<u>Scientific Computing Lab</u>   <u>MA – 322</u>   <u>Lab – 12</u>

Name – Rasesh Srivastava

Roll Number – 210123072

Branch – Mathematics and Computing

For each part of all the questions, the five plots are as follows:

(i) t versus actual and approximate solutions

(ii) t versus absolute error

(iii) N versus Order of Convergence

(iv) loglog plot of t versus absolute error

(v) $\log(E_N)$ versus $\log(N)$

Where, N = (b-a)/h = Number of intervals

Order of convergence = $\log_2(E_N/E_{2N})$, where E_N and E_{2N} and the maximum errors obtained during the computation for that specific value of N

Absolute error = absolute value of the difference between the approximation and the actual solution

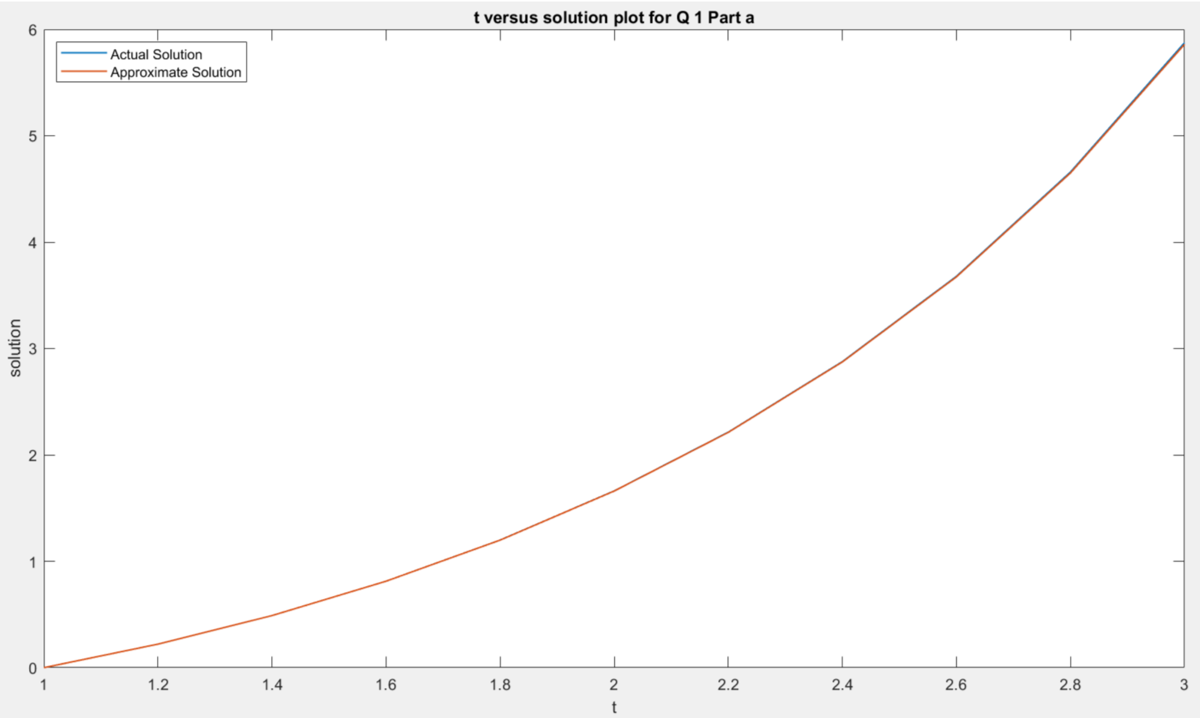
1)

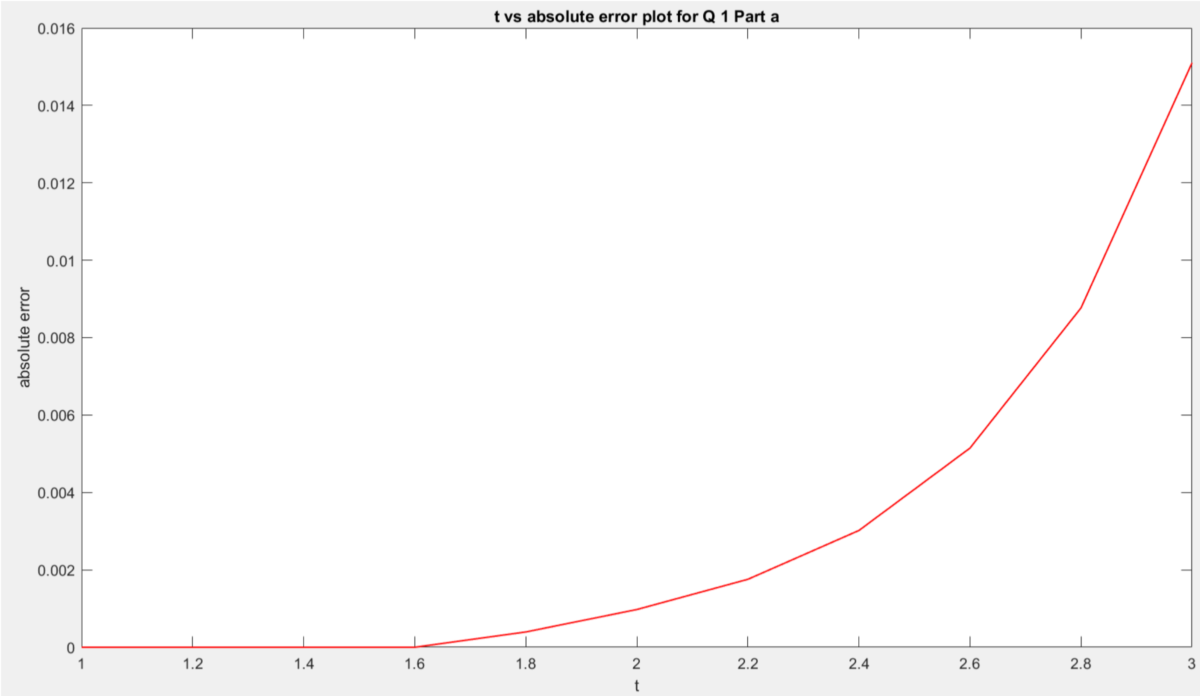
a)

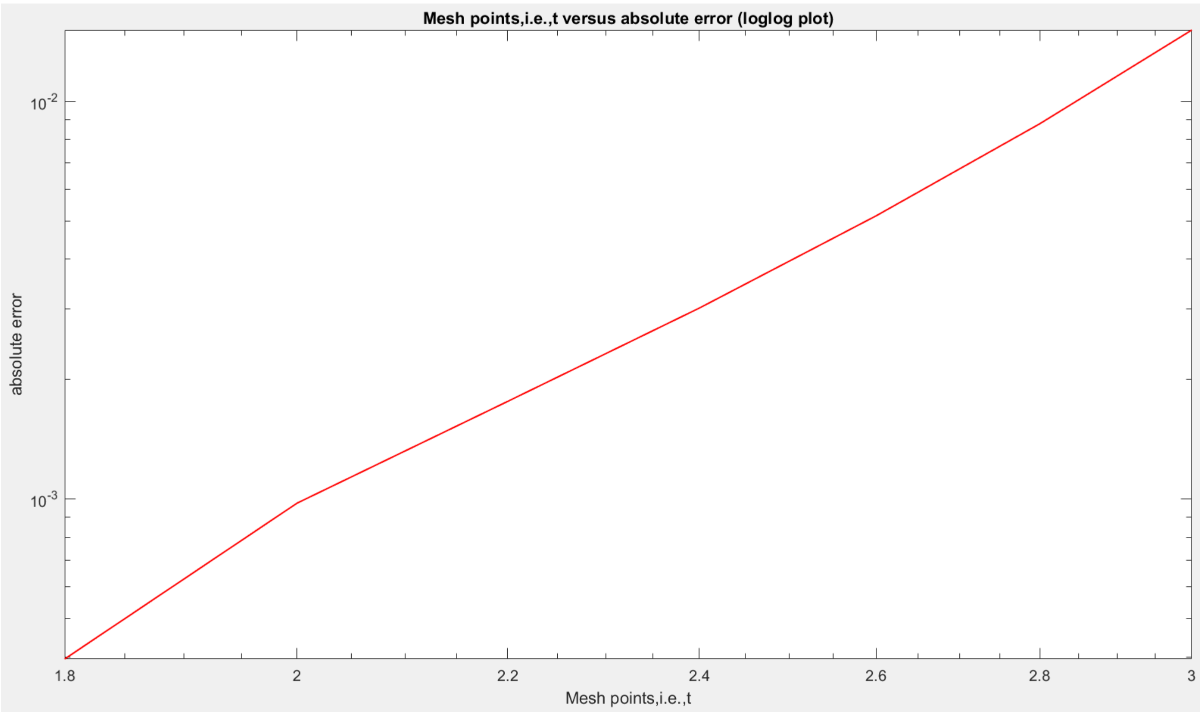
Using Adams-Bashforth method,

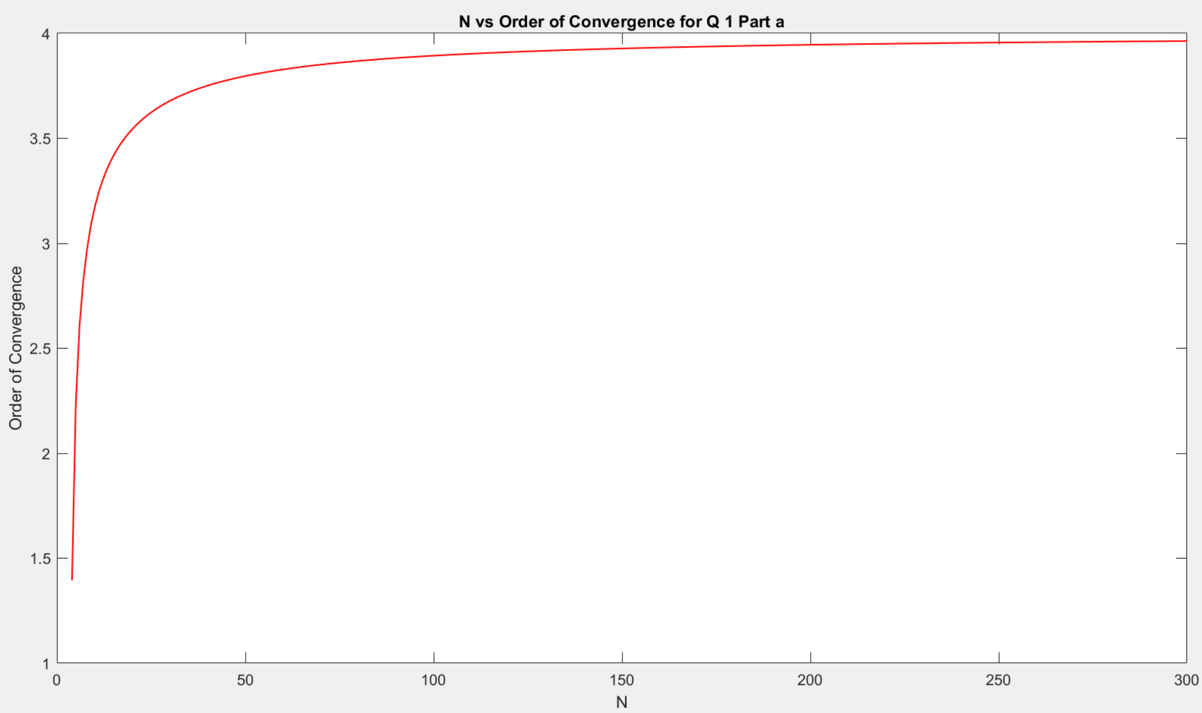
```
Question 1 Part a) using exact starting values,

Using Adams-Bashforth method for Question 1 Part a,
     t          Approximate Solution     Exact Solution  Absolute Error
1.000000         0.000000               0.000000         0.000000
1.200000         0.221243               0.221243         0.000000
1.400000         0.489682               0.489682         0.000000
1.600000         0.812753               0.812753         0.000000
1.800000         1.199044               1.199439         0.000395
2.000000         1.660307               1.661282         0.000975
2.200000         2.211746               2.213502         0.001756
2.400000         2.873534               2.876551         0.003017
2.600000         3.673329               3.678475         0.005146
2.800000         4.649897               4.658665         0.008768
3.000000         5.858999               5.874100         0.015101
```

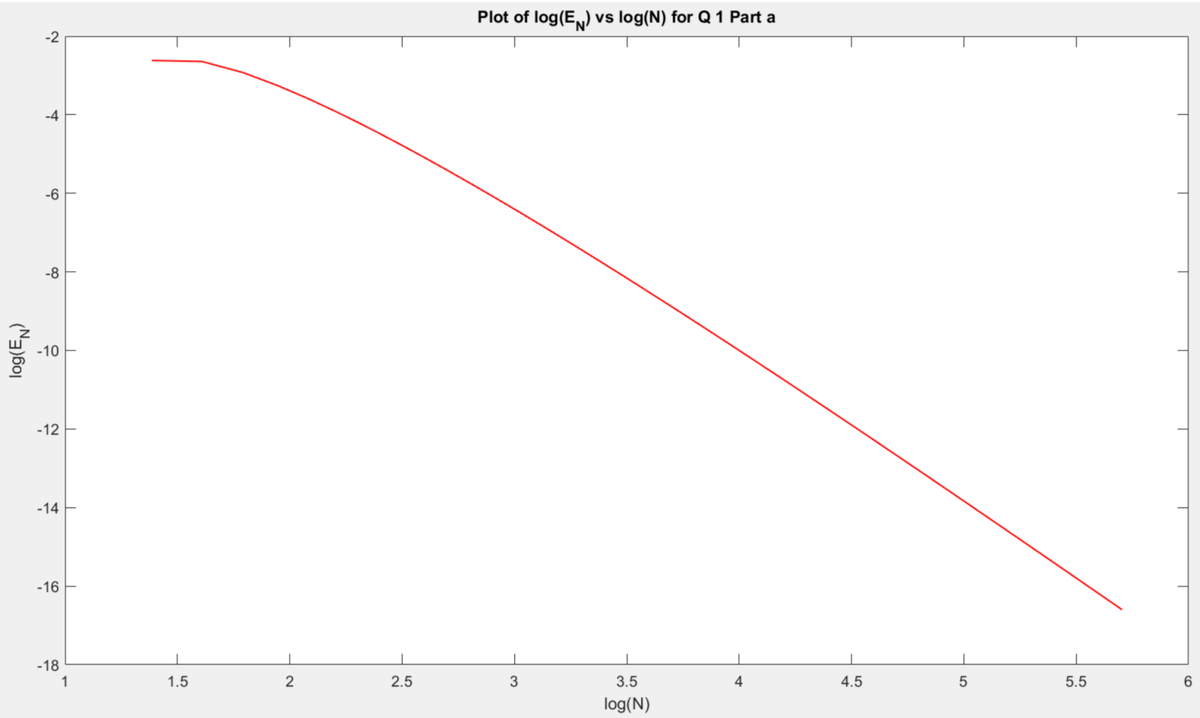
The graphs are as follows:




t vs absolute error plot for Q 1 Part a
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
absolute error
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
t


Mesh points,i.e.,t versus absolute error (loglog plot)
10^{-2}
10^{-3}
absolute error
1.8
2
2.2
2.4
2.6
2.8
3
Mesh points,i.e.,t

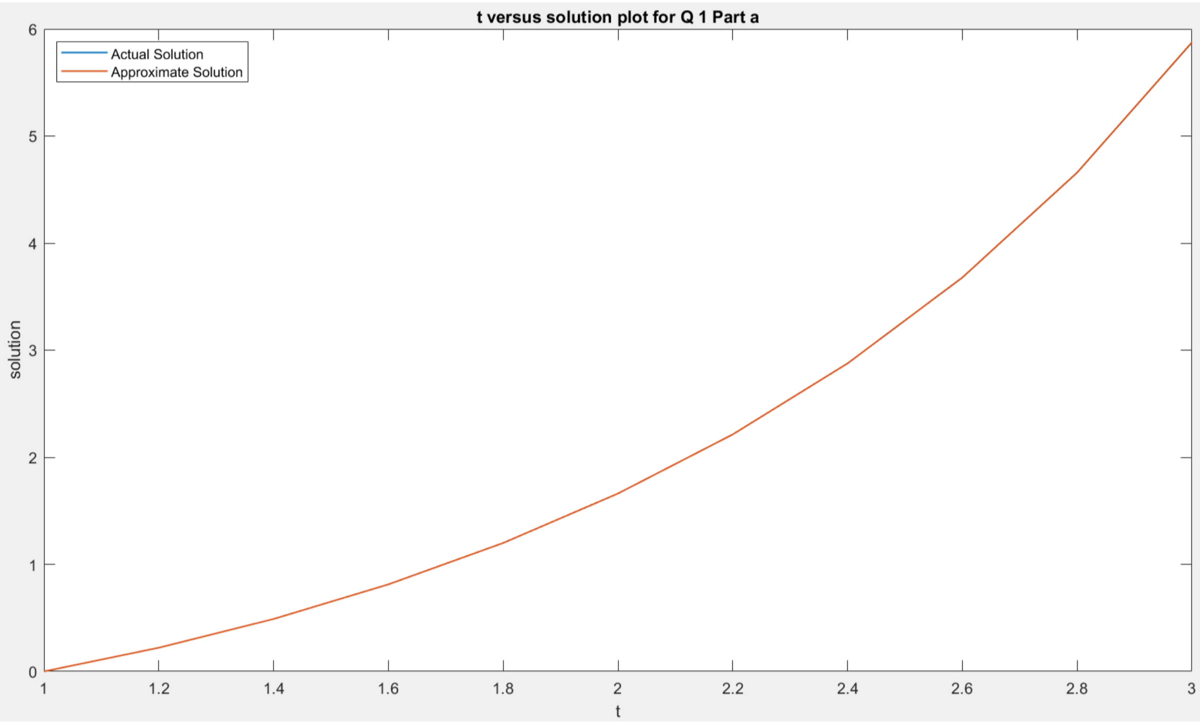


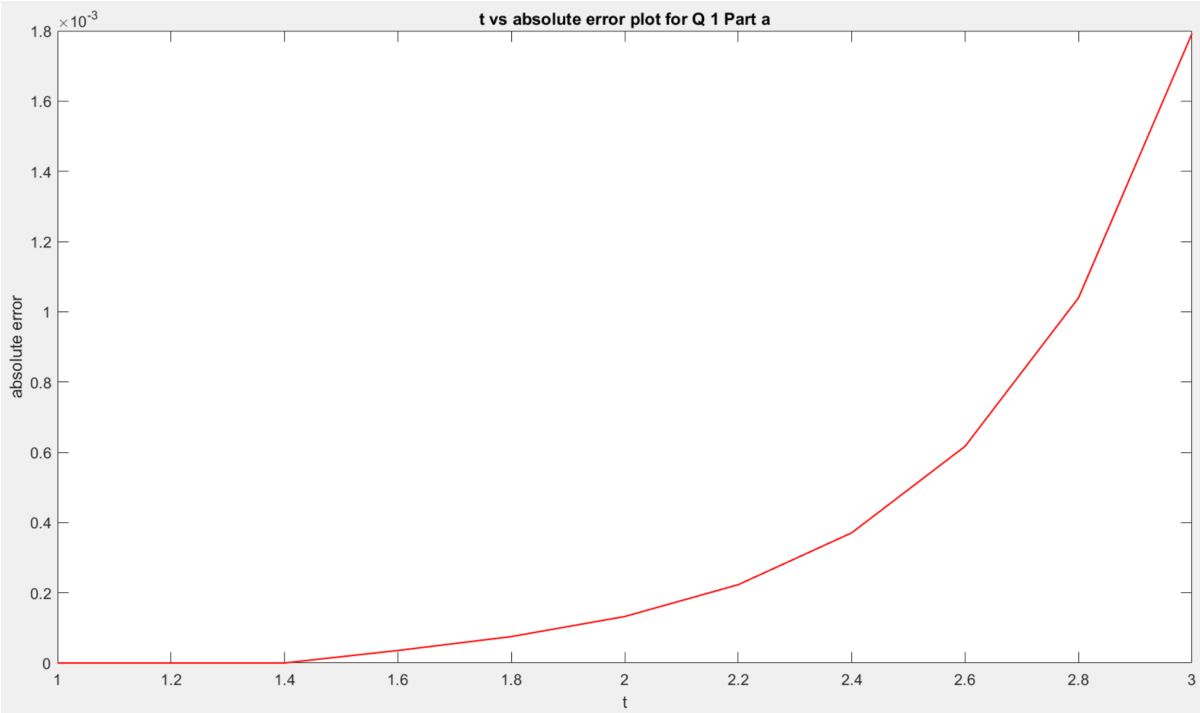


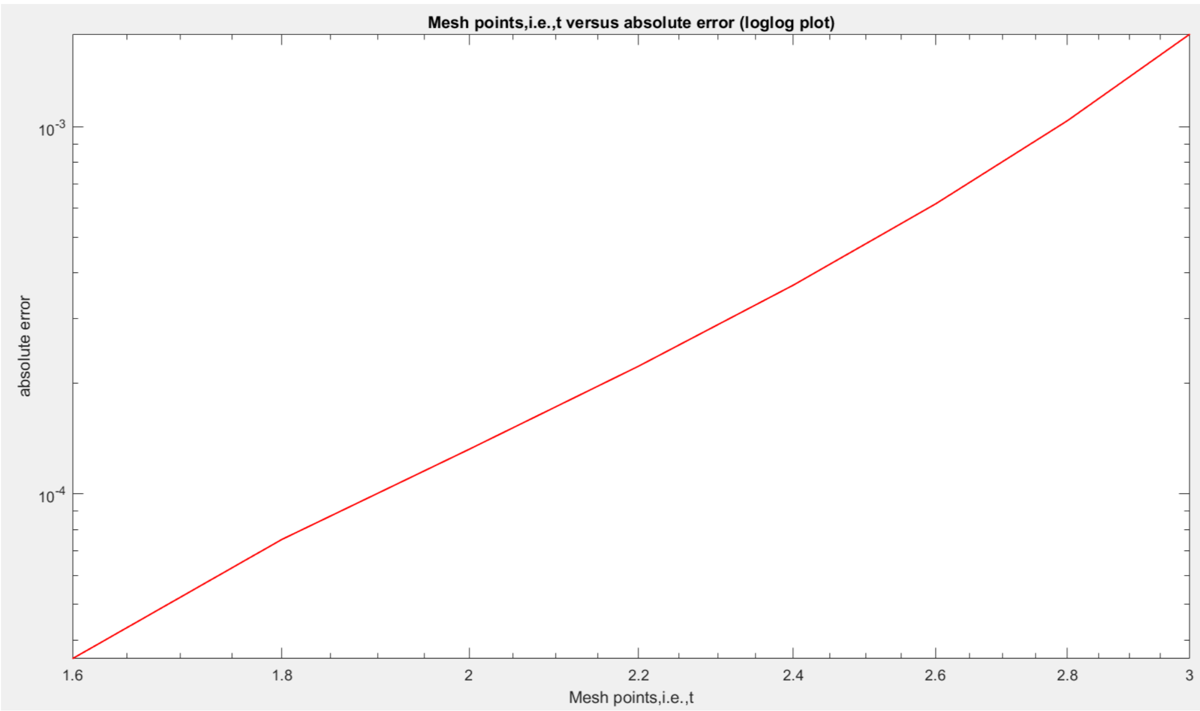
Using Admas-Moulton method,

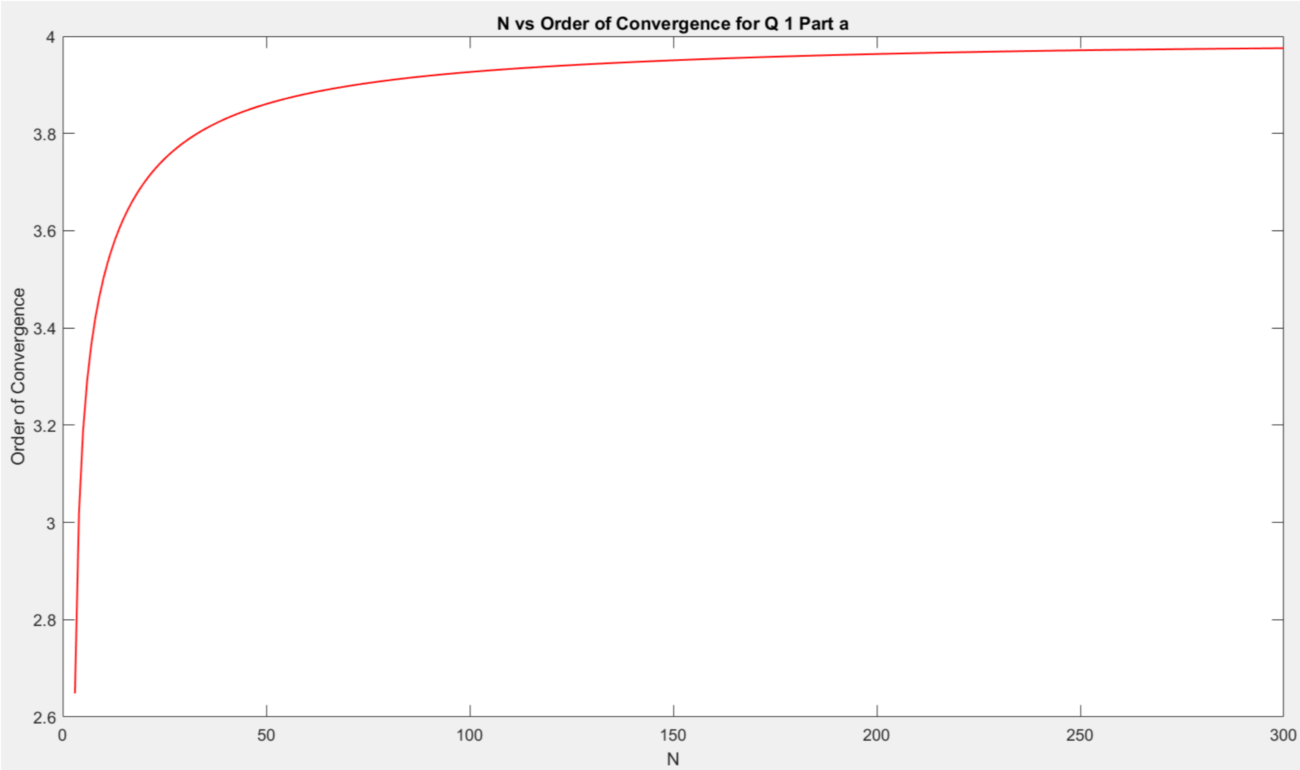
```
Question 1 Part a) using exact starting values,

Using Adams-Moulton method for Question 1 Part a,
     t         Approximate Solution     Exact Solution  Absolute Error
1.000000           0.000000               0.000000          0.000000
1.200000           0.221243               0.221243          0.000000
1.400000           0.489682               0.489682          0.000000
1.600000           0.812788               0.812753          0.000036
1.800000           1.199514               1.199439          0.000075
2.000000           1.661414               1.661282          0.000132
2.200000           2.213725               2.213502          0.000223
2.400000           2.876922               2.876551          0.000371
2.600000           3.679092               3.678475          0.000617
2.800000           4.659705               4.658665          0.001040
3.000000           5.875892               5.874100          0.001792
```

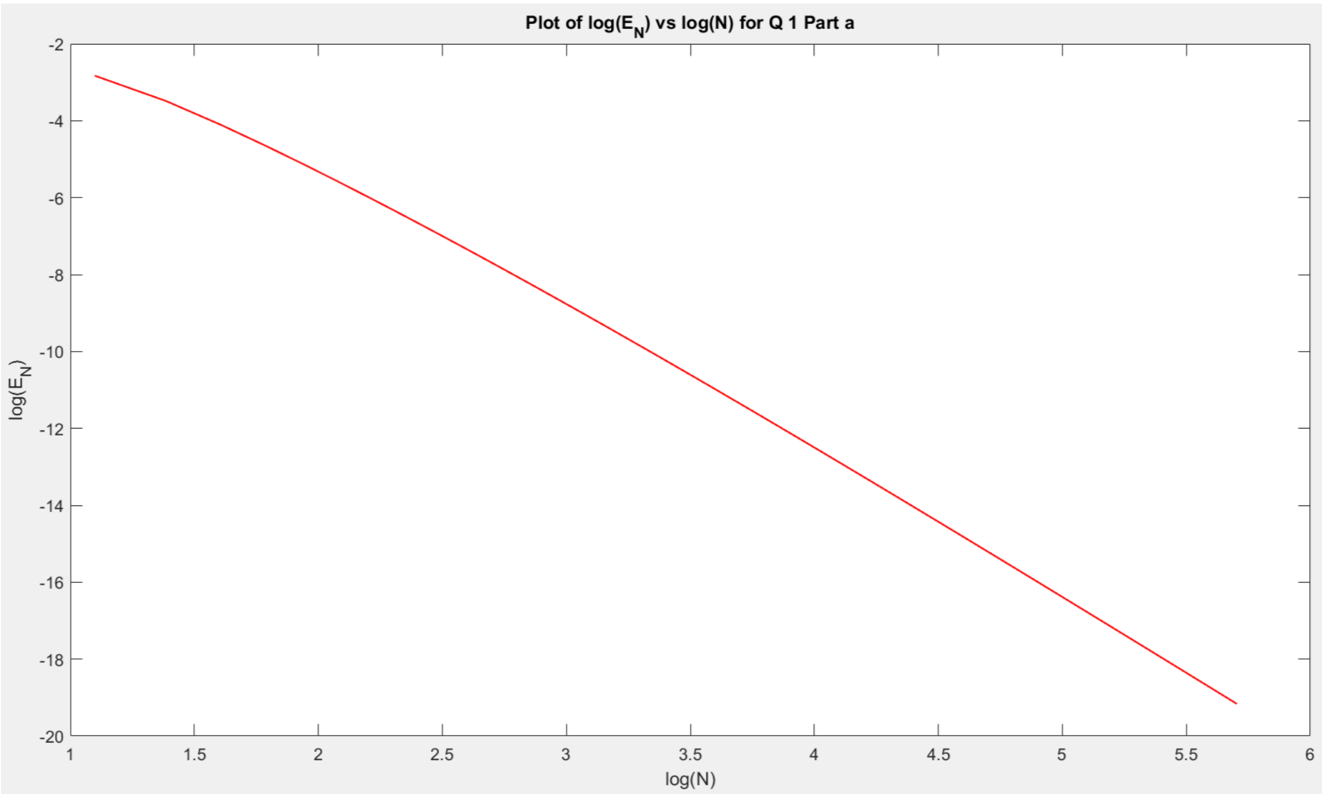
The graphs are as follows:




t vs absolute error plot for Q 1 Part a
×10-3
absolute error
t
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3


Mesh points,i.e.,t versus absolute error (loglog plot)
absolute error
Mesh points,i.e.,t
10-3
10-4
1.6
1.8
2
2.2
2.4
2.6
2.8
3





b)

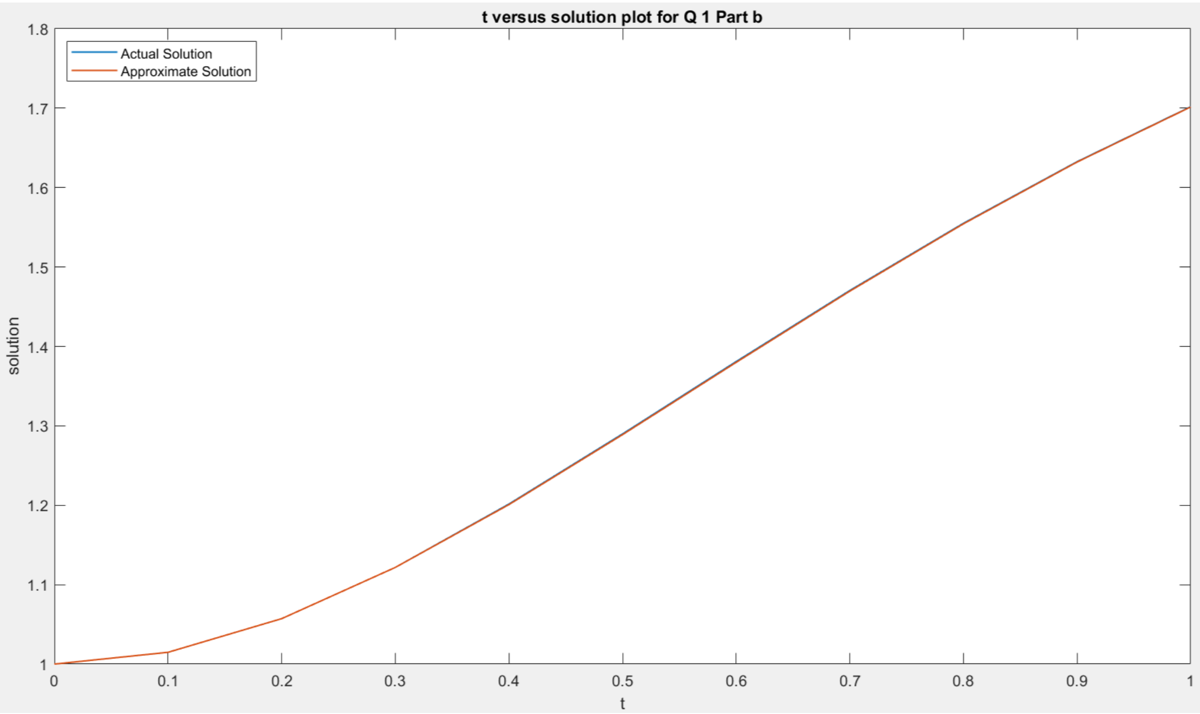
Using Admas-Bashforth method,

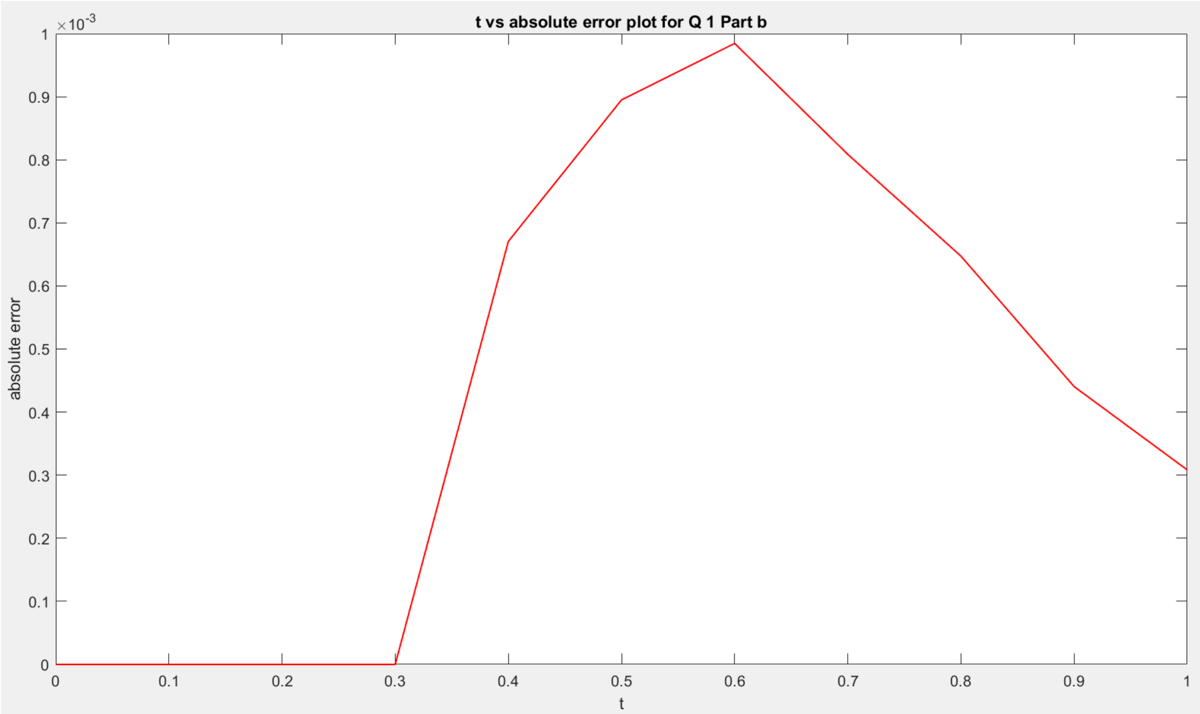
Question 1 Part b) using exact starting values,

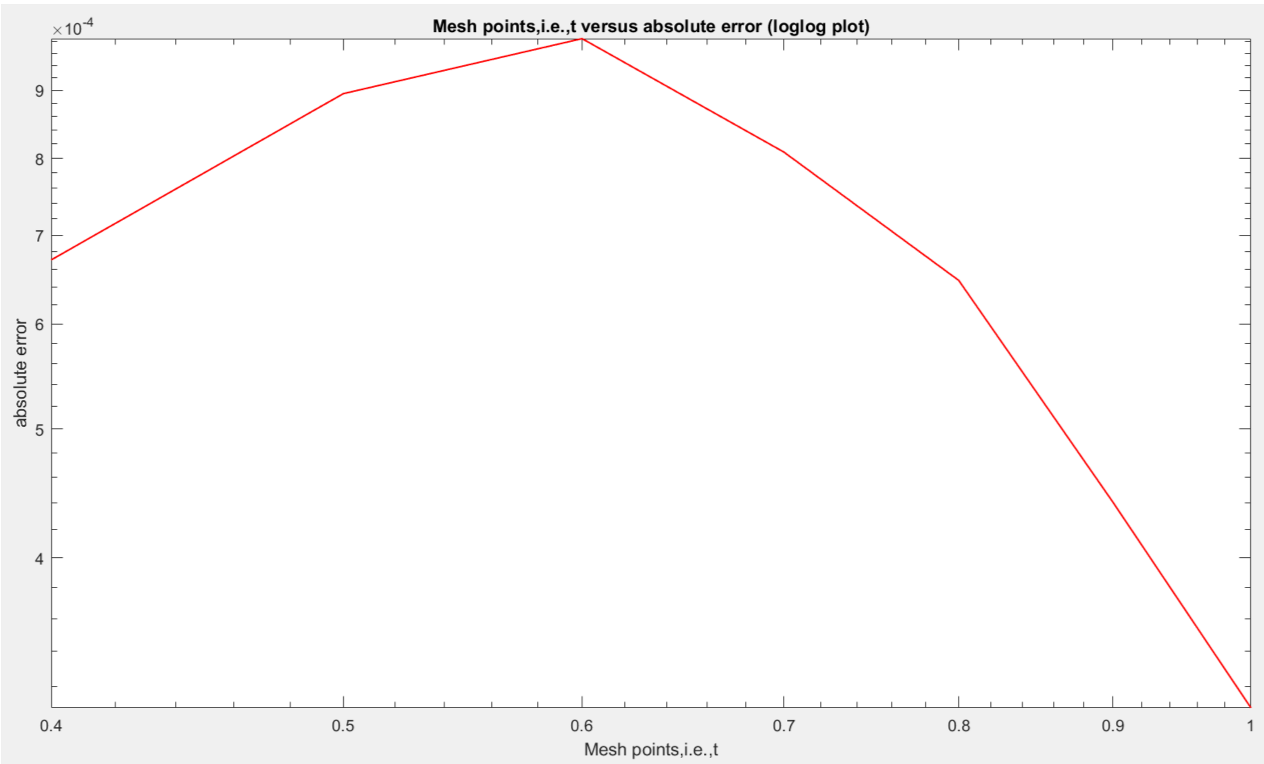
Using Adams-Bashforth method for Question 1 Part b,

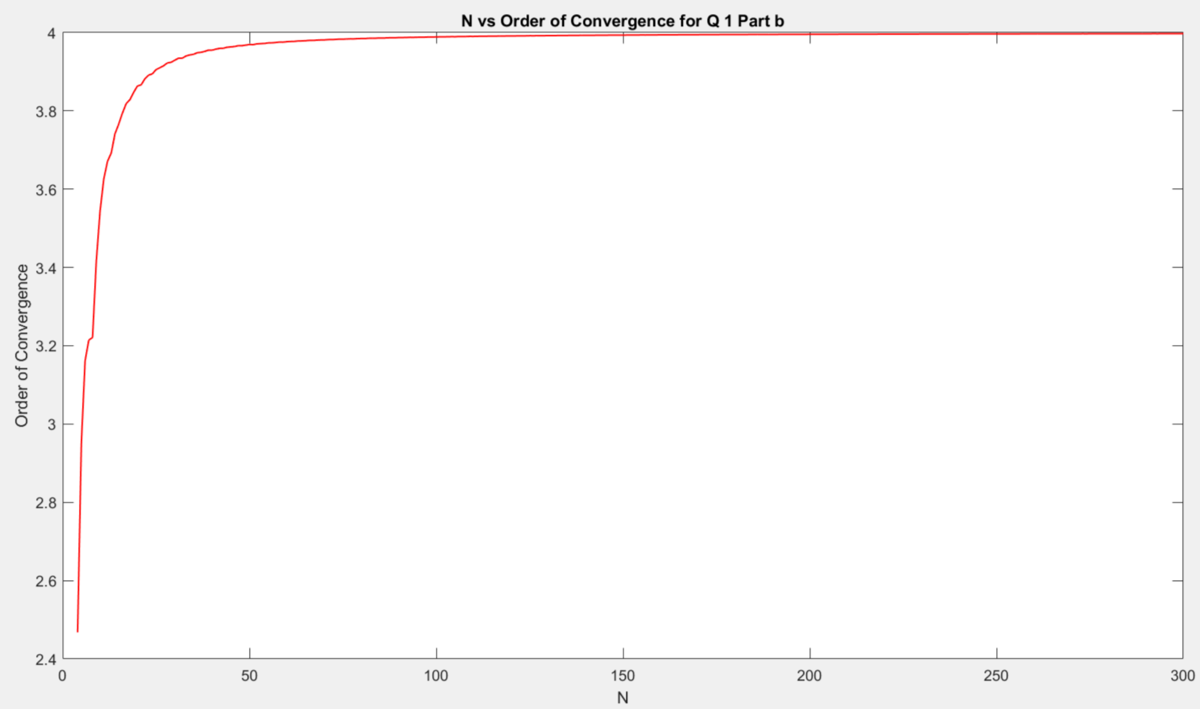
t	Approximate Solution	Exact Solution	Absolute Error
0.000000	1.000000	1.000000	0.000000
0.100000	1.014815	1.014815	0.000000
0.200000	1.057181	1.057181	0.000000
0.300000	1.121698	1.121698	0.000000
0.400000	1.200815	1.201486	0.000671
0.500000	1.288910	1.289805	0.000895
0.600000	1.379947	1.380931	0.000985
0.700000	1.469607	1.470415	0.000809
0.800000	1.554384	1.555031	0.000647
0.900000	1.632173	1.632613	0.000440
1.000000	1.701562	1.701870	0.000308

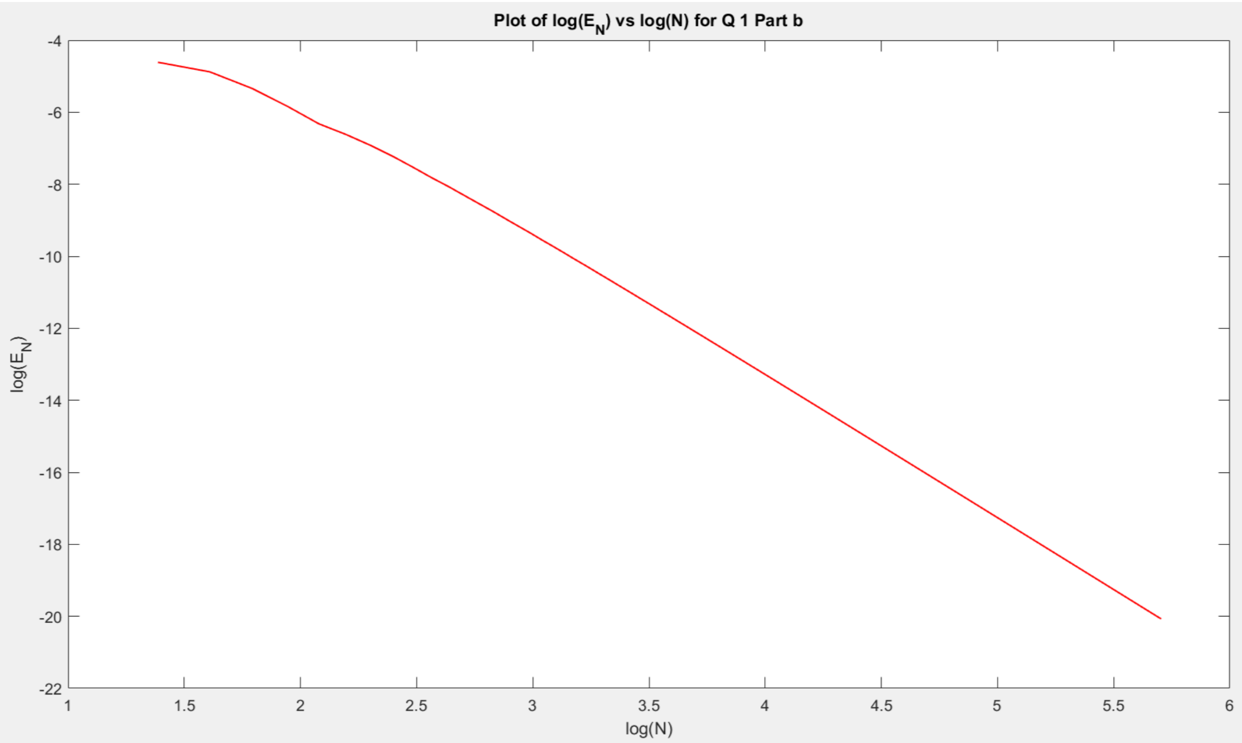
The graphs are as follows:




t vs absolute error plot for Q 1 Part b
×10^-3
absolute error
t
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Mesh points,i.e.,t versus absolute error (loglog plot)
×10^-4
absolute error
Mesh points,i.e.,t
9
8
7
6
5
4
0.4
0.5
0.6
0.7
0.8
0.9
1

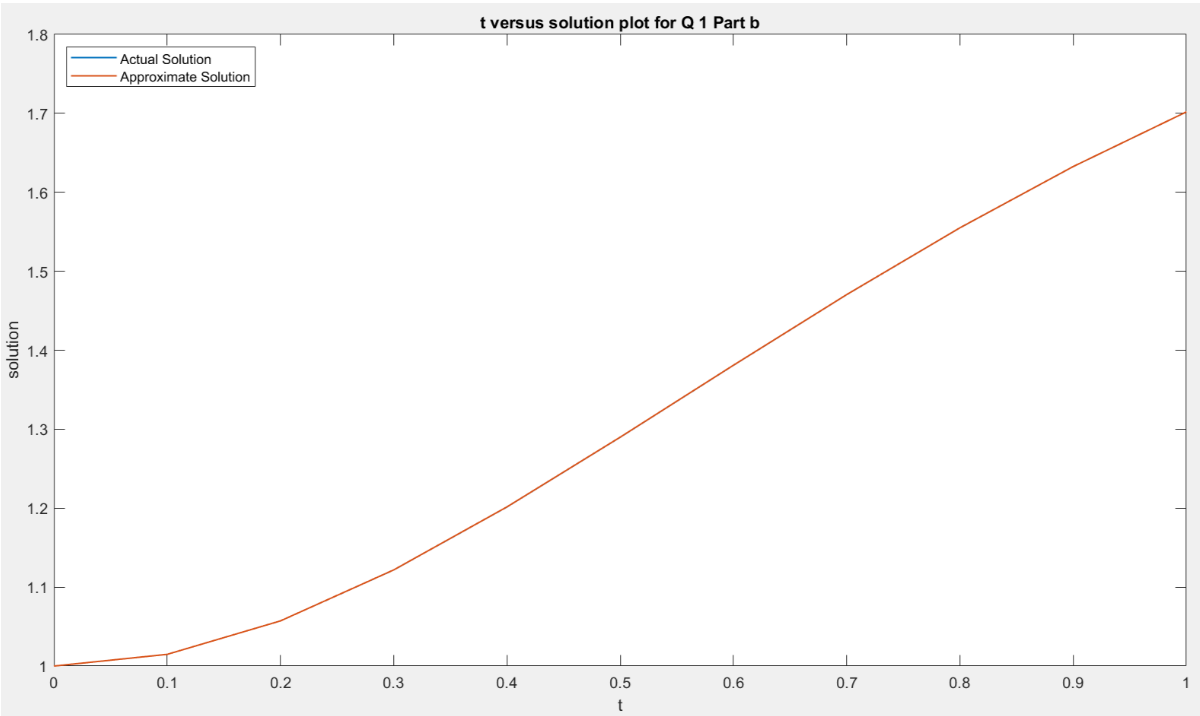


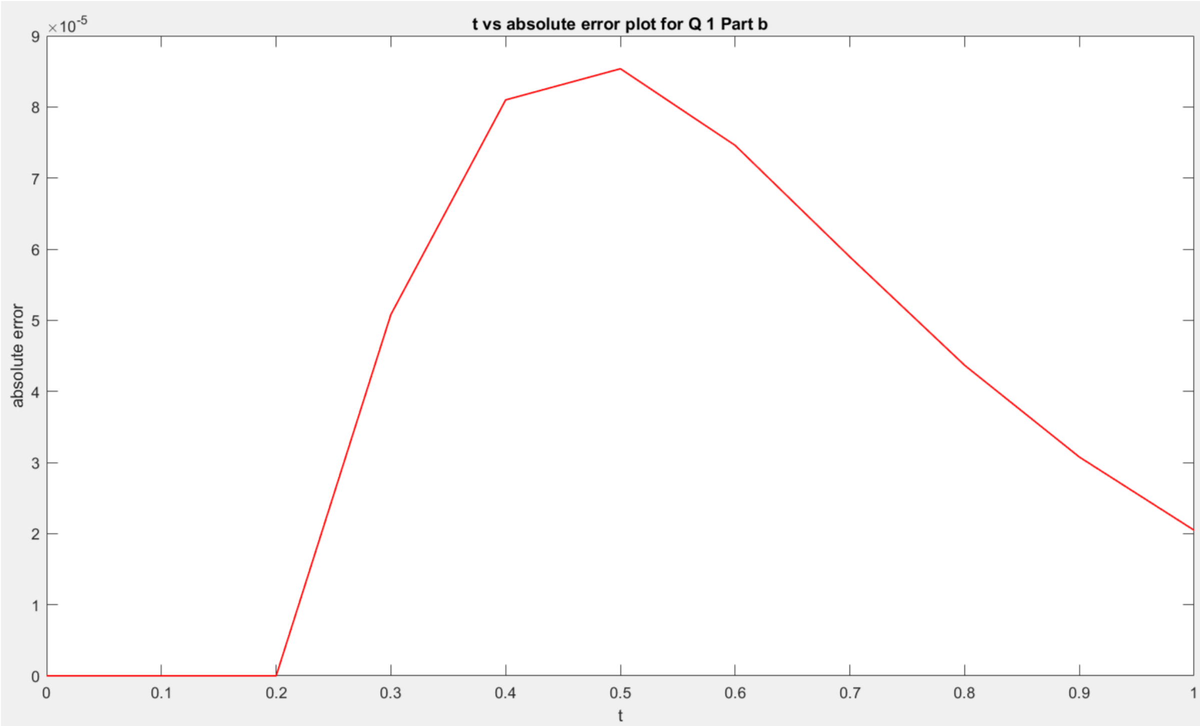


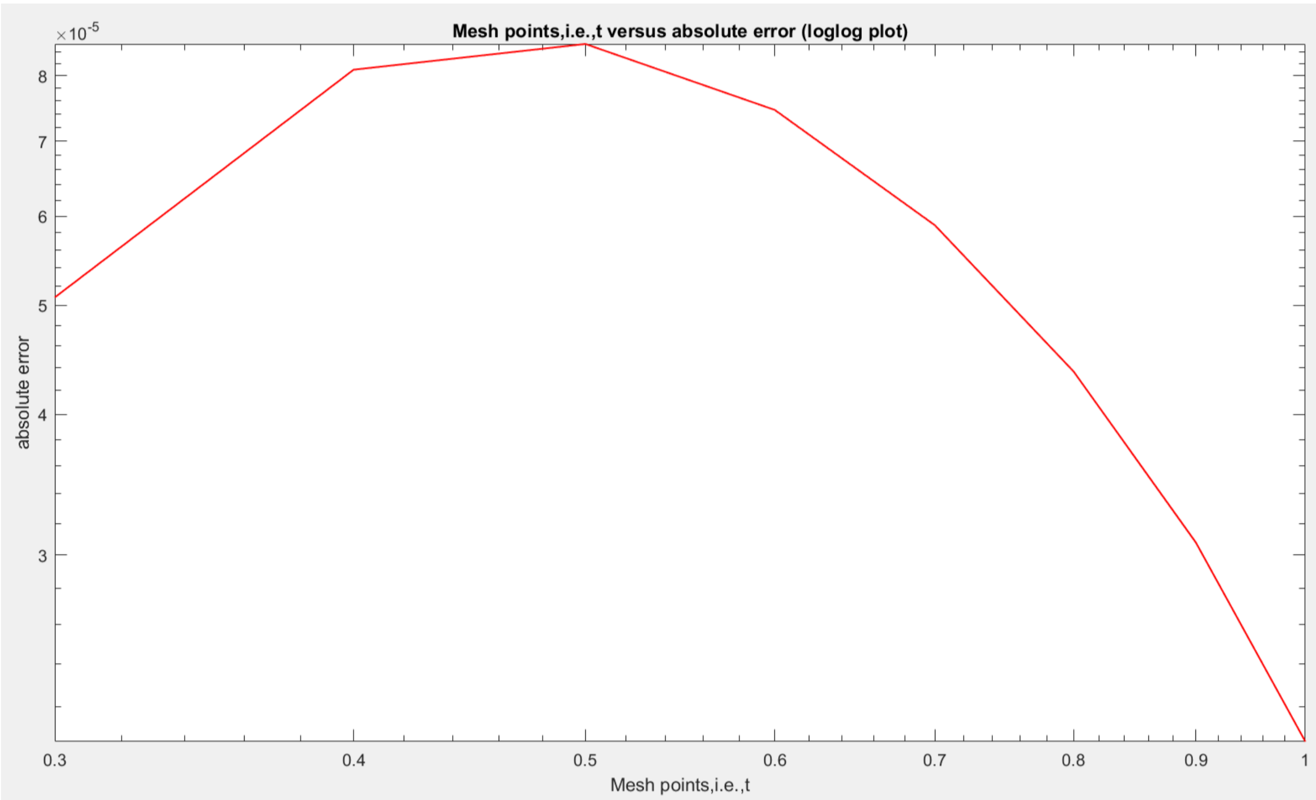
Using Admas-Moulton method,

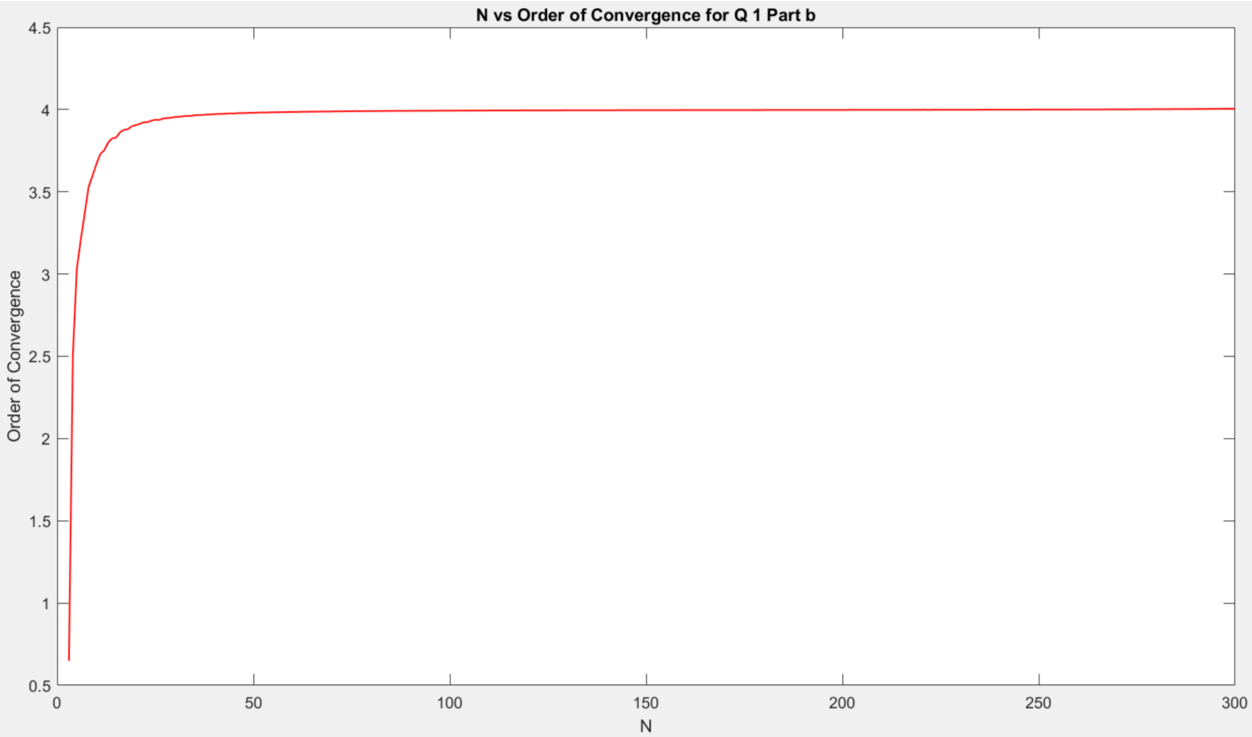
```
Question 1 Part b) using exact starting values,

Using Adams-Moulton method for Question 1 Part b,
     t         Approximate Solution     Exact Solution  Absolute Error
0.000000        1.000000                1.000000         0.000000
0.100000        1.014815                1.014815         0.000000
0.200000        1.057181                1.057181         0.000000
0.300000        1.121749                1.121698         0.000051
0.400000        1.201567                1.201486         0.000081
0.500000        1.289891                1.289805         0.000085
0.600000        1.381006                1.380931         0.000075
0.700000        1.470474                1.470415         0.000059
0.800000        1.555075                1.555031         0.000044
0.900000        1.632644                1.632613         0.000031
1.000000        1.701891                1.701870         0.000020
```

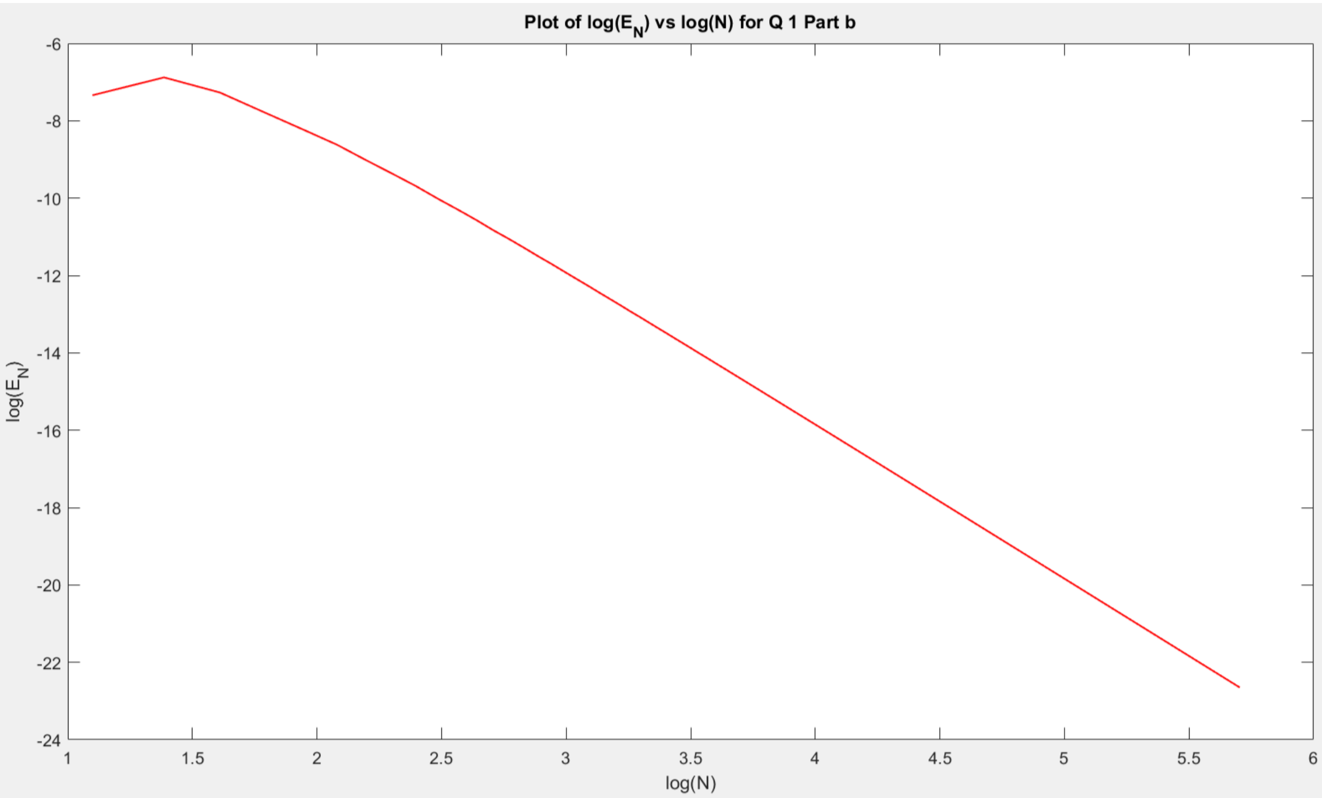
The graphs are as follows:




t vs absolute error plot for Q 1 Part b
absolute error
t
×10^-5
0
1
2
3
4
5
6
7
8
9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Mesh points,i.e.,t versus absolute error (loglog plot)
absolute error
Mesh points,i.e.,t
×10^-5
3
4
5
6
7
8
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1





2)

I have solved the given problem with the value of h = 0.25 for the three methods

a) Explicit Euler Method

b) Implicit Euler Method

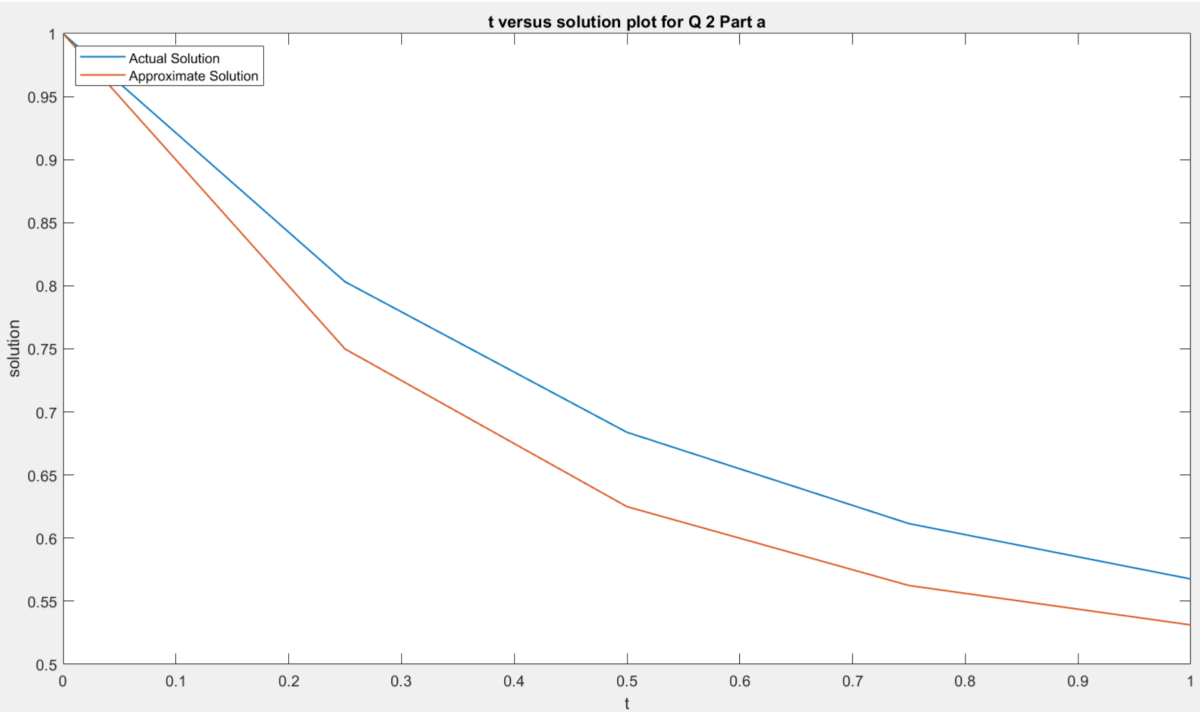
c) Central Difference method

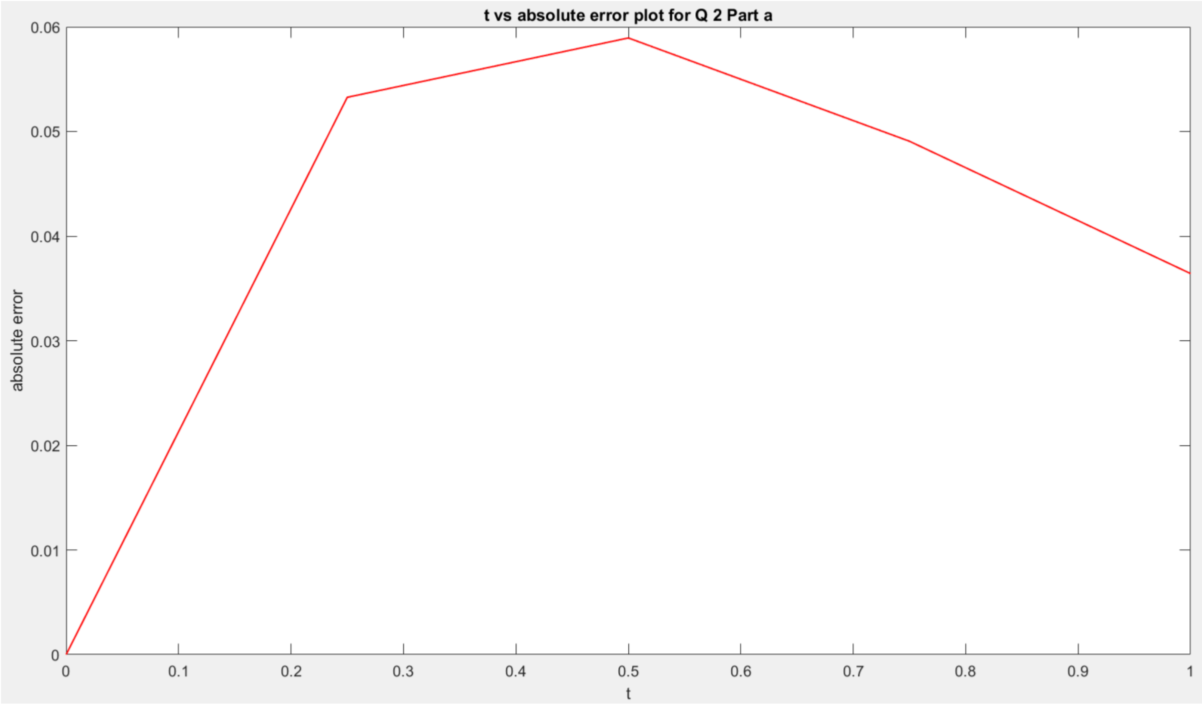
For checking the stability of each method, we vary the values of h and see whether the graphs are oscillatory or not and hence, the methods are unstable or stable.

a)

Using Explicit-Euler method,

```
Question 2
Part a) Using Explicit Euler Method,
Using Explicit-Euler method for Question 2 Part a
     t         Approximate Solution     Exact Solution  Absolute Error
0.000000           1.000000                1.000000        0.000000
0.250000           0.750000                0.803265        0.053265
0.500000           0.625000                0.683940        0.058940
0.750000           0.562500                0.611565        0.049065
1.000000           0.531250                0.567668        0.036418
```

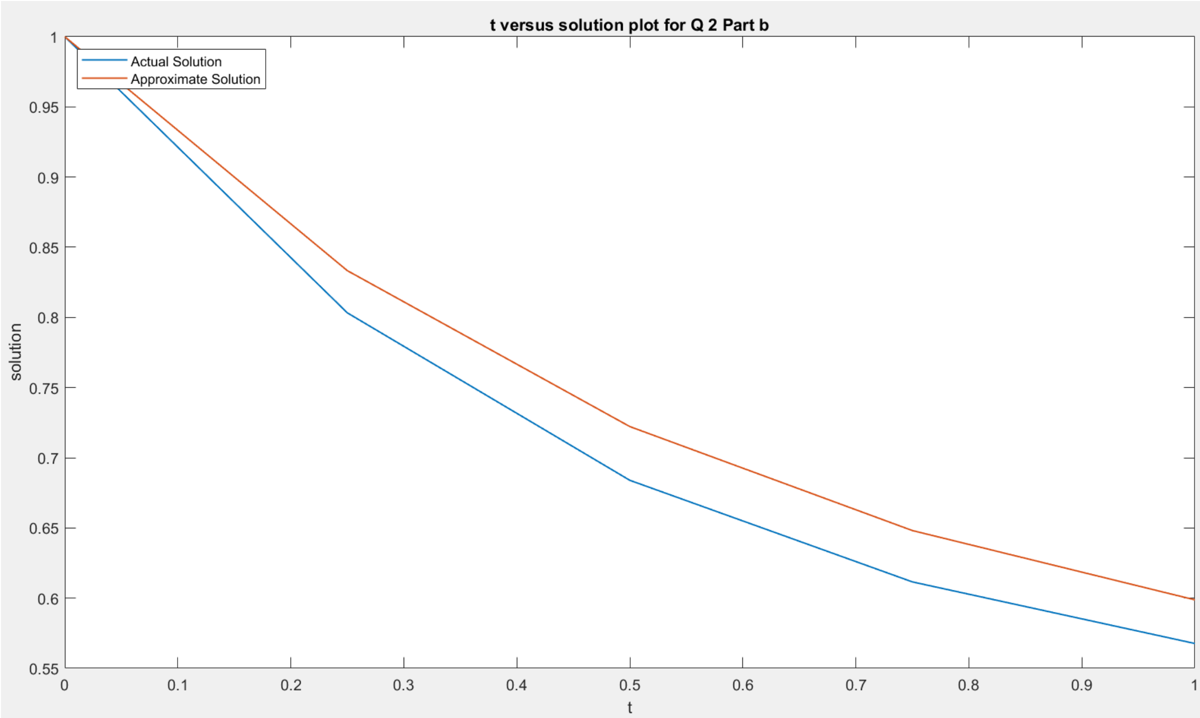


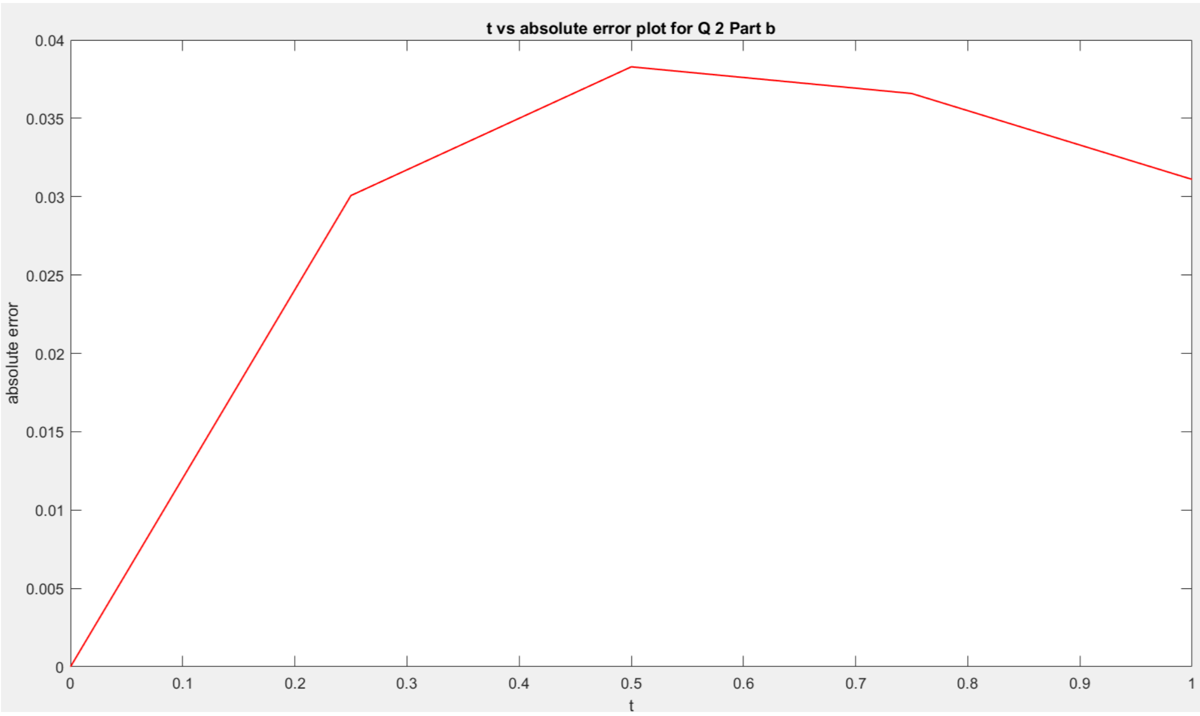


b)

Using Implicit-Euler method,

```
Part b) Using Implicit Euler Method,
Using Implicit-Euler method for Question 2 Part b
    t         Approximate Solution    Exact Solution   Absolute Error
0.000000          1.000000              1.000000          0.000000
0.250000          0.833336              0.803265          0.030071
0.500000          0.722226              0.683940          0.038286
0.750000          0.648148              0.611565          0.036583
1.000000          0.598768              0.567668          0.031101
```

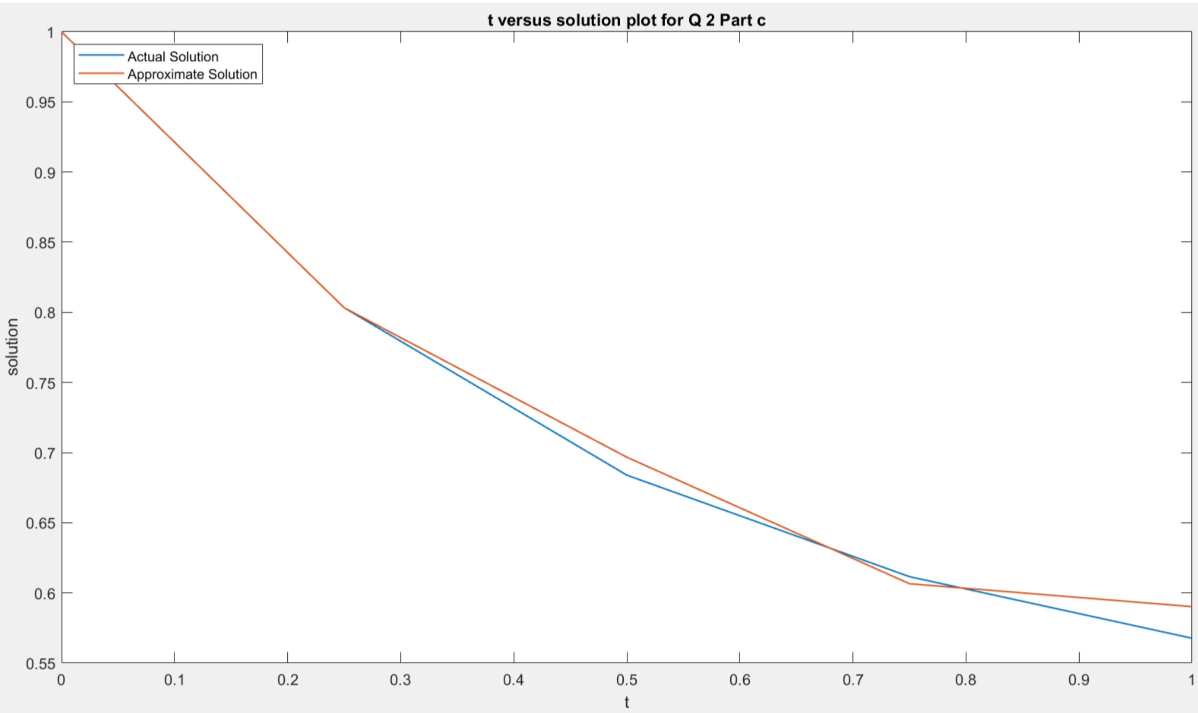


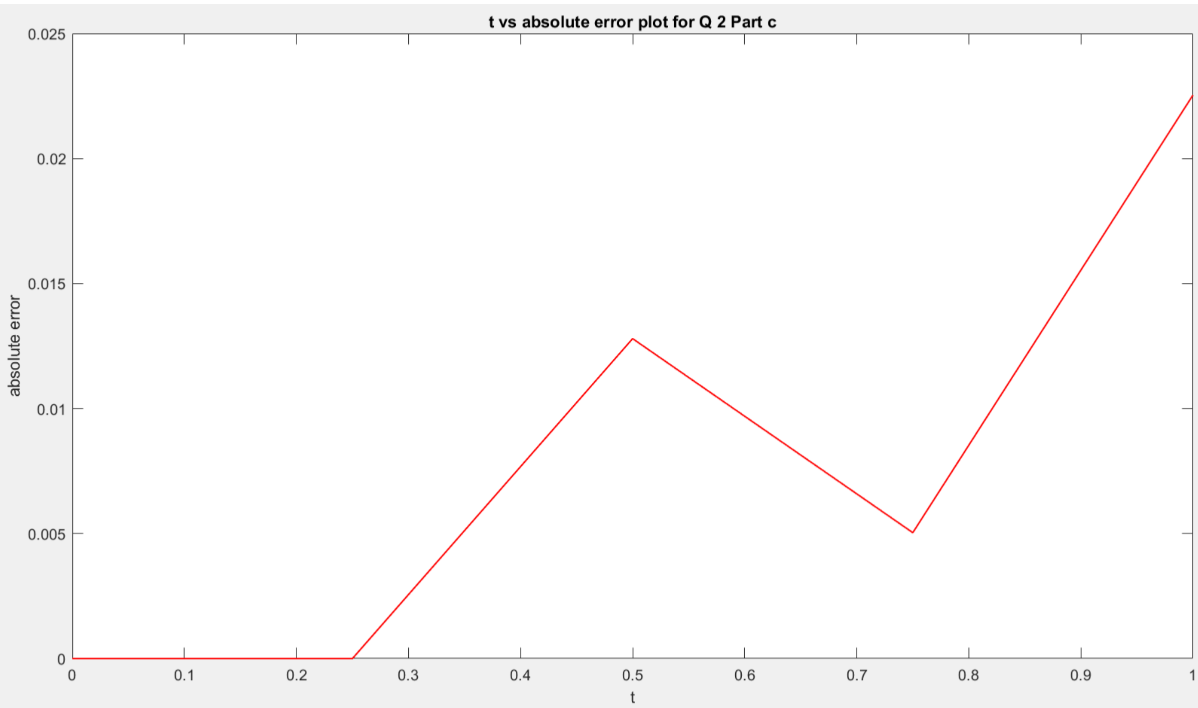


c)

Using Central differences method,

```
Part c),
Using Central difference method for Question 2 Part c
      t          Approximate Solution     Exact Solution   Absolute Error
0.000000            1.000000                1.000000          0.000000
0.250000            0.803265                0.803265          0.000000
0.500000            0.696735                0.683940          0.012795
0.750000            0.606531                0.611565          0.005034
1.000000            0.590204                0.567668          0.022536
```

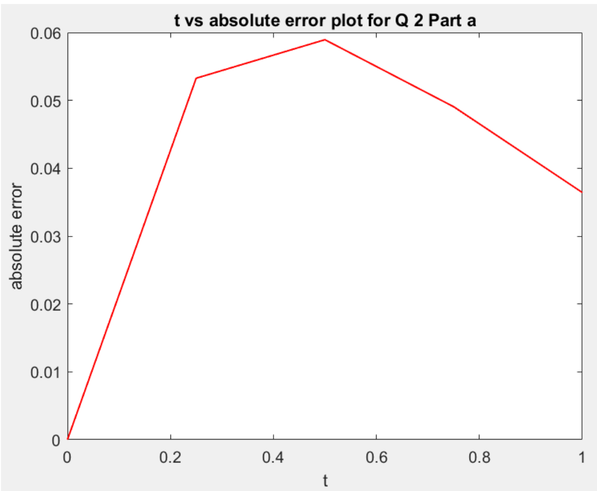


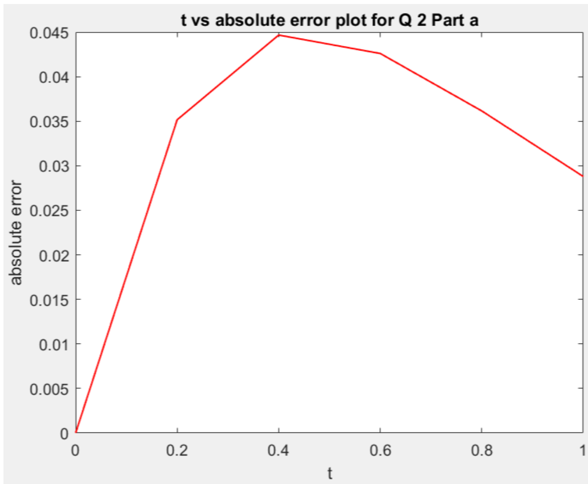


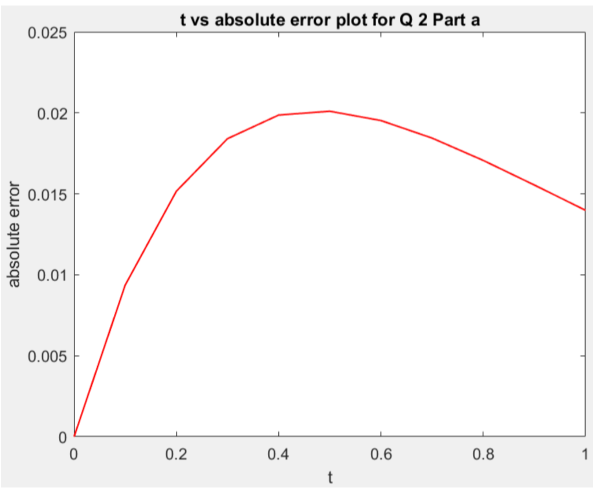
Stability Analysis: (This part is commented out in the output file, else too many graphs would be formed when we run the output_file.m . To run this part, please uncomment those lines in the output_file.m)

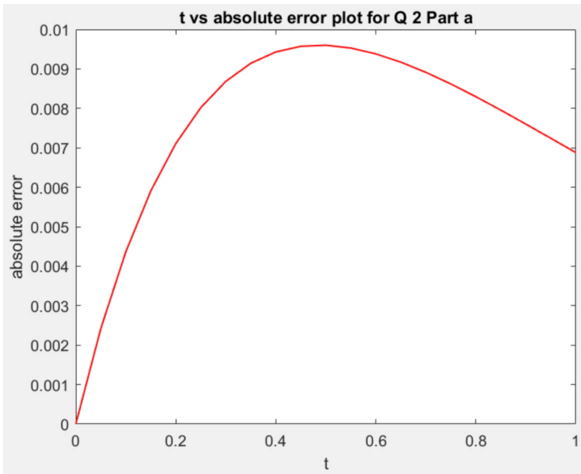
a) For Explicit Euler Method,

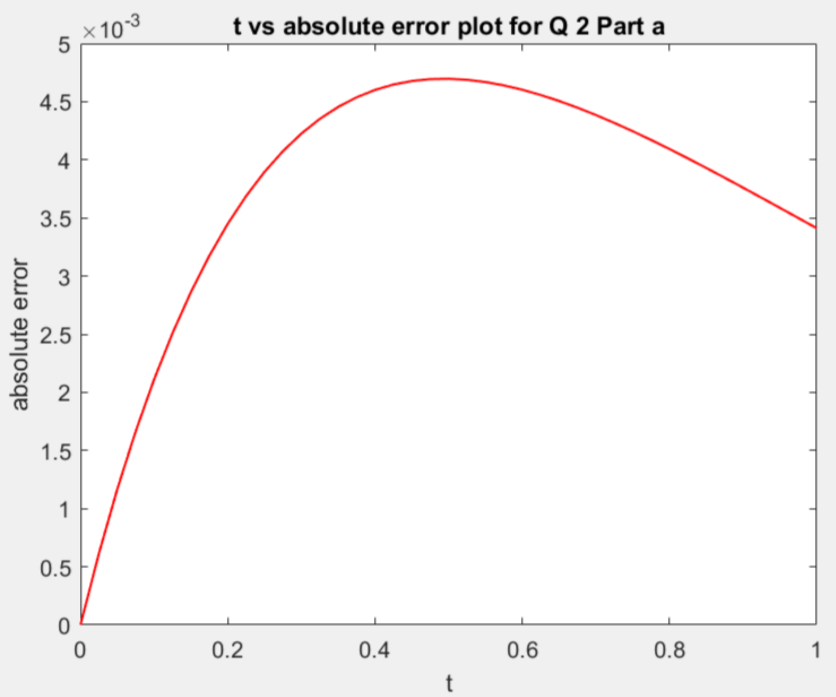
The Explicit Euler's method is conditionally stable for the IVP of the form y' = c*y, with the stability region of step size h is the interval (0,2/|c|). The given solution can be split into a homogeneous solution as well as a particular solution and for the homogeneous solution, the constant c = 2. So, the region of stability for the Explicit Euler Scheme is the interval (0,1) and the graphs for varying h suggest the same too. The given plots are plotted for 5 different values of h = 0.25, 0.2, 0.1, 0.05 and 0.025.











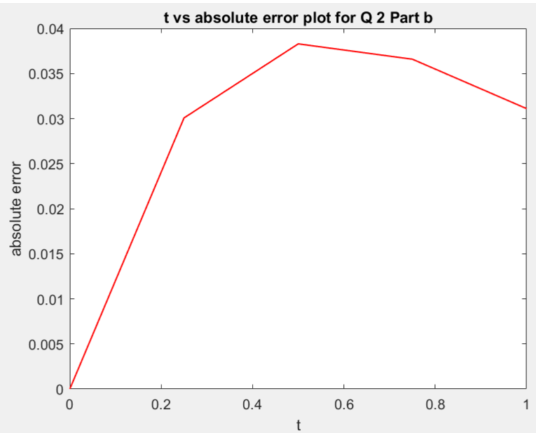
We see that for various values of h, the plots are similar in nature and become smoother as the value of h decreases. The method is stable conditionally for h belongs to (0,1).

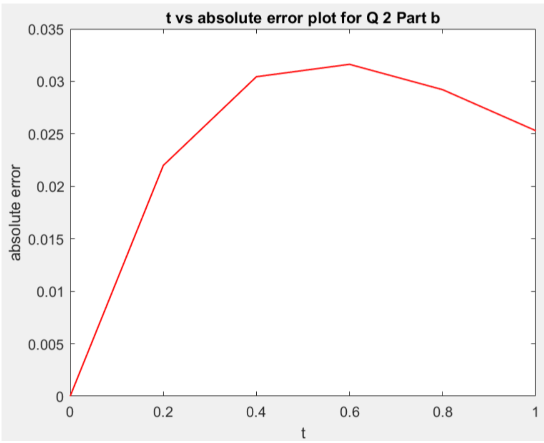
The Absolute error vs t plot is getting smoother with decrease in the value of h, the approximation is getting better, and error is not oscillating, this shows that the Explicit-Euler method is stable here.

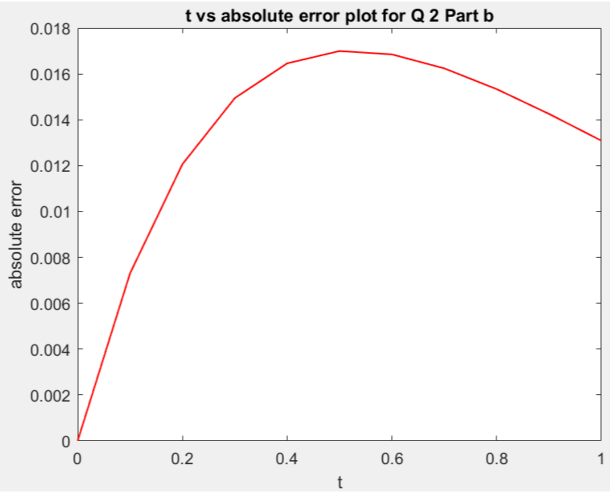
Implicit-Euler Method

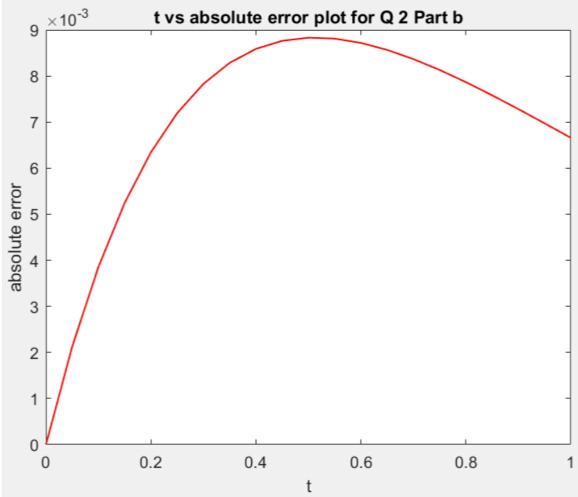
The Implicit Euler's method is unconditionally stable for the IVP of the form y' = c*y. So, the method is stable for all values of h from 0 to infinity.

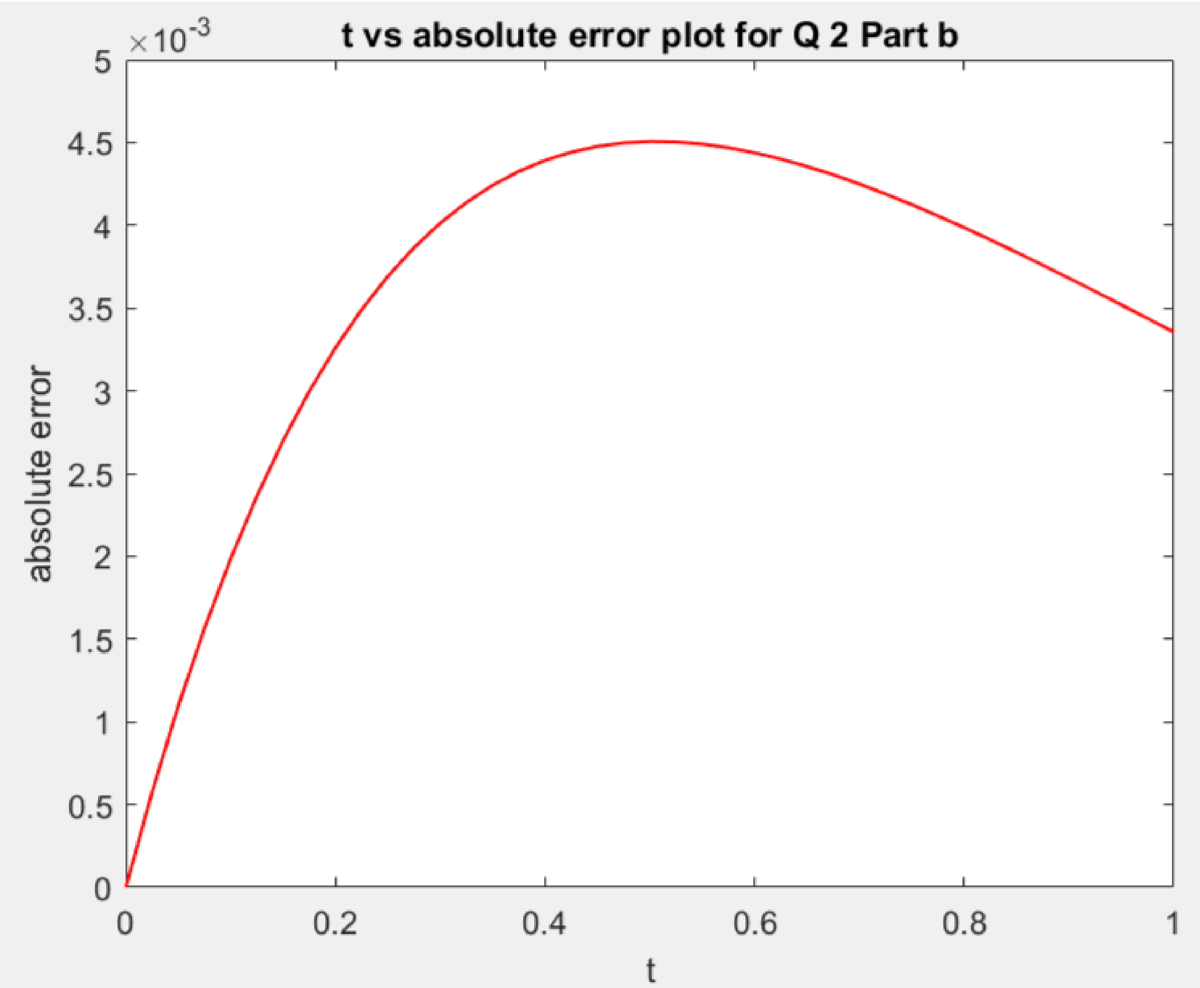
The given plots are plotted for 5 different values of h = 0.25, 0.2, 0.1, 0.05 and 0.025.









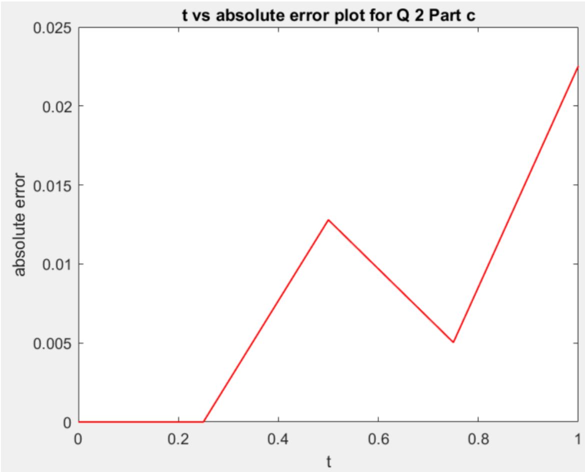


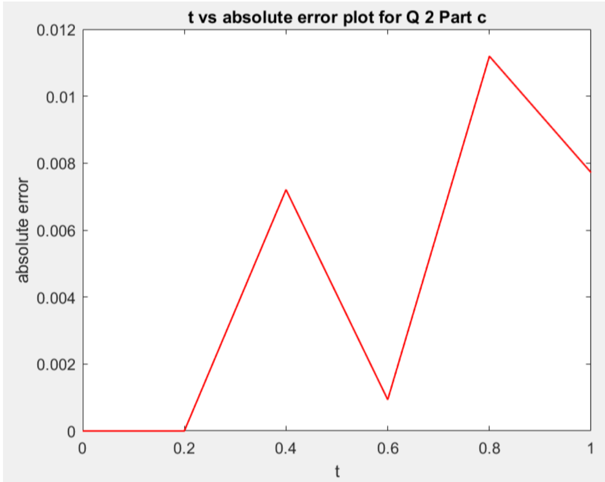
We see that for various values of h, the plots are similar in nature and become smoother as the value of h decreases. The method is stable unconditionally. Regardless of what h we choose, the method is stable.

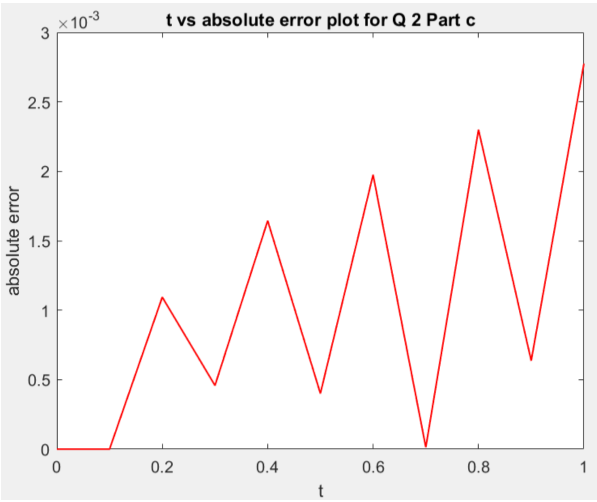
The Absolute error vs t plot is getting smoother with decrease in the value of h, the approximation is getting better, and error is not oscillating, this shows that the Implicit-Euler method is stable here.

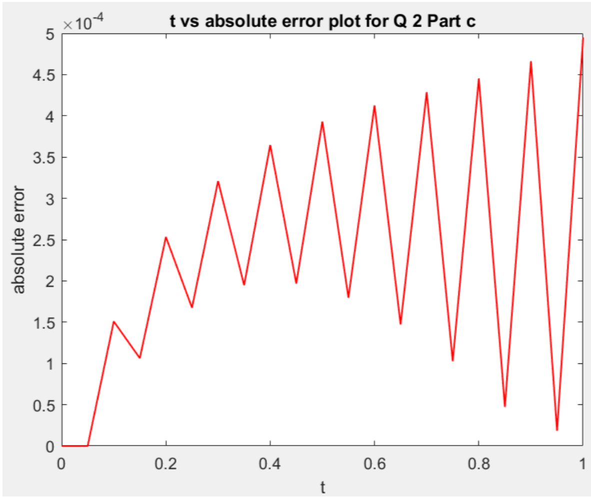
Central Difference Method:

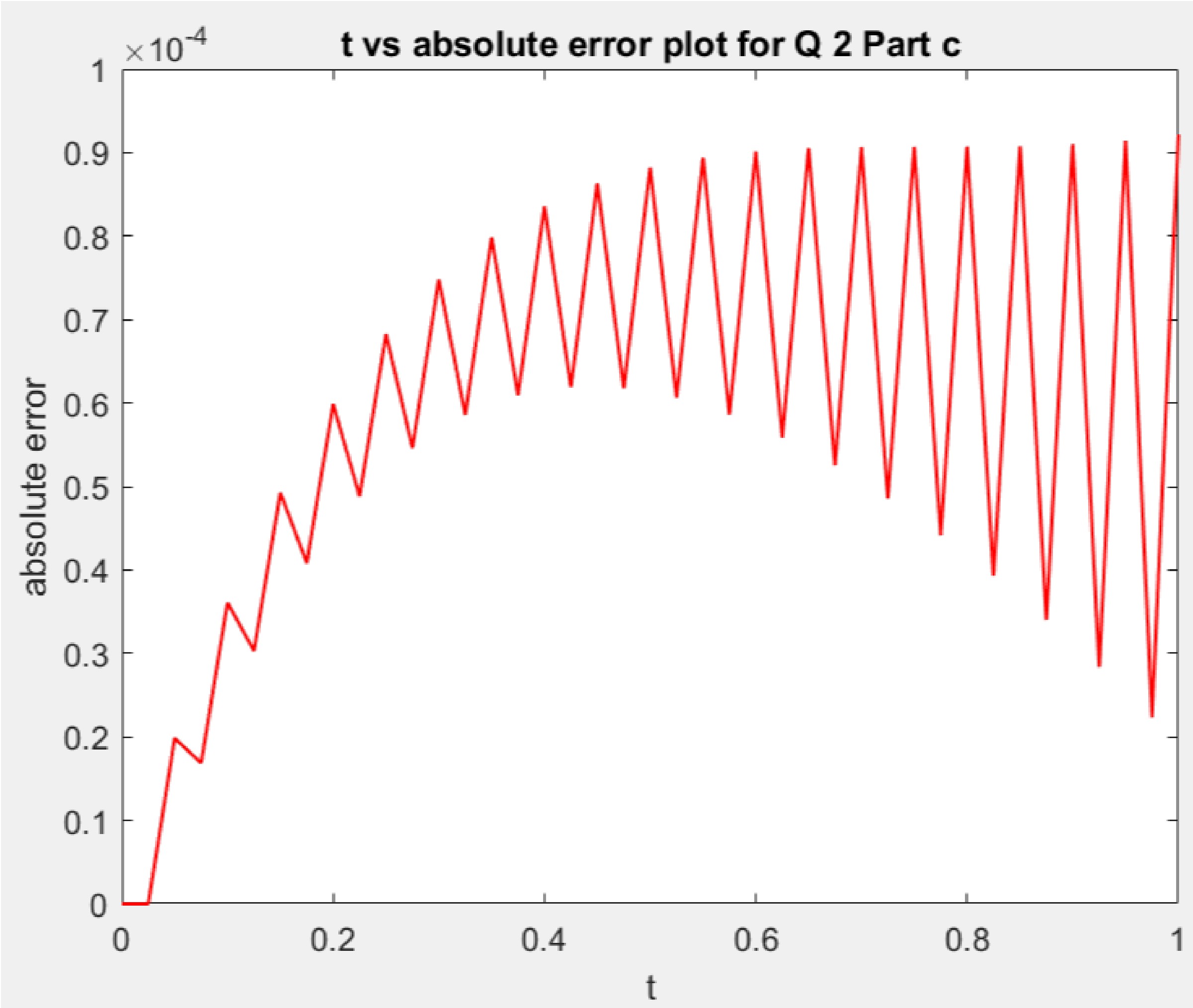
The Central Difference method is of order 2 and there is an extraneous root that is associated when we solve the IVP, which is oscillatory in nature and its amplitude is also increasing. The given plots are plotted for 5 different values of h = 0.25, 0.2, 0.1, 0.05 and 0.025.









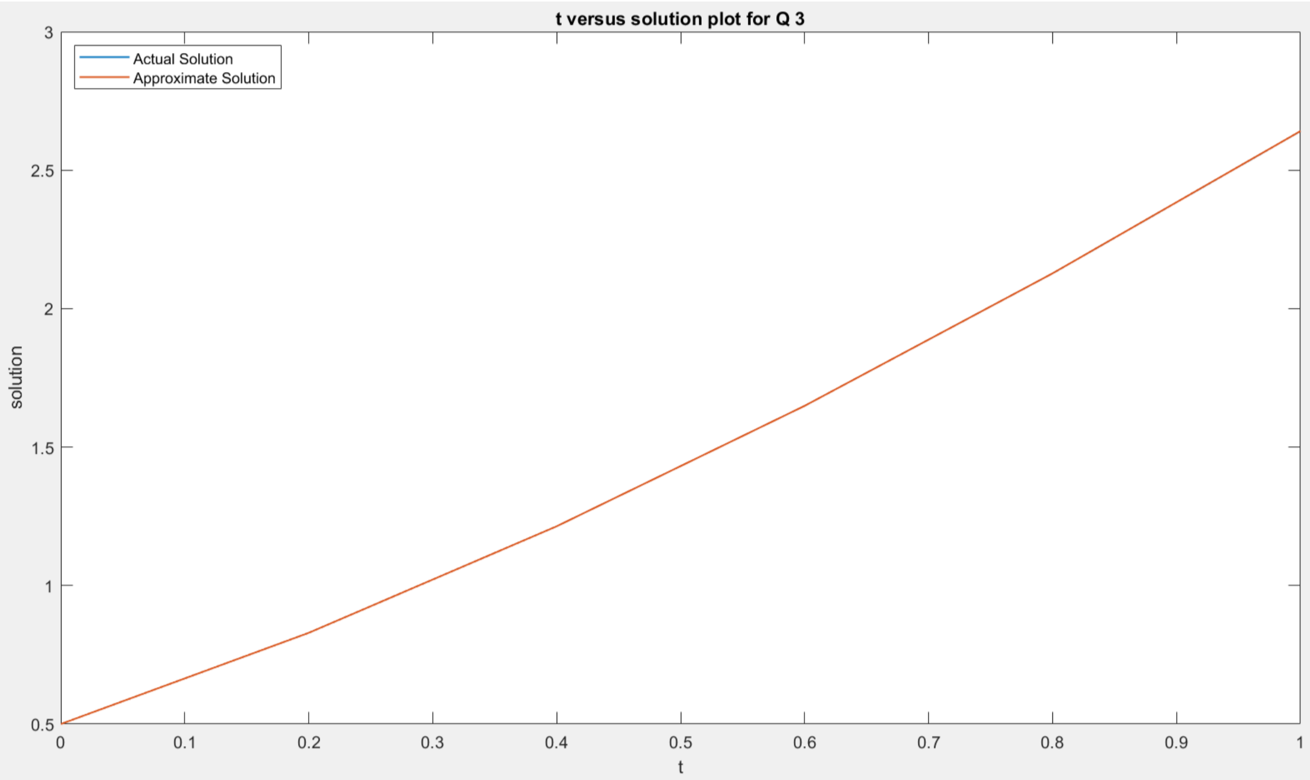


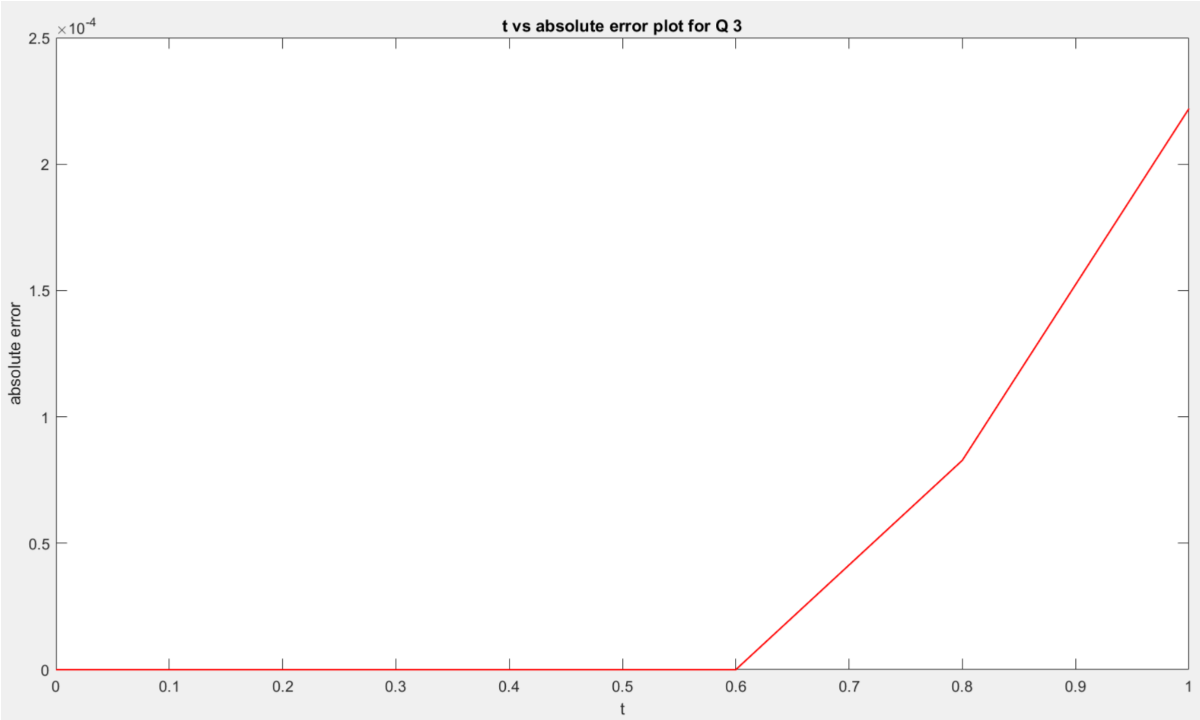
The approximation is getting better with decreasing value of h, but the absolute error is fluctuating a lot with t. This is due to the extra term we got while solving the difference equation. Since central difference method is a second order scheme which is being used to solve a first order IVP, it is resulting in some extra term which is making the error to oscillate, and amplitude of oscillations is increasing with decreasing value of h. This fluctuation in absolute error clearly shows that Central Difference Method is unstable here.

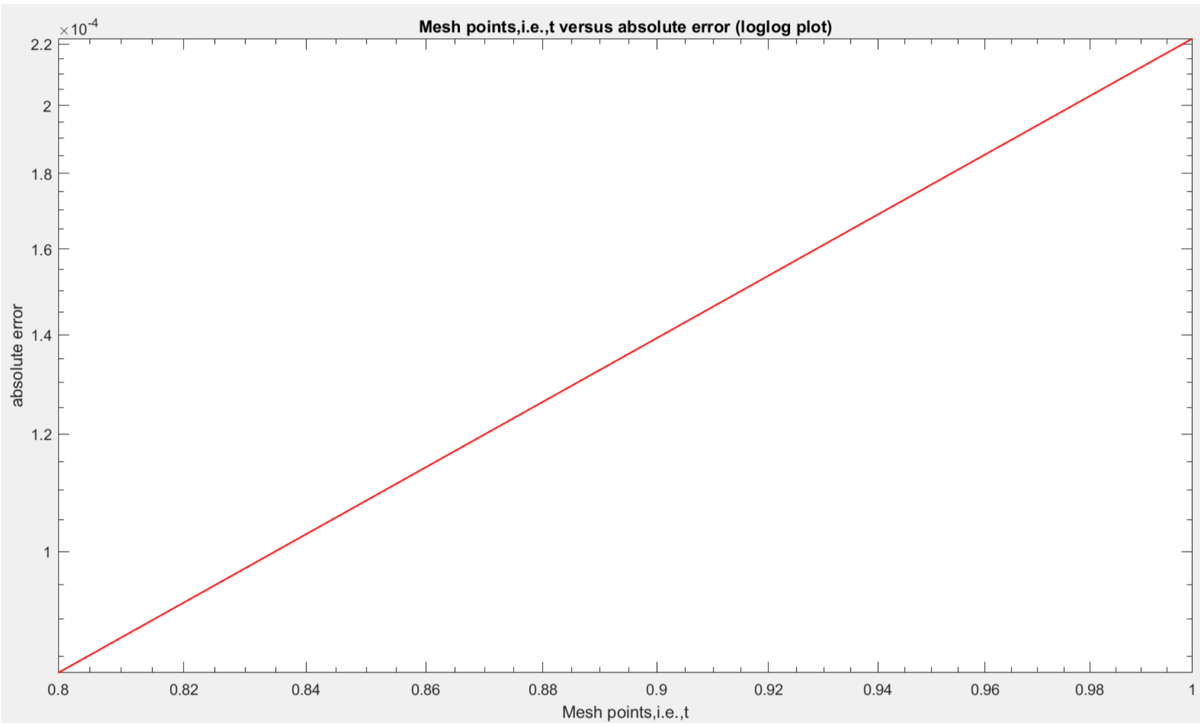
3)

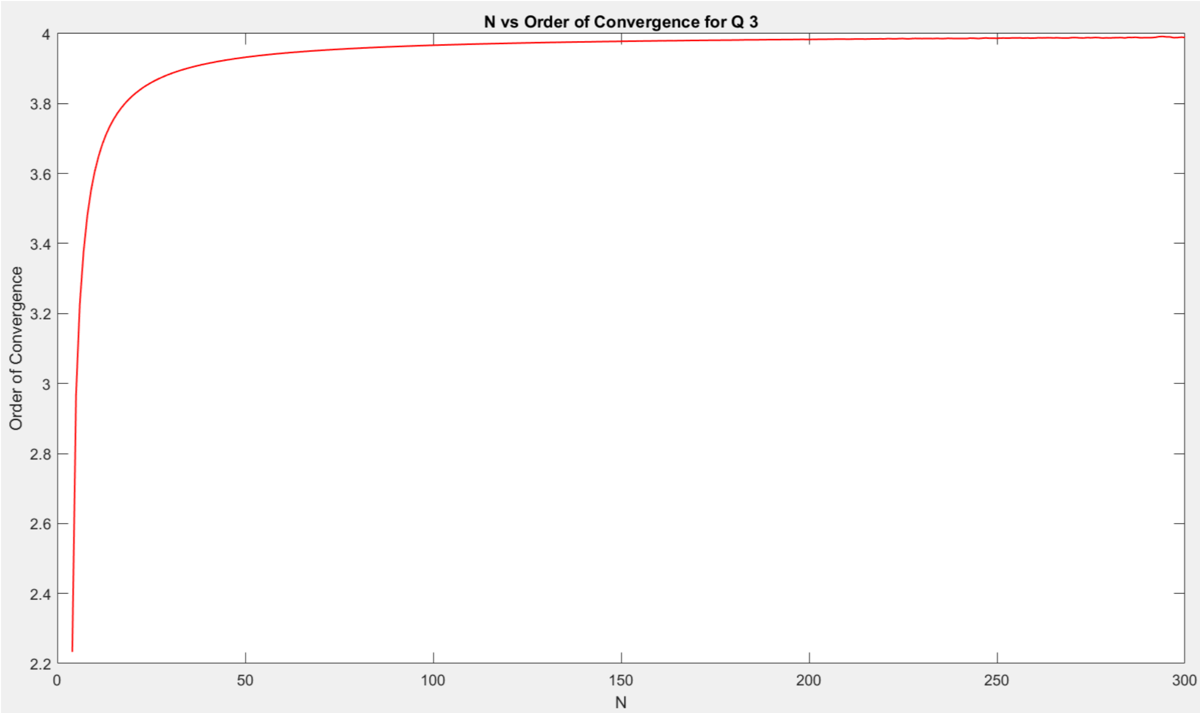
```
Using explicit Adams-Bashforth four-step method,
Using Adams-Bashforth method for Question 3,
    t          Approximate Solution    Exact Solution  Absolute Error
0.000000        0.500000                0.500000        0.000000
0.200000        0.829299                0.829299        0.000000
0.400000        1.214088                1.214088        0.000000
0.600000        1.648941                1.648941        0.000000
0.800000        2.127312                2.127230        0.000083
1.000000        2.641081                2.640859        0.000222
```

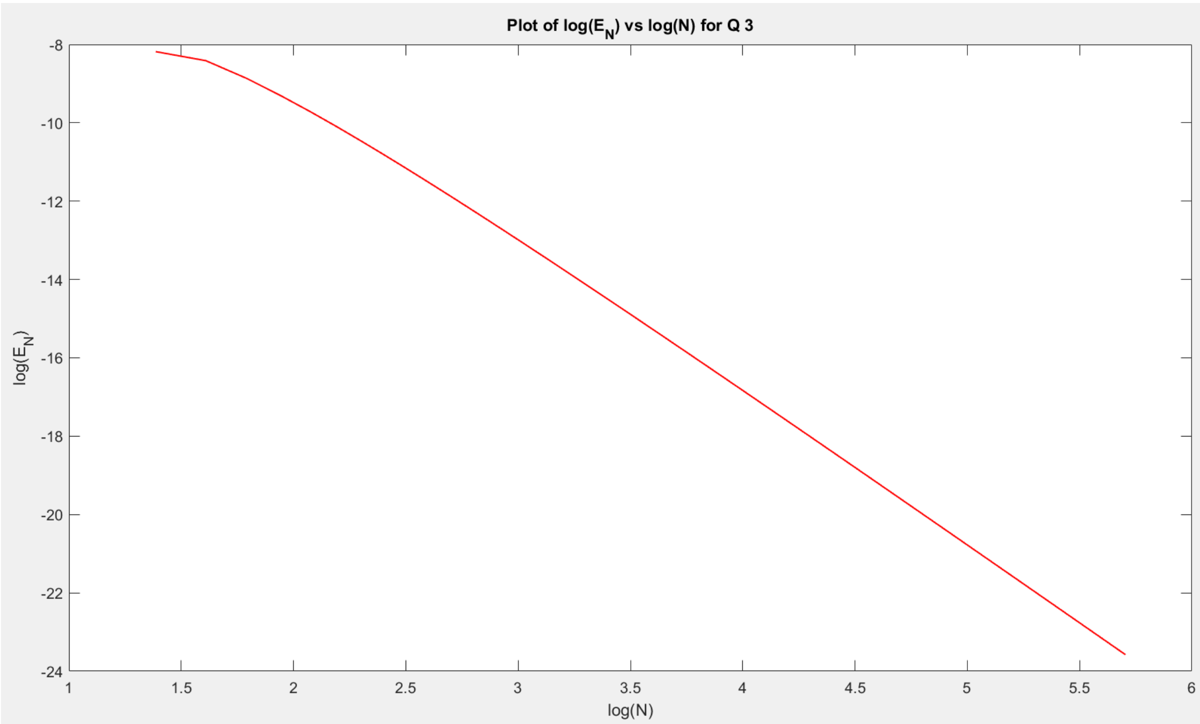
The graphs are as follows:




t vs absolute error plot for Q 3
$\times 10^{-4}$
2.5
2
1.5
1
0.5
0
absolute error
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

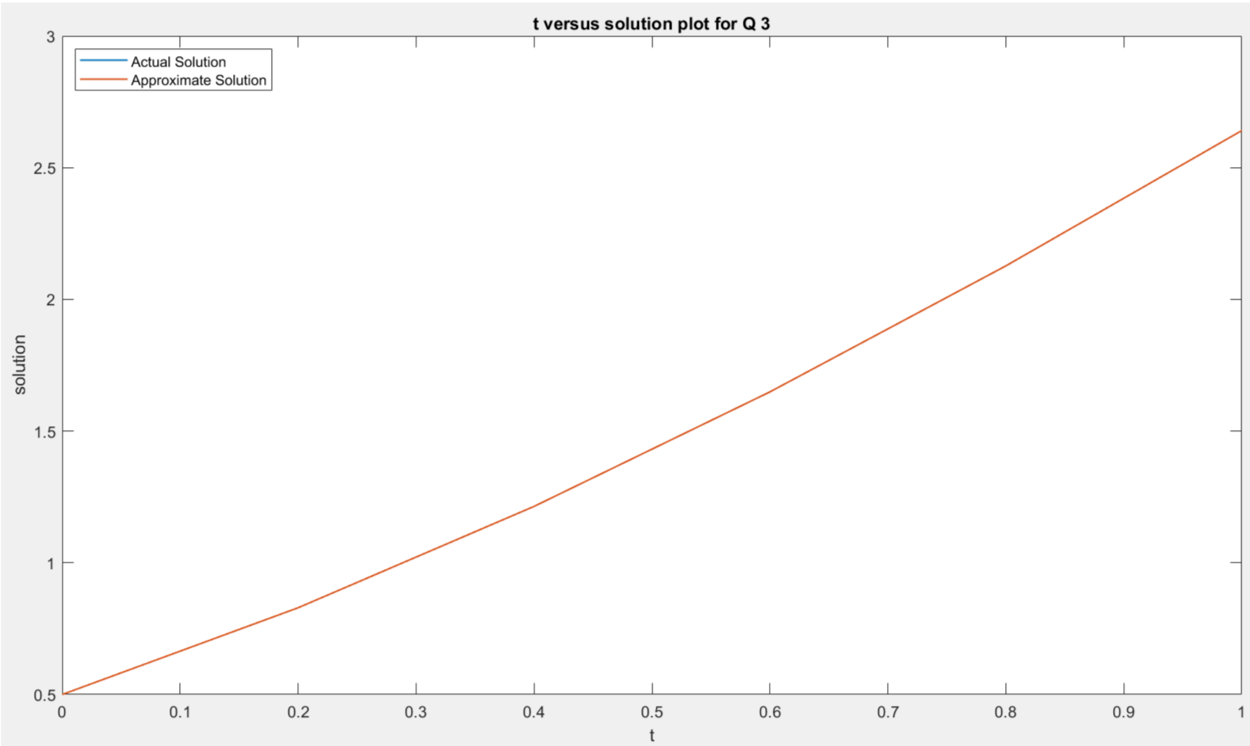

Mesh points,i.e.,t versus absolute error (loglog plot)
$\times 10^{-4}$
2.2
2
1.8
1.6
1.4
1.2
1
absolute error
0.8
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98
1
Mesh points,i.e.,t

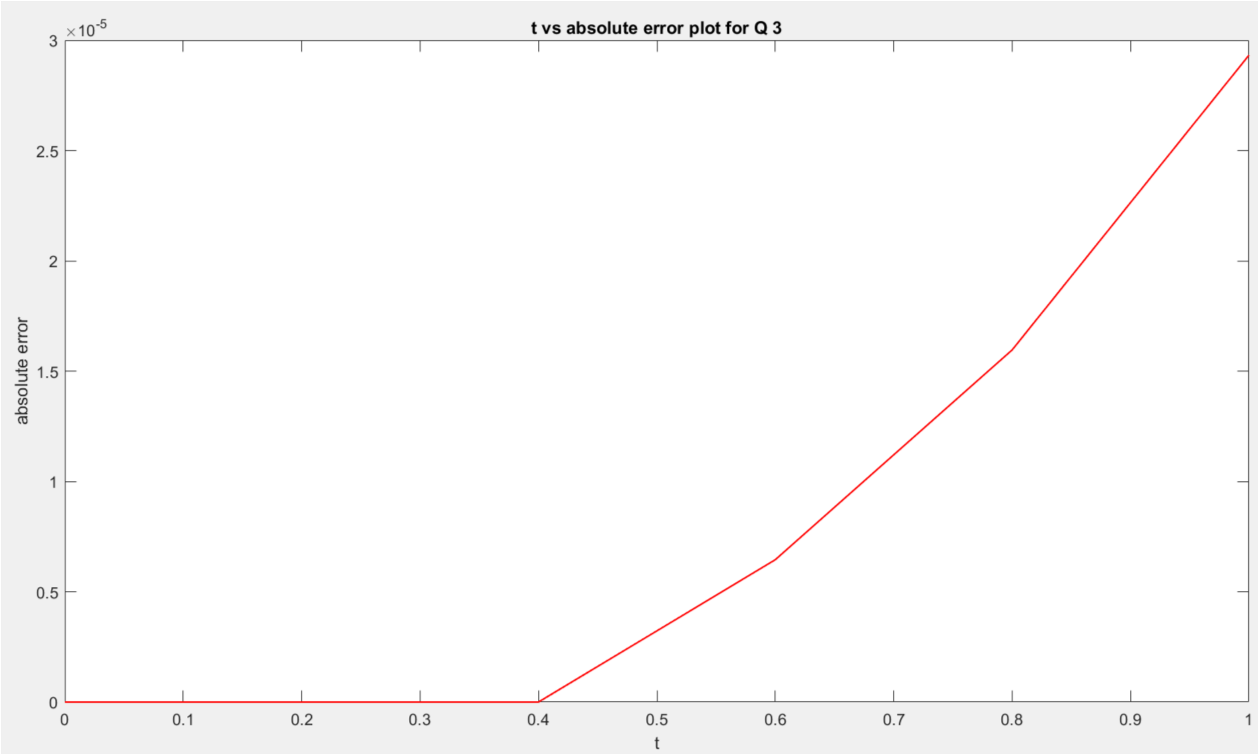

N vs Order of Convergence for Q 3
Order of Convergence
N
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
0
50
100
150
200
250
300

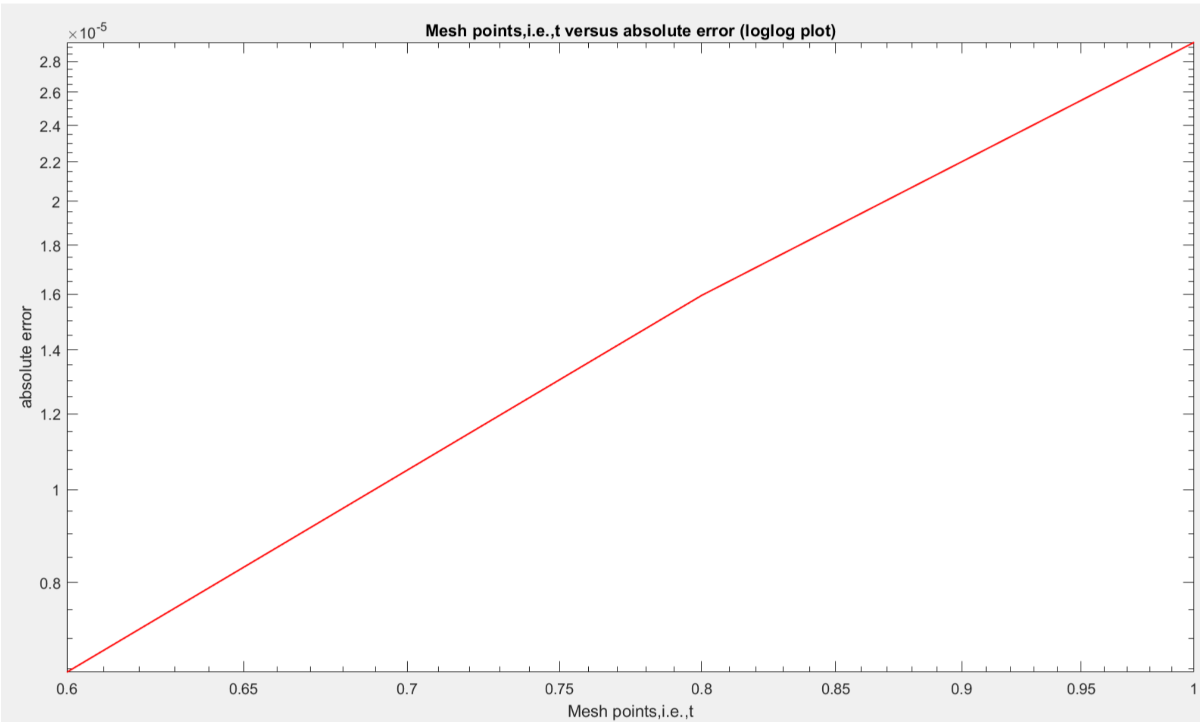

Plot of log(E_N) vs log(N) for Q 3
log(E_N)
log(N)
-8
-10
-12
-14
-16
-18
-20
-22
-24
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6

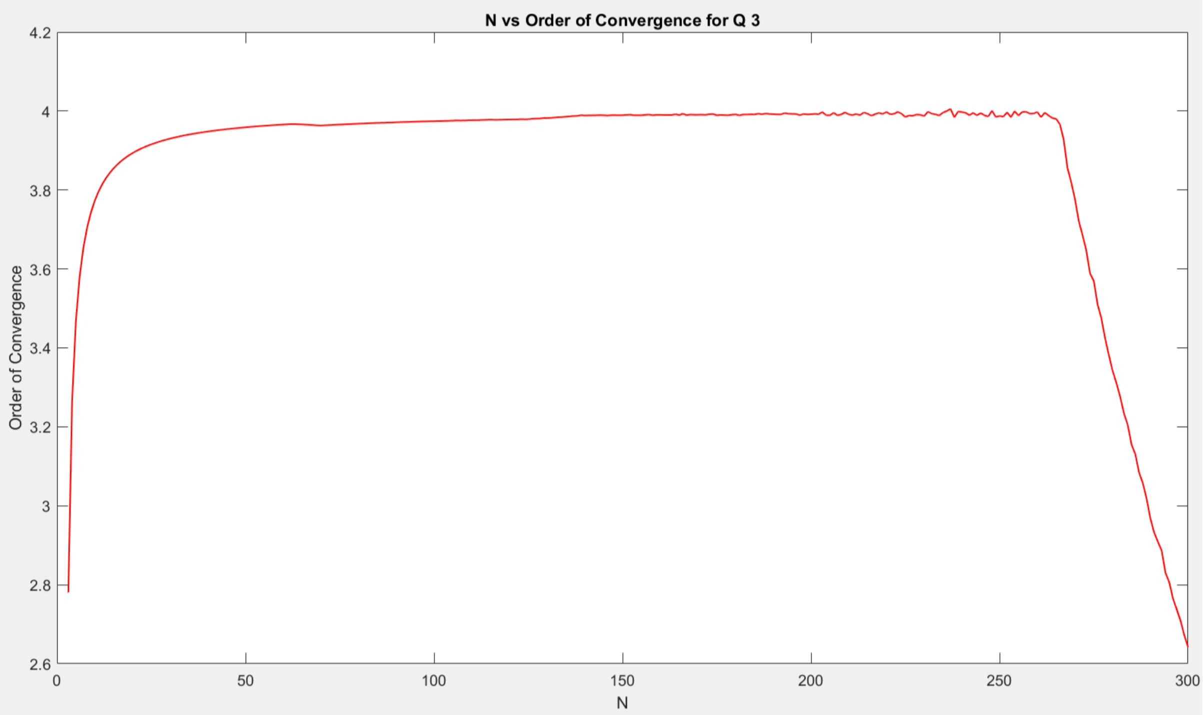
```
Using implicit Adams-Moulton three-step method,
Using Adams-Moulton method for Question 3,
     t          Approximate Solution     Exact Solution  Absolute Error
0.000000            0.500000                0.500000        0.000000
0.200000            0.829299                0.829299        0.000000
0.400000            1.214088                1.214088        0.000000
0.600000            1.648934                1.648941        0.000006
0.800000            2.127214                2.127230        0.000016
1.000000            2.640830                2.640859        0.000029
```

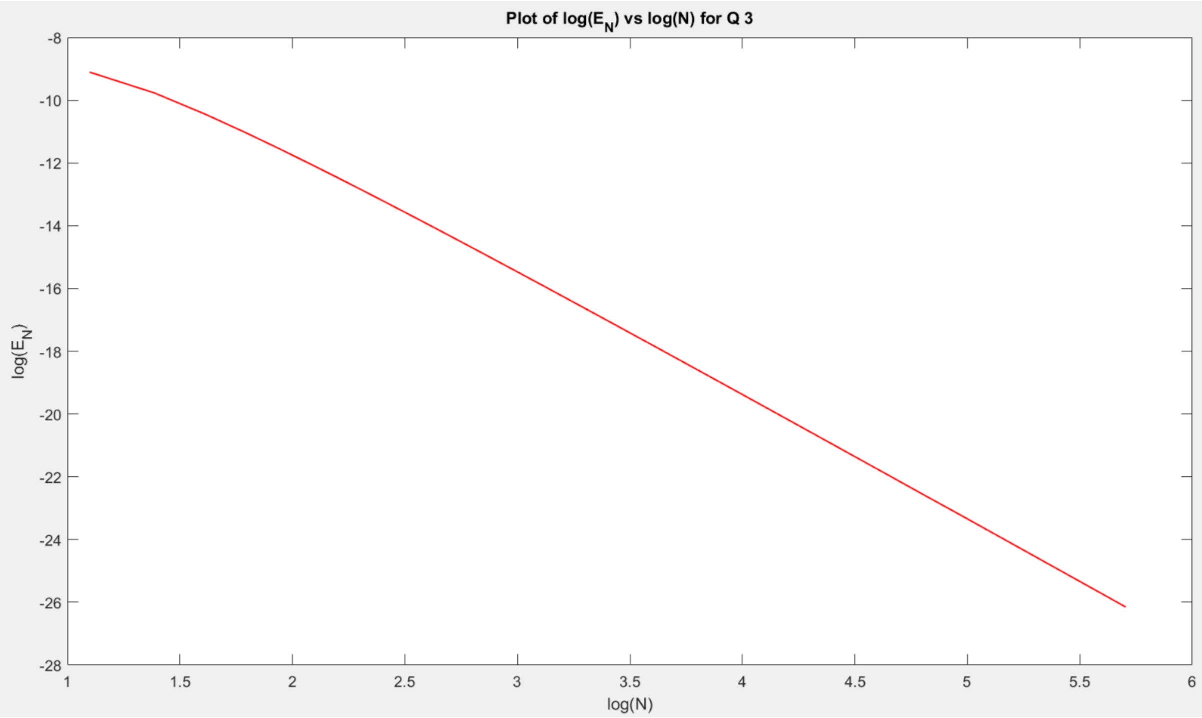
The graphs are as follows:




t vs absolute error plot for Q 3
×10^-5
absolute error
t
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0 0.5 1 1.5 2 2.5 3


Mesh points,i.e.,t versus absolute error (loglog plot)
×10^-5
absolute error
Mesh points,i.e.,t
0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 1
0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8



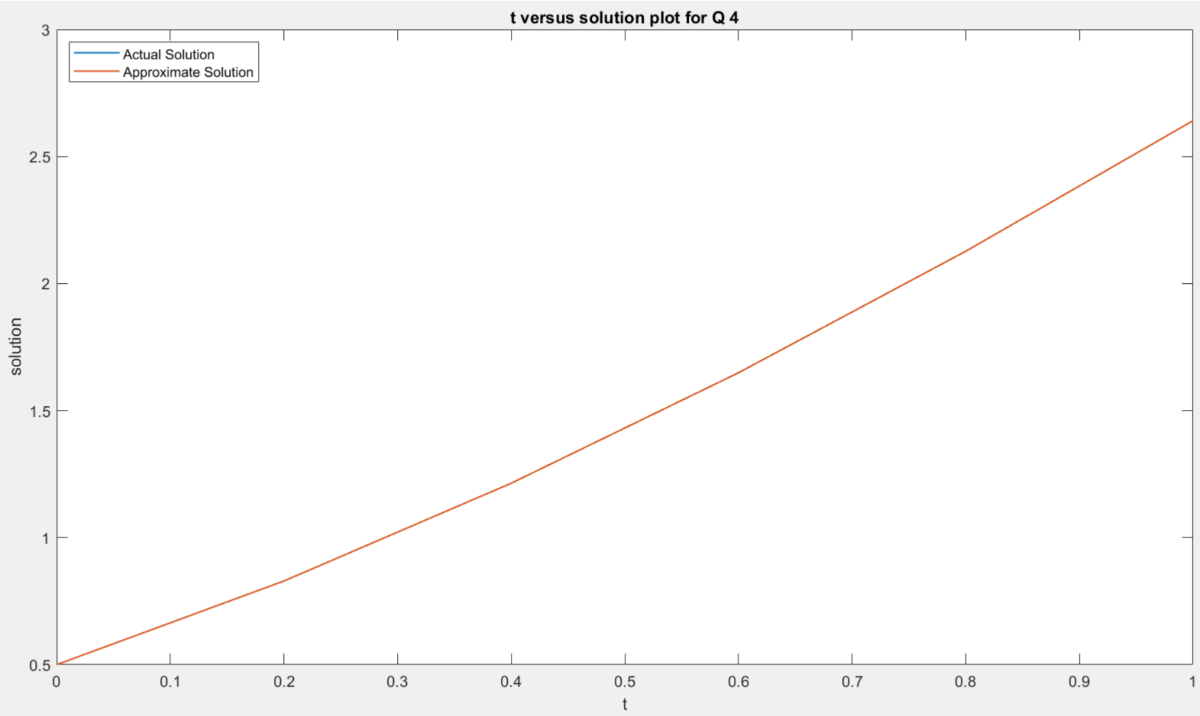


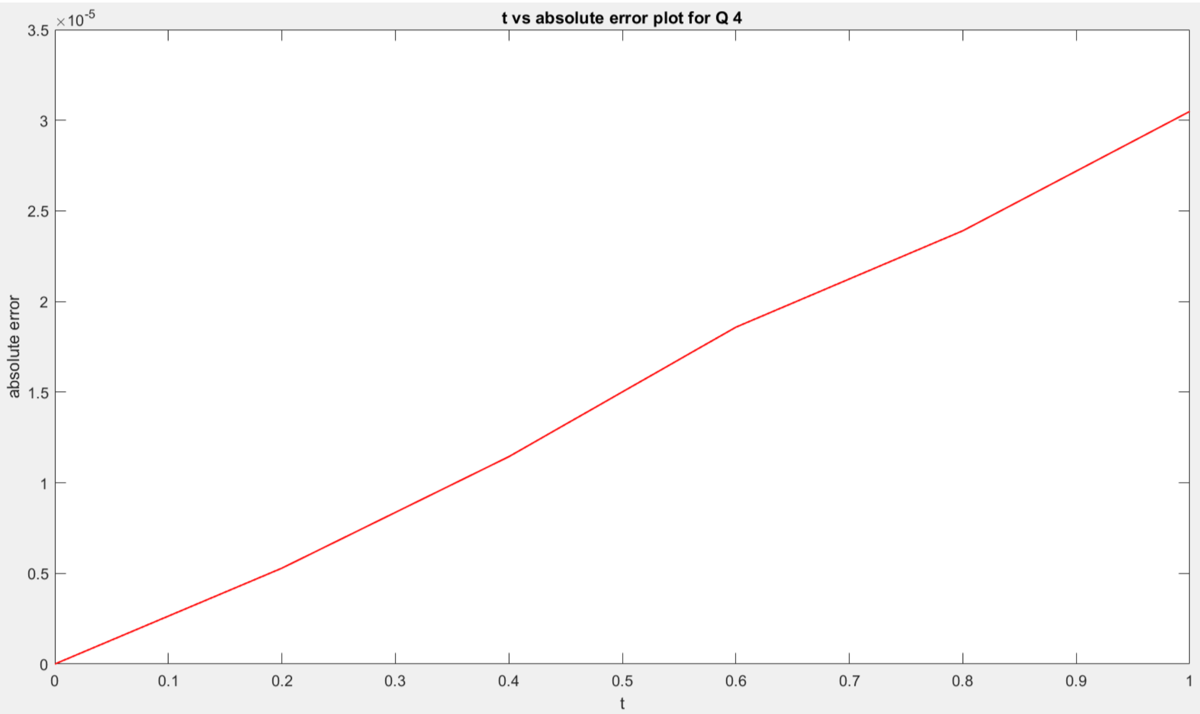
4)

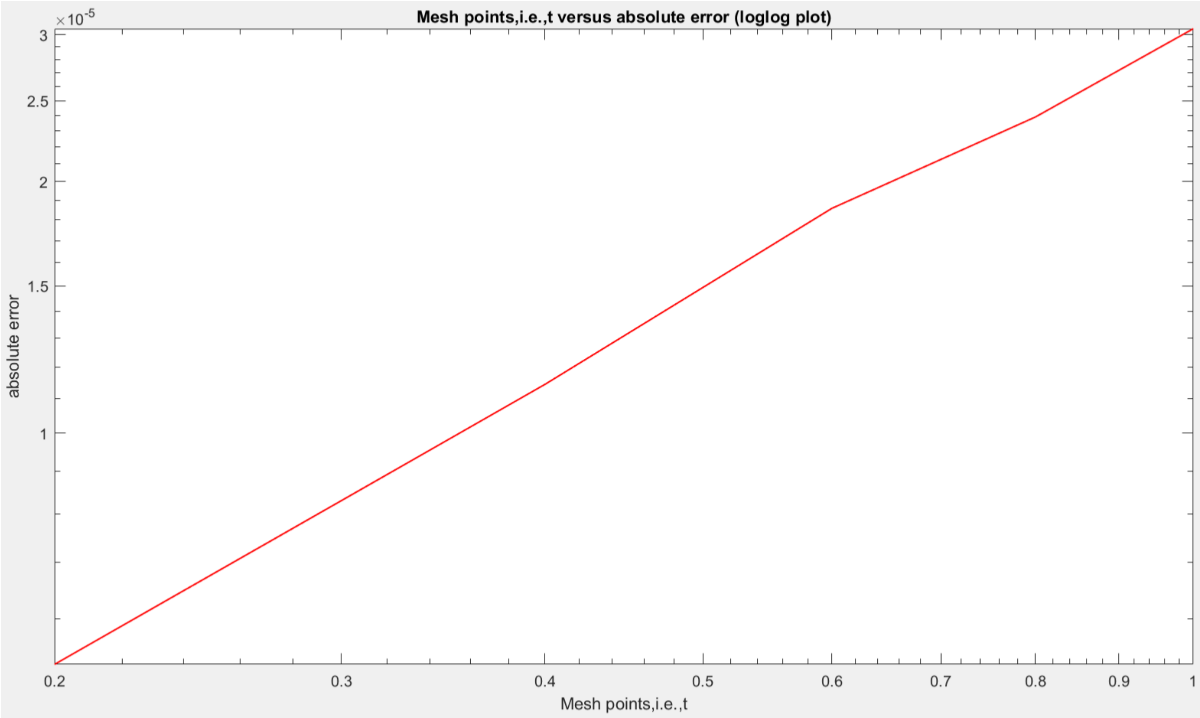
Using the fourth order Runge-Kutta Classical method for starting values,

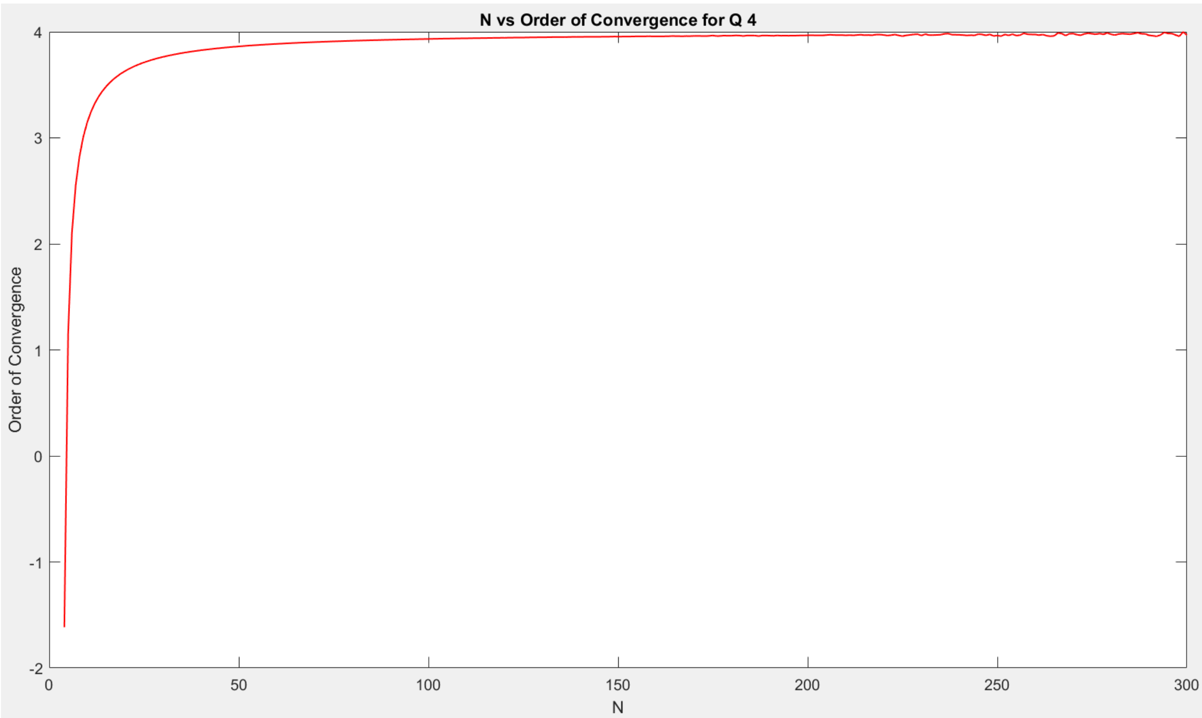
```
Question 4

Using Adams fourth-order predictor-corrector method with starting values from the Runge-Kutta fourth order method
to the IVP given in Question 3,
Using Predictor Corrector method for Question 4,
```

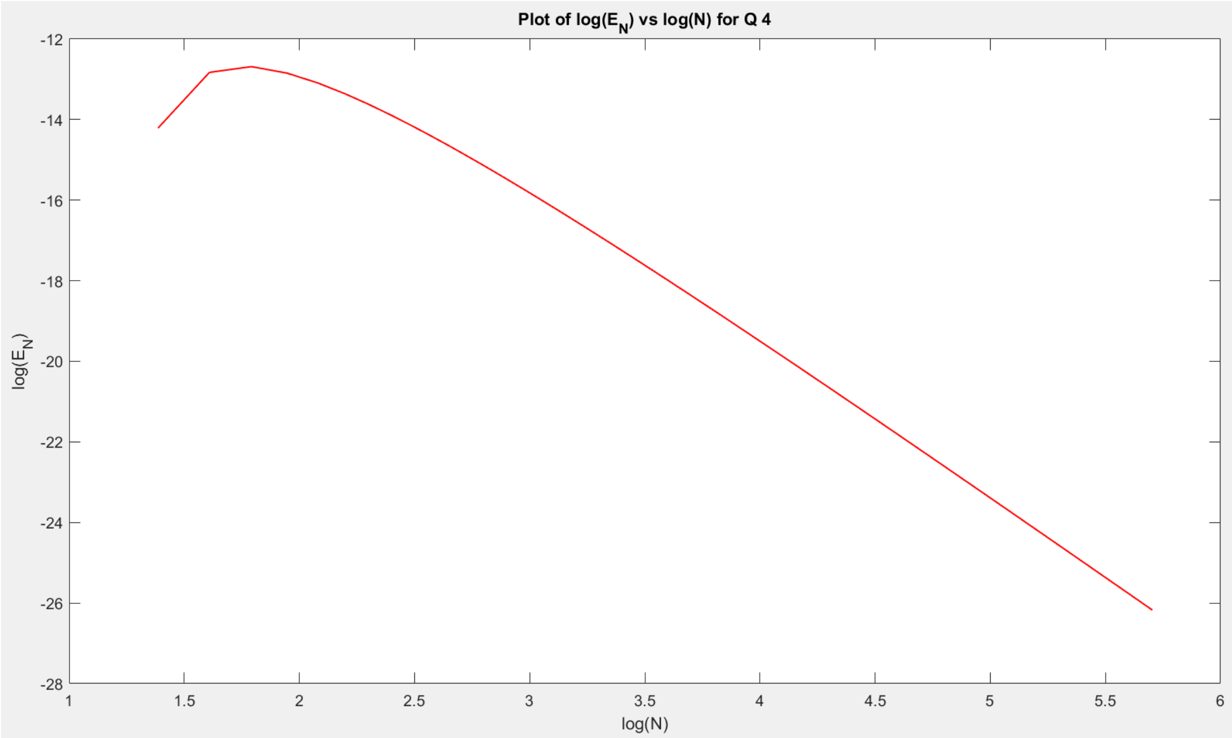
t	Approximate Solution	Exact Solution	Absolute Error
0.000000	0.500000	0.500000	0.000000
0.200000	0.829293	0.829299	0.000005
0.400000	1.214076	1.214088	0.000011
0.600000	1.648922	1.648941	0.000019
0.800000	2.127206	2.127230	0.000024
1.000000	2.640829	2.640859	0.000030






Mesh points,i.e.,t versus absolute error (loglog plot)
×10^-5
3
2.5
2
1.5
1
absolute error
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Mesh points,i.e.,t


N vs Order of Convergence for Q 4
4
3
2
1
0
-1
-2
Order of Convergence
0
50
100
150
200
250
300
N


Plot of log(E_N) vs log(N) for Q 4
log(E_N)
log(N)
-12
-14
-16
-18
-20
-22
-24
-26
-28
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6