University of Moratuwa

Faculty of Engineering

Department of Electronic & Telecommunication Engineering

14$^{\text{th}}$ of June, 2025

BM2102: Modelling and Analysis of Physiological Systems

A5: Compartmental Modelling

By: Fernando S.R.N.

220169V

Part 1

Question 1

Unit Step Input System

The file **glucose_insulin_step.m** contains a function that models the system in the form yp = $ax + b$ for a unit step input. This script solves the system and plots the glucose and insulin levels over time.

```
tspan = [0 4];
initial = [0 0];
[t, y] = ode23(@glucose_insulin_step, tspan, initial);

figure;
plot(t, y(:,1), 'b', t, y(:,2), 'r', 'LineWidth', 2);
xlabel('Time (h)');
ylabel('Deviation');
legend('Insulin (i)', 'Glucose (g)');
title('Glucose/Insulin Model: Step Input');
grid on;
```

Figure 1: MATLAB code for solving and plotting the system with unit step input.

Figure 2: Glucose/Insulin levels vs Time for unit step input.

Bolus Input System

The system code for a bolus input is available as a function in the file `glucose_insulin_boulus.m`. This script simulates the system's response and generates a plot showing glucose and insulin levels over time.

```
tspan = [0 4];
initial = [0 0];
[t, y] = ode23(@glucose_insulin_bolus, tspan, initial);

figure;
plot(t, y(:,1), 'b', t, y(:,2), 'r', 'LineWidth', 2);
xlabel('Time (h)');
ylabel('Deviation');
legend('Insulin (i)', 'Glucose (g)');
title('Glucose/Insulin Model: Bolus Input');
grid on;
```

Figure 3: MATLAB code for solving and plotting the system with bolus input.

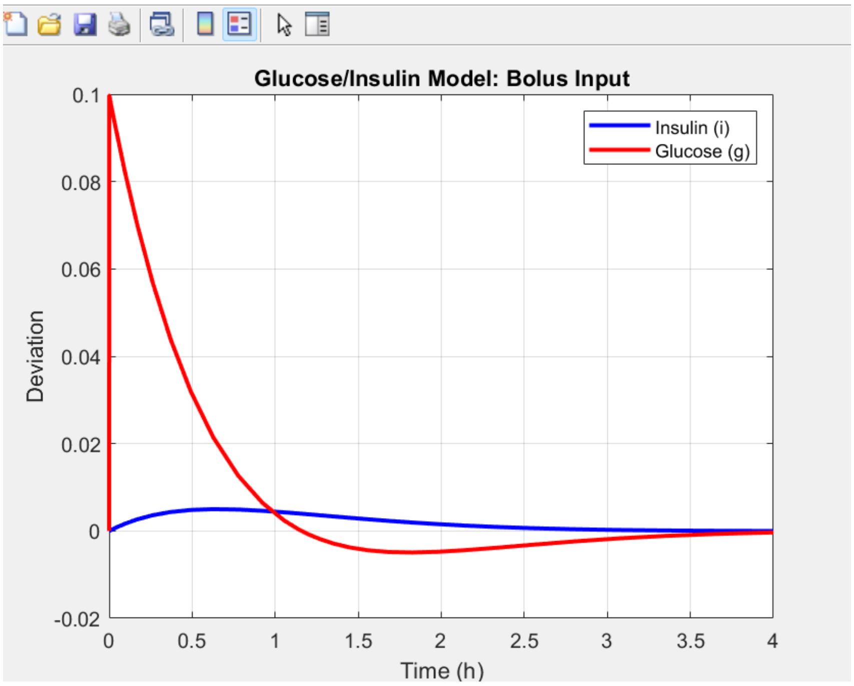


Figure 4: Glucose/Insulin levels vs Time for bolus input.

Diabetic Condition (Unit Step Input)

The system under diabetic conditions, modeled with a unit step input, is implemented as a function in the file `glucose_insulin_diabetics.m`. This code solves the system and plots the glucose and insulin levels over time.

```
tspan = [0 4];
initial = [0 0];
[t, y] = ode23(@glucose_insulin_diabetic, tspan, initial);

figure;
plot(t, y(:,1), 'b', t, y(:,2), 'r', 'LineWidth', 2);
xlabel('Time (h)');
ylabel('Deviation');
legend('Insulin (i)', 'Glucose (g)');
title('Diabetic Patient: Low Insulin, No Infusion');
grid on;
```

Figure 5: MATLAB code for diabetic system (unit step input).

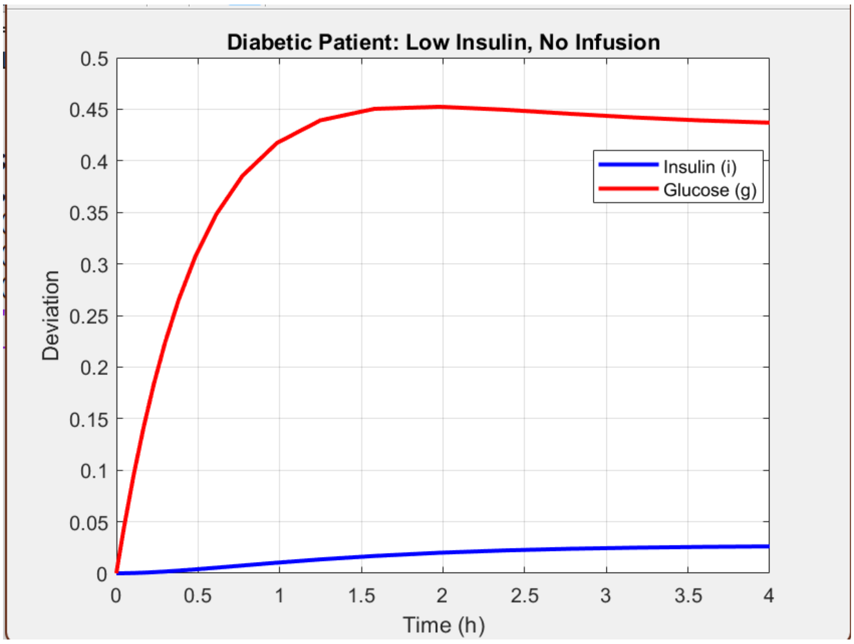


Figure 6: Glucose/Insulin levels vs Time for diabetic patient without insulin infusion.

Diabetic Condition with Insulin Infusion (Unit Step Input)

The system representing diabetic conditions with insulin infusion, using a unit step input, is defined as a function in the file `glucose_insulin_step.m`. This script solves the system and plots the glucose and insulin levels over time.

```
tspan = [0 4];
initial = [0 0];
[t, y] = ode23(@glucose_insulin_step, tspan, initial);

figure;
plot(t, y(:,1), 'b', t, y(:,2), 'r', 'LineWidth', 2);
xlabel('Time (h)');
ylabel('Deviation');
legend('Insulin (i)', 'Glucose (g)');
title('Glucose/Insulin Model: Step Input');
grid on;
```

Figure 7: MATLAB code for diabetic system with insulin infusion (unit step input).

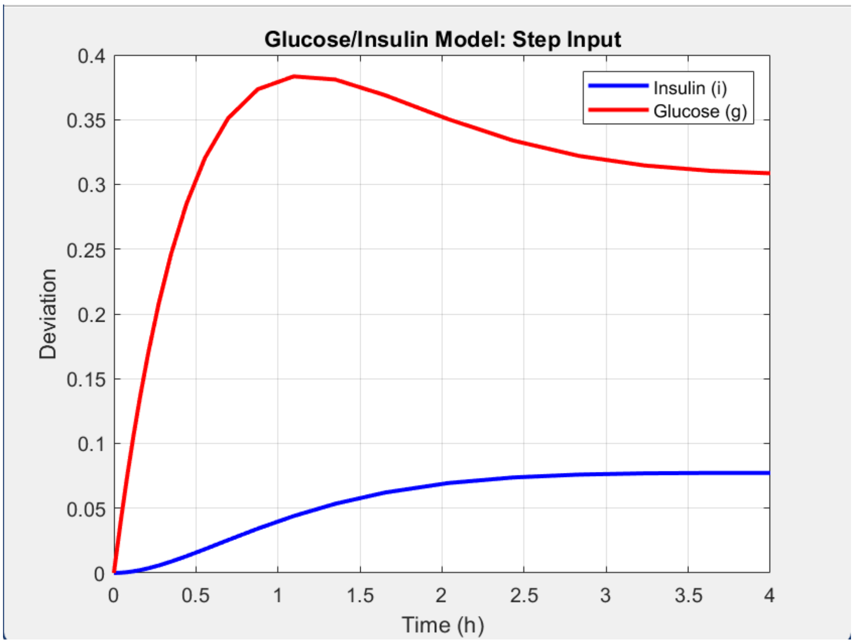


Figure 8: Glucose/Insulin levels vs Time for diabetic patient with insulin infusion.

Question 2

The Rigg's iodine model for a 150 µg iodine intake is implemented in the file `riggs_iodine_150.m`. The script simulates the model over a period of 10 days.

Riggs Iodine Model (150 μg Iodine Intake, 10 days)

```
figure;
[t,y] = ode23('riggs_iodine_150',[0, 10],[81.2, 6821, 682]');
plot(t,y); grid on;
xlabel('Time (hours)');
ylabel('Iodine level');
title('150ug Iodine intake (10 days)');
legend ('Plasma Iodine','Gland Iodine','Hormonal Iodine')
```

Figure 9: MATLAB code for Riggs iodine model (150 μg, 10 days).

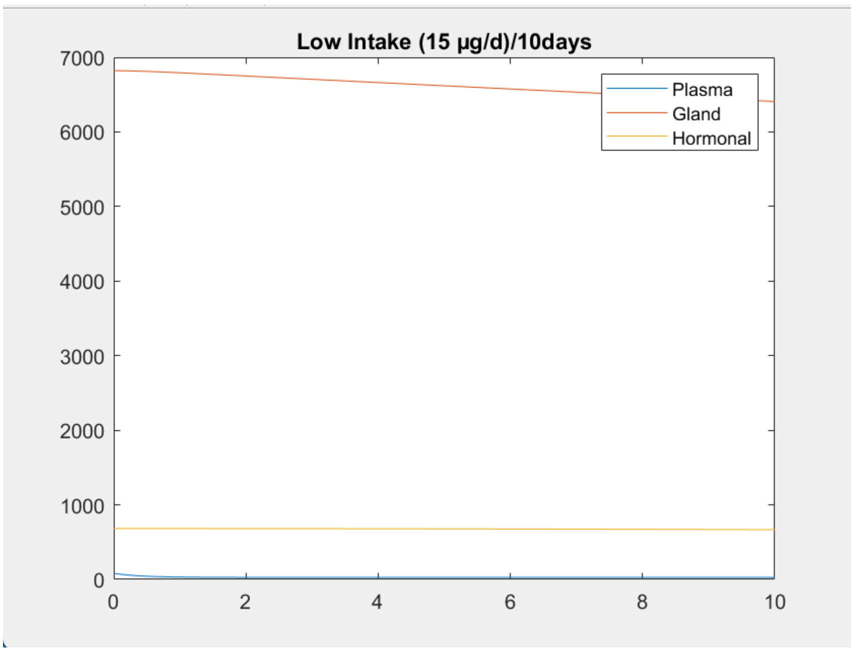


Figure 10: Iodine levels vs Time for 150 μg intake (10 days).

Riggs Iodine Model (15 μg Iodine Intake, 10 days)

The `riggs_low.m` file contains the Rigg's iodine model for a 15 µg iodine intake. It includes code to simulate the model for both 10 days and 300 days, with graphs showing how iodine levels change over time.

```
[t, y] = ode23(@riggs_low, [0 10], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Low Intake (15 µg/d)/10days')
```

Figure 11: MATLAB code for Riggs iodine model (15 μg, 10 days).

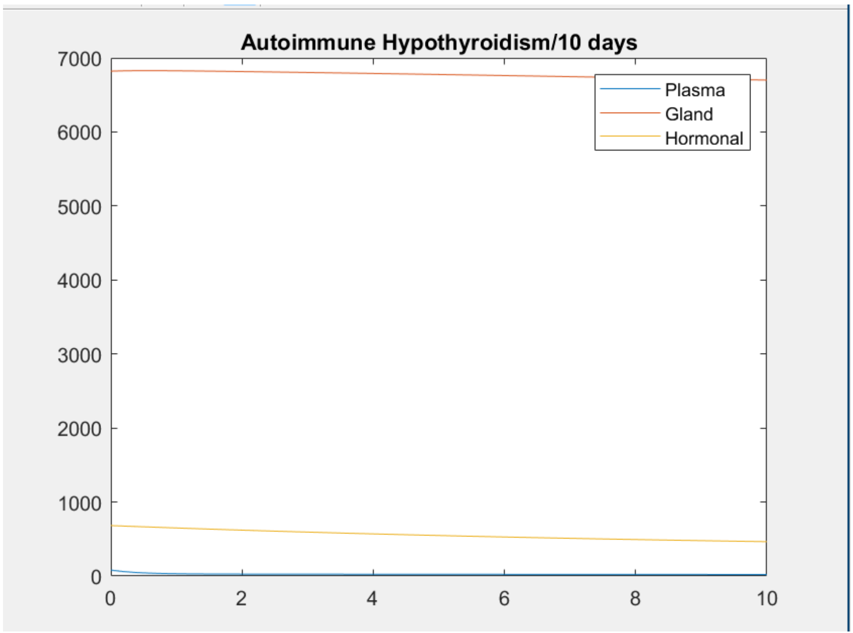


Figure 12: Iodine levels vs Time for 15 μg intake (10 days).

Riggs Iodine Model (15 μg Iodine Intake, 300 days)

```
[t, y] = ode23(@riggs_low, [0 300], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Low Intake (15 µg/d)/300days')
```

Figure 13: MATLAB code for Riggs iodine model (150 μg, 10 days).

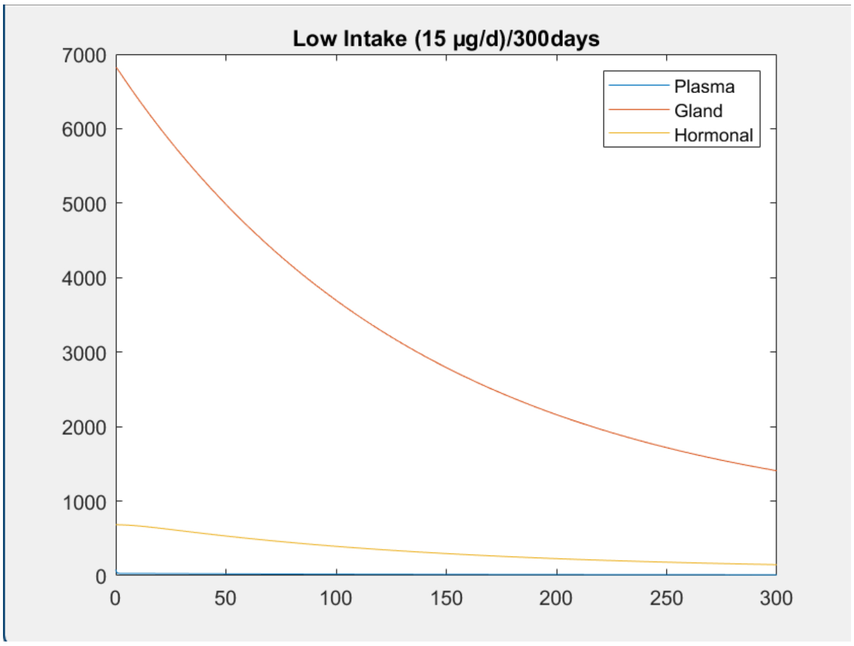


Figure 14: Iodine levels vs Time for 150 μg intake (300 days).

Hypothyroidism Scenario

Hypothyroidism is characterized by reduced thyroid function, leading to decreased iodine uptake and hormone production. The following codes and graphs illustrate the iodine kinetics for both short-term (10 days) and long-term (300 days) simulations.

10-Day Simulation

```
[t, y] = ode23(@riggs_hypo, [0 10], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Autoimmune Hypothyroidism/10 days')
```

Figure 15: MATLAB code for Riggs iodine model: Hypothyroidism scenario (10 days).

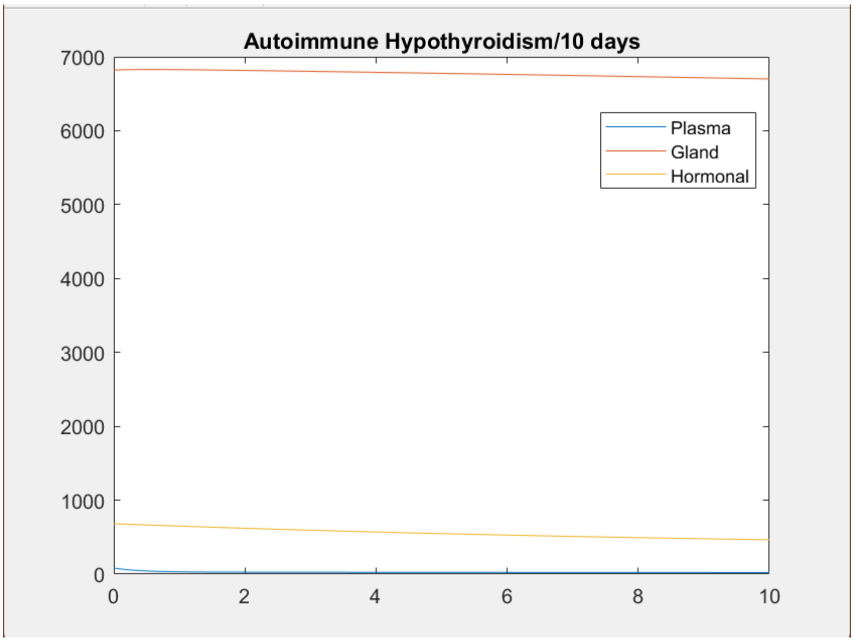


Figure 16: Iodine levels vs Time for Hypothyroidism scenario (10 days).

300-Day Simulation

```
[t, y] = ode23(@riggs_hypo, [0 300], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Autoimmune Hypothyroidism/300 days')
```

Figure 17: MATLAB code for Riggs iodine model: Hypothyroidism scenario (300 days).

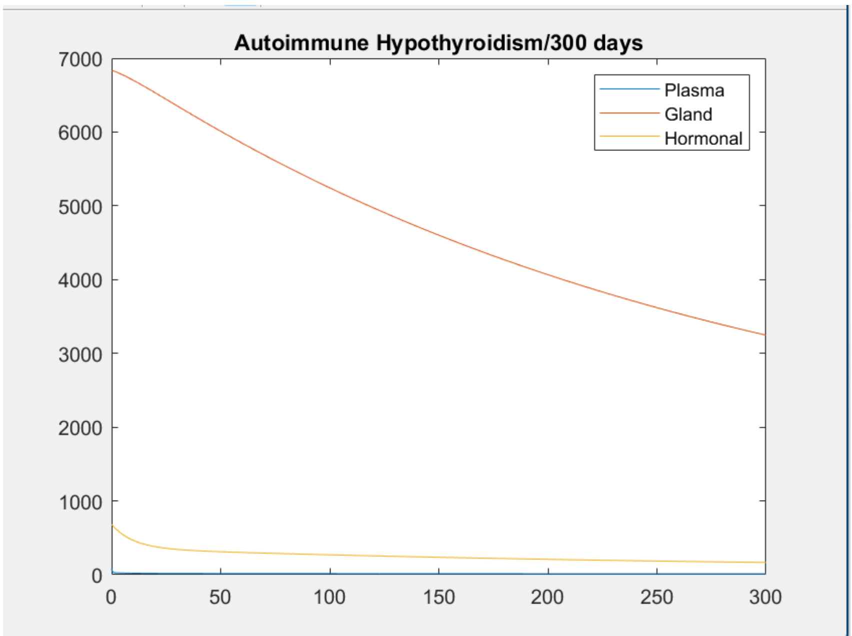


Figure 18: Iodine levels vs Time for Hypothyroidism scenario (300 days).

Low Iodine Intake Scenario

Low dietary iodine intake can lead to insufficient thyroid hormone synthesis. This scenario models the effect of reduced iodine input over both short and long durations.

10-Day Simulation

```
[t, y] = ode23(@riggs_hypolow, [0 10], [81.2; 6821; 682]);
figure;
plot(t, y)
xlabel('Time (days)')
ylabel('Iodine (\mug)')
legend('Plasma Iodine','Gland Iodine','Hormonal Iodine')
title('Hypothyroidism (Low Iodine Intake, 10 days)')
grid on
```

Figure 19: MATLAB code for Riggs iodine model: Low iodine intake scenario (10 days).

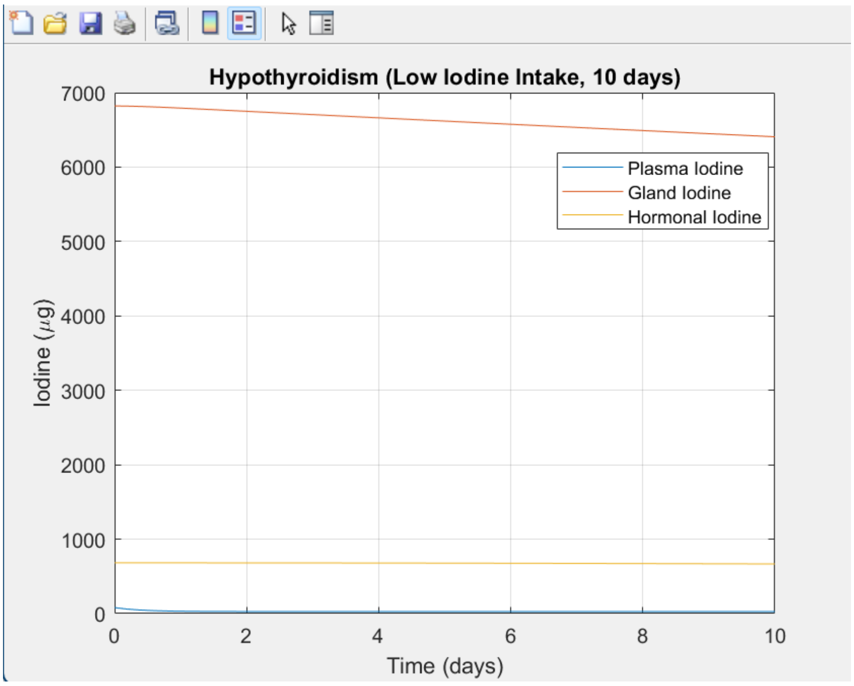


Figure 20: Iodine levels vs Time for Low iodine intake scenario (10 days).

300-Day Simulation

```
[t, y] = ode23(@riggs_hypolow, [0 300], [81.2; 6821; 682]);
figure;
plot(t, y)
xlabel('Time (days)')
ylabel('Iodine (\mug)')
legend('Plasma Iodine','Gland Iodine','Hormonal Iodine')
title('Hypothyroidism (Low Iodine Intake, 300 days)')
grid on
```

Figure 21: MATLAB code for Riggs iodine model: Low iodine intake scenario (300 days).

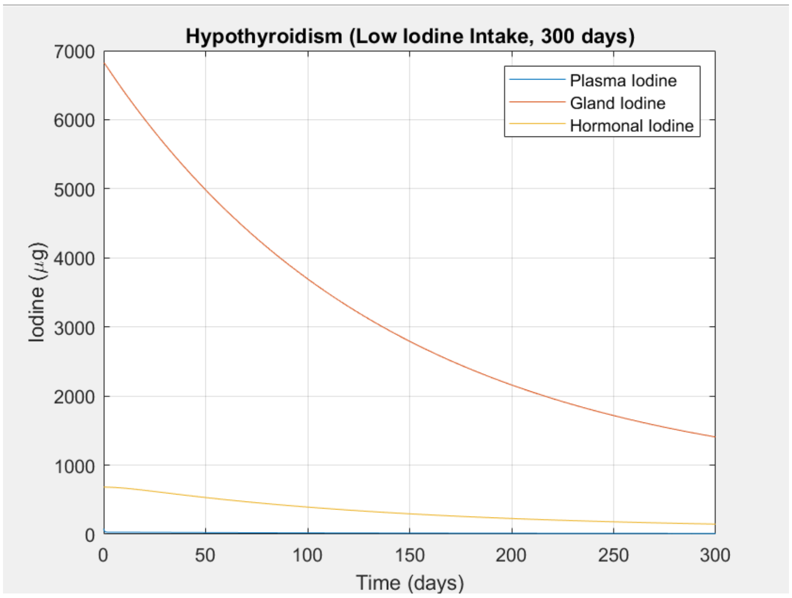


Figure 22: Iodine levels vs Time for Low iodine intake scenario (300 days).

Graves Disease Scenario

Graves disease is an autoimmune disorder that results in hyperthyroidism, or overactive thyroid function. The following simulations reflect increased iodine uptake and hormone release, for both 10-day and 300-day periods.

10-Day Simulation

```
[t, y] = ode23(@riggs_hyper, [0 10], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Graves Hyperthyroidism, 10 days')
```

Figure 23: MATLAB code for Riggs iodine model: Graves disease scenario (10 days).

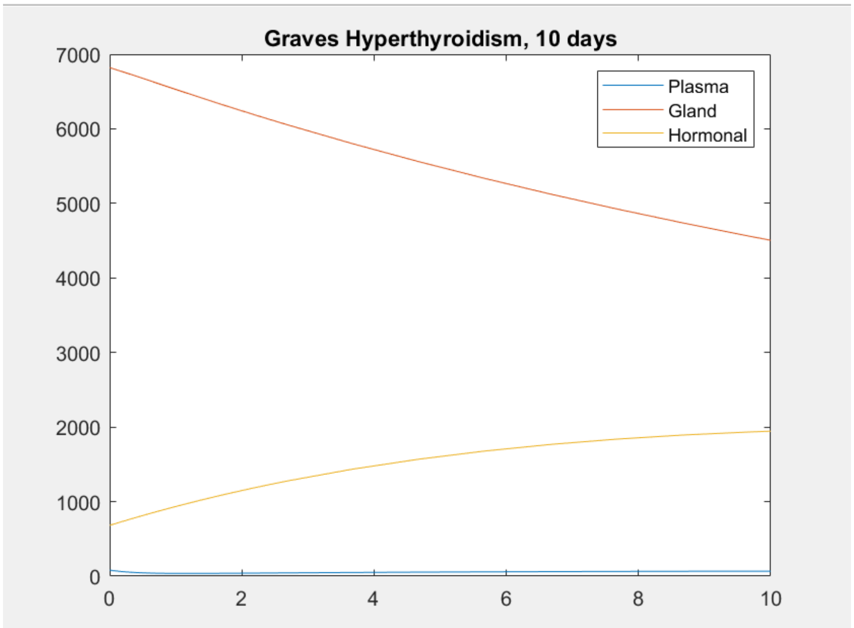


Figure 24: Iodine levels vs Time for Graves disease scenario (10 days).

300-Day Simulation

```
[t, y] = ode23(@riggs_hyper, [0 300], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Graves Hyperthyroidism, 300 days')
```

Figure 25: MATLAB code for Riggs iodine model: Graves disease scenario (300 days).

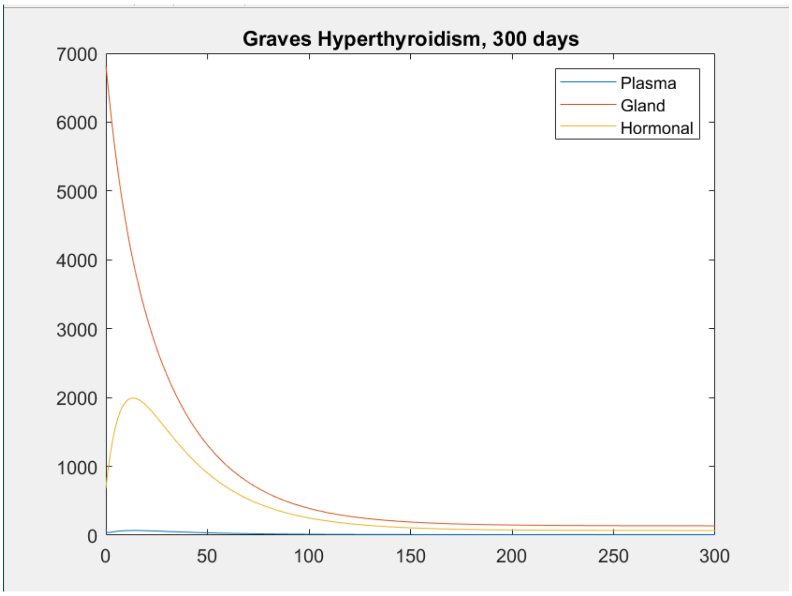


Figure 26: Iodine levels vs Time for Graves disease scenario (300 days).

Goiter Scenario

Goiter refers to an enlarged thyroid gland, often due to chronic iodine deficiency. The following results show the altered iodine kinetics associated with goiter for both timeframes.

10-Day Simulation

```
[t, y] = ode23(@riggs_goiter, [0 300], [81.2; 15000; 682]); % Initial gland = 15,000 µg
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Goiter Condition Simulation, 10 days')
```

Figure 27: MATLAB code for Riggs iodine model: Goiter scenario (10 days).

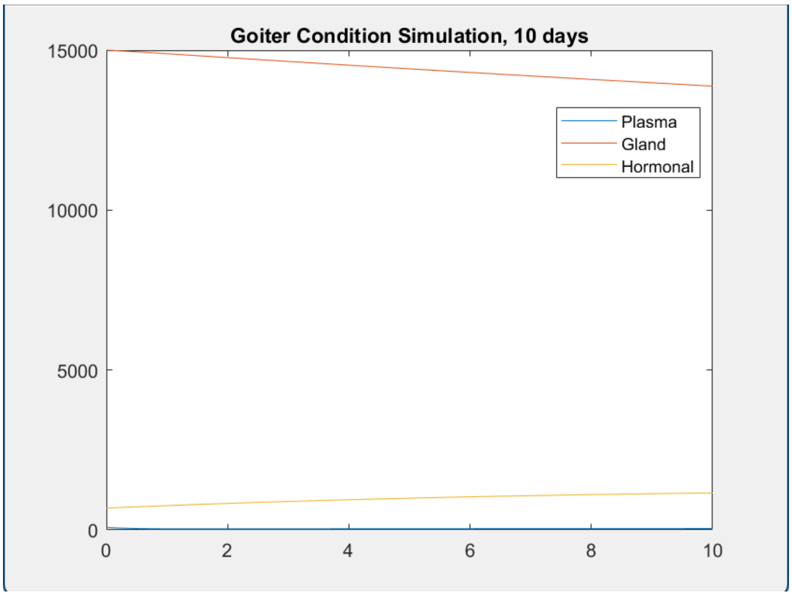


Figure 28: Iodine levels vs Time for Goiter scenario (10 days).

300-Day Simulation

```
[t, y] = ode23(@riggs_goiter, [0 300], [81.2; 15000; 682]); % Initial gland = 15,000 µg
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Goiter Condition Simulation, 300 days')
```

Figure 29: MATLAB code for Riggs iodine model: Goiter scenario (300 days).

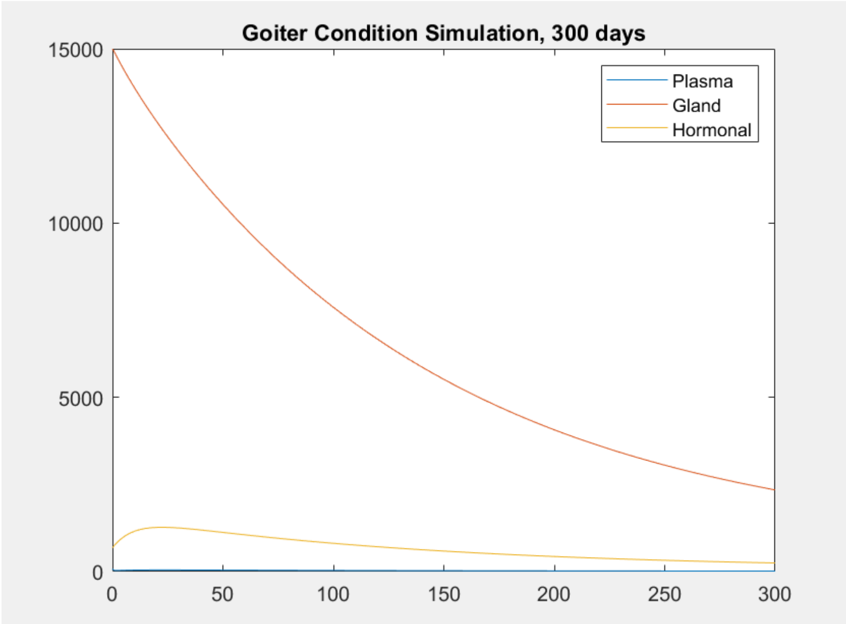


Figure 30: Iodine levels vs Time for Goiter scenario (300 days).

Thyroid Tumor Scenario

Thyroid tumors can disrupt normal iodine metabolism. The following code and graphs illustrate the simulated iodine dynamics in the presence of a tumor over 10 and 300 days.

10-Day Simulation

```
[t, y] = ode23(@riggs_tumor, [0 10], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Tumor Simulation, 10 days')
```

Figure 31: MATLAB code for Riggs iodine model: Tumor scenario (10 days).

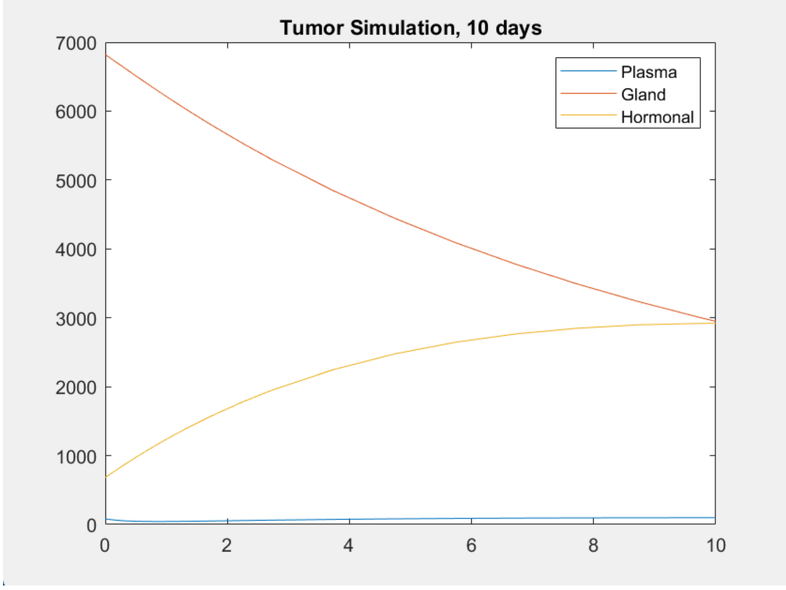


Figure 32: Iodine levels vs Time for Tumor scenario (10 days).

300-Day Simulation

```
[t, y] = ode23(@riggs_tumor, [0 300], [81.2; 6821; 682]);
plot(t, y)
legend('Plasma','Gland','Hormonal')
title('Tumor Simulation, 300 days')
```

Figure 33: MATLAB code for Riggs iodine model: Tumor scenario (300 days).

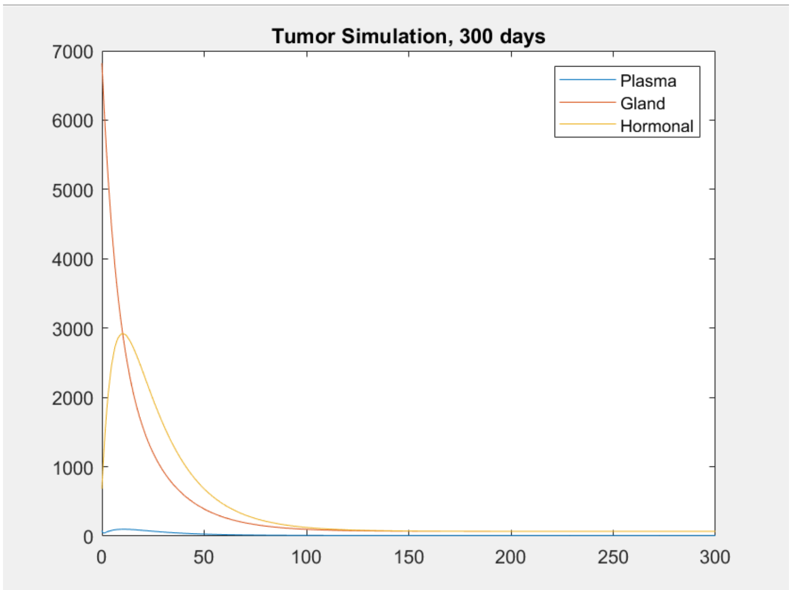


Figure 34: Iodine levels vs Time for Tumor scenario (300 days).

Part 2: Glucose-Insulin & Riggs Iodine Model Simulations Using Simulink

This section presents simulations of various glucose-insulin models using Simulink. For each scenario, the Simulink model diagram and the resulting glucose and insulin concentration graphs are shown.

Glucose-Insulin Model 1

This scenario simulates the basic glucose-insulin regulatory system using a Simulink block diagram.

Simulink Model

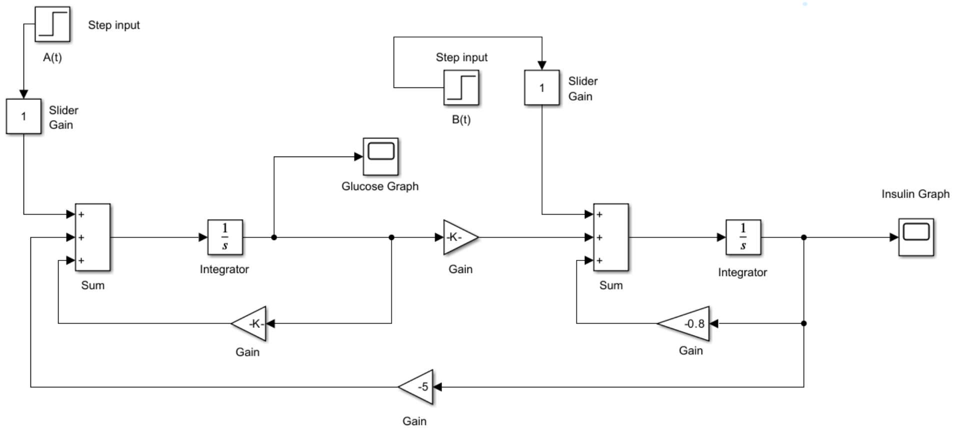


Figure 35: Simulink model for Glucose-Insulin Model 1.

Glucose Concentration Over Time

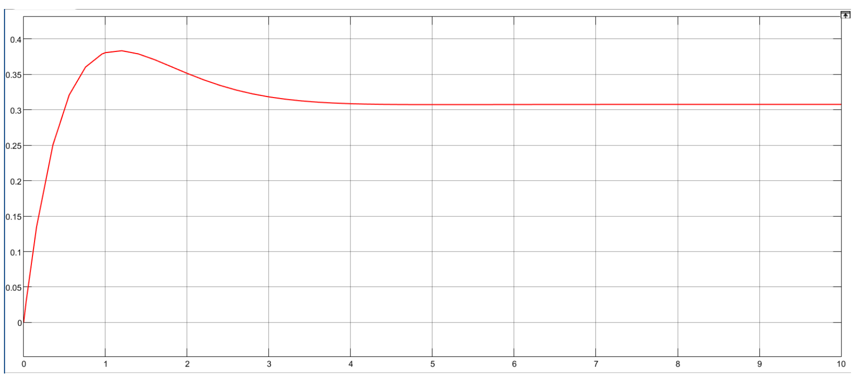


Figure 36: Glucose concentration vs. time for Model 1.

Insulin Concentration Over Time

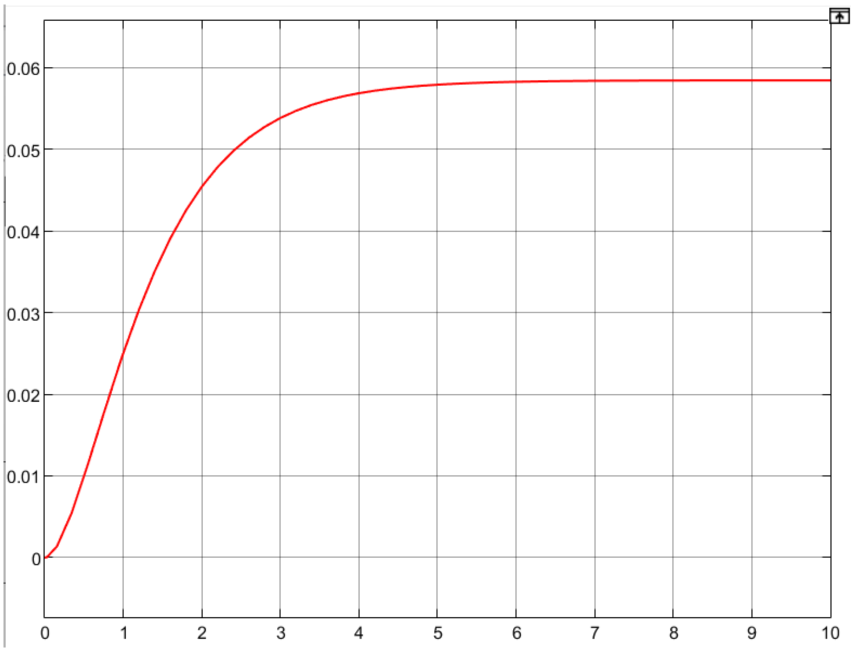


Figure 37: Insulin concentration vs. time for Model 1.

Glucose-Insulin Model 2

This scenario introduces a delay in the insulin response to glucose levels, modeled in Simulink.

Simulink Model

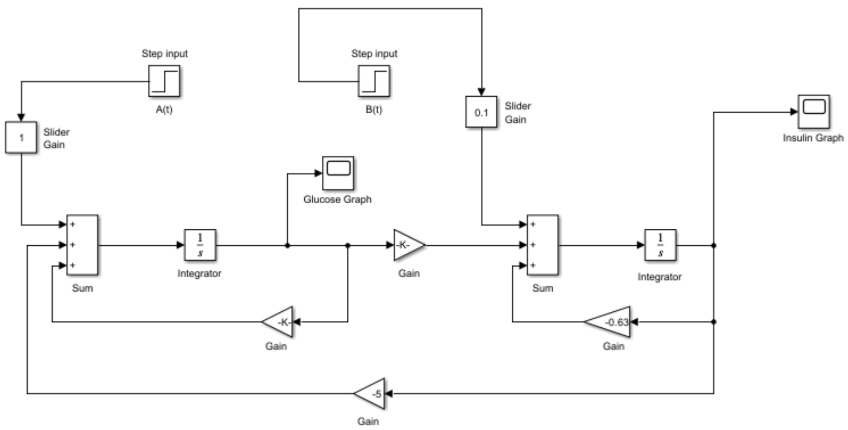


Figure 38: Simulink model for Glucose-Insulin Model 2 (delayed response).

Glucose Concentration Over Time

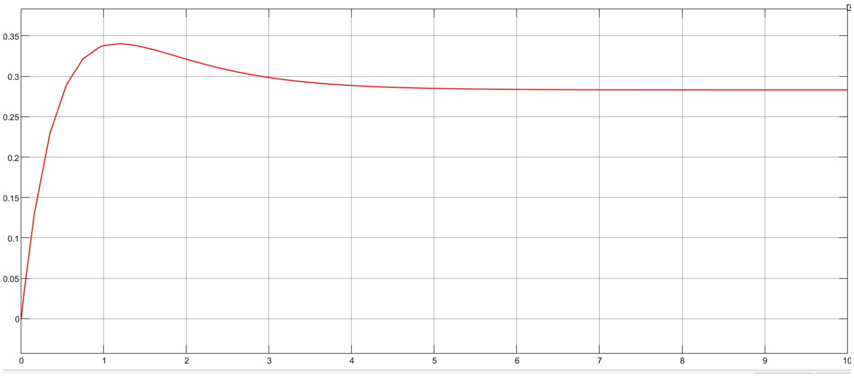


Figure 39: Glucose concentration vs. time for Model 2.

Insulin Concentration Over Time

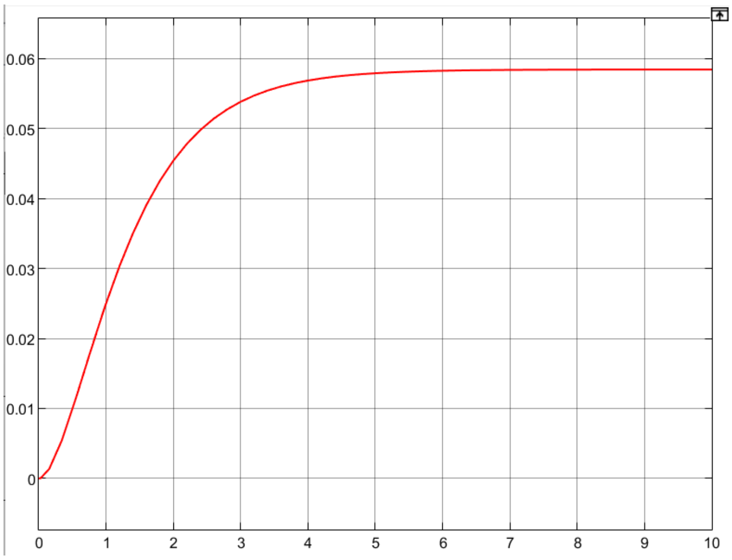


Figure 40: Insulin concentration vs. time for Model 2.

Glucose-Insulin Model 3 (Diabetic Condition)

Simulink Model

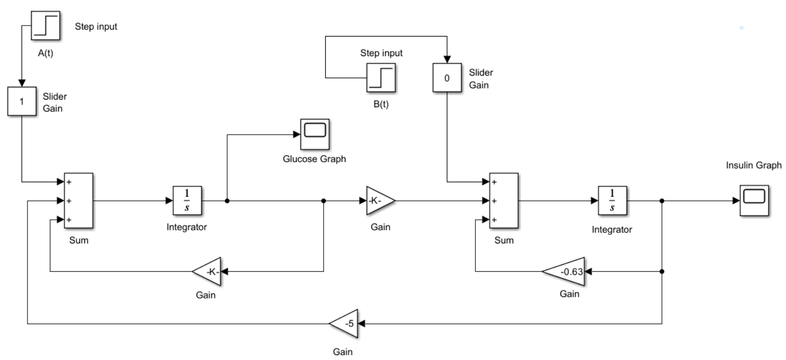


Figure 41: Simulink model for Glucose-Insulin Model 3 (diabetic condition).

Glucose Concentration Over Time

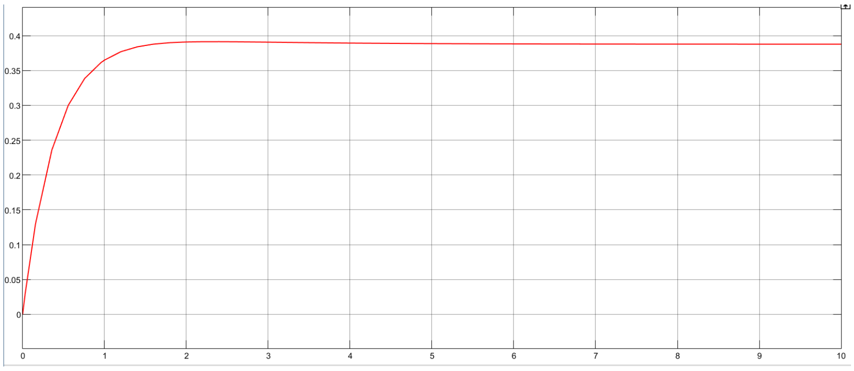


Figure 42: Glucose concentration vs. time for Model 3 (diabetic).

Insulin Concentration Over Time

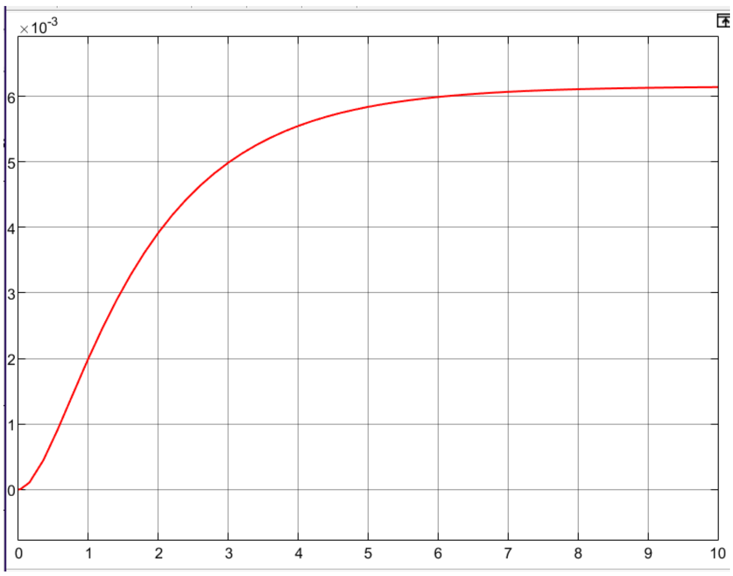


Figure 43: Insulin concentration vs. time for Model 3 (diabetic).

Riggs Iodine Model Simulation

Simulink Model

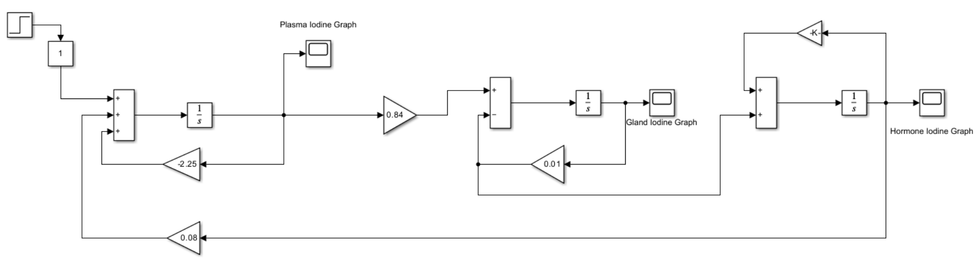


Figure 44: Simulink model for Riggs Iodine Model.

Gland Iodine Concentration Over Time

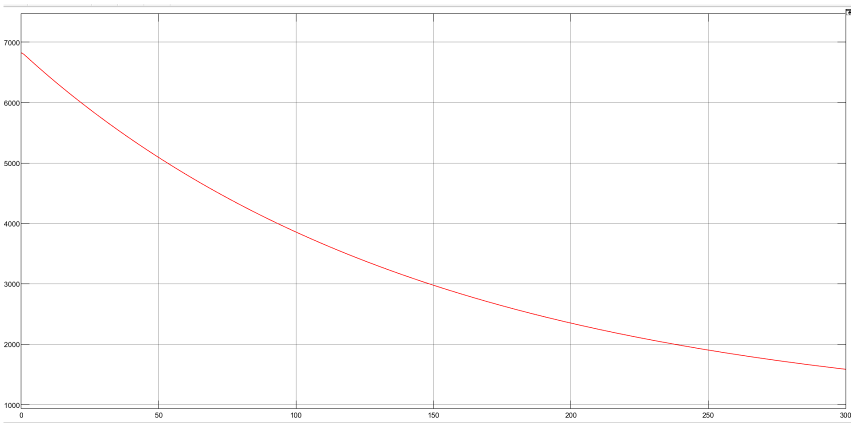


Figure 45: Gland iodine concentration vs. time.

Plasma Iodine Concentration Over Time

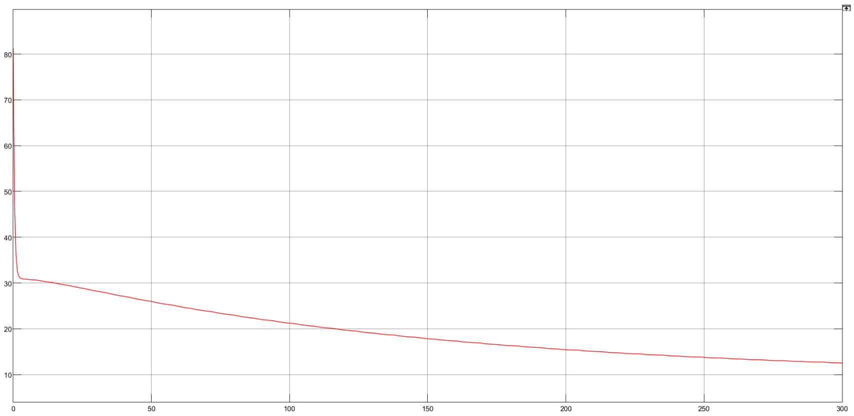


Figure 46: Plasma iodine concentration vs. time.

Hormone Iodine Concentration Over Time

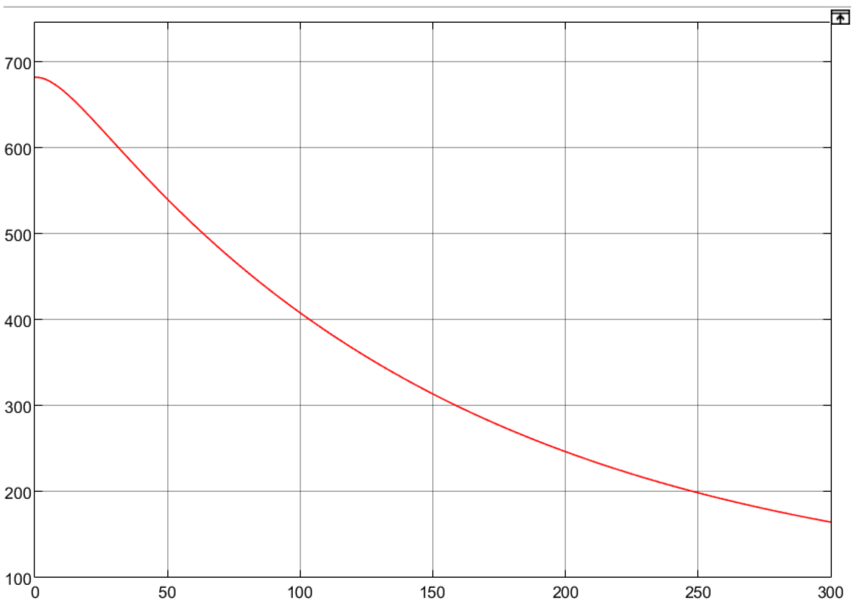


Figure 47: Hormone iodine concentration vs. time.

Part 3

1)
$$\frac{dg}{dt} = -k_4 g - k_6 i + A(t) \quad \text{——} ①$$

$$\frac{di}{dt} = k_3 g - k_1 i + B(t) \quad \text{——} ②$$

from ①

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_6 \frac{di}{dt} + \frac{d\,A(t)}{dt}$$

Assume $A(t) = au(t)$

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_6 \frac{di}{dt} + \frac{d}{dt}(au(t))$$

Assume $B(t) = 0$ and substitute ②,

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_6 [k_3 g - k_1 i + 0] + a \frac{d}{dt}(u(t))$$

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_3 k_6 g + k_1 k_6 i + a \frac{d}{dt}(u(t))$$

substitute for $k_6 i(t)$ from ①,

$$\frac{d^2g}{dt^2} = -k_4 \frac{dg}{dt} - k_3 k_6 g + k_1 \left[-\frac{dg}{dt} - k_4 g + au(t)\right] + a\frac{d}{dt}u(t)$$

$$\frac{d^2g}{dt^2} = (-k_4 + k_1)\frac{dg}{dt} - (k_1 k_4 + k_3 k_6) g - k_1 a + a\frac{d}{dt}u(t)$$

$$\frac{d^2g}{dt^2} + (k_1 + k_4)\frac{dg}{dt} + (k_1k_4 + k_3k_6)g = k_1a + a\frac{d}{dt}u(t)$$

substitute typical values,

$k_1 = 0.8h^{-1}$ $\qquad$ $k_6 = 5\,g\,lh/iu$

$k_4 = 2h^{-1}$ $\qquad$ $a = 1\,g/l/h$

$k_3 = 2\,iu/hg$ $\qquad$ $\frac{d}{dt}u(t) = 0$ for $t > 0$

$$\frac{d^2g}{dt^2} + 2.8\frac{dg}{dt} + 2.6g = 0.8$$

$g(t) = g_c(t) + g_p(t)$ type solution

<u>complementary solution</u>

$m^2 + 2.8m + 2.6 = 0$

$m = -1.4 \pm 0.8i$

$g_c(t) = c_1 e^{(-1.4+0.8i)t} + c_2 e^{(-1.4-0.8i)t}$

$g_c(t) = e^{-1.4t}\left(M\cos(0.8t) + N\sin(0.8t)\right)$

Assume $g_p(t) = k$

$0 + 0 + 2.6k = 0.8$

$k = \frac{4}{13}$

$\therefore\ g(t) = e^{-1.4t}\left(M\cos(0.8t) + N\sin(0.8t)\right) + \frac{4}{13}$

from initial conditions,

$g(0) = 0$

$M + \frac{4}{13} = 0 \longrightarrow M = \frac{-4}{13}$

$$g'(t) = -1.4e^{-1.4t}\left(M\cos(0.8t) + N\sin(0.8t)\right)$$
$$+e^{-1.4t}\left(-0.8M\sin(0.8t) + 0.8N\cos(0.8t)\right)$$

$$g'(0) = 1$$
$$g'(0) = -1.4 \times M + 0.8N$$
$$1 = -1.4 \times -\frac{4}{13} + 0.8N$$

$$N = \frac{37}{52}$$

$$\therefore \; g(t) = \frac{4}{13}u(t) + e^{-1.4t}\left(-\frac{4}{13}\cos(0.8t) + \frac{37}{52}\sin(0.8t)\right)$$

from ①,

$$i(t) = -\frac{1}{5}\frac{dg}{dt} - \frac{2}{5}g + \frac{1}{5}$$

$$i(t) = e^{-1.4t}\left(\frac{1}{13}\cos(0.8t) + \frac{7}{52}\sin(0.8t)\right) + \frac{1}{18}u(t)$$

```
t = 0:0.01:10;
% Analytical solutions
g_t = (exp((-1.4).*t)).*(-(4/13)*cos((0.8).*t) + (37/52)*sin((0.8).*t)) + 4/13;
i_t = (exp((-1.4).*t)).*(-(1/13)*cos((0.8).*t) - (7/52)*sin((0.8).*t)) + 1/13;

% Plot solutions
plot(t,g_t,t,i_t);
grid on;
legend('Glucose','Insulin',"Location","best")
xlabel ('Time (hours)');
ylabel ('Glucose/Insulin level');
title('Plots of g(t) and i(t)');
```

Figure 48: Hormone iodine concentration vs. time.

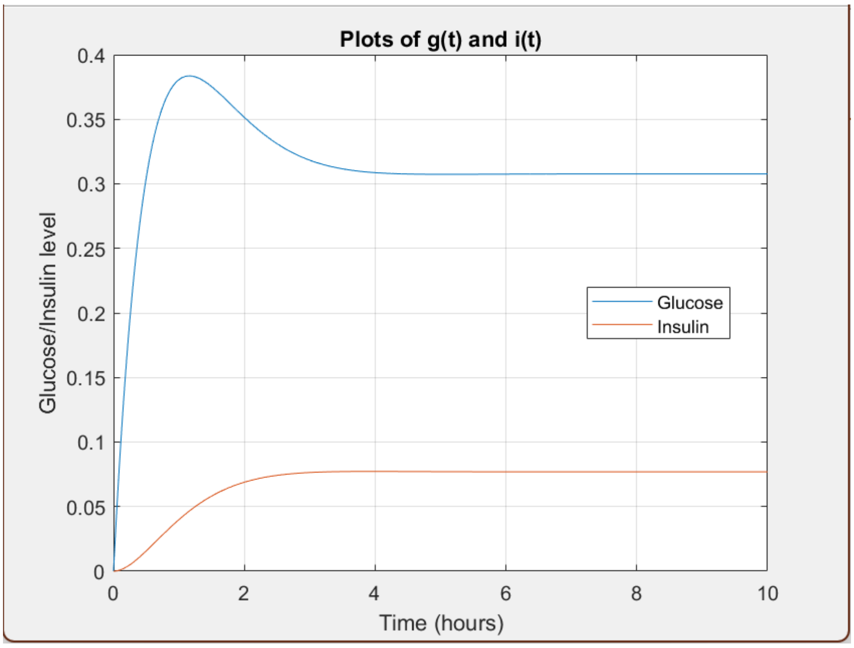


Figure 49: Hormone iodine concentration vs. time.

- The graph illustrates how insulin and glucagon interact to regulate blood sugar levels in response to glucose intake.
- After a significant glucose load, insulin levels rise sharply and peak shortly after glucose, reflecting the pancreas's delayed response to increased glucose.
- When glucagon is present, the glucose level remains elevated at the endpoint compared to when it is absent, because glucagon prompts the liver to release more glucose and counteracts insulin's effect.
- Even though insulin secretion increases in the presence of glucagon, the interplay between these hormones results in a higher final glucose level, which supports and validates Bolie's model of glucose regulation.

2) From bolie's plasma glucose model,

$$\frac{dG}{dt} = k_5 + A(t) - k_4 G - k_6 I + k_{10} G_n(t) \longrightarrow \text{Glucose}$$

$$\frac{dI}{dt} = k_2 + k_3 G + B(t) - k_1 I \longrightarrow \text{Insulin}$$

$$\frac{dG_n}{dt} = k_5 + C(t) + k_9 G - k_7 G_n \longrightarrow \text{Glucagon}$$

Glucose levels increase due to glucogon

Considering equlibrium state,

$$\frac{dG}{dt} = 0 \quad \Rightarrow \quad k_3 = k_4 G_0 + k_6 I_0 - k_{10} G_{n_0}$$

$$\frac{dI}{dt} = 0 \quad \Rightarrow \quad k_2 = k_1 I_0 - k_3 G_0$$

$$\frac{dG_n}{dt} = 0 \quad \Rightarrow \quad k_8 = k_7 G_{n_0} - k_4 G_0$$

substitute $i = I - I_0$ and $g = G - G_0$ and $g_n = G_n - G_{n_0}$

Assume $A(t) = a u(t)$

$B(t) = 0$

$C(t) = 0$

$$\frac{dg}{dt} = -k_4 g - k_6 i - k_{10} g_n + a u(t)$$

$$\frac{di}{dt} = k_3 g - k_1 i$$

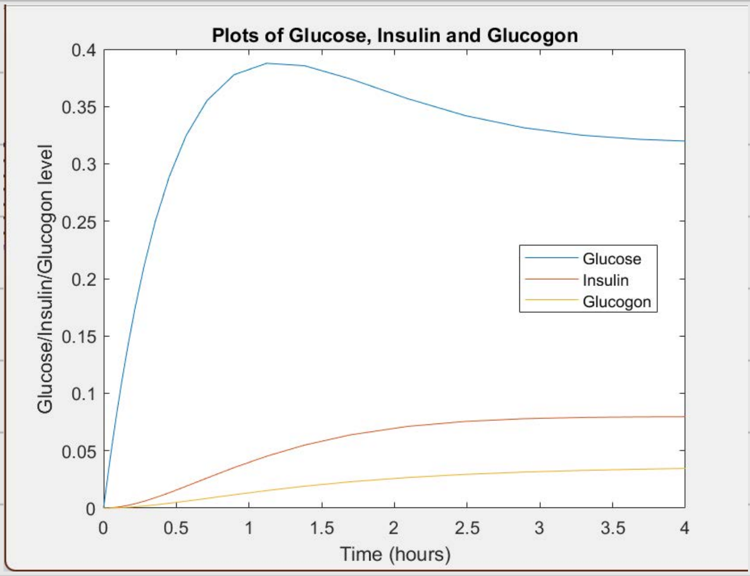
$$\frac{dg_n}{dt} = k_9 g - k_7 i$$

$$\begin{pmatrix} dg/dt \\ di/dt \\ dg_n/dt \end{pmatrix} = \begin{pmatrix} -k_4 & -k_6 & k_{10} \\ k_3 & -k_1 & 0 \\ k_9 & 0 & -k_8 \end{pmatrix} \begin{pmatrix} g \\ i \\ g_n \end{pmatrix} + \begin{pmatrix} a\,u(t) \\ 0 \\ 0 \end{pmatrix}$$

```
function yp = bolies_model(t, y)
    % Typical values for transfer rates
    k1 = 0.8; k3 = 0.2; k4 = 2; k6 = 5; a = 1;
    % Parameters for Glycogen
    k7 = 0.5; k9 = 0.06; k10 = 1;

    yp = [-k4 -k6 k10;
           k3 -k1 0;
           k9  0 -k7] * y + [a 0 0]';
end
```

```
[t, y] = ode23('bolies_model', [0 4], [0 0 0]);
plot(t, y)
legend('Glucose', 'Insulin', 'Glucogon', 'Location', 'best');
xlabel('Time (hours)');
ylabel('Glucose/Insulin/Glucogon level');
title('Plots of Glucose, Insulin and Glucogon')
```



The graph illustrates the dynamic interaction between insulin and glucagon in regulating blood glucose levels. Following a significant glucose intake, insulin levels rise sharply, peaking shortly after glucose levels due to a delay in pancreatic response. This delay occurs because pancreatic cells take time to detect the elevated glucose concentration.

When glucagon is present, the glucose level at the endpoint remains higher compared to when it is absent. This is because glucagon opposes the action of insulin by stimulating the liver to release glucose, thereby maintaining elevated blood sugar levels.

Even though insulin secretion increases in the presence of glucagon, the hormonal balance results in a higher final glucose level. This behavior aligns with the predictions made by Bolie's model.