Scientific Computing Lab MA – 322 Lab – 10

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Roll Number – 210123072

Branch – Mathematics and Computing

1)

a)

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Question 1 Part a
Using forward-difference formula, the table is:
x       f(x)        f'(x)
0.5     0.4794      0.8520
0.6     0.5646      0.7960
0.7     0.6442        --

Using backward-difference formula, the table is:
x       f(x)        f'(x)
0.5     0.4794        --
0.6     0.5646      0.8520
0.7     0.6442      0.7960
```

b)

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Question 1 Part b
Using forward-difference formula, the table is:
x       f(x)        f'(x)
1.0     1.0000      1.3125
1.2     1.2625      1.9850
1.4     1.6595        --

Using backward-difference formula, the table is:
x       f(x)        f'(x)
1.0     1.0000        --
1.2     1.2625      1.3125
1.4     1.6595      1.9850
```

2)

a)

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Question 2 Part a
In forward-difference table,
At x = 0.5,
Approximate value of f' = 0.8520000000
Exact value of f' = 0.8775825619
Absolute error = 0.0255825619
Error bound = 0.0282321237

At x = 0.6,
Approximate value of f' = 0.7960000000
Exact value of f' = 0.8253356149
Absolute error = 0.0293356149
Error bound = 0.0322108844


In backward-difference table,
At x = 0.6,
Approximate value of f' = 0.8520000000
Exact value of f' = 0.8253356149
Absolute error = 0.0266643851
Error bound = 0.0282321237

At x = 0.7,
Approximate value of f' = 0.7960000000
Exact value of f' = 0.7648421873
Absolute error = 0.0311578127
Error bound = 0.0322108844
```

b)

```
Question 2 Part b
In forward-difference table,
At x = 1.0,
Approximate value of f' = 1.3125000000
Exact value of f' = 1.0000000000
Absolute error = 0.3125000000
Error bound = 0.3364643114

At x = 1.2,
Approximate value of f' = 1.9850000000
Exact value of f' = 1.6375717363
Absolute error = 0.3474282637
Error bound = 0.3672944473


In backward-difference table,
At x = 1.2,
Approximate value of f' = 1.3125000000
Exact value of f' = 1.6375717363
Absolute error = 0.3250717363
Error bound = 0.3364643114

At x = 1.4,
Approximate value of f' = 1.9850000000
Exact value of f' = 2.3421222625
Absolute error = 0.3571222625
Error bound = 0.3672944473
```

3)

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Question 3
Approximate value of the voltage at t = 1.00 is: ε(1.00) = 2.400200 volt
Approximate value of the voltage at t = 1.01 is: ε(1.01) = 2.403040 volt
Approximate value of the voltage at t = 1.02 is: ε(1.02) = 4.365880 volt
Approximate value of the voltage at t = 1.03 is: ε(1.03) = 6.331560 volt
Approximate value of the voltage at t = 1.04 is: ε(1.04) = 6.340080 volt
```

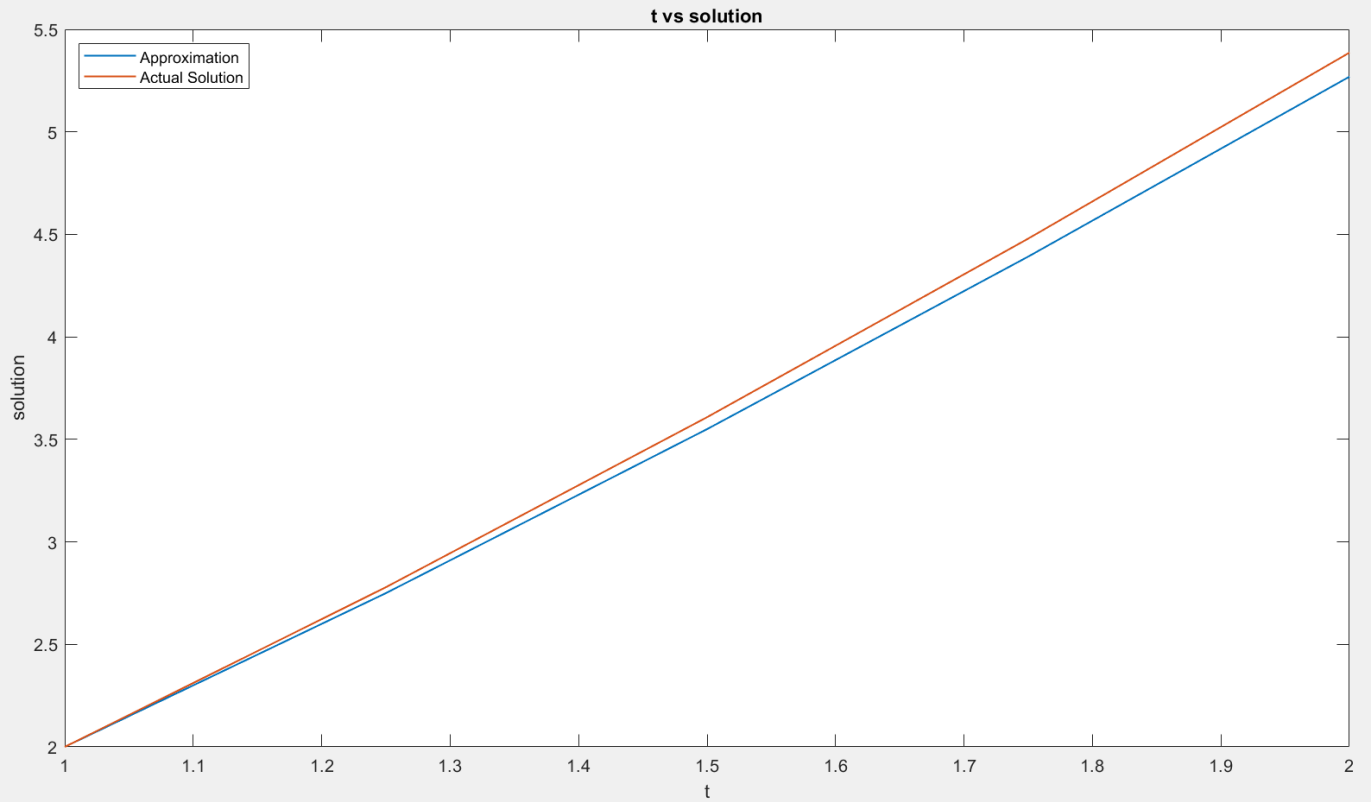
4)

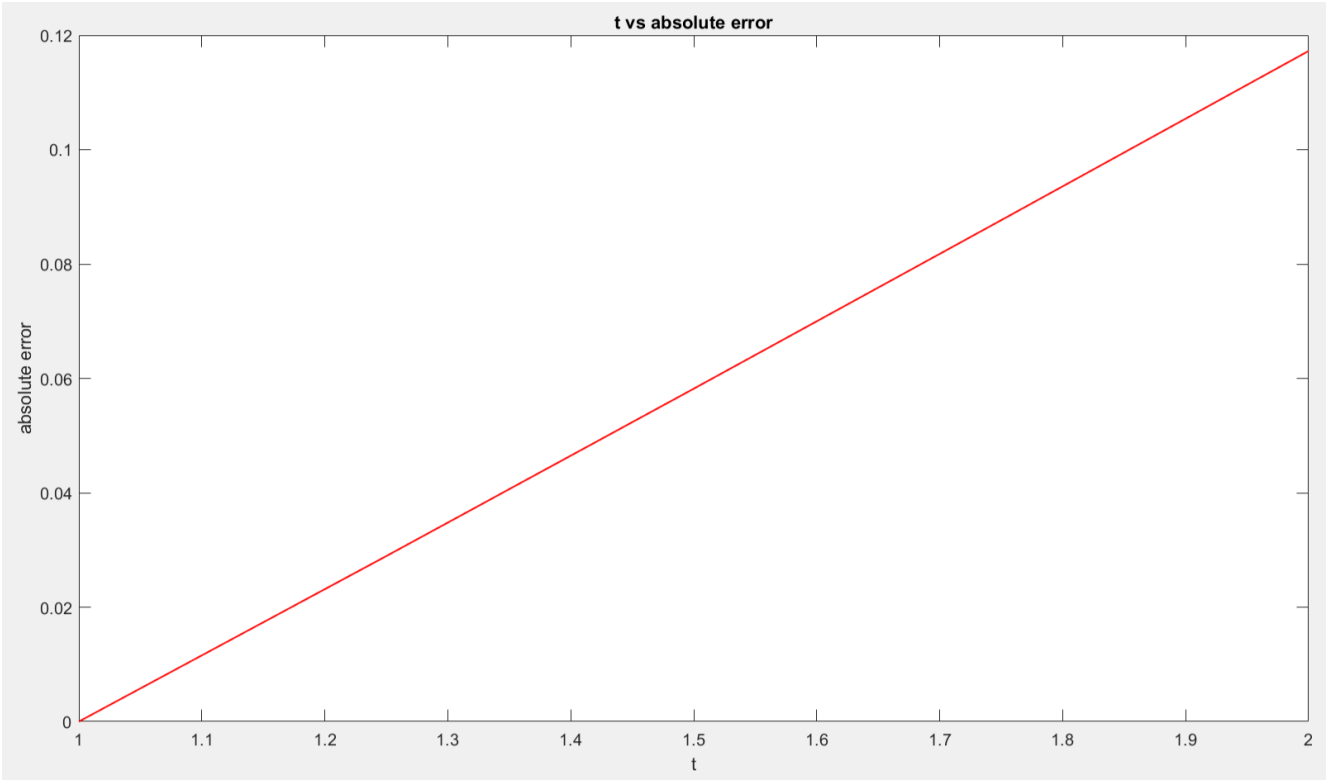
a)

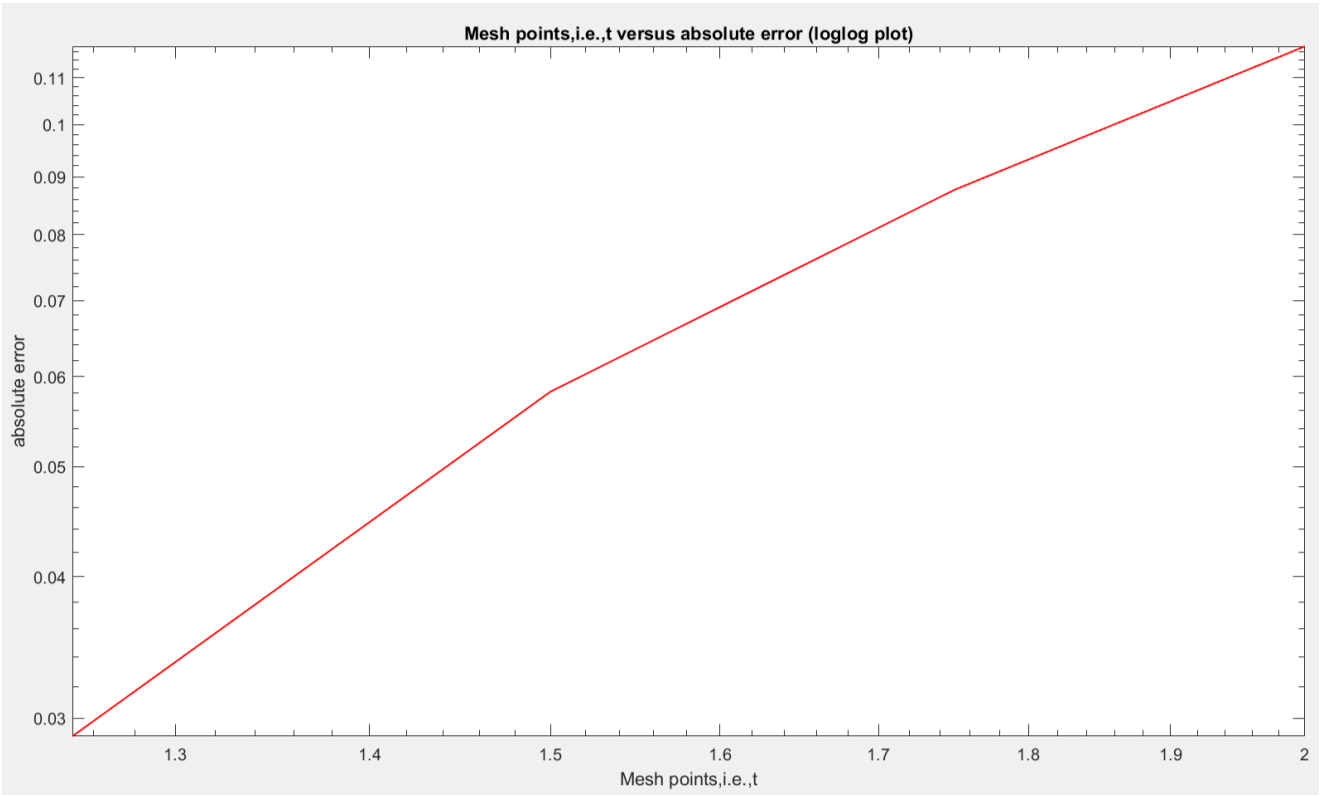
Question 4 Part a

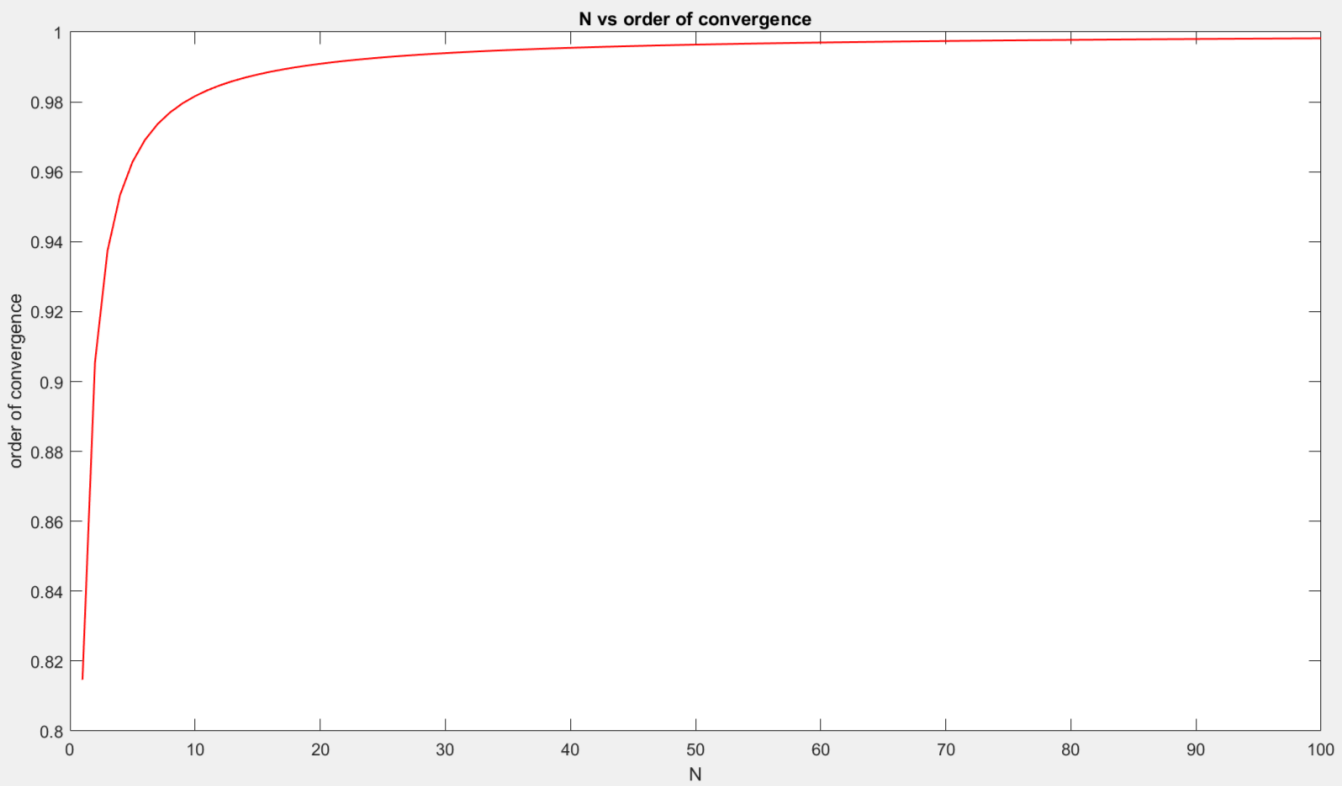
For h = 0.25,

t	Approximate Solution	Exact Solution	Absolute Error
1.000000	2.000000	2.000000	0.000000
1.250000	2.750000	2.778929	0.028929
1.500000	3.550000	3.608198	0.058198
1.750000	4.391667	4.479328	0.087661
2.000000	5.269048	5.386294	0.117247



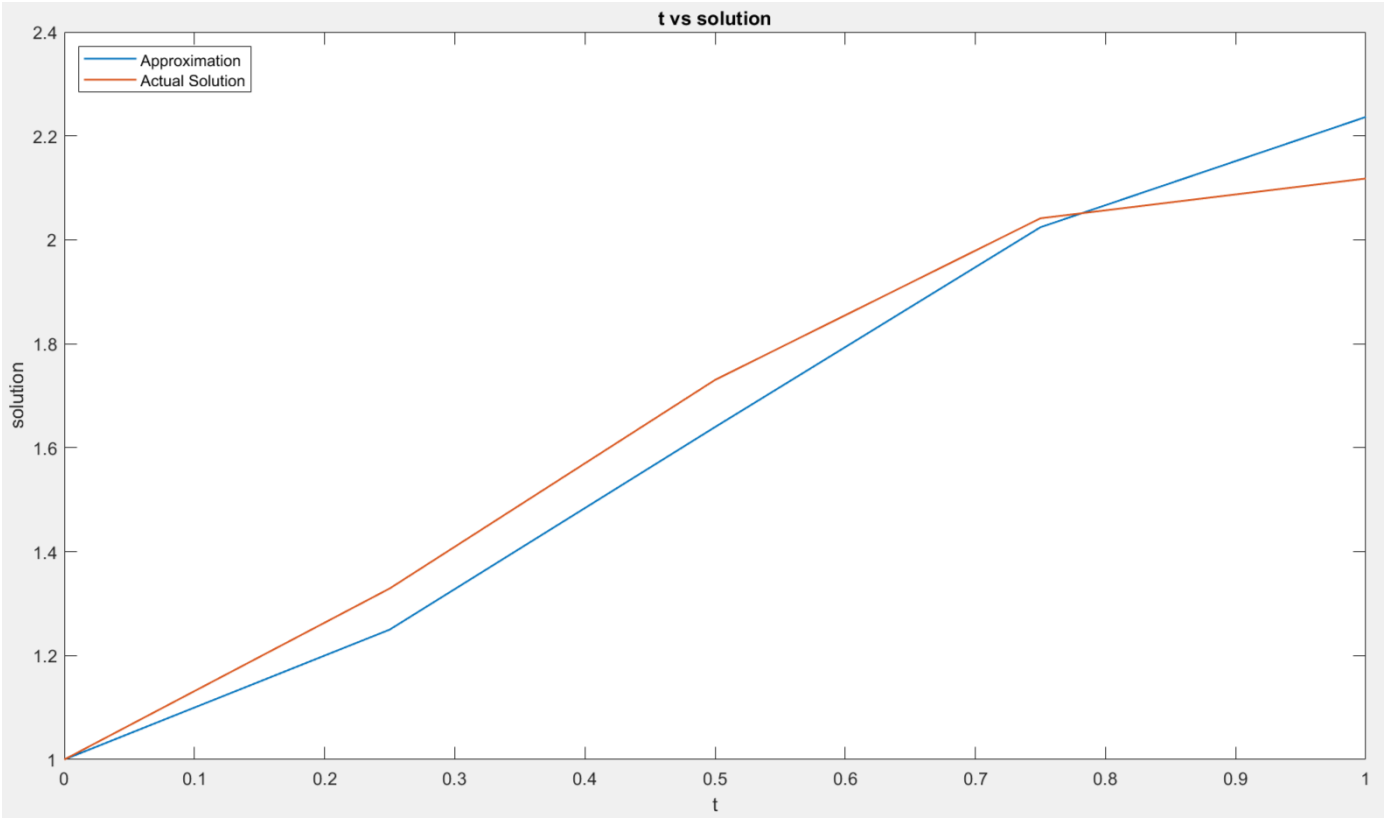

t vs absolute error
absolute error
t
0.12
0.1
0.08
0.06
0.04
0.02
0
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

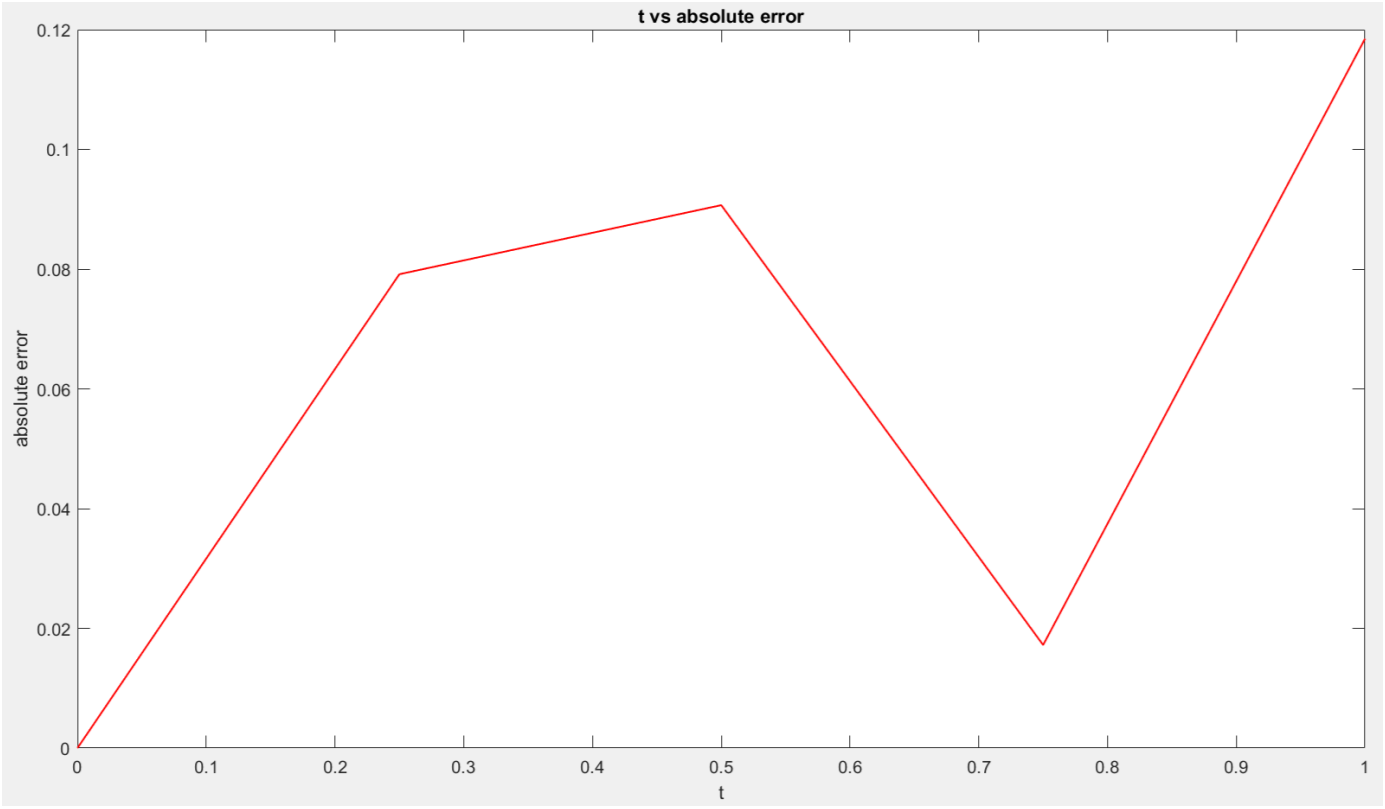

Mesh points,i.e.,t versus absolute error (loglog plot)
absolute error
Mesh points,i.e.,t
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

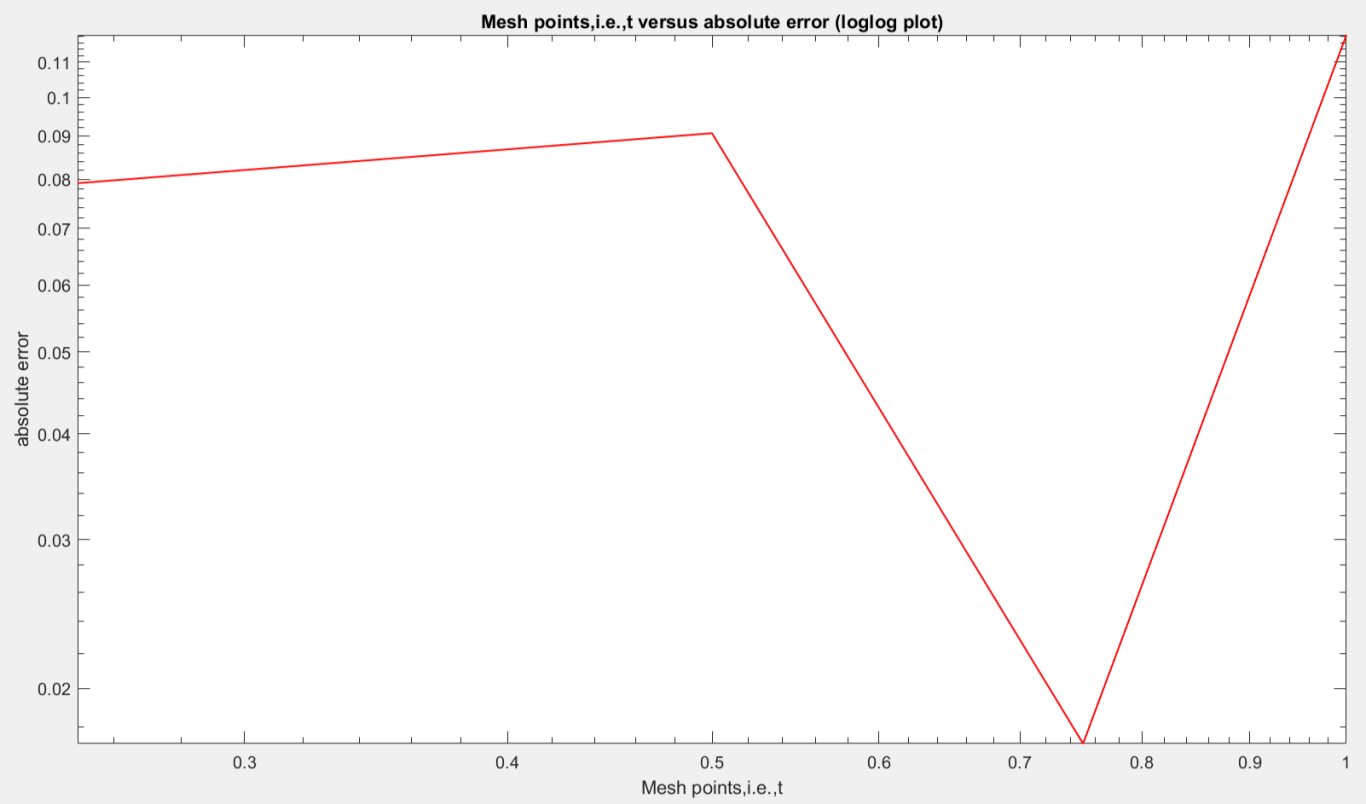


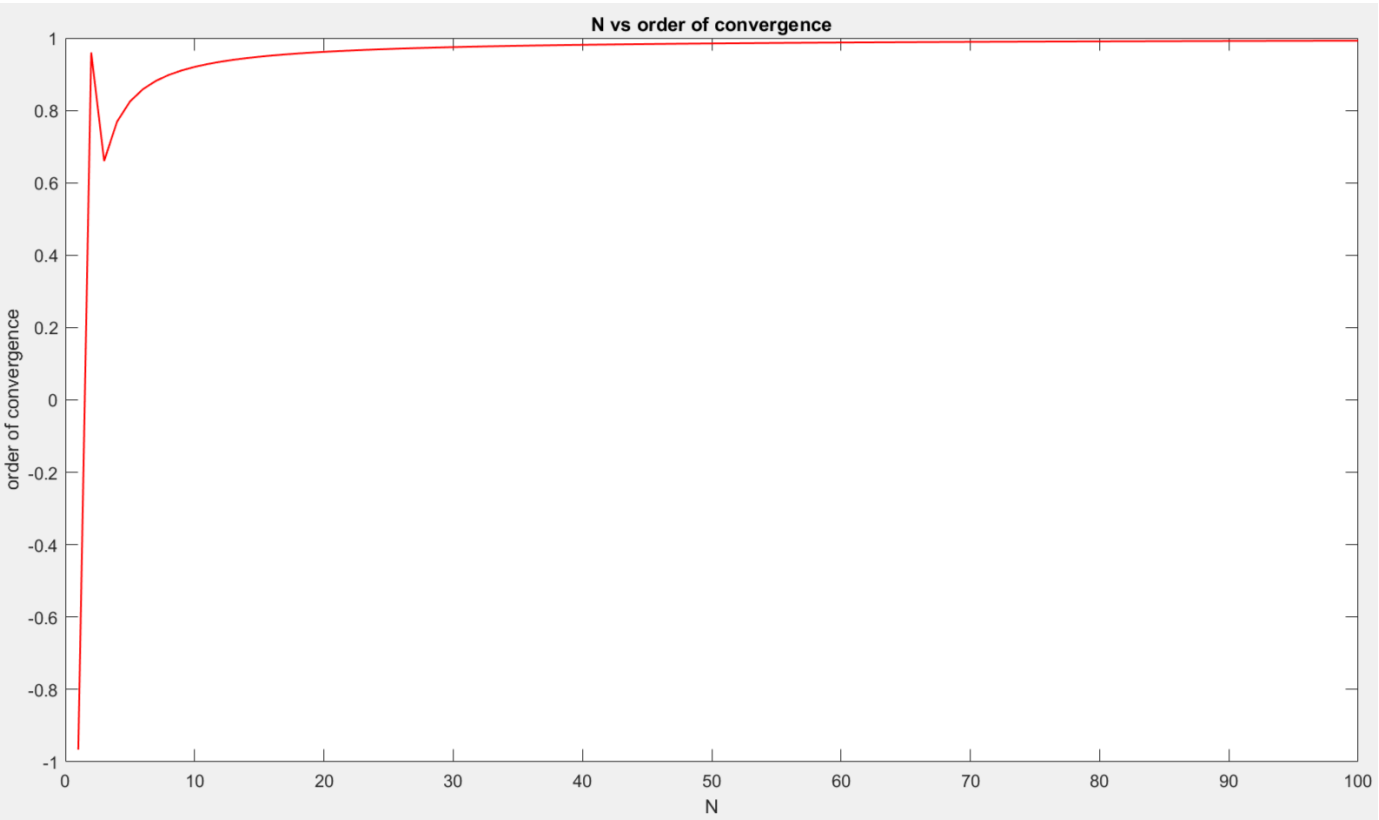
b)

```
Question 4 Part b
For h = 0.25,
   t          Approximate Solution     Exact Solution     Absolute Error
0.000000          1.000000              1.000000           0.000000
0.250000          1.250000              1.329150           0.079150
0.500000          1.639805              1.730490           0.090684
0.750000          2.024255              2.041472           0.017217
1.000000          2.236457              2.117980           0.118478
```


t vs solution
Approximation
Actual Solution
solution
t
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


t vs absolute error
absolute error
t
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Mesh points,i.e.,t versus absolute error (loglog plot)
absolute error
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Mesh points,i.e.,t

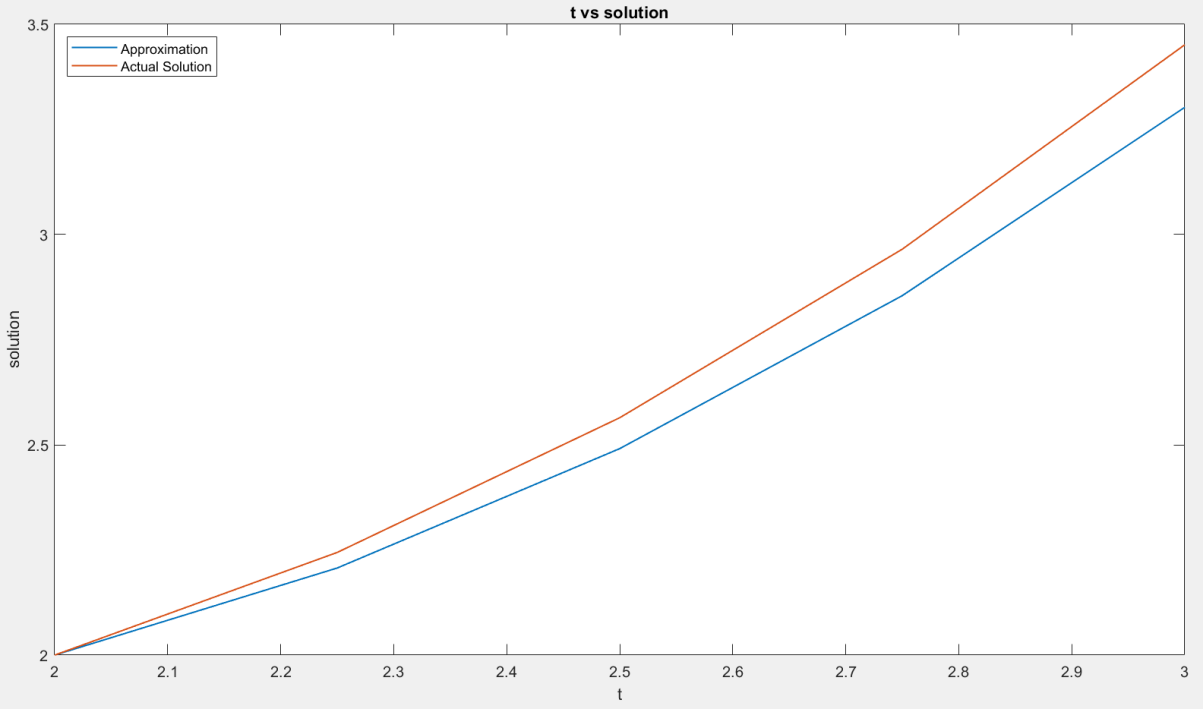

N vs order of convergence
order of convergence
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
10
20
30
40
50
60
70
80
90
100
N

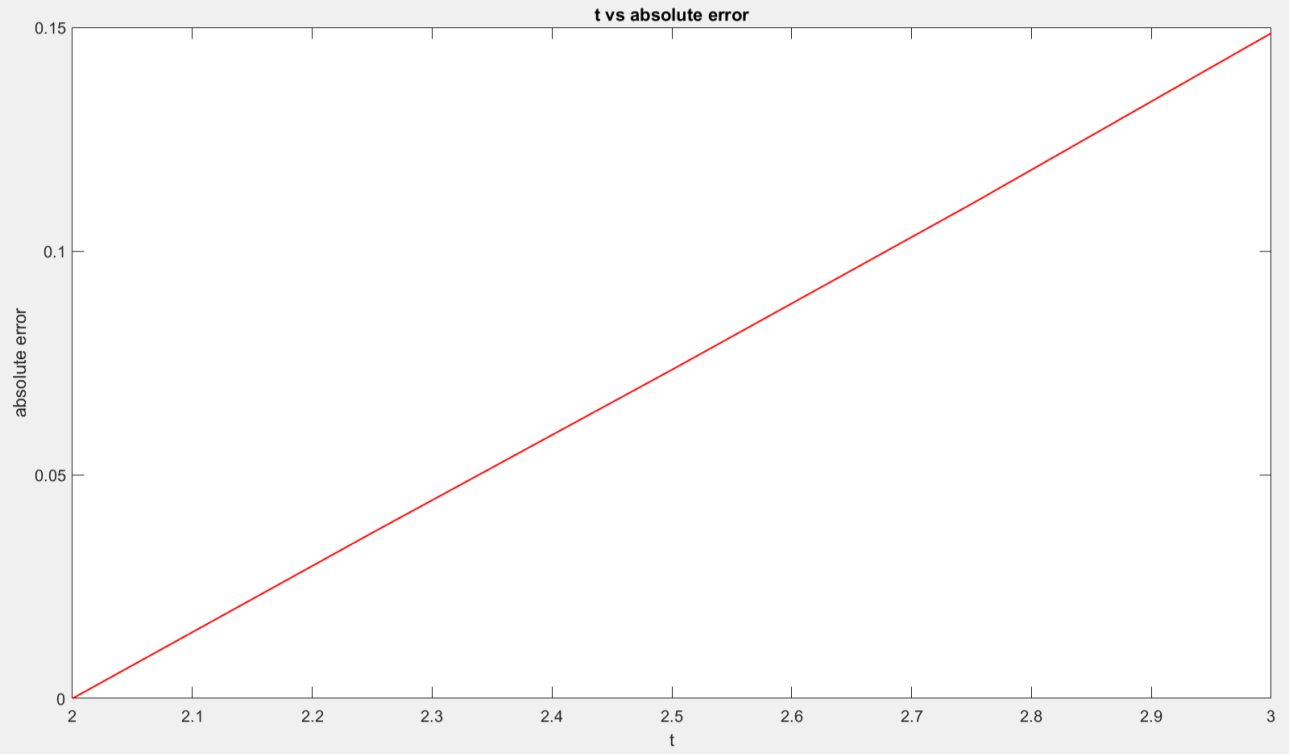
c)

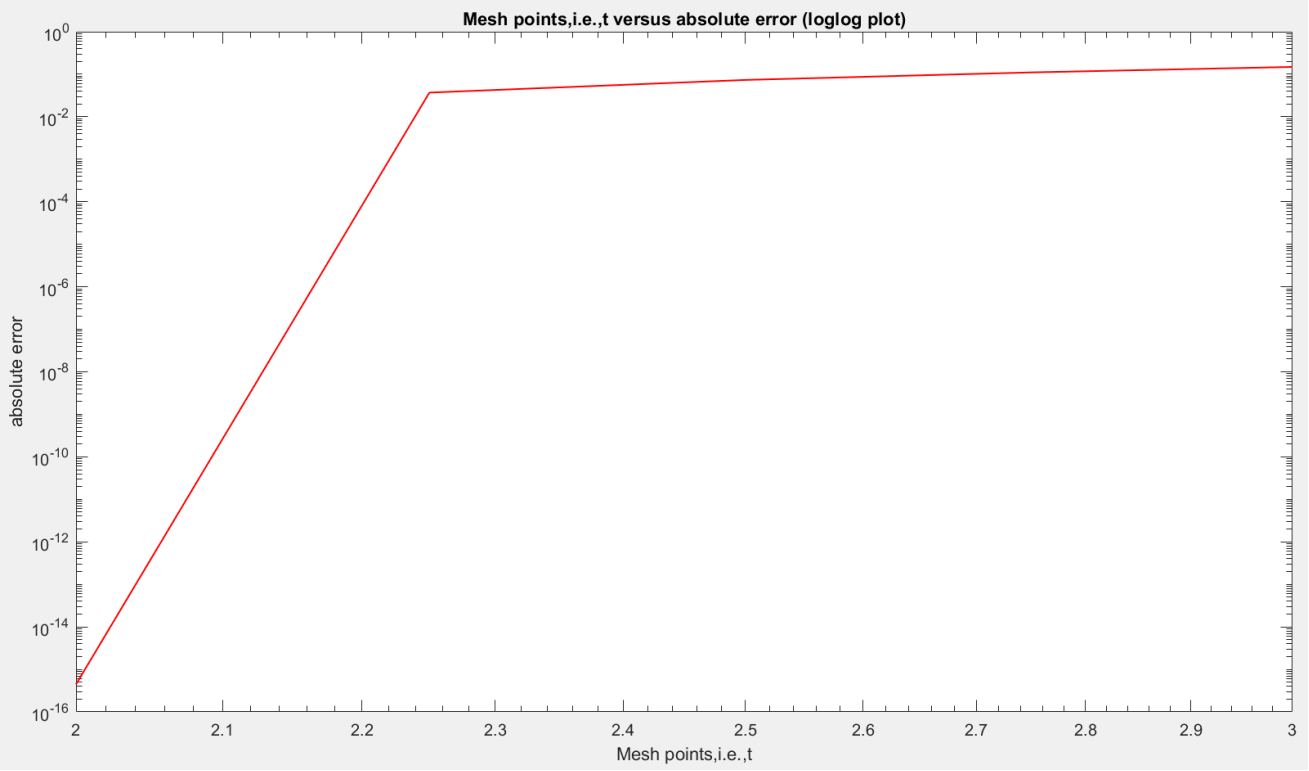
Question 4 Part c

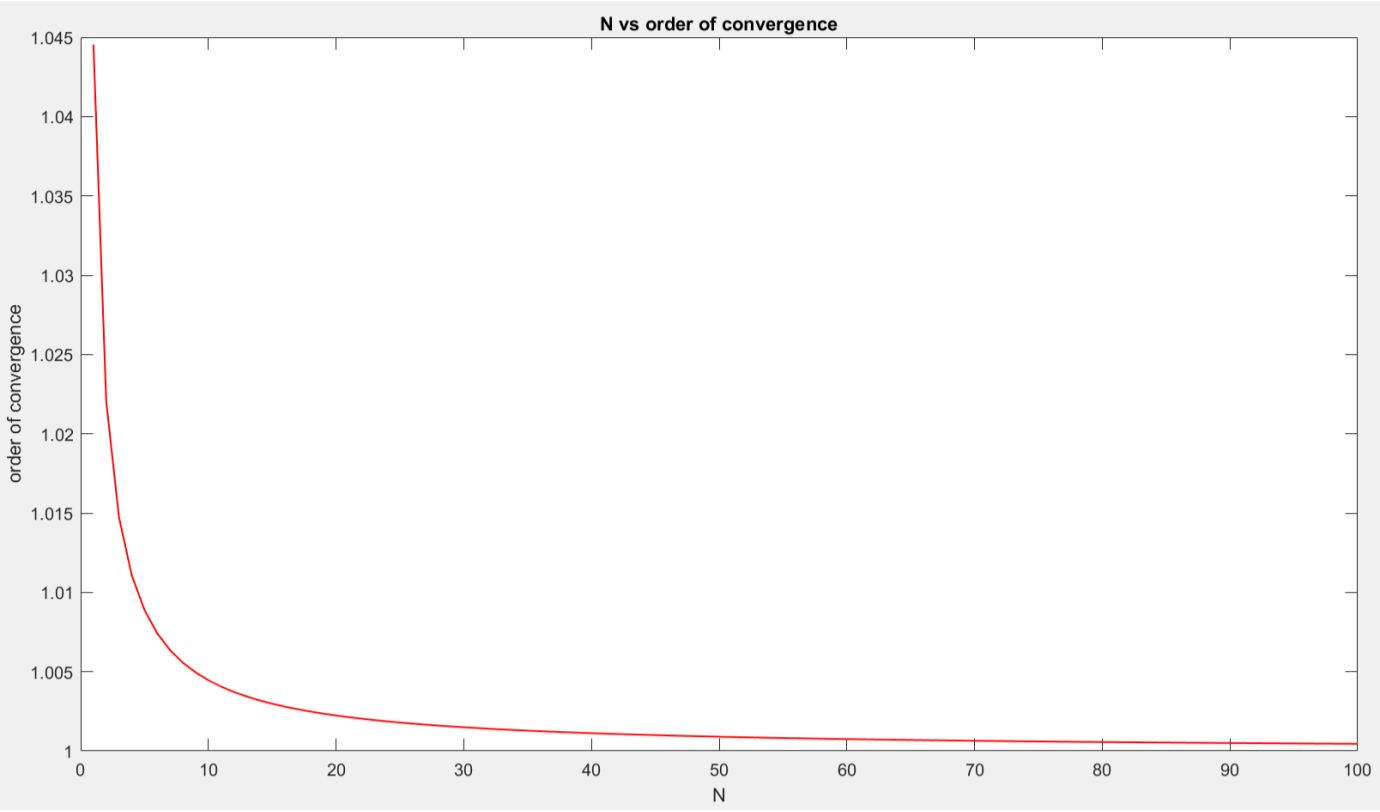
For h = 0.25,

t	Approximate Solution	Exact Solution	Absolute Error
2.000000	2.000000	2.000000	0.000000
2.250000	2.207107	2.244121	0.037014
2.500000	2.490999	2.564452	0.073453
2.750000	2.854680	2.965194	0.110513
3.000000	3.302596	3.451287	0.148690









5)

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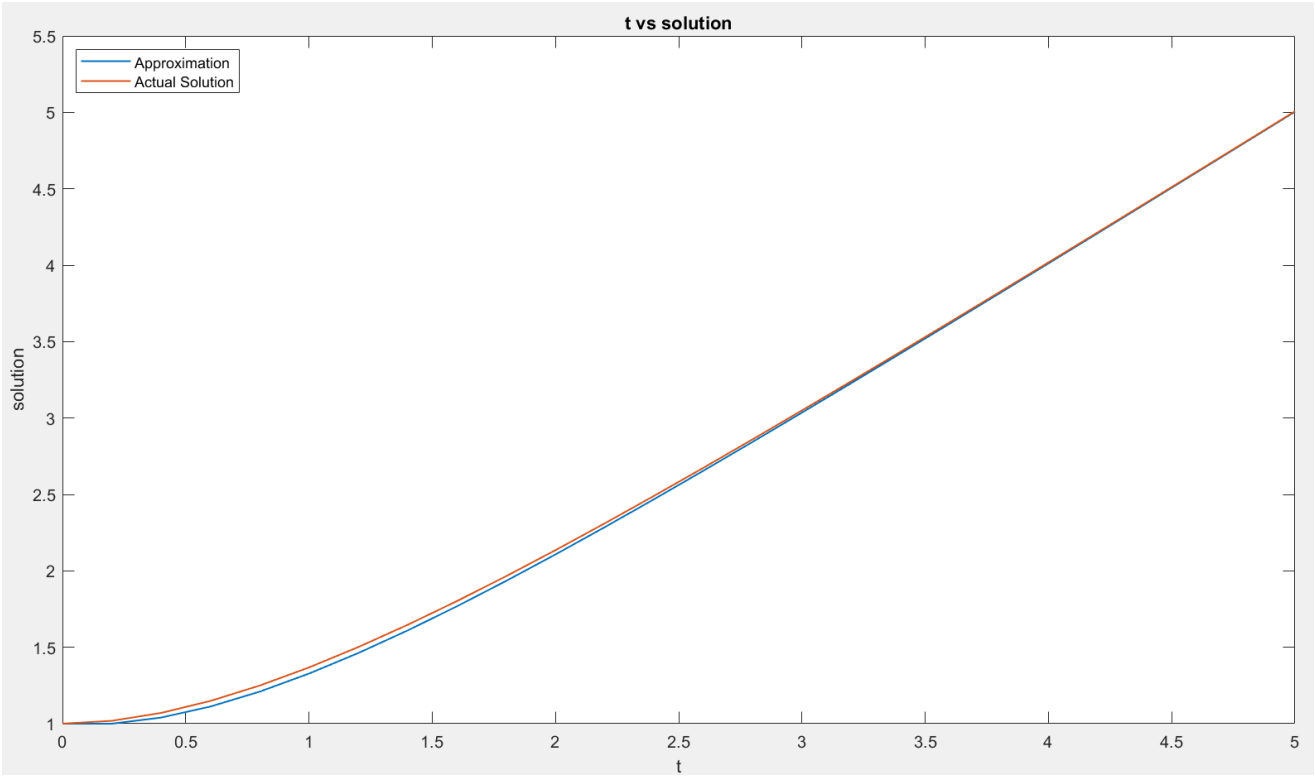
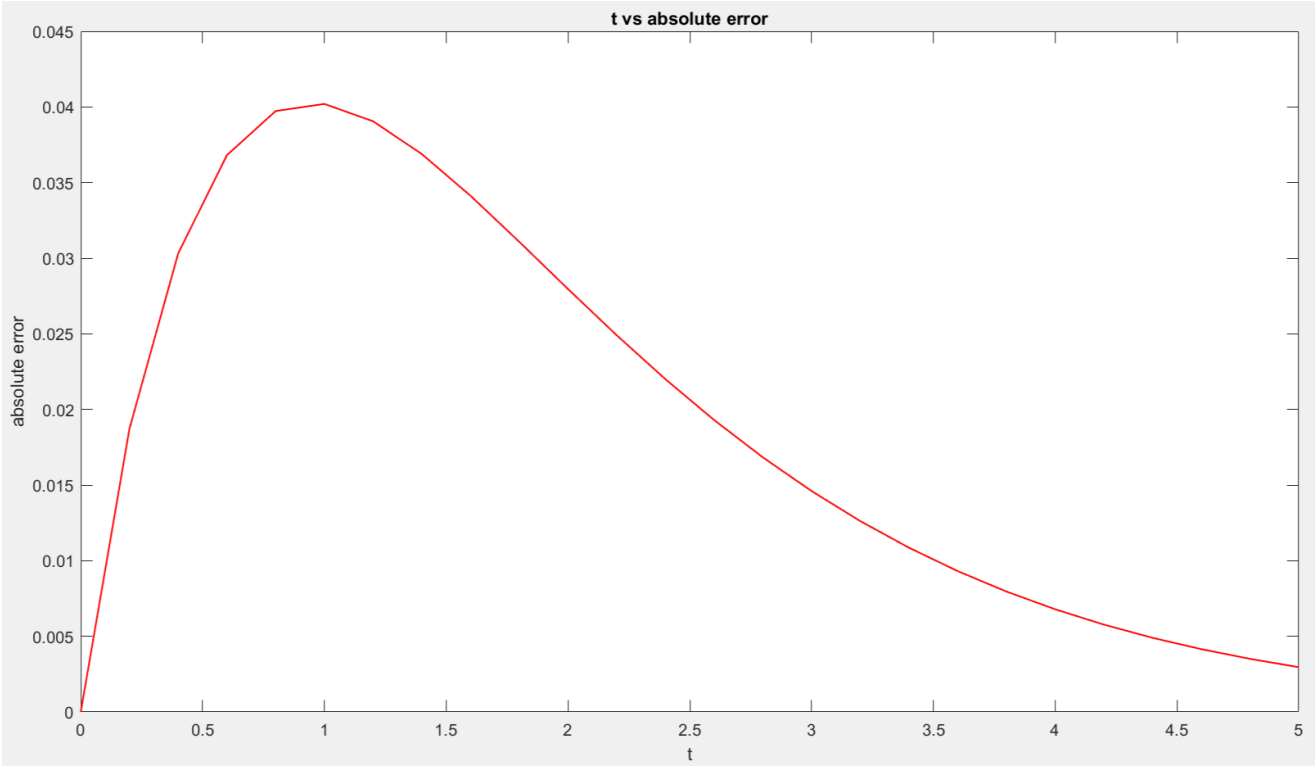
Question 5
The exact value of y(5) = 5.006738
```

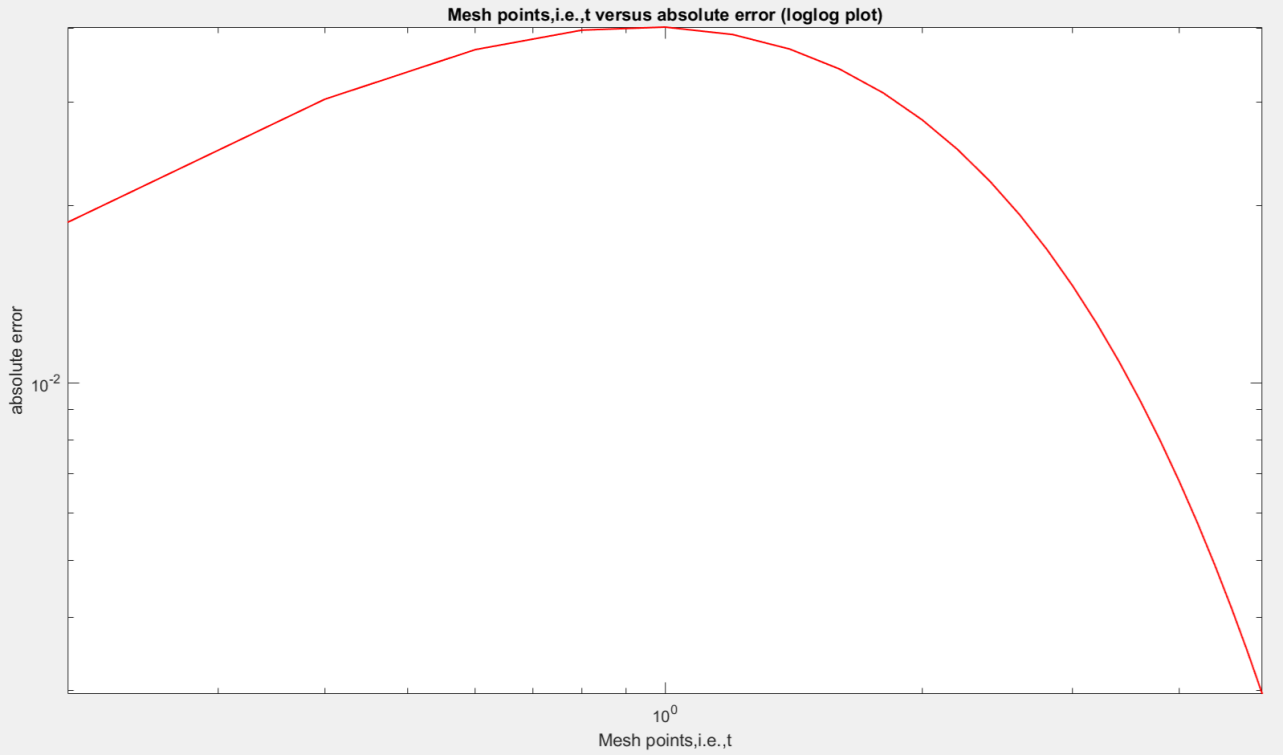
With h = 0.2,

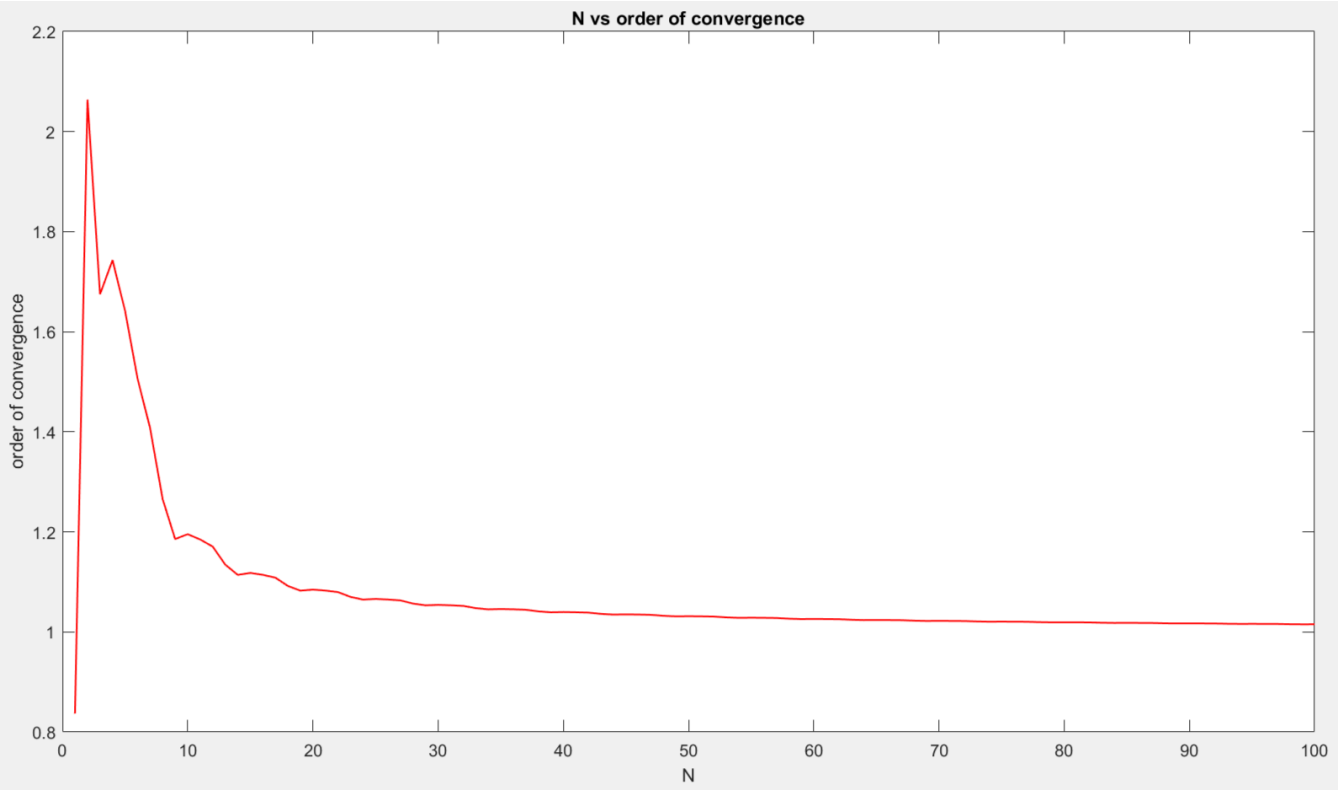
```
For h = 0.20,
   t         Approximate Solution     Exact Solution     Absolute Error
0.000000         1.000000               1.000000          0.000000
0.200000         1.000000               1.018731          0.018731
0.400000         1.040000               1.070320          0.030320
0.600000         1.112000               1.148812          0.036812
0.800000         1.209600               1.249329          0.039729
1.000000         1.327680               1.367879          0.040199
1.200000         1.462144               1.501194          0.039050
1.400000         1.609715               1.646597          0.036882
1.600000         1.767772               1.801897          0.034124
1.800000         1.934218               1.965299          0.031081
2.000000         2.107374               2.135335          0.027961
2.200000         2.285899               2.310803          0.024904
2.400000         2.468719               2.490718          0.021998
2.600000         2.654976               2.674274          0.019298
2.800000         2.843980               2.860810          0.016830
3.000000         3.035184               3.049787          0.014603
3.200000         3.228147               3.240762          0.012615
3.400000         3.422518               3.433373          0.010855
3.600000         3.618014               3.627324          0.009309
3.800000         3.814412               3.822371          0.007959
4.000000         4.011529               4.018316          0.006786
4.200000         4.209223               4.214996          0.005772
4.400000         4.407379               4.412277          0.004899
4.600000         4.605903               4.610052          0.004149
4.800000         4.804722               4.808230          0.003507
5.000000         5.003778               5.006738          0.002960
```

```
Approximate value of y(5) using Explicit-Euler's method for h = 0.20 is 5.003778
```


t vs solution
Approximation
Actual Solution
solution
t
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


t vs absolute error
absolute error
t
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Mesh points,i.e.,t versus absolute error (loglog plot)
absolute error
10^{-2}
10^{0}
Mesh points,i.e.,t


N vs order of convergence
order of convergence
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
10
20
30
40
50
60
70
80
90
100
N

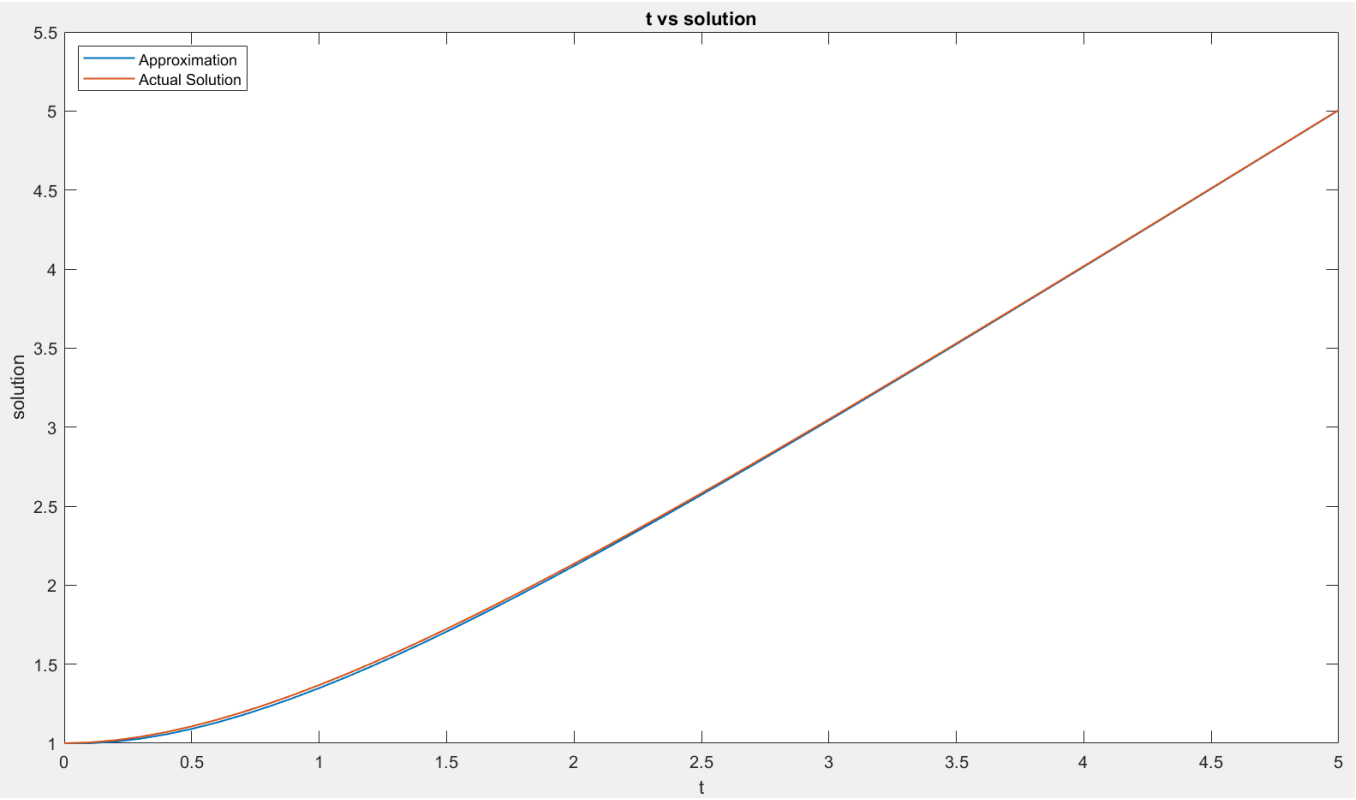
With h = 0.1,

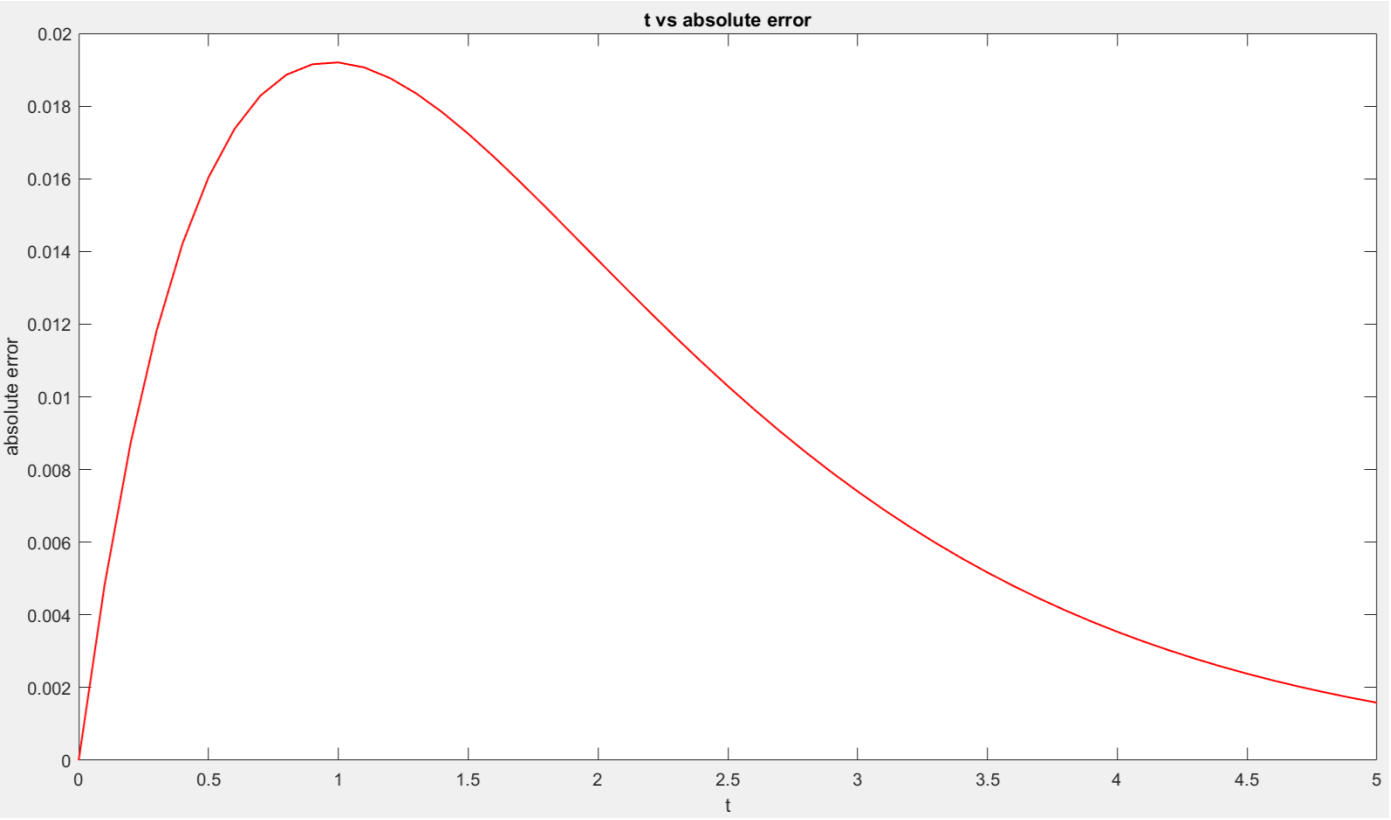
For h = 0.10,

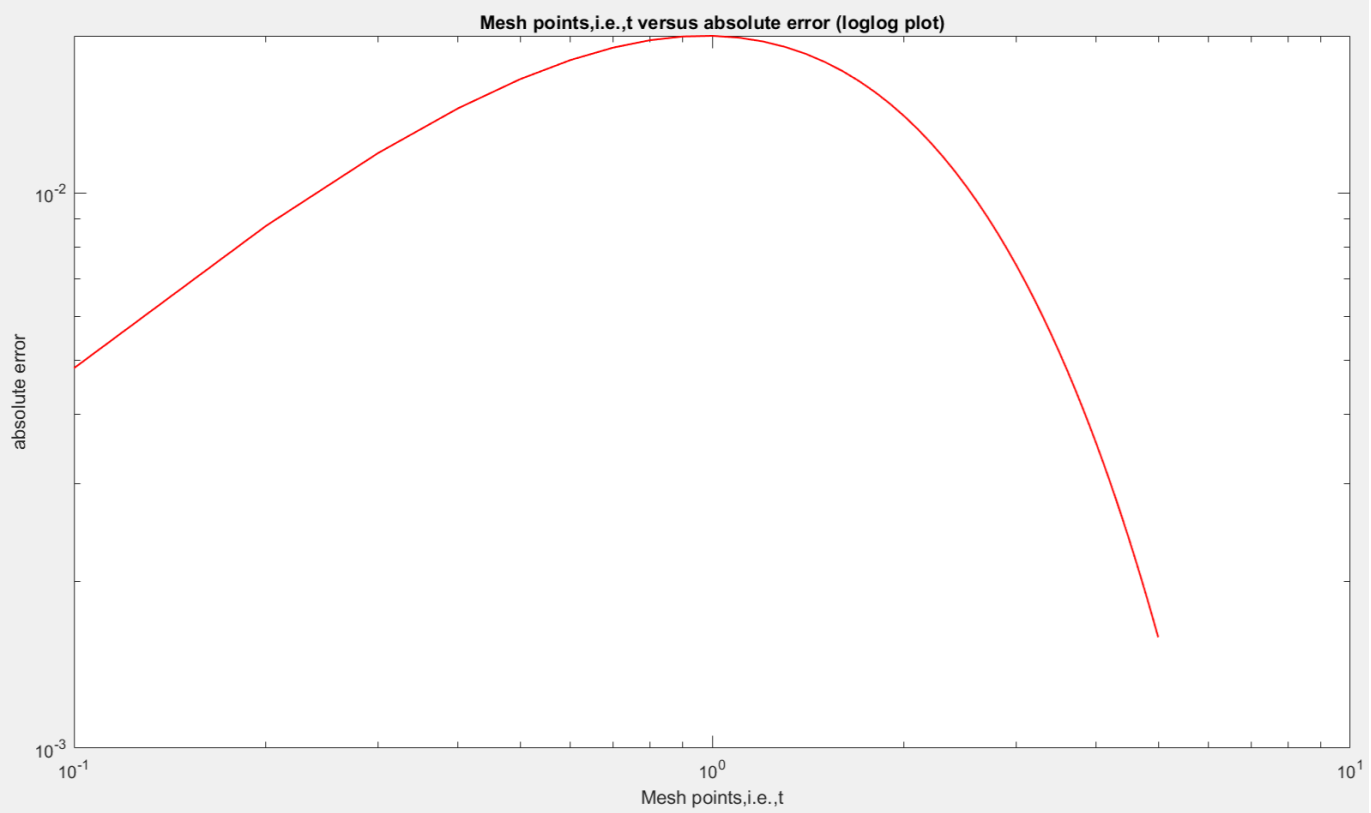
t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.000000	1.004837	0.004837
0.200000	1.010000	1.018731	0.008731
0.300000	1.029000	1.040818	0.011818
0.400000	1.056100	1.070320	0.014220
0.500000	1.090490	1.106531	0.016041
0.600000	1.131441	1.148812	0.017371
0.700000	1.178297	1.196585	0.018288
0.800000	1.230467	1.249329	0.018862
0.900000	1.287420	1.306570	0.019149
1.000000	1.348678	1.367879	0.019201
1.100000	1.413811	1.432871	0.019060
1.200000	1.482430	1.501194	0.018765
1.300000	1.554187	1.572532	0.018345
1.400000	1.628768	1.646597	0.017829
1.500000	1.705891	1.723130	0.017239
1.600000	1.785302	1.801897	0.016594
1.700000	1.866772	1.882684	0.015912
1.800000	1.950095	1.965299	0.015204
1.900000	2.035085	2.049569	0.014483
2.000000	2.121577	2.135335	0.013759
2.100000	2.209419	2.222456	0.013037
2.200000	2.298477	2.310803	0.012326
2.300000	2.388629	2.400259	0.011629
2.400000	2.479766	2.490718	0.010952

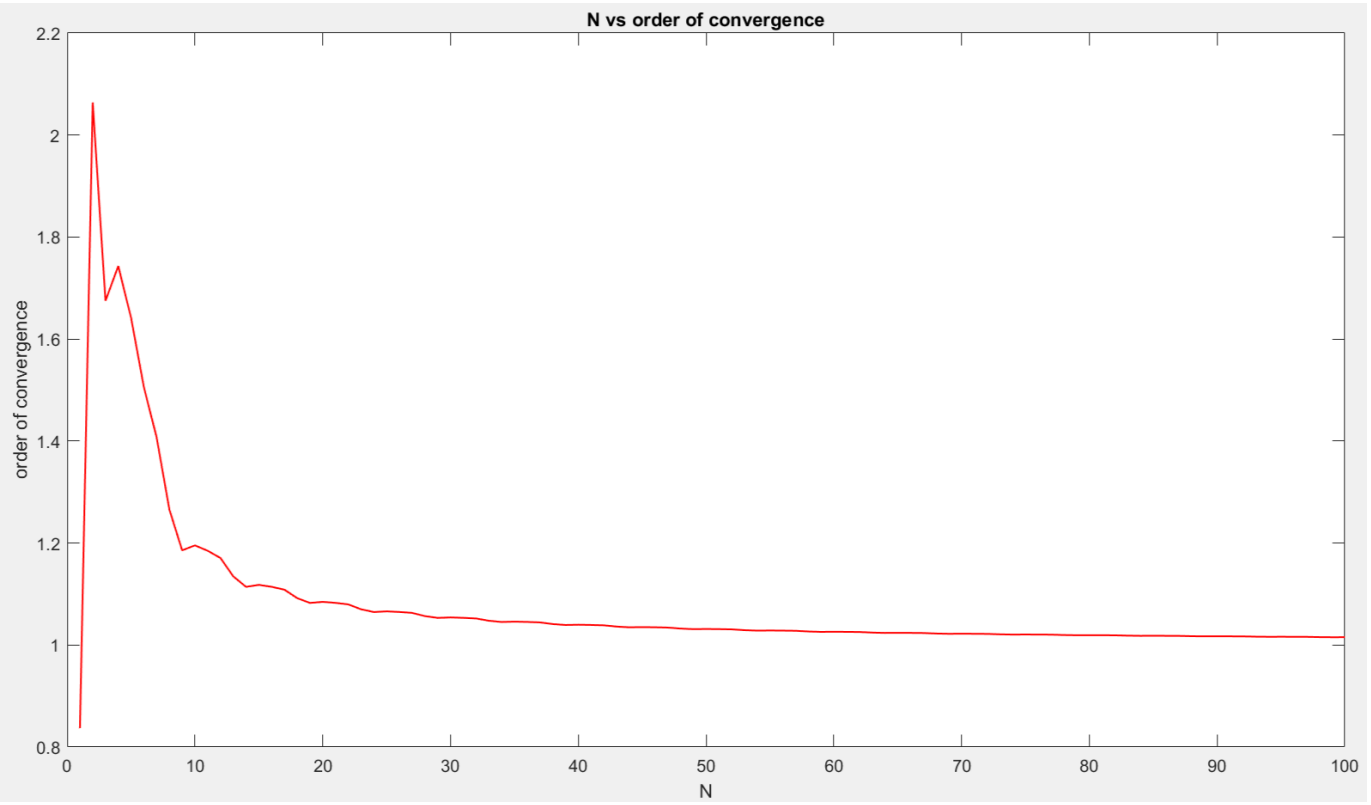
2.500000	2.571790	2.582085	0.010295
2.600000	2.664611	2.674274	0.009663
2.700000	2.758150	2.767206	0.009056
2.800000	2.852335	2.860810	0.008475
2.900000	2.947101	2.955023	0.007922
3.000000	3.042391	3.049787	0.007396
3.100000	3.138152	3.145049	0.006897
3.200000	3.234337	3.240762	0.006425
3.300000	3.330903	3.336883	0.005980
3.400000	3.427813	3.433373	0.005560
3.500000	3.525032	3.530197	0.005166
3.600000	3.622528	3.627324	0.004795
3.700000	3.720276	3.724724	0.004448
3.800000	3.818248	3.822371	0.004123
3.900000	3.916423	3.920242	0.003819
4.000000	4.014781	4.018316	0.003535
4.100000	4.113303	4.116573	0.003270
4.200000	4.211973	4.214996	0.003023
4.300000	4.310775	4.313569	0.002793
4.400000	4.409698	4.412277	0.002580
4.500000	4.508728	4.511109	0.002381
4.600000	4.607855	4.610052	0.002197
4.700000	4.707070	4.709095	0.002026
4.800000	4.806363	4.808230	0.001867
4.900000	4.905726	4.907447	0.001720
5.000000	5.005154	5.006738	0.001584

Approximate value of y(5) using Explicit-Euler's method for h = 0.10 is 5.005154


t vs solution
Approximation
Actual Solution
solution
t
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


t vs absolute error
absolute error
t
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Mesh points,i.e.,t versus absolute error (loglog plot)
absolute error
Mesh points,i.e.,t
10^{-2}
10^{-3}
10^{-1}
10^{0}
10^{1}


N vs order of convergence
order of convergence
N
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
10
20
30
40
50
60
70
80
90
100

With h = 0.05,

For h = 0.05,

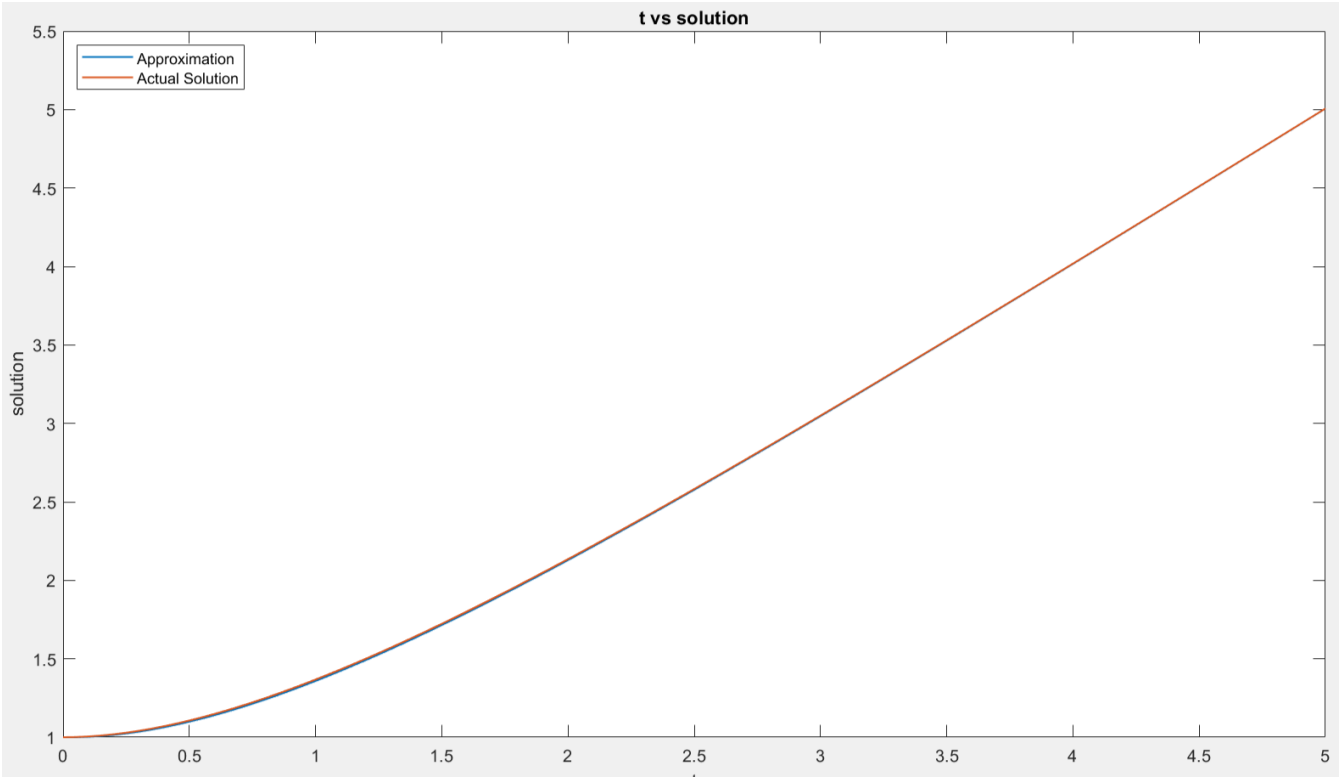
t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.050000	1.000000	1.001229	0.001229
0.100000	1.002500	1.004837	0.002337
0.150000	1.007375	1.010708	0.003333
0.200000	1.014506	1.018731	0.004225
0.250000	1.023781	1.028801	0.005020
0.300000	1.035092	1.040818	0.005726
0.350000	1.048337	1.054688	0.006351
0.400000	1.063420	1.070320	0.006900
0.450000	1.080249	1.087628	0.007379
0.500000	1.098737	1.106531	0.007794
0.550000	1.118800	1.126950	0.008150
0.600000	1.140360	1.148812	0.008452
0.650000	1.163342	1.172046	0.008704
0.700000	1.187675	1.196585	0.008910
0.750000	1.213291	1.222367	0.009075
0.800000	1.240127	1.249329	0.009202
0.850000	1.268120	1.277415	0.009295
0.900000	1.297214	1.306570	0.009355
0.950000	1.327354	1.336741	0.009387
1.000000	1.358486	1.367879	0.009394
1.050000	1.390562	1.399938	0.009376
1.100000	1.423534	1.432871	0.009338
1.150000	1.457357	1.466637	0.009280
1.200000	1.491989	1.501194	0.009205
1.250000	1.527390	1.536505	0.009115

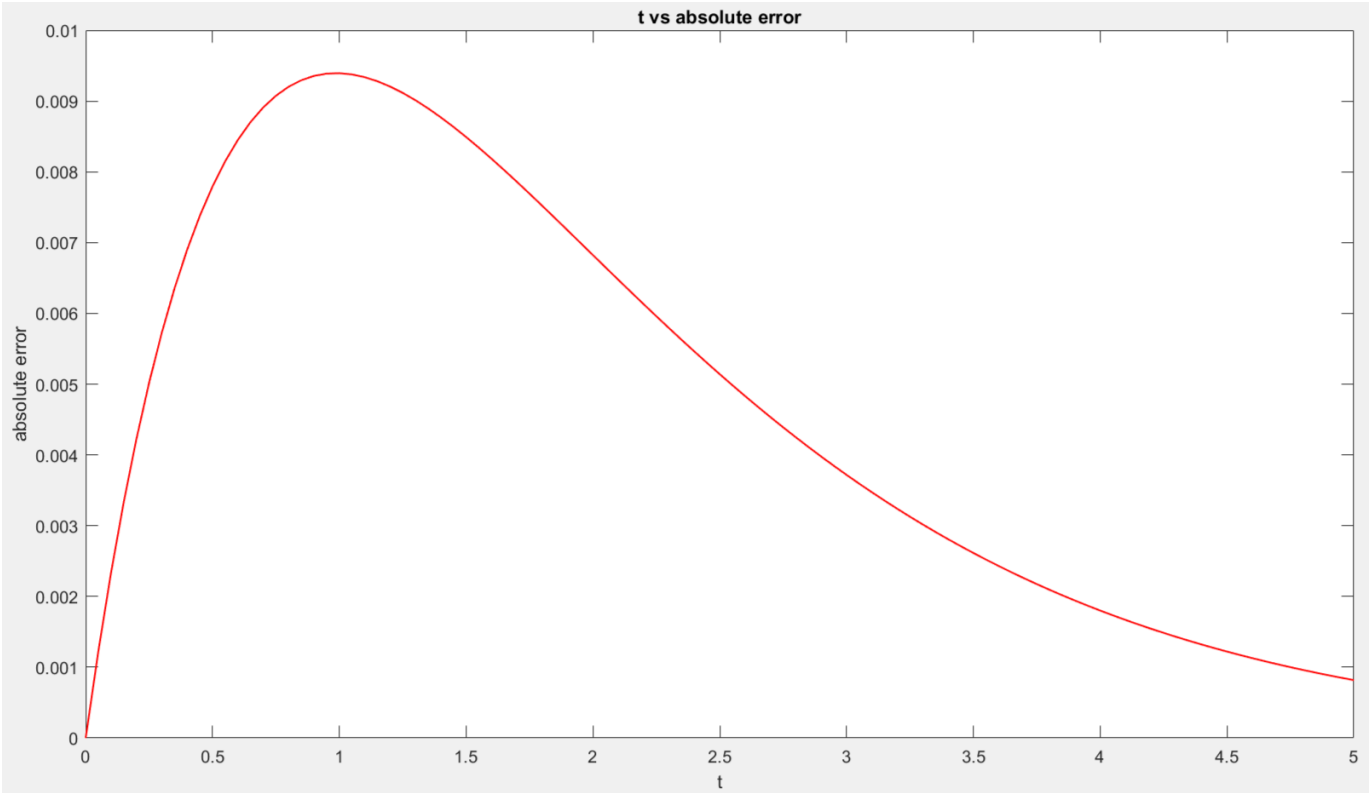
1.300000	1.563520	1.572532	0.009012
1.350000	1.600344	1.609240	0.008896
1.400000	1.637827	1.646597	0.008770
1.450000	1.675936	1.684570	0.008635
1.500000	1.714639	1.723130	0.008491
1.550000	1.753907	1.762248	0.008341
1.600000	1.793711	1.801897	0.008185
1.650000	1.834026	1.842050	0.008024
1.700000	1.874825	1.882684	0.007859
1.750000	1.916083	1.923774	0.007691
1.800000	1.957779	1.965299	0.007520
1.850000	1.999890	2.007237	0.007347
1.900000	2.042396	2.049569	0.007173
1.950000	2.085276	2.092274	0.006998
2.000000	2.128512	2.135335	0.006823
2.050000	2.172087	2.178735	0.006648
2.100000	2.215982	2.222456	0.006474
2.150000	2.260183	2.266484	0.006301
2.200000	2.304674	2.310803	0.006129
2.250000	2.349440	2.355399	0.005959
2.300000	2.394468	2.400259	0.005791
2.350000	2.439745	2.445369	0.005624
2.400000	2.485258	2.490718	0.005460

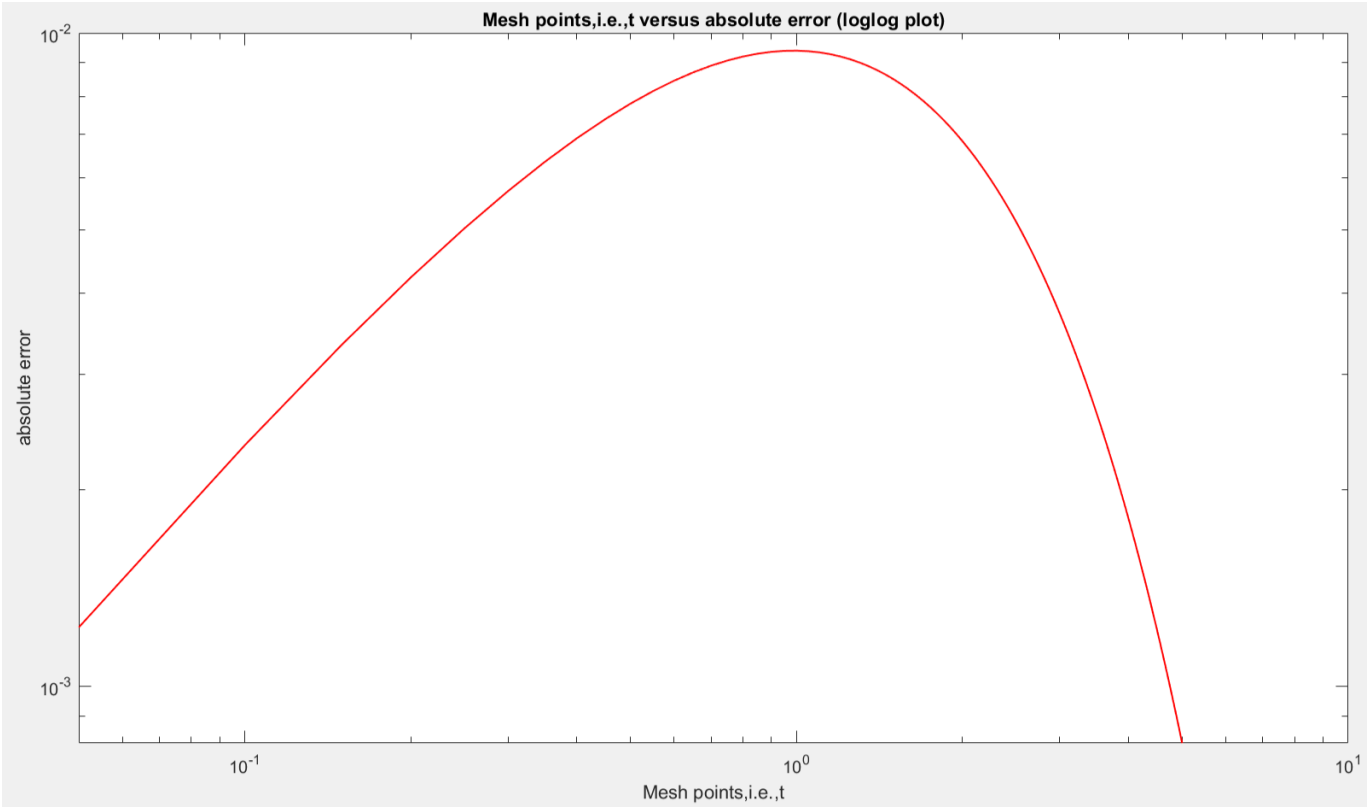
2.450000	2.530995	2.536294	0.005299
2.500000	2.576945	2.582085	0.005140
2.550000	2.623098	2.628082	0.004984
2.600000	2.669443	2.674274	0.004831
2.650000	2.715971	2.720651	0.004681
2.700000	2.762672	2.767206	0.004533
2.750000	2.809539	2.813928	0.004389
2.800000	2.856562	2.860810	0.004248
2.850000	2.903734	2.907844	0.004111
2.900000	2.951047	2.955023	0.003976
2.950000	2.998495	3.002340	0.003845
3.000000	3.046070	3.049787	0.003717
3.050000	3.093766	3.097359	0.003593
3.100000	3.141578	3.145049	0.003471
3.150000	3.189499	3.192852	0.003353
3.200000	3.237524	3.240762	0.003238
3.250000	3.285648	3.288774	0.003126
3.300000	3.333866	3.336883	0.003018
3.350000	3.382172	3.385084	0.002912
3.400000	3.430564	3.433373	0.002810
3.450000	3.479035	3.481746	0.002710
3.500000	3.527584	3.530197	0.002614
3.550000	3.576205	3.578725	0.002520
3.600000	3.624894	3.627324	0.002429

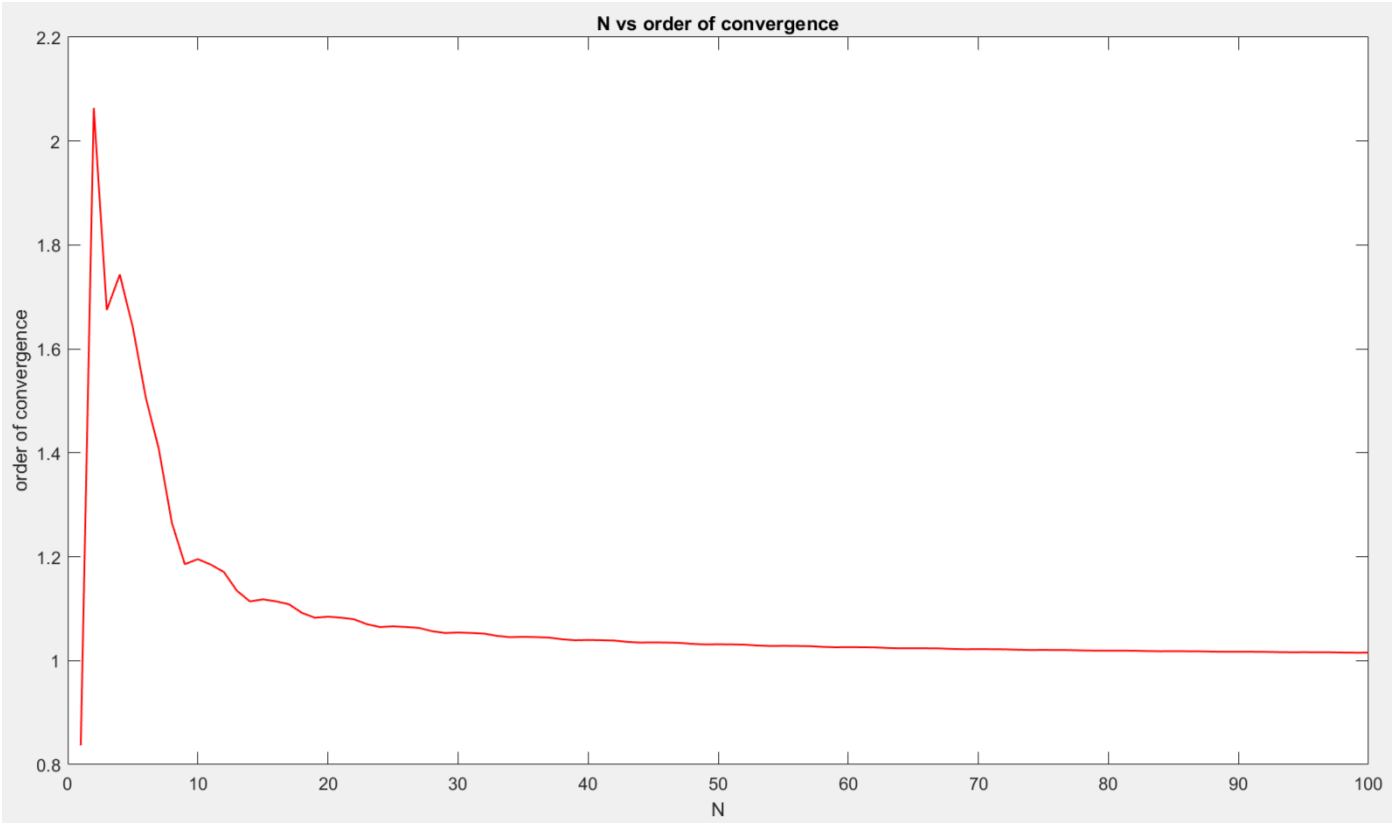
3.650000	3.673650	3.675991	0.002342
3.700000	3.722467	3.724724	0.002256
3.750000	3.771344	3.773518	0.002174
3.800000	3.820277	3.822371	0.002094
3.850000	3.869263	3.871280	0.002017
3.900000	3.918300	3.920242	0.001942
3.950000	3.967385	3.969255	0.001870
4.000000	4.016515	4.018316	0.001800
4.050000	4.065690	4.067422	0.001733
4.100000	4.114905	4.116573	0.001668
4.150000	4.164160	4.165764	0.001605
4.200000	4.213452	4.214996	0.001544
4.250000	4.262779	4.264264	0.001485
4.300000	4.312140	4.313569	0.001428
4.350000	4.361533	4.362907	0.001374
4.400000	4.410957	4.412277	0.001321
4.450000	4.460409	4.461679	0.001270
4.500000	4.509888	4.511109	0.001221
4.550000	4.559394	4.560567	0.001173
4.600000	4.608924	4.610052	0.001128
4.650000	4.658478	4.659562	0.001084
4.700000	4.708054	4.709095	0.001041
4.750000	4.757651	4.758652	0.001000
4.800000	4.807269	4.808230	0.000961
4.850000	4.856905	4.857828	0.000923
4.900000	4.906560	4.907447	0.000886
4.950000	4.956232	4.957083	0.000851
5.000000	5.005921	5.006738	0.000817

Approximate value of y(5) using Explicit-Euler's method for h = 0.05 is 5.005921


t vs solution
Approximation
Actual Solution
solution
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


t vs absolute error
absolute error
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t


Mesh points,i.e.,t versus absolute error (loglog plot)
absolute error
Mesh points,i.e.,t
10^{-2}
10^{-3}
10^{-1}
10^{0}
10^{1}


N vs order of convergence
order of convergence
N
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
10
20
30
40
50
60
70
80
90
100