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BM2102: Modelling and Analysis of Physiological Systems

A4: Hodgkin Huxley Model

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Contents

1 Introduction

This report explores the basic properties of the Hodgkin-Huxley model, which describes how nerve cells (neurons) generate and transmit electrical signals called action potentials. Using MATLAB, we simulate how a neuron responds to different types of electrical stimulation. The key features we study include the minimum current needed to trigger an action potential (threshold), how the neuron recovers after firing (refractory periods), how it fires repeatedly under continuous stimulation (repetitive activity), and how temperature affects its behavior. These simulations help us understand how real neurons work in the body.

2 Threshold

```
hhconst;
amp1 = 6;
width1 = 1;
hhmplot(0,50,0);
amp1 = 7;
hhmplot(0,50,1);
```

Figure 1: Code

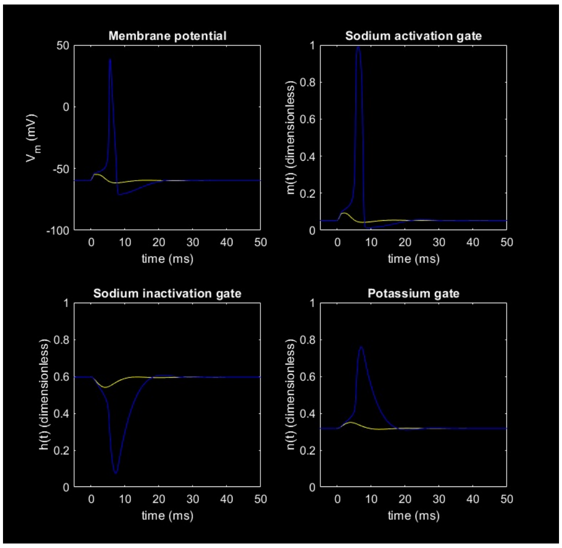

Figure 2: Membrane potential for stimulus currents of $6\,\mu\text{A/cm}^2$ (Yellow) and $7\,\mu\text{A/cm}^2$ (Blue)

Question 1

```
hhconst;

% Begin with a lower stimulus to check for absence of action potential
amp1 = 6;
width1 = 1;
hhmplot(0, 50, 0);

% Gradually increase stimulus amplitude in 0.1 steps
for step = 1:10
    amp1 = 6 + step * 0.1;
    hhmplot(0, 50, 1);
end
```

Figure 3: Code

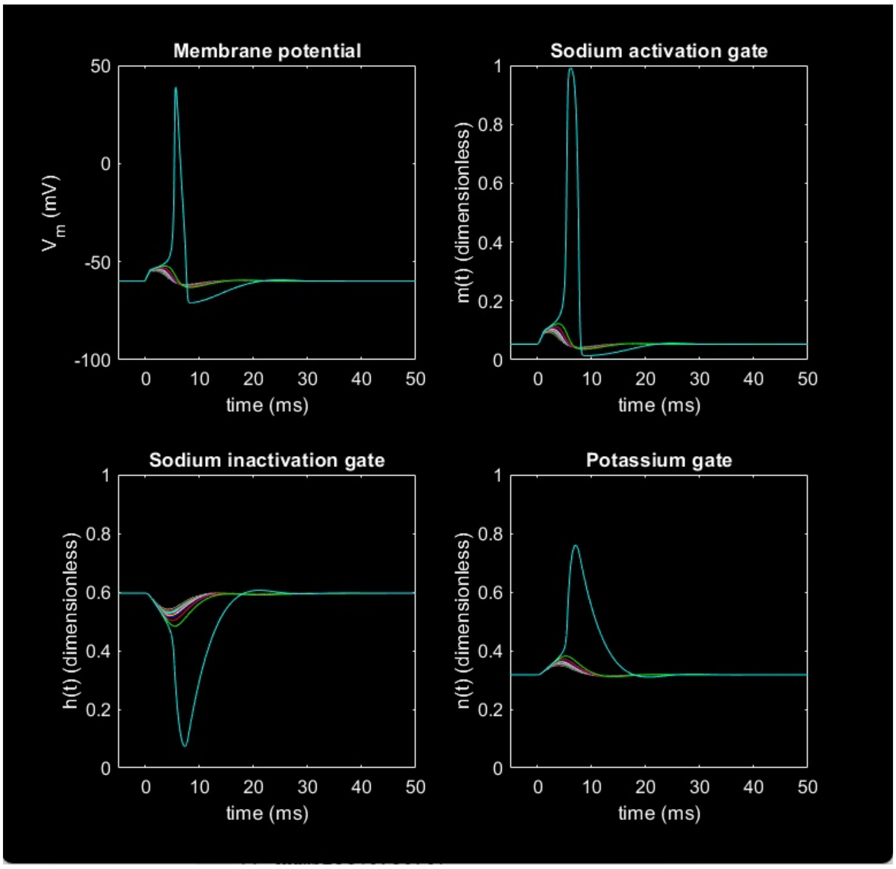

Figure 4: Representation of Model (Action potential occurs in last iteration)

From the above figure we can estimate that the threshold lies between $6.9\,\mu\text{A}/\text{cm}^2$ and $7.0\,\mu\text{A}/\text{cm}^2$.

```
hhconst;

% Narrow down search around expected threshold value
amp1 = 6.90;
width1 = 1;
hhmplot(0, 50, 0);

% Increase stimulus in 0.01 steps to identify precise threshold
for step = 1:10
    amp1 = 6.90 + step * 0.01;
    hhmplot(0, 50, step);
end
```

Figure 5: Code

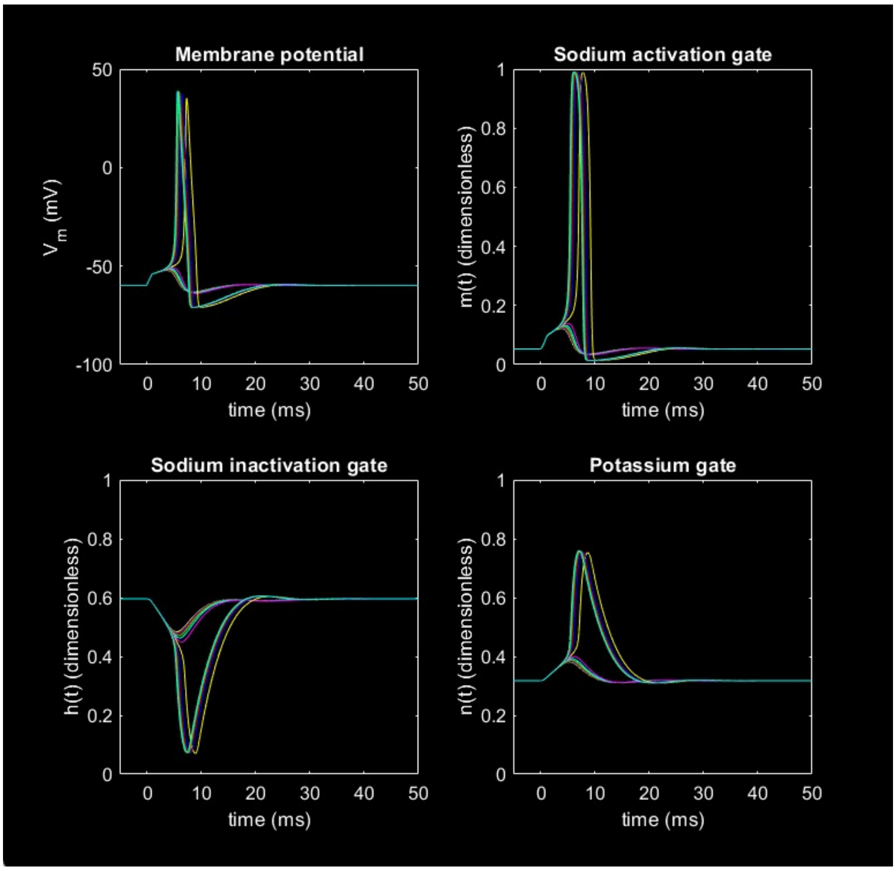

Figure 6: Representation of Model

After further bisecting the above mentioned estimated interval we can conclude the threshold is $6.96\,\mu\mathrm{A/cm}^2$.

Question 2

For amplitude greater than the threshold

```
hhconst;
amp1 = 7.10; % Intensity is greater than threshold
width1 = 1;
[qna,qk,ql]=hhsplot(0,50);

sum_Jk = qna + qk + ql % Check sum
```

Figure 7: Code

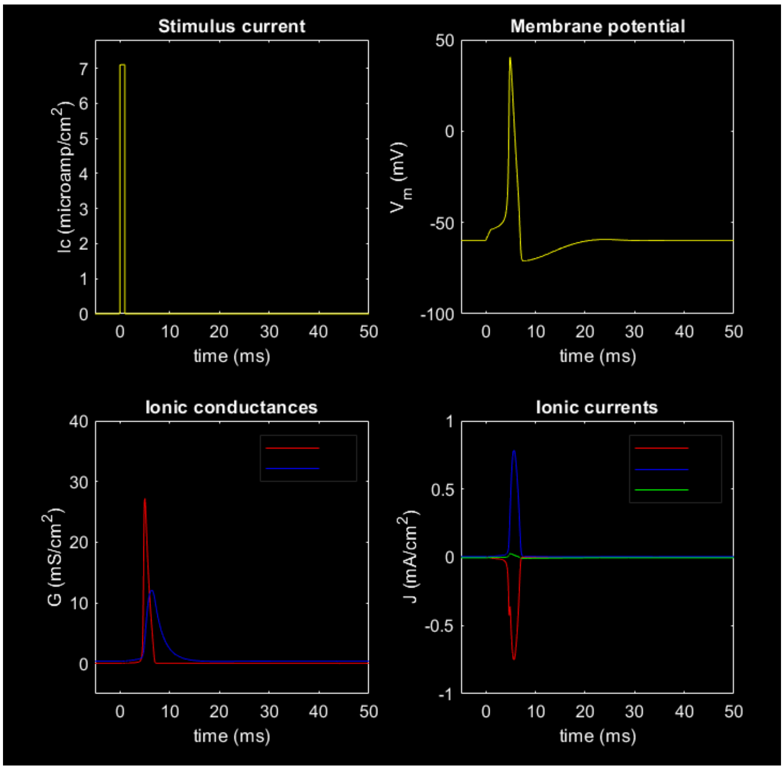

Figure 8: Representation of Model

sum_Jk = 7.1017

The above code calculates the sum $\int_{t_0}^{t_f} \sum_k J_k(t)\, dt$, which represents the total ionic current over the duration of the stimulus. The result is displayed as sum_Jk. We can observe that this value is approximately equal to the applied stimulus amplitude of $7.10\,\mu\text{A/cm}^2$, confirming the current balance in the model.

For amplitude less than the threshold

```
hhconst;
amp1 = 6.70; % Choose intensity lesser than threshold
width1 = 1;
[qna,qk,ql]=hhsplot(0,50);

sum_Jk = qna + qk + ql % Check the sum
```

Figure 9: Code

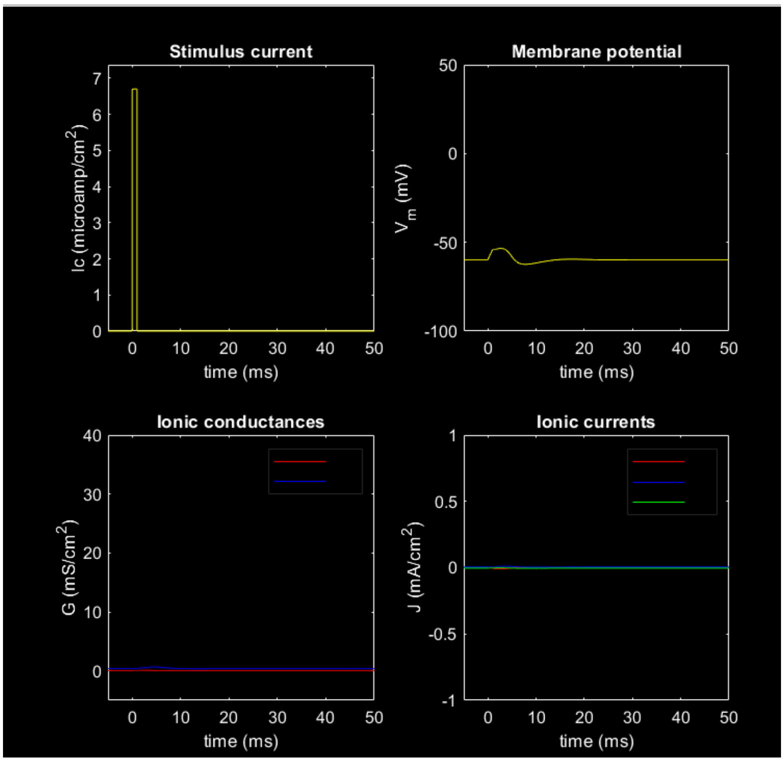

Figure 10: Representation of Model

sum_Jk = 6.6999 We can observe that the value of the sum of ionic currents is approximately equal to the applied stimulus amplitude of 6.70 μA/cm^2, confirming the current balance in the model.

3 Refractoriness

```
%refactoriness base case
amp1 = 27.4;
width1 = 0.5;
delay2 = 25;
amp2 = 13.7;
width2 = 0.5;
hhsplot(0,40);
```

Figure 11: Code

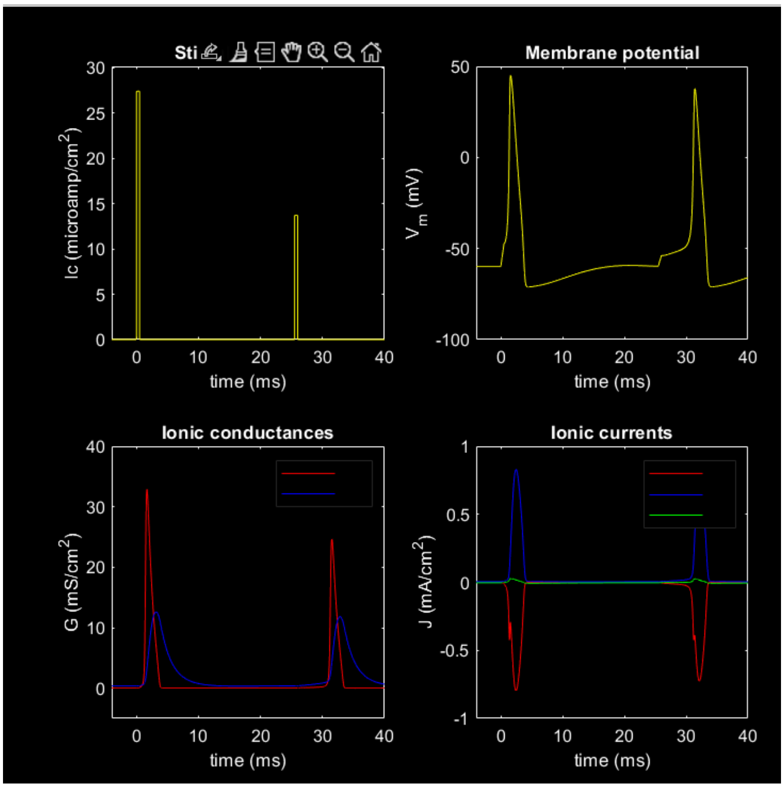

Figure 12: Representation of Model

Question 3

For delay = 20ms

```
%changing delays (20,18,16,14,12,10,8,6)
amp1 = 27.4;
width1 = 0.5;
amp2 = 11.0;
delay2 = 20;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:6
    amp2 = amp2 + 0.1;
    hhmplot(0,40,1);
end
```

Figure 13: Code

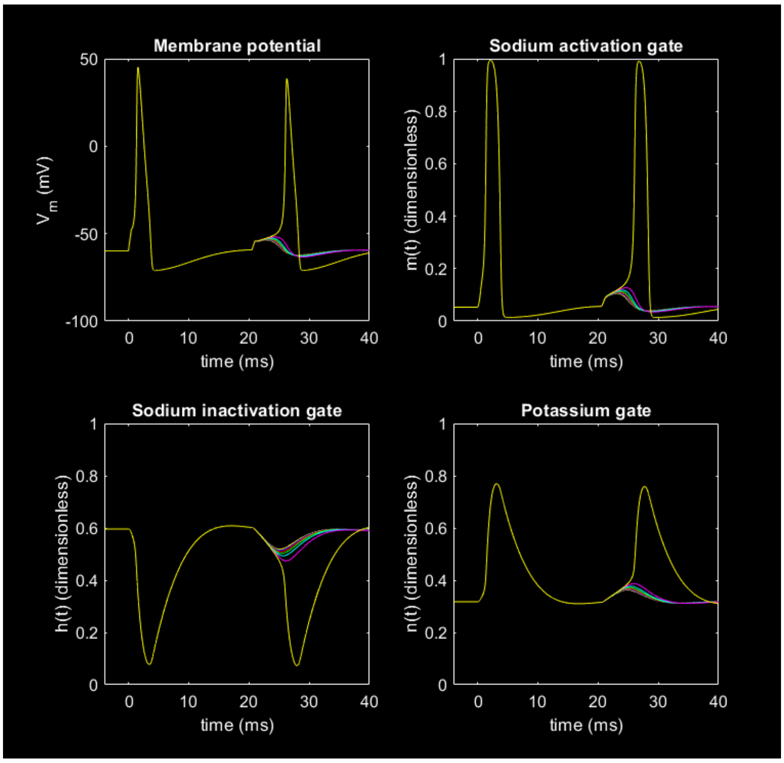

Figure 14: Representation of Model

An action potential was triggered at $I_{2\mathrm{th}} = 11.6\,\mu\mathrm{A/cm}^2$, which is the threshold amplitude for a second pulse.

For delay = 18ms

```
%changing delays (20,18,16,14,12,10
amp1 = 27.4;
width1 = 0.5;
amp2 = 11.0;
delay2 = 18;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:3
    amp2 = amp2 + 0.1;
    hhmplot(0,40,1);
end
```

Figure 15: Code

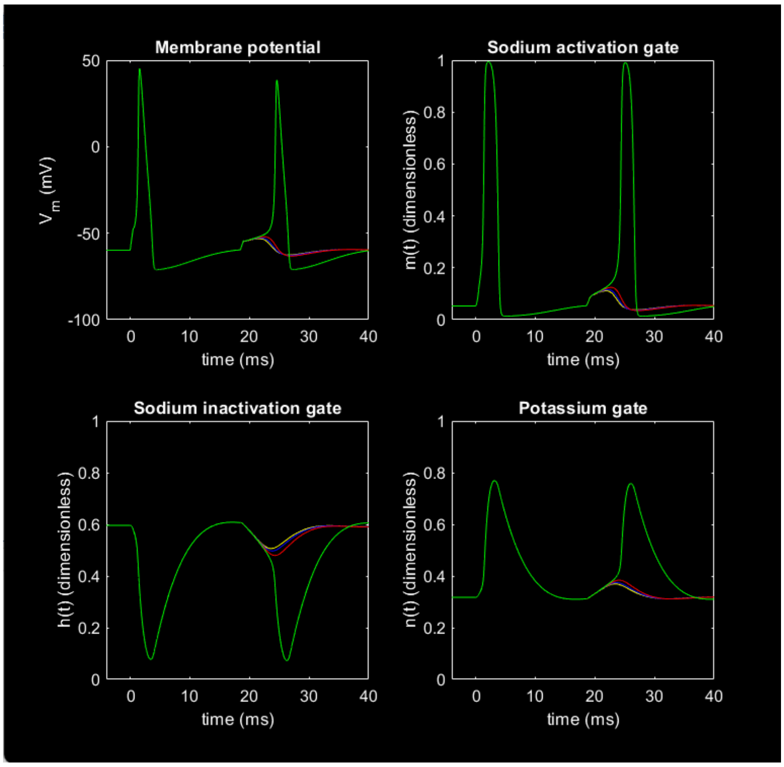

Figure 16: Representation of Model

An action potential was triggered at $I_{2\text{th}} = 11.3\,\mu\text{A/cm}^2$, which is the threshold amplitude for a second pulse.

For delay = 16ms

```
%changing delays (20,18,16,14,12,10,8,6)
amp1 = 27.4;
width1 = 0.5;
amp2 = 12.0;
delay2 = 16;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:7
    amp2 = amp2 + 0.1;
    hhmplot(0,40,1);
end
```

Figure 17: Code

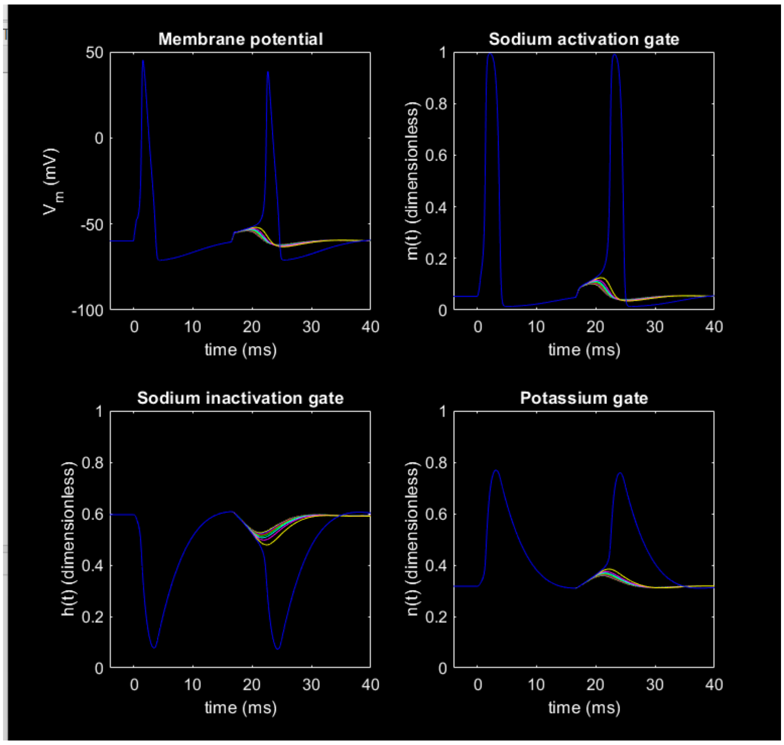

Figure 18: Representation of Model

An action potential was triggered at $I_{2\text{th}} = 12.7\,\mu\text{A/cm}^2$, which is the threshold amplitude for a second pulse.

For delay = 14ms

```
%changing delays (20,18,16,14,12,10,8,6)
amp1 = 27.4;
width1 = 0.5;
amp2 = 16.0; %starting value
delay2 = 14;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:9  %iterate
    amp2 = amp2 + 0.1;  %increment amplitue by 0.1
    hhmplot(0,40,1);
end
```

Figure 19: Code

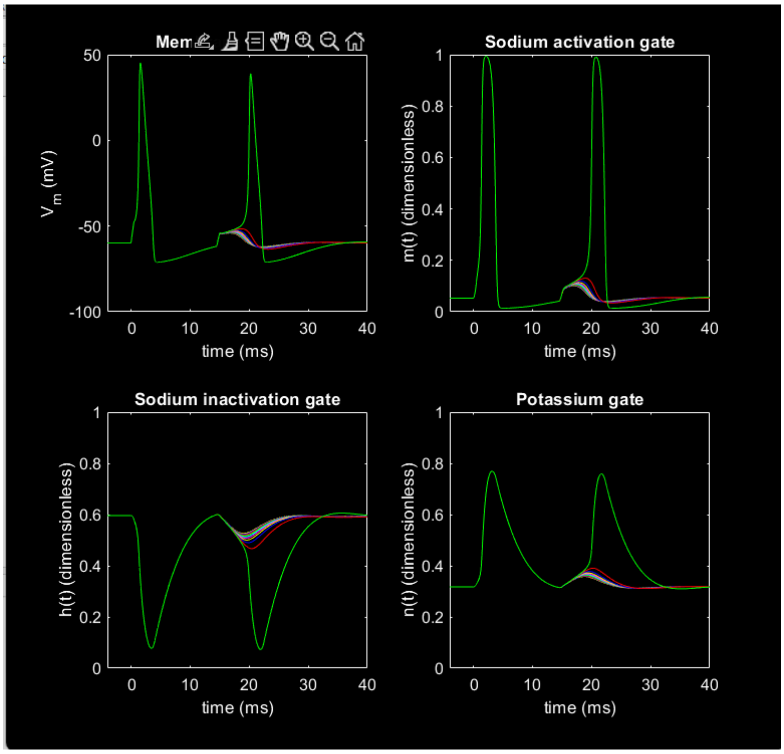

Figure 20: Representation of Model

An action potential was triggered at $I_{2\text{th}} = 16.9\,\mu\text{A}/\text{cm}^2$, which is the threshold amplitude for a second pulse.

For delay = 12ms

```
%changing delays (20,18,16,14,12,10,8,6)
amp1 = 27.4;
width1 = 0.5;
amp2 = 25.0; %starting value
delay2 = 12;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:3  %iterate
    amp2 = amp2 + 0.1;  %increment amplitude by 0.1
    hhmplot(0,40,1);
end
```

Figure 21: Code

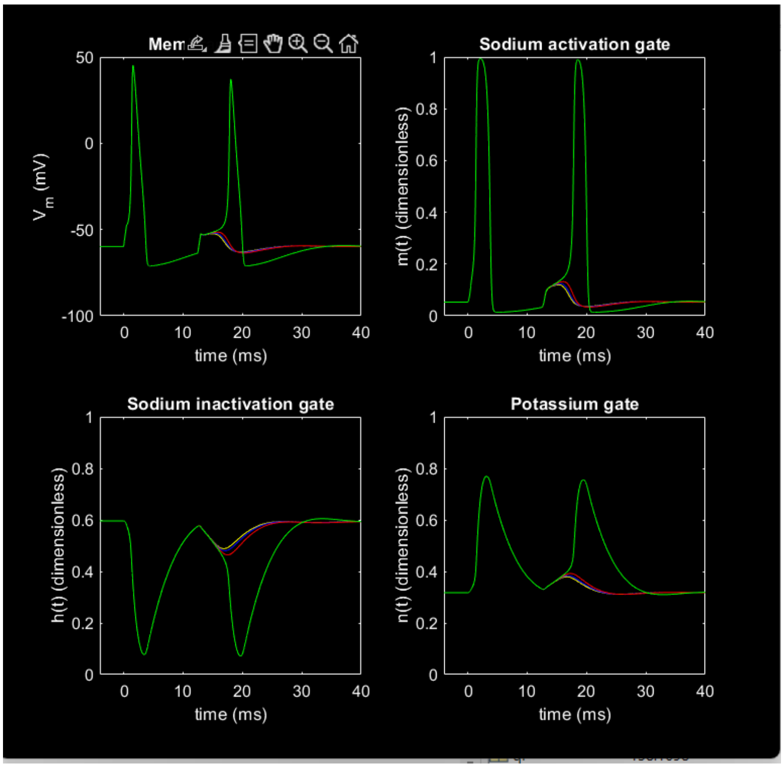

Figure 22: Representation of Model

An action potential was triggered at $I_{2\mathrm{th}} = 25.3\,\mu\mathrm{A/cm}^2$, which is the threshold amplitude for a second pulse.

For delay = 10ms

```
%changing delays (20,18,16,14,12,10,8,6)
amp1 = 27.4;
width1 = 0.5;
amp2 = 40.0; %starting value
delay2 = 10;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:5  %iterate
    amp2 = amp2 + 0.1;  %increment amplitue by 0.1
    hhmplot(0,40,1);
end
```

Figure 23: Code

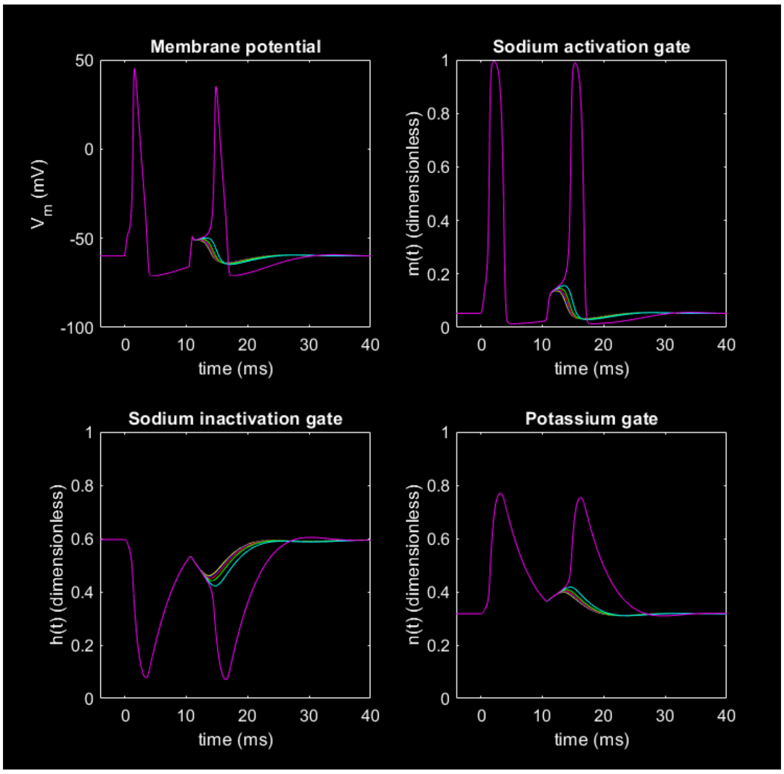

Figure 24: Representation of Model

An action potential was triggered at $I_{2\mathrm{th}} = 40.5\,\mu\mathrm{A/cm}^2$, which is the threshold amplitude for a second pulse.

For delay = 8ms

```
%changing delays (20,18,16,14,12,10,8,6)
amp1 = 27.4;
width1 = 0.5;
amp2 = 69.0; %starting value
delay2 = 8;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:6  %iterate
    amp2 = amp2 + 0.1;  %increment amplitue by 0.1
    hhmplot(0,40,1);
end
```

Figure 25: Code

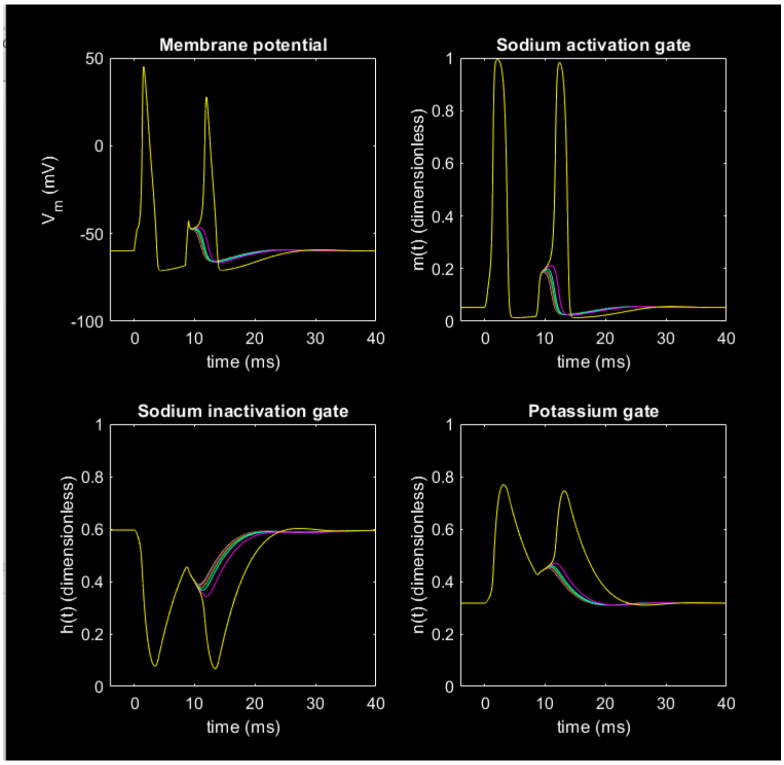

Figure 26: Representation of Model

An action potential was triggered at $I_{2\text{th}} = 69.6\,\mu\text{A/cm}^2$, which is the threshold amplitude for a second pulse.

For delay = 6ms

```
%changing delays (20,18,16,14,12,10,8,6)
amp1 = 27.4;
width1 = 0.5;
amp2 = 143; %starting value
delay2 = 6;
width2 = 0.5;
hhmplot(0,40,0);

for j = 1:5  %iterate
    amp2 = amp2 + 0.1;  %increment amplitue by 0.1
    hhmplot(0,40,1);
end
```

Figure 27: Code

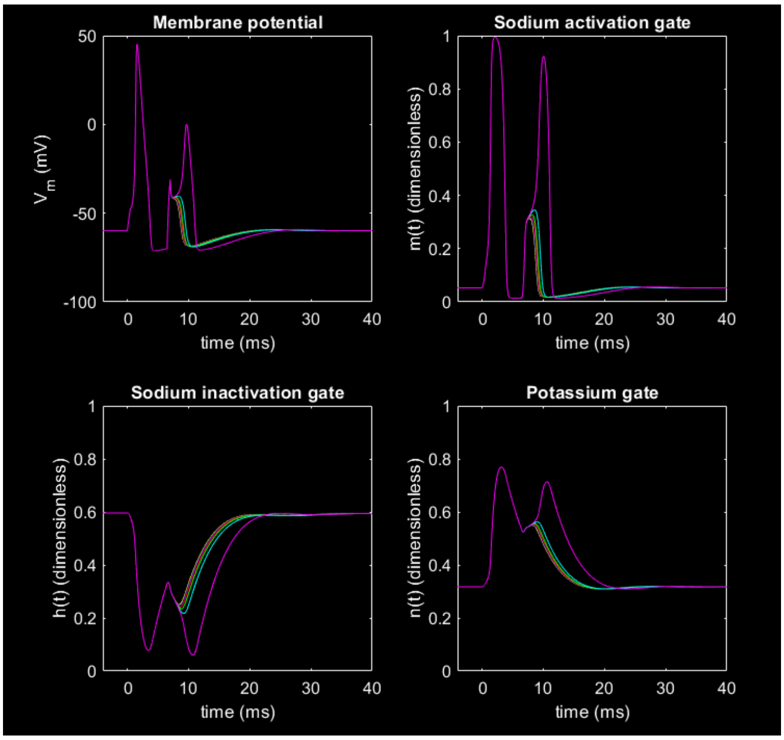

Figure 28: Representation of Model

An action potential was triggered at $I_{2\text{th}} = 143.5\,\mu\text{A}/\text{cm}^2$, which is the threshold amplitude for a second pulse.

Question 4

```
delay_values = [6, 8, 10, 12, 14, 16, 18, 20, 25];
threshold_amplitudes = [143.5, 69.6, 40.5, 25.3, 16.9, 12.7, 11.3, 11.6, 13.7];

% Compute the ratio of second pulse threshold to initial pulse amplitude
threshold_ratios = threshold_amplitudes / 26.8;


interp_delay = linspace(4, 25, 1000);
interp_ratio = spline(delay_values, threshold_ratios, interp_delay);

% Plot the interpolated curve
plot(interp_delay, interp_ratio, 'LineWidth', 2);
hold on;
yline(1, 'b--', 'LineWidth', 1);  % Reference line at ratio = 1

% Annotate the graph
xlabel('Delay (ms)');
ylabel('I_2 / I_1');
title('Threshold Ratio vs Inter-Stimulus Delay');
grid on;
```

Figure 29: Code

The graph below can be obtained from the above code.

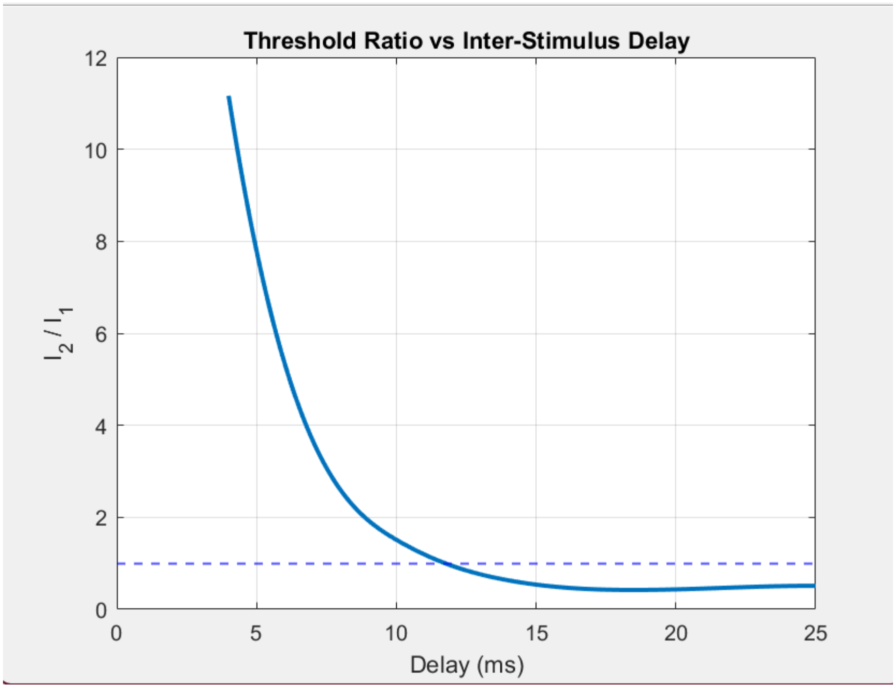

Figure 30: Graph

- For delays less than approximately 8 ms, the second stimulus must be several times stronger than the first in order to trigger a second action potential. This indicates that the neuron is in its absolute refractory period. Based on the data, we can estimate the absolute refractory period to be from **0 ms** to around **8 ms**.

- When the delay exceeds 12 ms, the second pulse successfully elicits an action potential even at amplitudes lower than or equal to the first stimulus ($I_1 = 13.7\,\mu\text{A/cm}^2$). This suggests that the membrane has largely recovered, marking the end of the relative refractory period. Therefore, the relative refractory period is estimated to lie between **8 ms** and **12 ms**.

4 Repetitive Activity

Question 5

For intensity of $5\,\mu\text{A/cm}^2$

```
% repetitive activity base
amp1 = 5; % Change
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 31: Code

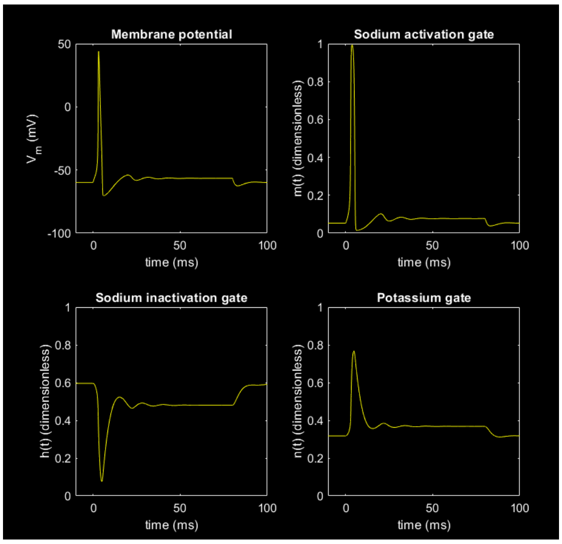

Figure 32: Representation of Membrane Potential

Only one action potential is triggered.

For intensity of 10 μA/cm^2

```
% repetitive activity base
amp1 = 10; % Change
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 33: Code

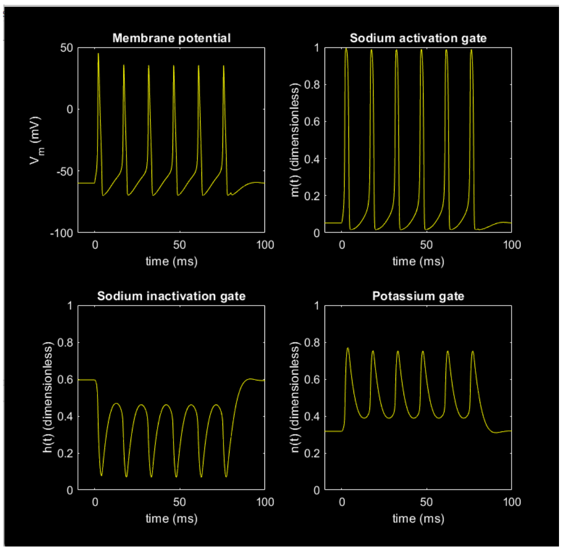

Figure 34: Representation of Membrane Potential

Number of action potentials triggered are 6.

For intensity of 20 μA/cm^2

```
% repetitive activity base
amp1 = 20; % Change
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 35: Code

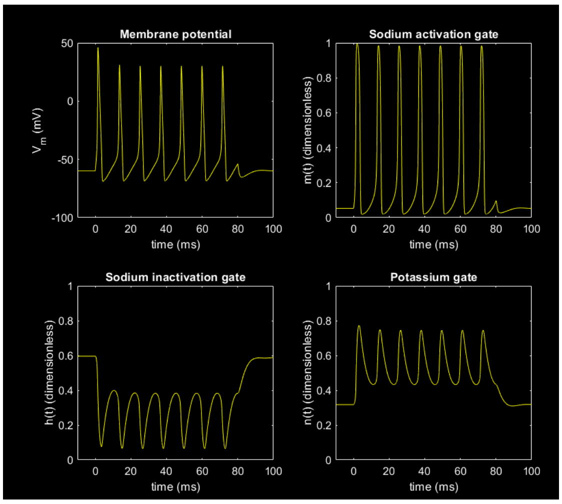

Figure 36: Representation of Membrane Potential

Number of action potentials triggered are 7.

For intensity of 30 μA/cm^2

```
% repetitive activity base
amp1 = 30; % Change
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 37: Code

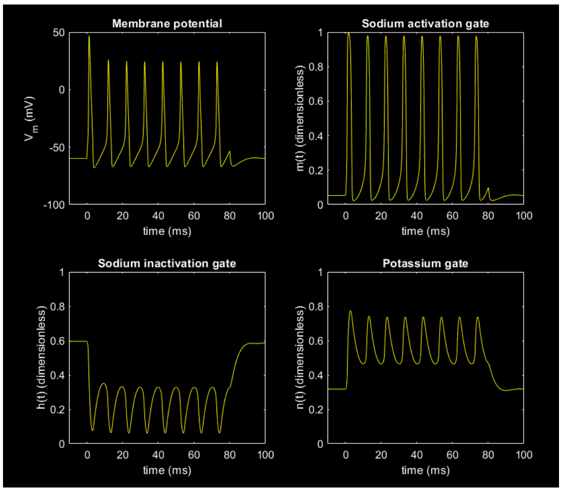

Figure 38: Representation of Membrane Potential

Number of action potentials triggered are 8.

For intensity of 50 μA/cm^2

```
% repetitive activity base
amp1 = 50; % Change
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 39: Code

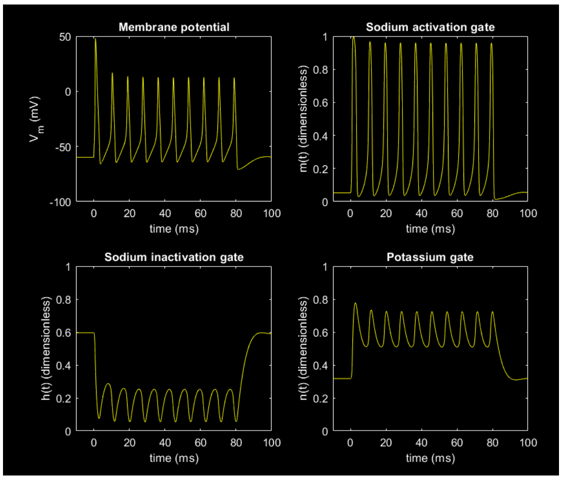

Figure 40: Representation of Membrane Potential

Number of action potentials triggered are 10.

For intensity of 70 μA/cm^2

```
% repetitive activity base
amp1 = 70; % Change
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 41: Code

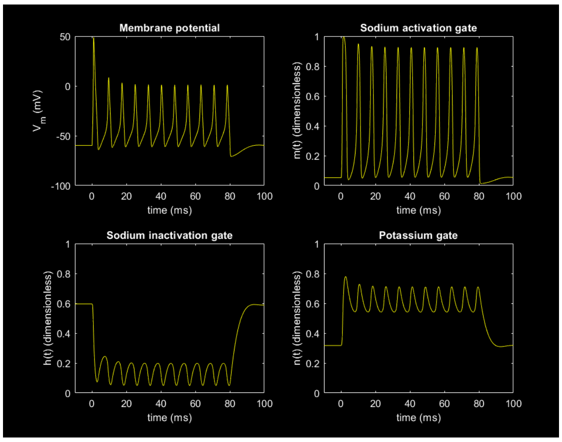

Figure 42: Representation of Membrane Potential

Number of action potentials triggered are 11.

For intensity of 100 μA/cm^2

```
% repetitive activity base
amp1 = 100; % Change
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 43: Code

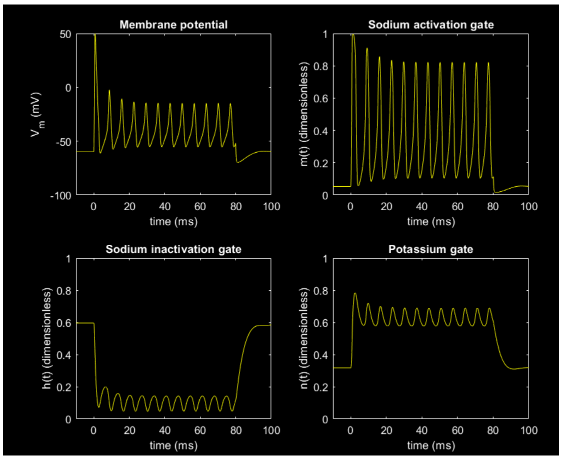

Figure 44: Representation of Membrane Potential

Number of action potentials triggered are 12.

Action potential frequency as a function of stimulating current amplitude

```
amplitudes = [5, 10, 20, 30, 50, 70, 100];
frequencies = [1, 6, 7, 8, 10, 11, 12];


x = linspace(0, 105, 1000);

% Interpolate frequency values using spline method
f = spline(amplitudes, frequencies, x);

% Plot the interpolated curve
plot(x, f, 'b-', 'LineWidth', 2);
hold on;

% Overlay the original data points
plot(amplitudes, frequencies, 'ro', 'MarkerSize', 8, 'LineWidth', 2);

% Customize the plot
xlabel('Amplitude (\muA/cm^2)');
ylabel('Frequency (No. of APs triggered)');
title('Stimulus Amplitude vs Action Potential Frequency');
ylim([0 15]);
grid on;
```

Figure 45: Representation of Model

From the above code the following graph can be obtained.

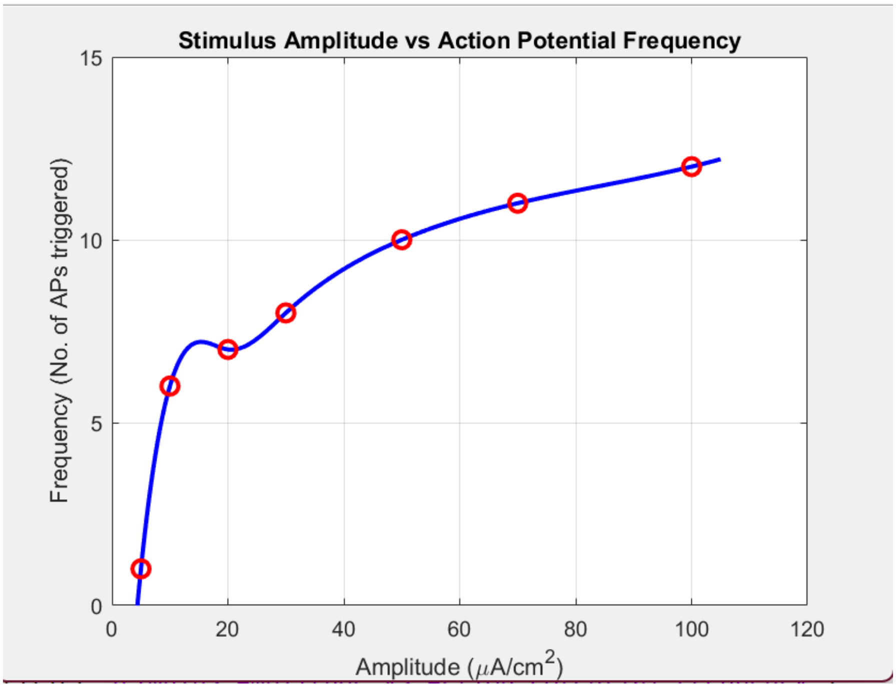

Figure 46: Representation of Model

From the above graph we can observe that the frequency of action potentials increases with the stimulus amplitude, showing a sharp rise at lower amplitudes followed by a more gradual increase.

Question 6

```
amp1 = 200;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

Figure 47: Code

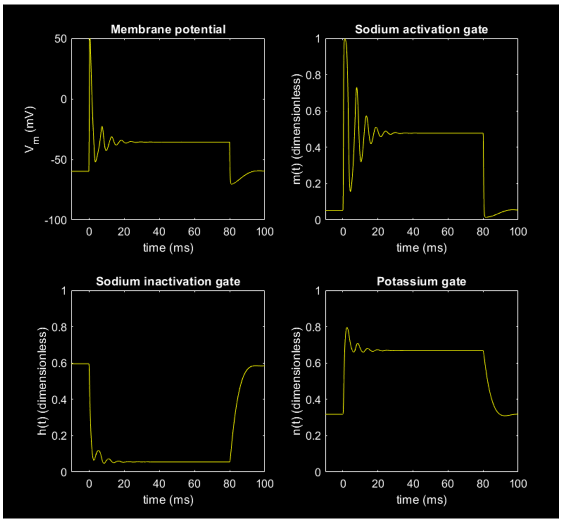

Figure 48: Representation of Membrane Potential

From the results above, the action potentials exhibit a decaying sinusoidal pattern, indicating high-frequency oscillatory behavior. As the stimulus amplitude increases, the amplitude of the action potentials tends to decrease.

This behavior can be explained by the ion channel dynamics described in the Hodgkin-Huxley model. Sodium (Na^+) channels, controlled by the gating variables m and h, open more with increased depolarization, resulting in a greater inward sodium current during the upstroke of the action potential. However, at very high stimulus levels, the rapid inactivation of these channels reduces the peak amplitude of the action potential.

Conversely, potassium (K^+) channels, represented by the gating variable n, also respond to depolarization by increasing their conductance. If this potassium conductance becomes too dominant, it can counterbalance the sodium influx too early, thereby dampening the amplitude of the action potential.

5 Temperature Dependence

Question 7

```matlab
vclamp = 0;
amp1 = 20;
width1 = 0.5;

temps = [0, 5, 10, 15, 20, 24, 25, 26, 30];

for i = 1:length(temps)
    tempc = temps(i);
    hhmplot(0,30,1);
    legend('show')
end
```

Figure 49: Representation of Model

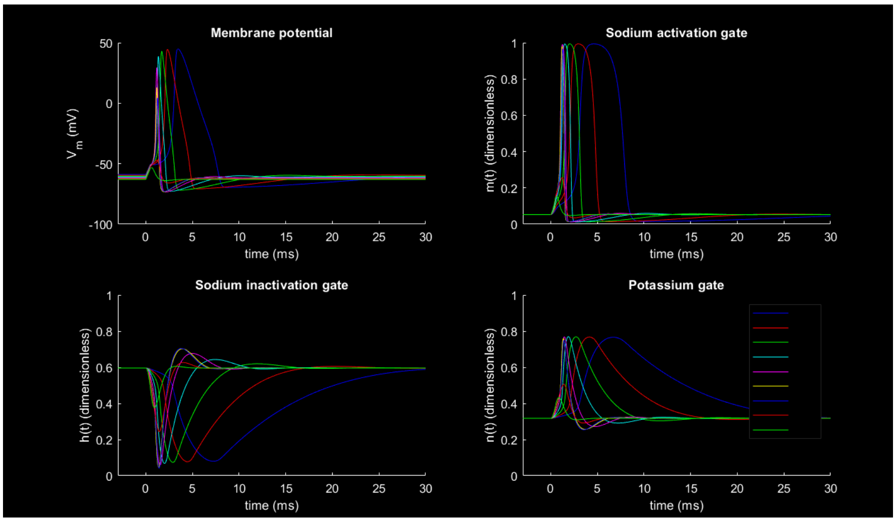

Figure 50: Representation of Model

As the temperature increases, the duration of the action potential decreases, indicating faster conduction velocities. Additionally, the peak of the membrane potential becomes lower at higher temperatures, reflecting a reduction in depolarization strength.

Furthermore, both the absolute and relative refractory periods are shortened due to the more rapid activation and inactivation of ion channels. This allows the membrane to return to its resting state more quickly, enabling the neuron to fire again sooner.