Name: Rashad Ahmed Bhuiyan Student #: 400 263 180

MECHTRON/SFWR ENG 3DX4 Tutorial Quiz 1 Wednesday AM: Laplace Transforms review

1. **Inverse Laplace Transforms** (10 marks)

   Laplace transform tables are on the back!

   a) (5 marks) Assume that you are given a system $G_1(s) = \frac{Y(s)}{R(s)} = \frac{s+5}{(s+1)(s+2)^2}$. What is the steady state value of $y(t)$ (i.e. $\lim_{t\to\infty} y(t)$), in response to a 5 unit step input (i.e. $r(t) = 5u(t)$)?

$(s+1)(s+2)^2 = 0 \rightarrow s+1=0 \;\&\; s+2=0 \rightarrow s=-1,\; s=-2$

↳ since both roots are non-positive, we can use final-value theorem

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0}\left[\frac{s+5}{(s+1)(s+2)^2}\right]\cdot r(t)$$

$$y(\infty) = \frac{5}{4}\cdot 5 \rightarrow \boxed{y(\infty) = \frac{25}{4}}$$

   b) (5 marks) What is the time domain output, $y(t)$ for the system $G_1(s)$, in response to a 5 unit step input (i.e. $r(t) = 5u(t)$)?

$$\frac{s+5}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$s+5 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

$$s+5 = As^2 + A4s + 4A + Bs^2 + B3s + 2B + Cs + C$$

$$s+5 = [A+B]s^2 + [4A+3B+C]s + [4A+2B+C]$$

$A+B=0 \rightarrow A=-B \rightarrow A=-(-4) \rightarrow \boxed{A=4}$

$4A+3B+C=1 \rightarrow -4B+3B+C=1 \rightarrow -4(-4)+3(-4)+C=1 \rightarrow 4+C=1 \rightarrow \boxed{C=-3}$

$4A+2B+C=5 \rightarrow -\;-4B+2B+C=5$

$\boxed{B=-4}$

$$\frac{s+5}{(s+1)(s+2)^2} = \frac{4}{s+1} - \frac{4}{s+2} - \frac{3}{(s+2)^2}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{4}{s+1} - \frac{4}{s+2} - \frac{3}{(s+2)^2}\right\}\cdot 5 \rightarrow y(t) = \left[\mathcal{L}^{-1}\left\{\frac{4}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{4}{s+2}\right\} - \mathcal{L}^{-1}\left\{\frac{3}{(s+2)^2}\right\}\right]\cdot 5$$

$$y(t) = [4e^{-t} - 4e^{-2t} - 3te^{-2t}]\cdot 5 \rightarrow \boxed{y(t) = 20e^{-t} - 20e^{-2t} - 15te^{-2t}}$$

Item no.	f(t)	F(s)
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).