**Scientific Computing Lab MA – 322 Lab – 13**

**Name –** Rasesh Srivastava

**Roll Number –** 210123072

**Branch –** Mathematics and Computing

For each part of all the questions, the four plots are as follows:

(i) x versus actual and approximate solutions

(ii) x versus absolute error

(iii) N versus Order of Convergence

(iv) loglog plot of N versus absolute error

Where, N = (b-a)/h = Number of intervals

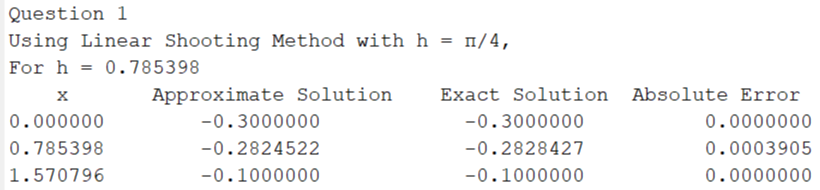
Order of convergence = log2(EN/E2N), where EN and E2N and the maximum errors obtained during the computation for that specific value of N

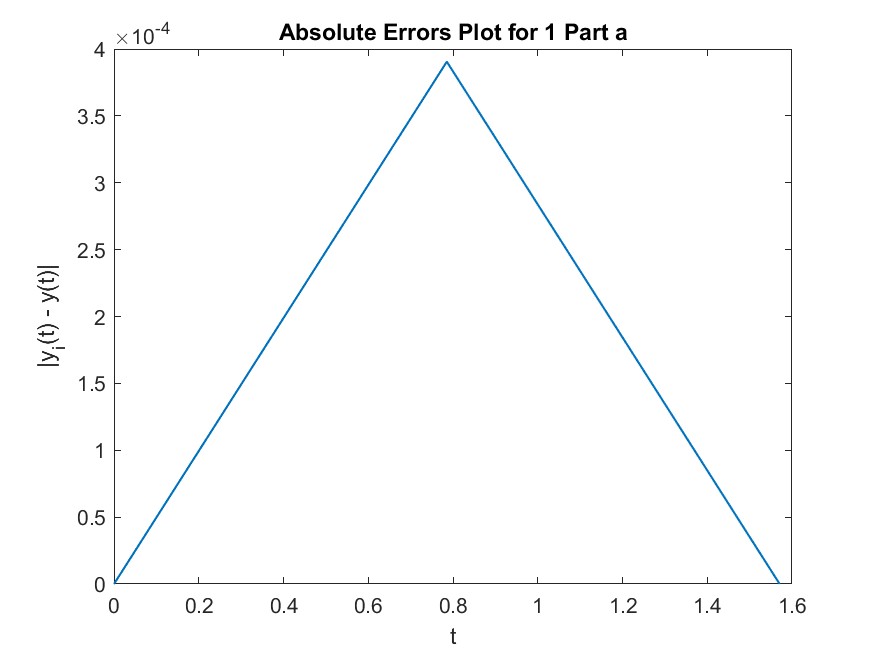
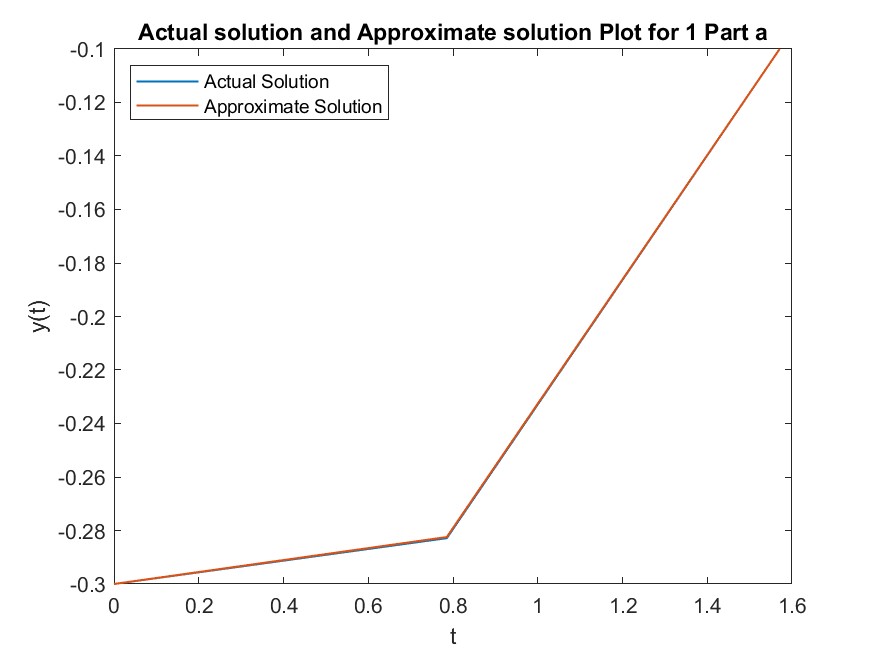
Absolute error = absolute value of the difference between the approximation and the actual solution

1)

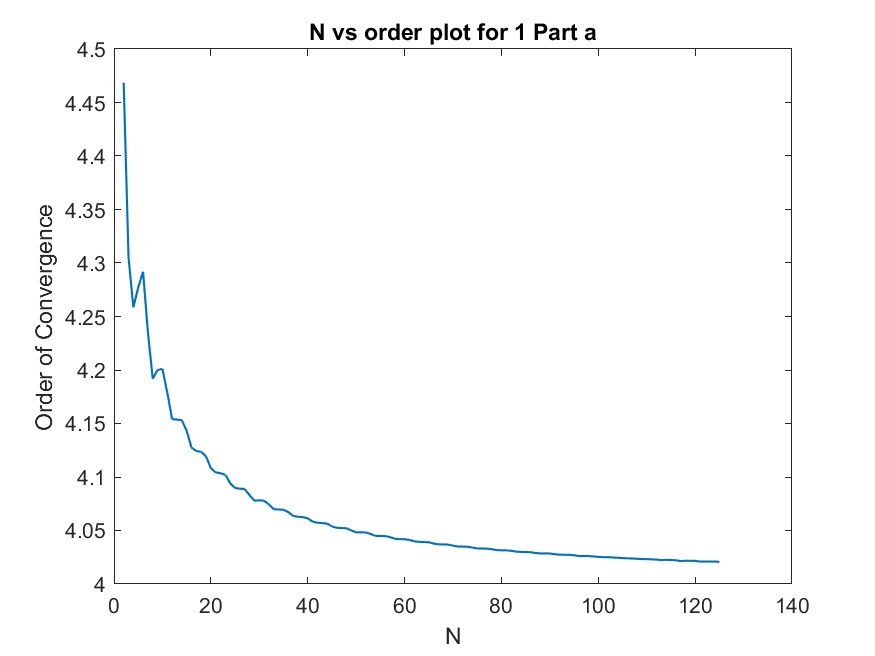
Using Linear Shooting method, we got a system of 2 ODEs, which we solved using the fourth order Runge-Kutta method for system of equations.

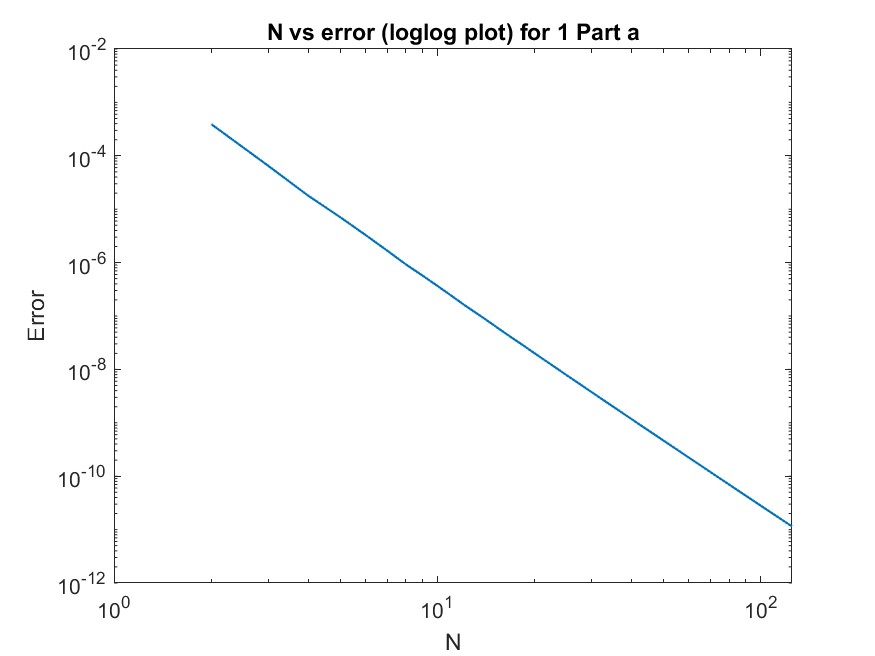
a)



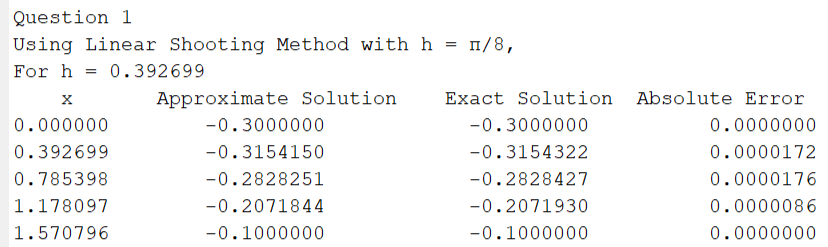


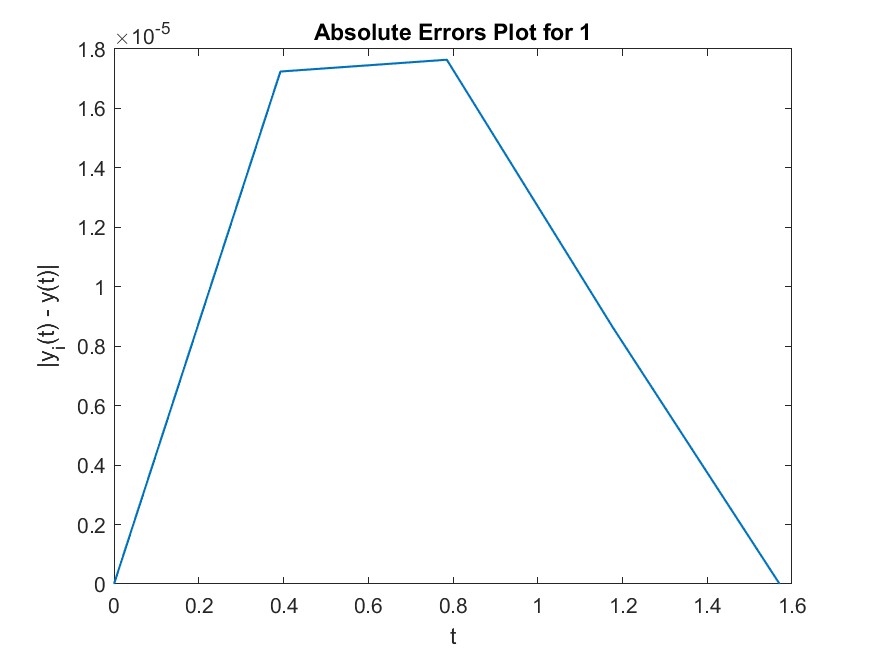
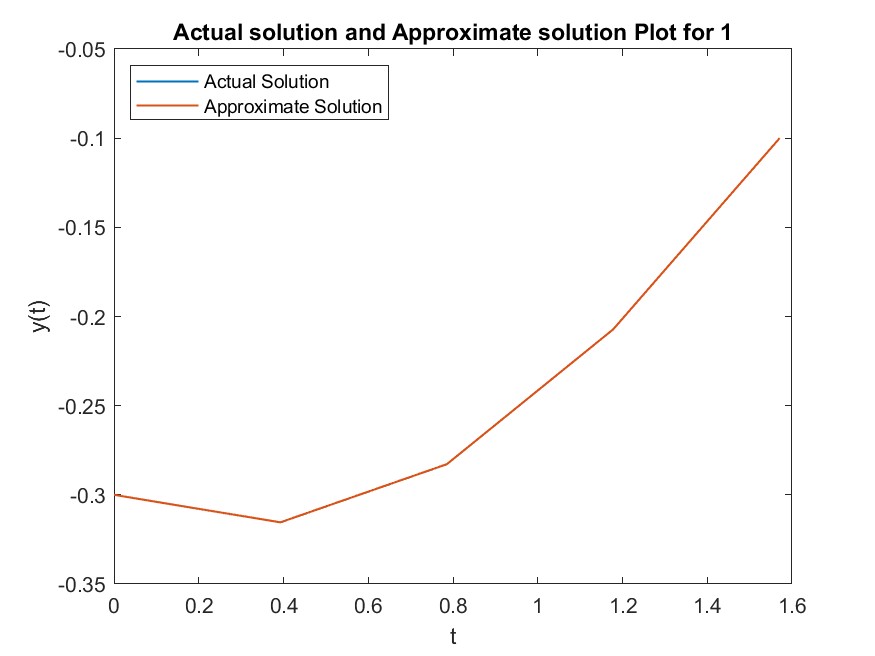
Since the Runge-Kutta method is used, the order of convergence should converge to 4.

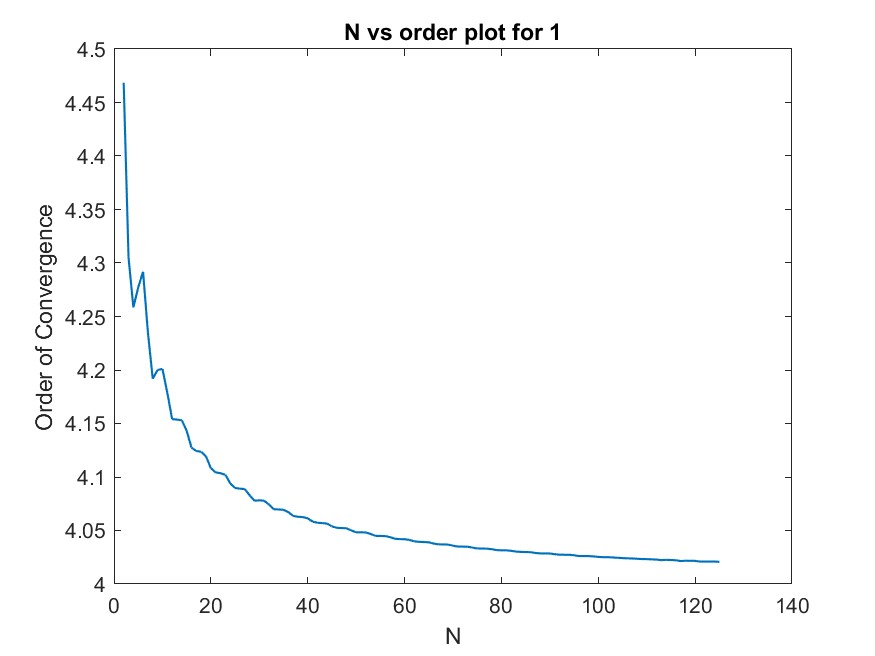


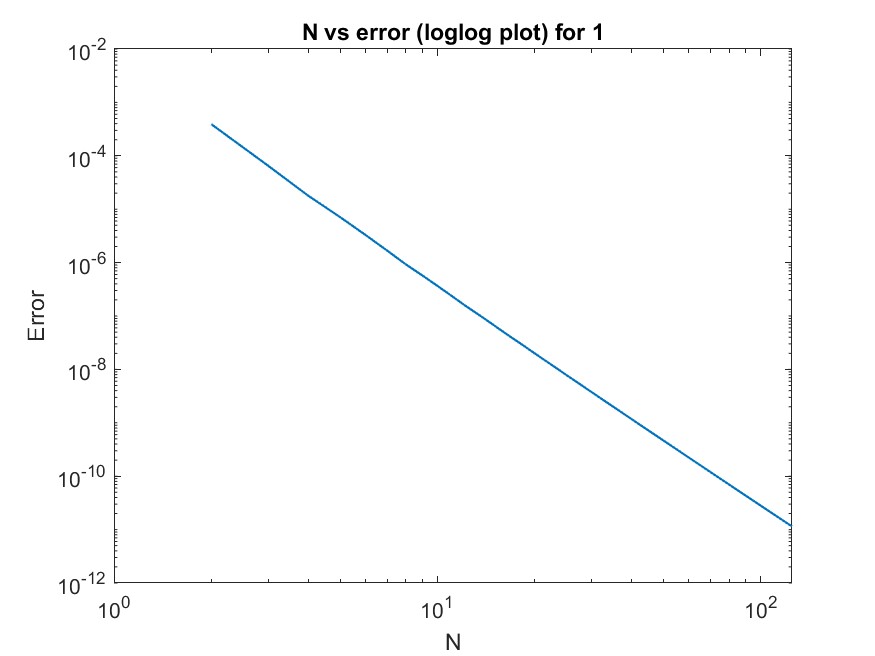


b)



Since the Runge-Kutta method is used, the order of convergence should converge to 4.





We can observe that the errors are very low.

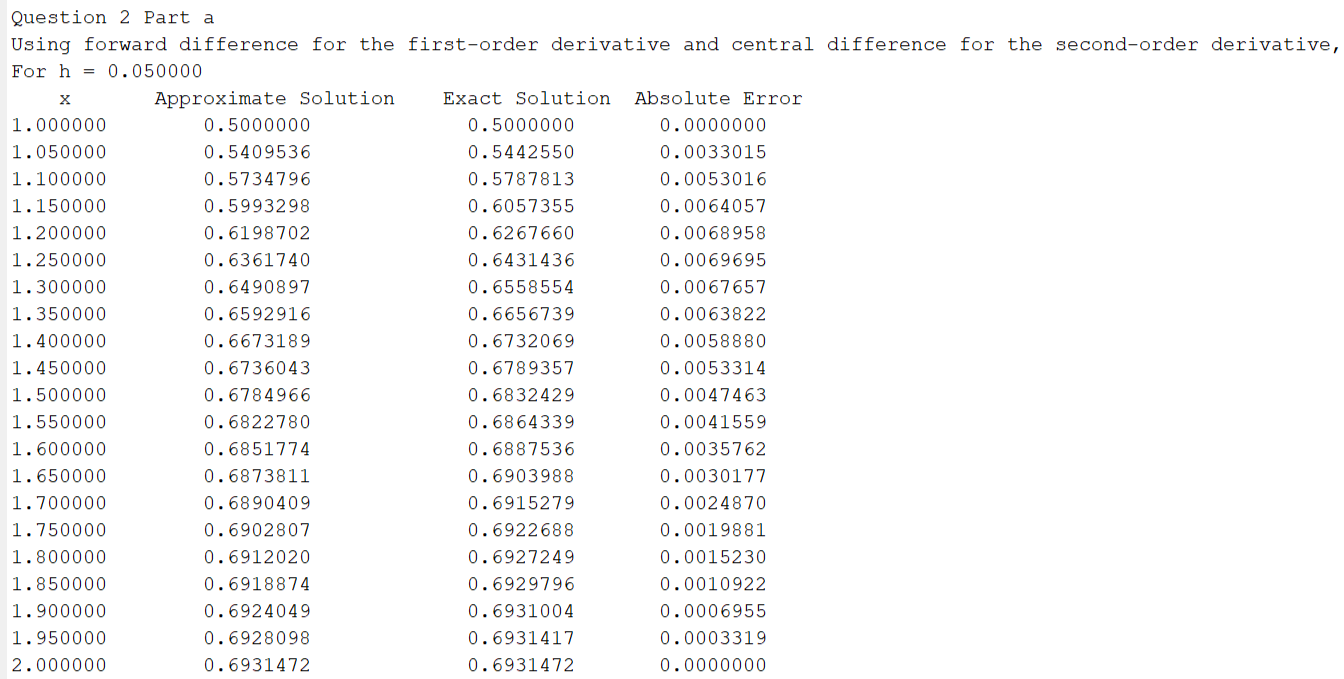
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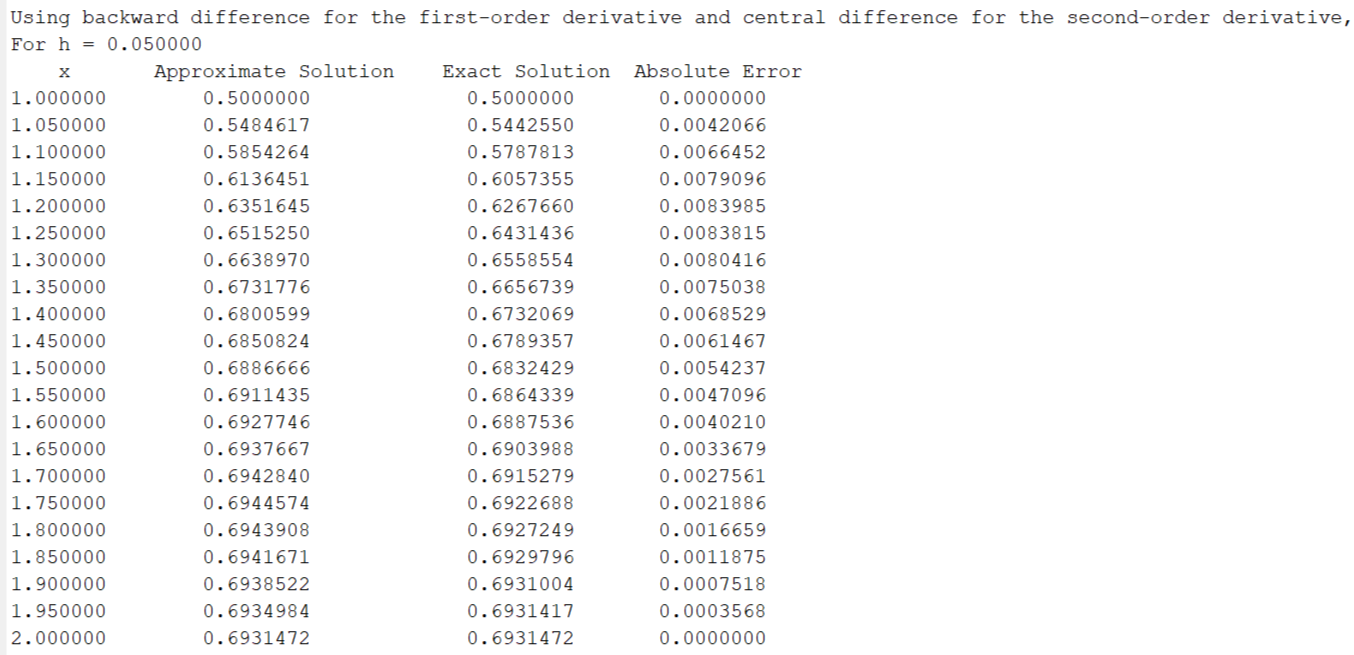
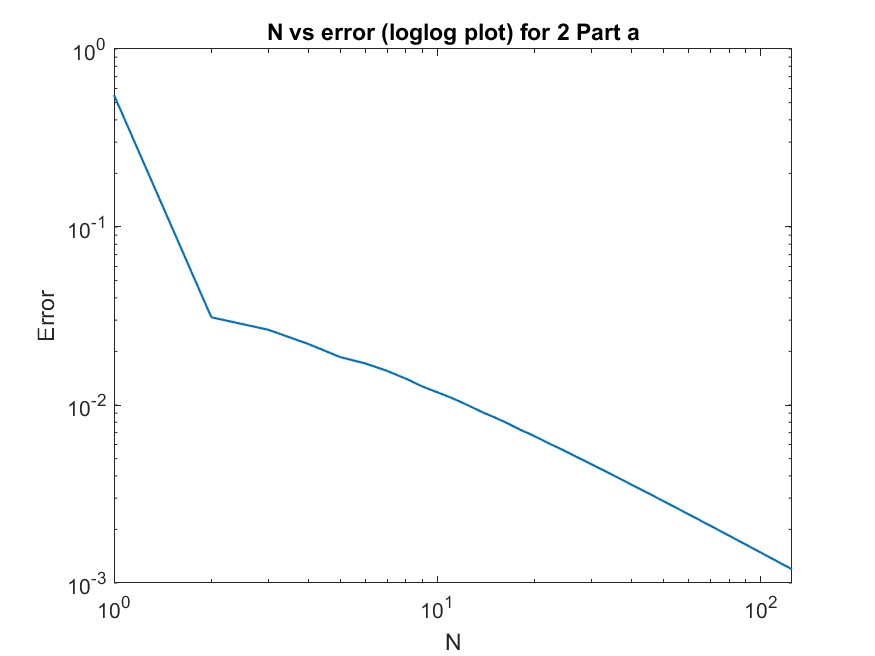
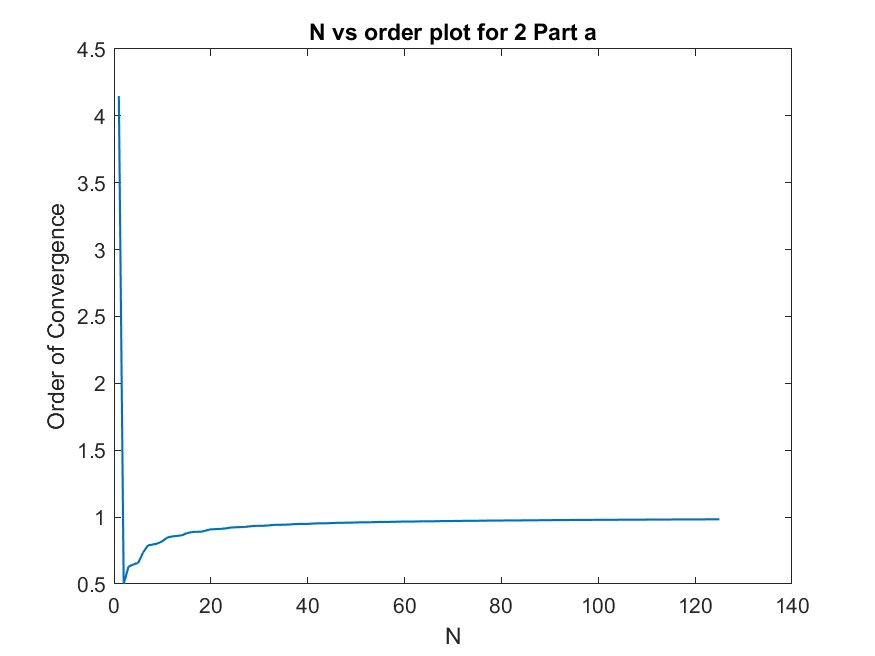
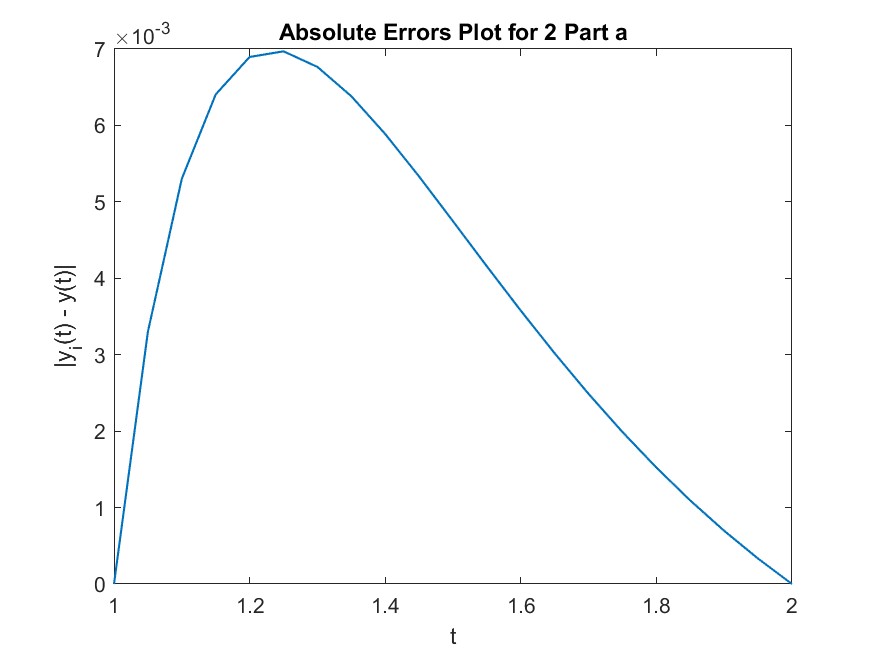
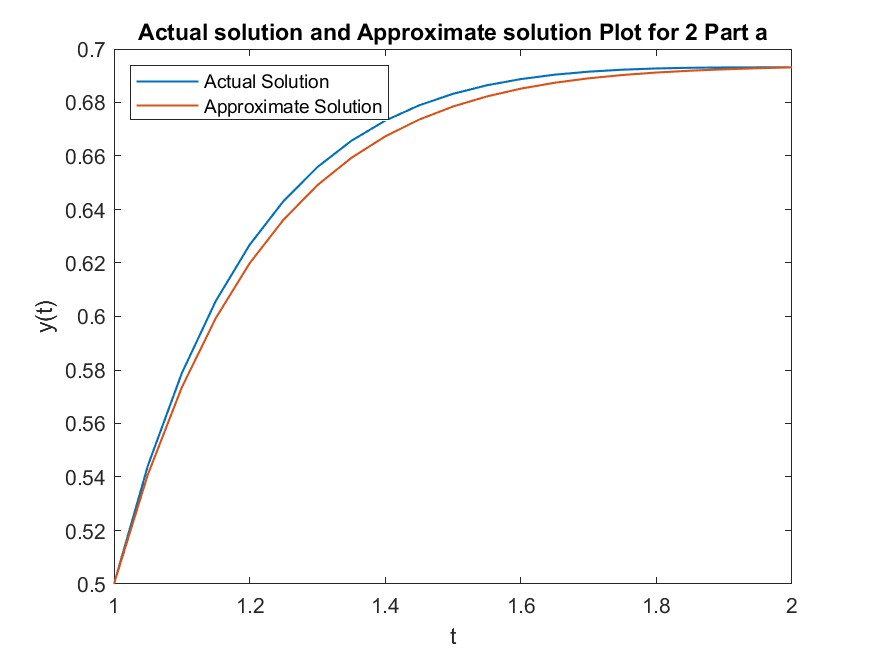
The six parts have in total 3 types of boundary conditions, which are as follows:

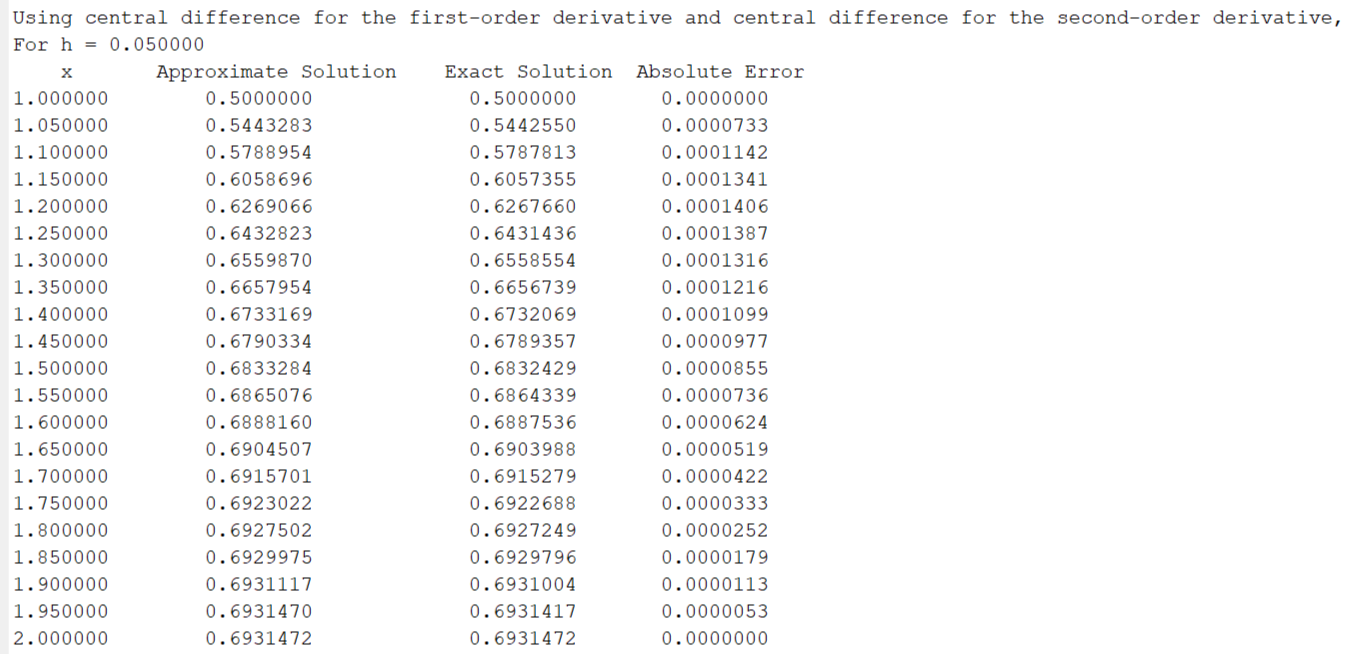
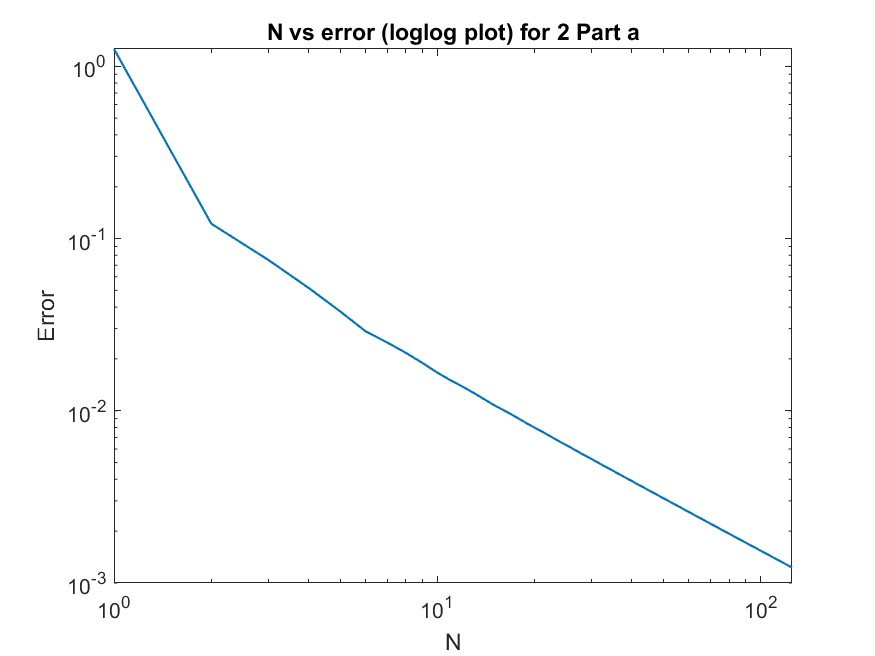
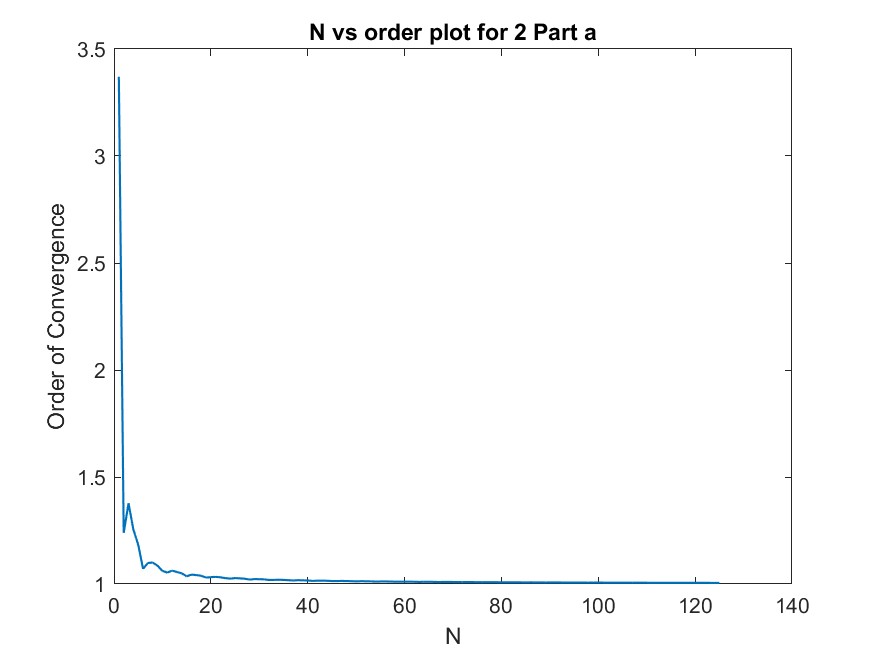
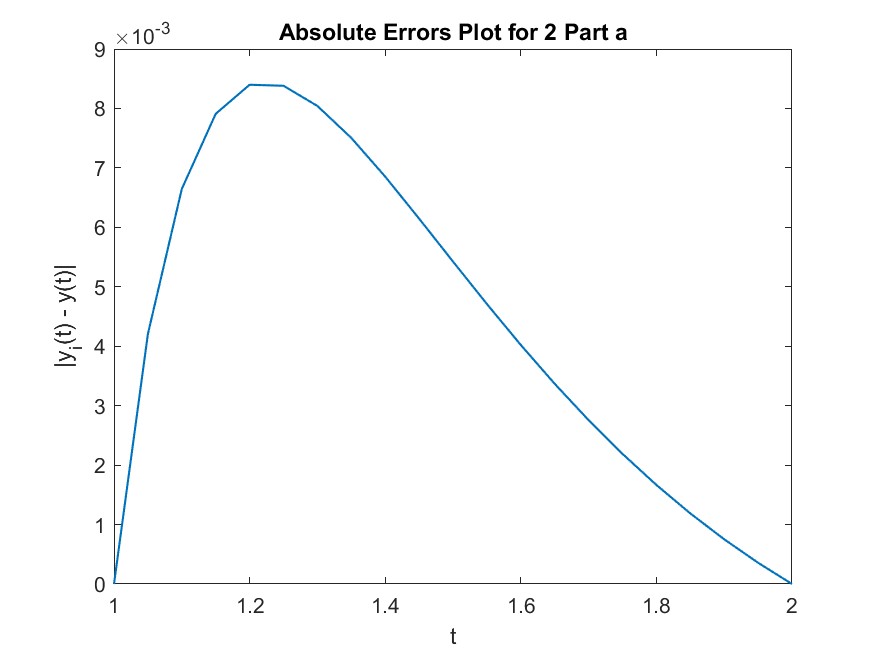
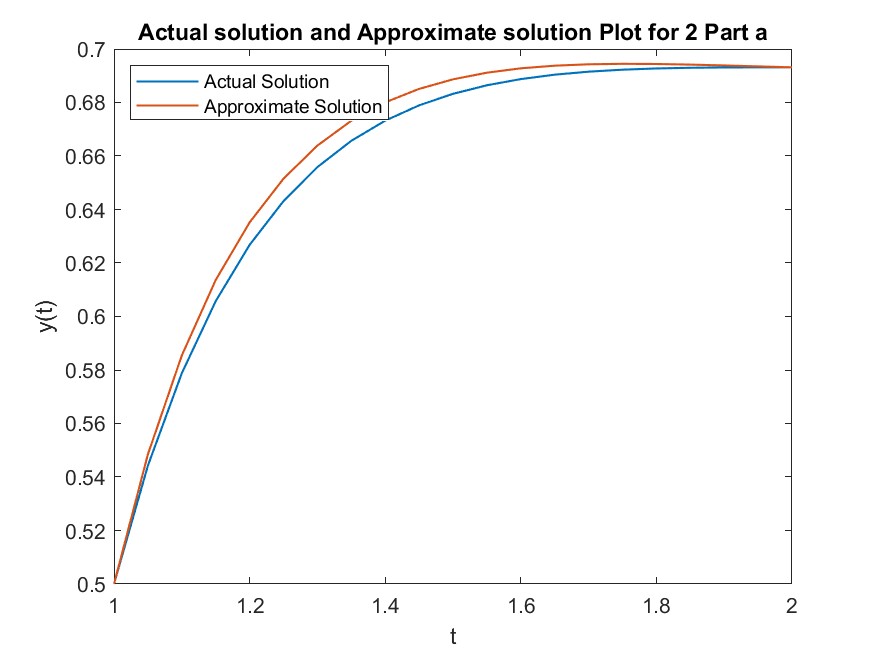
* For parts a and b, Dirichlet Conditions
* For parts c and d, Neumann Conditions
* For parts e and f, Robin or Mixed Conditions

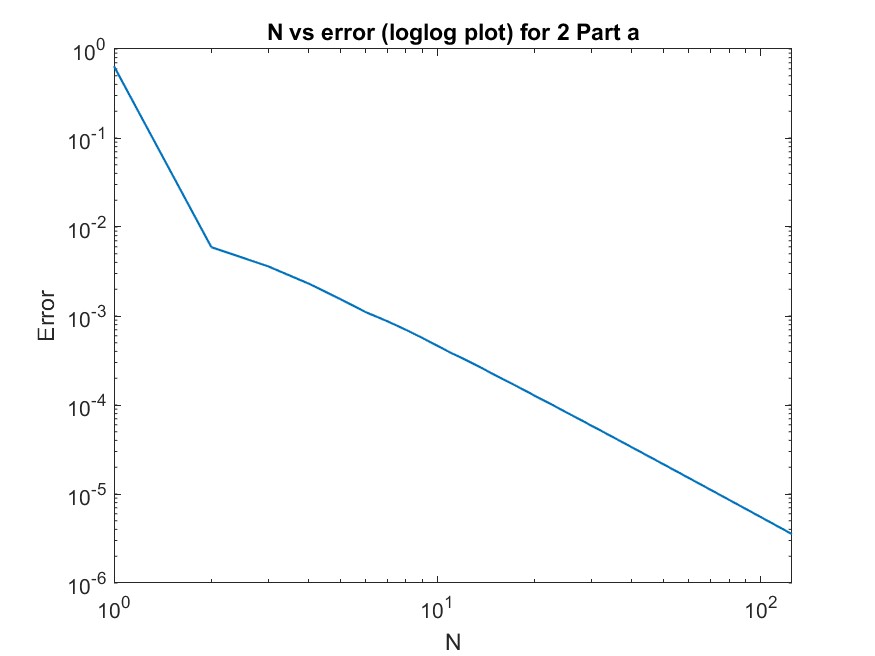
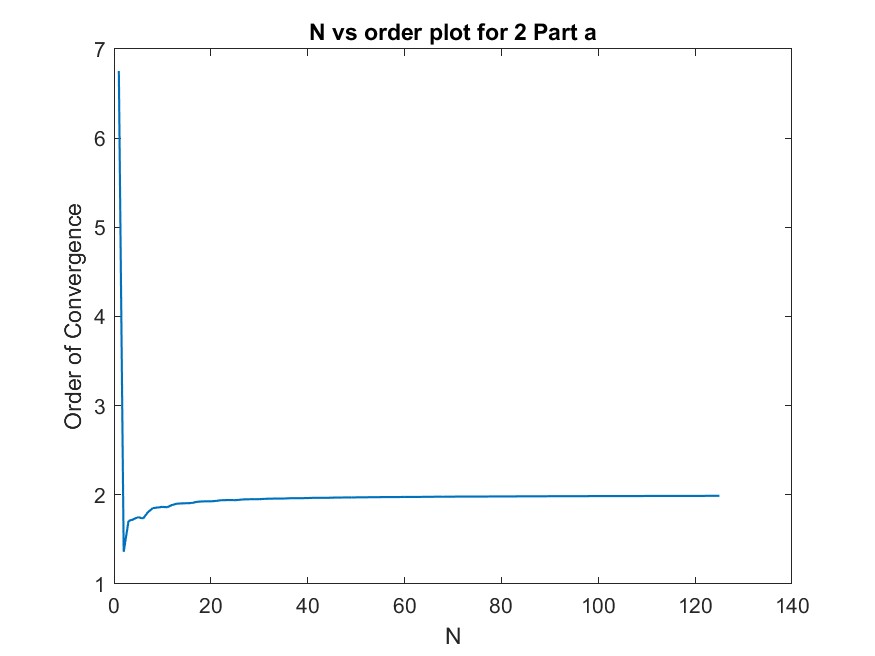
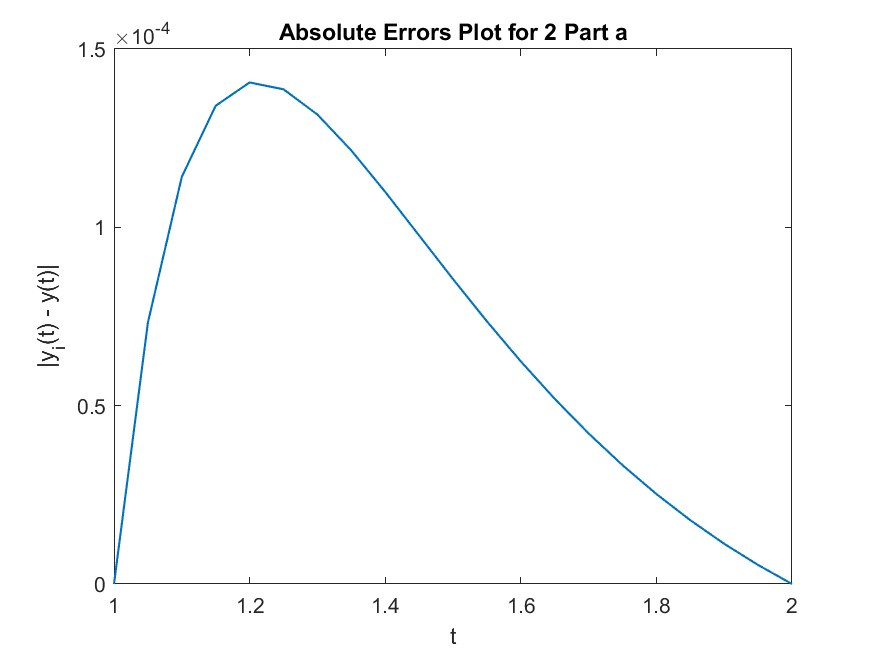
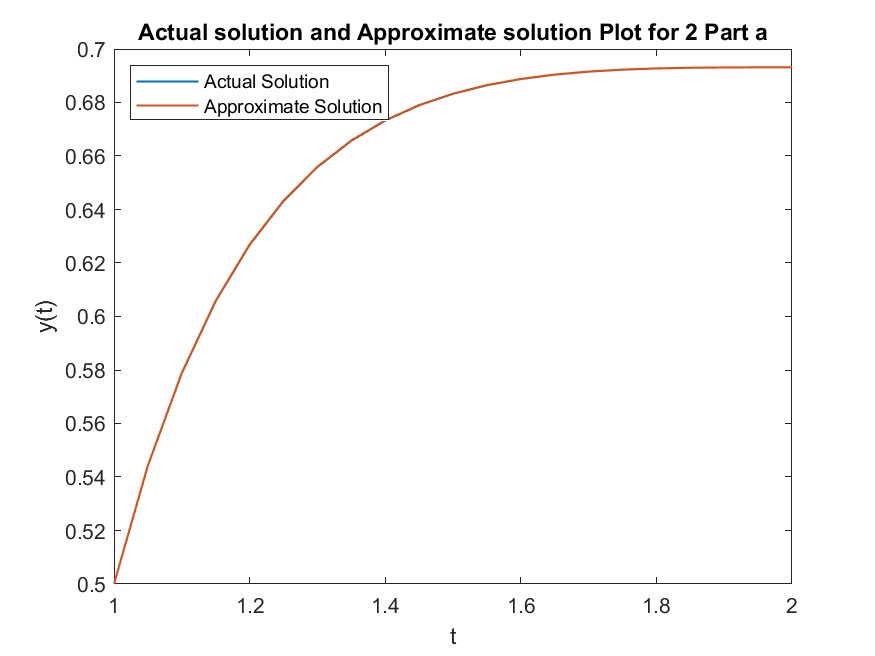
The second derivative is always replaced by the Central difference, while the first derivative is replaced by Forward difference, Backward Difference and Central Difference one by one.

a)

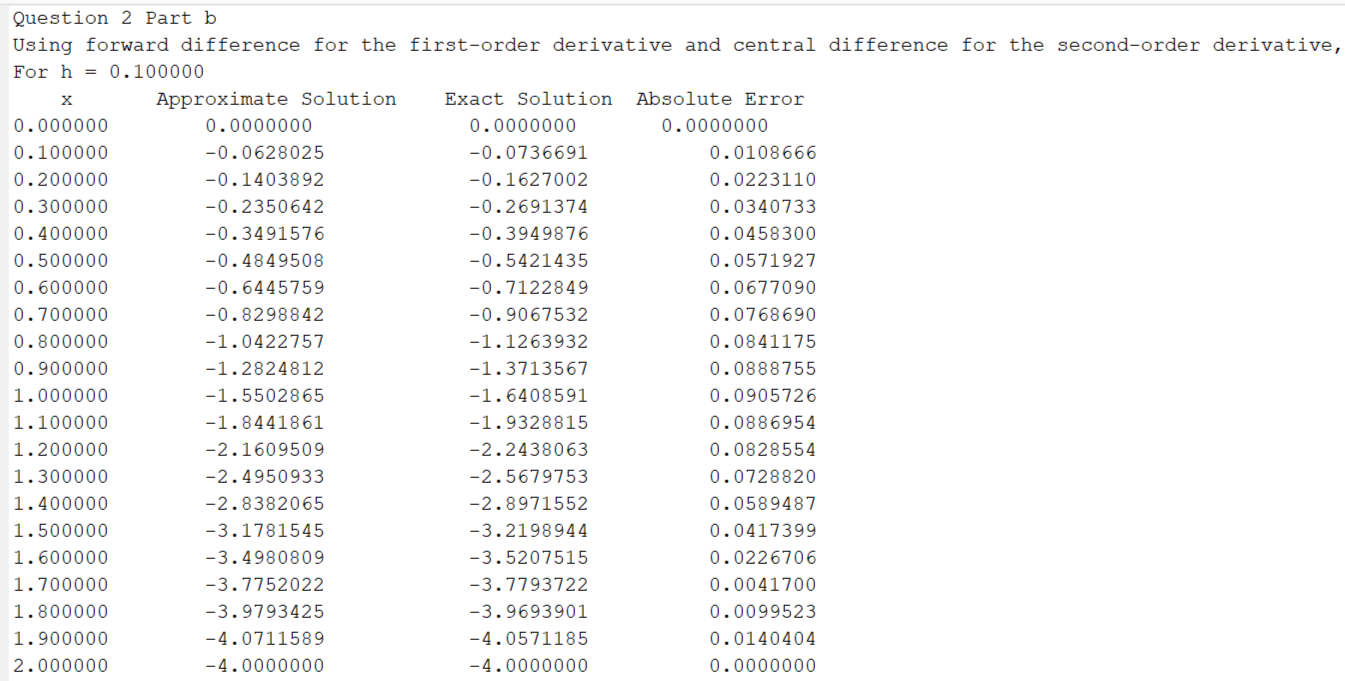


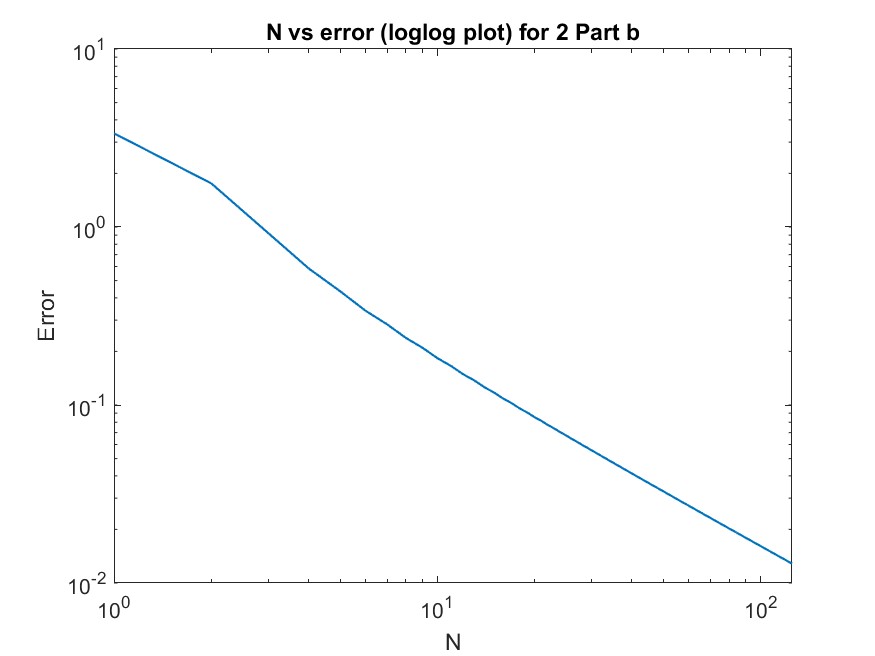
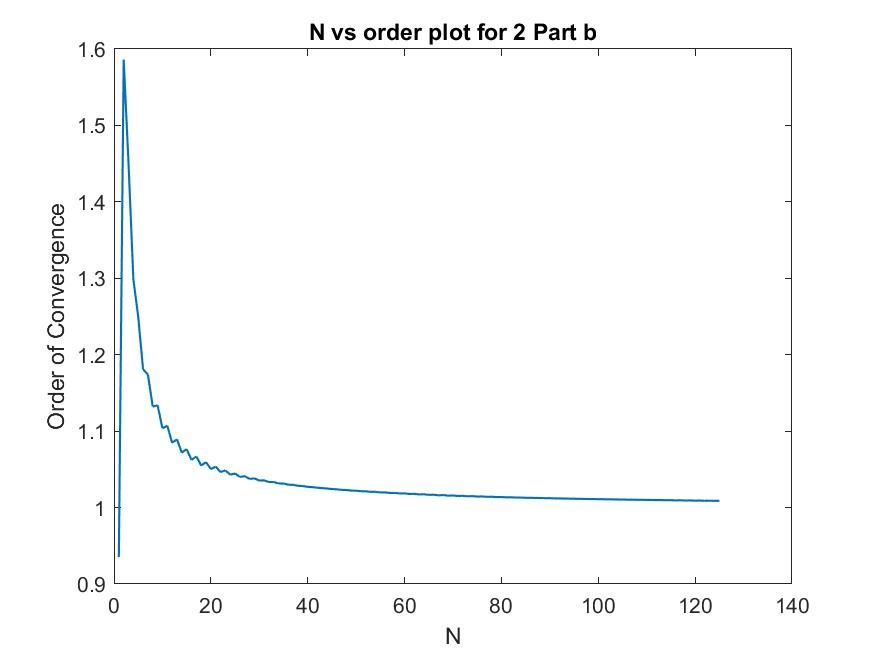
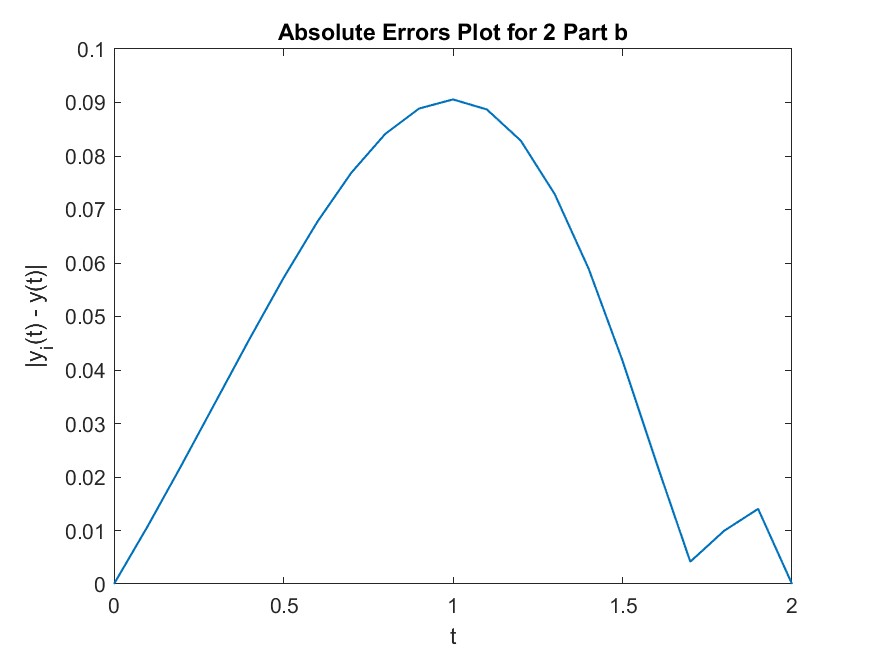
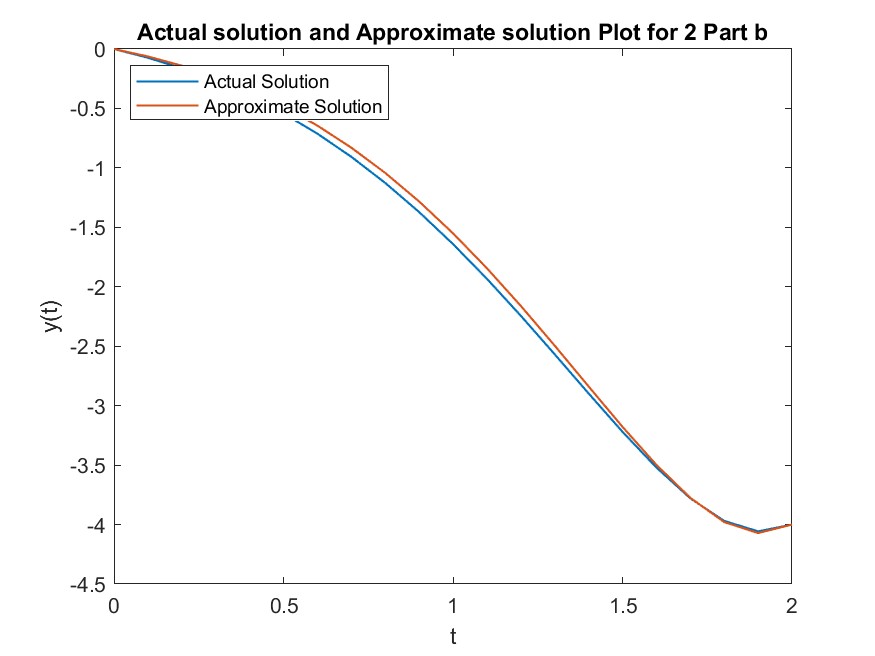


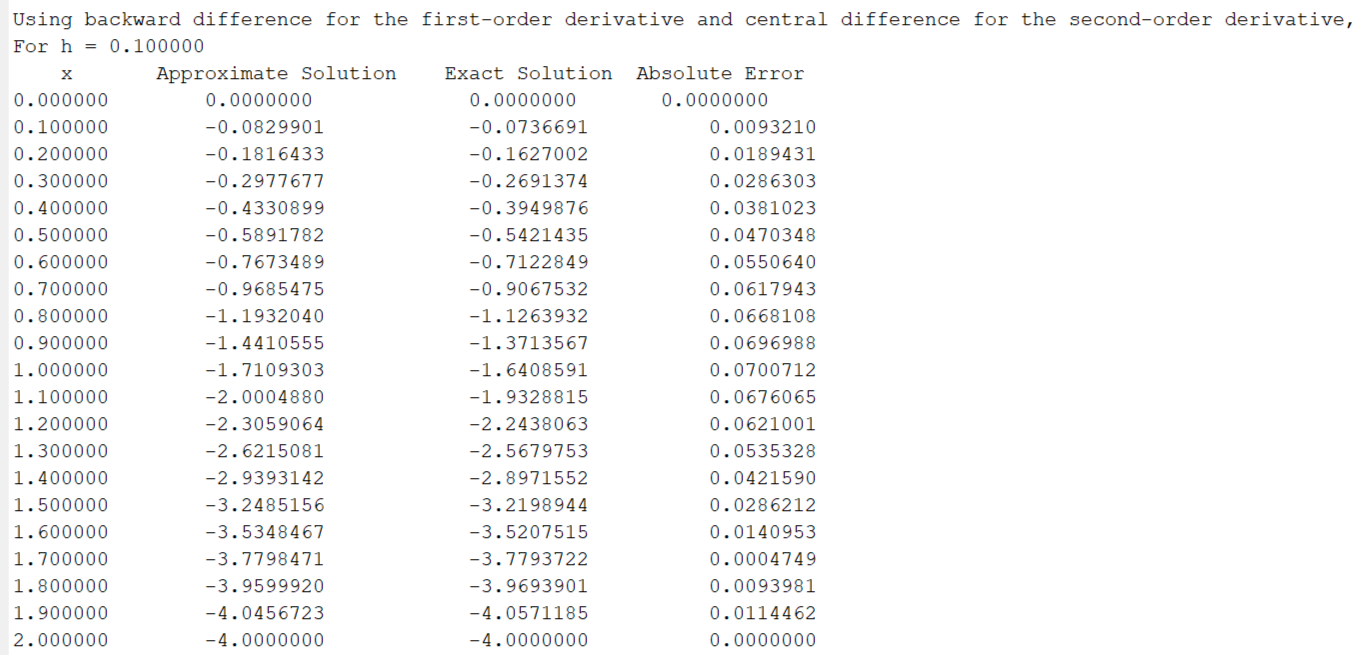


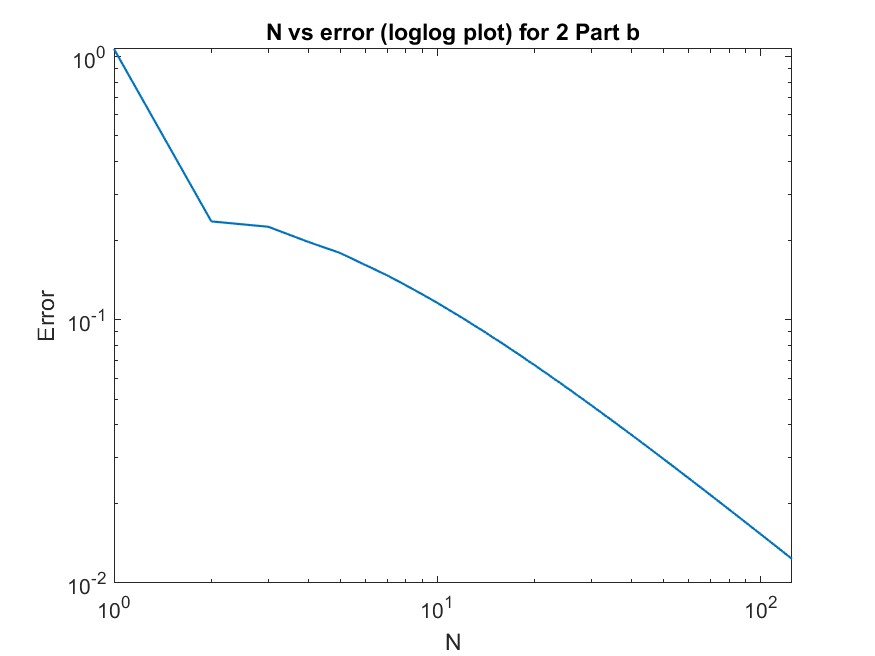
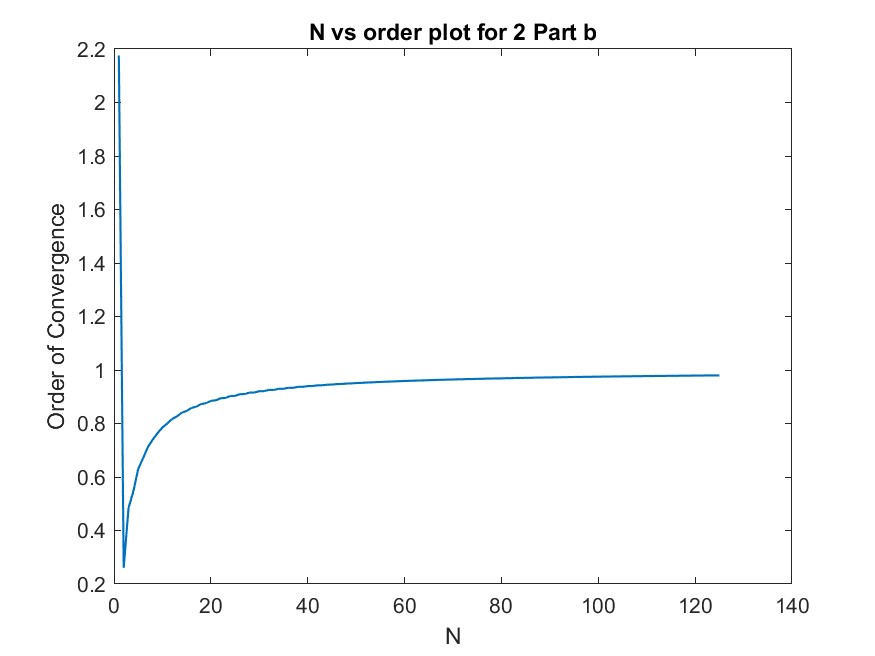
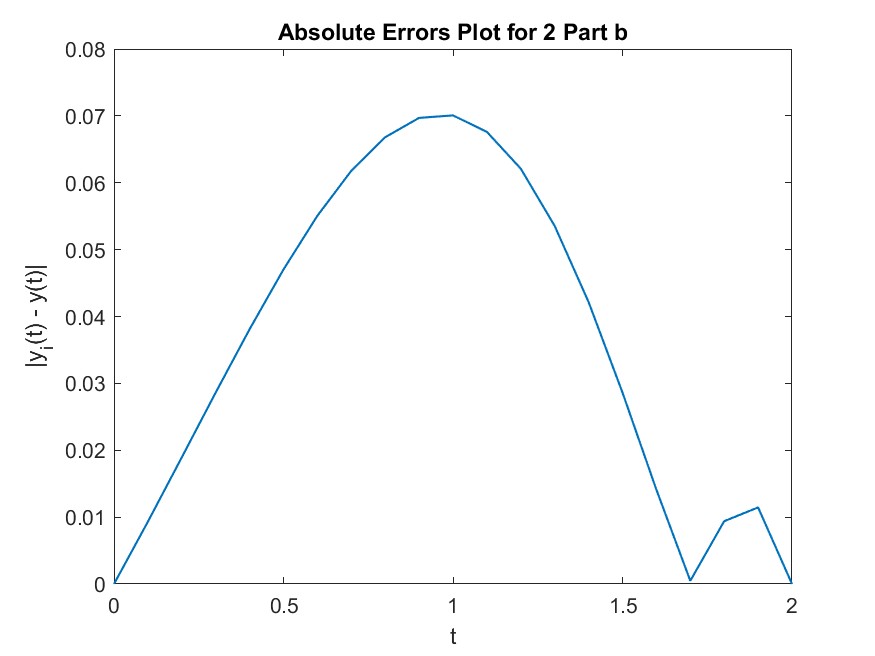
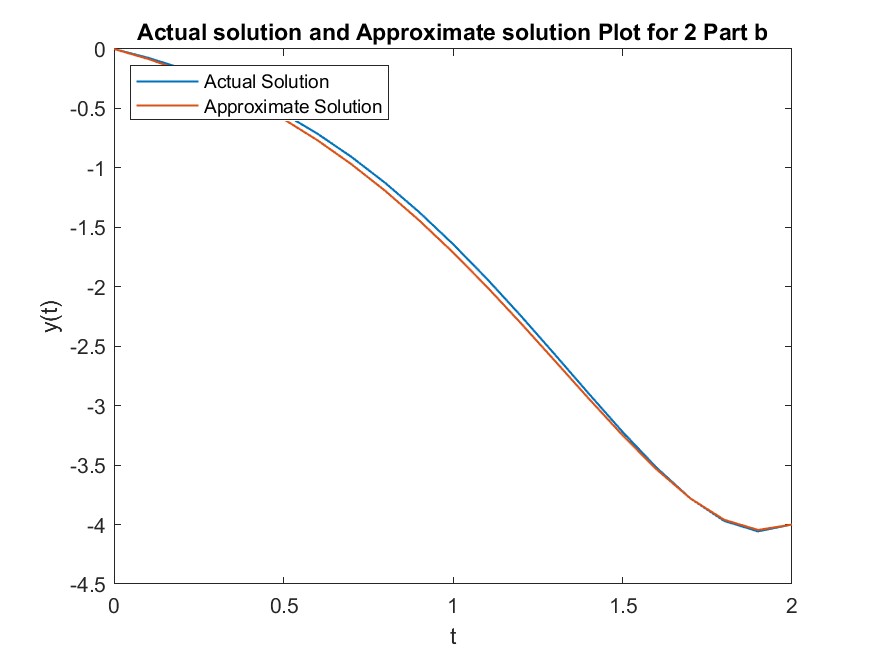


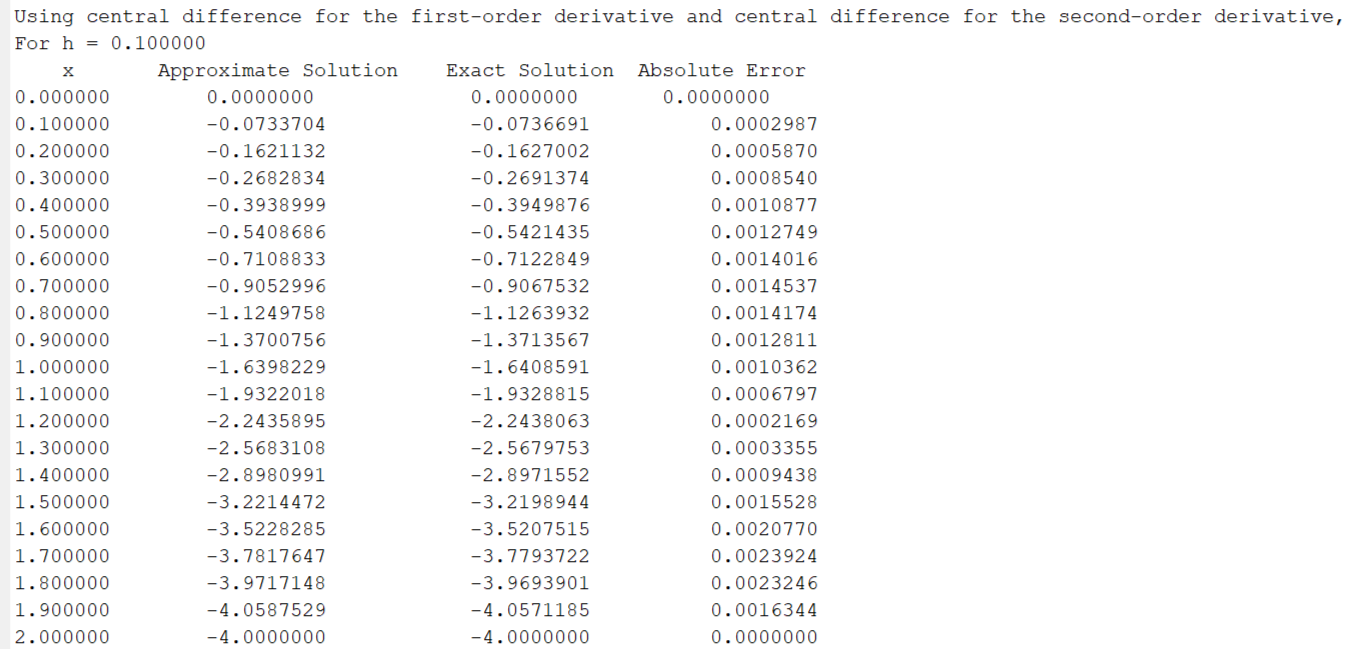
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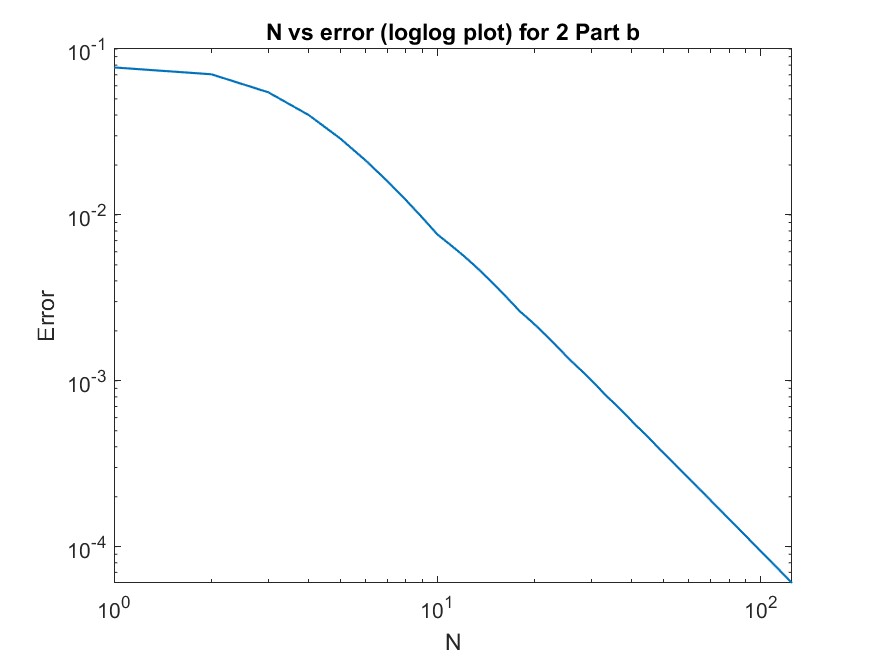
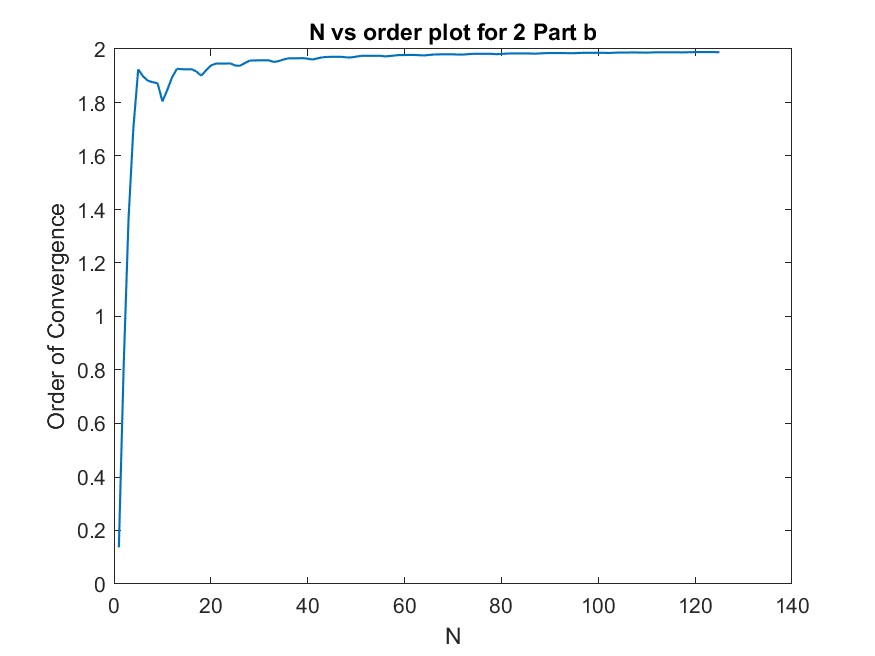
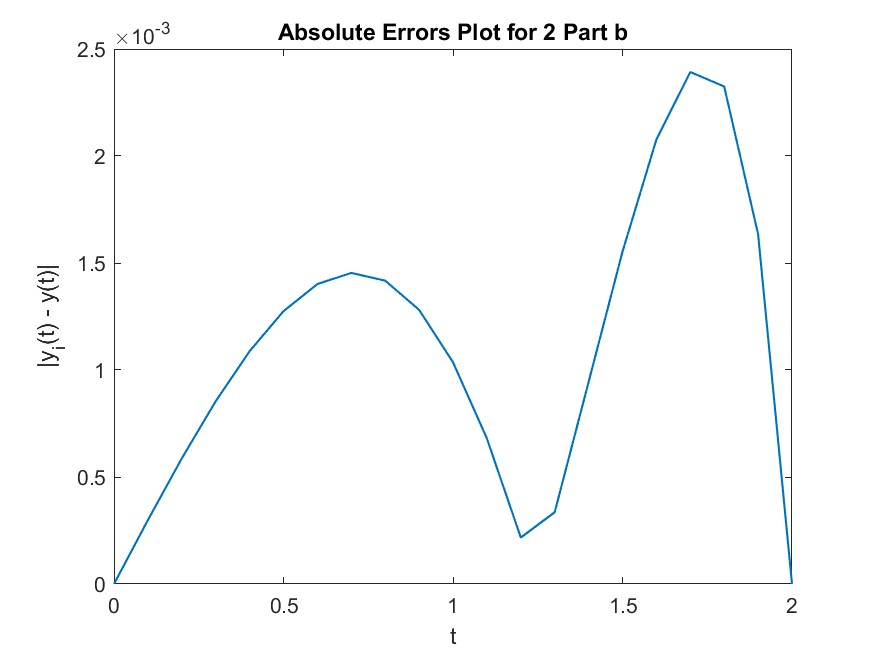
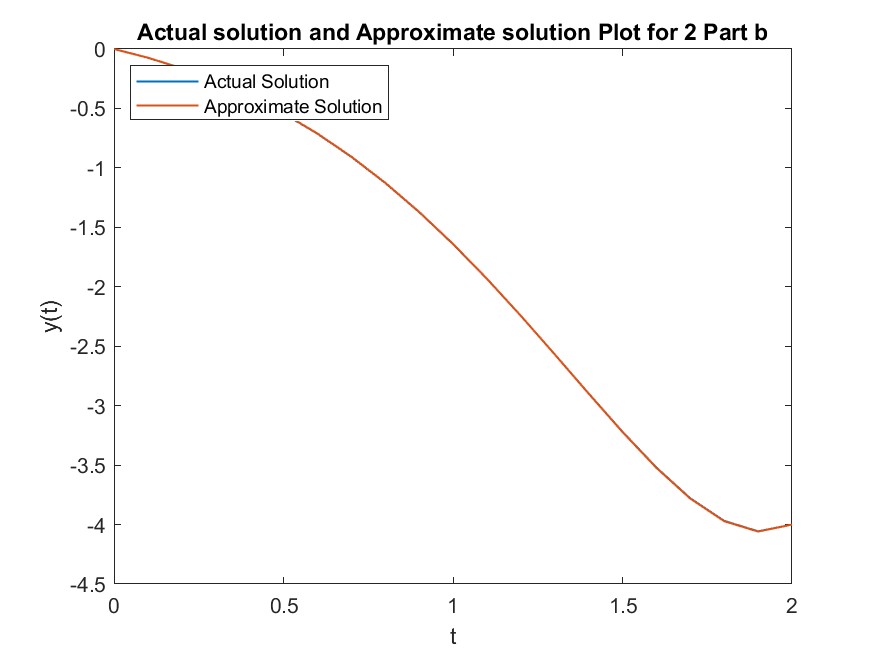




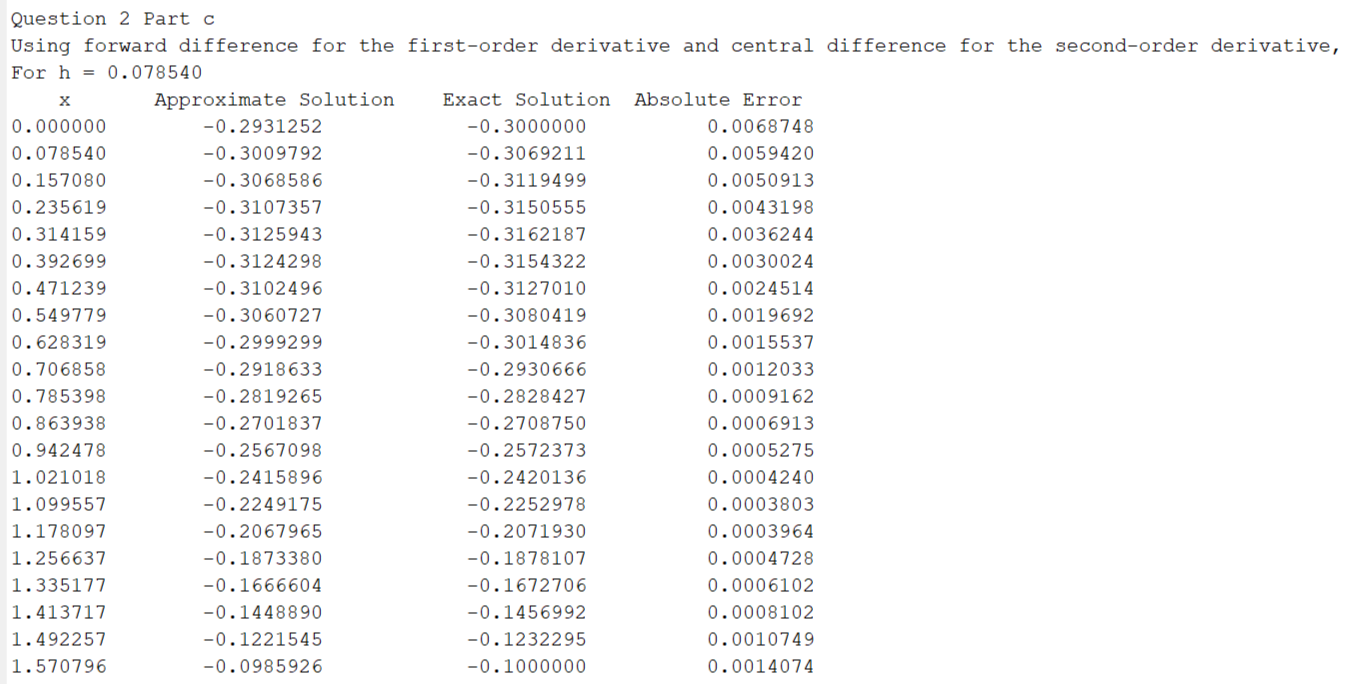


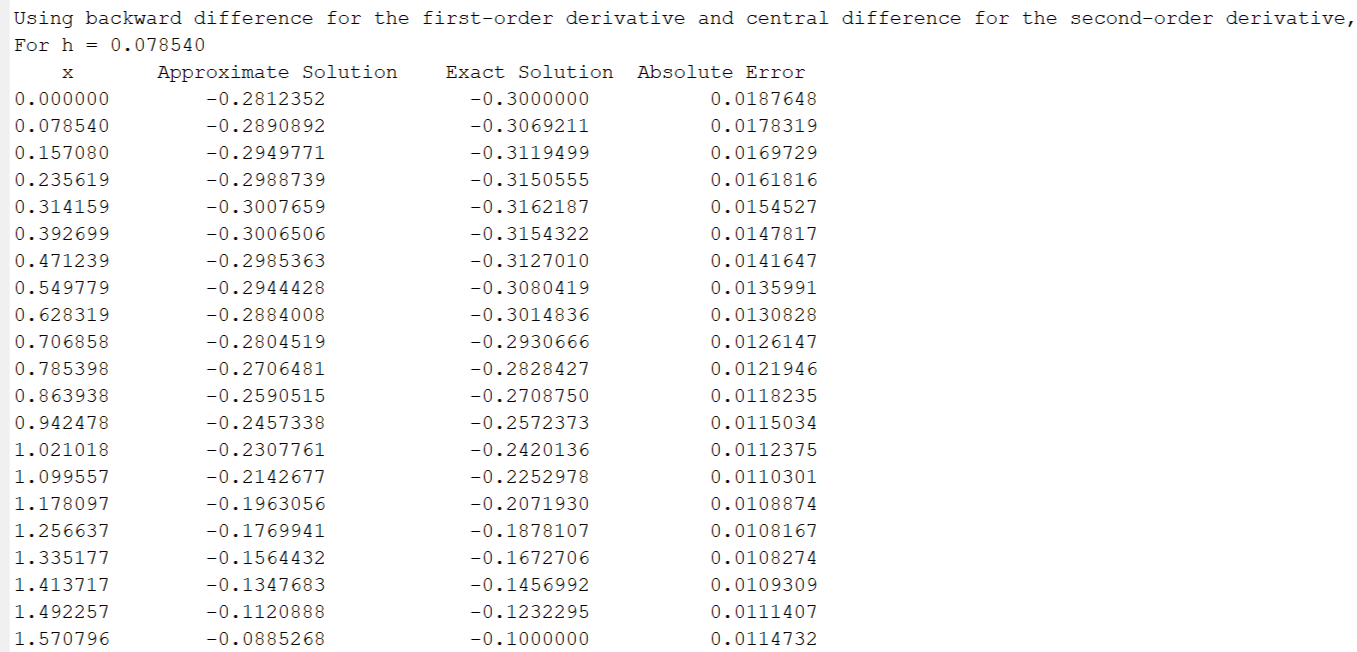
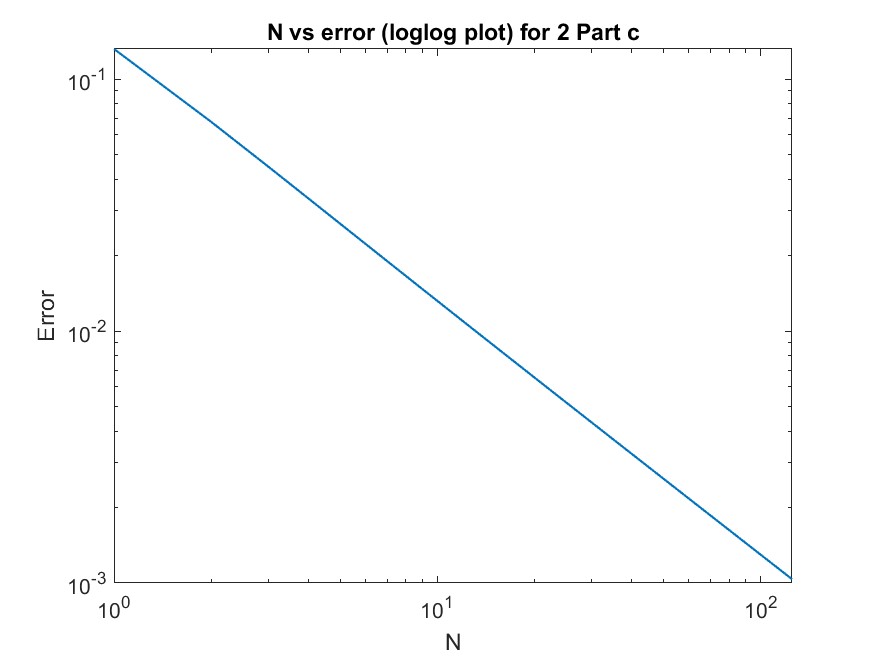
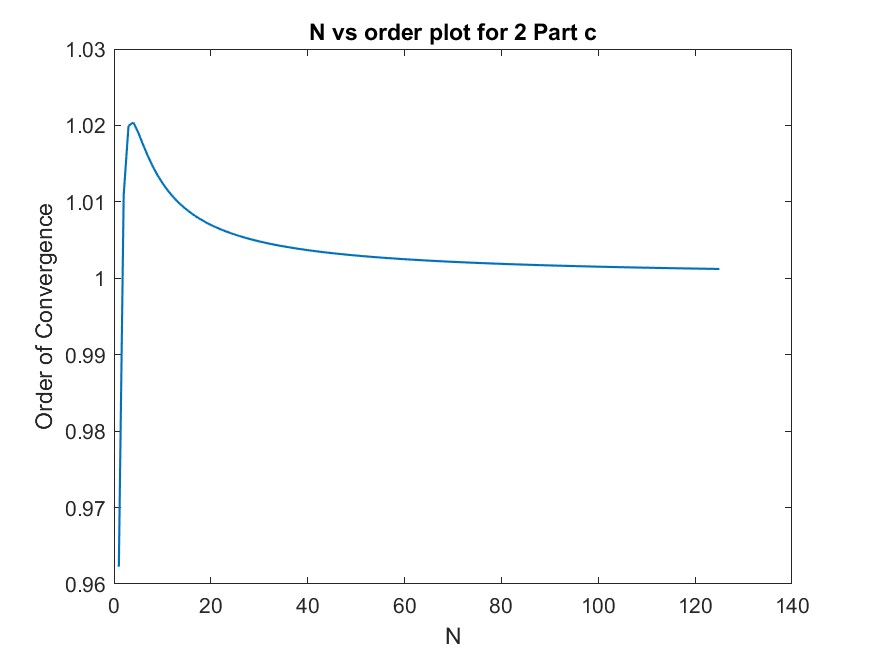
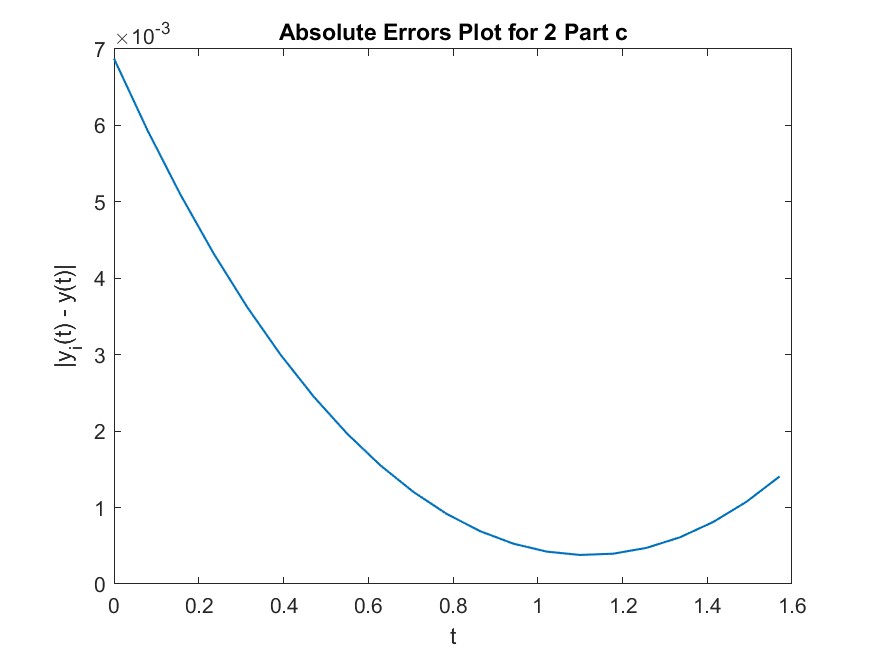
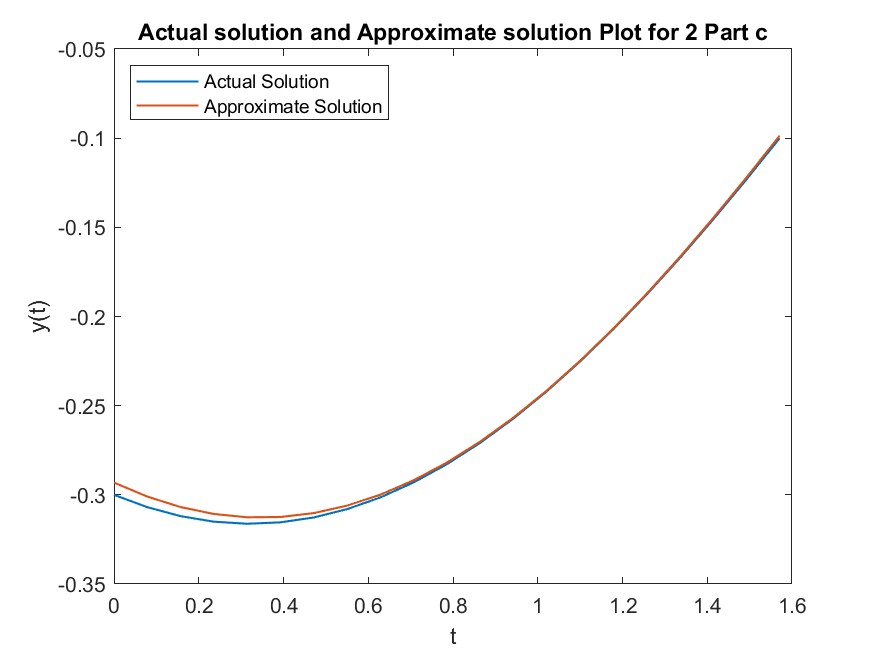


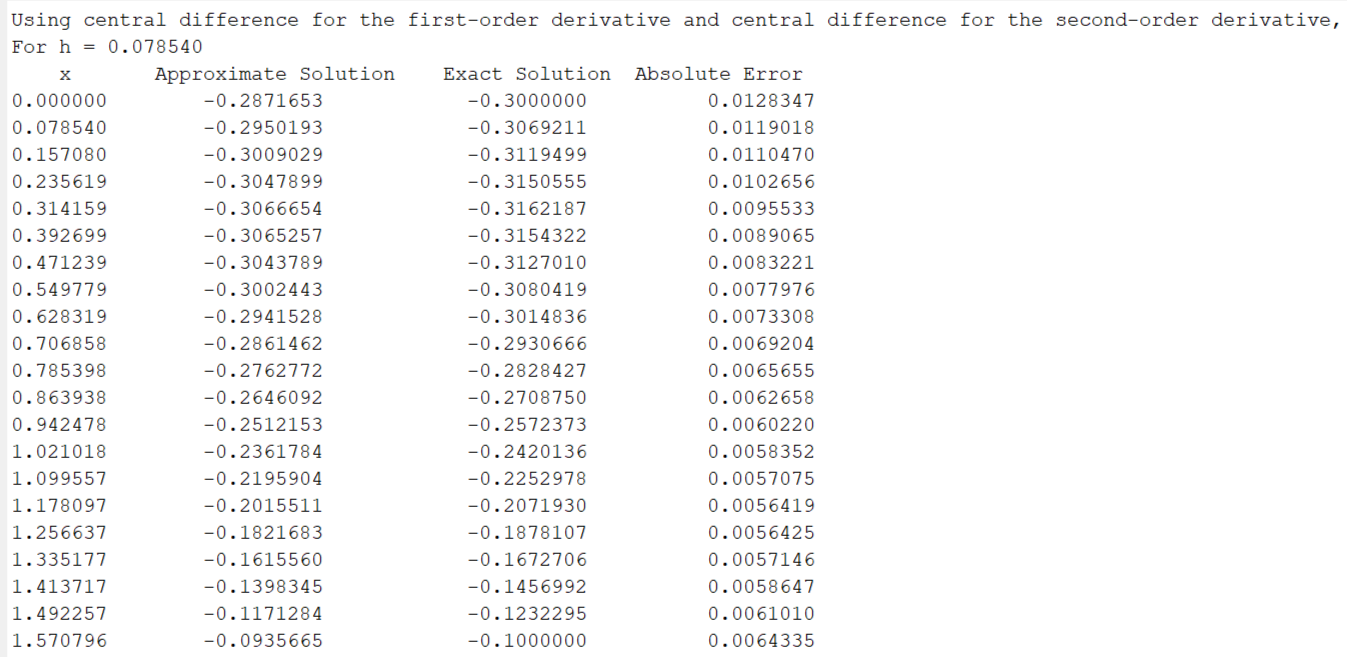
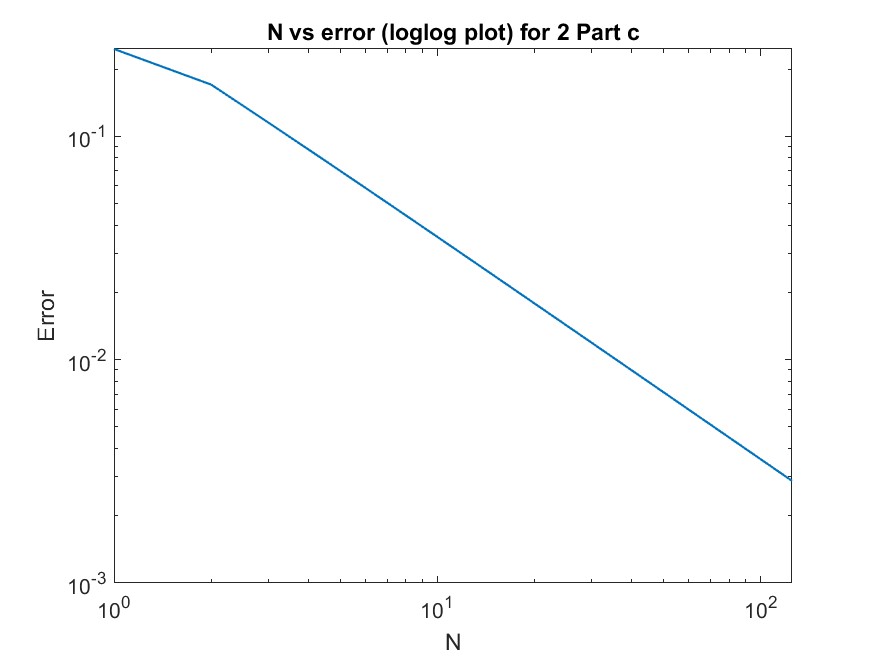
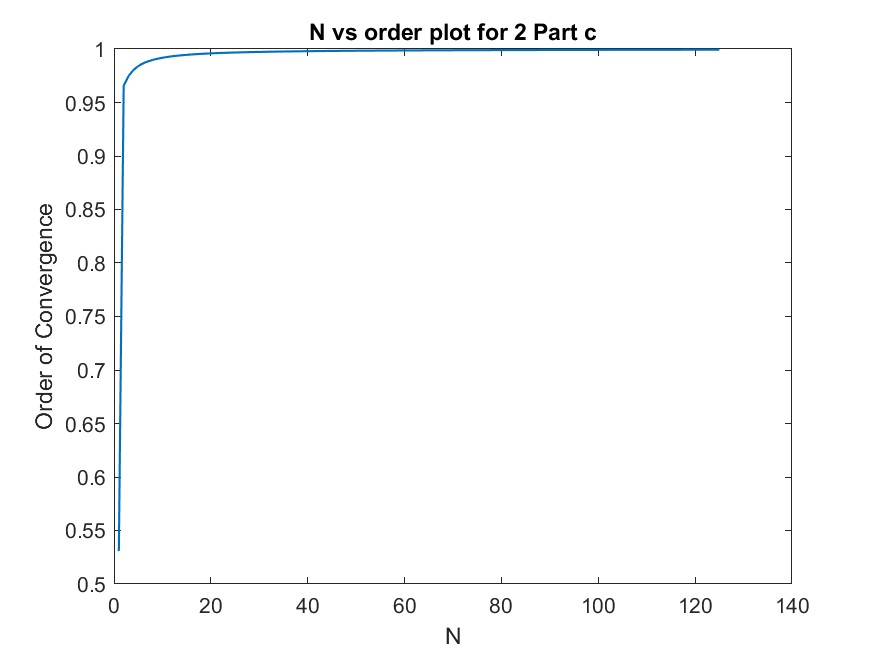
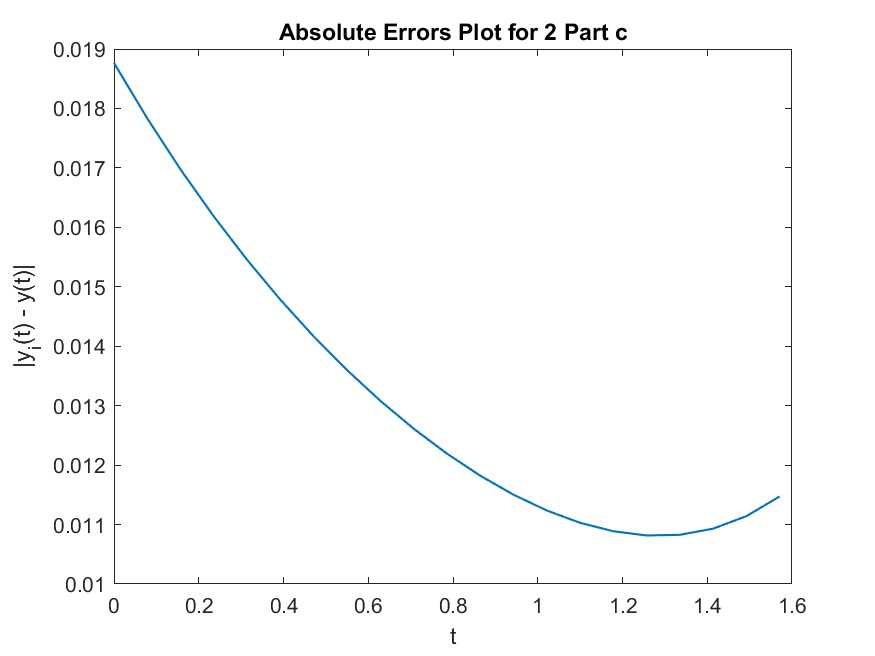
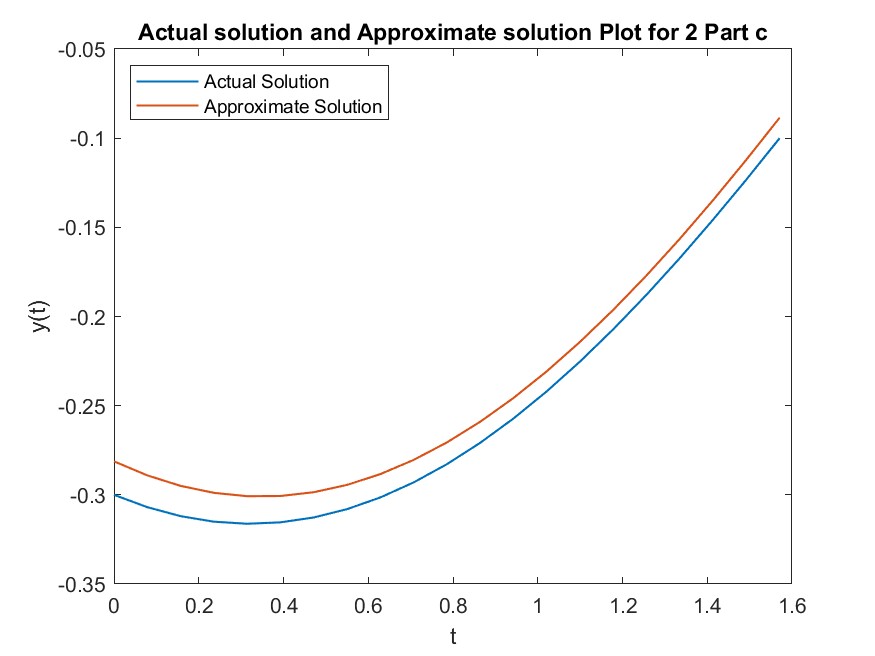


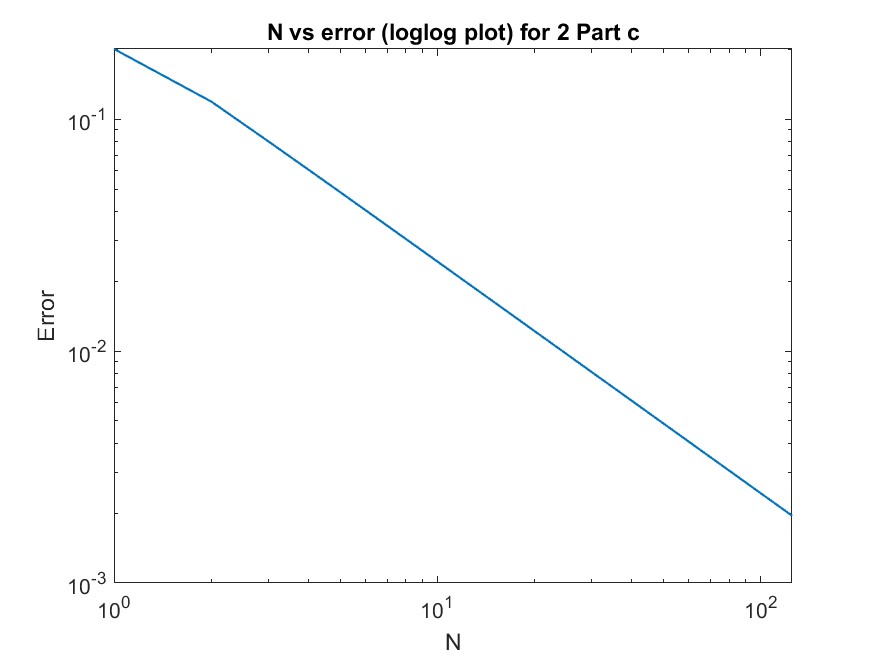
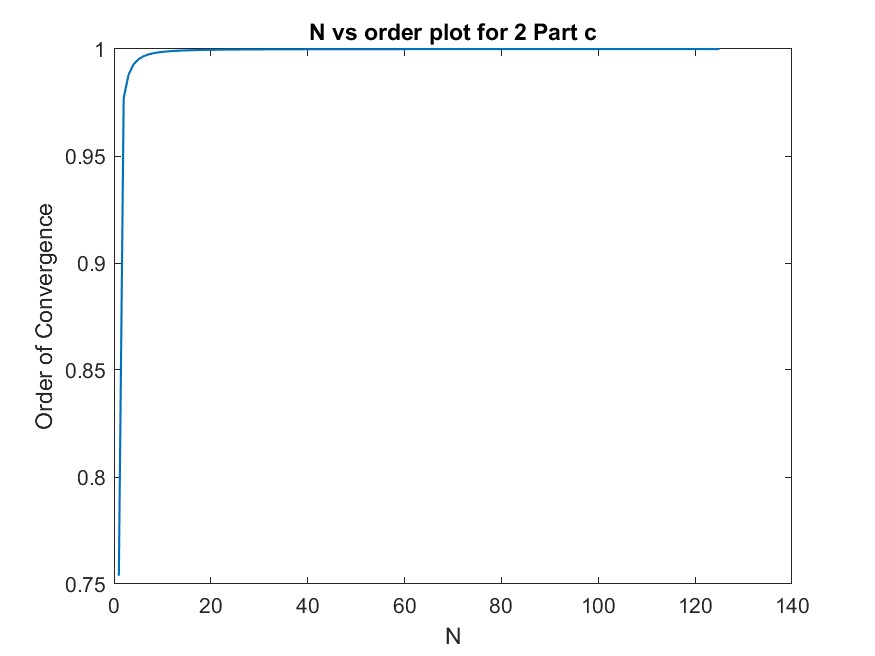
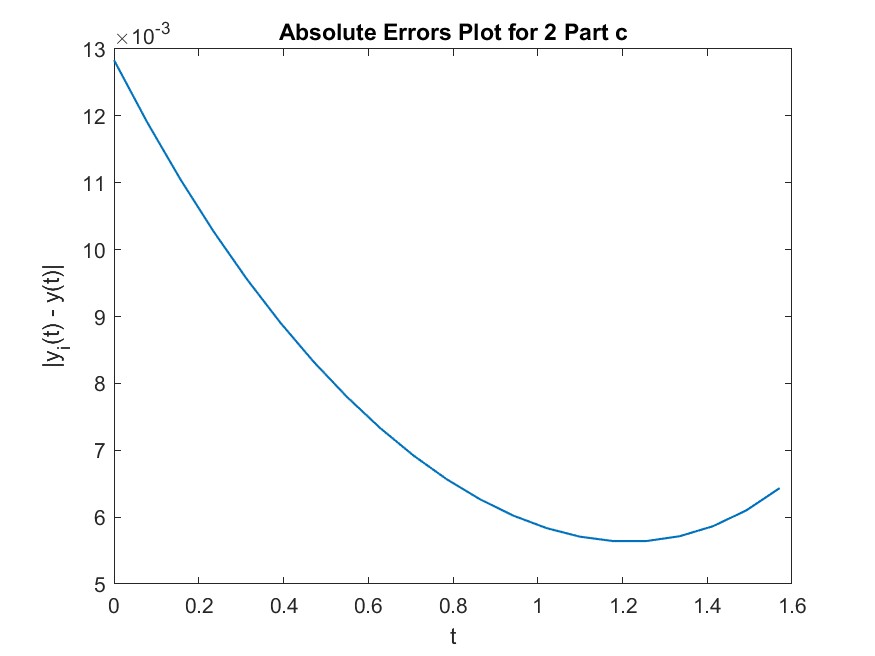
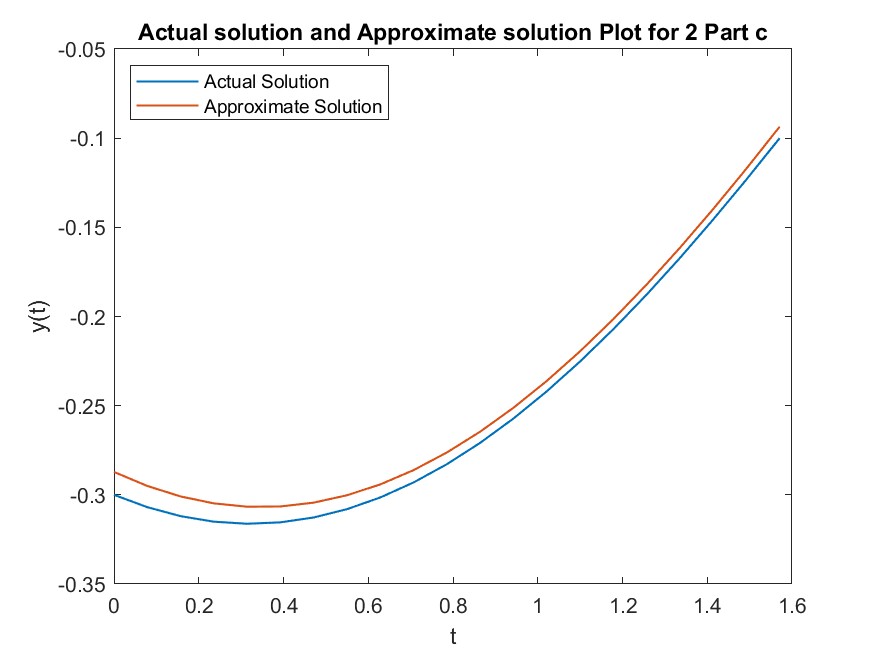


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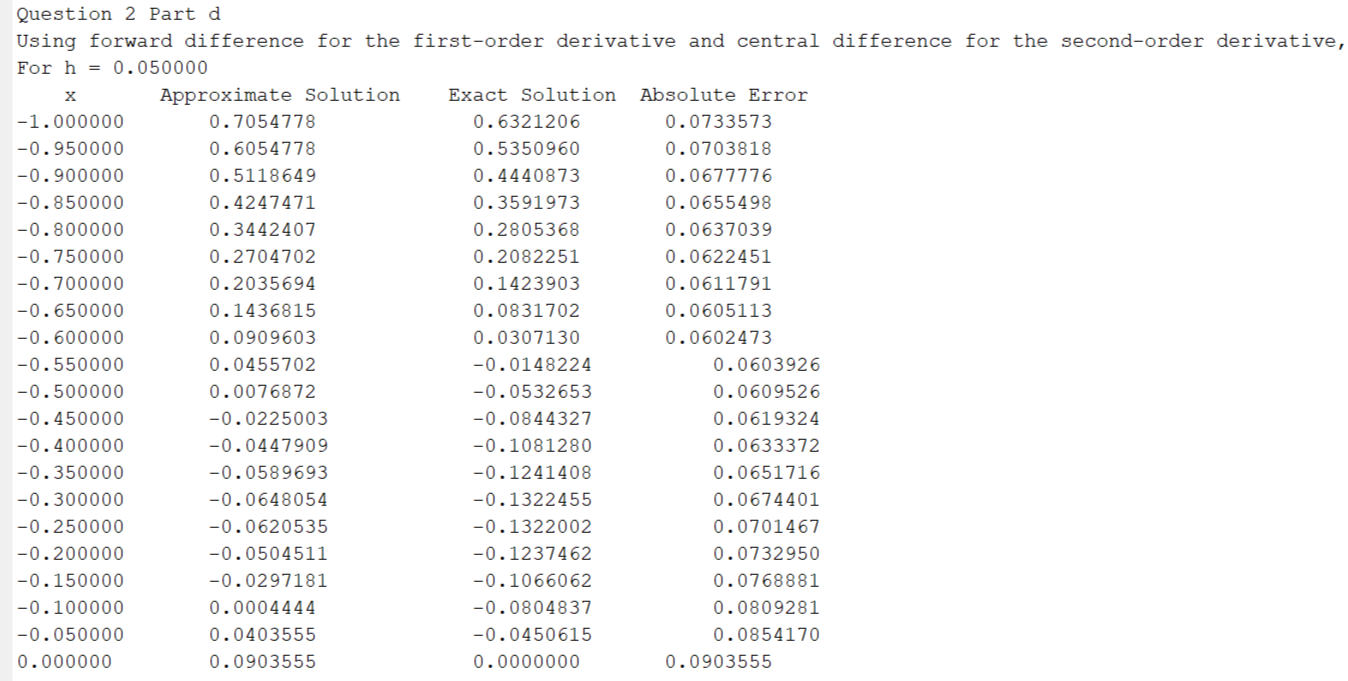


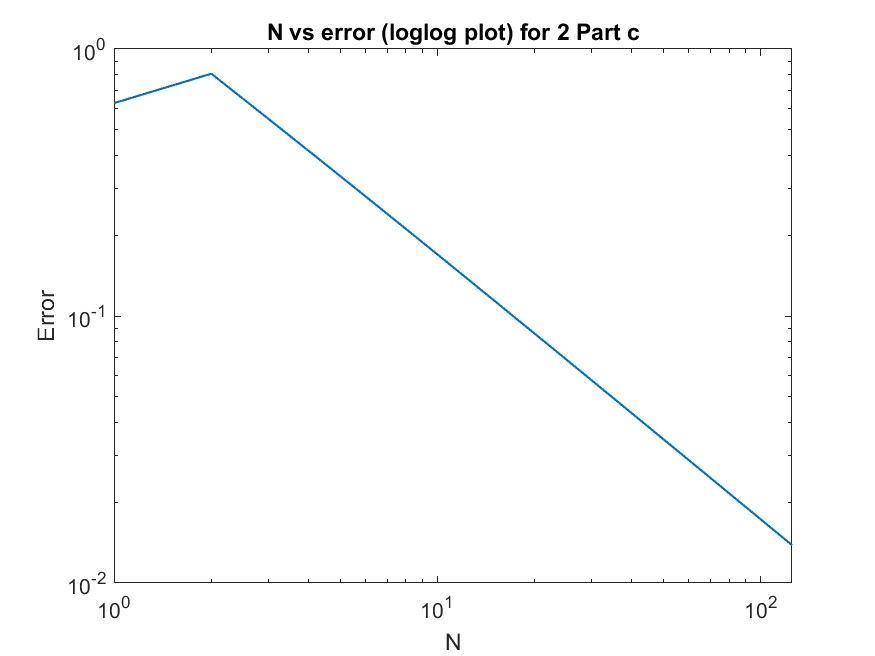
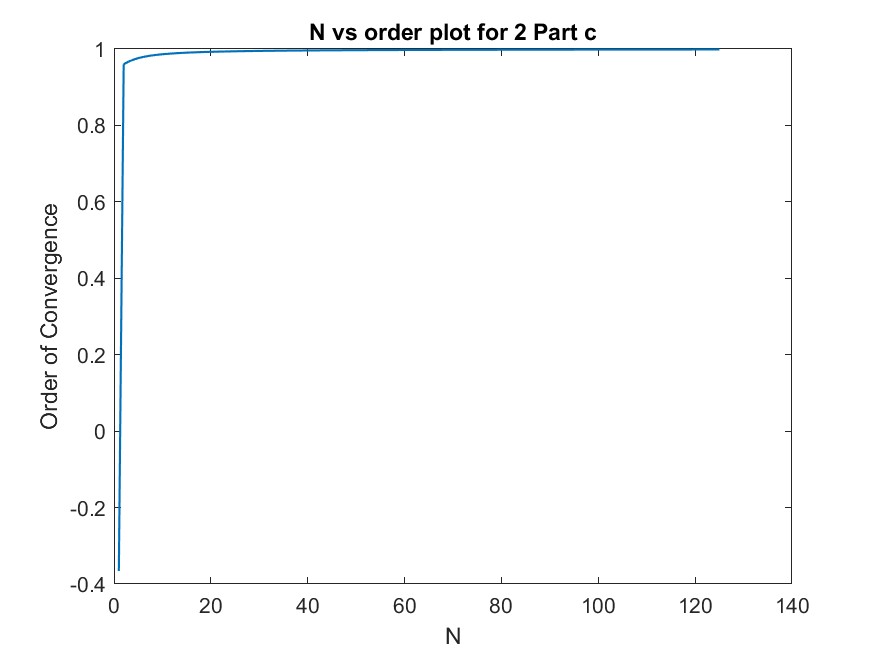
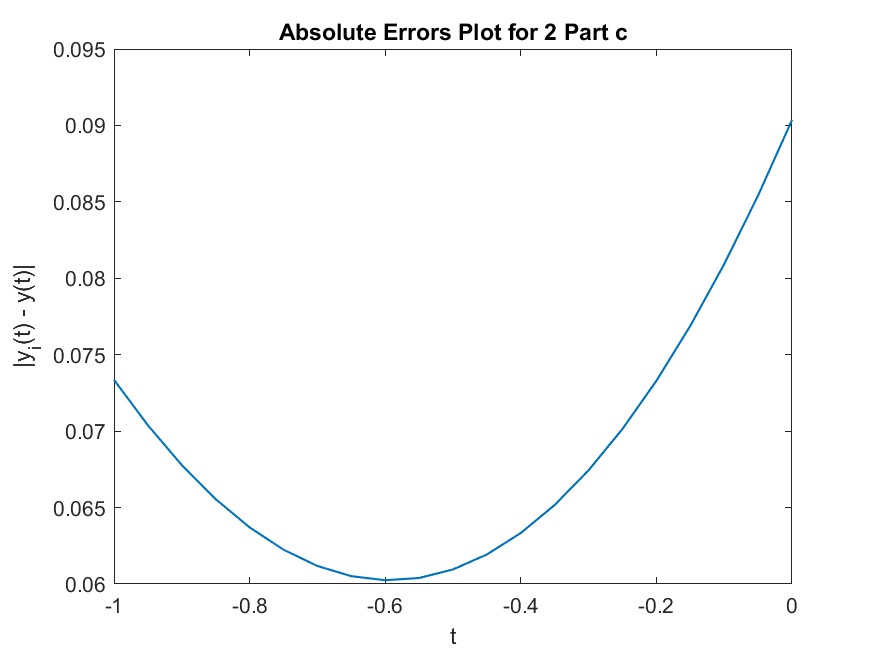
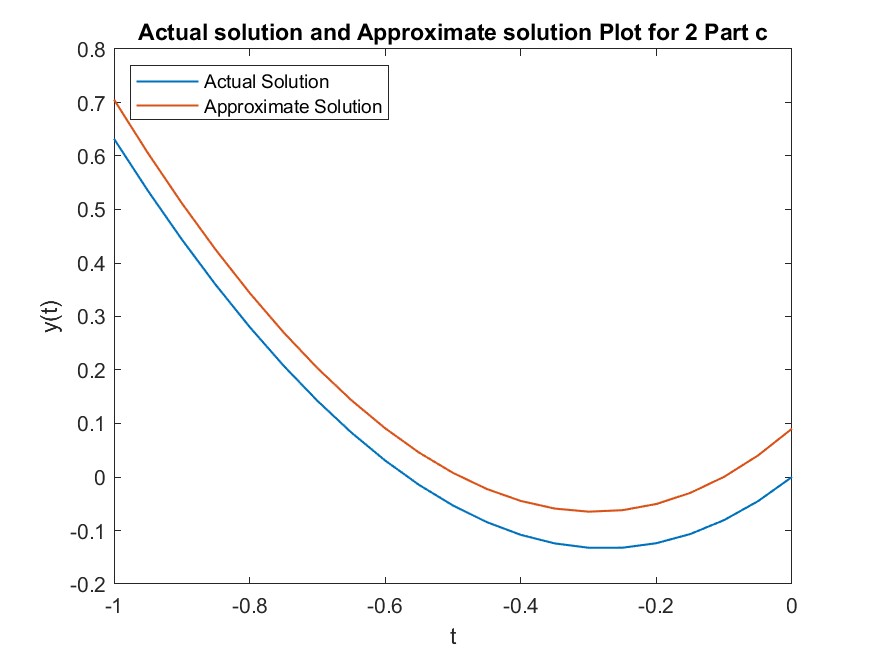


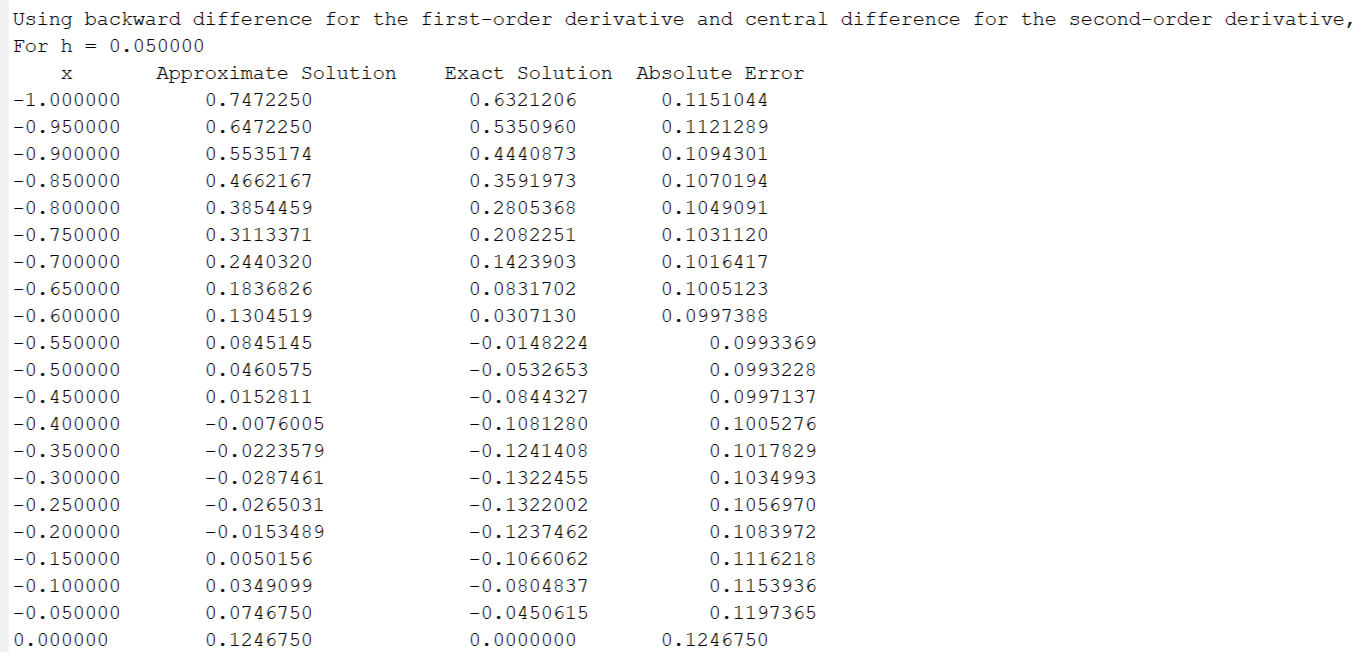


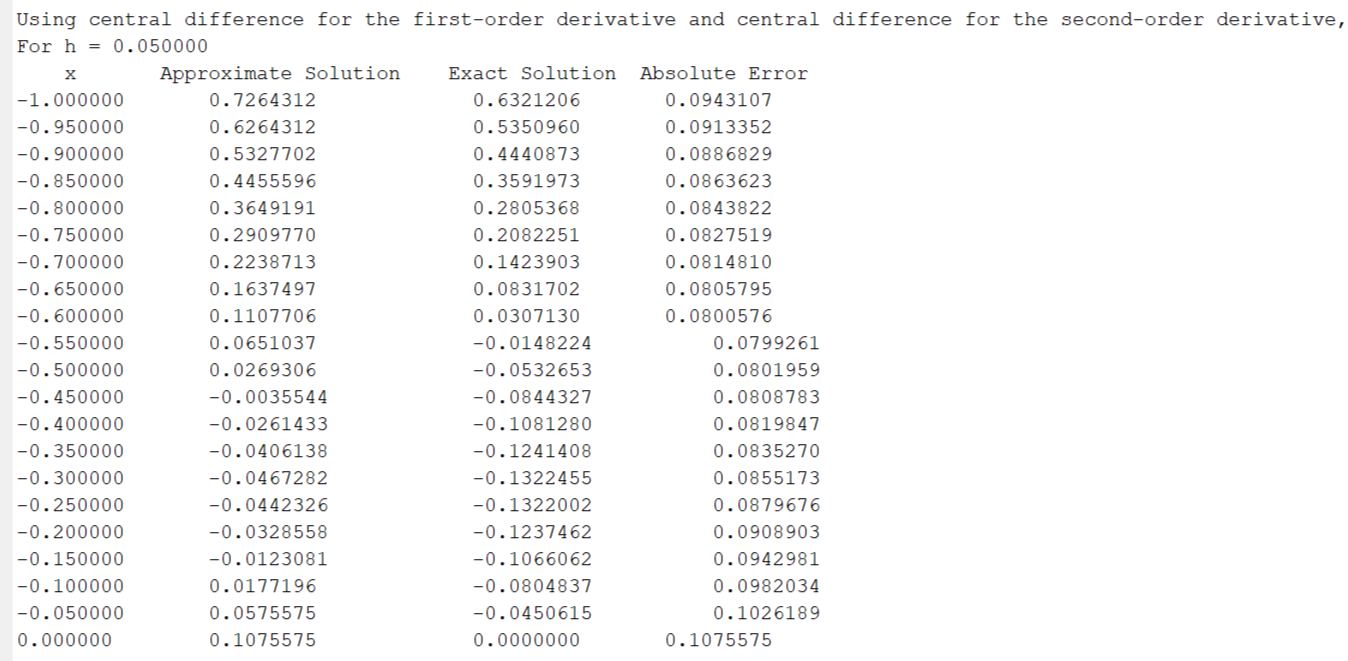
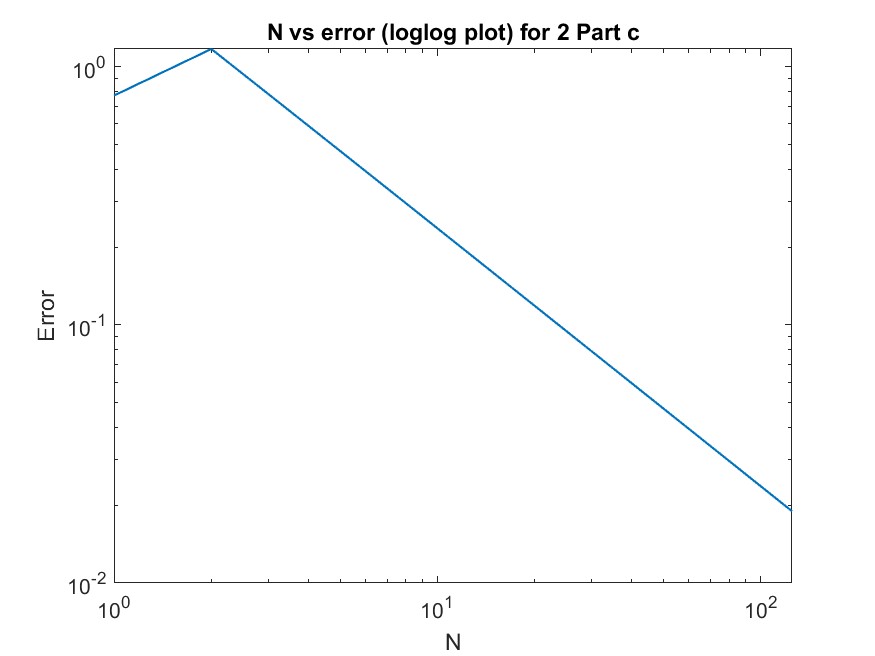
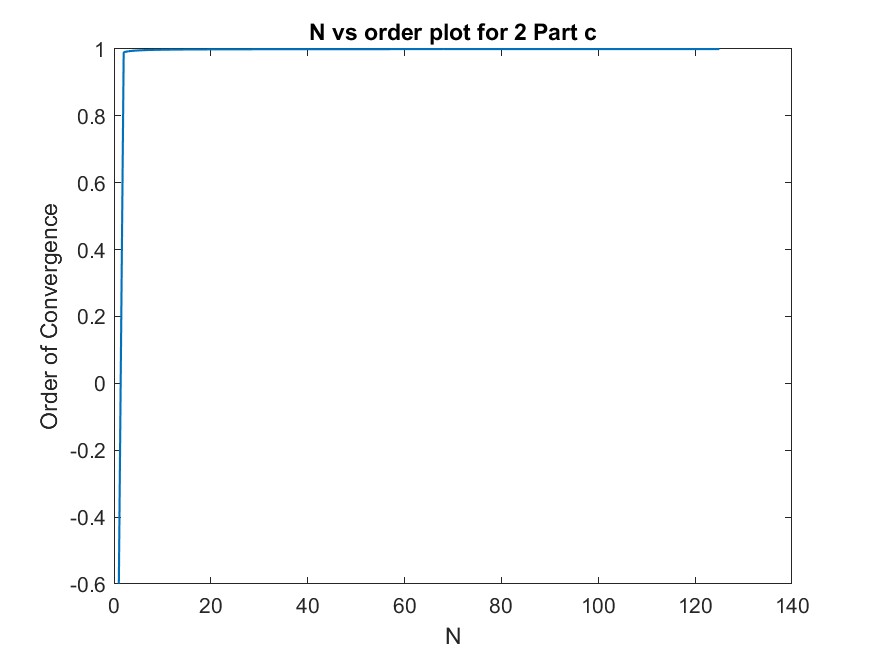
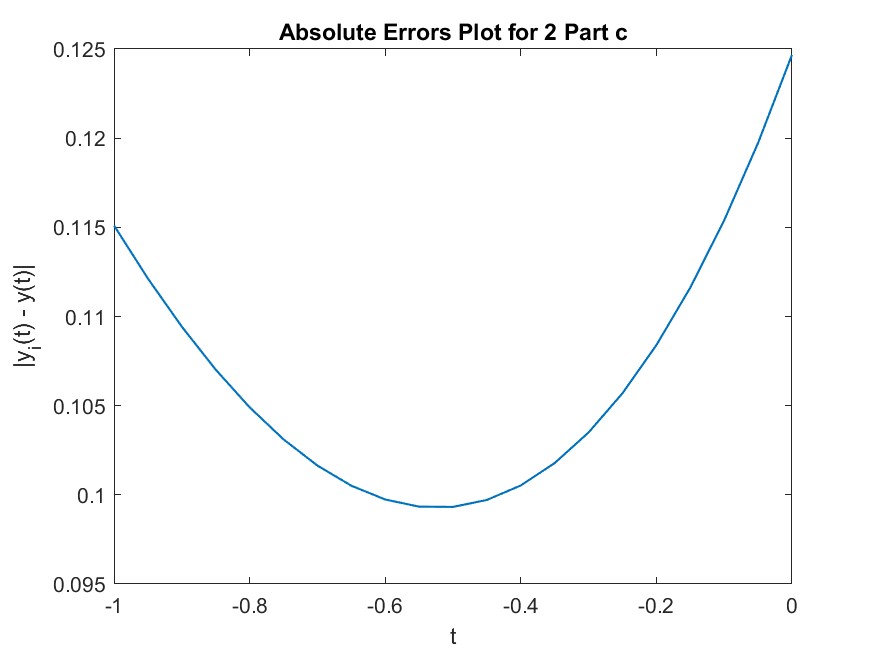
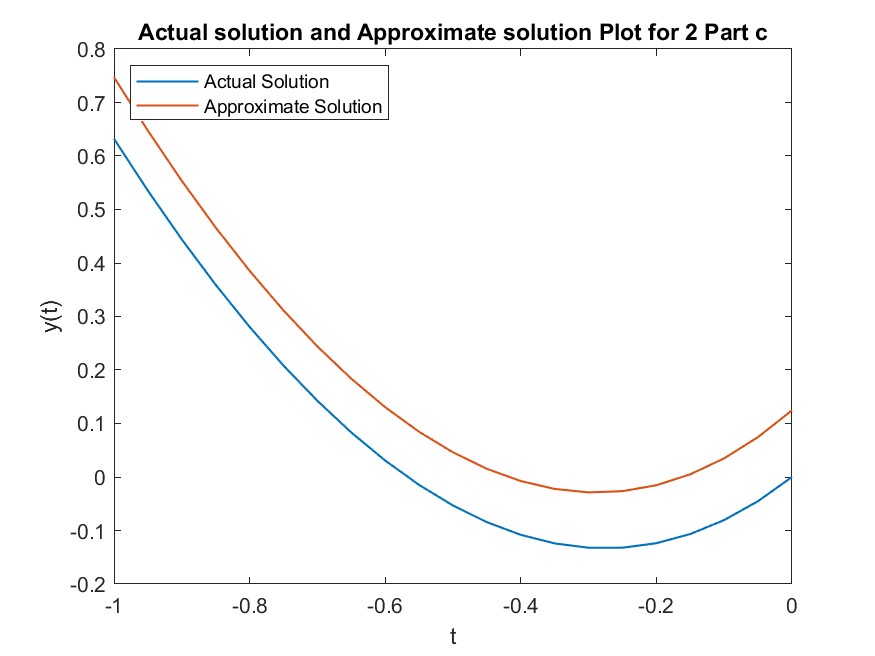


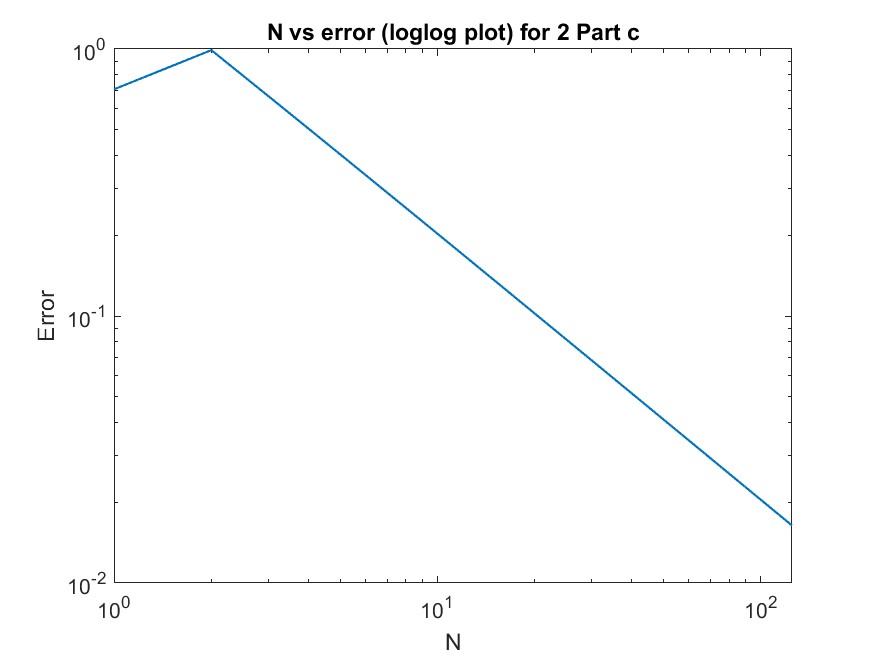
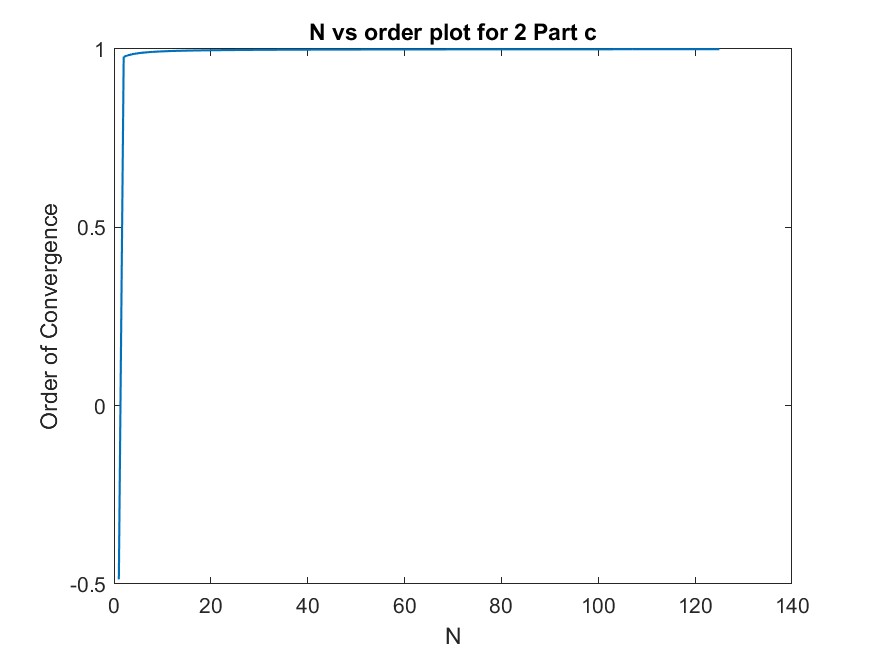
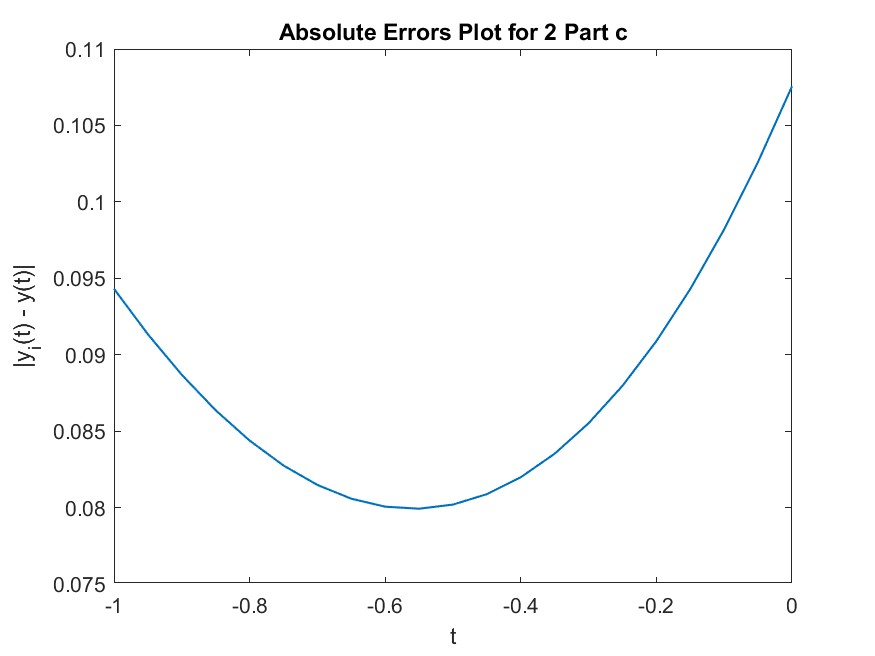
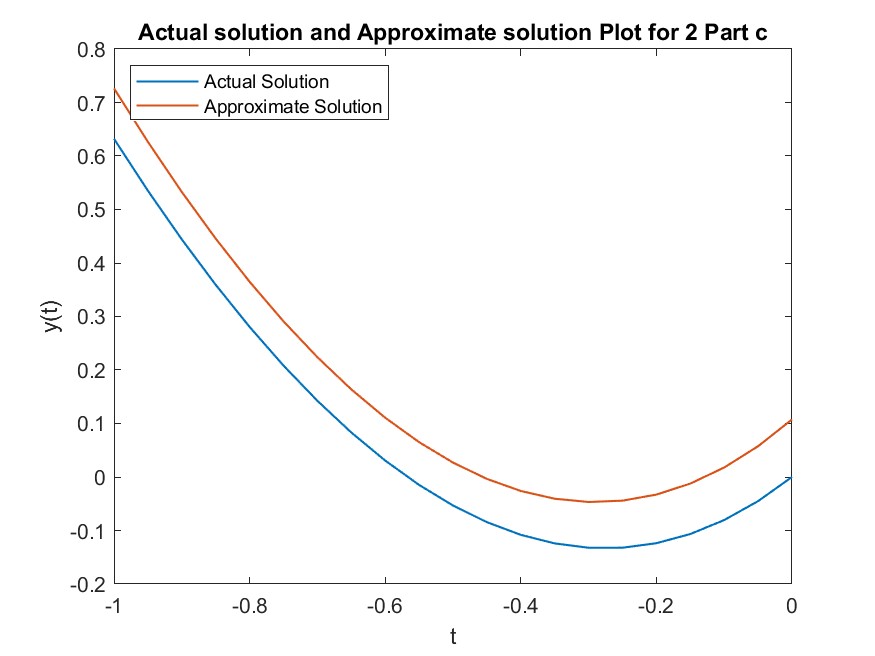
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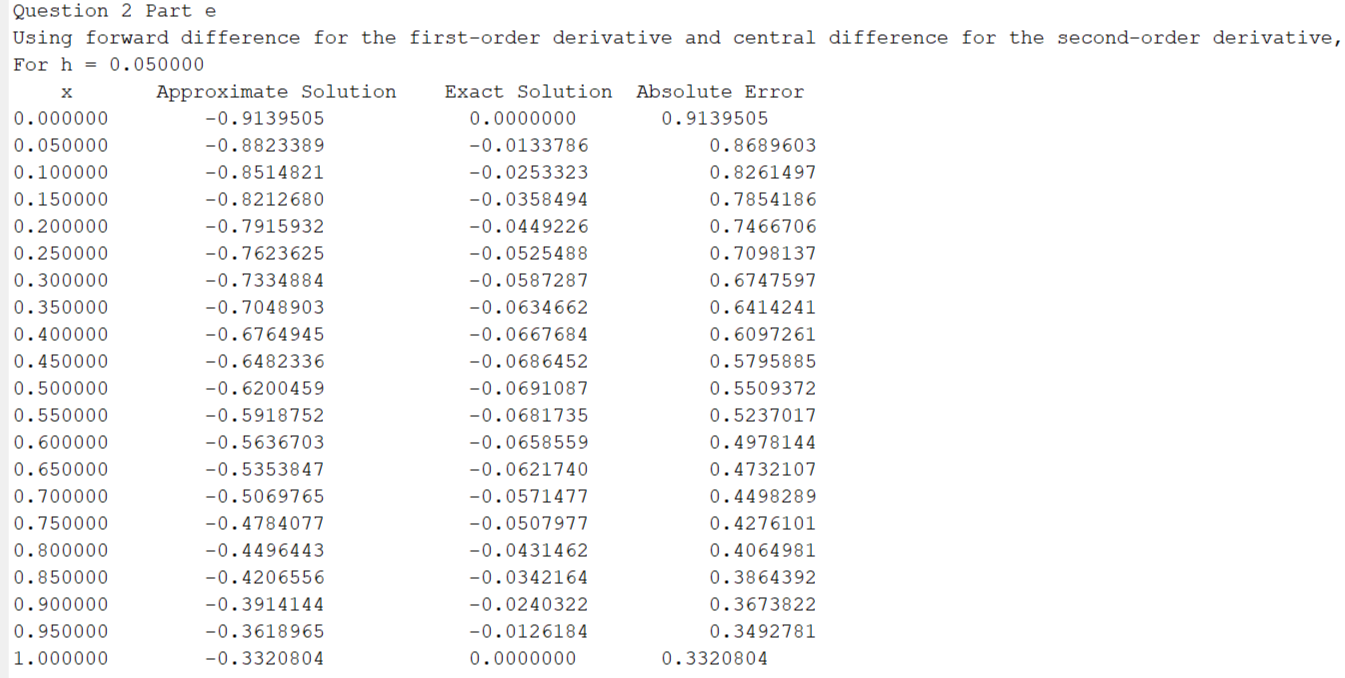


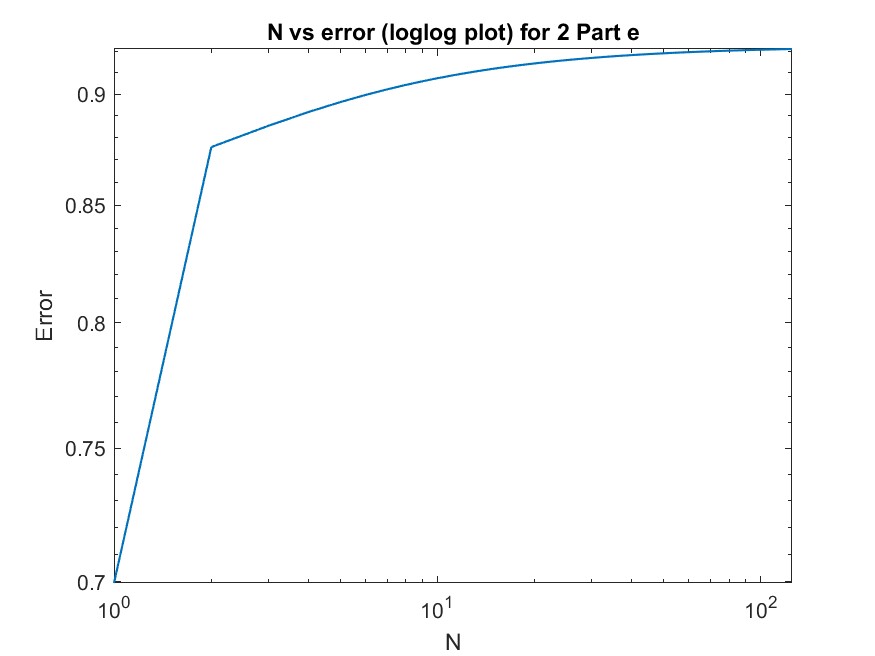
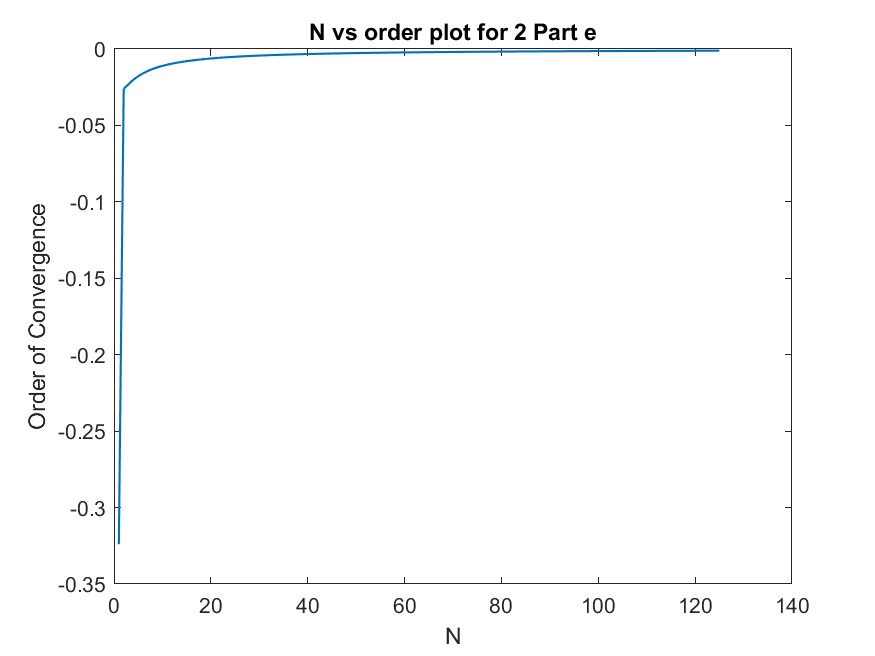
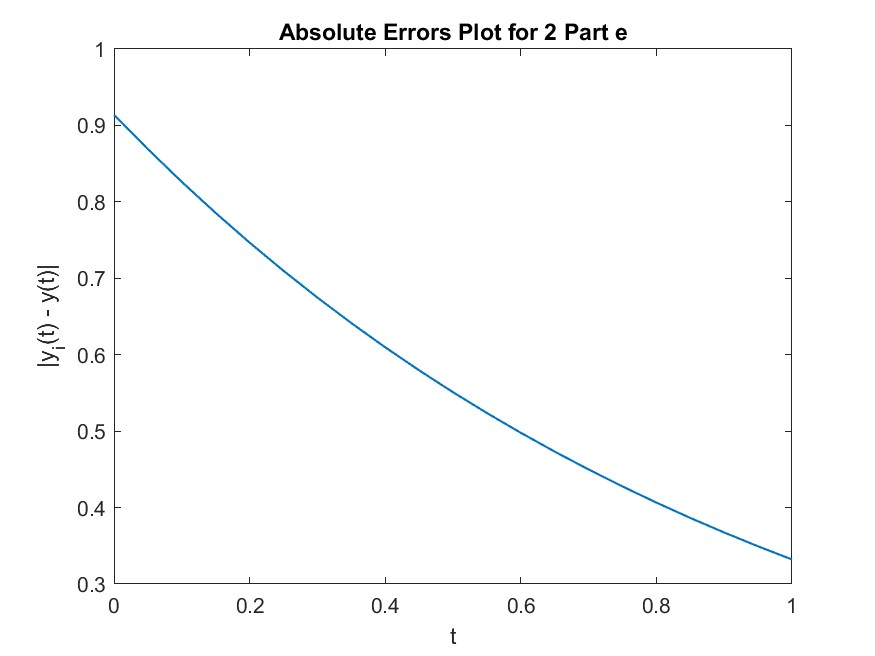
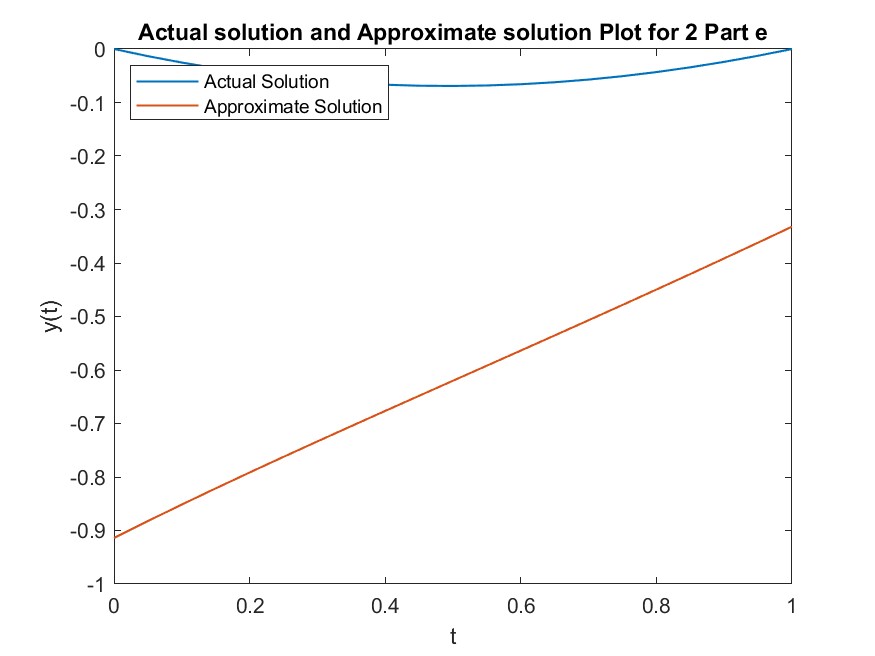


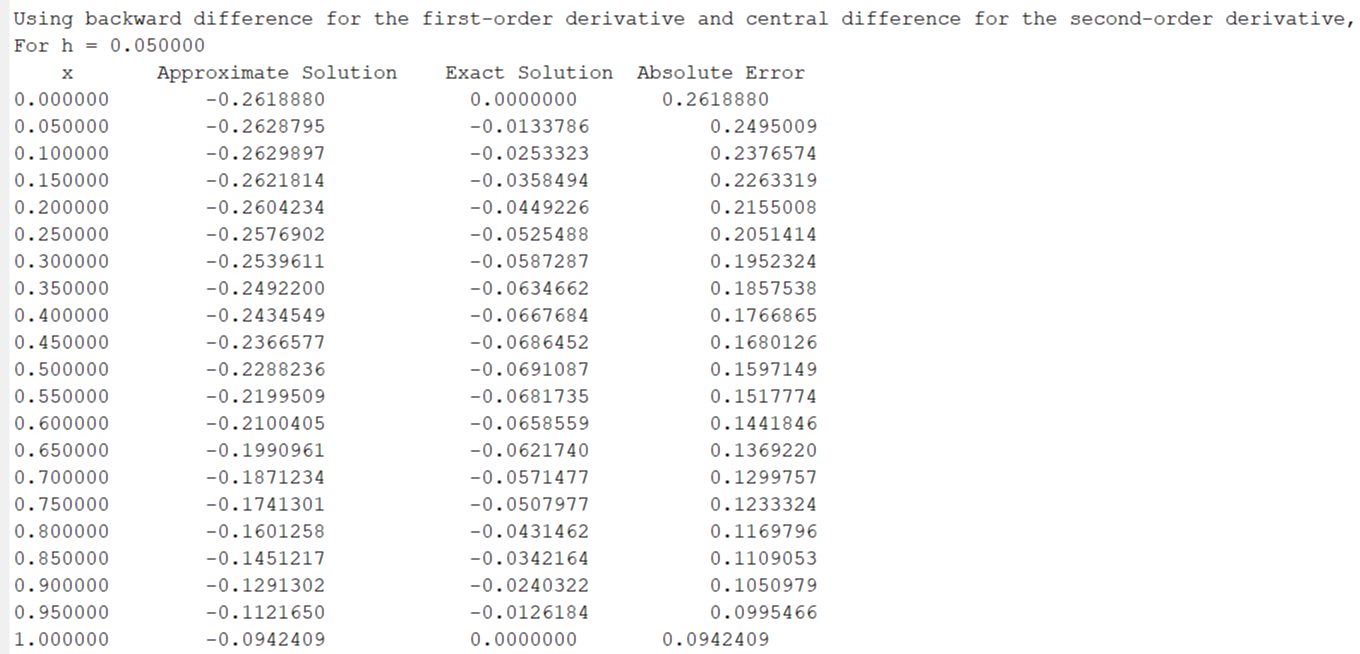


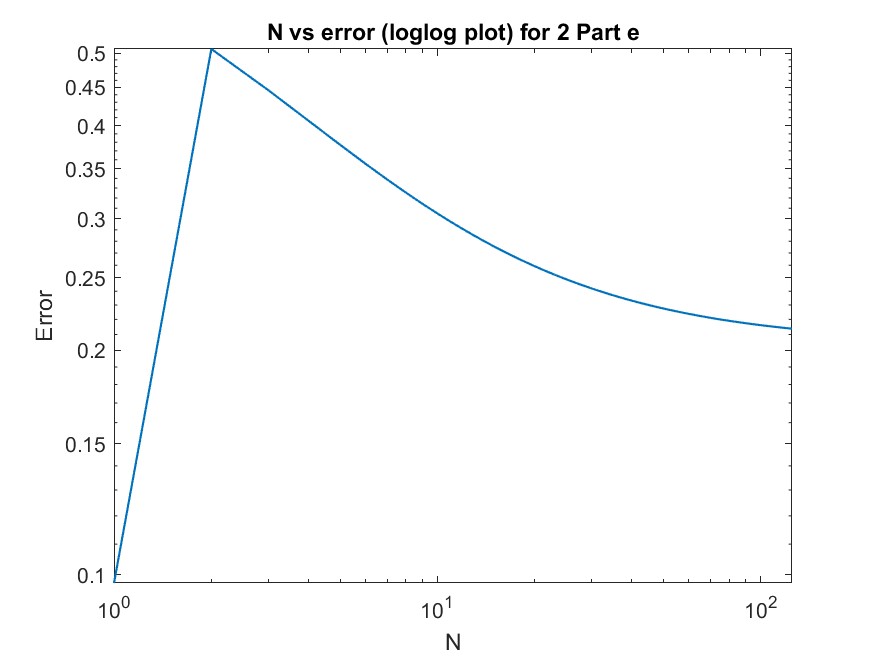
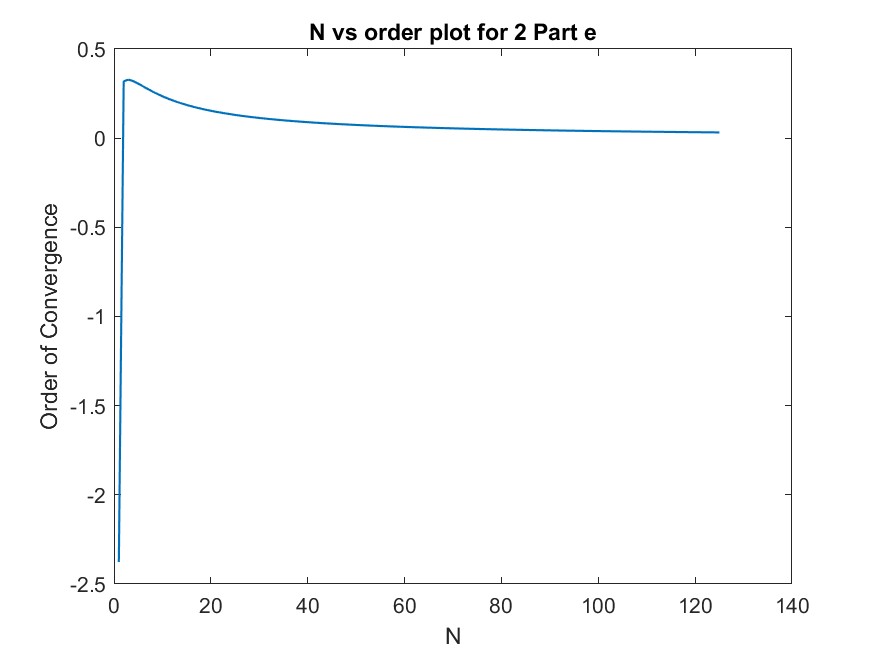
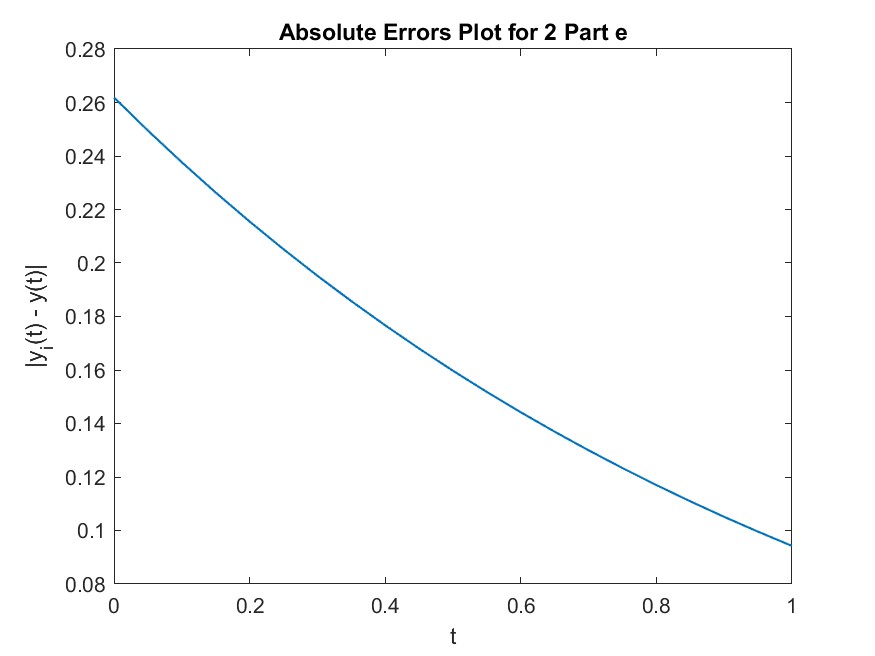
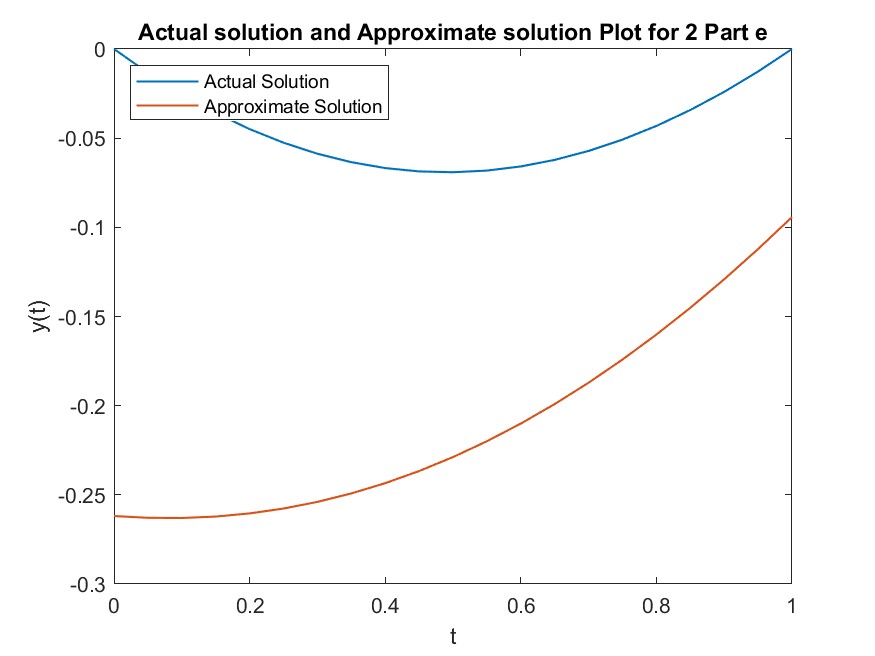


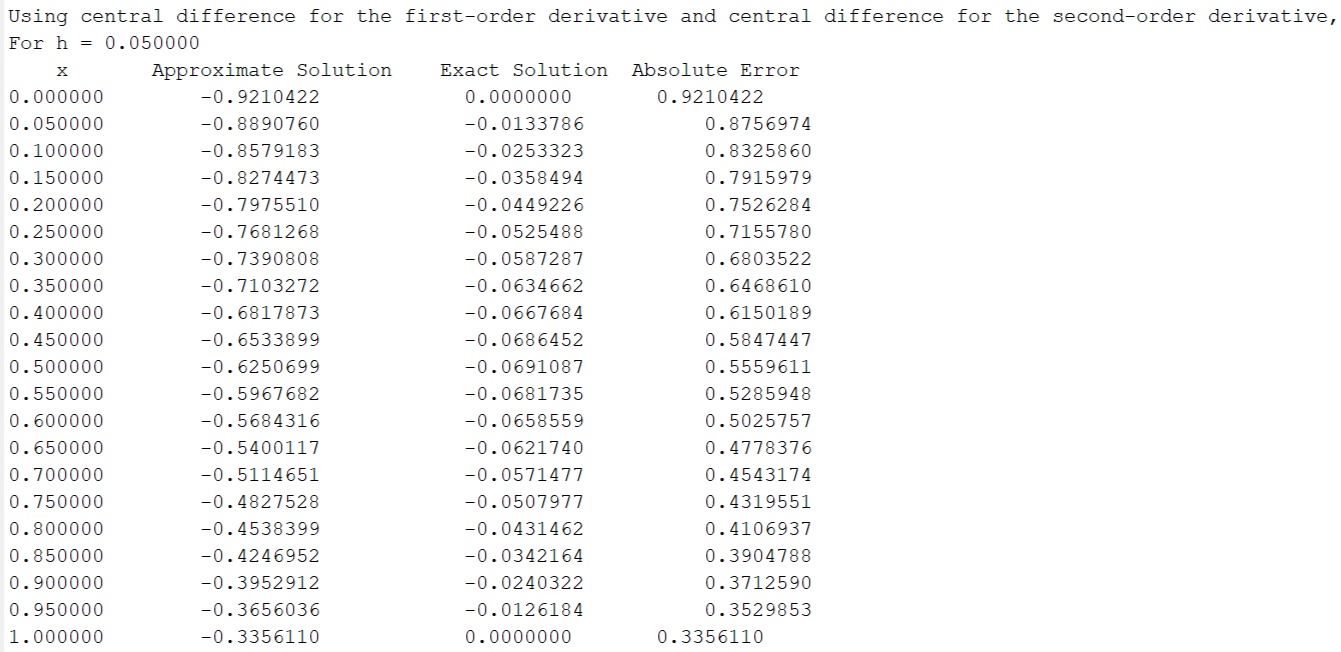
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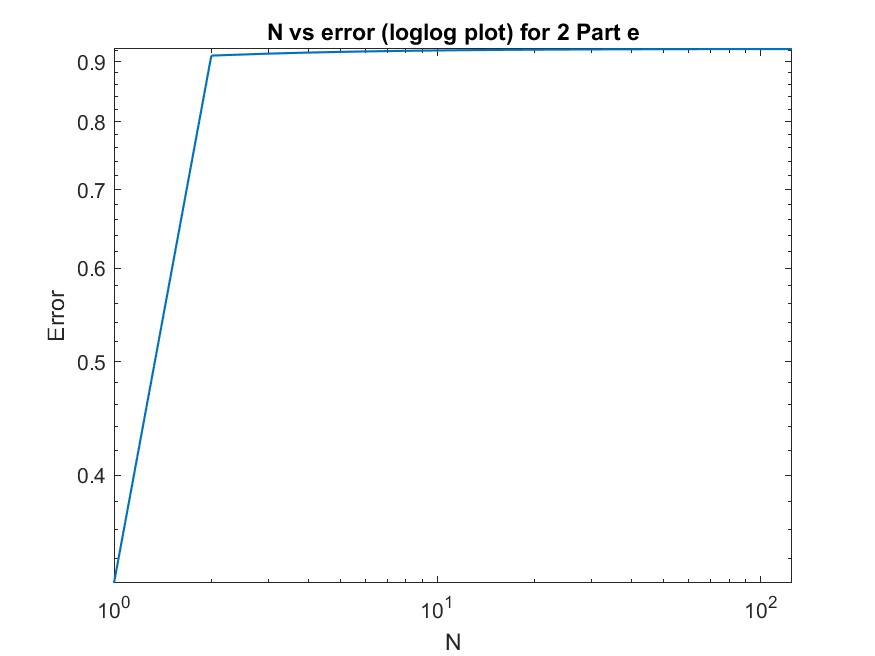
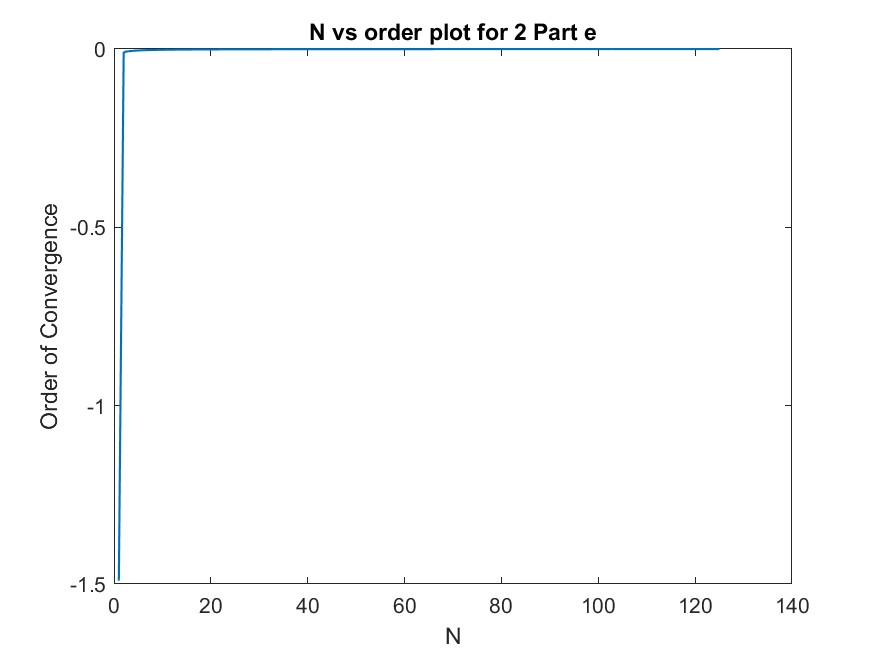
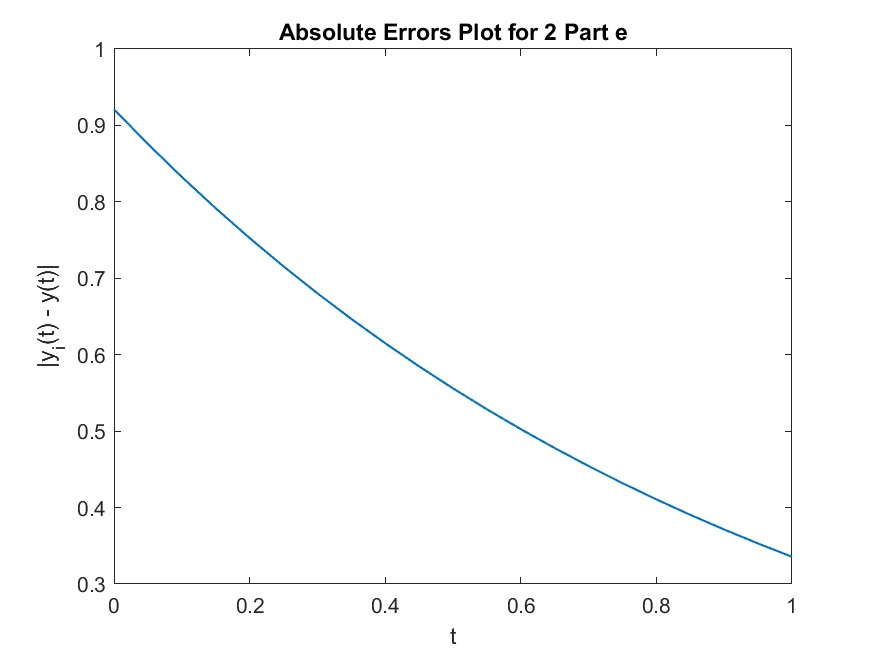
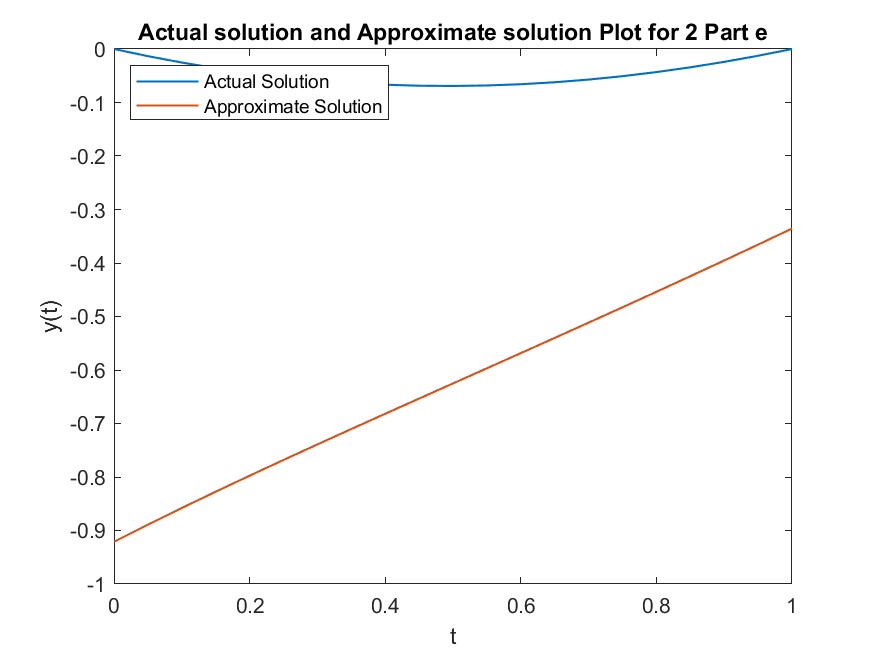




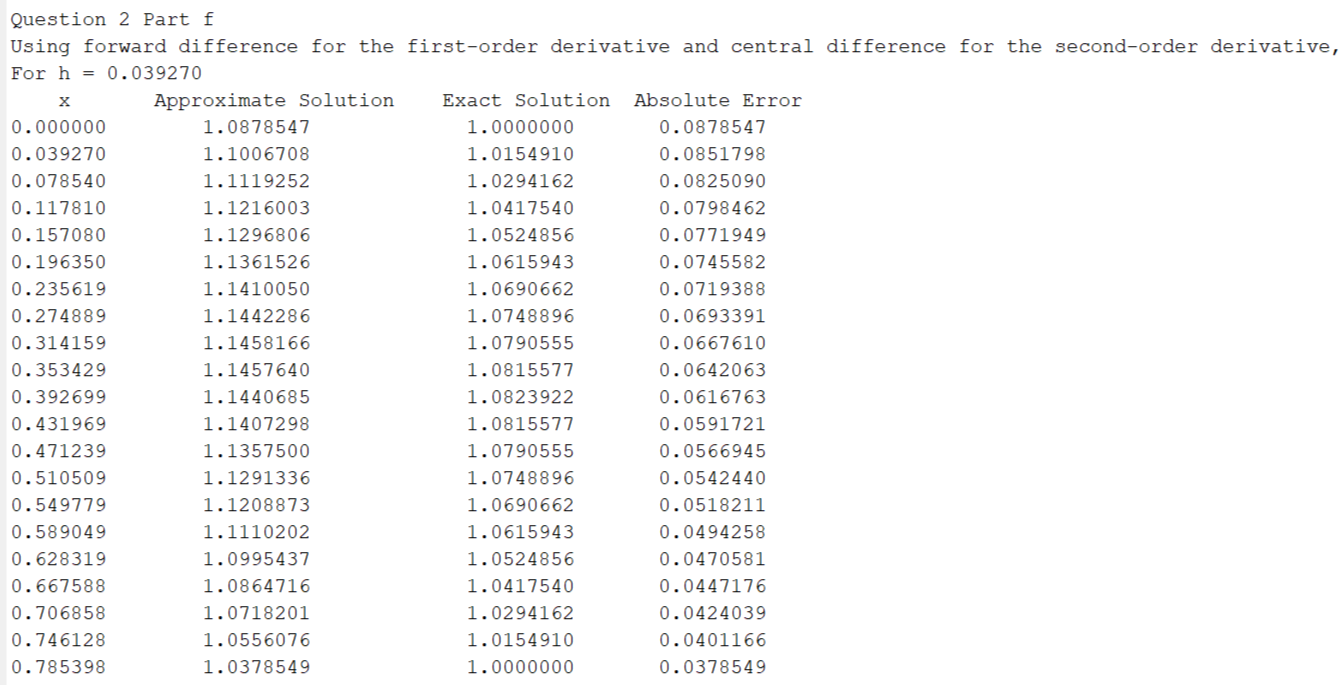


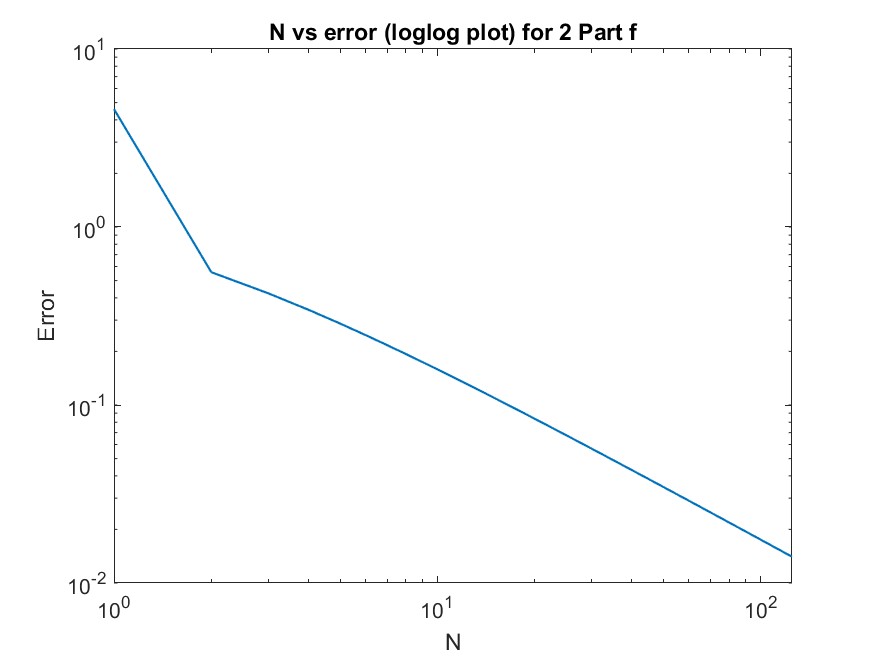
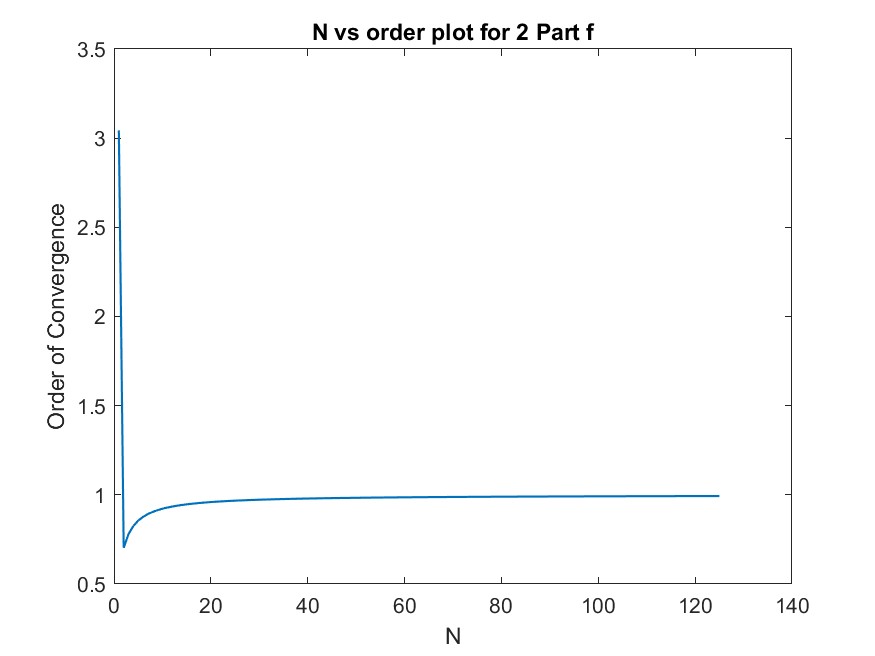
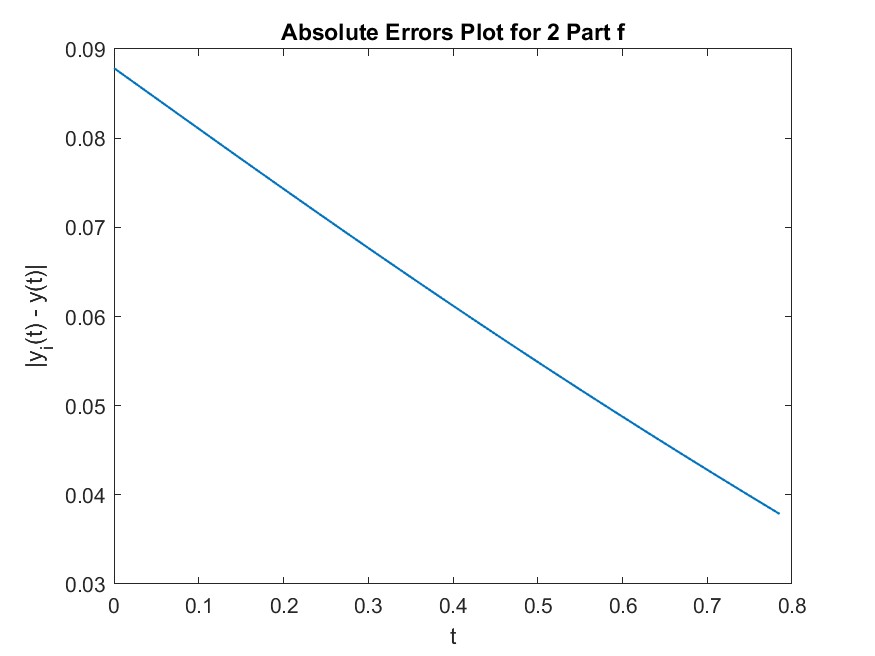
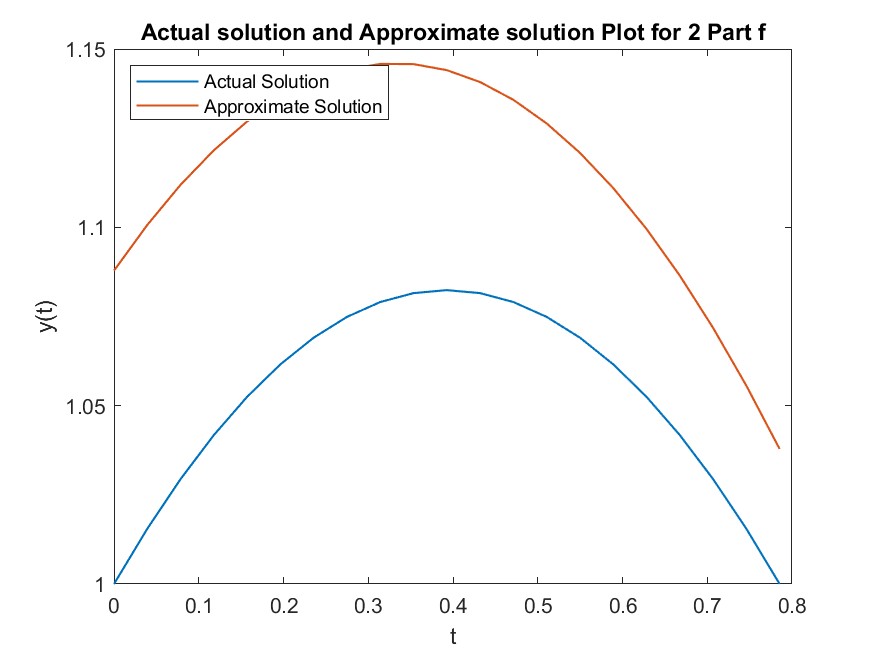


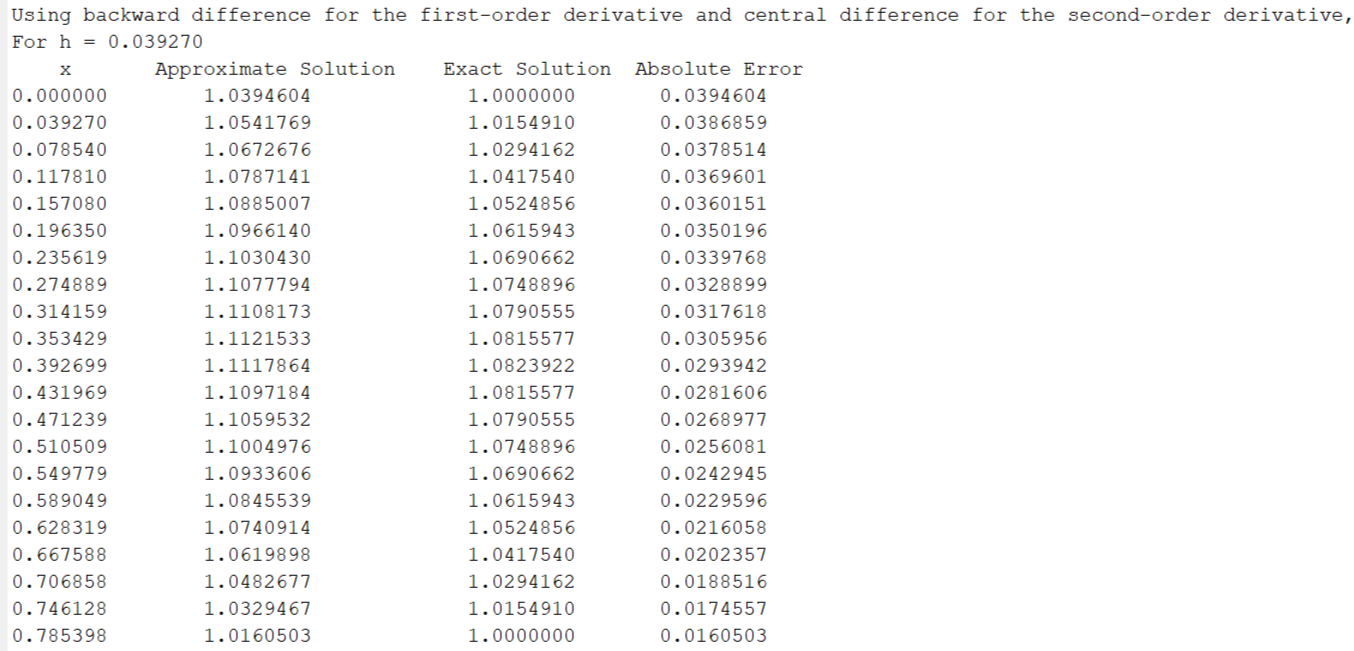


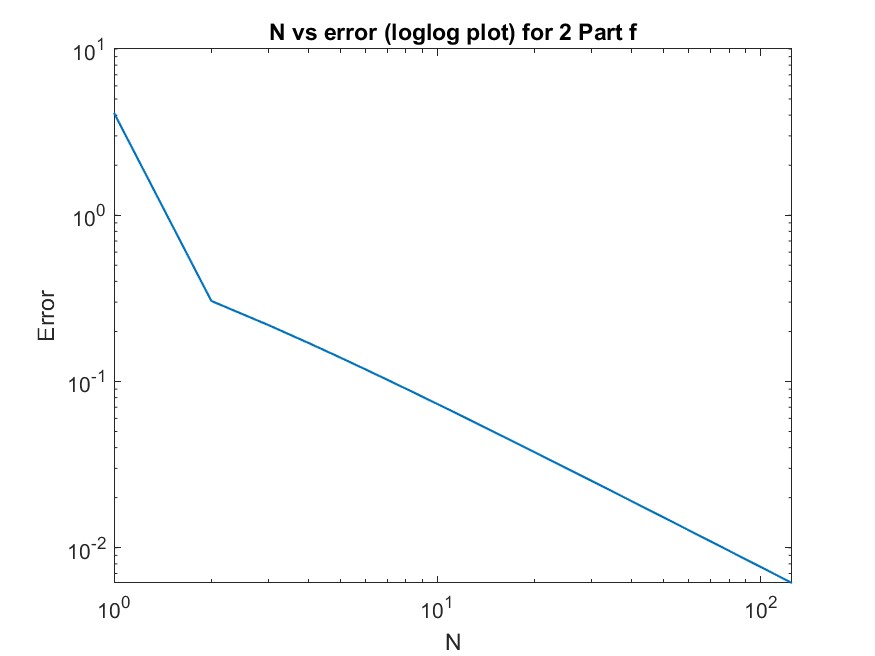
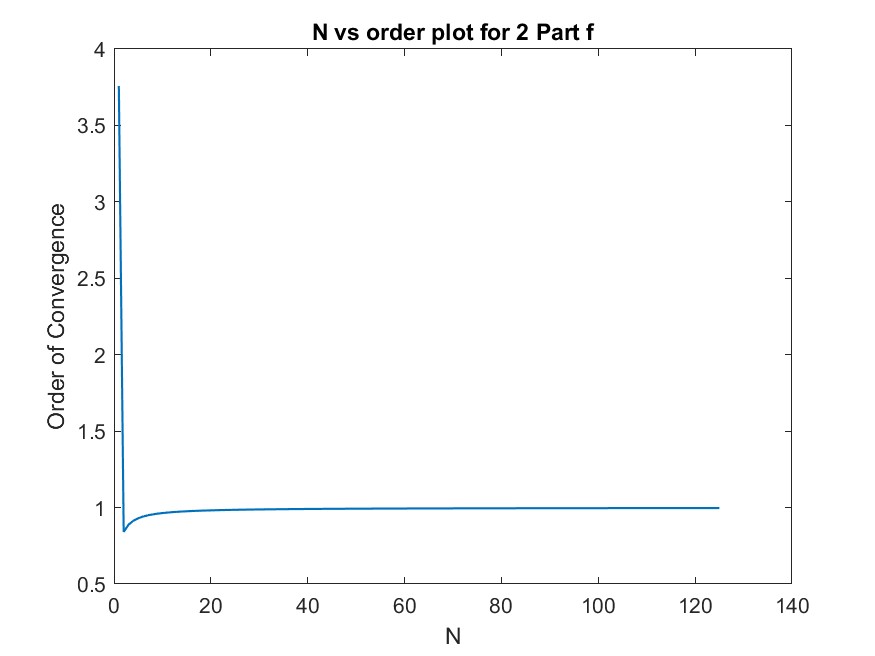
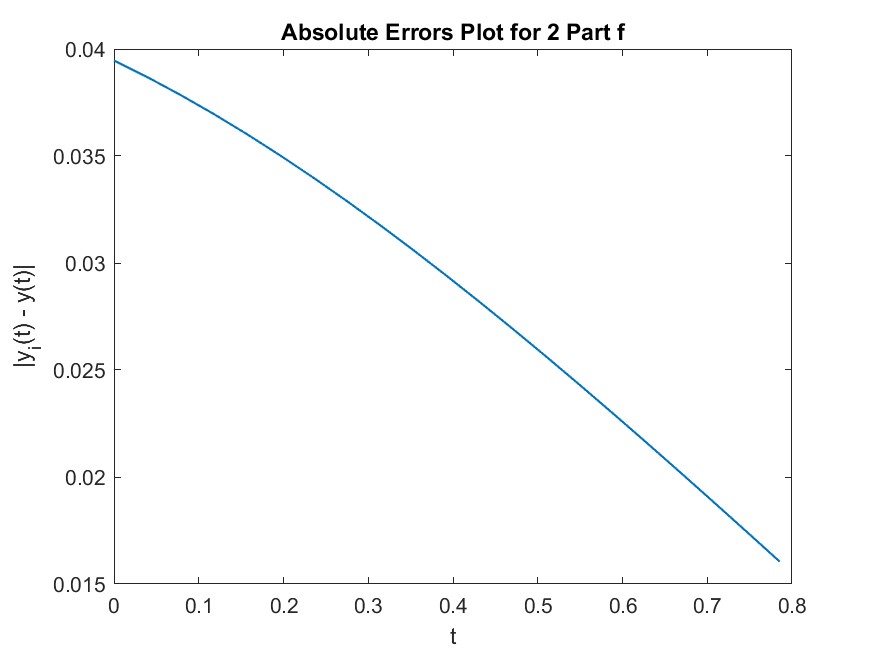
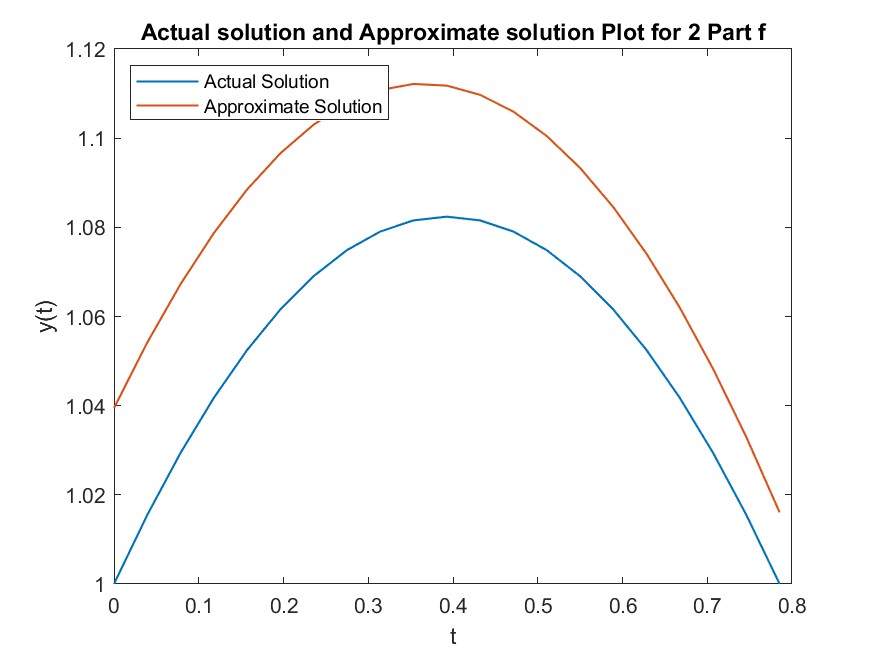


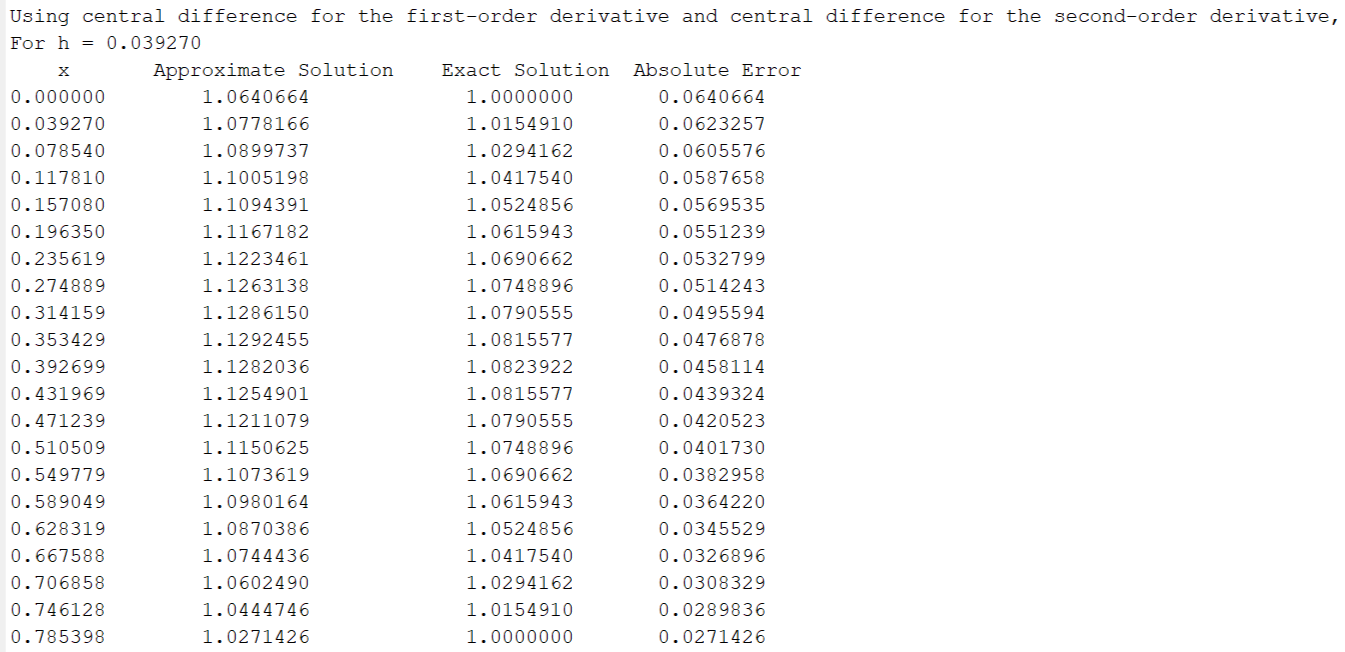
f)

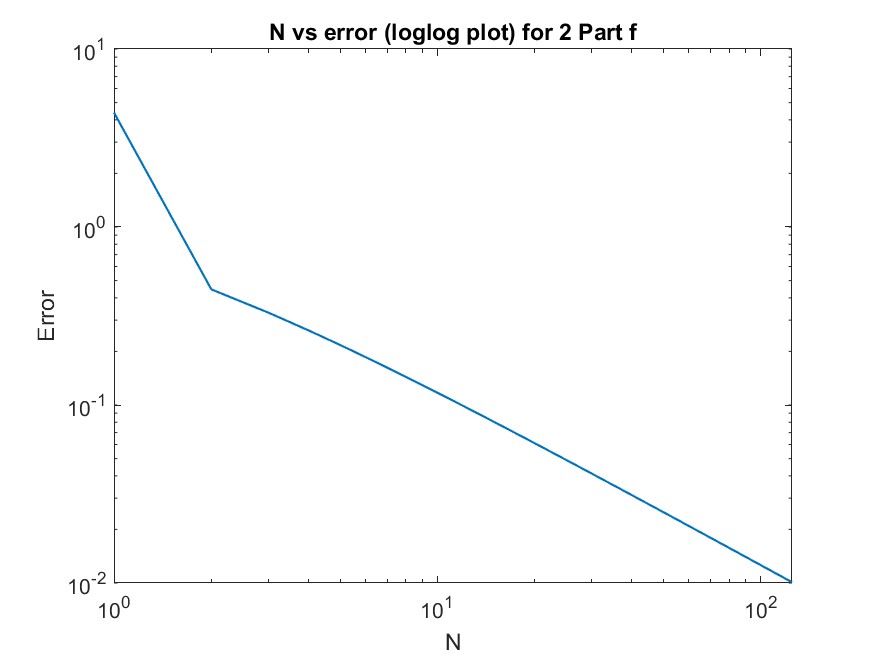
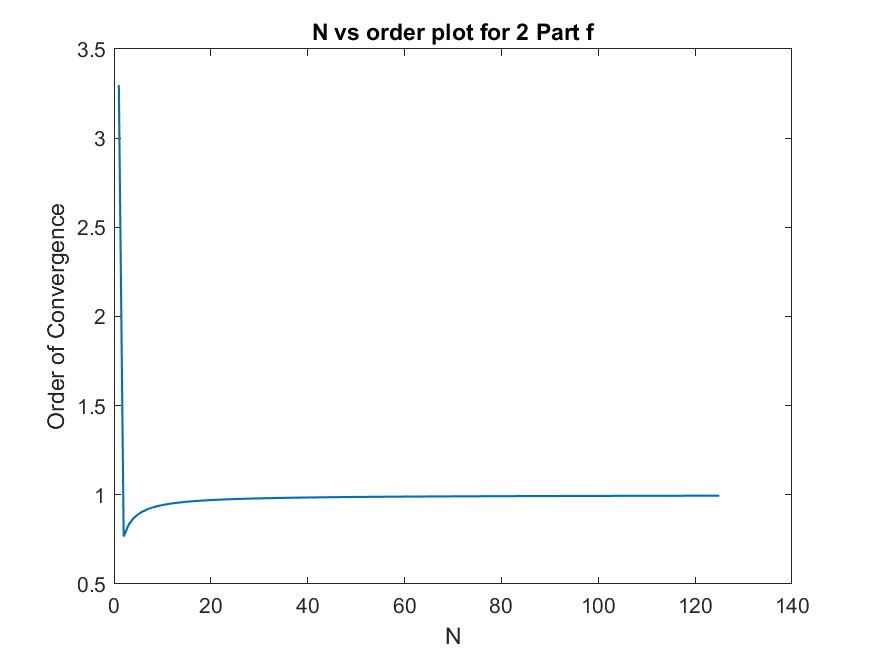
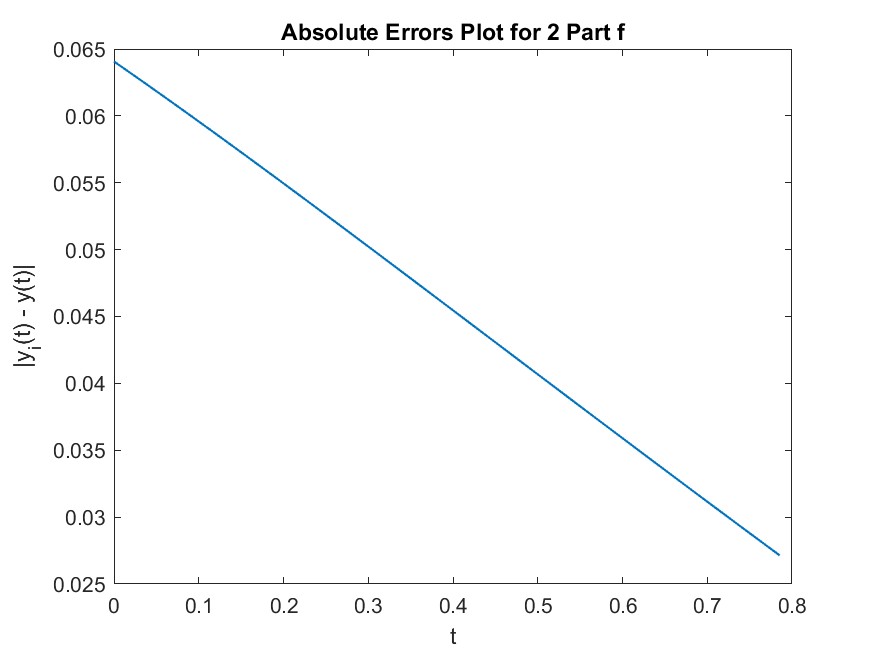
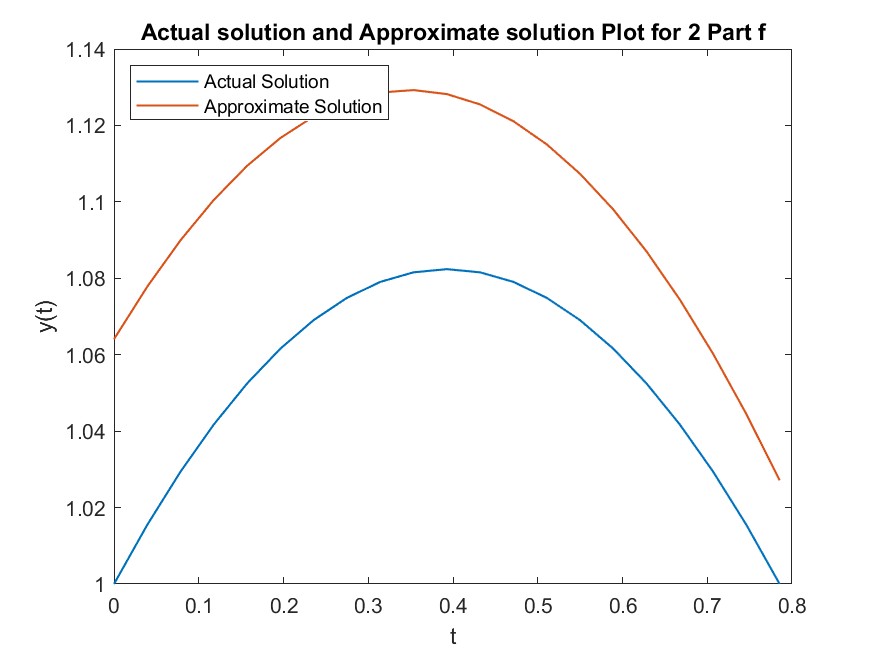












Observations:

* In part (e), the provided Boundary Value Problem (BVP) fails to converge for any of the three methods. This failure is evident in the plot of N versus order, where the convergence order diminishes towards zero as N, the number of discretization points, increases. Additionally, the log-log plot remains flat instead of exhibiting the typical decreasing trend observed in parts where the BVP converges with a discernible order based on the chosen method.
* The order of convergence of method 1(Using forward difference for first derivative) and method 2(Using backward difference for first derivative) is 1 for each boundary condition.
* The order of convergence of method 3(Using central difference for first derivative) is 2 for Dirichlet’s conditions, while the order of convergence is 1 for Neumann and Mixed/Robin conditions.