Rashad Ahmed Bhuiyan 2022 - 01 - 24

Prelab 1, SFWR ENG / MECHTRON 3DX4

Introduction to Simulink and Quanser's Quarc Library

Prelabs due the week of: Jan 24, 2022

Prelab Questions

These prelab questions are due at the beginning of your lab period, in the week of Jan 24, 2022.

The general open loop transfer function which models the angular velocity $\omega(t)$ of the motor is:

$$G_\omega(s) = \frac{\Omega(s)}{R(s)} = \frac{A}{\tau s + 1}$$

where A and τ are positive, real valued constants and $\Omega(s) = \mathcal{L}\{\omega(t)\}$.

1. What is the transfer function for the angular position of a motor $\theta(t)$?
2. What, if any, is the steady state value of ω in open loop response to a step input:

$$r(t) = U_o u(t) = \begin{cases} U_0, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

where $r(t) = \mathcal{L}^{-1}\{R(s)\}$ and $U_o \in \mathbb{R}$ is a constant voltage.

Prelab 1 Answers

2022 - 01 - 24

↳ all work in this prelab has been done solely by Rashad Ahmed Bhuiyan, bhuiyr2, 400 263 180, [signature]

↳ 1. $G_\omega(s) = \dfrac{\Omega(s)}{R(s)} = \dfrac{A}{\tau s + 1}$, A & τ are +ve real constants

$\Omega(s) = \mathcal{L}\{\omega(t)\}$

↳ $G_\omega(s) = \dfrac{\mathcal{L}\{\omega(t)\}}{R(s)} = \dfrac{A/\tau}{s + 1/\tau}$

↳ for unit step for some v_0 steps where $v_0 = U_\theta U_0$

↳ $\mathcal{L}\{\omega(t)\} = \left\{\dfrac{A/\tau}{s + 1/\tau}\right\} v_0$ → laplace for $U_0\,\omega(t)$ is $\dfrac{U_0}{s}$

↳ $\mathcal{L}\{\omega(t)\} = \left\{\dfrac{A/\tau}{s + 1/\tau}\right\}\left\{\dfrac{v_0}{s}\right\}$ → $\mathcal{L}\{\omega(t)\} = \dfrac{Av_0}{s} - \dfrac{Av_0}{s + 1/\tau}$

↳ take inverse laplace of both sides: $\omega(t) = Av_0 - Av_0 e^{-t/\tau}$ for $t > 0$ ①

↳ we know that angular velocity $= d\theta(t)/dt$

↳ $d\theta(t) = \omega(t)\,dt$ → $\theta = \int \omega(t)\,dt$

↳ $\theta(t) = \displaystyle\int_0^t (Av_0 - Av_0 e^{-t/\tau})\,dt = \left[Av_0 t\right]_0^t - Av_0\left[\dfrac{e^{-t/\tau}}{1/\tau}\right]_0^t$

↳ $\theta(t) = Av_0 + Av_0\tau(e^{-t/\tau} - 1)$ → $\boxed{\theta(t) = Av_0(1-\tau) + Av_0\tau e^{-t/\tau}}$

↳ 2. expression ① is the step response to the open-loop system:

↳ $\omega(t) = Av_0 - Av_0 e^{-t/\tau}$ for $t > 0$, where $v_0 = U_\theta U_0$

↳ steady state value of ω is $\omega(t)$ as $t \to \infty$

↳ $\lim_{t\to\infty} \omega(t) \Rightarrow \lim_{t\to\infty} [AU_\theta U_0 - AU_\theta U_0 e^{-t/\tau}]$

↳ $\lim_{t\to\infty} \omega(t) = AU_\theta U_0 - AU_\theta U_0 \cdot \left[\lim_{t\to\infty} e^{-t/\tau}\right]$ → goes to 0

↳ $\lim_{t\to\infty} \omega(t) = AU_\theta U_0$ → $\boxed{\text{steady state value of } \omega(t) = AU_\theta U_0}$