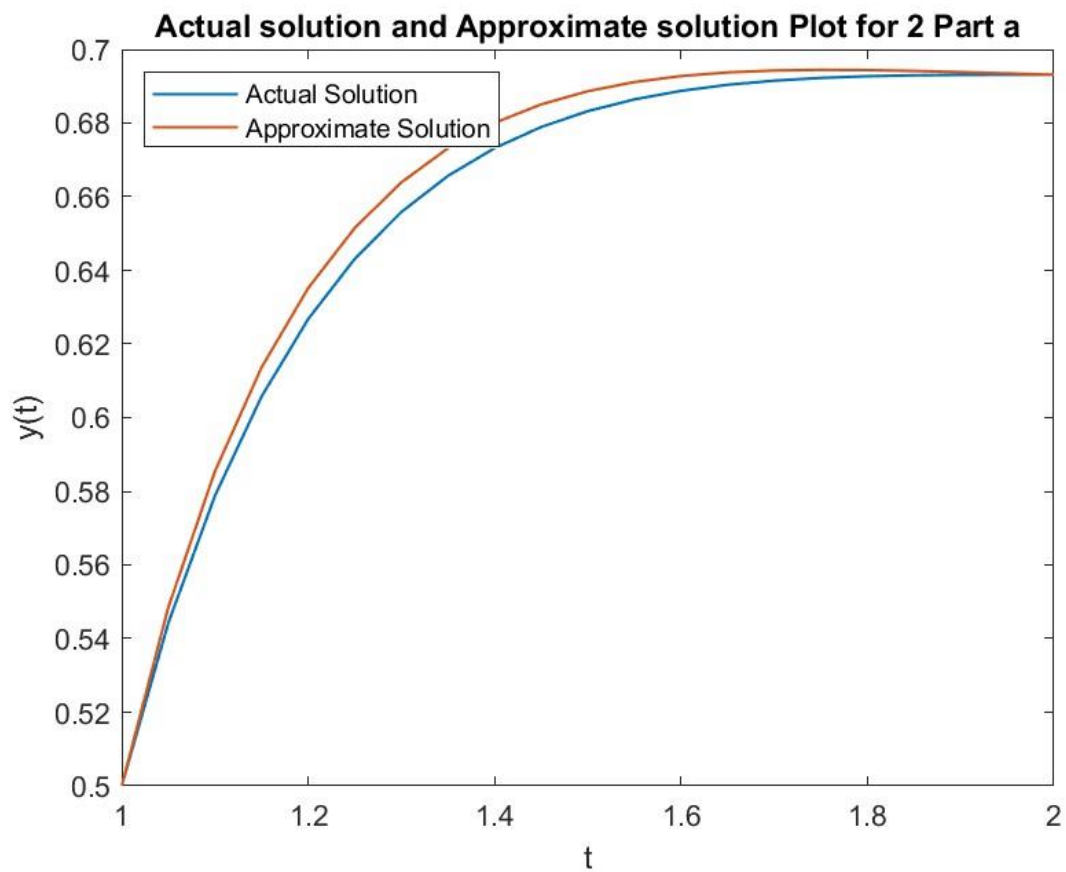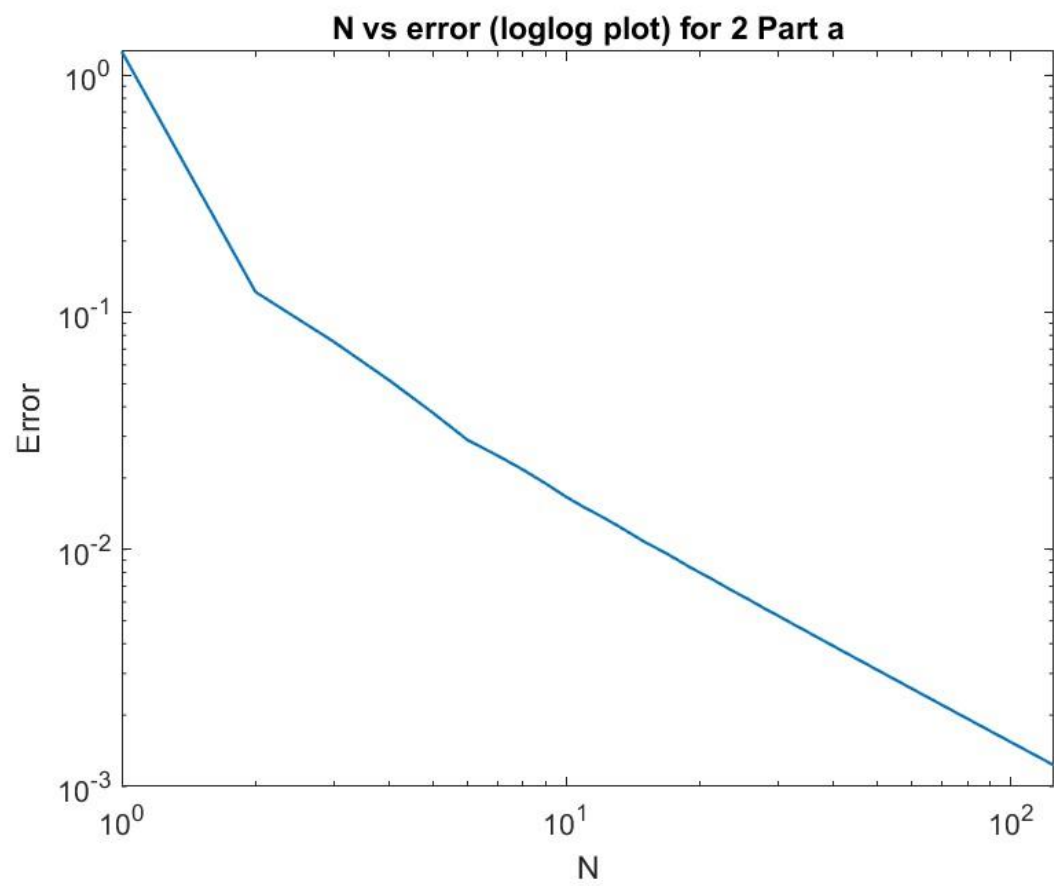For h = 0.050000

x	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.5000000	0.5000000	0.0000000
1.050000	0.5484617	0.5442550	0.0042066
1.100000	0.5854264	0.5787813	0.0066452
1.150000	0.6136451	0.6057355	0.0079096
1.200000	0.6351645	0.6267660	0.0083985
1.250000	0.6515250	0.6431436	0.0083815
1.300000	0.6638970	0.6558554	0.0080416
1.350000	0.6731776	0.6656739	0.0075038
1.400000	0.6800599	0.6732069	0.0068529
1.450000	0.6850824	0.6789357	0.0061467
1.500000	0.6886666	0.6832429	0.0054237
1.550000	0.6911435	0.6864339	0.0047096
1.600000	0.6927746	0.6887536	0.0040210
1.650000	0.6937667	0.6903988	0.0033679
1.700000	0.6942840	0.6915279	0.0027561
1.750000	0.6944574	0.6922688	0.0021886
1.800000	0.6943908	0.6927249	0.0016659
1.850000	0.6941671	0.6929796	0.0011875
1.900000	0.6938522	0.6931004	0.0007518
1.950000	0.6934984	0.6931417	0.0003568
2.000000	0.6931472	0.6931472	0.0000000

Actual solution and Approximate solution Plot for 2 Part a

Absolute Errors Plot for 2 Part a

N vs order plot for 2 Part a
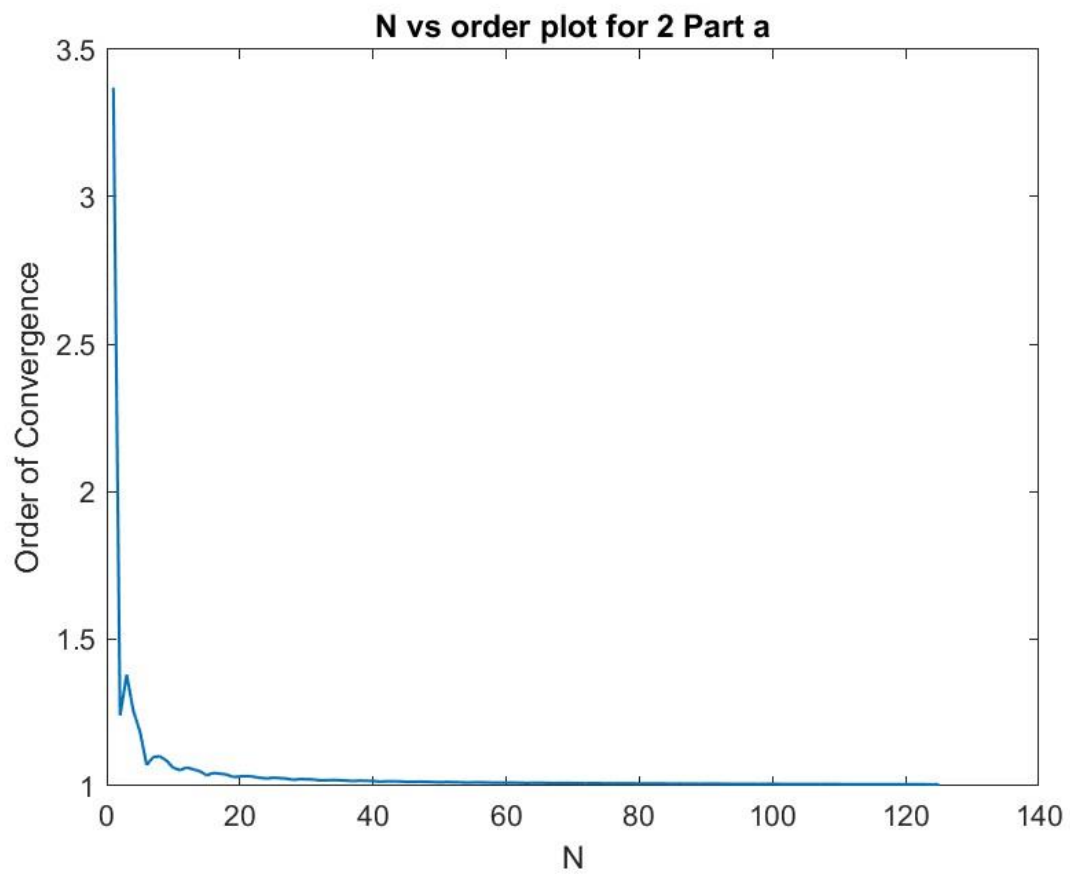
N vs error (loglog plot) for 2 Part a

Using central difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.050000

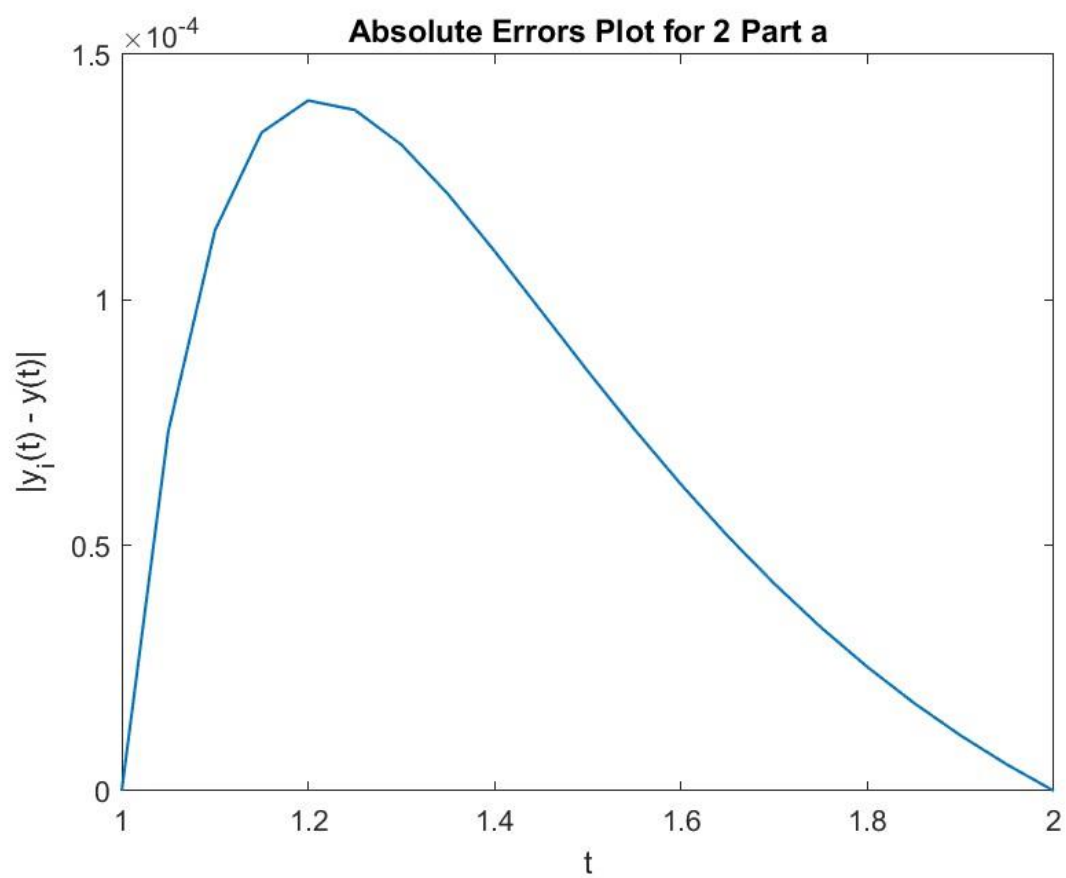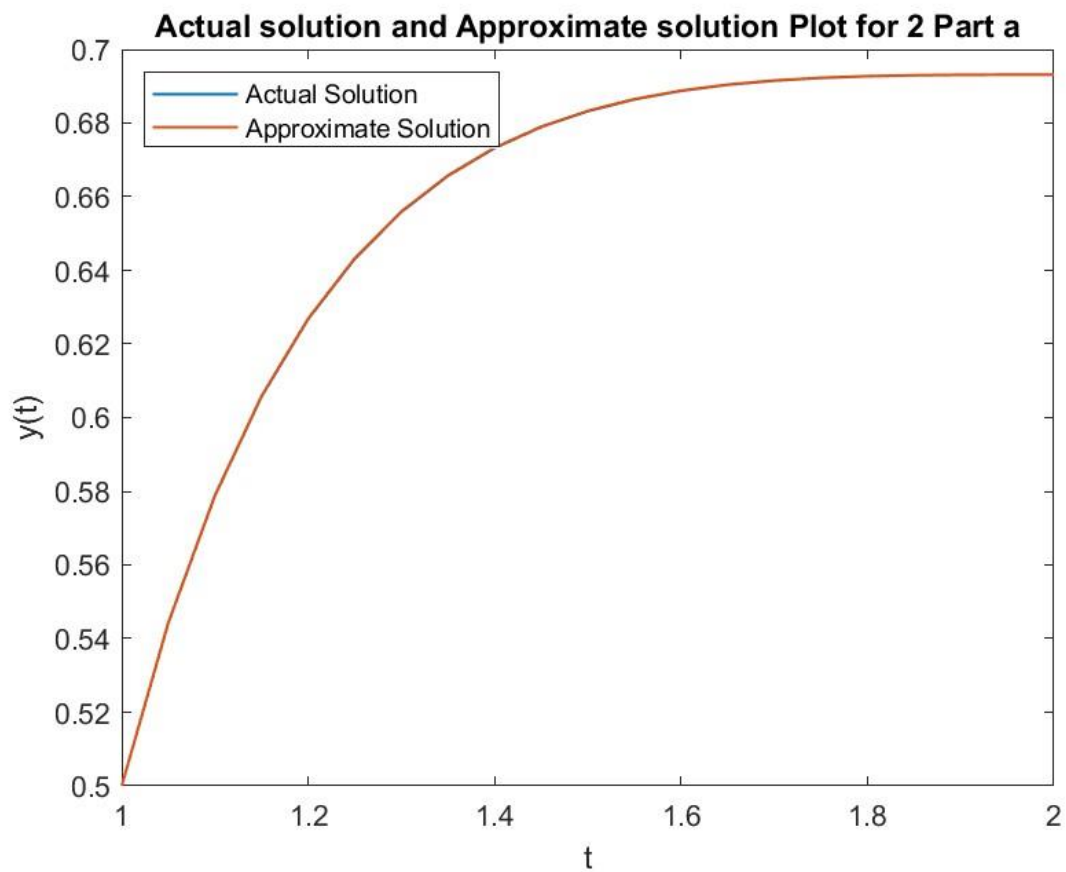| x	Approximate Solution	Exact Solution	Absolute Error
1.000000	0.5000000	0.5000000	0.0000000
1.050000	0.5443283	0.5442550	0.0000733
1.100000	0.5788954	0.5787813	0.0001142
1.150000	0.6058696	0.6057355	0.0001341
1.200000	0.6269066	0.6267660	0.0001406
1.250000	0.6432823	0.6431436	0.0001387
1.300000	0.6559870	0.6558554	0.0001316
1.350000	0.6657954	0.6656739	0.0001216
1.400000	0.6733169	0.6732069	0.0001099
1.450000	0.6790334	0.6789357	0.0000977
1.500000	0.6833284	0.6832429	0.0000855
1.550000	0.6865076	0.6864339	0.0000736
1.600000	0.6888160	0.6887536	0.0000624
1.650000	0.6904507	0.6903988	0.0000519
1.700000	0.6915701	0.6915279	0.0000422
1.750000	0.6923022	0.6922688	0.0000333
1.800000	0.6927502	0.6927249	0.0000252
1.850000	0.6929975	0.6929796	0.0000179
1.900000	0.6931117	0.6931004	0.0000113
1.950000	0.6931470	0.6931417	0.0000053
2.000000	0.6931472	0.6931472	0.0000000

Actual solution and Approximate solution Plot for 2 Part a

Absolute Errors Plot for 2 Part a

N vs order plot for 2 Part a

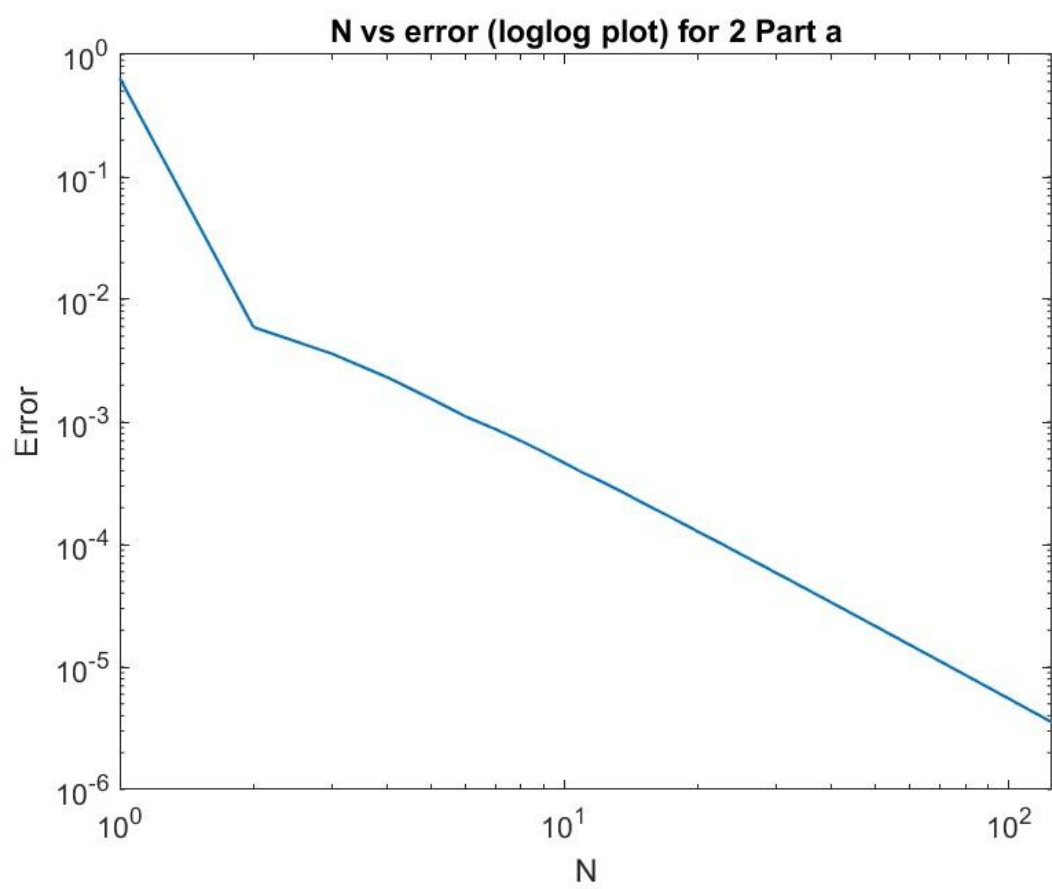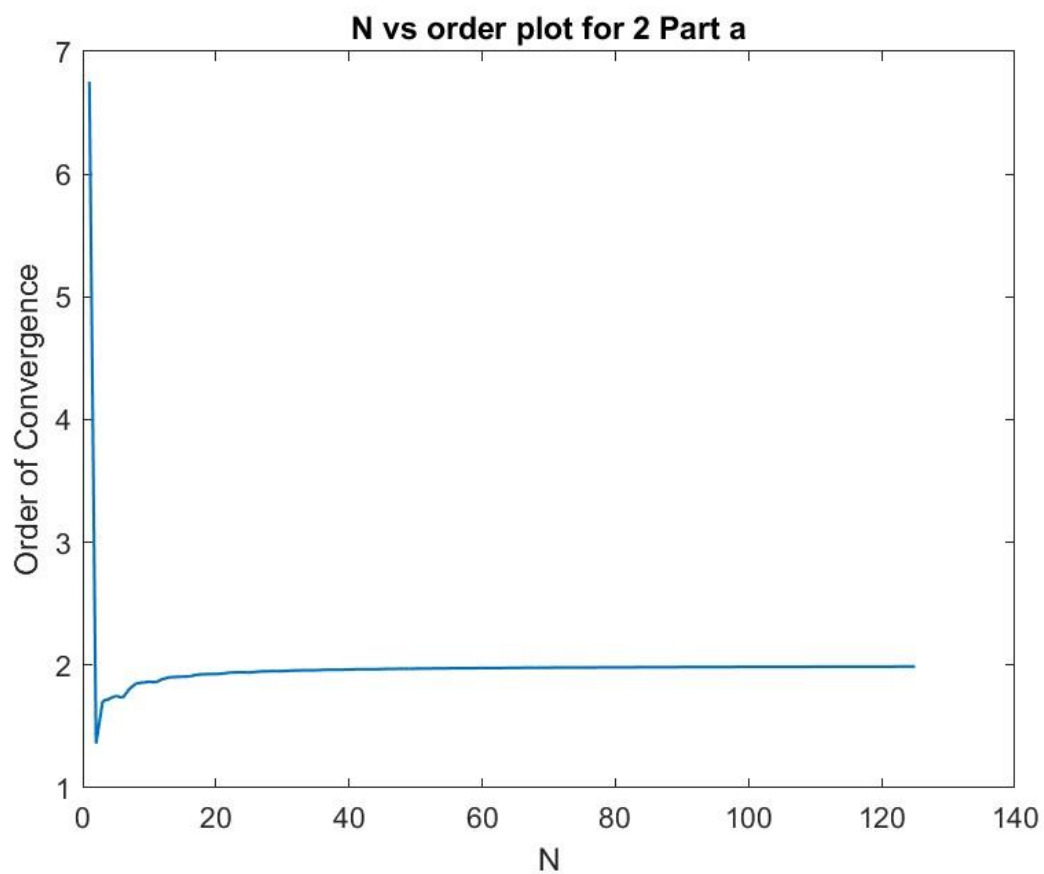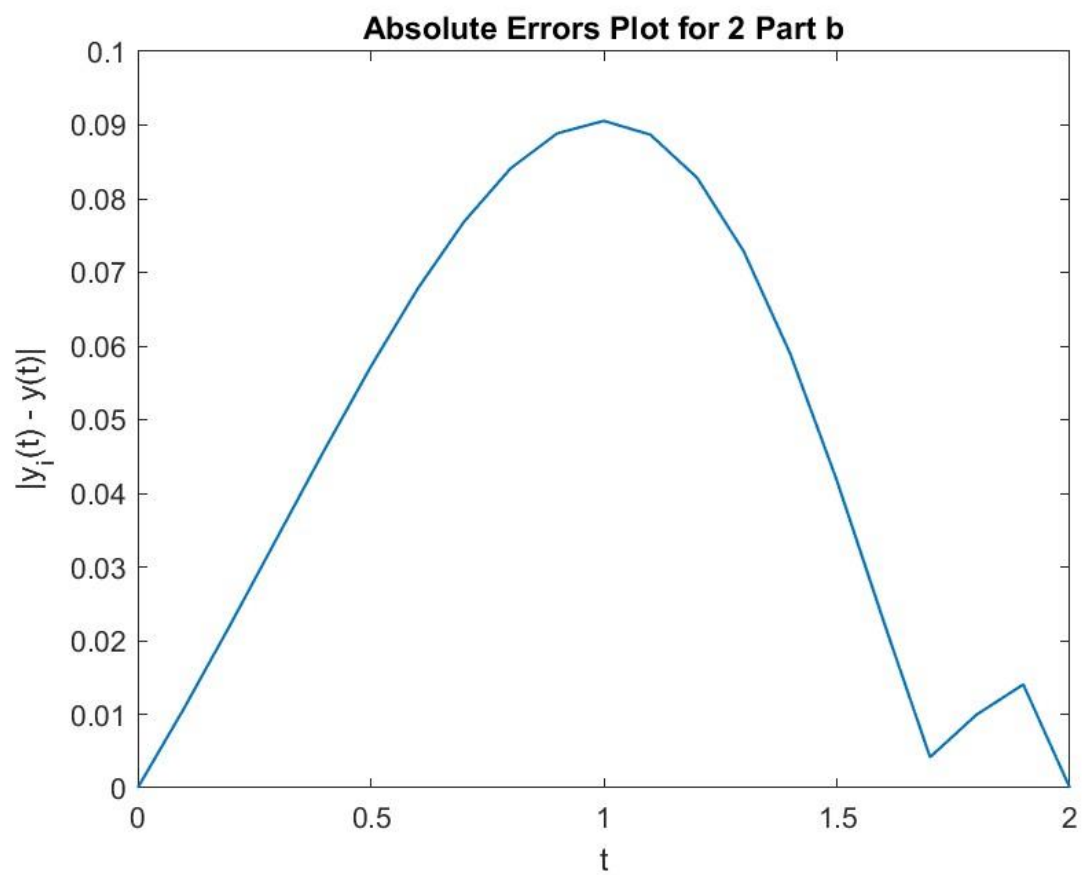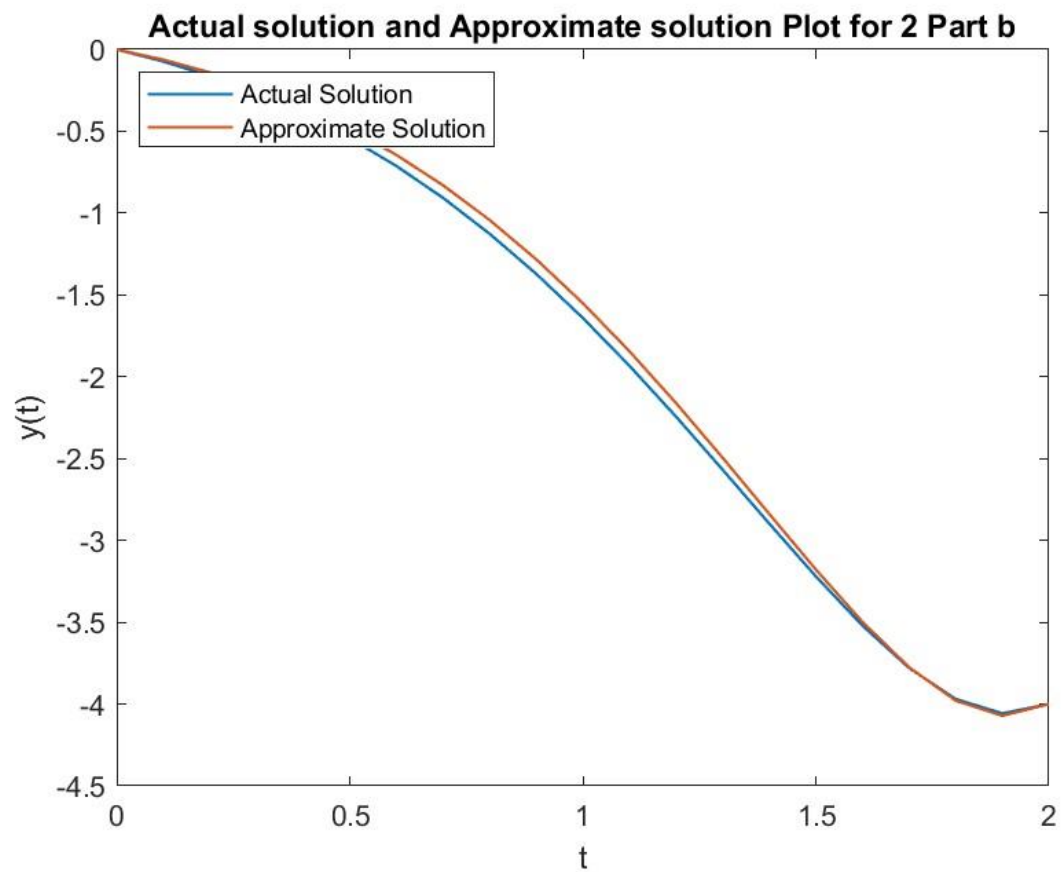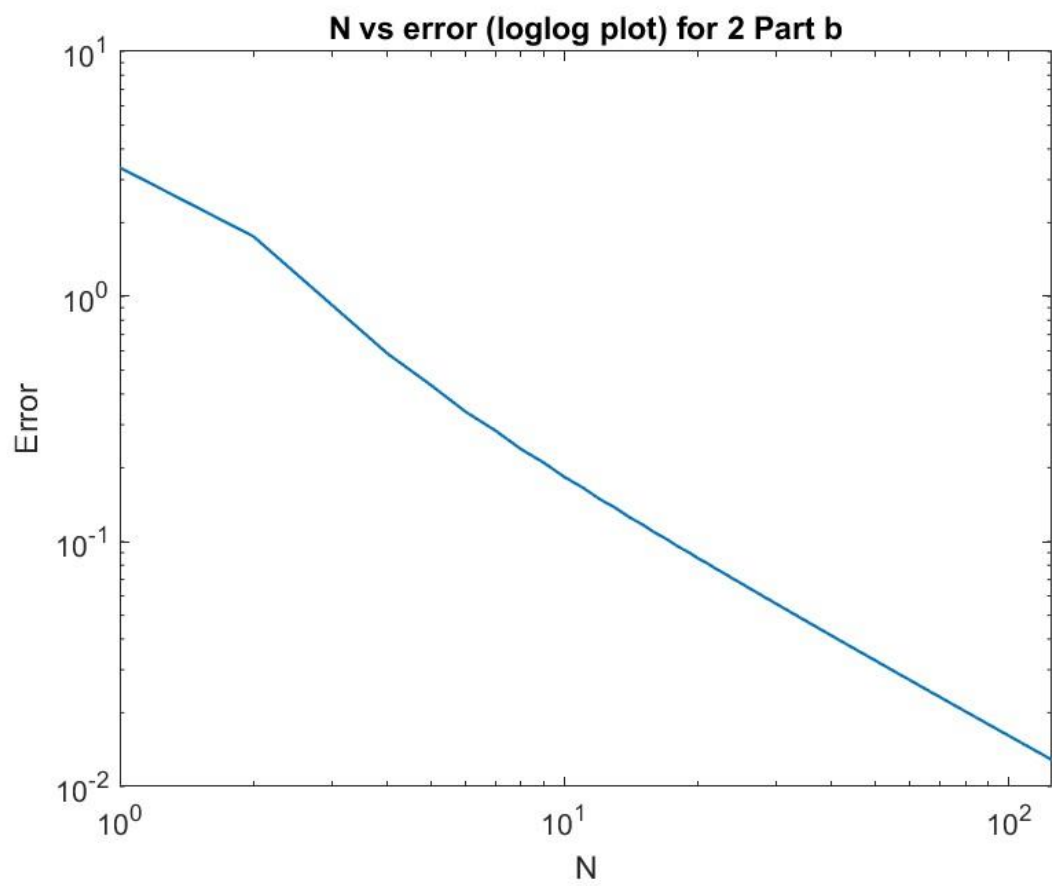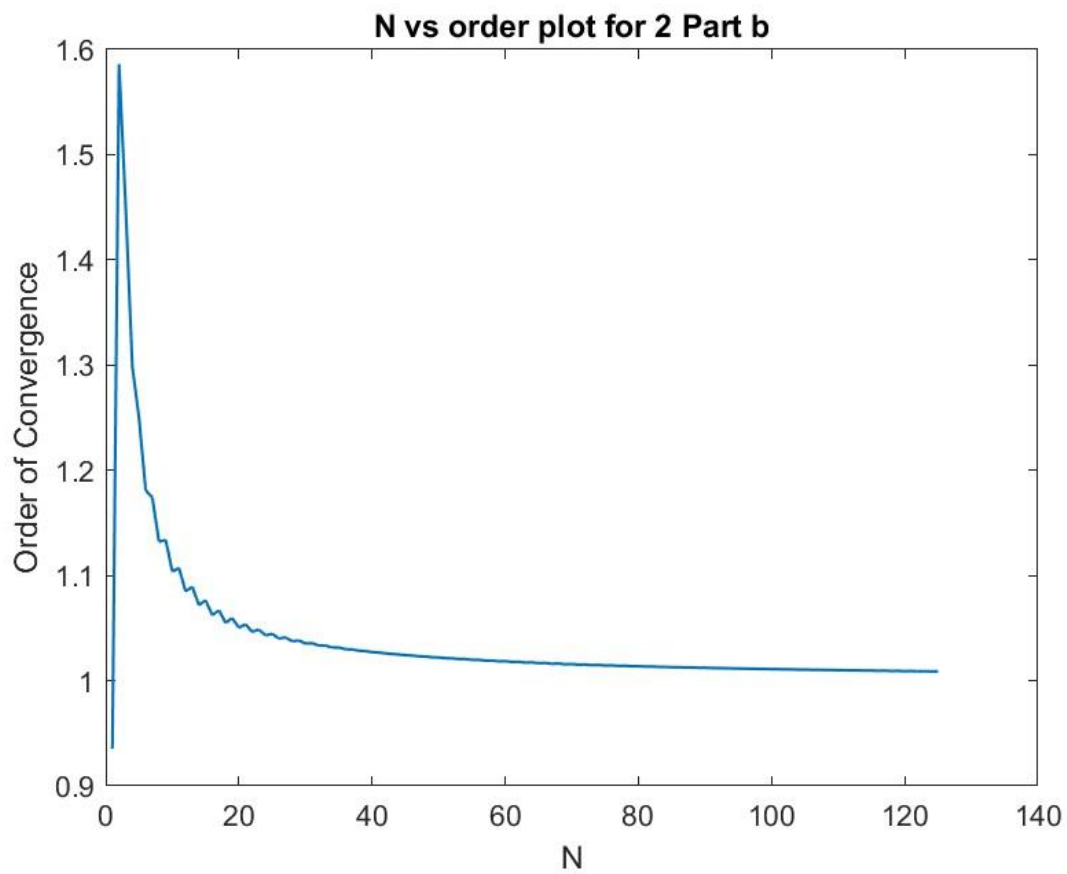N vs error (loglog plot) for 2 Part a

b)

```
Question 2 Part b
Using forward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.100000
    x         Approximate Solution    Exact Solution   Absolute Error
0.000000         0.0000000              0.0000000        0.0000000
0.100000        -0.0628025             -0.0736691        0.0108666
0.200000        -0.1403892             -0.1627002        0.0223110
0.300000        -0.2350642             -0.2691374        0.0340733
0.400000        -0.3491576             -0.3949876        0.0458300
0.500000        -0.4849508             -0.5421435        0.0571927
0.600000        -0.6445759             -0.7122849        0.0677090
0.700000        -0.8298842             -0.9067532        0.0768690
0.800000        -1.0422757             -1.1263932        0.0841175
0.900000        -1.2824812             -1.3713567        0.0888755
1.000000        -1.5502865             -1.6408591        0.0905726
1.100000        -1.8441861             -1.9328815        0.0886954
1.200000        -2.1609509             -2.2438063        0.0828554
1.300000        -2.4950933             -2.5679753        0.0728820
1.400000        -2.8382065             -2.8971552        0.0589487
1.500000        -3.1781545             -3.2198944        0.0417399
1.600000        -3.4980809             -3.5207515        0.0226706
1.700000        -3.7752022             -3.7793722        0.0041700
1.800000        -3.9793425             -3.9693901        0.0099523
1.900000        -4.0711589             -4.0571185        0.0140404
2.000000        -4.0000000             -4.0000000        0.0000000
```

Actual solution and Approximate solution Plot for 2 Part b

Absolute Errors Plot for 2 Part b

N vs order plot for 2 Part b

N vs error (loglog plot) for 2 Part b

Using backward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.100000

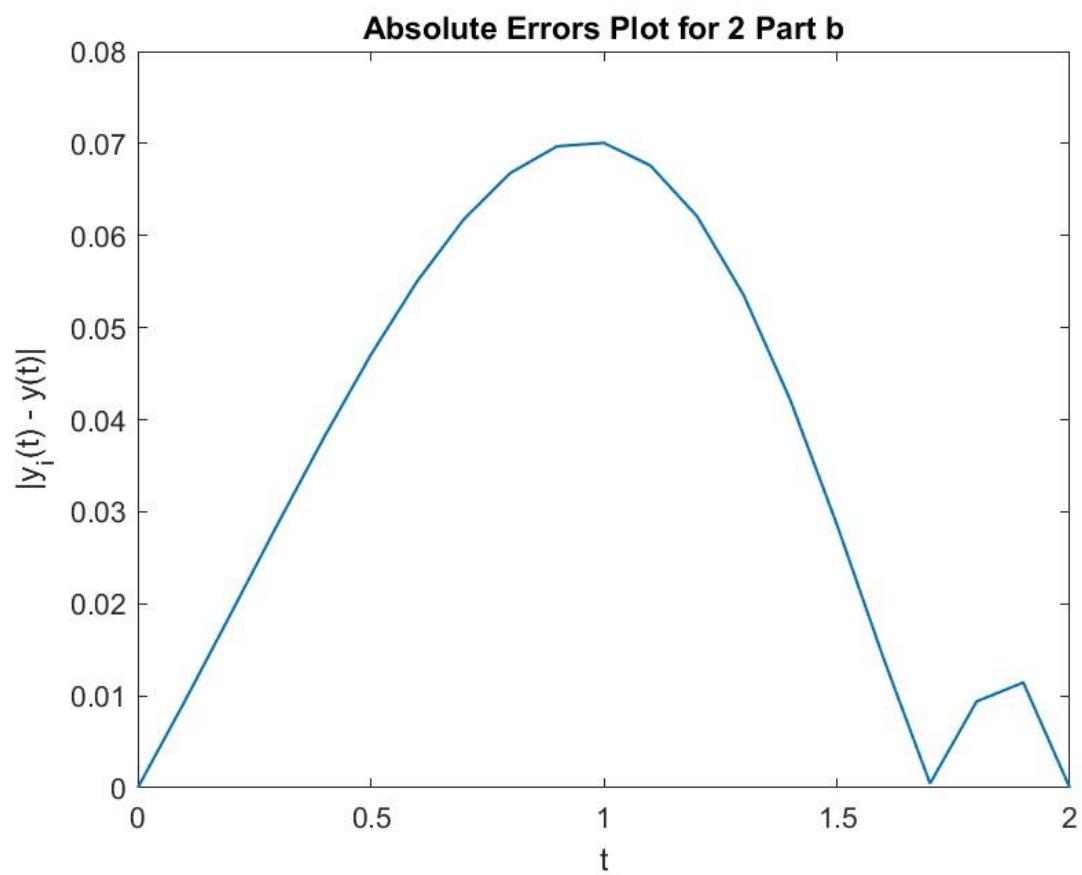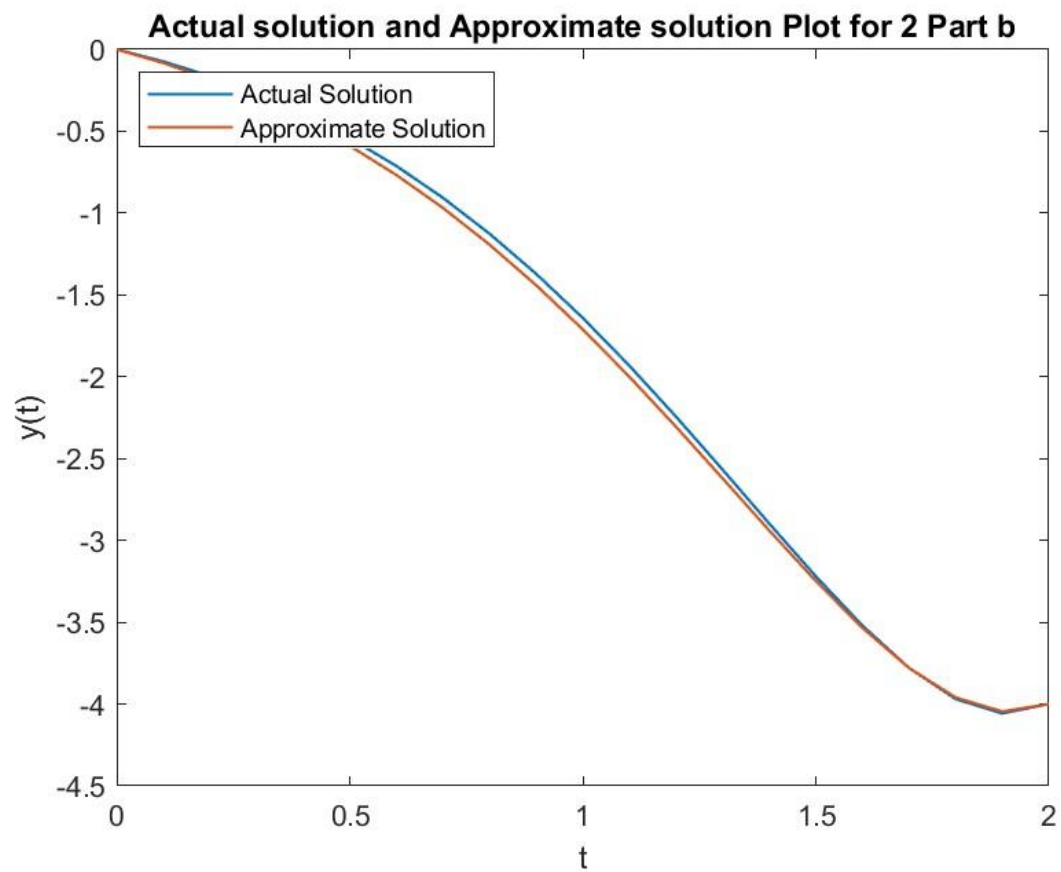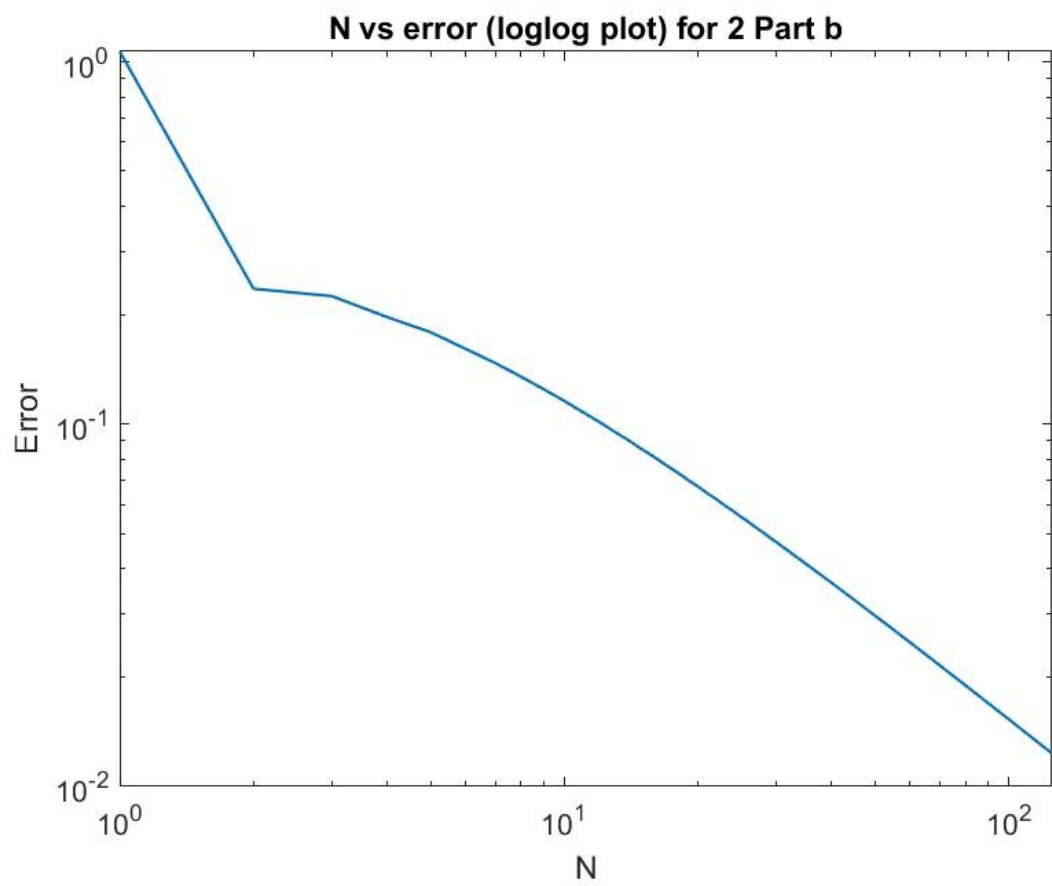| x	Approximate Solution	Exact Solution	Absolute Error
0.000000	0.0000000	0.0000000	0.0000000
0.100000	-0.0829901	-0.0736691	0.0093210
0.200000	-0.1816433	-0.1627002	0.0189431
0.300000	-0.2977677	-0.2691374	0.0286303
0.400000	-0.4330899	-0.3949876	0.0381023
0.500000	-0.5891782	-0.5421435	0.0470348
0.600000	-0.7673489	-0.7122849	0.0550640
0.700000	-0.9685475	-0.9067532	0.0617943
0.800000	-1.1932040	-1.1263932	0.0668108
0.900000	-1.4410555	-1.3713567	0.0696988
1.000000	-1.7109303	-1.6408591	0.0700712
1.100000	-2.0004880	-1.9328815	0.0676065
1.200000	-2.3059064	-2.2438063	0.0621001
1.300000	-2.6215081	-2.5679753	0.0535328
1.400000	-2.9393142	-2.8971552	0.0421590
1.500000	-3.2485156	-3.2198944	0.0286212
1.600000	-3.5348467	-3.5207515	0.0140953
1.700000	-3.7798471	-3.7793722	0.0004749
1.800000	-3.9599920	-3.9693901	0.0093981
1.900000	-4.0456723	-4.0571185	0.0114462
2.000000	-4.0000000	-4.0000000	0.0000000

Actual solution and Approximate solution Plot for 2 Part b

Absolute Errors Plot for 2 Part b

N vs order plot for 2 Part b
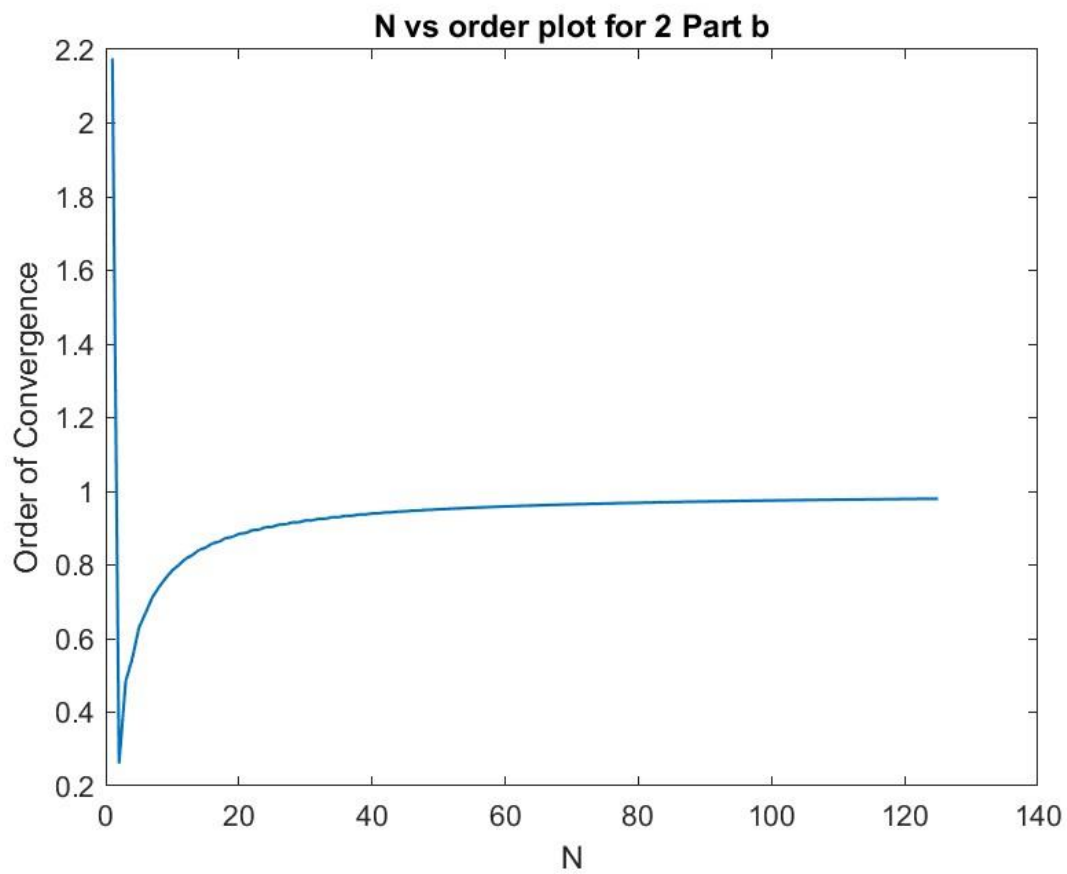
N vs error (loglog plot) for 2 Part b

Using central difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.100000

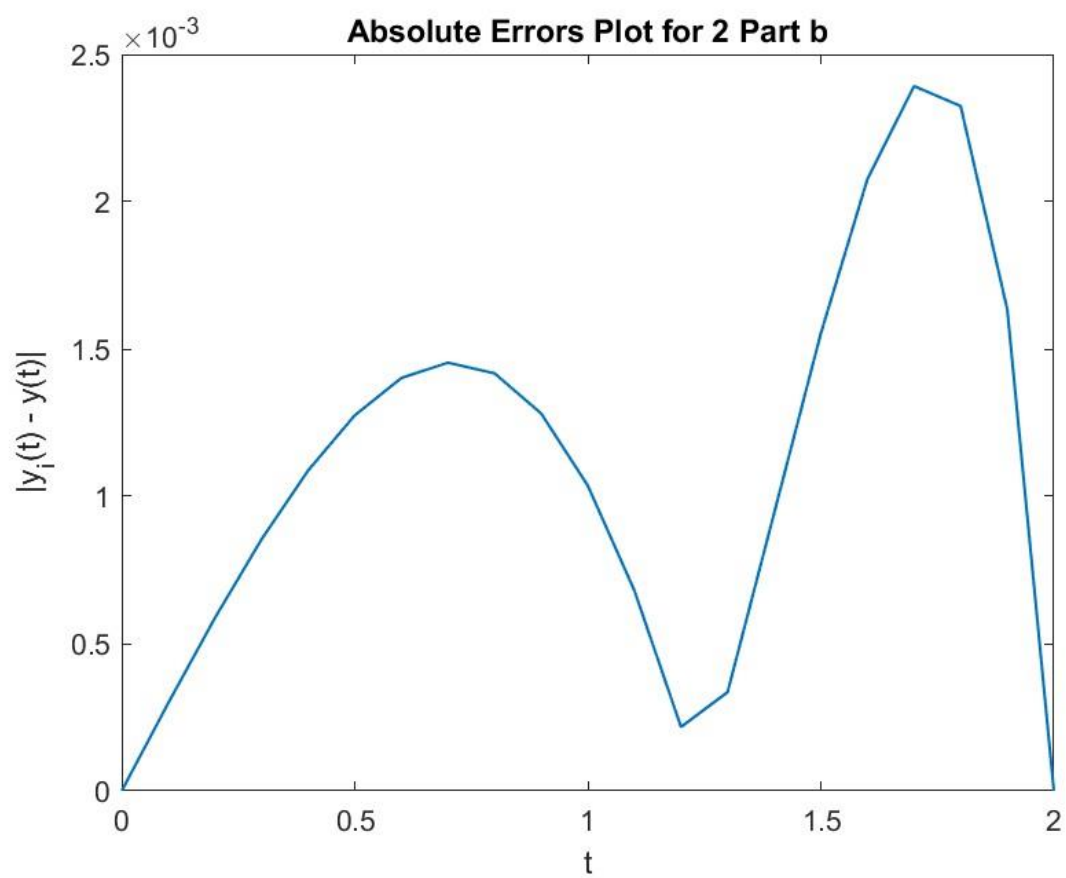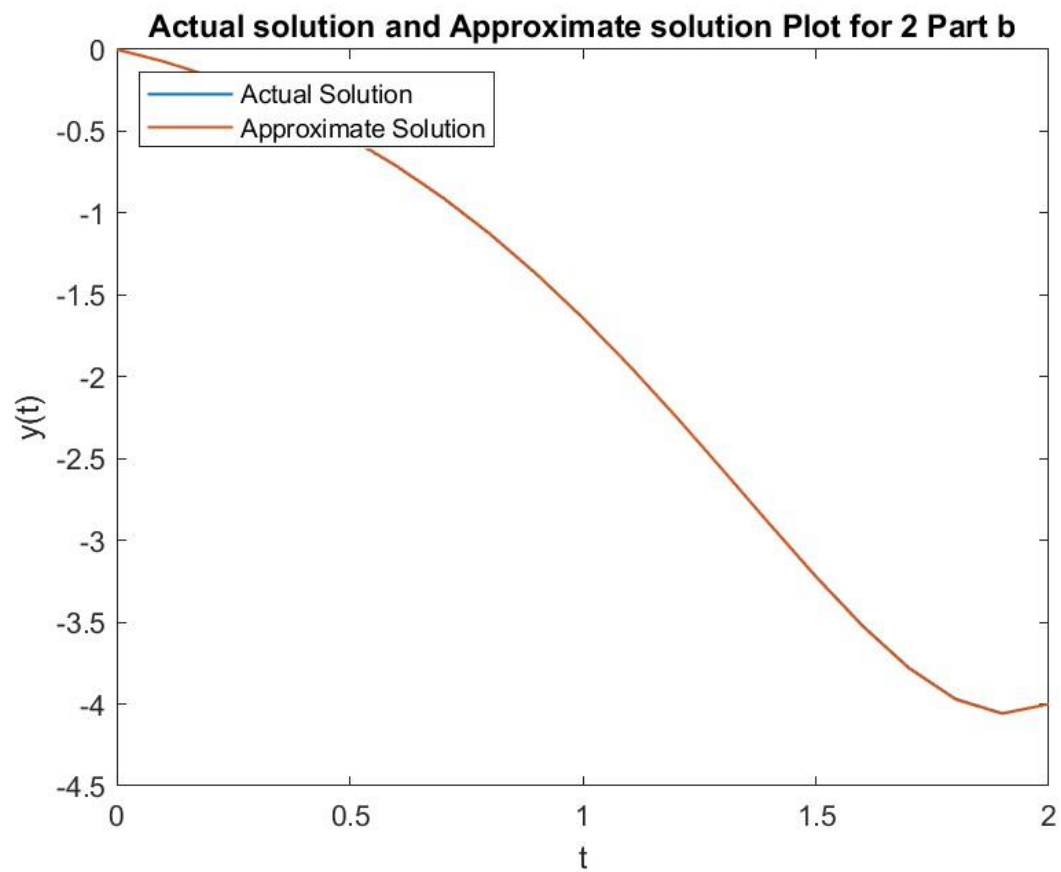| x	Approximate Solution	Exact Solution	Absolute Error
0.000000	0.0000000	0.0000000	0.0000000
0.100000	-0.0733704	-0.0736691	0.0002987
0.200000	-0.1621132	-0.1627002	0.0005870
0.300000	-0.2682834	-0.2691374	0.0008540
0.400000	-0.3938999	-0.3949876	0.0010877
0.500000	-0.5408686	-0.5421435	0.0012749
0.600000	-0.7108833	-0.7122849	0.0014016
0.700000	-0.9052996	-0.9067532	0.0014537
0.800000	-1.1249758	-1.1263932	0.0014174
0.900000	-1.3700756	-1.3713567	0.0012811
1.000000	-1.6398229	-1.6408591	0.0010362
1.100000	-1.9322018	-1.9328815	0.0006797
1.200000	-2.2435895	-2.2438063	0.0002169
1.300000	-2.5683108	-2.5679753	0.0003355
1.400000	-2.8980991	-2.8971552	0.0009438
1.500000	-3.2214472	-3.2198944	0.0015528
1.600000	-3.5228285	-3.5207515	0.0020770
1.700000	-3.7817647	-3.7793722	0.0023924
1.800000	-3.9717148	-3.9693901	0.0023246
1.900000	-4.0587529	-4.0571185	0.0016344
2.000000	-4.0000000	-4.0000000	0.0000000

Actual solution and Approximate solution Plot for 2 Part b

Absolute Errors Plot for 2 Part b

N vs order plot for 2 Part b

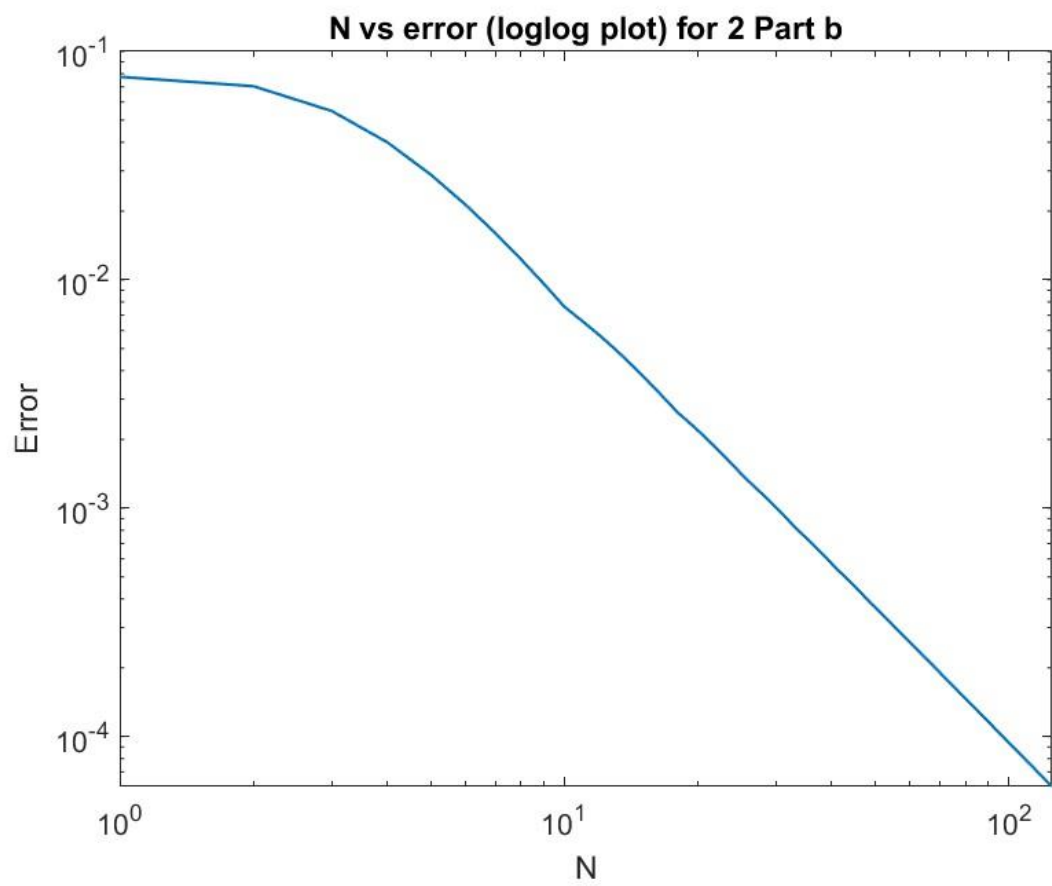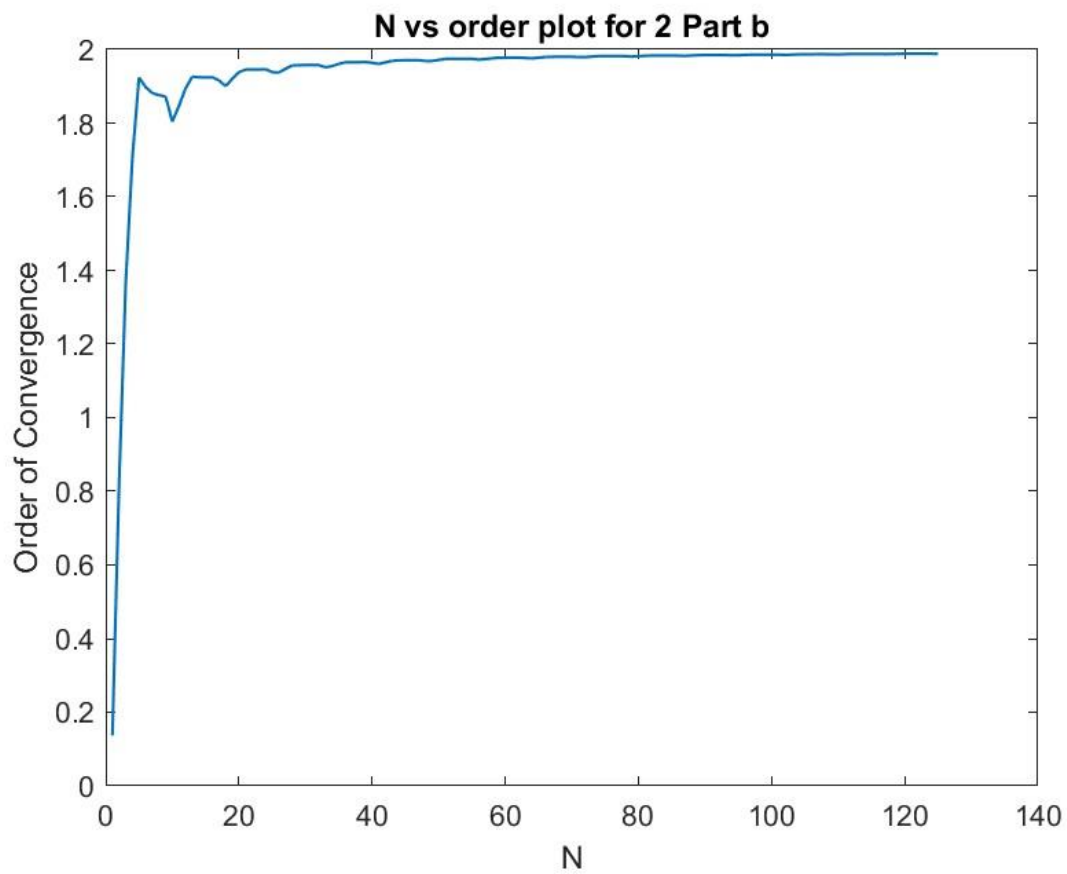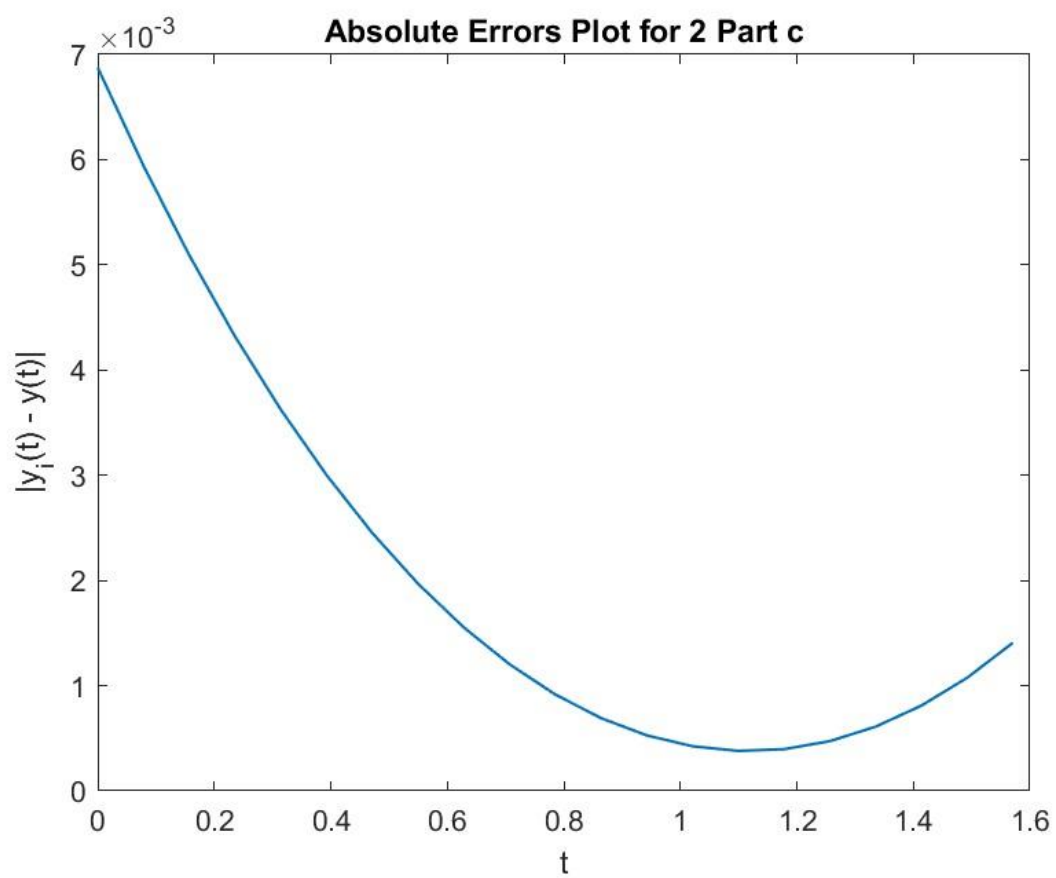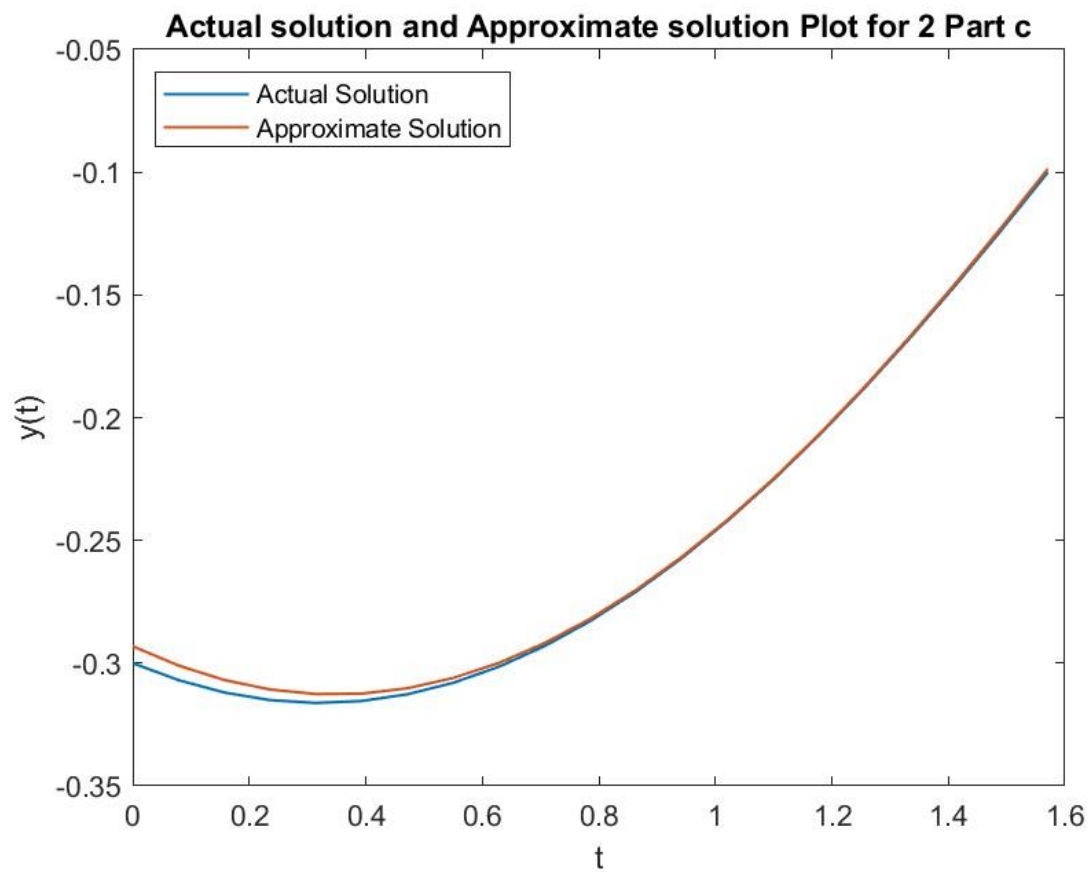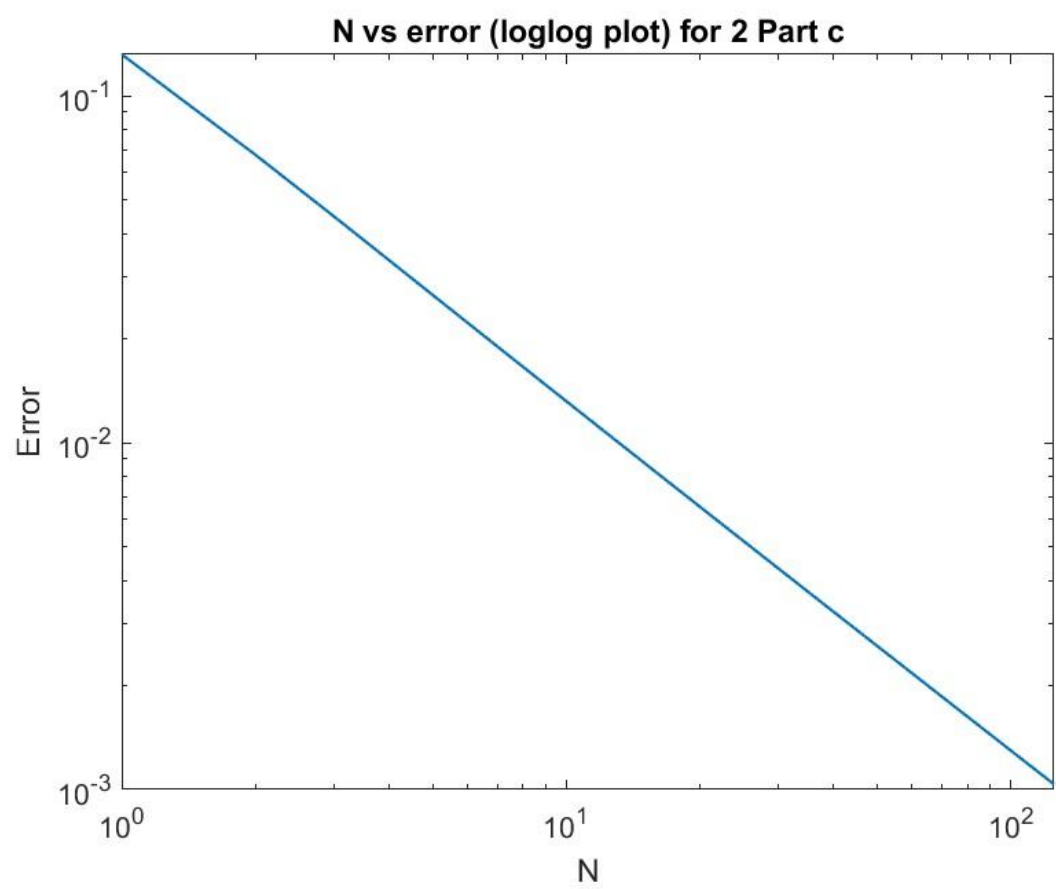N vs error (loglog plot) for 2 Part b

c)

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Question 2 Part c
Using forward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.078540
     x        Approximate Solution    Exact Solution   Absolute Error
0.000000        -0.2931252            -0.3000000         0.0068748
0.078540        -0.3009792            -0.3069211         0.0059420
0.157080        -0.3068586            -0.3119499         0.0050913
0.235619        -0.3107357            -0.3150555         0.0043198
0.314159        -0.3125943            -0.3162187         0.0036244
0.392699        -0.3124298            -0.3154322         0.0030024
0.471239        -0.3102496            -0.3127010         0.0024514
0.549779        -0.3060727            -0.3080419         0.0019692
0.628319        -0.2999299            -0.3014836         0.0015537
0.706858        -0.2918633            -0.2930666         0.0012033
0.785398        -0.2819265            -0.2828427         0.0009162
0.863938        -0.2701837            -0.2708750         0.0006913
0.942478        -0.2567098            -0.2572373         0.0005275
1.021018        -0.2415896            -0.2420136         0.0004240
1.099557        -0.2249175            -0.2252978         0.0003803
1.178097        -0.2067965            -0.2071930         0.0003964
1.256637        -0.1873380            -0.1878107         0.0004728
1.335177        -0.1666604            -0.1672706         0.0006102
1.413717        -0.1448890            -0.1456992         0.0008102
1.492257        -0.1221545            -0.1232295         0.0010749
1.570796        -0.0985926            -0.1000000         0.0014074
```

Actual solution and Approximate solution Plot for 2 Part c

Absolute Errors Plot for 2 Part c

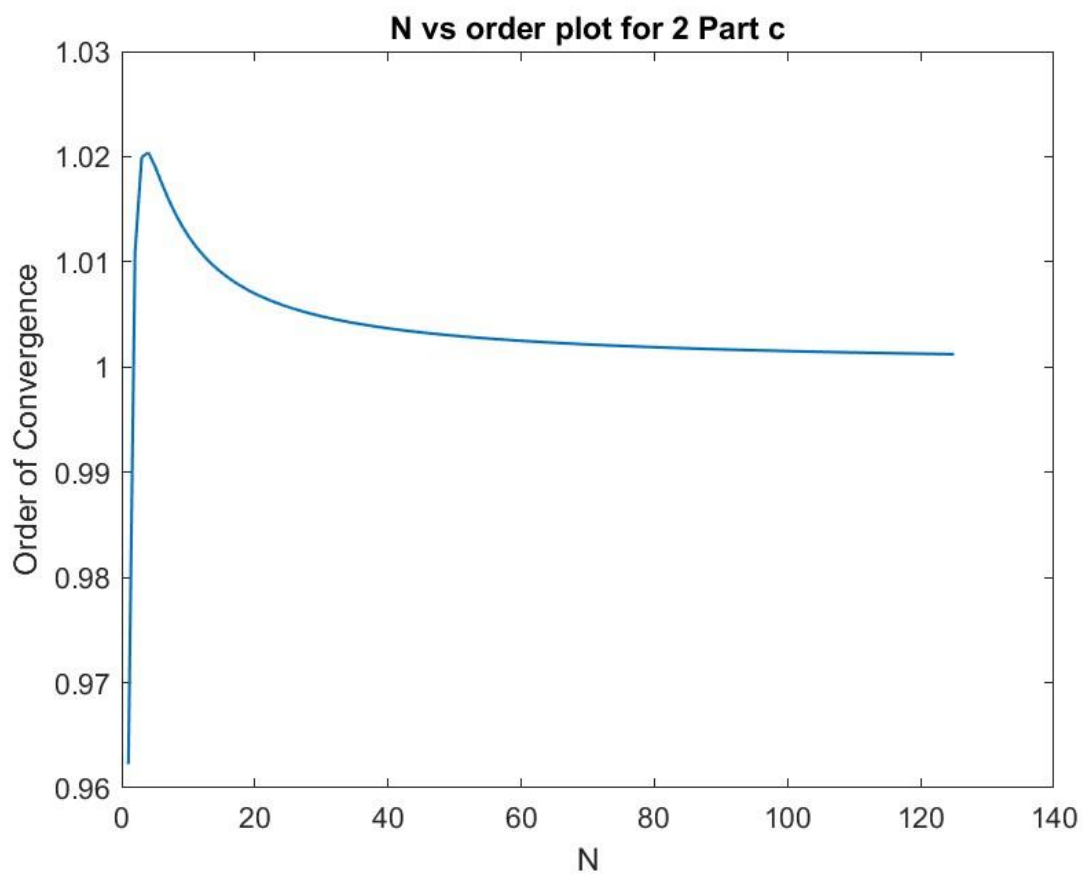**N vs order plot for 2 Part c**

N vs error (loglog plot) for 2 Part c

Using backward difference for the first-order derivative and central difference for the second-order derivative,
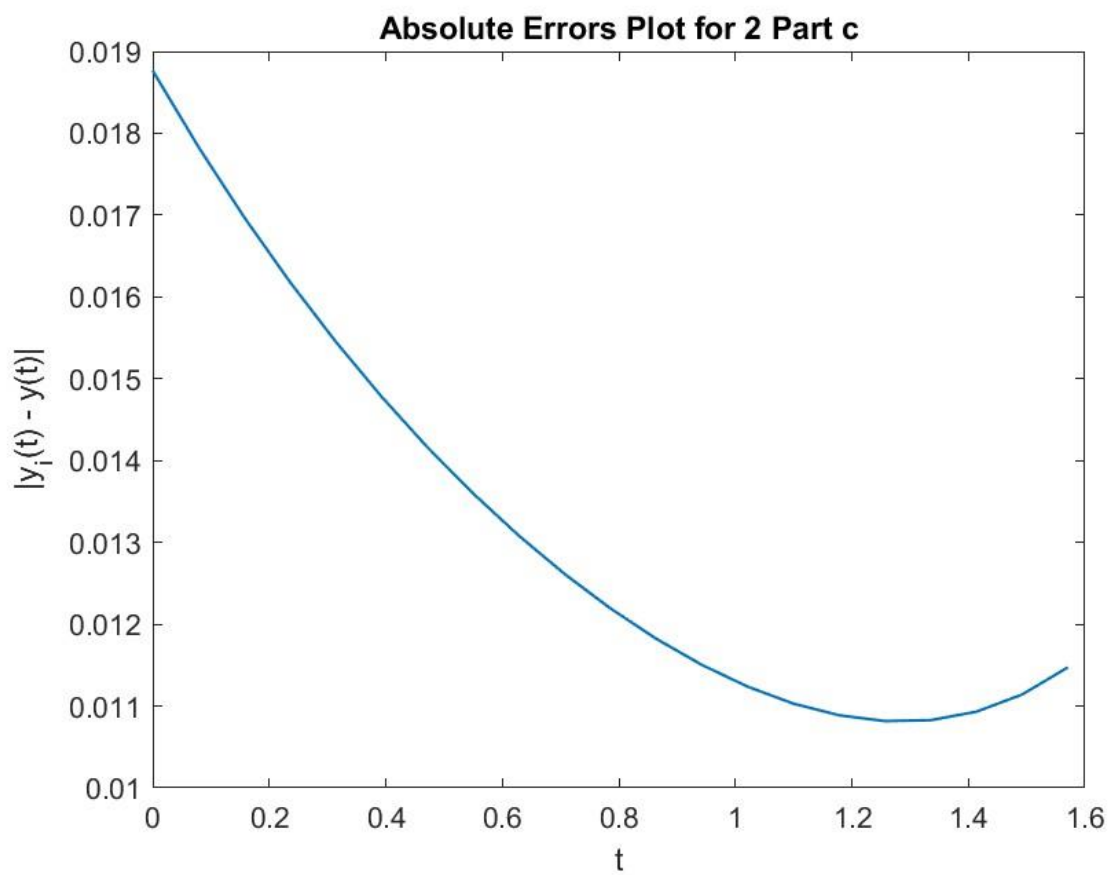For h = 0.078540

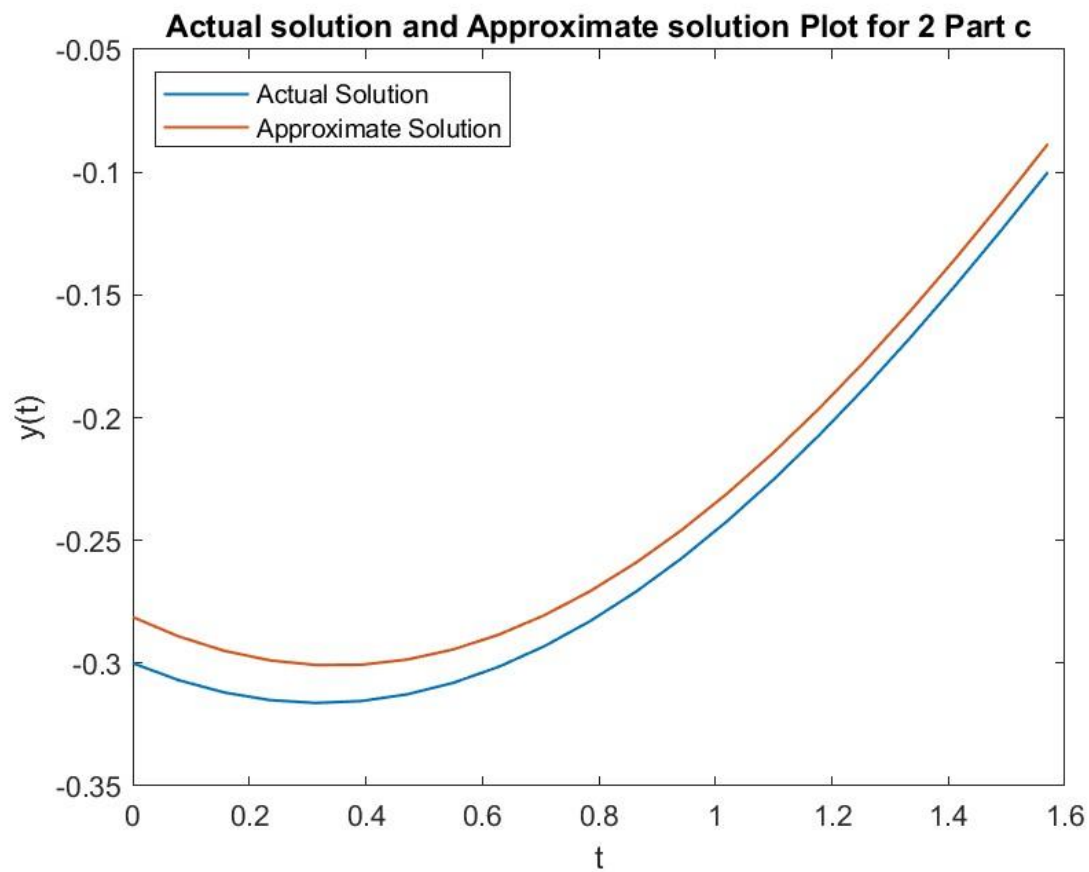| x	Approximate Solution	Exact Solution	Absolute Error
0.000000	-0.2812352	-0.3000000	0.0187648
0.078540	-0.2890892	-0.3069211	0.0178319
0.157080	-0.2949771	-0.3119499	0.0169729
0.235619	-0.2988739	-0.3150555	0.0161816
0.314159	-0.3007659	-0.3162187	0.0154527
0.392699	-0.3006506	-0.3154322	0.0147817
0.471239	-0.2985363	-0.3127010	0.0141647
0.549779	-0.2944428	-0.3080419	0.0135991
0.628319	-0.2884008	-0.3014836	0.0130828
0.706858	-0.2804519	-0.2930666	0.0126147
0.785398	-0.2706481	-0.2828427	0.0121946
0.863938	-0.2590515	-0.2708750	0.0118235
0.942478	-0.2457338	-0.2572373	0.0115034
1.021018	-0.2307761	-0.2420136	0.0112375
1.099557	-0.2142677	-0.2252978	0.0110301
1.178097	-0.1963056	-0.2071930	0.0108874
1.256637	-0.1769941	-0.1878107	0.0108167
1.335177	-0.1564432	-0.1672706	0.0108274
1.413717	-0.1347683	-0.1456992	0.0109309
1.492257	-0.1120888	-0.1232295	0.0111407
1.570796	-0.0885268	-0.1000000	0.0114732

Actual solution and Approximate solution Plot for 2 Part c

Absolute Errors Plot for 2 Part c

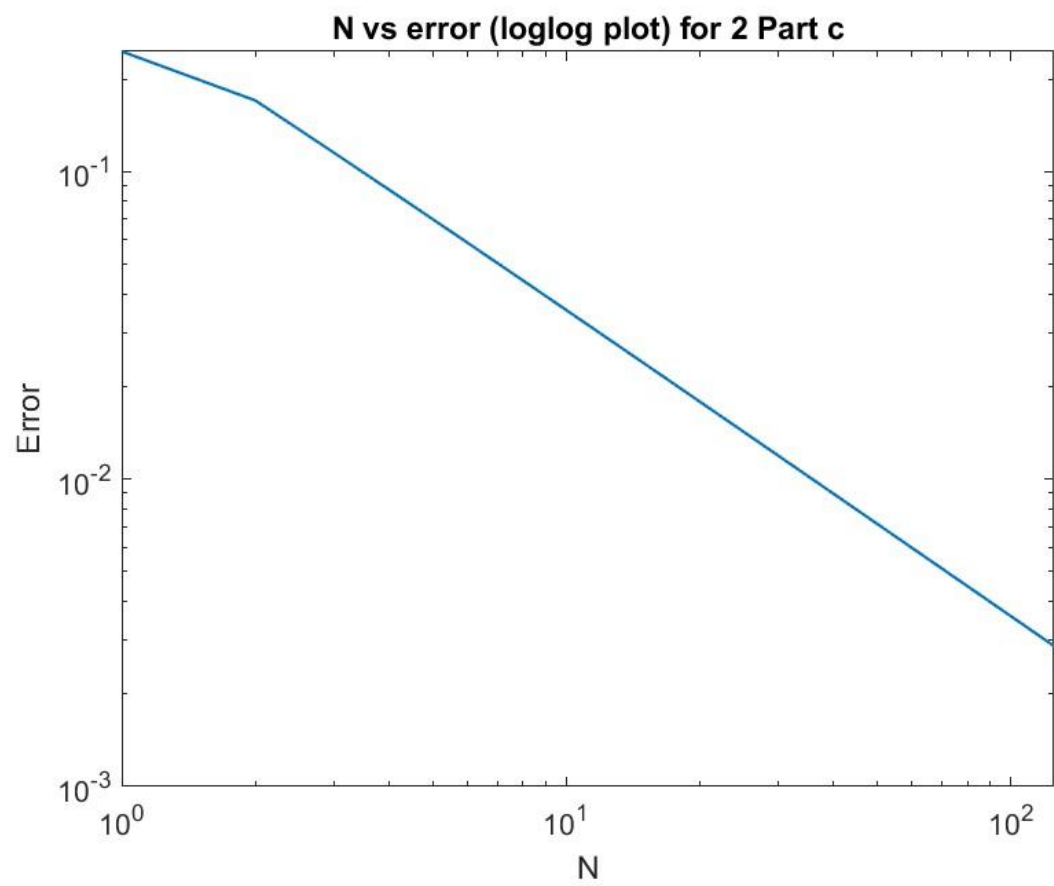**N vs order plot for 2 Part c**

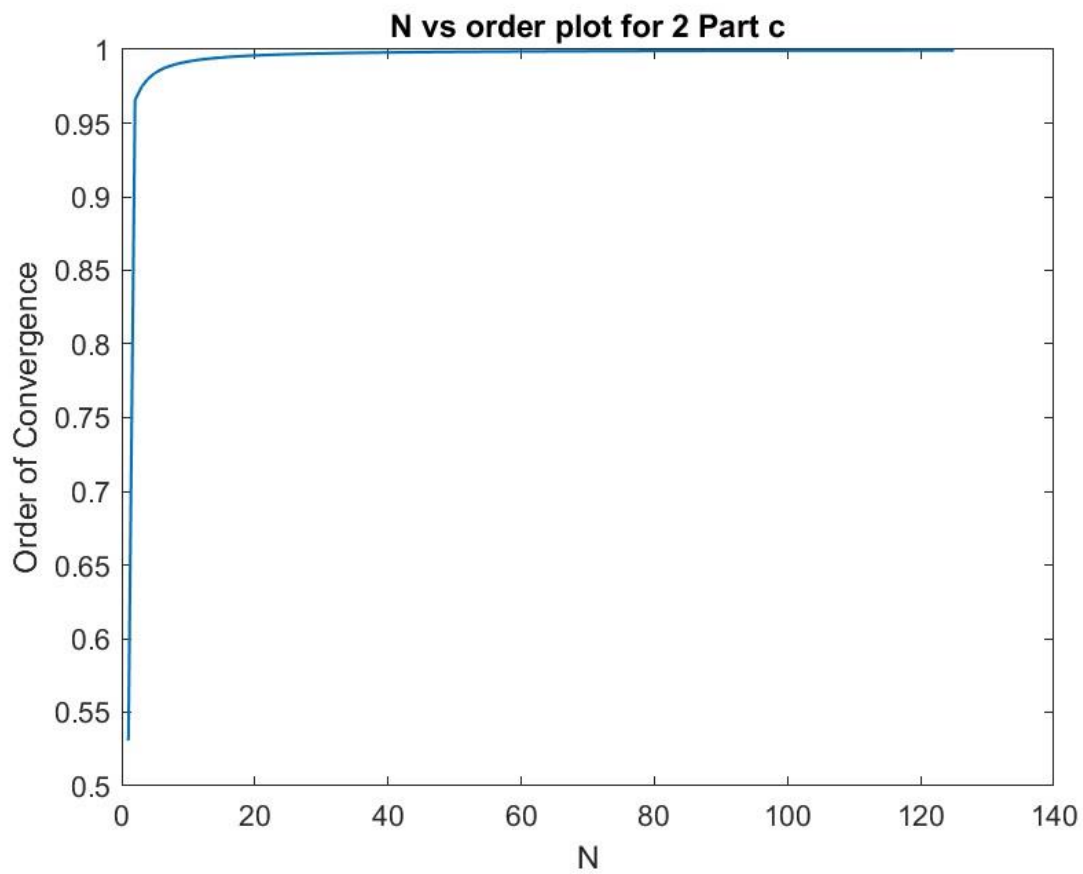**N vs error (loglog plot) for 2 Part c**

Using central difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.078540

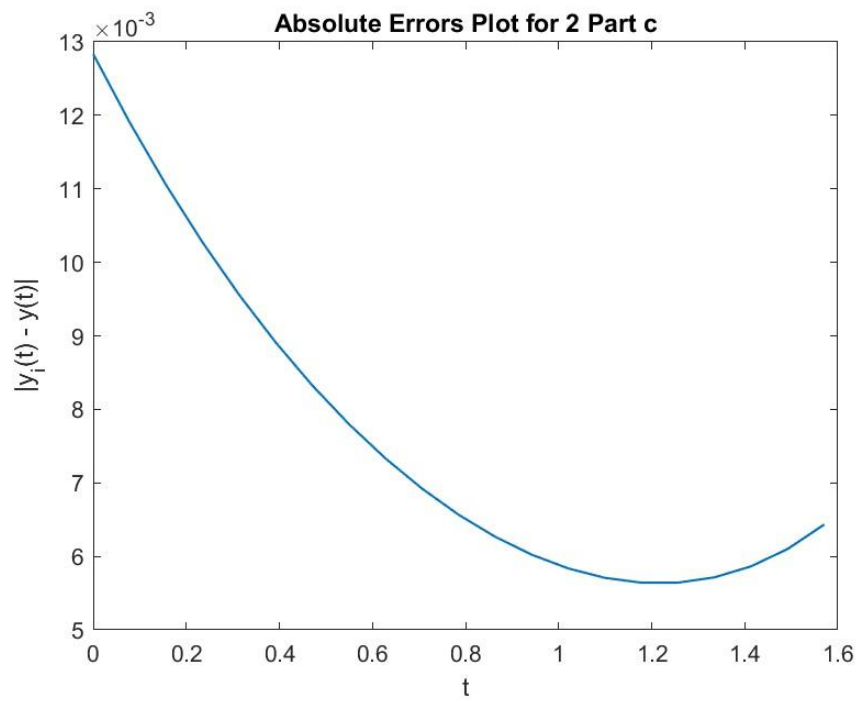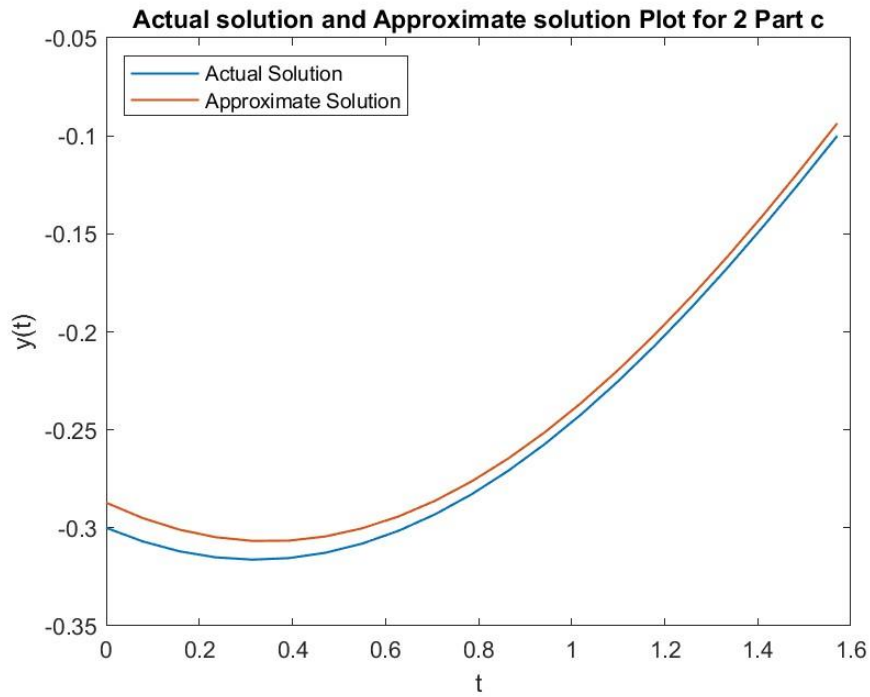| x	Approximate Solution	Exact Solution	Absolute Error
0.000000	-0.2871653	-0.3000000	0.0128347
0.078540	-0.2950193	-0.3069211	0.0119018
0.157080	-0.3009029	-0.3119499	0.0110470
0.235619	-0.3047899	-0.3150555	0.0102656
0.314159	-0.3066654	-0.3162187	0.0095533
0.392699	-0.3065257	-0.3154322	0.0089065
0.471239	-0.3043789	-0.3127010	0.0083221
0.549779	-0.3002443	-0.3080419	0.0077976
0.628319	-0.2941528	-0.3014836	0.0073308
0.706858	-0.2861462	-0.2930666	0.0069204
0.785398	-0.2762772	-0.2828427	0.0065655
0.863938	-0.2646092	-0.2708750	0.0062658
0.942478	-0.2512153	-0.2572373	0.0060220
1.021018	-0.2361784	-0.2420136	0.0058352
1.099557	-0.2195904	-0.2252978	0.0057075
1.178097	-0.2015511	-0.2071930	0.0056419
1.256637	-0.1821683	-0.1878107	0.0056425
1.335177	-0.1615560	-0.1672706	0.0057146
1.413717	-0.1398345	-0.1456992	0.0058647
1.492257	-0.1171284	-0.1232295	0.0061010
1.570796	-0.0935665	-0.1000000	0.0064335

Actual solution and Approximate solution Plot for 2 Part c

Actual Solution
Approximate Solution

Absolute Errors Plot for 2 Part c

N vs order plot for 2 Part c

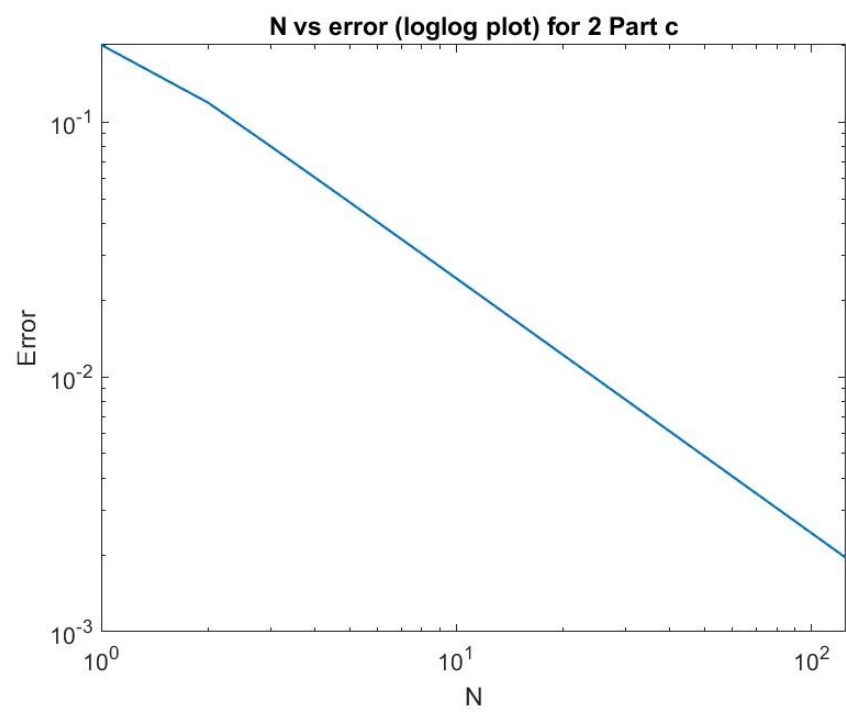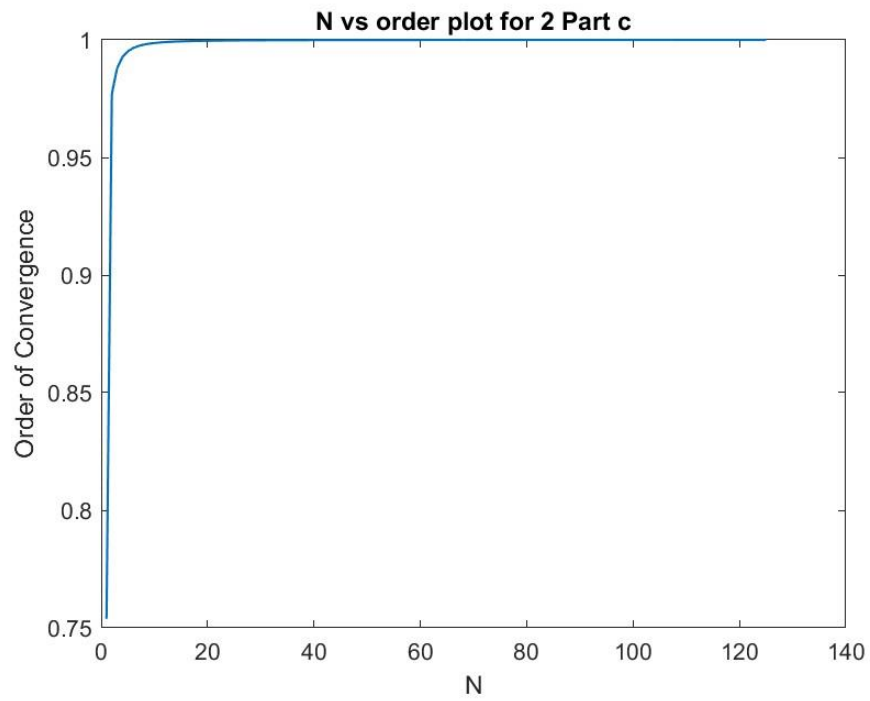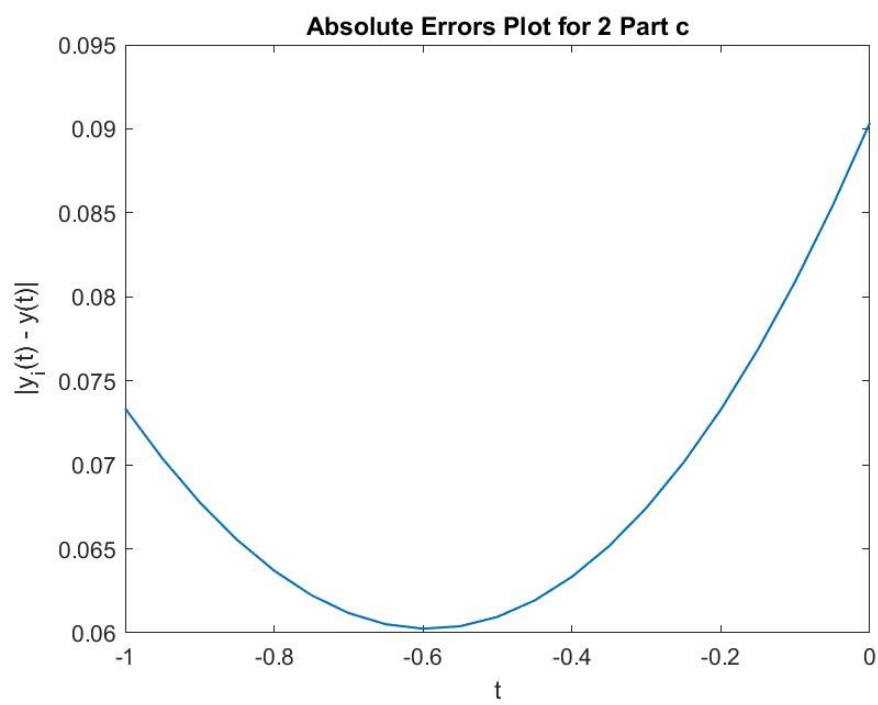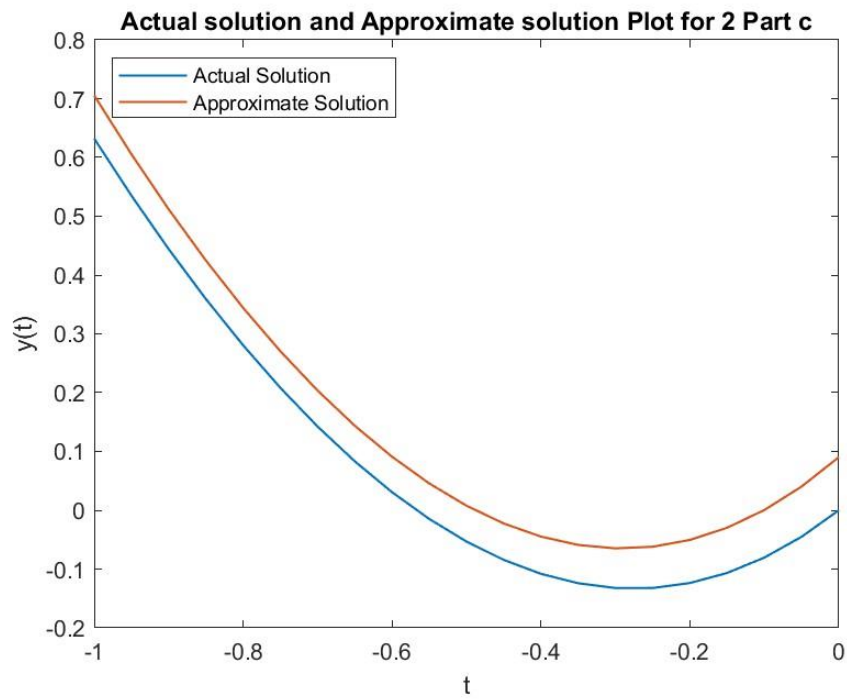**N vs error (loglog plot) for 2 Part c**

d)

```
Question 2 Part d
Using forward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.050000
     x        Approximate Solution    Exact Solution   Absolute Error
-1.000000        0.7054778              0.6321206        0.0733573
-0.950000        0.6054778              0.5350960        0.0703818
-0.900000        0.5118649              0.4440873        0.0677776
-0.850000        0.4247471              0.3591973        0.0655498
-0.800000        0.3442407              0.2805368        0.0637039
-0.750000        0.2704702              0.2082251        0.0622451
-0.700000        0.2035694              0.1423903        0.0611791
-0.650000        0.1436815              0.0831702        0.0605113
-0.600000        0.0909603              0.0307130        0.0602473
-0.550000        0.0455702             -0.0148224        0.0603926
-0.500000        0.0076872             -0.0532653        0.0609526
-0.450000       -0.0225003             -0.0844327        0.0619324
-0.400000       -0.0447909             -0.1081280        0.0633372
-0.350000       -0.0589693             -0.1241408        0.0651716
-0.300000       -0.0648054             -0.1322455        0.0674401
-0.250000       -0.0620535             -0.1322002        0.0701467
-0.200000       -0.0504511             -0.1237462        0.0732950
-0.150000       -0.0297181             -0.1066062        0.0768881
-0.100000        0.0004444             -0.0804837        0.0809281
-0.050000        0.0403555             -0.0450615        0.0854170
0.000000         0.0903555              0.0000000        0.0903555
```

Actual solution and Approximate solution Plot for 2 Part c

Legend:
- Actual Solution
- Approximate Solution

x-axis: t
y-axis: y(t)

Absolute Errors Plot for 2 Part c

x-axis: t
y-axis: $|y_i(t) - y(t)|$
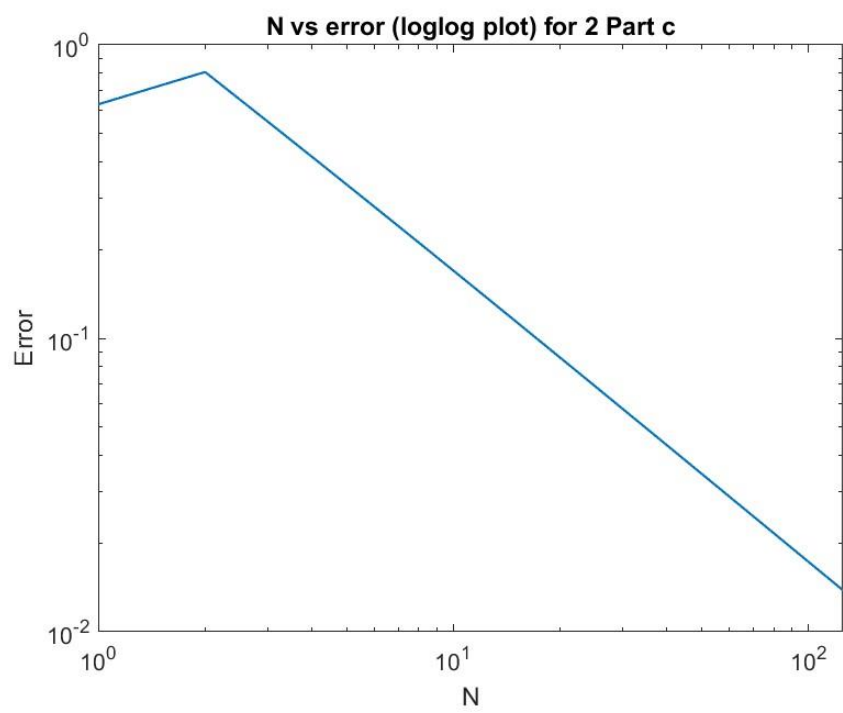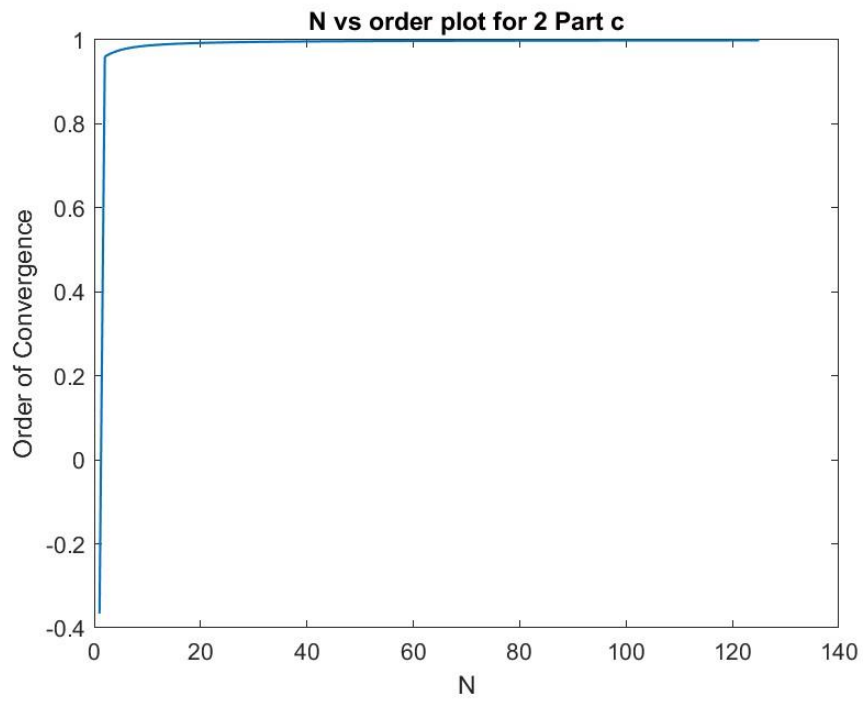
N vs order plot for 2 Part c

N vs error (loglog plot) for 2 Part c

Using backward difference for the first-order derivative and central difference for the second-order derivative,
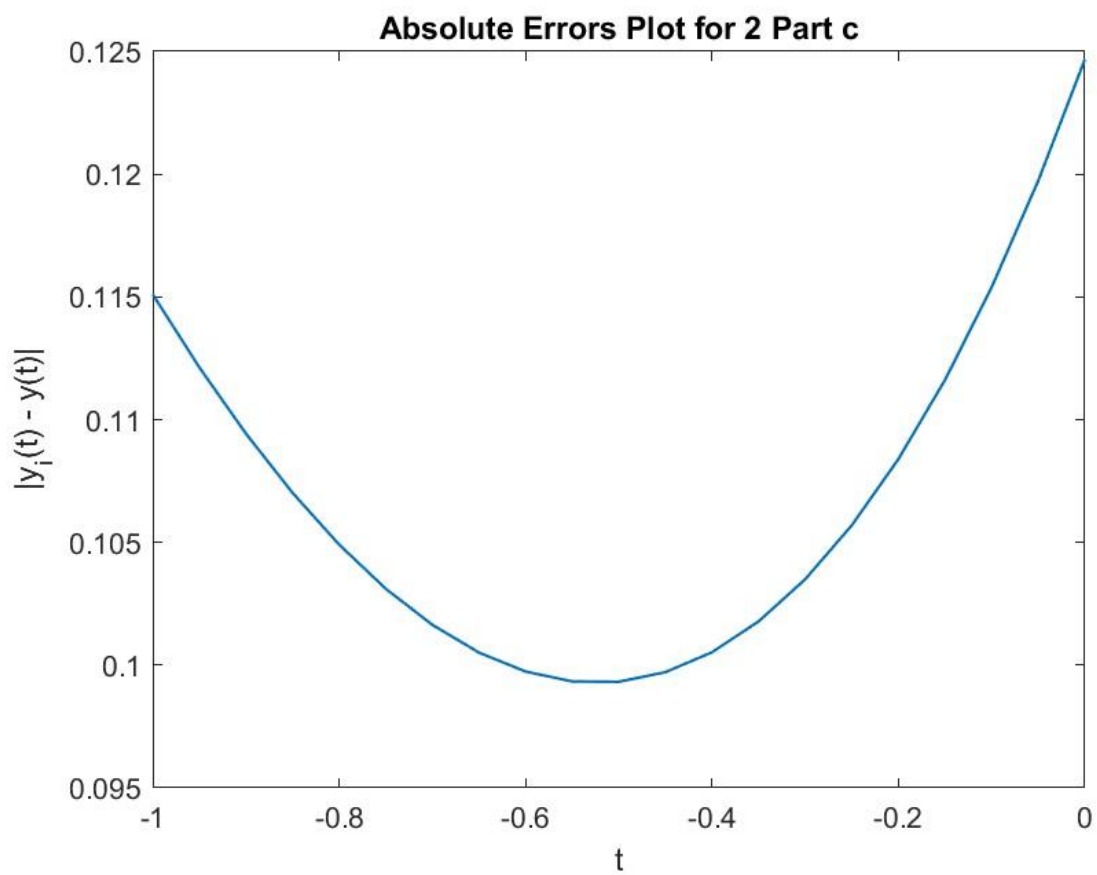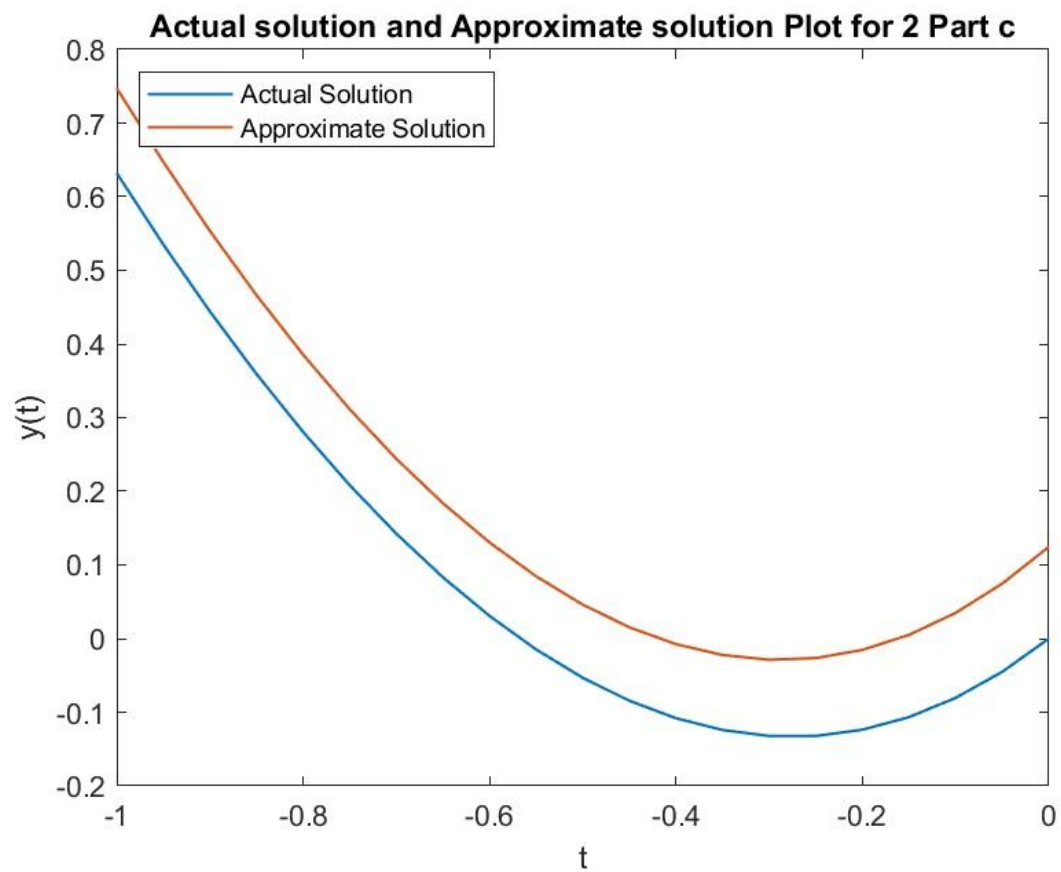For h = 0.050000

x	Approximate Solution	Exact Solution	Absolute Error
-1.000000	0.7472250	0.6321206	0.1151044
-0.950000	0.6472250	0.5350960	0.1121289
-0.900000	0.5535174	0.4440873	0.1094301
-0.850000	0.4662167	0.3591973	0.1070194
-0.800000	0.3854459	0.2805368	0.1049091
-0.750000	0.3113371	0.2082251	0.1031120
-0.700000	0.2440320	0.1423903	0.1016417
-0.650000	0.1836826	0.0831702	0.1005123
-0.600000	0.1304519	0.0307130	0.0997388
-0.550000	0.0845145	-0.0148224	0.0993369
-0.500000	0.0460575	-0.0532653	0.0993228
-0.450000	0.0152811	-0.0844327	0.0997137
-0.400000	-0.0076005	-0.1081280	0.1005276
-0.350000	-0.0223579	-0.1241408	0.1017829
-0.300000	-0.0287461	-0.1322455	0.1034993
-0.250000	-0.0265031	-0.1322002	0.1056970
-0.200000	-0.0153489	-0.1237462	0.1083972
-0.150000	0.0050156	-0.1066062	0.1116218
-0.100000	0.0349099	-0.0804837	0.1153936
-0.050000	0.0746750	-0.0450615	0.1197365
0.000000	0.1246750	0.0000000	0.1246750

Actual solution and Approximate solution Plot for 2 Part c

Legend: Actual Solution, Approximate Solution

y-axis: y(t)
x-axis: t

Absolute Errors Plot for 2 Part c

y-axis: $|y_i(t) - y(t)|$
x-axis: t

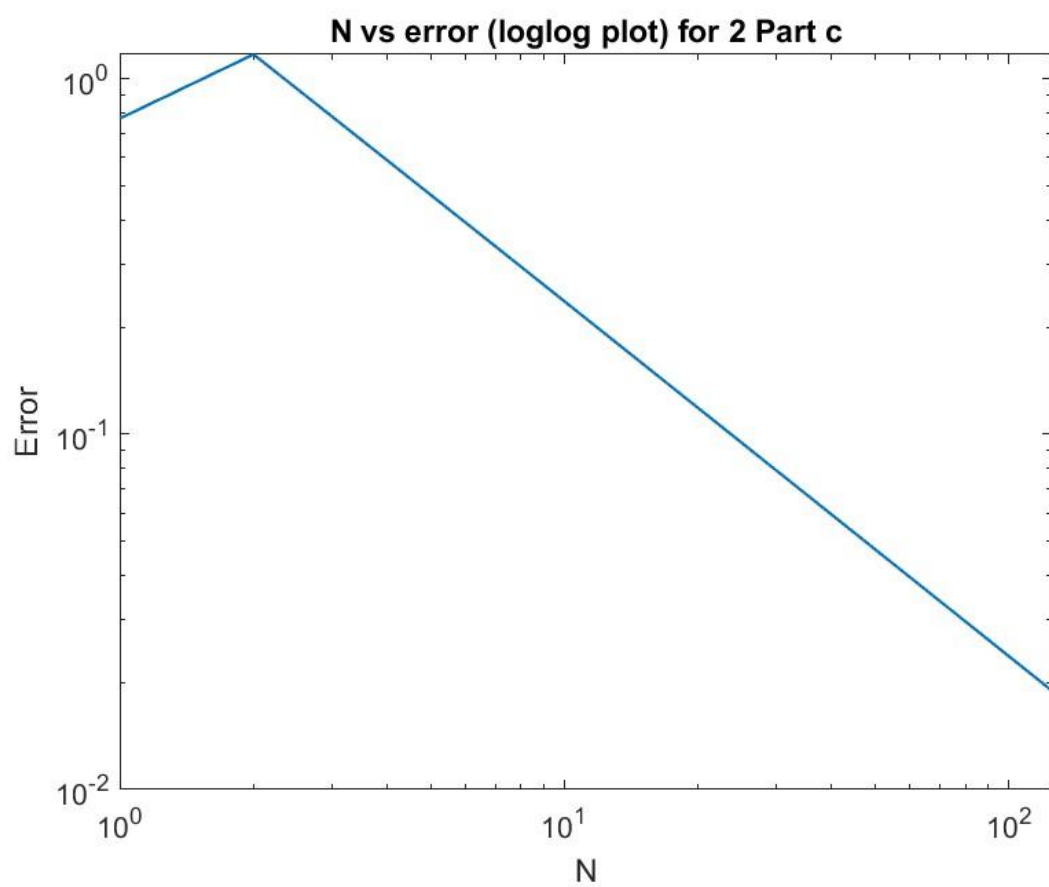N vs order plot for 2 Part c
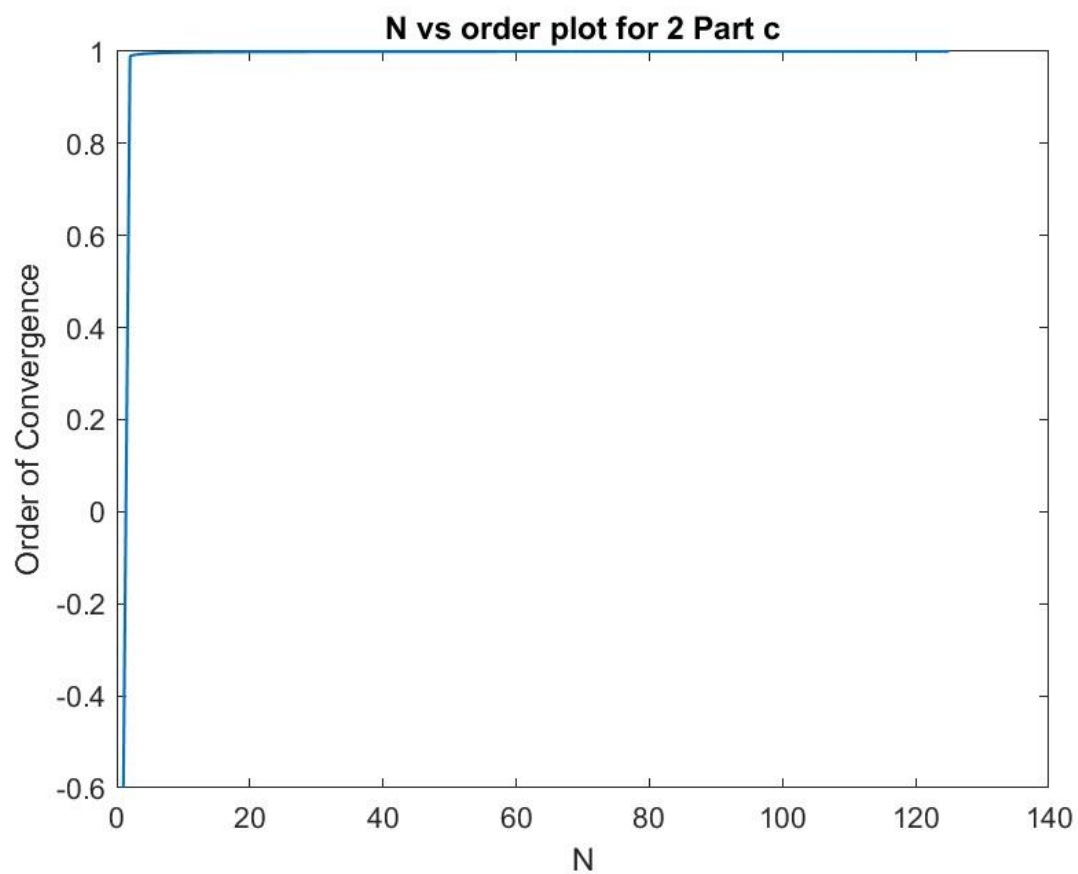
N vs error (loglog plot) for 2 Part c

```
Using central difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.050000
     x        Approximate Solution     Exact Solution  Absolute Error
-1.000000          0.7264312             0.6321206        0.0943107
-0.950000          0.6264312             0.5350960        0.0913352
-0.900000          0.5327702             0.4440873        0.0886829
-0.850000          0.4455596             0.3591973        0.0863623
-0.800000          0.3649191             0.2805368        0.0843822
-0.750000          0.2909770             0.2082251        0.0827519
-0.700000          0.2238713             0.1423903        0.0814810
-0.650000          0.1637497             0.0831702        0.0805795
-0.600000          0.1107706             0.0307130        0.0800576
-0.550000          0.0651037            -0.0148224        0.0799261
-0.500000          0.0269306            -0.0532653        0.0801959
-0.450000         -0.0035544            -0.0844327        0.0808783
-0.400000         -0.0261433            -0.1081280        0.0819847
-0.350000         -0.0406138            -0.1241408        0.0835270
-0.300000         -0.0467282            -0.1322455        0.0855173
-0.250000         -0.0442326            -0.1322002        0.0879676
-0.200000         -0.0328558            -0.1237462        0.0908903
-0.150000         -0.0123081            -0.1066062        0.0942981
-0.100000          0.0177196            -0.0804837        0.0982034
-0.050000          0.0575575            -0.0450615        0.1026189
0.000000           0.1075575             0.0000000        0.1075575
```
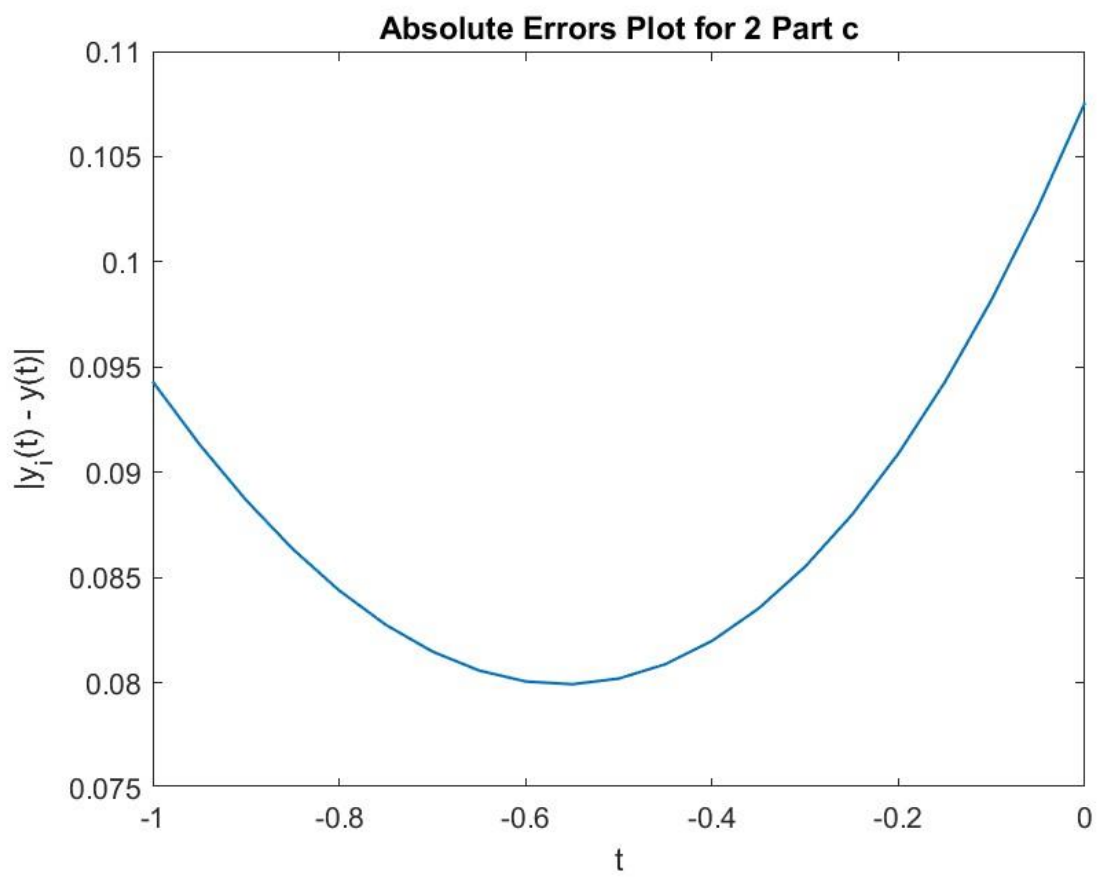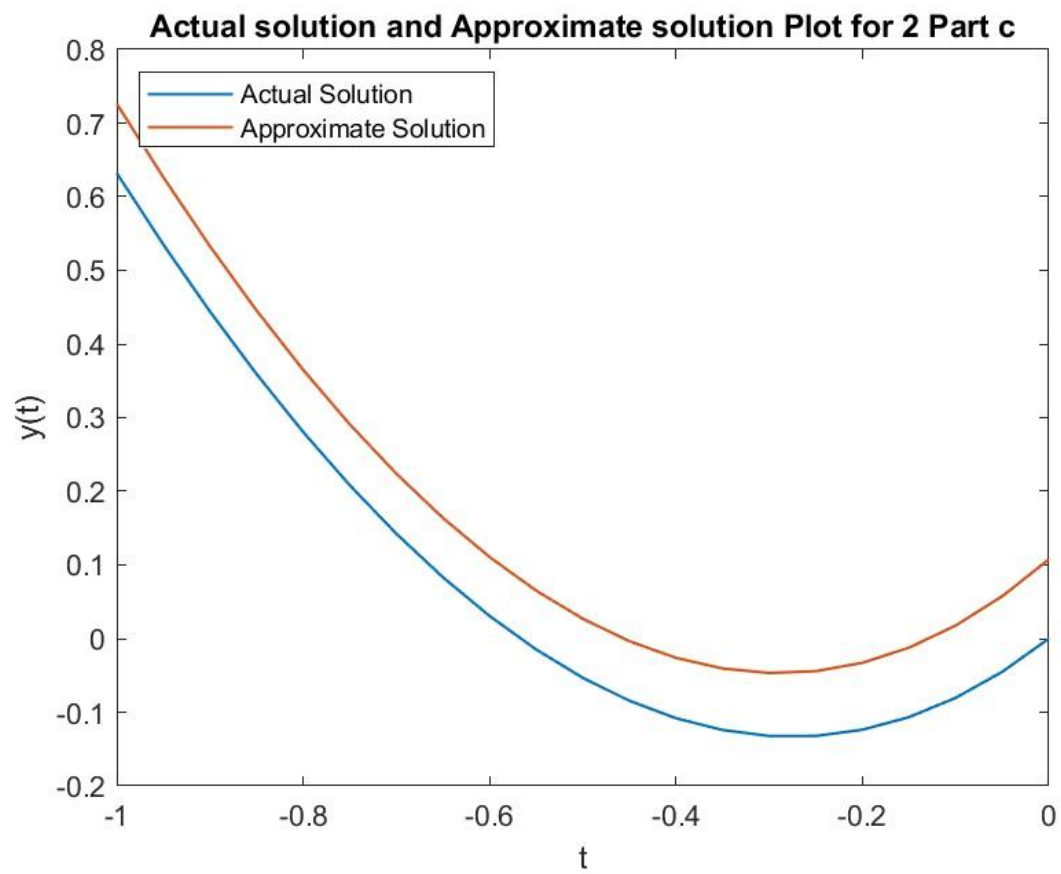
Actual solution and Approximate solution Plot for 2 Part c

Absolute Errors Plot for 2 Part c

N vs order plot for 2 Part c

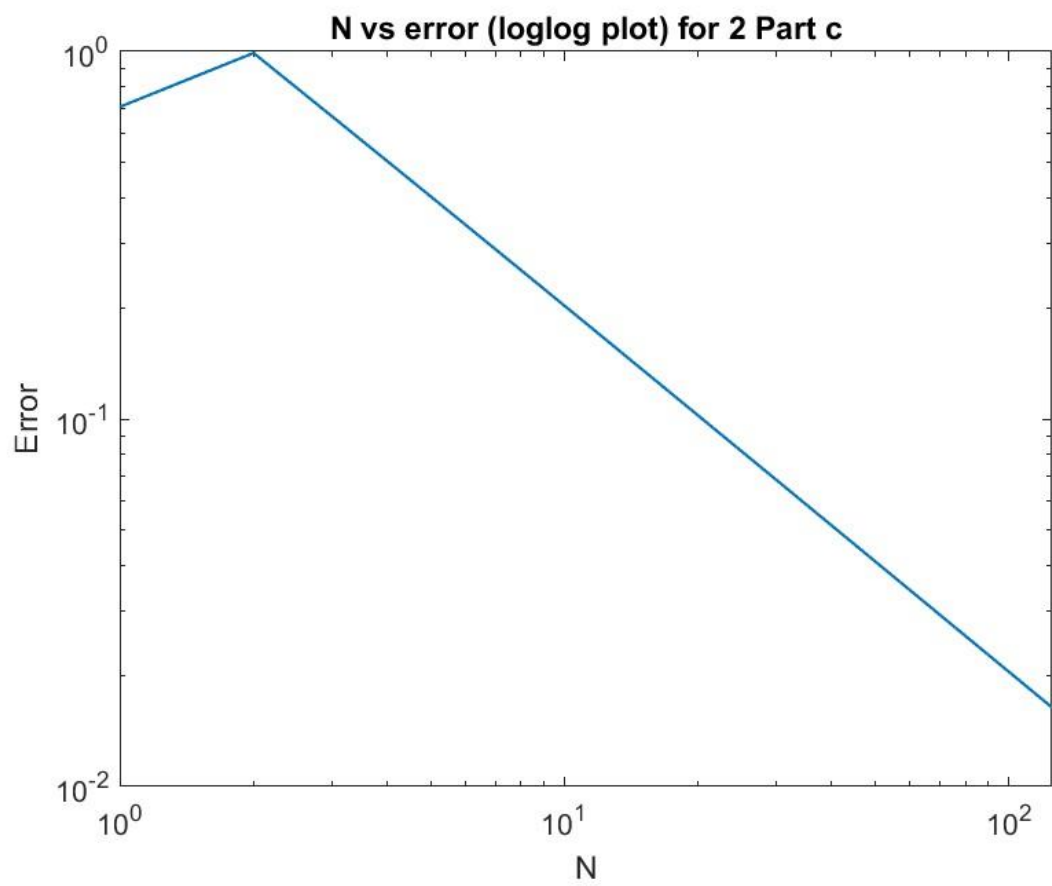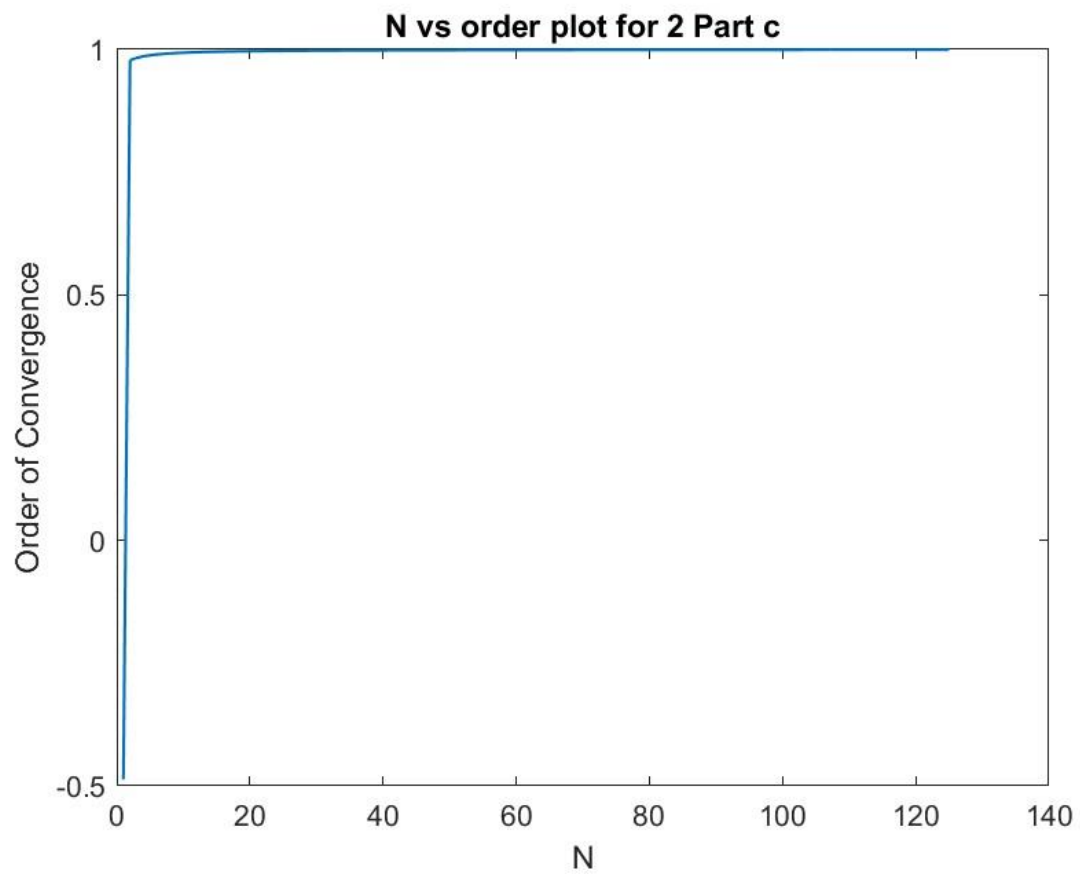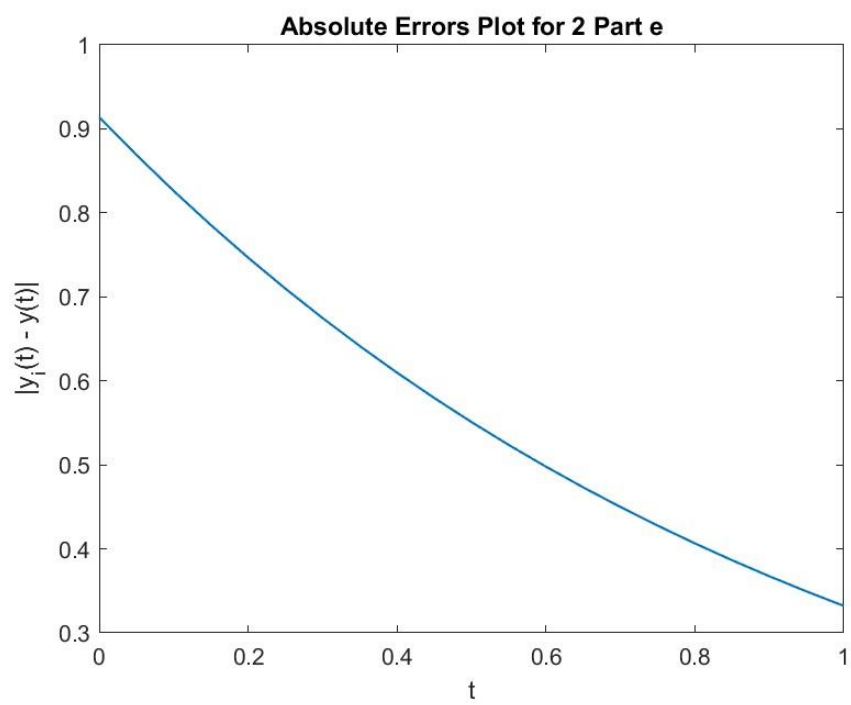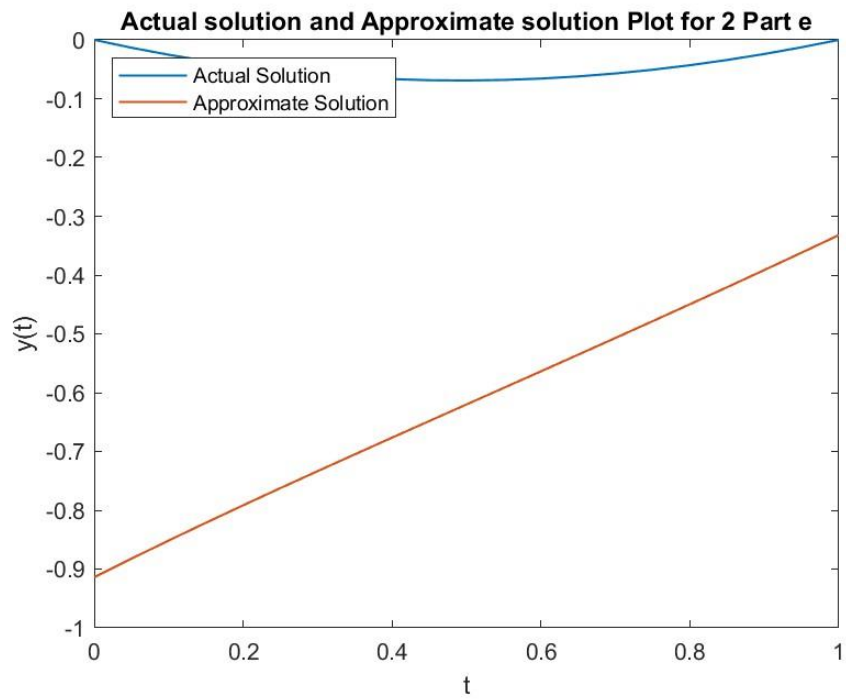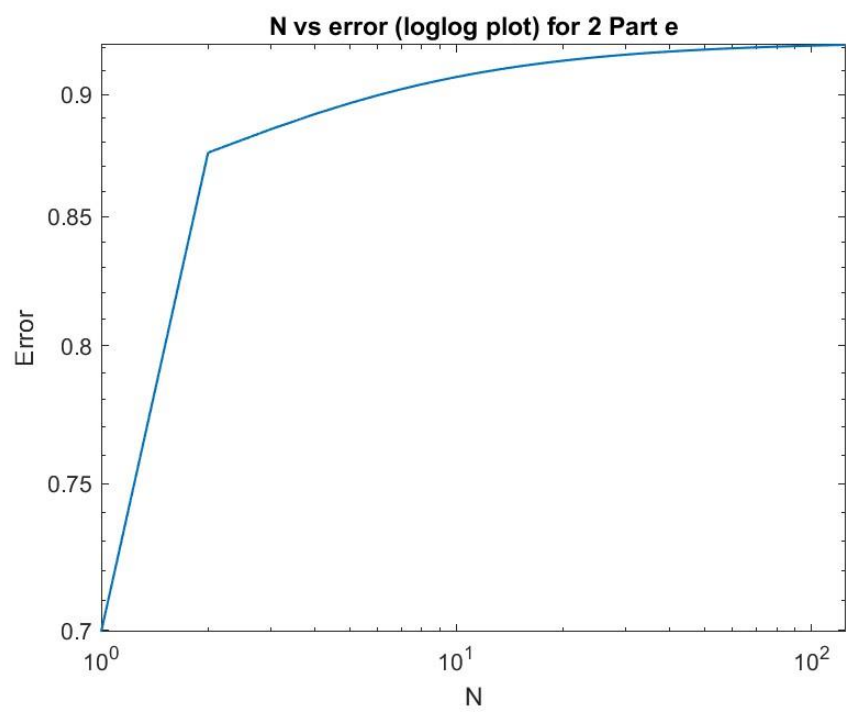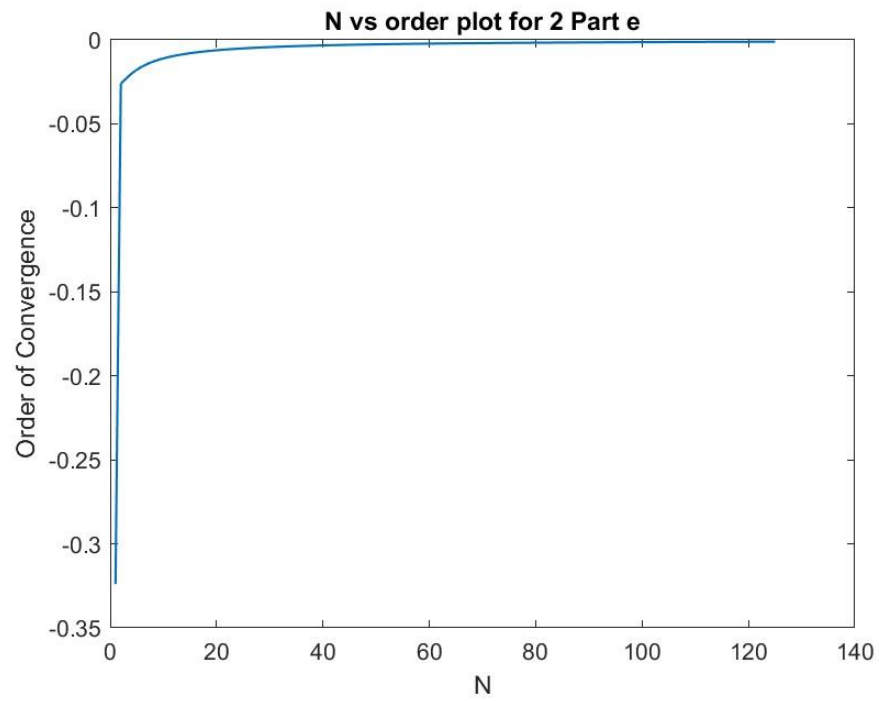N vs error (loglog plot) for 2 Part c

e)

```
Question 2 Part e
Using forward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.050000
     x        Approximate Solution    Exact Solution   Absolute Error
0.000000          -0.9139505            0.0000000        0.9139505
0.050000          -0.8823389           -0.0133786        0.8689603
0.100000          -0.8514821           -0.0253323        0.8261497
0.150000          -0.8212680           -0.0358494        0.7854186
0.200000          -0.7915932           -0.0449226        0.7466706
0.250000          -0.7623625           -0.0525488        0.7098137
0.300000          -0.7334884           -0.0587287        0.6747597
0.350000          -0.7048903           -0.0634662        0.6414241
0.400000          -0.6764945           -0.0667684        0.6097261
0.450000          -0.6482336           -0.0686452        0.5795885
0.500000          -0.6200459           -0.0691087        0.5509372
0.550000          -0.5918752           -0.0681735        0.5237017
0.600000          -0.5636703           -0.0658559        0.4978144
0.650000          -0.5353847           -0.0621740        0.4732107
0.700000          -0.5069765           -0.0571477        0.4498289
0.750000          -0.4784077           -0.0507977        0.4276101
0.800000          -0.4496443           -0.0431462        0.4064981
0.850000          -0.4206556           -0.0342164        0.3864392
0.900000          -0.3914144           -0.0240322        0.3673822
0.950000          -0.3618965           -0.0126184        0.3492781
1.000000          -0.3320804            0.0000000        0.3320804
```

Actual solution and Approximate solution Plot for 2 Part e

Absolute Errors Plot for 2 Part e

N vs order plot for 2 Part e

N vs error (loglog plot) for 2 Part e

Using backward difference for the first-order derivative and central difference for the second-order derivative,
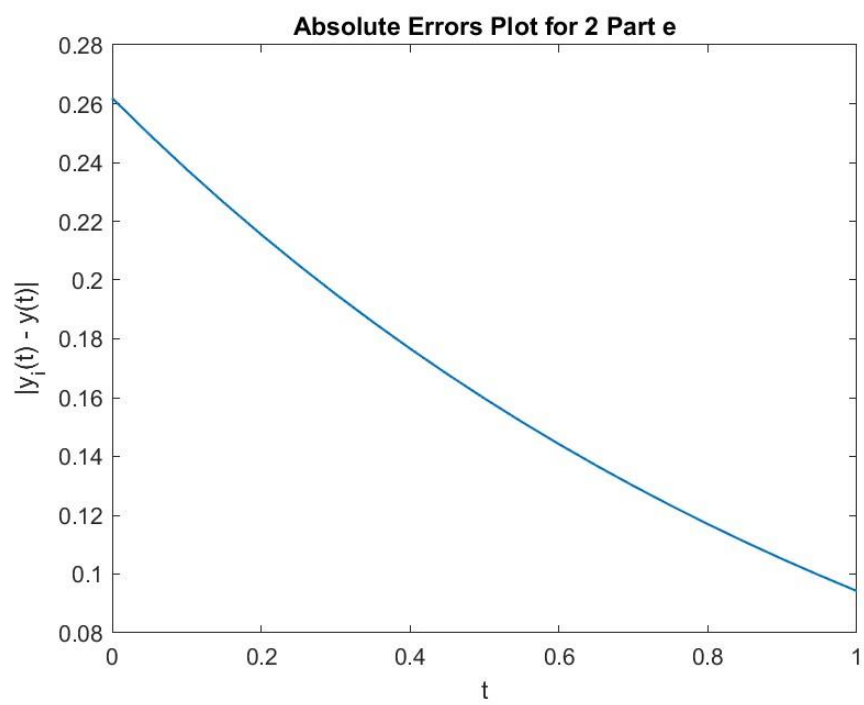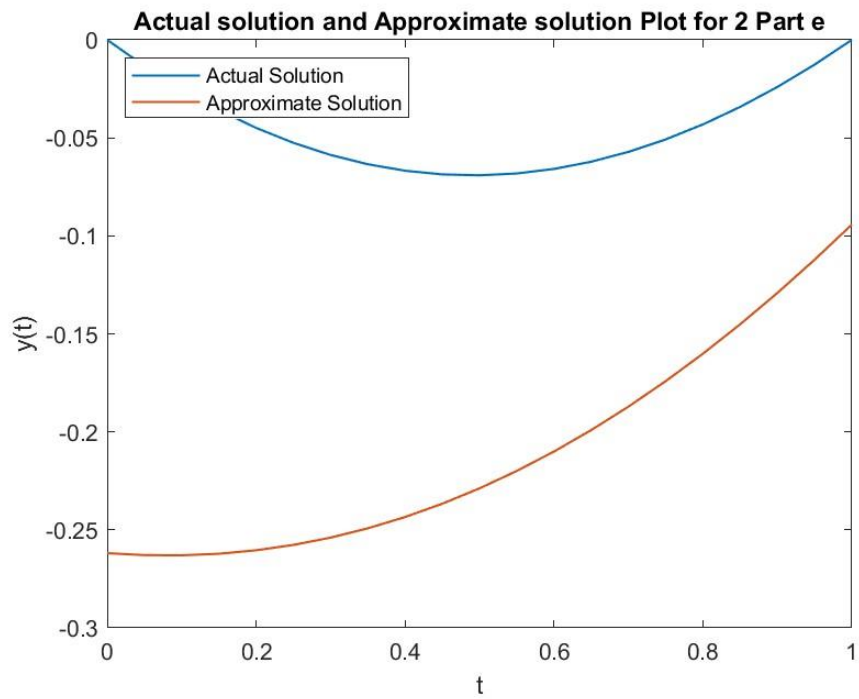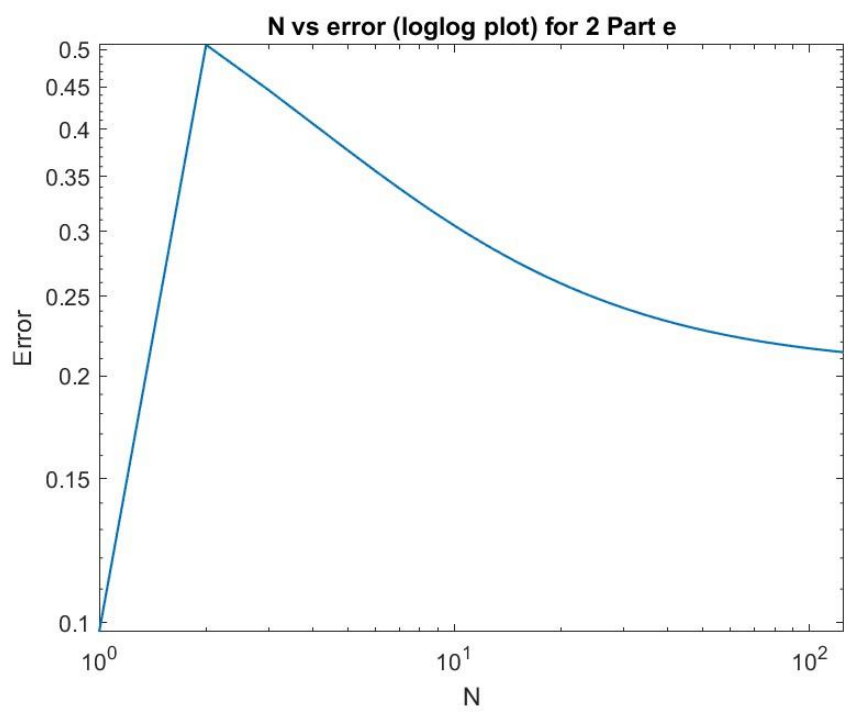For h = 0.050000

x	Approximate Solution	Exact Solution	Absolute Error
0.000000	-0.2618880	0.0000000	0.2618880
0.050000	-0.2628795	-0.0133786	0.2495009
0.100000	-0.2629897	-0.0253323	0.2376574
0.150000	-0.2621814	-0.0358494	0.2263319
0.200000	-0.2604234	-0.0449226	0.2155008
0.250000	-0.2576902	-0.0525488	0.2051414
0.300000	-0.2539611	-0.0587287	0.1952324
0.350000	-0.2492200	-0.0634662	0.1857538
0.400000	-0.2434549	-0.0667684	0.1766865
0.450000	-0.2366577	-0.0686452	0.1680126
0.500000	-0.2288236	-0.0691087	0.1597149
0.550000	-0.2199509	-0.0681735	0.1517774
0.600000	-0.2100405	-0.0658559	0.1441846
0.650000	-0.1990961	-0.0621740	0.1369220
0.700000	-0.1871234	-0.0571477	0.1299757
0.750000	-0.1741301	-0.0507977	0.1233324
0.800000	-0.1601258	-0.0431462	0.1169796
0.850000	-0.1451217	-0.0342164	0.1109053
0.900000	-0.1291302	-0.0240322	0.1050979
0.950000	-0.1121650	-0.0126184	0.0995466
1.000000	-0.0942409	0.0000000	0.0942409

Actual solution and Approximate solution Plot for 2 Part e

Absolute Errors Plot for 2 Part e

N vs order plot for 2 Part e

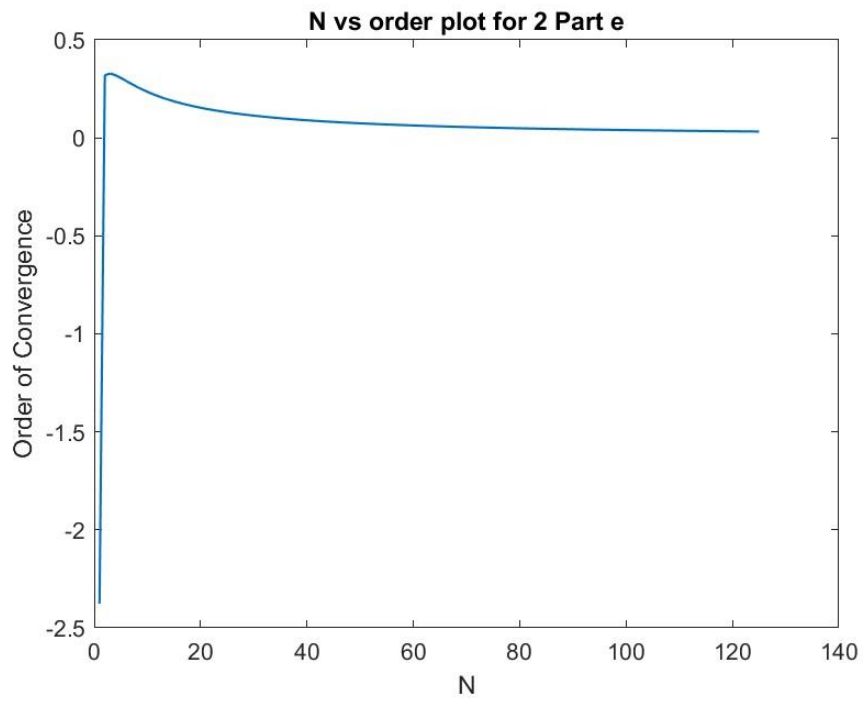**N vs error (loglog plot) for 2 Part e**

Using central difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.050000
```
     x         Approximate Solution    Exact Solution   Absolute Error
0.000000          -0.9210422             0.0000000        0.9210422
0.050000          -0.8890760            -0.0133786        0.8756974
0.100000          -0.8579183            -0.0253323        0.8325860
0.150000          -0.8274473            -0.0358494        0.7915979
0.200000          -0.7975510            -0.0449226        0.7526284
0.250000          -0.7681268            -0.0525488        0.7155780
0.300000          -0.7390808            -0.0587287        0.6803522
0.350000          -0.7103272            -0.0634662        0.6468610
0.400000          -0.6817873            -0.0667684        0.6150189
0.450000          -0.6533899            -0.0686452        0.5847447
0.500000          -0.6250699            -0.0691087        0.5559611
0.550000          -0.5967682            -0.0681735        0.5285948
0.600000          -0.5684316            -0.0658559        0.5025757
0.650000          -0.5400117            -0.0621740        0.4778376
0.700000          -0.5114651            -0.0571477        0.4543174
0.750000          -0.4827528            -0.0507977        0.4319551
0.800000          -0.4538399            -0.0431462        0.4106937
0.850000          -0.4246952            -0.0342164        0.3904788
0.900000          -0.3952912            -0.0240322        0.3712590
0.950000          -0.3656036            -0.0126184        0.3529853
1.000000          -0.3356110             0.0000000        0.3356110
```
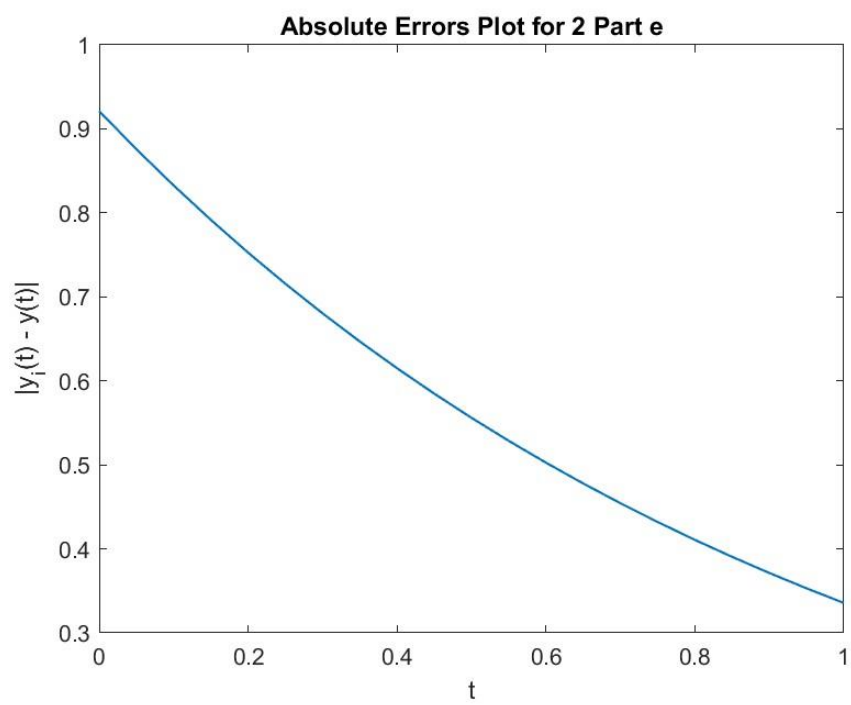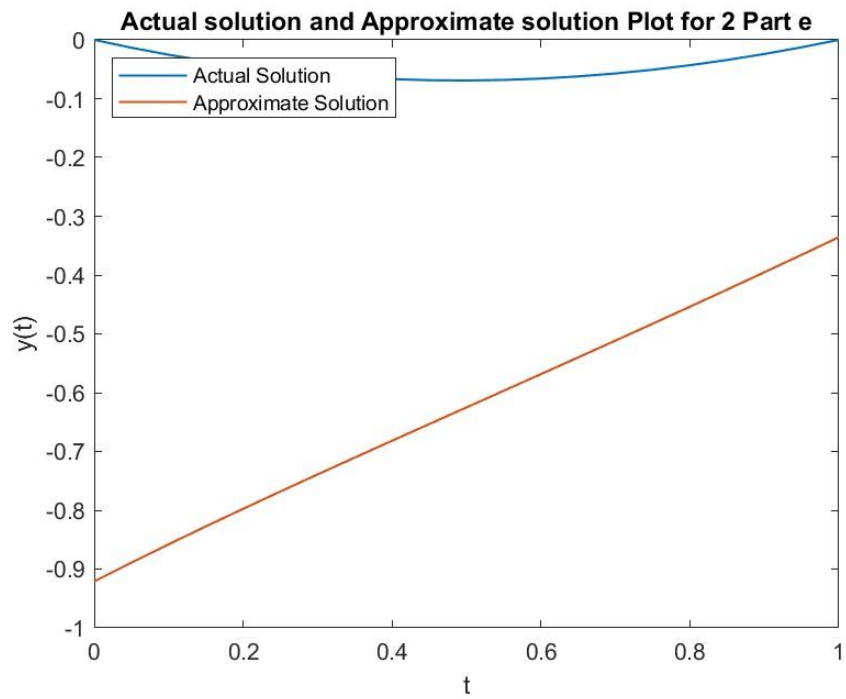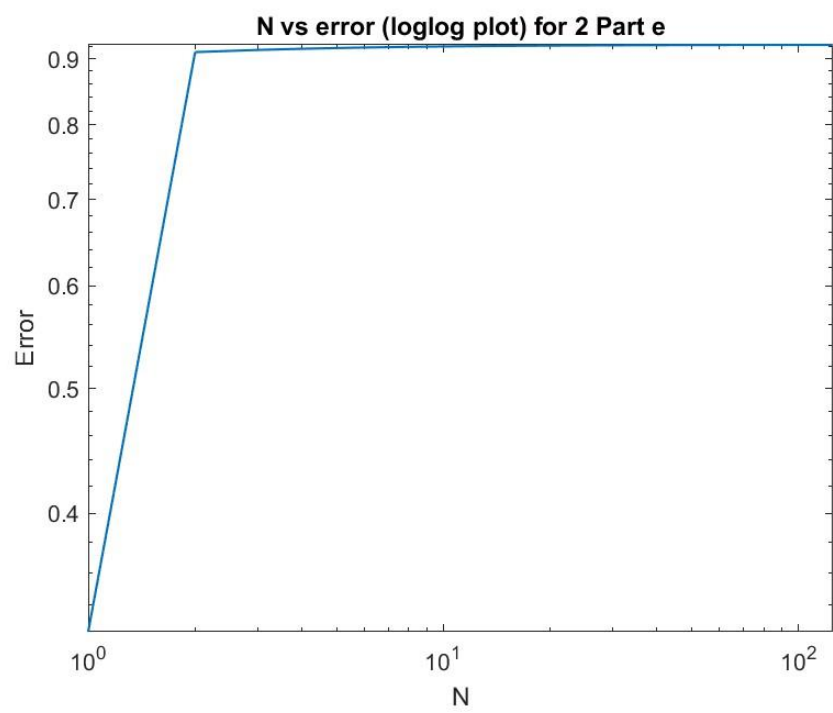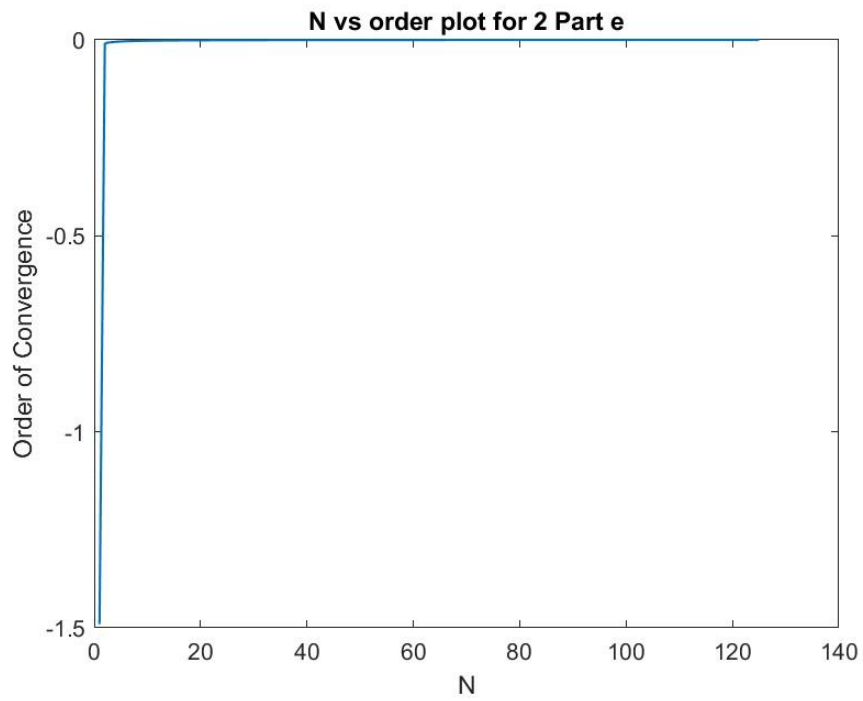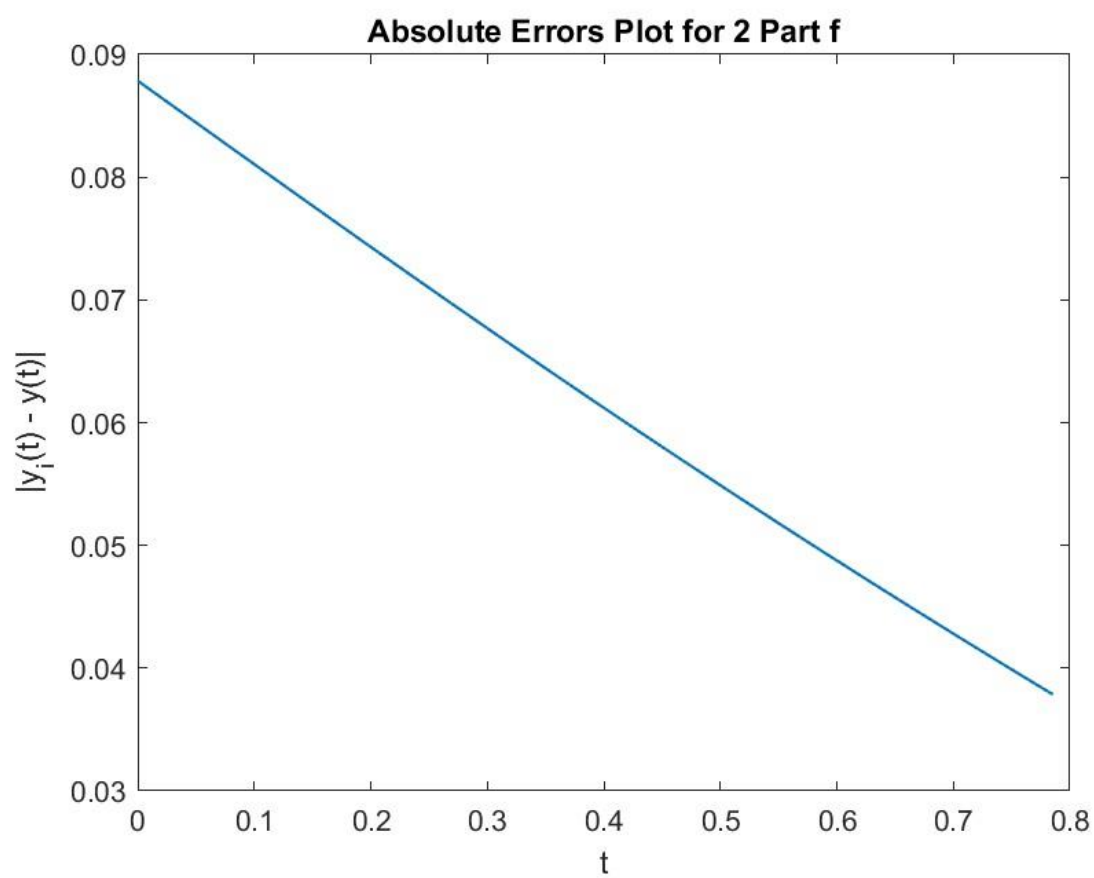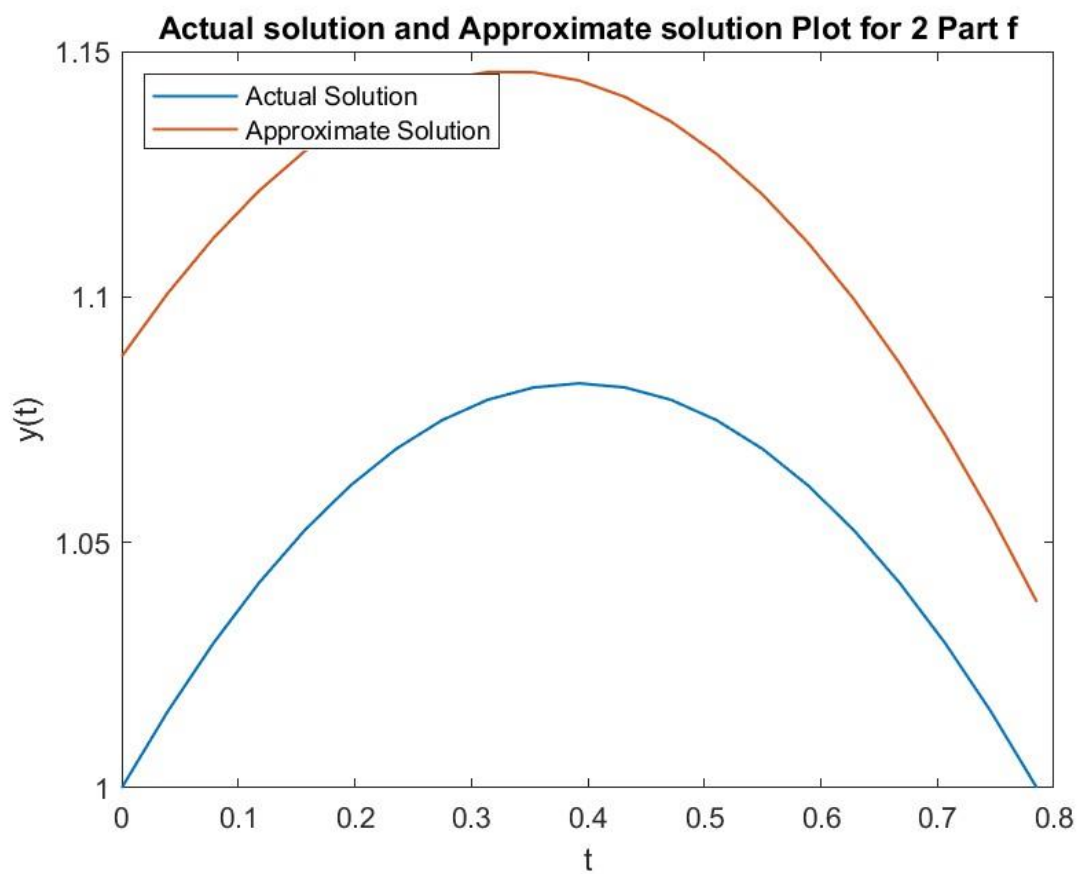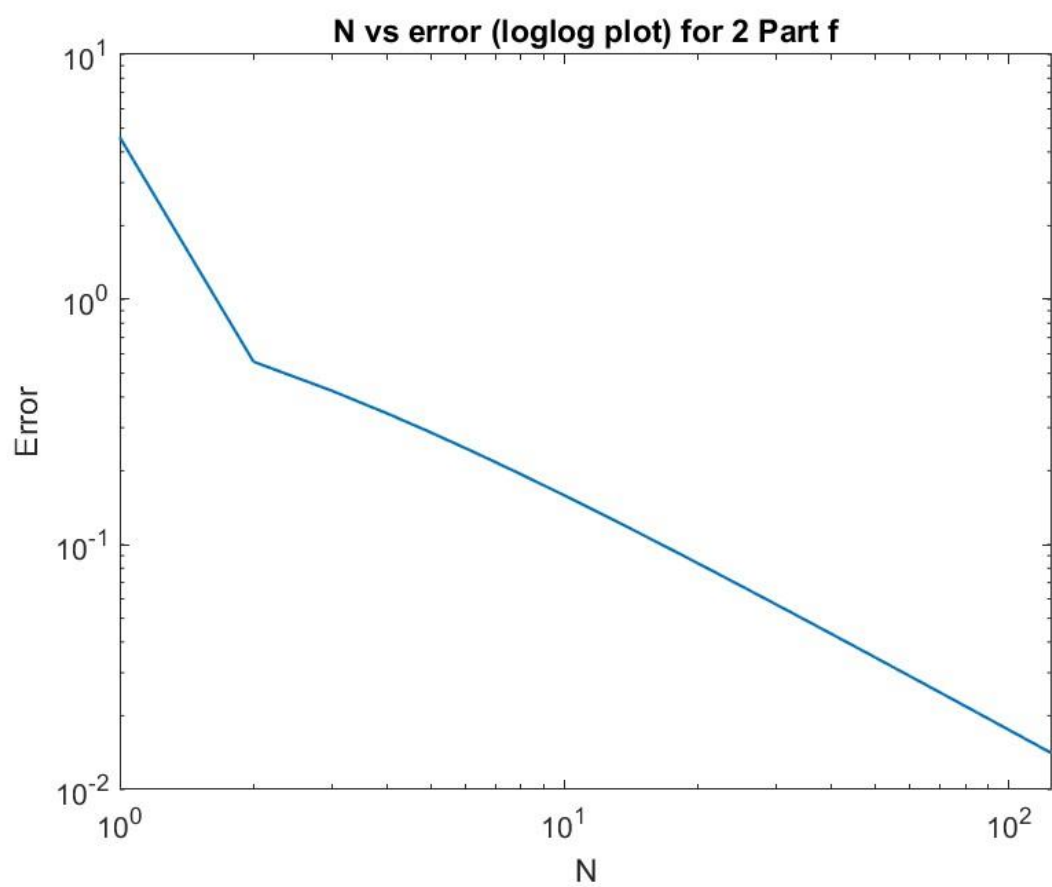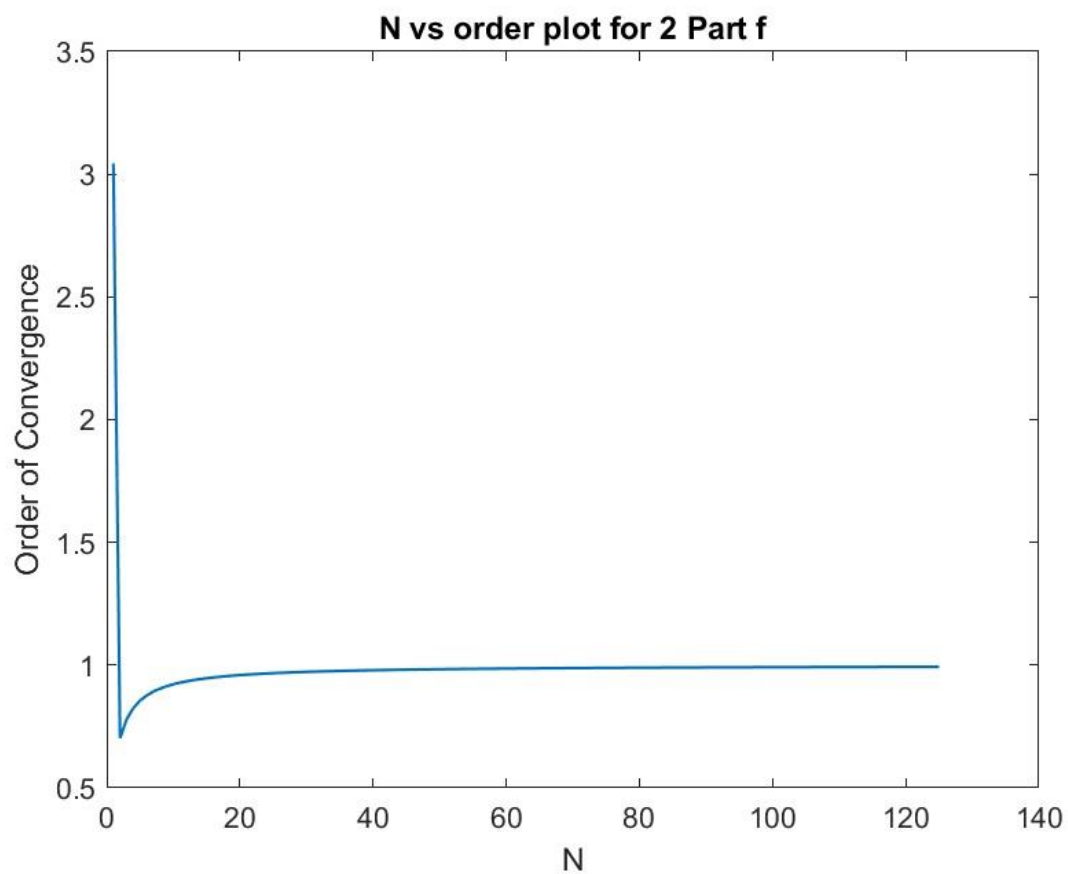
Actual solution and Approximate solution Plot for 2 Part e

Absolute Errors Plot for 2 Part e

N vs order plot for 2 Part e

N vs error (loglog plot) for 2 Part e

f)

```
Question 2 Part f
Using forward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.039270
     x        Approximate Solution     Exact Solution   Absolute Error
0.000000           1.0878547              1.0000000        0.0878547
0.039270           1.1006708              1.0154910        0.0851798
0.078540           1.1119252              1.0294162        0.0825090
0.117810           1.1216003              1.0417540        0.0798462
0.157080           1.1296806              1.0524856        0.0771949
0.196350           1.1361526              1.0615943        0.0745582
0.235619           1.1410050              1.0690662        0.0719388
0.274889           1.1442286              1.0748896        0.0693391
0.314159           1.1458166              1.0790555        0.0667610
0.353429           1.1457640              1.0815577        0.0642063
0.392699           1.1440685              1.0823922        0.0616763
0.431969           1.1407298              1.0815577        0.0591721
0.471239           1.1357500              1.0790555        0.0566945
0.510509           1.1291336              1.0748896        0.0542440
0.549779           1.1208873              1.0690662        0.0518211
0.589049           1.1110202              1.0615943        0.0494258
0.628319           1.0995437              1.0524856        0.0470581
0.667588           1.0864716              1.0417540        0.0447176
0.706858           1.0718201              1.0294162        0.0424039
0.746128           1.0556076              1.0154910        0.0401166
0.785398           1.0378549              1.0000000        0.0378549
```
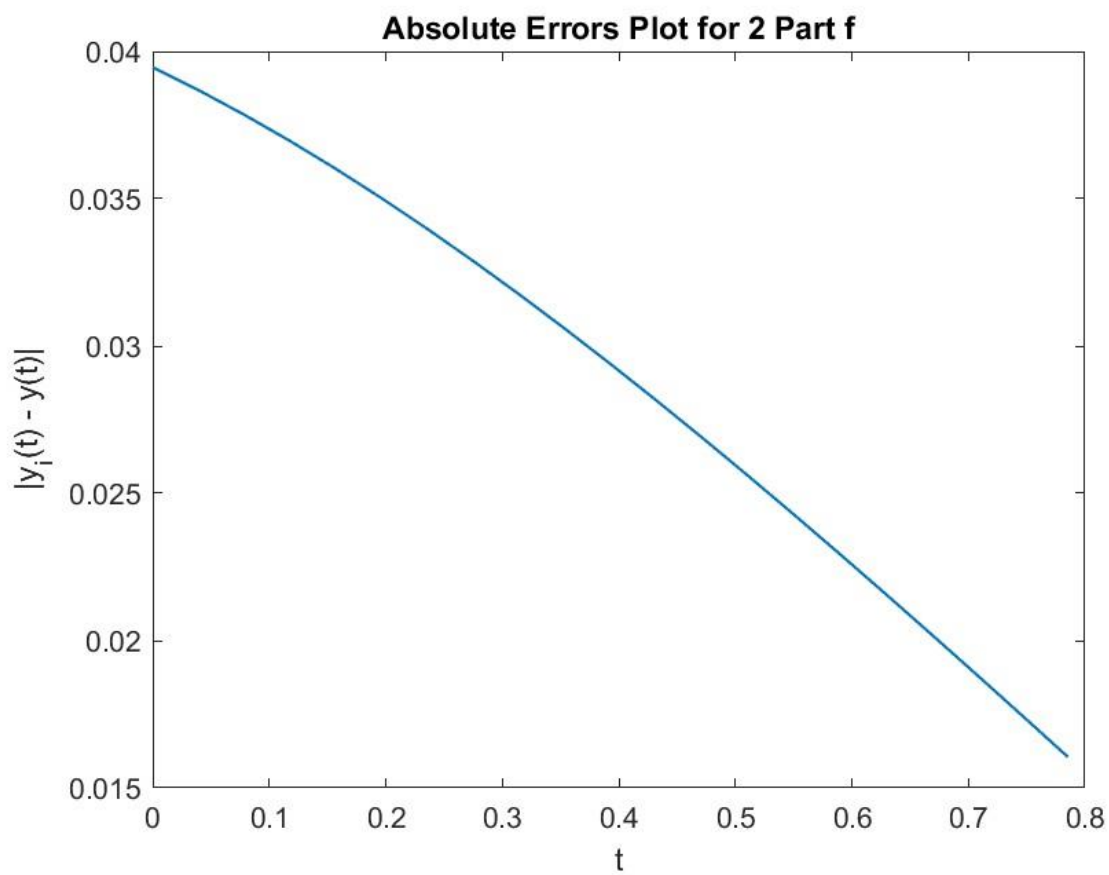
Actual solution and Approximate solution Plot for 2 Part f

Legend: Actual Solution, Approximate Solution

Absolute Errors Plot for 2 Part f

N vs order plot for 2 Part f

N vs error (loglog plot) for 2 Part f

```
Using backward difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.039270
      x        Approximate Solution     Exact Solution   Absolute Error
0.000000          1.0394604               1.0000000         0.0394604
0.039270          1.0541769               1.0154910         0.0386859
0.078540          1.0672676               1.0294162         0.0378514
0.117810          1.0787141               1.0417540         0.0369601
0.157080          1.0885007               1.0524856         0.0360151
0.196350          1.0966140               1.0615943         0.0350196
0.235619          1.1030430               1.0690662         0.0339768
0.274889          1.1077794               1.0748896         0.0328899
0.314159          1.1108173               1.0790555         0.0317618
0.353429          1.1121533               1.0815577         0.0305956
0.392699          1.1117864               1.0823922         0.0293942
0.431969          1.1097184               1.0815577         0.0281606
0.471239          1.1059532               1.0790555         0.0268977
0.510509          1.1004976               1.0748896         0.0256081
0.549779          1.0933606               1.0690662         0.0242945
0.589049          1.0845539               1.0615943         0.0229596
0.628319          1.0740914               1.0524856         0.0216058
0.667588          1.0619898               1.0417540         0.0202357
0.706858          1.0482677               1.0294162         0.0188516
0.746128          1.0329467               1.0154910         0.0174557
0.785398          1.0160503               1.0000000         0.0160503
```
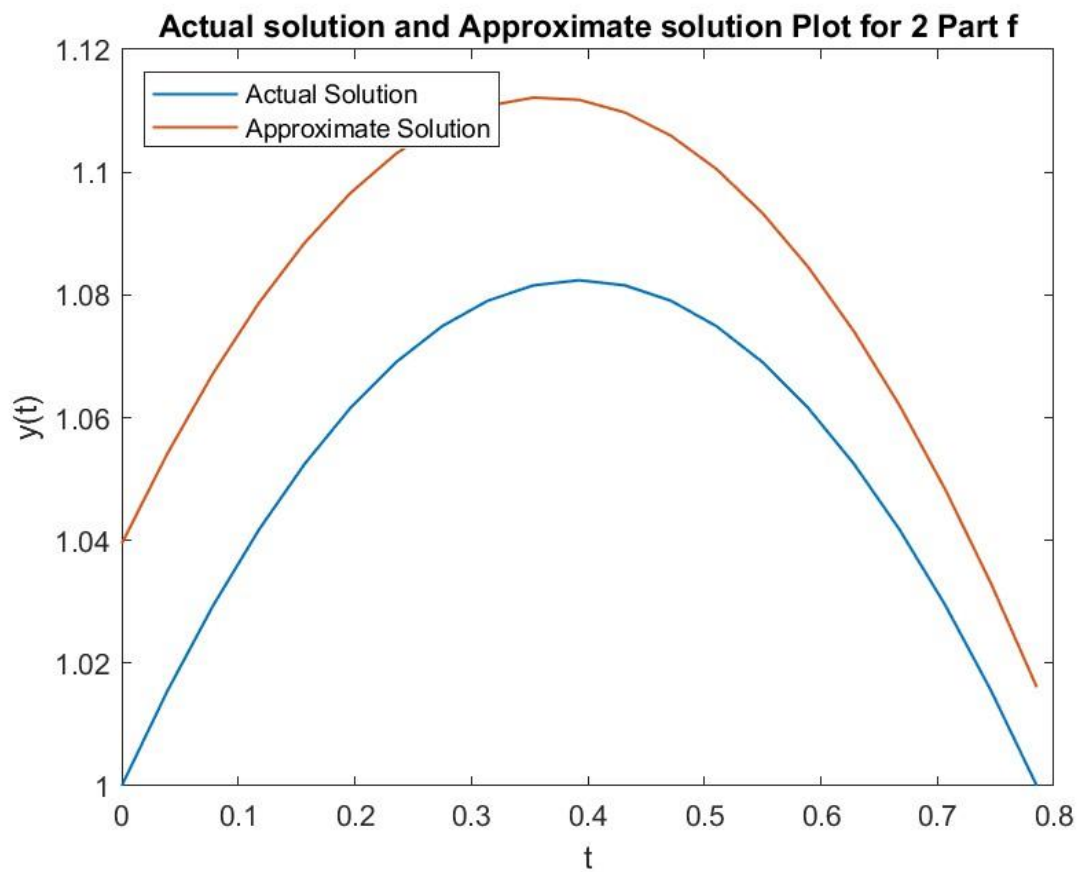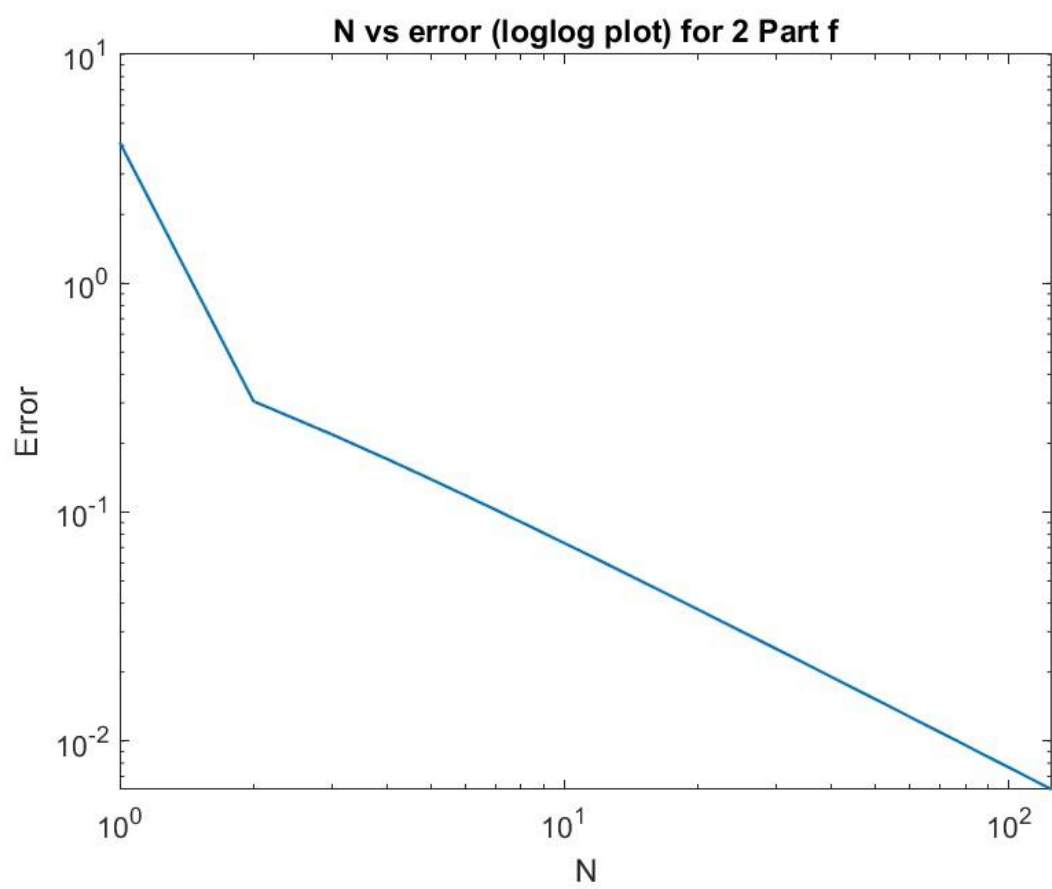
Actual solution and Approximate solution Plot for 2 Part f

Absolute Errors Plot for 2 Part f

N vs order plot for 2 Part f
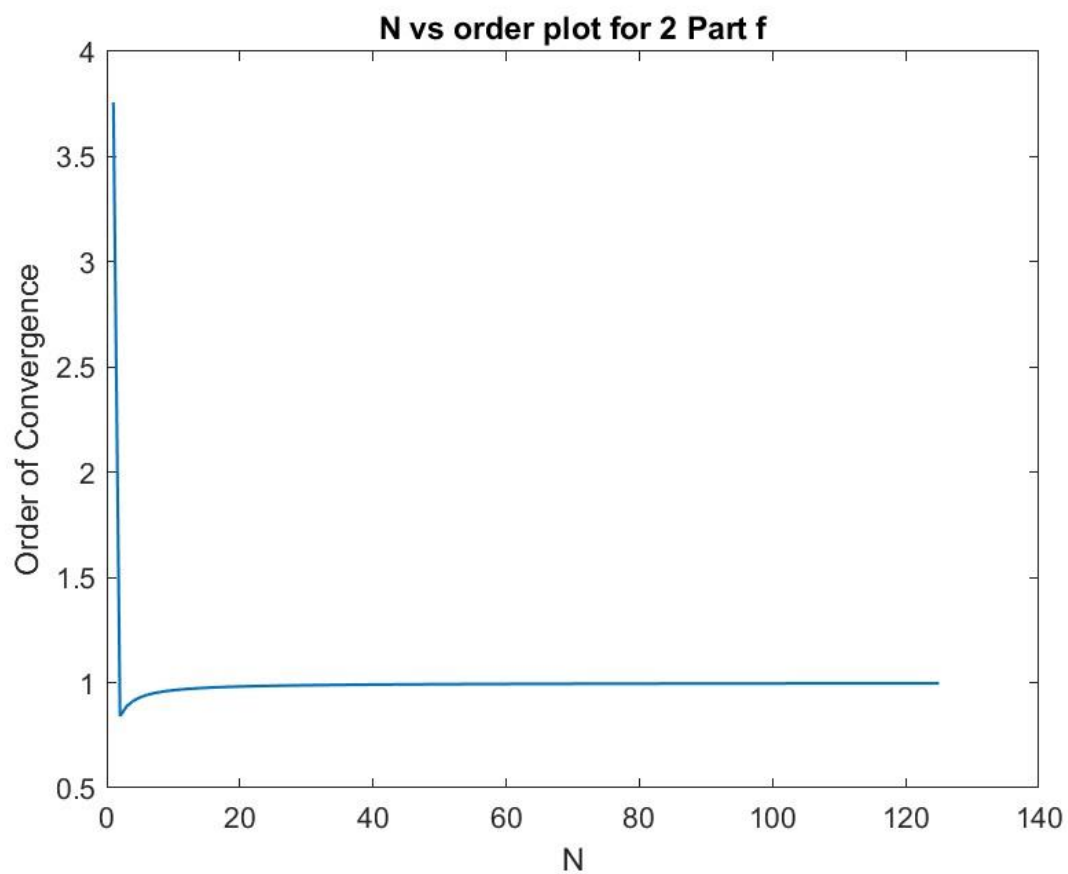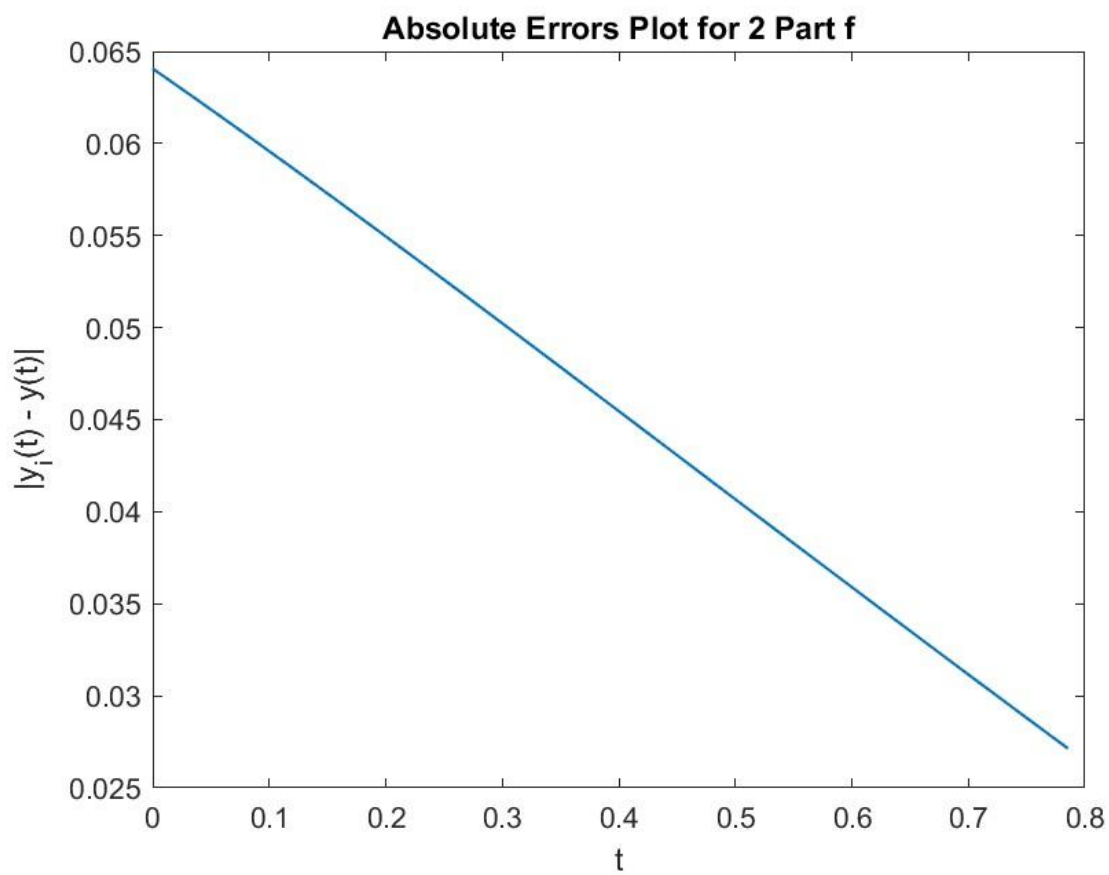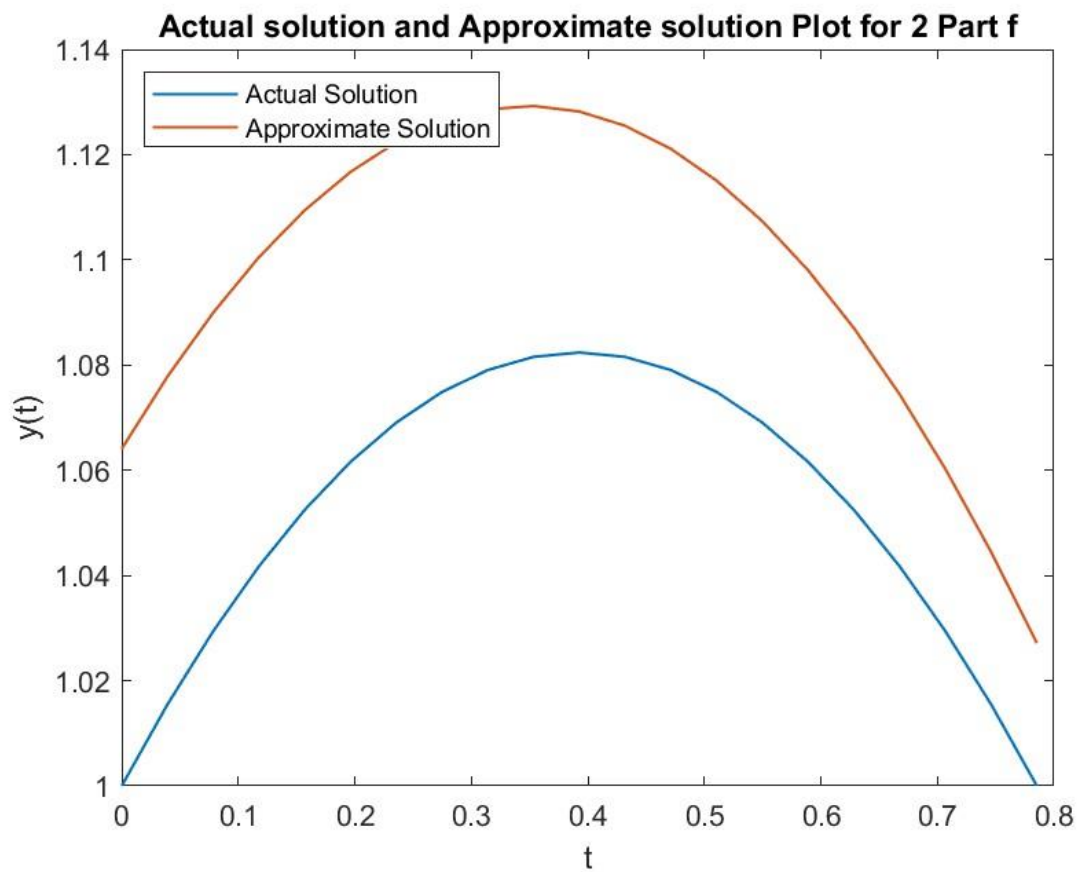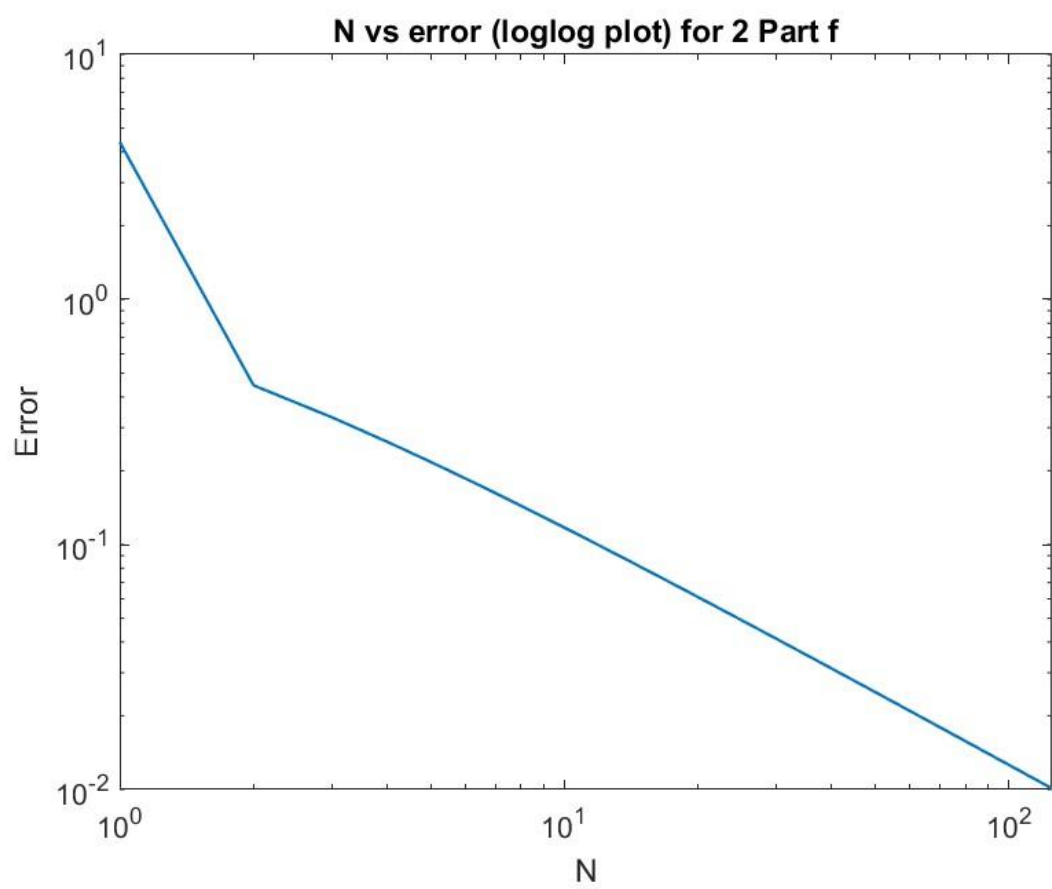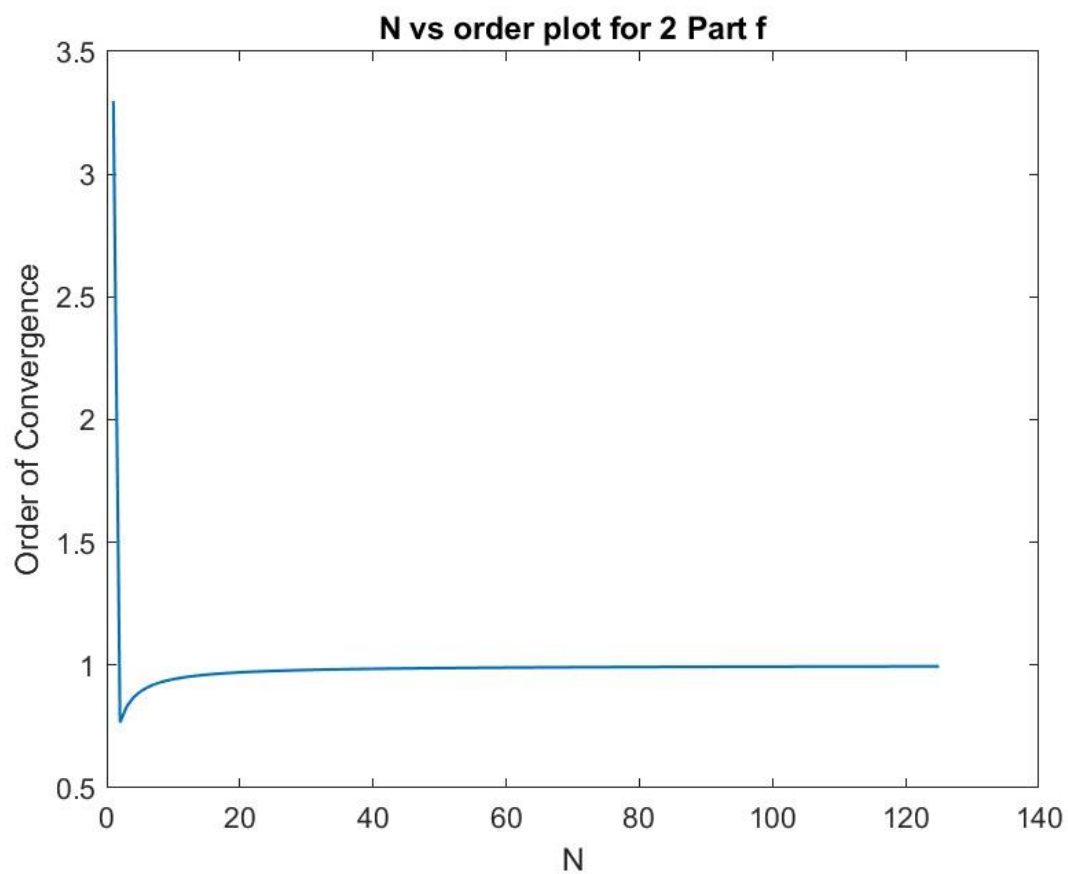
N vs error (loglog plot) for 2 Part f

```
Using central difference for the first-order derivative and central difference for the second-order derivative,
For h = 0.039270
     x        Approximate Solution     Exact Solution   Absolute Error
0.000000           1.0640664              1.0000000        0.0640664
0.039270           1.0778166              1.0154910        0.0623257
0.078540           1.0899737              1.0294162        0.0605576
0.117810           1.1005198              1.0417540        0.0587658
0.157080           1.1094391              1.0524856        0.0569535
0.196350           1.1167182              1.0615943        0.0551239
0.235619           1.1223461              1.0690662        0.0532799
0.274889           1.1263138              1.0748896        0.0514243
0.314159           1.1286150              1.0790555        0.0495594
0.353429           1.1292455              1.0815577        0.0476878
0.392699           1.1282036              1.0823922        0.0458114
0.431969           1.1254901              1.0815577        0.0439324
0.471239           1.1211079              1.0790555        0.0420523
0.510509           1.1150625              1.0748896        0.0401730
0.549779           1.1073619              1.0690662        0.0382958
0.589049           1.0980164              1.0615943        0.0364220
0.628319           1.0870386              1.0524856        0.0345529
0.667588           1.0744436              1.0417540        0.0326896
0.706858           1.0602490              1.0294162        0.0308329
0.746128           1.0444746              1.0154910        0.0289836
0.785398           1.0271426              1.0000000        0.0271426
```

Actual solution and Approximate solution Plot for 2 Part f

Absolute Errors Plot for 2 Part f

N vs order plot for 2 Part f

N vs error (loglog plot) for 2 Part f

Observations:

- In part (e), the provided Boundary Value Problem (BVP) fails to converge for any of the three methods. This failure is evident in the plot of N versus order, where the convergence order diminishes towards zero as N, the number of discretization points, increases. Additionally, the log-log plot remains flat instead of exhibiting the typical decreasing trend observed in parts where the BVP converges with a discernible order based on the chosen method.
- The order of convergence of method 1(Using forward difference for first derivative) and method 2(Using backward difference for first derivative) is 1 for each boundary condition.
- The order of convergence of method 3(Using central difference for first derivative) is 2 for Dirichlet's conditions, while the order of convergence is 1 for Neumann and Mixed/Robin conditions.