University of Moratuwa

Faculty of Engineering

Department of Electronic & Telecommunication Engineering

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BM2102: Modelling and Analysis of Physiological Systems

A2: Electrical Properties of Branching Dendrites

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220169V

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$$V_1(x) = A_1 e^{-x} + B_1 e^{x} \qquad 0 \leq x \leq L_1$$
$$V_{21}(x) = A_{21} e^{-x} + B_{21} e^{x} \quad \text{— ②} \qquad L_1 \leq x \leq L_{21}$$
$$V_{22}(x) = A_{22} e^{-x} + B_{22} e^{x} \qquad L_1 \leq x \leq L_{22}$$

$$\left.\frac{dV_1}{dx}\right|_{x=0} = -(r_i \lambda_c)_1 I_{app} \quad \text{— ③}$$

$$V_{21}(L_{21}) = V_{22}(L_{22}) = 0 \quad \text{— ④}$$
$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1) \quad \text{— ⑤}$$

$$\frac{-1}{(r_i \lambda_c)_1} \left.\frac{dV_1}{dx}\right|_{x=L_1} = \frac{-1}{(r_i \lambda_c)_{21}} \left.\frac{dV_{21}}{dx}\right|_{x=L_1} + \frac{-1}{(r_i \lambda_c)_{22}} \left.\frac{dV_{22}}{dx}\right|_{x=L_1} \quad \text{⑥}$$

from ③

$$\left.\frac{dV_1}{dx}\right|_{x=0} = -(r_i \lambda_c)_1 I_{app}$$

$$\left. -A_1 e^{-x} + B e^{x} \right|_{x=0} = -(r_i \lambda_c)_1 I_{app}$$
$$-A_1 + B_1 = -(r_i \lambda_c)_1 I_{app}$$
$$A_1 - B_1 = (r_i \lambda_c)_1 I_{app} \quad \text{— ①'}$$

from ④

$$V_{21}(L_{21}) = 0$$
$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \quad \text{— ②'}$$

$$V_{22}(L_{22}) = 0$$
$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \quad \text{— ③}$$

from ⑤

$$V_1(L_1) = V_{21}(L_1)$$

$$V_1(L_1) = A_1 e^{-L_1} + B_1 e^{L_1}$$

$$V_{21}(L_1) = A_{21} e^{-L_1} + B_{21} e^{L_1}$$

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1}$$

$$(A_1 - A_{21}) e^{-L_1} + (B_1 - B_{21}) e^{L_1} = 0$$

from ⑤

$$V_{21}(L_1) = V_{22}(L_1)$$

$$\therefore (A_{21} - A_{22}) e^{-L_1} + (B_{21} - B_{22}) e^{L_2} = 0$$

from ⑥

$$\frac{-1}{(r_i x_c)_1}\left(-A_1 e^{-L_1} + B_1 e^{L_1}\right) = \frac{-1}{(r_i x_c)_{21}}\left[-A_{21} e^{-L_1} + B_{21} e^{L_1}\right] + \frac{(-1)}{(r_i x_c)_{22}}\left[A_{22} e^{L_1} + B_{22} e^{L_1}\right]$$

$$\frac{-A_1}{(r_i x_c)_1} e^{-L_1} + \frac{B_1}{(r_i x_c)_1} e^{L_1} + \frac{A_{21}}{(r_i x_c)_{21}} e^{-L_1} - \frac{B_{21}}{(r_i x_c)_{21}} e^{-L_1} - \frac{B_{22}}{(r_i x_c)_{22}} e^{L_1} = 0$$

$\therefore$ equation ⑦ is valid.

②

equation ⑦ written in $Ax = b$ form
(given in eq ⑨)

$$
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\
0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\
e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\
0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\
\dfrac{e^{-L_1}}{(r_i \lambda_c)_1} & \dfrac{e^{L_1}}{(r_i \lambda_c)_1} & \dfrac{e^{-L_1}}{(r_i \lambda_c)_{21}} & \dfrac{-e^{L_1}}{(r_i \lambda_c)_{21}} & \dfrac{e^{-L_1}}{(r_i \lambda_c)_{22}} & \dfrac{-e^{L_1}}{(r_i \lambda_c)_{22}}
\end{bmatrix}
\begin{bmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{bmatrix}
=
\begin{bmatrix} (r_i \lambda_c) I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$A_1 - B_1 + 0 + 0 + 0 + 0 = (r_i \lambda_c)_1 I_{app}$$

$$A_1 - B_1 = (r_i \lambda_c) I_{app} \quad \text{—} \; (1)$$

$$0 + 0 + A_{21} e^{-L_1} + B_{21} e^{L_1} + 0 + 0 = 0$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \quad \text{—} \; (2)$$

$$0 + 0 + 0 + 0 + A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \quad \text{—} \; (3)$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0$$

$$A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{L_1} = 0 \quad \text{—} \; (4)$$

$$0 + 0 + A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0$$

$$A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} = 0 \quad \text{—} \; (5)$$

$$\frac{-A_1}{(r_i \lambda_c)_1} e^{-L_1} + \frac{B_1}{(r_i \lambda_c)_1} e^{L_1} + \frac{A_{21}}{(r_i \lambda_c)_{21}} e^{-L_1} - \frac{B_{21}}{(r_i \lambda_c)} e^{L_1}$$

$$+ \frac{A_{22}}{(r_i \lambda_c)_{22}} e^{-L_1} - \frac{B_{22}}{(r_i \lambda_c)_{22}} e^{L_1} = 0 \quad \text{—} \; (6)$$

$\therefore$ All the given equations can be obtained from equation 9

```
% electrical constants and derived quantities for typical
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;             % cm
d21 = 30e-4;             % cm
d22 = 15e-4;             % cm
%d21 = 47.2470e-4;       % E9 cm
%d22 = d21;               % E9 cm

l1 = 1.5;            % dimensionless
l21 = 3.0;            % dimensionless
l22 = 3.0;            % dimensionless

% Electrical properties of compartments

Rm = 6e3;            % Ohms cm^2
Rc = 90;            % Ohms cm
Rs = 1e6;            % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);        % Ohms
rl21 = c1*d21^(-3/2);        % Ohms
rl22 = c1*d22^(-3/2);        % Ohms


% Applied current

iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22
-rl1*exp(-l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';
```

Question 3

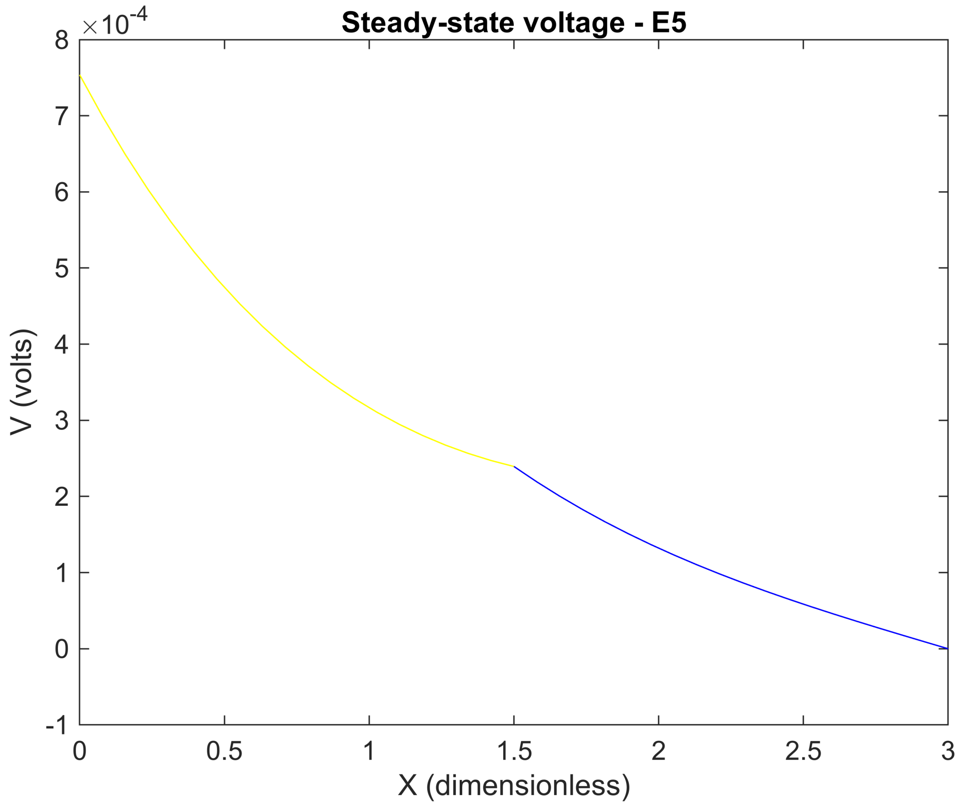
```
x=A\b;
display(x);
```

```
x = 6×1
    0.0007
    0.0000
    0.0011
```

```
   -0.0000
    0.0011
   -0.0000
```

Question 4

```
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5')
```



What do you note about the steady state voltage profile in the two daughter branches?

One daughter branch which is represented by the red line in the above graph overlaps with another daughter (blue) branch which indicates both branches have the same steady state voltage profile. This can also can be verified by observing the resulting column vector of Q3 where A21=A22 and B21=B22

Question 5

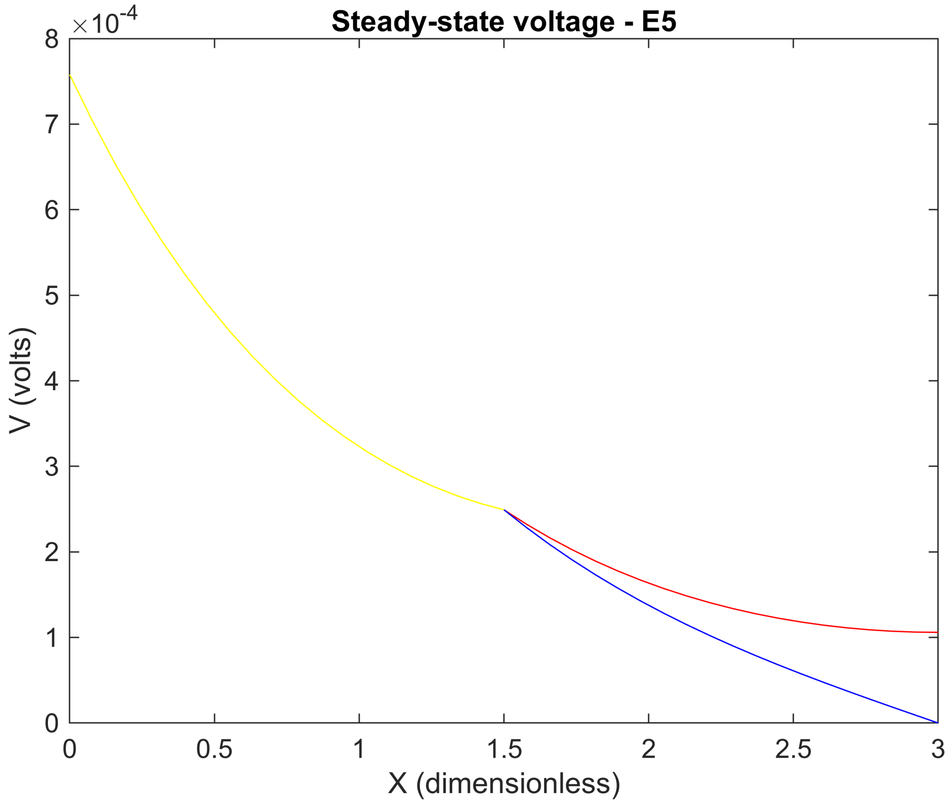
Part a)

```
% Update boundary condition
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
```

```
x = A\b;
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



Part b)

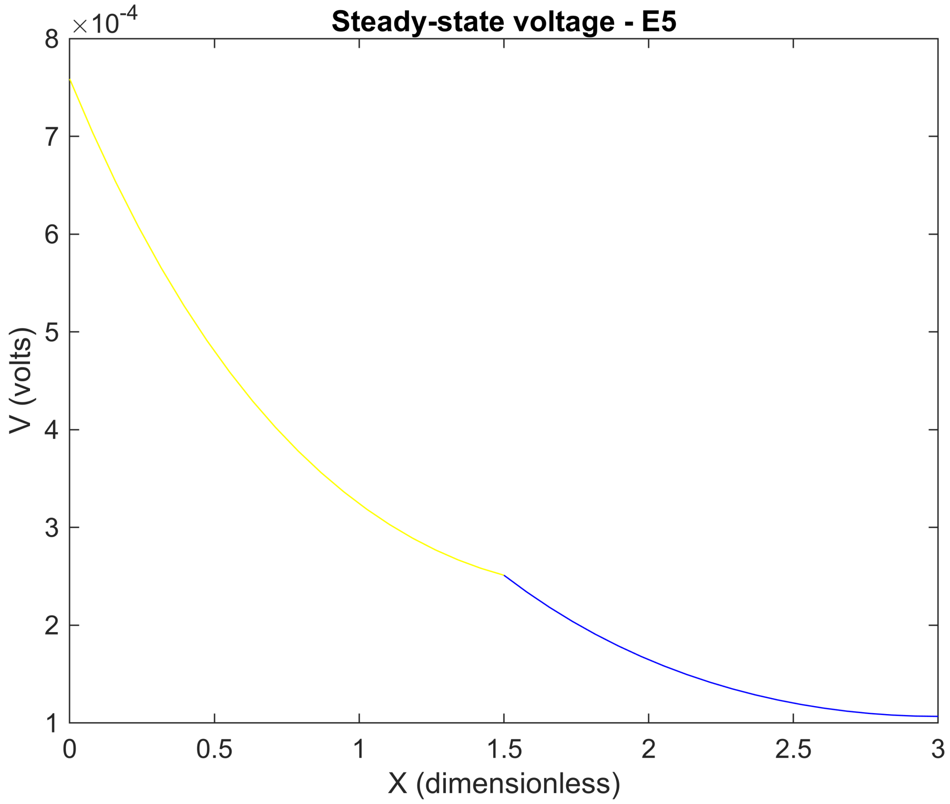
```
% Update boundary condition
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

x = A\b;
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
```

```
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



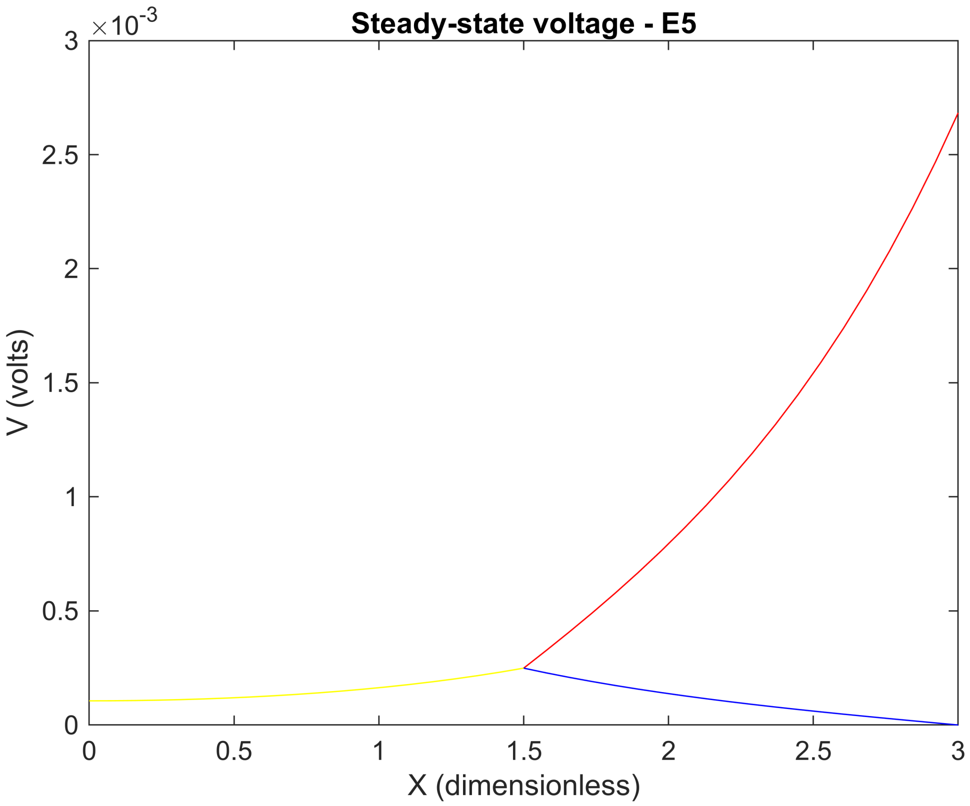
Part c)

```
% Update boundary condition
A(3,:) = [0 0 0 0 exp(-l22) exp(l22)];
b(1) = 0;
b(2) = rl21*iapp;

x = A\b;
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
title('Steady-state voltage - E5');
ylabel('V (volts)');
```



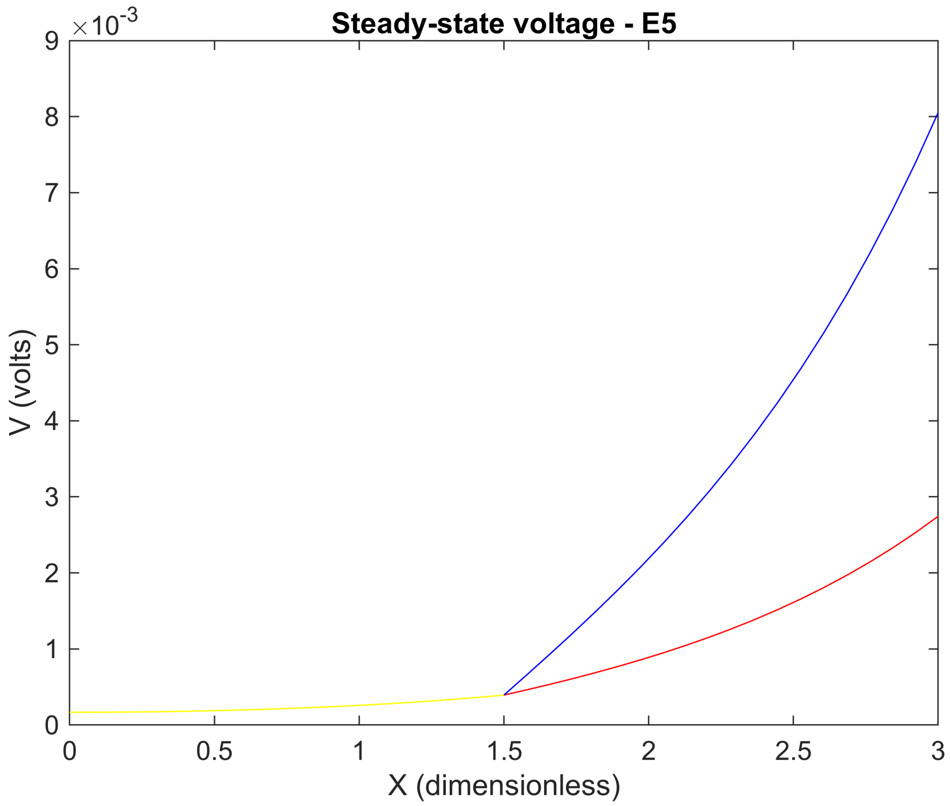
Part d)

```
% Update boundary condition
b(3)=  rl22*iapp;

x = A\b;
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
title('Steady-state voltage - E5');
ylabel('V (volts)');
```



What is the meaning of the positive right hand sides of $\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}}$ and $\left.\frac{dV_{22}}{dX}\right|_{X=L_{22}}$ in cases 2(c) and 2(d)?

In cases 2(c) and 2(d), a current is leaving the branch at the right-hand side. As the current flows across an impedance, it generates a positive membrane potential. This behavior represents the dendritic branches transmitting an electrical signal to another neuron. The resulting increase in membrane potential leads to a more positive voltage gradient at the right-hand ends of the branches.

Question 6

The parameters d_{21} and d_{22} are changed to $47.2470 \times 10^{-4}\,\mathrm{cm}$

```
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%d21 = 30e-4;             % cm
%d22 = 15e-4;             % cm
d21 = 47.2470e-4;       % E9 cm
d22 = d21;               % E9 cm

l1 = 1.5;             % dimensionless
l21 = 3.0;             % dimensionless
l22 = 3.0;             % dimensionless
```

```
% Electrical properties of compartments

Rm = 6e3;            % Ohms cm^2
Rc = 90;             % Ohms cm
Rs = 1e6;            % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);        % Ohms
rl21 = c1*d21^(-3/2);        % Ohms
rl22 = c1*d22^(-3/2);        % Ohms


% Applied current

iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22
-rl1*exp(l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';


x=A\b;
display(x);
```

```
x = 6×1
10^-3 ×
    0.7185
   -0.0018
    0.7185
   -0.0018
    0.7185
   -0.0018
```

Part b)

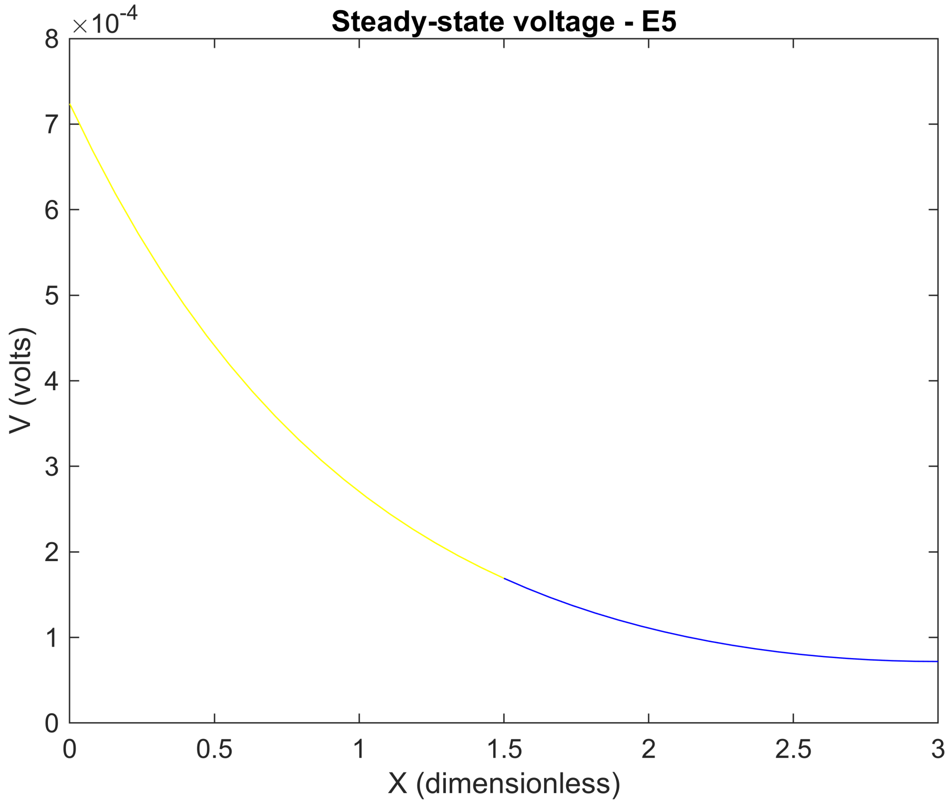
```
% Update boundary conditions
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
```

```
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



Part d)

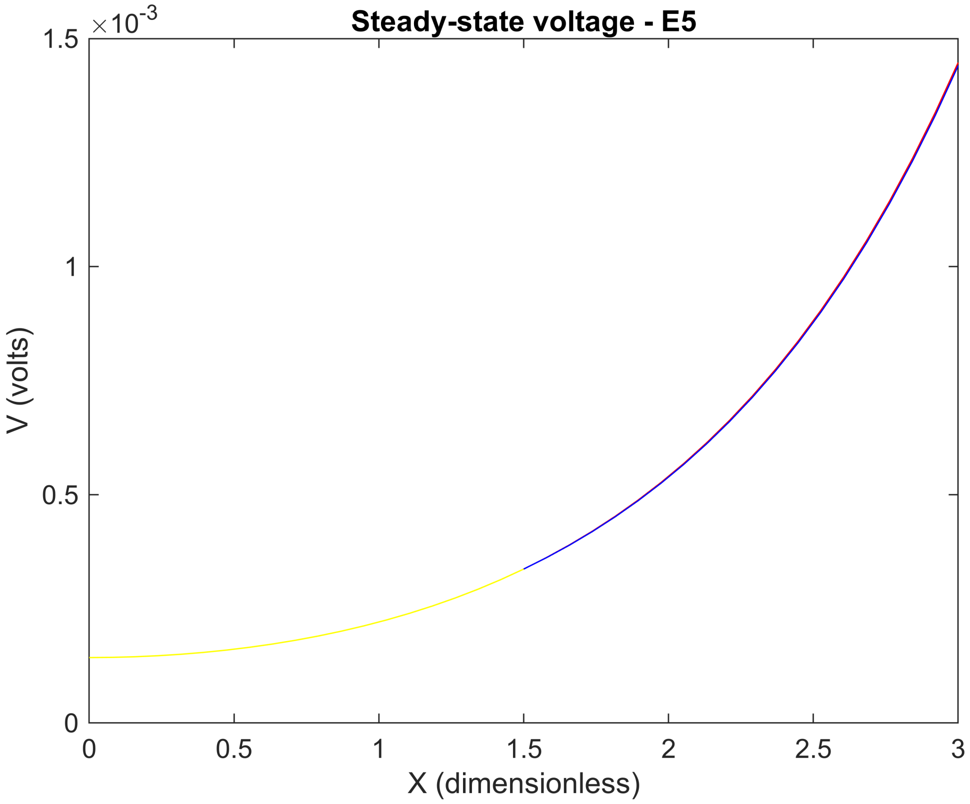
```
% Update the boundary conditions
A(3,:) = [0 0 0 0 exp(-l22) exp(l22)];
b(1) = 0; b(2) = rl21*iapp;
b(3) = rl22*iapp;

x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);

v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);

plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



What do you notice?

The graphs have become smoother at the branching point, with no sudden changes. The voltage profiles of both daughter branches are approximately the same in both cases. This is because the two daughter branches have equal diameters, allowing them to carry equal amounts of current.