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2022 - 02 - 08

Lab 2, SFWR ENG / MECHTRON 3DX4

Empirical Estimation of Transfer Functions for First Order Systems

PRELAB EXCERCISES

Prelabs due the week of: Feb. 7, 2022

In Lab 1, the model we used for our DC motor was derived theoretically from first principles. Such a derivation is not possible in all cases, so in Lab 2 we will be finding the transfer function modelling our physical plant using experimental methodology, and comparing our approximation to the system's actual behaviour.

Once again, our first order approximation of the transfer function of a DC electric motor with respect to angular velocity is:

$$G_\omega(s) = \frac{\Omega(s)}{V(s)} = \frac{A}{\tau s + 1}$$

where A and τ are positive, real valued constants. $\Omega(s)$ and $V(s)$ are angular velocity and voltage as functions of s in the Laplace domain. (Note: Ω is capital ω so we are using $\Omega(s) := \mathcal{L}\{\omega(t)\}$.)

1. We will now develop formula for the motor DC gain constant A in terms of a step input change ΔV and step output change $\Delta\omega$.

   a) Using the final value theorem, find an expression for the steady state value of $\omega(t)$ when a step input of amplitude V_x is applied. NOTE: $\tau > 0$ so the pole of $G_\omega(s)$ is at $s = -\frac{1}{\tau}$ is in the open Left Half Plane (LHP) so the system is stable! Therefore we can use the final value theorem.

   b) Give the expression for $\omega(t)$ in response to a step input V_x at time $t = 0$. Assume a nonzero initial condition for $\omega(t)$ - i.e. $\omega(0) = \omega_0$. Note that the response due to a non zero initial condition ω_0 can be modeled as the response due to input $v(t) = \omega_0\delta(t)$ where $\delta(t)$ in the impulse function. Since $G_\omega(s)$ is a linear system, the response to step input V_x with nonzero initial condition ω_0 is just the sum of the responses due to the step and the initial condition.

   c) For $\omega(t)$ you computed in (1b), what is the $\lim_{t\to\infty}\omega(t)$? How does it compare your answer from 1a?

   d) Now assume that you run the motor with an initial step input of V_{min} until time t_0 when the system has reached steady state before the step input is changed to V_{max} at time t_0. In other words, your system input will take the form

   $$v(t) = \begin{cases} V_{max}, & t \geq t_0 \\ V_{min}, & 0 \leq t < t_0 \end{cases} \tag{1}$$

   where $t_0 >> \tau$, and V_{min} and V_{max} may be non-zero.
   Use the results above to show that:

   $$A = \frac{\Delta\omega}{\Delta V}$$

   where

- $\Delta V := V_{max} - V_{min}$
- $\Delta\omega = \omega_{ss} - \omega_0$ where ω_0 is the steady state response to a step input V_{min} and ω_{ss} is the steady state step response to input (1).

2. Using the formula derived in question 1, use the following graphs to calculate A for this system.

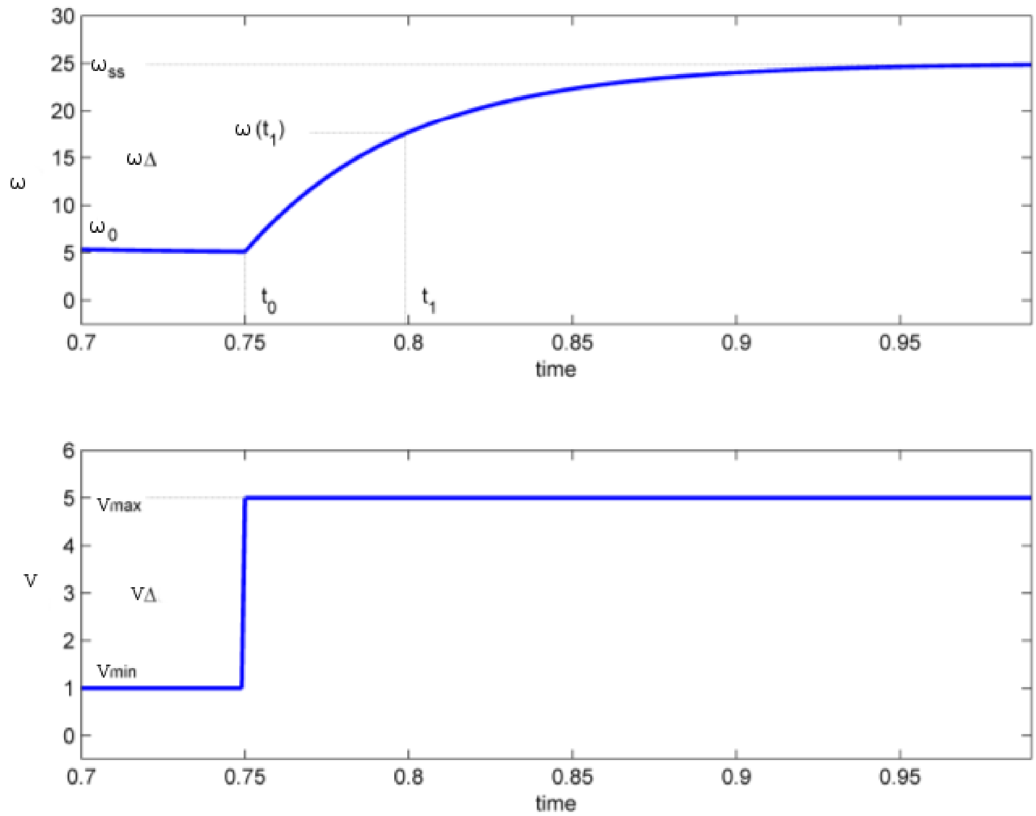

Figure 1: A First Order Response to Step Input

3. For a first order system, the time it takes a step response to reach 63.2% of its steady state value ($t_1 - t_0$ in Fig. 1) is the response's time constant τ. i.e., at time t_1, $\omega(t_1) = 0.632\Delta\omega + \omega_0$. Find the time constant τ for the above system in Fig. 1.

4. Using A and τ, give the transfer function $G_\omega(s)$ in terms of s.

5. Complete problem 21a) in Chapter 4 of Control Systems Engineering 8th ed. (problem 32(a) in Chapter 4 of Control Systems Engineering 7th ed)

6. In one or two paragraphs, Describe a situation in which it is preferable to derive a transfer function experimentally, and then design/calibrate a controller

using simulation software, rather than experimenting with the plant to design/calibrate the controller directly. Describe a situation in which this is not preferable. (Answer in complete sentences!)

Prelab 1 Answers

2022 - 02 - 08

↳ all work in this prelab has been done solely by Rashad Ahmed Bhuiyan, bhuiyr2, 400 263 180, [signature]

↳ 1. a) final value theorem: $f(\infty) = \lim_{s \to 0} s F(s)$

↳ $G_\omega(s) = \frac{\Omega(s)}{V(s)} = \frac{A}{\tau s + 1} \rightarrow \frac{\mathcal{L}\{\omega(t)\}}{V(s)} = \frac{A}{\tau s + 1}$

↳ for step input of amplitude V_x

↳ $V(s) = \frac{V_x}{s} \longrightarrow \mathcal{L}\{\omega(t)\} = \frac{V_x}{s} \cdot \frac{A}{\tau s + 1}$

↳ $\omega(\infty) = \lim_{s \to 0} \cancel{s} \cdot \frac{V_x}{\cancel{s}} \cdot \frac{A}{\tau s + 1}$

↳ $\boxed{\omega(\infty) = A V_x}$

b) given non-zero initial condition, ω_0 modelled as $v(t) = \omega_0 \delta(t)$

↳ $v(t) = \omega_0 \delta(t) + V_x \rightarrow V_s = \omega_0 + \frac{V_x}{s}$

↳ $\mathcal{L}\{\omega(t)\} = \frac{A}{\tau s + 1} \cdot \left[\omega_0 + \frac{V_x}{s}\right]$

↳ $\omega(t) = \mathcal{L}^{-1}\left\{\frac{A\omega_0}{\tau s + 1}\right\} + \mathcal{L}^{-1}\left\{\frac{AV_x}{s[\tau s + 1]}\right\}$

↳ $\omega(t) = \frac{A\omega_0}{\tau} \cdot \mathcal{L}^{-1}\left\{\frac{1}{s + 1/\tau}\right\} + \frac{AV_x}{\tau} \cdot \left[\mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{1}{s + 1/\tau}\right\}\right]$

↳ $\omega(t) = \frac{A\omega_0 e^{-t/\tau}}{\tau} + \frac{AV_x}{\tau} - \frac{AV_x e^{-t/\tau}}{\tau}$

↳ $\omega(t) = \left[A/\tau\, \omega_0 - A/\tau\, V_x\right] e^{-t/\tau} + A/\tau\, V_x$

↳ $\omega(0) = A/\tau\, \omega_0 - A/\tau\, V_x + A/\tau\, V_x \rightarrow \boxed{\omega(0) = \frac{A}{\tau}\omega_0}$

c) $\omega(\infty) = \left[\frac{A}{\tau}\omega_0 - \frac{A}{\tau}V_x\right] e^{-\infty/\tau} + \frac{A}{\tau}V_x$

$\boxed{\omega(\infty) = A/\tau\, \omega_0}$ → differs greatly based on ω_0 or V_x

d) $\int_0^{\infty} \mathcal{L}\{w(t)\} = \int_0^{t_0} \frac{A\,V(s)}{s[\tau s+1]} + \int_{t_0}^{\infty} \frac{A\,V(s)}{s[\tau s+1]}$

↳ $\int_0^{\infty} w(t) = \mathcal{L}^{-1}\left\{\frac{A V_{min}}{s[\tau s+1]}\right\} + \mathcal{L}^{-1}\left\{\frac{A V_{max}}{s[\tau s+1}\right\}$

↳ $\Delta w = \frac{A V_{min} e^{-t/\tau}}{\tau} + \frac{A V_{max} e^{-t/\tau}}{\tau} \rightarrow \boxed{A = \frac{\Delta w}{\Delta V}}$

↳ 2. $w_{ss} \approx 25$, $w_0 \approx 5$, $V_{min} \approx 1$, $V_{max} \approx 5$

↳ $\Delta w = w_{ss} - w_0 = 25 - 5 = 20$, $\Delta V = V_{max} - V_{min} = 5 - 1 = 4$

↳ $A = \Delta w / \Delta V = 20/4 \rightarrow \boxed{A = 5}$

↳ 3. $t_0 = 0.75$, $t_1 \approx 0.79$

↳ $\tau = t_1 - t_0 = 0.79 - 0.75 \rightarrow \boxed{\tau = 0.04}$

↳ 4. $G_w(s) = \frac{5}{0.04s + 1} \rightarrow \boxed{G_w(s) = \frac{125}{s + 25}}$

↳ 5. $\Delta w = 2 - 0 \rightarrow \Delta w = 2$ → $w(t) = 1.264$ at approx. 0.025

$0.632(2) = 1.264$

↳ $\therefore \tau = 0.025$, $K = w(\infty)/\tau \rightarrow K = 2/0.025 \rightarrow K = 80$

↳ $\therefore \boxed{G(s) = \frac{80}{s + 40}}$

↳ 6. One situation where it would be preferable to derive a transfer function experimentally is when there are multiple different values of step inputs at differing times. This would create a set of first order response graphs that are difficult to discern the needed constants and form. It is not preferable when doing low-constant step inputs that result in a lot of noise for the graph as it becomes difficult to experimentally derive the constants from all the noise.